Computer Algebra
Independent Integration Tests

Summer 2024

4-Trig-functions/4.7-Trig-exponential/251-4.7.1

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**118**]. This is test number [251].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	96.61 (114)	3.39 (4)
Mathematica	96.61 (114)	3.39 (4)
Fricas	77.97 (92)	22.03 (26)
Giac	74.58 (88)	25.42 (30)
Maxima	71.19 (84)	28.81 (34)
Maple	57.63 (68)	42.37 (50)
Mupad	47.46 (56)	52.54 (62)
Sympy	45.76 (54)	54.24 (64)
Reduce	24.58 (29)	75.42 (89)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

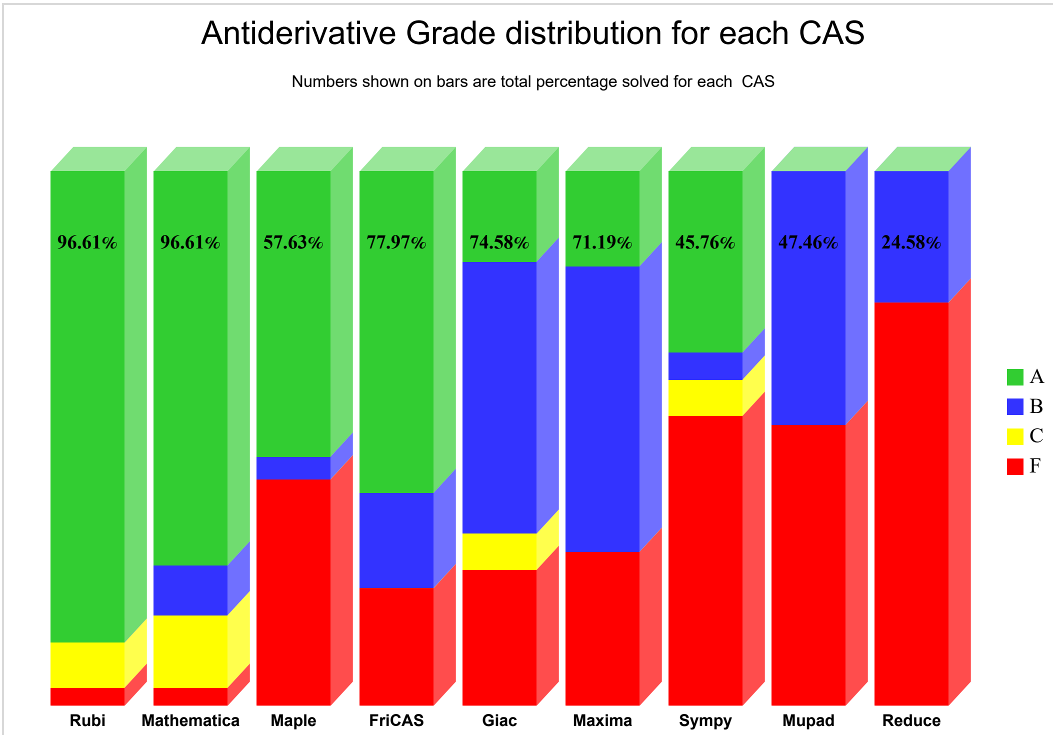
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

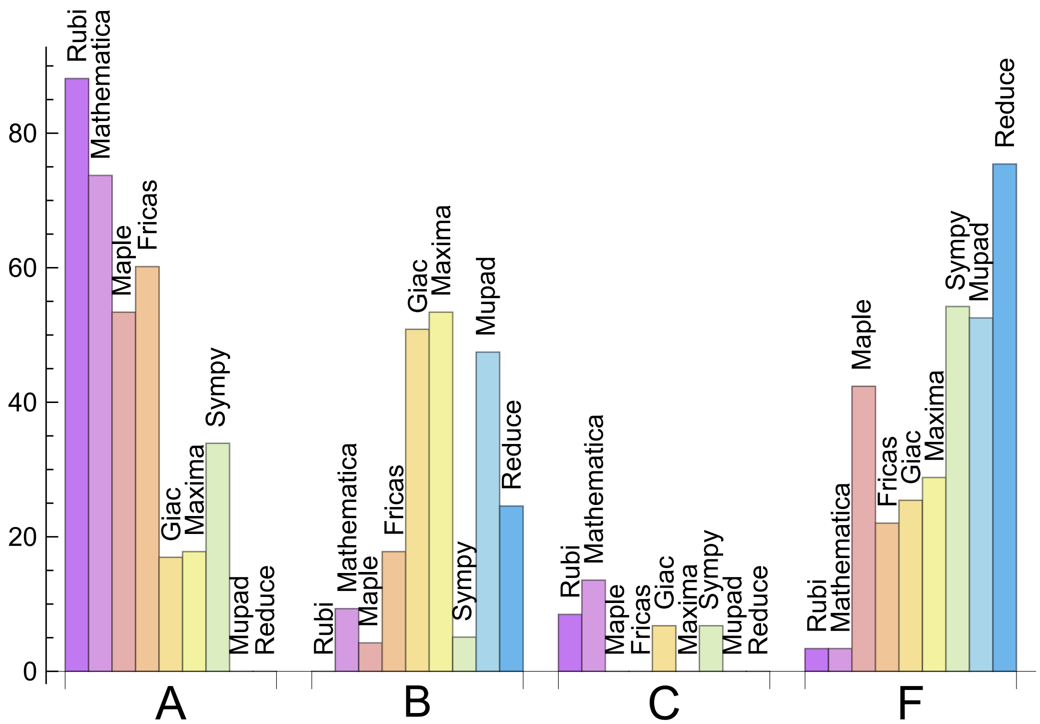
System	% A grade	% B grade	% C grade	% F grade
Rubi	88.136	0.000	8.475	3.390
Mathematica	73.729	9.322	13.559	3.390
Fricas	60.169	17.797	0.000	22.034
Maple	53.390	4.237	0.000	42.373
Sympy	33.898	5.085	6.780	54.237
Maxima	17.797	53.390	0.000	28.814
Giac	16.949	50.847	6.780	25.424
Mupad	0.000	47.458	0.000	52.542
Reduce	0.000	24.576	0.000	75.424

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	4	100.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Fricas	26	100.00	0.00	0.00
Giac	30	100.00	0.00	0.00
Maxima	34	100.00	0.00	0.00
Maple	50	100.00	0.00	0.00
Mupad	62	0.00	100.00	0.00
Sympy	64	100.00	0.00	0.00
Reduce	89	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.08
Reduce	0.19
Rubi	0.38
Mathematica	0.55
Maple	0.68
Giac	1.43
Sympy	1.50
Maxima	8.02
Mupad	13.95

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Reduce	64.90	0.87	61.00	0.72
Maple	81.09	0.83	65.50	0.74
Mathematica	126.24	1.12	105.00	0.94
Rubi	135.37	1.11	119.50	1.02
Mupad	149.27	1.28	120.50	1.13
Fricas	171.65	1.11	90.00	1.03
Sympy	314.37	2.53	155.00	1.46
Giac	1159.16	6.30	178.00	1.53
Maxima	1969.35	9.01	356.00	4.39

