Computer Algebra
Independent Integration Tests

Summer 2024

4-Trig-functions/4.6-Cosecant/250-4.6.11

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**84**]. This is test number [250].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (84)	0.00 (0)
Mathematica	95.24 (80)	4.76 (4)
Fricas	76.19 (64)	23.81 (20)
Maple	61.90 (52)	38.10 (32)
Maxima	60.71 (51)	39.29 (33)
Reduce	57.14 (48)	42.86 (36)
Mupad	55.95 (47)	44.05 (37)
Giac	52.38 (44)	47.62 (40)
Sympy	44.05 (37)	55.95 (47)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

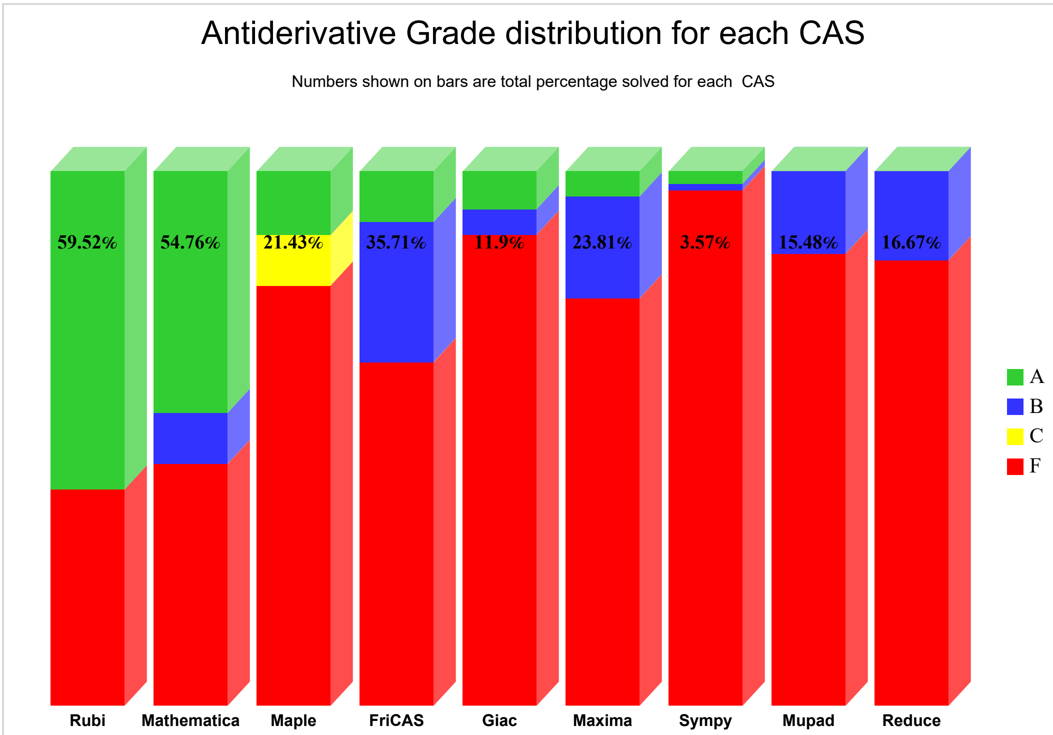
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

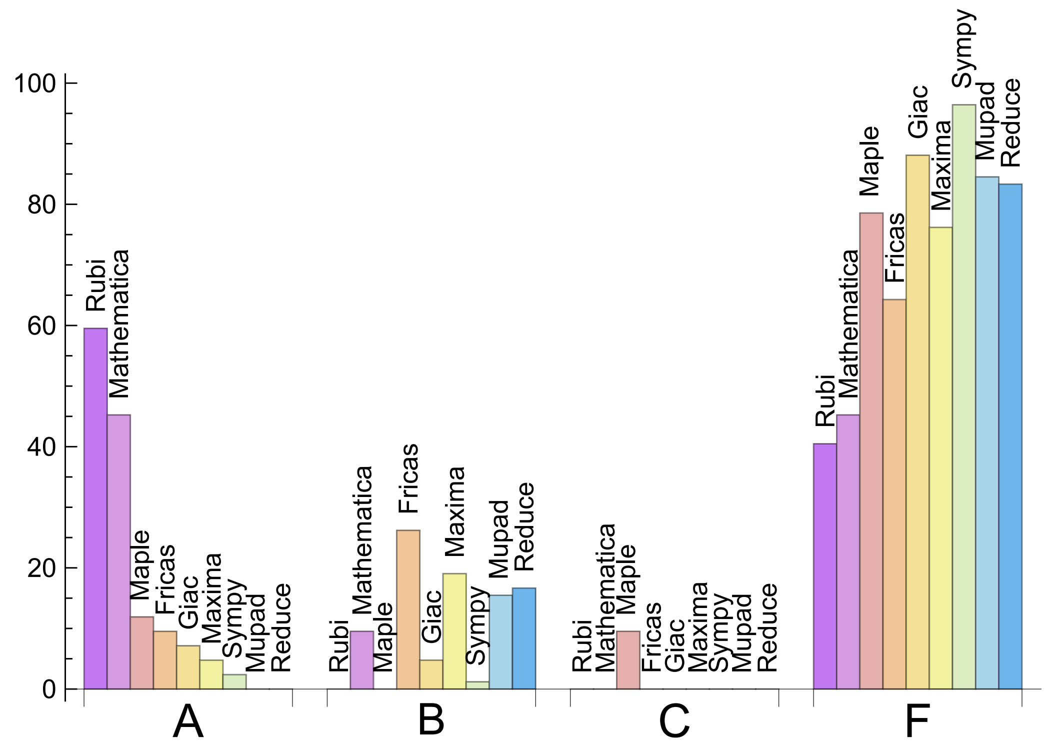
System	% A grade	% B grade	% C grade	% F grade
Rubi	59.524	0.000	0.000	40.476
Mathematica	45.238	9.524	0.000	45.238
Maple	11.905	0.000	9.524	78.571
Fricas	9.524	26.190	0.000	64.286
Giac	7.143	4.762	0.000	88.095
Maxima	4.762	19.048	0.000	76.190
Sympy	2.381	1.190	0.000	96.429
Mupad	0.000	15.476	0.000	84.524
Reduce	0.000	16.667	0.000	83.333

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Fricas	20	100.00	0.00	0.00
Maxima	33	45.45	15.15	39.39
Maple	32	100.00	0.00	0.00
Reduce	36	100.00	0.00	0.00
Mupad	37	0.00	100.00	0.00
Giac	40	100.00	0.00	0.00
Sympy	47	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.11
Reduce	0.19
Maple	0.21
Giac	0.34
Rubi	0.67
Maxima	1.18
Sympy	4.79
Mathematica	10.87
Mupad	14.85

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	21.24	1.01	17.00	0.94
Giac	38.32	1.18	20.00	1.11
Reduce	64.04	1.86	38.00	1.52
Mupad	170.23	2.46	22.00	1.22
Maple	175.42	1.44	18.00	1.00
Rubi	301.33	1.00	75.00	1.00
Fricas	392.12	2.26	45.00	2.11
Mathematica	406.84	1.34	73.50	1.11
Maxima	1090.16	33.15	310.00	6.75

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

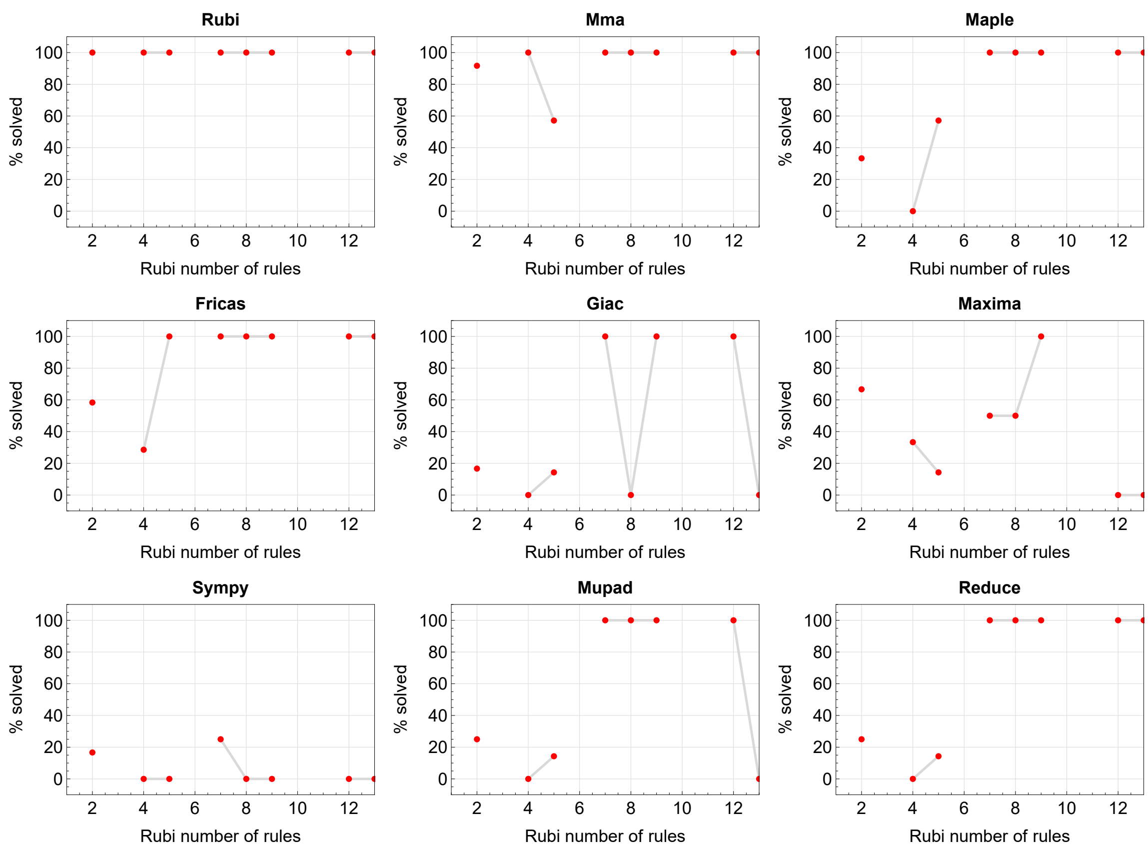


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

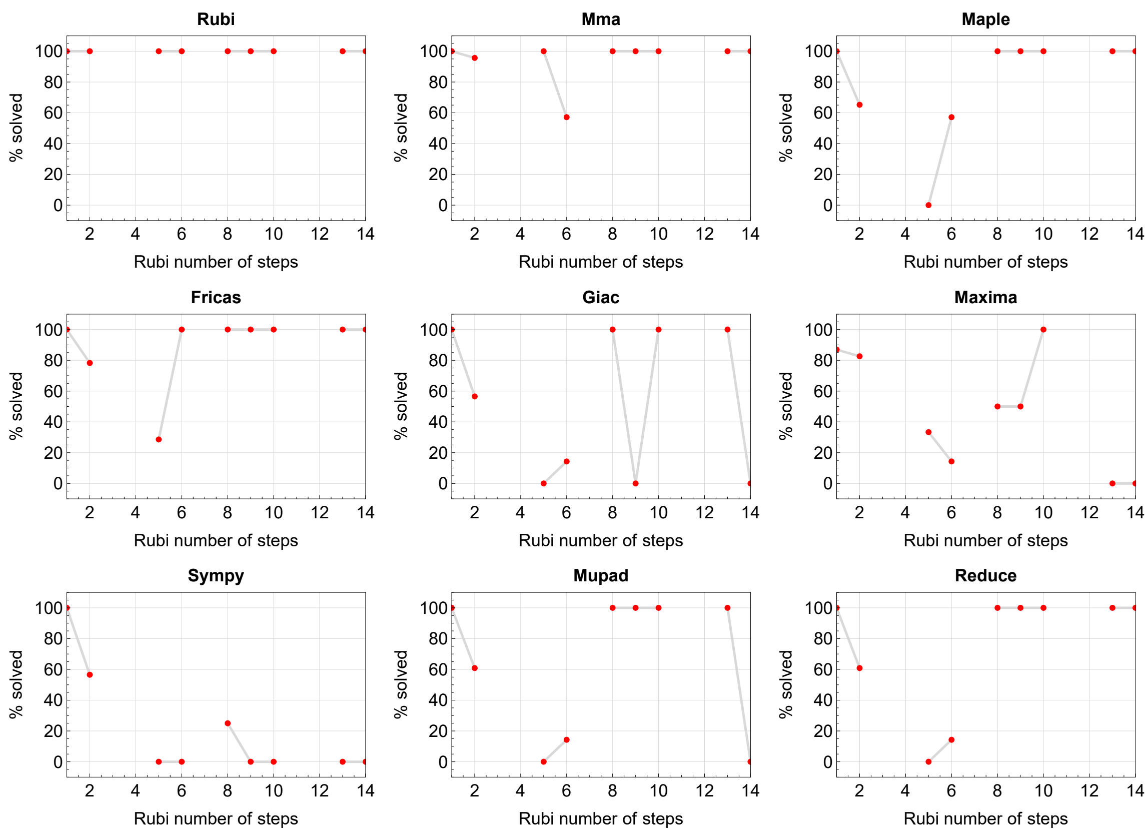


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

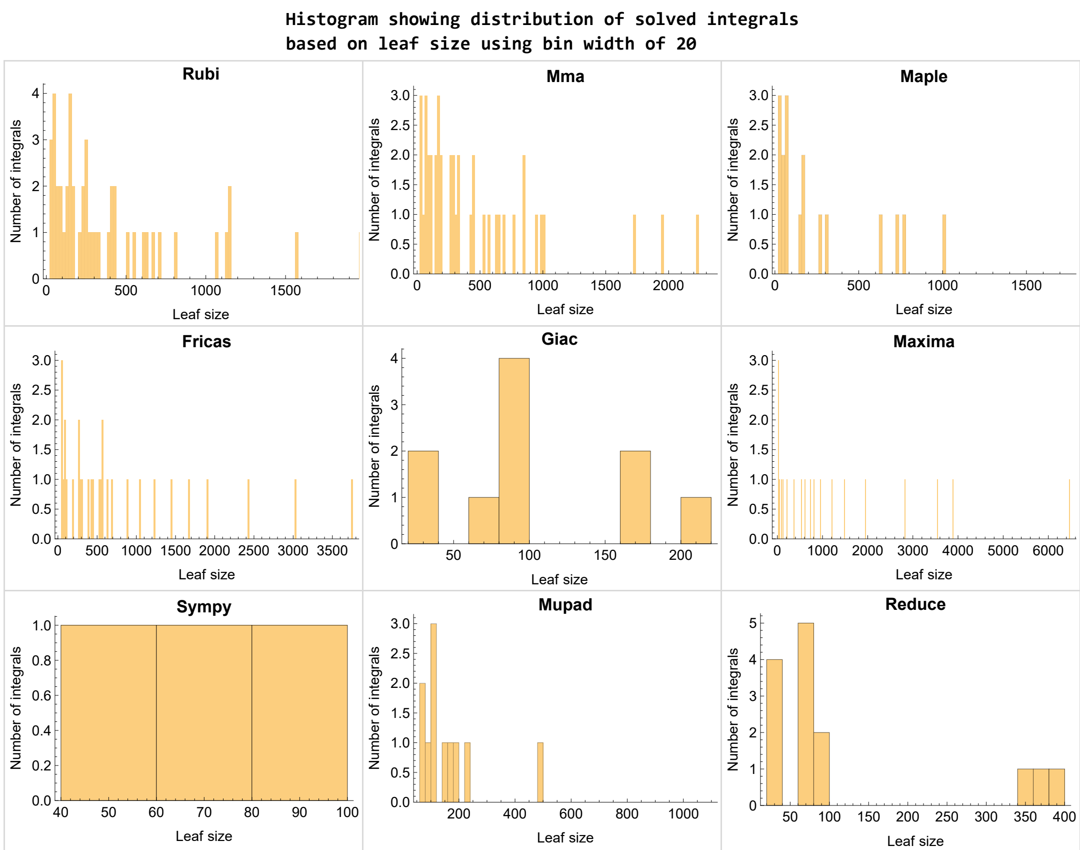


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

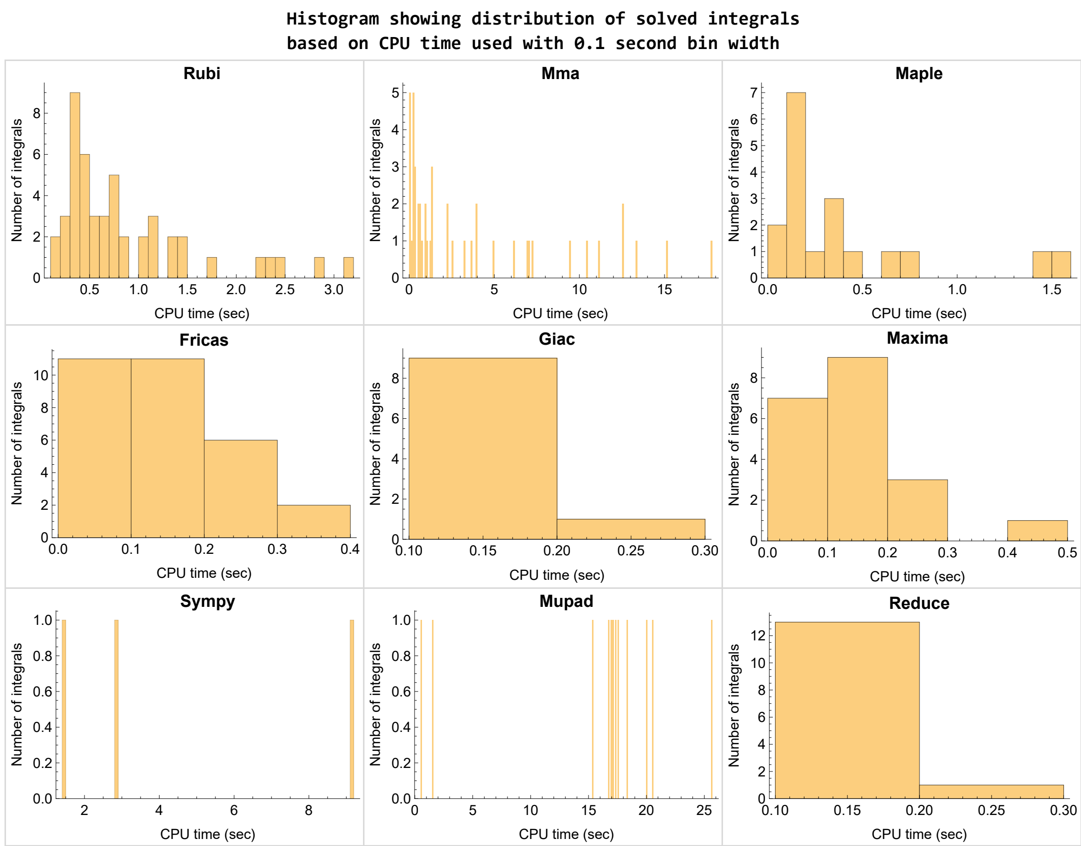


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

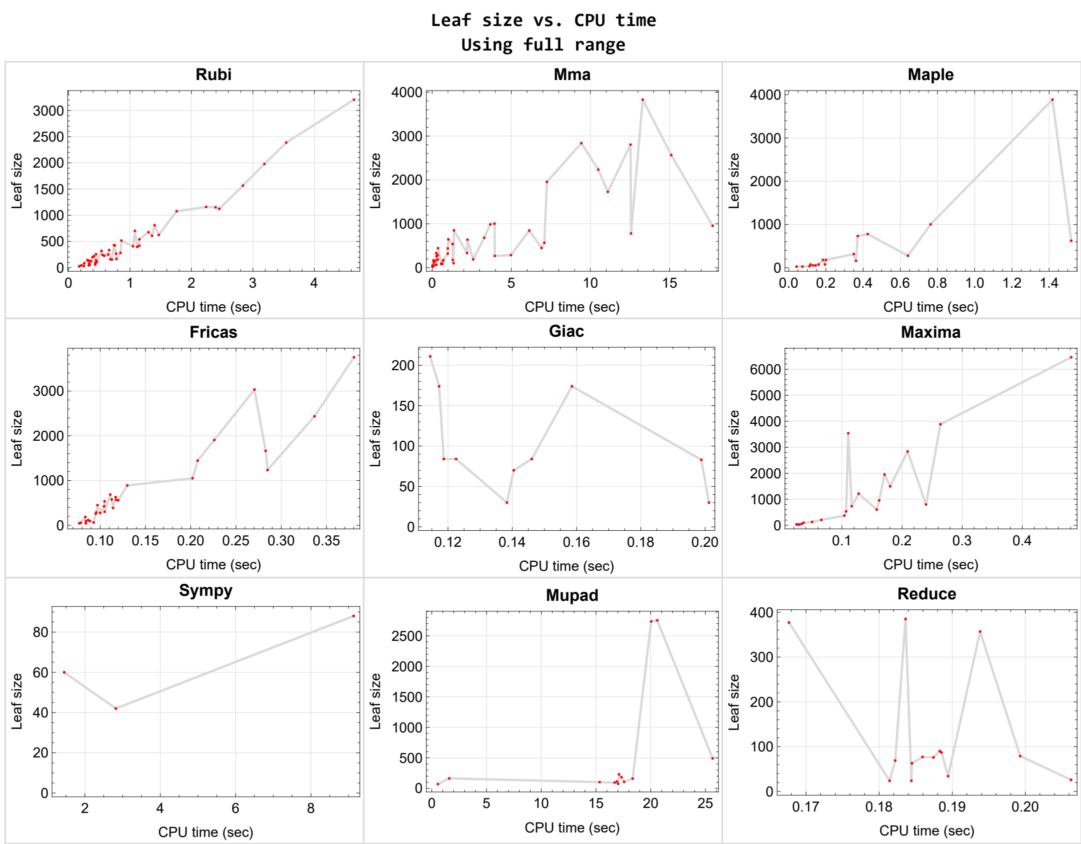


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{2, 4, 6, 7, 9, 11, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 34, 35, 39, 40, 44, 45, 49, 50, 54, 55, 59, 60, 65, 66, 70, 71, 72\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{10, 18, 23, 25, 46, 47, 48, 67, 68, 77, 80, 83\}$

Maple $\{73, 74, 76, 77, 79, 80, 82, 83\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

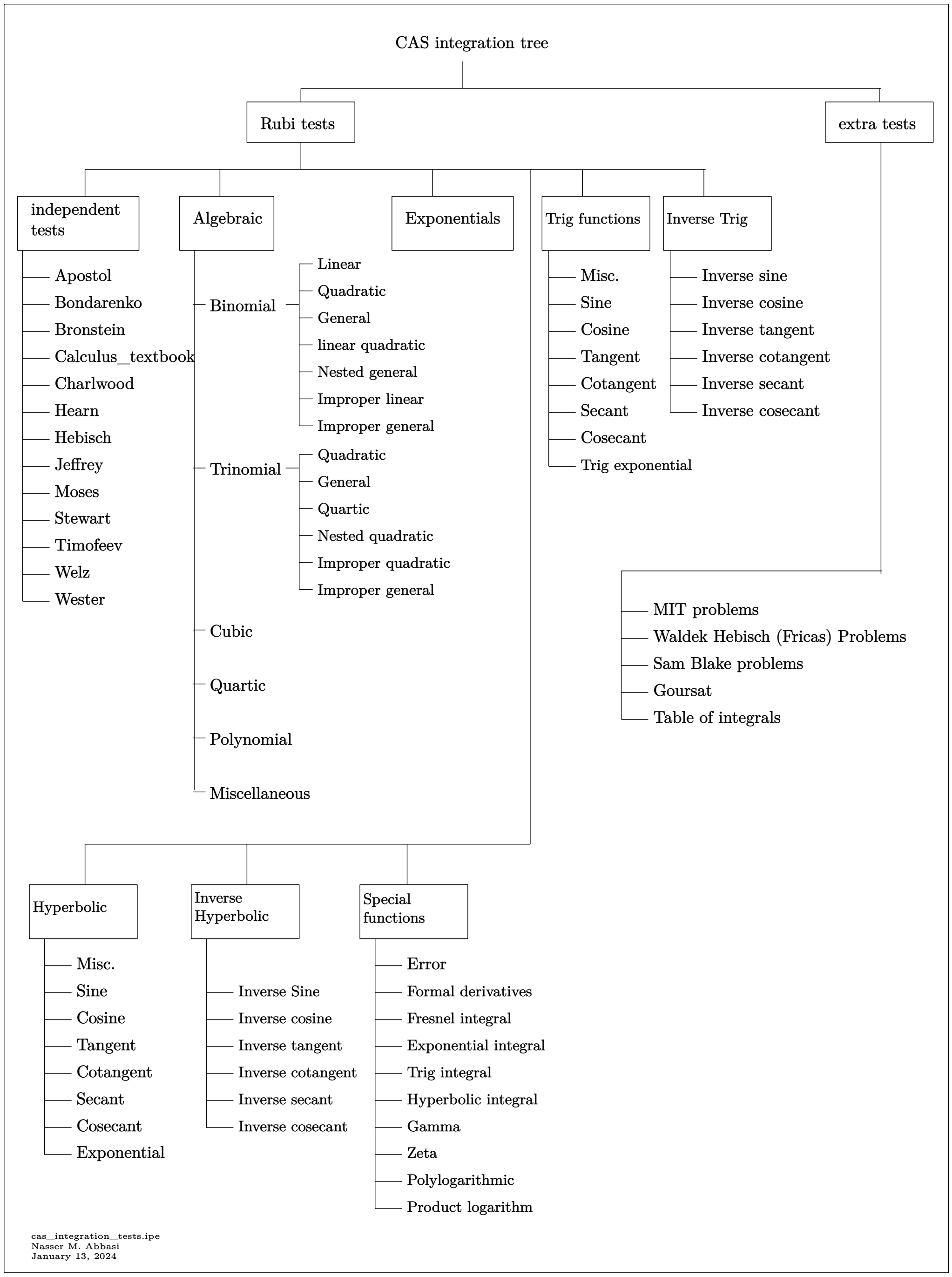
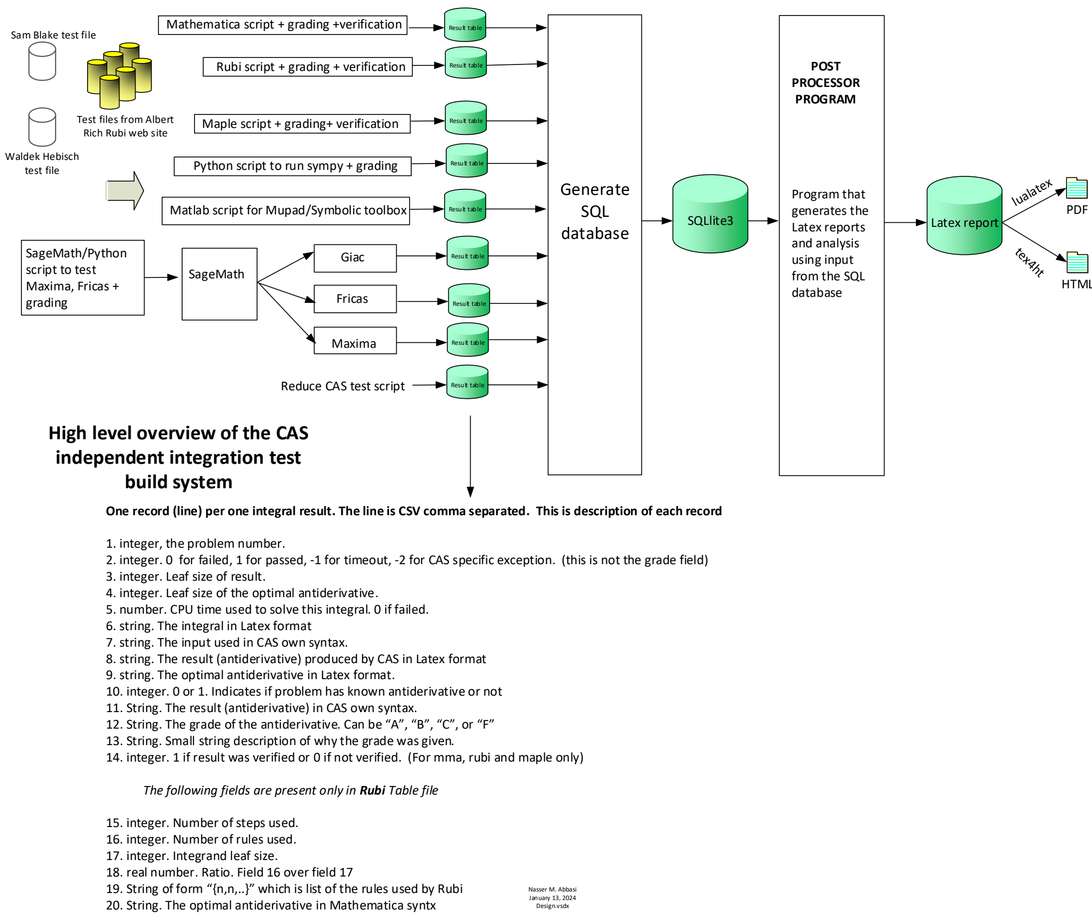


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 3, 5, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 53, 56, 57, 58, 61, 62, 63, 64, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 3, 5, 12, 15, 16, 20, 23, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 53, 56, 58, 62, 63, 64, 67, 68, 69, 73, 74, 76, 77, 79, 82 }

B grade { 8, 10, 18, 25, 57, 61, 80, 83 }

C grade { }

F normal fail { 75, 78, 81, 84 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 5, 12, 15, 20, 27, 53, 58, 61, 64, 69 }

B grade { }

C grade { 73, 74, 76, 77, 79, 80, 82, 83 }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 62, 63, 67, 68, 75, 78, 81, 84 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 5, 20, 53, 64, 73, 76, 79, 82 }

B grade { 1, 3, 8, 10, 12, 15, 16, 18, 23, 25, 27, 58, 61, 69, 74, 75, 77, 78, 80, 81, 83, 84 }

C grade { }

F normal fail { 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 62, 63, 67, 68 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 5, 53, 58, 61 }

B grade { 8, 10, 12, 15, 31, 32, 33, 36, 37, 38, 51, 52, 56, 57, 73, 76 }

C grade { }

F normal fail { 1, 3, 16, 18, 23, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84 }

F(-1) timedout fail { 20, 27, 60, 70, 71 }

F(-2) exception fail { 25, 41, 42, 43, 46, 47, 48, 62, 63, 64, 67, 68, 69 }

Giac

A grade { 5, 20, 27, 53, 64, 69 }

B grade { 12, 15, 58, 61 }

C grade { }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 62, 63, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 5, 12, 15, 20, 27, 53, 58, 61, 64, 69, 73, 76, 79 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 62, 63, 67, 68, 74, 75, 77, 78, 80, 81, 82, 83, 84 }

F(-2) exception fail { }

Sympy

A grade { 53, 58 }

B grade { 5 }

C grade { }

F normal fail { 1, 3, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 63, 64, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 5, 12, 15, 20, 27, 53, 58, 61, 64, 69, 73, 76, 79, 82 }

C grade { }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 62, 63, 67, 68, 74, 75, 77, 78, 80, 81, 83, 84 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	159	0	0	425	0	0	23	0
N.S.	1	1.00	1.23	0.00	0.00	3.29	0.00	0.00	0.18	0.00
time (sec)	N/A	0.335	0.215	0.000	0.000	0.104	0.000	0.000	0.178	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	106	21	15	18	23	20
N.S.	1	1.00	1.12	1.00	6.62	1.31	0.94	1.12	1.44	1.25
time (sec)	N/A	0.180	2.886	0.036	0.101	0.068	2.365	0.186	0.182	15.091

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	118	0	0	288	0	0	23	0
N.S.	1	1.00	1.40	0.00	0.00	3.43	0.00	0.00	0.27	0.00
time (sec)	N/A	0.256	0.104	0.000	0.000	0.096	0.000	0.000	0.185	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	106	21	15	18	23	20
N.S.	1	1.00	1.12	1.00	6.62	1.31	0.94	1.12	1.44	1.25
time (sec)	N/A	0.177	2.298	0.039	0.094	0.069	1.964	0.139	0.205	14.550

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	26	31	44	42	30	26	69
N.S.	1	1.00	1.00	1.00	1.19	1.69	1.62	1.15	1.00	2.65
time (sec)	N/A	0.176	0.013	0.074	0.029	0.077	2.820	0.138	0.206	0.556

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	108	18	14	18	21	20
N.S.	1	1.00	1.12	1.00	6.75	1.12	0.88	1.12	1.31	1.25
time (sec)	N/A	0.174	2.162	0.041	0.099	0.070	0.702	0.143	0.176	15.771

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	126	18	15	18	25	20
N.S.	1	1.00	1.12	1.00	7.88	1.12	0.94	1.12	1.56	1.25
time (sec)	N/A	0.175	2.186	0.045	0.101	0.071	0.468	0.213	0.180	15.711

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	228	227	639	0	800	687	0	0	47	0
N.S.	1	1.00	2.80	0.00	3.51	3.01	0.00	0.00	0.21	0.00
time (sec)	N/A	0.581	2.237	0.000	0.240	0.111	0.000	0.000	0.183	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	310	42	17	20	47	22
N.S.	1	1.00	1.11	1.00	17.22	2.33	0.94	1.11	2.61	1.22
time (sec)	N/A	0.179	17.094	0.079	0.254	0.080	2.905	0.598	0.183	15.465

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	124	268	0	604	451	0	0	134	0
N.S.	1	0.99	2.14	0.00	4.83	3.61	0.00	0.00	1.07	0.00
time (sec)	N/A	0.369	3.960	0.000	0.158	0.097	0.000	0.000	0.185	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	300	42	17	20	102	22
N.S.	1	1.00	1.11	1.00	16.67	2.33	0.94	1.11	5.67	1.22
time (sec)	N/A	0.179	27.156	0.076	0.217	0.074	2.323	0.630	0.188	15.616

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	44	86	51	98	94	0	84	69	102
N.S.	1	0.98	1.91	1.13	2.18	2.09	0.00	1.87	1.53	2.27
time (sec)	N/A	0.331	0.651	0.131	0.036	0.084	0.000	0.146	0.182	15.333

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	360	36	15	20	64	22
N.S.	1	1.00	1.11	1.00	20.00	2.00	0.83	1.11	3.56	1.22
time (sec)	N/A	0.180	68.782	0.076	0.220	0.072	2.473	0.207	0.184	14.848

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	366	36	17	20	50	22
N.S.	1	1.00	1.11	1.00	20.33	2.00	0.94	1.11	2.78	1.22
time (sec)	N/A	0.181	39.426	0.077	0.248	0.070	0.767	0.657	0.174	14.810

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	104	167	73	3543	183	0	211	90	491
N.S.	1	1.16	1.86	0.81	39.37	2.03	0.00	2.34	1.00	5.46
time (sec)	N/A	0.461	0.083	0.195	0.111	0.084	0.000	0.114	0.188	25.657

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	396	397	319	0	0	1445	0	0	20	0
N.S.	1	1.00	0.81	0.00	0.00	3.65	0.00	0.00	0.05	0.00
time (sec)	N/A	1.118	0.985	0.000	0.000	0.208	0.000	0.000	0.186	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	253	20	15	20	20	22
N.S.	1	1.00	1.11	1.00	14.06	1.11	0.83	1.11	1.11	1.22
time (sec)	N/A	0.182	1.701	0.058	0.241	0.067	0.518	0.218	0.187	15.783

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	266	987	0	0	1050	0	0	20	0
N.S.	1	0.98	3.64	0.00	0.00	3.87	0.00	0.00	0.07	0.00
time (sec)	N/A	0.775	3.671	0.000	0.000	0.202	0.000	0.000	0.183	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	253	20	15	20	20	22
N.S.	1	1.00	1.11	1.00	14.06	1.11	0.83	1.11	1.11	1.22
time (sec)	N/A	0.182	1.410	0.061	0.235	0.064	0.435	0.178	0.194	15.164

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	74	66	73	0	261	0	84	79	163
N.S.	1	1.17	1.05	1.16	0.00	4.14	0.00	1.33	1.25	2.59
time (sec)	N/A	0.336	0.226	0.117	0.000	0.095	0.000	0.119	0.199	1.594

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	250	19	15	20	19	22
N.S.	1	1.00	1.11	1.00	13.89	1.06	0.83	1.11	1.06	1.22
time (sec)	N/A	0.187	1.647	0.059	0.233	0.070	0.899	0.175	0.186	14.690

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	126	18	15	18	25	20
N.S.	1	1.00	1.12	1.00	7.88	1.12	0.94	1.12	1.56	1.25
time (sec)	N/A	0.172	0.137	0.000	0.100	0.082	0.469	0.236	0.175	0.003

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1124	1123	1956	0	0	3032	0	0	38	0
N.S.	1	1.00	1.74	0.00	0.00	2.70	0.00	0.00	0.03	0.00
time (sec)	N/A	2.454	7.254	0.000	0.000	0.270	0.000	0.000	0.172	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	1286	38	17	20	38	22
N.S.	1	1.00	1.11	1.00	71.44	2.11	0.94	1.11	2.11	1.22
time (sec)	N/A	0.167	8.111	0.089	0.679	0.079	1.361	0.222	0.180	15.470

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	616	607	2566	0	0	1906	0	0	38	0
N.S.	1	0.99	4.17	0.00	0.00	3.09	0.00	0.00	0.06	0.00
time (sec)	N/A	1.359	15.111	0.000	0.000	0.226	0.000	0.000	0.186	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	1265	38	17	20	38	22
N.S.	1	1.00	1.11	1.00	70.28	2.11	0.94	1.11	2.11	1.22
time (sec)	N/A	0.176	7.175	0.088	0.638	0.083	1.080	0.292	0.175	15.734

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	154	158	179	0	536	0	174	377	2755
N.S.	1	1.28	1.32	1.49	0.00	4.47	0.00	1.45	3.14	22.96
time (sec)	N/A	0.706	0.689	0.184	0.000	0.105	0.000	0.117	0.168	20.598

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	4629	38	17	20	38	22
N.S.	1	1.00	1.11	1.00	257.17	2.11	0.94	1.11	2.11	1.22
time (sec)	N/A	0.174	12.691	0.081	5.096	0.079	1.348	0.656	0.160	15.040

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	4560	44	19	20	55	22
N.S.	1	1.00	1.11	1.00	253.33	2.44	1.06	1.11	3.06	1.22
time (sec)	N/A	0.179	9.711	0.087	5.203	0.078	1.230	0.262	0.183	15.093

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	3530	44	19	20	55	22
N.S.	1	1.00	1.11	1.00	196.11	2.44	1.06	1.11	3.06	1.22
time (sec)	N/A	0.178	10.854	0.088	5.208	0.080	1.762	0.861	0.186	15.609

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	445	0	1498	0	0	0	22	0
N.S.	1	1.00	1.03	0.00	3.47	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.741	0.373	0.000	0.180	0.000	0.000	0.000	0.183	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	333	0	956	0	0	0	22	0
N.S.	1	1.00	1.05	0.00	3.03	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.537	0.253	0.000	0.162	0.000	0.000	0.000	0.179	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	260	0	534	0	0	0	20	0
N.S.	1	1.00	1.30	0.00	2.67	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.391	0.284	0.000	0.107	0.000	0.000	0.000	0.201	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	105	18	15	18	20	20
N.S.	1	1.00	1.11	0.89	5.83	1.00	0.83	1.00	1.11	1.11
time (sec)	N/A	0.173	7.445	0.102	0.325	0.075	1.943	0.169	0.181	16.426

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	111	18	17	18	24	20
N.S.	1	1.00	1.11	0.89	6.17	1.00	0.94	1.00	1.33	1.11
time (sec)	N/A	0.175	11.305	0.112	0.403	0.072	2.016	0.199	0.180	15.215

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	695	703	954	0	6462	0	0	0	45	0
N.S.	1	1.01	1.37	0.00	9.30	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.081	17.725	0.000	0.481	0.000	0.000	0.000	0.184	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	513	517	779	0	3885	0	0	0	45	0
N.S.	1	1.01	1.52	0.00	7.57	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.861	12.566	0.000	0.264	0.000	0.000	0.000	0.182	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	337	449	0	1950	0	0	0	41	0
N.S.	1	1.01	1.35	0.00	5.86	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.662	6.911	0.000	0.171	0.000	0.000	0.000	0.180	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	386	36	17	20	70	22
N.S.	1	1.00	1.10	0.90	19.30	1.80	0.85	1.00	3.50	1.10
time (sec)	N/A	0.177	90.817	0.206	0.665	0.075	12.870	0.303	0.186	15.258

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	388	36	19	20	48	22
N.S.	1	1.00	1.10	0.90	19.40	1.80	0.95	1.00	2.40	1.10
time (sec)	N/A	0.178	57.942	0.202	0.844	0.076	3.920	0.396	0.178	15.249

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1075	1077	850	0	0	0	0	0	19	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	1.759	1.376	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	807	809	644	0	0	0	0	0	19	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	1.401	1.023	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	539	541	438	0	0	0	0	0	17	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	1.154	0.992	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	242	19	17	20	18	22
N.S.	1	1.00	1.10	0.90	12.10	0.95	0.85	1.00	0.90	1.10
time (sec)	N/A	0.185	3.126	0.136	0.611	0.074	3.310	0.142	0.187	15.645

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	111	18	17	18	24	20
N.S.	1	1.00	1.11	0.89	6.17	1.00	0.94	1.00	1.33	1.11
time (sec)	N/A	0.176	0.145	0.000	0.384	0.074	2.050	0.177	0.180	0.002

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	3205	3207	3831	0	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.644	13.309	0.000	0.000	0.000	0.000	0.000	0.817	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	2385	2387	2806	0	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.543	12.536	0.000	0.000	0.000	0.000	0.000	0.496	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1565	1567	1729	0	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.837	11.103	0.000	0.000	0.000	0.000	0.000	0.337	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	4411	38	19	20	36	22
N.S.	1	1.00	1.10	0.90	220.55	1.90	0.95	1.00	1.80	1.10
time (sec)	N/A	0.169	43.663	0.199	12.430	0.085	4.662	0.499	0.196	15.162

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	4411	44	20	20	42	22
N.S.	1	1.00	1.10	0.90	220.55	2.20	1.00	1.00	2.10	1.10
time (sec)	N/A	0.168	39.709	0.200	18.005	0.090	9.662	0.932	0.192	15.227

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	286	0	730	0	0	0	24	0
N.S.	1	1.00	1.11	0.00	2.83	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.449	0.353	0.000	0.116	0.000	0.000	0.000	0.187	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	191	0	370	0	0	0	21	0
N.S.	1	1.00	1.33	0.00	2.57	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.309	2.594	0.000	0.104	0.000	0.000	0.000	0.182	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	32	31	43	60	30	24	73
N.S.	1	1.00	1.00	1.23	1.19	1.65	2.31	1.15	0.92	2.81
time (sec)	N/A	0.177	0.050	0.112	0.027	0.084	1.451	0.201	0.184	17.027

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	113	25	19	18	30	20
N.S.	1	1.00	1.10	0.80	5.65	1.25	0.95	0.90	1.50	1.00
time (sec)	N/A	0.172	16.667	0.109	0.394	0.073	1.226	0.225	0.185	15.450

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	115	25	19	18	36	20
N.S.	1	1.00	1.10	0.80	5.75	1.25	0.95	0.90	1.80	1.00
time (sec)	N/A	0.169	25.668	0.113	0.431	0.081	7.545	0.262	0.185	15.559

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	421	423	567	0	2836	0	0	0	47	0
N.S.	1	1.00	1.35	0.00	6.74	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.754	7.089	0.000	0.209	0.000	0.000	0.000	0.199	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	241	243	681	0	1217	0	0	0	43	0
N.S.	1	1.01	2.83	0.00	5.05	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.554	3.291	0.000	0.128	0.000	0.000	0.000	0.198	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	48	93	51	52	94	88	83	63	111
N.S.	1	1.02	1.98	1.09	1.11	2.00	1.87	1.77	1.34	2.36
time (sec)	N/A	0.344	0.578	0.145	0.033	0.089	9.135	0.199	0.184	16.914

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	794	46	20	20	59	22
N.S.	1	1.00	1.09	0.82	36.09	2.09	0.91	0.91	2.68	1.00
time (sec)	N/A	0.184	50.411	0.206	0.959	0.075	2.355	0.258	0.183	15.504

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	46	20	20	66	22
N.S.	1	1.00	1.09	0.82	0.00	2.09	0.91	0.91	3.00	1.00
time (sec)	N/A	0.186	57.062	0.208	0.000	0.074	9.062	0.244	0.205	15.125

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	30	57	24	34	56	0	70	24	94
N.S.	1	1.25	2.38	1.00	1.42	2.33	0.00	2.92	1.00	3.92
time (sec)	N/A	0.252	0.033	0.042	0.025	0.079	0.000	0.140	0.181	16.705

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	675	677	539	0	0	0	0	0	19	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	1.301	1.295	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	407	409	333	0	0	0	0	0	18	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.050	2.204	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	76	68	73	0	275	0	84	76	159
N.S.	1	1.15	1.03	1.11	0.00	4.17	0.00	1.27	1.15	2.41
time (sec)	N/A	0.341	0.267	0.161	0.000	0.100	0.000	0.123	0.187	18.352

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	244	27	20	20	30	22
N.S.	1	1.00	1.09	0.82	11.09	1.23	0.91	0.91	1.36	1.00
time (sec)	N/A	0.177	3.063	0.134	0.681	0.073	2.634	0.230	0.192	15.713

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	245	27	20	20	26	22
N.S.	1	1.00	1.09	0.82	11.14	1.23	0.91	0.91	1.18	1.00
time (sec)	N/A	0.178	3.128	0.137	0.872	0.071	8.496	0.274	0.179	15.043

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1977	1979	2236	0	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.187	10.497	0.000	0.000	0.000	0.000	0.000	0.408	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1157	1159	846	0	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.239	6.136	0.000	0.000	0.000	0.000	0.000	0.278	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	160	172	179	0	576	0	174	357	2737
N.S.	1	1.28	1.38	1.43	0.00	4.61	0.00	1.39	2.86	21.90
time (sec)	N/A	0.682	0.707	0.200	0.000	0.113	0.000	0.159	0.194	20.039

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	48	22	20	42	22
N.S.	1	1.00	1.09	0.82	0.00	2.18	1.00	0.91	1.91	1.00
time (sec)	N/A	0.175	32.321	0.204	0.000	0.082	6.217	0.939	0.194	16.038

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	48	22	20	48	22
N.S.	1	1.00	1.09	0.82	0.00	2.18	1.00	0.91	2.18	1.00
time (sec)	N/A	0.179	37.788	0.201	0.000	0.080	32.693	1.750	0.190	16.094

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	24	24
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.20	1.20
time (sec)	N/A	0.233	2.952	0.154	1.735	0.080	30.090	0.558	0.210	16.131

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	39	158	128	62	0	0	34	106
N.S.	1	1.00	0.87	3.51	2.84	1.38	0.00	0.00	0.76	2.36
time (sec)	N/A	0.210	0.081	0.362	0.050	0.093	0.000	0.000	0.189	17.570

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	185	731	0	384	0	0	44	0
N.S.	1	1.00	1.31	5.18	0.00	2.72	0.00	0.00	0.31	0.00
time (sec)	N/A	0.320	0.352	0.371	0.000	0.114	0.000	0.000	0.190	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	557	0	0	44	0
N.S.	1	1.00	0.00	0.00	0.00	2.52	0.00	0.00	0.20	0.00
time (sec)	N/A	0.415	0.000	0.000	0.000	0.120	0.000	0.000	0.183	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	80	57	102	275	207	116	0	0	77	182
N.S.	1	0.71	1.28	3.44	2.59	1.45	0.00	0.00	0.96	2.28
time (sec)	N/A	0.434	1.357	0.641	0.066	0.087	0.000	0.000	0.186	17.331

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	214	141	286	1002	0	568	0	0	153	0
N.S.	1	0.66	1.34	4.68	0.00	2.65	0.00	0.00	0.71	0.00
time (sec)	N/A	0.449	4.979	0.763	0.000	0.117	0.000	0.000	0.193	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	377	250	0	0	0	890	0	0	74	0
N.S.	1	0.66	0.00	0.00	0.00	2.36	0.00	0.00	0.20	0.00
time (sec)	N/A	0.645	0.000	0.000	0.000	0.130	0.000	0.000	0.187	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	85	87	79	315	0	301	0	0	87	229
N.S.	1	1.02	0.93	3.71	0.00	3.54	0.00	0.00	1.02	2.69
time (sec)	N/A	0.436	0.591	0.350	0.000	0.105	0.000	0.000	0.189	17.102

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	338	283	1003	777	0	1235	0	0	34	0
N.S.	1	0.84	2.97	2.30	0.00	3.65	0.00	0.00	0.10	0.00
time (sec)	N/A	0.846	3.942	0.425	0.000	0.285	0.000	0.000	0.192	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	499	418	0	0	0	1663	0	0	34	0
N.S.	1	0.84	0.00	0.00	0.00	3.33	0.00	0.00	0.07	0.00
time (sec)	N/A	1.152	0.000	0.000	0.000	0.283	0.000	0.000	0.194	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	156	167	176	622	0	630	0	0	385	0
N.S.	1	1.07	1.13	3.99	0.00	4.04	0.00	0.00	2.47	0.00
time (sec)	N/A	0.781	1.301	1.519	0.000	0.117	0.000	0.000	0.184	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	778	624	2839	3887	0	2435	0	0	698	0
N.S.	1	0.80	3.65	5.00	0.00	3.13	0.00	0.00	0.90	0.00
time (sec)	N/A	1.470	9.428	1.418	0.000	0.337	0.000	0.000	0.270	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1417	1152	0	0	0	3755	0	0	0	0
N.S.	1	0.81	0.00	0.00	0.00	2.65	0.00	0.00	0.00	0.00
time (sec)	N/A	2.393	0.000	0.000	0.000	0.380	0.000	0.000	0.323	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [15] had the largest ratio of [.7500000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	16	0.125
2	N/A	2	0	1.00	16	0.000
3	A	2	2	1.00	16	0.125
4	N/A	2	0	1.00	16	0.000
5	A	2	2	1.00	14	0.143
6	N/A	2	0	1.00	16	0.000
7	N/A	2	0	1.00	16	0.000
8	A	5	4	1.00	18	0.222
9	N/A	1	0	1.00	18	0.000
10	A	5	4	0.99	18	0.222
11	N/A	1	0	1.00	18	0.000
12	A	8	7	0.98	16	0.438
13	N/A	1	0	1.00	18	0.000
14	N/A	1	0	1.00	18	0.000
15	A	10	9	1.16	12	0.750
16	A	5	4	1.00	18	0.222
17	N/A	1	0	1.00	18	0.000
18	A	5	4	0.98	18	0.222
19	N/A	1	0	1.00	18	0.000
20	A	8	7	1.17	16	0.438

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	N/A	1	0	1.00	18	0.000
22	N/A	2	0	1.00	16	0.000
23	A	5	4	1.00	18	0.222
24	N/A	1	0	1.00	18	0.000
25	A	5	4	0.99	18	0.222
26	N/A	1	0	1.00	18	0.000
27	A	13	12	1.28	16	0.750
28	N/A	1	0	1.00	18	0.000
29	N/A	1	0	1.00	18	0.000
30	N/A	1	0	1.00	18	0.000
31	A	2	2	1.00	18	0.111
32	A	2	2	1.00	18	0.111
33	A	2	2	1.00	16	0.125
34	N/A	2	0	1.00	18	0.000
35	N/A	2	0	1.00	18	0.000
36	A	5	4	1.01	20	0.200
37	A	5	4	1.01	20	0.200
38	A	5	4	1.01	18	0.222
39	N/A	1	0	1.00	20	0.000
40	N/A	1	0	1.00	20	0.000
41	A	5	4	1.00	20	0.200
42	A	5	4	1.00	20	0.200
43	A	5	4	1.00	18	0.222
44	N/A	1	0	1.00	20	0.000
45	N/A	2	0	1.00	18	0.000
46	A	5	4	1.00	20	0.200
47	A	5	4	1.00	20	0.200
48	A	5	4	1.00	18	0.222
49	N/A	1	0	1.00	20	0.000
50	N/A	1	0	1.00	20	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
51	A	2	2	1.00	20	0.100
52	A	2	2	1.00	20	0.100
53	A	2	2	1.00	20	0.100
54	N/A	2	0	1.00	20	0.000
55	N/A	2	0	1.00	20	0.000
56	A	5	4	1.00	22	0.182
57	A	5	4	1.01	22	0.182
58	A	8	7	1.02	22	0.318
59	N/A	1	0	1.00	22	0.000
60	N/A	1	0	1.00	22	0.000
61	A	6	5	1.25	14	0.357
62	A	5	4	1.00	22	0.182
63	A	5	4	1.00	22	0.182
64	A	8	7	1.15	22	0.318
65	N/A	1	0	1.00	22	0.000
66	N/A	1	0	1.00	22	0.000
67	A	5	4	1.00	22	0.182
68	A	5	4	1.00	22	0.182
69	A	13	12	1.28	22	0.545
70	N/A	1	0	1.00	22	0.000
71	N/A	1	0	1.00	22	0.000
72	N/A	2	0	1.00	20	0.000
73	A	2	2	1.00	20	0.100
74	A	2	2	1.00	22	0.091
75	A	2	2	1.00	22	0.091
76	A	9	8	0.71	22	0.364
77	A	6	5	0.66	24	0.208
78	A	6	5	0.66	24	0.208
79	A	9	8	1.02	22	0.364
80	A	6	5	0.84	24	0.208
81	A	6	5	0.84	24	0.208

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
82	A	14	13	1.07	22	0.591
83	A	6	5	0.80	24	0.208
84	A	6	5	0.81	24	0.208

CHAPTER 3

Listing of integrals

3.1 $\int x^5(a+b\csc(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 129

$$\int x^5(a+b\csc(c+dx^2))\,dx=\frac{ax^6}{6}-\frac{bx^4\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d}+\frac{ibx^2\operatorname{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2}-\frac{ibx^2\operatorname{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2}-\frac{b\operatorname{PolyLog}\left(3,-e^{i(c+dx^2)}\right)}{d^3}+\frac{b\operatorname{PolyLog}\left(3,e^{i(c+dx^2)}\right)}{d^3}$$

output
```
1/6*a*x^6-b*x^4*arctanh(exp(I*(d*x^2+c)))/d+I*b*x^2*polylog(2,-exp(I*(d*x^
2+c)))/d^2-I*b*x^2*polylog(2,exp(I*(d*x^2+c)))/d^2-b*polylog(3,-exp(I*(d*x
^2+c)))/d^3+b*polylog(3,exp(I*(d*x^2+c)))/d^3
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.23

$$\int x^5 \left(a+b\csc\left(c+dx^2\right)\right)\,dx = \frac{ax^6}{6}$$
$$-\frac{b(d^2x^4\text{arctanh}(\cos\left(c+dx^2\right)+i\sin\left(c+dx^2\right))-idx^2\,\text{PolyLog}\left(2,-\cos\left(c+dx^2\right)-i\sin\left(c+dx^2\right)\right)+}{}$$

input
```
Integrate[x^5*(a + b*Csc[c + d*x^2]),x]
```

output
```
(a*x^6)/6 - (b*(d^2*x^4*ArcTanh[Cos[c + d*x^2] + I*Sin[c + d*x^2]] - I*d*x
^2*PolyLog[2, -Cos[c + d*x^2] - I*Sin[c + d*x^2]] + I*d*x^2*PolyLog[2, Cos
[c + d*x^2] + I*Sin[c + d*x^2]] + PolyLog[3, -Cos[c + d*x^2] - I*Sin[c + d
*x^2]] - PolyLog[3, Cos[c + d*x^2] + I*Sin[c + d*x^2]]))/d^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b\csc\left(c+dx^2\right)\right)\,dx$$
$$\Big\downarrow \text{2010}$$
$$\int \left(ax^5+bx^5\csc\left(c+dx^2\right)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{ax^6}{6}-\frac{bx^4\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d}-\frac{b\,\text{PolyLog}\left(3,-e^{i(dx^2+c)}\right)}{d^3}+\frac{b\,\text{PolyLog}\left(3,e^{i(dx^2+c)}\right)}{d^3}+\frac{ibx^2\,\text{PolyLog}\left(2,-e^{i(dx^2+c)}\right)}{d^2}-\frac{ibx^2\,\text{PolyLog}\left(2,e^{i(dx^2+c)}\right)}{d^2}$$

input
```
Int[x^5*(a + b*Csc[c + d*x^2]),x]
```

output
```
(a*x^6)/6 - (b*x^4*ArcTanh[E^(I*(c + d*x^2))])/d + (I*b*x^2*PolyLog[2, -E^
(I*(c + d*x^2))])/d^2 - (I*b*x^2*PolyLog[2, E^(I*(c + d*x^2))])/d^2 - (b*P
olyLog[3, -E^(I*(c + d*x^2))])/d^3 + (b*PolyLog[3, E^(I*(c + d*x^2))])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int x^5 (a + b\csc(d\,x^2 + c))\,dx$$

input
```
int(x^5*(a+b*csc(d*x^2+c)),x)
```

output
```
int(x^5*(a+b*csc(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(111) = 222$.

Time = 0.10 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.29

$$\int x^5 (a + b\csc(c + dx^2))\ dx$$
$$= \frac{2\,ad^3x^6 - 3\,bd^2x^4\log(\cos(dx^2 + c) + i\,\sin(dx^2 + c) + 1) - 3\,bd^2x^4\log(\cos(dx^2 + c) - i\,\sin(dx^2 + c) +}{}$$

input
```
integrate(x^5*(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

output
```
1/12*(2*a*d^3*x^6 - 3*b*d^2*x^4*log(cos(d*x^2 + c) + I*sin(d*x^2 + c) + 1)
 - 3*b*d^2*x^4*log(cos(d*x^2 + c) - I*sin(d*x^2 + c) + 1) - 6*I*b*d*x^2*di
log(cos(d*x^2 + c) + I*sin(d*x^2 + c)) + 6*I*b*d*x^2*dilog(cos(d*x^2 + c)
- I*sin(d*x^2 + c)) - 6*I*b*d*x^2*dilog(-cos(d*x^2 + c) + I*sin(d*x^2 + c)
) + 6*I*b*d*x^2*dilog(-cos(d*x^2 + c) - I*sin(d*x^2 + c)) + 3*b*c^2*log(-1
/2*cos(d*x^2 + c) + 1/2*I*sin(d*x^2 + c) + 1/2) + 3*b*c^2*log(-1/2*cos(d*x
^2 + c) - 1/2*I*sin(d*x^2 + c) + 1/2) + 3*(b*d^2*x^4 - b*c^2)*log(-cos(d*x
^2 + c) + I*sin(d*x^2 + c) + 1) + 3*(b*d^2*x^4 - b*c^2)*log(-cos(d*x^2 + c
) - I*sin(d*x^2 + c) + 1) + 6*b*polylog(3, cos(d*x^2 + c) + I*sin(d*x^2 +
c)) + 6*b*polylog(3, cos(d*x^2 + c) - I*sin(d*x^2 + c)) - 6*b*polylog(3, -
cos(d*x^2 + c) + I*sin(d*x^2 + c)) - 6*b*polylog(3, -cos(d*x^2 + c) - I*si
n(d*x^2 + c)))/d^3
```

Sympy [F]

$$\int x^5 \left(a + b\csc\left(c + dx^2\right)\right)\,dx = \int x^5 \left(a + b\csc\left(c + dx^2\right)\right)\,dx$$

input
```
integrate(x**5*(a+b*csc(d*x**2+c)),x)
```

output
```
Integral(x**5*(a + b*csc(c + d*x**2)), x)
```

Maxima [F]

$$\int x^5 \left(a + b\csc\left(c + dx^2\right)\right)\,dx = \int \left(b\csc\left(dx^2 + c\right) + a\right)x^5\,dx$$

input
```
integrate(x^5*(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + b*(integrate(x^5*sin(d*x^2 + c)/(cos(d*x^2 + c)^2 + sin(d*x^2
+ c)^2 + 2*cos(d*x^2 + c) + 1), x) + integrate(x^5*sin(d*x^2 + c)/(cos(d*x
^2 + c)^2 + sin(d*x^2 + c)^2 - 2*cos(d*x^2 + c) + 1), x))
```

Giac [F]

$$\int x^5 \left(a + b\csc\left(c + dx^2\right)\right) dx = \int \left(b\csc\left(dx^2 + c\right) + a\right)x^5 \, dx$$

input
```
integrate(x^5*(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^2 + c) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a + b\csc\left(c + dx^2\right)\right) dx = \int x^5 \left(a + \frac{b}{\sin\left(d\,x^2 + c\right)}\right) dx$$

input
```
int(x^5*(a + b/sin(c + d*x^2)),x)
```

output
```
int(x^5*(a + b/sin(c + d*x^2)), x)
```

Reduce [F]

$$\int x^5 \left(a + b\csc\left(c + dx^2\right)\right) dx = \left(\int \csc\left(d\,x^2 + c\right) x^5 dx\right) b + \frac{a\,x^6}{6}$$

input
```
int(x^5*(a+b*csc(d*x^2+c)),x)
```

output
```
(6*int(csc(c + d*x**2)*x***5,x)*b + a*x***6)/6
```

3.2 $\int x^4(a + b\csc(c + dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{ax^5}{5}+b\text{Int}\left(x^4\csc\left(c+dx^2\right),x\right)$$

output
```
1/5*a*x^5+b*Defer(Int)(x^4*csc(d*x^2+c),x)
```

Mathematica [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx$$

input
```
Integrate[x^4*(a + b*Csc[c + d*x^2]),x]
```

output
```
Integrate[x^4*(a + b*Csc[c + d*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a + b\csc(c + dx^2)) \, dx$$

$$\Big\downarrow \text{2010}$$

$$\int (ax^4 + bx^4\csc(c + dx^2)) \, dx$$

$$\Big\downarrow \text{2009}$$

$$b\int x^4\csc(dx^2 + c) \, dx + \frac{ax^5}{5}$$

input
```
Int[x^4*(a + b*Csc[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^{4}\left(a+b\csc\left(d\,x^{2}+c\right)\right)dx$$

input
```
int(x^4*(a+b*csc(d*x^2+c)),x)
```

output
```
int(x^4*(a+b*csc(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\int\left(b\csc\left(dx^2+c\right)+a\right)x^4\,dx$$

input
```
integrate(x^4*(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

output
```
integral(b*x^4*csc(d*x^2 + c) + a*x^4, x)
```

Sympy [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx$$

input
```
integrate(x**4*(a+b*csc(d*x**2+c)),x)
```

output
```
Integral(x**4*(a + b*csc(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 6.62

$$\int x^4(a+b\csc(c+dx^2))\,dx=\int (b\csc(dx^2+c)+a)x^4\,dx$$

input
```
integrate(x^4*(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + b*(integrate(x^4*sin(d*x^2 + c)/(cos(d*x^2 + c)^2 + sin(d*x^2
+ c)^2 + 2*cos(d*x^2 + c) + 1), x) + integrate(x^4*sin(d*x^2 + c)/(cos(d*x
^2 + c)^2 + sin(d*x^2 + c)^2 - 2*cos(d*x^2 + c) + 1), x))
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4(a+b\csc(c+dx^2))\,dx=\int (b\csc(dx^2+c)+a)x^4\,dx$$

input
```
integrate(x^4*(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^2 + c) + a)*x^4, x)
```

Mupad [N/A]

Not integrable

Time = 15.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^4(a+b\csc(c+dx^2))\,dx=\int x^4\left(a+\frac{b}{\sin(d\,x^2+c)}\right)\,dx$$

input
```
int(x^4*(a + b/sin(c + d*x^2)),x)
```

output
```
int(x^4*(a + b/sin(c + d*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int x^4(a+b\csc(c+dx^2))\,dx=\left(\int \csc\left(d\,x^2+c\right)x^4dx\right)b+\frac{a\,x^5}{5}$$

input
```
int(x^4*(a+b*csc(d*x^2+c)),x)
```

output
```
(5*int(csc(c + d*x**2)*x**4,x)*b + a*x**5)/5
```

3.3 $\int x^3(a+b\csc(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int x^3\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{ax^4}{4}-\frac{bx^2\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d}+\frac{ib\,\text{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{2d^2}-\frac{ib\,\text{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{2d^2}$$

output
```
1/4*a*x^4-b*x^2*arctanh(exp(I*(d*x^2+c)))/d+1/2*I*b*polylog(2,-exp(I*(d*x^
2+c)))/d^2-1/2*I*b*polylog(2,exp(I*(d*x^2+c)))/d^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int x^3\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{ax^4}{4}+\frac{b\left(\left(c+dx^2\right)\left(\log\left(1-e^{i(c+dx^2)}\right)-\log\left(1+e^{i(c+dx^2)}\right)\right)-c\log\left(\tan\left(\frac{1}{2}(c+dx^2)\right)\right)+i\left(\text{PolyLog}\left(2,-\right.\right.\right.}{2d^2}$$

input
```
Integrate[x^3*(a + b*Csc[c + d*x^2]),x]
```

output
```
(a*x^4)/4 + (b*((c + d*x^2)*(Log[1 - E^(I*(c + d*x^2))] - Log[1 + E^(I*(c
+ d*x^2))]) - c*Log[Tan[(c + d*x^2)/2]] + I*(PolyLog[2, -E^(I*(c + d*x^2))
] - PolyLog[2, E^(I*(c + d*x^2))])))/(2*d^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\csc\left(c+dx^2\right)\right)\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(ax^3+bx^3\csc\left(c+dx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^4}{4}-\frac{bx^2\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d}+\frac{ib\,\text{PolyLog}\left(2,-e^{i(dx^2+c)}\right)}{2d^2}-\frac{ib\,\text{PolyLog}\left(2,e^{i(dx^2+c)}\right)}{2d^2}$$

input
```
Int[x^3*(a + b*Csc[c + d*x^2]),x]
```

output
```
(a*x^4)/4 - (b*x^2*ArcTanh[E^(I*(c + d*x^2))])/d + ((I/2)*b*PolyLog[2, -E^
(I*(c + d*x^2))])/d^2 - ((I/2)*b*PolyLog[2, E^(I*(c + d*x^2))])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int x^{3}\left(a+b\csc\left(d\,x^{2}+c\right)\right)dx$$

input
```
int(x^3*(a+b*csc(d*x^2+c)),x)
```

output
```
int(x^3*(a+b*csc(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(66) = 132.

Time = 0.10 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.43

$$\int x^{3}\left(a+b\csc\left(c+dx^{2}\right)\right)\,dx$$
$$=\frac{ad^{2}x^{4}-bdx^{2}\log\left(\cos\left(dx^{2}+c\right)+i\,\sin\left(dx^{2}+c\right)+1\right)-bdx^{2}\log\left(\cos\left(dx^{2}+c\right)-i\,\sin\left(dx^{2}+c\right)+1\right)-b}{}$$

input
```
integrate(x^3*(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

output
```
1/4*(a*d^2*x^4 - b*d*x^2*log(cos(d*x^2 + c) + I*sin(d*x^2 + c) + 1) - b*d*
x^2*log(cos(d*x^2 + c) - I*sin(d*x^2 + c) + 1) - b*c*log(-1/2*cos(d*x^2 +
c) + 1/2*I*sin(d*x^2 + c) + 1/2) - b*c*log(-1/2*cos(d*x^2 + c) - 1/2*I*sin
(d*x^2 + c) + 1/2) - I*b*dilog(cos(d*x^2 + c) + I*sin(d*x^2 + c)) + I*b*di
log(cos(d*x^2 + c) - I*sin(d*x^2 + c)) - I*b*dilog(-cos(d*x^2 + c) + I*sin
(d*x^2 + c)) + I*b*dilog(-cos(d*x^2 + c) - I*sin(d*x^2 + c)) + (b*d*x^2 +
b*c)*log(-cos(d*x^2 + c) + I*sin(d*x^2 + c) + 1) + (b*d*x^2 + b*c)*log(-co
s(d*x^2 + c) - I*sin(d*x^2 + c) + 1))/d^2
```

Sympy [F]

$$\int x^3(a+b\csc(c+dx^2))\,dx=\int x^3(a+b\csc(c+dx^2))\,dx$$

input
```
integrate(x**3*(a+b*csc(d*x**2+c)),x)
```

output
```
Integral(x**3*(a + b*csc(c + d*x**2)), x)
```

Maxima [F]

$$\int x^3(a+b\csc(c+dx^2))\,dx=\int (b\csc(dx^2+c)+a)x^3\,dx$$

input
```
integrate(x^3*(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + b*(integrate(x^3*sin(d*x^2 + c)/(cos(d*x^2 + c)^2 + sin(d*x^2
+ c)^2 + 2*cos(d*x^2 + c) + 1), x) + integrate(x^3*sin(d*x^2 + c)/(cos(d*x
^2 + c)^2 + sin(d*x^2 + c)^2 - 2*cos(d*x^2 + c) + 1), x))
```

Giac [F]

$$\int x^3 \left(a+b\csc \left(c+dx^2\right)\right)\,dx=\int \left(b\csc \left(dx^2+c\right)+a\right)x^3\,dx$$

input
```
integrate(x^3*(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^2 + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\csc \left(c+dx^2\right)\right)\,dx=\int x^3 \left(a+\frac{b}{\sin \left(d\,x^2+c\right)}\right)\,dx$$

input
```
int(x^3*(a + b/sin(c + d*x^2)),x)
```

output
```
int(x^3*(a + b/sin(c + d*x^2)), x)
```

Reduce [F]

$$\int x^3 \left(a+b\csc \left(c+dx^2\right)\right)\,dx=\left(\int \csc \left(d\,x^2+c\right)x^3dx\right)b+\frac{a\,x^4}{4}$$

input
```
int(x^3*(a+b*csc(d*x^2+c)),x)
```

output
```
(4*int(csc(c + d*x**2)*x**3,x)*b + a*x**4)/4
```

3.4 $\int x^2(a+b\csc(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^2\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{ax^3}{3}+b\mathrm{Int}\left(x^2\csc\left(c+dx^2\right),x\right)$$

output
```
1/3*a*x^3+b*Defer(Int)(x^2*csc(d*x^2+c),x)
```

Mathematica [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\int x^2\left(a+b\csc\left(c+dx^2\right)\right)\,dx$$

input
```
Integrate[x^2*(a + b*Csc[c + d*x^2]),x]
```

output
```
Integrate[x^2*(a + b*Csc[c + d*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \csc \left(c+d x^2\right)\right) \, dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(a x^2+b x^2 \csc \left(c+d x^2\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$b \int x^2 \csc \left(d x^2+c\right)dx+\frac{a x^3}{3}$$

input
```
Int[x^2*(a + b*Csc[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^2 \left(a + b \csc \left(d\, x^2 + c\right)\right) dx$$

input
```
int(x^2*(a+b*csc(d*x^2+c)),x)
```

output
```
int(x^2*(a+b*csc(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^2 \left(a + b \csc \left(c + dx^2\right)\right)\, dx = \int \left(b \csc \left(dx^2 + c\right) + a\right) x^2\, dx$$

input
```
integrate(x^2*(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

output
```
integral(b*x^2*csc(d*x^2 + c) + a*x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^2 \left(a + b \csc \left(c + dx^2\right)\right)\, dx = \int x^2 \left(a + b \csc \left(c + dx^2\right)\right)\, dx$$

input
```
integrate(x**2*(a+b*csc(d*x**2+c)),x)
```

output
```
Integral(x**2*(a + b*csc(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 6.62

$$\int x^2 \left(a + b \csc \left(c + dx^2\right)\right) \, dx = \int \left(b \csc \left(dx^2 + c\right) + a\right) x^2 \, dx$$

input
```
integrate(x^2*(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + b*(integrate(x^2*sin(d*x^2 + c)/(cos(d*x^2 + c)^2 + sin(d*x^2
+ c)^2 + 2*cos(d*x^2 + c) + 1), x) + integrate(x^2*sin(d*x^2 + c)/(cos(d*x
^2 + c)^2 + sin(d*x^2 + c)^2 - 2*cos(d*x^2 + c) + 1), x))
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2 \left(a + b \csc \left(c + dx^2\right)\right) \, dx = \int \left(b \csc \left(dx^2 + c\right) + a\right) x^2 \, dx$$

input
```
integrate(x^2*(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^2 + c) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 14.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^2 \left(a + b \csc \left(c + dx^2\right)\right) \, dx = \int x^2 \left(a + \frac{b}{\sin \left(d\, x^2 + c\right)}\right) \, dx$$

input
```
int(x^2*(a + b/sin(c + d*x^2)),x)
```

output
```
int(x^2*(a + b/sin(c + d*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int x^2(a+b\csc(c+dx^2))\,dx = \left(\int \csc\left(d\,x^2+c\right)x^2dx\right)b+\frac{a\,x^3}{3}$$

input
```
int(x^2*(a+b*csc(d*x^2+c)),x)
```

output
```
(3*int(csc(c + d*x**2)*x**2,x)*b + a*x**3)/3
```

3.5 $\int x(a+b\csc(c+dx^2))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}-\frac{b\text{arctanh}(\cos\left(c+dx^2\right))}{2d}$$

output
```
1/2*a*x^2-1/2*b*arctanh(cos(d*x^2+c))/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}-\frac{b\text{arctanh}(\cos\left(c+dx^2\right))}{2d}$$

input
```
Integrate[x*(a + b*Csc[c + d*x^2]),x]
```

output
```
(a*x^2)/2 - (b*ArcTanh[Cos[c + d*x^2]])/(2*d)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)\,dx$$
$$\Big\downarrow \text{ 2010}$$
$$\int \left(ax+bx\csc\left(c+dx^2\right)\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{ax^2}{2}-\frac{b\,\text{arctanh}\left(\cos\left(c+dx^2\right)\right)}{2d}$$

input
```
Int[x*(a + b*Csc[c + d*x^2]),x]
```

output
```
(a*x^2)/2 - (b*ArcTanh[Cos[c + d*x^2]])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
norman	$\frac{a\,x^{2}}{2}+\frac{b\ln\left(\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{2d}$	26
parallelrisch	$\frac{a d\,x^{2}+b\ln\left(\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{2d}$	27
parts	$\frac{a\,x^{2}}{2}-\frac{b\ln\left(\csc\left(d\,x^{2}+c\right)+\cot\left(d\,x^{2}+c\right)\right)}{2d}$	32
derivativedivides	$\frac{\left(d\,x^{2}+c\right)a-b\ln\left(\csc\left(d\,x^{2}+c\right)+\cot\left(d\,x^{2}+c\right)\right)}{2d}$	37
default	$\frac{\left(d\,x^{2}+c\right)a-b\ln\left(\csc\left(d\,x^{2}+c\right)+\cot\left(d\,x^{2}+c\right)\right)}{2d}$	37
risch	$\frac{a\,x^{2}}{2}+\frac{b\ln\left(\mathrm{e}^{i\left(d\,x^{2}+c\right)}-1\right)}{2d}-\frac{b\ln\left(\mathrm{e}^{i\left(d\,x^{2}+c\right)}+1\right)}{2d}$	48

input
```
int(x*(a+b*csc(d*x^2+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/2*b/d*ln(tan(1/2*d*x^2+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)\,dx = \frac{2\,adx^2-b\log\left(\frac{1}{2}\,\cos\left(dx^2+c\right)+\frac{1}{2}\right)+b\log\left(-\frac{1}{2}\,\cos\left(dx^2+c\right)+\frac{1}{2}\right)}{4\,d}$$

input
```
integrate(x*(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

output
```
1/4*(2*a*d*x^2 - b*log(1/2*cos(d*x^2 + c) + 1/2) + b*log(-1/2*cos(d*x^2 +
c) + 1/2))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 2.82 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\begin{cases}\frac{a(c+dx^2)-b\log\left(\cot\left(c+dx^2\right)+\csc\left(c+dx^2\right)\right)}{2d} & \text{for } d\neq 0\\ \frac{x^2(a+b\csc(c))}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(a+b*csc(d*x**2+c)),x)
```

output
```
Piecewise(((a*(c + d*x**2) - b*log(cot(c + d*x**2) + csc(c + d*x**2)))/(2*
d), Ne(d, 0)), (x**2*(a + b*csc(c))/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{1}{2}\,ax^2-\frac{b\log\left(\cot\left(dx^2+c\right)+\csc\left(dx^2+c\right)\right)}{2\,d}$$

input
```
integrate(x*(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 - 1/2*b*log(cot(d*x^2 + c) + csc(d*x^2 + c))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{(dx^2+c)a+b\log\left(\left|\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)\right|\right)}{2\,d}$$

input
```
integrate(x*(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

output
```
1/2*((d*x^2 + c)*a + b*log(abs(tan(1/2*d*x^2 + 1/2*c))))/d
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.65

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,\ln\left(-b\,x\,2\mathrm{i}-b\,x\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}\right)}{2\,d}+\frac{b\,\ln\left(b\,x\,2\mathrm{i}-b\,x\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}\right)}{2\,d}$$

input
```
int(x*(a + b/sin(c + d*x^2)),x)
```

output
```
(a*x^2)/2 - (b*log(- b*x*2i - b*x*exp(d*x^2*1i)*exp(c*1i)*2i))/(2*d) + (b*
log(b*x*2i - b*x*exp(d*x^2*1i)*exp(c*1i)*2i))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{\log\left(\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)b+ad\,x^2}{2d}$$

input
```
int(x*(a+b*csc(d*x^2+c)),x)
```

output
```
(log(tan((c + d*x**2)/2))*b + a*d*x**2)/(2*d)
```

3.6 $\int \frac{a+b\csc(c+dx^2)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\csc(c+dx^2)}{x}\,dx = a\log(x) + b\text{Int}\left(\frac{\csc(c+dx^2)}{x},x\right)$$

output
```
a*ln(x)+b*Defer(Int)(csc(d*x^2+c)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc(c+dx^2)}{x}\,dx = \int \frac{a+b\csc(c+dx^2)}{x}\,dx$$

input
```
Integrate[(a + b*Csc[c + d*x^2])/x,x]
```

output
```
Integrate[(a + b*Csc[c + d*x^2])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x}dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x}+\frac{b\csc\left(c+dx^2\right)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$b\int \frac{\csc\left(dx^2+c\right)}{x}dx+a\log(x)$$

input
```
Int[(a + b*Csc[c + d*x^2])/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc\left(d\,x^{2}+c\right)}{x}dx$$

input
```
int((a+b*csc(d*x^2+c))/x,x)
```

output
```
int((a+b*csc(d*x^2+c))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc\left(c+dx^{2}\right)}{x}\,dx=\int \frac{b\csc\left(dx^{2}+c\right)+a}{x}\,dx$$

input
```
integrate((a+b*csc(d*x^2+c))/x,x, algorithm="fricas")
```

output
```
integral((b*csc(d*x^2 + c) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a+b\csc\left(c+dx^{2}\right)}{x}\,dx=\int \frac{a+b\csc\left(c+dx^{2}\right)}{x}\,dx$$

input
```
integrate((a+b*csc(d*x**2+c))/x,x)
```

output
```
Integral((a + b*csc(c + d*x**2))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 6.75

$$\int \frac{a+b\csc(c+dx^2)}{x}\,dx = \int \frac{b\csc(dx^2+c)+a}{x}\,dx$$

input
```
integrate((a+b*csc(d*x^2+c))/x,x, algorithm="maxima")
```

output
```
b*(integrate(sin(d*x^2 + c)/(x*cos(d*x^2 + c)^2 + x*sin(d*x^2 + c)^2 + 2*x
*cos(d*x^2 + c) + x), x) + integrate(sin(d*x^2 + c)/(x*cos(d*x^2 + c)^2 +
x*sin(d*x^2 + c)^2 - 2*x*cos(d*x^2 + c) + x), x)) + a*log(x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc(c+dx^2)}{x}\,dx = \int \frac{b\csc(dx^2+c)+a}{x}\,dx$$

input
```
integrate((a+b*csc(d*x^2+c))/x,x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^2 + c) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 15.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a+b\csc(c+dx^2)}{x}\,dx = \int \frac{a+\frac{b}{\sin(d\,x^2+c)}}{x}\,dx$$

input
```
int((a + b/sin(c + d*x^2))/x,x)
```

output
```
int((a + b/sin(c + d*x^2))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x}\,dx=\left(\int \frac{\csc\left(d\,x^2+c\right)}{x}dx\right)b+\log(x)\,a$$

input
```
int((a+b*csc(d*x^2+c))/x,x)
```

output
```
int(csc(c + d*x**2)/x,x)*b + log(x)*a
```

3.7 $\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = -\frac{a}{x}+b\text{Int}\left(\frac{\csc(c+dx^2)}{x^2},x\right)$$

output
```
-a/x+b*Defer(Int)(csc(d*x^2+c)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = \int \frac{a+b\csc(c+dx^2)}{x^2}\,dx$$

input
```
Integrate[(a + b*Csc[c + d*x^2])/x^2,x]
```

output
```
Integrate[(a + b*Csc[c + d*x^2])/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}dx$$

$$\Big\downarrow \ 2010$$

$$\int \left(\frac{a}{x^2}+\frac{b\csc\left(c+dx^2\right)}{x^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$b\int \frac{\csc\left(dx^2+c\right)}{x^2}dx-\frac{a}{x}$$

input
```
Int[(a + b*Csc[c + d*x^2])/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a + b\csc\left(d\, x^{2} + c\right)}{x^{2}} dx$$

input
```
int((a+b*csc(d*x^2+c))/x^2,x)
```

output
```
int((a+b*csc(d*x^2+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\csc\left(c + dx^{2}\right)}{x^{2}}\, dx = \int \frac{b\csc\left(dx^{2} + c\right) + a}{x^{2}}\, dx$$

input
```
integrate((a+b*csc(d*x^2+c))/x^2,x, algorithm="fricas")
```

output
```
integral((b*csc(d*x^2 + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a + b\csc\left(c + dx^{2}\right)}{x^{2}}\, dx = \int \frac{a + b\csc\left(c + dx^{2}\right)}{x^{2}}\, dx$$

input
```
integrate((a+b*csc(d*x**2+c))/x**2,x)
```

output
```
Integral((a + b*csc(c + d*x**2))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 7.88

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx=\int \frac{b\csc\left(dx^2+c\right)+a}{x^2}\,dx$$

input
```
integrate((a+b*csc(d*x^2+c))/x^2,x, algorithm="maxima")
```

output
```
b*(integrate(sin(d*x^2 + c)/(x^2*cos(d*x^2 + c)^2 + x^2*sin(d*x^2 + c)^2 +
 2*x^2*cos(d*x^2 + c) + x^2), x) + integrate(sin(d*x^2 + c)/(x^2*cos(d*x^2
 + c)^2 + x^2*sin(d*x^2 + c)^2 - 2*x^2*cos(d*x^2 + c) + x^2), x)) - a/x
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx=\int \frac{b\csc\left(dx^2+c\right)+a}{x^2}\,dx$$

input
```
integrate((a+b*csc(d*x^2+c))/x^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^2 + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 15.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx=\int \frac{a+\frac{b}{\sin\left(d\,x^2+c\right)}}{x^2}\,dx$$

input
```
int((a + b/sin(c + d*x^2))/x^2,x)
```

output
```
int((a + b/sin(c + d*x^2))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx=\frac{\left(\int \frac{\csc(d\,x^2+c)}{x^2}dx\right)bx-a}{x}$$

input
```
int((a+b*csc(d*x^2+c))/x^2,x)
```

output
```
(int(csc(c + d*x**2)/x**2,x)*b*x - a)/x
```

3.8 $\int x^5(a+b\csc(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 228

$$\begin{aligned}\int x^5(a+b\csc(c+dx^2))^2\,dx &= -\frac{ib^2x^4}{2d}+\frac{a^2x^6}{6}-\frac{2abx^4\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d} \\ &\quad -\frac{b^2x^4\cot(c+dx^2)}{2d}+\frac{b^2x^2\log\left(1-e^{2i(c+dx^2)}\right)}{d^2} \\ &\quad +\frac{2iabx^2\operatorname{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2} \\ &\quad -\frac{2iabx^2\operatorname{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2} \\ &\quad -\frac{ib^2\operatorname{PolyLog}\left(2,e^{2i(c+dx^2)}\right)}{2d^3} \\ &\quad -\frac{2ab\operatorname{PolyLog}\left(3,-e^{i(c+dx^2)}\right)}{d^3} \\ &\quad +\frac{2ab\operatorname{PolyLog}\left(3,e^{i(c+dx^2)}\right)}{d^3}\end{aligned}$$

output
```
-1/2*I*b^2*x^4/d+1/6*a^2*x^6-2*a*b*x^4*arctanh(exp(I*(d*x^2+c)))/d-1/2*b^2
*x^4*cot(d*x^2+c)/d+b^2*x^2*ln(1-exp(2*I*(d*x^2+c)))/d^2+2*I*a*b*x^2*polyl
og(2,-exp(I*(d*x^2+c)))/d^2-2*I*a*b*x^2*polylog(2,exp(I*(d*x^2+c)))/d^2-1/
2*I*b^2*polylog(2,exp(2*I*(d*x^2+c)))/d^3-2*a*b*polylog(3,-exp(I*(d*x^2+c)
))/d^3+2*a*b*polylog(3,exp(I*(d*x^2+c)))/d^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 639 vs. $2(228) = 456$.

Time = 2.24 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.80

$$\int x^5 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx$$
$$= \frac{-12ib^2d^2x^4 - 2a^2d^3x^6 + 2a^2d^3e^{2ic}x^6 - 12b^2dx^2 \log \left(1 - e^{-i(c+dx^2)}\right) + 12b^2de^{2ic}x^2 \log \left(1 - e^{-i(c+dx^2)}\right) -}{}$$

input
```
Integrate[x^5*(a + b*Csc[c + d*x^2])^2,x]
```

output
```
((-12*I)*b^2*d^2*x^4 - 2*a^2*d^3*x^6 + 2*a^2*d^3*E^((2*I)*c)*x^6 - 12*b^2*
d*x^2*Log[1 - E^((-I)*(c + d*x^2))] + 12*b^2*d*E^((2*I)*c)*x^2*Log[1 - E^(
(-I)*(c + d*x^2))] - 12*a*b*d^2*x^4*Log[1 - E^((-I)*(c + d*x^2))] + 12*a*b
*d^2*E^((2*I)*c)*x^4*Log[1 - E^((-I)*(c + d*x^2))] - 12*b^2*d*x^2*Log[1 +
E^((-I)*(c + d*x^2))] + 12*b^2*d*E^((2*I)*c)*x^2*Log[1 + E^((-I)*(c + d*x^
2))] + 12*a*b*d^2*x^4*Log[1 + E^((-I)*(c + d*x^2))] - 12*a*b*d^2*E^((2*I)*
c)*x^4*Log[1 + E^((-I)*(c + d*x^2))] + (12*I)*b*(-1 + E^((2*I)*c))*(b - 2*
a*d*x^2)*PolyLog[2, -E^((-I)*(c + d*x^2))] + (12*I)*b*(-1 + E^((2*I)*c))*(
b + 2*a*d*x^2)*PolyLog[2, E^((-I)*(c + d*x^2))] + 24*a*b*PolyLog[3, -E^((-
I)*(c + d*x^2))] - 24*a*b*E^((2*I)*c)*PolyLog[3, -E^((-I)*(c + d*x^2))] -
24*a*b*PolyLog[3, E^((-I)*(c + d*x^2))] + 24*a*b*E^((2*I)*c)*PolyLog[3, E^
((-I)*(c + d*x^2))] - 3*b^2*d^2*x^4*Csc[c/2]*Csc[(c + d*x^2)/2]*Sin[(d*x^2
)/2] + 3*b^2*d^2*E^((2*I)*c)*x^4*Csc[c/2]*Csc[(c + d*x^2)/2]*Sin[(d*x^2)/2
] - 3*b^2*d^2*x^4*Sec[c/2]*Sec[(c + d*x^2)/2]*Sin[(d*x^2)/2] + 3*b^2*d^2*E
^((2*I)*c)*x^4*Sec[c/2]*Sec[(c + d*x^2)/2]*Sin[(d*x^2)/2])/(12*d^3*(-1 + E
^((2*I)*c)))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4693, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b\csc\left(c+dx^2\right)\right)^2 \, dx$$

$$\Big\downarrow \; 4693$$

$$\frac{1}{2}\int x^4 \left(a+b\csc\left(dx^2+c\right)\right)^2 dx^2$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\int x^4 \left(a+b\csc\left(dx^2+c\right)\right)^2 dx^2$$

$$\Big\downarrow \; 4678$$

$$\frac{1}{2}\int \left(a^2x^4+b^2\csc^2\left(dx^2+c\right)x^4+2ab\csc\left(dx^2+c\right)x^4\right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{a^2x^6}{3}-\frac{4abx^4\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d}-\frac{4ab\,\text{PolyLog}\left(3,-e^{i(dx^2+c)}\right)}{d^3}+\frac{4ab\,\text{PolyLog}\left(3,e^{i(dx^2+c)}\right)}{d^3}+\frac{4iabx^2\,\text{Pol}\right.$$

input
```
Int[x^5*(a + b*Csc[c + d*x^2])^2,x]
```

output
```
(((-I)*b^2*x^4)/d + (a^2*x^6)/3 - (4*a*b*x^4*ArcTanh[E^(I*(c + d*x^2))])/d
 - (b^2*x^4*Cot[c + d*x^2])/d + (2*b^2*x^2*Log[1 - E^((2*I)*(c + d*x^2))])
/d^2 + ((4*I)*a*b*x^2*PolyLog[2, -E^(I*(c + d*x^2))])/d^2 - ((4*I)*a*b*x^2
*PolyLog[2, E^(I*(c + d*x^2))])/d^2 - (I*b^2*PolyLog[2, E^((2*I)*(c + d*x^
2))])/d^3 - (4*a*b*PolyLog[3, -E^(I*(c + d*x^2))])/d^3 + (4*a*b*PolyLog[3,
 E^(I*(c + d*x^2))])/d^3)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int x^5 \left(a+b\csc\left(d\,x^2+c\right)\right)^2 dx$$

input
```
int(x^5*(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(x^5*(a+b*csc(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 687 vs. 2(195) = 390.

Time = 0.11 (sec) , antiderivative size = 687, normalized size of antiderivative = 3.01

$$\int x^5 \left(a+b\csc\left(c+dx^2\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
1/6*(a^2*d^3*x^6*sin(d*x^2 + c) - 3*b^2*d^2*x^4*cos(d*x^2 + c) + 6*a*b*pol
ylog(3, cos(d*x^2 + c) + I*sin(d*x^2 + c))*sin(d*x^2 + c) + 6*a*b*polylog(
3, cos(d*x^2 + c) - I*sin(d*x^2 + c))*sin(d*x^2 + c) - 6*a*b*polylog(3, -c
os(d*x^2 + c) + I*sin(d*x^2 + c))*sin(d*x^2 + c) - 6*a*b*polylog(3, -cos(d
*x^2 + c) - I*sin(d*x^2 + c))*sin(d*x^2 + c) - 3*(2*I*a*b*d*x^2 + I*b^2)*d
ilog(cos(d*x^2 + c) + I*sin(d*x^2 + c))*sin(d*x^2 + c) - 3*(-2*I*a*b*d*x^2
 - I*b^2)*dilog(cos(d*x^2 + c) - I*sin(d*x^2 + c))*sin(d*x^2 + c) - 3*(2*I
*a*b*d*x^2 - I*b^2)*dilog(-cos(d*x^2 + c) + I*sin(d*x^2 + c))*sin(d*x^2 +
c) - 3*(-2*I*a*b*d*x^2 + I*b^2)*dilog(-cos(d*x^2 + c) - I*sin(d*x^2 + c))*
sin(d*x^2 + c) - 3*(a*b*d^2*x^4 - b^2*d*x^2)*log(cos(d*x^2 + c) + I*sin(d*
x^2 + c) + 1)*sin(d*x^2 + c) - 3*(a*b*d^2*x^4 - b^2*d*x^2)*log(cos(d*x^2 +
 c) - I*sin(d*x^2 + c) + 1)*sin(d*x^2 + c) + 3*(a*b*c^2 - b^2*c)*log(-1/2*
cos(d*x^2 + c) + 1/2*I*sin(d*x^2 + c) + 1/2)*sin(d*x^2 + c) + 3*(a*b*c^2 -
 b^2*c)*log(-1/2*cos(d*x^2 + c) - 1/2*I*sin(d*x^2 + c) + 1/2)*sin(d*x^2 +
c) + 3*(a*b*d^2*x^4 + b^2*d*x^2 - a*b*c^2 + b^2*c)*log(-cos(d*x^2 + c) + I
*sin(d*x^2 + c) + 1)*sin(d*x^2 + c) + 3*(a*b*d^2*x^4 + b^2*d*x^2 - a*b*c^2
 + b^2*c)*log(-cos(d*x^2 + c) - I*sin(d*x^2 + c) + 1)*sin(d*x^2 + c))/(d^3
*sin(d*x^2 + c))
```

Sympy [F]

$$\int x^5\left(a + b\csc\left(c + dx^2\right)\right)^2\,dx = \int x^5\left(a + b\csc\left(c + dx^2\right)\right)^2\,dx$$

input
```
integrate(x**5*(a+b*csc(d*x**2+c))**2,x)
```

output
```
Integral(x**5*(a + b*csc(c + d*x**2))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(195) = 390$.

Time = 0.24 (sec) , antiderivative size = 800, normalized size of antiderivative = 3.51

$$\int x^5 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
1/6*a^2*x^6 - (2*b^2*d^2*x^4*cos(2*d*x^2 + 2*c) + 2*I*b^2*d^2*x^4*sin(2*d*
x^2 + 2*c) - 2*(a*b*d^2*x^4 - b^2*d*x^2 - (a*b*d^2*x^4 - b^2*d*x^2)*cos(2*
d*x^2 + 2*c) + (-I*a*b*d^2*x^4 + I*b^2*d*x^2)*sin(2*d*x^2 + 2*c))*arctan2(
sin(d*x^2 + c), cos(d*x^2 + c) + 1) - 2*(a*b*d^2*x^4 + b^2*d*x^2 - (a*b*d^
2*x^4 + b^2*d*x^2)*cos(2*d*x^2 + 2*c) + (-I*a*b*d^2*x^4 - I*b^2*d*x^2)*sin
(2*d*x^2 + 2*c))*arctan2(sin(d*x^2 + c), -cos(d*x^2 + c) + 1) + 2*(2*a*b*d
*x^2 - b^2 - (2*a*b*d*x^2 - b^2)*cos(2*d*x^2 + 2*c) - (2*I*a*b*d*x^2 - I*b
^2)*sin(2*d*x^2 + 2*c))*dilog(-e^(I*d*x^2 + I*c)) - 2*(2*a*b*d*x^2 + b^2 -
 (2*a*b*d*x^2 + b^2)*cos(2*d*x^2 + 2*c) + (-2*I*a*b*d*x^2 - I*b^2)*sin(2*d
*x^2 + 2*c))*dilog(e^(I*d*x^2 + I*c)) + (I*a*b*d^2*x^4 - I*b^2*d*x^2 + (-I
*a*b*d^2*x^4 + I*b^2*d*x^2)*cos(2*d*x^2 + 2*c) + (a*b*d^2*x^4 - b^2*d*x^2)
*sin(2*d*x^2 + 2*c))*log(cos(d*x^2 + c)^2 + sin(d*x^2 + c)^2 + 2*cos(d*x^2
 + c) + 1) + (-I*a*b*d^2*x^4 - I*b^2*d*x^2 + (I*a*b*d^2*x^4 + I*b^2*d*x^2)
*cos(2*d*x^2 + 2*c) - (a*b*d^2*x^4 + b^2*d*x^2)*sin(2*d*x^2 + 2*c))*log(co
s(d*x^2 + c)^2 + sin(d*x^2 + c)^2 - 2*cos(d*x^2 + c) + 1) - 4*(I*a*b*cos(2
*d*x^2 + 2*c) - a*b*sin(2*d*x^2 + 2*c) - I*a*b)*polylog(3, -e^(I*d*x^2 + I
*c)) - 4*(-I*a*b*cos(2*d*x^2 + 2*c) + a*b*sin(2*d*x^2 + 2*c) + I*a*b)*poly
log(3, e^(I*d*x^2 + I*c)))/(-2*I*d^3*cos(2*d*x^2 + 2*c) + 2*d^3*sin(2*d*x^
2 + 2*c) + 2*I*d^3)
```

Giac [F]

$$\int x^5\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int \left(b\csc\left(dx^2+c\right)+a\right)^2x^5\,dx$$

input
```
integrate(x^5*(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^2 + c) + a)^2*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int x^5\left(a+\frac{b}{\sin\left(d\,x^2+c\right)}\right)^2\,dx$$

input
```
int(x^5*(a + b/sin(c + d*x^2))^2,x)
```

output
```
int(x^5*(a + b/sin(c + d*x^2))^2, x)
```

Reduce [F]

$$\int x^5\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=2\left(\int \csc\left(d\,x^2+c\right)x^5dx\right)ab+\left(\int \csc\left(d\,x^2+c\right)^2x^5dx\right)b^2+\frac{a^2x^6}{6}$$

input
```
int(x^5*(a+b*csc(d*x^2+c))^2,x)
```

output
```
(12*int(csc(c + d*x**2)*x**5,x)*a*b + 6*int(csc(c + d*x**2)**2*x**5,x)*b**
2 + a**2*x**6)/6
```

3.9 $\int x^4 (a+b\csc(c+dx^2))^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^4 (a+b\csc(c+dx^2))^2 \, dx = \text{Int}\left(x^4 (a+b\csc(c+dx^2))^2, x\right)$$

output
```
Defer(Int)(x^4*(a+b*csc(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 17.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4 (a+b\csc(c+dx^2))^2 \, dx = \int x^4 (a+b\csc(c+dx^2))^2 \, dx$$

input
```
Integrate[x^4*(a + b*Csc[c + d*x^2])^2,x]
```

output
```
Integrate[x^4*(a + b*Csc[c + d*x^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$

$$\Big\downarrow \ 4695$$

$$\int x^4\left(a+b\csc\left(c+dx^2\right)\right)^2dx$$

input
```
Int[x^4*(a + b*Csc[c + d*x^2])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^4\left(a+b\csc\left(d\,x^2+c\right)\right)^2dx$$

input
```
int(x^4*(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(x^4*(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^4 \left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int \left(b\csc\left(dx^2+c\right)+a\right)^2 x^4\,dx$$

input
```
integrate(x^4*(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^4*csc(d*x^2 + c)^2 + 2*a*b*x^4*csc(d*x^2 + c) + a^2*x^4, x)
```

Sympy [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^4 \left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int x^4 \left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$

input
```
integrate(x**4*(a+b*csc(d*x**2+c))**2,x)
```

output
```
Integral(x**4*(a + b*csc(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 310, normalized size of antiderivative = 17.22

$$\int x^4 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = \int \left(b \csc \left(dx^2 + c\right) + a\right)^2 x^4 \, dx$$

input
```
integrate(x^4*(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*x^5 - (b^2*x^3*sin(2*d*x^2 + 2*c) - (d*cos(2*d*x^2 + 2*c)^2 + d*si
n(2*d*x^2 + 2*c)^2 - 2*d*cos(2*d*x^2 + 2*c) + d)*integrate(1/2*(4*a*b*d*x^
4 - 3*b^2*x^2)*sin(d*x^2 + c)/(d*cos(d*x^2 + c)^2 + d*sin(d*x^2 + c)^2 + 2
*d*cos(d*x^2 + c) + d), x) - (d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c
)^2 - 2*d*cos(2*d*x^2 + 2*c) + d)*integrate(1/2*(4*a*b*d*x^4 + 3*b^2*x^2)*
sin(d*x^2 + c)/(d*cos(d*x^2 + c)^2 + d*sin(d*x^2 + c)^2 - 2*d*cos(d*x^2 +
c) + d), x))/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 - 2*d*cos(2*
d*x^2 + 2*c) + d)
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = \int \left(b \csc \left(dx^2 + c\right) + a\right)^2 x^4 \, dx$$

input
```
integrate(x^4*(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^2 + c) + a)^2*x^4, x)
```

Mupad [N/A]

Not integrable

Time = 15.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^4 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = \int x^4 \left(a + \frac{b}{\sin \left(d\,x^2 + c\right)}\right)^2 dx$$

input
```
int(x^4*(a + b/sin(c + d*x^2))^2,x)
```

output
```
int(x^4*(a + b/sin(c + d*x^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.61

$$\int x^4 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = 2\left(\int \csc \left(d\,x^2 + c\right) x^4 dx\right) ab + \left(\int \csc \left(d\,x^2 + c\right)^2 x^4 dx\right) b^2 + \frac{a^2 x^5}{5}$$

input
```
int(x^4*(a+b*csc(d*x^2+c))^2,x)
```

output
```
(10*int(csc(c + d*x**2)*x**4,x)*a*b + 5*int(csc(c + d*x**2)**2*x**4,x)*b**
2 + a**2*x**5)/5
```

3.10 $\int x^3(a+b\csc(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 125

$$\int x^3\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\frac{a^2x^4}{4}-\frac{2abx^2\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d}-\frac{b^2x^2\cot\left(c+dx^2\right)}{2d}+\frac{b^2\log\left(\sin\left(c+dx^2\right)\right)}{2d^2}+\frac{iab\,\mathrm{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2}-\frac{iab\,\mathrm{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2}$$

output

```
1/4*a^2*x^4-2*a*b*x^2*arctanh(exp(I*(d*x^2+c)))/d-1/2*b^2*x^2*cot(d*x^2+c)
/d+1/2*b^2*ln(sin(d*x^2+c))/d^2+I*a*b*polylog(2,-exp(I*(d*x^2+c)))/d^2-I*a
*b*polylog(2,exp(I*(d*x^2+c)))/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 268 vs. 2(125) = 250.

Time = 3.96 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.14

$$\int x^3 (a + b \csc (c + dx^2))^2 \, dx$$

$$= \frac{2b^2dx^2 \cot(c) + dx^2(a^2dx^2 - 2b^2 \cot(c)) - 2b^2(dx^2 \cot(c) - \log (\sin (c + dx^2))) + 4ab\Big(2 \arctan(\tan(c))a}{}$$

input
```
Integrate[x^3*(a + b*Csc[c + d*x^2])^2,x]
```

output
```
(2*b^2*d*x^2*Cot[c] + d*x^2*(a^2*d*x^2 - 2*b^2*Cot[c]) - 2*b^2*(d*x^2*Cot[
c] - Log[Sin[c + d*x^2]]) + 4*a*b*(2*ArcTan[Tan[c]]*ArcTanh[Cos[c] - Sin[c
]*Tan[(d*x^2)/2]] + (((d*x^2 + ArcTan[Tan[c]])*(Log[1 - E^(I*(d*x^2 + ArcT
an[Tan[c]]))] - Log[1 + E^(I*(d*x^2 + ArcTan[Tan[c]]))]) + I*PolyLog[2, -E
^(I*(d*x^2 + ArcTan[Tan[c]]))] - I*PolyLog[2, E^(I*(d*x^2 + ArcTan[Tan[c]]
))])*Sec[c])/Sqrt[Sec[c]^2]) + b^2*d*x^2*Csc[c/2]*Csc[(c + d*x^2)/2]*Sin[(
d*x^2)/2] + b^2*d*x^2*Sec[c/2]*Sec[(c + d*x^2)/2]*Sin[(d*x^2)/2])/(4*d^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4693, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a + b \csc (c + dx^2))^2 \, dx$$

$$\Big\downarrow \text{4693}$$

$$\frac{1}{2} \int x^2 (a + b \csc (dx^2 + c))^2 \, dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int x^2\left(a+b\csc\left(dx^2+c\right)\right)^2dx^2$$

$$\Big\downarrow \text{4678}$$

$$\frac{1}{2}\int \left(a^2x^2+b^2\csc^2\left(dx^2+c\right)x^2+2ab\csc\left(dx^2+c\right)x^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^2x^4}{2}-\frac{4abx^2\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d}+\frac{2iab\,\text{PolyLog}\left(2,-e^{i(dx^2+c)}\right)}{d^2}-\frac{2iab\,\text{PolyLog}\left(2,e^{i(dx^2+c)}\right)}{d^2}+\frac{b^2\log\left(\sin\right.}{}\right.$$

input
```
Int[x^3*(a + b*Csc[c + d*x^2])^2,x]
```

output
```
((a^2*x^4)/2 - (4*a*b*x^2*ArcTanh[E^(I*(c + d*x^2))])/d - (b^2*x^2*Cot[c +
 d*x^2])/d + (b^2*Log[Sin[c + d*x^2]])/d^2 + ((2*I)*a*b*PolyLog[2, -E^(I*(
c + d*x^2))])/d^2 - ((2*I)*a*b*PolyLog[2, E^(I*(c + d*x^2))])/d^2)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int x^3 \left(a+b\csc \left(d\,x^2+c\right)\right)^2 dx$$

input
```
int(x^3*(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(x^3*(a+b*csc(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(107) = 214.

Time = 0.10 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.61

$$\int x^3 \left(a+b\csc \left(c+dx^2\right)\right)^2 \, dx$$
$$= \frac{a^2d^2x^4 \sin \left(dx^2+c\right) - 2\,b^2dx^2 \cos \left(dx^2+c\right) - 2i\,ab\mathrm{Li}_2(\cos \left(dx^2+c\right) + i\,\sin \left(dx^2+c\right)) \sin \left(dx^2+c\right) + 2i}{}$$

input
```
integrate(x^3*(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
1/4*(a^2*d^2*x^4*sin(d*x^2 + c) - 2*b^2*d*x^2*cos(d*x^2 + c) - 2*I*a*b*dil
og(cos(d*x^2 + c) + I*sin(d*x^2 + c))*sin(d*x^2 + c) + 2*I*a*b*dilog(cos(d
*x^2 + c) - I*sin(d*x^2 + c))*sin(d*x^2 + c) - 2*I*a*b*dilog(-cos(d*x^2 +
c) + I*sin(d*x^2 + c))*sin(d*x^2 + c) + 2*I*a*b*dilog(-cos(d*x^2 + c) - I*
sin(d*x^2 + c))*sin(d*x^2 + c) - (2*a*b*d*x^2 - b^2)*log(cos(d*x^2 + c) +
I*sin(d*x^2 + c) + 1)*sin(d*x^2 + c) - (2*a*b*d*x^2 - b^2)*log(cos(d*x^2 +
 c) - I*sin(d*x^2 + c) + 1)*sin(d*x^2 + c) - (2*a*b*c - b^2)*log(-1/2*cos(
d*x^2 + c) + 1/2*I*sin(d*x^2 + c) + 1/2)*sin(d*x^2 + c) - (2*a*b*c - b^2)*
log(-1/2*cos(d*x^2 + c) - 1/2*I*sin(d*x^2 + c) + 1/2)*sin(d*x^2 + c) + 2*(
a*b*d*x^2 + a*b*c)*log(-cos(d*x^2 + c) + I*sin(d*x^2 + c) + 1)*sin(d*x^2 +
 c) + 2*(a*b*d*x^2 + a*b*c)*log(-cos(d*x^2 + c) - I*sin(d*x^2 + c) + 1)*si
n(d*x^2 + c))/(d^2*sin(d*x^2 + c))
```

Sympy [F]

$$\int x^3 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = \int x^3 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx$$

input
```
integrate(x**3*(a+b*csc(d*x**2+c))**2,x)
```

output
```
Integral(x**3*(a + b*csc(c + d*x**2))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 604 vs. 2(107) = 214.

Time = 0.16 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.83

$$\int x^3 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*x^4 - (4*b^2*d*x^2*cos(2*d*x^2 + 2*c) + 4*I*b^2*d*x^2*sin(2*d*x^2
+ 2*c) - 2*(2*a*b*d*x^2 - b^2 - (2*a*b*d*x^2 - b^2)*cos(2*d*x^2 + 2*c) + (
-2*I*a*b*d*x^2 + I*b^2)*sin(2*d*x^2 + 2*c))*arctan2(sin(d*x^2 + c), cos(d*
x^2 + c) + 1) - 2*(b^2*cos(2*d*x^2 + 2*c) + I*b^2*sin(2*d*x^2 + 2*c) - b^2
)*arctan2(sin(d*x^2 + c), cos(d*x^2 + c) - 1) + 4*(a*b*d*x^2*cos(2*d*x^2 +
 2*c) + I*a*b*d*x^2*sin(2*d*x^2 + 2*c) - a*b*d*x^2)*arctan2(sin(d*x^2 + c)
, -cos(d*x^2 + c) + 1) - 4*(a*b*cos(2*d*x^2 + 2*c) + I*a*b*sin(2*d*x^2 + 2
*c) - a*b)*dilog(-e^(I*d*x^2 + I*c)) + 4*(a*b*cos(2*d*x^2 + 2*c) + I*a*b*s
in(2*d*x^2 + 2*c) - a*b)*dilog(e^(I*d*x^2 + I*c)) + (2*I*a*b*d*x^2 - I*b^2
 + (-2*I*a*b*d*x^2 + I*b^2)*cos(2*d*x^2 + 2*c) + (2*a*b*d*x^2 - b^2)*sin(2
*d*x^2 + 2*c))*log(cos(d*x^2 + c)^2 + sin(d*x^2 + c)^2 + 2*cos(d*x^2 + c)
+ 1) + (-2*I*a*b*d*x^2 - I*b^2 + (2*I*a*b*d*x^2 + I*b^2)*cos(2*d*x^2 + 2*c
) - (2*a*b*d*x^2 + b^2)*sin(2*d*x^2 + 2*c))*log(cos(d*x^2 + c)^2 + sin(d*x
^2 + c)^2 - 2*cos(d*x^2 + c) + 1))/(-4*I*d^2*cos(2*d*x^2 + 2*c) + 4*d^2*si
n(2*d*x^2 + 2*c) + 4*I*d^2)
```

Giac [F]

$$\int x^3\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int\left(b\csc\left(dx^2+c\right)+a\right)^2x^3\,dx$$

input
```
integrate(x^3*(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^2 + c) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int x^3\left(a+\frac{b}{\sin\left(d\,x^2+c\right)}\right)^2\,dx$$

input
```
int(x^3*(a + b/sin(c + d*x^2))^2,x)
```

output
```
int(x^3*(a + b/sin(c + d*x^2))^2, x)
```

Reduce [F]

$$\int x^3 \left(a + b\csc\left(c + dx^2\right)\right)^2 \, dx$$

$$= \frac{-2\cos\left(d\,x^2 + c\right) b^2 d\,x^2 + 8\left(\int \csc\left(d\,x^2 + c\right) x^3 dx\right) \sin\left(d\,x^2 + c\right) ab\,d^2 - 2\log\left(\tan\left(\frac{d\,x^2}{2} + \frac{c}{2}\right)^2 + 1\right) \sin\left(\right.}{4\sin\left(d\,x^2 + c\right) d^2}$$

input

```
int(x^3*(a+b*csc(d*x^2+c))^2,x)
```

output

```
( - 2*cos(c + d*x**2)*b**2*d*x**2 + 8*int(csc(c + d*x**2)*x**3,x)*sin(c +
d*x**2)*a*b*d**2 - 2*log(tan((c + d*x**2)/2)**2 + 1)*sin(c + d*x**2)*b**2
+ 2*log(tan((c + d*x**2)/2))*sin(c + d*x**2)*b**2 + sin(c + d*x**2)*a**2*d
**2*x**4)/(4*sin(c + d*x**2)*d**2)
```

3.11 $\int x^2(a+b\csc(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^2\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\text{Int}\left(x^2\left(a+b\csc\left(c+dx^2\right)\right)^2,x\right)$$

output
```
Defer(Int)(x^2*(a+b*csc(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 27.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int x^2\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$

input
```
Integrate[x^2*(a + b*Csc[c + d*x^2])^2,x]
```

output
```
Integrate[x^2*(a + b*Csc[c + d*x^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \csc \left(c+d x^2\right)\right)^2 \, dx$$

$$\Big\downarrow \text{4695}$$

$$\int x^2 \left(a+b \csc \left(c+d x^2\right)\right)^2 dx$$

input
```
Int[x^2*(a + b*Csc[c + d*x^2])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2 \left(a+b \csc \left(d\, x^2+c\right)\right)^2 dx$$

input
```
int(x^2*(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(x^2*(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^2 \left(a+b\csc\left(c+dx^2\right)\right)^2 \,dx=\int \left(b\csc\left(dx^2+c\right)+a\right)^2 x^2 \,dx$$

input
```
integrate(x^2*(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*csc(d*x^2 + c)^2 + 2*a*b*x^2*csc(d*x^2 + c) + a^2*x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^2 \left(a+b\csc\left(c+dx^2\right)\right)^2 \,dx=\int x^2 \left(a+b\csc\left(c+dx^2\right)\right)^2 \,dx$$

input
```
integrate(x**2*(a+b*csc(d*x**2+c))**2,x)
```

output
```
Integral(x**2*(a + b*csc(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 300, normalized size of antiderivative = 16.67

$$\int x^2 \left(a+b\csc\left(c+dx^2\right)\right)^2 \, dx = \int \left(b\csc\left(dx^2+c\right)+a\right)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 - (b^2*x*sin(2*d*x^2 + 2*c) - (d*cos(2*d*x^2 + 2*c)^2 + d*sin(
2*d*x^2 + 2*c)^2 - 2*d*cos(2*d*x^2 + 2*c) + d)*integrate(1/2*(4*a*b*d*x^2
- b^2)*sin(d*x^2 + c)/(d*cos(d*x^2 + c)^2 + d*sin(d*x^2 + c)^2 + 2*d*cos(d
*x^2 + c) + d), x) - (d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 - 2*
d*cos(2*d*x^2 + 2*c) + d)*integrate(1/2*(4*a*b*d*x^2 + b^2)*sin(d*x^2 + c)
/(d*cos(d*x^2 + c)^2 + d*sin(d*x^2 + c)^2 - 2*d*cos(d*x^2 + c) + d), x))/(
d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 - 2*d*cos(2*d*x^2 + 2*c) +
 d)
```

Giac [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2 \left(a+b\csc\left(c+dx^2\right)\right)^2 \, dx = \int \left(b\csc\left(dx^2+c\right)+a\right)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^2 + c) + a)^2*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 15.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^2(a+b\csc(c+dx^2))^2\,dx=\int x^2\left(a+\frac{b}{\sin(d\,x^2+c)}\right)^2dx$$

input
```
int(x^2*(a + b/sin(c + d*x^2))^2,x)
```

output
```
int(x^2*(a + b/sin(c + d*x^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.67

$$\int x^2(a+b\csc(c+dx^2))^2\,dx = \frac{3\left(\int\frac{x^2}{\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2}dx\right)b^2d-3\left(\int\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)dx\right)b^2+24\left(\int\csc(d\,x^2+c)\,x^2dx\right)abd+3\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}{12d}$$

input
```
int(x^2*(a+b*csc(d*x^2+c))^2,x)
```

output
```
(3*int(x**2/tan((c + d*x**2)/2)**2,x)*b**2*d - 3*int(tan((c + d*x**2)/2),x
)*b**2 + 24*int(csc(c + d*x**2)*x**2,x)*a*b*d + 3*tan((c + d*x**2)/2)*b**2
*x + 4*a**2*d*x**3 + b**2*d*x**3)/(12*d)
```

3.12 $\int x(a+b\csc(c+dx^2))^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\frac{a^2x^2}{2}-\frac{ab\text{arctanh}(\cos\left(c+dx^2\right))}{d}-\frac{b^2\cot\left(c+dx^2\right)}{2d}$$

output
```
1/2*a^2*x^2-a*b*arctanh(cos(d*x^2+c))/d-1/2*b^2*cot(d*x^2+c)/d
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.91

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{-b^2\cot\left(\frac{1}{2}(c+dx^2)\right)+2a\left(ac+adx^2-2b\log\left(\cos\left(\frac{1}{2}(c+dx^2)\right)\right)+2b\log\left(\sin\left(\frac{1}{2}(c+dx^2)\right)\right)\right)+b^2\tan\left(\right.}{4d}$$

input
```
Integrate[x*(a + b*Csc[c + d*x^2])^2,x]
```

output
```
(-(b^2*Cot[(c + d*x^2)/2]) + 2*a*(a*c + a*d*x^2 - 2*b*Log[Cos[(c + d*x^2)/
2]] + 2*b*Log[Sin[(c + d*x^2)/2]]) + b^2*Tan[(c + d*x^2)/2])/(4*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4693, 3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2dx$$

$$\Big\downarrow \text{4693}$$

$$\frac{1}{2}\int \left(a+b\csc\left(dx^2+c\right)\right)^2dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \left(a+b\csc\left(dx^2+c\right)\right)^2dx^2$$

$$\Big\downarrow \text{4260}$$

$$\frac{1}{2}\left(2ab\int \csc\left(dx^2+c\right)dx^2+b^2\int \csc^2\left(dx^2+c\right)dx^2+a^2x^2\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(2ab\int \csc\left(dx^2+c\right)dx^2+b^2\int \csc\left(dx^2+c\right)^2dx^2+a^2x^2\right)$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{2}\left(2ab\int \csc\left(dx^2+c\right)dx^2-\frac{b^2\int 1d\cot\left(dx^2+c\right)}{d}+a^2x^2\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(2ab\int \csc\left(dx^2+c\right)dx^2+a^2x^2-\frac{b^2\cot\left(c+dx^2\right)}{d}\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\left(a^2x^2-\frac{2ab\text{arctanh}\left(\cos\left(c+dx^2\right)\right)}{d}-\frac{b^2\cot\left(c+dx^2\right)}{d}\right)$$

input
```
Int[x*(a + b*Csc[c + d*x^2])^2,x]
```

output
```
(a^2*x^2 - (2*a*b*ArcTanh[Cos[c + d*x^2]])/d - (b^2*Cot[c + d*x^2])/d)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4260
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Simp[2*a*b   Int[Csc[c + d*x], x], x] + Simp[b^2   Int[Csc[c + d*x]^2, x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

method	result	size
parts	$\frac{a^2x^2}{2}-\frac{b^2\cot(d\,x^2+c)}{2d}-\frac{ba\ln(\csc(d\,x^2+c)+\cot(d\,x^2+c))}{d}$	51
derivativedivides	$\frac{a^2(d\,x^2+c)+2ba\ln(\csc(d\,x^2+c)-\cot(d\,x^2+c))-b^2\cot(d\,x^2+c)}{2d}$	55
default	$\frac{a^2(d\,x^2+c)+2ba\ln(\csc(d\,x^2+c)-\cot(d\,x^2+c))-b^2\cot(d\,x^2+c)}{2d}$	55
parallelrisch	$\frac{2a^2d\,x^2+4\ln\left(\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)ab-b^2\left(-\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)+\cot\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{4d}$	62
risch	$\frac{a^2x^2}{2}-\frac{ib^2}{d\left(e^{2i(d\,x^2+c)}-1\right)}+\frac{ba\ln\left(e^{i(d\,x^2+c)}-1\right)}{d}-\frac{ba\ln\left(e^{i(d\,x^2+c)}+1\right)}{d}$	75
norman	$\frac{-\frac{b^2}{4d}+\frac{a^2x^2\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}{2}+\frac{b^2\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)^2}{4d}}{\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)}+\frac{ba\ln\left(\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{d}$	83

input
```
int(x*(a+b*csc(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2-1/2*b^2*cot(d*x^2+c)/d-b*a/d*ln(csc(d*x^2+c)+cot(d*x^2+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(41) = 82.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.09

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{a^2dx^2\sin\left(dx^2+c\right)-ab\log\left(\frac{1}{2}\cos\left(dx^2+c\right)+\frac{1}{2}\right)\sin\left(dx^2+c\right)+ab\log\left(-\frac{1}{2}\cos\left(dx^2+c\right)+\frac{1}{2}\right)\sin\left(dx^2\right.}{2\,d\sin\left(dx^2+c\right)}$$

input
```
integrate(x*(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
1/2*(a^2*d*x^2*sin(d*x^2 + c) - a*b*log(1/2*cos(d*x^2 + c) + 1/2)*sin(d*x^
2 + c) + a*b*log(-1/2*cos(d*x^2 + c) + 1/2)*sin(d*x^2 + c) - b^2*cos(d*x^2
 + c))/(d*sin(d*x^2 + c))
```

Sympy [F]

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$

input
```
integrate(x*(a+b*csc(d*x**2+c))**2,x)
```

output
```
Integral(x*(a + b*csc(c + d*x**2))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.18

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx = \frac{1}{2}\,a^2x^2-\frac{ab\log\left(\cot\left(dx^2+c\right)+\csc\left(dx^2+c\right)\right)}{d} - \frac{b^2\sin\left(2\,dx^2+2\,c\right)}{d\cos\left(2\,dx^2+2\,c\right)^2+d\sin\left(2\,dx^2+2\,c\right)^2-2\,d\cos\left(2\,dx^2+2\,c\right)+d}$$

input
```
integrate(x*(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*x^2 - a*b*log(cot(d*x^2 + c) + csc(d*x^2 + c))/d - b^2*sin(2*d*x^2
 + 2*c)/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 - 2*d*cos(2*d*x^2
 + 2*c) + d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(41) = 82.

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.87

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{2\left(dx^2+c\right)a^2+4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)\right|\right)+b^2\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)-\frac{4\,ab\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+b^2}{\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)}}{4\,d}$$

input
```
integrate(x*(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

output
```
1/4*(2*(d*x^2 + c)*a^2 + 4*a*b*log(abs(tan(1/2*d*x^2 + 1/2*c))) + b^2*tan(
1/2*d*x^2 + 1/2*c) - (4*a*b*tan(1/2*d*x^2 + 1/2*c) + b^2)/tan(1/2*d*x^2 +
1/2*c))/d
```

Mupad [B] (verification not implemented)

Time = 15.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.27

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\frac{a^2\,x^2}{2}-\frac{b^2\,1\mathrm{i}}{d\,\left(\mathrm{e}^{2\mathrm{i}\,d\,x^2+c\,2\mathrm{i}}-1\right)}-\frac{a\,b\,\ln\left(-a\,b\,x\,4\mathrm{i}-a\,b\,x\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,4\mathrm{i}\right)}{d}+\frac{a\,b\,\ln\left(a\,b\,x\,4\mathrm{i}-a\,b\,x\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,4\mathrm{i}\right)}{d}$$

input
```
int(x*(a + b/sin(c + d*x^2))^2,x)
```

output
```
(a^2*x^2)/2 - (b^2*1i)/(d*(exp(c*2i + d*x^2*2i) - 1)) - (a*b*log(- a*b*x*4
i - a*b*x*exp(d*x^2*1i)*exp(c*1i)*4i))/d + (a*b*log(a*b*x*4i - a*b*x*exp(d
*x^2*1i)*exp(c*1i)*4i))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.53

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx = \frac{-\cos\left(d\,x^2+c\right)b^2+2\log\left(\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)\sin\left(d\,x^2+c\right)ab+\sin\left(d\,x^2+c\right)a^2d\,x^2}{2\sin\left(d\,x^2+c\right)d}$$

input
```
int(x*(a+b*csc(d*x^2+c))^2,x)
```

output
```
( - cos(c + d*x**2)*b**2 + 2*log(tan((c + d*x**2)/2))*sin(c + d*x**2)*a*b
+ sin(c + d*x**2)*a**2*d*x**2)/(2*sin(c + d*x**2)*d)
```

3.13 $\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx = \text{Int}\left(\frac{(a+b\csc(c+dx^2))^2}{x},x\right)$$

output
```
Defer(Int)((a+b*csc(d*x^2+c))^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 68.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx = \int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx$$

input
```
Integrate[(a + b*Csc[c + d*x^2])^2/x,x]
```

output
```
Integrate[(a + b*Csc[c + d*x^2])^2/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx$$

$$\Big\downarrow \ 4695$$

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}dx$$

input
```
Int[(a + b*Csc[c + d*x^2])^2/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\csc(d\,x^2+c))^2}{x}dx$$

input
```
int((a+b*csc(d*x^2+c))^2/x,x)
```

output
```
int((a+b*csc(d*x^2+c))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx = \int \frac{(b\csc(dx^2+c)+a)^2}{x}\,dx$$

input
```
integrate((a+b*csc(d*x^2+c))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*csc(d*x^2 + c)^2 + 2*a*b*csc(d*x^2 + c) + a^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx = \int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx$$

input
```
integrate((a+b*csc(d*x**2+c))**2/x,x)
```

output
```
Integral((a + b*csc(c + d*x**2))**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 360, normalized size of antiderivative = 20.00

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx = \int \frac{(b\csc(dx^2+c)+a)^2}{x}\,dx$$

input
```
integrate((a+b*csc(d*x^2+c))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) - (b^2*sin(2*d*x^2 + 2*c) - (d*x^2*cos(2*d*x^2 + 2*c)^2 + d*x^2
*sin(2*d*x^2 + 2*c)^2 - 2*d*x^2*cos(2*d*x^2 + 2*c) + d*x^2)*integrate((2*a
*b*d*x^2 + b^2)*sin(d*x^2 + c)/(d*x^3*cos(d*x^2 + c)^2 + d*x^3*sin(d*x^2 +
 c)^2 + 2*d*x^3*cos(d*x^2 + c) + d*x^3), x) - (d*x^2*cos(2*d*x^2 + 2*c)^2
+ d*x^2*sin(2*d*x^2 + 2*c)^2 - 2*d*x^2*cos(2*d*x^2 + 2*c) + d*x^2)*integra
te((2*a*b*d*x^2 - b^2)*sin(d*x^2 + c)/(d*x^3*cos(d*x^2 + c)^2 + d*x^3*sin(
d*x^2 + c)^2 - 2*d*x^3*cos(d*x^2 + c) + d*x^3), x))/(d*x^2*cos(2*d*x^2 + 2
*c)^2 + d*x^2*sin(2*d*x^2 + 2*c)^2 - 2*d*x^2*cos(2*d*x^2 + 2*c) + d*x^2)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx = \int \frac{(b\csc(dx^2+c)+a)^2}{x}\,dx$$

input
```
integrate((a+b*csc(d*x^2+c))^2/x,x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^2 + c) + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 14.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a + b\csc(c + dx^2))^2}{x}\,dx = \int \frac{\left(a + \frac{b}{\sin(d\,x^2+c)}\right)^2}{x}\,dx$$

input
```
int((a + b/sin(c + d*x^2))^2/x,x)
```

output
```
int((a + b/sin(c + d*x^2))^2/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.56

$$\int \frac{(a + b\csc(c + dx^2))^2}{x}\,dx = \left(\int \frac{1}{\sin(d\,x^2+c)^2\,x}dx\right)b^2 + 2\left(\int \frac{1}{\sin(d\,x^2+c)\,x}dx\right)ab - \frac{\left(\int \frac{1}{x}dx\right)b^2}{2} + \log(x)\,a^2 + \frac{\log(x)\,b^2}{2}$$

input
```
int((a+b*csc(d*x^2+c))^2/x,x)
```

output
```
(2*int(1/(sin(c + d*x**2)**2*x),x)*b**2 + 4*int(1/(sin(c + d*x**2)*x),x)*a
*b - int(1/x,x)*b**2 + 2*log(x)*a**2 + log(x)*b**2)/2
```

3.14 $\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx = \text{Int}\left(\frac{(a+b\csc(c+dx^2))^2}{x^2},x\right)$$

output
```
Defer(Int)((a+b*csc(d*x^2+c))^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 39.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx = \int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx$$

input
```
Integrate[(a + b*Csc[c + d*x^2])^2/x^2,x]
```

output
```
Integrate[(a + b*Csc[c + d*x^2])^2/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx$$
$$\Big\downarrow \; 4695$$
$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}dx$$

input
```
Int[(a + b*Csc[c + d*x^2])^2/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\csc(d\,x^2+c))^2}{x^2}dx$$

input
```
int((a+b*csc(d*x^2+c))^2/x^2,x)
```

output
```
int((a+b*csc(d*x^2+c))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx=\int \frac{(b\csc(dx^2+c)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*csc(d*x^2+c))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*csc(d*x^2 + c)^2 + 2*a*b*csc(d*x^2 + c) + a^2)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx=\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx$$

input
```
integrate((a+b*csc(d*x**2+c))**2/x**2,x)
```

output
```
Integral((a + b*csc(c + d*x**2))**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 366, normalized size of antiderivative = 20.33

$$\int \frac{(a + b\csc(c + dx^2))^2}{x^2} dx = \int \frac{(b\csc(dx^2 + c) + a)^2}{x^2} dx$$

input
```
integrate((a+b*csc(d*x^2+c))^2/x^2,x, algorithm="maxima")
```

output
```
-a^2/x - (b^2*sin(2*d*x^2 + 2*c) - (d*x^3*cos(2*d*x^2 + 2*c)^2 + d*x^3*sin
(2*d*x^2 + 2*c)^2 - 2*d*x^3*cos(2*d*x^2 + 2*c) + d*x^3)*integrate(1/2*(4*a
*b*d*x^2 + 3*b^2)*sin(d*x^2 + c)/(d*x^4*cos(d*x^2 + c)^2 + d*x^4*sin(d*x^2
 + c)^2 + 2*d*x^4*cos(d*x^2 + c) + d*x^4), x) - (d*x^3*cos(2*d*x^2 + 2*c)^
2 + d*x^3*sin(2*d*x^2 + 2*c)^2 - 2*d*x^3*cos(2*d*x^2 + 2*c) + d*x^3)*integ
rate(1/2*(4*a*b*d*x^2 - 3*b^2)*sin(d*x^2 + c)/(d*x^4*cos(d*x^2 + c)^2 + d*
x^4*sin(d*x^2 + c)^2 - 2*d*x^4*cos(d*x^2 + c) + d*x^4), x))/(d*x^3*cos(2*d
*x^2 + 2*c)^2 + d*x^3*sin(2*d*x^2 + 2*c)^2 - 2*d*x^3*cos(2*d*x^2 + 2*c) +
d*x^3)
```

Giac [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a + b\csc(c + dx^2))^2}{x^2} dx = \int \frac{(b\csc(dx^2 + c) + a)^2}{x^2} dx$$

input
```
integrate((a+b*csc(d*x^2+c))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^2 + c) + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 14.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx = \int \frac{\left(a+\frac{b}{\sin(d\,x^2+c)}\right)^2}{x^2}\,dx$$

input
```
int((a + b/sin(c + d*x^2))^2/x^2,x)
```

output
```
int((a + b/sin(c + d*x^2))^2/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx = \frac{2\left(\int \frac{\csc(d\,x^2+c)}{x^2}dx\right)abx+\left(\int \frac{\csc(d\,x^2+c)^2}{x^2}dx\right)b^2x-a^2}{x}$$

input
```
int((a+b*csc(d*x^2+c))^2/x^2,x)
```

output
```
(2*int(csc(c + d*x**2)/x**2,x)*a*b*x + int(csc(c + d*x**2)**2/x**2,x)*b**2
*x - a**2)/x
```

3.15 $\int x \csc^7(a+bx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int x \csc^7\left(a+bx^2\right)\,dx = -\frac{5\text{arctanh}(\cos(a+bx^2))}{32b} - \frac{5\cot(a+bx^2)\csc(a+bx^2)}{32b} - \frac{5\cot(a+bx^2)\csc^3(a+bx^2)}{48b} - \frac{\cot(a+bx^2)\csc^5(a+bx^2)}{12b}$$

output
```
-5/32*arctanh(cos(b*x^2+a))/b-5/32*cot(b*x^2+a)*csc(b*x^2+a)/b-5/48*cot(b*
x^2+a)*csc(b*x^2+a)^3/b-1/12*cot(b*x^2+a)*csc(b*x^2+a)^5/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.86

$$\int x \csc^7\left(a+bx^2\right)\,dx = -\frac{5\csc^2\left(\frac{1}{2}(a+bx^2)\right)}{128b} - \frac{\csc^4\left(\frac{1}{2}(a+bx^2)\right)}{128b} - \frac{\csc^6\left(\frac{1}{2}(a+bx^2)\right)}{768b} - \frac{5\log\left(\cos\left(\frac{1}{2}(a+bx^2)\right)\right)}{32b} + \frac{5\log\left(\sin\left(\frac{1}{2}(a+bx^2)\right)\right)}{32b} + \frac{5\sec^2\left(\frac{1}{2}(a+bx^2)\right)}{128b} + \frac{\sec^4\left(\frac{1}{2}(a+bx^2)\right)}{128b} + \frac{\sec^6\left(\frac{1}{2}(a+bx^2)\right)}{768b}$$

input
```
Integrate[x*Csc[a + b*x^2]^7,x]
```

output
```
(-5*Csc[(a + b*x^2)/2]^2)/(128*b) - Csc[(a + b*x^2)/2]^4/(128*b) - Csc[(a
+ b*x^2)/2]^6/(768*b) - (5*Log[Cos[(a + b*x^2)/2]])/(32*b) + (5*Log[Sin[(a
 + b*x^2)/2]])/(32*b) + (5*Sec[(a + b*x^2)/2]^2)/(128*b) + Sec[(a + b*x^2)
/2]^4/(128*b) + Sec[(a + b*x^2)/2]^6/(768*b)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4693, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \csc^7\left(a+bx^2\right) dx$$

$$\Big\downarrow \text{4693}$$

$$\frac{1}{2}\int \csc^7\left(bx^2+a\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc\left(bx^2+a\right)^7 dx^2$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{2}\left(\frac{5}{6}\int \csc^5\left(bx^2+a\right)dx^2-\frac{\cot\left(a+bx^2\right)\csc^5\left(a+bx^2\right)}{6b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5}{6}\int \csc\left(bx^2+a\right)^5 dx^2-\frac{\cot\left(a+bx^2\right)\csc^5\left(a+bx^2\right)}{6b}\right)$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{2}\left(\frac{5}{6}\left(\frac{3}{4}\int \csc^3\left(bx^2+a\right)dx^2-\frac{\cot\left(a+bx^2\right)\csc^3\left(a+bx^2\right)}{4b}\right)-\frac{\cot\left(a+bx^2\right)\csc^5\left(a+bx^2\right)}{6b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5}{6}\left(\frac{3}{4}\int \csc\left(bx^2+a\right)^3dx^2-\frac{\cot\left(a+bx^2\right)\csc^3\left(a+bx^2\right)}{4b}\right)-\frac{\cot\left(a+bx^2\right)\csc^5\left(a+bx^2\right)}{6b}\right)$$

$\downarrow$ 4255

$$\frac{1}{2}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(bx^2+a\right)dx^2-\frac{\cot\left(a+bx^2\right)\csc\left(a+bx^2\right)}{2b}\right)-\frac{\cot\left(a+bx^2\right)\csc^3\left(a+bx^2\right)}{4b}\right)-\frac{\cot\left(a+bx^2\right.}{}\right.$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(bx^2+a\right)dx^2-\frac{\cot\left(a+bx^2\right)\csc\left(a+bx^2\right)}{2b}\right)-\frac{\cot\left(a+bx^2\right)\csc^3\left(a+bx^2\right)}{4b}\right)-\frac{\cot\left(a+bx^2\right.}{}\right.$$

$\downarrow$ 4257

$$\frac{1}{2}\left(\frac{5}{6}\left(\frac{3}{4}\left(-\frac{\text{arctanh}\left(\cos\left(a+bx^2\right)\right)}{2b}-\frac{\cot\left(a+bx^2\right)\csc\left(a+bx^2\right)}{2b}\right)-\frac{\cot\left(a+bx^2\right)\csc^3\left(a+bx^2\right)}{4b}\right)-\frac{\cot\left(a+\right.}{}\right.$$

input
```
Int[x*Csc[a + b*x^2]^7,x]
```

output
```
(-1/6*(Cot[a + b*x^2]*Csc[a + b*x^2]^5)/b + (5*(-1/4*(Cot[a + b*x^2]*Csc[a
 + b*x^2]^3)/b + (3*(-1/2*ArcTanh[Cos[a + b*x^2]]/b - (Cot[a + b*x^2]*Csc[
a + b*x^2])/(2*b)))/4))/6)/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\left(-\frac{\csc(bx^2+a)^5}{6}-\frac{5\csc(bx^2+a)^3}{24}-\frac{5\csc(bx^2+a)}{16}\right)\cot(bx^2+a)+\frac{5\ln(\csc(bx^2+a)-\cot(bx^2+a))}{16}}{2b}$
default	$\frac{\left(-\frac{\csc(bx^2+a)^5}{6}-\frac{5\csc(bx^2+a)^3}{24}-\frac{5\csc(bx^2+a)}{16}\right)\cot(bx^2+a)+\frac{5\ln(\csc(bx^2+a)-\cot(bx^2+a))}{16}}{2b}$
parallelrisch	$\frac{-\cot\left(\frac{a}{2}+\frac{bx^2}{2}\right)^6+\tan\left(\frac{a}{2}+\frac{bx^2}{2}\right)^6-9\cot\left(\frac{a}{2}+\frac{bx^2}{2}\right)^4+9\tan\left(\frac{a}{2}+\frac{bx^2}{2}\right)^4-45\cot\left(\frac{a}{2}+\frac{bx^2}{2}\right)^2+45\tan\left(\frac{a}{2}+\frac{bx^2}{2}\right)^2+120\,\mathrm{l}}{768b}$
risch	$\frac{15\,e^{11i(bx^2+a)}-85\,e^{9i(bx^2+a)}+198\,e^{7i(bx^2+a)}+198\,e^{5i(bx^2+a)}-85\,e^{3i(bx^2+a)}+15\,e^{i(bx^2+a)}}{48b\left(e^{2i(bx^2+a)}-1\right)^6}+\frac{5\ln\left(e^{i(bx^2+a)}\right.}{32b}$

input
```
int(x*csc(b*x^2+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/2/b*((-1/6*csc(b*x^2+a)^5-5/24*csc(b*x^2+a)^3-5/16*csc(b*x^2+a))*cot(b*x
^2+a)+5/16*ln(csc(b*x^2+a)-cot(b*x^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(82) = 164$.

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.03

$$\int x\csc^7\left(a+bx^2\right)\,dx = \frac{30\,\cos\left(bx^2+a\right)^5-80\,\cos\left(bx^2+a\right)^3-15\left(\cos\left(bx^2+a\right)^6-3\,\cos\left(bx^2+a\right)^4+3\,\cos\left(bx^2+a\right)^2-1\right)\log}{192\left(b\cos\left(bx^2+a\right.\right.}$$

input
```
integrate(x*csc(b*x^2+a)^7,x, algorithm="fricas")
```

output
```
1/192*(30*cos(b*x^2 + a)^5 - 80*cos(b*x^2 + a)^3 - 15*(cos(b*x^2 + a)^6 -
3*cos(b*x^2 + a)^4 + 3*cos(b*x^2 + a)^2 - 1)*log(1/2*cos(b*x^2 + a) + 1/2)
 + 15*(cos(b*x^2 + a)^6 - 3*cos(b*x^2 + a)^4 + 3*cos(b*x^2 + a)^2 - 1)*log
(-1/2*cos(b*x^2 + a) + 1/2) + 66*cos(b*x^2 + a))/(b*cos(b*x^2 + a)^6 - 3*b
*cos(b*x^2 + a)^4 + 3*b*cos(b*x^2 + a)^2 - b)
```

Sympy [F]

$$\int x\csc^7\left(a+bx^2\right)\,dx = \int x\csc^7\left(a+bx^2\right)dx$$

input
```
integrate(x*csc(b*x**2+a)**7,x)
```

output
```
Integral(x*csc(a + b*x**2)**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3543 vs. $2(82) = 164$.

Time = 0.11 (sec) , antiderivative size = 3543, normalized size of antiderivative = 39.37

$$\int x\csc^7\left(a+bx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*csc(b*x^2+a)^7,x, algorithm="maxima")
```

output
```
1/192*(4*(15*cos(11*b*x^2 + 11*a) - 85*cos(9*b*x^2 + 9*a) + 198*cos(7*b*x^
2 + 7*a) + 198*cos(5*b*x^2 + 5*a) - 85*cos(3*b*x^2 + 3*a) + 15*cos(b*x^2 +
 a))*cos(12*b*x^2 + 12*a) - 60*(6*cos(10*b*x^2 + 10*a) - 15*cos(8*b*x^2 +
8*a) + 20*cos(6*b*x^2 + 6*a) - 15*cos(4*b*x^2 + 4*a) + 6*cos(2*b*x^2 + 2*a
) - 1)*cos(11*b*x^2 + 11*a) + 24*(85*cos(9*b*x^2 + 9*a) - 198*cos(7*b*x^2
+ 7*a) - 198*cos(5*b*x^2 + 5*a) + 85*cos(3*b*x^2 + 3*a) - 15*cos(b*x^2 + a
))*cos(10*b*x^2 + 10*a) - 340*(15*cos(8*b*x^2 + 8*a) - 20*cos(6*b*x^2 + 6*
a) + 15*cos(4*b*x^2 + 4*a) - 6*cos(2*b*x^2 + 2*a) + 1)*cos(9*b*x^2 + 9*a)
+ 60*(198*cos(7*b*x^2 + 7*a) + 198*cos(5*b*x^2 + 5*a) - 85*cos(3*b*x^2 + 3
*a) + 15*cos(b*x^2 + a))*cos(8*b*x^2 + 8*a) - 792*(20*cos(6*b*x^2 + 6*a) -
 15*cos(4*b*x^2 + 4*a) + 6*cos(2*b*x^2 + 2*a) - 1)*cos(7*b*x^2 + 7*a) - 80
*(198*cos(5*b*x^2 + 5*a) - 85*cos(3*b*x^2 + 3*a) + 15*cos(b*x^2 + a))*cos(
6*b*x^2 + 6*a) + 792*(15*cos(4*b*x^2 + 4*a) - 6*cos(2*b*x^2 + 2*a) + 1)*co
s(5*b*x^2 + 5*a) - 300*(17*cos(3*b*x^2 + 3*a) - 3*cos(b*x^2 + a))*cos(4*b*
x^2 + 4*a) + 340*(6*cos(2*b*x^2 + 2*a) - 1)*cos(3*b*x^2 + 3*a) - 360*cos(2
*b*x^2 + 2*a)*cos(b*x^2 + a) + 15*(2*(6*cos(10*b*x^2 + 10*a) - 15*cos(8*b*
x^2 + 8*a) + 20*cos(6*b*x^2 + 6*a) - 15*cos(4*b*x^2 + 4*a) + 6*cos(2*b*x^2
 + 2*a) - 1)*cos(12*b*x^2 + 12*a) - cos(12*b*x^2 + 12*a)^2 + 12*(15*cos(8*
b*x^2 + 8*a) - 20*cos(6*b*x^2 + 6*a) + 15*cos(4*b*x^2 + 4*a) - 6*cos(2*b*x
^2 + 2*a) + 1)*cos(10*b*x^2 + 10*a) - 36*cos(10*b*x^2 + 10*a)^2 + 30*(2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(82) = 164$.

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.34

$$\int x\csc^7\left(a+bx^2\right)\,dx = -\frac{\frac{\left(\frac{9\left(\cos\left(bx^2+a\right)-1\right)}{\cos\left(bx^2+a\right)+1}-\frac{45\left(\cos\left(bx^2+a\right)-1\right)^2}{\left(\cos\left(bx^2+a\right)+1\right)^2}+\frac{110\left(\cos\left(bx^2+a\right)-1\right)^3}{\left(\cos\left(bx^2+a\right)+1\right)^3}-1\right)\left(\cos(bx^2+a)+1\right)^3}{\left(\cos(bx^2+a)-1\right)^3}+\frac{45\left(\cos(bx^2+a)-1\right)}{\cos(bx^2+a)+1}-\frac{9\left(\cos(bx^2+a)-1\right)^2}{\left(\cos(bx^2+a)+1\right)^2}+}{768\,b}$$

input
```
integrate(x*csc(b*x^2+a)^7,x, algorithm="giac")
```

output
```
-1/768*((9*(cos(b*x^2 + a) - 1)/(cos(b*x^2 + a) + 1) - 45*(cos(b*x^2 + a)
- 1)^2/(cos(b*x^2 + a) + 1)^2 + 110*(cos(b*x^2 + a) - 1)^3/(cos(b*x^2 + a)
 + 1)^3 - 1)*(cos(b*x^2 + a) + 1)^3/(cos(b*x^2 + a) - 1)^3 + 45*(cos(b*x^2
 + a) - 1)/(cos(b*x^2 + a) + 1) - 9*(cos(b*x^2 + a) - 1)^2/(cos(b*x^2 + a)
 + 1)^2 + (cos(b*x^2 + a) - 1)^3/(cos(b*x^2 + a) + 1)^3 - 60*log(-(cos(b*x
^2 + a) - 1)/(cos(b*x^2 + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 25.66 (sec) , antiderivative size = 491, normalized size of antiderivative = 5.46

$$\int x\csc^7\left(a+bx^2\right)\,dx = \text{Too large to display}$$

input
```
int(x/sin(a + b*x^2)^7,x)
```

output
```
(5*log((x*5i)/8 - (x*exp(a*1i)*exp(b*x^2*1i)*5i)/8))/(32*b) - (5*log(- (x*
5i)/8 - (x*exp(a*1i)*exp(b*x^2*1i)*5i)/8))/(32*b) + (8*exp(a*3i + b*x^2*3i
))/(3*b*(5*exp(a*2i + b*x^2*2i) - 10*exp(a*4i + b*x^2*4i) + 10*exp(a*6i +
b*x^2*6i) - 5*exp(a*8i + b*x^2*8i) + exp(a*10i + b*x^2*10i) - 1)) + exp(a*
1i + b*x^2*1i)/(6*b*(3*exp(a*2i + b*x^2*2i) - 3*exp(a*4i + b*x^2*4i) + exp
(a*6i + b*x^2*6i) - 1)) + (5*exp(a*1i + b*x^2*1i))/(16*b*(exp(a*2i + b*x^2
*2i) - 1)) + (16*exp(a*5i + b*x^2*5i))/(3*b*(15*exp(a*4i + b*x^2*4i) - 6*e
xp(a*2i + b*x^2*2i) - 20*exp(a*6i + b*x^2*6i) + 15*exp(a*8i + b*x^2*8i) -
6*exp(a*10i + b*x^2*10i) + exp(a*12i + b*x^2*12i) + 1)) + exp(a*1i + b*x^2
*1i)/(b*(6*exp(a*4i + b*x^2*4i) - 4*exp(a*2i + b*x^2*2i) - 4*exp(a*6i + b*
x^2*6i) + exp(a*8i + b*x^2*8i) + 1)) - (5*exp(a*1i + b*x^2*1i))/(24*b*(exp
(a*4i + b*x^2*4i) - 2*exp(a*2i + b*x^2*2i) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int x\csc^7\left(a+bx^2\right)\,dx$$
$$= \frac{-15\cos\left(b\,x^2+a\right)\sin\left(b\,x^2+a\right)^4-10\cos\left(b\,x^2+a\right)\sin\left(b\,x^2+a\right)^2-8\cos\left(b\,x^2+a\right)+15\log\left(\tan\left(\frac{b\,x^2}{2}+\right.\right.}{96\sin\left(b\,x^2+a\right)^6 b}$$

input
```
int(x*csc(b*x^2+a)^7,x)
```

output
```
( - 15*cos(a + b*x**2)*sin(a + b*x**2)**4 - 10*cos(a + b*x**2)*sin(a + b*x
**2)**2 - 8*cos(a + b*x**2) + 15*log(tan((a + b*x**2)/2))*sin(a + b*x**2)*
*6)/(96*sin(a + b*x**2)**6*b)
```

3.16 $\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 396

$$\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx = \frac{x^6}{6a}+\frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}-\frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}+\frac{bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

output
```
1/6*x^6/a+1/2*I*b*x^4*ln(1-I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a/(-
a^2+b^2)^(1/2)/d-1/2*I*b*x^4*ln(1-I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)
))/a/(-a^2+b^2)^(1/2)/d+b*x^2*polylog(2,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)
^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2-b*x^2*polylog(2,I*a*exp(I*(d*x^2+c))/(b+(-
a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2+I*b*polylog(3,I*a*exp(I*(d*x^2+c))
/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^3-I*b*polylog(3,I*a*exp(I*(d*x
^2+c))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx$$

$$= \frac{\sqrt{a^2-b^2}d^3x^6 - 3bd^2x^4\log\left(1-\frac{ae^{i(c+dx^2)}}{-ib+\sqrt{a^2-b^2}}\right) + 3bd^2x^4\log\left(1+\frac{ae^{i(c+dx^2)}}{ib+\sqrt{a^2-b^2}}\right) + 6ibdx^2\operatorname{PolyLog}\left(2,\frac{ae^{i(c-}}{-ib+\sqrt{}}\right.}{6a\sqrt{a^2}}$$

input
```
Integrate[x^5/(a + b*Csc[c + d*x^2]),x]
```

output
```
(Sqrt[a^2 - b^2]*d^3*x^6 - 3*b*d^2*x^4*Log[1 - (a*E^(I*(c + d*x^2)))/((-I)
*b + Sqrt[a^2 - b^2])] + 3*b*d^2*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(I*b +
Sqrt[a^2 - b^2])] + (6*I)*b*d*x^2*PolyLog[2, (a*E^(I*(c + d*x^2)))/((-I)*b
 + Sqrt[a^2 - b^2])] - (6*I)*b*d*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(I
*b + Sqrt[a^2 - b^2]))] - 6*b*PolyLog[3, (a*E^(I*(c + d*x^2)))/((-I)*b + S
qrt[a^2 - b^2])] + 6*b*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(I*b + Sqrt[a^2
- b^2]))])/(6*a*Sqrt[a^2 - b^2]*d^3)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx$$

$$\Big\downarrow \ 4693$$

$$\frac{1}{2}\int \frac{x^4}{a+b\csc(dx^2+c)}\,dx^2$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\int \frac{x^4}{a+b\csc\left(dx^2+c\right)}dx^2$$

$$\Big\downarrow \text{4679}$$

$$\frac{1}{2}\int \left(\frac{x^4}{a}-\frac{bx^4}{a\left(b+a\sin\left(dx^2+c\right)\right)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2ib\,\text{PolyLog}\left(3,\frac{iae^{i\left(dx^2+c\right)}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}-\frac{2ib\,\text{PolyLog}\left(3,\frac{iae^{i\left(dx^2+c\right)}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}+\frac{2bx^2\,\text{PolyLog}\left(2,\frac{iae^{i\left(dx^2+c\right)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}-\frac{2bx^2\,\text{PolyLog}}{ad^2\sqrt{}}\right.$$

input
```
Int[x^5/(a + b*Csc[c + d*x^2]),x]
```

output
```
(x^6/(3*a) + (I*b*x^4*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2
])])/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^4*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b
+ Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (2*b*x^2*PolyLog[2, (I*a*E^
(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) - (2*b*
x^2*PolyLog[2, (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a
^2 + b^2]*d^2) + ((2*I)*b*PolyLog[3, (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^
2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3) - ((2*I)*b*PolyLog[3, (I*a*E^(I*(c +
d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x^5}{a + b\csc\left(d\,x^2 + c\right)}dx$$

input
```
int(x^5/(a+b*csc(d*x^2+c)),x)
```

output
```
int(x^5/(a+b*csc(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1445 vs. 2(332) = 664.

Time = 0.21 (sec) , antiderivative size = 1445, normalized size of antiderivative = 3.65

$$\int \frac{x^5}{a + b\csc\left(c + dx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

output
```
1/12*(2*(a^2 - b^2)*d^3*x^6 + 6*I*a*b*d*x^2*sqrt((a^2 - b^2)/a^2)*dilog((I
*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 +
 c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) - 6*I*a*b*d*x^2*sqrt((a^2 - b^2)/a^
2)*dilog((I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*
sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) - 6*I*a*b*d*x^2*sqrt((a^
2 - b^2)/a^2)*dilog((-I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2
 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) + 6*I*a*b*d*
x^2*sqrt((a^2 - b^2)/a^2)*dilog((-I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) -
(a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1)
+ 3*a*b*c^2*sqrt((a^2 - b^2)/a^2)*log(2*a*cos(d*x^2 + c) + 2*I*a*sin(d*x^2
 + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) + 3*a*b*c^2*sqrt((a^2 - b^2)/a^
2)*log(2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^
2) - 2*I*b) - 3*a*b*c^2*sqrt((a^2 - b^2)/a^2)*log(-2*a*cos(d*x^2 + c) + 2*
I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) - 3*a*b*c^2*sqrt((
a^2 - b^2)/a^2)*log(-2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(
(a^2 - b^2)/a^2) - 2*I*b) + 6*a*b*sqrt((a^2 - b^2)/a^2)*polylog(3, -(I*b*c
os(d*x^2 + c) + b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))
*sqrt((a^2 - b^2)/a^2))/a) - 6*a*b*sqrt((a^2 - b^2)/a^2)*polylog(3, -(I*b*
cos(d*x^2 + c) + b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c)
)*sqrt((a^2 - b^2)/a^2))/a) + 6*a*b*sqrt((a^2 - b^2)/a^2)*polylog(3, -(...
```

Sympy [F]

$$\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx$$

input
```
integrate(x**5/(a+b*csc(d*x**2+c)),x)
```

output
```
Integral(x**5/(a + b*csc(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^5}{b\csc(dx^2+c)+a}\,dx$$

input
```
integrate(x^5/(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/6*(x^6 - 12*a*b*integrate((2*b*x^5*cos(d*x^2 + c)^2 + a*x^5*cos(d*x^2 +
c)*sin(2*d*x^2 + 2*c) - a*x^5*cos(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^5*
sin(d*x^2 + c)^2 + a*x^5*sin(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b
^2*cos(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + a^3*sin(
2*d*x^2 + 2*c)^2 + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*sin(d*x^2 + c) + a^3
 - 2*(2*a^2*b*sin(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a
```

Giac [F]

$$\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^5}{b\csc(dx^2+c)+a}\,dx$$

input
```
integrate(x^5/(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate(x^5/(b*csc(d*x^2 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^5}{a+\frac{b}{\sin(dx^2+c)}}\,dx$$

input
```
int(x^5/(a + b/sin(c + d*x^2)),x)
```

output
```
int(x^5/(a + b/sin(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^5}{a + b\csc(c + dx^2)}\,dx = \int \frac{x^5}{\csc(d\,x^2 + c)\,b + a}dx$$

input
```
int(x^5/(a+b*csc(d*x^2+c)),x)
```

output
```
int(x**5/(csc(c + d*x**2)*b + a),x)
```

3.17 $\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx=\text{Int}\left(\frac{x^4}{a+b\csc(c+dx^2)},x\right)$$

output
```
Defer(Int)(x^4/(a+b*csc(d*x^2+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx$$

input
```
Integrate[x^4/(a + b*Csc[c + d*x^2]),x]
```

output
```
Integrate[x^4/(a + b*Csc[c + d*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx$$
$$\Big\downarrow \text{4695}$$
$$\int \frac{x^4}{a+b\csc(c+dx^2)}dx$$

input
```
Int[x^4/(a + b*Csc[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a+b\csc(d\,x^2+c)}dx$$

input
```
int(x^4/(a+b*csc(d*x^2+c)),x)
```

output
```
int(x^4/(a+b*csc(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^4}{b\csc(dx^2+c)+a}\,dx$$

input
```
integrate(x^4/(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

output
```
integral(x^4/(b*csc(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx$$

input
```
integrate(x**4/(a+b*csc(d*x**2+c)),x)
```

output
```
Integral(x**4/(a + b*csc(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 253, normalized size of antiderivative = 14.06

$$\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^4}{b\csc(dx^2+c)+a}\,dx$$

input
```
integrate(x^4/(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/5*(x^5 - 10*a*b*integrate((2*b*x^4*cos(d*x^2 + c)^2 + a*x^4*cos(d*x^2 +
c)*sin(2*d*x^2 + 2*c) - a*x^4*cos(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^4*
sin(d*x^2 + c)^2 + a*x^4*sin(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b
^2*cos(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + a^3*sin(
2*d*x^2 + 2*c)^2 + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*sin(d*x^2 + c) + a^3
 - 2*(2*a^2*b*sin(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^4}{b\csc(dx^2+c)+a}\,dx$$

input
```
integrate(x^4/(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate(x^4/(b*csc(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 15.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{a + b\csc(c + dx^2)}\,dx = \int \frac{x^4}{a + \frac{b}{\sin(d\,x^2+c)}}\,dx$$

input
```
int(x^4/(a + b/sin(c + d*x^2)),x)
```

output
```
int(x^4/(a + b/sin(c + d*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a + b\csc(c + dx^2)}\,dx = \int \frac{x^4}{\csc(d\,x^2 + c)\,b + a}dx$$

input
```
int(x^4/(a+b*csc(d*x^2+c)),x)
```

output
```
int(x**4/(csc(c + d*x**2)*b + a),x)
```

3.18 $\int \frac{x^3}{a+b\csc(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 271

$$\int \frac{x^3}{a+b\csc(c+dx^2)}\,dx = \frac{x^4}{4a} + \frac{ibx^2\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{ibx^2\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{b\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2} - \frac{b\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2}$$

output
```
1/4*x^4/a+1/2*I*b*x^2*ln(1-I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a/(-
a^2+b^2)^(1/2)/d-1/2*I*b*x^2*ln(1-I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)
))/a/(-a^2+b^2)^(1/2)/d+1/2*b*polylog(2,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)
^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2-1/2*b*polylog(2,I*a*exp(I*(d*x^2+c))/(b+(-
a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 987 vs. $2(271) = 542$.

Time = 3.67 (sec) , antiderivative size = 987, normalized size of antiderivative = 3.64

$$\int \frac{x^3}{a + b \csc(c + dx^2)} \, dx = \text{Too large to display}$$

input
```
Integrate[x^3/(a + b*Csc[c + d*x^2]),x]
```

output
```
(Csc[c + d*x^2]*(x^4 - (2*b*((Pi*ArcTan[(a + b*Tan[(c + d*x^2)/2])/Sqrt[-a
^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(c - ArcCos[-(b/a)])*ArcTanh[((a - b)*Co
t[(2*c + Pi + 2*d*x^2)/4])/Sqrt[a^2 - b^2]] + (-2*c + Pi - 2*d*x^2)*ArcTan
h[((a + b)*Tan[(2*c + Pi + 2*d*x^2)/4])/Sqrt[a^2 - b^2]] - (ArcCos[-(b/a)]
 - (2*I)*ArcTanh[((a - b)*Cot[(2*c + Pi + 2*d*x^2)/4])/Sqrt[a^2 - b^2]])*L
og[((a + b)*(a - b - I*Sqrt[a^2 - b^2])*(1 + I*Cot[(2*c + Pi + 2*d*x^2)/4]
))/(a*(a + b + Sqrt[a^2 - b^2]*Cot[(2*c + Pi + 2*d*x^2)/4]))] + (ArcCos[-(
b/a)] + (2*I)*(-ArcTanh[((a - b)*Cot[(2*c + Pi + 2*d*x^2)/4])/Sqrt[a^2 - b
^2]] + ArcTanh[((a + b)*Tan[(2*c + Pi + 2*d*x^2)/4])/Sqrt[a^2 - b^2]]))*Lo
g[((-1)^(1/4)*Sqrt[a^2 - b^2])/(Sqrt[2]*Sqrt[a]*E^((I/2)*(c + d*x^2))*Sqrt
[b + a*Sin[c + d*x^2]])] + (ArcCos[-(b/a)] + (2*I)*ArcTanh[((a - b)*Cot[(2
*c + Pi + 2*d*x^2)/4])/Sqrt[a^2 - b^2]] - (2*I)*ArcTanh[((a + b)*Tan[(2*c
+ Pi + 2*d*x^2)/4])/Sqrt[a^2 - b^2]])*Log[-(((-1)^(3/4)*Sqrt[a^2 - b^2]*E^
((I/2)*(c + d*x^2)))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Sin[c + d*x^2]]))] - (Arc
Cos[-(b/a)] + (2*I)*ArcTanh[((a - b)*Cot[(2*c + Pi + 2*d*x^2)/4])/Sqrt[a^2
 - b^2]])*Log[1 + (I*(I*b + Sqrt[a^2 - b^2])*(a + b + Sqrt[a^2 - b^2]*Tan[
(2*c - Pi + 2*d*x^2)/4]))/(a*(a + b + Sqrt[a^2 - b^2]*Cot[(2*c + Pi + 2*d*
x^2)/4]))] + I*(PolyLog[2, ((b - I*Sqrt[a^2 - b^2])*(a + b + Sqrt[a^2 - b^
2]*Tan[(2*c - Pi + 2*d*x^2)/4]))/(a*(a + b + Sqrt[a^2 - b^2]*Cot[(2*c + Pi
 + 2*d*x^2)/4]))] - PolyLog[2, ((b + I*Sqrt[a^2 - b^2])*(a + b + Sqrt[a...
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\csc(c+dx^2)}dx$$

$$\Big\downarrow \text{4693}$$

$$\frac{1}{2}\int \frac{x^2}{a+b\csc(dx^2+c)}dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{x^2}{a+b\csc(dx^2+c)}dx^2$$

$$\Big\downarrow \text{4679}$$

$$\frac{1}{2}\int \left(\frac{x^2}{a}-\frac{bx^2}{a(b+a\sin(dx^2+c))}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{b\,\text{PolyLog}\left(2,\frac{iae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}-\frac{b\,\text{PolyLog}\left(2,\frac{iae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}+\frac{ibx^2\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{ad\sqrt{b^2-a^2}}-\frac{ibx^2\log\left(1-\frac{iae^{i(c+d}}{\sqrt{b^2-a^2}}\right.}{ad\sqrt{b^2-a^2}}\right.$$

input
```
Int[x^3/(a + b*Csc[c + d*x^2]),x]
```

output
```
(x^4/(2*a) + (I*b*x^2*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2
])])/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^2*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b
+ Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (b*PolyLog[2, (I*a*E^(I*(c
+ d*x^2)))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) - (b*PolyLog[
2, (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^
2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x^3}{a + b\csc(d\,x^2 + c)}dx$$

input
```
int(x^3/(a+b*csc(d*x^2+c)),x)
```

output
```
int(x^3/(a+b*csc(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(223) = 446$.