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

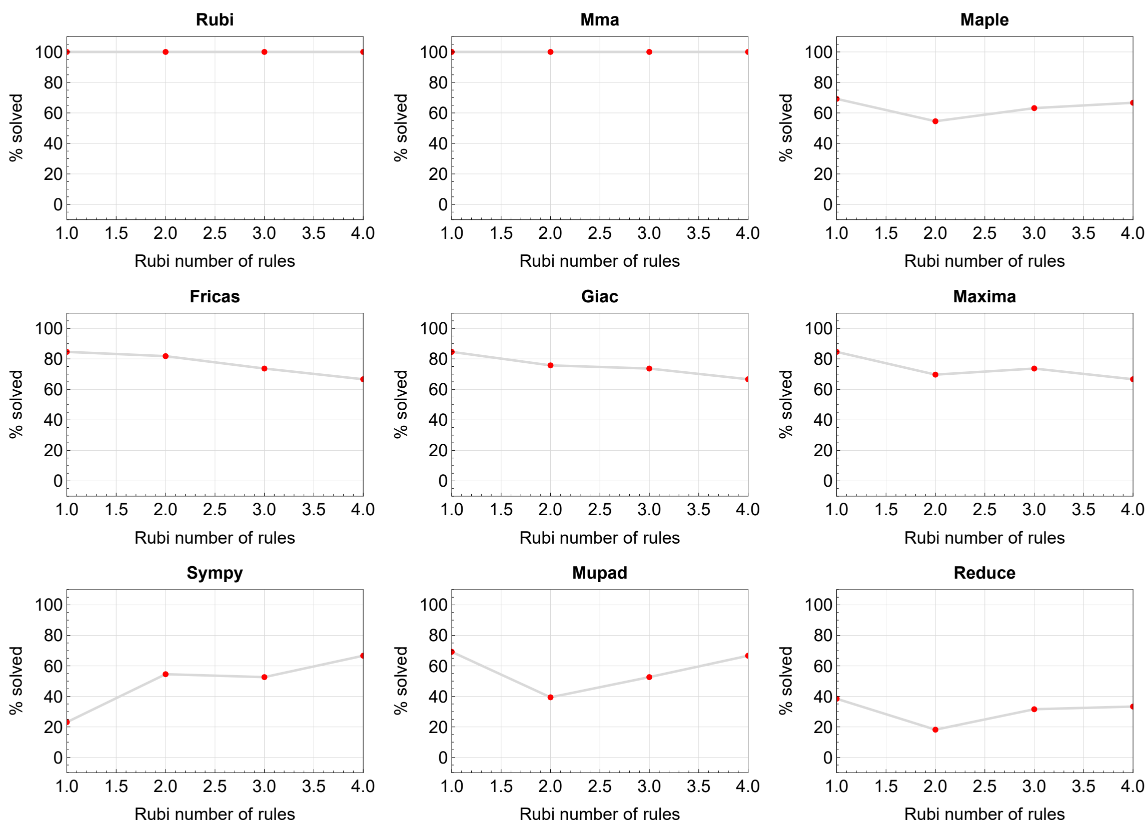


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

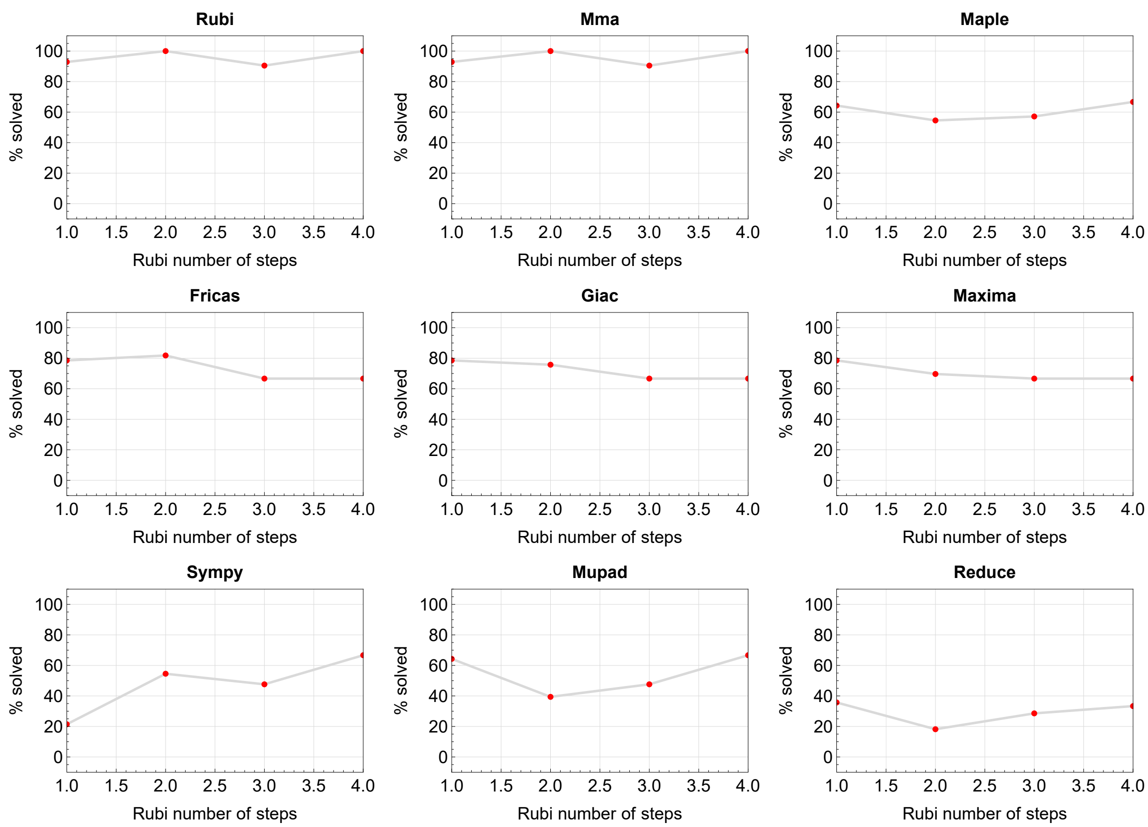


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

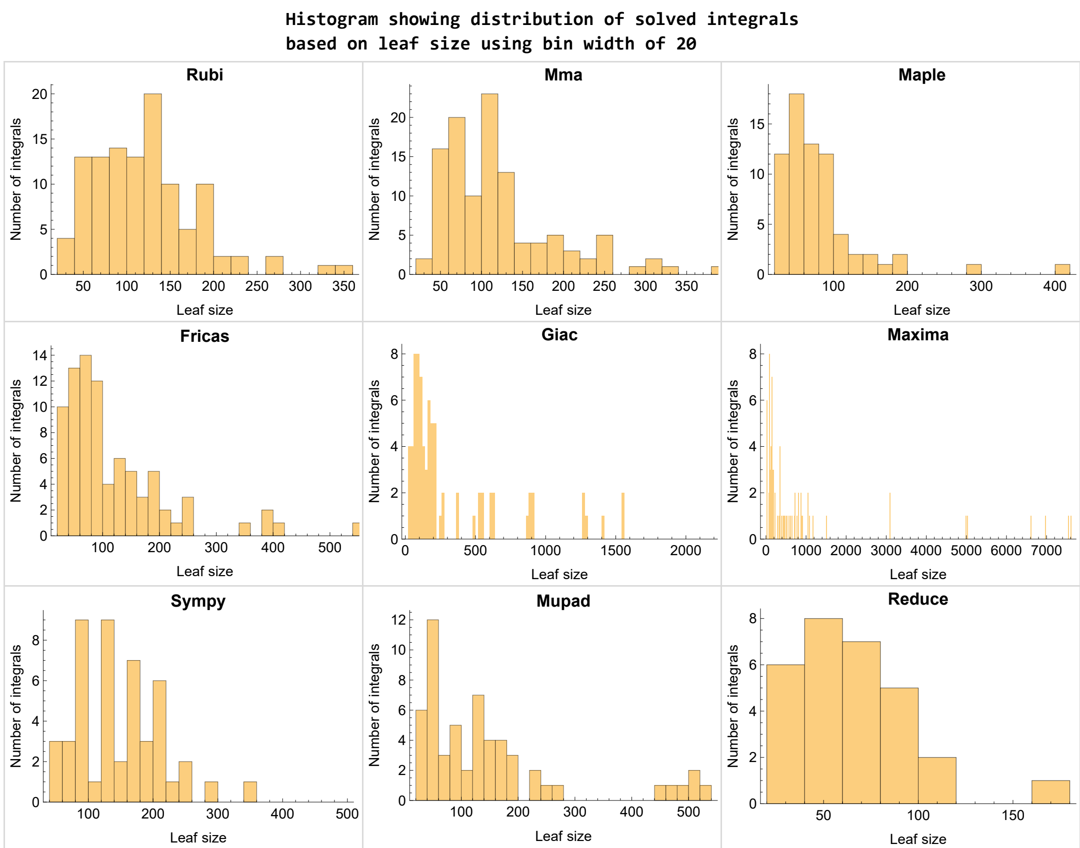


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

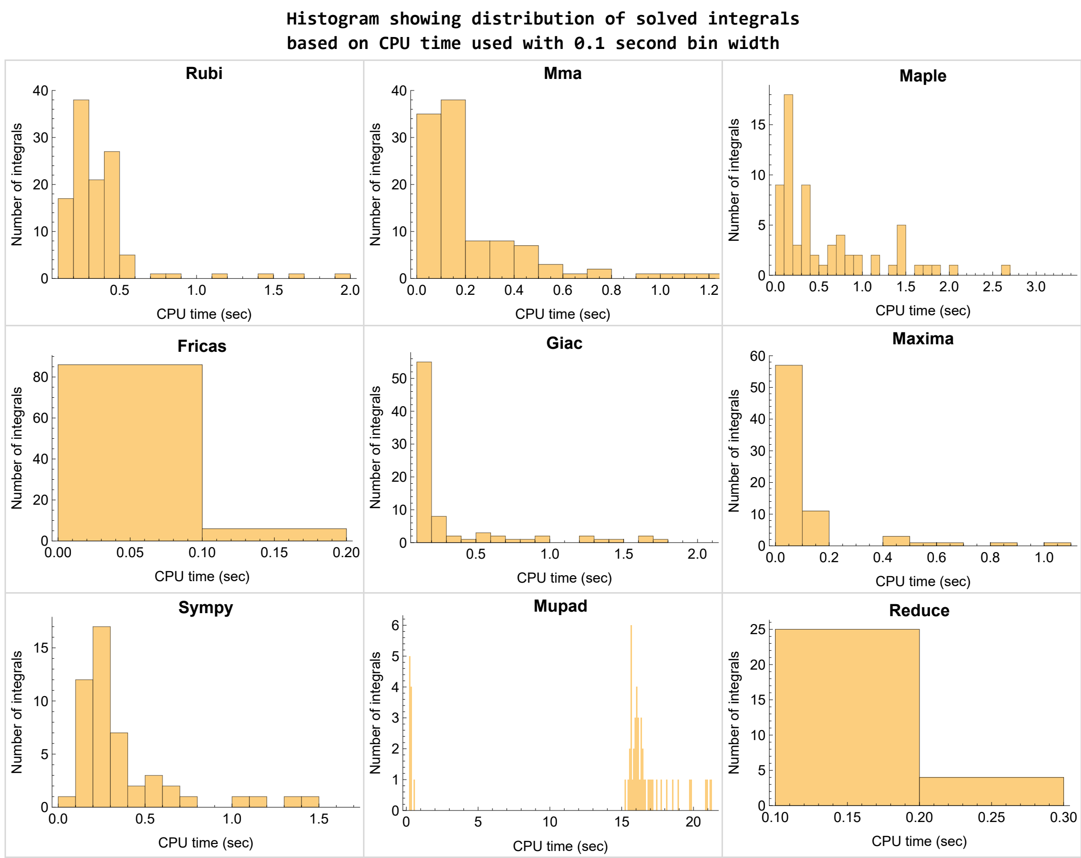


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

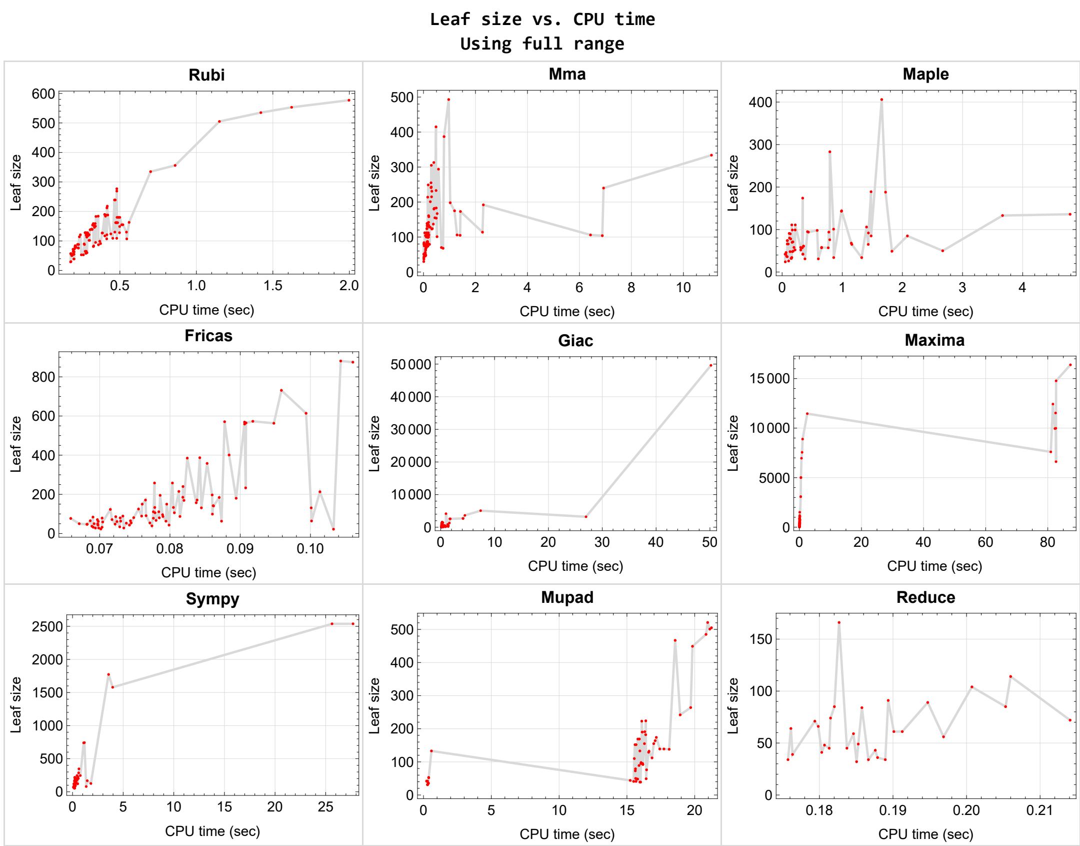


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{99, 101, 105, 117, 118\}$

Mathematica $\{115\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

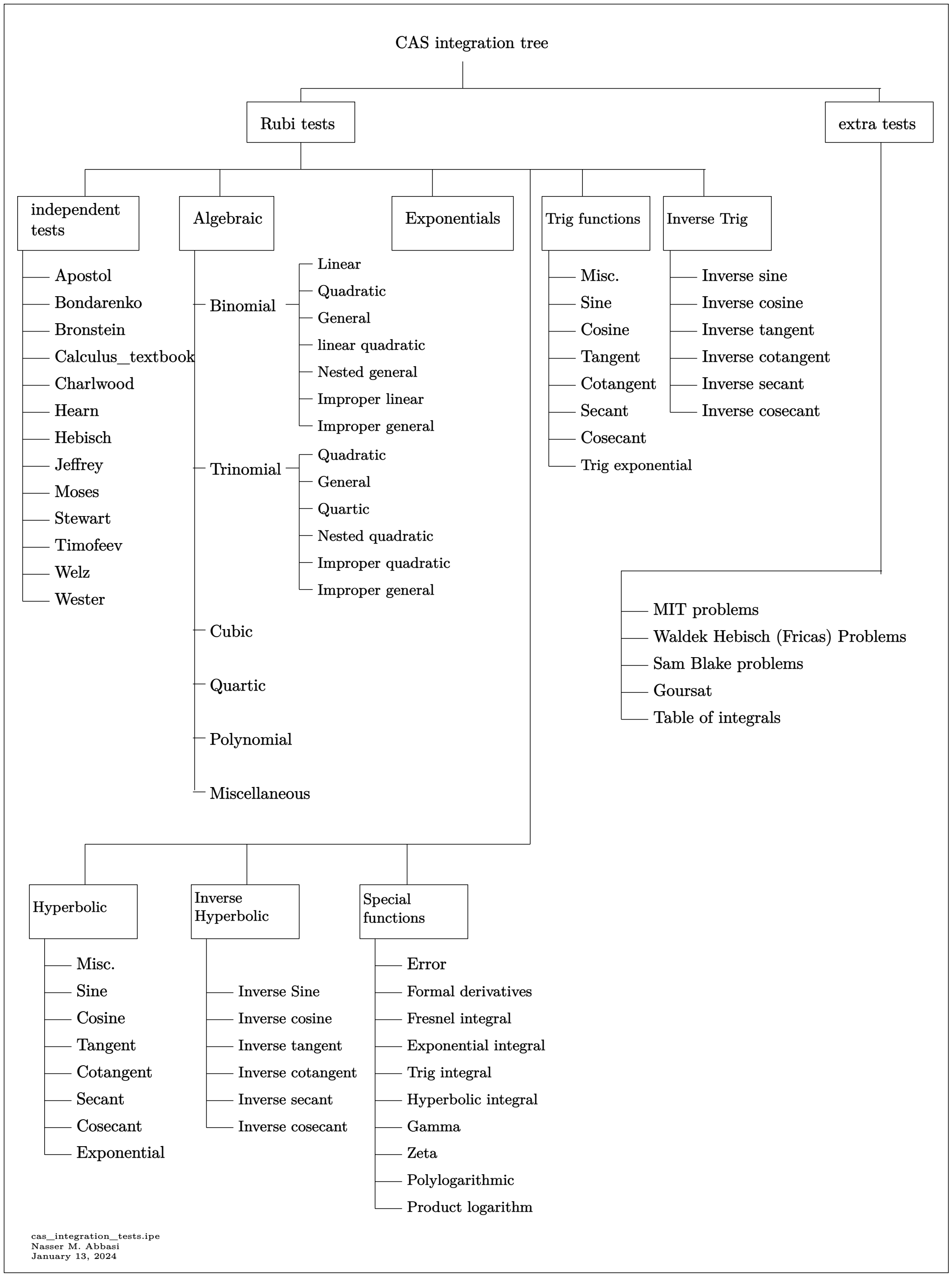
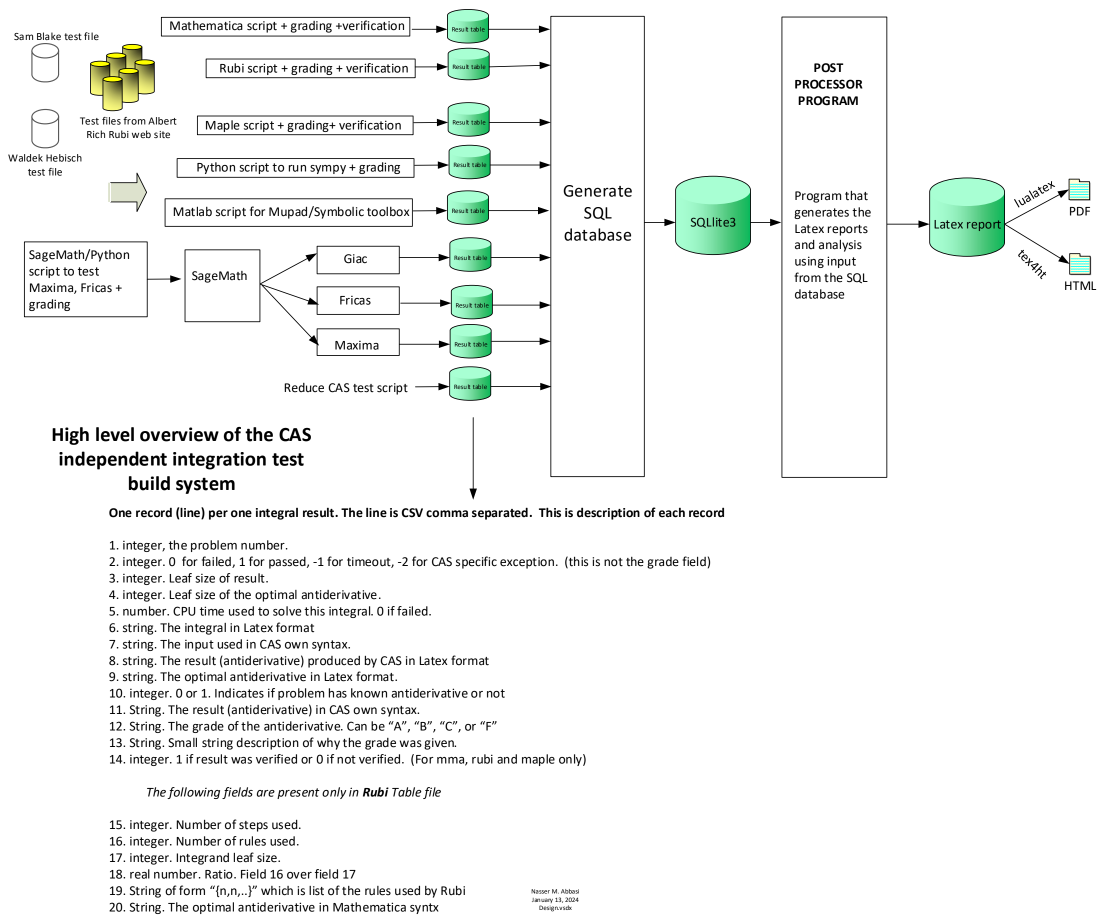


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 112, 113, 114, 115, 116, 117, 118 }

B grade { }

C grade { 85, 86, 87, 88, 89, 107, 108, 109, 110, 111 }

F normal fail { 56, 57, 73, 74 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 47, 48, 52, 53, 54, 55, 58, 59, 60, 61, 64, 65, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 112, 113, 115, 116, 117, 118 }

B grade { 45, 46, 62, 63, 97, 102, 103, 104, 105, 106, 114 }

C grade { 49, 50, 51, 66, 67, 68, 85, 86, 87, 88, 89, 107, 108, 109, 110, 111 }

F normal fail { 56, 57, 73, 74 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 2, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 47, 48, 58, 59, 60, 61, 62, 63, 64, 65, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

B grade { 1, 3, 6, 8, 21 }

C grade { }

F normal fail { 17, 18, 19, 20, 37, 38, 39, 40, 49, 50, 51, 52, 53, 54, 55, 56, 57, 66, 67, 68, 69, 70, 71, 72, 73, 74, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 62, 63, 64, 65, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 95, 96, 97, 99, 100, 101, 102, 103, 104, 106, 107, 117, 118 }

B grade { 49, 50, 51, 58, 59, 60, 61, 66, 67, 68, 83, 86, 87, 88, 89, 98, 105, 108, 109, 110, 111 }

C grade { }

F normal fail { 17, 18, 37, 38, 52, 53, 54, 55, 56, 57, 69, 70, 71, 72, 73, 74, 90, 91, 92, 93, 94, 112, 113, 114, 115, 116 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 21, 22, 23, 24, 25, 26, 27, 28, 29 }

B grade { 13, 14, 15, 16, 30, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

C grade { }

F normal fail { 17, 18, 19, 20, 37, 38, 39, 40, 52, 53, 54, 55, 56, 57, 69, 70, 71, 72, 73, 74, 90, 91, 92, 93, 94, 95, 96, 112, 113, 114, 115, 116, 117, 118 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 41, 45, 46, 47, 48, 58, 62, 63, 64, 65, 75, 77, 79, 80, 81, 97, 99, 101, 102, 103 }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 39, 40, 42, 43, 44, 49, 50, 51, 59, 60, 61, 66, 67, 68, 76, 78, 82, 83, 84, 85, 86, 87, 88, 89, 98, 100, 104, 105, 106, 107, 108, 109, 110, 111 }

C grade { 13, 14, 15, 16, 33, 34, 35, 36 }

F normal fail { 17, 18, 37, 38, 52, 53, 54, 55, 56, 57, 69, 70, 71, 72, 73, 74, 90, 91, 92, 93, 94, 95, 96, 112, 113, 114, 115, 116, 117, 118 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 41, 42, 45, 46, 49, 50, 58, 59, 62, 63, 66, 67, 75, 76, 80, 81, 85, 86, 97, 98, 102, 103, 107, 108 }

C grade { }

F normal fail { }

F(-1) timedout fail { 17, 18, 19, 20, 37, 38, 39, 40, 43, 44, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 64, 65, 68, 69, 70, 71, 72, 73, 74, 77, 78, 79, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

F(-2) exception fail { }

Sympy

A grade { 1, 3, 6, 8, 9, 10, 11, 12, 21, 23, 25, 26, 27, 28, 29, 30, 31, 32, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68 }

B grade { 2, 4, 5, 7, 22, 24 }

C grade { 13, 14, 15, 16, 33, 34, 35, 36 }

F normal fail { 17, 18, 19, 20, 37, 38, 39, 40, 52, 53, 54, 55, 56, 57, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 4, 5, 6, 7, 9, 10, 11, 13, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 77, 79, 83, 99, 101, 105 }

C grade { }

F normal fail { 3, 8, 12, 14, 15, 16, 17, 18, 19, 20, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	39	50	70	25	28	71	65	41	42
N.S.	1	0.93	1.19	1.67	0.60	0.67	1.69	1.55	0.98	1.00
time (sec)	N/A	0.201	0.055	0.166	0.024	0.069	0.090	0.155	0.180	0.300

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	76	63	57	71	47	121	100	56	39
N.S.	1	0.93	0.77	0.70	0.87	0.57	1.48	1.22	0.68	0.48
time (sec)	N/A	0.228	0.081	0.335	0.034	0.068	0.155	0.164	0.197	16.008

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	102	112	283	98	59	167	142	21	110
N.S.	1	1.11	1.22	3.08	1.07	0.64	1.82	1.54	0.23	1.20
time (sec)	N/A	0.302	0.126	0.794	0.035	0.070	0.191	0.117	0.193	15.544

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	139	124	68	124	71	202	172	85	155
N.S.	1	1.02	0.91	0.50	0.91	0.52	1.49	1.26	0.62	1.14
time (sec)	N/A	0.317	0.132	1.150	0.040	0.072	0.245	0.172	0.205	16.291

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	53	49	32	32	32	83	87	32	31
N.S.	1	0.93	0.86	0.56	0.56	0.56	1.46	1.53	0.56	0.54
time (sec)	N/A	0.183	0.052	0.151	0.024	0.069	0.131	0.104	0.185	0.299