Time = 0.20 (sec) , antiderivative size = 1050, normalized size of antiderivative = 3.87

$$\int \frac{x^3}{a + b\csc(c + dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

output
```
1/4*((a^2 - b^2)*d^2*x^4 - a*b*c*sqrt((a^2 - b^2)/a^2)*log(2*a*cos(d*x^2 +
 c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) - a*b*c*sq
rt((a^2 - b^2)/a^2)*log(2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sq
rt((a^2 - b^2)/a^2) - 2*I*b) + a*b*c*sqrt((a^2 - b^2)/a^2)*log(-2*a*cos(d*
x^2 + c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) + a*b
*c*sqrt((a^2 - b^2)/a^2)*log(-2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) +
2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b) + I*a*b*sqrt((a^2 - b^2)/a^2)*dilog((I*
b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 +
c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) - I*a*b*sqrt((a^2 - b^2)/a^2)*dilog(
(I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2
 + c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) - I*a*b*sqrt((a^2 - b^2)/a^2)*dil
og((-I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d
*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) + I*a*b*sqrt((a^2 - b^2)/a^2)
*dilog((-I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*s
in(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) - (a*b*d*x^2 + a*b*c)*sqr
t((a^2 - b^2)/a^2)*log(-(I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*
x^2 + c) + I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a) + (a*b*d*x^2
+ a*b*c)*sqrt((a^2 - b^2)/a^2)*log(-(I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c)
 - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a) -
 (a*b*d*x^2 + a*b*c)*sqrt((a^2 - b^2)/a^2)*log(-(-I*b*cos(d*x^2 + c) - ...
```

Sympy [F]

$$\int \frac{x^3}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^3}{a+b\csc(c+dx^2)}\,dx$$

input
```
integrate(x**3/(a+b*csc(d*x**2+c)),x)
```

output
```
Integral(x**3/(a + b*csc(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^3}{b\csc(dx^2+c)+a}\,dx$$

input
```
integrate(x^3/(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/4*(x^4 - 8*a*b*integrate((2*b*x^3*cos(d*x^2 + c)^2 + a*x^3*cos(d*x^2 + c
)*sin(2*d*x^2 + 2*c) - a*x^3*cos(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^3*s
in(d*x^2 + c)^2 + a*x^3*sin(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^
2*cos(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + a^3*sin(2
*d*x^2 + 2*c)^2 + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*sin(d*x^2 + c) + a^3
- 2*(2*a^2*b*sin(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a
```

Giac [F]

$$\int \frac{x^3}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^3}{b\csc(dx^2+c)+a}\,dx$$

input
```
integrate(x^3/(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate(x^3/(b*csc(d*x^2 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^3}{a+\frac{b}{\sin(d\,x^2+c)}}\,dx$$

input
```
int(x^3/(a + b/sin(c + d*x^2)),x)
```

output
```
int(x^3/(a + b/sin(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^3}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^3}{\csc(d\,x^2+c)\,b+a}dx$$

input
```
int(x^3/(a+b*csc(d*x^2+c)),x)
```

output
```
int(x**3/(csc(c + d*x**2)*b + a),x)
```

3.19 $\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx = \text{Int}\left(\frac{x^2}{a+b\csc(c+dx^2)},x\right)$$

output
```
Defer(Int)(x^2/(a+b*csc(d*x^2+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^2}{a+b\csc(c+dx^2)}\,dx$$

input
```
Integrate[x^2/(a + b*Csc[c + d*x^2]),x]
```

output
```
Integrate[x^2/(a + b*Csc[c + d*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\csc\left(c+dx^2\right)}\,dx$$
$$\Big\downarrow \; 4695$$
$$\int \frac{x^2}{a+b\csc\left(c+dx^2\right)}dx$$

input
```
Int[x^2/(a + b*Csc[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\csc\left(d\,x^2+c\right)}dx$$

input
```
int(x^2/(a+b*csc(d*x^2+c)),x)
```

output
```
int(x^2/(a+b*csc(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^2}{b\csc(dx^2+c)+a}\,dx$$

input
```
integrate(x^2/(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

output
```
integral(x^2/(b*csc(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx$$

input
```
integrate(x**2/(a+b*csc(d*x**2+c)),x)
```

output
```
Integral(x**2/(a + b*csc(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 253, normalized size of antiderivative = 14.06

$$\int \frac{x^2}{a + b\csc(c + dx^2)}\,dx = \int \frac{x^2}{b\csc(dx^2 + c) + a}\,dx$$

input
```
integrate(x^2/(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/3*(x^3 - 6*a*b*integrate((2*b*x^2*cos(d*x^2 + c)^2 + a*x^2*cos(d*x^2 + c
)*sin(2*d*x^2 + 2*c) - a*x^2*cos(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^2*s
in(d*x^2 + c)^2 + a*x^2*sin(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^
2*cos(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + a^3*sin(2
*d*x^2 + 2*c)^2 + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*sin(d*x^2 + c) + a^3
- 2*(2*a^2*b*sin(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a + b\csc(c + dx^2)}\,dx = \int \frac{x^2}{b\csc(dx^2 + c) + a}\,dx$$

input
```
integrate(x^2/(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate(x^2/(b*csc(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 15.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^2}{a+\frac{b}{\sin(d\,x^2+c)}}\,dx$$

input
```
int(x^2/(a + b/sin(c + d*x^2)),x)
```

output
```
int(x^2/(a + b/sin(c + d*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^2}{\csc(d\,x^2+c)\,b+a}dx$$

input
```
int(x^2/(a+b*csc(d*x^2+c)),x)
```

output
```
int(x**2/(csc(c + d*x**2)*b + a),x)
```

3.20 $\int \frac{x}{a+b\csc(c+dx^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 63

$$\int \frac{x}{a+b\csc(c+dx^2)}\,dx=\frac{x^2}{2a}+\frac{b\,\mathrm{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d}$$

output
```
1/2*x^2/a+b*arctanh((a+b*tan(1/2*d*x^2+1/2*c))/(a^2-b^2)^(1/2))/a/(a^2-b^2
)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{x}{a+b\csc(c+dx^2)}\,dx=\frac{\frac{c}{d}+x^2-\frac{2b\arctan\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d}}{2a}$$

input
```
Integrate[x/(a + b*Csc[c + d*x^2]),x]
```

output
```
(c/d + x^2 - (2*b*ArcTan[(a + b*Tan[(c + d*x^2)/2])/Sqrt[-a^2 + b^2]])/(Sq
rt[-a^2 + b^2]*d))/(2*a)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4693, 3042, 4270, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\csc(c+dx^2)}dx$$

$$\Big\downarrow \text{4693}$$

$$\frac{1}{2}\int \frac{1}{a+b\csc(dx^2+c)}dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{1}{a+b\csc(dx^2+c)}dx^2$$

$$\Big\downarrow \text{4270}$$

$$\frac{1}{2}\left(\frac{x^2}{a}-\frac{\int \frac{1}{\frac{a\sin(dx^2+c)}{b}+1}dx^2}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{x^2}{a}-\frac{\int \frac{1}{\frac{a\sin(dx^2+c)}{b}+1}dx^2}{a}\right)$$

$$\Big\downarrow \text{3139}$$

$$\frac{1}{2}\left(\frac{x^2}{a}-\frac{2\int \frac{1}{x^4+\frac{2a\tan(\frac{1}{2}(dx^2+c))}{b}+1}d\tan\left(\frac{1}{2}(dx^2+c)\right)}{ad}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{4\int \frac{1}{-x^4-4\left(1-\frac{a^2}{b^2}\right)}d\left(\frac{2a}{b}+2\tan\left(\frac{1}{2}(dx^2+c)\right)\right)}{ad}+\frac{x^2}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2b\,\text{arctanh}\left(\frac{b\left(\frac{2a}{b}+2\tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}+\frac{x^2}{a}\right)$$

input
```
Int[x/(a + b*Csc[c + d*x^2]),x]
```

output
```
(x^2/a + (2*b*ArcTanh[(b*((2*a)/b + 2*Tan[(c + d*x^2)/2]))/(2*Sqrt[a^2 - b
^2])])/(a*Sqrt[a^2 - b^2]*d))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
 - Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{-\frac{2b\arctan\left(\frac{2b\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}+\frac{2\arctan\left(\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{a}}{2d}$	73
default	$\frac{-\frac{2b\arctan\left(\frac{2b\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}+\frac{2\arctan\left(\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)\right)}{a}}{2d}$	73
risch	$\frac{x^2}{2a}+\frac{b\ln\left(\mathrm{e}^{i\left(d\,x^2+c\right)}+\frac{ib\sqrt{a^2-b^2}+a^2-b^2}{a\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,da}-\frac{b\ln\left(\mathrm{e}^{i\left(d\,x^2+c\right)}+\frac{ib\sqrt{a^2-b^2}-a^2+b^2}{a\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,da}$	154

input
```
int(x/(a+b*csc(d*x^2+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(-2/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*d*x^2+1/2*c)+2*a)/(
-a^2+b^2)^(1/2))+2/a*arctan(tan(1/2*d*x^2+1/2*c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 261, normalized size of antiderivative = 4.14

$$\int \frac{x}{a+b\csc(c+dx^2)}\,dx$$
$$= \left[\frac{2\,(a^2-b^2)dx^2+\sqrt{a^2-b^2}b\log\left(\frac{(a^2-2\,b^2)\cos(dx^2+c)^2+2\,ab\sin(dx^2+c)+a^2+b^2+2\,(b\cos(dx^2+c)\sin(dx^2+c)+a\cos(dx^2+c))}{a^2\cos(dx^2+c)^2-2\,ab\sin(dx^2+c)-a^2-b^2}\right.}{4\,(a^3-ab^2)d}\right.$$

input
```
integrate(x/(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2 - b^2)*d*x^2 + sqrt(a^2 - b^2)*b*log(((a^2 - 2*b^2)*cos(d*x^2
 + c)^2 + 2*a*b*sin(d*x^2 + c) + a^2 + b^2 + 2*(b*cos(d*x^2 + c)*sin(d*x^2
 + c) + a*cos(d*x^2 + c))*sqrt(a^2 - b^2))/(a^2*cos(d*x^2 + c)^2 - 2*a*b*s
in(d*x^2 + c) - a^2 - b^2)))/((a^3 - a*b^2)*d), 1/2*((a^2 - b^2)*d*x^2 + s
qrt(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*(b*sin(d*x^2 + c) + a)/((a^2 -
b^2)*cos(d*x^2 + c))))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{x}{a + b\csc(c + dx^2)}\,dx = \int \frac{x}{a + b\csc(c + dx^2)}\,dx$$

input
```
integrate(x/(a+b*csc(d*x**2+c)),x)
```

output
```
Integral(x/(a + b*csc(c + d*x**2)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x}{a + b\csc(c + dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x/(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.33

$$\int \frac{x}{a+b\csc(c+dx^2)}\,dx = -\frac{\left(\pi\left\lfloor\frac{dx^2+c}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)+a}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}ad}+\frac{dx^2+c}{2ad}$$

input
```
integrate(x/(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

output
```
-(pi*floor(1/2*(d*x^2 + c)/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*d*x^2 + 1/
2*c) + a)/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2)*a*d) + 1/2*(d*x^2 + c)/(a
*d)
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.59

$$\int \frac{x}{a+b\csc(c+dx^2)}\,dx = \frac{x^2}{2a} - \frac{b\ln\left(b\,x\,e^{d\,x^2\,1i}\,e^{c\,1i}\,2i - \frac{2\,b\,x\,\left(a\,1i+b\,e^{d\,x^2\,1i}\,e^{c\,1i}\right)}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{2\,a\,d\,\sqrt{a+b}\,\sqrt{a-b}} + \frac{b\ln\left(b\,x\,e^{d\,x^2\,1i}\,e^{c\,1i}\,2i + \frac{2\,b\,x\,\left(a\,1i+b\,e^{d\,x^2\,1i}\,e^{c\,1i}\right)}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{2\,a\,d\,\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(x/(a + b/sin(c + d*x^2)),x)
```

output
```
x^2/(2*a) - (b*log(b*x*exp(d*x^2*1i)*exp(c*1i)*2i - (2*b*x*(a*1i + b*exp(d
*x^2*1i)*exp(c*1i)))/((a + b)^(1/2)*(a - b)^(1/2))))/(2*a*d*(a + b)^(1/2)*
(a - b)^(1/2)) + (b*log(b*x*exp(d*x^2*1i)*exp(c*1i)*2i + (2*b*x*(a*1i + b*
exp(d*x^2*1i)*exp(c*1i)))/((a + b)^(1/2)*(a - b)^(1/2))))/(2*a*d*(a + b)^(
1/2)*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25

$$\int \frac{x}{a+b\csc(c+dx^2)}\,dx = \frac{2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)b+a^2d\,x^2-b^2d\,x^2}{2ad\,(a^2-b^2)}$$

input
```
int(x/(a+b*csc(d*x^2+c)),x)
```

output
```
(2*sqrt( - a**2 + b**2)*atan((tan((c + d*x**2)/2)*b + a)/sqrt( - a**2 + b*
*2))*b + a**2*d*x**2 - b**2*d*x**2)/(2*a*d*(a**2 - b**2))
```

3.21 $\int \frac{1}{x(a+b\csc(c+dx^2))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)},x\right)$$

output
```
Defer(Int)(1/x/(a+b*csc(d*x^2+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx$$

input
```
Integrate[1/(x*(a + b*Csc[c + d*x^2])),x]
```

output
```
Integrate[1/(x*(a + b*Csc[c + d*x^2])), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{4695\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx$$
$$\Big\downarrow \; 4695$$
$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}dx$$

input
```
Int[1/(x*(a + b*Csc[c + d*x^2])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\csc\left(d\,x^2+c\right)\right)}dx$$

input
```
int(1/x/(a+b*csc(d*x^2+c)),x)
```

output
```
int(1/x/(a+b*csc(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

output
```
integral(1/(b*x*csc(d*x^2 + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx$$

input
```
integrate(1/x/(a+b*csc(d*x**2+c)),x)
```

output
```
Integral(1/(x*(a + b*csc(c + d*x**2))), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 250, normalized size of antiderivative = 13.89

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

output
```
-(2*a*b*integrate((2*b*cos(d*x^2 + c)^2 + a*cos(d*x^2 + c)*sin(2*d*x^2 + 2
*c) - a*cos(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*sin(d*x^2 + c)^2 + a*sin(d
*x^2 + c))/(a^3*x*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*x*cos(d*x^2 + c)^2 + 4*a^
2*b*x*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + a^3*x*sin(2*d*x^2 + 2*c)^2 + 4*a
*b^2*x*sin(d*x^2 + c)^2 + 4*a^2*b*x*sin(d*x^2 + c) + a^3*x - 2*(2*a^2*b*x*
sin(d*x^2 + c) + a^3*x)*cos(2*d*x^2 + 2*c)), x) - log(x))/a
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*csc(d*x^2 + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 14.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x (a + b \csc (c + dx^2))} \, dx = \int \frac{1}{x \left(a + \frac{b}{\sin(d\,x^2+c)}\right)} \, dx$$

input
```
int(1/(x*(a + b/sin(c + d*x^2))),x)
```

output
```
int(1/(x*(a + b/sin(c + d*x^2))), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x (a + b \csc (c + dx^2))} \, dx = \int \frac{1}{\csc (d\,x^2 + c)\, bx + ax} dx$$

input
```
int(1/x/(a+b*csc(d*x^2+c)),x)
```

output
```
int(1/(csc(c + d*x**2)*b*x + a*x),x)
```

3.22 $\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = -\frac{a}{x}+b\text{Int}\left(\frac{\csc(c+dx^2)}{x^2},x\right)$$

output
```
-a/x+b*Defer(Int)(csc(d*x^2+c)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = \int \frac{a+b\csc(c+dx^2)}{x^2}\,dx$$

input
```
Integrate[(a + b*Csc[c + d*x^2])/x^2,x]
```

output
```
Integrate[(a + b*Csc[c + d*x^2])/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\csc\left(c + dx^2\right)}{x^2} \, dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x^2} + \frac{b\csc\left(c + dx^2\right)}{x^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$b\int \frac{\csc\left(dx^2 + c\right)}{x^2} dx - \frac{a}{x}$$

input
```
Int[(a + b*Csc[c + d*x^2])/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc(d\,x^2+c)}{x^2}dx$$

input
```
int((a+b*csc(d*x^2+c))/x^2,x)
```

output
```
int((a+b*csc(d*x^2+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = \int \frac{b\csc(dx^2+c)+a}{x^2}\,dx$$

input
```
integrate((a+b*csc(d*x^2+c))/x^2,x, algorithm="fricas")
```

output
```
integral((b*csc(d*x^2 + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = \int \frac{a+b\csc(c+dx^2)}{x^2}\,dx$$

input
```
integrate((a+b*csc(d*x**2+c))/x**2,x)
```

output
```
Integral((a + b*csc(c + d*x**2))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 7.88

$$\int \frac{a + b\csc(c + dx^2)}{x^2}\,dx = \int \frac{b\csc(dx^2 + c) + a}{x^2}\,dx$$

input
```
integrate((a+b*csc(d*x^2+c))/x^2,x, algorithm="maxima")
```

output
```
b*(integrate(sin(d*x^2 + c)/(x^2*cos(d*x^2 + c)^2 + x^2*sin(d*x^2 + c)^2 +
 2*x^2*cos(d*x^2 + c) + x^2), x) + integrate(sin(d*x^2 + c)/(x^2*cos(d*x^2
 + c)^2 + x^2*sin(d*x^2 + c)^2 - 2*x^2*cos(d*x^2 + c) + x^2), x)) - a/x
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\csc(c + dx^2)}{x^2}\,dx = \int \frac{b\csc(dx^2 + c) + a}{x^2}\,dx$$

input
```
integrate((a+b*csc(d*x^2+c))/x^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^2 + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a + b\csc(c + dx^2)}{x^2}\,dx = \int \frac{a + \frac{b}{\sin(d\,x^2+c)}}{x^2}\,dx$$

input
```
int((a + b/sin(c + d*x^2))/x^2,x)
```

output
```
int((a + b/sin(c + d*x^2))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx=\frac{\left(\int \frac{\csc\left(d\,x^2+c\right)}{x^2}dx\right)bx-a}{x}$$

input
```
int((a+b*csc(d*x^2+c))/x^2,x)
```

output
```
(int(csc(c + d*x**2)/x**2,x)*b*x - a)/x
```

3.23 $\int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1124

$$\int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx = \text{Too large to display}$$

output
```
2*I*b*polylog(3,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^
(1/2)/d^3+1/6*x^6/a^2+b^2*x^2*ln(1+a*exp(I*(d*x^2+c))/(I*b-(a^2-b^2)^(1/2)
))/a^2/(a^2-b^2)/d^2+b^2*x^2*ln(1+a*exp(I*(d*x^2+c))/(I*b+(a^2-b^2)^(1/2))
)/a^2/(a^2-b^2)/d^2-I*b^2*polylog(2,-a*exp(I*(d*x^2+c))/(I*b+(a^2-b^2)^(1/
2)))/a^2/(a^2-b^2)/d^3+I*b^3*polylog(3,I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^
(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-1/2*I*b^3*x^4*ln(1-I*a*exp(I*(d*x^2+c))/(
b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+I*b*x^4*ln(1-I*a*exp(I*(d*x^2+
c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d-2*I*b*polylog(3,I*a*exp(I
*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^3+1/2*I*b^3*x^4*l
n(1-I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-b^3*
x^2*polylog(2,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3
/2)/d^2+2*b*x^2*polylog(2,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(
-a^2+b^2)^(1/2)/d^2+b^3*x^2*polylog(2,I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(
1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-2*b*x^2*polylog(2,I*a*exp(I*(d*x^2+c))/(b+
(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2-I*b*x^4*ln(1-I*a*exp(I*(d*x^2+
c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d-I*b^3*polylog(3,I*a*exp(I
*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-I*b^2*polylog(2
,-a*exp(I*(d*x^2+c))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-1/2*I*b^2*x^
4/a^2/(a^2-b^2)/d-1/2*b^2*x^4*cos(d*x^2+c)/a/(a^2-b^2)/d/(b+a*sin(d*x^2+c)
)
```

Mathematica [A] (warning: unable to verify)

Time = 7.25 (sec) , antiderivative size = 1956, normalized size of antiderivative = 1.74

$$\int \frac{x^5}{(a + b\csc(c + dx^2))^2} \, dx = \text{Too large to display}$$

input
```
Integrate[x^5/(a + b*Csc[c + d*x^2])^2,x]
```

output
```
(Csc[c + d*x^2]^2*(b + a*Sin[c + d*x^2])*((6*b^2*x^4*Csc[c]*(b*Cos[c] + a*
Sin[d*x^2]))/((a - b)*(a + b)*d) + 2*x^6*(b + a*Sin[c + d*x^2]) - (6*b*E^(
(2*I)*c)*((2*I)*b*d^2*E^((2*I)*c)*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^4 + 2*b*
d*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^2*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I*b*E
^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 2*b*d*E^((2*I)*c)*Sqrt[(a^2 - b
^2)*E^((2*I)*c)]*x^2*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) - Sqrt[(
a^2 - b^2)*E^((2*I)*c)])] - 2*a^2*d^2*E^(I*c)*x^4*Log[1 + (a*E^(I*(2*c + d
*x^2)))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + b^2*d^2*E^(I*c)*x
^4*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I
)*c)])] + 2*a^2*d^2*E^((3*I)*c)*x^4*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I*b*E
^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - b^2*d^2*E^((3*I)*c)*x^4*Log[1 +
 (a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] +
2*b*d*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^2*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I
*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 2*b*d*E^((2*I)*c)*Sqrt[(a^2
 - b^2)*E^((2*I)*c)]*x^2*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) + Sq
rt[(a^2 - b^2)*E^((2*I)*c)])] + 2*a^2*d^2*E^(I*c)*x^4*Log[1 + (a*E^(I*(2*c
 + d*x^2)))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - b^2*d^2*E^(I*
c)*x^4*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^(
(2*I)*c)])] - 2*a^2*d^2*E^((3*I)*c)*x^4*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I
*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + b^2*d^2*E^((3*I)*c)*x^4*...
```

Rubi [A] (verified)

Time = 2.45 (sec) , antiderivative size = 1123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+b\csc(c+dx^2))^2}dx$$

$$\Big\downarrow 4693$$

$$\frac{1}{2}\int \frac{x^4}{(a+b\csc(dx^2+c))^2}dx^2$$

$$\Big\downarrow 3042$$

$$\frac{1}{2}\int \frac{x^4}{\left(a+b\csc\left(dx^2+c\right)\right)^2}dx^2$$

$$\Big\downarrow \; 4679$$

$$\frac{1}{2}\int \left(-\frac{2bx^4}{a^2\left(b+a\sin\left(dx^2+c\right)\right)}+\frac{x^4}{a^2}+\frac{b^2x^4}{a^2\left(b+a\sin\left(dx^2+c\right)\right)^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{x^6}{3a^2}+\frac{2ib\log\left(1-\frac{iae^{i\left(dx^2+c\right)}}{b-\sqrt{b^2-a^2}}\right)x^4}{a^2\sqrt{b^2-a^2}d}-\frac{ib^3\log\left(1-\frac{iae^{i\left(dx^2+c\right)}}{b-\sqrt{b^2-a^2}}\right)x^4}{a^2\left(b^2-a^2\right)^{3/2}d}-\frac{2ib\log\left(1-\frac{iae^{i\left(dx^2+c\right)}}{b+\sqrt{b^2-a^2}}\right)x^4}{a^2\sqrt{b^2-a^2}d}+\frac{ib^3\log\left(1\right.}{a^2\left(b\right.}\right.$$

input

```
Int[x^5/(a + b*Csc[c + d*x^2])^2,x]
```

output

```
(((-I)*b^2*x^4)/(a^2*(a^2 - b^2)*d) + x^6/(3*a^2) + (2*b^2*x^2*Log[1 + (a*
E^(I*(c + d*x^2)))/(I*b - Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) + (2*b^
2*x^2*Log[1 + (a*E^(I*(c + d*x^2)))/(I*b + Sqrt[a^2 - b^2])])/(a^2*(a^2 -
b^2)*d^2) - (I*b^3*x^4*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^
2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^4*Log[1 - (I*a*E^(I*(c + d*x
^2)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) + (I*b^3*x^4*Log[1
 - (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2
)*d) - ((2*I)*b*x^4*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])
])/(a^2*Sqrt[-a^2 + b^2]*d) - ((2*I)*b^2*PolyLog[2, -((a*E^(I*(c + d*x^2))
)/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - ((2*I)*b^2*PolyLog[2,
 -((a*E^(I*(c + d*x^2)))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3)
- (2*b^3*x^2*PolyLog[2, (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])])/(
a^2*(-a^2 + b^2)^(3/2)*d^2) + (4*b*x^2*PolyLog[2, (I*a*E^(I*(c + d*x^2)))/
(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (2*b^3*x^2*PolyLog[2
, (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)
*d^2) - (4*b*x^2*PolyLog[2, (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])
])/(a^2*Sqrt[-a^2 + b^2]*d^2) - ((2*I)*b^3*PolyLog[3, (I*a*E^(I*(c + d*x^2
)))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((4*I)*b*PolyL
og[3, (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^
2]*d^3) + ((2*I)*b^3*PolyLog[3, (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x^5}{\left(a+b\csc\left(d\,x^2+c\right)\right)^2}dx$$

input
```
int(x^5/(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(x^5/(a+b*csc(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3032 vs. 2(966) = 1932.

Time = 0.27 (sec) , antiderivative size = 3032, normalized size of antiderivative = 2.70

$$\int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx$$

input
```
integrate(x**5/(a+b*csc(d*x**2+c))**2,x)
```

output
```
Integral(x**5/(a + b*csc(c + d*x**2))**2, x)
```

Maxima [F]

$$\int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^5}{(b\csc(dx^2+c)+a)^2}\,dx$$

input
```
integrate(x^5/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
1/6*((a^4 - a^2*b^2)*d*x^6*cos(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^6*
cos(d*x^2 + c)^2 + (a^4 - a^2*b^2)*d*x^6*sin(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2
 - b^4)*d*x^6*sin(d*x^2 + c)^2 - 6*a*b^3*x^4*cos(d*x^2 + c) + 4*(a^3*b - a
*b^3)*d*x^6*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^6 - 2*(3*a*b^3*x^4*cos(d*
x^2 + c) + 2*(a^3*b - a*b^3)*d*x^6*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^6)
*cos(2*d*x^2 + 2*c) - 6*((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b
^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c)*si
n(2*d*x^2 + 2*c) + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a
^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 -
 a^4*b^2)*d - 2*(2*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d)
*cos(2*d*x^2 + 2*c))*integrate(2*(2*(2*a^2*b^2 - b^4)*d*x^5*cos(d*x^2 + c)
^2 + 2*(2*a^2*b^2 - b^4)*d*x^5*sin(d*x^2 + c)^2 - 2*a*b^3*x^3*cos(d*x^2 +
c) + (2*a^3*b - a*b^3)*d*x^5*sin(d*x^2 + c) - (2*a*b^3*x^3*cos(d*x^2 + c)
+ (2*a^3*b - a*b^3)*d*x^5*sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + ((2*a^3*b -
 a*b^3)*d*x^5*cos(d*x^2 + c) - 2*a*b^3*x^3*sin(d*x^2 + c) - 2*a^2*b^2*x^3)
*sin(2*d*x^2 + 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2
- a^2*b^4)*d*cos(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c)*sin(2
*d*x^2 + 2*c) + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*
b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^
4*b^2)*d - 2*(2*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d)...
```

Giac [F]

$$\int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^5}{(b\csc(dx^2+c)+a)^2}\,dx$$

input
```
integrate(x^5/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate(x^5/(b*csc(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^5}{\left(a+\frac{b}{\sin(d\,x^2+c)}\right)^2}\,dx$$

input
```
int(x^5/(a + b/sin(c + d*x^2))^2,x)
```

output
```
int(x^5/(a + b/sin(c + d*x^2))^2, x)
```

Reduce [F]

$$\int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^5}{\csc(d\,x^2+c)^2 b^2+2\csc(d\,x^2+c)\,ab+a^2}dx$$

input
```
int(x^5/(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(x**5/(csc(c + d*x**2)**2*b**2 + 2*csc(c + d*x**2)*a*b + a**2),x)
```

3.24 $\int \frac{x^4}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx = \text{Int}\left(\frac{x^4}{(a+b\csc(c+dx^2))^2},x\right)$$

output
```
Defer(Int)(x^4/(a+b*csc(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 8.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx$$

input
```
Integrate[x^4/(a + b*Csc[c + d*x^2])^2,x]
```

output
```
Integrate[x^4/(a + b*Csc[c + d*x^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}dx$$
$$\Big\downarrow \; 4695$$
$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}dx$$

input
```
Int[x^4/(a + b*Csc[c + d*x^2])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+b\csc(d\,x^2+c))^2}dx$$

input
```
int(x^4/(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(x^4/(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^4}{(a + b\csc(c + dx^2))^2}\,dx = \int \frac{x^4}{(b\csc(dx^2 + c) + a)^2}\,dx$$

input
```
integrate(x^4/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
integral(x^4/(b^2*csc(d*x^2 + c)^2 + 2*a*b*csc(d*x^2 + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{(a + b\csc(c + dx^2))^2}\,dx = \int \frac{x^4}{(a + b\csc(c + dx^2))^2}\,dx$$

input
```
integrate(x**4/(a+b*csc(d*x**2+c))**2,x)
```

output
```
Integral(x**4/(a + b*csc(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 1286, normalized size of antiderivative = 71.44

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^4}{(b\csc(dx^2+c)+a)^2}\,dx$$

input
```
integrate(x^4/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
1/5*((a^4 - a^2*b^2)*d*x^5*cos(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^5*
cos(d*x^2 + c)^2 + (a^4 - a^2*b^2)*d*x^5*sin(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2
 - b^4)*d*x^5*sin(d*x^2 + c)^2 - 5*a*b^3*x^3*cos(d*x^2 + c) + 4*(a^3*b - a
*b^3)*d*x^5*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^5 - (5*a*b^3*x^3*cos(d*x^
2 + c) + 4*(a^3*b - a*b^3)*d*x^5*sin(d*x^2 + c) + 2*(a^4 - a^2*b^2)*d*x^5)
*cos(2*d*x^2 + 2*c) - 5*((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b
^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c)*si
n(2*d*x^2 + 2*c) + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a
^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 -
 a^4*b^2)*d - 2*(2*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d)
*cos(2*d*x^2 + 2*c))*integrate((4*(2*a^2*b^2 - b^4)*d*x^4*cos(d*x^2 + c)^2
 + 4*(2*a^2*b^2 - b^4)*d*x^4*sin(d*x^2 + c)^2 - 3*a*b^3*x^2*cos(d*x^2 + c)
 + 2*(2*a^3*b - a*b^3)*d*x^4*sin(d*x^2 + c) - (3*a*b^3*x^2*cos(d*x^2 + c)
+ 2*(2*a^3*b - a*b^3)*d*x^4*sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (2*(2*a^3
*b - a*b^3)*d*x^4*cos(d*x^2 + c) - 3*a*b^3*x^2*sin(d*x^2 + c) - 3*a^2*b^2*
x^2)*sin(2*d*x^2 + 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*
b^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c)*s
in(2*d*x^2 + 2*c) + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 -
a^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6
- a^4*b^2)*d - 2*(2*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2...
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^4}{(b\csc(dx^2+c)+a)^2}\,dx$$

input
```
integrate(x^4/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate(x^4/(b*csc(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 15.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^4}{\left(a+\frac{b}{\sin(d\,x^2+c)}\right)^2}\,dx$$

input
```
int(x^4/(a + b/sin(c + d*x^2))^2,x)
```

output
```
int(x^4/(a + b/sin(c + d*x^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^4}{\csc(d\,x^2+c)^2 b^2+2\csc(d\,x^2+c)\,ab+a^2}dx$$

input
```
int(x^4/(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(x**4/(csc(c + d*x**2)**2*b**2 + 2*csc(c + d*x**2)*a*b + a**2),x)
```

3.25 $\int \frac{x^3}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 616

$$\begin{aligned}
\int \frac{x^3}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx &= \frac{x^4}{4a^2}-\frac{ib^3x^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} \\
&+\frac{ibx^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}+\frac{ib^3x^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} \\
&-\frac{ibx^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}+\frac{b^2\log\left(b+a\sin\left(c+dx^2\right)\right)}{2a^2\left(a^2-b^2\right)d^2} \\
&-\frac{b^3\operatorname{PolyLog}\left(2,\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d^2} \\
&+\frac{b\operatorname{PolyLog}\left(2,\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}+\frac{b^3\operatorname{PolyLog}\left(2,\frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d^2} \\
&-\frac{b\operatorname{PolyLog}\left(2,\frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&-\frac{b^2x^2\cos\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\sin\left(c+dx^2\right)\right)}
\end{aligned}$$

output
```
1/4*x^4/a^2-1/2*I*b^3*x^2*ln(1-I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/
a^2/(-a^2+b^2)^(3/2)/d+I*b*x^2*ln(1-I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/
2)))/a^2/(-a^2+b^2)^(1/2)/d+1/2*I*b^3*x^2*ln(1-I*a*exp(I*(d*x^2+c))/(b+(-a
^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-I*b*x^2*ln(1-I*a*exp(I*(d*x^2+c))/(
b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d+1/2*b^2*ln(b+a*sin(d*x^2+c))/a
^2/(a^2-b^2)/d^2-1/2*b^3*polylog(2,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2
)))/a^2/(-a^2+b^2)^(3/2)/d^2+b*polylog(2,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2
)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2+1/2*b^3*polylog(2,I*a*exp(I*(d*x^2+c))/
(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-b*polylog(2,I*a*exp(I*(d*x^
2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2-1/2*b^2*x^2*cos(d*x^2
+c)/a/(a^2-b^2)/d/(b+a*sin(d*x^2+c))
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2566 vs. $2(616) = 1232$.

Time = 15.11 (sec) , antiderivative size = 2566, normalized size of antiderivative = 4.17

$$\int \frac{x^3}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^3/(a + b*Csc[c + d*x^2])^2,x]
```

output
```
((-(b^2*c*Cos[c + d*x^2]) + b^2*(c + d*x^2)*Cos[c + d*x^2])*Csc[c + d*x^2]
^2*(b + a*Sin[c + d*x^2]))/(2*a*(-a + b)*(a + b)*d^2*(a + b*Csc[c + d*x^2]
)^2) + ((-c + d*x^2)*(c + d*x^2)*Csc[c + d*x^2]^2*(b + a*Sin[c + d*x^2])^2
)/(4*a^2*d^2*(a + b*Csc[c + d*x^2])^2) + (Csc[c + d*x^2]^2*(-2*a*b*ArcTanh
[(a + b*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]] + 2*(a*b + 2*a^2*c - b^2*c)*A
rcTanh[(a + b*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]] + b*Sqrt[a^2 - b^2]*Log
[Sec[(c + d*x^2)/2]^2] - b*Sqrt[a^2 - b^2]*Log[Sec[(c + d*x^2)/2]^2*(b + a
*Sin[c + d*x^2])] + I*(2*a^2 - b^2)*Log[1 - I*Tan[(c + d*x^2)/2]]*Log[(a -
 Sqrt[a^2 - b^2] + b*Tan[(c + d*x^2)/2])/(a - I*b - Sqrt[a^2 - b^2])] - I*
(2*a^2 - b^2)*Log[1 + I*Tan[(c + d*x^2)/2]]*Log[(a - Sqrt[a^2 - b^2] + b*T
an[(c + d*x^2)/2])/(a + I*b - Sqrt[a^2 - b^2])] - I*(2*a^2 - b^2)*Log[1 -
I*Tan[(c + d*x^2)/2]]*Log[(a + Sqrt[a^2 - b^2] + b*Tan[(c + d*x^2)/2])/(a
- I*b + Sqrt[a^2 - b^2])] + I*(2*a^2 - b^2)*Log[1 + I*Tan[(c + d*x^2)/2]]*
Log[(a + Sqrt[a^2 - b^2] + b*Tan[(c + d*x^2)/2])/(a + I*b + Sqrt[a^2 - b^2
])] - I*(2*a^2 - b^2)*PolyLog[2, (b*(1 + I*Tan[(c + d*x^2)/2]))/((-I)*a +
b + I*Sqrt[a^2 - b^2])] + I*(2*a^2 - b^2)*PolyLog[2, (b*(1 + I*Tan[(c + d*
x^2)/2]))/(b - I*(a + Sqrt[a^2 - b^2]))] - I*(2*a^2 - b^2)*PolyLog[2, -((b
*(I + Tan[(c + d*x^2)/2]))/(a - I*b + Sqrt[a^2 - b^2]))] + I*(2*a^2 - b^2)
*PolyLog[2, (b*(I + Tan[(c + d*x^2)/2]))/(-a + I*b + Sqrt[a^2 - b^2])])*(b
 + a*Sin[c + d*x^2])^2*((2*b*c)/((a^2 - b^2)*d*(b + a*Sin[c + d*x^2])) ...
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 607, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx$$

$$\Big\downarrow \text{4693}$$

$$\frac{1}{2}\int \frac{x^2}{(a+b\csc(dx^2+c))^2}dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{x^2}{(a+b\csc(dx^2+c))^2}dx^2$$
$$\Big\downarrow \text{4679}$$
$$\frac{1}{2}\int \left(-\frac{2bx^2}{a^2(b+a\sin(dx^2+c))}+\frac{x^2}{a^2}+\frac{b^2x^2}{a^2(b+a\sin(dx^2+c))^2}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{2b\,\text{PolyLog}\left(2,\frac{iae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}}-\frac{2b\,\text{PolyLog}\left(2,\frac{iae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}}+\frac{b^2\log(a\sin(c+dx^2)+b)}{a^2d^2(a^2-b^2)}+\frac{2ibx^2\log\left(1-\frac{iae}{b-}\right.}{a^2d\sqrt{b^2-}}\right.$$

input
```
Int[x^3/(a + b*Csc[c + d*x^2])^2,x]
```

output
```
(x^4/(2*a^2) - (I*b^3*x^2*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 +
 b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^2*Log[1 - (I*a*E^(I*(c +
d*x^2)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) + (I*b^3*x^2*Lo
g[1 - (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(
3/2)*d) - ((2*I)*b*x^2*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^
2])])/(a^2*Sqrt[-a^2 + b^2]*d) + (b^2*Log[b + a*Sin[c + d*x^2]])/(a^2*(a^2
 - b^2)*d^2) - (b^3*PolyLog[2, (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^
2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (2*b*PolyLog[2, (I*a*E^(I*(c + d*x^2)
))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (b^3*PolyLog[2, (
I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^
2) - (2*b*PolyLog[2, (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2
*Sqrt[-a^2 + b^2]*d^2) - (b^2*x^2*Cos[c + d*x^2])/(a*(a^2 - b^2)*d*(b + a*
Sin[c + d*x^2])))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x^3}{(a+b\csc(d\,x^2+c))^2}dx$$

input
```
int(x^3/(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(x^3/(a+b*csc(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1906 vs. $2(526) = 1052$.

Time = 0.23 (sec) , antiderivative size = 1906, normalized size of antiderivative = 3.09

$$\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
1/4*((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4*sin(d*x^2 + c) + (a^4*b - 2*a^2*b^3
 + b^5)*d^2*x^4 - 2*(a^3*b^2 - a*b^4)*d*x^2*cos(d*x^2 + c) + (2*I*a^3*b^2
- I*a*b^4 + (2*I*a^4*b - I*a^2*b^3)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2)*
dilog((I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin
(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) + (-2*I*a^3*b^2 + I*a*b^4 +
 (-2*I*a^4*b + I*a^2*b^3)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2)*dilog((I*b
*cos(d*x^2 + c) - b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c
))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) + (-2*I*a^3*b^2 + I*a*b^4 + (-2*I*a^4
*b + I*a^2*b^3)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2)*dilog((-I*b*cos(d*x^
2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((
a^2 - b^2)/a^2) - a)/a + 1) + (2*I*a^3*b^2 - I*a*b^4 + (2*I*a^4*b - I*a^2*
b^3)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2)*dilog((-I*b*cos(d*x^2 + c) - b*
sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/
a^2) - a)/a + 1) - ((2*a^3*b^2 - a*b^4)*d*x^2 + (2*a^3*b^2 - a*b^4)*c + ((
2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*sin(d*x^2 + c))*sqrt((a^
2 - b^2)/a^2)*log(-(I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 +
 c) + I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a) + ((2*a^3*b^2 - a*
b^4)*d*x^2 + (2*a^3*b^2 - a*b^4)*c + ((2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b
 - a^2*b^3)*c)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2)*log(-(I*b*cos(d*x^2 +
 c) - b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt((...
```

Sympy [F]

$$\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx=\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx$$

input
```
integrate(x**3/(a+b*csc(d*x**2+c))**2,x)
```

output
```
Integral(x**3/(a + b*csc(c + d*x**2))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^3}{(b\csc(dx^2+c)+a)^2}\,dx$$

input
```
integrate(x^3/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate(x^3/(b*csc(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^3}{\left(a+\frac{b}{\sin(d\,x^2+c)}\right)^2}\,dx$$

input
```
int(x^3/(a + b/sin(c + d*x^2))^2,x)
```

output
```
int(x^3/(a + b/sin(c + d*x^2))^2, x)
```

Reduce [F]

$$\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^3}{\csc(d\,x^2+c)^2\,b^2+2\csc(d\,x^2+c)\,ab+a^2}dx$$

input
```
int(x^3/(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(x**3/(csc(c + d*x**2)**2*b**2 + 2*csc(c + d*x**2)*a*b + a**2),x)
```

3.26 $\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx = \text{Int}\left(\frac{x^2}{(a+b\csc(c+dx^2))^2},x\right)$$

output
```
Defer(Int)(x^2/(a+b*csc(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 7.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx$$

input
```
Integrate[x^2/(a + b*Csc[c + d*x^2])^2,x]
```

output
```
Integrate[x^2/(a + b*Csc[c + d*x^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}dx$$
$$\Big\downarrow \text{4695}$$
$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}dx$$

input
```
Int[x^2/(a + b*Csc[c + d*x^2])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+b\csc(d\,x^2+c))^2}dx$$

input
```
int(x^2/(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(x^2/(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\csc(dx^2+c)+a)^2}\,dx$$

input
```
integrate(x^2/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
integral(x^2/(b^2*csc(d*x^2 + c)^2 + 2*a*b*csc(d*x^2 + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx$$

input
```
integrate(x**2/(a+b*csc(d*x**2+c))**2,x)
```

output
```
Integral(x**2/(a + b*csc(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 1265, normalized size of antiderivative = 70.28

$$\int \frac{x^2}{(a + b \csc (c + dx^2))^2} \, dx = \int \frac{x^2}{(b \csc (dx^2 + c) + a)^2} \, dx$$

input
```
integrate(x^2/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
1/3*((a^4 - a^2*b^2)*d*x^3*cos(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^3*
cos(d*x^2 + c)^2 + (a^4 - a^2*b^2)*d*x^3*sin(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2
 - b^4)*d*x^3*sin(d*x^2 + c)^2 - 3*a*b^3*x*cos(d*x^2 + c) + 4*(a^3*b - a*b
^3)*d*x^3*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^3 - (3*a*b^3*x*cos(d*x^2 +
c) + 4*(a^3*b - a*b^3)*d*x^3*sin(d*x^2 + c) + 2*(a^4 - a^2*b^2)*d*x^3)*cos
(2*d*x^2 + 2*c) - 3*((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 -
 a^2*b^4)*d*cos(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c)*sin(2*
d*x^2 + 2*c) + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b
^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4
*b^2)*d - 2*(2*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos
(2*d*x^2 + 2*c))*integrate((4*(2*a^2*b^2 - b^4)*d*x^2*cos(d*x^2 + c)^2 + 4
*(2*a^2*b^2 - b^4)*d*x^2*sin(d*x^2 + c)^2 - a*b^3*cos(d*x^2 + c) + 2*(2*a^
3*b - a*b^3)*d*x^2*sin(d*x^2 + c) - (a*b^3*cos(d*x^2 + c) + 2*(2*a^3*b - a
*b^3)*d*x^2*sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (2*(2*a^3*b - a*b^3)*d*x^
2*cos(d*x^2 + c) - a*b^3*sin(d*x^2 + c) - a^2*b^2)*sin(2*d*x^2 + 2*c))/((a
^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*cos(d*x^2 +
 c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^6 - a
^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*sin(d*x^2 + c)^2
+ 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d - 2*(2*(a^5*b -
 a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 + 2*c)), x)...
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\csc(dx^2+c)+a)^2}\,dx$$

input
```
integrate(x^2/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate(x^2/(b*csc(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 15.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^2}{\left(a+\frac{b}{\sin(d\,x^2+c)}\right)^2}\,dx$$

input
```
int(x^2/(a + b/sin(c + d*x^2))^2,x)
```

output
```
int(x^2/(a + b/sin(c + d*x^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^2}{\csc(d\,x^2+c)^2\,b^2+2\csc(d\,x^2+c)\,ab+a^2}\,dx$$

input
```
int(x^2/(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(x**2/(csc(c + d*x**2)**2*b**2 + 2*csc(c + d*x**2)*a*b + a**2),x)
```

3.27 $\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 120

$$\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx = \frac{x^2}{2a^2} + \frac{b(2a^2-b^2)\operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{3/2}d} - \frac{b^2\cot(c+dx^2)}{2a(a^2-b^2)d(a+b\csc(c+dx^2))}$$

output
```
1/2*x^2/a^2+b*(2*a^2-b^2)*arctanh((a+b*tan(1/2*d*x^2+1/2*c))/(a^2-b^2)^(1/
2))/a^2/(a^2-b^2)^(3/2)/d-1/2*b^2*cot(d*x^2+c)/a/(a^2-b^2)/d/(a+b*csc(d*x^
2+c))
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.32

$$\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx = \frac{\csc(c+dx^2)\left(\frac{ab^2\cot(c+dx^2)}{(-a+b)(a+b)} + (c+dx^2)(a+b\csc(c+dx^2)) - \frac{2b(-2a^2+b^2)\arctan\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{-a^2+b^2}}\right)(a+b\csc(c+}{(-a^2+b^2)^{3/2}}\right.}{2a^2d(a+b\csc(c+dx^2))^2}$$

input
```
Integrate[x/(a + b*Csc[c + d*x^2])^2,x]
```

output
```
(Csc[c + d*x^2]*((a*b^2*Cot[c + d*x^2])/((-a + b)*(a + b)) + (c + d*x^2)*(
a + b*Csc[c + d*x^2]) - (2*b*(-2*a^2 + b^2)*ArcTan[(a + b*Tan[(c + d*x^2)/
2])/Sqrt[-a^2 + b^2]]*(a + b*Csc[c + d*x^2]))/(-a^2 + b^2)^(3/2))*(b + a*S
in[c + d*x^2]))/(2*a^2*d*(a + b*Csc[c + d*x^2])^2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.28, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4693, 3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\csc(c+dx^2))^2}dx$$

$$\Big\downarrow \text{4693}$$

$$\frac{1}{2}\int \frac{1}{(a+b\csc(dx^2+c))^2}dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{1}{(a+b\csc(dx^2+c))^2}dx^2$$

$$\Big\downarrow \text{4272}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{a^2-b\csc(dx^2+c)a-b^2}{a+b\csc(dx^2+c)}dx^2}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^2)}{ad(a^2-b^2)(a+b\csc(c+dx^2))}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{a^2-b\csc(dx^2+c)a-b^2}{a+b\csc(dx^2+c)}dx^2}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^2)}{ad(a^2-b^2)(a+b\csc(c+dx^2))}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\int \frac{a^2-b\csc(dx^2+c)a-b^2}{a+b\csc(dx^2+c)}dx^2}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^2)}{ad(a^2-b^2)(a+b\csc(c+dx^2))}\right)$$

$$\Big\downarrow \text{4407}$$

$$\frac{1}{2}\left(\frac{\frac{x^2(a^2-b^2)}{a}-\frac{b(2a^2-b^2)\int \frac{\csc(dx^2+c)}{a+b\csc(dx^2+c)}dx^2}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^2)}{ad(a^2-b^2)(a+b\csc(c+dx^2))}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\frac{x^2(a^2-b^2)}{a}-\frac{b(2a^2-b^2)\int \frac{\csc(dx^2+c)}{a+b\csc(dx^2+c)}dx^2}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^2)}{ad(a^2-b^2)(a+b\csc(c+dx^2))}\right)$$

$$\Big\downarrow \text{4318}$$

$$\frac{1}{2}\left(\frac{\frac{x^2(a^2-b^2)}{a}-\frac{(2a^2-b^2)\int \frac{1}{\frac{a\sin(dx^2+c)}{b}+1}dx^2}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^2)}{ad(a^2-b^2)(a+b\csc(c+dx^2))}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\frac{x^2(a^2-b^2)}{a}-\frac{(2a^2-b^2)\int \frac{1}{\frac{a\sin(dx^2+c)}{b}+1}dx^2}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^2)}{ad(a^2-b^2)(a+b\csc(c+dx^2))}\right)$$

$$\Big\downarrow \text{3139}$$

$$\frac{1}{2}\left(\frac{\frac{x^2(a^2-b^2)}{a}-\frac{2(2a^2-b^2)\int \frac{1}{x^4+\frac{2a\tan\left(\frac{1}{2}(dx^2+c)\right)}{b}+1}d\tan\left(\frac{1}{2}(dx^2+c)\right)}{ad}}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^2)}{ad(a^2-b^2)(a+b\csc(c+dx^2))}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{\frac{4(2a^2-b^2)\int \frac{1}{-x^4-4\left(1-\frac{a^2}{b^2}\right)}d\left(\frac{2a}{b}+2\tan\left(\frac{1}{2}(dx^2+c)\right)\right)}{ad}+\frac{x^2(a^2-b^2)}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^2)}{ad(a^2-b^2)(a+b\csc(c+dx^2))}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\frac{2b(2a^2-b^2)\text{arctanh}\left(\frac{b\left(\frac{2a}{b}+2\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)\right)}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}+\frac{x^2(a^2-b^2)}{a}}{a\left(a^2-b^2\right)}-\frac{b^2\cot\left(c+dx^2\right)}{ad\left(a^2-b^2\right)\left(a+b\csc\left(c+dx^2\right)\right)}\right)$$

input
```
Int[x/(a + b*Csc[c + d*x^2])^2,x]
```

output
```
((((a^2 - b^2)*x^2)/a + (2*b*(2*a^2 - b^2)*ArcTanh[(b*((2*a)/b + 2*Tan[(c
+ d*x^2)/2]))/(2*Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d))/(a*(a^2 - b^2))
 - (b^2*Cot[c + d*x^2])/(a*(a^2 - b^2)*d*(a + b*Csc[c + d*x^2])))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{-\frac{2b\left(\frac{\frac{\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)a^2}{2a^2-2b^2}+\frac{ba}{2a^2-2b^2}}{\frac{\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2 b}{2}+a\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+\frac{b}{2}}+\frac{2(2a^2-b^2)\arctan\left(\frac{2b\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{(2a^2-2b^2)\sqrt{-a^2+b^2}}\right)}{a^2}+\frac{2\arctan\left(\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)}{a^2}}{2d}$
default	$\frac{-\frac{2b\left(\frac{\frac{\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)a^2}{2a^2-2b^2}+\frac{ba}{2a^2-2b^2}}{\frac{\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2 b}{2}+a\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+\frac{b}{2}}+\frac{2(2a^2-b^2)\arctan\left(\frac{2b\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{(2a^2-2b^2)\sqrt{-a^2+b^2}}\right)}{a^2}+\frac{2\arctan\left(\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)}{a^2}}{2d}$
risch	$\frac{x^2}{2a^2}-\frac{ib^2\left(ia+b\,e^{i(dx^2+c)}\right)}{a^2(-a^2+b^2)d\left(2b\,e^{i(dx^2+c)}-ia\,e^{2i(dx^2+c)}+ia\right)}+\frac{b\ln\left(e^{i(dx^2+c)}+\frac{ib\sqrt{a^2-b^2}+a^2-b^2}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{b^3\ln\left(e^{i(dx^2}\right.}{2\sqrt{a^2-}}$

input

```
int(x/(a+b*csc(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

output

```
1/2/d*(-2/a^2*b*((1/2*a^2/(a^2-b^2)*tan(1/2*d*x^2+1/2*c)+1/2*b*a/(a^2-b^2)
)/(1/2*tan(1/2*d*x^2+1/2*c)^2*b+a*tan(1/2*d*x^2+1/2*c)+1/2*b)+2*(2*a^2-b^2
)/(2*a^2-2*b^2)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*d*x^2+1/2*c)+2*a)
/(-a^2+b^2)^(1/2)))+2/a^2*arctan(tan(1/2*d*x^2+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(111) = 222$.

Time = 0.10 (sec) , antiderivative size = 536, normalized size of antiderivative = 4.47

$$\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx$$
$$= \left[\frac{2(a^5-2a^3b^2+ab^4)dx^2\sin(dx^2+c)+2(a^4b-2a^2b^3+b^5)dx^2+(2a^2b^2-b^4+(2a^3b-ab^3)\sin(dx^2}{4((a^7-2a^5b^2+a}\right.$$

input

```
integrate(x/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x^2*sin(d*x^2 + c) + 2*(a^4*b - 2*a^2*
b^3 + b^5)*d*x^2 + (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*sin(d*x^2 + c))*sq
rt(a^2 - b^2)*log(((a^2 - 2*b^2)*cos(d*x^2 + c)^2 + 2*a*b*sin(d*x^2 + c) +
 a^2 + b^2 + 2*(b*cos(d*x^2 + c)*sin(d*x^2 + c) + a*cos(d*x^2 + c))*sqrt(a
^2 - b^2))/(a^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2)) - 2*
(a^3*b^2 - a*b^4)*cos(d*x^2 + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*sin(d*x^2
 + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), 1/2*((a^5 - 2*a^3*b^2 + a*b^4)*d
*x^2*sin(d*x^2 + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*x^2 + (2*a^2*b^2 - b^4 +
 (2*a^3*b - a*b^3)*sin(d*x^2 + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^
2)*(b*sin(d*x^2 + c) + a)/((a^2 - b^2)*cos(d*x^2 + c))) - (a^3*b^2 - a*b^4
)*cos(d*x^2 + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*sin(d*x^2 + c) + (a^6*b -
 2*a^4*b^3 + a^2*b^5)*d)]
```

Sympy [F]

$$\int \frac{x}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{x}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

input
```
integrate(x/(a+b*csc(d*x**2+c))**2,x)
```

output
```
Integral(x/(a + b*csc(c + d*x**2))**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.45

$$\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx$$
$$= -\frac{(2a^2b-b^3)\left(\pi\left\lfloor\frac{dx^2+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)+a}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4d-a^2b^2d)\sqrt{-a^2+b^2}} - \frac{ab\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)+b^2}{(a^3d-ab^2d)\left(b\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)^2+2a\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)+b\right)} + \frac{dx^2+c}{2a^2d}$$

input
```
integrate(x/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

output
```
-(2*a^2*b - b^3)*(pi*floor(1/2*(d*x^2 + c)/pi + 1/2)*sgn(b) + arctan((b*ta
n(1/2*d*x^2 + 1/2*c) + a)/sqrt(-a^2 + b^2)))/((a^4*d - a^2*b^2*d)*sqrt(-a^
2 + b^2)) - (a*b*tan(1/2*d*x^2 + 1/2*c) + b^2)/((a^3*d - a*b^2*d)*(b*tan(1
/2*d*x^2 + 1/2*c)^2 + 2*a*tan(1/2*d*x^2 + 1/2*c) + b)) + 1/2*(d*x^2 + c)/(
a^2*d)
```

Mupad [B] (verification not implemented)

Time = 20.60 (sec) , antiderivative size = 2755, normalized size of antiderivative = 22.96

$$\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx = \text{Too large to display}$$

input
```
int(x/(a + b/sin(c + d*x^2))^2,x)
```

output
```
- atan((8*a^3*b^3*tan(c/2 + (d*x^2)/2))/((8*a^3*b^9)/(a^6 + a^2*b^4 - 2*a^
4*b^2) - (24*a^5*b^7)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (16*a^7*b^5)/(a^6 + a^
2*b^4 - 2*a^4*b^2) + (8*a^9*b^3)/(a^6 + a^2*b^4 - 2*a^4*b^2) - (8*a^11*b)/
(a^6 + a^2*b^4 - 2*a^4*b^2)) - (8*a*b^5*tan(c/2 + (d*x^2)/2))/((8*a^3*b^9)
/(a^6 + a^2*b^4 - 2*a^4*b^2) - (24*a^5*b^7)/(a^6 + a^2*b^4 - 2*a^4*b^2) +
(16*a^7*b^5)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (8*a^9*b^3)/(a^6 + a^2*b^4 - 2*
a^4*b^2) - (8*a^11*b)/(a^6 + a^2*b^4 - 2*a^4*b^2)) + (8*a^5*b*tan(c/2 + (d
*x^2)/2))/((8*a^3*b^9)/(a^6 + a^2*b^4 - 2*a^4*b^2) - (24*a^5*b^7)/(a^6 + a
^2*b^4 - 2*a^4*b^2) + (16*a^7*b^5)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (8*a^9*b^
3)/(a^6 + a^2*b^4 - 2*a^4*b^2) - (8*a^11*b)/(a^6 + a^2*b^4 - 2*a^4*b^2)))/
(a^2*d) - (b^2/(a*(a^2 - b^2)) + (b*tan(c/2 + (d*x^2)/2))/(a^2 - b^2))/(d*
(b + b*tan(c/2 + (d*x^2)/2)^2 + 2*a*tan(c/2 + (d*x^2)/2))) - (b*atan(((b*(
2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x^2)/2)*(2*a*b^7
 - 2*a^7*b - 8*a^3*b^5 + 9*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (4*(2*a
*b^6 - 4*a^3*b^4 + 2*a^5*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (b*(2*a^2 - b
^2)*((a + b)^3*(a - b)^3)^(1/2)*((4*(4*a^8*b - 4*a^6*b^3))/(a^6 + a^2*b^4
- 2*a^4*b^2) + (8*tan(c/2 + (d*x^2)/2)*(4*a^4*b^6 - 12*a^6*b^4 + 8*a^8*b^2
))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (b*((4*(8*a^5*b^6 - 16*a^7*b^4 + 8*a^9*b^
2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (8*tan(c/2 + (d*x^2)/2)*(12*a^11*b - 8*a
^5*b^7 + 28*a^7*b^5 - 32*a^9*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(2*a^2 ...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.14

$$\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx$$

$$= \frac{4\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)\sin\left(d\,x^2+c\right)a^3b-2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)\sin\left(d\,x^2+c\right)a}{}$$

input
```
int(x/(a+b*csc(d*x^2+c))^2,x)
```

output
```
(4*sqrt( - a**2 + b**2)*atan((tan((c + d*x**2)/2)*b + a)/sqrt( - a**2 + b*
*2))*sin(c + d*x**2)*a**3*b - 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x**2
)/2)*b + a)/sqrt( - a**2 + b**2))*sin(c + d*x**2)*a*b**3 + 4*sqrt( - a**2
+ b**2)*atan((tan((c + d*x**2)/2)*b + a)/sqrt( - a**2 + b**2))*a**2*b**2 -
 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x**2)/2)*b + a)/sqrt( - a**2 + b*
*2))*b**4 - cos(c + d*x**2)*a**3*b**2 + cos(c + d*x**2)*a*b**4 + sin(c + d
*x**2)*a**5*d*x**2 - 2*sin(c + d*x**2)*a**3*b**2*d*x**2 + sin(c + d*x**2)*
a*b**4*d*x**2 + a**4*b*d*x**2 - 2*a**2*b**3*d*x**2 + b**5*d*x**2)/(2*a**2*
d*(sin(c + d*x**2)*a**5 - 2*sin(c + d*x**2)*a**3*b**2 + sin(c + d*x**2)*a*
b**4 + a**4*b - 2*a**2*b**3 + b**5))
```

3.28 $\int \frac{1}{x(a+b\csc(c+dx^2))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\,(a+b\csc(c+dx^2))^2}\,dx = \mathrm{Int}\left(\frac{1}{x\,(a+b\csc(c+dx^2))^2},x\right)$$

output
```
Defer(Int)(1/x/(a+b*csc(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 12.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\,(a+b\csc(c+dx^2))^2}\,dx = \int \frac{1}{x\,(a+b\csc(c+dx^2))^2}\,dx$$

input
```
Integrate[1/(x*(a + b*Csc[c + d*x^2])^2),x]
```

output
```
Integrate[1/(x*(a + b*Csc[c + d*x^2])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

$$\Big\downarrow \ 4695$$

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}dx$$

input
```
Int[1/(x*(a + b*Csc[c + d*x^2])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\csc\left(d\,x^{2}+c\right)\right)^{2}}dx$$

input
```
int(1/x/(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(1/x/(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x*csc(d*x^2 + c)^2 + 2*a*b*x*csc(d*x^2 + c) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

input
```
integrate(1/x/(a+b*csc(d*x**2+c))**2,x)
```

output
```
Integral(1/(x*(a + b*csc(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 5.10 (sec) , antiderivative size = 4629, normalized size of antiderivative = 257.17

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
(a^6*d*x^2*cos(2*d*x^2 + 2*c)^2*log(x) + a^6*d*x^2*log(x)*sin(2*d*x^2 + 2*
c)^2 + (a^2*b^4*cos(2*c)^2 + a^2*b^4*sin(2*c)^2)*d*x^2*cos(2*d*x^2)^2*log(
x) + 4*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(c)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)
*sin(c)^2)*d*x^2*cos(d*x^2)^2*log(x) + (a^2*b^4*cos(2*c)^2 + a^2*b^4*sin(2
*c)^2)*d*x^2*log(x)*sin(2*d*x^2)^2 + 4*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(c)
^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(c)^2)*d*x^2*log(x)*sin(d*x^2)^2 + (a^
6 - 2*a^4*b^2 + a^2*b^4)*d*x^2*log(x) + (a^2*b^4*sin(2*c) - 4*((a^3*b^3 -
a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^2*cos(d*x^
2)*log(x) + 2*(a^4*b^2 - a^2*b^4)*d*x^2*cos(2*c)*log(x) + 4*((a^3*b^3 - a*
b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*log(x)*sin
(d*x^2))*cos(2*d*x^2) - (2*a^4*b^2*d*x^2*cos(2*d*x^2)*cos(2*c)*log(x) - 2*
a^4*b^2*d*x^2*log(x)*sin(2*d*x^2)*sin(2*c) + a^3*b^3*cos(d*x^2 + c) + 4*(a
^5*b - a^3*b^3)*d*x^2*cos(c)*log(x)*sin(d*x^2) + 4*(a^5*b - a^3*b^3)*d*x^2
*cos(d*x^2)*log(x)*sin(c) + 2*(a^6 - a^4*b^2)*d*x^2*log(x))*cos(2*d*x^2 +
2*c) + (a*b^5*cos(2*d*x^2)*cos(2*c) - a*b^5*sin(2*d*x^2)*sin(2*c) + a^3*b^
3 - a*b^5 + 2*(a^2*b^4 - b^6)*cos(c)*sin(d*x^2) + 2*(a^2*b^4 - b^6)*cos(d*
x^2)*sin(c))*cos(d*x^2 + c) + 2*(2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*log(x
)*sin(c) - (a^3*b^3 - a*b^5)*cos(c))*cos(d*x^2) - (a^8*d*x^2*cos(2*d*x^2 +
 2*c)^2 + a^8*d*x^2*sin(2*d*x^2 + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*s
in(2*c)^2)*d*x^2*cos(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*co...
```

Giac [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*csc(d*x^2 + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 15.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+\frac{b}{\sin\left(d\,x^2+c\right)}\right)^2}\,dx$$

input
```
int(1/(x*(a + b/sin(c + d*x^2))^2),x)
```

output
```
int(1/(x*(a + b/sin(c + d*x^2))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\csc\left(d\,x^2+c\right)^2b^2x+2\csc\left(d\,x^2+c\right)abx+a^2x}dx$$

input
```
int(1/x/(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(1/(csc(c + d*x**2)**2*b**2*x + 2*csc(c + d*x**2)*a*b*x + a**2*x),x)
```

3.29 $\int \frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*csc(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 9.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

input
```
Integrate[1/(x^2*(a + b*Csc[c + d*x^2])^2),x]
```

output
```
Integrate[1/(x^2*(a + b*Csc[c + d*x^2])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+b\csc(c+dx^2))^2} \, dx$$
$$\Big\downarrow \text{4695}$$
$$\int \frac{1}{x^2 (a+b\csc(c+dx^2))^2} dx$$

input
```
Int[1/(x^2*(a + b*Csc[c + d*x^2])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a+b\csc(d\,x^2+c))^2} dx$$

input
```
int(1/x^2/(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(1/x^2/(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^2*csc(d*x^2 + c)^2 + 2*a*b*x^2*csc(d*x^2 + c) + a^2*x^2)
, x)
```

Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

input
```
integrate(1/x**2/(a+b*csc(d*x**2+c))**2,x)
```

output
```
Integral(1/(x**2*(a + b*csc(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 5.20 (sec) , antiderivative size = 4560, normalized size of antiderivative = 253.33

$$\int \frac{1}{x^2 (a+b\csc(c+dx^2))^2} \, dx = \int \frac{1}{(b\csc(dx^2+c)+a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
-((a^6 - a^4*b^2)*d*x^2*cos(2*d*x^2 + 2*c)^2 + (a^6 - a^4*b^2)*d*x^2*sin(2
*d*x^2 + 2*c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*d*x^2 - (a^2*b^4*sin(2*c) -
(a^4*b^2 - a^2*b^4)*d*x^2*cos(2*c))*cos(2*d*x^2) + (a^3*b^3*cos(d*x^2 + c)
 - (a^4*b^2 - a^2*b^4)*d*x^2*cos(2*d*x^2)*cos(2*c) - 2*(a^5*b - 2*a^3*b^3
+ a*b^5)*d*x^2*cos(c)*sin(d*x^2) + (a^4*b^2 - a^2*b^4)*d*x^2*sin(2*d*x^2)*
sin(2*c) - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(d*x^2)*sin(c) - 2*(a^5*
b - a^3*b^3)*d*x^2*sin(d*x^2 + c) - (2*a^6 - 3*a^4*b^2 + a^2*b^4)*d*x^2)*c
os(2*d*x^2 + 2*c) - (a^3*b^3 - a*b^5 + (a*b^5*cos(2*c) + 2*(a^3*b^3 - a*b^
5)*d*x^2*sin(2*c))*cos(2*d*x^2) - 2*(2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2*c
os(c) - (a^2*b^4 - b^6)*sin(c))*cos(d*x^2) - (a*b^5*sin(2*c) - 2*(a^3*b^3
- a*b^5)*d*x^2*cos(2*c))*sin(2*d*x^2) + 2*(2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d
*x^2*sin(c) + (a^2*b^4 - b^6)*cos(c))*sin(d*x^2))*cos(d*x^2 + c) + 2*((a^5
*b - 2*a^3*b^3 + a*b^5)*d*x^2*sin(c) + (a^3*b^3 - a*b^5)*cos(c))*cos(d*x^2
) + (a^8*d*x^3*cos(2*d*x^2 + 2*c)^2 + a^8*d*x^3*sin(2*d*x^2 + 2*c)^2 + (a^
4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^3*cos(2*d*x^2)^2 + 4*((a^6*b^2
- 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2
)*d*x^3*cos(d*x^2)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^3*sin
(2*d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*cos(c)*sin(d*x^2) + 4*
((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6
)*sin(c)^2)*d*x^3*sin(d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*...
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b \csc(c + dx^2))^2} \, dx = \int \frac{1}{(b \csc(dx^2 + c) + a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*csc(d*x^2 + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 15.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^2 (a + b \csc(c + dx^2))^2} \, dx = \int \frac{1}{x^2 \left(a + \frac{b}{\sin(d x^2 + c)}\right)^2} \, dx$$

input
```
int(1/(x^2*(a + b/sin(c + d*x^2))^2),x)
```

output
```
int(1/(x^2*(a + b/sin(c + d*x^2))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.06

$$\int \frac{1}{x^2 (a + b \csc(c + dx^2))^2} \, dx = \int \frac{\sin(d x^2 + c)^2}{\sin(d x^2 + c)^2 a^2 x^2 + 2 \sin(d x^2 + c) a b x^2 + b^2 x^2} dx$$

input
```
int(1/x^2/(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(sin(c + d*x**2)**2/(sin(c + d*x**2)**2*a**2*x**2 + 2*sin(c + d*x**2)*a
*b*x**2 + b**2*x**2),x)
```

3.30 $\int \frac{1}{x^3\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^3\left(a+b\csc\left(c+dx^2\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/x^3/(a+b*csc(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 10.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

input
```
Integrate[1/(x^3*(a + b*Csc[c + d*x^2])^2),x]
```

output
```
Integrate[1/(x^3*(a + b*Csc[c + d*x^2])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+b \csc (c+dx^2))^2} \, dx$$
$$\Big\downarrow \text{4695}$$
$$\int \frac{1}{x^3 (a+b \csc (c+dx^2))^2}dx$$

input
```
Int[1/(x^3*(a + b*Csc[c + d*x^2])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a+b \csc (d\,x^2+c))^2}dx$$

input
```
int(1/x^3/(a+b*csc(d*x^2+c))^2,x)
```

output
```
int(1/x^3/(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3 \left(a+b\csc\left(c+dx^2\right)\right)^2}\, dx = \int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)^2 x^3}\, dx$$

input
```
integrate(1/x^3/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^3*csc(d*x^2 + c)^2 + 2*a*b*x^3*csc(d*x^2 + c) + a^2*x^3)
, x)
```

Sympy [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 \left(a+b\csc\left(c+dx^2\right)\right)^2}\, dx = \int \frac{1}{x^3 \left(a+b\csc\left(c+dx^2\right)\right)^2}\, dx$$

input
```
integrate(1/x**3/(a+b*csc(d*x**2+c))**2,x)
```

output
```
Integral(1/(x**3*(a + b*csc(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 5.21 (sec) , antiderivative size = 3530, normalized size of antiderivative = 196.11

$$\int \frac{1}{x^3 (a + b \csc (c + dx^2))^2} \, dx = \int \frac{1}{(b \csc (dx^2 + c) + a)^2 x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

output
```
-1/2*((a^4 - a^2*b^2)*d*x^2 - ((a^4 - a^2*b^2)*d*x^2*cos(2*c) - 2*a^2*b^2*
sin(2*c))*cos(2*d*x^2) + ((a^4 - a^2*b^2)*d*x^2*cos(2*d*x^2)*cos(2*c) - 2*
(a^3*b - a*b^3)*d*x^2*cos(c)*sin(d*x^2) - (a^4 - a^2*b^2)*d*x^2*sin(2*d*x^
2)*sin(2*c) - 2*(a^3*b - a*b^3)*d*x^2*cos(d*x^2)*sin(c) - (a^4 - a^2*b^2)*
d*x^2)*cos(2*d*x^2 + 2*c) - 2*(a*b^3 - (a*b^3*cos(2*c) + (a^3*b - a*b^3)*d
*x^2*sin(2*c))*cos(2*d*x^2) - 2*((a^2*b^2 - b^4)*d*x^2*cos(c) - b^4*sin(c)
)*cos(d*x^2) - ((a^3*b - a*b^3)*d*x^2*cos(2*c) - a*b^3*sin(2*c))*sin(2*d*x
^2) + 2*(b^4*cos(c) + (a^2*b^2 - b^4)*d*x^2*sin(c))*sin(d*x^2))*cos(d*x^2
+ c) + 2*(2*a*b^3*cos(c) + (a^3*b - a*b^3)*d*x^2*sin(c))*cos(d*x^2) + 2*((
(a^6 - a^4*b^2)*cos(2*c)^2 + (a^6 - a^4*b^2)*sin(2*c)^2)*d*x^4*cos(2*d*x^2
)^2 + 4*((a^4*b^2 - a^2*b^4)*cos(c)^2 + (a^4*b^2 - a^2*b^4)*sin(c)^2)*d*x^
4*cos(d*x^2)^2 + ((a^6 - a^4*b^2)*cos(2*c)^2 + (a^6 - a^4*b^2)*sin(2*c)^2)
*d*x^4*sin(2*d*x^2)^2 + 4*(a^5*b - a^3*b^3)*d*x^4*cos(c)*sin(d*x^2) + 4*((
a^4*b^2 - a^2*b^4)*cos(c)^2 + (a^4*b^2 - a^2*b^4)*sin(c)^2)*d*x^4*sin(d*x^
2)^2 + 4*(a^5*b - a^3*b^3)*d*x^4*cos(d*x^2)*sin(c) + (a^6 - a^4*b^2)*d*x^4
 + 2*(2*((a^5*b - a^3*b^3)*cos(c)*sin(2*c) - (a^5*b - a^3*b^3)*cos(2*c)*si
n(c))*d*x^4*cos(d*x^2) - (a^6 - a^4*b^2)*d*x^4*cos(2*c) - 2*((a^5*b - a^3*
b^3)*cos(2*c)*cos(c) + (a^5*b - a^3*b^3)*sin(2*c)*sin(c))*d*x^4*sin(d*x^2)
)*cos(2*d*x^2) + 2*(2*((a^5*b - a^3*b^3)*cos(2*c)*cos(c) + (a^5*b - a^3*b^
3)*sin(2*c)*sin(c))*d*x^4*cos(d*x^2) + 2*((a^5*b - a^3*b^3)*cos(c)*sin(...
```

Giac [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a+b\csc(c+dx^2))^2}\,dx = \int \frac{1}{(b\csc(dx^2+c)+a)^2 x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*csc(d*x^2 + c) + a)^2*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 15.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3 (a+b\csc(c+dx^2))^2}\,dx = \int \frac{1}{x^3 \left(a+\frac{b}{\sin(d\,x^2+c)}\right)^2}\,dx$$

input
```
int(1/(x^3*(a + b/sin(c + d*x^2))^2),x)
```

output
```
int(1/(x^3*(a + b/sin(c + d*x^2))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.06

$$\int \frac{1}{x^3 (a+b\csc(c+dx^2))^2}\,dx = \int \frac{\sin(d\,x^2+c)^2}{\sin(d\,x^2+c)^2 a^2x^3+2\sin(d\,x^2+c)\,ab\,x^3+b^2x^3}dx$$

input
```
int(1/x^3/(a+b*csc(d*x^2+c))^2,x)
```

output

```
int(sin(c + d*x**2)**2/(sin(c + d*x**2)**2*a**2*x**3 + 2*sin(c + d*x**2)*a
*b*x**3 + b**2*x**3),x)
```

3.31 $\int x^3 \left(a+b\csc\left(c+d\sqrt{x}\right)\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 432

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^4}{4}-\frac{4bx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{14ibx^3\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{14ibx^3\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{84bx^{5/2}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}-\frac{420ibx^2\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{420ibx^2\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5}+\frac{5040ibx\operatorname{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{5040ibx\operatorname{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,-e^{i(c+d\sqrt{x})}\right)}{d^7}+\frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,e^{i(c+d\sqrt{x})}\right)}{d^7}-\frac{10080ib\operatorname{PolyLog}\left(8,-e^{i(c+d\sqrt{x})}\right)}{d^8}+\frac{10080ib\operatorname{PolyLog}\left(8,e^{i(c+d\sqrt{x})}\right)}{d^8}$$

output
```
1/4*a*x^4-4*b*x^(7/2)*arctanh(exp(I*(c+d*x^(1/2))))/d+5040*I*b*x*polylog(6
,-exp(I*(c+d*x^(1/2))))/d^6+420*I*b*x^2*polylog(4,exp(I*(c+d*x^(1/2))))/d^
4-84*b*x^(5/2)*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3+84*b*x^(5/2)*polylog(3
,exp(I*(c+d*x^(1/2))))/d^3-5040*I*b*x*polylog(6,exp(I*(c+d*x^(1/2))))/d^6-
10080*I*b*polylog(8,-exp(I*(c+d*x^(1/2))))/d^8+1680*b*x^(3/2)*polylog(5,-e
xp(I*(c+d*x^(1/2))))/d^5-1680*b*x^(3/2)*polylog(5,exp(I*(c+d*x^(1/2))))/d^
5+14*I*b*x^3*polylog(2,-exp(I*(c+d*x^(1/2))))/d^2-14*I*b*x^3*polylog(2,exp
(I*(c+d*x^(1/2))))/d^2-10080*b*x^(1/2)*polylog(7,-exp(I*(c+d*x^(1/2))))/d^
7+10080*b*x^(1/2)*polylog(7,exp(I*(c+d*x^(1/2))))/d^7-420*I*b*x^2*polylog(
4,-exp(I*(c+d*x^(1/2))))/d^4+10080*I*b*polylog(8,exp(I*(c+d*x^(1/2))))/d^8
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.03

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^4}{4}$$
$$+\frac{2b\left(d^7x^{7/2}\log\left(1-e^{i(c+d\sqrt{x})}\right)-d^7x^{7/2}\log\left(1+e^{i(c+d\sqrt{x})}\right)+7id^6x^3\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)-7id^6x^3}{}$$

input
```
Integrate[x^3*(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(a*x^4)/4 + (2*b*(d^7*x^(7/2)*Log[1 - E^(I*(c + d*Sqrt[x]))] - d^7*x^(7/2)
*Log[1 + E^(I*(c + d*Sqrt[x]))] + (7*I)*d^6*x^3*PolyLog[2, -E^(I*(c + d*Sq
rt[x]))] - (7*I)*d^6*x^3*PolyLog[2, E^(I*(c + d*Sqrt[x]))] - 42*d^5*x^(5/2
)*PolyLog[3, -E^(I*(c + d*Sqrt[x]))] + 42*d^5*x^(5/2)*PolyLog[3, E^(I*(c +
 d*Sqrt[x]))] - (210*I)*d^4*x^2*PolyLog[4, -E^(I*(c + d*Sqrt[x]))] + (210*
I)*d^4*x^2*PolyLog[4, E^(I*(c + d*Sqrt[x]))] + 840*d^3*x^(3/2)*PolyLog[5,
-E^(I*(c + d*Sqrt[x]))] - 840*d^3*x^(3/2)*PolyLog[5, E^(I*(c + d*Sqrt[x]))
] + (2520*I)*d^2*x*PolyLog[6, -E^(I*(c + d*Sqrt[x]))] - (2520*I)*d^2*x*Pol
yLog[6, E^(I*(c + d*Sqrt[x]))] - 5040*d*Sqrt[x]*PolyLog[7, -E^(I*(c + d*Sq
rt[x]))] + 5040*d*Sqrt[x]*PolyLog[7, E^(I*(c + d*Sqrt[x]))] - (5040*I)*Pol
yLog[8, -E^(I*(c + d*Sqrt[x]))] + (5040*I)*PolyLog[8, E^(I*(c + d*Sqrt[x])
)]))/d^8
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b \csc \left(c+d \sqrt{x}\right)\right) \, dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(a x^3+b x^3 \csc \left(c+d \sqrt{x}\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a x^4}{4}-\frac{4 b x^{7/2} \text{arctanh}\left(e^{i \left(c+d \sqrt{x}\right)}\right)}{d}-\frac{10080 i b \operatorname{PolyLog}\left(8,-e^{i \left(c+d \sqrt{x}\right)}\right)}{d^8}+\frac{10080 i b \operatorname{PolyLog}\left(8,e^{i \left(c+d \sqrt{x}\right)}\right)}{d^8}-\frac{10080 b \sqrt{x} \operatorname{PolyLog}\left(7,-e^{i \left(c+d \sqrt{x}\right)}\right)}{d^7}+\frac{10080 b \sqrt{x} \operatorname{PolyLog}\left(7,e^{i \left(c+d \sqrt{x}\right)}\right)}{d^7}+\frac{5040 i b x \operatorname{PolyLog}\left(6,-e^{i \left(c+d \sqrt{x}\right)}\right)}{d^6}-\frac{5040 i b x \operatorname{PolyLog}\left(6,e^{i \left(c+d \sqrt{x}\right)}\right)}{d^6}+\frac{1680 b x^{3/2} \operatorname{PolyLog}\left(5,-e^{i \left(c+d \sqrt{x}\right)}\right)}{d^5}-\frac{1680 b x^{3/2} \operatorname{PolyLog}\left(5,e^{i \left(c+d \sqrt{x}\right)}\right)}{d^5}-\frac{420 i b x^2 \operatorname{PolyLog}\left(4,-e^{i \left(c+d \sqrt{x}\right)}\right)}{d^4}+\frac{420 i b x^2 \operatorname{PolyLog}\left(4,e^{i \left(c+d \sqrt{x}\right)}\right)}{d^4}-\frac{84 b x^{5/2} \operatorname{PolyLog}\left(3,-e^{i \left(c+d \sqrt{x}\right)}\right)}{d^3}+\frac{84 b x^{5/2} \operatorname{PolyLog}\left(3,e^{i \left(c+d \sqrt{x}\right)}\right)}{d^3}+\frac{14 i b x^3 \operatorname{PolyLog}\left(2,-e^{i \left(c+d \sqrt{x}\right)}\right)}{d^2}-\frac{14 i b x^3 \operatorname{PolyLog}\left(2,e^{i \left(c+d \sqrt{x}\right)}\right)}{d^2}$$

input
```
Int[x^3*(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(a*x^4)/4 - (4*b*x^(7/2)*ArcTanh[E^(I*(c + d*Sqrt[x]))])/d + ((14*I)*b*x^3
*PolyLog[2, -E^(I*(c + d*Sqrt[x]))])/d^2 - ((14*I)*b*x^3*PolyLog[2, E^(I*(
c + d*Sqrt[x]))])/d^2 - (84*b*x^(5/2)*PolyLog[3, -E^(I*(c + d*Sqrt[x]))])/
d^3 + (84*b*x^(5/2)*PolyLog[3, E^(I*(c + d*Sqrt[x]))])/d^3 - ((420*I)*b*x^
2*PolyLog[4, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((420*I)*b*x^2*PolyLog[4, E^(I
*(c + d*Sqrt[x]))])/d^4 + (1680*b*x^(3/2)*PolyLog[5, -E^(I*(c + d*Sqrt[x])
)])/d^5 - (1680*b*x^(3/2)*PolyLog[5, E^(I*(c + d*Sqrt[x]))])/d^5 + ((5040*
I)*b*x*PolyLog[6, -E^(I*(c + d*Sqrt[x]))])/d^6 - ((5040*I)*b*x*PolyLog[6,
E^(I*(c + d*Sqrt[x]))])/d^6 - (10080*b*Sqrt[x]*PolyLog[7, -E^(I*(c + d*Sqr
t[x]))])/d^7 + (10080*b*Sqrt[x]*PolyLog[7, E^(I*(c + d*Sqrt[x]))])/d^7 - (
(10080*I)*b*PolyLog[8, -E^(I*(c + d*Sqrt[x]))])/d^8 + ((10080*I)*b*PolyLog
[8, E^(I*(c + d*Sqrt[x]))])/d^8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)dx$$

input
```
int(x^3*(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(x^3*(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^3 \left(a + b\csc\left(c + d\sqrt{x}\right)\right)\, dx = \int \left(b\csc\left(d\sqrt{x} + c\right) + a\right)x^3\, dx$$

input
```
integrate(x^3*(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
integral(b*x^3*csc(d*sqrt(x) + c) + a*x^3, x)
```

Sympy [F]

$$\int x^3 \left(a + b\csc\left(c + d\sqrt{x}\right)\right)\, dx = \int x^3 \left(a + b\csc\left(c + d\sqrt{x}\right)\right)\, dx$$

input
```
integrate(x**3*(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(x**3*(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1498 vs. $2(338) = 676$.

Time = 0.18 (sec) , antiderivative size = 1498, normalized size of antiderivative = 3.47

$$\int x^3 \left(a + b\csc\left(c + d\sqrt{x}\right)\right)\, dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
1/4*((d*sqrt(x) + c)^8*a - 8*(d*sqrt(x) + c)^7*a*c + 28*(d*sqrt(x) + c)^6*
a*c^2 - 56*(d*sqrt(x) + c)^5*a*c^3 + 70*(d*sqrt(x) + c)^4*a*c^4 - 56*(d*sq
rt(x) + c)^3*a*c^5 + 28*(d*sqrt(x) + c)^2*a*c^6 - 8*(d*sqrt(x) + c)*a*c^7
+ 8*b*c^7*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c)) + 8*(-I*(d*sqrt(x)
+ c)^7*b + 7*I*(d*sqrt(x) + c)^6*b*c - 21*I*(d*sqrt(x) + c)^5*b*c^2 + 35*I
*(d*sqrt(x) + c)^4*b*c^3 - 35*I*(d*sqrt(x) + c)^3*b*c^4 + 21*I*(d*sqrt(x)
+ c)^2*b*c^5 - 7*I*(d*sqrt(x) + c)*b*c^6)*arctan2(sin(d*sqrt(x) + c), cos(
d*sqrt(x) + c) + 1) + 8*(-I*(d*sqrt(x) + c)^7*b + 7*I*(d*sqrt(x) + c)^6*b*
c - 21*I*(d*sqrt(x) + c)^5*b*c^2 + 35*I*(d*sqrt(x) + c)^4*b*c^3 - 35*I*(d*
sqrt(x) + c)^3*b*c^4 + 21*I*(d*sqrt(x) + c)^2*b*c^5 - 7*I*(d*sqrt(x) + c)*
b*c^6)*arctan2(sin(d*sqrt(x) + c), -cos(d*sqrt(x) + c) + 1) + 56*(I*(d*sqr
t(x) + c)^6*b - 6*I*(d*sqrt(x) + c)^5*b*c + 15*I*(d*sqrt(x) + c)^4*b*c^2 -
 20*I*(d*sqrt(x) + c)^3*b*c^3 + 15*I*(d*sqrt(x) + c)^2*b*c^4 - 6*I*(d*sqrt
(x) + c)*b*c^5 + I*b*c^6)*dilog(-e^(I*d*sqrt(x) + I*c)) + 56*(-I*(d*sqrt(x
) + c)^6*b + 6*I*(d*sqrt(x) + c)^5*b*c - 15*I*(d*sqrt(x) + c)^4*b*c^2 + 20
*I*(d*sqrt(x) + c)^3*b*c^3 - 15*I*(d*sqrt(x) + c)^2*b*c^4 + 6*I*(d*sqrt(x)
 + c)*b*c^5 - I*b*c^6)*dilog(e^(I*d*sqrt(x) + I*c)) - 4*((d*sqrt(x) + c)^7
*b - 7*(d*sqrt(x) + c)^6*b*c + 21*(d*sqrt(x) + c)^5*b*c^2 - 35*(d*sqrt(x)
+ c)^4*b*c^3 + 35*(d*sqrt(x) + c)^3*b*c^4 - 21*(d*sqrt(x) + c)^2*b*c^5 + 7
*(d*sqrt(x) + c)*b*c^6)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2...
```

Giac [F]

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^3\,dx$$

input
```
integrate(x^3*(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int x^3\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)\,dx$$

input
```
int(x^3*(a + b/sin(c + d*x^(1/2))),x)
```

output
```
int(x^3*(a + b/sin(c + d*x^(1/2))), x)
```

Reduce [F]

$$\int x^3 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\left(\int \csc\left(\sqrt{x}\,d+c\right)x^3dx\right)b+\frac{a\,x^4}{4}$$

input
```
int(x^3*(a+b*csc(c+d*x^(1/2))),x)
```

output
```
(4*int(csc(sqrt(x)*d + c)*x**3,x)*b + a*x**4)/4
```

3.32 $\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 316

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^3}{3}-\frac{4bx^{5/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{10ibx^2\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{10ibx^2\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{40bx^{3/2}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}-\frac{120ibx\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{120ibx\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5}+\frac{240ib\operatorname{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{240ib\operatorname{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6}$$

output
```
1/3*a*x^3-4*b*x^(5/2)*arctanh(exp(I*(c+d*x^(1/2))))/d+10*I*b*x^2*polylog(2
,-exp(I*(c+d*x^(1/2))))/d^2-10*I*b*x^2*polylog(2,exp(I*(c+d*x^(1/2))))/d^2
-40*b*x^(3/2)*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3+40*b*x^(3/2)*polylog(3,
exp(I*(c+d*x^(1/2))))/d^3-120*I*b*x*polylog(4,-exp(I*(c+d*x^(1/2))))/d^4+1
20*I*b*x*polylog(4,exp(I*(c+d*x^(1/2))))/d^4+240*b*x^(1/2)*polylog(5,-exp(
I*(c+d*x^(1/2))))/d^5-240*b*x^(1/2)*polylog(5,exp(I*(c+d*x^(1/2))))/d^5+24
0*I*b*polylog(6,-exp(I*(c+d*x^(1/2))))/d^6-240*I*b*polylog(6,exp(I*(c+d*x^
(1/2))))/d^6
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.05

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^3}{3}+\frac{2b\left(d^5x^{5/2}\log\left(1-e^{i(c+d\sqrt{x})}\right)-d^5x^{5/2}\log\left(1+e^{i(c+d\sqrt{x})}\right)+5id^4x^2\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)-5id^4x^2\right.}{}$$

input
```
Integrate[x^2*(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(a*x^3)/3 + (2*b*(d^5*x^(5/2)*Log[1 - E^(I*(c + d*Sqrt[x]))] - d^5*x^(5/2)
*Log[1 + E^(I*(c + d*Sqrt[x]))] + (5*I)*d^4*x^2*PolyLog[2, -E^(I*(c + d*Sq
rt[x]))] - (5*I)*d^4*x^2*PolyLog[2, E^(I*(c + d*Sqrt[x]))] - 20*d^3*x^(3/2
)*PolyLog[3, -E^(I*(c + d*Sqrt[x]))] + 20*d^3*x^(3/2)*PolyLog[3, E^(I*(c +
 d*Sqrt[x]))] - (60*I)*d^2*x*PolyLog[4, -E^(I*(c + d*Sqrt[x]))] + (60*I)*d
^2*x*PolyLog[4, E^(I*(c + d*Sqrt[x]))] + 120*d*Sqrt[x]*PolyLog[5, -E^(I*(c
 + d*Sqrt[x]))] - 120*d*Sqrt[x]*PolyLog[5, E^(I*(c + d*Sqrt[x]))] + (120*I
)*PolyLog[6, -E^(I*(c + d*Sqrt[x]))] - (120*I)*PolyLog[6, E^(I*(c + d*Sqrt
[x]))]))/d^6
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(ax^2+bx^2\csc\left(c+d\sqrt{x}\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^3}{3} - \frac{4bx^{5/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{240ib\,\text{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{240ib\,\text{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6} + \frac{240b\sqrt{x}\,\text{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{240b\sqrt{x}\,\text{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{120ibx\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{120ibx\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} - \frac{40bx^{3/2}\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{40bx^{3/2}\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{10ibx^2\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{10ibx^2\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}$$

input
```
Int[x^2*(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(a*x^3)/3 - (4*b*x^(5/2)*ArcTanh[E^(I*(c + d*Sqrt[x]))])/d + ((10*I)*b*x^2
*PolyLog[2, -E^(I*(c + d*Sqrt[x]))])/d^2 - ((10*I)*b*x^2*PolyLog[2, E^(I*(
c + d*Sqrt[x]))])/d^2 - (40*b*x^(3/2)*PolyLog[3, -E^(I*(c + d*Sqrt[x]))])/
d^3 + (40*b*x^(3/2)*PolyLog[3, E^(I*(c + d*Sqrt[x]))])/d^3 - ((120*I)*b*x*
PolyLog[4, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((120*I)*b*x*PolyLog[4, E^(I*(c
+ d*Sqrt[x]))])/d^4 + (240*b*Sqrt[x]*PolyLog[5, -E^(I*(c + d*Sqrt[x]))])/d
^5 - (240*b*Sqrt[x]*PolyLog[5, E^(I*(c + d*Sqrt[x]))])/d^5 + ((240*I)*b*Po
lyLog[6, -E^(I*(c + d*Sqrt[x]))])/d^6 - ((240*I)*b*PolyLog[6, E^(I*(c + d*
Sqrt[x]))])/d^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right) dx$$

input
```
int(x^2*(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(x^2*(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)\, dx = \int \left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^2\, dx$$

input
```
integrate(x^2*(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
integral(b*x^2*csc(d*sqrt(x) + c) + a*x^2, x)
```

Sympy [F]

$$\int x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)\, dx = \int x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)\, dx$$

input
```
integrate(x**2*(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(x**2*(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 956 vs. $2(246) = 492$.

Time = 0.16 (sec) , antiderivative size = 956, normalized size of antiderivative = 3.03

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^2*(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
1/3*((d*sqrt(x) + c)^6*a - 6*(d*sqrt(x) + c)^5*a*c + 15*(d*sqrt(x) + c)^4*
a*c^2 - 20*(d*sqrt(x) + c)^3*a*c^3 + 15*(d*sqrt(x) + c)^2*a*c^4 - 6*(d*sqr
t(x) + c)*a*c^5 + 6*b*c^5*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c)) + 6
*(-I*(d*sqrt(x) + c)^5*b + 5*I*(d*sqrt(x) + c)^4*b*c - 10*I*(d*sqrt(x) + c
)^3*b*c^2 + 10*I*(d*sqrt(x) + c)^2*b*c^3 - 5*I*(d*sqrt(x) + c)*b*c^4)*arct
an2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) + 1) + 6*(-I*(d*sqrt(x) + c)^5*
b + 5*I*(d*sqrt(x) + c)^4*b*c - 10*I*(d*sqrt(x) + c)^3*b*c^2 + 10*I*(d*sqr
t(x) + c)^2*b*c^3 - 5*I*(d*sqrt(x) + c)*b*c^4)*arctan2(sin(d*sqrt(x) + c),
 -cos(d*sqrt(x) + c) + 1) + 30*(I*(d*sqrt(x) + c)^4*b - 4*I*(d*sqrt(x) + c
)^3*b*c + 6*I*(d*sqrt(x) + c)^2*b*c^2 - 4*I*(d*sqrt(x) + c)*b*c^3 + I*b*c^
4)*dilog(-e^(I*d*sqrt(x) + I*c)) + 30*(-I*(d*sqrt(x) + c)^4*b + 4*I*(d*sqr
t(x) + c)^3*b*c - 6*I*(d*sqrt(x) + c)^2*b*c^2 + 4*I*(d*sqrt(x) + c)*b*c^3
- I*b*c^4)*dilog(e^(I*d*sqrt(x) + I*c)) - 3*((d*sqrt(x) + c)^5*b - 5*(d*sq
rt(x) + c)^4*b*c + 10*(d*sqrt(x) + c)^3*b*c^2 - 10*(d*sqrt(x) + c)^2*b*c^3
 + 5*(d*sqrt(x) + c)*b*c^4)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^
2 + 2*cos(d*sqrt(x) + c) + 1) + 3*((d*sqrt(x) + c)^5*b - 5*(d*sqrt(x) + c)
^4*b*c + 10*(d*sqrt(x) + c)^3*b*c^2 - 10*(d*sqrt(x) + c)^2*b*c^3 + 5*(d*sq
rt(x) + c)*b*c^4)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(
d*sqrt(x) + c) + 1) + 720*I*b*polylog(6, -e^(I*d*sqrt(x) + I*c)) - 720*I*b
*polylog(6, e^(I*d*sqrt(x) + I*c)) + 720*((d*sqrt(x) + c)*b - b*c)*poly...
```

Giac [F]

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^2\,dx$$

input
```
integrate(x^2*(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int x^2\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)\,dx$$

input
```
int(x^2*(a + b/sin(c + d*x^(1/2))),x)
```

output
```
int(x^2*(a + b/sin(c + d*x^(1/2))), x)
```

Reduce [F]

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\left(\int\csc\left(\sqrt{x}\,d+c\right)x^2dx\right)b+\frac{a\,x^3}{3}$$

input
```
int(x^2*(a+b*csc(c+d*x^(1/2))),x)
```

output
```
(3*int(csc(sqrt(x)*d + c)*x**2,x)*b + a*x**3)/3
```

3.33 $\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 200

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^2}{2}-\frac{4bx^{3/2}\operatorname{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}+\frac{6ibx\operatorname{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{6ibx\operatorname{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}-\frac{12ib\operatorname{PolyLog}\left(4,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{12ib\operatorname{PolyLog}\left(4,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}$$

output
```
1/2*a*x^2-4*b*x^(3/2)*arctanh(exp(I*(c+d*x^(1/2))))/d+6*I*b*x*polylog(2,-e
xp(I*(c+d*x^(1/2))))/d^2-6*I*b*x*polylog(2,exp(I*(c+d*x^(1/2))))/d^2-12*b*
x^(1/2)*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3+12*b*x^(1/2)*polylog(3,exp(I*
(c+d*x^(1/2))))/d^3-12*I*b*polylog(4,-exp(I*(c+d*x^(1/2))))/d^4+12*I*b*pol
ylog(4,exp(I*(c+d*x^(1/2))))/d^4
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.30

$$\int x(a+b\csc(c+d\sqrt{x}))\,dx=\frac{ax^2}{2}$$
$$-\frac{2b(2d^3x^{3/2}\text{arctanh}(\cos(c+d\sqrt{x})+i\sin(c+d\sqrt{x}))-3id^2x\,\text{PolyLog}(2,-\cos(c+d\sqrt{x})-i\sin(c$$

input
```
Integrate[x*(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(a*x^2)/2 - (2*b*(2*d^3*x^(3/2)*ArcTanh[Cos[c + d*Sqrt[x]] + I*Sin[c + d*S
qrt[x]]] - (3*I)*d^2*x*PolyLog[2, -Cos[c + d*Sqrt[x]] - I*Sin[c + d*Sqrt[x
]]] + (3*I)*d^2*x*PolyLog[2, Cos[c + d*Sqrt[x]] + I*Sin[c + d*Sqrt[x]]] +
6*d*Sqrt[x]*PolyLog[3, -Cos[c + d*Sqrt[x]] - I*Sin[c + d*Sqrt[x]]] - 6*d*S
qrt[x]*PolyLog[3, Cos[c + d*Sqrt[x]] + I*Sin[c + d*Sqrt[x]]] + (6*I)*PolyL
og[4, -Cos[c + d*Sqrt[x]] - I*Sin[c + d*Sqrt[x]]] - (6*I)*PolyLog[4, Cos[c
 + d*Sqrt[x]] + I*Sin[c + d*Sqrt[x]]]))/d^4
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\csc(c+d\sqrt{x}))\,dx$$

$$\Big\downarrow \; 2010$$

$$\int \left(ax + bx\csc\left(c + d\sqrt{x}\right)\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{ax^2}{2} - \frac{4bx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{12ib\,\text{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{12ib\,\text{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{6ibx\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ibx\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2}$$

input
```
Int[x*(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(a*x^2)/2 - (4*b*x^(3/2)*ArcTanh[E^(I*(c + d*Sqrt[x]))])/d + ((6*I)*b*x*Po
lyLog[2, -E^(I*(c + d*Sqrt[x]))])/d^2 - ((6*I)*b*x*PolyLog[2, E^(I*(c + d*
Sqrt[x]))])/d^2 - (12*b*Sqrt[x]*PolyLog[3, -E^(I*(c + d*Sqrt[x]))])/d^3 +
(12*b*Sqrt[x]*PolyLog[3, E^(I*(c + d*Sqrt[x]))])/d^3 - ((12*I)*b*PolyLog[4
, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((12*I)*b*PolyLog[4, E^(I*(c + d*Sqrt[x])
)])/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)dx$$

input
```
int(x*(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(x*(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x\,dx$$

input
```
integrate(x*(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
integral(b*x*csc(d*sqrt(x) + c) + a*x, x)
```

Sympy [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$$

input
```
integrate(x*(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(x*(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(154) = 308$.

Time = 0.11 (sec) , antiderivative size = 534, normalized size of antiderivative = 2.67

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
1/2*((d*sqrt(x) + c)^4*a - 4*(d*sqrt(x) + c)^3*a*c + 6*(d*sqrt(x) + c)^2*a
*c^2 - 4*(d*sqrt(x) + c)*a*c^3 + 4*b*c^3*log(cot(d*sqrt(x) + c) + csc(d*sq
rt(x) + c)) + 4*(-I*(d*sqrt(x) + c)^3*b + 3*I*(d*sqrt(x) + c)^2*b*c - 3*I*
(d*sqrt(x) + c)*b*c^2)*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) + 1)
 + 4*(-I*(d*sqrt(x) + c)^3*b + 3*I*(d*sqrt(x) + c)^2*b*c - 3*I*(d*sqrt(x)
+ c)*b*c^2)*arctan2(sin(d*sqrt(x) + c), -cos(d*sqrt(x) + c) + 1) + 12*(I*(
d*sqrt(x) + c)^2*b - 2*I*(d*sqrt(x) + c)*b*c + I*b*c^2)*dilog(-e^(I*d*sqrt
(x) + I*c)) + 12*(-I*(d*sqrt(x) + c)^2*b + 2*I*(d*sqrt(x) + c)*b*c - I*b*c
^2)*dilog(e^(I*d*sqrt(x) + I*c)) - 2*((d*sqrt(x) + c)^3*b - 3*(d*sqrt(x) +
 c)^2*b*c + 3*(d*sqrt(x) + c)*b*c^2)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt
(x) + c)^2 + 2*cos(d*sqrt(x) + c) + 1) + 2*((d*sqrt(x) + c)^3*b - 3*(d*sqr
t(x) + c)^2*b*c + 3*(d*sqrt(x) + c)*b*c^2)*log(cos(d*sqrt(x) + c)^2 + sin(
d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1) - 24*I*b*polylog(4, -e^(I*d*s
qrt(x) + I*c)) + 24*I*b*polylog(4, e^(I*d*sqrt(x) + I*c)) - 24*((d*sqrt(x)
 + c)*b - b*c)*polylog(3, -e^(I*d*sqrt(x) + I*c)) + 24*((d*sqrt(x) + c)*b
- b*c)*polylog(3, e^(I*d*sqrt(x) + I*c)))/d^4
```

Giac [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx = \int \left(b\csc\left(d\sqrt{x}+c\right)+a\right)x\,dx$$

input
```
integrate(x*(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int x\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)\,dx$$

input
```
int(x*(a + b/sin(c + d*x^(1/2))),x)
```

output
```
int(x*(a + b/sin(c + d*x^(1/2))), x)
```

Reduce [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\left(\int\csc\left(\sqrt{x}\,d+c\right)x dx\right)b+\frac{a\,x^{2}}{2}$$

input
```
int(x*(a+b*csc(c+d*x^(1/2))),x)
```

output
```
(2*int(csc(sqrt(x)*d + c)*x,x)*b + a*x**2)/2
```

3.34 $\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx = a\log(x)+b\text{Int}\left(\frac{\csc\left(c+d\sqrt{x}\right)}{x},x\right)$$

output
```
a*ln(x)+b*Defer(Int)(csc(c+d*x^(1/2))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 7.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx = \int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx$$

input
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])/x,x]
```

output
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx$$
$$\Big\downarrow \; 2010$$
$$\int \left(\frac{a}{x}+\frac{b\csc\left(c+d\sqrt{x}\right)}{x}\right)dx$$
$$\Big\downarrow \; 2009$$
$$b\int \frac{\csc\left(c+d\sqrt{x}\right)}{x}dx+a\log(x)$$

input
```
Int[(a + b*Csc[c + d*Sqrt[x]])/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x}dx$$

input
```
int((a+b*csc(c+d*x^(1/2)))/x,x)
```

output
```
int((a+b*csc(c+d*x^(1/2)))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x}\,dx = \int \frac{b\csc\left(d\sqrt{x} + c\right) + a}{x}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x,x, algorithm="fricas")
```

output
```
integral((b*csc(d*sqrt(x) + c) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x}\,dx = \int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x}\,dx$$

input
```
integrate((a+b*csc(c+d*x**(1/2)))/x,x)
```

output
```
Integral((a + b*csc(c + d*sqrt(x)))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.83

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x}\,dx = \int \frac{b\csc\left(d\sqrt{x} + c\right) + a}{x}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x,x, algorithm="maxima")
```

output
```
b*integrate(sin(d*sqrt(x) + c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)
^2 + 2*cos(d*sqrt(x) + c) + 1)*x), x) + b*integrate(sin(d*sqrt(x) + c)/((c
os(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1)*x),
 x) + a*log(x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x}\,dx = \int \frac{b\csc\left(d\sqrt{x} + c\right) + a}{x}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x,x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 16.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx = \int \frac{a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}}{x}\,dx$$

input
```
int((a + b/sin(c + d*x^(1/2)))/x,x)
```

output
```
int((a + b/sin(c + d*x^(1/2)))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx = \left(\int \frac{\csc\left(\sqrt{x}\,d+c\right)}{x}dx\right)b+\log(x)\,a$$

input
```
int((a+b*csc(c+d*x^(1/2)))/x,x)
```

output
```
int(csc(sqrt(x)*d + c)/x,x)*b + log(x)*a
```

3.35 $\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=-\frac{a}{x}+b\text{Int}\left(\frac{\csc\left(c+d\sqrt{x}\right)}{x^2},x\right)$$

output
```
-a/x+b*Defer(Int)(csc(c+d*x^(1/2))/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 11.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

input
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^2,x]
```

output
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}dx$$
$$\Big\downarrow \text{2010}$$
$$\int \left(\frac{a}{x^2}+\frac{b\csc\left(c+d\sqrt{x}\right)}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$b\int \frac{\csc\left(c+d\sqrt{x}\right)}{x^2}dx-\frac{a}{x}$$

input
```
Int[(a + b*Csc[c + d*Sqrt[x]])/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x^2} dx$$

input
```
int((a+b*csc(c+d*x^(1/2)))/x^2,x)
```

output
```
int((a+b*csc(c+d*x^(1/2)))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x^2} dx = \int \frac{b\csc\left(d\sqrt{x} + c\right) + a}{x^2} dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^2,x, algorithm="fricas")
```

output
```
integral((b*csc(d*sqrt(x) + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x^2} dx = \int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x^2} dx$$

input
```
integrate((a+b*csc(c+d*x**(1/2)))/x**2,x)
```

output
```
Integral((a + b*csc(c + d*sqrt(x)))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 6.17

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x^2}\,dx = \int \frac{b\csc\left(d\sqrt{x} + c\right) + a}{x^2}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^2,x, algorithm="maxima")
```

output
```
((b*integrate(sin(d*sqrt(x) + c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) +
c)^2 + 2*cos(d*sqrt(x) + c) + 1)*x^2), x) + b*integrate(sin(d*sqrt(x) + c)
/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1)
*x^2), x))*x - a)/x
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x^2}\,dx = \int \frac{b\csc\left(d\sqrt{x} + c\right) + a}{x^2}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 15.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x^2}\,dx = \int \frac{a + \frac{b}{\sin\left(c+d\sqrt{x}\right)}}{x^2}\,dx$$

input
```
int((a + b/sin(c + d*x^(1/2)))/x^2,x)
```

output
```
int((a + b/sin(c + d*x^(1/2)))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x^2}\,dx = \frac{\left(\int \frac{\csc\left(\sqrt{x}\,d+c\right)}{x^2}dx\right)bx - a}{x}$$

input
```
int((a+b*csc(c+d*x^(1/2)))/x^2,x)
```

output
```
(int(csc(sqrt(x)*d + c)/x**2,x)*b*x - a)/x
```

3.36 $\int x^3 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 695

$$\int x^3 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx = \text{Too large to display}$$

output
```
-3360*a*b*x^(3/2)*polylog(5,exp(I*(c+d*x^(1/2))))/d^5+168*a*b*x^(5/2)*poly
log(3,exp(I*(c+d*x^(1/2))))/d^3+3360*a*b*x^(3/2)*polylog(5,-exp(I*(c+d*x^(
1/2))))/d^5-168*a*b*x^(5/2)*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3-8*a*b*x^(
7/2)*arctanh(exp(I*(c+d*x^(1/2))))/d-20160*a*b*x^(1/2)*polylog(7,-exp(I*(c
+d*x^(1/2))))/d^7+20160*a*b*x^(1/2)*polylog(7,exp(I*(c+d*x^(1/2))))/d^7-42
*I*b^2*x^(5/2)*polylog(2,exp(2*I*(c+d*x^(1/2))))/d^3-20160*I*a*b*polylog(8
,-exp(I*(c+d*x^(1/2))))/d^8-315*I*b^2*x^(1/2)*polylog(6,exp(2*I*(c+d*x^(1/
2))))/d^7+210*I*b^2*x^(3/2)*polylog(4,exp(2*I*(c+d*x^(1/2))))/d^5+20160*I*
a*b*polylog(8,exp(I*(c+d*x^(1/2))))/d^8-840*I*a*b*x^2*polylog(4,-exp(I*(c+
d*x^(1/2))))/d^4-10080*I*a*b*x*polylog(6,exp(I*(c+d*x^(1/2))))/d^6-28*I*a*
b*x^3*polylog(2,exp(I*(c+d*x^(1/2))))/d^2+10080*I*a*b*x*polylog(6,-exp(I*(
c+d*x^(1/2))))/d^6+28*I*a*b*x^3*polylog(2,-exp(I*(c+d*x^(1/2))))/d^2+840*I
*a*b*x^2*polylog(4,exp(I*(c+d*x^(1/2))))/d^4+315/2*b^2*polylog(7,exp(2*I*(
c+d*x^(1/2))))/d^8-2*I*b^2*x^(7/2)/d+1/4*a^2*x^4-315*b^2*x*polylog(5,exp(2
*I*(c+d*x^(1/2))))/d^6+105*b^2*x^2*polylog(3,exp(2*I*(c+d*x^(1/2))))/d^4+1
4*b^2*x^3*ln(1-exp(2*I*(c+d*x^(1/2))))/d^2-2*b^2*x^(7/2)*cot(c+d*x^(1/2))/
d
```

Mathematica [A] (verified)

Time = 17.73 (sec) , antiderivative size = 954, normalized size of antiderivative = 1.37

$$\int x^3 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx = \text{Too large to display}$$

input
```
Integrate[x^3*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
(a^2*x^4*(a + b*Csc[c + d*Sqrt[x]])^2*Sin[c + d*Sqrt[x]]^2)/(4*(b + a*Sin[
c + d*Sqrt[x]])^2) - ((I/2)*b*(a + b*Csc[c + d*Sqrt[x]])^2*((8*b*d^7*E^((2
*I)*c)*x^(7/2))/(-1 + E^((2*I)*c)) + (8*I)*a*d^7*x^(7/2)*Log[1 - E^(I*(c +
 d*Sqrt[x]))] - (8*I)*a*d^7*x^(7/2)*Log[1 + E^(I*(c + d*Sqrt[x]))] + (28*I
)*b*d^6*x^3*Log[1 - E^((2*I)*(c + d*Sqrt[x]))] - 56*a*d^6*x^3*PolyLog[2, -
E^(I*(c + d*Sqrt[x]))] + 56*a*d^6*x^3*PolyLog[2, E^(I*(c + d*Sqrt[x]))] +
84*b*d^5*x^(5/2)*PolyLog[2, E^((2*I)*(c + d*Sqrt[x]))] - (336*I)*a*d^5*x^(
5/2)*PolyLog[3, -E^(I*(c + d*Sqrt[x]))] + (336*I)*a*d^5*x^(5/2)*PolyLog[3,
 E^(I*(c + d*Sqrt[x]))] + (210*I)*b*d^4*x^2*PolyLog[3, E^((2*I)*(c + d*Sqr
t[x]))] + 1680*a*d^4*x^2*PolyLog[4, -E^(I*(c + d*Sqrt[x]))] - 1680*a*d^4*x
^2*PolyLog[4, E^(I*(c + d*Sqrt[x]))] - 420*b*d^3*x^(3/2)*PolyLog[4, E^((2*
I)*(c + d*Sqrt[x]))] + (6720*I)*a*d^3*x^(3/2)*PolyLog[5, -E^(I*(c + d*Sqrt
[x]))] - (6720*I)*a*d^3*x^(3/2)*PolyLog[5, E^(I*(c + d*Sqrt[x]))] - (630*I
)*b*d^2*x*PolyLog[5, E^((2*I)*(c + d*Sqrt[x]))] - 20160*a*d^2*x*PolyLog[6,
 -E^(I*(c + d*Sqrt[x]))] + 20160*a*d^2*x*PolyLog[6, E^(I*(c + d*Sqrt[x]))]
 + 630*b*d*Sqrt[x]*PolyLog[6, E^((2*I)*(c + d*Sqrt[x]))] - (40320*I)*a*d*S
qrt[x]*PolyLog[7, -E^(I*(c + d*Sqrt[x]))] + (40320*I)*a*d*Sqrt[x]*PolyLog[
7, E^(I*(c + d*Sqrt[x]))] + (315*I)*b*PolyLog[7, E^((2*I)*(c + d*Sqrt[x]))
] + 40320*a*PolyLog[8, -E^(I*(c + d*Sqrt[x]))] - 40320*a*PolyLog[8, E^(I*(
c + d*Sqrt[x]))])*Sin[c + d*Sqrt[x]]^2)/(d^8*(b + a*Sin[c + d*Sqrt[x]])...
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4693, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$$

$$\Big\downarrow \; 4693$$

$$2\int x^{7/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 d\sqrt{x}$$

$$\Big\downarrow \; 3042$$

$$2\int x^{7/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 d\sqrt{x}$$

$$\Big\downarrow \; 4678$$

$$2\int \left(a^2x^{7/2}+b^2\csc^2\left(c+d\sqrt{x}\right)x^{7/2}+2ab\csc\left(c+d\sqrt{x}\right)x^{7/2}\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{a^2x^4}{8}-\frac{4abx^{7/2}\text{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}-\frac{10080iab\,\text{PolyLog}\left(8,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^8}+\frac{10080iab\,\text{PolyLog}\left(8,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^8}\right.$$

input
```
Int[x^3*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
2*(((-I)*b^2*x^(7/2))/d + (a^2*x^4)/8 - (4*a*b*x^(7/2)*ArcTanh[E^(I*(c + d
*Sqrt[x]))])/d - (b^2*x^(7/2)*Cot[c + d*Sqrt[x]])/d + (7*b^2*x^3*Log[1 - E
^((2*I)*(c + d*Sqrt[x]))])/d^2 + ((14*I)*a*b*x^3*PolyLog[2, -E^(I*(c + d*S
qrt[x]))])/d^2 - ((14*I)*a*b*x^3*PolyLog[2, E^(I*(c + d*Sqrt[x]))])/d^2 -
((21*I)*b^2*x^(5/2)*PolyLog[2, E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (84*a*b*x
^(5/2)*PolyLog[3, -E^(I*(c + d*Sqrt[x]))])/d^3 + (84*a*b*x^(5/2)*PolyLog[3
, E^(I*(c + d*Sqrt[x]))])/d^3 + (105*b^2*x^2*PolyLog[3, E^((2*I)*(c + d*Sq
rt[x]))])/(2*d^4) - ((420*I)*a*b*x^2*PolyLog[4, -E^(I*(c + d*Sqrt[x]))])/d
^4 + ((420*I)*a*b*x^2*PolyLog[4, E^(I*(c + d*Sqrt[x]))])/d^4 + ((105*I)*b^
2*x^(3/2)*PolyLog[4, E^((2*I)*(c + d*Sqrt[x]))])/d^5 + (1680*a*b*x^(3/2)*P
olyLog[5, -E^(I*(c + d*Sqrt[x]))])/d^5 - (1680*a*b*x^(3/2)*PolyLog[5, E^(I
*(c + d*Sqrt[x]))])/d^5 - (315*b^2*x*PolyLog[5, E^((2*I)*(c + d*Sqrt[x]))]
)/(2*d^6) + ((5040*I)*a*b*x*PolyLog[6, -E^(I*(c + d*Sqrt[x]))])/d^6 - ((50
40*I)*a*b*x*PolyLog[6, E^(I*(c + d*Sqrt[x]))])/d^6 - (((315*I)/2)*b^2*Sqrt
[x]*PolyLog[6, E^((2*I)*(c + d*Sqrt[x]))])/d^7 - (10080*a*b*Sqrt[x]*PolyLo
g[7, -E^(I*(c + d*Sqrt[x]))])/d^7 + (10080*a*b*Sqrt[x]*PolyLog[7, E^(I*(c
+ d*Sqrt[x]))])/d^7 + (315*b^2*PolyLog[7, E^((2*I)*(c + d*Sqrt[x]))])/(4*d
^8) - ((10080*I)*a*b*PolyLog[8, -E^(I*(c + d*Sqrt[x]))])/d^8 + ((10080*I)*
a*b*PolyLog[8, E^(I*(c + d*Sqrt[x]))])/d^8)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int x^3 \left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2 dx$$

input
```
int(x^3*(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(x^3*(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x^3 \left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b\csc\left(d\sqrt{x} + c\right) + a\right)^2 x^3 \, dx$$

input
```
integrate(x^3*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^3*csc(d*sqrt(x) + c)^2 + 2*a*b*x^3*csc(d*sqrt(x) + c) + a^2
*x^3, x)
```

Sympy [F]

$$\int x^3 \left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int x^3 \left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2 \, dx$$

input
```
integrate(x**3*(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(x**3*(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6462 vs. $2(550) = 1100$.

Time = 0.48 (sec) , antiderivative size = 6462, normalized size of antiderivative = 9.30

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
1/4*((d*sqrt(x) + c)^8*a^2 - 8*(d*sqrt(x) + c)^7*a^2*c + 28*(d*sqrt(x) + c
)^6*a^2*c^2 - 56*(d*sqrt(x) + c)^5*a^2*c^3 + 70*(d*sqrt(x) + c)^4*a^2*c^4
- 56*(d*sqrt(x) + c)^3*a^2*c^5 + 28*(d*sqrt(x) + c)^2*a^2*c^6 - 8*(d*sqrt(
x) + c)*a^2*c^7 + 16*a*b*c^7*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c))
+ 8*(4*b^2*c^7 + 2*(2*(d*sqrt(x) + c)^7*a*b - 7*b^2*c^6 - 7*(2*a*b*c + b^2
)*(d*sqrt(x) + c)^6 + 42*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^5 - 35*(2*a*b*c
^3 + 3*b^2*c^2)*(d*sqrt(x) + c)^4 + 70*(a*b*c^4 + 2*b^2*c^3)*(d*sqrt(x) +
c)^3 - 21*(2*a*b*c^5 + 5*b^2*c^4)*(d*sqrt(x) + c)^2 + 14*(a*b*c^6 + 3*b^2*
c^5)*(d*sqrt(x) + c) - (2*(d*sqrt(x) + c)^7*a*b - 7*b^2*c^6 - 7*(2*a*b*c +
 b^2)*(d*sqrt(x) + c)^6 + 42*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^5 - 35*(2*a
*b*c^3 + 3*b^2*c^2)*(d*sqrt(x) + c)^4 + 70*(a*b*c^4 + 2*b^2*c^3)*(d*sqrt(x
) + c)^3 - 21*(2*a*b*c^5 + 5*b^2*c^4)*(d*sqrt(x) + c)^2 + 14*(a*b*c^6 + 3*
b^2*c^5)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-2*I*(d*sqrt(x) + c)^7
*a*b + 7*I*b^2*c^6 + 7*(2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^6 + 42*(-I*a*b*
c^2 - I*b^2*c)*(d*sqrt(x) + c)^5 + 35*(2*I*a*b*c^3 + 3*I*b^2*c^2)*(d*sqrt(
x) + c)^4 + 70*(-I*a*b*c^4 - 2*I*b^2*c^3)*(d*sqrt(x) + c)^3 + 21*(2*I*a*b*
c^5 + 5*I*b^2*c^4)*(d*sqrt(x) + c)^2 + 14*(-I*a*b*c^6 - 3*I*b^2*c^5)*(d*sq
rt(x) + c))*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt
(x) + c) + 1) + 14*(b^2*c^6*cos(2*d*sqrt(x) + 2*c) + I*b^2*c^6*sin(2*d*sqr
t(x) + 2*c) - b^2*c^6)*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) -...
```

Giac [F]

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x^3\,dx$$

input
```
integrate(x^3*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x^3\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2\,dx$$

input
```
int(x^3*(a + b/sin(c + d*x^(1/2)))^2,x)
```

output
```
int(x^3*(a + b/sin(c + d*x^(1/2)))^2, x)
```

Reduce [F]

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=2\left(\int\csc\left(\sqrt{x}\,d+c\right)x^3dx\right)ab+\left(\int\csc\left(\sqrt{x}\,d+c\right)^2x^3dx\right)b^2+\frac{a^2x^4}{4}$$

input
```
int(x^3*(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
(8*int(csc(sqrt(x)*d + c)*x**3,x)*a*b + 4*int(csc(sqrt(x)*d + c)**2*x**3,x
)*b**2 + a**2*x**4)/4
```

3.37 $\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 513

$$\int x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx = -\frac{2ib^2x^{5/2}}{d}+\frac{a^2x^3}{3}-\frac{8abx^{5/2}\text{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d} -\frac{2b^2x^{5/2}\cot\left(c+d\sqrt{x}\right)}{d}+\frac{10b^2x^2\log\left(1-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^2} +\frac{20iabx^2\,\text{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} -\frac{20iabx^2\,\text{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} -\frac{20ib^2x^{3/2}\,\text{PolyLog}\left(2,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^3} -\frac{80abx^{3/2}\,\text{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3} +\frac{80abx^{3/2}\,\text{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3} +\frac{30b^2x\,\text{PolyLog}\left(3,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^4} -\frac{240iabx\,\text{PolyLog}\left(4,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4} +\frac{240iabx\,\text{PolyLog}\left(4,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4} +\frac{30ib^2\sqrt{x}\,\text{PolyLog}\left(4,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^5} +\frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5} -\frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5} -\frac{15b^2\,\text{PolyLog}\left(5,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^6} +\frac{480iab\,\text{PolyLog}\left(6,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6} -\frac{480iab\,\text{PolyLog}\left(6,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}$$

output
```
30*I*b^2*x^(1/2)*polylog(4,exp(2*I*(c+d*x^(1/2))))/d^5+1/3*a^2*x^3-8*a*b*x
^(5/2)*arctanh(exp(I*(c+d*x^(1/2))))/d-2*b^2*x^(5/2)*cot(c+d*x^(1/2))/d+10
*b^2*x^2*ln(1-exp(2*I*(c+d*x^(1/2))))/d^2+20*I*a*b*x^2*polylog(2,-exp(I*(c
+d*x^(1/2))))/d^2+480*I*a*b*polylog(6,-exp(I*(c+d*x^(1/2))))/d^6+240*I*a*b
*x*polylog(4,exp(I*(c+d*x^(1/2))))/d^4-80*a*b*x^(3/2)*polylog(3,-exp(I*(c+
d*x^(1/2))))/d^3+80*a*b*x^(3/2)*polylog(3,exp(I*(c+d*x^(1/2))))/d^3+30*b^2
*x*polylog(3,exp(2*I*(c+d*x^(1/2))))/d^4-480*I*a*b*polylog(6,exp(I*(c+d*x^
(1/2))))/d^6-20*I*a*b*x^2*polylog(2,exp(I*(c+d*x^(1/2))))/d^2-2*I*b^2*x^(5
/2)/d+480*a*b*x^(1/2)*polylog(5,-exp(I*(c+d*x^(1/2))))/d^5-480*a*b*x^(1/2)
*polylog(5,exp(I*(c+d*x^(1/2))))/d^5-15*b^2*polylog(5,exp(2*I*(c+d*x^(1/2)
)))/d^6-20*I*b^2*x^(3/2)*polylog(2,exp(2*I*(c+d*x^(1/2))))/d^3-240*I*a*b*x
*polylog(4,-exp(I*(c+d*x^(1/2))))/d^4
```

Mathematica [A] (verified)

Time = 12.57 (sec) , antiderivative size = 779, normalized size of antiderivative = 1.52

$$\int x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 dx = \text{Too large to display}$$

input
```
Integrate[x^2*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
(a^2*x^3*(a + b*Csc[c + d*Sqrt[x]])^2*Sin[c + d*Sqrt[x]]^2)/(3*(b + a*Sin[
c + d*Sqrt[x]])^2) - (I*b*(a + b*Csc[c + d*Sqrt[x]])^2*((4*b*d^5*E^((2*I)*
c)*x^(5/2))/(-1 + E^((2*I)*c)) - 20*a*d^4*x^2*PolyLog[2, -E^(I*(c + d*Sqrt
[x]))] + 20*a*d^4*x^2*PolyLog[2, E^(I*(c + d*Sqrt[x]))] + I*(4*a*d^5*x^(5/
2)*Log[1 - E^(I*(c + d*Sqrt[x]))] - 4*a*d^5*x^(5/2)*Log[1 + E^(I*(c + d*Sq
rt[x]))] + 10*b*d^4*x^2*Log[1 - E^((2*I)*(c + d*Sqrt[x]))] - (20*I)*b*d^3*
x^(3/2)*PolyLog[2, E^((2*I)*(c + d*Sqrt[x]))] - 80*a*d^3*x^(3/2)*PolyLog[3
, -E^(I*(c + d*Sqrt[x]))] + 80*a*d^3*x^(3/2)*PolyLog[3, E^(I*(c + d*Sqrt[x
]))] + 30*b*d^2*x*PolyLog[3, E^((2*I)*(c + d*Sqrt[x]))] - (240*I)*a*d^2*x*
PolyLog[4, -E^(I*(c + d*Sqrt[x]))] + (240*I)*a*d^2*x*PolyLog[4, E^(I*(c +
d*Sqrt[x]))] + (30*I)*b*d*Sqrt[x]*PolyLog[4, E^((2*I)*(c + d*Sqrt[x]))] +
480*a*d*Sqrt[x]*PolyLog[5, -E^(I*(c + d*Sqrt[x]))] - 480*a*d*Sqrt[x]*PolyL
og[5, E^(I*(c + d*Sqrt[x]))] - 15*b*PolyLog[5, E^((2*I)*(c + d*Sqrt[x]))]
+ (480*I)*a*PolyLog[6, -E^(I*(c + d*Sqrt[x]))] - (480*I)*a*PolyLog[6, E^(I
*(c + d*Sqrt[x]))]))*Sin[c + d*Sqrt[x]]^2)/(d^6*(b + a*Sin[c + d*Sqrt[x]])
^2) + (b^2*x^(5/2)*Csc[c/2]*Csc[c/2 + (d*Sqrt[x])/2]*(a + b*Csc[c + d*Sqrt
[x]])^2*Sin[c + d*Sqrt[x]]^2*Sin[(d*Sqrt[x])/2])/(d*(b + a*Sin[c + d*Sqrt[
x]])^2) + (b^2*x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])^2*Sec[c/2]*Sec[c/2 + (d*
Sqrt[x])/2]*Sin[c + d*Sqrt[x]]^2*Sin[(d*Sqrt[x])/2])/(d*(b + a*Sin[c + d*S
qrt[x]])^2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4693, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2 \, dx$$

$$\Big\downarrow \text{ 4693}$$

$$2\int x^{5/2}\left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2 d\sqrt{x}$$

$$\Big\downarrow \text{ 3042}$$

$$2\int x^{5/2}\left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2 d\sqrt{x}$$

$$\downarrow 4678$$

$$2\int \left(a^2x^{5/2}+b^2\csc^2\left(c+d\sqrt{x}\right)x^{5/2}+2ab\csc\left(c+d\sqrt{x}\right)x^{5/2}\right)d\sqrt{x}$$

$$\downarrow 2009$$

$$2\left(\frac{a^2x^3}{6}-\frac{4abx^{5/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{240iab\,\text{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{240iab\,\text{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6}+\frac{2}{}\right.$$

input
```
Int[x^2*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
2*(((-I)*b^2*x^(5/2))/d + (a^2*x^3)/6 - (4*a*b*x^(5/2)*ArcTanh[E^(I*(c + d
*Sqrt[x]))])/d - (b^2*x^(5/2)*Cot[c + d*Sqrt[x]])/d + (5*b^2*x^2*Log[1 - E
^((2*I)*(c + d*Sqrt[x]))])/d^2 + ((10*I)*a*b*x^2*PolyLog[2, -E^(I*(c + d*S
qrt[x]))])/d^2 - ((10*I)*a*b*x^2*PolyLog[2, E^(I*(c + d*Sqrt[x]))])/d^2 -
((10*I)*b^2*x^(3/2)*PolyLog[2, E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (40*a*b*x
^(3/2)*PolyLog[3, -E^(I*(c + d*Sqrt[x]))])/d^3 + (40*a*b*x^(3/2)*PolyLog[3
, E^(I*(c + d*Sqrt[x]))])/d^3 + (15*b^2*x*PolyLog[3, E^((2*I)*(c + d*Sqrt[
x]))])/d^4 - ((120*I)*a*b*x*PolyLog[4, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((12
0*I)*a*b*x*PolyLog[4, E^(I*(c + d*Sqrt[x]))])/d^4 + ((15*I)*b^2*Sqrt[x]*Po
lyLog[4, E^((2*I)*(c + d*Sqrt[x]))])/d^5 + (240*a*b*Sqrt[x]*PolyLog[5, -E^
(I*(c + d*Sqrt[x]))])/d^5 - (240*a*b*Sqrt[x]*PolyLog[5, E^(I*(c + d*Sqrt[x
]))])/d^5 - (15*b^2*PolyLog[5, E^((2*I)*(c + d*Sqrt[x]))])/(2*d^6) + ((240
*I)*a*b*PolyLog[6, -E^(I*(c + d*Sqrt[x]))])/d^6 - ((240*I)*a*b*PolyLog[6,
E^(I*(c + d*Sqrt[x]))])/d^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int x^2 \left(a+b\csc \left(c+d\sqrt{x}\right)\right)^2 dx$$

input
```
int(x^2*(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(x^2*(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x^2 \left(a+b\csc \left(c+d\sqrt{x}\right)\right)^2 \, dx=\int \left(b\csc \left(d\sqrt{x}+c\right)+a\right)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*csc(d*sqrt(x) + c)^2 + 2*a*b*x^2*csc(d*sqrt(x) + c) + a^2
*x^2, x)
```

Sympy [F]

$$\int x^2 \left(a + b \csc \left(c + d\sqrt{x}\right)\right)^2 \, dx = \int x^2 \left(a + b \csc \left(c + d\sqrt{x}\right)\right)^2 \, dx$$

input
```
integrate(x**2*(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(x**2*(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3885 vs. $2(406) = 812$.

Time $= 0.26$ (sec) , antiderivative size $= 3885$, normalized size of antiderivative $= 7.57$

$$\int x^2 \left(a + b \csc \left(c + d\sqrt{x}\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
1/3*((d*sqrt(x) + c)^6*a^2 - 6*(d*sqrt(x) + c)^5*a^2*c + 15*(d*sqrt(x) + c
)^4*a^2*c^2 - 20*(d*sqrt(x) + c)^3*a^2*c^3 + 15*(d*sqrt(x) + c)^2*a^2*c^4
- 6*(d*sqrt(x) + c)*a^2*c^5 + 12*a*b*c^5*log(cot(d*sqrt(x) + c) + csc(d*sq
rt(x) + c)) + 6*(4*b^2*c^5 + 2*(2*(d*sqrt(x) + c)^5*a*b - 5*b^2*c^4 - 5*(2
*a*b*c + b^2)*(d*sqrt(x) + c)^4 + 20*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^3 -
 10*(2*a*b*c^3 + 3*b^2*c^2)*(d*sqrt(x) + c)^2 + 10*(a*b*c^4 + 2*b^2*c^3)*(
d*sqrt(x) + c) - (2*(d*sqrt(x) + c)^5*a*b - 5*b^2*c^4 - 5*(2*a*b*c + b^2)*
(d*sqrt(x) + c)^4 + 20*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^3 - 10*(2*a*b*c^3
 + 3*b^2*c^2)*(d*sqrt(x) + c)^2 + 10*(a*b*c^4 + 2*b^2*c^3)*(d*sqrt(x) + c)
)*cos(2*d*sqrt(x) + 2*c) + (-2*I*(d*sqrt(x) + c)^5*a*b + 5*I*b^2*c^4 + 5*(
2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^4 + 20*(-I*a*b*c^2 - I*b^2*c)*(d*sqrt(x
) + c)^3 + 10*(2*I*a*b*c^3 + 3*I*b^2*c^2)*(d*sqrt(x) + c)^2 + 10*(-I*a*b*c
^4 - 2*I*b^2*c^3)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(d*s
qrt(x) + c), cos(d*sqrt(x) + c) + 1) + 10*(b^2*c^4*cos(2*d*sqrt(x) + 2*c)
+ I*b^2*c^4*sin(2*d*sqrt(x) + 2*c) - b^2*c^4)*arctan2(sin(d*sqrt(x) + c),
cos(d*sqrt(x) + c) - 1) + 2*(2*(d*sqrt(x) + c)^5*a*b - 5*(2*a*b*c - b^2)*(
d*sqrt(x) + c)^4 + 20*(a*b*c^2 - b^2*c)*(d*sqrt(x) + c)^3 - 10*(2*a*b*c^3
- 3*b^2*c^2)*(d*sqrt(x) + c)^2 + 10*(a*b*c^4 - 2*b^2*c^3)*(d*sqrt(x) + c)
- (2*(d*sqrt(x) + c)^5*a*b - 5*(2*a*b*c - b^2)*(d*sqrt(x) + c)^4 + 20*(a*b
*c^2 - b^2*c)*(d*sqrt(x) + c)^3 - 10*(2*a*b*c^3 - 3*b^2*c^2)*(d*sqrt(x)...
```

Giac [F]

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int x^2 \left(a + \frac{b}{\sin\left(c + d\sqrt{x}\right)}\right)^2 \, dx$$

input
```
int(x^2*(a + b/sin(c + d*x^(1/2)))^2,x)
```

output
```
int(x^2*(a + b/sin(c + d*x^(1/2)))^2, x)
```

Reduce [F]

$$\int x^2 \left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2 \, dx = 2\left(\int \csc\left(\sqrt{x}\, d + c\right) x^2 dx\right) ab + \left(\int \csc\left(\sqrt{x}\, d + c\right)^2 x^2 dx\right) b^2 + \frac{a^2 x^3}{3}$$

input
```
int(x^2*(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
(6*int(csc(sqrt(x)*d + c)*x**2,x)*a*b + 3*int(csc(sqrt(x)*d + c)**2*x**2,x
)*b**2 + a**2*x**3)/3
```

3.38 $\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 333

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=-\frac{2ib^2x^{3/2}}{d}+\frac{a^2x^2}{2}-\frac{8abx^{3/2}\operatorname{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}-\frac{2b^2x^{3/2}\cot\left(c+d\sqrt{x}\right)}{d}+\frac{6b^2x\log\left(1-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^2}+\frac{12iabx\,\mathrm{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{12iabx\,\mathrm{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{6ib^2\sqrt{x}\,\mathrm{PolyLog}\left(2,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^3}-\frac{24ab\sqrt{x}\,\mathrm{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{24ab\sqrt{x}\,\mathrm{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{3b^2\,\mathrm{PolyLog}\left(3,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{24iab\,\mathrm{PolyLog}\left(4,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{24iab\,\mathrm{PolyLog}\left(4,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}$$

output
```
-2*I*b^2*x^(3/2)/d+1/2*a^2*x^2-8*a*b*x^(3/2)*arctanh(exp(I*(c+d*x^(1/2))))
/d-2*b^2*x^(3/2)*cot(c+d*x^(1/2))/d+6*b^2*x*ln(1-exp(2*I*(c+d*x^(1/2))))/d
^2+12*I*a*b*x*polylog(2,-exp(I*(c+d*x^(1/2))))/d^2-12*I*a*b*x*polylog(2,ex
p(I*(c+d*x^(1/2))))/d^2-6*I*b^2*x^(1/2)*polylog(2,exp(2*I*(c+d*x^(1/2))))/
d^3-24*a*b*x^(1/2)*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3+24*a*b*x^(1/2)*pol
ylog(3,exp(I*(c+d*x^(1/2))))/d^3+3*b^2*polylog(3,exp(2*I*(c+d*x^(1/2))))/d
^4-24*I*a*b*polylog(4,-exp(I*(c+d*x^(1/2))))/d^4+24*I*a*b*polylog(4,exp(I*
(c+d*x^(1/2))))/d^4
```

Mathematica [A] (verified)

Time = 6.91 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.35

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{a^2x^2}{2}$$
$$-\frac{2ib\left(\frac{2bd^3e^{2ic}x^{3/2}}{-1+e^{2ic}}+\left(6bd\sqrt{x}-6ad^2x\right)\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)+i\left(3bd^2x\log\left(1-e^{i(c+d\sqrt{x})}\right)+2ad^3x^{3/}\right.\right.}{}$$
$$+\frac{b^2x^{3/2}\csc\left(\frac{c}{2}\right)\csc\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\sin\left(\frac{d\sqrt{x}}{2}\right)}{d}$$
$$+\frac{b^2x^{3/2}\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\sin\left(\frac{d\sqrt{x}}{2}\right)}{d}$$

input
```
Integrate[x*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
(a^2*x^2)/2 - ((2*I)*b*((2*b*d^3*E^((2*I)*c)*x^(3/2))/(-1 + E^((2*I)*c)) +
 (6*b*d*Sqrt[x] - 6*a*d^2*x)*PolyLog[2, -E^(I*(c + d*Sqrt[x]))] + I*(3*b*d
^2*x*Log[1 - E^(I*(c + d*Sqrt[x]))] + 2*a*d^3*x^(3/2)*Log[1 - E^(I*(c + d*
Sqrt[x]))] + 3*b*d^2*x*Log[1 + E^(I*(c + d*Sqrt[x]))] - 2*a*d^3*x^(3/2)*Lo
g[1 + E^(I*(c + d*Sqrt[x]))] - (6*I)*(b*d*Sqrt[x] + a*d^2*x)*PolyLog[2, E^
(I*(c + d*Sqrt[x]))] + 6*(b - 2*a*d*Sqrt[x])*PolyLog[3, -E^(I*(c + d*Sqrt[
x]))] + 6*b*PolyLog[3, E^(I*(c + d*Sqrt[x]))] + 12*a*d*Sqrt[x]*PolyLog[3,
E^(I*(c + d*Sqrt[x]))] - (12*I)*a*PolyLog[4, -E^(I*(c + d*Sqrt[x]))] + (12
*I)*a*PolyLog[4, E^(I*(c + d*Sqrt[x]))])))/d^4 + (b^2*x^(3/2)*Csc[c/2]*Csc
[(c + d*Sqrt[x])/2]*Sin[(d*Sqrt[x])/2])/d + (b^2*x^(3/2)*Sec[c/2]*Sec[(c +
 d*Sqrt[x])/2]*Sin[(d*Sqrt[x])/2])/d
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {4693, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$$

$$\Big\downarrow \text{4693}$$

$$2\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2d\sqrt{x}$$

$$\Big\downarrow \text{4678}$$

$$2\int\left(x^{3/2}a^2+2bx^{3/2}\csc\left(c+d\sqrt{x}\right)a+b^2x^{3/2}\csc^2\left(c+d\sqrt{x}\right)\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^2x^2}{4}-\frac{4abx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{12iab\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{12iab\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}-\frac{12a}{}\right.$$

input

```
Int[x*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output

```
2*(((-I)*b^2*x^(3/2))/d + (a^2*x^2)/4 - (4*a*b*x^(3/2)*ArcTanh[E^(I*(c + d
*Sqrt[x]))])/d - (b^2*x^(3/2)*Cot[c + d*Sqrt[x]])/d + (3*b^2*x*Log[1 - E^(
(2*I)*(c + d*Sqrt[x]))])/d^2 + ((6*I)*a*b*x*PolyLog[2, -E^(I*(c + d*Sqrt[x
]))])/d^2 - ((6*I)*a*b*x*PolyLog[2, E^(I*(c + d*Sqrt[x]))])/d^2 - ((3*I)*b
^2*Sqrt[x]*PolyLog[2, E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (12*a*b*Sqrt[x]*Po
lyLog[3, -E^(I*(c + d*Sqrt[x]))])/d^3 + (12*a*b*Sqrt[x]*PolyLog[3, E^(I*(c
 + d*Sqrt[x]))])/d^3 + (3*b^2*PolyLog[3, E^((2*I)*(c + d*Sqrt[x]))])/(2*d^
4) - ((12*I)*a*b*PolyLog[4, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((12*I)*a*b*Pol
yLog[4, E^(I*(c + d*Sqrt[x]))])/d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}dx$$

input
```
int(x*(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(x*(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^{2}x\,dx$$

input
```
integrate(x*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x*csc(d*sqrt(x) + c)^2 + 2*a*b*x*csc(d*sqrt(x) + c) + a^2*x,
x)
```

Sympy [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$$

input
```
integrate(x*(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(x*(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1950 vs. $2(262) = 524$.

Time = 0.17 (sec) , antiderivative size = 1950, normalized size of antiderivative = 5.86

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(x*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
1/2*((d*sqrt(x) + c)^4*a^2 - 4*(d*sqrt(x) + c)^3*a^2*c + 6*(d*sqrt(x) + c)
^2*a^2*c^2 - 4*(d*sqrt(x) + c)*a^2*c^3 + 8*a*b*c^3*log(cot(d*sqrt(x) + c)
+ csc(d*sqrt(x) + c)) + 4*(4*b^2*c^3 + 2*(2*(d*sqrt(x) + c)^3*a*b - 3*b^2*
c^2 - 3*(2*a*b*c + b^2)*(d*sqrt(x) + c)^2 + 6*(a*b*c^2 + b^2*c)*(d*sqrt(x)
 + c) - (2*(d*sqrt(x) + c)^3*a*b - 3*b^2*c^2 - 3*(2*a*b*c + b^2)*(d*sqrt(x
) + c)^2 + 6*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (
-2*I*(d*sqrt(x) + c)^3*a*b + 3*I*b^2*c^2 + 3*(2*I*a*b*c + I*b^2)*(d*sqrt(x
) + c)^2 + 6*(-I*a*b*c^2 - I*b^2*c)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c
))*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) + 1) + 6*(b^2*c^2*cos(2*
d*sqrt(x) + 2*c) + I*b^2*c^2*sin(2*d*sqrt(x) + 2*c) - b^2*c^2)*arctan2(sin
(d*sqrt(x) + c), cos(d*sqrt(x) + c) - 1) + 2*(2*(d*sqrt(x) + c)^3*a*b - 3*
(2*a*b*c - b^2)*(d*sqrt(x) + c)^2 + 6*(a*b*c^2 - b^2*c)*(d*sqrt(x) + c) -
(2*(d*sqrt(x) + c)^3*a*b - 3*(2*a*b*c - b^2)*(d*sqrt(x) + c)^2 + 6*(a*b*c^
2 - b^2*c)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-2*I*(d*sqrt(x) + c)
^3*a*b + 3*(2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^2 + 6*(-I*a*b*c^2 + I*b^2*c
)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(d*sqrt(x) + c), -co
s(d*sqrt(x) + c) + 1) - 4*((d*sqrt(x) + c)^3*b^2 - 3*(d*sqrt(x) + c)^2*b^2
*c + 3*(d*sqrt(x) + c)*b^2*c^2)*cos(2*d*sqrt(x) + 2*c) - 12*((d*sqrt(x) +
c)^2*a*b + a*b*c^2 + b^2*c - (2*a*b*c + b^2)*(d*sqrt(x) + c) - ((d*sqrt(x)
 + c)^2*a*b + a*b*c^2 + b^2*c - (2*a*b*c + b^2)*(d*sqrt(x) + c))*cos(2*...
```

Giac [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2\,dx$$

input
```
int(x*(a + b/sin(c + d*x^(1/2)))^2,x)
```

output
```
int(x*(a + b/sin(c + d*x^(1/2)))^2, x)
```

Reduce [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=2\left(\int\csc\left(\sqrt{x}\,d+c\right)xdx\right)ab+\left(\int\csc\left(\sqrt{x}\,d+c\right)^2xdx\right)b^2+\frac{a^2x^2}{2}$$

input
```
int(x*(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
(4*int(csc(sqrt(x)*d + c)*x,x)*a*b + 2*int(csc(sqrt(x)*d + c)**2*x,x)*b**2
 + a**2*x**2)/2
```

3.39 $\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx = \text{Int}\left(\frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x},x\right)$$

output
```
Defer(Int)((a+b*csc(c+d*x^(1/2)))^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 90.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx = \int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

input
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x,x]
```

output
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}dx$$

$$\Big\downarrow \text{4695}$$

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}dx$$

input
```
Int[(a + b*Csc[c + d*Sqrt[x]])^2/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}dx$$

input
```
int((a+b*csc(c+d*x^(1/2)))^2/x,x)
```

output
```
int((a+b*csc(c+d*x^(1/2)))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 12.87 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

input
```
integrate((a+b*csc(c+d*x**(1/2)))**2/x,x)
```

output
```
Integral((a + b*csc(c + d*sqrt(x)))**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 386, normalized size of antiderivative = 19.30

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x,x, algorithm="maxima")
```

output
```
-(4*b^2*sqrt(x)*sin(2*d*sqrt(x) + 2*c) - (d*cos(2*d*sqrt(x) + 2*c)^2 + d*s
in(2*d*sqrt(x) + 2*c)^2 - 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x*integrate((2*a
*b*d*x*sin(d*sqrt(x) + c) + b^2*sqrt(x)*sin(d*sqrt(x) + c))/((d*cos(d*sqrt
(x) + c)^2 + d*sin(d*sqrt(x) + c)^2 + 2*d*cos(d*sqrt(x) + c) + d)*x^2), x)
 + (d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 - 2*d*cos(2*d*
sqrt(x) + 2*c) + d)*x*integrate(-(2*a*b*d*x*sin(d*sqrt(x) + c) - b^2*sqrt(
x)*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c)^2 -
 2*d*cos(d*sqrt(x) + c) + d)*x^2), x) - (a^2*d*cos(2*d*sqrt(x) + 2*c)^2 +
a^2*d*sin(2*d*sqrt(x) + 2*c)^2 - 2*a^2*d*cos(2*d*sqrt(x) + 2*c) + a^2*d)*x
*log(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 - 2*d*c
os(2*d*sqrt(x) + 2*c) + d)*x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x,x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 15.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}\right)^2}{x}\,dx$$

input
```
int((a + b/sin(c + d*x^(1/2)))^2/x,x)
```

output
```
int((a + b/sin(c + d*x^(1/2)))^2/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\left(\int \frac{1}{\sin\left(\sqrt{x}\,d+c\right)^2 x}dx\right)b^2+2\left(\int \frac{1}{\sin\left(\sqrt{x}\,d+c\right)x}dx\right)ab-\frac{\left(\int \frac{1}{x}dx\right)b^2}{2}+2\log\left(\sqrt{x}\right)b^2+\log(x)\,a^2-\frac{\log(x)\,b^2}{2}$$

input
```
int((a+b*csc(c+d*x^(1/2)))^2/x,x)
```

output
```
(2*int(1/(sin(sqrt(x)*d + c)**2*x),x)*b**2 + 4*int(1/(sin(sqrt(x)*d + c)*x
),x)*a*b - int(1/x,x)*b**2 + 4*log(sqrt(x))*b**2 + 2*log(x)*a**2 - log(x)*
b**2)/2
```