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	65	67	174	82	43	119	104	74	49
N.S.	1	0.84	0.87	2.26	1.06	0.56	1.55	1.35	0.96	0.64
time (sec)	N/A	0.280	0.087	0.342	0.033	0.074	0.129	0.135	0.182	16.416

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	120	99	58	117	62	177	170	84	129
N.S.	1	1.06	0.88	0.51	1.04	0.55	1.57	1.50	0.74	1.14
time (sec)	N/A	0.285	0.171	0.657	0.040	0.070	0.229	0.162	0.186	16.597

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	127	132	406	143	80	218	184	24	152
N.S.	1	0.93	0.96	2.96	1.04	0.58	1.59	1.34	0.18	1.11
time (sec)	N/A	0.387	0.145	1.656	0.037	0.078	0.214	0.161	0.189	15.676

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	57	53	34	33	32	95	85	34	41
N.S.	1	0.88	0.82	0.52	0.51	0.49	1.46	1.31	0.52	0.63
time (sec)	N/A	0.178	0.067	0.161	0.027	0.070	0.184	0.133	0.187	0.258

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	88	70	42	88	53	148	114	45	41
N.S.	1	0.85	0.68	0.41	0.85	0.51	1.44	1.11	0.44	0.40
time (sec)	N/A	0.230	0.104	0.344	0.036	0.074	0.254	0.128	0.181	15.496

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	128	108	57	122	67	201	170	61	133
N.S.	1	0.99	0.84	0.44	0.95	0.52	1.56	1.32	0.47	1.03
time (sec)	N/A	0.281	0.165	0.656	0.038	0.078	0.375	0.170	0.191	15.955

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	159	134	65	148	81	246	194	22	169
N.S.	1	0.93	0.78	0.38	0.87	0.47	1.44	1.13	0.13	0.99
time (sec)	N/A	0.339	0.172	1.159	0.038	0.075	0.444	0.170	0.176	15.807

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	73	48	49	194	49	286	634	49	50
N.S.	1	1.49	0.98	1.00	3.96	1.00	5.84	12.94	1.00	1.02
time (sec)	N/A	0.203	0.090	0.183	0.044	0.070	0.604	0.196	0.185	15.641

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	128	86	95	356	89	740	915	23	95
N.S.	1	1.25	0.84	0.93	3.49	0.87	7.25	8.97	0.23	0.93
time (sec)	N/A	0.289	0.142	0.422	0.046	0.073	1.099	0.208	0.182	16.029

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	184	154	143	813	171	1773	1275	23	190
N.S.	1	1.42	1.18	1.10	6.25	1.32	13.64	9.81	0.18	1.46
time (sec)	N/A	0.358	0.491	0.986	0.075	0.084	3.560	0.220	0.185	16.124

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	239	198	189	1046	233	2538	1554	23	223
N.S.	1	1.23	1.02	0.97	5.36	1.19	13.02	7.97	0.12	1.14
time (sec)	N/A	0.470	1.028	1.479	0.093	0.091	25.626	0.262	0.182	16.095

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	107	104	0	0	0	0	0	59	0
N.S.	1	1.27	1.24	0.00	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.545	6.879	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	109	112	0	0	0	0	0	69	0
N.S.	1	0.99	1.02	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.479	0.048	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	162	75	0	0	111	0	550	170	0
N.S.	1	1.69	0.78	0.00	0.00	1.16	0.00	5.73	1.77	0.00
time (sec)	N/A	0.485	0.108	0.000	0.000	0.078	0.000	0.613	0.194	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	162	87	0	0	111	0	550	178	0
N.S.	1	1.74	0.94	0.00	0.00	1.19	0.00	5.91	1.91	0.00
time (sec)	N/A	0.477	0.160	0.000	0.000	0.078	0.000	0.579	0.189	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	39	49	71	25	28	66	64	43	44
N.S.	1	0.93	1.17	1.69	0.60	0.67	1.57	1.52	1.02	1.05
time (sec)	N/A	0.199	0.050	0.180	0.036	0.073	0.101	0.151	0.188	15.241

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	76	63	61	71	47	121	100	59	39
N.S.	1	0.93	0.77	0.74	0.87	0.57	1.48	1.22	0.72	0.48
time (sec)	N/A	0.229	0.081	0.355	0.033	0.068	0.168	0.144	0.185	15.957

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	102	112	76	96	61	162	136	71	112
N.S.	1	1.06	1.17	0.79	1.00	0.64	1.69	1.42	0.74	1.17
time (sec)	N/A	0.298	0.122	0.792	0.035	0.072	0.198	0.139	0.179	16.850

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	139	124	106	124	71	202	172	85	155
N.S.	1	1.02	0.91	0.78	0.91	0.52	1.49	1.26	0.62	1.14
time (sec)	N/A	0.319	0.130	1.404	0.035	0.070	0.272	0.132	0.182	16.977

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	53	49	32	32	34	85	84	34	31
N.S.	1	0.87	0.80	0.52	0.52	0.56	1.39	1.38	0.56	0.51
time (sec)	N/A	0.186	0.052	0.152	0.024	0.073	0.141	0.154	0.176	0.295

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	65	67	61	82	43	121	104	64	49
N.S.	1	0.84	0.87	0.79	1.06	0.56	1.57	1.35	0.83	0.64
time (sec)	N/A	0.273	0.086	0.357	0.035	0.080	0.160	0.174	0.176	15.698

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	120	105	94	117	64	177	163	89	133
N.S.	1	0.99	0.87	0.78	0.97	0.53	1.46	1.35	0.74	1.10
time (sec)	N/A	0.279	0.125	0.777	0.034	0.100	0.266	0.136	0.195	0.587

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	127	132	92	142	80	218	184	91	152
N.S.	1	0.93	0.96	0.67	1.04	0.58	1.59	1.34	0.66	1.11
time (sec)	N/A	0.384	0.141	1.435	0.036	0.073	0.247	0.130	0.189	15.596

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	57	53	34	33	28	97	82	34	42
N.S.	1	0.88	0.82	0.52	0.51	0.43	1.49	1.26	0.52	0.65
time (sec)	N/A	0.181	0.066	0.160	0.025	0.070	0.250	0.141	0.189	0.227

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	88	70	42	88	43	148	114	45	41
N.S.	1	0.91	0.72	0.43	0.91	0.44	1.53	1.18	0.46	0.42
time (sec)	N/A	0.224	0.109	0.346	0.033	0.070	0.356	0.153	0.184	15.673

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	128	112	57	122	54	201	162	61	73
N.S.	1	0.99	0.87	0.44	0.95	0.42	1.56	1.26	0.47	0.57
time (sec)	N/A	0.273	0.143	0.767	0.035	0.074	0.571	0.132	0.190	15.618

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	159	134	65	148	65	246	194	72	89
N.S.	1	0.99	0.83	0.40	0.92	0.40	1.53	1.20	0.45	0.55
time (sec)	N/A	0.326	0.198	1.432	0.037	0.069	0.768	0.128	0.214	15.891

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	72	47	48	192	48	347	631	48	48
N.S.	1	1.53	1.00	1.02	4.09	1.02	7.38	13.43	1.02	1.02
time (sec)	N/A	0.194	0.079	0.170	0.042	0.072	0.634	0.142	0.181	15.789

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	128	85	94	356	78	743	915	23	98
N.S.	1	1.28	0.85	0.94	3.56	0.78	7.43	9.15	0.23	0.98
time (sec)	N/A	0.273	0.134	0.437	0.045	0.078	1.172	0.158	0.192	16.044

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	183	155	144	813	142	1579	1271	23	191
N.S.	1	1.44	1.22	1.13	6.40	1.12	12.43	10.01	0.18	1.50
time (sec)	N/A	0.343	0.437	0.990	0.073	0.086	3.961	0.167	0.178	16.315

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	239	233	188	1046	180	2538	1554	23	224
N.S.	1	1.24	1.21	0.98	5.45	0.94	13.22	8.09	0.12	1.17
time (sec)	N/A	0.465	0.485	1.720	0.091	0.089	27.698	0.253	0.184	16.391

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	107	106	0	0	0	0	0	59	0
N.S.	1	1.27	1.26	0.00	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.414	6.425	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	109	112	0	0	0	0	0	68	0
N.S.	1	0.99	1.02	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.454	0.049	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	162	74	0	0	106	0	525	169	0
N.S.	1	1.74	0.80	0.00	0.00	1.14	0.00	5.65	1.82	0.00
time (sec)	N/A	0.479	0.106	0.000	0.000	0.081	0.000	0.662	0.184	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	162	87	0	0	106	0	525	174	0
N.S.	1	1.74	0.94	0.00	0.00	1.14	0.00	5.65	1.87	0.00
time (sec)	N/A	0.472	0.150	0.000	0.000	0.078	0.000	0.764	0.176	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	53	47	37	329	54	53	54	19	139
N.S.	1	1.06	0.94	0.74	6.58	1.08	1.06	1.08	0.38	2.78
time (sec)	N/A	0.248	0.072	0.075	0.152	0.077	0.190	0.183	0.172	17.730

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	81	129	66	472	99	92	134	21	174
N.S.	1	0.86	1.37	0.70	5.02	1.05	0.98	1.43	0.22	1.85
time (sec)	N/A	0.326	0.219	0.102	0.156	0.086	0.284	0.530	0.161	17.165

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	117	123	78	617	157	122	199	21	0
N.S.	1	0.94	0.98	0.62	4.94	1.26	0.98	1.59	0.17	0.00
time (sec)	N/A	0.402	0.320	0.143	0.165	0.084	0.282	0.945	0.185	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	149	231	99	763	197	184	486	21	0
N.S.	1	0.84	1.30	0.56	4.29	1.11	1.03	2.73	0.12	0.00
time (sec)	N/A	0.499	0.322	0.217	0.174	0.086	0.327	1.426	0.166	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	139	42	144	39	60	39	22	52
N.S.	1	1.00	2.40	0.72	2.48	0.67	1.03	0.67	0.38	0.90
time (sec)	N/A	0.278	0.139	0.062	0.039	0.077	0.167	0.122	0.180	0.388

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	87	201	74	355	89	95	99	24	92
N.S.	1	0.91	2.09	0.77	3.70	0.93	0.99	1.03	0.25	0.96
time (sec)	N/A	0.360	0.262	0.089	0.048	0.076	0.269	0.524	0.176	16.180

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	124	241	90	535	125	133	150	24	0
N.S.	1	1.02	1.99	0.74	4.42	1.03	1.10	1.24	0.20	0.00
time (sec)	N/A	0.441	0.290	0.136	0.061	0.076	0.263	0.898	0.190	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	155	313	111	718	169	184	201	24	0
N.S.	1	0.95	1.91	0.68	4.38	1.03	1.12	1.23	0.15	0.00
time (sec)	N/A	0.512	0.397	0.219	0.086	0.082	0.329	1.390	0.179	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	B	B	A	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	204	356	100	0	7602	401	80	3205	20	467
N.S.	1	1.75	0.49	0.00	37.26	1.97	0.39	15.71	0.10	2.29
time (sec)	N/A	0.861	0.108	0.000	80.967	0.088	1.349	26.985	0.177	18.557

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	B	B	A	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	535	158	0	9984	573	178	4151	22	501
N.S.	1	2.00	0.59	0.00	37.25	2.14	0.66	15.49	0.08	1.87
time (sec)	N/A	1.421	0.205	0.000	82.585	0.092	0.445	0.977	0.179	21.108

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	B	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	305	577	180	0	12429	731	168	1290	22	0
N.S.	1	1.89	0.59	0.00	40.75	2.40	0.55	4.23	0.07	0.00
time (sec)	N/A	1.997	0.401	0.000	81.646	0.096	1.452	1.604	0.189	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	70	0	0	0	0	0	21	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.266	0.689	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	150	106	0	0	0	0	0	23	0
N.S.	1	1.34	0.95	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.340	1.291	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	218	175	0	0	0	0	0	23	0
N.S.	1	1.27	1.02	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.418	1.194	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	277	240	0	0	0	0	0	23	0
N.S.	1	1.18	1.02	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.480	6.928	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	79	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	0	0	0	0	0	0	0	25	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	53	46	36	188	87	58	58	19	132
N.S.	1	1.08	0.94	0.73	3.84	1.78	1.18	1.18	0.39	2.69
time (sec)	N/A	0.261	0.069	0.085	0.043	0.081	0.209	0.170	0.185	16.630

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	83	128	66	423	141	85	127	21	182
N.S.	1	0.86	1.33	0.69	4.41	1.47	0.89	1.32	0.22	1.90
time (sec)	N/A	0.329	0.257	0.086	0.049	0.086	0.240	0.148	0.182	16.384

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	121	124	80	658	184	134	211	21	0
N.S.	1	0.93	0.95	0.62	5.06	1.42	1.03	1.62	0.16	0.00
time (sec)	N/A	0.415	0.340	0.124	0.064	0.087	0.325	0.198	0.185	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	155	167	99	895	240	175	270	21	0
N.S.	1	0.84	0.91	0.54	4.86	1.30	0.95	1.47	0.11	0.00
time (sec)	N/A	0.520	0.527	0.170	0.087	0.082	0.322	0.276	0.184	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	138	42	223	37	56	37	22	53
N.S.	1	1.00	2.30	0.70	3.72	0.62	0.93	0.62	0.37	0.88
time (sec)	N/A	0.285	0.142	0.050	0.042	0.069	0.216	0.154	0.172	0.380

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	91	215	74	569	91	95	99	24	95
N.S.	1	0.91	2.15	0.74	5.69	0.91	0.95	0.99	0.24	0.95
time (sec)	N/A	0.374	0.311	0.085	0.059	0.077	0.218	0.174	0.174	16.156

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	130	243	91	869	123	134	148	24	0
N.S.	1	1.02	1.91	0.72	6.84	0.97	1.06	1.17	0.19	0.00
time (sec)	N/A	0.467	0.300	0.118	0.072	0.072	0.285	0.192	0.176	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	163	294	111	1177	171	184	201	24	0
N.S.	1	0.95	1.71	0.65	6.84	0.99	1.07	1.17	0.14	0.00
time (sec)	N/A	0.560	0.577	0.161	0.107	0.077	0.301	0.269	0.175	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	B	B	A	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	208	335	103	0	3081	358	121	49619	20	449
N.S.	1	1.61	0.50	0.00	14.81	1.72	0.58	238.55	0.10	2.16
time (sec)	N/A	0.701	0.107	0.000	0.433	0.085	0.510	50.189	0.183	19.834

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	B	B	A	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	272	505	161	0	5024	563	126	2700	22	521
N.S.	1	1.86	0.59	0.00	18.47	2.07	0.46	9.93	0.08	1.92
time (sec)	N/A	1.151	0.197	0.000	0.508	0.095	1.812	4.123	0.185	20.952

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	B	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	313	553	182	0	6962	614	223	5078	22	0
N.S.	1	1.77	0.58	0.00	22.24	1.96	0.71	16.22	0.07	0.00
time (sec)	N/A	1.623	0.414	0.000	0.685	0.099	0.536	7.412	0.187	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	0	0	0	0	0	21	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.269	0.753	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	146	105	0	0	0	0	0	23	0
N.S.	1	1.30	0.94	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.334	1.409	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	212	173	0	0	0	0	0	23	0
N.S.	1	1.22	0.99	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.415	1.421	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	269	192	0	0	0	0	0	23	0
N.S.	1	1.13	0.80	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.480	2.303	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	21	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	0	0	0	0	0	0	0	25	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	26	107	23	0	23	293	36
N.S.	1	1.00	1.00	0.87	3.57	0.77	0.00	0.77	9.77	1.20
time (sec)	N/A	0.180	0.011	0.108	0.038	0.070	0.000	0.151	0.204	0.360

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	79	56	52	446	87	0	120	0	164
N.S.	1	1.14	0.81	0.75	6.46	1.26	0.00	1.74	0.00	2.38
time (sec)	N/A	0.208	0.064	0.310	0.164	0.079	0.000	0.138	0.221	17.082

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	55	37	31	90	50	0	49	104	0
N.S.	1	1.38	0.92	0.78	2.25	1.25	0.00	1.22	2.60	0.00
time (sec)	N/A	0.194	0.016	0.602	0.038	0.067	0.000	0.143	0.201	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	138	86	85	736	185	0	272	0	0
N.S.	1	0.91	0.57	0.56	4.87	1.23	0.00	1.80	0.00	0.00
time (sec)	N/A	0.305	0.136	2.083	0.187	0.082	0.000	0.154	0.222	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	118	52	50	166	86	0	83	166	0
N.S.	1	1.23	0.54	0.52	1.73	0.90	0.00	0.86	1.73	0.00
time (sec)	N/A	0.293	0.035	2.670	0.046	0.072	0.000	0.159	0.183	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	66	44	48	358	63	0	67	304	138
N.S.	1	1.12	0.75	0.81	6.07	1.07	0.00	1.14	5.15	2.34
time (sec)	N/A	0.196	0.020	0.132	0.158	0.079	0.000	0.114	0.185	18.116

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	114	75	58	293	63	0	74	0	76
N.S.	1	1.73	1.14	0.88	4.44	0.95	0.00	1.12	0.00	1.15
time (sec)	N/A	0.235	0.026	0.308	0.044	0.073	0.000	0.140	0.202	16.449

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	123	80	101	735	150	0	212	0	0
N.S.	1	1.05	0.68	0.86	6.28	1.28	0.00	1.81	0.00	0.00
time (sec)	N/A	0.294	0.125	0.856	0.164	0.080	0.000	0.121	0.223	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	63	41	34	156	77	0	76	114	0
N.S.	1	1.40	0.91	0.76	3.47	1.71	0.00	1.69	2.53	0.00
time (sec)	N/A	0.201	0.014	1.323	0.040	0.066	0.000	0.150	0.206	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	190	106	136	1095	258	0	374	0	0
N.S.	1	0.90	0.50	0.64	5.19	1.22	0.00	1.77	0.00	0.00
time (sec)	N/A	0.400	0.209	4.788	0.200	0.080	0.000	0.155	0.251	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	B	A	F	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	54	51	0	6613	213	0	2578	161	264
N.S.	1	0.32	0.30	0.00	38.67	1.25	0.00	15.08	0.94	1.54
time (sec)	N/A	0.195	0.016	0.000	82.651	0.101	0.000	1.692	0.194	19.702

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	B	B	F	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	54	51	0	9954	559	0	880	0	485
N.S.	1	0.23	0.22	0.00	42.54	2.39	0.00	3.76	0.00	2.07
time (sec)	N/A	0.192	0.012	0.000	82.278	0.091	0.000	0.307	0.252	20.832

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	117	77	0	11528	385	0	610	0	0
N.S.	1	0.49	0.32	0.00	48.23	1.61	0.00	2.55	0.00	0.00
time (sec)	N/A	0.283	0.067	0.000	82.515	0.082	0.000	0.281	0.228	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	121	79	0	14770	875	0	1418	0	0
N.S.	1	0.36	0.24	0.00	43.96	2.60	0.00	4.22	0.00	0.00
time (sec)	N/A	0.299	0.070	0.000	82.673	0.106	0.000	0.402	0.252	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	337	188	100	0	16387	569	0	880	0	0
N.S.	1	0.56	0.30	0.00	48.63	1.69	0.00	2.61	0.00	0.00
time (sec)	N/A	0.409	0.196	0.000	87.293	0.091	0.000	0.330	0.273	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	0	0	0	21	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.208	0.019	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	80	80	0	0	0	0	0	23	0
N.S.	1	0.95	0.95	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.210	0.014	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	154	112	0	0	0	0	0	23	0
N.S.	1	1.83	1.33	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.321	0.187	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	95	74	0	0	0	0	0	21	0
N.S.	1	1.28	1.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.341	0.061	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	128	104	0	0	0	0	0	25	0
N.S.	1	1.29	1.05	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.498	0.074	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	180	79	0	0	131	0	0	40	0
N.S.	1	1.28	0.56	0.00	0.00	0.93	0.00	0.00	0.28	0.00
time (sec)	N/A	0.491	0.232	0.000	0.000	0.084	0.000	0.000	0.204	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	180	74	0	0	131	0	0	41	0
N.S.	1	1.28	0.52	0.00	0.00	0.93	0.00	0.00	0.29	0.00
time (sec)	N/A	0.482	0.148	0.000	0.000	0.100	0.000	0.000	0.207	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	75	24	170	22	0	22	19	35
N.S.	1	1.00	2.59	0.83	5.86	0.76	0.00	0.76	0.66	1.21
time (sec)	N/A	0.178	0.084	0.053	0.044	0.103	0.000	0.132	0.188	0.364

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	81	108	52	397	96	0	115	21	169
N.S.	1	1.14	1.52	0.73	5.59	1.35	0.00	1.62	0.30	2.38
time (sec)	N/A	0.211	0.144	0.215	0.049	0.079	0.000	0.119	0.182	15.921

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	55	78	31	92	50	0	49	36	0
N.S.	1	1.38	1.95	0.78	2.30	1.25	0.00	1.22	0.90	0.00
time (sec)	N/A	0.200	0.091	0.380	0.038	0.069	0.000	0.188	0.188	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	140	137	85	870	195	0	258	21	0
N.S.	1	0.89	0.87	0.54	5.54	1.24	0.00	1.64	0.13	0.00
time (sec)	N/A	0.314	0.379	1.480	0.083	0.079	0.000	0.177	0.187	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	118	103	49	168	84	0	81	66	0
N.S.	1	1.23	1.07	0.51	1.75	0.88	0.00	0.84	0.69	0.00
time (sec)	N/A	0.284	0.123	1.826	0.045	0.069	0.000	0.129	0.180	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	64	249	46	232	63	0	67	22	139
N.S.	1	1.16	4.53	0.84	4.22	1.15	0.00	1.22	0.40	2.53
time (sec)	N/A	0.195	0.181	0.066	0.043	0.087	0.000	0.111	0.175	17.420

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	121	183	58	507	65	0	74	24	79
N.S.	1	1.73	2.61	0.83	7.24	0.93	0.00	1.06	0.34	1.13
time (sec)	N/A	0.242	0.489	0.197	0.047	0.074	0.000	0.124	0.178	15.630

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	121	387	98	902	150	0	212	24	0
N.S.	1	1.05	3.37	0.85	7.84	1.30	0.00	1.84	0.21	0.00
time (sec)	N/A	0.287	0.791	0.583	0.071	0.076	0.000	0.165	0.181	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	63	105	34	156	79	0	76	39	0
N.S.	1	1.34	2.23	0.72	3.32	1.68	0.00	1.62	0.83	0.00
time (sec)	N/A	0.199	0.127	0.858	0.040	0.070	0.000	0.207	0.176	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	188	493	133	1510	258	0	374	24	0
N.S.	1	0.90	2.36	0.64	7.22	1.23	0.00	1.79	0.11	0.00
time (sec)	N/A	0.421	0.969	3.665	0.114	0.078	0.000	0.149	0.178	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	B	A	F	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	50	214	0	3086	215	0	2572	20	242
N.S.	1	0.31	1.32	0.00	19.05	1.33	0.00	15.88	0.12	1.49
time (sec)	N/A	0.186	0.162	0.000	0.434	0.081	0.000	1.778	0.165	18.925

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	B	B	F	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	52	142	0	4994	565	0	2675	22	505
N.S.	1	0.22	0.60	0.00	20.98	2.37	0.00	11.24	0.09	2.12
time (sec)	N/A	0.188	0.195	0.000	0.491	0.091	0.000	4.159	0.168	21.233

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	233	113	305	0	7558	387	0	604	22	0
N.S.	1	0.48	1.31	0.00	32.44	1.66	0.00	2.59	0.09	0.00
time (sec)	N/A	0.284	0.302	0.000	0.872	0.084	0.000	1.288	0.174	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	346	119	255	0	8894	881	0	3636	22	0
N.S.	1	0.34	0.74	0.00	25.71	2.55	0.00	10.51	0.06	0.00
time (sec)	N/A	0.288	0.289	0.000	1.045	0.104	0.000	4.486	0.174	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	331	184	415	0	11459	571	0	874	22	0
N.S.	1	0.56	1.25	0.00	34.62	1.73	0.00	2.64	0.07	0.00
time (sec)	N/A	0.406	0.481	0.000	2.551	0.088	0.000	1.289	0.177	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	114	0	0	0	0	0	21	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.207	2.270	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	78	101	0	0	0	0	0	23	0
N.S.	1	0.95	1.23	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.213	0.521	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	152	334	0	0	0	0	0	23	0
N.S.	1	1.83	4.02	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.330	11.079	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	99	76	0	0	0	0	0	21	0
N.S.	1	1.34	1.03	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.368	0.077	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	130	104	0	0	0	0	0	25	0
N.S.	1	1.31	1.05	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.543	0.092	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	180	81	0	0	133	0	0	40	0
N.S.	1	1.28	0.57	0.00	0.00	0.94	0.00	0.00	0.28	0.00
time (sec)	N/A	0.492	0.188	0.000	0.000	0.080	0.000	0.000	0.185	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	180	74	0	0	133	0	0	44	0
N.S.	1	1.28	0.52	0.00	0.00	0.94	0.00	0.00	0.31	0.00
time (sec)	N/A	0.499	0.170	0.000	0.000	0.078	0.000	0.000	0.189	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [17] had the largest ratio of [.2105259999999999991]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	0.93	17	0.118
2	A	2	2	0.93	19	0.105
3	A	3	3	1.11	19	0.158
4	A	3	3	1.02	19	0.158
5	A	1	1	0.93	19	0.053
6	A	3	3	0.84	21	0.143
7	A	2	2	1.06	21	0.095
8	A	4	4	0.93	21	0.190
9	A	1	1	0.88	21	0.048
10	A	2	2	0.85	23	0.087
11	A	2	2	0.99	23	0.087
12	A	3	3	0.93	23	0.130
13	A	1	1	1.49	16	0.062
14	A	2	2	1.25	18	0.111
15	A	2	2	1.42	18	0.111
16	A	3	3	1.23	18	0.167
17	A	4	4	1.27	19	0.211
18	A	3	3	0.99	20	0.150
19	A	2	2	1.69	31	0.065
20	A	2	2	1.74	31	0.065
21	A	2	2	0.93	17	0.118

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	0.93	19	0.105
23	A	3	3	1.06	19	0.158
24	A	3	3	1.02	19	0.158
25	A	1	1	0.87	19	0.053
26	A	3	3	0.84	21	0.143
27	A	2	2	0.99	21	0.095
28	A	4	4	0.93	21	0.190
29	A	1	1	0.88	21	0.048
30	A	2	2	0.91	23	0.087
31	A	2	2	0.99	23	0.087
32	A	3	3	0.99	23	0.130
33	A	1	1	1.53	16	0.062
34	A	2	2	1.28	18	0.111
35	A	2	2	1.44	18	0.111
36	A	3	3	1.24	18	0.167
37	A	3	3	1.27	19	0.158
38	A	3	3	0.99	20	0.150
39	A	2	2	1.74	31	0.065
40	A	2	2	1.74	31	0.065
41	A	2	2	1.06	17	0.118
42	A	2	2	0.86	19	0.105
43	A	2	2	0.94	19	0.105
44	A	2	2	0.84	19	0.105
45	A	2	2	1.00	19	0.105
46	A	2	2	0.91	21	0.095
47	A	2	2	1.02	21	0.095
48	A	2	2	0.95	21	0.095
49	A	2	2	1.75	21	0.095
50	A	2	2	2.00	23	0.087
51	A	2	2	1.89	23	0.087
52	A	2	2	1.00	16	0.125
53	A	2	2	1.34	18	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.27	18	0.111
55	A	2	2	1.18	18	0.111
56	F	0	0	N/A	0.000	N/A
57	F	0	0	N/A	0.000	N/A
58	A	2	2	1.08	17	0.118
59	A	2	2	0.86	19	0.105
60	A	2	2	0.93	19	0.105
61	A	2	2	0.84	19	0.105
62	A	2	2	1.00	19	0.105
63	A	2	2	0.91	21	0.095
64	A	2	2	1.02	21	0.095
65	A	2	2	0.95	21	0.095
66	A	2	2	1.61	21	0.095
67	A	2	2	1.86	23	0.087
68	A	2	2	1.77	23	0.087
69	A	2	2	1.00	16	0.125
70	A	2	2	1.30	18	0.111
71	A	2	2	1.22	18	0.111
72	A	2	2	1.13	18	0.111
73	F	0	0	N/A	0.000	N/A
74	F	0	0	N/A	0.000	N/A
75	A	1	1	1.00	17	0.059
76	A	1	1	1.14	19	0.053
77	A	1	1	1.38	19	0.053
78	A	2	2	0.91	19	0.105
79	A	2	2	1.23	19	0.105
80	A	1	1	1.12	19	0.053
81	A	1	1	1.73	21	0.048
82	A	2	2	1.05	21	0.095
83	A	1	1	1.40	21	0.048
84	A	3	3	0.90	21	0.143
85	C	1	1	0.32	21	0.048

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	C	1	1	0.23	23	0.043
87	C	2	2	0.49	23	0.087
88	C	2	2	0.36	23	0.087
89	C	3	3	0.56	23	0.130
90	A	1	1	1.00	16	0.062
91	A	1	1	0.95	18	0.056
92	A	2	2	1.83	18	0.111
93	A	2	2	1.28	19	0.105
94	A	3	3	1.29	20	0.150
95	A	2	2	1.28	31	0.065
96	A	2	2	1.28	31	0.065
97	A	1	1	1.00	17	0.059
98	A	1	1	1.14	19	0.053
99	A	1	1	1.38	19	0.053
100	A	2	2	0.89	19	0.105
101	A	2	2	1.23	19	0.105
102	A	1	1	1.16	19	0.053
103	A	1	1	1.73	21	0.048
104	A	2	2	1.05	21	0.095
105	A	1	1	1.34	21	0.048
106	A	3	3	0.90	21	0.143
107	C	1	1	0.31	21	0.048
108	C	1	1	0.22	23	0.043
109	C	2	2	0.48	23	0.087
110	C	2	2	0.34	23	0.087
111	C	3	3	0.56	23	0.130
112	A	1	1	1.00	16	0.062
113	A	1	1	0.95	18	0.056
114	A	2	2	1.83	18	0.111
115	A	2	2	1.34	19	0.105
116	A	3	3	1.31	20	0.150
117	A	2	2	1.28	31	0.065

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.28	31	0.065

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int e^{a+ibx}\sin(d+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int e^{a+ibx}\sin(d+bx)\,dx=-\frac{e^{a-id+2i(d+bx)}}{4b}+\frac{1}{2}ie^{a-id}x$$

output
```
-1/4*exp(a-I*d+2*I*(b*x+d))/b+1/2*I*exp(a-I*d)*x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int e^{a+ibx}\sin(d+bx)\,dx=\frac{e^a\left(-\left(\left(e^{2ibx}-2ibx\right)\cos(d)\right)+\left(-ie^{2ibx}+2bx\right)\sin(d)\right)}{4b}$$

input
```
Integrate[E^(a + I*b*x)*Sin[d + b*x],x]
```

output
```
(E^a*(-((E^((2*I)*b*x) - (2*I)*b*x)*Cos[d]) + ((-I)*E^((2*I)*b*x) + 2*b*x)
*Sin[d]))/(4*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\sin(bx+d)\,dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(\frac{1}{2}ie^{a-id}-\frac{1}{2}ie^{a+2ibx+id}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}ixe^{a-id}-\frac{e^{a+2ibx+id}}{4b}$$

input
```
Int[E^(a + I*b*x)*Sin[d + b*x],x]
```

output
```
-1/4*E^(a + I*d + (2*I)*b*x)/b + (I/2)*E^(a - I*d)*x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(32) = 64.

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

method	result	size
orering	$-\frac{(-2bx+i)\mathrm{e}^{ibx+a}\sin(bx+d)}{2b}+\frac{ix\left(ib\,\mathrm{e}^{ibx+a}\sin(bx+d)+\mathrm{e}^{ibx+a}b\cos(bx+d)\right)}{2b}$	70
norman	$\frac{x\,\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)-\frac{i\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{b}+\frac{ix\,\mathrm{e}^{ibx+a}}{2}-\frac{ix\,\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}}{2}}{1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}}$	95

input
```
int(exp(a+I*b*x)*sin(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*b*x+I)/b*exp(a+I*b*x)*sin(b*x+d)+1/2*I/b*x*(I*b*exp(a+I*b*x)*sin(
b*x+d)+exp(a+I*b*x)*b*cos(b*x+d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int e^{a+ibx}\sin(d+bx)\,dx=\frac{2i\,bxe^{(a-i\,d)}-e^{(2i\,bx+a+i\,d)}}{4\,b}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d),x, algorithm="fricas")
```

output
```
1/4*(2*I*b*x*e^(a - I*d) - e^(2*I*b*x + a + I*d))/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int e^{a+ibx}\sin(d+bx)\,dx=\frac{ixe^{a}e^{-id}}{2}+\begin{cases}-\frac{e^{a}e^{id}e^{2ibx}}{4b} & \text{for } b\neq 0\\ x\left(\frac{(-ie^{a}e^{2id}+ie^{a})e^{-id}}{2}-\frac{ie^{a}e^{-id}}{2}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d),x)
```

output
```
I*x*exp(a)*exp(-I*d)/2 + Piecewise((-exp(a)*exp(I*d)*exp(2*I*b*x)/(4*b), N
e(b, 0)), (x*((-I*exp(a)*exp(2*I*d) + I*exp(a))*exp(-I*d)/2 - I*exp(a)*exp
(-I*d)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

$$\int e^{a+ibx}\sin(d+bx)\,dx=\frac{1}{2}i\,xe^{(a-i\,d)}-\frac{e^{(2i\,bx+a+i\,d)}}{4\,b}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d),x, algorithm="maxima")
```

output
```
1/2*I*x*e^(a - I*d) - 1/4*e^(2*I*b*x + a + I*d)/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(25) = 50.