3.40 $\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\text{Int}\left(\frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2},x\right)$$

output
```
Defer(Int)((a+b*csc(c+d*x^(1/2)))^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 57.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

input
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x^2,x]
```

output
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$
$$\Big\downarrow \; 4695$$
$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}dx$$

input
```
Int[(a + b*Csc[c + d*Sqrt[x]])^2/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}dx$$

input
```
int((a+b*csc(c+d*x^(1/2)))^2/x^2,x)
```

output
```
int((a+b*csc(c+d*x^(1/2)))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^2)/x^2,
x)
```

Sympy [N/A]

Not integrable

Time = 3.92 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*csc(c+d*x**(1/2)))**2/x**2,x)
```

output
```
Integral((a + b*csc(c + d*sqrt(x)))**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 388, normalized size of antiderivative = 19.40

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x^2,x, algorithm="maxima")
```

output
```
((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 - 2*d*cos(2*d*sq
rt(x) + 2*c) + d)*x^2*integrate((2*a*b*d*x*sin(d*sqrt(x) + c) + 3*b^2*sqrt
(x)*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c)^2
+ 2*d*cos(d*sqrt(x) + c) + d)*x^3), x) - (d*cos(2*d*sqrt(x) + 2*c)^2 + d*s
in(2*d*sqrt(x) + 2*c)^2 - 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^2*integrate(-(
2*a*b*d*x*sin(d*sqrt(x) + c) - 3*b^2*sqrt(x)*sin(d*sqrt(x) + c))/((d*cos(d
*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c)^2 - 2*d*cos(d*sqrt(x) + c) + d)*x^3
), x) - 4*b^2*sqrt(x)*sin(2*d*sqrt(x) + 2*c) - (a^2*d*cos(2*d*sqrt(x) + 2*
c)^2 + a^2*d*sin(2*d*sqrt(x) + 2*c)^2 - 2*a^2*d*cos(2*d*sqrt(x) + 2*c) + a
^2*d)*x)/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 - 2*d*c
os(2*d*sqrt(x) + 2*c) + d)*x^2)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 15.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}\right)^2}{x^2}\,dx$$

input
```
int((a + b/sin(c + d*x^(1/2)))^2/x^2,x)
```

output
```
int((a + b/sin(c + d*x^(1/2)))^2/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\frac{2\left(\int \frac{\csc\left(\sqrt{x}\,d+c\right)}{x^2}dx\right)abx+\left(\int \frac{\csc\left(\sqrt{x}\,d+c\right)^2}{x^2}dx\right)b^2x-a^2}{x}$$

input
```
int((a+b*csc(c+d*x^(1/2)))^2/x^2,x)
```

output
```
(2*int(csc(sqrt(x)*d + c)/x**2,x)*a*b*x + int(csc(sqrt(x)*d + c)**2/x**2,x
)*b**2*x - a**2)/x
```

3.41 $\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 1075

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \text{Too large to display}$$

output
```
1/4*x^4/a-84*I*b*x^(5/2)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^
(1/2)))/a/(-a^2+b^2)^(1/2)/d^3+10080*I*b*x^(1/2)*polylog(7,I*a*exp(I*(c+d*
x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^7+14*b*x^3*polylog(2,
I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2-14*b
*x^3*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)
^(1/2)/d^2+2*I*b*x^(7/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)
))/a/(-a^2+b^2)^(1/2)/d-10080*I*b*x^(1/2)*polylog(7,I*a*exp(I*(c+d*x^(1/2)
))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^7-420*b*x^2*polylog(4,I*a*ex
p(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^4+420*b*x^2*
polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2
)/d^4-2*I*b*x^(7/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/
(-a^2+b^2)^(1/2)/d+84*I*b*x^(5/2)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-
a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^3+5040*b*x*polylog(6,I*a*exp(I*(c+d*
x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^6-5040*b*x*polylog(6,
I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^6+1680
*I*b*x^(3/2)*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-
a^2+b^2)^(1/2)/d^5-1680*I*b*x^(3/2)*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b-
(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^5-10080*b*polylog(8,I*a*exp(I*(c+d
*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^8+10080*b*polylog(8,
I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^8
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 850, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$$
$$= \frac{\sqrt{a^2-b^2}d^8x^4 - 8bd^7x^{7/2}\log\left(1-\frac{ae^{i(c+d\sqrt{x})}}{-ib+\sqrt{a^2-b^2}}\right) + 8bd^7x^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right) + 56ibd^6x^3\operatorname{PolyLog}\left(2,\right.}{}$$

input
```
Integrate[x^3/(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(Sqrt[a^2 - b^2]*d^8*x^4 - 8*b*d^7*x^(7/2)*Log[1 - (a*E^(I*(c + d*Sqrt[x])
))/((-I)*b + Sqrt[a^2 - b^2])] + 8*b*d^7*x^(7/2)*Log[1 + (a*E^(I*(c + d*Sq
rt[x])))/(I*b + Sqrt[a^2 - b^2])] + (56*I)*b*d^6*x^3*PolyLog[2, (a*E^(I*(c
 + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - (56*I)*b*d^6*x^3*PolyLog[2,
-((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - 336*b*d^5*x^(5/2)*
PolyLog[3, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] + 336*b*d
^5*x^(5/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))
] - (1680*I)*b*d^4*x^2*PolyLog[4, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt
[a^2 - b^2])] + (1680*I)*b*d^4*x^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/
(I*b + Sqrt[a^2 - b^2]))] + 6720*b*d^3*x^(3/2)*PolyLog[5, (a*E^(I*(c + d*S
qrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - 6720*b*d^3*x^(3/2)*PolyLog[5, -((a
*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] + (20160*I)*b*d^2*x*Poly
Log[6, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - (20160*I)*b
*d^2*x*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] -
40320*b*d*Sqrt[x]*PolyLog[7, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2
- b^2])] + 40320*b*d*Sqrt[x]*PolyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(I*b +
 Sqrt[a^2 - b^2]))] - (40320*I)*b*PolyLog[8, (a*E^(I*(c + d*Sqrt[x])))/((-
I)*b + Sqrt[a^2 - b^2])] + (40320*I)*b*PolyLog[8, -((a*E^(I*(c + d*Sqrt[x]
)))/(I*b + Sqrt[a^2 - b^2]))])/(4*a*Sqrt[a^2 - b^2]*d^8)
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 1077, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$$

$$\Big\downarrow \text{4693}$$

$$2\int \frac{x^{7/2}}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{x^{7/2}}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}$$

$$\Big\downarrow \text{4679}$$

$$2\int \left(\frac{x^{7/2}}{a}-\frac{bx^{7/2}}{a\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{x^4}{8a}+\frac{ib\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{7/2}}{a\sqrt{b^2-a^2}d}-\frac{ib\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{7/2}}{a\sqrt{b^2-a^2}d}+\frac{7b\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^3}{a\sqrt{b^2-a^2}d^2}-\frac{7b\operatorname{PolyLo}}{a}\right.$$

input
```
Int[x^3/(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
2*(x^4/(8*a) + (I*b*x^(7/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[
-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^(7/2)*Log[1 - (I*a*E^(I*(c
+ d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (7*b*x^3*
PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a
^2 + b^2]*d^2) - (7*b*x^3*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt
[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) + ((42*I)*b*x^(5/2)*PolyLog[3, (I
*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3
) - ((42*I)*b*x^(5/2)*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^
2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3) - (210*b*x^2*PolyLog[4, (I*a*E^(I*(c
+ d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^4) + (210*b*
x^2*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqr
t[-a^2 + b^2]*d^4) - ((840*I)*b*x^(3/2)*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x
])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^5) + ((840*I)*b*x^(3/2
)*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[
-a^2 + b^2]*d^5) + (2520*b*x*PolyLog[6, (I*a*E^(I*(c + d*Sqrt[x])))/(b - S
qrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^6) - (2520*b*x*PolyLog[6, (I*a*E^
(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^6) + (
(5040*I)*b*Sqrt[x]*PolyLog[7, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 +
 b^2])])/(a*Sqrt[-a^2 + b^2]*d^7) - ((5040*I)*b*Sqrt[x]*PolyLog[7, (I*a*E^
(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^7) ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}dx$$

input
```
int(x^3/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(x^3/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^3}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

input
```
integrate(x^3/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
integral(x^3/(b*csc(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$$

input
```
integrate(x**3/(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(x**3/(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^3/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^3}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

input
```
integrate(x^3/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
integrate(x^3/(b*csc(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^3}{a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}}\,dx$$

input
```
int(x^3/(a + b/sin(c + d*x^(1/2))),x)
```

output
```
int(x^3/(a + b/sin(c + d*x^(1/2))), x)
```

Reduce [F]

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^3}{\csc\left(\sqrt{x}\,d+c\right)b+a}dx$$

input
```
int(x^3/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(x**3/(csc(sqrt(x)*d + c)*b + a),x)
```

3.42 $\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 807

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \frac{x^3}{3a}+\frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{10bx^2\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{10bx^2\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{40ibx^{3/2}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{40ibx^{3/2}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{120bx\,\mathrm{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}+\frac{120bx\,\mathrm{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}-\frac{240ib\sqrt{x}\,\mathrm{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}+\frac{240ib\sqrt{x}\,\mathrm{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}+\frac{240b\,\mathrm{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}-\frac{240b\,\mathrm{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$

output
```
1/3*x^3/a+2*I*b*x^(5/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2))
)/a/(-a^2+b^2)^(1/2)/d-2*I*b*x^(5/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^
2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d+10*b*x^2*polylog(2,I*a*exp(I*(c+d*x^(1
/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2-10*b*x^2*polylog(2,I*a*
exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2+40*I*b*x
^(3/2)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^
2)^(1/2)/d^3-40*I*b*x^(3/2)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^
2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^3-120*b*x*polylog(4,I*a*exp(I*(c+d*x^(1/2)
))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^4+120*b*x*polylog(4,I*a*exp(
I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^4-240*I*b*x^(1
/2)*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^
(1/2)/d^5+240*I*b*x^(1/2)*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)
^(1/2)))/a/(-a^2+b^2)^(1/2)/d^5+240*b*polylog(6,I*a*exp(I*(c+d*x^(1/2)))/(
b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^6-240*b*polylog(6,I*a*exp(I*(c+d
*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^6
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 644, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \frac{\sqrt{a^2-b^2}d^6x^3-6bd^5x^{5/2}\log\left(1-\frac{ae^{i(c+d\sqrt{x})}}{-ib+\sqrt{a^2-b^2}}\right)+6bd^5x^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)+30ibd^4x^2\,\mathrm{PolyLog}\left(2,\ldots\right.}{}$$

input
```
Integrate[x^2/(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(Sqrt[a^2 - b^2]*d^6*x^3 - 6*b*d^5*x^(5/2)*Log[1 - (a*E^(I*(c + d*Sqrt[x])
))/((-I)*b + Sqrt[a^2 - b^2])] + 6*b*d^5*x^(5/2)*Log[1 + (a*E^(I*(c + d*Sq
rt[x])))/(I*b + Sqrt[a^2 - b^2])] + (30*I)*b*d^4*x^2*PolyLog[2, (a*E^(I*(c
 + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - (30*I)*b*d^4*x^2*PolyLog[2,
-((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - 120*b*d^3*x^(3/2)*
PolyLog[3, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] + 120*b*d
^3*x^(3/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))
] - (360*I)*b*d^2*x*PolyLog[4, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^
2 - b^2])] + (360*I)*b*d^2*x*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(I*b +
 Sqrt[a^2 - b^2]))] + 720*b*d*Sqrt[x]*PolyLog[5, (a*E^(I*(c + d*Sqrt[x])))
/((-I)*b + Sqrt[a^2 - b^2])] - 720*b*d*Sqrt[x]*PolyLog[5, -((a*E^(I*(c + d
*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] + (720*I)*b*PolyLog[6, (a*E^(I*(c +
d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - (720*I)*b*PolyLog[6, -((a*E^(I*
(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(3*a*Sqrt[a^2 - b^2]*d^6)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$$

$$\Big\downarrow \text{4693}$$

$$2\int \frac{x^{5/2}}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{x^{5/2}}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}$$

$$\Big\downarrow \text{4679}$$

$$2\int \left(\frac{x^{5/2}}{a}-\frac{bx^{5/2}}{a\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}\right)d\sqrt{x}$$

$$\downarrow \text{2009}$$

$$2\left(\frac{x^3}{6a}+\frac{ib\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{5/2}}{a\sqrt{b^2-a^2}d}-\frac{ib\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{5/2}}{a\sqrt{b^2-a^2}d}+\frac{5b\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^2}{a\sqrt{b^2-a^2}d^2}-\frac{5b\operatorname{PolyLo}}{a}\right.$$

input
```
Int[x^2/(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
2*(x^3/(6*a) + (I*b*x^(5/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[
-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^(5/2)*Log[1 - (I*a*E^(I*(c
+ d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (5*b*x^2*
PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a
^2 + b^2]*d^2) - (5*b*x^2*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt
[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) + ((20*I)*b*x^(3/2)*PolyLog[3, (I
*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3
) - ((20*I)*b*x^(3/2)*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^
2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3) - (60*b*x*PolyLog[4, (I*a*E^(I*(c + d
*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^4) + (60*b*x*Po
lyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2
 + b^2]*d^4) - ((120*I)*b*Sqrt[x]*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x])))/(
b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^5) + ((120*I)*b*Sqrt[x]*Poly
Log[5, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 +
 b^2]*d^5) + (120*b*PolyLog[6, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2
+ b^2])])/(a*Sqrt[-a^2 + b^2]*d^6) - (120*b*PolyLog[6, (I*a*E^(I*(c + d*Sq
rt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^6))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}dx$$

input
```
int(x^2/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(x^2/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^2}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

input
```
integrate(x^2/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
integral(x^2/(b*csc(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x^2}{a + b \csc\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^2}{a + b \csc\left(c + d\sqrt{x}\right)}\,dx$$

input
```
integrate(x**2/(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(x**2/(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a + b \csc\left(c + d\sqrt{x}\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x^2}{a + b \csc\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^2}{b \csc\left(d\sqrt{x} + c\right) + a}\,dx$$

input
```
integrate(x^2/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
integrate(x^2/(b*csc(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \int \frac{x^2}{a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}}\,dx$$

input
```
int(x^2/(a + b/sin(c + d*x^(1/2))),x)
```

output
```
int(x^2/(a + b/sin(c + d*x^(1/2))), x)
```

Reduce [F]

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \int \frac{x^2}{\csc\left(\sqrt{x}\,d+c\right)b+a}dx$$

input
```
int(x^2/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(x**2/(csc(sqrt(x)*d + c)*b + a),x)
```

3.43 $\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 539

$$
\begin{aligned}
\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = {} & \frac{x^2}{2a}+\frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d} \\
& -\frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d} \\
& +\frac{6bx\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^2} \\
& -\frac{6bx\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^2} \\
& +\frac{12ib\sqrt{x}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^3} \\
& -\frac{12ib\sqrt{x}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^3} \\
& -\frac{12b\,\mathrm{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^4} \\
& +\frac{12b\,\mathrm{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^4}
\end{aligned}
$$

output
```
1/2*x^2/a+2*I*b*x^(3/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2))
)/a/(-a^2+b^2)^(1/2)/d-2*I*b*x^(3/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^
2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d+6*b*x*polylog(2,I*a*exp(I*(c+d*x^(1/2)
))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2-6*b*x*polylog(2,I*a*exp(I*
(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2+12*I*b*x^(1/2)
*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/
2)/d^3-12*I*b*x^(1/2)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/
2)))/a/(-a^2+b^2)^(1/2)/d^3-12*b*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a
^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^4+12*b*polylog(4,I*a*exp(I*(c+d*x^(1/
2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^4
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.81

$$\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$$

$$= \frac{\sqrt{a^2-b^2}\,d^4x^2 - 4bd^3x^{3/2}\log\left(1-\frac{ae^{i(c+d\sqrt{x})}}{-ib+\sqrt{a^2-b^2}}\right) + 4bd^3x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right) + 12ibd^2x\,\mathrm{PolyLog}\left(2,\frac{ae}{-i}\right.}{}$$

input
```
Integrate[x/(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(Sqrt[a^2 - b^2]*d^4*x^2 - 4*b*d^3*x^(3/2)*Log[1 - (a*E^(I*(c + d*Sqrt[x])
))/((-I)*b + Sqrt[a^2 - b^2])] + 4*b*d^3*x^(3/2)*Log[1 + (a*E^(I*(c + d*Sq
rt[x])))/(I*b + Sqrt[a^2 - b^2])] + (12*I)*b*d^2*x*PolyLog[2, (a*E^(I*(c +
 d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - (12*I)*b*d^2*x*PolyLog[2, -((a
*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - 24*b*d*Sqrt[x]*PolyLog
[3, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] + 24*b*d*Sqrt[x]
*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - (24*I)
*b*PolyLog[4, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] + (24*
I)*b*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(2*
a*Sqrt[a^2 - b^2]*d^4)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}dx$$

$$\Big\downarrow \text{ 4693}$$

$$2\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}$$

$$\Big\downarrow \text{ 4679}$$

$$2\int \left(\frac{x^{3/2}}{a}-\frac{bx^{3/2}}{a\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{6b\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}}+\frac{6b\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}}+\frac{6ib\sqrt{x}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}-\frac{6ib\sqrt{x}\,\text{PolyLo}}{ad^3\sqrt{}}\right.$$

input
```
Int[x/(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
2*(x^2/(4*a) + (I*b*x^(3/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[
-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^(3/2)*Log[1 - (I*a*E^(I*(c
+ d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (3*b*x*Po
lyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2
 + b^2]*d^2) - (3*b*x*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^
2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) + ((6*I)*b*Sqrt[x]*PolyLog[3, (I*a*E^
(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3) - (
(6*I)*b*Sqrt[x]*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^
2])])/(a*Sqrt[-a^2 + b^2]*d^3) - (6*b*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])
))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^4) + (6*b*PolyLog[4, (I*
a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^4)
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}dx$$

input
```
int(x/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(x/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \int \frac{x}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

input
```
integrate(x/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
integral(x/(b*csc(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$$

input
```
integrate(x/(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(x/(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \int \frac{x}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

input
```
integrate(x/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
integrate(x/(b*csc(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \int \frac{x}{a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}}\,dx$$

input
```
int(x/(a + b/sin(c + d*x^(1/2))),x)
```

output
```
int(x/(a + b/sin(c + d*x^(1/2))), x)
```

Reduce [F]

$$\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x}{\csc\left(\sqrt{x}\,d+c\right)b+a}dx$$

input
```
int(x/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(x/(csc(sqrt(x)*d + c)*b + a),x)
```

3.44 $\int \frac{1}{x(a+b\csc(c+d\sqrt{x}))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx = \text{Int}\left(\frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)},x\right)$$

output
```
Defer(Int)(1/x/(a+b*csc(c+d*x^(1/2))),x)
```

Mathematica [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx = \int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

input
```
Integrate[1/(x*(a + b*Csc[c + d*Sqrt[x]])),x]
```

output
```
Integrate[1/(x*(a + b*Csc[c + d*Sqrt[x]])), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

$$\Big\downarrow \text{4695}$$

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}dx$$

input
```
Int[1/(x*(a + b*Csc[c + d*Sqrt[x]])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}dx$$

input
```
int(1/x/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(1/x/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
integral(1/(b*x*csc(d*sqrt(x) + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

input
```
integrate(1/x/(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(1/(x*(a + b*csc(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 242, normalized size of antiderivative = 12.10

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
-(2*a*b*integrate((2*b*cos(d*sqrt(x) + c)^2 + a*cos(d*sqrt(x) + c)*sin(2*d
*sqrt(x) + 2*c) - a*cos(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 2*b*sin(d*
sqrt(x) + c)^2 + a*sin(d*sqrt(x) + c))/((a^3*cos(2*d*sqrt(x) + 2*c)^2 + 4*
a*b^2*cos(d*sqrt(x) + c)^2 + 4*a^2*b*cos(d*sqrt(x) + c)*sin(2*d*sqrt(x) +
2*c) + a^3*sin(2*d*sqrt(x) + 2*c)^2 + 4*a*b^2*sin(d*sqrt(x) + c)^2 + 4*a^2
*b*sin(d*sqrt(x) + c) + a^3 - 2*(2*a^2*b*sin(d*sqrt(x) + c) + a^3)*cos(2*d
*sqrt(x) + 2*c))*x), x) - log(x))/a
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
integrate(1/((b*csc(d*sqrt(x) + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 15.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}\right)}\,dx$$

input
```
int(1/(x*(a + b/sin(c + d*x^(1/2)))),x)
```

output
```
int(1/(x*(a + b/sin(c + d*x^(1/2)))), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\csc\left(\sqrt{x}\,d+c\right)bx+ax}dx$$

input
```
int(1/x/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(1/(csc(sqrt(x)*d + c)*b*x + a*x),x)
```

3.45 $\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx = -\frac{a}{x} + b\text{Int}\left(\frac{\csc\left(c+d\sqrt{x}\right)}{x^2},x\right)$$

output
```
-a/x+b*Defer(Int)(csc(c+d*x^(1/2))/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx = \int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

input
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^2,x]
```

output
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(\frac{a}{x^2}+\frac{b\csc\left(c+d\sqrt{x}\right)}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$b\int \frac{\csc\left(c+d\sqrt{x}\right)}{x^2}dx-\frac{a}{x}$$

input
```
Int[(a + b*Csc[c + d*Sqrt[x]])/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}dx$$

input
```
int((a+b*csc(c+d*x^(1/2)))/x^2,x)
```

output
```
int((a+b*csc(c+d*x^(1/2)))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx = \int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^2,x, algorithm="fricas")
```

output
```
integral((b*csc(d*sqrt(x) + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx = \int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

input
```
integrate((a+b*csc(c+d*x**(1/2)))/x**2,x)
```

output
```
Integral((a + b*csc(c + d*sqrt(x)))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 111, normalized size of antiderivative = 6.17

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^2,x, algorithm="maxima")
```

output
```
((b*integrate(sin(d*sqrt(x) + c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) +
c)^2 + 2*cos(d*sqrt(x) + c) + 1)*x^2), x) + b*integrate(sin(d*sqrt(x) + c)
/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1)
*x^2), x))*x - a)/x
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}}{x^2}\,dx$$

input
```
int((a + b/sin(c + d*x^(1/2)))/x^2,x)
```

output
```
int((a + b/sin(c + d*x^(1/2)))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\frac{\left(\int \frac{\csc\left(\sqrt{x}\,d+c\right)}{x^2}dx\right)bx-a}{x}$$

input
```
int((a+b*csc(c+d*x^(1/2)))/x^2,x)
```

output
```
(int(csc(sqrt(x)*d + c)/x**2,x)*b*x - a)/x
```

3.46 $\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 3205

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Too large to display}$$

output
```
-2*I*b^2*x^(7/2)/a^2/(a^2-b^2)/d+14*b^2*x^3*ln(1+a*exp(I*(c+d*x^(1/2)))/(I
*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+14*b^2*x^3*ln(1+a*exp(I*(c+d*x^(1/2
)))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2-5040*b^3*x*polylog(6,I*a*exp(
I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^6+420*b^2*x^
2*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d
^4-5040*b^2*x*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2
/(a^2-b^2)/d^6-14*b^3*x^3*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)
^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+5040*b^3*x*polylog(6,I*a*exp(I*(c+d*x^(1
/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^6+420*b^2*x^2*polylog(3
,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4-5040*b^2
*x*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/
d^6+10080*b*x*polylog(6,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2
/(-a^2+b^2)^(1/2)/d^6+28*b*x^3*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2
+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2-10080*b*x*polylog(6,I*a*exp(I*(c+d*
x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^6-28*b*x^3*polylog(
2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2+
840*b*x^2*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a
^2+b^2)^(1/2)/d^4-840*b*x^2*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^
2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^4+14*b^3*x^3*polylog(2,I*a*exp(I*(c+d*x^
(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-420*b^3*x^2*poly...
```

Mathematica [A] (warning: unable to verify)

Time = 13.31 (sec) , antiderivative size = 3831, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^3/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
(x^4*Csc[c + d*Sqrt[x]]^2*(b + a*Sin[c + d*Sqrt[x]])^2)/(4*a^2*(a + b*Csc[
c + d*Sqrt[x]])^2) - ((2*I)*b*E^(I*c)*Csc[c + d*Sqrt[x]]^2*(2*b*E^(I*c)*x^
(7/2) - ((-1 + E^((2*I)*c))*((-7*I)*b*d^6*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^
3*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((
2*I)*c)])] + (2*I)*a^2*d^7*E^(I*c)*x^(7/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x
])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - I*b^2*d^7*E^(I*c)*x^
(7/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*
E^((2*I)*c)])] - (7*I)*b*d^6*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^3*Log[1 + (a*
E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] -
(2*I)*a^2*d^7*E^(I*c)*x^(7/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(
I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + I*b^2*d^7*E^(I*c)*x^(7/2)*Log[1 +
 (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])
] - 7*d^5*(6*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b
^2*d*E^(I*c)*Sqrt[x])*x^(5/2)*PolyLog[2, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*
E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 7*d^5*(-6*b*Sqrt[(a^2 - b^2)
*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*x^(5/2)*P
olyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^
((2*I)*c)]))] - (210*I)*b*d^4*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^2*PolyLog[3,
 (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)
])] + (84*I)*a^2*d^5*E^(I*c)*x^(5/2)*PolyLog[3, (I*a*E^(I*(2*c + d*Sqrt...
```

Rubi [A] (verified)

Time = 4.64 (sec) , antiderivative size = 3207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

$$\Big\downarrow \text{4693}$$

$$2\int \frac{x^{7/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{x^{7/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{4679}$$

$$2\int \left(-\frac{2bx^{7/2}}{a^2\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}+\frac{x^{7/2}}{a^2}+\frac{b^2x^{7/2}}{a^2\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{x^4}{8a^2}+\frac{2ib\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{7/2}}{a^2\sqrt{b^2-a^2}d}-\frac{ib^3\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{7/2}}{a^2\left(b^2-a^2\right)^{3/2}d}-\frac{2ib\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{7/2}}{a^2\sqrt{b^2-a^2}d}+\frac{ib^3\log\left(\right.}{a}\right.$$

input
```
Int[x^3/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
2*(((-I)*b^2*x^(7/2))/(a^2*(a^2 - b^2)*d) + x^4/(8*a^2) + (7*b^2*x^3*Log[1
 + (a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^
2) + (7*b^2*x^3*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2])]
)/(a^2*(a^2 - b^2)*d^2) - (I*b^3*x^(7/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])
))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^(7/2)*
Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^
2 + b^2]*d) + (I*b^3*x^(7/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt
[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x^(7/2)*Log[1 - (I*a
*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d)
- ((42*I)*b^2*x^(5/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a
^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - ((42*I)*b^2*x^(5/2)*PolyLog[2, -((a*
E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) -
(7*b^3*x^3*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])
/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (14*b*x^3*PolyLog[2, (I*a*E^(I*(c + d*Sqrt
[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (7*b^3*x^3*Po
lyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 +
 b^2)^(3/2)*d^2) - (14*b*x^3*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + S
qrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (210*b^2*x^2*PolyLog[3, -(
(a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4)
 + (210*b^2*x^2*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

input
```
int(x^3/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(x^3/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^3}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

input
```
integrate(x^3/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(x^3/(b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^2),
x)
```

Sympy [F]

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
integrate(x**3/(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(x**3/(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^3}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

input
```
integrate(x^3/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate(x^3/(b*csc(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^3}{\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

input
```
int(x^3/(a + b/sin(c + d*x^(1/2)))^2,x)
```

output
```
int(x^3/(a + b/sin(c + d*x^(1/2)))^2, x)
```

Reduce [F]

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{too large to display}$$

input
```
int(x^3/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
(117573120*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt(
- a**2 + b**2))*sin(sqrt(x)*d + c)*a**9 - 143700480*sqrt( - a**2 + b**2)*a
tan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*sin(sqrt(x)*d + c
)*a**7*b**2 + 50077440*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b
 + a)/sqrt( - a**2 + b**2))*sin(sqrt(x)*d + c)*a**5*b**4 - 5241600*sqrt( -
 a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*si
n(sqrt(x)*d + c)*a**3*b**6 + 80640*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)
*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*sin(sqrt(x)*d + c)*a*b**8 + 117573
120*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2
+ b**2))*a**8*b - 143700480*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)
/2)*b + a)/sqrt( - a**2 + b**2))*a**6*b**3 + 50077440*sqrt( - a**2 + b**2)
*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*a**4*b**5 - 524
1600*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2
 + b**2))*a**2*b**7 + 80640*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)
/2)*b + a)/sqrt( - a**2 + b**2))*b**9 + 58786560*sqrt(x)*cos(sqrt(x)*d + c
)*a**10*d + 1088640*sqrt(x)*cos(sqrt(x)*d + c)*a**8*b**2*d**3*x - 10124352
0*sqrt(x)*cos(sqrt(x)*d + c)*a**8*b**2*d + 6048*sqrt(x)*cos(sqrt(x)*d + c)
*a**6*b**4*d**5*x**2 - 1632960*sqrt(x)*cos(sqrt(x)*d + c)*a**6*b**4*d**3*x
 + 51166080*sqrt(x)*cos(sqrt(x)*d + c)*a**6*b**4*d + 16*sqrt(x)*cos(sqrt(x
)*d + c)*a**4*b**6*d**7*x**3 - 7728*sqrt(x)*cos(sqrt(x)*d + c)*a**4*b**...
```

3.47 $\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 2385

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Too large to display}$$

output

```
-2*I*b^2*x^(5/2)/a^2/(a^2-b^2)/d+120*b^3*x*polylog(4,I*a*exp(I*(c+d*x^(1/2
)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4+10*b^3*x^2*polylog(2,I*
a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-10*b
^3*x^2*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+
b^2)^(3/2)/d^2+10*b^2*x^2*ln(1+a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)
))/a^2/(a^2-b^2)/d^2+10*b^2*x^2*ln(1+a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)
^(1/2)))/a^2/(a^2-b^2)/d^2+120*b^2*x*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(I*
b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4-120*b^3*x*polylog(4,I*a*exp(I*(c+d*x
^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4+120*b^2*x*polylog(
3,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4-240*b*x
*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(
1/2)/d^4-20*b*x^2*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))
/a^2/(-a^2+b^2)^(1/2)/d^2+20*b*x^2*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b-(
-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2+240*b*x*polylog(4,I*a*exp(I*(c+
d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^4-2*b^2*x^(5/2)*c
os(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*sin(c+d*x^(1/2)))-480*I*b*x^(1/2)*polyl
og(5,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d
^5-240*I*b^3*x^(1/2)*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2
)))/a^2/(-a^2+b^2)^(3/2)/d^5-4*I*b*x^(5/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(
b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d-40*I*b^3*x^(3/2)*polylog(3,...
```

Mathematica [A] (warning: unable to verify)

Time = 12.54 (sec) , antiderivative size = 2806, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Result too large to show}$$

input

```
Integrate[x^2/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
(x^3*Csc[c + d*Sqrt[x]]^2*(b + a*Sin[c + d*Sqrt[x]])^2)/(3*a^2*(a + b*Csc[
c + d*Sqrt[x]])^2) - ((2*I)*b*Csc[c + d*Sqrt[x]]^2*((2*b*d^5*E^((2*I)*c)*x
^(5/2))/(-1 + E^((2*I)*c)) + ((5*I)*b*d^4*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^
2*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((
2*I)*c)])] - (2*I)*a^2*d^5*E^(I*c)*x^(5/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x
])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + I*b^2*d^5*E^(I*c)*x^
(5/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*
E^((2*I)*c)])] + (5*I)*b*d^4*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^2*Log[1 + (a*
E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] +
(2*I)*a^2*d^5*E^(I*c)*x^(5/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(
I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - I*b^2*d^5*E^(I*c)*x^(5/2)*Log[1 +
 (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])
] + 5*d^3*(4*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b
^2*d*E^(I*c)*Sqrt[x])*x^(3/2)*PolyLog[2, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*
E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 5*d^3*(4*b*Sqrt[(a^2 - b^2)*
E^((2*I)*c)] + 2*a^2*d*E^(I*c)*Sqrt[x] - b^2*d*E^(I*c)*Sqrt[x])*x^(3/2)*Po
lyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^(
(2*I)*c)]))] + (60*I)*b*d^2*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x*PolyLog[3, (I*
a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])]
- (40*I)*a^2*d^3*E^(I*c)*x^(3/2)*PolyLog[3, (I*a*E^(I*(2*c + d*Sqrt[x])...
```

Rubi [A] (verified)

Time = 3.54 (sec) , antiderivative size = 2387, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

$$\Big\downarrow \text{4693}$$

$$2\int \frac{x^{5/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{x^{5/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{4679}$$

$$2\int \left(-\frac{2bx^{5/2}}{a^2\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}+\frac{x^{5/2}}{a^2}+\frac{b^2x^{5/2}}{a^2\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{ix^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}+\frac{ix^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}-\frac{5x^2\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}+\frac{5x^2\operatorname{PolyL}}{a^2(}\right.$$

input
```
Int[x^2/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
2*(((-I)*b^2*x^(5/2))/(a^2*(a^2 - b^2)*d) + x^3/(6*a^2) + (5*b^2*x^2*Log[1
 + (a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^
2) + (5*b^2*x^2*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2])]
)/(a^2*(a^2 - b^2)*d^2) - (I*b^3*x^(5/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])
))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^(5/2)*
Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^
2 + b^2]*d) + (I*b^3*x^(5/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt
[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x^(5/2)*Log[1 - (I*a
*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d)
- ((20*I)*b^2*x^(3/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a
^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - ((20*I)*b^2*x^(3/2)*PolyLog[2, -((a*
E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) -
(5*b^3*x^2*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])
/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (10*b*x^2*PolyLog[2, (I*a*E^(I*(c + d*Sqrt
[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (5*b^3*x^2*Po
lyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 +
 b^2)^(3/2)*d^2) - (10*b*x^2*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + S
qrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (60*b^2*x*PolyLog[3, -((a*
E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) +
(60*b^2*x*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

input
```
int(x^2/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(x^2/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

input
```
integrate(x^2/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(x^2/(b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^2),
x)
```

Sympy [F]

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
integrate(x**2/(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(x**2/(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

input
```
integrate(x^2/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate(x^2/(b*csc(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

input
```
int(x^2/(a + b/sin(c + d*x^(1/2)))^2,x)
```

output
```
int(x^2/(a + b/sin(c + d*x^(1/2)))^2, x)
```

Reduce [F]

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{too large to display}$$

input
```
int(x^2/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
(2*(233280*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt(
- a**2 + b**2))*sin(sqrt(x)*d + c)*a**7 - 233280*sqrt( - a**2 + b**2)*atan
((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*sin(sqrt(x)*d + c)*a
**5*b**2 + 50400*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/
sqrt( - a**2 + b**2))*sin(sqrt(x)*d + c)*a**3*b**4 - 1440*sqrt( - a**2 + b
**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*sin(sqrt(x)
*d + c)*a*b**6 + 233280*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*
b + a)/sqrt( - a**2 + b**2))*a**6*b - 233280*sqrt( - a**2 + b**2)*atan((ta
n((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*a**4*b**3 + 50400*sqrt(
- a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*a
**2*b**5 - 1440*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/s
qrt( - a**2 + b**2))*b**7 + 116640*sqrt(x)*cos(sqrt(x)*d + c)*a**8*d + 216
0*sqrt(x)*cos(sqrt(x)*d + c)*a**6*b**2*d**3*x - 174960*sqrt(x)*cos(sqrt(x)
*d + c)*a**6*b**2*d + 12*sqrt(x)*cos(sqrt(x)*d + c)*a**4*b**4*d**5*x**2 -
2760*sqrt(x)*cos(sqrt(x)*d + c)*a**4*b**4*d**3*x + 64080*sqrt(x)*cos(sqrt(
x)*d + c)*a**4*b**4*d - 12*sqrt(x)*cos(sqrt(x)*d + c)*a**2*b**6*d**5*x**2
+ 600*sqrt(x)*cos(sqrt(x)*d + c)*a**2*b**6*d**3*x - 5760*sqrt(x)*cos(sqrt(
x)*d + c)*a**2*b**6*d - 19440*cos(sqrt(x)*d + c)*a**7*b*d**2*x - 180*cos(s
qrt(x)*d + c)*a**5*b**3*d**4*x**2 + 27000*cos(sqrt(x)*d + c)*a**5*b**3*d**
2*x + 210*cos(sqrt(x)*d + c)*a**3*b**5*d**4*x**2 - 7920*cos(sqrt(x)*d +...
```

3.48 $\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1565

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Too large to display}$$

output
```
-2*I*b^2*x^(3/2)/a^2/(a^2-b^2)/d-6*b^3*x*polylog(2,I*a*exp(I*(c+d*x^(1/2))
)/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+12*b*x*polylog(2,I*a*exp(
I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2-12*b*x*pol
ylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)
/d^2+6*b^3*x*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/
(-a^2+b^2)^(3/2)/d^2+6*b^2*x*ln(1+a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1
/2)))/a^2/(a^2-b^2)/d^2+6*b^2*x*ln(1+a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)
^(1/2)))/a^2/(a^2-b^2)/d^2+2*I*b^3*x^(3/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(
b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-2*b^2*x^(3/2)*cos(c+d*x^(1/2))
/a/(a^2-b^2)/d/(b+a*sin(c+d*x^(1/2)))-2*I*b^3*x^(3/2)*ln(1-I*a*exp(I*(c+d*
x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-4*I*b*x^(3/2)*ln(1-
I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d-12*I
*b^2*x^(1/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2/
(a^2-b^2)/d^3-12*I*b^2*x^(1/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2
-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-12*I*b^3*x^(1/2)*polylog(3,I*a*exp(I*(c+d*
x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-24*I*b*x^(1/2)*po
lylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2
)/d^3+4*I*b*x^(3/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^
2/(-a^2+b^2)^(1/2)/d+12*I*b^3*x^(1/2)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(
b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+24*I*b*x^(1/2)*polylog(3,...
```

Mathematica [A] (warning: unable to verify)

Time = 11.10 (sec) , antiderivative size = 1729, normalized size of antiderivative = 1.10

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[x/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
(Csc[c + d*Sqrt[x]]^2*(b + a*Sin[c + d*Sqrt[x]])*(x^2*(b + a*Sin[c + d*Sqr
t[x]]) - ((4*I)*b*((2*b*d^3*E^((2*I)*c)*x^(3/2))/(-1 + E^((2*I)*c)) + ((3*
I)*b*d^2*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])
))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (2*I)*a^2*d^3*E^(I*c)*
x^(3/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2
)*E^((2*I)*c)])] + I*b^2*d^3*E^(I*c)*x^(3/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt
[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (3*I)*b*d^2*Sqrt[(
a^2 - b^2)*E^((2*I)*c)]*x*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c)
 + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (2*I)*a^2*d^3*E^(I*c)*x^(3/2)*Log[1 +
 (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])
] - I*b^2*d^3*E^(I*c)*x^(3/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(
I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 3*d*(2*b*Sqrt[(a^2 - b^2)*E^((2*I
)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*Sqrt[x]*PolyLog[2
, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c
)])] + 3*d*(2*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] + 2*a^2*d*E^(I*c)*Sqrt[x] -
b^2*d*E^(I*c)*Sqrt[x])*Sqrt[x]*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(I
*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + (6*I)*b*Sqrt[(a^2 - b^2)*E
^((2*I)*c)]*PolyLog[3, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(
a^2 - b^2)*E^((2*I)*c)])] - (12*I)*a^2*d*E^(I*c)*Sqrt[x]*PolyLog[3, (I*a*E
^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] ...
```

Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 1567, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

$$\Big\downarrow \text{4693}$$

$$2\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{4679}$$

$$2\int \left(\frac{x^{3/2}b^2}{a^2\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}-\frac{2x^{3/2}b}{a^2\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}+\frac{x^{3/2}}{a^2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{ix^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}+\frac{ix^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}-\frac{3x\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}+\frac{3x\,\text{PolyLog}}{a^2\left(b^2\right.}\right.$$

input
```
Int[x/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
2*(((-I)*b^2*x^(3/2))/(a^2*(a^2 - b^2)*d) + x^2/(4*a^2) + (3*b^2*x*Log[1 +
 (a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2)
 + (3*b^2*x*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2])])/(a
^2*(a^2 - b^2)*d^2) - (I*b^3*x^(3/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(
b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^(3/2)*Log[
1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 +
b^2]*d) + (I*b^3*x^(3/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^
2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x^(3/2)*Log[1 - (I*a*E^(
I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((
6*I)*b^2*Sqrt[x]*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 -
b^2]))])/(a^2*(a^2 - b^2)*d^3) - ((6*I)*b^2*Sqrt[x]*PolyLog[2, -((a*E^(I*(
c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (3*b^3
*x*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-
a^2 + b^2)^(3/2)*d^2) + (6*b*x*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b -
 Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (3*b^3*x*PolyLog[2, (I*a
*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d
^2) - (6*b*x*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])
])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (6*b^2*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x]
)))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) + (6*b^2*PolyLog[3, -
((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

input
```
int(x/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(x/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

input
```
integrate(x/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(x/(b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^2), x)
```

Sympy [F]

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
integrate(x/(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(x/(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

input
```
integrate(x/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate(x/(b*csc(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

input
```
int(x/(a + b/sin(c + d*x^(1/2)))^2,x)
```

output
```
int(x/(a + b/sin(c + d*x^(1/2)))^2, x)
```

Reduce [F]

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{too large to display}$$

input
```
int(x/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output

```
(864*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2
 + b**2))*sin(sqrt(x)*d + c)*a**5 - 672*sqrt( - a**2 + b**2)*atan((tan((sq
rt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*sin(sqrt(x)*d + c)*a**3*b**2
+ 48*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2
 + b**2))*sin(sqrt(x)*d + c)*a*b**4 + 864*sqrt( - a**2 + b**2)*atan((tan((
sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*a**4*b - 672*sqrt( - a**2 +
 b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*a**2*b**3
 + 48*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**
2 + b**2))*b**5 + 432*sqrt(x)*cos(sqrt(x)*d + c)*a**6*d + 8*sqrt(x)*cos(sq
rt(x)*d + c)*a**4*b**2*d**3*x - 552*sqrt(x)*cos(sqrt(x)*d + c)*a**4*b**2*d
 - 8*sqrt(x)*cos(sqrt(x)*d + c)*a**2*b**4*d**3*x + 120*sqrt(x)*cos(sqrt(x)
*d + c)*a**2*b**4*d - 72*cos(sqrt(x)*d + c)*a**5*b*d**2*x + 84*cos(sqrt(x)
*d + c)*a**3*b**3*d**2*x - 12*cos(sqrt(x)*d + c)*a*b**5*d**2*x - 12*sqrt(x
)*sin(sqrt(x)*d + c)*a**5*b*d**3*x + 16*sqrt(x)*sin(sqrt(x)*d + c)*a**3*b*
*3*d**3*x - 4*sqrt(x)*sin(sqrt(x)*d + c)*a*b**5*d**3*x + 432*sqrt(x)*a**6*
d - 4*sqrt(x)*a**4*b**2*d**3*x - 768*sqrt(x)*a**4*b**2*d + 4*sqrt(x)*a**2*
b**4*d**3*x + 360*sqrt(x)*a**2*b**4*d - 24*sqrt(x)*b**6*d - 72*int(sqrt(x)
/(tan((sqrt(x)*d + c)/2)**4*b**2 + 4*tan((sqrt(x)*d + c)/2)**3*a*b + 4*tan
((sqrt(x)*d + c)/2)**2*a**2 + 2*tan((sqrt(x)*d + c)/2)**2*b**2 + 4*tan((sq
rt(x)*d + c)/2)*a*b + b**2),x)*sin(sqrt(x)*d + c)*a**7*b*d**3 + 96*int(...
```

3.49 $\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/x/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 43.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
Integrate[1/(x*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

output
```
Integrate[1/(x*(a + b*Csc[c + d*Sqrt[x]])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$\Big\downarrow \text{4695}$$

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

input
```
Int[1/(x*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

input
```
int(1/x/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(1/x/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2 x}\,dx$$

input
```
integrate(1/x/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x*csc(d*sqrt(x) + c)^2 + 2*a*b*x*csc(d*sqrt(x) + c) + a^2*
x), x)
```

Sympy [N/A]

Not integrable

Time = 4.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
integrate(1/x/(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(1/(x*(a + b*csc(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 12.43 (sec) , antiderivative size = 4411, normalized size of antiderivative = 220.55

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
-((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4*
b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 -
2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*
d*cos(d*sqrt(x))^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*sin(2*d*s
qrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(c)*sin(d*sqrt(x)) + 4*((
a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*
sin(c)^2)*d*sin(d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqr
t(x))*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*c
os(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x)) - (
a^6*b^2 - a^4*b^4)*d*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (
a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*sin(d*sqrt(x)))*cos(2*d*sqrt(x)) - 2
*(a^6*b^2*d*cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c
) + 2*(a^7*b - a^5*b^3)*d*cos(c)*sin(d*sqrt(x)) + 2*(a^7*b - a^5*b^3)*d*co
s(d*sqrt(x))*sin(c) + (a^8 - a^6*b^2)*d)*cos(2*d*sqrt(x) + 2*c) - 2*(2*((a
^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d
*cos(d*sqrt(x)) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*
b^5)*cos(2*c)*sin(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*sin(2*c))*s
in(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^6*b^2*d*cos(2
*d*sqrt(x))*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a^
7*b - a^5*b^3)*d*sin(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x*integ...
```

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*csc(d*sqrt(x) + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 15.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

input
```
int(1/(x*(a + b/sin(c + d*x^(1/2)))^2),x)
```

output
```
int(1/(x*(a + b/sin(c + d*x^(1/2)))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\csc\left(\sqrt{x}\,d+c\right)^2b^2x+2\csc\left(\sqrt{x}\,d+c\right)abx+a^2x}dx$$

input
```
int(1/x/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(1/(csc(sqrt(x)*d + c)**2*b**2*x + 2*csc(sqrt(x)*d + c)*a*b*x + a**2*x)
,x)
```

3.50 $\int \frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 39.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
Integrate[1/(x^2*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

output
```
Integrate[1/(x^2*(a + b*Csc[c + d*Sqrt[x]])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2} \, dx$$

$$\Big\downarrow \text{ 4695}$$

$$\int \frac{1}{x^2 \left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2}dx$$

input
```
Int[1/(x^2*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2 \left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2}dx$$

input
```
int(1/x^2/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(1/x^2/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2 x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^2*csc(d*sqrt(x) + c)^2 + 2*a*b*x^2*csc(d*sqrt(x) + c) +
a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 9.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
integrate(1/x**2/(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(1/(x**2*(a + b*csc(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 18.00 (sec) , antiderivative size = 4411, normalized size of antiderivative = 220.55

$$\int \frac{1}{x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
-((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4*
b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 -
2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*
d*cos(d*sqrt(x))^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*sin(2*d*s
qrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(c)*sin(d*sqrt(x)) + 4*((
a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*
sin(c)^2)*d*sin(d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqr
t(x))*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*c
os(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x)) - (
a^6*b^2 - a^4*b^4)*d*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (
a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*sin(d*sqrt(x)))*cos(2*d*sqrt(x)) - 2
*(a^6*b^2*d*cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c
) + 2*(a^7*b - a^5*b^3)*d*cos(c)*sin(d*sqrt(x)) + 2*(a^7*b - a^5*b^3)*d*co
s(d*sqrt(x))*sin(c) + (a^8 - a^6*b^2)*d)*cos(2*d*sqrt(x) + 2*c) - 2*(2*((a
^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d
*cos(d*sqrt(x)) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*
b^5)*cos(2*c)*sin(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*sin(2*c))*s
in(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^6*b^2*d*cos(2
*d*sqrt(x))*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a^
7*b - a^5*b^3)*d*sin(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x^2*int...
```

Giac [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(a+b\csc \left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{\left(b\csc \left(d\sqrt{x}+c\right)+a\right)^2 x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*csc(d*sqrt(x) + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 15.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2 \left(a+b\csc \left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{x^2 \left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

input
```
int(1/(x^2*(a + b/sin(c + d*x^(1/2)))^2),x)
```

output
```
int(1/(x^2*(a + b/sin(c + d*x^(1/2)))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{1}{x^2 \left(a+b\csc \left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{\csc \left(\sqrt{x}\,d+c\right)^2 b^2 x^2+2\csc \left(\sqrt{x}\,d+c\right) ab\,x^2+a^2x^2}dx$$

input
```
int(1/x^2/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(1/(csc(sqrt(x)*d + c)**2*b**2*x**2 + 2*csc(sqrt(x)*d + c)*a*b*x**2 + a
**2*x**2),x)
```

3.51 $\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 258

$$\begin{aligned}\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx &= \frac{2}{5}ax^{5/2}-\frac{4bx^2\operatorname{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}\\
&+\frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\
&-\frac{24bx\operatorname{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{24bx\operatorname{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\
&-\frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}\\
&+\frac{48b\operatorname{PolyLog}\left(5,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{48b\operatorname{PolyLog}\left(5,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}\end{aligned}$$

output
```
2/5*a*x^(5/2)-4*b*x^2*arctanh(exp(I*(c+d*x^(1/2))))/d+8*I*b*x^(3/2)*polylo
g(2,-exp(I*(c+d*x^(1/2))))/d^2-8*I*b*x^(3/2)*polylog(2,exp(I*(c+d*x^(1/2))
))/d^2-24*b*x*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3+24*b*x*polylog(3,exp(I*
(c+d*x^(1/2))))/d^3-48*I*b*x^(1/2)*polylog(4,-exp(I*(c+d*x^(1/2))))/d^4+48
*I*b*x^(1/2)*polylog(4,exp(I*(c+d*x^(1/2))))/d^4+48*b*polylog(5,-exp(I*(c+
d*x^(1/2))))/d^5-48*b*polylog(5,exp(I*(c+d*x^(1/2))))/d^5
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.11

$$\int x^{3/2}(a+b\csc(c+d\sqrt{x}))\,dx=\frac{2\left(ad^5x^{5/2}+5bd^4x^2\log\left(1-e^{i(c+d\sqrt{x})}\right)-5bd^4x^2\log\left(1+e^{i(c+d\sqrt{x})}\right)+20ibd^3x^{3/2}\,\mathrm{I}\right.}{}$$

input
```
Integrate[x^(3/2)*(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(2*(a*d^5*x^(5/2) + 5*b*d^4*x^2*Log[1 - E^(I*(c + d*Sqrt[x]))] - 5*b*d^4*x
^2*Log[1 + E^(I*(c + d*Sqrt[x]))] + (20*I)*b*d^3*x^(3/2)*PolyLog[2, -E^(I*
(c + d*Sqrt[x]))] - (20*I)*b*d^3*x^(3/2)*PolyLog[2, E^(I*(c + d*Sqrt[x]))]
 - 60*b*d^2*x*PolyLog[3, -E^(I*(c + d*Sqrt[x]))] + 60*b*d^2*x*PolyLog[3, E
^(I*(c + d*Sqrt[x]))] - (120*I)*b*d*Sqrt[x]*PolyLog[4, -E^(I*(c + d*Sqrt[x
]))] + (120*I)*b*d*Sqrt[x]*PolyLog[4, E^(I*(c + d*Sqrt[x]))] + 120*b*PolyL
og[5, -E^(I*(c + d*Sqrt[x]))] - 120*b*PolyLog[5, E^(I*(c + d*Sqrt[x]))]))/
(5*d^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+b\csc(c+d\sqrt{x}))\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(ax^{3/2}+bx^{3/2}\csc(c+d\sqrt{x})\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}ax^{5/2} - \frac{4bx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{48b\,\text{PolyLog}\left(5, -e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{48b\,\text{PolyLog}\left(5, e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{48ib\sqrt{x}\,\text{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{48ib\sqrt{x}\,\text{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} - \frac{24bx\,\text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24bx\,\text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2}$$

input
```
Int[x^(3/2)*(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(2*a*x^(5/2))/5 - (4*b*x^2*ArcTanh[E^(I*(c + d*Sqrt[x]))])/d + ((8*I)*b*x^
(3/2)*PolyLog[2, -E^(I*(c + d*Sqrt[x]))])/d^2 - ((8*I)*b*x^(3/2)*PolyLog[2
, E^(I*(c + d*Sqrt[x]))])/d^2 - (24*b*x*PolyLog[3, -E^(I*(c + d*Sqrt[x]))]
)/d^3 + (24*b*x*PolyLog[3, E^(I*(c + d*Sqrt[x]))])/d^3 - ((48*I)*b*Sqrt[x]
*PolyLog[4, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((48*I)*b*Sqrt[x]*PolyLog[4, E^
(I*(c + d*Sqrt[x]))])/d^4 + (48*b*PolyLog[5, -E^(I*(c + d*Sqrt[x]))])/d^5
- (48*b*PolyLog[5, E^(I*(c + d*Sqrt[x]))])/d^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int x^{\frac{3}{2}} \left(a + b \csc \left(c + d\sqrt{x}\right)\right) dx$$

input
```
int(x^(3/2)*(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(x^(3/2)*(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^{3/2} \left(a + b \csc \left(c + d\sqrt{x}\right)\right) \, dx = \int \left(b \csc \left(d\sqrt{x} + c\right) + a\right) x^{\frac{3}{2}} \, dx$$

input
```
integrate(x^(3/2)*(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
integral(b*x^(3/2)*csc(d*sqrt(x) + c) + a*x^(3/2), x)
```

Sympy [F]

$$\int x^{3/2} \left(a + b \csc \left(c + d\sqrt{x}\right)\right) \, dx = \int x^{\frac{3}{2}} \left(a + b \csc \left(c + d\sqrt{x}\right)\right) \, dx$$

input
```
integrate(x**(3/2)*(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(x**(3/2)*(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(200) = 400$.

Time = 0.12 (sec) , antiderivative size = 730, normalized size of antiderivative = 2.83

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^(3/2)*(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
1/5*(2*(d*sqrt(x) + c)^5*a - 10*(d*sqrt(x) + c)^4*a*c + 20*(d*sqrt(x) + c)
^3*a*c^2 - 20*(d*sqrt(x) + c)^2*a*c^3 + 10*(d*sqrt(x) + c)*a*c^4 - 10*b*c^
4*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c)) + 10*(-I*(d*sqrt(x) + c)^4*
b + 4*I*(d*sqrt(x) + c)^3*b*c - 6*I*(d*sqrt(x) + c)^2*b*c^2 + 4*I*(d*sqrt(
x) + c)*b*c^3)*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) + 1) + 10*(-
I*(d*sqrt(x) + c)^4*b + 4*I*(d*sqrt(x) + c)^3*b*c - 6*I*(d*sqrt(x) + c)^2*
b*c^2 + 4*I*(d*sqrt(x) + c)*b*c^3)*arctan2(sin(d*sqrt(x) + c), -cos(d*sqrt
(x) + c) + 1) + 40*(I*(d*sqrt(x) + c)^3*b - 3*I*(d*sqrt(x) + c)^2*b*c + 3*
I*(d*sqrt(x) + c)*b*c^2 - I*b*c^3)*dilog(-e^(I*d*sqrt(x) + I*c)) + 40*(-I*
(d*sqrt(x) + c)^3*b + 3*I*(d*sqrt(x) + c)^2*b*c - 3*I*(d*sqrt(x) + c)*b*c^
2 + I*b*c^3)*dilog(e^(I*d*sqrt(x) + I*c)) - 5*((d*sqrt(x) + c)^4*b - 4*(d*
sqrt(x) + c)^3*b*c + 6*(d*sqrt(x) + c)^2*b*c^2 - 4*(d*sqrt(x) + c)*b*c^3)*
log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*cos(d*sqrt(x) + c) + 1
) + 5*((d*sqrt(x) + c)^4*b - 4*(d*sqrt(x) + c)^3*b*c + 6*(d*sqrt(x) + c)^2
*b*c^2 - 4*(d*sqrt(x) + c)*b*c^3)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x)
 + c)^2 - 2*cos(d*sqrt(x) + c) + 1) + 240*b*polylog(5, -e^(I*d*sqrt(x) + I
*c)) - 240*b*polylog(5, e^(I*d*sqrt(x) + I*c)) + 240*(-I*(d*sqrt(x) + c)*b
 + I*b*c)*polylog(4, -e^(I*d*sqrt(x) + I*c)) + 240*(I*(d*sqrt(x) + c)*b -
I*b*c)*polylog(4, e^(I*d*sqrt(x) + I*c)) - 120*((d*sqrt(x) + c)^2*b - 2*(d
*sqrt(x) + c)*b*c + b*c^2)*polylog(3, -e^(I*d*sqrt(x) + I*c)) + 120*((d...
```

Giac [F]

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int x^{3/2}\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)\,dx$$

input
```
int(x^(3/2)*(a + b/sin(c + d*x^(1/2))),x)
```

output
```
int(x^(3/2)*(a + b/sin(c + d*x^(1/2))), x)
```

Reduce [F]

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2\sqrt{x}\,a\,x^{2}}{5}+\left(\int\sqrt{x}\,\csc\left(\sqrt{x}\,d+c\right)xdx\right)b$$

input
```
int(x^(3/2)*(a+b*csc(c+d*x^(1/2))),x)
```

output
```
(2*sqrt(x)*a*x**2 + 5*int(sqrt(x)*csc(sqrt(x)*d + c)*x,x)*b)/5
```

3.52 $\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2}{3}ax^{3/2}-\frac{4bx\operatorname{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}+\frac{4ib\sqrt{x}\operatorname{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{4ib\sqrt{x}\operatorname{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{4b\operatorname{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{4b\operatorname{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}$$

output

```
2/3*a*x^(3/2)-4*b*x*arctanh(exp(I*(c+d*x^(1/2))))/d+4*I*b*x^(1/2)*polylog(
2,-exp(I*(c+d*x^(1/2))))/d^2-4*I*b*x^(1/2)*polylog(2,exp(I*(c+d*x^(1/2))))
/d^2-4*b*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3+4*b*polylog(3,exp(I*(c+d*x^(
1/2))))/d^3
```

Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.33

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$$
$$=\frac{2\left(ad^3x^{3/2}-6bd^2x\text{arctanh}\left(\cos\left(c+d\sqrt{x}\right)+i\sin\left(c+d\sqrt{x}\right)\right)+6ibd\sqrt{x}\,\text{PolyLog}\left(2,-\cos\left(c+d\sqrt{x}\right)-\right.\right.}{}$$

input
```
Integrate[Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(2*(a*d^3*x^(3/2) - 6*b*d^2*x*ArcTanh[Cos[c + d*Sqrt[x]] + I*Sin[c + d*Sqr
t[x]]] + (6*I)*b*d*Sqrt[x]*PolyLog[2, -Cos[c + d*Sqrt[x]] - I*Sin[c + d*Sq
rt[x]]] - (6*I)*b*d*Sqrt[x]*PolyLog[2, Cos[c + d*Sqrt[x]] + I*Sin[c + d*Sq
rt[x]]] - 6*b*PolyLog[3, -Cos[c + d*Sqrt[x]] - I*Sin[c + d*Sqrt[x]]] + 6*b
*PolyLog[3, Cos[c + d*Sqrt[x]] + I*Sin[c + d*Sqrt[x]]]))/(3*d^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$$
$$\Big\downarrow \text{2010}$$
$$\int \left(a\sqrt{x}+b\sqrt{x}\csc\left(c+d\sqrt{x}\right)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{3}ax^{3/2}-\frac{4bx\text{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}-\frac{4b\,\text{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{4b\,\text{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}$$

input
```
Int[Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(2*a*x^(3/2))/3 - (4*b*x*ArcTanh[E^(I*(c + d*Sqrt[x]))])/d + ((4*I)*b*Sqrt
[x]*PolyLog[2, -E^(I*(c + d*Sqrt[x]))])/d^2 - ((4*I)*b*Sqrt[x]*PolyLog[2,
E^(I*(c + d*Sqrt[x]))])/d^2 - (4*b*PolyLog[3, -E^(I*(c + d*Sqrt[x]))])/d^3
 + (4*b*PolyLog[3, E^(I*(c + d*Sqrt[x]))])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)dx$$

input
```
int(x^(1/2)*(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(x^(1/2)*(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
integral(b*sqrt(x)*csc(d*sqrt(x) + c) + a*sqrt(x), x)
```

Sympy [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$$

input
```
integrate(x**(1/2)*(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(sqrt(x)*(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(110) = 220.

Time = 0.10 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.57

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$$
$$=\frac{2\left(d\sqrt{x}+c\right)^{3}a-6\left(d\sqrt{x}+c\right)^{2}ac+6\left(d\sqrt{x}+c\right)ac^{2}-6\,bc^{2}\log\left(\cot\left(d\sqrt{x}+c\right)+\csc\left(d\sqrt{x}+c\right)\right)+6\left(-\right.}{}$$

input
```
integrate(x^(1/2)*(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
1/3*(2*(d*sqrt(x) + c)^3*a - 6*(d*sqrt(x) + c)^2*a*c + 6*(d*sqrt(x) + c)*a
*c^2 - 6*b*c^2*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c)) + 6*(-I*(d*sqr
t(x) + c)^2*b + 2*I*(d*sqrt(x) + c)*b*c)*arctan2(sin(d*sqrt(x) + c), cos(d
*sqrt(x) + c) + 1) + 6*(-I*(d*sqrt(x) + c)^2*b + 2*I*(d*sqrt(x) + c)*b*c)*
arctan2(sin(d*sqrt(x) + c), -cos(d*sqrt(x) + c) + 1) + 12*(I*(d*sqrt(x) +
c)*b - I*b*c)*dilog(-e^(I*d*sqrt(x) + I*c)) + 12*(-I*(d*sqrt(x) + c)*b + I
*b*c)*dilog(e^(I*d*sqrt(x) + I*c)) - 3*((d*sqrt(x) + c)^2*b - 2*(d*sqrt(x)
 + c)*b*c)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*cos(d*sqrt(
x) + c) + 1) + 3*((d*sqrt(x) + c)^2*b - 2*(d*sqrt(x) + c)*b*c)*log(cos(d*s
qrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1) - 12*b*po
lylog(3, -e^(I*d*sqrt(x) + I*c)) + 12*b*polylog(3, e^(I*d*sqrt(x) + I*c)))
/d^3
```

Giac [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int \sqrt{x}\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)\,dx$$

input
```
int(x^(1/2)*(a + b/sin(c + d*x^(1/2))),x)
```

output
```
int(x^(1/2)*(a + b/sin(c + d*x^(1/2))), x)
```

Reduce [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2\sqrt{x}\,ax}{3}+\left(\int \sqrt{x}\,\csc\left(\sqrt{x}\,d+c\right)dx\right)b$$

input
```
int(x^(1/2)*(a+b*csc(c+d*x^(1/2))),x)
```

output
```
(2*sqrt(x)*a*x + 3*int(sqrt(x)*csc(sqrt(x)*d + c),x)*b)/3
```

3.53 $\int \frac{a+b \csc \left(c+d \sqrt{x}\right)}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \frac{a+b \csc \left(c+d \sqrt{x}\right)}{\sqrt{x}} \, dx = 2a\sqrt{x} - \frac{2b\text{arctanh}\left(\cos \left(c+d\sqrt{x}\right)\right)}{d}$$

output
```
2*a*x^(1/2)-2*b*arctanh(cos(c+d*x^(1/2)))/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b \csc \left(c+d \sqrt{x}\right)}{\sqrt{x}} \, dx = 2a\sqrt{x} - \frac{2b\text{arctanh}\left(\cos \left(c+d\sqrt{x}\right)\right)}{d}$$

input
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])/Sqrt[x],x]
```

output
```
2*a*Sqrt[x] - (2*b*ArcTanh[Cos[c + d*Sqrt[x]]])/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{\sqrt{x}}+\frac{b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a\sqrt{x}-\frac{2b\text{arctanh}\left(\cos\left(c+d\sqrt{x}\right)\right)}{d}$$

input
```
Int[(a + b*Csc[c + d*Sqrt[x]])/Sqrt[x],x]
```

output
```
2*a*Sqrt[x] - (2*b*ArcTanh[Cos[c + d*Sqrt[x]]])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$2a\sqrt{x}-\frac{2b\ln\left(\csc\left(c+d\sqrt{x}\right)+\cot\left(c+d\sqrt{x}\right)\right)}{d}$	32
default	$2a\sqrt{x}-\frac{2b\ln\left(\csc\left(c+d\sqrt{x}\right)+\cot\left(c+d\sqrt{x}\right)\right)}{d}$	32
parts	$2a\sqrt{x}-\frac{2b\ln\left(\csc\left(c+d\sqrt{x}\right)+\cot\left(c+d\sqrt{x}\right)\right)}{d}$	32

input
```
int((a+b*csc(c+d*x^(1/2)))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*x^(1/2)-2*b/d*ln(csc(c+d*x^(1/2))+cot(c+d*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\,ad\sqrt{x}-b\log\left(\frac{1}{2}\,\cos\left(d\sqrt{x}+c\right)+\frac{1}{2}\right)+b\log\left(-\frac{1}{2}\,\cos\left(d\sqrt{x}+c\right)+\frac{1}{2}\right)}{d}$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^(1/2),x, algorithm="fricas")
```

output
```
(2*a*d*sqrt(x) - b*log(1/2*cos(d*sqrt(x) + c) + 1/2) + b*log(-1/2*cos(d*sq
rt(x) + c) + 1/2))/d
```

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = 2a\sqrt{x}+2b\left(\begin{cases}\frac{\sqrt{x}\left(\cot\left(c\right)\csc\left(c\right)+\csc^{2}\left(c\right)\right)}{\cot\left(c\right)+\csc\left(c\right)} & \text{for } d=0\\ -\frac{\log\left(\cot\left(c+d\sqrt{x}\right)+\csc\left(c+d\sqrt{x}\right)\right)}{d} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b*csc(c+d*x**(1/2)))/x**(1/2),x)
```

output
```
2*a*sqrt(x) + 2*b*Piecewise((sqrt(x)*(cot(c)*csc(c) + csc(c)**2)/(cot(c) +
 csc(c)), Eq(d, 0)), (-log(cot(c + d*sqrt(x)) + csc(c + d*sqrt(x)))/d, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,a\sqrt{x}-\frac{2\,b\log\left(\cot\left(d\sqrt{x}+c\right)+\csc\left(d\sqrt{x}+c\right)\right)}{d}$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")
```

output
```
2*a*sqrt(x) - 2*b*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\left(\left(d\sqrt{x}+c\right)a+b\log\left(\left|\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)\right|\right)\right)}{d}$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^(1/2),x, algorithm="giac")
```

output
```
2*((d*sqrt(x) + c)*a + b*log(abs(tan(1/2*d*sqrt(x) + 1/2*c))))/d
```

Mupad [B] (verification not implemented)

Time = 17.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.81

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx=2\,a\,\sqrt{x}+\frac{2\,b\,\ln\left(\frac{b\,2\mathrm{i}-b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}}{\sqrt{x}}\right)}{d}-\frac{2\,b\,\ln\left(\frac{b\,2\mathrm{i}+b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}}{\sqrt{x}}\right)}{d}$$

input
```
int((a + b/sin(c + d*x^(1/2)))/x^(1/2),x)
```

output
```
2*a*x^(1/2) + (2*b*log((b*2i - b*exp(d*x^(1/2)*1i)*exp(c*1i)*2i)/x^(1/2)))
/d - (2*b*log((b*2i + b*exp(d*x^(1/2)*1i)*exp(c*1i)*2i)/x^(1/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx=\frac{2\sqrt{x}\,ad+2\log\left(\tan\left(\frac{\sqrt{x}\,d}{2}+\frac{c}{2}\right)\right)b}{d}$$

input
```
int((a+b*csc(c+d*x^(1/2)))/x^(1/2),x)
```

output
```
(2*(sqrt(x)*a*d + log(tan((sqrt(x)*d + c)/2))*b))/d
```

3.54 $\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2a}{\sqrt{x}} + b\text{Int}\left(\frac{\csc\left(c+d\sqrt{x}\right)}{x^{3/2}},x\right)$$

output
```
-2*a/x^(1/2)+b*Defer(Int)(csc(c+d*x^(1/2))/x^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 16.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx = \int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx$$

input
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^(3/2),x]
```

output
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{a}{x^{3/2}}+\frac{b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$b\int \frac{\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}dx-\frac{2a}{\sqrt{x}}$$

input
```
Int[(a + b*Csc[c + d*Sqrt[x]])/x^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{\frac{3}{2}}}dx$$

input
```
int((a+b*csc(c+d*x^(1/2)))/x^(3/2),x)
```

output
```
int((a+b*csc(c+d*x^(1/2)))/x^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx = \int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sqrt(x)*csc(d*sqrt(x) + c) + a*sqrt(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx = \int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x**(1/2)))/x**(3/2),x)
```

output
```
Integral((a + b*csc(c + d*sqrt(x)))/x**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 5.65

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^(3/2),x, algorithm="maxima")
```

output
```
((b*integrate(sin(d*sqrt(x) + c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) +
c)^2 + 2*cos(d*sqrt(x) + c) + 1)*x^(3/2)), x) + b*integrate(sin(d*sqrt(x)
+ c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c)
+ 1)*x^(3/2)), x))*sqrt(x) - 2*a)/sqrt(x)
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^(3/2),x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)/x^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 15.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc \left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}}{x^{3/2}}\,dx$$

input
```
int((a + b/sin(c + d*x^(1/2)))/x^(3/2),x)
```

output
```
int((a + b/sin(c + d*x^(1/2)))/x^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{a+b\csc \left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\frac{\sqrt{x}\left(\int \frac{\csc\left(\sqrt{x}\,d+c\right)}{\sqrt{x}\,x}dx\right)b-2a}{\sqrt{x}}$$

input
```
int((a+b*csc(c+d*x^(1/2)))/x^(3/2),x)
```

output
```
(sqrt(x)*int(csc(sqrt(x)*d + c)/(sqrt(x)*x),x)*b - 2*a)/sqrt(x)
```

3.55 $\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=-\frac{2a}{3x^{3/2}}+b\mathrm{Int}\left(\frac{\csc\left(c+d\sqrt{x}\right)}{x^{5/2}},x\right)$$

output
```
-2/3*a/x^(3/2)+b*Defer(Int)(csc(c+d*x^(1/2))/x^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 25.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx$$

input
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^(5/2),x]
```

output
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx$$

$$\Big\downarrow \ 2010$$

$$\int \left(\frac{a}{x^{5/2}}+\frac{b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$b\int \frac{\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}dx-\frac{2a}{3x^{3/2}}$$

input
```
Int[(a + b*Csc[c + d*Sqrt[x]])/x^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{\frac{5}{2}}}dx$$

input
```
int((a+b*csc(c+d*x^(1/2)))/x^(5/2),x)
```

output
```
int((a+b*csc(c+d*x^(1/2)))/x^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^(5/2),x, algorithm="fricas")
```

output
```
integral((b*sqrt(x)*csc(d*sqrt(x) + c) + a*sqrt(x))/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 7.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x**(1/2)))/x**(5/2),x)
```

output
```
Integral((a + b*csc(c + d*sqrt(x)))/x**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 5.75

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x^{5/2}}\,dx = \int \frac{b\csc\left(d\sqrt{x} + c\right) + a}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^(5/2),x, algorithm="maxima")
```

output
```
1/3*(3*(b*integrate(sin(d*sqrt(x) + c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt
(x) + c)^2 + 2*cos(d*sqrt(x) + c) + 1)*x^(5/2)), x) + b*integrate(sin(d*sq
rt(x) + c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x)
 + c) + 1)*x^(5/2)), x))*x^(3/2) - 2*a)/x^(3/2)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x^{5/2}}\,dx = \int \frac{b\csc\left(d\sqrt{x} + c\right) + a}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))/x^(5/2),x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)/x^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 15.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}}{x^{5/2}}\,dx$$

input
```
int((a + b/sin(c + d*x^(1/2)))/x^(5/2),x)
```

output
```
int((a + b/sin(c + d*x^(1/2)))/x^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\frac{3\sqrt{x}\left(\int \frac{\csc\left(\sqrt{x}\,d+c\right)}{\sqrt{x}\,x^2}dx\right)bx-2a}{3\sqrt{x}\,x}$$

input
```
int((a+b*csc(c+d*x^(1/2)))/x^(5/2),x)
```

output
```
(3*sqrt(x)*int(csc(sqrt(x)*d + c)/(sqrt(x)*x**2),x)*b*x - 2*a)/(3*sqrt(x)*
x)
```

3.56 $\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 421

$$\begin{aligned}\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx&=-\frac{2ib^2x^2}{d}+\frac{2}{5}a^2x^{5/2}\\&-\frac{8abx^2\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{2b^2x^2\cot\left(c+d\sqrt{x}\right)}{d}\\&+\frac{8b^2x^{3/2}\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2}+\frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}\\&-\frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{12ib^2x\operatorname{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3}\\&-\frac{48abx\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{48abx\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}\\&+\frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4}-\frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4}\\&+\frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{6ib^2\operatorname{PolyLog}\left(4,e^{2i(c+d\sqrt{x})}\right)}{d^5}\\&+\frac{96ab\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{96ab\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5}\end{aligned}$$

output
```
96*I*a*b*x^(1/2)*polylog(4,exp(I*(c+d*x^(1/2))))/d^4+2/5*a^2*x^(5/2)-8*a*b
*x^2*arctanh(exp(I*(c+d*x^(1/2))))/d-2*b^2*x^2*cot(c+d*x^(1/2))/d+8*b^2*x^
(3/2)*ln(1-exp(2*I*(c+d*x^(1/2))))/d^2+16*I*a*b*x^(3/2)*polylog(2,-exp(I*(
c+d*x^(1/2))))/d^2-2*I*b^2*x^2/d-12*I*b^2*x*polylog(2,exp(2*I*(c+d*x^(1/2)
)))/d^3-48*a*b*x*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3+48*a*b*x*polylog(3,e
xp(I*(c+d*x^(1/2))))/d^3+12*b^2*x^(1/2)*polylog(3,exp(2*I*(c+d*x^(1/2))))/
d^4-96*I*a*b*x^(1/2)*polylog(4,-exp(I*(c+d*x^(1/2))))/d^4+6*I*b^2*polylog(
4,exp(2*I*(c+d*x^(1/2))))/d^5-16*I*a*b*x^(3/2)*polylog(2,exp(I*(c+d*x^(1/2
))))/d^2+96*a*b*polylog(5,-exp(I*(c+d*x^(1/2))))/d^5-96*a*b*polylog(5,exp(
I*(c+d*x^(1/2))))/d^5
```

Mathematica [A] (verified)

Time = 7.09 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.35

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{2}{5}a^2x^{5/2}$$

$$+\frac{4b\left(-\frac{ibd^4x^2}{-1+e^{2ic}}+2bd^3x^{3/2}\log\left(1-e^{-i\left(c+d\sqrt{x}\right)}\right)+ad^4x^2\log\left(1-e^{-i\left(c+d\sqrt{x}\right)}\right)+2bd^3x^{3/2}\log\left(1+e^{-i\left(c+d\sqrt{x}\right)}\right)\right.}{}$$

$$+\frac{b^2x^2\csc\left(\frac{c}{2}\right)\csc\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\sin\left(\frac{d\sqrt{x}}{2}\right)}{d}+\frac{b^2x^2\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\sin\left(\frac{d\sqrt{x}}{2}\right)}{d}$$

input
```
Integrate[x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
(2*a^2*x^(5/2))/5 + (4*b*(((-I)*b*d^4*x^2)/(-1 + E^((2*I)*c)) + 2*b*d^3*x^
(3/2)*Log[1 - E^((-I)*(c + d*Sqrt[x]))] + a*d^4*x^2*Log[1 - E^((-I)*(c + d
*Sqrt[x]))] + 2*b*d^3*x^(3/2)*Log[1 + E^((-I)*(c + d*Sqrt[x]))] - a*d^4*x^
2*Log[1 + E^((-I)*(c + d*Sqrt[x]))] - (2*I)*d^2*(-3*b + 2*a*d*Sqrt[x])*x*P
olyLog[2, -E^((-I)*(c + d*Sqrt[x]))] + (2*I)*d^2*(3*b + 2*a*d*Sqrt[x])*x*P
olyLog[2, E^((-I)*(c + d*Sqrt[x]))] + 12*b*d*Sqrt[x]*PolyLog[3, -E^((-I)*(
c + d*Sqrt[x]))] - 12*a*d^2*x*PolyLog[3, -E^((-I)*(c + d*Sqrt[x]))] + 12*b
*d*Sqrt[x]*PolyLog[3, E^((-I)*(c + d*Sqrt[x]))] + 12*a*d^2*x*PolyLog[3, E^
((-I)*(c + d*Sqrt[x]))] - (12*I)*b*PolyLog[4, -E^((-I)*(c + d*Sqrt[x]))] +
 (24*I)*a*d*Sqrt[x]*PolyLog[4, -E^((-I)*(c + d*Sqrt[x]))] - (12*I)*b*PolyL
og[4, E^((-I)*(c + d*Sqrt[x]))] - (24*I)*a*d*Sqrt[x]*PolyLog[4, E^((-I)*(c
 + d*Sqrt[x]))] + 24*a*PolyLog[5, -E^((-I)*(c + d*Sqrt[x]))] - 24*a*PolyLo
g[5, E^((-I)*(c + d*Sqrt[x]))]))/d^5 + (b^2*x^2*Csc[c/2]*Csc[(c + d*Sqrt[x
])/2]*Sin[(d*Sqrt[x])/2])/d + (b^2*x^2*Sec[c/2]*Sec[(c + d*Sqrt[x])/2]*Sin
[(d*Sqrt[x])/2])/d
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4693, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2dx$$

$$\Big\downarrow \text{4693}$$

$$2\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2d\sqrt{x}$$

$$\Big\downarrow \text{4678}$$

$$2\int \left(a^2x^2+b^2\csc^2\left(c+d\sqrt{x}\right)x^2+2ab\csc\left(c+d\sqrt{x}\right)x^2\right)d\sqrt{x}$$

$\downarrow$ 2009

$$2\left(\frac{1}{5}a^2x^{5/2}-\frac{4abx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{48ab\,\text{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{48ab\,\text{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{48ia\right.$$

input
```
Int[x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
2*(((-I)*b^2*x^2)/d + (a^2*x^(5/2))/5 - (4*a*b*x^2*ArcTanh[E^(I*(c + d*Sqr
t[x]))])/d - (b^2*x^2*Cot[c + d*Sqrt[x]])/d + (4*b^2*x^(3/2)*Log[1 - E^((2
*I)*(c + d*Sqrt[x]))])/d^2 + ((8*I)*a*b*x^(3/2)*PolyLog[2, -E^(I*(c + d*Sq
rt[x]))])/d^2 - ((8*I)*a*b*x^(3/2)*PolyLog[2, E^(I*(c + d*Sqrt[x]))])/d^2
- ((6*I)*b^2*x*PolyLog[2, E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (24*a*b*x*Poly
Log[3, -E^(I*(c + d*Sqrt[x]))])/d^3 + (24*a*b*x*PolyLog[3, E^(I*(c + d*Sqr
t[x]))])/d^3 + (6*b^2*Sqrt[x]*PolyLog[3, E^((2*I)*(c + d*Sqrt[x]))])/d^4 -
 ((48*I)*a*b*Sqrt[x]*PolyLog[4, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((48*I)*a*b
*Sqrt[x]*PolyLog[4, E^(I*(c + d*Sqrt[x]))])/d^4 + ((3*I)*b^2*PolyLog[4, E^
((2*I)*(c + d*Sqrt[x]))])/d^5 + (48*a*b*PolyLog[5, -E^(I*(c + d*Sqrt[x]))]
)/d^5 - (48*a*b*PolyLog[5, E^(I*(c + d*Sqrt[x]))])/d^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int x^{\frac{3}{2}} \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 dx$$

input
```
int(x^(3/2)*(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(x^(3/2)*(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int \left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^(3/2)*csc(d*sqrt(x) + c)^2 + 2*a*b*x^(3/2)*csc(d*sqrt(x) +
c) + a^2*x^(3/2), x)
```

Sympy [F]

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x^{\frac{3}{2}}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$$

input
```
integrate(x**(3/2)*(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(x**(3/2)*(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2836 vs. $2(334) = 668$.