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.55

$$\int e^{a+ibx}\sin(d+bx)\,dx=-\frac{-4i\,(bx+d)\cos(d)\,e^{a}-4\,(bx+d)e^{a}\sin(d)+\left(e^{(2i\,bx+i\,d)}-e^{(-2i\,bx-i\,d)}\right)e^{a}+2\,\cos(-2\,bx-d)\,e^{a}}{8\,b}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d),x, algorithm="giac")
```

output
```
-1/8*(-4*I*(b*x + d)*cos(d)*e^a - 4*(b*x + d)*e^a*sin(d) + (e^(2*I*b*x + I
*d) - e^(-2*I*b*x - I*d))*e^a + 2*cos(-2*b*x - d)*e^a)/b
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int e^{a+ibx}\sin(d+bx)\,dx = \frac{e^a\,(2\,x\,\sin(d)+x\,\cos(d)\,2i)}{4} - \frac{e^a\,(\cos(d+2\,b\,x)+\sin(d+2\,b\,x)\,1i)}{4\,b}$$

input
```
int(exp(a + b*x*1i)*sin(d + b*x),x)
```

output
```
(exp(a)*(x*cos(d)*2i + 2*x*sin(d)))/4 - (exp(a)*(cos(d + 2*b*x) + sin(d +
2*b*x)*1i))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int e^{a+ibx}\sin(d+bx)\,dx = \frac{e^{bix+a}(\cos(bx+d)\,bix-\cos(bx+d)+\sin(bx+d)\,bx)}{2b}$$

input
```
int(exp(a+I*b*x)*sin(b*x+d),x)
```

output
```
(e**(a + b*i*x)*(cos(b*x + d)*b*i*x - cos(b*x + d) + sin(b*x + d)*b*x))/(2
*b)
```

3.2 $\int e^{a+ibx}\sin^2(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int e^{a+ibx}\sin^2(d+bx)\,dx=-\frac{ie^{a-id-i(d+bx)}}{4b}-\frac{ie^{a-id+i(d+bx)}}{2b}+\frac{ie^{a-id+3i(d+bx)}}{12b}$$

output
```
-1/4*I*exp(a-I*d-I*(b*x+d))/b-1/2*I*exp(a-I*d+I*(b*x+d))/b+1/12*I*exp(a-I*
d+3*I*(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int e^{a+ibx}\sin^2(d+bx)\,dx=\frac{ie^{a-ibx}\left(-6e^{2ibx}+\left(-3+e^{4ibx}\right)\cos(2d)+i\left(3+e^{4ibx}\right)\sin(2d)\right)}{12b}$$

input
```
Integrate[E^(a + I*b*x)*Sin[d + b*x]^2,x]
```

output
```
((I/12)*E^(a - I*b*x)*(-6*E^((2*I)*b*x) + (-3 + E^((4*I)*b*x))*Cos[2*d] +
I*(3 + E^((4*I)*b*x))*Sin[2*d]))/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4934, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx} \sin^2(bx+d) \, dx$$

$$\Big\downarrow \text{4934}$$

$$\frac{2}{3} \int e^{a+ibx} dx + \frac{ie^{a+ibx} \sin^2(bx+d)}{3b} - \frac{2e^{a+ibx} \sin(bx+d) \cos(bx+d)}{3b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{ie^{a+ibx} \sin^2(bx+d)}{3b} - \frac{2e^{a+ibx} \sin(bx+d) \cos(bx+d)}{3b} - \frac{2ie^{a+ibx}}{3b}$$

input
```
Int[E^(a + I*b*x)*Sin[d + b*x]^2,x]
```

output
```
(((-2*I)/3)*E^(a + I*b*x))/b - (2*E^(a + I*b*x)*Cos[d + b*x]*Sin[d + b*x])
/(3*b) + ((I/3)*E^(a + I*b*x)*Sin[d + b*x]^2)/b
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{(-3i+4i\cos(bx+d)-i\cos(2bx+2d)+4\sin(bx+d)-2\sin(2bx+2d))\,\mathrm{e}^{ibx+a}}{6b}$
default	$-\frac{i\mathrm{e}^{ibx+a}}{2b}-\frac{i\mathrm{e}^{ibx+a}\cos(2bx+2d)}{6b}-\frac{\mathrm{e}^{ibx+a}\sin(2bx+2d)}{3b}$
norman	$\frac{-\frac{4i\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^4}{3b}+\frac{8\,\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^3}{3b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2\right)^2}$
orering	$-\frac{i\mathrm{e}^{ibx+a}\sin(bx+d)^2}{3b}-\frac{ib\,\mathrm{e}^{ibx+a}\sin(bx+d)^2+2\,\mathrm{e}^{ibx+a}\sin(bx+d)b\cos(bx+d)}{b^2}-\frac{i\left(-3b^2\mathrm{e}^{ibx+a}\sin(bx+d)^2+4ib^2\mathrm{e}^{ibx+a}\,\mathrm{s}\right.}{}$

input
```
int(exp(a+I*b*x)*sin(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(-3*I+4*I*cos(b*x+d)-I*cos(2*b*x+2*d)+4*sin(b*x+d)-2*sin(2*b*x+2*d))*e
xp(a+I*b*x)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int e^{a+ibx}\sin^2(d+bx)\,dx=\frac{\left(i\,e^{(4i\,bx+a+3i\,d)}-6i\,e^{(2i\,bx+a+i\,d)}-3i\,e^{(a-i\,d)}\right)e^{(-i\,bx-i\,d)}}{12\,b}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/12*(I*e^(4*I*b*x + a + 3*I*d) - 6*I*e^(2*I*b*x + a + I*d) - 3*I*e^(a - I
*d))*e^(-I*b*x - I*d)/b
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(56) = 112$.

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.48

$$\int e^{a+ibx}\sin^2(d+bx)\,dx = \begin{cases} \frac{(8ib^2e^ae^{4id}e^{3ibx}-48ib^2e^ae^{2id}e^{ibx}-24ib^2e^ae^{-ibx})e^{-2id}}{96b^3} & \text{for } b^3e^{2id}\neq 0 \\ \frac{x(-e^ae^{4id}+2e^ae^{2id}-e^a)e^{-2id}}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d)**2,x)
```

output
```
Piecewise(((8*I*b**2*exp(a)*exp(4*I*d)*exp(3*I*b*x) - 48*I*b**2*exp(a)*exp
(2*I*d)*exp(I*b*x) - 24*I*b**2*exp(a)*exp(-I*b*x))*exp(-2*I*d)/(96*b**3),
Ne(b**3*exp(2*I*d), 0)), (x*(-exp(a)*exp(4*I*d) + 2*exp(a)*exp(2*I*d) - ex
p(a))*exp(-2*I*d)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int e^{a+ibx}\sin^2(d+bx)\,dx = -\frac{-i\cos(3bx+2d)e^a+3i\cos(bx+2d)e^a+6i\cos(bx)e^a+e^a\sin(3bx+2d)+3e^a\sin(bx+2d)-}{12b}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d)^2,x, algorithm="maxima")
```

output
```
-1/12*(-I*cos(3*b*x + 2*d)*e^a + 3*I*cos(b*x + 2*d)*e^a + 6*I*cos(b*x)*e^a
 + e^a*sin(3*b*x + 2*d) + 3*e^a*sin(b*x + 2*d) - 6*e^a*sin(b*x))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(43) = 86.

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int e^{a+ibx}\sin^2(d+bx)\,dx = -\frac{-i\left(e^{(3i\,bx+2i\,d)}+e^{(-3i\,bx-2i\,d)}\right)e^a+3i\left(e^{(i\,bx+2i\,d)}+e^{(-i\,bx-2i\,d)}\right)e^a+6i\left(e^{(i\,bx)}+e^{(-i\,bx)}\right)e^a+6\,e^a\sin\left(bx\right.}{24\,b}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d)^2,x, algorithm="giac")
```

output
```
-1/24*(-I*(e^(3*I*b*x + 2*I*d) + e^(-3*I*b*x - 2*I*d))*e^a + 3*I*(e^(I*b*x
 + 2*I*d) + e^(-I*b*x - 2*I*d))*e^a + 6*I*(e^(I*b*x) + e^(-I*b*x))*e^a + 6
*e^a*sin(b*x + 2*d) - 12*e^a*sin(b*x) - 2*e^a*sin(-3*b*x - 2*d))/b
```

Mupad [B] (verification not implemented)

Time = 16.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.48

$$\int e^{a+ibx}\sin^2(d+bx)\,dx = -\frac{\mathrm{e}^{a+b\,x\,1\mathrm{i}}\left(\cos\left(2\,d+2\,b\,x\right)\,1\mathrm{i}+2\,\sin\left(2\,d+2\,b\,x\right)+3\mathrm{i}\right)}{6\,b}$$

input
```
int(exp(a + b*x*1i)*sin(d + b*x)^2,x)
```

output
```
-(exp(a + b*x*1i)*(cos(2*d + 2*b*x)*1i + 2*sin(2*d + 2*b*x) + 3i))/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

$$\int e^{a+ibx}\sin^2(d+bx)\,dx=\frac{4e^{bix+a}\left(-2\tan\left(\frac{bx}{2}+\frac{d}{2}\right)-i\right)}{3b\left(\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^4+2\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2+1\right)}$$

input
```
int(exp(a+I*b*x)*sin(b*x+d)^2,x)
```

output
```
(4*e**(a + b*i*x)*( - 2*tan((b*x + d)/2) - i))/(3*b*(tan((b*x + d)/2)**4 +
 2*tan((b*x + d)/2)**2 + 1))
```

3.3 $\int e^{a+ibx}\sin^3(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 92

$$\int e^{a+ibx}\sin^3(d+bx)\,dx=\frac{e^{a-id-2i(d+bx)}}{16b}-\frac{3e^{a-id+2i(d+bx)}}{16b}+\frac{e^{a-id+4i(d+bx)}}{32b}+\frac{3}{8}ie^{a-id}x$$

output
```
1/16*exp(a-I*d-2*I*(b*x+d))/b-3/16*exp(a-I*d+2*I*(b*x+d))/b+1/32*exp(a-I*d
+4*I*(b*x+d))/b+3/8*I*exp(a-I*d)*x
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.22

$$\int e^{a+ibx}\sin^3(d+bx)\,dx$$
$$=\frac{e^{a-2ibx}\left(-6e^{2ibx}\left(e^{2ibx}-2ibx\right)\cos(d)+\left(2+e^{6ibx}\right)\cos(3d)-6ie^{4ibx}\sin(d)+12be^{2ibx}x\sin(d)-2i\sin(3d)}{32b}$$

input
```
Integrate[E^(a + I*b*x)*Sin[d + b*x]^3,x]
```

output
```
(E^(a - (2*I)*b*x)*(-6*E^((2*I)*b*x)*(E^((2*I)*b*x) - (2*I)*b*x)*Cos[d] +
(2 + E^((6*I)*b*x))*Cos[3*d] - (6*I)*E^((4*I)*b*x)*Sin[d] + 12*b*E^((2*I)*
b*x)*x*Sin[d] - (2*I)*Sin[3*d] + I*E^((6*I)*b*x)*Sin[3*d]))/(32*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4934, 4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\sin^3(bx+d)\,dx$$

$$\Big\downarrow \text{ 4934}$$

$$\frac{3}{4}\int e^{a+ibx}\sin(d+bx)dx+\frac{ie^{a+ibx}\sin^3(bx+d)}{8b}-\frac{3e^{a+ibx}\sin^2(bx+d)\cos(bx+d)}{8b}$$

$$\Big\downarrow \text{ 4975}$$

$$\frac{3}{4}\int\left(\frac{1}{2}ie^{a-id}-\frac{1}{2}ie^{a+id+2ibx}\right)dx+\frac{ie^{a+ibx}\sin^3(bx+d)}{8b}-\frac{3e^{a+ibx}\sin^2(bx+d)\cos(bx+d)}{8b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{4}\left(\frac{1}{2}ixe^{a-id}-\frac{e^{a+2ibx+id}}{4b}\right)+\frac{ie^{a+ibx}\sin^3(bx+d)}{8b}-\frac{3e^{a+ibx}\sin^2(bx+d)\cos(bx+d)}{8b}$$

input
```
Int[E^(a + I*b*x)*Sin[d + b*x]^3,x]
```

output
```
(3*(-1/4*E^(a + I*d + (2*I)*b*x)/b + (I/2)*E^(a - I*d)*x))/4 - (3*E^(a + I
*b*x)*Cos[d + b*x]*Sin[d + b*x]^2)/(8*b) + ((I/8)*E^(a + I*b*x)*Sin[d + b*
x]^3)/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(72) = 144$.

Time = 0.79 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.08

method	result
orering	$-\frac{(-4bx+i)e^{ibx+a}\sin(bx+d)^3}{4b}+\frac{i(bx+i)\left(ib\,e^{ibx+a}\sin(bx+d)^3+3\,e^{ibx+a}\sin(bx+d)^2 b\cos(bx+d)\right)}{4b^2}-\frac{(-4bx+i)\left(-4\sin(bx+d)^3\right.}{}$
norman	$\frac{\frac{3x\,e^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{4}+\frac{3x\,e^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^3}{2}+\frac{3x\,e^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^5}{4}+\frac{3ix\,e^{ibx+a}}{8}-\frac{3\,e^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2}{2b}+\frac{3\,e^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^4}{2b}+}{\left(1+\right.}$

input
```
int(exp(a+I*b*x)*sin(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-4*b*x+I)/b*exp(a+I*b*x)*sin(b*x+d)^3+1/4*I*(b*x+I)/b^2*(I*b*exp(a+I
*b*x)*sin(b*x+d)^3+3*exp(a+I*b*x)*sin(b*x+d)^2*b*cos(b*x+d))-1/16*(-4*b*x+
I)/b^3*(-4*sin(b*x+d)^3*exp(a+I*b*x)*b^2+6*I*sin(b*x+d)^2*cos(b*x+d)*exp(a
+I*b*x)*b^2+6*sin(b*x+d)*cos(b*x+d)^2*exp(a+I*b*x)*b^2)+1/16*I/b^3*x*(-30*
sin(b*x+d)^2*cos(b*x+d)*exp(a+I*b*x)*b^3-10*I*sin(b*x+d)^3*exp(a+I*b*x)*b^
3+18*I*sin(b*x+d)*cos(b*x+d)^2*exp(a+I*b*x)*b^3+6*cos(b*x+d)^3*exp(a+I*b*x
)*b^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.64

$$\int e^{a+ibx}\sin^3(d+bx)\,dx = \frac{\left(12i\,bxe^{(2i\,bx+a+i\,d)}+e^{(6i\,bx+a+5i\,d)}-6\,e^{(4i\,bx+a+3i\,d)}+2\,e^{(a-i\,d)}\right)e^{(-2i\,bx-2i\,d)}}{32\,b}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/32*(12*I*b*x*e^(2*I*b*x + a + I*d) + e^(6*I*b*x + a + 5*I*d) - 6*e^(4*I*
b*x + a + 3*I*d) + 2*e^(a - I*d))*e^(-2*I*b*x - 2*I*d)/b
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.82

$$\int e^{a+ibx}\sin^3(d+bx)\,dx = \frac{3ixe^ae^{-id}}{8}+\begin{cases}\frac{(256b^2e^ae^{6id}e^{4ibx}-1536b^2e^ae^{4id}e^{2ibx}+512b^2e^ae^{-2ibx})e^{-3id}}{8192b^3} & \text{for } b^3e^{3id}\neq 0\\ x\left(\frac{(ie^ae^{6id}-3ie^ae^{4id}+3ie^ae^{2id}-ie^a)e^{-3id}}{8}-\frac{3ie^ae^{-id}}{8}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d)**3,x)
```

output
```
3*I*x*exp(a)*exp(-I*d)/8 + Piecewise(((256*b**2*exp(a)*exp(6*I*d)*exp(4*I*
b*x) - 1536*b**2*exp(a)*exp(4*I*d)*exp(2*I*b*x) + 512*b**2*exp(a)*exp(-2*I
*b*x))*exp(-3*I*d)/(8192*b**3), Ne(b**3*exp(3*I*d), 0)), (x*((I*exp(a)*exp
(6*I*d) - 3*I*exp(a)*exp(4*I*d) + 3*I*exp(a)*exp(2*I*d) - I*exp(a))*exp(-3
*I*d)/8 - 3*I*exp(a)*exp(-I*d)/8), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07

$$\int e^{a+ibx}\sin^3(d+bx)\,dx = -\frac{12\,(-i\,b\cos{(d)}\,e^a - be^a\sin{(d)})x - \cos{(4\,bx+3\,d)}\,e^a - 2\,\cos{(2\,bx+3\,d)}\,e^a + 6\,\cos{(2\,bx+d)}\,e^a - i\,e}{32\,b}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d)^3,x, algorithm="maxima")
```

output
```
-1/32*(12*(-I*b*cos(d)*e^a - b*e^a*sin(d))*x - cos(4*b*x + 3*d)*e^a - 2*co
s(2*b*x + 3*d)*e^a + 6*cos(2*b*x + d)*e^a - I*e^a*sin(4*b*x + 3*d) + 2*I*e
^a*sin(2*b*x + 3*d) + 6*I*e^a*sin(2*b*x + d))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(55) = 110.

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.54

$$\int e^{a+ibx}\sin^3(d+bx)\,dx = -\frac{-24i\,(bx+d)\cos{(d)}\,e^a - 24\,(bx+d)e^a\sin{(d)} - \left(e^{(4i\,bx+3i\,d)} - e^{(-4i\,bx-3i\,d)}\right)e^a + 2\,\left(e^{(2i\,bx+3i\,d)} - e^{(-2i\,b}\right.}{}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/64*(-24*I*(b*x + d)*cos(d)*e^a - 24*(b*x + d)*e^a*sin(d) - (e^(4*I*b*x
+ 3*I*d) - e^(-4*I*b*x - 3*I*d))*e^a + 2*(e^(2*I*b*x + 3*I*d) - e^(-2*I*b*
x - 3*I*d))*e^a + 6*(e^(2*I*b*x + I*d) - e^(-2*I*b*x - I*d))*e^a - 4*cos(2
*b*x + 3*d)*e^a + 12*cos(-2*b*x - d)*e^a - 2*cos(-4*b*x - 3*d)*e^a)/b
```