Time = 0.21 (sec) , antiderivative size = 2836, normalized size of antiderivative = 6.74

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
2/5*((d*sqrt(x) + c)^5*a^2 - 5*(d*sqrt(x) + c)^4*a^2*c + 10*(d*sqrt(x) + c
)^3*a^2*c^2 - 10*(d*sqrt(x) + c)^2*a^2*c^3 + 5*(d*sqrt(x) + c)*a^2*c^4 - 1
0*a*b*c^4*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c)) - 5*(2*b^2*c^4 - 2*
((d*sqrt(x) + c)^4*a*b + 2*b^2*c^3 - 2*(2*a*b*c + b^2)*(d*sqrt(x) + c)^3 +
 6*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^2 - 2*(2*a*b*c^3 + 3*b^2*c^2)*(d*sqrt
(x) + c) - ((d*sqrt(x) + c)^4*a*b + 2*b^2*c^3 - 2*(2*a*b*c + b^2)*(d*sqrt(
x) + c)^3 + 6*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^2 - 2*(2*a*b*c^3 + 3*b^2*c
^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-I*(d*sqrt(x) + c)^4*a*b -
2*I*b^2*c^3 + 2*(2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^3 + 6*(-I*a*b*c^2 - I*
b^2*c)*(d*sqrt(x) + c)^2 + 2*(2*I*a*b*c^3 + 3*I*b^2*c^2)*(d*sqrt(x) + c))*
sin(2*d*sqrt(x) + 2*c))*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) + 1
) + 4*(b^2*c^3*cos(2*d*sqrt(x) + 2*c) + I*b^2*c^3*sin(2*d*sqrt(x) + 2*c) -
 b^2*c^3)*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) - 1) - 2*((d*sqrt
(x) + c)^4*a*b - 2*(2*a*b*c - b^2)*(d*sqrt(x) + c)^3 + 6*(a*b*c^2 - b^2*c)
*(d*sqrt(x) + c)^2 - 2*(2*a*b*c^3 - 3*b^2*c^2)*(d*sqrt(x) + c) - ((d*sqrt(
x) + c)^4*a*b - 2*(2*a*b*c - b^2)*(d*sqrt(x) + c)^3 + 6*(a*b*c^2 - b^2*c)*
(d*sqrt(x) + c)^2 - 2*(2*a*b*c^3 - 3*b^2*c^2)*(d*sqrt(x) + c))*cos(2*d*sqr
t(x) + 2*c) + (-I*(d*sqrt(x) + c)^4*a*b + 2*(2*I*a*b*c - I*b^2)*(d*sqrt(x)
 + c)^3 + 6*(-I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) + c)^2 + 2*(2*I*a*b*c^3 - 3*
I*b^2*c^2)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(d*sqrt(...
```

Giac [F]

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)^2*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x^{3/2}\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2\,dx$$

input
```
int(x^(3/2)*(a + b/sin(c + d*x^(1/2)))^2,x)
```

output
```
int(x^(3/2)*(a + b/sin(c + d*x^(1/2)))^2, x)
```

Reduce [F]

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{2\sqrt{x}\,a^2x^2}{5}+2\left(\int\sqrt{x}\,\csc\left(\sqrt{x}\,d+c\right)xdx\right)ab+\left(\int\sqrt{x}\,\csc\left(\sqrt{x}\,d+c\right)^2xdx\right)b^2$$

input
```
int(x^(3/2)*(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
(2*sqrt(x)*a**2*x**2 + 10*int(sqrt(x)*csc(sqrt(x)*d + c)*x,x)*a*b + 5*int(
sqrt(x)*csc(sqrt(x)*d + c)**2*x,x)*b**2)/5
```

3.57 $\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 241

$$\begin{aligned}\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx &= -\frac{2ib^2x}{d}+\frac{2}{3}a^2x^{3/2}-\frac{8abx\operatorname{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}\\
&\quad-\frac{2b^2x\cot\left(c+d\sqrt{x}\right)}{d}+\frac{4b^2\sqrt{x}\log\left(1-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\
&\quad+\frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\
&\quad-\frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\
&\quad-\frac{2ib^2\operatorname{PolyLog}\left(2,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\
&\quad-\frac{8ab\operatorname{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\
&\quad+\frac{8ab\operatorname{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}\end{aligned}$$

output
```
-2*I*b^2*x/d+2/3*a^2*x^(3/2)-8*a*b*x*arctanh(exp(I*(c+d*x^(1/2))))/d-2*b^2
*x*cot(c+d*x^(1/2))/d+4*b^2*x^(1/2)*ln(1-exp(2*I*(c+d*x^(1/2))))/d^2+8*I*a
*b*x^(1/2)*polylog(2,-exp(I*(c+d*x^(1/2))))/d^2-8*I*a*b*x^(1/2)*polylog(2,
exp(I*(c+d*x^(1/2))))/d^2-2*I*b^2*polylog(2,exp(2*I*(c+d*x^(1/2))))/d^3-8*
a*b*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3+8*a*b*polylog(3,exp(I*(c+d*x^(1/2
))))/d^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 681 vs. 2(241) = 482.

Time = 3.29 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.83

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$$

$$=\frac{-12ib^2d^2x-2a^2d^3x^{3/2}+2a^2d^3e^{2ic}x^{3/2}-12b^2d\sqrt{x}\log\left(1-e^{-i(c+d\sqrt{x})}\right)+12b^2de^{2ic}\sqrt{x}\log\left(1-e^{-i(c+d\sqrt{x})}\right.}{}$$

input
```
Integrate[Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
((-12*I)*b^2*d^2*x - 2*a^2*d^3*x^(3/2) + 2*a^2*d^3*E^((2*I)*c)*x^(3/2) - 1
2*b^2*d*Sqrt[x]*Log[1 - E^((-I)*(c + d*Sqrt[x]))] + 12*b^2*d*E^((2*I)*c)*S
qrt[x]*Log[1 - E^((-I)*(c + d*Sqrt[x]))] - 12*a*b*d^2*x*Log[1 - E^((-I)*(c
 + d*Sqrt[x]))] + 12*a*b*d^2*E^((2*I)*c)*x*Log[1 - E^((-I)*(c + d*Sqrt[x])
)] - 12*b^2*d*Sqrt[x]*Log[1 + E^((-I)*(c + d*Sqrt[x]))] + 12*b^2*d*E^((2*I
)*c)*Sqrt[x]*Log[1 + E^((-I)*(c + d*Sqrt[x]))] + 12*a*b*d^2*x*Log[1 + E^((
-I)*(c + d*Sqrt[x]))] - 12*a*b*d^2*E^((2*I)*c)*x*Log[1 + E^((-I)*(c + d*Sq
rt[x]))] + (12*I)*b*(-1 + E^((2*I)*c))*(b - 2*a*d*Sqrt[x])*PolyLog[2, -E^(
(-I)*(c + d*Sqrt[x]))] + (12*I)*b*(-1 + E^((2*I)*c))*(b + 2*a*d*Sqrt[x])*P
olyLog[2, E^((-I)*(c + d*Sqrt[x]))] + 24*a*b*PolyLog[3, -E^((-I)*(c + d*Sq
rt[x]))] - 24*a*b*E^((2*I)*c)*PolyLog[3, -E^((-I)*(c + d*Sqrt[x]))] - 24*a
*b*PolyLog[3, E^((-I)*(c + d*Sqrt[x]))] + 24*a*b*E^((2*I)*c)*PolyLog[3, E^
((-I)*(c + d*Sqrt[x]))] - 3*b^2*d^2*x*Csc[c/2]*Csc[(c + d*Sqrt[x])/2]*Sin[
(d*Sqrt[x])/2] + 3*b^2*d^2*E^((2*I)*c)*x*Csc[c/2]*Csc[(c + d*Sqrt[x])/2]*S
in[(d*Sqrt[x])/2] - 3*b^2*d^2*x*Sec[c/2]*Sec[(c + d*Sqrt[x])/2]*Sin[(d*Sqr
t[x])/2] + 3*b^2*d^2*E^((2*I)*c)*x*Sec[c/2]*Sec[(c + d*Sqrt[x])/2]*Sin[(d*
Sqrt[x])/2])/(3*d^3*(-1 + E^((2*I)*c)))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4693, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+b \csc \left(c+d\sqrt{x}\right)\right)^2 dx$$

$$\Big\downarrow \text{ 4693}$$

$$2\int x\left(a+b \csc \left(c+d\sqrt{x}\right)\right)^2 d\sqrt{x}$$

$$\Big\downarrow \text{ 3042}$$

$$2\int x\left(a+b \csc \left(c+d\sqrt{x}\right)\right)^2 d\sqrt{x}$$

$$\Big\downarrow \text{ 4678}$$

$$2\int \left(xa^2+2bx \csc \left(c+d\sqrt{x}\right) a+b^2 x \csc ^2\left(c+d\sqrt{x}\right)\right) d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{1}{3}a^2x^{3/2}-\frac{4abx\text{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}-\frac{4ab\operatorname{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{4ab\operatorname{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{4iab\sqrt{x}\right.$$

input
```
Int[Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
2*(((-I)*b^2*x)/d + (a^2*x^(3/2))/3 - (4*a*b*x*ArcTanh[E^(I*(c + d*Sqrt[x]
))])/d - (b^2*x*Cot[c + d*Sqrt[x]])/d + (2*b^2*Sqrt[x]*Log[1 - E^((2*I)*(c
 + d*Sqrt[x]))])/d^2 + ((4*I)*a*b*Sqrt[x]*PolyLog[2, -E^(I*(c + d*Sqrt[x])
)])/d^2 - ((4*I)*a*b*Sqrt[x]*PolyLog[2, E^(I*(c + d*Sqrt[x]))])/d^2 - (I*b
^2*PolyLog[2, E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (4*a*b*PolyLog[3, -E^(I*(c
 + d*Sqrt[x]))])/d^3 + (4*a*b*PolyLog[3, E^(I*(c + d*Sqrt[x]))])/d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2dx$$

input
```
int(x^(1/2)*(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(x^(1/2)*(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(b^2*sqrt(x)*csc(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*csc(d*sqrt(x) +
c) + a^2*sqrt(x), x)
```

Sympy [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$$

input
```
integrate(x**(1/2)*(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(sqrt(x)*(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1217 vs. $2(190) = 380$.

Time = 0.13 (sec) , antiderivative size = 1217, normalized size of antiderivative = 5.05

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(x^(1/2)*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
2/3*((d*sqrt(x) + c)^3*a^2 - 3*(d*sqrt(x) + c)^2*a^2*c + 3*(d*sqrt(x) + c)
*a^2*c^2 - 6*a*b*c^2*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c)) - 3*(2*b
^2*c^2 - 2*((d*sqrt(x) + c)^2*a*b + b^2*c - (2*a*b*c + b^2)*(d*sqrt(x) + c
) - ((d*sqrt(x) + c)^2*a*b + b^2*c - (2*a*b*c + b^2)*(d*sqrt(x) + c))*cos(
2*d*sqrt(x) + 2*c) + (-I*(d*sqrt(x) + c)^2*a*b - I*b^2*c + (2*I*a*b*c + I*
b^2)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(d*sqrt(x) + c),
cos(d*sqrt(x) + c) + 1) + 2*(b^2*c*cos(2*d*sqrt(x) + 2*c) + I*b^2*c*sin(2*
d*sqrt(x) + 2*c) - b^2*c)*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) -
 1) - 2*((d*sqrt(x) + c)^2*a*b - (2*a*b*c - b^2)*(d*sqrt(x) + c) - ((d*sqr
t(x) + c)^2*a*b - (2*a*b*c - b^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c)
+ (-I*(d*sqrt(x) + c)^2*a*b + (2*I*a*b*c - I*b^2)*(d*sqrt(x) + c))*sin(2*d
*sqrt(x) + 2*c))*arctan2(sin(d*sqrt(x) + c), -cos(d*sqrt(x) + c) + 1) + 2*
((d*sqrt(x) + c)^2*b^2 - 2*(d*sqrt(x) + c)*b^2*c)*cos(2*d*sqrt(x) + 2*c) +
 2*(2*(d*sqrt(x) + c)*a*b - 2*a*b*c - b^2 - (2*(d*sqrt(x) + c)*a*b - 2*a*b
*c - b^2)*cos(2*d*sqrt(x) + 2*c) - (2*I*(d*sqrt(x) + c)*a*b - 2*I*a*b*c -
I*b^2)*sin(2*d*sqrt(x) + 2*c))*dilog(-e^(I*d*sqrt(x) + I*c)) - 2*(2*(d*sqr
t(x) + c)*a*b - 2*a*b*c + b^2 - (2*(d*sqrt(x) + c)*a*b - 2*a*b*c + b^2)*co
s(2*d*sqrt(x) + 2*c) + (-2*I*(d*sqrt(x) + c)*a*b + 2*I*a*b*c - I*b^2)*sin(
2*d*sqrt(x) + 2*c))*dilog(e^(I*d*sqrt(x) + I*c)) + (I*(d*sqrt(x) + c)^2*a*
b + I*b^2*c + (-2*I*a*b*c - I*b^2)*(d*sqrt(x) + c) + (-I*(d*sqrt(x) + c...
```

Giac [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)^2*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 dx = \int \sqrt{x}\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2 dx$$

input
```
int(x^(1/2)*(a + b/sin(c + d*x^(1/2)))^2,x)
```

output
```
int(x^(1/2)*(a + b/sin(c + d*x^(1/2)))^2, x)
```

Reduce [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 dx = \frac{2\sqrt{x}\,a^2x}{3} + 2\left(\int \sqrt{x}\,\csc\left(\sqrt{x}\,d+c\right)dx\right)ab + \left(\int \sqrt{x}\,\csc\left(\sqrt{x}\,d+c\right)^2 dx\right)b^2$$

input
```
int(x^(1/2)*(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
(2*sqrt(x)*a**2*x + 6*int(sqrt(x)*csc(sqrt(x)*d + c),x)*a*b + 3*int(sqrt(x
)*csc(sqrt(x)*d + c)**2,x)*b**2)/3
```

3.58 $\int \frac{(a+b\csc(c+d\sqrt{x}))^2}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx=2a^2\sqrt{x}-\frac{4ab\operatorname{arctanh}\left(\cos\left(c+d\sqrt{x}\right)\right)}{d}-\frac{2b^2\cot\left(c+d\sqrt{x}\right)}{d}$$

output
```
2*a^2*x^(1/2)-4*a*b*arctanh(cos(c+d*x^(1/2)))/d-2*b^2*cot(c+d*x^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx$$
$$=\frac{-b^2\cot\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)+2a\left(ac+ad\sqrt{x}-2b\log\left(\cos\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)+2b\log\left(\sin\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)\right)+b^2}{d}$$

input
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/Sqrt[x],x]
```

output
```
(-(b^2*Cot[(c + d*Sqrt[x])/2]) + 2*a*(a*c + a*d*Sqrt[x] - 2*b*Log[Cos[(c +
 d*Sqrt[x])/2]] + 2*b*Log[Sin[(c + d*Sqrt[x])/2]]) + b^2*Tan[(c + d*Sqrt[x
])/2])/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4693, 3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}dx$$

$$\Big\downarrow \text{4693}$$

$$2\int \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2d\sqrt{x}$$

$$\Big\downarrow \text{4260}$$

$$2\left(2ab\int \csc\left(c+d\sqrt{x}\right)d\sqrt{x}+b^2\int \csc^2\left(c+d\sqrt{x}\right)d\sqrt{x}+a^2\sqrt{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(2ab\int \csc\left(c+d\sqrt{x}\right)d\sqrt{x}+b^2\int \csc\left(c+d\sqrt{x}\right)^2d\sqrt{x}+a^2\sqrt{x}\right)$$

$$\Big\downarrow \text{4254}$$

$$2\left(2ab\int \csc\left(c+d\sqrt{x}\right)d\sqrt{x}-\frac{b^2\int 1d\cot\left(c+d\sqrt{x}\right)}{d}+a^2\sqrt{x}\right)$$

$$\Big\downarrow \text{24}$$

$$2\left(2ab\int \csc\left(c+d\sqrt{x}\right)d\sqrt{x}+a^2\sqrt{x}-\frac{b^2\cot\left(c+d\sqrt{x}\right)}{d}\right)$$

$$\Big\downarrow \text{4257}$$

$$2\left(a^2\sqrt{x}-\frac{2ab\text{arctanh}\left(\cos\left(c+d\sqrt{x}\right)\right)}{d}-\frac{b^2\cot\left(c+d\sqrt{x}\right)}{d}\right)$$

input
```
Int[(a + b*Csc[c + d*Sqrt[x]])^2/Sqrt[x],x]
```

output
```
2*(a^2*Sqrt[x] - (2*a*b*ArcTanh[Cos[c + d*Sqrt[x]]])/d - (b^2*Cot[c + d*Sq
rt[x]])/d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4260
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Simp[2*a*b   Int[Csc[c + d*x], x], x] + Simp[b^2   Int[Csc[c + d*x]^2, x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

method	result	size
parts	$2a^2\sqrt{x}-\frac{2b^2\cot\left(c+d\sqrt{x}\right)}{d}-\frac{4ba\ln\left(\csc\left(c+d\sqrt{x}\right)+\cot\left(c+d\sqrt{x}\right)\right)}{d}$	51
derivativedivides	$\frac{2a^2\left(c+d\sqrt{x}\right)+4ba\ln\left(\csc\left(c+d\sqrt{x}\right)-\cot\left(c+d\sqrt{x}\right)\right)-2b^2\cot\left(c+d\sqrt{x}\right)}{d}$	55
default	$\frac{2a^2\left(c+d\sqrt{x}\right)+4ba\ln\left(\csc\left(c+d\sqrt{x}\right)-\cot\left(c+d\sqrt{x}\right)\right)-2b^2\cot\left(c+d\sqrt{x}\right)}{d}$	55

input
```
int((a+b*csc(c+d*x^(1/2)))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2*x^(1/2)-2*b^2*cot(c+d*x^(1/2))/d-4*b*a/d*ln(csc(c+d*x^(1/2))+cot(c+d
*x^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(41) = 82.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.00

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx$$
$$=\frac{2\left(a^2d\sqrt{x}\sin\left(d\sqrt{x}+c\right)-ab\log\left(\frac{1}{2}\cos\left(d\sqrt{x}+c\right)+\frac{1}{2}\right)\sin\left(d\sqrt{x}+c\right)+ab\log\left(-\frac{1}{2}\cos\left(d\sqrt{x}+c\right)+\frac{1}{2}\right)\right.}{d\sin\left(d\sqrt{x}+c\right)}$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="fricas")
```

output
```
2*(a^2*d*sqrt(x)*sin(d*sqrt(x) + c) - a*b*log(1/2*cos(d*sqrt(x) + c) + 1/2
)*sin(d*sqrt(x) + c) + a*b*log(-1/2*cos(d*sqrt(x) + c) + 1/2)*sin(d*sqrt(x
) + c) - b^2*cos(d*sqrt(x) + c))/(d*sin(d*sqrt(x) + c))
```

Sympy [A] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = \begin{cases} \frac{2a^2\left(c+d\sqrt{x}\right)-4ab\log\left(\cot\left(c+d\sqrt{x}\right)+\csc\left(c+d\sqrt{x}\right)\right)-2b^2\cot\left(c+d\sqrt{x}\right)}{d} & \text{for } d\neq 0 \\ -\sqrt{x}(-2a^2-4ab\csc\left(c\right)-2b^2\csc^2\left(c\right)) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*csc(c+d*x**(1/2)))**2/x**(1/2),x)
```

output
```
Piecewise(((2*a**2*(c + d*sqrt(x)) - 4*a*b*log(cot(c + d*sqrt(x)) + csc(c
+ d*sqrt(x))) - 2*b**2*cot(c + d*sqrt(x)))/d, Ne(d, 0)), (-sqrt(x)*(-2*a**
2 - 4*a*b*csc(c) - 2*b**2*csc(c)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = 2\,a^2\sqrt{x} - \frac{4\,ab\log\left(\cot\left(d\sqrt{x}+c\right)+\csc\left(d\sqrt{x}+c\right)\right)}{d} - \frac{2\,b^2}{d\tan\left(d\sqrt{x}+c\right)}$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")
```

output
```
2*a^2*sqrt(x) - 4*a*b*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c))/d - 2*b
^2/(d*tan(d*sqrt(x) + c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(41) = 82$.

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.77

$$\int \frac{\left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = \frac{2\left(d\sqrt{x} + c\right)a^2 + 4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,d\sqrt{x} + \frac{1}{2}\,c\right)\right|\right) + b^2\tan\left(\frac{1}{2}\,d\sqrt{x} + \frac{1}{2}\,c\right) - \frac{4\,ab\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)+b^2}{\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)}}{d}$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="giac")
```

output
```
(2*(d*sqrt(x) + c)*a^2 + 4*a*b*log(abs(tan(1/2*d*sqrt(x) + 1/2*c))) + b^2*
tan(1/2*d*sqrt(x) + 1/2*c) - (4*a*b*tan(1/2*d*sqrt(x) + 1/2*c) + b^2)/tan(
1/2*d*sqrt(x) + 1/2*c))/d
```

Mupad [B] (verification not implemented)

Time = 16.91 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.36

$$\int \frac{\left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = 2\,a^2\,\sqrt{x} - \frac{b^2\,4\mathrm{i}}{d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,\sqrt{x}\,2\mathrm{i}} - 1\right)} - \frac{4\,a\,b\,\ln\left(-\frac{a\,b\,4\mathrm{i}}{\sqrt{x}} - \frac{a\,b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,4\mathrm{i}}{\sqrt{x}}\right)}{d} + \frac{4\,a\,b\,\ln\left(\frac{a\,b\,4\mathrm{i}}{\sqrt{x}} - \frac{a\,b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,4\mathrm{i}}{\sqrt{x}}\right)}{d}$$

input
```
int((a + b/sin(c + d*x^(1/2)))^2/x^(1/2),x)
```

output
```
2*a^2*x^(1/2) - (b^2*4i)/(d*(exp(c*2i + d*x^(1/2)*2i) - 1)) - (4*a*b*log(-
 (a*b*4i)/x^(1/2) - (a*b*exp(d*x^(1/2)*1i)*exp(c*1i)*4i)/x^(1/2)))/d + (4*
a*b*log((a*b*4i)/x^(1/2) - (a*b*exp(d*x^(1/2)*1i)*exp(c*1i)*4i)/x^(1/2)))/
d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx$$
$$=\frac{-2\cos\left(\sqrt{x}\,d+c\right)b^2+2\sqrt{x}\,\sin\left(\sqrt{x}\,d+c\right)a^2d+4\log\left(\tan\left(\frac{\sqrt{x}\,d}{2}+\frac{c}{2}\right)\right)\sin\left(\sqrt{x}\,d+c\right)ab}{\sin\left(\sqrt{x}\,d+c\right)d}$$

input
```
int((a+b*csc(c+d*x^(1/2)))^2/x^(1/2),x)
```

output
```
(2*( - cos(sqrt(x)*d + c)*b**2 + sqrt(x)*sin(sqrt(x)*d + c)*a**2*d + 2*log
(tan((sqrt(x)*d + c)/2))*sin(sqrt(x)*d + c)*a*b))/(sin(sqrt(x)*d + c)*d)
```

3.59 $\int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^{3/2}}\,dx = \text{Int}\left(\frac{(a+b\csc(c+d\sqrt{x}))^2}{x^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*csc(c+d*x^(1/2)))^2/x^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 50.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^{3/2}}\,dx = \int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^{3/2}}\,dx$$

input
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x^(3/2),x]
```

output
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$$
$$\Big\downarrow \text{4695}$$
$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}dx$$

input
```
Int[(a + b*Csc[c + d*Sqrt[x]])^2/x^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{3}{2}}}dx$$

input
```
int((a+b*csc(c+d*x^(1/2)))^2/x^(3/2),x)
```

output
```
int((a+b*csc(c+d*x^(1/2)))^2/x^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*sqrt(x)*csc(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*csc(d*sqrt(x) +
 c) + a^2*sqrt(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x**(1/2)))**2/x**(3/2),x)
```

output
```
Integral((a + b*csc(c + d*sqrt(x)))**2/x**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 794, normalized size of antiderivative = 36.09

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="maxima")
```

output
```
-(4*b^2*sin(2*d*sqrt(x) + 2*c) - ((d*integrate(2*(a*b*d*sqrt(x)*sin(d*sqrt
(x) + c) + b^2*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt
(x) + c)^2 + 2*d*cos(d*sqrt(x) + c) + d)*x^2), x) + d*integrate(2*(a*b*d*s
qrt(x)*sin(d*sqrt(x) + c) - b^2*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)
^2 + d*sin(d*sqrt(x) + c)^2 - 2*d*cos(d*sqrt(x) + c) + d)*x^2), x))*cos(2*
d*sqrt(x) + 2*c)^2 + (d*integrate(2*(a*b*d*sqrt(x)*sin(d*sqrt(x) + c) + b^
2*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c)^2 +
2*d*cos(d*sqrt(x) + c) + d)*x^2), x) + d*integrate(2*(a*b*d*sqrt(x)*sin(d*
sqrt(x) + c) - b^2*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*
sqrt(x) + c)^2 - 2*d*cos(d*sqrt(x) + c) + d)*x^2), x))*sin(2*d*sqrt(x) + 2
*c)^2 - 2*(d*integrate(2*(a*b*d*sqrt(x)*sin(d*sqrt(x) + c) + b^2*sin(d*sqr
t(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c)^2 + 2*d*cos(d*s
qrt(x) + c) + d)*x^2), x) + d*integrate(2*(a*b*d*sqrt(x)*sin(d*sqrt(x) + c
) - b^2*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c
)^2 - 2*d*cos(d*sqrt(x) + c) + d)*x^2), x))*cos(2*d*sqrt(x) + 2*c) + d*int
egrate(2*(a*b*d*sqrt(x)*sin(d*sqrt(x) + c) + b^2*sin(d*sqrt(x) + c))/((d*c
os(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c)^2 + 2*d*cos(d*sqrt(x) + c) + d)
*x^2), x) + d*integrate(2*(a*b*d*sqrt(x)*sin(d*sqrt(x) + c) - b^2*sin(d*sq
rt(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c)^2 - 2*d*cos(d*
sqrt(x) + c) + d)*x^2), x))*x + 2*(a^2*d*cos(2*d*sqrt(x) + 2*c)^2 + a^2...
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)^2/x^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 15.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}{x^{3/2}}\,dx$$

input
```
int((a + b/sin(c + d*x^(1/2)))^2/x^(3/2),x)
```

output
```
int((a + b/sin(c + d*x^(1/2)))^2/x^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\frac{2\sqrt{x}\left(\int \frac{\csc\left(\sqrt{x}\,d+c\right)}{\sqrt{x}\,x}dx\right)ab+\sqrt{x}\left(\int \frac{\csc\left(\sqrt{x}\,d+c\right)^2}{\sqrt{x}\,x}dx\right)b^2-2a^2}{\sqrt{x}}$$

input
```
int((a+b*csc(c+d*x^(1/2)))^2/x^(3/2),x)
```

output
```
(2*sqrt(x)*int(csc(sqrt(x)*d + c)/(sqrt(x)*x),x)*a*b + sqrt(x)*int(csc(sqr
t(x)*d + c)**2/(sqrt(x)*x),x)*b**2 - 2*a**2)/sqrt(x)
```

3.60 $\int \frac{\left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2}{x^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2}{x^{5/2}} \, dx = \text{Int}\left(\frac{\left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2}{x^{5/2}},x\right)$$

output
```
Defer(Int)((a+b*csc(c+d*x^(1/2)))^2/x^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 57.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2}{x^{5/2}} \, dx = \int \frac{\left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2}{x^{5/2}} \, dx$$

input
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x^(5/2),x]
```

output
```
Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx$$
$$\Big\downarrow \text{4695}$$
$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}dx$$

input
```
Int[(a + b*Csc[c + d*Sqrt[x]])^2/x^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{5}{2}}}dx$$

input
```
int((a+b*csc(c+d*x^(1/2)))^2/x^(5/2),x)
```

output
```
int((a+b*csc(c+d*x^(1/2)))^2/x^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sqrt(x)*csc(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*csc(d*sqrt(x) +
 c) + a^2*sqrt(x))/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 9.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x**(1/2)))**2/x**(5/2),x)
```

output
```
Integral((a + b*csc(c + d*sqrt(x)))**2/x**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*csc(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="giac")
```

output
```
integrate((b*csc(d*sqrt(x) + c) + a)^2/x^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 15.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}{x^{5/2}}\,dx$$

input
```
int((a + b/sin(c + d*x^(1/2)))^2/x^(5/2),x)
```

output
```
int((a + b/sin(c + d*x^(1/2)))^2/x^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.00

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx = \frac{6\sqrt{x}\left(\int \frac{\csc(\sqrt{x}\,d+c)}{\sqrt{x}\,x^2}dx\right)abx+3\sqrt{x}\left(\int \frac{\csc(\sqrt{x}\,d+c)^2}{\sqrt{x}\,x^2}dx\right)b^2x-2a^2}{3\sqrt{x}\,x}$$

input
```
int((a+b*csc(c+d*x^(1/2)))^2/x^(5/2),x)
```

output
```
(6*sqrt(x)*int(csc(sqrt(x)*d + c)/(sqrt(x)*x**2),x)*a*b*x + 3*sqrt(x)*int(
csc(sqrt(x)*d + c)**2/(sqrt(x)*x**2),x)*b**2*x - 2*a**2)/(3*sqrt(x)*x)
```

3.61 $\int \frac{\csc^3(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\operatorname{arctanh}\left(\cos\left(\sqrt{x}\right)\right) - \cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)$$

output
```
-arctanh(cos(x^(1/2)))-cot(x^(1/2))*csc(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(24) = 48.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{1}{4}\csc^2\left(\frac{\sqrt{x}}{2}\right) - \log\left(\cos\left(\frac{\sqrt{x}}{2}\right)\right) + \log\left(\sin\left(\frac{\sqrt{x}}{2}\right)\right) + \frac{1}{4}\sec^2\left(\frac{\sqrt{x}}{2}\right)$$

input
```
Integrate[Csc[Sqrt[x]]^3/Sqrt[x],x]
```

output
```
-1/4*Csc[Sqrt[x]/2]^2 - Log[Cos[Sqrt[x]/2]] + Log[Sin[Sqrt[x]/2]] + Sec[Sq
rt[x]/2]^2/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4693, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{4693}$$

$$2\int \csc^3\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \csc\left(\sqrt{x}\right)^3d\sqrt{x}$$

$$\Big\downarrow \text{4255}$$

$$2\left(\frac{1}{2}\int \csc\left(\sqrt{x}\right)d\sqrt{x}-\frac{1}{2}\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{1}{2}\int \csc\left(\sqrt{x}\right)d\sqrt{x}-\frac{1}{2}\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{4257}$$

$$2\left(-\frac{1}{2}\text{arctanh}\left(\cos\left(\sqrt{x}\right)\right)-\frac{1}{2}\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)\right)$$

input
```
Int[Csc[Sqrt[x]]^3/Sqrt[x],x]
```

output
```
2*(-1/2*ArcTanh[Cos[Sqrt[x]]] - (Cot[Sqrt[x]]*Csc[Sqrt[x]])/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)+\ln\left(\csc\left(\sqrt{x}\right)-\cot\left(\sqrt{x}\right)\right)$	24
default	$-\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)+\ln\left(\csc\left(\sqrt{x}\right)-\cot\left(\sqrt{x}\right)\right)$	24

input
```
int(csc(x^(1/2))^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cot(x^(1/2))*csc(x^(1/2))+ln(csc(x^(1/2))-cot(x^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{\left(\cos\left(\sqrt{x}\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(\sqrt{x}\right)+\frac{1}{2}\right)-\left(\cos\left(\sqrt{x}\right)^2-1\right)\log\left(-\frac{1}{2}\cos\left(\sqrt{x}\right)+\frac{1}{2}\right)-2\cos\left(\sqrt{x}\right)}{2\left(\cos\left(\sqrt{x}\right)^2-1\right)}$$

input
```
integrate(csc(x^(1/2))^3/x^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((cos(sqrt(x))^2 - 1)*log(1/2*cos(sqrt(x)) + 1/2) - (cos(sqrt(x))^2 -
 1)*log(-1/2*cos(sqrt(x)) + 1/2) - 2*cos(sqrt(x)))/(cos(sqrt(x))^2 - 1)
```

Sympy [F]

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$$

input
```
integrate(csc(x**(1/2))**3/x**(1/2),x)
```

output
```
Integral(csc(sqrt(x))**3/sqrt(x), x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{\cos\left(\sqrt{x}\right)}{\cos\left(\sqrt{x}\right)^2-1} - \frac{1}{2}\log\left(\cos\left(\sqrt{x}\right)+1\right) + \frac{1}{2}\log\left(\cos\left(\sqrt{x}\right)-1\right)$$

input
```
integrate(csc(x^(1/2))^3/x^(1/2),x, algorithm="maxima")
```

output
```
cos(sqrt(x))/(cos(sqrt(x))^2 - 1) - 1/2*log(cos(sqrt(x)) + 1) + 1/2*log(co
s(sqrt(x)) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(18) = 36$.

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.92

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{\left(\frac{2\left(\cos\left(\sqrt{x}\right)-1\right)}{\cos\left(\sqrt{x}\right)+1}-1\right)\left(\cos\left(\sqrt{x}\right)+1\right)}{4\left(\cos\left(\sqrt{x}\right)-1\right)} - \frac{\cos\left(\sqrt{x}\right)-1}{4\left(\cos\left(\sqrt{x}\right)+1\right)} + \frac{1}{2}\log\left(-\frac{\cos\left(\sqrt{x}\right)-1}{\cos\left(\sqrt{x}\right)+1}\right)$$

input
```
integrate(csc(x^(1/2))^3/x^(1/2),x, algorithm="giac")
```

output
```
-1/4*(2*(cos(sqrt(x)) - 1)/(cos(sqrt(x)) + 1) - 1)*(cos(sqrt(x)) + 1)/(cos
(sqrt(x)) - 1) - 1/4*(cos(sqrt(x)) - 1)/(cos(sqrt(x)) + 1) + 1/2*log(-(cos
(sqrt(x)) - 1)/(cos(sqrt(x)) + 1))
```

Mupad [B] (verification not implemented)

Time = 16.70 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.92

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\ln\left(-\frac{e^{\sqrt{x}\,1i}\,1i}{\sqrt{x}} - \frac{1i}{\sqrt{x}}\right) + \ln\left(-\frac{e^{\sqrt{x}\,1i}\,1i}{\sqrt{x}} + \frac{1i}{\sqrt{x}}\right) + \frac{4\,e^{\sqrt{x}\,1i}}{1+e^{\sqrt{x}\,4i}-2\,e^{\sqrt{x}\,2i}} + \frac{2\,e^{\sqrt{x}\,1i}}{e^{\sqrt{x}\,2i}-1}$$

input
```
int(1/(x^(1/2)*sin(x^(1/2))^3),x)
```

output
```
log(1i/x^(1/2) - (exp(x^(1/2)*1i)*1i)/x^(1/2)) - log(- (exp(x^(1/2)*1i)*1i
)/x^(1/2) - 1i/x^(1/2)) + (4*exp(x^(1/2)*1i))/(exp(x^(1/2)*4i) - 2*exp(x^(
1/2)*2i) + 1) + (2*exp(x^(1/2)*1i))/(exp(x^(1/2)*2i) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(\sqrt{x})}{\sqrt{x}}\,dx = \frac{-\cos(\sqrt{x}) + \log\left(\tan\left(\frac{\sqrt{x}}{2}\right)\right)\sin(\sqrt{x})^2}{\sin(\sqrt{x})^2}$$

input
```
int(csc(x^(1/2))^3/x^(1/2),x)
```

output
```
( - cos(sqrt(x)) + log(tan(sqrt(x)/2))*sin(sqrt(x))**2)/sin(sqrt(x))**2
```

3.62 $\int \frac{x^{3/2}}{a+b\csc(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 675

$$\begin{aligned}\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx &= \frac{2x^{5/2}}{5a}+\frac{2ibx^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\
&-\frac{2ibx^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{8bx^{3/2}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}\\
&-\frac{8bx^{3/2}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{24ibx\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}\\
&-\frac{24ibx\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{48b\sqrt{x}\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}\\
&+\frac{48b\sqrt{x}\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}\\
&-\frac{48ib\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}+\frac{48ib\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}\end{aligned}$$

output
```
2/5*x^(5/2)/a+2*I*b*x^2*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2))
)/a/(-a^2+b^2)^(1/2)/d-2*I*b*x^2*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^
2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d+8*b*x^(3/2)*polylog(2,I*a*exp(I*(c+d*x^(1/
2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2-8*b*x^(3/2)*polylog(2,I*
a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2+24*I*b
*x*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(
1/2)/d^3-24*I*b*x*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))
/a/(-a^2+b^2)^(1/2)/d^3-48*b*x^(1/2)*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b
-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^4+48*b*x^(1/2)*polylog(4,I*a*exp(
I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^4-48*I*b*polyl
og(5,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^5
+48*I*b*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b
^2)^(1/2)/d^5
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 539, normalized size of antiderivative = 0.80

$$\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \frac{2\left(\sqrt{a^2-b^2}d^5x^{5/2}-5bd^4x^2\log\left(1-\frac{ae^{i(c+d\sqrt{x})}}{-ib+\sqrt{a^2-b^2}}\right)+5bd^4x^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)\right.}{}$$

input
```
Integrate[x^(3/2)/(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(2*(Sqrt[a^2 - b^2]*d^5*x^(5/2) - 5*b*d^4*x^2*Log[1 - (a*E^(I*(c + d*Sqrt[
x])))/((-I)*b + Sqrt[a^2 - b^2])] + 5*b*d^4*x^2*Log[1 + (a*E^(I*(c + d*Sqr
t[x])))/(I*b + Sqrt[a^2 - b^2])] + (20*I)*b*d^3*x^(3/2)*PolyLog[2, (a*E^(I
*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - (20*I)*b*d^3*x^(3/2)*Poly
Log[2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - 60*b*d^2*x*
PolyLog[3, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] + 60*b*d^
2*x*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - (12
0*I)*b*d*Sqrt[x]*PolyLog[4, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 -
 b^2])] + (120*I)*b*d*Sqrt[x]*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(I*b
+ Sqrt[a^2 - b^2]))] + 120*b*PolyLog[5, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b
+ Sqrt[a^2 - b^2])] - 120*b*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(I*b +
Sqrt[a^2 - b^2]))]))/(5*a*Sqrt[a^2 - b^2]*d^5)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 677, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}dx$$

$$\Big\downarrow \text{ 4693}$$

$$2\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}$$

$$\Big\downarrow \text{ 4679}$$

$$2\int \left(\frac{x^2}{a}-\frac{bx^2}{a\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{24ib\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}}+\frac{24ib\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}}-\frac{24b\sqrt{x}\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}}+\frac{24b\sqrt{x}\,\text{Po}}{}\right.$$

input
```
Int[x^(3/2)/(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
2*(x^(5/2)/(5*a) + (I*b*x^2*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[
-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^2*Log[1 - (I*a*E^(I*(c + d*
Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (4*b*x^(3/2)*
PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a
^2 + b^2]*d^2) - (4*b*x^(3/2)*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b +
Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) + ((12*I)*b*x*PolyLog[3, (I*a
*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3)
- ((12*I)*b*x*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]
)])/(a*Sqrt[-a^2 + b^2]*d^3) - (24*b*Sqrt[x]*PolyLog[4, (I*a*E^(I*(c + d*S
qrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^4) + (24*b*Sqrt[x
]*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[
-a^2 + b^2]*d^4) - ((24*I)*b*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x])))/(b - S
qrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^5) + ((24*I)*b*PolyLog[5, (I*a*E^
(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^5))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{a+b\csc\left(c+d\sqrt{x}\right)}dx$$

input
```
int(x^(3/2)/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(x^(3/2)/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

input
```
integrate(x^(3/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
integral(x^(3/2)/(b*csc(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$$

input
```
integrate(x**(3/2)/(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(x**(3/2)/(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^(3/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

input
```
integrate(x^(3/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
integrate(x^(3/2)/(b*csc(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{3/2}}{a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}}\,dx$$

input
```
int(x^(3/2)/(a + b/sin(c + d*x^(1/2))),x)
```

output
```
int(x^(3/2)/(a + b/sin(c + d*x^(1/2))), x)
```

Reduce [F]

$$\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{\sqrt{x}\,x}{\csc\left(\sqrt{x}\,d+c\right)b+a}dx$$

input
```
int(x^(3/2)/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int((sqrt(x)*x)/(csc(sqrt(x)*d + c)*b + a),x)
```

3.63 $\int \frac{\sqrt{x}}{a+b\csc(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 407

$$\begin{aligned}
\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx &= \frac{2x^{3/2}}{3a}+\frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d} \\
&\quad -\frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d} \\
&\quad +\frac{4b\sqrt{x}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^2} \\
&\quad -\frac{4b\sqrt{x}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^2} \\
&\quad +\frac{4ib\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^3} \\
&\quad -\frac{4ib\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^3}
\end{aligned}$$

output
```
2/3*x^(3/2)/a+2*I*b*x*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/
a/(-a^2+b^2)^(1/2)/d-2*I*b*x*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(
1/2)))/a/(-a^2+b^2)^(1/2)/d+4*b*x^(1/2)*polylog(2,I*a*exp(I*(c+d*x^(1/2)))
/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2-4*b*x^(1/2)*polylog(2,I*a*ex
p(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2+4*I*b*poly
log(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^
3-4*I*b*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b
^2)^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \frac{2\left(\sqrt{a^2-b^2}d^3x^{3/2}-3bd^2x\log\left(1-\frac{ae^{i(c+d\sqrt{x})}}{-ib+\sqrt{a^2-b^2}}\right)+3bd^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)+6ibd\sqrt{x}\,\mathrm{PolyLog}\left(2,\frac{ae^{i\cdots}}{-ib+\cdots}\right.\right.\cdots}{3a\sqrt{a\cdots}}$$

input
```
Integrate[Sqrt[x]/(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
(2*(Sqrt[a^2 - b^2]*d^3*x^(3/2) - 3*b*d^2*x*Log[1 - (a*E^(I*(c + d*Sqrt[x]
)))/((-I)*b + Sqrt[a^2 - b^2])] + 3*b*d^2*x*Log[1 + (a*E^(I*(c + d*Sqrt[x]
)))/(I*b + Sqrt[a^2 - b^2])] + (6*I)*b*d*Sqrt[x]*PolyLog[2, (a*E^(I*(c + d
*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - (6*I)*b*d*Sqrt[x]*PolyLog[2, -((
a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - 6*b*PolyLog[3, (a*E^(
I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] + 6*b*PolyLog[3, -((a*E^(I
*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))]))/(3*a*Sqrt[a^2 - b^2]*d^3)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}dx$$

$$\Big\downarrow \text{ 4693}$$

$$2\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}$$

$$\Big\downarrow \text{ 4679}$$

$$2\int \left(\frac{x}{a}-\frac{bx}{a\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{2ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}-\frac{2ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}+\frac{2b\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}-\frac{2b\sqrt{x}\,\text{PolyLog}}{ad^2\sqrt{}}\right.$$

input
```
Int[Sqrt[x]/(a + b*Csc[c + d*Sqrt[x]]),x]
```

output
```
2*(x^(3/2)/(3*a) + (I*b*x*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a
^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x*Log[1 - (I*a*E^(I*(c + d*Sqrt
[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (2*b*Sqrt[x]*Poly
Log[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 +
 b^2]*d^2) - (2*b*Sqrt[x]*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt
[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) + ((2*I)*b*PolyLog[3, (I*a*E^(I*(
c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3) - ((2*I
)*b*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqr
t[-a^2 + b^2]*d^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}dx$$

input
```
int(x^(1/2)/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(x^(1/2)/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{\sqrt{x}}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

input
```
integrate(x^(1/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
integral(sqrt(x)/(b*csc(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$$

input
```
integrate(x**(1/2)/(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(sqrt(x)/(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^(1/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \int \frac{\sqrt{x}}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

input
```
integrate(x^(1/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/(b*csc(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \int \frac{\sqrt{x}}{a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}}\,dx$$

input
```
int(x^(1/2)/(a + b/sin(c + d*x^(1/2))),x)
```

output
```
int(x^(1/2)/(a + b/sin(c + d*x^(1/2))), x)
```

Reduce [F]

$$\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{\sqrt{x}}{\csc\left(\sqrt{x}\,d+c\right)b+a}dx$$

input
```
int(x^(1/2)/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(sqrt(x)/(csc(sqrt(x)*d + c)*b + a),x)
```

3.64 $\int \frac{1}{\sqrt{x}(a+b\csc(c+d\sqrt{x}))} dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\frac{2\sqrt{x}}{a}+\frac{4b\text{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d}$$

output
```
2*x^(1/2)/a+4*b*arctanh((a+b*tan(1/2*c+1/2*d*x^(1/2)))/(a^2-b^2)^(1/2))/a/
(a^2-b^2)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\frac{2\left(\frac{c}{d}+\sqrt{x}-\frac{2b\arctan\left(\frac{a+b\tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}d}\right)}{a}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]])),x]
```

output
```
(2*(c/d + Sqrt[x] - (2*b*ArcTan[(a + b*Tan[(c + d*Sqrt[x])/2])/Sqrt[-a^2 +
 b^2]])/(Sqrt[-a^2 + b^2]*d)))/a
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4693, 3042, 4270, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}dx$$

$$\Big\downarrow \text{4693}$$

$$2\int \frac{1}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{1}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}$$

$$\Big\downarrow \text{4270}$$

$$2\left(\frac{\sqrt{x}}{a}-\frac{\int \frac{1}{\frac{a\sin\left(c+d\sqrt{x}\right)}{b}+1}d\sqrt{x}}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{\sqrt{x}}{a}-\frac{\int \frac{1}{\frac{a\sin\left(c+d\sqrt{x}\right)}{b}+1}d\sqrt{x}}{a}\right)$$

$$\Big\downarrow \text{3139}$$

$$2\left(\frac{\sqrt{x}}{a}-\frac{2\int \frac{1}{x+\frac{2a\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{b}+1}d\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{ad}\right)$$

$$\Big\downarrow \text{1083}$$

$$2\left(\frac{4\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x}d\left(\frac{2a}{b}+2\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{ad}+\frac{\sqrt{x}}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{2b\text{arctanh}\left(\frac{b\left(\frac{2a}{b}+2\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}+\frac{\sqrt{x}}{a}\right)$$

input
```
Int[1/(Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]])),x]
```

output
```
2*(Sqrt[x]/a + (2*b*ArcTanh[(b*((2*a)/b + 2*Tan[(c + d*Sqrt[x])/2]))/(2*Sq
rt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{-\frac{4b\arctan\left(\frac{2b\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}+\frac{4\arctan\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{a}}{d}$	73
default	$\frac{-\frac{4b\arctan\left(\frac{2b\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}+\frac{4\arctan\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{a}}{d}$	73

input
```
int(1/x^(1/2)/(a+b*csc(c+d*x^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-2/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*c+1/2*d*x^(1/2))+2*a)
/(-a^2+b^2)^(1/2))+2/a*arctan(tan(1/2*c+1/2*d*x^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 275, normalized size of antiderivative = 4.17

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

$$= \left[\frac{2\,(a^2-b^2)d\sqrt{x}+\sqrt{a^2-b^2}b\log\left(\frac{(a^2-2\,b^2)\cos\left(d\sqrt{x}+c\right)^2+2\,\sqrt{a^2-b^2}a\cos\left(d\sqrt{x}+c\right)+a^2+b^2+2\left(\sqrt{a^2-b^2}b\cos\left(d\sqrt{x}+c\right)+ab\right)\sin}{a^2\cos\left(d\sqrt{x}+c\right)^2-2\,ab\sin\left(d\sqrt{x}+c\right)-a^2-b^2}\right.}{(a^3-ab^2)d}\right.$$

input
```
integrate(1/x^(1/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
[(2*(a^2 - b^2)*d*sqrt(x) + sqrt(a^2 - b^2)*b*log(((a^2 - 2*b^2)*cos(d*sqr
t(x) + c)^2 + 2*sqrt(a^2 - b^2)*a*cos(d*sqrt(x) + c) + a^2 + b^2 + 2*(sqrt
(a^2 - b^2)*b*cos(d*sqrt(x) + c) + a*b)*sin(d*sqrt(x) + c))/(a^2*cos(d*sqr
t(x) + c)^2 - 2*a*b*sin(d*sqrt(x) + c) - a^2 - b^2)))/((a^3 - a*b^2)*d), 2
*((a^2 - b^2)*d*sqrt(x) + sqrt(-a^2 + b^2)*b*arctan(-(sqrt(-a^2 + b^2)*b*s
in(d*sqrt(x) + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*cos(d*sqrt(x) + c)))
/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

input
```
integrate(1/x**(1/2)/(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(1/(sqrt(x)*(a + b*csc(c + d*sqrt(x)))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x^(1/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx = -\frac{4\left(\pi\left\lfloor\frac{d\sqrt{x}+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)+a}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}ad}+\frac{2\left(d\sqrt{x}+c\right)}{ad}$$

input
```
integrate(1/x^(1/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
-4*(pi*floor(1/2*(d*sqrt(x) + c)/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*d*sq
rt(x) + 1/2*c) + a)/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2)*a*d) + 2*(d*sqr
t(x) + c)/(a*d)
```

Mupad [B] (verification not implemented)

Time = 18.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx = \frac{2\sqrt{x}}{a}-\frac{2\,b\,\ln\left(b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}-\frac{2\,b\,\left(a\,1\mathrm{i}+b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{a-b}}+\frac{2\,b\,\ln\left(b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}+\frac{2\,b\,\left(a\,1\mathrm{i}+b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(1/(x^(1/2)*(a + b/sin(c + d*x^(1/2)))),x)
```

output
```
(2*x^(1/2))/a - (2*b*log(b*exp(d*x^(1/2)*1i)*exp(c*1i)*2i - (2*b*(a*1i + b
*exp(d*x^(1/2)*1i)*exp(c*1i)))/((a + b)^(1/2)*(a - b)^(1/2))))/(a*d*(a + b
)^(1/2)*(a - b)^(1/2)) + (2*b*log(b*exp(d*x^(1/2)*1i)*exp(c*1i)*2i + (2*b*
(a*1i + b*exp(d*x^(1/2)*1i)*exp(c*1i)))/((a + b)^(1/2)*(a - b)^(1/2))))/(a
*d*(a + b)^(1/2)*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx = \frac{4\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{\sqrt{x}\,d}{2}+\frac{c}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)b+2\sqrt{x}\,a^2d-2\sqrt{x}\,b^2d}{ad\left(a^2-b^2\right)}$$

input
```
int(1/x^(1/2)/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
(2*(2*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**
2 + b**2))*b + sqrt(x)*a**2*d - sqrt(x)*b**2*d))/(a*d*(a**2 - b**2))
```

3.65 $\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx = \text{Int}\left(\frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)},x\right)$$

output
```
Defer(Int)(1/x^(3/2)/(a+b*csc(c+d*x^(1/2))),x)
```

Mathematica [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx = \int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

input
```
Integrate[1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])),x]
```

output
```
Integrate[1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)} \, dx$$

$$\Big\downarrow \text{4695}$$

$$\int \frac{1}{x^{3/2} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)}dx$$

input
```
Int[1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{3}{2}} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)}dx$$

input
```
int(1/x^(3/2)/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(1/x^(3/2)/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
integral(sqrt(x)/(b*x^2*csc(d*sqrt(x) + c) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{\frac{3}{2}}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

input
```
integrate(1/x**(3/2)/(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(1/(x**(3/2)*(a + b*csc(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 244, normalized size of antiderivative = 11.09

$$\int \frac{1}{x^{3/2} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)} \, dx=\int \frac{1}{\left(b \csc \left(d \sqrt{x}+c\right)+a\right) x^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x^(3/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
-2*(a*b*sqrt(x)*integrate((2*b*cos(d*sqrt(x) + c)^2 + a*cos(d*sqrt(x) + c)
*sin(2*d*sqrt(x) + 2*c) - a*cos(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 2*
b*sin(d*sqrt(x) + c)^2 + a*sin(d*sqrt(x) + c))/((a^3*cos(2*d*sqrt(x) + 2*c
)^2 + 4*a*b^2*cos(d*sqrt(x) + c)^2 + 4*a^2*b*cos(d*sqrt(x) + c)*sin(2*d*sq
rt(x) + 2*c) + a^3*sin(2*d*sqrt(x) + 2*c)^2 + 4*a*b^2*sin(d*sqrt(x) + c)^2
 + 4*a^2*b*sin(d*sqrt(x) + c) + a^3 - 2*(2*a^2*b*sin(d*sqrt(x) + c) + a^3)
*cos(2*d*sqrt(x) + 2*c))*x^(3/2)), x) + 1)/(a*sqrt(x))
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)} \, dx=\int \frac{1}{\left(b \csc \left(d \sqrt{x}+c\right)+a\right) x^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x^(3/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
integrate(1/((b*csc(d*sqrt(x) + c) + a)*x^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 15.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{3/2}\,\left(a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}\right)}\,dx$$

input
```
int(1/(x^(3/2)*(a + b/sin(c + d*x^(1/2)))),x)
```

output
```
int(1/(x^(3/2)*(a + b/sin(c + d*x^(1/2)))), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{\sin\left(\sqrt{x}\,d+c\right)}{\sqrt{x}\,\sin\left(\sqrt{x}\,d+c\right)ax+\sqrt{x}\,bx}dx$$

input
```
int(1/x^(3/2)/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(sin(sqrt(x)*d + c)/(sqrt(x)*sin(sqrt(x)*d + c)*a*x + sqrt(x)*b*x),x)
```

3.66 $\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\text{Int}\left(\frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)},x\right)$$

output
```
Defer(Int)(1/x^(5/2)/(a+b*csc(c+d*x^(1/2))),x)
```

Mathematica [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

input
```
Integrate[1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])),x]
```

output
```
Integrate[1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{4695\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)} \, dx$$
$$\Big\downarrow \text{ 4695}$$
$$\int \frac{1}{x^{5/2} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)}dx$$

input
```
Int[1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{5}{2}} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)}dx$$

input
```
int(1/x^(5/2)/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(1/x^(5/2)/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

output
```
integral(sqrt(x)/(b*x^3*csc(d*sqrt(x) + c) + a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 8.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{\frac{5}{2}}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

input
```
integrate(1/x**(5/2)/(a+b*csc(c+d*x**(1/2))),x)
```

output
```
Integral(1/(x**(5/2)*(a + b*csc(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 245, normalized size of antiderivative = 11.14

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

output
```
-2/3*(3*a*b*x^(3/2)*integrate((2*b*cos(d*sqrt(x) + c)^2 + a*cos(d*sqrt(x)
+ c)*sin(2*d*sqrt(x) + 2*c) - a*cos(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c)
+ 2*b*sin(d*sqrt(x) + c)^2 + a*sin(d*sqrt(x) + c))/((a^3*cos(2*d*sqrt(x) +
 2*c)^2 + 4*a*b^2*cos(d*sqrt(x) + c)^2 + 4*a^2*b*cos(d*sqrt(x) + c)*sin(2*
d*sqrt(x) + 2*c) + a^3*sin(2*d*sqrt(x) + 2*c)^2 + 4*a*b^2*sin(d*sqrt(x) +
c)^2 + 4*a^2*b*sin(d*sqrt(x) + c) + a^3 - 2*(2*a^2*b*sin(d*sqrt(x) + c) +
a^3)*cos(2*d*sqrt(x) + 2*c))*x^(5/2)), x) + 1)/(a*x^(3/2))
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

output
```
integrate(1/((b*csc(d*sqrt(x) + c) + a)*x^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 15.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{5/2}\,\left(a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}\right)}\,dx$$

input
```
int(1/(x^(5/2)*(a + b/sin(c + d*x^(1/2)))),x)
```

output
```
int(1/(x^(5/2)*(a + b/sin(c + d*x^(1/2)))), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\sqrt{x}\,\csc\left(\sqrt{x}\,d+c\right)b\,x^{2}+\sqrt{x}\,a\,x^{2}}dx$$

input
```
int(1/x^(5/2)/(a+b*csc(c+d*x^(1/2))),x)
```

output
```
int(1/(sqrt(x)*csc(sqrt(x)*d + c)*b*x**2 + sqrt(x)*a*x**2),x)
```

3.67 $\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1977

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Too large to display}$$

output
```
-2*b^2*x^2*cos(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*sin(c+d*x^(1/2)))-24*I*b^2*
x*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d
^3-24*I*b^2*x*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/a^2
/(a^2-b^2)/d^3-2*I*b^3*x^2*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/
2)))/a^2/(-a^2+b^2)^(3/2)/d-24*I*b^3*x*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/
(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-4*I*b*x^2*ln(1-I*a*exp(I*(c
+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d-48*I*b*x*polylog
(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^3
+2*I*b^3*x^2*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2
+b^2)^(3/2)/d+24*I*b^3*x*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^
(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+4*I*b*x^2*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(
b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d+48*I*b*x*polylog(3,I*a*exp(I*(
c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^3-2*I*b^2*x^2/a
^2/(a^2-b^2)/d-8*b^3*x^(3/2)*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b
^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-96*I*b*polylog(5,I*a*exp(I*(c+d*x^(1/
2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^5-48*I*b^3*polylog(5,I*a
*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^5+96*I*
b*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^
(1/2)/d^5+48*I*b^2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2))
)/a^2/(a^2-b^2)/d^5+48*I*b^2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^...
```

Mathematica [A] (warning: unable to verify)

Time = 10.50 (sec) , antiderivative size = 2236, normalized size of antiderivative = 1.13

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^(3/2)/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
(Csc[c + d*Sqrt[x]]^2*(b + a*Sin[c + d*Sqrt[x]])*(2*x^(5/2)*(b + a*Sin[c +
 d*Sqrt[x]]) - ((10*I)*b*E^(I*c)*(2*b*E^(I*c)*x^2 + ((-1 + E^((2*I)*c))*((
4*I)*b*d^3*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^(3/2)*Log[1 + (a*E^(I*(2*c + d*
Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (2*I)*a^2*d^4*
E^(I*c)*x^2*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 -
 b^2)*E^((2*I)*c)])] + I*b^2*d^4*E^(I*c)*x^2*Log[1 + (a*E^(I*(2*c + d*Sqrt
[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (4*I)*b*d^3*Sqrt[(
a^2 - b^2)*E^((2*I)*c)]*x^(3/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E
^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (2*I)*a^2*d^4*E^(I*c)*x^2*Log[1
 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)
])] - I*b^2*d^4*E^(I*c)*x^2*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*
c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 4*d^2*(3*b*Sqrt[(a^2 - b^2)*E^((2*I
)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*x*PolyLog[2, (I*a
*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] +
 4*d^2*(3*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] + 2*a^2*d*E^(I*c)*Sqrt[x] - b^2*
d*E^(I*c)*Sqrt[x])*x*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c)
 + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + (24*I)*b*d*Sqrt[(a^2 - b^2)*E^((2*I)
*c)]*Sqrt[x]*PolyLog[3, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[
(a^2 - b^2)*E^((2*I)*c)])] - (24*I)*a^2*d^2*E^(I*c)*x*PolyLog[3, (I*a*E^(I
*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (...
```

Rubi [A] (verified)

Time = 3.19 (sec) , antiderivative size = 1979, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 4693}$$

$$2\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{4679}$$

$$2\int \left(-\frac{2bx^2}{a^2\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}+\frac{x^2}{a^2}+\frac{b^2x^2}{a^2\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{ix^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}+\frac{ix^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}-\frac{4x^{3/2}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}+\frac{4x^{3/2}\operatorname{PolyL}}{a^2\left(b\right.}\right.$$

input
```
Int[x^(3/2)/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
2*(((-I)*b^2*x^2)/(a^2*(a^2 - b^2)*d) + x^(5/2)/(5*a^2) + (4*b^2*x^(3/2)*L
og[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2
)*d^2) + (4*b^2*x^(3/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2
- b^2])])/(a^2*(a^2 - b^2)*d^2) - (I*b^3*x^2*Log[1 - (I*a*E^(I*(c + d*Sqrt
[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^2*
Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^
2 + b^2]*d) + (I*b^3*x^2*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^
2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x^2*Log[1 - (I*a*E^(I*(c
 + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((12*I
)*b^2*x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))])/
(a^2*(a^2 - b^2)*d^3) - ((12*I)*b^2*x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])
))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (4*b^3*x^(3/2)*PolyL
og[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^
2)^(3/2)*d^2) + (8*b*x^(3/2)*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - S
qrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (4*b^3*x^(3/2)*PolyLog[2,
(I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/
2)*d^2) - (8*b*x^(3/2)*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a
^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (24*b^2*Sqrt[x]*PolyLog[3, -((a*
E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) +
(24*b^2*Sqrt[x]*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}}dx$$

input
```
int(x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

input
```
integrate(x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(x^(3/2)/(b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^
2), x)
```

Sympy [F]

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
integrate(x**(3/2)/(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(x**(3/2)/(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

input
```
integrate(x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate(x^(3/2)/(b*csc(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^{3/2}}{\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

input
```
int(x^(3/2)/(a + b/sin(c + d*x^(1/2)))^2,x)
```

output
```
int(x^(3/2)/(a + b/sin(c + d*x^(1/2)))^2, x)
```

Reduce [F]

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{too large to display}$$

input
```
int(x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output

```
(4*( - 12960*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt
( - a**2 + b**2))*sin(sqrt(x)*d + c)*a**6 + 11520*sqrt( - a**2 + b**2)*ata
n((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*sin(sqrt(x)*d + c)*
a**4*b**2 - 1680*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/
sqrt( - a**2 + b**2))*sin(sqrt(x)*d + c)*a**2*b**4 - 12960*sqrt( - a**2 +
b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*a**5*b + 1
1520*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2
 + b**2))*a**3*b**3 - 1680*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/
2)*b + a)/sqrt( - a**2 + b**2))*a*b**5 - 6480*sqrt(x)*cos(sqrt(x)*d + c)*a
**7*d - 120*sqrt(x)*cos(sqrt(x)*d + c)*a**5*b**2*d**3*x + 9000*sqrt(x)*cos
(sqrt(x)*d + c)*a**5*b**2*d + 140*sqrt(x)*cos(sqrt(x)*d + c)*a**3*b**4*d**
3*x - 2640*sqrt(x)*cos(sqrt(x)*d + c)*a**3*b**4*d - 20*sqrt(x)*cos(sqrt(x)
*d + c)*a*b**6*d**3*x + 120*sqrt(x)*cos(sqrt(x)*d + c)*a*b**6*d + 1080*cos
(sqrt(x)*d + c)*a**6*b*d**2*x + 10*cos(sqrt(x)*d + c)*a**4*b**3*d**4*x**2
- 1380*cos(sqrt(x)*d + c)*a**4*b**3*d**2*x - 10*cos(sqrt(x)*d + c)*a**2*b*
*5*d**4*x**2 + 300*cos(sqrt(x)*d + c)*a**2*b**5*d**2*x + 180*sqrt(x)*sin(s
qrt(x)*d + c)*a**6*b*d**3*x + sqrt(x)*sin(sqrt(x)*d + c)*a**4*b**3*d**5*x*
*2 - 260*sqrt(x)*sin(sqrt(x)*d + c)*a**4*b**3*d**3*x - sqrt(x)*sin(sqrt(x)
*d + c)*a**2*b**5*d**5*x**2 + 80*sqrt(x)*sin(sqrt(x)*d + c)*a**2*b**5*d**3
*x - 6480*sqrt(x)*a**7*d + 60*sqrt(x)*a**5*b**2*d**3*x + 12240*sqrt(x)*...
```

3.68 $\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1157

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Too large to display}$$

output
```
4*I*b*x*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)
^(1/2)/d+2/3*x^(3/2)/a^2+4*b^2*x^(1/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(I*b-(a
^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+4*b^2*x^(1/2)*ln(1+a*exp(I*(c+d*x^(1/2))
)/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+2*I*b^3*x*ln(1-I*a*exp(I*(c+d*x
^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+8*I*b*polylog(3,I*a*
exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^3-4*I*b^
2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d
^3-4*I*b*x*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b
^2)^(1/2)/d-4*I*b^3*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)
))/a^2/(-a^2+b^2)^(3/2)/d^3-4*I*b^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(I*b
-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-4*b^3*x^(1/2)*polylog(2,I*a*exp(I*(c+
d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+8*b*x^(1/2)*pol
ylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)
/d^2+4*b^3*x^(1/2)*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2))
)/a^2/(-a^2+b^2)^(3/2)/d^2-8*b*x^(1/2)*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/
(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2-2*I*b^2*x/a^2/(a^2-b^2)/d-2
*I*b^3*x*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2
)^(3/2)/d-8*I*b*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a
^2/(-a^2+b^2)^(1/2)/d^3+4*I*b^3*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^
2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-2*b^2*x*cos(c+d*x^(1/2))/a/(a^2...
```

Mathematica [A] (warning: unable to verify)

Time = 6.14 (sec) , antiderivative size = 846, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$= \frac{\csc^2\left(c+d\sqrt{x}\right)\left(b+a\sin\left(c+d\sqrt{x}\right)\right)\left(2x^{3/2}\left(b+a\sin\left(c+d\sqrt{x}\right)\right)-\frac{6ib\left(\frac{2bd^2e^{2ic}x}{-1+e^{2ic}}+\frac{2\left(b\sqrt{\left(a^2-b^2\right)e^{2ic}}-2a^2de^{ic}\sqrt{x}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sqrt[x]/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
(Csc[c + d*Sqrt[x]]^2*(b + a*Sin[c + d*Sqrt[x]])*(2*x^(3/2)*(b + a*Sin[c +
 d*Sqrt[x]]) - ((6*I)*b*((2*b*d^2*E^((2*I)*c)*x)/(-1 + E^((2*I)*c)) + (2*(
b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*
Sqrt[x])*PolyLog[2, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2
 - b^2)*E^((2*I)*c)])] + 2*(b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] + 2*a^2*d*E^(I
*c)*Sqrt[x] - b^2*d*E^(I*c)*Sqrt[x])*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x]
)))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + I*(d*Sqrt[x]*((2*b*S
qrt[(a^2 - b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqr
t[x])*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*
E^((2*I)*c)])] + (2*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] + 2*a^2*d*E^(I*c)*Sqrt
[x] - b^2*d*E^(I*c)*Sqrt[x])*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I
*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])]) - 2*(2*a^2 - b^2)*E^(I*c)*PolyLog[3
, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c
)])] + 2*(2*a^2 - b^2)*E^(I*c)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(I
*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))]))/Sqrt[(a^2 - b^2)*E^((2*I)*
c)])*(b + a*Sin[c + d*Sqrt[x]]))/((a^2 - b^2)*d^3) + (6*b^2*x*Csc[c]*(b*Co
s[c] + a*Sin[d*Sqrt[x]]))/((a - b)*(a + b)*d)))/(3*a^2*(a + b*Csc[c + d*Sq
rt[x]])^2)
```

Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 1159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

$$\Big\downarrow \text{4693}$$

$$2\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{4679}$$

$$2\int \left(\frac{xb^2}{a^2\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}-\frac{2xb}{a^2\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}+\frac{x}{a^2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{ix\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}+\frac{ix\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}-\frac{2\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}+\frac{2\sqrt{x}\,\text{PolyLog}\left(\right.}{a^2\left(b^2-\right.}\right.$$

input
```
Int[Sqrt[x]/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

output
```
2*(((-I)*b^2*x)/(a^2*(a^2 - b^2)*d) + x^(3/2)/(3*a^2) + (2*b^2*Sqrt[x]*Log
[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*
d^2) + (2*b^2*Sqrt[x]*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 -
b^2])])/(a^2*(a^2 - b^2)*d^2) - (I*b^3*x*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])
))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x*Log[1
- (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^
2]*d) + (I*b^3*x*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]
)])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x*Log[1 - (I*a*E^(I*(c + d*Sqrt[
x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((2*I)*b^2*PolyL
og[2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b
^2)*d^3) - ((2*I)*b^2*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a
^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (2*b^3*Sqrt[x]*PolyLog[2, (I*a*E^(I*
(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) +
(4*b*Sqrt[x]*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])
])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (2*b^3*Sqrt[x]*PolyLog[2, (I*a*E^(I*(c + d
*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (4*b*S
qrt[x]*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^
2*Sqrt[-a^2 + b^2]*d^2) - ((2*I)*b^3*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x]))
)/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((4*I)*b*PolyLog
[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}}dx$$

input
```
int(x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{\sqrt{x}}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

input
```
integrate(x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(x)/(b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^
2), x)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
integrate(x**(1/2)/(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(sqrt(x)/(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{\sqrt{x}}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

input
```
integrate(x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x)/(b*csc(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{\sqrt{x}}{\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

input
```
int(x^(1/2)/(a + b/sin(c + d*x^(1/2)))^2,x)
```

output
```
int(x^(1/2)/(a + b/sin(c + d*x^(1/2)))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{too large to display}$$

input
```
int(x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output

```
(4*( - 72*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( -
 a**2 + b**2))*sin(sqrt(x)*d + c)*a**4 + 48*sqrt( - a**2 + b**2)*atan((tan
((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*sin(sqrt(x)*d + c)*a**2*b
**2 - 72*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( -
a**2 + b**2))*a**3*b + 48*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2
)*b + a)/sqrt( - a**2 + b**2))*a*b**3 - 36*sqrt(x)*cos(sqrt(x)*d + c)*a**5
*d + 42*sqrt(x)*cos(sqrt(x)*d + c)*a**3*b**2*d - 6*sqrt(x)*cos(sqrt(x)*d +
 c)*a*b**4*d + 6*cos(sqrt(x)*d + c)*a**4*b*d**2*x - 6*cos(sqrt(x)*d + c)*a
**2*b**3*d**2*x + sqrt(x)*sin(sqrt(x)*d + c)*a**4*b*d**3*x - sqrt(x)*sin(s
qrt(x)*d + c)*a**2*b**3*d**3*x - 36*sqrt(x)*a**5*d + sqrt(x)*a**3*b**2*d**
3*x + 60*sqrt(x)*a**3*b**2*d - sqrt(x)*a*b**4*d**3*x - 24*sqrt(x)*a*b**4*d
 + 6*int(sqrt(x)/(tan((sqrt(x)*d + c)/2)**4*b**2 + 4*tan((sqrt(x)*d + c)/2
)**3*a*b + 4*tan((sqrt(x)*d + c)/2)**2*a**2 + 2*tan((sqrt(x)*d + c)/2)**2*
b**2 + 4*tan((sqrt(x)*d + c)/2)*a*b + b**2),x)*sin(sqrt(x)*d + c)*a**6*b*d
**3 - 9*int(sqrt(x)/(tan((sqrt(x)*d + c)/2)**4*b**2 + 4*tan((sqrt(x)*d + c
)/2)**3*a*b + 4*tan((sqrt(x)*d + c)/2)**2*a**2 + 2*tan((sqrt(x)*d + c)/2)*
*2*b**2 + 4*tan((sqrt(x)*d + c)/2)*a*b + b**2),x)*sin(sqrt(x)*d + c)*a**4*
b**3*d**3 + 3*int(sqrt(x)/(tan((sqrt(x)*d + c)/2)**4*b**2 + 4*tan((sqrt(x)
*d + c)/2)**3*a*b + 4*tan((sqrt(x)*d + c)/2)**2*a**2 + 2*tan((sqrt(x)*d +
c)/2)**2*b**2 + 4*tan((sqrt(x)*d + c)/2)*a*b + b**2),x)*sin(sqrt(x)*d +...
```