Mupad [B] (verification not implemented)

Time = 15.54 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

$$\int e^{a+ibx}\sin^3(d+bx)\,dx = \frac{x\,\mathrm{e}^a\,(\cos(d)-\sin(d)\,1\mathrm{i})\,3\mathrm{i}}{8} + \frac{\mathrm{e}^a\,(\cos(2\,b\,x)-\sin(2\,b\,x)\,1\mathrm{i})\,(\cos(3\,d)-\sin(3\,d)\,1\mathrm{i})}{16\,b} + \frac{\mathrm{e}^a\,(\cos(4\,b\,x)+\sin(4\,b\,x)\,1\mathrm{i})\,(\cos(3\,d)+\sin(3\,d)\,1\mathrm{i})}{32\,b} - \frac{3\,\mathrm{e}^a\,(\cos(2\,b\,x)+\sin(2\,b\,x)\,1\mathrm{i})\,(\cos(d)+\sin(d)\,1\mathrm{i})}{16\,b}$$

input
```
int(exp(a + b*x*1i)*sin(d + b*x)^3,x)
```

output
```
(x*exp(a)*(cos(d) - sin(d)*1i)*3i)/8 + (exp(a)*(cos(2*b*x) - sin(2*b*x)*1i
)*(cos(3*d) - sin(3*d)*1i))/(16*b) + (exp(a)*(cos(4*b*x) + sin(4*b*x)*1i)*
(cos(3*d) + sin(3*d)*1i))/(32*b) - (3*exp(a)*(cos(2*b*x) + sin(2*b*x)*1i)*
(cos(d) + sin(d)*1i))/(16*b)
```

Reduce [F]

$$\int e^{a+ibx}\sin^3(d+bx)\,dx = e^a\left(\int e^{bix}\sin(bx+d)^3\,dx\right)$$

input
```
int(exp(a+I*b*x)*sin(b*x+d)^3,x)
```

output
```
e**a*int(e**(b*i*x)*sin(b*x + d)**3,x)
```

3.4 $\int e^{a+ibx} \sin^4(d+bx)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\int e^{a+ibx} \sin^4(d+bx)\, dx = -\frac{ie^{a-id-i(d+bx)}}{4b} - \frac{3ie^{a-id+i(d+bx)}}{8b} + \frac{ie^{a-id-3i(d+bx)}}{48b} + \frac{ie^{a-id+3i(d+bx)}}{12b} - \frac{ie^{a-id+5i(d+bx)}}{80b}$$

output
```
-1/4*I*exp(a-I*d-I*(b*x+d))/b-3/8*I*exp(a-I*d+I*(b*x+d))/b+1/48*I*exp(a-I*
d-3*I*(b*x+d))/b+1/12*I*exp(a-I*d+3*I*(b*x+d))/b-1/80*I*exp(a-I*d+5*I*(b*x
+d))/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int e^{a+ibx} \sin^4(d+bx)\, dx = \frac{e^{a-3ibx}\left(-90ie^{4ibx} + 20ie^{2ibx}\left(-3+e^{4ibx}\right)\cos(2d) - i\left(-5+3e^{8ibx}\right)\cos(4d) - 60e^{2ibx}\sin(2d) - 20e^{6ibx}\sin(}{240b}$$

input
```
Integrate[E^(a + I*b*x)*Sin[d + b*x]^4,x]
```

output
```
(E^(a - (3*I)*b*x)*((-90*I)*E^((4*I)*b*x) + (20*I)*E^((2*I)*b*x)*(-3 + E^(
(4*I)*b*x))*Cos[2*d] - I*(-5 + 3*E^((8*I)*b*x))*Cos[4*d] - 60*E^((2*I)*b*x
)*Sin[2*d] - 20*E^((6*I)*b*x)*Sin[2*d] + 5*Sin[4*d] + 3*E^((8*I)*b*x)*Sin[
4*d]))/(240*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4934, 4934, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\sin^4(bx+d)\,dx$$

$$\Big\downarrow \text{ 4934}$$

$$\frac{4}{5}\int e^{a+ibx}\sin^2(d+bx)dx+\frac{ie^{a+ibx}\sin^4(bx+d)}{15b}-\frac{4e^{a+ibx}\sin^3(bx+d)\cos(bx+d)}{15b}$$

$$\Big\downarrow \text{ 4934}$$

$$\frac{4}{5}\left(\frac{2}{3}\int e^{a+ibx}dx+\frac{ie^{a+ibx}\sin^2(bx+d)}{3b}-\frac{2e^{a+ibx}\sin(bx+d)\cos(bx+d)}{3b}\right)+\frac{ie^{a+ibx}\sin^4(bx+d)}{15b}-\frac{4e^{a+ibx}\sin^3(bx+d)\cos(bx+d)}{15b}$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{ie^{a+ibx}\sin^4(bx+d)}{15b}-\frac{4e^{a+ibx}\sin^3(bx+d)\cos(bx+d)}{15b}+\frac{4}{5}\left(\frac{ie^{a+ibx}\sin^2(bx+d)}{3b}-\frac{2e^{a+ibx}\sin(bx+d)\cos(bx+d)}{3b}-\frac{2ie^{a+ibx}}{3b}\right)$$

input
```
Int[E^(a + I*b*x)*Sin[d + b*x]^4,x]
```

output
```
(-4*E^(a + I*b*x)*Cos[d + b*x]*Sin[d + b*x]^3)/(15*b) + ((I/15)*E^(a + I*b
*x)*Sin[d + b*x]^4)/b + (4*((((-2*I)/3)*E^(a + I*b*x))/b - (2*E^(a + I*b*x
)*Cos[d + b*x]*Sin[d + b*x])/(3*b) + ((I/3)*E^(a + I*b*x)*Sin[d + b*x]^2)/
b))/5
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

method	result
parallelrisch	$-\frac{32\,\mathrm{e}^{ibx+a}\left(i\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}+3\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}+\frac{i}{2}+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)\right)}{15b\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}\right)^{4}}$
norman	$\frac{-\frac{16i\mathrm{e}^{ibx+a}}{15b}-\frac{32i\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}}{15b}-\frac{32\,\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{15b}-\frac{32\,\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}}{5b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}\right)^{4}}$
default	$-\frac{3i\mathrm{e}^{ibx+a}}{8b}+\frac{i\mathrm{e}^{ibx+a}\cos(4bx+4d)}{120b}+\frac{\mathrm{e}^{ibx+a}\sin(4bx+4d)}{30b}-\frac{i\mathrm{e}^{ibx+a}\cos(2bx+2d)}{6b}-\frac{\mathrm{e}^{ibx+a}\sin(2bx+2d)}{3b}$
orering	$-\frac{i\mathrm{e}^{ibx+a}\sin(bx+d)^{4}}{5b}-\frac{10\left(ib\,\mathrm{e}^{ibx+a}\sin(bx+d)^{4}+4\,\mathrm{e}^{ibx+a}\sin(bx+d)^{3}b\cos(bx+d)\right)}{9b^{2}}-\frac{2i\left(-5b^{2}\mathrm{e}^{ibx+a}\sin(bx+d)^{4}+8ib^{2}\right.}{}$

input
```
int(exp(a+I*b*x)*sin(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-32/15*exp(a+I*b*x)*(I*tan(1/2*b*x+1/2*d)^2+3*tan(1/2*b*x+1/2*d)^3+1/2*I+t
an(1/2*b*x+1/2*d))/b/(1+tan(1/2*b*x+1/2*d)^2)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.52

$$\int e^{a+ibx}\sin^4(d+bx)\,dx = \frac{\left(-3i\,e^{(8i\,bx+a+7i\,d)}+20i\,e^{(6i\,bx+a+5i\,d)}-90i\,e^{(4i\,bx+a+3i\,d)}-60i\,e^{(2i\,bx+a+i\,d)}+5i\,e^{(a-i\,d)}\right)e^{(-3i\,bx-3i\,d)}}{240\,b}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d)^4,x, algorithm="fricas")
```

output
```
1/240*(-3*I*e^(8*I*b*x + a + 7*I*d) + 20*I*e^(6*I*b*x + a + 5*I*d) - 90*I*
e^(4*I*b*x + a + 3*I*d) - 60*I*e^(2*I*b*x + a + I*d) + 5*I*e^(a - I*d))*e^
(-3*I*b*x - 3*I*d)/b
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(99) = 198.

Time = 0.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.49

$$\int e^{a+ibx}\sin^4(d+bx)\,dx = \begin{cases} \frac{\left(-18432ib^4e^ae^{10id}e^{5ibx}+122880ib^4e^ae^{8id}e^{3ibx}-552960ib^4e^ae^{6id}e^{ibx}-368640ib^4e^ae^{4id}e^{-ibx}+30720ib^4e^ae^{2id}e^{-3ibx}\right)e^{-6id}}{1474560b^5} & \text{for } b^5e^{6id} \\ \frac{x\left(e^ae^{8id}-4e^ae^{6id}+6e^ae^{4id}-4e^ae^{2id}+e^a\right)e^{-4id}}{16} & \text{otherwis} \end{cases}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d)**4,x)
```

output
```
Piecewise(((-18432*I*b**4*exp(a)*exp(10*I*d)*exp(5*I*b*x) + 122880*I*b**4*
exp(a)*exp(8*I*d)*exp(3*I*b*x) - 552960*I*b**4*exp(a)*exp(6*I*d)*exp(I*b*x
) - 368640*I*b**4*exp(a)*exp(4*I*d)*exp(-I*b*x) + 30720*I*b**4*exp(a)*exp(
2*I*d)*exp(-3*I*b*x))*exp(-6*I*d)/(1474560*b**5), Ne(b**5*exp(6*I*d), 0)),
 (x*(exp(a)*exp(8*I*d) - 4*exp(a)*exp(6*I*d) + 6*exp(a)*exp(4*I*d) - 4*exp
(a)*exp(2*I*d) + exp(a))*exp(-4*I*d)/16, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int e^{a+ibx}\sin^4(d+bx)\,dx$$
$$= \frac{-3i\cos(5bx+4d)e^a + 5i\cos(3bx+4d)e^a + 20i\cos(3bx+2d)e^a - 60i\cos(bx+2d)e^a - 90i\cos(\ldots}{}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d)^4,x, algorithm="maxima")
```

output
```
1/240*(-3*I*cos(5*b*x + 4*d)*e^a + 5*I*cos(3*b*x + 4*d)*e^a + 20*I*cos(3*b
*x + 2*d)*e^a - 60*I*cos(b*x + 2*d)*e^a - 90*I*cos(b*x)*e^a + 3*e^a*sin(5*
b*x + 4*d) + 5*e^a*sin(3*b*x + 4*d) - 20*e^a*sin(3*b*x + 2*d) - 60*e^a*sin
(b*x + 2*d) + 90*e^a*sin(b*x))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(73) = 146.

Time = 0.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int e^{a+ibx}\sin^4(d+bx)\,dx =$$
$$-\frac{3i\left(e^{(5ibx+4id)} + e^{(-5ibx-4id)}\right)e^a - 5i\left(e^{(3ibx+4id)} + e^{(-3ibx-4id)}\right)e^a - 20i\left(e^{(3ibx+2id)} + e^{(-3ibx-2id)}\right)e^a + \ldots}{}$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+d)^4,x, algorithm="giac")
```

output
```
-1/480*(3*I*(e^(5*I*b*x + 4*I*d) + e^(-5*I*b*x - 4*I*d))*e^a - 5*I*(e^(3*I
*b*x + 4*I*d) + e^(-3*I*b*x - 4*I*d))*e^a - 20*I*(e^(3*I*b*x + 2*I*d) + e^
(-3*I*b*x - 2*I*d))*e^a + 60*I*(e^(I*b*x + 2*I*d) + e^(-I*b*x - 2*I*d))*e^
a + 90*I*(e^(I*b*x) + e^(-I*b*x))*e^a - 10*e^a*sin(3*b*x + 4*d) + 120*e^a*
sin(b*x + 2*d) - 180*e^a*sin(b*x) - 40*e^a*sin(-3*b*x - 2*d) + 6*e^a*sin(-
5*b*x - 4*d))/b
```

Mupad [B] (verification not implemented)

Time = 16.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.14

$$\int e^{a+ibx}\sin^4(d+bx)\,dx = -\frac{e^a\left(\cos(b\,x)+\sin(b\,x)\ 1i\right)\ 3i}{8\,b} - \frac{e^a\left(\cos(b\,x)-\sin(b\,x)\ 1i\right)\left(\cos(2\,d)-\sin(2\,d)\ 1i\right)\ 1i}{4\,b} + \frac{e^a\left(\cos(3\,b\,x)+\sin(3\,b\,x)\ 1i\right)\left(\cos(2\,d)+\sin(2\,d)\ 1i\right)\ 1i}{12\,b} + \frac{e^a\left(\cos(3\,b\,x)-\sin(3\,b\,x)\ 1i\right)\left(\cos(4\,d)-\sin(4\,d)\ 1i\right)\ 1i}{48\,b} - \frac{e^a\left(\cos(5\,b\,x)+\sin(5\,b\,x)\ 1i\right)\left(\cos(4\,d)+\sin(4\,d)\ 1i\right)\ 1i}{80\,b}$$

input
```
int(exp(a + b*x*1i)*sin(d + b*x)^4,x)
```

output
```
(exp(a)*(cos(3*b*x) + sin(3*b*x)*1i)*(cos(2*d) + sin(2*d)*1i)*1i)/(12*b) -
 (exp(a)*(cos(b*x) - sin(b*x)*1i)*(cos(2*d) - sin(2*d)*1i)*1i)/(4*b) - (ex
p(a)*(cos(b*x) + sin(b*x)*1i)*3i)/(8*b) + (exp(a)*(cos(3*b*x) - sin(3*b*x)
*1i)*(cos(4*d) - sin(4*d)*1i)*1i)/(48*b) - (exp(a)*(cos(5*b*x) + sin(5*b*x
)*1i)*(cos(4*d) + sin(4*d)*1i)*1i)/(80*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62

$$\int e^{a+ibx}\sin^4(d+bx)\,dx$$
$$= \frac{e^{bix+a}\left(-4\cos(bx+d)\sin(bx+d)^3-8\cos(bx+d)\sin(bx+d)-8\cos(bx+d)\,i+\sin(bx+d)^4\,i+4\,\mathrm{si}\right.}{15b}$$

input
```
int(exp(a+I*b*x)*sin(b*x+d)^4,x)
```

output
```
(e**(a + b*i*x)*( - 4*cos(b*x + d)*sin(b*x + d)**3 - 8*cos(b*x + d)*sin(b*
x + d) - 8*cos(b*x + d)*i + sin(b*x + d)**4*i + 4*sin(b*x + d)**2*i - 8*si
n(b*x + d) - 8*i))/(15*b)
```

3.5 $\int e^{2(a+ibx)}\sin(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int e^{2(a+ibx)}\sin(d+bx)\,dx=\frac{e^{2(a-id)+i(d+bx)}}{2b}-\frac{e^{2(a-id)+3i(d+bx)}}{6b}$$

output
```
1/2*exp(2*a-2*I*d+I*(b*x+d))/b-1/6*exp(2*a-2*I*d+3*I*(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int e^{2(a+ibx)}\sin(d+bx)\,dx=-\frac{e^{2a+ibx}\left(\left(-3+e^{2ibx}\right)\cos(d)+i\left(3+e^{2ibx}\right)\sin(d)\right)}{6b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Sin[d + b*x],x]
```

output
```
-1/6*(E^(2*a + I*b*x)*((-3 + E^((2*I)*b*x))*Cos[d] + I*(3 + E^((2*I)*b*x))
*Sin[d]))/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\sin(bx+d)\,dx$$

$$\Big\downarrow \; 4932$$

$$\frac{e^{2(a+ibx)}\cos(bx+d)}{3b}-\frac{2ie^{2(a+ibx)}\sin(bx+d)}{3b}$$

input
```
Int[E^(2*(a + I*b*x))*Sin[d + b*x],x]
```

output
```
(E^(2*(a + I*b*x))*Cos[d + b*x])/(3*b) - (((2*I)/3)*E^(2*(a + I*b*x))*Sin[
d + b*x])/b
```

Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

method	result	size
parallelrisch	$\frac{(-2i\sin(bx+d)+\cos(bx+d))e^{2ibx+2a}}{3b}$	32
default	$\frac{e^{2ibx+2a}\cos(bx+d)}{3b}-\frac{2ie^{2ibx+2a}\sin(bx+d)}{3b}$	45
orering	$-\frac{4ie^{2ibx+2a}\sin(bx+d)}{3b}+\frac{2ib\,e^{2ibx+2a}\sin(bx+d)+e^{2ibx+2a}b\cos(bx+d)}{3b^2}$	68
norman	$\frac{\frac{e^{2ibx+2a}}{3b}-\frac{e^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2}{3b}-\frac{4ie^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{3b}}{1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2}$	84

input
```
int(exp(2*a+2*I*b*x)*sin(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-2*I*sin(b*x+d)+cos(b*x+d))*exp(2*a+2*I*b*x)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

$$\int e^{2(a+ibx)}\sin(d+bx)\,dx=-\frac{e^{(3i\,bx+2\,a+i\,d)}-3\,e^{(i\,bx+2\,a-i\,d)}}{6\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d),x, algorithm="fricas")
```

output
```
-1/6*(e^(3*I*b*x + 2*a + I*d) - 3*e^(I*b*x + 2*a - I*d))/b
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(41) = 82.