3.69 $\int \frac{1}{\sqrt{x}(a+b\csc(c+d\sqrt{x}))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2\sqrt{x}}{a^2} + \frac{4b(2a^2-b^2)\operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}d} - \frac{2b^2\cot\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}$$

output
```
2*x^(1/2)/a^2+4*b*(2*a^2-b^2)*arctanh((a+b*tan(1/2*c+1/2*d*x^(1/2)))/(a^2-
b^2)^(1/2))/a^2/(a^2-b^2)^(3/2)/d-2*b^2*cot(c+d*x^(1/2))/a/(a^2-b^2)/d/(a+
b*csc(c+d*x^(1/2)))
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$= \frac{2\csc\left(c+d\sqrt{x}\right)\left(\frac{ab^2\cot(c+d\sqrt{x})}{(-a+b)(a+b)} + \left(c+d\sqrt{x}\right)\left(a+b\csc\left(c+d\sqrt{x}\right)\right) - \frac{2b(-2a^2+b^2)\arctan\left(\frac{a+b\tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{-a^2+b^2}}\right)(a}{(-a^2+b^2)^{3/2}}\right.}{a^2d\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

output
```
(2*Csc[c + d*Sqrt[x]]*((a*b^2*Cot[c + d*Sqrt[x]])/((-a + b)*(a + b)) + (c
+ d*Sqrt[x])*(a + b*Csc[c + d*Sqrt[x]]) - (2*b*(-2*a^2 + b^2)*ArcTan[(a +
b*Tan[(c + d*Sqrt[x])/2])/Sqrt[-a^2 + b^2]]*(a + b*Csc[c + d*Sqrt[x]]))/(-
a^2 + b^2)^(3/2))*(b + a*Sin[c + d*Sqrt[x]]))/(a^2*d*(a + b*Csc[c + d*Sqrt
[x]])^2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.28, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {4693, 3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

$$\Big\downarrow \text{4693}$$

$$2\int \frac{1}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{1}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{4272}$$

$$2\left(-\frac{\int -\frac{a^2-b\csc\left(c+d\sqrt{x}\right)a-b^2}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}}{a\left(a^2-b^2\right)}-\frac{b^2\cot\left(c+d\sqrt{x}\right)}{ad\left(a^2-b^2\right)\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\int \frac{a^2-b\csc\left(c+d\sqrt{x}\right)a-b^2}{a+b\csc\left(c+d\sqrt{x}\right)}d\sqrt{x}}{a\left(a^2-b^2\right)}-\frac{b^2\cot\left(c+d\sqrt{x}\right)}{ad\left(a^2-b^2\right)\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{\int \frac{a^2-b\csc(c+d\sqrt{x})a-b^2}{a+b\csc(c+d\sqrt{x})}d\sqrt{x}}{a(a^2-b^2)}-\frac{b^2\cot(c+d\sqrt{x})}{ad(a^2-b^2)(a+b\csc(c+d\sqrt{x}))}\right)$$

$$\Big\downarrow \text{4407}$$

$$2\left(\frac{\frac{\sqrt{x}(a^2-b^2)}{a}-\frac{b(2a^2-b^2)\int \frac{\csc(c+d\sqrt{x})}{a+b\csc(c+d\sqrt{x})}d\sqrt{x}}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+d\sqrt{x})}{ad(a^2-b^2)(a+b\csc(c+d\sqrt{x}))}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{\frac{\sqrt{x}(a^2-b^2)}{a}-\frac{b(2a^2-b^2)\int \frac{\csc(c+d\sqrt{x})}{a+b\csc(c+d\sqrt{x})}d\sqrt{x}}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+d\sqrt{x})}{ad(a^2-b^2)(a+b\csc(c+d\sqrt{x}))}\right)$$

$$\Big\downarrow \text{4318}$$

$$2\left(\frac{\frac{\sqrt{x}(a^2-b^2)}{a}-\frac{(2a^2-b^2)\int \frac{1}{\frac{a\sin(c+d\sqrt{x})}{b}+1}d\sqrt{x}}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+d\sqrt{x})}{ad(a^2-b^2)(a+b\csc(c+d\sqrt{x}))}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{\frac{\sqrt{x}(a^2-b^2)}{a}-\frac{(2a^2-b^2)\int \frac{1}{\frac{a\sin(c+d\sqrt{x})}{b}+1}d\sqrt{x}}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+d\sqrt{x})}{ad(a^2-b^2)(a+b\csc(c+d\sqrt{x}))}\right)$$

$$\Big\downarrow \text{3139}$$

$$2\left(\frac{\frac{\sqrt{x}(a^2-b^2)}{a}-\frac{2(2a^2-b^2)\int \frac{1}{x+\frac{2a\tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{b}+1}d\tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{ad}}{a(a^2-b^2)}-\frac{b^2\cot(c+d\sqrt{x})}{ad(a^2-b^2)(a+b\csc(c+d\sqrt{x}))}\right)$$

$$\Big\downarrow \text{1083}$$

$$2\left(\frac{\frac{4(2a^2-b^2)\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x}d\left(\frac{2a}{b}+2\tan\left(\frac{1}{2}(c+d\sqrt{x})\right)\right)}{ad}+\frac{\sqrt{x}(a^2-b^2)}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+d\sqrt{x})}{ad(a^2-b^2)(a+b\csc(c+d\sqrt{x}))}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{\frac{2b(2a^2-b^2)\text{arctanh}\left(\frac{b\left(\frac{2a}{b}+2\tan\left(\frac{1}{2}(c+d\sqrt{x})\right)\right)}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}+\frac{\sqrt{x}(a^2-b^2)}{a}}{a\left(a^2-b^2\right)}-\frac{b^2\cot\left(c+d\sqrt{x}\right)}{ad\left(a^2-b^2\right)\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\right)$$

input
```
Int[1/(Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

output
```
2*((((a^2 - b^2)*Sqrt[x])/a + (2*b*(2*a^2 - b^2)*ArcTanh[(b*((2*a)/b + 2*T
an[(c + d*Sqrt[x])/2]))/(2*Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d))/(a*(a
^2 - b^2)) - (b^2*Cot[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(a + b*Csc[c + d*Sq
rt[x]])))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.43

method	result	s
derivativedivides	$\frac{-\frac{4b\left(\frac{\frac{a^2\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{2a^2-2b^2}+\frac{ba}{2a^2-2b^2}}{\frac{\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^2 b}{2}+a\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+\frac{b}{2}}+\frac{2(2a^2-b^2)\arctan\left(\frac{2b\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{(2a^2-2b^2)\sqrt{-a^2+b^2}}\right)}{a^2}+\frac{4\arctan\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{a^2}}{d}$	1
default	$\frac{-\frac{4b\left(\frac{\frac{a^2\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{2a^2-2b^2}+\frac{ba}{2a^2-2b^2}}{\frac{\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^2 b}{2}+a\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+\frac{b}{2}}+\frac{2(2a^2-b^2)\arctan\left(\frac{2b\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{(2a^2-2b^2)\sqrt{-a^2+b^2}}\right)}{a^2}+\frac{4\arctan\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{a^2}}{d}$	1

input
```
int(1/x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-2/a^2*b*((1/2*a^2/(a^2-b^2)*tan(1/2*c+1/2*d*x^(1/2))+1/2*b*a/(a^2-b^
2))/(1/2*tan(1/2*c+1/2*d*x^(1/2))^2*b+a*tan(1/2*c+1/2*d*x^(1/2))+1/2*b)+2*
(2*a^2-b^2)/(2*a^2-2*b^2)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*c+1/2*d
*x^(1/2))+2*a)/(-a^2+b^2)^(1/2)))+2/a^2*arctan(tan(1/2*c+1/2*d*x^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(112) = 224.

Time = 0.11 (sec) , antiderivative size = 576, normalized size of antiderivative = 4.61

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$
$$= \left[\frac{2\left(a^5-2a^3b^2+ab^4\right)d\sqrt{x}\sin\left(d\sqrt{x}+c\right)+2\left(a^4b-2a^2b^3+b^5\right)d\sqrt{x}-2\left(a^3b^2-ab^4\right)\cos\left(d\sqrt{x}+c\right)+(}{(a^7-2a}\right.$$

input
```
integrate(1/x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
[(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*sin(d*sqrt(x) + c) + 2*(a^4*b - 2*
a^2*b^3 + b^5)*d*sqrt(x) - 2*(a^3*b^2 - a*b^4)*cos(d*sqrt(x) + c) + ((2*a^
3*b - a*b^3)*sqrt(a^2 - b^2)*sin(d*sqrt(x) + c) + (2*a^2*b^2 - b^4)*sqrt(a
^2 - b^2))*log(((a^2 - 2*b^2)*cos(d*sqrt(x) + c)^2 + 2*sqrt(a^2 - b^2)*a*c
os(d*sqrt(x) + c) + a^2 + b^2 + 2*(sqrt(a^2 - b^2)*b*cos(d*sqrt(x) + c) +
a*b)*sin(d*sqrt(x) + c))/(a^2*cos(d*sqrt(x) + c)^2 - 2*a*b*sin(d*sqrt(x) +
 c) - a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*sin(d*sqrt(x) + c) + (a^
6*b - 2*a^4*b^3 + a^2*b^5)*d), 2*((a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*sin(
d*sqrt(x) + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*sqrt(x) + ((2*a^3*b - a*b^3)*
sqrt(-a^2 + b^2)*sin(d*sqrt(x) + c) + (2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2))*
arctan(-(sqrt(-a^2 + b^2)*b*sin(d*sqrt(x) + c) + sqrt(-a^2 + b^2)*a)/((a^2
 - b^2)*cos(d*sqrt(x) + c))) - (a^3*b^2 - a*b^4)*cos(d*sqrt(x) + c))/((a^7
 - 2*a^5*b^2 + a^3*b^4)*d*sin(d*sqrt(x) + c) + (a^6*b - 2*a^4*b^3 + a^2*b^
5)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
integrate(1/x**(1/2)/(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(1/(sqrt(x)*(a + b*csc(c + d*sqrt(x)))**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.39

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$
$$= -\frac{4\left(2a^2b-b^3\right)\left(\pi\left\lfloor\frac{d\sqrt{x}+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}d\sqrt{x}+\frac{1}{2}c\right)+a}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4d-a^2b^2d\right)\sqrt{-a^2+b^2}}$$
$$-\frac{4\left(ab\tan\left(\frac{1}{2}d\sqrt{x}+\frac{1}{2}c\right)+b^2\right)}{\left(a^3d-ab^2d\right)\left(b\tan\left(\frac{1}{2}d\sqrt{x}+\frac{1}{2}c\right)^2+2a\tan\left(\frac{1}{2}d\sqrt{x}+\frac{1}{2}c\right)+b\right)}+\frac{2\left(d\sqrt{x}+c\right)}{a^2d}$$

input
```
integrate(1/x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
-4*(2*a^2*b - b^3)*(pi*floor(1/2*(d*sqrt(x) + c)/pi + 1/2)*sgn(b) + arctan
((b*tan(1/2*d*sqrt(x) + 1/2*c) + a)/sqrt(-a^2 + b^2)))/((a^4*d - a^2*b^2*d
)*sqrt(-a^2 + b^2)) - 4*(a*b*tan(1/2*d*sqrt(x) + 1/2*c) + b^2)/((a^3*d - a
*b^2*d)*(b*tan(1/2*d*sqrt(x) + 1/2*c)^2 + 2*a*tan(1/2*d*sqrt(x) + 1/2*c) +
 b)) + 2*(d*sqrt(x) + c)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 20.04 (sec) , antiderivative size = 2737, normalized size of antiderivative = 21.90

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(1/2)*(a + b/sin(c + d*x^(1/2)))^2),x)
```

output
```
- (4*atan((512*a^3*b^3*tan(c/2 + (d*x^(1/2))/2))/((512*a^3*b^9)/(a^6 + a^2
*b^4 - 2*a^4*b^2) - (1536*a^5*b^7)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (1024*a^7
*b^5)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (512*a^9*b^3)/(a^6 + a^2*b^4 - 2*a^4*b
^2) - (512*a^11*b)/(a^6 + a^2*b^4 - 2*a^4*b^2)) - (512*a*b^5*tan(c/2 + (d*
x^(1/2))/2))/((512*a^3*b^9)/(a^6 + a^2*b^4 - 2*a^4*b^2) - (1536*a^5*b^7)/(
a^6 + a^2*b^4 - 2*a^4*b^2) + (1024*a^7*b^5)/(a^6 + a^2*b^4 - 2*a^4*b^2) +
(512*a^9*b^3)/(a^6 + a^2*b^4 - 2*a^4*b^2) - (512*a^11*b)/(a^6 + a^2*b^4 -
2*a^4*b^2)) + (512*a^5*b*tan(c/2 + (d*x^(1/2))/2))/((512*a^3*b^9)/(a^6 + a
^2*b^4 - 2*a^4*b^2) - (1536*a^5*b^7)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (1024*a
^7*b^5)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (512*a^9*b^3)/(a^6 + a^2*b^4 - 2*a^4
*b^2) - (512*a^11*b)/(a^6 + a^2*b^4 - 2*a^4*b^2))))/(a^2*d) - ((4*b^2)/(a*
(a^2 - b^2)) + (4*b*tan(c/2 + (d*x^(1/2))/2))/(a^2 - b^2))/(d*(b + b*tan(c
/2 + (d*x^(1/2))/2)^2 + 2*a*tan(c/2 + (d*x^(1/2))/2))) - (b*atan(((b*(2*a^
2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x^(1/2))/2)*(8*a*b^
7 - 8*a^7*b - 32*a^3*b^5 + 36*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (32*
(4*a*b^6 - 8*a^3*b^4 + 4*a^5*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (2*b*(2*a
^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*(2*a^8*b - 2*a^6*b^3))/(a^6 + a
^2*b^4 - 2*a^4*b^2) + (32*tan(c/2 + (d*x^(1/2))/2)*(4*a^4*b^6 - 12*a^6*b^4
 + 8*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (2*b*((32*(a^5*b^6 - 2*a^7*b^
4 + a^9*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (32*tan(c/2 + (d*x^(1/2))/2...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.86

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$= \frac{8\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{\sqrt{x}\,d}{2}+\frac{c}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)\sin\left(\sqrt{x}\,d+c\right)a^3b-4\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{\sqrt{x}\,d}{2}+\frac{c}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)\sin\left(\sqrt{x}\,d+\right.}{}$$

input
```
int(1/x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
(2*(4*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**
2 + b**2))*sin(sqrt(x)*d + c)*a**3*b - 2*sqrt( - a**2 + b**2)*atan((tan((s
qrt(x)*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*sin(sqrt(x)*d + c)*a*b**3 +
4*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b + a)/sqrt( - a**2 +
b**2))*a**2*b**2 - 2*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*b +
 a)/sqrt( - a**2 + b**2))*b**4 - cos(sqrt(x)*d + c)*a**3*b**2 + cos(sqrt(x
)*d + c)*a*b**4 + sqrt(x)*sin(sqrt(x)*d + c)*a**5*d - 2*sqrt(x)*sin(sqrt(x
)*d + c)*a**3*b**2*d + sqrt(x)*sin(sqrt(x)*d + c)*a*b**4*d + sqrt(x)*a**4*
b*d - 2*sqrt(x)*a**2*b**3*d + sqrt(x)*b**5*d))/(a**2*d*(sin(sqrt(x)*d + c)
*a**5 - 2*sin(sqrt(x)*d + c)*a**3*b**2 + sin(sqrt(x)*d + c)*a*b**4 + a**4*
b - 2*a**2*b**3 + b**5))
```

3.70 $\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 32.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
Integrate[1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

output
```
Integrate[1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2} \, dx$$

$$\Big\downarrow \text{ 4695}$$

$$\int \frac{1}{x^{3/2} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2}dx$$

input
```
Int[1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{3}{2}} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2}dx$$

input
```
int(1/x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(1/x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{3/2} \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2 x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(x)/(b^2*x^2*csc(d*sqrt(x) + c)^2 + 2*a*b*x^2*csc(d*sqrt(x) +
 c) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 6.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2} \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{x^{\frac{3}{2}} \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
integrate(1/x**(3/2)/(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(1/(x**(3/2)*(a + b*csc(c + d*sqrt(x)))**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2 x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*csc(d*sqrt(x) + c) + a)^2*x^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 16.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{3/2}\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

input
```
int(1/(x^(3/2)*(a + b/sin(c + d*x^(1/2)))^2),x)
```

output
```
int(1/(x^(3/2)*(a + b/sin(c + d*x^(1/2)))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{1}{x^{3/2} \left(a+b\csc \left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\sqrt{x}\,\csc \left(\sqrt{x}\,d+c\right)^2 b^2x+2\sqrt{x}\,\csc \left(\sqrt{x}\,d+c\right)abx+\sqrt{x}\,a^2x}dx$$

input
```
int(1/x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(1/(sqrt(x)*csc(sqrt(x)*d + c)**2*b**2*x + 2*sqrt(x)*csc(sqrt(x)*d + c)
*a*b*x + sqrt(x)*a**2*x),x)
```

3.71 $\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/x^(5/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 37.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
Integrate[1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

output
```
Integrate[1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2} \, dx$$

$$\Big\downarrow \text{ 4695}$$

$$\int \frac{1}{x^{5/2} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2}dx$$

input
```
Int[1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{5}{2}} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2}dx$$

input
```
int(1/x^(5/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(1/x^(5/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{5/2} \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2 x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(x)/(b^2*x^3*csc(d*sqrt(x) + c)^2 + 2*a*b*x^3*csc(d*sqrt(x) +
 c) + a^2*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 32.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2} \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{x^{\frac{5}{2}} \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

input
```
integrate(1/x**(5/2)/(a+b*csc(c+d*x**(1/2)))**2,x)
```

output
```
Integral(1/(x**(5/2)*(a + b*csc(c + d*sqrt(x)))**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/x^(5/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*csc(d*sqrt(x) + c) + a)^2*x^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 16.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{5/2}\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

input
```
int(1/(x^(5/2)*(a + b/sin(c + d*x^(1/2)))^2),x)
```

output
```
int(1/(x^(5/2)*(a + b/sin(c + d*x^(1/2)))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{5/2} \left(a + b \csc \left(c + d\sqrt{x}\right)\right)^2} \, dx = \int \frac{1}{\sqrt{x}\, \csc \left(\sqrt{x}\, d + c\right)^2 b^2 x^2 + 2\sqrt{x}\, \csc \left(\sqrt{x}\, d + c\right) ab\, x^2 + \sqrt{x}\, a^2 x^2} dx$$

input
```
int(1/x^(5/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

output
```
int(1/(sqrt(x)*csc(sqrt(x)*d + c)**2*b**2*x**2 + 2*sqrt(x)*csc(sqrt(x)*d +
 c)*a*b*x**2 + sqrt(x)*a**2*x**2),x)
```

3.72 $\int (ex)^m (a+b\csc(c+dx^n))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m (a+b\csc(c+dx^n))^p \, dx = x^{-m}(ex)^m \mathrm{Int}(x^m(a+b\csc(c+dx^n))^p, x)$$

output
```
(e*x)^m*Defer(Int)(x^m*(a+b*csc(c+d*x^n))^p,x)/(x^m)
```

Mathematica [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\csc(c+dx^n))^p \, dx = \int (ex)^m (a+b\csc(c+dx^n))^p \, dx$$

input
```
Integrate[(e*x)^m*(a + b*Csc[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^m*(a + b*Csc[c + d*x^n])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4697, 4695}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a + b\csc(c + dx^n))^p \, dx$$

$$\Big\downarrow \text{4697}$$

$$x^{-m}(ex)^m \int x^m(a + b\csc(dx^n + c))^p dx$$

$$\Big\downarrow \text{4695}$$

$$x^{-m}(ex)^m \int x^m(a + b\csc(dx^n + c))^p dx$$

input
```
Int[(e*x)^m*(a + b*Csc[c + d*x^n])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4695
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Unintegrable[x^m*(a + b*Csc[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m,
 n, p}, x]
```

rule 4697
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m (a + b\csc(c + d\,x^n))^p dx$$

input
```
int((e*x)^m*(a+b*csc(c+d*x^n))^p,x)
```

output
```
int((e*x)^m*(a+b*csc(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\csc(c + dx^n))^p \, dx = \int (ex)^m (b\csc(dx^n + c) + a)^p \, dx$$

input
```
integrate((e*x)^m*(a+b*csc(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*(b*csc(d*x^n + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 30.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m (a + b\csc(c + dx^n))^p \, dx = \int (ex)^m (a + b\csc(c + dx^n))^p \, dx$$

input
```
integrate((e*x)**m*(a+b*csc(c+d*x**n))**p,x)
```

output
```
Integral((e*x)**m*(a + b*csc(c + d*x**n))**p, x)
```

Maxima [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\csc(c+dx^n))^p \, dx = \int (ex)^m (b\csc(dx^n+c)+a)^p \, dx$$

input
```
integrate((e*x)^m*(a+b*csc(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(b*csc(d*x^n + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\csc(c+dx^n))^p \, dx = \int (ex)^m (b\csc(dx^n+c)+a)^p \, dx$$

input
```
integrate((e*x)^m*(a+b*csc(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(b*csc(d*x^n + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 16.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (ex)^m (a+b\csc(c+dx^n))^p \, dx = \int \left(a+\frac{b}{\sin(c+d\,x^n)}\right)^p (e\,x)^m \, dx$$

input
```
int((a + b/sin(c + d*x^n))^p*(e*x)^m,x)
```

output
```
int((a + b/sin(c + d*x^n))^p*(e*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (ex)^m (a + b\csc (c + dx^n))^p \, dx = e^m \left(\int x^m (\csc (x^n d + c) b + a)^p dx \right)$$

input
```
int((e*x)^m*(a+b*csc(c+d*x^n))^p,x)
```

output
```
e**m*int(x**m*(csc(x**n*d + c)*b + a)**p,x)
```

3.73 $\int (ex)^{-1+n} (a+b\csc(c+dx^n))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int (ex)^{-1+n} (a+b\csc(c+dx^n))\,dx = \frac{a(ex)^n}{en} - \frac{bx^{-n}(ex)^n \operatorname{arctanh}(\cos(c+dx^n))}{den}$$

output
```
a*(e*x)^n/e/n-b*(e*x)^n*arctanh(cos(c+d*x^n))/d/e/n/(x^n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int (ex)^{-1+n} (a+b\csc(c+dx^n))\,dx = \frac{x^{-n}(ex)^n (adx^n - b\operatorname{arctanh}(\cos(c+dx^n)))}{den}$$

input
```
Integrate[(e*x)^(-1 + n)*(a + b*Csc[c + d*x^n]),x]
```

output
```
((e*x)^n*(a*d*x^n - b*ArcTanh[Cos[c + d*x^n]]))/(d*e*n*x^n)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1}(a+b\csc(c+dx^n))\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(a(ex)^{n-1}+b(ex)^{n-1}\csc(c+dx^n)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(ex)^n}{en}-\frac{bx^{-n}(ex)^n\text{arctanh}(\cos(c+dx^n))}{den}$$

input
```
Int[(e*x)^(-1 + n)*(a + b*Csc[c + d*x^n]),x]
```

output
```
(a*(e*x)^n)/(e*n) - (b*(e*x)^n*ArcTanh[Cos[c + d*x^n]])/(d*e*n*x^n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.51

method	result
risch	$\frac{ax\,\mathrm{e}^{\frac{(-1+n)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2\pi-i\,\mathrm{csgn}(iex)^3\pi+2\ln(x)+2\ln(e)\right)}{2}}}{n} - \frac{2\,\mathrm{arctanh}\left(\mathrm{e}^{i}\right.}{}$

input
```
int((e*x)^(-1+n)*(a+b*csc(c+d*x^n)),x,method=_RETURNVERBOSE)
```

output
```
a/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*cs
gn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(x)+2*l
n(e)))-2*arctanh(exp(I*(c+d*x^n)))/d/e*e^n/n*b*exp(1/2*I*Pi*csgn(I*e*x)*(-
1+n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int (ex)^{-1+n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx = \frac{2\,ade^{n-1}x^n - be^{n-1}\log\left(\frac{1}{2}\,\cos\left(dx^n+c\right)+\frac{1}{2}\right) + be^{n-1}\log\left(-\frac{1}{2}\,\cos\left(dx^n+c\right)+\frac{1}{2}\right)}{2\,dn}$$

input
```
integrate((e*x)^(-1+n)*(a+b*csc(c+d*x^n)),x, algorithm="fricas")
```

output
```
1/2*(2*a*d*e^(n - 1)*x^n - b*e^(n - 1)*log(1/2*cos(d*x^n + c) + 1/2) + b*e
^(n - 1)*log(-1/2*cos(d*x^n + c) + 1/2))/(d*n)
```

Sympy [F]

$$\int (ex)^{-1+n}(a+b\csc(c+dx^n))\,dx=\int (ex)^{n-1}(a+b\csc(c+dx^n))\,dx$$

input
```
integrate((e*x)**(-1+n)*(a+b*csc(c+d*x**n)),x)
```

output
```
Integral((e*x)**(n - 1)*(a + b*csc(c + d*x**n)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(45) = 90.

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.84

$$\int (ex)^{-1+n}(a+b\csc(c+dx^n))\,dx=\frac{(ex)^n a}{en} - \frac{\left(e^n\log\left(\cos(dx^n)^2+2\cos(dx^n)\cos(c)+\cos(c)^2+\sin(dx^n)^2-2\sin(dx^n)\sin(c)+\sin(c)^2\right)-e^n\,\mathrm{lo}\right.}{2\,den}$$

input
```
integrate((e*x)^(-1+n)*(a+b*csc(c+d*x^n)),x, algorithm="maxima")
```

output
```
(e*x)^n*a/(e*n) - 1/2*(e^n*log(cos(d*x^n)^2 + 2*cos(d*x^n)*cos(c) + cos(c)
^2 + sin(d*x^n)^2 - 2*sin(d*x^n)*sin(c) + sin(c)^2) - e^n*log(cos(d*x^n)^2
 - 2*cos(d*x^n)*cos(c) + cos(c)^2 + sin(d*x^n)^2 + 2*sin(d*x^n)*sin(c) + s
in(c)^2))*b/(d*e*n)
```

Giac [F]

$$\int (ex)^{-1+n} (a + b \csc(c + dx^n)) \, dx = \int (b \csc(dx^n + c) + a)(ex)^{n-1} \, dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*csc(c+d*x^n)),x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^n + c) + a)*(e*x)^(n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 17.57 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.36

$$\int (ex)^{-1+n} (a + b \csc(c + dx^n)) \, dx$$
$$= \frac{(e\,x)^n \left(a\,d\,x^n + b\,\ln\left(b\,(e\,x)^{n-1}\,2\mathrm{i} - b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\,(e\,x)^{n-1}\,2\mathrm{i}\right) - b\,\ln\left(-b\,(e\,x)^{n-1}\,2\mathrm{i} - b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\,(e\,x)^{n-1}\right.\right.}{d\,e\,n\,x^n}$$

input
```
int((a + b/sin(c + d*x^n))*(e*x)^(n - 1),x)
```

output
```
((e*x)^n*(b*log(b*(e*x)^(n - 1)*2i - b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n -
1)*2i) - b*log(- b*(e*x)^(n - 1)*2i - b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n -
 1)*2i) + a*d*x^n))/(d*e*n*x^n)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int (ex)^{-1+n} (a + b \csc(c + dx^n)) \, dx = \frac{e^n \left(x^n a d + \log\left(\tan\left(\frac{x^n d}{2} + \frac{c}{2}\right)\right) b\right)}{den}$$

input
```
int((e*x)^(-1+n)*(a+b*csc(c+d*x^n)),x)
```

output
```
(e**n*(x**n*a*d + log(tan((x**n*d + c)/2))*b))/(d*e*n)
```

3.74 $\int (ex)^{-1+2n}(a+b\csc(c+dx^n))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 141

$$\int (ex)^{-1+2n}(a+b\csc(c+dx^n))\,dx = \frac{a(ex)^{2n}}{2en} - \frac{2bx^{-n}(ex)^{2n}\operatorname{arctanh}\left(e^{i(c+dx^n)}\right)}{den} + \frac{ibx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} - \frac{ibx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en}$$

output
```
1/2*a*(e*x)^(2*n)/e/n-2*b*(e*x)^(2*n)*arctanh(exp(I*(c+d*x^n)))/d/e/n/(x^n
)+I*b*(e*x)^(2*n)*polylog(2,-exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-I*b*(e*x)
^(2*n)*polylog(2,exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.31

$$\int (ex)^{-1+2n}(a+b\csc(c+dx^n))\,dx$$
$$= \frac{x^{-2n}(ex)^{2n}\left(ad^2x^{2n}+2bc\log\left(1-e^{i(c+dx^n)}\right)+2bdx^n\log\left(1-e^{i(c+dx^n)}\right)-2bc\log\left(1+e^{i(c+dx^n)}\right)-2bdx^n\right.}{2d}$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*Csc[c + d*x^n]),x]
```

output
```
((e*x)^(2*n)*(a*d^2*x^(2*n) + 2*b*c*Log[1 - E^(I*(c + d*x^n))] + 2*b*d*x^n
*Log[1 - E^(I*(c + d*x^n))] - 2*b*c*Log[1 + E^(I*(c + d*x^n))] - 2*b*d*x^n
*Log[1 + E^(I*(c + d*x^n))] - 2*b*c*Log[Tan[(c + d*x^n)/2]] + (2*I)*b*Poly
Log[2, -E^(I*(c + d*x^n))] - (2*I)*b*PolyLog[2, E^(I*(c + d*x^n))]))/(2*d^
2*e*n*x^(2*n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1}(a+b\csc(c+dx^n))\,dx$$

$$\Big\downarrow \ 2010$$

$$\int \left(a(ex)^{2n-1}+b(ex)^{2n-1}\csc(c+dx^n)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a(ex)^{2n}}{2en}-\frac{2bx^{-n}(ex)^{2n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den}+\frac{ibx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-e^{i(dx^n+c)}\right)}{d^2en}-\frac{ibx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,e^{i(dx^n+c)}\right)}{d^2en}$$

input
```
Int[(e*x)^(-1 + 2*n)*(a + b*Csc[c + d*x^n]),x]
```

output
```
(a*(e*x)^(2*n))/(2*e*n) - (2*b*(e*x)^(2*n)*ArcTanh[E^(I*(c + d*x^n))])/(d*
e*n*x^n) + (I*b*(e*x)^(2*n)*PolyLog[2, -E^(I*(c + d*x^n))])/(d^2*e*n*x^(2*
n)) - (I*b*(e*x)^(2*n)*PolyLog[2, E^(I*(c + d*x^n))])/(d^2*e*n*x^(2*n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 731, normalized size of antiderivative = 5.18

method	result	size
risch	Expression too large to display	731

input
```
int((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/n*x*exp(1/2*(-1+2*n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I
*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(
x)+2*ln(e)))+1/d/n/e*(e^n)^2*b*ln(1-exp(I*(c+d*x^n)))*x^n*(-1)^(-1/2*csgn(
I*x)*csgn(I*e*x)^2)*(-1)^(-1/2*csgn(I*e)*csgn(I*e*x)^2)*(-1)^(1/2*csgn(I*e
)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e)*csgn(I*x)*
n+2*csgn(I*e)*csgn(I*e*x)*n+2*n*csgn(I*x)*csgn(I*e*x)-2*n*csgn(I*e*x)^2+cs
gn(I*e*x)^2))-1/d/n/e*(e^n)^2*b*ln(exp(I*(c+d*x^n))+1)*x^n*(-1)^(-1/2*csgn
(I*x)*csgn(I*e*x)^2)*(-1)^(-1/2*csgn(I*e)*csgn(I*e*x)^2)*(-1)^(1/2*csgn(I*
e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e)*csgn(I*x)
*n+2*csgn(I*e)*csgn(I*e*x)*n+2*n*csgn(I*x)*csgn(I*e*x)-2*n*csgn(I*e*x)^2+c
sgn(I*e*x)^2))-I/d^2/n/e*(e^n)^2*b*dilog(1-exp(I*(c+d*x^n)))*(-1)^(-1/2*cs
gn(I*x)*csgn(I*e*x)^2)*(-1)^(-1/2*csgn(I*e)*csgn(I*e*x)^2)*(-1)^(1/2*csgn(
I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e)*csgn(I*
x)*n+2*csgn(I*e)*csgn(I*e*x)*n+2*n*csgn(I*x)*csgn(I*e*x)-2*n*csgn(I*e*x)^2
+csgn(I*e*x)^2))+I/d^2/n/e*(e^n)^2*b*dilog(exp(I*(c+d*x^n))+1)*(-1)^(-1/2*
csgn(I*x)*csgn(I*e*x)^2)*(-1)^(-1/2*csgn(I*e)*csgn(I*e*x)^2)*(-1)^(1/2*csg
n(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e)*csgn(
I*x)*n+2*csgn(I*e)*csgn(I*e*x)*n+2*n*csgn(I*x)*csgn(I*e*x)-2*n*csgn(I*e*x)
^2+csgn(I*e*x)^2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(131) = 262$.

Time = 0.11 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.72

$$\int (ex)^{-1+2n} (a + b \csc(c + dx^n)) \, dx$$
$$= \frac{ad^2e^{2n-1}x^{2n} - bde^{2n-1}x^n \log(\cos(dx^n + c) + i \sin(dx^n + c) + 1) - bde^{2n-1}x^n \log(\cos(dx^n + c) - i \sin$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n)),x, algorithm="fricas")
```

output
```
1/2*(a*d^2*e^(2*n - 1)*x^(2*n) - b*d*e^(2*n - 1)*x^n*log(cos(d*x^n + c) +
I*sin(d*x^n + c) + 1) - b*d*e^(2*n - 1)*x^n*log(cos(d*x^n + c) - I*sin(d*x
^n + c) + 1) - b*c*e^(2*n - 1)*log(-1/2*cos(d*x^n + c) + 1/2*I*sin(d*x^n +
 c) + 1/2) - b*c*e^(2*n - 1)*log(-1/2*cos(d*x^n + c) - 1/2*I*sin(d*x^n + c
) + 1/2) - I*b*e^(2*n - 1)*dilog(cos(d*x^n + c) + I*sin(d*x^n + c)) + I*b*
e^(2*n - 1)*dilog(cos(d*x^n + c) - I*sin(d*x^n + c)) - I*b*e^(2*n - 1)*dil
og(-cos(d*x^n + c) + I*sin(d*x^n + c)) + I*b*e^(2*n - 1)*dilog(-cos(d*x^n
+ c) - I*sin(d*x^n + c)) + (b*d*e^(2*n - 1)*x^n + b*c*e^(2*n - 1))*log(-co
s(d*x^n + c) + I*sin(d*x^n + c) + 1) + (b*d*e^(2*n - 1)*x^n + b*c*e^(2*n -
 1))*log(-cos(d*x^n + c) - I*sin(d*x^n + c) + 1))/(d^2*n)
```

Sympy [F]

$$\int (ex)^{-1+2n}(a+b\csc(c+dx^n))\,dx=\int (ex)^{2n-1}(a+b\csc(c+dx^n))\,dx$$

input
```
integrate((e*x)**(-1+2*n)*(a+b*csc(c+d*x**n)),x)
```

output
```
Integral((e*x)**(2*n - 1)*(a + b*csc(c + d*x**n)), x)
```

Maxima [F]

$$\int (ex)^{-1+2n}(a+b\csc(c+dx^n))\,dx=\int (b\csc(dx^n+c)+a)(ex)^{2n-1}\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n)),x, algorithm="maxima")
```

output
```
(e^(2*n + 1)*integrate(x^(2*n)*sin(d*x^n + c)/(e^2*x*cos(d*x^n + c)^2 + e^
2*x*sin(d*x^n + c)^2 + 2*e^2*x*cos(d*x^n + c) + e^2*x), x) + e^(2*n + 1)*i
ntegrate(x^(2*n)*sin(d*x^n + c)/(e^2*x*cos(d*x^n + c)^2 + e^2*x*sin(d*x^n
+ c)^2 - 2*e^2*x*cos(d*x^n + c) + e^2*x), x))*b + 1/2*(e*x)^(2*n)*a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+2n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx=\int (b\csc\left(dx^n+c\right)+a)(ex)^{2n-1}\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n)),x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^n + c) + a)*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx=\int \left(a+\frac{b}{\sin\left(c+d\,x^n\right)}\right)\,(e\,x)^{2n-1}\,dx$$

input
```
int((a + b/sin(c + d*x^n))*(e*x)^(2*n - 1),x)
```

output
```
int((a + b/sin(c + d*x^n))*(e*x)^(2*n - 1), x)
```

Reduce [F]

$$\int (ex)^{-1+2n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx=\frac{e^{2n}\left(x^{2n}a+2\left(\int \frac{x^{2n}\csc(x^n d+c)}{x}dx\right)bn\right)}{2en}$$

input
```
int((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n)),x)
```

output
```
(e**(2*n)*(x**(2*n)*a + 2*int((x**(2*n)*csc(x**n*d + c))/x,x)*b*n))/(2*e*n
)
```

3.75 $\int (ex)^{-1+3n}(a+b\csc(c+dx^n))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 221

$$\int (ex)^{-1+3n}(a+b\csc(c+dx^n))\,dx = \frac{a(ex)^{3n}}{3en} - \frac{2bx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} + \frac{2ibx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} - \frac{2ibx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en} - \frac{2bx^{-3n}(ex)^{3n}\text{PolyLog}\left(3,-e^{i(c+dx^n)}\right)}{d^3en} + \frac{2bx^{-3n}(ex)^{3n}\text{PolyLog}\left(3,e^{i(c+dx^n)}\right)}{d^3en}$$

output
```
1/3*a*(e*x)^(3*n)/e/n-2*b*(e*x)^(3*n)*arctanh(exp(I*(c+d*x^n)))/d/e/n/(x^n
)+2*I*b*(e*x)^(3*n)*polylog(2,-exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-2*I*b*(
e*x)^(3*n)*polylog(2,exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-2*b*(e*x)^(3*n)*p
olylog(3,-exp(I*(c+d*x^n)))/d^3/e/n/(x^(3*n))+2*b*(e*x)^(3*n)*polylog(3,ex
p(I*(c+d*x^n)))/d^3/e/n/(x^(3*n))
```

Mathematica [F]

$$\int (ex)^{-1+3n}(a+b\csc(c+dx^n))\,dx=\int (ex)^{-1+3n}(a+b\csc(c+dx^n))\,dx$$

input
```
Integrate[(e*x)^(-1 + 3*n)*(a + b*Csc[c + d*x^n]),x]
```

output
```
Integrate[(e*x)^(-1 + 3*n)*(a + b*Csc[c + d*x^n]), x]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3n-1}(a+b\csc(c+dx^n))\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(a(ex)^{3n-1}+b(ex)^{3n-1}\csc(c+dx^n)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(ex)^{3n}}{3en}-\frac{2bx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den}-\frac{2bx^{-3n}(ex)^{3n}\text{PolyLog}\left(3,-e^{i(dx^n+c)}\right)}{d^3en}+\frac{2bx^{-3n}(ex)^{3n}\text{PolyLog}\left(3,e^{i(dx^n+c)}\right)}{d^3en}+\frac{2ibx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,-e^{i(dx^n+c)}\right)}{d^2en}-\frac{2ibx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,e^{i(dx^n+c)}\right)}{d^2en}$$

input
```
Int[(e*x)^(-1 + 3*n)*(a + b*Csc[c + d*x^n]),x]
```

output
```
(a*(e*x)^(3*n))/(3*e*n) - (2*b*(e*x)^(3*n)*ArcTanh[E^(I*(c + d*x^n))])/(d*
e*n*x^n) + ((2*I)*b*(e*x)^(3*n)*PolyLog[2, -E^(I*(c + d*x^n))])/(d^2*e*n*x
^(2*n)) - ((2*I)*b*(e*x)^(3*n)*PolyLog[2, E^(I*(c + d*x^n))])/(d^2*e*n*x^(
2*n)) - (2*b*(e*x)^(3*n)*PolyLog[3, -E^(I*(c + d*x^n))])/(d^3*e*n*x^(3*n))
 + (2*b*(e*x)^(3*n)*PolyLog[3, E^(I*(c + d*x^n))])/(d^3*e*n*x^(3*n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int (ex)^{-1+3n} (a + b\csc(c + d\,x^n))\,dx$$

input
```
int((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n)),x)
```

output
```
int((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(211) = 422.

Time = 0.12 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.52

$$\int (ex)^{-1+3n} (a + b\csc(c + dx^n))\ dx$$
$$= \frac{2\,ad^3e^{3\,n-1}x^{3\,n} - 3\,bd^2e^{3\,n-1}x^{2\,n}\log(\cos(dx^n + c) + i\,\sin(dx^n + c) + 1) - 3\,bd^2e^{3\,n-1}x^{2\,n}\log(\cos(dx^n +}{}$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n)),x, algorithm="fricas")
```

output
```
1/6*(2*a*d^3*e^(3*n - 1)*x^(3*n) - 3*b*d^2*e^(3*n - 1)*x^(2*n)*log(cos(d*x
^n + c) + I*sin(d*x^n + c) + 1) - 3*b*d^2*e^(3*n - 1)*x^(2*n)*log(cos(d*x^
n + c) - I*sin(d*x^n + c) + 1) - 6*I*b*d*e^(3*n - 1)*x^n*dilog(cos(d*x^n +
 c) + I*sin(d*x^n + c)) + 6*I*b*d*e^(3*n - 1)*x^n*dilog(cos(d*x^n + c) - I
*sin(d*x^n + c)) - 6*I*b*d*e^(3*n - 1)*x^n*dilog(-cos(d*x^n + c) + I*sin(d
*x^n + c)) + 6*I*b*d*e^(3*n - 1)*x^n*dilog(-cos(d*x^n + c) - I*sin(d*x^n +
 c)) + 3*b*c^2*e^(3*n - 1)*log(-1/2*cos(d*x^n + c) + 1/2*I*sin(d*x^n + c)
+ 1/2) + 3*b*c^2*e^(3*n - 1)*log(-1/2*cos(d*x^n + c) - 1/2*I*sin(d*x^n + c
) + 1/2) + 6*b*e^(3*n - 1)*polylog(3, cos(d*x^n + c) + I*sin(d*x^n + c)) +
 6*b*e^(3*n - 1)*polylog(3, cos(d*x^n + c) - I*sin(d*x^n + c)) - 6*b*e^(3*
n - 1)*polylog(3, -cos(d*x^n + c) + I*sin(d*x^n + c)) - 6*b*e^(3*n - 1)*po
lylog(3, -cos(d*x^n + c) - I*sin(d*x^n + c)) + 3*(b*d^2*e^(3*n - 1)*x^(2*n
) - b*c^2*e^(3*n - 1))*log(-cos(d*x^n + c) + I*sin(d*x^n + c) + 1) + 3*(b*
d^2*e^(3*n - 1)*x^(2*n) - b*c^2*e^(3*n - 1))*log(-cos(d*x^n + c) - I*sin(d
*x^n + c) + 1))/(d^3*n)
```

Sympy [F]

$$\int (ex)^{-1+3n}(a+b\csc(c+dx^n))\,dx=\int (ex)^{3n-1}(a+b\csc(c+dx^n))\,dx$$

input
```
integrate((e*x)**(-1+3*n)*(a+b*csc(c+d*x**n)),x)
```

output
```
Integral((e*x)**(3*n - 1)*(a + b*csc(c + d*x**n)), x)
```

Maxima [F]

$$\int (ex)^{-1+3n} (a + b\csc(c + dx^n))\, dx = \int (b\csc(dx^n + c) + a)(ex)^{3\,n-1}\, dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n)),x, algorithm="maxima")
```

output
```
(e^(3*n + 1)*integrate(x^(3*n)*sin(d*x^n + c)/(e^2*x*cos(d*x^n + c)^2 + e^
2*x*sin(d*x^n + c)^2 + 2*e^2*x*cos(d*x^n + c) + e^2*x), x) + e^(3*n + 1)*i
ntegrate(x^(3*n)*sin(d*x^n + c)/(e^2*x*cos(d*x^n + c)^2 + e^2*x*sin(d*x^n
+ c)^2 - 2*e^2*x*cos(d*x^n + c) + e^2*x), x))*b + 1/3*(e*x)^(3*n)*a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+3n} (a + b\csc(c + dx^n))\, dx = \int (b\csc(dx^n + c) + a)(ex)^{3\,n-1}\, dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n)),x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^n + c) + a)*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} (a + b\csc(c + dx^n))\, dx = \int \left(a + \frac{b}{\sin(c + d\,x^n)}\right) (e\,x)^{3\,n-1}\, dx$$

input
```
int((a + b/sin(c + d*x^n))*(e*x)^(3*n - 1),x)
```

output
```
int((a + b/sin(c + d*x^n))*(e*x)^(3*n - 1), x)
```

Reduce [F]

$$\int (ex)^{-1+3n}(a+b\csc(c+dx^n))\,dx=\frac{e^{3n}\left(x^{3n}a+3\left(\int\frac{x^{3n}\csc(x^n d+c)}{x}dx\right)bn\right)}{3en}$$

input
```
int((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n)),x)
```

output
```
(e**(3*n)*(x**(3*n)*a + 3*int((x**(3*n)*csc(x**n*d + c))/x,x)*b*n))/(3*e*n
)
```

3.76 $\int (ex)^{-1+n} (a+b\csc(c+dx^n))^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int (ex)^{-1+n} (a+b\csc(c+dx^n))^2 \, dx = \frac{a^2(ex)^n}{en} - \frac{2abx^{-n}(ex)^n \text{arctanh}(\cos(c+dx^n))}{den} - \frac{b^2x^{-n}(ex)^n \cot(c+dx^n)}{den}$$

output
```
a^2*(e*x)^n/e/n-2*a*b*(e*x)^n*arctanh(cos(c+d*x^n))/d/e/n/(x^n)-b^2*(e*x)^
n*cot(c+d*x^n)/d/e/n/(x^n)
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.28

$$\int (ex)^{-1+n} (a+b\csc(c+dx^n))^2 \, dx = \frac{x^{-n}(ex)^n \left(-b^2 \cot\left(\frac{1}{2}(c+dx^n)\right) + 2a\left(ac+adx^n - 2b\log\left(\cos\left(\frac{1}{2}(c+dx^n)\right)\right) + 2b\log\left(\sin\left(\frac{1}{2}(c+dx^n)\right)\right)\right.\right.}{2den}$$

input
```
Integrate[(e*x)^(-1 + n)*(a + b*Csc[c + d*x^n])^2,x]
```

output
```
((e*x)^n*(-(b^2*Cot[(c + d*x^n)/2]) + 2*a*(a*c + a*d*x^n - 2*b*Log[Cos[(c
+ d*x^n)/2]] + 2*b*Log[Sin[(c + d*x^n)/2]]) + b^2*Tan[(c + d*x^n)/2]))/(2*
d*e*n*x^n)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4697, 4693, 3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} (a+b\csc(c+dx^n))^2 \, dx$$

$$\Big\downarrow \text{4697}$$

$$\frac{x^{-n}(ex)^n \int x^{n-1}(a+b\csc(dx^n+c))^2 dx}{e}$$

$$\Big\downarrow \text{4693}$$

$$\frac{x^{-n}(ex)^n \int (a+b\csc(dx^n+c))^2 dx^n}{en}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^{-n}(ex)^n \int (a+b\csc(dx^n+c))^2 dx^n}{en}$$

$$\Big\downarrow \text{4260}$$

$$\frac{x^{-n}(ex)^n \left(2ab\int \csc(dx^n+c)\, dx^n + b^2\int \csc^2(dx^n+c)\, dx^n + a^2x^n\right)}{en}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^{-n}(ex)^n \left(2ab\int \csc(dx^n+c)\, dx^n + b^2\int \csc(dx^n+c)^2 dx^n + a^2x^n\right)}{en}$$

$$\Big\downarrow \text{4254}$$

$$\frac{x^{-n}(ex)^n \left(2ab\int \csc(dx^n+c)\, dx^n - \frac{b^2\int 1d\cot(dx^n+c)}{d} + a^2x^n\right)}{en}$$

$$\Big\downarrow \text{24}$$

$$\frac{x^{-n}(ex)^n \left(2ab \int \csc\left(dx^n+c\right)dx^n + a^2x^n - \frac{b^2\cot(c+dx^n)}{d}\right)}{en}$$

$$\Big\downarrow \text{4257}$$

$$\frac{x^{-n}(ex)^n \left(a^2x^n - \frac{2ab\text{arctanh}(\cos(c+dx^n))}{d} - \frac{b^2\cot(c+dx^n)}{d}\right)}{en}$$

input
```
Int[(e*x)^(-1 + n)*(a + b*Csc[c + d*x^n])^2,x]
```

output
```
((e*x)^n*(a^2*x^n - (2*a*b*ArcTanh[Cos[c + d*x^n]])/d - (b^2*Cot[c + d*x^n
])/d))/(e*n*x^n)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4260
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Simp[2*a*b   Int[Csc[c + d*x], x], x] + Simp[b^2   Int[Csc[c + d*x]^2, x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

rule 4697
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.44

method	result
risch	$\frac{a^{2} x \,\mathrm{e}^{\frac{(-1+n)\left(-i \,\mathrm{csgn}(i e)\,\mathrm{csgn}(i x)\,\mathrm{csgn}(i e x)\pi+i\,\mathrm{csgn}(i e)\,\mathrm{csgn}(i e x)^{2}\pi+i\,\mathrm{csgn}(i x)\,\mathrm{csgn}(i e x)^{2}\pi-i\,\mathrm{csgn}(i e x)^{3}\pi+2\ln(x)+2\ln(e)\right)}{2}}}{n}-\frac{2 i x\, b^{2} \mathrm{e}^{\frac{(-1+}{}}}{}$

input
```
int((e*x)^(-1+n)*(a+b*csc(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*
csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(x)+2
*ln(e)))-2*I*x*b^2*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I
*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*P
i+2*ln(x)+2*ln(e)))/d/n/(x^n)/(exp(2*I*(c+d*x^n))-1)-4*arctanh(exp(I*(c+d*
x^n)))/d/e*e^n/n*b*a*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(csgn(I*e*x)-csgn(I*x
))*(-csgn(I*e*x)+csgn(I*e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int (ex)^{-1+n} (a + b\csc(c + dx^n))^2 \, dx$$
$$= \frac{a^2de^{n-1}x^n\sin(dx^n + c) - abe^{n-1}\log\left(\frac{1}{2}\cos(dx^n + c) + \frac{1}{2}\right)\sin(dx^n + c) + abe^{n-1}\log\left(-\frac{1}{2}\cos(dx^n + c)\right.}{dn\sin(dx^n + c)}$$

input
```
integrate((e*x)^(-1+n)*(a+b*csc(c+d*x^n))^2,x, algorithm="fricas")
```

output
```
(a^2*d*e^(n - 1)*x^n*sin(d*x^n + c) - a*b*e^(n - 1)*log(1/2*cos(d*x^n + c)
 + 1/2)*sin(d*x^n + c) + a*b*e^(n - 1)*log(-1/2*cos(d*x^n + c) + 1/2)*sin(
d*x^n + c) - b^2*e^(n - 1)*cos(d*x^n + c))/(d*n*sin(d*x^n + c))
```

Sympy [F]

$$\int (ex)^{-1+n} (a + b\csc(c + dx^n))^2 \, dx = \int (ex)^{n-1} (a + b\csc(c + dx^n))^2 \, dx$$

input
```
integrate((e*x)**(-1+n)*(a+b*csc(c+d*x**n))**2,x)
```

output
```
Integral((e*x)**(n - 1)*(a + b*csc(c + d*x**n))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(80) = 160.

Time = 0.07 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.59

$$\int (ex)^{-1+n} (a + b\csc(c + dx^n))^2 \, dx$$
$$= -\frac{2 b^2 e^n \sin(2 dx^n + 2 c)}{den \cos(2 dx^n + 2 c)^2 + den \sin(2 dx^n + 2 c)^2 - 2 den \cos(2 dx^n + 2 c) + den}$$
$$+ \frac{(ex)^n a^2}{en}$$
$$- \frac{(e^n \log(\cos(dx^n)^2 + 2 \cos(dx^n)\cos(c) + \cos(c)^2 + \sin(dx^n)^2 - 2 \sin(dx^n)\sin(c) + \sin(c)^2) - e^n \log}{den}$$

input
```
integrate((e*x)^(-1+n)*(a+b*csc(c+d*x^n))^2,x, algorithm="maxima")
```

output
```
-2*b^2*e^n*sin(2*d*x^n + 2*c)/(d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*
x^n + 2*c)^2 - 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n) + (e*x)^n*a^2/(e*n) - (
e^n*log(cos(d*x^n)^2 + 2*cos(d*x^n)*cos(c) + cos(c)^2 + sin(d*x^n)^2 - 2*s
in(d*x^n)*sin(c) + sin(c)^2) - e^n*log(cos(d*x^n)^2 - 2*cos(d*x^n)*cos(c)
+ cos(c)^2 + sin(d*x^n)^2 + 2*sin(d*x^n)*sin(c) + sin(c)^2))*a*b/(d*e*n)
```

Giac [F]

$$\int (ex)^{-1+n} (a + b\csc(c + dx^n))^2 \, dx = \int (b\csc(dx^n + c) + a)^2 (ex)^{n-1} \, dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*csc(c+d*x^n))^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^n + c) + a)^2*(e*x)^(n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 17.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.28

$$\int (ex)^{-1+n}\left(a+b\csc\left(c+dx^n\right)\right)^2\,dx$$
$$= \frac{a^2\,x\,(e\,x)^{n-1}}{n}-\frac{b^2\,x\,(e\,x)^{n-1}\,2\mathrm{i}}{d\,n\,x^n\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x^n\,2\mathrm{i}}-1\right)}$$
$$-\frac{2\,a\,b\,x\,\ln\left(-a\,b\,(e\,x)^{n-1}\,4\mathrm{i}-a\,b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\,(e\,x)^{n-1}\,4\mathrm{i}\right)\,(e\,x)^{n-1}}{d\,n\,x^n}$$
$$+\frac{2\,a\,b\,x\,\ln\left(a\,b\,(e\,x)^{n-1}\,4\mathrm{i}-a\,b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\,(e\,x)^{n-1}\,4\mathrm{i}\right)\,(e\,x)^{n-1}}{d\,n\,x^n}$$

input
```
int((a + b/sin(c + d*x^n))^2*(e*x)^(n - 1),x)
```

output
```
(a^2*x*(e*x)^(n - 1))/n - (b^2*x*(e*x)^(n - 1)*2i)/(d*n*x^n*(exp(c*2i + d*
x^n*2i) - 1)) - (2*a*b*x*log(- a*b*(e*x)^(n - 1)*4i - a*b*exp(c*1i)*exp(d*
x^n*1i)*(e*x)^(n - 1)*4i)*(e*x)^(n - 1))/(d*n*x^n) + (2*a*b*x*log(a*b*(e*x
)^(n - 1)*4i - a*b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1)*4i)*(e*x)^(n - 1)
)/(d*n*x^n)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int (ex)^{-1+n}\left(a+b\csc\left(c+dx^n\right)\right)^2\,dx$$
$$= \frac{e^n\left(-\cos\left(x^nd+c\right)b^2+x^n\sin\left(x^nd+c\right)a^2d+2\log\left(\tan\left(\frac{x^nd}{2}+\frac{c}{2}\right)\right)\sin\left(x^nd+c\right)ab\right)}{\sin\left(x^nd+c\right)den}$$

input
```
int((e*x)^(-1+n)*(a+b*csc(c+d*x^n))^2,x)
```

output
```
(e**n*( - cos(x**n*d + c)*b**2 + x**n*sin(x**n*d + c)*a**2*d + 2*log(tan((
x**n*d + c)/2))*sin(x**n*d + c)*a*b))/(sin(x**n*d + c)*d*e*n)
```

3.77 $\int (ex)^{-1+2n} (a+b\csc(c+dx^n))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\int (ex)^{-1+2n}(a+b\csc(c+dx^n))^2\,dx = \frac{a^2(ex)^{2n}}{2en} - \frac{4abx^{-n}(ex)^{2n}\operatorname{arctanh}\left(e^{i(c+dx^n)}\right)}{den} - \frac{b^2x^{-n}(ex)^{2n}\cot(c+dx^n)}{den} + \frac{b^2x^{-2n}(ex)^{2n}\log(\sin(c+dx^n))}{d^2en} + \frac{2iabx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} - \frac{2iabx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en}$$

output
```
1/2*a^2*(e*x)^(2*n)/e/n-4*a*b*(e*x)^(2*n)*arctanh(exp(I*(c+d*x^n)))/d/e/n/
(x^n)-b^2*(e*x)^(2*n)*cot(c+d*x^n)/d/e/n/(x^n)+b^2*(e*x)^(2*n)*ln(sin(c+d*
x^n))/d^2/e/n/(x^(2*n))+2*I*a*b*(e*x)^(2*n)*polylog(2,-exp(I*(c+d*x^n)))/d
^2/e/n/(x^(2*n))-2*I*a*b*(e*x)^(2*n)*polylog(2,exp(I*(c+d*x^n)))/d^2/e/n/(
x^(2*n))
```

Mathematica [A] (warning: unable to verify)

Time = 4.98 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.34

$$\int (ex)^{-1+2n}(a+b\csc(c+dx^n))^2\,dx$$

$$= \frac{x^{-2n}(ex)^{2n}\left(2b^2dx^n\cot(c)+dx^n(a^2dx^n-2b^2\cot(c))-2b^2(dx^n\cot(c)-\log(\sin(c+dx^n)))+4ab\left(2\,\mathrm{ar}\right.\right.}{}$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*Csc[c + d*x^n])^2,x]
```

output
```
((e*x)^(2*n)*(2*b^2*d*x^n*Cot[c] + d*x^n*(a^2*d*x^n - 2*b^2*Cot[c]) - 2*b^
2*(d*x^n*Cot[c] - Log[Sin[c + d*x^n]]) + 4*a*b*(2*ArcTan[Tan[c]]*ArcTanh[C
os[c] - Sin[c]*Tan[(d*x^n)/2]] + (((d*x^n + ArcTan[Tan[c]])*(Log[1 - E^(I*
(d*x^n + ArcTan[Tan[c]]))] - Log[1 + E^(I*(d*x^n + ArcTan[Tan[c]]))]) + I*
PolyLog[2, -E^(I*(d*x^n + ArcTan[Tan[c]]))] - I*PolyLog[2, E^(I*(d*x^n + A
rcTan[Tan[c]]))])*Sec[c])/Sqrt[Sec[c]^2]) + b^2*d*x^n*Csc[c/2]*Csc[(c + d*
x^n)/2]*Sin[(d*x^n)/2] + b^2*d*x^n*Sec[c/2]*Sec[(c + d*x^n)/2]*Sin[(d*x^n)
/2]))/(2*d^2*e*n*x^(2*n))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4697, 4693, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1}(a+b\csc(c+dx^n))^2\,dx$$

$$\Big\downarrow \text{4697}$$

$$\frac{x^{-2n}(ex)^{2n}\int x^{2n-1}(a+b\csc(dx^n+c))^2dx}{e}$$

$$\Big\downarrow \text{4693}$$

$$\frac{x^{-2n}(ex)^{2n}\int x^n(a+b\csc(dx^n+c))^2\,dx^n}{en}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^{-2n}(ex)^{2n}\int x^n(a+b\csc(dx^n+c))^2\,dx^n}{en}$$

$$\Big\downarrow \text{4678}$$

$$\frac{x^{-2n}(ex)^{2n}\int\left(a^2x^n+b^2\csc^2(dx^n+c)\,x^n+2ab\csc(dx^n+c)\,x^n\right)dx^n}{en}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{-2n}(ex)^{2n}\left(\frac{1}{2}a^2x^{2n}-\frac{4abx^n\operatorname{arctanh}\left(e^{i(c+dx^n)}\right)}{d}+\frac{2iab\operatorname{PolyLog}\left(2,-e^{i(dx^n+c)}\right)}{d^2}-\frac{2iab\operatorname{PolyLog}\left(2,e^{i(dx^n+c)}\right)}{d^2}+\frac{b^2\log(\sin(c+d}{d^2}\right.}{en}$$

input
```
Int[(e*x)^(-1 + 2*n)*(a + b*Csc[c + d*x^n])^2,x]
```

output
```
((e*x)^(2*n)*((a^2*x^(2*n))/2 - (4*a*b*x^n*ArcTanh[E^(I*(c + d*x^n))])/d -
 (b^2*x^n*Cot[c + d*x^n])/d + (b^2*Log[Sin[c + d*x^n]])/d^2 + ((2*I)*a*b*P
olyLog[2, -E^(I*(c + d*x^n))])/d^2 - ((2*I)*a*b*PolyLog[2, E^(I*(c + d*x^n
))])/d^2))/(e*n*x^(2*n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

rule 4697
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 1002, normalized size of antiderivative = 4.68

method	result	size
risch	Expression too large to display	1002

input
```
int((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2/n*x*exp(1/2*(-1+2*n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn
(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*l
n(x)+2*ln(e)))-2*I*x*b^2*exp(1/2*(-1+2*n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e
*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*
e*x)^3*Pi+2*ln(x)+2*ln(e)))/d/n/(x^n)/(exp(2*I*(c+d*x^n))-1)-2*b^2/d^2/n*(
e^n)^2/e*exp(1/2*I*csgn(I*e*x)*Pi*(-1+2*n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(
I*e*x)+csgn(I*e)))*ln(exp(I*x^n*d))+1/n/d^2*b^2*(e^n)^2/e*exp(1/2*I*csgn(I
*e*x)*Pi*(-1+2*n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))*ln(exp
(2*I*(c+d*x^n))-1)+2/n/d*b*a/e*(e^n)^2*ln(1-exp(I*(c+d*x^n)))*x^n*(-1)^(-1
/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^(-1/2*csgn(I*e)*csgn(I*e*x)^2)*(-1)^(1/2*
csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e)*cs
gn(I*x)*n+2*csgn(I*e)*csgn(I*e*x)*n+2*n*csgn(I*x)*csgn(I*e*x)-2*n*csgn(I*e
*x)^2+csgn(I*e*x)^2))-2/n/d*b*a/e*(e^n)^2*ln(exp(I*(c+d*x^n))+1)*x^n*(-1)^
(-1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^(-1/2*csgn(I*e)*csgn(I*e*x)^2)*(-1)^(1
/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e)
*csgn(I*x)*n+2*csgn(I*e)*csgn(I*e*x)*n+2*n*csgn(I*x)*csgn(I*e*x)-2*n*csgn(
I*e*x)^2+csgn(I*e*x)^2))-2*I/n/d^2*b*a/e*(e^n)^2*dilog(1-exp(I*(c+d*x^n)))
*(-1)^(-1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^(-1/2*csgn(I*e)*csgn(I*e*x)^2)*(
-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csg
n(I*e)*csgn(I*x)*n+2*csgn(I*e)*csgn(I*e*x)*n+2*n*csgn(I*x)*csgn(I*e*x)-...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(206) = 412$.

Time = 0.12 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.65

$$\int (ex)^{-1+2n} (a + b\csc(c + dx^n))^2 \, dx = \frac{a^2d^2e^{2n-1}x^{2n}\sin(dx^n+c) - 2b^2de^{2n-1}x^n\cos(dx^n+c) - 2i\,abe^{2n-1}\mathrm{Li}_2(\cos(dx^n+c) + i\sin(dx^n+c)}{}$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n))^2,x, algorithm="fricas")
```

output
```
1/2*(a^2*d^2*e^(2*n - 1)*x^(2*n)*sin(d*x^n + c) - 2*b^2*d*e^(2*n - 1)*x^n*
cos(d*x^n + c) - 2*I*a*b*e^(2*n - 1)*dilog(cos(d*x^n + c) + I*sin(d*x^n +
c))*sin(d*x^n + c) + 2*I*a*b*e^(2*n - 1)*dilog(cos(d*x^n + c) - I*sin(d*x^
n + c))*sin(d*x^n + c) - 2*I*a*b*e^(2*n - 1)*dilog(-cos(d*x^n + c) + I*sin
(d*x^n + c))*sin(d*x^n + c) + 2*I*a*b*e^(2*n - 1)*dilog(-cos(d*x^n + c) -
I*sin(d*x^n + c))*sin(d*x^n + c) - (2*a*b*c - b^2)*e^(2*n - 1)*log(-1/2*co
s(d*x^n + c) + 1/2*I*sin(d*x^n + c) + 1/2)*sin(d*x^n + c) - (2*a*b*c - b^2
)*e^(2*n - 1)*log(-1/2*cos(d*x^n + c) - 1/2*I*sin(d*x^n + c) + 1/2)*sin(d*
x^n + c) - (2*a*b*d*e^(2*n - 1)*x^n - b^2*e^(2*n - 1))*log(cos(d*x^n + c)
+ I*sin(d*x^n + c) + 1)*sin(d*x^n + c) - (2*a*b*d*e^(2*n - 1)*x^n - b^2*e^
(2*n - 1))*log(cos(d*x^n + c) - I*sin(d*x^n + c) + 1)*sin(d*x^n + c) + 2*(
a*b*d*e^(2*n - 1)*x^n + a*b*c*e^(2*n - 1))*log(-cos(d*x^n + c) + I*sin(d*x
^n + c) + 1)*sin(d*x^n + c) + 2*(a*b*d*e^(2*n - 1)*x^n + a*b*c*e^(2*n - 1)
)*log(-cos(d*x^n + c) - I*sin(d*x^n + c) + 1)*sin(d*x^n + c))/(d^2*n*sin(d
*x^n + c))
```

Sympy [F]

$$\int (ex)^{-1+2n} (a + b\csc(c + dx^n))^2 \, dx = \int (ex)^{2n-1} (a + b\csc(c + dx^n))^2 \, dx$$

input
```
integrate((e*x)**(-1+2*n)*(a+b*csc(c+d*x**n))**2,x)
```

output
```
Integral((e*x)**(2*n - 1)*(a + b*csc(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+2n} (a + b\csc(c + dx^n))^2 \, dx = \int (b\csc(dx^n + c) + a)^2 (ex)^{2n-1} \, dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n))^2,x, algorithm="maxima")
```

output
```
1/2*(e*x)^(2*n)*a^2/(e*n) - (2*b^2*e^(2*n)*x^n*sin(2*d*x^n + 2*c) - (d*e*n
*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 - 2*d*e*n*cos(2*d*x^n +
 2*c) + d*e*n)*integrate((2*a*b*d*e^(2*n)*x^(2*n) - b^2*e^(2*n)*x^n)*sin(d
*x^n + c)/(d*e*x*cos(d*x^n + c)^2 + d*e*x*sin(d*x^n + c)^2 + 2*d*e*x*cos(d
*x^n + c) + d*e*x), x) - (d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n +
 2*c)^2 - 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n)*integrate((2*a*b*d*e^(2*n)*x
^(2*n) + b^2*e^(2*n)*x^n)*sin(d*x^n + c)/(d*e*x*cos(d*x^n + c)^2 + d*e*x*s
in(d*x^n + c)^2 - 2*d*e*x*cos(d*x^n + c) + d*e*x), x))/(d*e*n*cos(2*d*x^n
+ 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 - 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n
)
```

Giac [F]

$$\int (ex)^{-1+2n}\,(a+b\csc(c+dx^n))^2\,dx=\int (b\csc(dx^n+c)+a)^2(ex)^{2n-1}\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n))^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^n + c) + a)^2*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}\,(a+b\csc(c+dx^n))^2\,dx=\int \left(a+\frac{b}{\sin(c+d\,x^n)}\right)^2(e\,x)^{2\,n-1}\,dx$$

input
```
int((a + b/sin(c + d*x^n))^2*(e*x)^(2*n - 1),x)
```

output
```
int((a + b/sin(c + d*x^n))^2*(e*x)^(2*n - 1), x)
```

Reduce [F]

$$\int (ex)^{-1+2n} (a+b\csc(c+dx^n))^2 \, dx$$
$$= \frac{e^{2n}\left(-2x^n\cos(x^nd+c)\,b^2d+x^{2n}\sin(x^nd+c)\,a^2d^2+4\left(\int \frac{x^{2n}\csc(x^nd+c)}{x}dx\right)\sin(x^nd+c)\,ab\,d^2n-2\log\left(\right.\right.}{2\sin(x^nd+c)\,d^2en}$$

input
```
int((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n))^2,x)
```

output
```
(e**(2*n)*( - 2*x**n*cos(x**n*d + c)*b**2*d + x**(2*n)*sin(x**n*d + c)*a**
2*d**2 + 4*int((x**(2*n)*csc(x**n*d + c))/x,x)*sin(x**n*d + c)*a*b*d**2*n
- 2*log(tan((x**n*d + c)/2)**2 + 1)*sin(x**n*d + c)*b**2 + 2*log(tan((x**n
*d + c)/2))*sin(x**n*d + c)*b**2))/(2*sin(x**n*d + c)*d**2*e*n)
```

3.78 $\int (ex)^{-1+3n}\left(a+b\csc\left(c+dx^n\right)\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 377

$$\begin{aligned}\int (ex)^{-1+3n}\left(a+b\csc\left(c+dx^n\right)\right)^2\,dx &= \frac{a^2(ex)^{3n}}{3en}-\frac{ib^2x^{-n}(ex)^{3n}}{den}\\
&-\frac{4abx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den}\\
&-\frac{b^2x^{-n}(ex)^{3n}\cot\left(c+dx^n\right)}{den}\\
&+\frac{2b^2x^{-2n}(ex)^{3n}\log\left(1-e^{2i(c+dx^n)}\right)}{d^2en}\\
&+\frac{4iabx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en}\\
&-\frac{4iabx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en}\\
&-\frac{ib^2x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{2i(c+dx^n)}\right)}{d^3en}\\
&-\frac{4abx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-e^{i(c+dx^n)}\right)}{d^3en}\\
&+\frac{4abx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,e^{i(c+dx^n)}\right)}{d^3en}\end{aligned}$$

output
```
1/3*a^2*(e*x)^(3*n)/e/n-I*b^2*(e*x)^(3*n)/d/e/n/(x^n)-4*a*b*(e*x)^(3*n)*ar
ctanh(exp(I*(c+d*x^n)))/d/e/n/(x^n)-b^2*(e*x)^(3*n)*cot(c+d*x^n)/d/e/n/(x^
n)+2*b^2*(e*x)^(3*n)*ln(1-exp(2*I*(c+d*x^n)))/d^2/e/n/(x^(2*n))+4*I*a*b*(e
*x)^(3*n)*polylog(2,-exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-4*I*a*b*(e*x)^(3*
n)*polylog(2,exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-I*b^2*(e*x)^(3*n)*polylog
(2,exp(2*I*(c+d*x^n)))/d^3/e/n/(x^(3*n))-4*a*b*(e*x)^(3*n)*polylog(3,-exp(
I*(c+d*x^n)))/d^3/e/n/(x^(3*n))+4*a*b*(e*x)^(3*n)*polylog(3,exp(I*(c+d*x^n
)))/d^3/e/n/(x^(3*n))
```

Mathematica [F]

$$\int (ex)^{-1+3n}\left(a+b\csc\left(c+dx^n\right)\right)^2\,dx=\int (ex)^{-1+3n}\left(a+b\csc\left(c+dx^n\right)\right)^2\,dx$$

input
```
Integrate[(e*x)^(-1 + 3*n)*(a + b*Csc[c + d*x^n])^2,x]
```

output
```
Integrate[(e*x)^(-1 + 3*n)*(a + b*Csc[c + d*x^n])^2, x]
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4697, 4693, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3n-1}\left(a+b\csc\left(c+dx^n\right)\right)^2\,dx$$

$$\Big\downarrow \; 4697$$

$$\frac{x^{-3n}(ex)^{3n}\int x^{3n-1}(a+b\csc(dx^n+c))^2dx}{e}$$

$$\Big\downarrow \; 4693$$

$$\frac{x^{-3n}(ex)^{3n}\int x^{2n}(a+b\csc(dx^n+c))^2\,dx^n}{en}$$

$$\Big\downarrow \; 3042$$

$$\frac{x^{-3n}(ex)^{3n}\int x^{2n}(a+b\csc(dx^n+c))^2\,dx^n}{en}$$

$$\Big\downarrow \; 4678$$

$$\frac{x^{-3n}(ex)^{3n}\int \left(a^2x^{2n}+b^2\csc^2(dx^n+c)\,x^{2n}+2ab\csc(dx^n+c)\,x^{2n}\right)dx^n}{en}$$

$$\Big\downarrow \; 2009$$

$$x^{-3n}(ex)^{3n}\left(\frac{1}{3}a^2x^{3n}-\frac{4abx^{2n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{d}-\frac{4ab\,\text{PolyLog}\left(3,-e^{i(dx^n+c)}\right)}{d^3}+\frac{4ab\,\text{PolyLog}\left(3,e^{i(dx^n+c)}\right)}{d^3}+\frac{4iabx^n\,\text{PolyLo}}{}\right.$$

input
```
Int[(e*x)^(-1 + 3*n)*(a + b*Csc[c + d*x^n])^2,x]
```

output
```
((e*x)^(3*n)*(((-I)*b^2*x^(2*n))/d + (a^2*x^(3*n))/3 - (4*a*b*x^(2*n)*ArcT
anh[E^(I*(c + d*x^n))])/d - (b^2*x^(2*n)*Cot[c + d*x^n])/d + (2*b^2*x^n*Lo
g[1 - E^((2*I)*(c + d*x^n))])/d^2 + ((4*I)*a*b*x^n*PolyLog[2, -E^(I*(c + d
*x^n))])/d^2 - ((4*I)*a*b*x^n*PolyLog[2, E^(I*(c + d*x^n))])/d^2 - (I*b^2*
PolyLog[2, E^((2*I)*(c + d*x^n))])/d^3 - (4*a*b*PolyLog[3, -E^(I*(c + d*x^
n))])/d^3 + (4*a*b*PolyLog[3, E^(I*(c + d*x^n))])/d^3))/(e*n*x^(3*n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

rule 4697
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Maple [F]

$$\int (ex)^{-1+3n} (a + b\csc(c + d\,x^n))^2 dx$$

input
```
int((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n))^2,x)
```

output
```
int((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 890 vs. 2(362) = 724.

Time = 0.13 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.36

$$\int (ex)^{-1+3n} (a + b\csc(c + dx^n))^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n))^2,x, algorithm="fricas")
```

output
```
1/3*(a^2*d^3*e^(3*n - 1)*x^(3*n)*sin(d*x^n + c) - 3*b^2*d^2*e^(3*n - 1)*x^
(2*n)*cos(d*x^n + c) + 6*a*b*e^(3*n - 1)*polylog(3, cos(d*x^n + c) + I*sin
(d*x^n + c))*sin(d*x^n + c) + 6*a*b*e^(3*n - 1)*polylog(3, cos(d*x^n + c)
- I*sin(d*x^n + c))*sin(d*x^n + c) - 6*a*b*e^(3*n - 1)*polylog(3, -cos(d*x
^n + c) + I*sin(d*x^n + c))*sin(d*x^n + c) - 6*a*b*e^(3*n - 1)*polylog(3,
-cos(d*x^n + c) - I*sin(d*x^n + c))*sin(d*x^n + c) + 3*(a*b*c^2 - b^2*c)*e
^(3*n - 1)*log(-1/2*cos(d*x^n + c) + 1/2*I*sin(d*x^n + c) + 1/2)*sin(d*x^n
 + c) + 3*(a*b*c^2 - b^2*c)*e^(3*n - 1)*log(-1/2*cos(d*x^n + c) - 1/2*I*si
n(d*x^n + c) + 1/2)*sin(d*x^n + c) - 3*(2*I*a*b*d*e^(3*n - 1)*x^n + I*b^2*
e^(3*n - 1))*dilog(cos(d*x^n + c) + I*sin(d*x^n + c))*sin(d*x^n + c) - 3*(
-2*I*a*b*d*e^(3*n - 1)*x^n - I*b^2*e^(3*n - 1))*dilog(cos(d*x^n + c) - I*s
in(d*x^n + c))*sin(d*x^n + c) - 3*(2*I*a*b*d*e^(3*n - 1)*x^n - I*b^2*e^(3*
n - 1))*dilog(-cos(d*x^n + c) + I*sin(d*x^n + c))*sin(d*x^n + c) - 3*(-2*I
*a*b*d*e^(3*n - 1)*x^n + I*b^2*e^(3*n - 1))*dilog(-cos(d*x^n + c) - I*sin(
d*x^n + c))*sin(d*x^n + c) - 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) - b^2*d*e^(3*n
 - 1)*x^n)*log(cos(d*x^n + c) + I*sin(d*x^n + c) + 1)*sin(d*x^n + c) - 3*(
a*b*d^2*e^(3*n - 1)*x^(2*n) - b^2*d*e^(3*n - 1)*x^n)*log(cos(d*x^n + c) -
I*sin(d*x^n + c) + 1)*sin(d*x^n + c) + 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) + b^
2*d*e^(3*n - 1)*x^n - (a*b*c^2 - b^2*c)*e^(3*n - 1))*log(-cos(d*x^n + c) +
 I*sin(d*x^n + c) + 1)*sin(d*x^n + c) + 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) ...
```

Sympy [F]

$$\int (ex)^{-1+3n}(a+b\csc(c+dx^n))^2\,dx=\int (ex)^{3n-1}(a+b\csc(c+dx^n))^2\,dx$$

input
```
integrate((e*x)**(-1+3*n)*(a+b*csc(c+d*x**n))**2,x)
```

output
```
Integral((e*x)**(3*n - 1)*(a + b*csc(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+3n} (a + b\csc(c + dx^n))^2 \, dx = \int (b\csc(dx^n + c) + a)^2 (ex)^{3n-1} \, dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n))^2,x, algorithm="maxima")
```

output
```
1/3*(e*x)^(3*n)*a^2/(e*n) - (2*b^2*e^(3*n)*x^(2*n)*sin(2*d*x^n + 2*c) - (d
*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 - 2*d*e*n*cos(2*d*x
^n + 2*c) + d*e*n)*integrate(2*(a*b*d*e^(3*n)*x^(3*n) - b^2*e^(3*n)*x^(2*n
))*sin(d*x^n + c)/(d*e*x*cos(d*x^n + c)^2 + d*e*x*sin(d*x^n + c)^2 + 2*d*e
*x*cos(d*x^n + c) + d*e*x), x) - (d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2
*d*x^n + 2*c)^2 - 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n)*integrate(2*(a*b*d*e
^(3*n)*x^(3*n) + b^2*e^(3*n)*x^(2*n))*sin(d*x^n + c)/(d*e*x*cos(d*x^n + c)
^2 + d*e*x*sin(d*x^n + c)^2 - 2*d*e*x*cos(d*x^n + c) + d*e*x), x))/(d*e*n*
cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 - 2*d*e*n*cos(2*d*x^n +
2*c) + d*e*n)
```

Giac [F]

$$\int (ex)^{-1+3n} (a + b\csc(c + dx^n))^2 \, dx = \int (b\csc(dx^n + c) + a)^2 (ex)^{3n-1} \, dx$$

input
```
integrate((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n))^2,x, algorithm="giac")
```

output
```
integrate((b*csc(d*x^n + c) + a)^2*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}(a+b\csc(c+dx^n))^2\,dx=\int \left(a+\frac{b}{\sin(c+d\,x^n)}\right)^2(e\,x)^{3\,n-1}\,dx$$

input
```
int((a + b/sin(c + d*x^n))^2*(e*x)^(3*n - 1),x)
```

output
```
int((a + b/sin(c + d*x^n))^2*(e*x)^(3*n - 1), x)
```

Reduce [F]

$$\int (ex)^{-1+3n}(a+b\csc(c+dx^n))^2\,dx$$
$$=\frac{e^{3n}\left(x^{3n}a^2+6\left(\int \frac{x^{3n}\csc(x^n d+c)}{x}dx\right)abn+3\left(\int \frac{x^{3n}\csc(x^n d+c)^2}{x}dx\right)b^2n\right)}{3en}$$

input
```
int((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n))^2,x)
```

output
```
(e**(3*n)*(x**(3*n)*a**2 + 6*int((x**(3*n)*csc(x**n*d + c))/x,x)*a*b*n + 3
*int((x**(3*n)*csc(x**n*d + c)**2)/x,x)*b**2*n))/(3*e*n)
```

3.79 $\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx=\frac{(ex)^n}{aen}+\frac{2bx^{-n}(ex)^n\text{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}den}$$

output
```
(e*x)^n/a/e/n+2*b*(e*x)^n*arctanh((a+b*tan(1/2*c+1/2*d*x^n))/(a^2-b^2)^(1/
2))/a/(a^2-b^2)^(1/2)/d/e/n/(x^n)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx=\frac{(ex)^n\left(d+cx^{-n}-\frac{2bx^{-n}\arctan\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}\right)}{aden}$$

input
```
Integrate[(e*x)^(-1 + n)/(a + b*Csc[c + d*x^n]),x]
```

output
```
((e*x)^n*(d + c/x^n - (2*b*ArcTan[(a + b*Tan[(c + d*x^n)/2])/Sqrt[-a^2 + b
^2]])/(Sqrt[-a^2 + b^2]*x^n)))/(a*d*e*n)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4697, 4693, 3042, 4270, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{n-1}}{a+b\csc\left(c+dx^n\right)}dx$$

$$\Big\downarrow \text{4697}$$

$$\frac{x^{-n}(ex)^n\int \frac{x^{n-1}}{a+b\csc(dx^n+c)}dx}{e}$$

$$\Big\downarrow \text{4693}$$

$$\frac{x^{-n}(ex)^n\int \frac{1}{a+b\csc(dx^n+c)}dx^n}{en}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^{-n}(ex)^n\int \frac{1}{a+b\csc(dx^n+c)}dx^n}{en}$$

$$\Big\downarrow \text{4270}$$

$$\frac{x^{-n}(ex)^n\left(\frac{x^n}{a}-\frac{\int \frac{1}{\frac{a\sin(dx^n+c)}{b}+1}dx^n}{a}\right)}{en}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^{-n}(ex)^n\left(\frac{x^n}{a}-\frac{\int \frac{1}{\frac{a\sin(dx^n+c)}{b}+1}dx^n}{a}\right)}{en}$$

$$\Big\downarrow \text{3139}$$

$$\frac{x^{-n}(ex)^n\left(\frac{x^n}{a}-\frac{2\int \frac{1}{x^{2n}+\frac{2a\tan\left(\frac{1}{2}(dx^n+c)\right)}{b}+1}d\tan\left(\frac{1}{2}(dx^n+c)\right)}{ad}\right)}{en}$$

$$\Big\downarrow \text{1083}$$

$$\frac{x^{-n}(ex)^n\left(\frac{4\int \frac{1}{-x^{2n}-4\left(1-\frac{a^2}{b^2}\right)}d\left(\frac{2a}{b}+2\tan\left(\frac{1}{2}(dx^n+c)\right)\right)}{ad}+\frac{x^n}{a}\right)}{en}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^{-n}(ex)^n\left(\frac{2b\,\text{arctanh}\left(\frac{b\left(\frac{2a}{b}+2\tan\left(\frac{1}{2}(c+dx^n)\right)\right)}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}+\frac{x^n}{a}\right)}{en}$$

input
```
Int[(e*x)^(-1 + n)/(a + b*Csc[c + d*x^n]),x]
```

output
```
((e*x)^n*(x^n/a + (2*b*ArcTanh[(b*((2*a)/b + 2*Tan[(c + d*x^n)/2]))/(2*Sqr
t[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d)))/(e*n*x^n)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

rule 4697
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.71

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(-1+n)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^{2}\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^{2}\pi-i\,\mathrm{csgn}(iex)^{3}\pi+2\ln(x)+2\ln(e)\right)}{2}}}{an} - \frac{2i\arctan\left(\frac{2ia\,\ldots}{2\ldots}\right.}{\ldots}$

input
```
int((e*x)^(-1+n)/(a+b*csc(c+d*x^n)),x,method=_RETURNVERBOSE)
```

output
```
1/a/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*
csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(x)+2
*ln(e)))-2*I*arctan(1/2*(2*I*a*exp(I*(d*x^n+2*c))-2*exp(I*c)*b)/(a^2*exp(2
*I*c)-exp(2*I*c)*b^2)^(1/2))/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)/d/e*e^n
/n/a*b*exp(1/2*I*(-Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*n*csgn(I*e)*csg
n(I*e*x)^2+Pi*n*csgn(I*x)*csgn(I*e*x)^2-Pi*n*csgn(I*e*x)^3+Pi*csgn(I*e)*cs
gn(I*x)*csgn(I*e*x)-Pi*csgn(I*e)*csgn(I*e*x)^2-Pi*csgn(I*x)*csgn(I*e*x)^2+
Pi*csgn(I*e*x)^3+2*c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.54

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx = \left[\frac{2\left(a^2-b^2\right)de^{n-1}x^n+\sqrt{a^2-b^2}be^{n-1}\log\left(\frac{\left(a^2-2\,b^2\right)\cos(dx^n+c)^2+2\,\sqrt{a^2-b^2}a\cos(dx^n+c)+a^2+b^2+2\left(\sqrt{a^2-b^2}b\cos(dx^n+c)\right.}{a^2\cos(dx^n+c)^2-2\,ab\sin(dx^n+c)-a^2-b^2}\right.}{2\left(a^3-ab^2\right)dn}\right.$$

input
```
integrate((e*x)^(-1+n)/(a+b*csc(c+d*x^n)),x, algorithm="fricas")
```

output
```
[1/2*(2*(a^2 - b^2)*d*e^(n - 1)*x^n + sqrt(a^2 - b^2)*b*e^(n - 1)*log(((a^
2 - 2*b^2)*cos(d*x^n + c)^2 + 2*sqrt(a^2 - b^2)*a*cos(d*x^n + c) + a^2 + b
^2 + 2*(sqrt(a^2 - b^2)*b*cos(d*x^n + c) + a*b)*sin(d*x^n + c))/(a^2*cos(d
*x^n + c)^2 - 2*a*b*sin(d*x^n + c) - a^2 - b^2)))/((a^3 - a*b^2)*d*n), ((a
^2 - b^2)*d*e^(n - 1)*x^n + sqrt(-a^2 + b^2)*b*e^(n - 1)*arctan(-(sqrt(-a^
2 + b^2)*b*sin(d*x^n + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*cos(d*x^n + c
))))/((a^3 - a*b^2)*d*n)]
```

Sympy [F]

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx=\int \frac{(ex)^{n-1}}{a+b\csc(c+dx^n)}\,dx$$

input
```
integrate((e*x)**(-1+n)/(a+b*csc(c+d*x**n)),x)
```

output
```
Integral((e*x)**(n - 1)/(a + b*csc(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx=\int \frac{(ex)^{n-1}}{b\csc(dx^n+c)+a}\,dx$$

input
```
integrate((e*x)^(-1+n)/(a+b*csc(c+d*x^n)),x, algorithm="maxima")
```

output
```
-(2*a*b*e^(n + 1)*n*integrate((2*b*x^n*cos(d*x^n + c)^2 + a*x^n*cos(d*x^n
+ c)*sin(2*d*x^n + 2*c) - a*x^n*cos(2*d*x^n + 2*c)*sin(d*x^n + c) + 2*b*x^
n*sin(d*x^n + c)^2 + a*x^n*sin(d*x^n + c))/(a^3*e*x*cos(2*d*x^n + 2*c)^2 +
 4*a*b^2*e*x*cos(d*x^n + c)^2 + 4*a^2*b*e*x*cos(d*x^n + c)*sin(2*d*x^n + 2
*c) + a^3*e*x*sin(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x*sin(d*x^n + c)^2 + 4*a^2*
b*e*x*sin(d*x^n + c) + a^3*e*x - 2*(2*a^2*b*e*x*sin(d*x^n + c) + a^3*e*x)*
cos(2*d*x^n + 2*c)), x) - e^n*x^n)/(a*e*n)
```

Giac [F]

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx=\int \frac{(ex)^{n-1}}{b\csc(dx^n+c)+a}\,dx$$

input
```
integrate((e*x)^(-1+n)/(a+b*csc(c+d*x^n)),x, algorithm="giac")
```

output
```
integrate((e*x)^(n - 1)/(b*csc(d*x^n + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 17.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.69

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx$$
$$= \frac{x\,(e\,x)^{n-1}}{a\,n} - \frac{b\,x\,\ln\left(b\,x\,e^{c\,1i}\,e^{d\,x^n\,1i}\,(e\,x)^{n-1}\,2i - \frac{2\,b\,x\,(e\,x)^{n-1}\left(a\,1i+b\,e^{c\,1i}\,e^{d\,x^n\,1i}\right)}{\sqrt{a+b}\,\sqrt{a-b}}\right)\,(e\,x)^{n-1}}{a\,d\,n\,x^n\,\sqrt{a+b}\,\sqrt{a-b}}$$
$$+ \frac{b\,x\,\ln\left(b\,x\,e^{c\,1i}\,e^{d\,x^n\,1i}\,(e\,x)^{n-1}\,2i + \frac{2\,b\,x\,(e\,x)^{n-1}\left(a\,1i+b\,e^{c\,1i}\,e^{d\,x^n\,1i}\right)}{\sqrt{a+b}\,\sqrt{a-b}}\right)\,(e\,x)^{n-1}}{a\,d\,n\,x^n\,\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int((e*x)^(n - 1)/(a + b/sin(c + d*x^n)),x)
```

output
```
(x*(e*x)^(n - 1))/(a*n) - (b*x*log(b*x*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n -
1)*2i - (2*b*x*(e*x)^(n - 1)*(a*1i + b*exp(c*1i)*exp(d*x^n*1i)))/((a + b)^
(1/2)*(a - b)^(1/2)))*(e*x)^(n - 1))/(a*d*n*x^n*(a + b)^(1/2)*(a - b)^(1/2
)) + (b*x*log(b*x*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1)*2i + (2*b*x*(e*x)^
(n - 1)*(a*1i + b*exp(c*1i)*exp(d*x^n*1i)))/((a + b)^(1/2)*(a - b)^(1/2)))
*(e*x)^(n - 1))/(a*d*n*x^n*(a + b)^(1/2)*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx = \frac{e^n\left(2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{x^n d}{2}+\frac{c}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)b + x^n a^2 d - x^n b^2 d\right)}{aden\,(a^2-b^2)}$$

input
```
int((e*x)^(-1+n)/(a+b*csc(c+d*x^n)),x)
```

output
```
(e**n*(2*sqrt( - a**2 + b**2)*atan((tan((x**n*d + c)/2)*b + a)/sqrt( - a**
2 + b**2))*b + x**n*a**2*d - x**n*b**2*d))/(a*d*e*n*(a**2 - b**2))
```

3.80 $\int \frac{(ex)^{-1+2n}}{a+b\csc(c+dx^n)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 338

$$\begin{aligned}\int \frac{(ex)^{-1+2n}}{a+b\csc(c+dx^n)}\,dx &= \frac{(ex)^{2n}}{2aen}+\frac{ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} \\ &\quad -\frac{ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} \\ &\quad +\frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} \\ &\quad -\frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}\end{aligned}$$

output
```
1/2*(e*x)^(2*n)/a/e/n+I*b*(e*x)^(2*n)*ln(1-I*a*exp(I*(c+d*x^n))/(b-(-a^2+b
^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d/e/n/(x^n)-I*b*(e*x)^(2*n)*ln(1-I*a*exp(I*
(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d/e/n/(x^n)+b*(e*x)^(2
*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2
)/d^2/e/n/(x^(2*n))-b*(e*x)^(2*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b+(-a^2+
b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2/e/n/(x^(2*n))
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1003 vs. $2(338) = 676$.

Time = 3.94 (sec) , antiderivative size = 1003, normalized size of antiderivative = 2.97

$$\int \frac{(ex)^{-1+2n}}{a + b\csc(c + dx^n)}\,dx = \text{Too large to display}$$

input
```
Integrate[(e*x)^(-1 + 2*n)/(a + b*Csc[c + d*x^n]),x]
```

output
```
((e*x)^(2*n)*Csc[c + d*x^n]*(1 - (2*b*((Pi*ArcTan[(a + b*Tan[(c + d*x^n)/2
])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(c - ArcCos[-(b/a)])*ArcTanh[(
(a - b)*Cot[(2*c + Pi + 2*d*x^n)/4])/Sqrt[a^2 - b^2]] + (-2*c + Pi - 2*d*x
^n)*ArcTanh[((a + b)*Tan[(2*c + Pi + 2*d*x^n)/4])/Sqrt[a^2 - b^2]] - (ArcC
os[-(b/a)] - (2*I)*ArcTanh[((a - b)*Cot[(2*c + Pi + 2*d*x^n)/4])/Sqrt[a^2
- b^2]])*Log[((a + b)*(a - b - I*Sqrt[a^2 - b^2])*(1 + I*Cot[(2*c + Pi + 2
*d*x^n)/4]))/(a*(a + b + Sqrt[a^2 - b^2]*Cot[(2*c + Pi + 2*d*x^n)/4]))] +
(ArcCos[-(b/a)] + (2*I)*(-ArcTanh[((a - b)*Cot[(2*c + Pi + 2*d*x^n)/4])/Sq
rt[a^2 - b^2]] + ArcTanh[((a + b)*Tan[(2*c + Pi + 2*d*x^n)/4])/Sqrt[a^2 -
b^2]]))*Log[((-1)^(1/4)*Sqrt[a^2 - b^2])/(Sqrt[2]*Sqrt[a]*E^((I/2)*(c + d*
x^n))*Sqrt[b + a*Sin[c + d*x^n]])] + (ArcCos[-(b/a)] + (2*I)*ArcTanh[((a -
 b)*Cot[(2*c + Pi + 2*d*x^n)/4])/Sqrt[a^2 - b^2]] - (2*I)*ArcTanh[((a + b)
*Tan[(2*c + Pi + 2*d*x^n)/4])/Sqrt[a^2 - b^2]])*Log[-(((-1)^(3/4)*Sqrt[a^2
 - b^2]*E^((I/2)*(c + d*x^n)))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Sin[c + d*x^n]]
))] - (ArcCos[-(b/a)] + (2*I)*ArcTanh[((a - b)*Cot[(2*c + Pi + 2*d*x^n)/4]
)/Sqrt[a^2 - b^2]])*Log[1 + (I*(I*b + Sqrt[a^2 - b^2])*(a + b + Sqrt[a^2 -
 b^2]*Tan[(2*c - Pi + 2*d*x^n)/4]))/(a*(a + b + Sqrt[a^2 - b^2]*Cot[(2*c +
 Pi + 2*d*x^n)/4]))] + I*(PolyLog[2, ((b - I*Sqrt[a^2 - b^2])*(a + b + Sqr
t[a^2 - b^2]*Tan[(2*c - Pi + 2*d*x^n)/4]))/(a*(a + b + Sqrt[a^2 - b^2]*Cot
[(2*c + Pi + 2*d*x^n)/4]))] - PolyLog[2, ((b + I*Sqrt[a^2 - b^2])*(a + ...
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4697, 4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{2n-1}}{a+b\csc(c+dx^n)}dx$$

$$\Big\downarrow \text{4697}$$

$$\frac{x^{-2n}(ex)^{2n}\int \frac{x^{2n-1}}{a+b\csc(dx^n+c)}dx}{e}$$

$$\Big\downarrow \text{4693}$$

$$\frac{x^{-2n}(ex)^{2n}\int \frac{x^n}{a+b\csc(dx^n+c)}dx^n}{en}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^{-2n}(ex)^{2n}\int \frac{x^n}{a+b\csc(dx^n+c)}dx^n}{en}$$

$$\Big\downarrow \text{4679}$$

$$\frac{x^{-2n}(ex)^{2n}\int \left(\frac{x^n}{a}-\frac{bx^n}{a(b+a\sin(dx^n+c))}\right)dx^n}{en}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{-2n}(ex)^{2n}\left(\frac{b\,\text{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}-\frac{b\,\text{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}+\frac{ibx^n\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{ad\sqrt{b^2-a^2}}-\frac{ibx^n\log\left(1-\frac{iae^{i(c+dx^n)}}{\sqrt{b^2-a^2}+b}\right)}{ad\sqrt{b^2-a^2}}+\right.}{en}$$

input
```
Int[(e*x)^(-1 + 2*n)/(a + b*Csc[c + d*x^n]),x]
```

output
```
((e*x)^(2*n)*(x^(2*n)/(2*a) + (I*b*x^n*Log[1 - (I*a*E^(I*(c + d*x^n)))/(b
 - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^n*Log[1 - (I*a*E^(I*
(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (b*PolyLog
[2, (I*a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d
^2) - (b*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a*Sq
rt[-a^2 + b^2]*d^2)))/(e*n*x^(2*n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

rule 4697
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 777, normalized size of antiderivative = 2.30

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(-1+2n)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2\pi-i\,\mathrm{csgn}(iex)^3\pi+2\ln(x)+2\ln(e)\right)}{2}}}{2an}+\frac{\left(-d\,x^n\ln\left(\frac{ie}{}\right.\right.}{}$

input
```
int((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n)),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/n*x*exp(1/2*(-1+2*n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I
*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(
x)+2*ln(e)))+1/(a^2-b^2)*(-d*x^n*ln((I*exp(I*c)*b+a*exp(I*(d*x^n+2*c))-(a^
2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))/(I*exp(I*c)*b-(a^2*exp(2*I*c)-exp(2*I*
c)*b^2)^(1/2)))+d*x^n*ln((I*exp(I*c)*b+a*exp(I*(d*x^n+2*c))+(a^2*exp(2*I*c
)-exp(2*I*c)*b^2)^(1/2))/(I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/
2)))+I*dilog(I/(I*exp(I*c)*b-(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*b*exp(
I*c)+a/(I*exp(I*c)*b-(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*exp(I*(d*x^n+2
*c))-1/(I*exp(I*c)*b-(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*(a^2*exp(2*I*c
)-exp(2*I*c)*b^2)^(1/2))-I*dilog(I/(I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c
)*b^2)^(1/2))*b*exp(I*c)+a/(I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(
1/2))*exp(I*(d*x^n+2*c))+1/(I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(
1/2))*(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)))*(a^2*exp(2*I*c)-exp(2*I*c)*b
^2)^(1/2)/d^2/n/e*(e^n)^2/a*b*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(
I*e*x)-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*
n*csgn(I*e*x)^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x
)^2+Pi*csgn(I*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1235 vs. 2(302) = 604.

Time = 0.28 (sec) , antiderivative size = 1235, normalized size of antiderivative = 3.65

$$\int \frac{(ex)^{-1+2n}}{a+b\csc(c+dx^n)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n)),x, algorithm="fricas")
```

output
```
-1/2*(a*b*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) + 2*I
*a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) + a*b*c*e^(2*n - 1)
*sqrt((a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n + c) + 2*a
*sqrt((a^2 - b^2)/a^2) - 2*I*b) - a*b*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)*
log(-2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2)
 + 2*I*b) - a*b*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)*log(-2*a*cos(d*x^n + c
) - 2*I*a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b) - (a^2 - b^2
)*d^2*e^(2*n - 1)*x^(2*n) - I*a*b*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)*dilog(
((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) + (I*a*sqrt((a^2 - b^2)/a^
2) - b)*sin(d*x^n + c) - a)/a + 1) - I*a*b*e^(2*n - 1)*sqrt((a^2 - b^2)/a^
2)*dilog(-((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) - (I*a*sqrt((a^2
 - b^2)/a^2) - b)*sin(d*x^n + c) + a)/a + 1) + I*a*b*e^(2*n - 1)*sqrt((a^2
 - b^2)/a^2)*dilog(((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) + (-I*a
*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) - a)/a + 1) + I*a*b*e^(2*n - 1)
*sqrt((a^2 - b^2)/a^2)*dilog(-((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n +
 c) - (-I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) + a)/a + 1) + (a*b*d
*e^(2*n - 1)*x^n*sqrt((a^2 - b^2)/a^2) + a*b*c*e^(2*n - 1)*sqrt((a^2 - b^2
)/a^2))*log(-((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) + (I*a*sqrt((
a^2 - b^2)/a^2) - b)*sin(d*x^n + c) - a)/a) - (a*b*d*e^(2*n - 1)*x^n*sqrt(
(a^2 - b^2)/a^2) + a*b*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2))*log(((a*sqr...
```

Sympy [F]

$$\int \frac{(ex)^{-1+2n}}{a+b\csc(c+dx^n)}\,dx=\int \frac{(ex)^{2n-1}}{a+b\csc(c+dx^n)}\,dx$$

input
```
integrate((e*x)**(-1+2*n)/(a+b*csc(c+d*x**n)),x)
```

output
```
Integral((e*x)**(2*n - 1)/(a + b*csc(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+2n}}{a+b\csc(c+dx^n)}\,dx=\int \frac{(ex)^{2n-1}}{b\csc(dx^n+c)+a}\,dx$$

input
```
integrate((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n)),x, algorithm="maxima")
```

output
```
-1/2*(4*a*b*e^(2*n + 1)*n*integrate((2*b*x^(2*n)*cos(d*x^n + c)^2 + a*x^(2
*n)*cos(d*x^n + c)*sin(2*d*x^n + 2*c) - a*x^(2*n)*cos(2*d*x^n + 2*c)*sin(d
*x^n + c) + 2*b*x^(2*n)*sin(d*x^n + c)^2 + a*x^(2*n)*sin(d*x^n + c))/(a^3*
e*x*cos(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x*cos(d*x^n + c)^2 + 4*a^2*b*e*x*cos(
d*x^n + c)*sin(2*d*x^n + 2*c) + a^3*e*x*sin(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x
*sin(d*x^n + c)^2 + 4*a^2*b*e*x*sin(d*x^n + c) + a^3*e*x - 2*(2*a^2*b*e*x*
sin(d*x^n + c) + a^3*e*x)*cos(2*d*x^n + 2*c)), x) - e^(2*n)*x^(2*n))/(a*e*
n)
```

Giac [F]

$$\int \frac{(ex)^{-1+2n}}{a+b\csc(c+dx^n)}\,dx=\int \frac{(ex)^{2n-1}}{b\csc(dx^n+c)+a}\,dx$$

input
```
integrate((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n)),x, algorithm="giac")
```

output
```
integrate((e*x)^(2*n - 1)/(b*csc(d*x^n + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+2n}}{a+b\csc(c+dx^n)}\,dx=\int \frac{(e\,x)^{2\,n-1}}{a+\frac{b}{\sin(c+d\,x^n)}}\,dx$$

input
```
int((e*x)^(2*n - 1)/(a + b/sin(c + d*x^n)),x)
```

output
```
int((e*x)^(2*n - 1)/(a + b/sin(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{(ex)^{-1+2n}}{a + b\csc(c + dx^n)}\,dx = \frac{e^{2n}\left(\int \frac{x^{2n}}{\csc(x^n d+c)bx+ax}dx\right)}{e}$$

input
```
int((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n)),x)
```

output
```
(e**(2*n)*int(x**(2*n)/(csc(x**n*d + c)*b*x + a*x),x))/e
```

3.81 $\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 499

$$\begin{aligned}\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx &= \frac{(ex)^{3n}}{3aen}+\frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}\\
&\quad-\frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}\\
&\quad+\frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}\\
&\quad-\frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}\\
&\quad+\frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en}\\
&\quad-\frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en}\end{aligned}$$

output
```
1/3*(e*x)^(3*n)/a/e/n+I*b*(e*x)^(3*n)*ln(1-I*a*exp(I*(c+d*x^n))/(b-(-a^2+b
^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d/e/n/(x^n)-I*b*(e*x)^(3*n)*ln(1-I*a*exp(I*
(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d/e/n/(x^n)+2*b*(e*x)^
(3*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1
/2)/d^2/e/n/(x^(2*n))-2*b*(e*x)^(3*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b+(-
a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2/e/n/(x^(2*n))+2*I*b*(e*x)^(3*n)*po
lylog(3,I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^3/
e/n/(x^(3*n))-2*I*b*(e*x)^(3*n)*polylog(3,I*a*exp(I*(c+d*x^n))/(b+(-a^2+b^
2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^3/e/n/(x^(3*n))
```

Mathematica [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx=\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx$$

input
```
Integrate[(e*x)^(-1 + 3*n)/(a + b*Csc[c + d*x^n]),x]
```

output
```
Integrate[(e*x)^(-1 + 3*n)/(a + b*Csc[c + d*x^n]), x]
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4697, 4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3n-1}}{a+b\csc(c+dx^n)}\,dx$$

$$\Big\downarrow \text{4697}$$

$$\frac{x^{-3n}(ex)^{3n}\int \frac{x^{3n-1}}{a+b\csc(dx^n+c)}dx}{e}$$

$$\Big\downarrow \text{4693}$$

$$\frac{x^{-3n}(ex)^{3n}\int \frac{x^{2n}}{a+b\csc(dx^n+c)}dx^n}{en}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^{-3n}(ex)^{3n}\int \frac{x^{2n}}{a+b\csc(dx^n+c)}dx^n}{en}$$

$$\Big\downarrow \text{4679}$$

$$\frac{x^{-3n}(ex)^{3n}\int \left(\frac{x^{2n}}{a}-\frac{bx^{2n}}{a(b+a\sin(dx^n+c))}\right)dx^n}{en}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{-3n}(ex)^{3n}\left(\frac{2ib\,\text{PolyLog}\left(3,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}-\frac{2ib\,\text{PolyLog}\left(3,\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}+\frac{2bx^n\,\text{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}-\frac{2bx^n\,\text{PolyLog}\left(2,\frac{iae^{i(c}}{b+\sqrt{}}\right.}{ad^2\sqrt{b^2-a^2}}\right.}{en}$$

input
```
Int[(e*x)^(-1 + 3*n)/(a + b*Csc[c + d*x^n]),x]
```

output
```
((e*x)^(3*n)*(x^(3*n)/(3*a) + (I*b*x^(2*n)*Log[1 - (I*a*E^(I*(c + d*x^n)))
/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^(2*n)*Log[1 - (I
*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (2
*b*x^n*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt
[-a^2 + b^2]*d^2) - (2*b*x^n*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[
-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) + ((2*I)*b*PolyLog[3, (I*a*E^(I*(c
 + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3) - ((2*I)*b*P
olyLog[3, (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 +
b^2]*d^3)))/(e*n*x^(3*n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

rule 4697
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Maple [F]

$$\int \frac{(ex)^{-1+3n}}{a + b\csc(c + d\,x^n)}dx$$

input
```
int((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n)),x)
```

output
```
int((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1663 vs. $2(447) = 894$.

Time = 0.28 (sec) , antiderivative size = 1663, normalized size of antiderivative = 3.33

$$\int \frac{(ex)^{-1+3n}}{a + b\csc(c + dx^n)} \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n)),x, algorithm="fricas")
```

output
```
1/6*(6*I*a*b*d*e^(3*n - 1)*x^n*sqrt((a^2 - b^2)/a^2)*dilog(((a*sqrt((a^2 -
 b^2)/a^2) + I*b)*cos(d*x^n + c) + (I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x
^n + c) - a)/a + 1) + 6*I*a*b*d*e^(3*n - 1)*x^n*sqrt((a^2 - b^2)/a^2)*dilo
g(-((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) - (I*a*sqrt((a^2 - b^2)
/a^2) - b)*sin(d*x^n + c) + a)/a + 1) - 6*I*a*b*d*e^(3*n - 1)*x^n*sqrt((a^
2 - b^2)/a^2)*dilog(((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) + (-I*
a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) - a)/a + 1) - 6*I*a*b*d*e^(3*n
 - 1)*x^n*sqrt((a^2 - b^2)/a^2)*dilog(-((a*sqrt((a^2 - b^2)/a^2) - I*b)*co
s(d*x^n + c) - (-I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) + a)/a + 1)
 + 3*a*b*c^2*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) + 2*
I*a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) + 3*a*b*c^2*e^(3*n
 - 1)*sqrt((a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n + c)
+ 2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b) - 3*a*b*c^2*e^(3*n - 1)*sqrt((a^2 - b
^2)/a^2)*log(-2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt((a^2 -
b^2)/a^2) + 2*I*b) - 3*a*b*c^2*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2)*log(-2*a*
cos(d*x^n + c) - 2*I*a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b)
 + 2*(a^2 - b^2)*d^3*e^(3*n - 1)*x^(3*n) + 6*a*b*e^(3*n - 1)*sqrt((a^2 - b
^2)/a^2)*polylog(3, -((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) + (-I
*a*sqrt((a^2 - b^2)/a^2) + b)*sin(d*x^n + c))/a) - 6*a*b*e^(3*n - 1)*sqrt(
(a^2 - b^2)/a^2)*polylog(3, ((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n ...
```

Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx=\int \frac{(ex)^{3n-1}}{a+b\csc(c+dx^n)}\,dx$$

input
```
integrate((e*x)**(-1+3*n)/(a+b*csc(c+d*x**n)),x)
```

output
```
Integral((e*x)**(3*n - 1)/(a + b*csc(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx=\int \frac{(ex)^{3n-1}}{b\csc(dx^n+c)+a}\,dx$$

input
```
integrate((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n)),x, algorithm="maxima")
```

output
```
-1/3*(6*a*b*e^(3*n + 1)*n*integrate((2*b*x^(3*n)*cos(d*x^n + c)^2 + a*x^(3
*n)*cos(d*x^n + c)*sin(2*d*x^n + 2*c) - a*x^(3*n)*cos(2*d*x^n + 2*c)*sin(d
*x^n + c) + 2*b*x^(3*n)*sin(d*x^n + c)^2 + a*x^(3*n)*sin(d*x^n + c))/(a^3*
e*x*cos(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x*cos(d*x^n + c)^2 + 4*a^2*b*e*x*cos(
d*x^n + c)*sin(2*d*x^n + 2*c) + a^3*e*x*sin(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x
*sin(d*x^n + c)^2 + 4*a^2*b*e*x*sin(d*x^n + c) + a^3*e*x - 2*(2*a^2*b*e*x*
sin(d*x^n + c) + a^3*e*x)*cos(2*d*x^n + 2*c)), x) - e^(3*n)*x^(3*n))/(a*e*
n)
```

Giac [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx=\int \frac{(ex)^{3n-1}}{b\csc(dx^n+c)+a}\,dx$$

input
```
integrate((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n)),x, algorithm="giac")
```

output
```
integrate((e*x)^(3*n - 1)/(b*csc(d*x^n + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx = \int \frac{(e\,x)^{3\,n-1}}{a+\frac{b}{\sin(c+d\,x^n)}}\,dx$$

input
```
int((e*x)^(3*n - 1)/(a + b/sin(c + d*x^n)),x)
```

output
```
int((e*x)^(3*n - 1)/(a + b/sin(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx = \frac{e^{3n}\left(\int \frac{x^{3n}}{\csc(x^n d+c)bx+ax}dx\right)}{e}$$

input
```
int((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n)),x)
```

output
```
(e**(3*n)*int(x**(3*n)/(csc(x**n*d + c)*b*x + a*x),x))/e
```

3.82 $\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx = \frac{(ex)^n}{a^2en} + \frac{2b(2a^2-b^2)\,x^{-n}(ex)^n\operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}den} - \frac{b^2x^{-n}(ex)^n\cot(c+dx^n)}{a\left(a^2-b^2\right)den\left(a+b\csc(c+dx^n)\right)}$$

output
```
(e*x)^n/a^2/e/n+2*b*(2*a^2-b^2)*(e*x)^n*arctanh((a+b*tan(1/2*c+1/2*d*x^n))
/(a^2-b^2)^(1/2))/a^2/(a^2-b^2)^(3/2)/d/e/n/(x^n)-b^2*(e*x)^n*cot(c+d*x^n)
/a/(a^2-b^2)/d/e/n/(x^n)/(a+b*csc(c+d*x^n))
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.13

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx = \frac{x^{-n}(ex)^n\left(2b(-2a^2+b^2)\arctan\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{-a^2+b^2}}\right)(a+b\csc(c+dx^n)) + \sqrt{-a^2+b^2}(-ab^2\cot(c+dx^n\right.}{a^2(a-b)(a+b)\sqrt{-a^2+b^2}den\left(a+b\csc(c+dx^n)\right)}$$

input
```
Integrate[(e*x)^(-1 + n)/(a + b*Csc[c + d*x^n])^2,x]
```

output
```
((e*x)^n*(2*b*(-2*a^2 + b^2)*ArcTan[(a + b*Tan[(c + d*x^n)/2])/Sqrt[-a^2 +
 b^2]]*(a + b*Csc[c + d*x^n]) + Sqrt[-a^2 + b^2]*(-(a*b^2*Cot[c + d*x^n])
+ (a^2 - b^2)*(c + d*x^n)*(a + b*Csc[c + d*x^n]))))/(a^2*(a - b)*(a + b)*S
qrt[-a^2 + b^2]*d*e*n*x^n*(a + b*Csc[c + d*x^n]))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {4697, 4693, 3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{n-1}}{(a+b\csc(c+dx^n))^2}dx$$

$$\Big\downarrow \text{4697}$$

$$\frac{x^{-n}(ex)^n\int \frac{x^{n-1}}{(a+b\csc(dx^n+c))^2}dx}{e}$$

$$\Big\downarrow \text{4693}$$

$$\frac{x^{-n}(ex)^n\int \frac{1}{(a+b\csc(dx^n+c))^2}dx^n}{en}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^{-n}(ex)^n\int \frac{1}{(a+b\csc(dx^n+c))^2}dx^n}{en}$$

$$\Big\downarrow \text{4272}$$

$$\frac{x^{-n}(ex)^n\left(-\frac{\int -\frac{a^2-b\csc(dx^n+c)a-b^2}{a+b\csc(dx^n+c)}dx^n}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^n)}{ad(a^2-b^2)(a+b\csc(c+dx^n))}\right)}{en}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^{-n}(ex)^n\left(\frac{\int \frac{a^2-b\csc(dx^n+c)a-b^2}{a+b\csc(dx^n+c)}dx^n}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^n)}{ad(a^2-b^2)(a+b\csc(c+dx^n))}\right)}{en}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^{-n}(ex)^n\left(\frac{\int \frac{a^2-b\csc(dx^n+c)a-b^2}{a+b\csc(dx^n+c)}dx^n}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^n)}{ad(a^2-b^2)(a+b\csc(c+dx^n))}\right)}{en}$$

$$\Big\downarrow \text{ 4407}$$

$$\frac{x^{-n}(ex)^n\left(\frac{\frac{(a^2-b^2)x^n}{a}-\frac{b(2a^2-b^2)\int\frac{\csc(dx^n+c)}{a+b\csc(dx^n+c)}dx^n}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^n)}{ad(a^2-b^2)(a+b\csc(c+dx^n))}\right)}{en}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^{-n}(ex)^n\left(\frac{\frac{(a^2-b^2)x^n}{a}-\frac{b(2a^2-b^2)\int\frac{\csc(dx^n+c)}{a+b\csc(dx^n+c)}dx^n}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^n)}{ad(a^2-b^2)(a+b\csc(c+dx^n))}\right)}{en}$$

$$\Big\downarrow \text{ 4318}$$

$$\frac{x^{-n}(ex)^n\left(\frac{\frac{(a^2-b^2)x^n}{a}-\frac{(2a^2-b^2)\int\frac{1}{\frac{a\sin(dx^n+c)}{b}+1}dx^n}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^n)}{ad(a^2-b^2)(a+b\csc(c+dx^n))}\right)}{en}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^{-n}(ex)^n\left(\frac{\frac{(a^2-b^2)x^n}{a}-\frac{(2a^2-b^2)\int\frac{1}{\frac{a\sin(dx^n+c)}{b}+1}dx^n}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^n)}{ad(a^2-b^2)(a+b\csc(c+dx^n))}\right)}{en}$$

$$\Big\downarrow \text{ 3139}$$

$$\frac{x^{-n}(ex)^n\left(\frac{\frac{(a^2-b^2)x^n}{a}-\frac{2(2a^2-b^2)\int\frac{1}{x^{2n}+\frac{2a\tan\left(\frac{1}{2}(dx^n+c)\right)}{b}+1}d\tan\left(\frac{1}{2}(dx^n+c)\right)}{ad}}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^n)}{ad(a^2-b^2)(a+b\csc(c+dx^n))}\right)}{en}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{x^{-n}(ex)^n\left(\frac{\frac{4\left(2a^2-b^2\right)\int\frac{1}{-x^{2n}-4\left(1-\frac{a^2}{b^2}\right)}d\left(\frac{2a}{b}+2\tan\left(\frac{1}{2}(dx^n+c)\right)\right)}{ad}+\frac{\left(a^2-b^2\right)x^n}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^n)}{ad(a^2-b^2)(a+b\csc(c+dx^n))}\right)}{en}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^{-n}(ex)^n\left(\frac{\frac{2b\left(2a^2-b^2\right)\text{arctanh}\left(\frac{b\left(\frac{2a}{b}+2\tan\left(\frac{1}{2}(c+dx^n)\right)\right)}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}+\frac{\left(a^2-b^2\right)x^n}{a}}{a(a^2-b^2)}-\frac{b^2\cot(c+dx^n)}{ad(a^2-b^2)(a+b\csc(c+dx^n))}\right)}{en}$$

input
```
Int[(e*x)^(-1 + n)/(a + b*Csc[c + d*x^n])^2,x]
```

output
```
((e*x)^n*((((a^2 - b^2)*x^n)/a + (2*b*(2*a^2 - b^2)*ArcTanh[(b*((2*a)/b +
2*Tan[(c + d*x^n)/2]))/(2*Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d))/(a*(a^
2 - b^2)) - (b^2*Cot[c + d*x^n])/(a*(a^2 - b^2)*d*(a + b*Csc[c + d*x^n])))
)/(e*n*x^n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

rule 4697
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 622, normalized size of antiderivative = 3.99

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(-1+n)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^{2}\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^{2}\pi-i\,\mathrm{csgn}(iex)^{3}\pi+2\ln(x)+2\ln(e)\right)}{2}}}{a^{2}n} - \frac{2ib^{2}e^{n}(-1)^{-\mathrm{cs}}}{}$

input
```
int((e*x)^(-1+n)/(a+b*csc(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e
)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(x)
+2*ln(e)))-2*I*b^2/a^2/(-a^2+b^2)/d/n/(2*b*exp(I*(c+d*x^n))-I*a*exp(2*I*(c
+d*x^n))+I*a)*e^n*(-1)^(-1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^(-1/2*csgn(I*e)
*csgn(I*e*x)^2)*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(b*exp(1/2*I*(-
Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*n*csgn(I*e)*csgn(I*e*x)^2+Pi*n*csg
n(I*x)*csgn(I*e*x)^2-Pi*n*csgn(I*e*x)^3+Pi*csgn(I*e*x)^3+2*d*x^n+2*c))+I*e
xp(1/2*I*Pi*csgn(I*e*x)*(-csgn(I*e)*csgn(I*x)*n+csgn(I*e)*csgn(I*e*x)*n+n*
csgn(I*x)*csgn(I*e*x)-n*csgn(I*e*x)^2+csgn(I*e*x)^2))*a)/e-2*I*arctan(1/2*
(2*I*a*exp(I*(d*x^n+2*c))-2*exp(I*c)*b)/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1
/2))/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)/d/e*e^n/n/a^2/(-a^2+b^2)*(-2*a^
2+b^2)*b*exp(1/2*I*(-Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*n*csgn(I*e)*c
sgn(I*e*x)^2+Pi*n*csgn(I*x)*csgn(I*e*x)^2-Pi*n*csgn(I*e*x)^3+Pi*csgn(I*e)*
csgn(I*x)*csgn(I*e*x)-Pi*csgn(I*e)*csgn(I*e*x)^2-Pi*csgn(I*x)*csgn(I*e*x)^
2+Pi*csgn(I*e*x)^3+2*c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 630, normalized size of antiderivative = 4.04

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx$$

$$= \left[\frac{2\,(a^5-2\,a^3b^2+ab^4)de^{n-1}x^n\sin(dx^n+c)+2\,(a^4b-2\,a^2b^3+b^5)de^{n-1}x^n-2\,(a^3b^2-ab^4)e^{n-1}\cos(dx^n\cdots}{2\,((\cdots}\right.$$

input
```
integrate((e*x)^(-1+n)/(a+b*csc(c+d*x^n))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*e^(n - 1)*x^n*sin(d*x^n + c) + 2*(a^4*
b - 2*a^2*b^3 + b^5)*d*e^(n - 1)*x^n - 2*(a^3*b^2 - a*b^4)*e^(n - 1)*cos(d
*x^n + c) + ((2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*e^(n - 1)*sin(d*x^n + c) +
(2*a^2*b^2 - b^4)*sqrt(a^2 - b^2)*e^(n - 1))*log(((a^2 - 2*b^2)*cos(d*x^n
+ c)^2 + 2*sqrt(a^2 - b^2)*a*cos(d*x^n + c) + a^2 + b^2 + 2*(sqrt(a^2 - b^
2)*b*cos(d*x^n + c) + a*b)*sin(d*x^n + c))/(a^2*cos(d*x^n + c)^2 - 2*a*b*s
in(d*x^n + c) - a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*sin(d*x^n +
c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d*n), ((a^5 - 2*a^3*b^2 + a*b^4)*d*e^(n
 - 1)*x^n*sin(d*x^n + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*e^(n - 1)*x^n - (a^
3*b^2 - a*b^4)*e^(n - 1)*cos(d*x^n + c) + ((2*a^3*b - a*b^3)*sqrt(-a^2 + b
^2)*e^(n - 1)*sin(d*x^n + c) + (2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*e^(n - 1
))*arctan(-(sqrt(-a^2 + b^2)*b*sin(d*x^n + c) + sqrt(-a^2 + b^2)*a)/((a^2
- b^2)*cos(d*x^n + c))))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*sin(d*x^n + c) +
 (a^6*b - 2*a^4*b^3 + a^2*b^5)*d*n)]
```

Sympy [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(a+b\csc(c+dx^n))^2}\,dx$$

input
```
integrate((e*x)**(-1+n)/(a+b*csc(c+d*x**n))**2,x)
```

output
```
Integral((e*x)**(n - 1)/(a + b*csc(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(b\csc(dx^n+c)+a)^2}\,dx$$

input
```
integrate((e*x)^(-1+n)/(a+b*csc(c+d*x^n))^2,x, algorithm="maxima")
```

output
```
((a^4 - a^2*b^2)*d*e^n*x^n*cos(2*d*x^n + 2*c)^2 - 2*a*b^3*e^n*cos(d*x^n +
c) + 4*(a^2*b^2 - b^4)*d*e^n*x^n*cos(d*x^n + c)^2 + (a^4 - a^2*b^2)*d*e^n*
x^n*sin(2*d*x^n + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*e^n*x^n*sin(d*x^n + c)^2 +
4*(a^3*b - a*b^3)*d*e^n*x^n*sin(d*x^n + c) + (a^4 - a^2*b^2)*d*e^n*x^n - 2
*(a*b^3*e^n*cos(d*x^n + c) + 2*(a^3*b - a*b^3)*d*e^n*x^n*sin(d*x^n + c) +
(a^4 - a^2*b^2)*d*e^n*x^n)*cos(2*d*x^n + 2*c) + 2*((2*a^8*b - 3*a^6*b^3 +
a^4*b^5)*d*e^(n + 1)*n*cos(2*d*x^n + 2*c)^2*sin(c) + 4*(2*a^6*b^3 - 3*a^4*
b^5 + a^2*b^7)*d*e^(n + 1)*n*cos(d*x^n + c)^2*sin(c) + 4*(2*a^7*b^2 - 3*a^
5*b^4 + a^3*b^6)*d*e^(n + 1)*n*cos(d*x^n + c)*sin(2*d*x^n + 2*c)*sin(c) +
(2*a^8*b - 3*a^6*b^3 + a^4*b^5)*d*e^(n + 1)*n*sin(2*d*x^n + 2*c)^2*sin(c)
+ 4*(2*a^6*b^3 - 3*a^4*b^5 + a^2*b^7)*d*e^(n + 1)*n*sin(d*x^n + c)^2*sin(c
) + 4*(2*a^7*b^2 - 3*a^5*b^4 + a^3*b^6)*d*e^(n + 1)*n*sin(d*x^n + c)*sin(c
) + (2*a^8*b - 3*a^6*b^3 + a^4*b^5)*d*e^(n + 1)*n*sin(c) - 2*(2*(2*a^7*b^2
 - 3*a^5*b^4 + a^3*b^6)*d*e^(n + 1)*n*sin(d*x^n + c)*sin(c) + (2*a^8*b - 3
*a^6*b^3 + a^4*b^5)*d*e^(n + 1)*n*sin(c))*cos(2*d*x^n + 2*c))*integrate((a
^3*x^n*cos(2*d*x^n + 2*c)*cos(d*x^n) + a^3*x^n*sin(2*d*x^n + 2*c)*sin(d*x^
n) - 2*(a^2*b - b^3)*x^n*cos(d*x^n)^2*sin(c) - 2*(a^2*b - b^3)*x^n*sin(d*x
^n)^2*sin(c) - (a^3 - a*b^2)*x^n*cos(d*x^n) - (a*b^2*x^n*cos(d*x^n)*cos(2*
c) + a*b^2*x^n*sin(d*x^n)*sin(2*c))*cos(2*d*x^n) - (a*b^2*x^n*cos(2*c)*sin
(d*x^n) - a*b^2*x^n*cos(d*x^n)*sin(2*c))*sin(2*d*x^n))/(a^8*e*x*cos(2*d...
```

Giac [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx=\int \frac{(ex)^{n-1}}{(b\csc(dx^n+c)+a)^2}\,dx$$

input
```
integrate((e*x)^(-1+n)/(a+b*csc(c+d*x^n))^2,x, algorithm="giac")
```

output
```
integrate((e*x)^(n - 1)/(b*csc(d*x^n + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(e\,x)^{n-1}}{\left(a+\frac{b}{\sin(c+d\,x^n)}\right)^2}\,dx$$

input
```
int((e*x)^(n - 1)/(a + b/sin(c + d*x^n))^2,x)
```

output
```
int((e*x)^(n - 1)/(a + b/sin(c + d*x^n))^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.47

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx$$
$$= \frac{e^n\left(4\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{x^n d}{2}+\frac{c}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)\sin(x^n d+c)\,a^3b-2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{x^n d}{2}+\frac{c}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)\sin(x^n d+\right.}{}$$

input
```
int((e*x)^(-1+n)/(a+b*csc(c+d*x^n))^2,x)
```

output
```
(e**n*(4*sqrt( - a**2 + b**2)*atan((tan((x**n*d + c)/2)*b + a)/sqrt( - a**
2 + b**2))*sin(x**n*d + c)*a**3*b - 2*sqrt( - a**2 + b**2)*atan((tan((x**n
*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*sin(x**n*d + c)*a*b**3 + 4*sqrt( -
 a**2 + b**2)*atan((tan((x**n*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*a**2*
b**2 - 2*sqrt( - a**2 + b**2)*atan((tan((x**n*d + c)/2)*b + a)/sqrt( - a**
2 + b**2))*b**4 - cos(x**n*d + c)*a**3*b**2 + cos(x**n*d + c)*a*b**4 + x**
n*sin(x**n*d + c)*a**5*d - 2*x**n*sin(x**n*d + c)*a**3*b**2*d + x**n*sin(x
**n*d + c)*a*b**4*d + x**n*a**4*b*d - 2*x**n*a**2*b**3*d + x**n*b**5*d))/(
a**2*d*e*n*(sin(x**n*d + c)*a**5 - 2*sin(x**n*d + c)*a**3*b**2 + sin(x**n*
d + c)*a*b**4 + a**4*b - 2*a**2*b**3 + b**5))
```

3.83 $\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 778

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx = \frac{(ex)^{2n}}{2a^2en} - \frac{ib^3x^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} + \frac{2ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{ib^3x^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} - \frac{2ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{b^2x^{-2n}(ex)^{2n}\log(b+a\sin(c+dx^n))}{a^2(a^2-b^2)\,d^2en} - \frac{b^3x^{-2n}(ex)^{2n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} + \frac{2bx^{-2n}(ex)^{2n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{b^3x^{-2n}(ex)^{2n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} - \frac{2bx^{-2n}(ex)^{2n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} - \frac{b^2x^{-n}(ex)^{2n}\cos(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\sin(c+dx^n))}$$

output
```
1/2*(e*x)^(2*n)/a^2/e/n-I*b^3*(e*x)^(2*n)*ln(1-I*a*exp(I*(c+d*x^n))/(b-(-a
^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)+2*I*b*(e*x)^(2*n)*ln(1-I*
a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d/e/n/(x^n)+
I*b^3*(e*x)^(2*n)*ln(1-I*a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^
2+b^2)^(3/2)/d/e/n/(x^n)-2*I*b*(e*x)^(2*n)*ln(1-I*a*exp(I*(c+d*x^n))/(b+(-
a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d/e/n/(x^n)+b^2*(e*x)^(2*n)*ln(b+a*s
in(c+d*x^n))/a^2/(a^2-b^2)/d^2/e/n/(x^(2*n))-b^3*(e*x)^(2*n)*polylog(2,I*a
*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e/n/(x^(2
*n))+2*b*(e*x)^(2*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/
a^2/(-a^2+b^2)^(1/2)/d^2/e/n/(x^(2*n))+b^3*(e*x)^(2*n)*polylog(2,I*a*exp(I
*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))-2
*b*(e*x)^(2*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-
a^2+b^2)^(1/2)/d^2/e/n/(x^(2*n))-b^2*(e*x)^(2*n)*cos(c+d*x^n)/a/(a^2-b^2)/
d/e/n/(x^n)/(b+a*sin(c+d*x^n))
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2839 vs. $2(778) = 1556$.

Time = 9.43 (sec) , antiderivative size = 2839, normalized size of antiderivative = 3.65

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(e*x)^(-1 + 2*n)/(a + b*Csc[c + d*x^n])^2,x]
```

output
```
-1/2*(b^2*x^(1 - n)*(e*x)^(-1 + 2*n)*Csc[c/2]*Csc[c + d*x^n]^2*Sec[c/2]*(b
*Cos[c] + a*Sin[d*x^n])*(b + a*Sin[c + d*x^n]))/(a^2*(-a + b)*(a + b)*d*n*
(a + b*Csc[c + d*x^n])^2) - (b^2*x^(1 - n)*(e*x)^(-1 + 2*n)*Cot[c]*Csc[c +
 d*x^n]^2*(b + a*Sin[c + d*x^n])^2)/(a^2*(-a^2 + b^2)*d*n*(a + b*Csc[c + d
*x^n])^2) + (2*b^3*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*ArcTanh[(a*Cos[c + d*x^n]
+ I*(b + a*Sin[c + d*x^n]))/Sqrt[a^2 - b^2]]*Cot[c]*Csc[c + d*x^n]^2*(b +
a*Sin[c + d*x^n])^2)/(a^2*(a^2 - b^2)^(3/2)*d^2*n*(a + b*Csc[c + d*x^n])^2
) - (2*b*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*Csc[c + d*x^n]^2*((Pi*ArcTan[(a + b*
Tan[(c + d*x^n)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(-c + Pi/2 -
d*x^n)*ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x^n)/2])/Sqrt[a^2 - b^2]] - 2*(
-c + ArcCos[-(b/a)])*ArcTanh[((a - b)*Tan[(-c + Pi/2 - d*x^n)/2])/Sqrt[a^2
 - b^2]] + (ArcCos[-(b/a)] - (2*I)*(ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x^
n)/2])/Sqrt[a^2 - b^2]] - ArcTanh[((a - b)*Tan[(-c + Pi/2 - d*x^n)/2])/Sqr
t[a^2 - b^2]]))*Log[Sqrt[a^2 - b^2]/(Sqrt[2]*Sqrt[a]*E^((I/2)*(-c + Pi/2 -
 d*x^n))*Sqrt[b + a*Sin[c + d*x^n]])] + (ArcCos[-(b/a)] + (2*I)*(ArcTanh[(
(a + b)*Cot[(-c + Pi/2 - d*x^n)/2])/Sqrt[a^2 - b^2]] - ArcTanh[((a - b)*Ta
n[(-c + Pi/2 - d*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[(Sqrt[a^2 - b^2]*E^((I/2)
*(-c + Pi/2 - d*x^n)))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Sin[c + d*x^n]])] - (Ar
cCos[-(b/a)] + (2*I)*ArcTanh[((a - b)*Tan[(-c + Pi/2 - d*x^n)/2])/Sqrt[a^2
 - b^2]])*Log[1 - ((b - I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan...
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 624, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4697, 4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{2n-1}}{(a+b\csc(c+dx^n))^2}dx$$

$$\Big\downarrow \text{4697}$$

$$\frac{x^{-2n}(ex)^{2n}\int \frac{x^{2n-1}}{(a+b\csc(dx^n+c))^2}dx}{e}$$

$$\Big\downarrow \text{4693}$$

$$\frac{x^{-2n}(ex)^{2n}\int \frac{x^n}{(a+b\csc(dx^n+c))^2}dx^n}{en}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^{-2n}(ex)^{2n}\int \frac{x^n}{(a+b\csc(dx^n+c))^2}dx^n}{en}$$

$$\Big\downarrow \text{4679}$$

$$\frac{x^{-2n}(ex)^{2n}\int \left(-\frac{2bx^n}{a^2(b+a\sin(dx^n+c))}+\frac{x^n}{a^2}+\frac{b^2x^n}{a^2(b+a\sin(dx^n+c))^2}\right)dx^n}{en}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{-2n}(ex)^{2n}\left(\frac{2b\,\text{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}}-\frac{2b\,\text{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}}+\frac{b^2\log(a\sin(c+dx^n)+b)}{a^2d^2(a^2-b^2)}+\frac{2ibx^n\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{a^2d\sqrt{b^2-a^2}}-\right.}{}$$

input

```
Int[(e*x)^(-1 + 2*n)/(a + b*Csc[c + d*x^n])^2,x]
```

output

```
((e*x)^(2*n)*(x^(2*n)/(2*a^2) - (I*b^3*x^n*Log[1 - (I*a*E^(I*(c + d*x^n)))
/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^n*Log[1
- (I*a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d
) + (I*b^3*x^n*Log[1 - (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a
^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x^n*Log[1 - (I*a*E^(I*(c + d*x^n)))/(b
 + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) + (b^2*Log[b + a*Sin[c + d
*x^n]])/(a^2*(a^2 - b^2)*d^2) - (b^3*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b
 - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (2*b*PolyLog[2, (I*a
*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) +
(b^3*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^
2 + b^2)^(3/2)*d^2) - (2*b*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a
^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) - (b^2*x^n*Cos[c + d*x^n])/(a*(a^2
 - b^2)*d*(b + a*Sin[c + d*x^n]))))/(e*n*x^(2*n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

rule 4697
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.42 (sec) , antiderivative size = 3887, normalized size of antiderivative = 5.00

method	result	size
risch	Expression too large to display	3887

input
```
int((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2/n*x*exp(1/2*(-1+2*n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn
(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*l
n(x)+2*ln(e)))-2*I*b^2/a^2/(-a^2+b^2)/d/n*x*x^n/(2*b*exp(I*(c+d*x^n))-I*a*ex
p(2*I*(c+d*x^n))+I*a)*(e^n)^2*(-1)^(-1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^(-1
/2*csgn(I*e)*csgn(I*e*x)^2)*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(b*
exp(1/2*I*(-2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+2*Pi*n*csgn(I*e)*csgn(I
*e*x)^2+2*Pi*n*csgn(I*x)*csgn(I*e*x)^2-2*Pi*n*csgn(I*e*x)^3+Pi*csgn(I*e*x)
^3+2*d*x^n+2*c))+I*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e)*csgn(I*x)*n+2*cs
gn(I*e)*csgn(I*e*x)*n+2*n*csgn(I*x)*csgn(I*e*x)-2*n*csgn(I*e*x)^2+csgn(I*e
*x)^2))*a)/e-2/d/(a^2-b^2)^2*b*(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)/n/e*(
e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi*n*csgn(I*e)
*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x)^3-Pi*csgn
(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn(I
*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*ln((I*exp(I*c)*b+a*exp(I*(d*x^n+2*c))-(a^2*
exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))/(I*exp(I*c)*b-(a^2*exp(2*I*c)-exp(2*I*c)
*b^2)^(1/2)))*x^n+1/a^2/d/(a^2-b^2)^2*b^3*(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^
(1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi*
n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x
)^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(
I*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*ln((I*exp(I*c)*b+a*exp(I*(d*x...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2435 vs. $2(710) = 1420$.

Time = 0.34 (sec) , antiderivative size = 2435, normalized size of antiderivative = 3.13

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n))^2,x, algorithm="fricas")
```

output

```
1/2*((a^5 - 2*a^3*b^2 + a*b^4)*d^2*e^(2*n - 1)*x^(2*n)*sin(d*x^n + c) + (a
^4*b - 2*a^2*b^3 + b^5)*d^2*e^(2*n - 1)*x^(2*n) - 2*(a^3*b^2 - a*b^4)*d*e^
(2*n - 1)*x^n*cos(d*x^n + c) + ((2*I*a^4*b - I*a^2*b^3)*e^(2*n - 1)*sqrt((
a^2 - b^2)/a^2)*sin(d*x^n + c) + (2*I*a^3*b^2 - I*a*b^4)*e^(2*n - 1)*sqrt(
(a^2 - b^2)/a^2))*dilog(((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) +
(I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) - a)/a + 1) + ((2*I*a^4*b -
 I*a^2*b^3)*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)*sin(d*x^n + c) + (2*I*a^3*b^
2 - I*a*b^4)*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2))*dilog(-((a*sqrt((a^2 - b^2
)/a^2) + I*b)*cos(d*x^n + c) - (I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n +
 c) + a)/a + 1) + ((-2*I*a^4*b + I*a^2*b^3)*e^(2*n - 1)*sqrt((a^2 - b^2)/a
^2)*sin(d*x^n + c) + (-2*I*a^3*b^2 + I*a*b^4)*e^(2*n - 1)*sqrt((a^2 - b^2)
/a^2))*dilog(((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) + (-I*a*sqrt(
(a^2 - b^2)/a^2) - b)*sin(d*x^n + c) - a)/a + 1) + ((-2*I*a^4*b + I*a^2*b^
3)*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)*sin(d*x^n + c) + (-2*I*a^3*b^2 + I*a*
b^4)*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2))*dilog(-((a*sqrt((a^2 - b^2)/a^2) -
 I*b)*cos(d*x^n + c) - (-I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) + a
)/a + 1) + ((a^3*b^2 - a*b^4 - (2*a^4*b - a^2*b^3)*c*sqrt((a^2 - b^2)/a^2)
)*e^(2*n - 1)*sin(d*x^n + c) + (a^2*b^3 - b^5 - (2*a^3*b^2 - a*b^4)*c*sqrt
((a^2 - b^2)/a^2))*e^(2*n - 1))*log(2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n +
 c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) + ((a^3*b^2 - a*b^4 - (2*a^4*b...
```

Sympy [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(ex)^{2n-1}}{(a+b\csc(c+dx^n))^2}\,dx$$

input

```
integrate((e*x)**(-1+2*n)/(a+b*csc(c+d*x**n))**2,x)
```

output

```
Integral((e*x)**(2*n - 1)/(a + b*csc(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx=\int \frac{(ex)^{2n-1}}{(b\csc(dx^n+c)+a)^2}\,dx$$

input
```
integrate((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n))^2,x, algorithm="maxima")
```

output
```
-1/2*(4*a*b^3*e^(2*n)*x^n*cos(d*x^n + c) - (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*
n)*cos(2*d*x^n + 2*c)^2 - 4*(a^2*b^2 - b^4)*d*e^(2*n)*x^(2*n)*cos(d*x^n +
c)^2 - (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n)*sin(2*d*x^n + 2*c)^2 - 4*(a^2*b^2
 - b^4)*d*e^(2*n)*x^(2*n)*sin(d*x^n + c)^2 - 4*(a^3*b - a*b^3)*d*e^(2*n)*x
^(2*n)*sin(d*x^n + c) - (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n) + 2*(2*a*b^3*e^(
2*n)*x^n*cos(d*x^n + c) + 2*(a^3*b - a*b^3)*d*e^(2*n)*x^(2*n)*sin(d*x^n +
c) + (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n))*cos(2*d*x^n + 2*c) - 2*((a^6 - a^4
*b^2)*d*e*n*cos(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*cos(d*x^n +
 c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c)*sin(2*d*x^n + 2*c) + (a^6
 - a^4*b^2)*d*e*n*sin(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*sin(d
*x^n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*sin(d*x^n + c) + (a^6 - a^4*b^2)*d
*e*n - 2*(2*(a^5*b - a^3*b^3)*d*e*n*sin(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n
)*cos(2*d*x^n + 2*c))*integrate(-2*(a^2*b^4*e^(2*n)*x^n*cos(2*c)*sin(2*d*x
^n) + a^2*b^4*e^(2*n)*x^n*cos(2*d*x^n)*sin(2*c) - 2*(a^3*b^3 - a*b^5)*e^(2
*n)*x^n*cos(d*x^n)*cos(c) + 2*(a^3*b^3 - a*b^5)*e^(2*n)*x^n*sin(d*x^n)*sin
(c) - (a^3*b^3*e^(2*n)*x^n*cos(d*x^n + c) + (2*a^5*b - a^3*b^3)*d*e^(2*n)*
x^(2*n)*sin(d*x^n + c))*cos(2*d*x^n + 2*c) + ((a^3*b^3 - a*b^5)*e^(2*n)*x^
n + (a*b^5*e^(2*n)*x^n*cos(2*c) - (2*a^3*b^3 - a*b^5)*d*e^(2*n)*x^(2*n)*si
n(2*c))*cos(2*d*x^n) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*e^(2*n)*x^(2*n)*
cos(c) + (a^2*b^4 - b^6)*e^(2*n)*x^n*sin(c))*cos(d*x^n) - (a*b^5*e^(2*n...
```

Giac [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx=\int \frac{(ex)^{2n-1}}{(b\csc(dx^n+c)+a)^2}\,dx$$

input
```
integrate((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n))^2,x, algorithm="giac")
```

output
```
integrate((e*x)^(2*n - 1)/(b*csc(d*x^n + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(e\,x)^{2\,n-1}}{\left(a+\frac{b}{\sin(c+d\,x^n)}\right)^2}\,dx$$

input
```
int((e*x)^(2*n - 1)/(a + b/sin(c + d*x^n))^2,x)
```

output
```
int((e*x)^(2*n - 1)/(a + b/sin(c + d*x^n))^2, x)
```

Reduce [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx = \text{Too large to display}$$

input
```
int((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n))^2,x)
```

output
```
(e**(2*n)*(4*x***n*cos(x***n*d + c)*a*d + x**(2*n)*sin(x***n*d + c)*a*d**2 +
x**(2*n)*b*d**2 + 4*int(x**(2*n)/(sin(x***n*d + c)**2*a**2*x + 2*sin(x***n*d
 + c)*a*b*x + b**2*x),x)*sin(x***n*d + c)*a**3*d**2*n - 2*int(x**(2*n)/(sin
(x***n*d + c)**2*a**2*x + 2*sin(x***n*d + c)*a*b*x + b**2*x),x)*sin(x***n*d +
 c)*a*b**2*d**2*n + 4*int(x**(2*n)/(sin(x***n*d + c)**2*a**2*x + 2*sin(x***n
*d + c)*a*b*x + b**2*x),x)*a**2*b*d**2*n - 2*int(x**(2*n)/(sin(x***n*d + c)
**2*a**2*x + 2*sin(x***n*d + c)*a*b*x + b**2*x),x)*b**3*d**2*n - 2*int((x**
(2*n)*sin(x***n*d + c)**2)/(sin(x***n*d + c)**2*a**2*x + 2*sin(x***n*d + c)*a
*b*x + b**2*x),x)*sin(x***n*d + c)*a**3*d**2*n + int((x**(2*n)*sin(x***n*d +
 c)**2)/(sin(x***n*d + c)**2*a**2*x + 2*sin(x***n*d + c)*a*b*x + b**2*x),x)*
sin(x***n*d + c)*a*b**2*d**2*n - 2*int((x**(2*n)*sin(x***n*d + c)**2)/(sin(x
**n*d + c)**2*a**2*x + 2*sin(x***n*d + c)*a*b*x + b**2*x),x)*a**2*b*d**2*n
+ int((x**(2*n)*sin(x***n*d + c)**2)/(sin(x***n*d + c)**2*a**2*x + 2*sin(x**
n*d + c)*a*b*x + b**2*x),x)*b**3*d**2*n + 4*log(tan((x***n*d + c)/2)**2 + 1
)*sin(x***n*d + c)*a + 4*log(tan((x***n*d + c)/2)**2 + 1)*b - 4*log(tan((x**
n*d + c)/2)**2*b + 2*tan((x***n*d + c)/2)*a + b)*sin(x***n*d + c)*a - 4*log(
tan((x***n*d + c)/2)**2*b + 2*tan((x***n*d + c)/2)*a + b)*b))/(b**2*d**2*e*n
*(sin(x***n*d + c)*a + b))
```

3.84 $\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1417

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx = \text{Too large to display}$$

output
```
1/3*(e*x)^(3*n)/a^2/e/n-4*I*b*(e*x)^(3*n)*polylog(3,I*a*exp(I*(c+d*x^n))/(
b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^3/e/n/(x^(3*n))+2*b^2*(e*x)^(3
*n)*ln(1+a*exp(I*(c+d*x^n))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2/e/n/(
x^(2*n))+2*b^2*(e*x)^(3*n)*ln(1+a*exp(I*(c+d*x^n))/(I*b+(a^2-b^2)^(1/2)))/
a^2/(a^2-b^2)/d^2/e/n/(x^(2*n))-I*b^3*(e*x)^(3*n)*ln(1-I*a*exp(I*(c+d*x^n)
)/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)-2*I*b^2*(e*x)^(3*
n)*polylog(2,-a*exp(I*(c+d*x^n))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3/
e/n/(x^(3*n))+2*I*b*(e*x)^(3*n)*ln(1-I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1
/2)))/a^2/(-a^2+b^2)^(1/2)/d/e/n/(x^n)+4*I*b*(e*x)^(3*n)*polylog(3,I*a*exp
(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^3/e/n/(x^(3*n))
-2*I*b^3*(e*x)^(3*n)*polylog(3,I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/
a^2/(-a^2+b^2)^(3/2)/d^3/e/n/(x^(3*n))-2*I*b^2*(e*x)^(3*n)*polylog(2,-a*ex
p(I*(c+d*x^n))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3/e/n/(x^(3*n))-2*b^
3*(e*x)^(3*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a
^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))+4*b*(e*x)^(3*n)*polylog(2,I*a*exp(I*(c+d*x
^n))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2/e/n/(x^(2*n))+2*b^3*(e
*x)^(3*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b
^2)^(3/2)/d^2/e/n/(x^(2*n))-4*b*(e*x)^(3*n)*polylog(2,I*a*exp(I*(c+d*x^n))
/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2/e/n/(x^(2*n))+2*I*b^3*(e*x
)^(3*n)*polylog(3,I*a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+...
```

Mathematica [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx$$

input
```
Integrate[(e*x)^(-1 + 3*n)/(a + b*Csc[c + d*x^n])^2,x]
```

output
```
Integrate[(e*x)^(-1 + 3*n)/(a + b*Csc[c + d*x^n])^2, x]
```

Rubi [A] (verified)

Time = 2.39 (sec) , antiderivative size = 1152, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4697, 4693, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3n-1}}{(a+b\csc(c+dx^n))^2}dx$$

$$\Big\downarrow \text{4697}$$

$$\frac{x^{-3n}(ex)^{3n}\int \frac{x^{3n-1}}{(a+b\csc(dx^n+c))^2}dx}{e}$$

$$\Big\downarrow \text{4693}$$

$$\frac{x^{-3n}(ex)^{3n}\int \frac{x^{2n}}{(a+b\csc(dx^n+c))^2}dx^n}{en}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^{-3n}(ex)^{3n}\int \frac{x^{2n}}{(a+b\csc(dx^n+c))^2}dx^n}{en}$$

$$\Big\downarrow \text{4679}$$

$$\frac{x^{-3n}(ex)^{3n}\int \left(-\frac{2bx^{2n}}{a^2(b+a\sin(dx^n+c))}+\frac{x^{2n}}{a^2}+\frac{b^2x^{2n}}{a^2(b+a\sin(dx^n+c))^2}\right)dx^n}{en}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{-3n}(ex)^{3n}\left(\frac{2b^2\log\left(\frac{e^{i(dx^n+c)}a}{ib-\sqrt{a^2-b^2}}+1\right)x^n}{a^2(a^2-b^2)d^2}+\frac{2b^2\log\left(\frac{e^{i(dx^n+c)}a}{ib+\sqrt{a^2-b^2}}+1\right)x^n}{a^2(a^2-b^2)d^2}+\frac{4b\,\text{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)x^n}{a^2\sqrt{b^2-a^2}d^2}-\frac{2b^3\,\text{PolyLog}\left(2,\frac{iae^{i(dx^n+c}}{b-\sqrt{b^2-a^2}}\right.}{a^2(b^2-a^2)^{3/2}d^2}\right.}{}$$

input
```
Int[(e*x)^(-1 + 3*n)/(a + b*Csc[c + d*x^n])^2,x]
```

output
```
((e*x)^(3*n)*(((-I)*b^2*x^(2*n))/(a^2*(a^2 - b^2)*d) + x^(3*n)/(3*a^2) + (
2*b^2*x^n*Log[1 + (a*E^(I*(c + d*x^n)))/(I*b - Sqrt[a^2 - b^2])])/(a^2*(a^
2 - b^2)*d^2) + (2*b^2*x^n*Log[1 + (a*E^(I*(c + d*x^n)))/(I*b + Sqrt[a^2 -
 b^2])])/(a^2*(a^2 - b^2)*d^2) - (I*b^3*x^(2*n)*Log[1 - (I*a*E^(I*(c + d*x
^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^(2*
n)*Log[1 - (I*a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2
 + b^2]*d) + (I*b^3*x^(2*n)*Log[1 - (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2
 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x^(2*n)*Log[1 - (I*a*E^(I
*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((2*I)*
b^2*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^
2 - b^2)*d^3) - ((2*I)*b^2*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(I*b + Sqrt[
a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (2*b^3*x^n*PolyLog[2, (I*a*E^(I*(c
+ d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (4*b*x^
n*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a
^2 + b^2]*d^2) + (2*b^3*x^n*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-
a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (4*b*x^n*PolyLog[2, (I*a*E^(I
*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) - ((2*I
)*b^3*PolyLog[3, (I*a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a
^2 + b^2)^(3/2)*d^3) + ((4*I)*b*PolyLog[3, (I*a*E^(I*(c + d*x^n)))/(b - Sq
rt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^3) + ((2*I)*b^3*PolyLog[3, (I...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

rule 4697
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Maple [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+d\,x^{n}))^{2}}dx$$

input
```
int((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n))^2,x)
```

output
```
int((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3755 vs. 2(1291) = 2582.

Time = 0.38 (sec) , antiderivative size = 3755, normalized size of antiderivative = 2.65

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^{n}))^{2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(ex)^{3n-1}}{(a+b\csc(c+dx^n))^2}\,dx$$

input
```
integrate((e*x)**(-1+3*n)/(a+b*csc(c+d*x**n))**2,x)
```

output
```
Integral((e*x)**(3*n - 1)/(a + b*csc(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(ex)^{3n-1}}{(b\csc(dx^n+c)+a)^2}\,dx$$

input
```
integrate((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n))^2,x, algorithm="maxima")
```

output
```
-1/3*(6*a*b^3*e^(3*n)*x^(2*n)*cos(d*x^n + c) - (a^4 - a^2*b^2)*d*e^(3*n)*x
^(3*n)*cos(2*d*x^n + 2*c)^2 - 4*(a^2*b^2 - b^4)*d*e^(3*n)*x^(3*n)*cos(d*x^
n + c)^2 - (a^4 - a^2*b^2)*d*e^(3*n)*x^(3*n)*sin(2*d*x^n + 2*c)^2 - 4*(a^2
*b^2 - b^4)*d*e^(3*n)*x^(3*n)*sin(d*x^n + c)^2 - 4*(a^3*b - a*b^3)*d*e^(3*
n)*x^(3*n)*sin(d*x^n + c) - (a^4 - a^2*b^2)*d*e^(3*n)*x^(3*n) + 2*(3*a*b^3
*e^(3*n)*x^(2*n)*cos(d*x^n + c) + 2*(a^3*b - a*b^3)*d*e^(3*n)*x^(3*n)*sin(
d*x^n + c) + (a^4 - a^2*b^2)*d*e^(3*n)*x^(3*n))*cos(2*d*x^n + 2*c) - 3*((a
^6 - a^4*b^2)*d*e*n*cos(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*cos
(d*x^n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c)*sin(2*d*x^n + 2*c
) + (a^6 - a^4*b^2)*d*e*n*sin(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e
*n*sin(d*x^n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*sin(d*x^n + c) + (a^6 - a^
4*b^2)*d*e*n - 2*(2*(a^5*b - a^3*b^3)*d*e*n*sin(d*x^n + c) + (a^6 - a^4*b^
2)*d*e*n)*cos(2*d*x^n + 2*c))*integrate(-2*(2*a^2*b^4*e^(3*n)*x^(2*n)*cos(
2*c)*sin(2*d*x^n) + 2*a^2*b^4*e^(3*n)*x^(2*n)*cos(2*d*x^n)*sin(2*c) - 4*(a
^3*b^3 - a*b^5)*e^(3*n)*x^(2*n)*cos(d*x^n)*cos(c) + 4*(a^3*b^3 - a*b^5)*e^
(3*n)*x^(2*n)*sin(d*x^n)*sin(c) - (2*a^3*b^3*e^(3*n)*x^(2*n)*cos(d*x^n + c
) + (2*a^5*b - a^3*b^3)*d*e^(3*n)*x^(3*n)*sin(d*x^n + c))*cos(2*d*x^n + 2*
c) + (2*(a^3*b^3 - a*b^5)*e^(3*n)*x^(2*n) + (2*a*b^5*e^(3*n)*x^(2*n)*cos(2
*c) - (2*a^3*b^3 - a*b^5)*d*e^(3*n)*x^(3*n)*sin(2*c))*cos(2*d*x^n) + 2*((2
*a^4*b^2 - 3*a^2*b^4 + b^6)*d*e^(3*n)*x^(3*n)*cos(c) + 2*(a^2*b^4 - b^6...
```

Giac [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx=\int \frac{(ex)^{3n-1}}{(b\csc(dx^n+c)+a)^2}\,dx$$

input
```
integrate((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n))^2,x, algorithm="giac")
```

output
```
integrate((e*x)^(3*n - 1)/(b*csc(d*x^n + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(e\,x)^{3\,n-1}}{\left(a+\frac{b}{\sin(c+d\,x^n)}\right)^2}\,dx$$

input
```
int((e*x)^(3*n - 1)/(a + b/sin(c + d*x^n))^2,x)
```

output
```
int((e*x)^(3*n - 1)/(a + b/sin(c + d*x^n))^2, x)
```

Reduce [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx = \text{too large to display}$$

input
```
int((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n))^2,x)
```

output

```
(2*e**(3*n)*( - 72*sqrt( - a**2 + b**2)*atan((tan((x**n*d + c)/2)*b + a)/s
qrt( - a**2 + b**2))*sin(x**n*d + c)*a**4 + 48*sqrt( - a**2 + b**2)*atan((
tan((x**n*d + c)/2)*b + a)/sqrt( - a**2 + b**2))*sin(x**n*d + c)*a**2*b**2
 - 72*sqrt( - a**2 + b**2)*atan((tan((x**n*d + c)/2)*b + a)/sqrt( - a**2 +
 b**2))*a**3*b + 48*sqrt( - a**2 + b**2)*atan((tan((x**n*d + c)/2)*b + a)/
sqrt( - a**2 + b**2))*a*b**3 + 6*x**(2*n)*cos(x**n*d + c)*a**4*b*d**2 - 6*
x**(2*n)*cos(x**n*d + c)*a**2*b**3*d**2 - 36*x**n*cos(x**n*d + c)*a**5*d +
 42*x**n*cos(x**n*d + c)*a**3*b**2*d - 6*x**n*cos(x**n*d + c)*a*b**4*d + x
**(3*n)*sin(x**n*d + c)*a**4*b*d**3 - x**(3*n)*sin(x**n*d + c)*a**2*b**3*d
**3 + x**(3*n)*a**3*b**2*d**3 - x**(3*n)*a*b**4*d**3 - 9*x**(2*n)*sin(x**n
*d + c)*a**5*d**2 + 12*x**(2*n)*sin(x**n*d + c)*a**3*b**2*d**2 - 3*x**(2*n
)*sin(x**n*d + c)*a*b**4*d**2 - 3*x**(2*n)*a**4*b*d**2 + 3*x**(2*n)*a**2*b
**3*d**2 - 36*x**n*a**5*d + 60*x**n*a**3*b**2*d - 24*x**n*a*b**4*d + 12*in
t(x**(3*n)/(tan((x**n*d + c)/2)**4*b**2*x + 4*tan((x**n*d + c)/2)**3*a*b*x
 + 4*tan((x**n*d + c)/2)**2*a**2*x + 2*tan((x**n*d + c)/2)**2*b**2*x + 4*t
an((x**n*d + c)/2)*a*b*x + b**2*x),x)*sin(x**n*d + c)*a**6*b*d**3*n - 18*i
nt(x**(3*n)/(tan((x**n*d + c)/2)**4*b**2*x + 4*tan((x**n*d + c)/2)**3*a*b*
x + 4*tan((x**n*d + c)/2)**2*a**2*x + 2*tan((x**n*d + c)/2)**2*b**2*x + 4*
tan((x**n*d + c)/2)*a*b*x + b**2*x),x)*sin(x**n*d + c)*a**4*b**3*d**3*n +
6*int(x**(3*n)/(tan((x**n*d + c)/2)**4*b**2*x + 4*tan((x**n*d + c)/2)**...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                   ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file