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

$$\int e^{2(a+ibx)}\sin(d+bx)\,dx=\begin{cases}\frac{(-2be^{2a}e^{2id}e^{3ibx}+6be^{2a}e^{ibx})e^{-id}}{12b^2} & \text{for } b^2e^{id}\neq 0\\ \frac{x(-ie^{2a}e^{2id}+ie^{2a})e^{-id}}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d),x)
```

output
```
Piecewise(((-2*b*exp(2*a)*exp(2*I*d)*exp(3*I*b*x) + 6*b*exp(2*a)*exp(I*b*x
))*exp(-I*d)/(12*b**2), Ne(b**2*exp(I*d), 0)), (x*(-I*exp(2*a)*exp(2*I*d)
+ I*exp(2*a))*exp(-I*d)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

$$\int e^{2(a+ibx)}\sin(d+bx)\,dx=\frac{(b\cos(bx+d)-2i\,b\sin(bx+d))e^{(2i\,bx+2\,a)}}{3\,b^2}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d),x, algorithm="maxima")
```

output
```
1/3*(b*cos(b*x + d) - 2*I*b*sin(b*x + d))*e^(2*I*b*x + 2*a)/b^2
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(35) = 70.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.53

$$\int e^{2(a+ibx)}\sin(d+bx)\,dx= -\frac{\left(e^{(3i\,bx+i\,d)}-e^{(-3i\,bx-i\,d)}\right)e^{(2\,a)}-3\left(e^{(i\,bx-i\,d)}-e^{(-i\,bx+i\,d)}\right)e^{(2\,a)}-6\,\cos(-bx+d)\,e^{(2\,a)}+2\,\cos(-3\,bx-}{12\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d),x, algorithm="giac")
```

output
```
-1/12*((e^(3*I*b*x + I*d) - e^(-3*I*b*x - I*d))*e^(2*a) - 3*(e^(I*b*x - I*
d) - e^(-I*b*x + I*d))*e^(2*a) - 6*cos(-b*x + d)*e^(2*a) + 2*cos(-3*b*x -
d)*e^(2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54

$$\int e^{2(a+ibx)}\sin(d+bx)\,dx=\frac{\mathrm{e}^{2\,a+b\,x\,2\mathrm{i}}\,\left(\cos\left(d+b\,x\right)-\sin\left(d+b\,x\right)\,2\mathrm{i}\right)}{3\,b}$$

input
```
int(exp(2*a + b*x*2i)*sin(d + b*x),x)
```

output
```
(exp(2*a + b*x*2i)*(cos(d + b*x) - sin(d + b*x)*2i))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

$$\int e^{2(a+ibx)}\sin(d+bx)\,dx=\frac{e^{2bix+2a}(\cos\left(bx+d\right)-2\sin\left(bx+d\right)i)}{3b}$$

input
```
int(exp(2*a+2*I*b*x)*sin(b*x+d),x)
```

output
```
(e**(2*a + 2*b*i*x)*(cos(b*x + d) - 2*sin(b*x + d)*i))/(3*b)
```

3.6 $\int e^{2(a+ibx)}\sin^2(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int e^{2(a+ibx)}\sin^2(d+bx)\,dx=-\frac{ie^{2(a-id)+2i(d+bx)}}{4b}+\frac{ie^{2(a-id)+4i(d+bx)}}{16b}-\frac{1}{4}e^{2a-2id}x$$

output
```
-1/4*I*exp(2*a-2*I*d+2*I*(b*x+d))/b+1/16*I*exp(2*a-2*I*d+4*I*(b*x+d))/b-1/
4*exp(2*a-2*I*d)*x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int e^{2(a+ibx)}\sin^2(d+bx)\,dx$$
$$=\frac{ie^{2a}\left(-4e^{2ibx}+\left(e^{4ibx}+4ibx\right)\cos(2d)+\left(ie^{4ibx}+4bx\right)\sin(2d)\right)}{16b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Sin[d + b*x]^2,x]
```

output
```
((I/16)*E^(2*a)*(-4*E^((2*I)*b*x) + (E^((4*I)*b*x) + (4*I)*b*x)*Cos[2*d] +
 (I*E^((4*I)*b*x) + 4*b*x)*Sin[2*d]))/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4967, 4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\sin^2(bx+d)\,dx$$

$$\Big\downarrow \text{4967}$$

$$\int e^{2a+2ibx}\sin^2(bx+d)dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(-\frac{1}{4}e^{2a+4ibx+2id}+\frac{1}{2}e^{2a+2ibx}-\frac{1}{4}e^{2(a-id)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ie^{2(a+id)+4ibx}}{16b}-\frac{ie^{2a+2ibx}}{4b}-\frac{1}{4}xe^{2a-2id}$$

input
```
Int[E^(2*(a + I*b*x))*Sin[d + b*x]^2,x]
```

output
```
((-1/4*I)*E^(2*a + (2*I)*b*x))/b + ((I/16)*E^(2*(a + I*d) + (4*I)*b*x))/b
- (E^(2*a - (2*I)*d)*x)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4967
```
Int[(F_)^((c_.)*(u_))*(G_)[v_]^(n_.), x_Symbol] :> Int[F^(c*ExpandToSum[u,
x])*G[ExpandToSum[v, x]]^n, x] /; FreeQ[{F, c, n}, x] && TrigQ[G] && Linear
Q[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(59) = 118$.

Time = 0.34 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.26

method	result
orering	$-\frac{(-4bx+3i)e^{2ibx+2a}\sin(bx+d)^2}{4b} - \frac{i(-6bx+i)\left(2ib\,e^{2ibx+2a}\sin(bx+d)^2+2\,e^{2ibx+2a}\sin(bx+d)b\cos(bx+d)\right)}{8b^2} - \frac{x\left(-6\sin(bx+d\right.}{}$
norman	$\frac{ix\,e^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)-\frac{x\,e^{2ibx+2a}}{4}+\frac{3x\,e^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2}{2}-\frac{x\,e^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^4}{4}-\frac{2ie^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2}{b}-ix\,e^{2ibx+2a}\tan}{\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2\right)^2}$

input
```
int(exp(2*a+2*I*b*x)*sin(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-4*b*x+3*I)/b*exp(2*a+2*I*b*x)*sin(b*x+d)^2-1/8*I*(-6*b*x+I)/b^2*(2*
I*b*exp(2*a+2*I*b*x)*sin(b*x+d)^2+2*exp(2*a+2*I*b*x)*sin(b*x+d)*b*cos(b*x+
d))-1/8/b^2*x*(-6*sin(b*x+d)^2*exp(2*a+2*I*b*x)*b^2+8*I*b^2*exp(2*a+2*I*b*
x)*sin(b*x+d)*cos(b*x+d)+2*cos(b*x+d)^2*exp(2*a+2*I*b*x)*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.56

$$\int e^{2(a+ibx)}\sin^2(d+bx)\,dx = -\frac{4\,bxe^{(2\,a-2i\,d)} - i\,e^{(4i\,bx+2\,a+2i\,d)} + 4i\,e^{(2i\,bx+2\,a)}}{16\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*b*x*e^(2*a - 2*I*d) - I*e^(4*I*b*x + 2*a + 2*I*d) + 4*I*e^(2*I*b*
x + 2*a))/b
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.55

$$\int e^{2(a+ibx)}\sin^2(d+bx)\,dx = -\frac{xe^{2a}e^{-2id}}{4} + \begin{cases} \frac{4ibe^{2a}e^{2id}e^{4ibx}-16ibe^{2a}e^{2ibx}}{64b^2} & \text{for } b^2\neq 0 \\ x\left(\frac{(-e^{2a}e^{4id}+2e^{2a}e^{2id}-e^{2a})e^{-2id}}{4}+\frac{e^{2a}e^{-2id}}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d)**2,x)
```

output
```
-x*exp(2*a)*exp(-2*I*d)/4 + Piecewise(((4*I*b*exp(2*a)*exp(2*I*d)*exp(4*I*
b*x) - 16*I*b*exp(2*a)*exp(2*I*b*x))/(64*b**2), Ne(b**2, 0)), (x*((-exp(2*
a)*exp(4*I*d) + 2*exp(2*a)*exp(2*I*d) - exp(2*a))*exp(-2*I*d)/4 + exp(2*a)
*exp(-2*I*d)/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int e^{2(a+ibx)}\sin^2(d+bx)\,dx = -\frac{4\left(b\cos(2d)e^{(2a)}-i\,be^{(2a)}\sin(2d)\right)x+4i\cos(2bx)e^{(2a)}-i\cos(4bx+2d)e^{(2a)}-4e^{(2a)}\sin(2bx)}{16b}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d)^2,x, algorithm="maxima")
```

output
```
-1/16*(4*(b*cos(2*d)*e^(2*a) - I*b*e^(2*a)*sin(2*d))*x + 4*I*cos(2*b*x)*e^
(2*a) - I*cos(4*b*x + 2*d)*e^(2*a) - 4*e^(2*a)*sin(2*b*x) + e^(2*a)*sin(4*
b*x + 2*d))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(43) = 86$.

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int e^{2(a+ibx)}\sin^2(d+bx)\,dx = -\frac{8\,(bx+d)\cos\left(2\,d\right)e^{\left(2\,a\right)} - 8i\,(bx+d)e^{\left(2\,a\right)}\sin\left(2\,d\right) - i\left(e^{\left(4i\,bx+2i\,d\right)} + e^{\left(-4i\,bx-2i\,d\right)}\right)e^{\left(2\,a\right)} + 4i\left(e^{\left(2i\,bx\right)} + e\right.}{32\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d)^2,x, algorithm="giac")
```

output
```
-1/32*(8*(b*x + d)*cos(2*d)*e^(2*a) - 8*I*(b*x + d)*e^(2*a)*sin(2*d) - I*(
e^(4*I*b*x + 2*I*d) + e^(-4*I*b*x - 2*I*d))*e^(2*a) + 4*I*(e^(2*I*b*x) + e
^(-2*I*b*x))*e^(2*a) - 8*e^(2*a)*sin(2*b*x) - 2*e^(2*a)*sin(-4*b*x - 2*d))
/b
```

Mupad [B] (verification not implemented)

Time = 16.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int e^{2(a+ibx)}\sin^2(d+bx)\,dx = -\frac{x\,{\mathrm{e}}^{2\,a-d\,2{}\mathrm{i}}}{4} + \frac{{\mathrm{e}}^{2\,a+d\,2{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,1{}\mathrm{i}}{16\,b} - \frac{{\mathrm{e}}^{2\,a+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{4\,b}$$

input
```
int(exp(2*a + b*x*2i)*sin(d + b*x)^2,x)
```

output
```
(exp(2*a + d*2i + b*x*4i)*1i)/(16*b) - (x*exp(2*a - d*2i))/4 - (exp(2*a +
b*x*2i)*1i)/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int e^{2(a+ibx)}\sin^2(d+bx)\,dx$$
$$= \frac{e^{2bix+2a}(2\cos(bx+d)\sin(bx+d)\,bix+\cos(bx+d)\sin(bx+d)+2\sin(bx+d)^2\,bx-2\sin(bx+d)^2\,i\,\cdot}{4b}$$

input
```
int(exp(2*a+2*I*b*x)*sin(b*x+d)^2,x)
```

output
```
(e**(2*a + 2*b*i*x)*(2*cos(b*x + d)*sin(b*x + d)*b*i*x + cos(b*x + d)*sin(
b*x + d) + 2*sin(b*x + d)**2*b*x - 2*sin(b*x + d)**2*i - b*x))/(4*b)
```

3.7 $\int e^{2(a+ibx)}\sin^3(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int e^{2(a+ibx)}\sin^3(d+bx)\,dx=\frac{e^{2(a-id)-i(d+bx)}}{8b}+\frac{3e^{2(a-id)+i(d+bx)}}{8b}-\frac{e^{2(a-id)+3i(d+bx)}}{8b}+\frac{e^{2(a-id)+5i(d+bx)}}{40b}$$

output
```
1/8*exp(2*a-2*I*d-I*(b*x+d))/b+3/8*exp(2*a-2*I*d+I*(b*x+d))/b-1/8*exp(2*a-
2*I*d+3*I*(b*x+d))/b+1/40*exp(2*a-2*I*d+5*I*(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int e^{2(a+ibx)}\sin^3(d+bx)\,dx$$
$$=\frac{e^{2a-ibx}\left(-5e^{2ibx}\left(-3+e^{2ibx}\right)\cos(d)+\left(5+e^{6ibx}\right)\cos(3d)+i\left(-5e^{2ibx}\left(3+e^{2ibx}\right)\sin(d)+\left(-5+e^{6ibx}\right)\sin\right.\right.}{40b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Sin[d + b*x]^3,x]
```

output
```
(E^(2*a - I*b*x)*(-5*E^((2*I)*b*x)*(-3 + E^((2*I)*b*x))*Cos[d] + (5 + E^((
6*I)*b*x))*Cos[3*d] + I*(-5*E^((2*I)*b*x)*(3 + E^((2*I)*b*x))*Sin[d] + (-5
 + E^((6*I)*b*x))*Sin[3*d])))/(40*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4934, 4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)} \sin^3(bx+d) \, dx$$

$$\Big\downarrow \text{4934}$$

$$\frac{6}{5} \int e^{2(a+ibx)} \sin(d+bx)dx + \frac{2ie^{2(a+ibx)} \sin^3(bx+d)}{5b} - \frac{3e^{2(a+ibx)} \sin^2(bx+d) \cos(bx+d)}{5b}$$

$$\Big\downarrow \text{4932}$$

$$\frac{2ie^{2(a+ibx)} \sin^3(bx+d)}{5b} - \frac{3e^{2(a+ibx)} \sin^2(bx+d) \cos(bx+d)}{5b} + \frac{6}{5} \left(\frac{e^{2(a+ibx)} \cos(bx+d)}{3b} - \frac{2ie^{2(a+ibx)} \sin(bx+d)}{3b} \right)$$

input
```
Int[E^(2*(a + I*b*x))*Sin[d + b*x]^3,x]
```

output
```
(-3*E^(2*(a + I*b*x))*Cos[d + b*x]*Sin[d + b*x]^2)/(5*b) + (((2*I)/5)*E^(2
*(a + I*b*x))*Sin[d + b*x]^3)/b + (6*((E^(2*(a + I*b*x))*Cos[d + b*x])/(3*
b) - (((2*I)/3)*E^(2*(a + I*b*x))*Sin[d + b*x])/b))/5
```

Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.51

method	result
parallelrisch	$\frac{4\left(-4i\tan\left(\frac{bx}{2}+\frac{d}{2}\right)-5\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}+1\right)e^{2ibx+2a}}{5b\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}\right)^{3}}$
norman	$\frac{\frac{4\,e^{2ibx+2a}}{5b}-\frac{4\,e^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}}{b}-\frac{16i e^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{5b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}\right)^{3}}$
default	$\frac{3\,e^{2ibx+2a}\cos(3bx+3d)}{20b}-\frac{i e^{2ibx+2a}\sin(3bx+3d)}{10b}+\frac{e^{2ibx+2a}\cos(bx+d)}{4b}-\frac{i e^{2ibx+2a}\sin(bx+d)}{2b}$
orering	$-\frac{8i e^{2ibx+2a}\sin(bx+d)^{3}}{15b}-\frac{14\left(2ib\,e^{2ibx+2a}\sin(bx+d)^{3}+3\,e^{2ibx+2a}\sin(bx+d)^{2}b\cos(bx+d)\right)}{15b^{2}}-\frac{8i\left(-7b^{2}e^{2ibx+2a}\sin(bx+\right.}{}$

input
```
int(exp(2*a+2*I*b*x)*sin(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
4/5*(-4*I*tan(1/2*b*x+1/2*d)-5*tan(1/2*b*x+1/2*d)^2+1)*exp(2*a+2*I*b*x)/b/
(1+tan(1/2*b*x+1/2*d)^2)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int e^{2(a+ibx)}\sin^3(d+bx)\,dx = \frac{\left(e^{(6i\,bx+2\,a+4i\,d)}-5\,e^{(4i\,bx+2\,a+2i\,d)}+15\,e^{(2i\,bx+2\,a)}+5\,e^{(2\,a-2i\,d)}\right)e^{(-i\,bx-i\,d)}}{40\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/40*(e^(6*I*b*x + 2*a + 4*I*d) - 5*e^(4*I*b*x + 2*a + 2*I*d) + 15*e^(2*I*
b*x + 2*a) + 5*e^(2*a - 2*I*d))*e^(-I*b*x - I*d)/b
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(85) = 170.

Time = 0.23 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.57

$$\int e^{2(a+ibx)}\sin^3(d+bx)\,dx = \begin{cases} \frac{(512b^3e^{2a}e^{7id}e^{5ibx}-2560b^3e^{2a}e^{5id}e^{3ibx}+7680b^3e^{2a}e^{3id}e^{ibx}+2560b^3e^{2a}e^{id}e^{-ibx})e^{-4id}}{20480b^4} & \text{for } b^4e^{4id}\neq 0 \\ \frac{x(ie^{2a}e^{6id}-3ie^{2a}e^{4id}+3ie^{2a}e^{2id}-ie^{2a})e^{-3id}}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d)**3,x)
```

output
```
Piecewise(((512*b**3*exp(2*a)*exp(7*I*d)*exp(5*I*b*x) - 2560*b**3*exp(2*a)
*exp(5*I*d)*exp(3*I*b*x) + 7680*b**3*exp(2*a)*exp(3*I*d)*exp(I*b*x) + 2560
*b**3*exp(2*a)*exp(I*d)*exp(-I*b*x))*exp(-4*I*d)/(20480*b**4), Ne(b**4*exp
(4*I*d), 0)), (x*(I*exp(2*a)*exp(6*I*d) - 3*I*exp(2*a)*exp(4*I*d) + 3*I*ex
p(2*a)*exp(2*I*d) - I*exp(2*a))*exp(-3*I*d)/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int e^{2(a+ibx)}\sin^3(d+bx)\,dx$$
$$= \frac{\cos(5\,bx+3\,d)\,e^{(2\,a)} - 5\,\cos(3\,bx+d)\,e^{(2\,a)} + 5\,\cos(bx+3\,d)\,e^{(2\,a)} + 15\,\cos(bx-d)\,e^{(2\,a)} + i\,e^{(2\,a)}\sin(5}{40\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d)^3,x, algorithm="maxima")
```

output
```
1/40*(cos(5*b*x + 3*d)*e^(2*a) - 5*cos(3*b*x + d)*e^(2*a) + 5*cos(b*x + 3*
d)*e^(2*a) + 15*cos(b*x - d)*e^(2*a) + I*e^(2*a)*sin(5*b*x + 3*d) - 5*I*e^
(2*a)*sin(3*b*x + d) - 5*I*e^(2*a)*sin(b*x + 3*d) + 15*I*e^(2*a)*sin(b*x -
 d))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(69) = 138.

Time = 0.16 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.50

$$\int e^{2(a+ibx)}\sin^3(d+bx)\,dx$$
$$= \frac{\left(e^{(5i\,bx+3i\,d)} - e^{(-5i\,bx-3i\,d)}\right)e^{(2\,a)} - 5\left(e^{(3i\,bx+i\,d)} - e^{(-3i\,bx-i\,d)}\right)e^{(2\,a)} - 5\left(e^{(i\,bx+3i\,d)} - e^{(-i\,bx-3i\,d)}\right)e^{(2\,a)} + 15}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d)^3,x, algorithm="giac")
```

output
```
1/80*((e^(5*I*b*x + 3*I*d) - e^(-5*I*b*x - 3*I*d))*e^(2*a) - 5*(e^(3*I*b*x
 + I*d) - e^(-3*I*b*x - I*d))*e^(2*a) - 5*(e^(I*b*x + 3*I*d) - e^(-I*b*x -
 3*I*d))*e^(2*a) + 15*(e^(I*b*x - I*d) - e^(-I*b*x + I*d))*e^(2*a) + 10*co
s(b*x + 3*d)*e^(2*a) + 30*cos(-b*x + d)*e^(2*a) - 10*cos(-3*b*x - d)*e^(2*
a) + 2*cos(-5*b*x - 3*d)*e^(2*a))/b
```

Mupad [B] (verification not implemented)

Time = 16.60 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

$$\int e^{2(a+ibx)}\sin^3(d+bx)\,dx = \frac{e^{2a}\left(\cos(bx)-\sin(bx)\ 1i\right)\left(\cos(3d)-\sin(3d)\ 1i\right)}{8b} + \frac{e^{2a}\left(\cos(5bx)+\sin(5bx)\ 1i\right)\left(\cos(3d)+\sin(3d)\ 1i\right)}{40b} + \frac{3e^{2a}\left(\cos(bx)+\sin(bx)\ 1i\right)\left(\cos(d)-\sin(d)\ 1i\right)}{8b} - \frac{e^{2a}\left(\cos(3bx)+\sin(3bx)\ 1i\right)\left(\cos(d)+\sin(d)\ 1i\right)}{8b}$$

input
```
int(exp(2*a + b*x*2i)*sin(d + b*x)^3,x)
```

output
```
(exp(2*a)*(cos(b*x) - sin(b*x)*1i)*(cos(3*d) - sin(3*d)*1i))/(8*b) + (exp(
2*a)*(cos(5*b*x) + sin(5*b*x)*1i)*(cos(3*d) + sin(3*d)*1i))/(40*b) + (3*ex
p(2*a)*(cos(b*x) + sin(b*x)*1i)*(cos(d) - sin(d)*1i))/(8*b) - (exp(2*a)*(c
os(3*b*x) + sin(3*b*x)*1i)*(cos(d) + sin(d)*1i))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int e^{2(a+ibx)}\sin^3(d+bx)\,dx = \frac{4e^{2bix+2a}\left(-5\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2-4\tan\left(\frac{bx}{2}+\frac{d}{2}\right)i+1\right)}{5b\left(\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^6+3\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^4+3\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2+1\right)}$$

input
```
int(exp(2*a+2*I*b*x)*sin(b*x+d)^3,x)
```

output
```
(4*e**(2*a + 2*b*i*x)*( - 5*tan((b*x + d)/2)**2 - 4*tan((b*x + d)/2)*i + 1
))/(5*b*(tan((b*x + d)/2)**6 + 3*tan((b*x + d)/2)**4 + 3*tan((b*x + d)/2)*
*2 + 1))
```

3.8 $\int e^{2(a+ibx)}\sin^4(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\int e^{2(a+ibx)}\sin^4(d+bx)\,dx=\frac{ie^{2(a-id)-2i(d+bx)}}{32b}-\frac{3ie^{2(a-id)+2i(d+bx)}}{16b}+\frac{ie^{2(a-id)+4i(d+bx)}}{16b}-\frac{ie^{2(a-id)+6i(d+bx)}}{96b}-\frac{1}{4}e^{2a-2id}x$$

output
```
1/32*I*exp(2*a-2*I*d-2*I*(b*x+d))/b-3/16*I*exp(2*a-2*I*d+2*I*(b*x+d))/b+1/
16*I*exp(2*a-2*I*d+4*I*(b*x+d))/b-1/96*I*exp(2*a-2*I*d+6*I*(b*x+d))/b-1/4*
exp(2*a-2*I*d)*x
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int e^{2(a+ibx)}\sin^4(d+bx)\,dx$$
$$=\frac{e^{2a-2ibx}\left(-18ie^{4ibx}+6ie^{2ibx}\left(e^{4ibx}+4ibx\right)\cos(2d)-i\left(-3+e^{8ibx}\right)\cos(4d)-6e^{6ibx}\sin(2d)+24ibe^{2ibx}x\,\mathrm{si}\right.}{96b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Sin[d + b*x]^4,x]
```

output
```
(E^(2*a - (2*I)*b*x)*((-18*I)*E^((4*I)*b*x) + (6*I)*E^((2*I)*b*x)*(E^((4*I
)*b*x) + (4*I)*b*x)*Cos[2*d] - I*(-3 + E^((8*I)*b*x))*Cos[4*d] - 6*E^((6*I
)*b*x)*Sin[2*d] + (24*I)*b*E^((2*I)*b*x)*x*Sin[2*d] + 3*Sin[4*d] + E^((8*I
)*b*x)*Sin[4*d]))/(96*b)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = $\{4934, 4967, 4975, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\sin^4(bx+d)\,dx$$

$$\Big\downarrow \text{4934}$$

$$\int e^{2(a+ibx)}\sin^2(d+bx)dx+\frac{ie^{2(a+ibx)}\sin^4(bx+d)}{6b}-\frac{e^{2(a+ibx)}\sin^3(bx+d)\cos(bx+d)}{3b}$$

$$\Big\downarrow \text{4967}$$

$$\int e^{2a+2ibx}\sin^2(d+bx)dx+\frac{ie^{2(a+ibx)}\sin^4(bx+d)}{6b}-\frac{e^{2(a+ibx)}\sin^3(bx+d)\cos(bx+d)}{3b}$$

$$\Big\downarrow \text{4975}$$

$$\int \left(-\frac{1}{4}e^{2(a-id)}+\frac{1}{2}e^{2a+2ibx}-\frac{1}{4}e^{2a+2id+4ibx}\right)dx+\frac{ie^{2(a+ibx)}\sin^4(bx+d)}{6b}-\frac{e^{2(a+ibx)}\sin^3(bx+d)\cos(bx+d)}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ie^{2(a+id)+4ibx}}{16b}+\frac{ie^{2(a+ibx)}\sin^4(bx+d)}{6b}-\frac{e^{2(a+ibx)}\sin^3(bx+d)\cos(bx+d)}{3b}-\frac{ie^{2a+2ibx}}{4b}-\frac{1}{4}xe^{2a-2id}$$

input
```
Int[E^(2*(a + I*b*x))*Sin[d + b*x]^4,x]
```

output
```
((-1/4*I)*E^(2*a + (2*I)*b*x))/b + ((I/16)*E^(2*(a + I*d) + (4*I)*b*x))/b
- (E^(2*a - (2*I)*d)*x)/4 - (E^(2*(a + I*b*x))*Cos[d + b*x]*Sin[d + b*x]^3
)/(3*b) + ((I/6)*E^(2*(a + I*b*x))*Sin[d + b*x]^4)/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

rule 4967
```
Int[(F_)^((c_.)*(u_))*(G_)[v_]^(n_.), x_Symbol] :> Int[F^(c*ExpandToSum[u,
x])*G[ExpandToSum[v, x]]^n, x] /; FreeQ[{F, c, n}, x] && TrigQ[G] && Linear
Q[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(105) = 210$.

Time = 1.66 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.96

method	result
norman	$\frac{x\,\mathrm{e}^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}+x\,\mathrm{e}^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{6}-\frac{4i\mathrm{e}^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{4}}{3b}+ix\,\mathrm{e}^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}-\frac{x\,\mathrm{e}^{2ibx+2a}}{4}+\frac{5x\,\mathrm{e}^{2ibx+2a}}{}}{}$
orering	$-\frac{(-12bx+5i)\mathrm{e}^{2ibx+2a}\sin(bx+d)^{4}}{12b}+\frac{5i(2bx+i)\left(2ib\,\mathrm{e}^{2ibx+2a}\sin(bx+d)^{4}+4\,\mathrm{e}^{2ibx+2a}\sin(bx+d)^{3}b\cos(bx+d)\right)}{24b^{2}}-\frac{5(-2bx+i)\left(-\right.}{}$

input
```
int(exp(2*a+2*I*b*x)*sin(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
(x*exp(2*a+2*I*b*x)*tan(1/2*b*x+1/2*d)^2+x*exp(2*a+2*I*b*x)*tan(1/2*b*x+1/
2*d)^6-4/3*I/b*exp(2*a+2*I*b*x)*tan(1/2*b*x+1/2*d)^4+I*x*exp(2*a+2*I*b*x)*
tan(1/2*b*x+1/2*d)^3-1/4*x*exp(2*a+2*I*b*x)+5/2*x*exp(2*a+2*I*b*x)*tan(1/2
*b*x+1/2*d)^4-1/4*x*exp(2*a+2*I*b*x)*tan(1/2*b*x+1/2*d)^8-2*I/b*exp(2*a+2*
I*b*x)*tan(1/2*b*x+1/2*d)^6-2*I/b*exp(2*a+2*I*b*x)*tan(1/2*b*x+1/2*d)^2-I*
x*exp(2*a+2*I*b*x)*tan(1/2*b*x+1/2*d)^7-I*x*exp(2*a+2*I*b*x)*tan(1/2*b*x+1
/2*d)^5+I*x*exp(2*a+2*I*b*x)*tan(1/2*b*x+1/2*d)+1/2/b*exp(2*a+2*I*b*x)*tan
(1/2*b*x+1/2*d)-1/2/b*exp(2*a+2*I*b*x)*tan(1/2*b*x+1/2*d)^7-13/6/b*exp(2*a
+2*I*b*x)*tan(1/2*b*x+1/2*d)^3+13/6/b*exp(2*a+2*I*b*x)*tan(1/2*b*x+1/2*d)^
5)/(1+tan(1/2*b*x+1/2*d)^2)^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.58

$$\int e^{2(a+ibx)}\sin^4(d+bx)\,dx = -\frac{\left(24\,bxe^{(2i\,bx+2\,a)}+i\,e^{(8i\,bx+2\,a+6i\,d)}-6i\,e^{(6i\,bx+2\,a+4i\,d)}+18i\,e^{(4i\,bx+2\,a+2i\,d)}-3i\,e^{(2\,a-2i\,d)}\right)e^{(-2i\,bx-2i\,d)}}{96\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d)^4,x, algorithm="fricas")
```

output
```
-1/96*(24*b*x*e^(2*I*b*x + 2*a) + I*e^(8*I*b*x + 2*a + 6*I*d) - 6*I*e^(6*I
*b*x + 2*a + 4*I*d) + 18*I*e^(4*I*b*x + 2*a + 2*I*d) - 3*I*e^(2*a - 2*I*d)
)*e^(-2*I*b*x - 2*I*d)/b
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.59

$$\int e^{2(a+ibx)}\sin^4(d+bx)\,dx = -\frac{xe^{2a}e^{-2id}}{4} + \begin{cases} \frac{\left(-8192ib^3e^{2a}e^{8id}e^{6ibx}+49152ib^3e^{2a}e^{6id}e^{4ibx}-147456ib^3e^{2a}e^{4id}e^{2ibx}+24576ib^3e^{2a}e^{-2ibx}\right)e^{-4id}}{786432b^4} & \text{for } b^4e^{4id}\neq 0 \\ x\left(\frac{\left(e^{2a}e^{8id}-4e^{2a}e^{6id}+6e^{2a}e^{4id}-4e^{2a}e^{2id}+e^{2a}\right)e^{-4id}}{16}+\frac{e^{2a}e^{-2id}}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d)**4,x)
```

output
```
-x*exp(2*a)*exp(-2*I*d)/4 + Piecewise(((-8192*I*b**3*exp(2*a)*exp(8*I*d)*e
xp(6*I*b*x) + 49152*I*b**3*exp(2*a)*exp(6*I*d)*exp(4*I*b*x) - 147456*I*b**
3*exp(2*a)*exp(4*I*d)*exp(2*I*b*x) + 24576*I*b**3*exp(2*a)*exp(-2*I*b*x))*
exp(-4*I*d)/(786432*b**4), Ne(b**4*exp(4*I*d), 0)), (x*((exp(2*a)*exp(8*I*
d) - 4*exp(2*a)*exp(6*I*d) + 6*exp(2*a)*exp(4*I*d) - 4*exp(2*a)*exp(2*I*d)
 + exp(2*a))*exp(-4*I*d)/16 + exp(2*a)*exp(-2*I*d)/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int e^{2(a+ibx)}\sin^4(d+bx)\,dx = -\frac{24\left(b\cos(2d)e^{(2a)}-ibe^{(2a)}\sin(2d)\right)x+18i\cos(2bx)e^{(2a)}+i\cos(6bx+4d)e^{(2a)}-6i\cos(4bx+$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d)^4,x, algorithm="maxima")
```

output
```
-1/96*(24*(b*cos(2*d)*e^(2*a) - I*b*e^(2*a)*sin(2*d))*x + 18*I*cos(2*b*x)*
e^(2*a) + I*cos(6*b*x + 4*d)*e^(2*a) - 6*I*cos(4*b*x + 2*d)*e^(2*a) - 3*I*
cos(2*b*x + 4*d)*e^(2*a) - 18*e^(2*a)*sin(2*b*x) - e^(2*a)*sin(6*b*x + 4*d
) + 6*e^(2*a)*sin(4*b*x + 2*d) - 3*e^(2*a)*sin(2*b*x + 4*d))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(77) = 154$.

Time = 0.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.34

$$\int e^{2(a+ibx)}\sin^4(d+bx)\,dx =$$
$$-\frac{48\,(bx+d)\cos\left(2\,d\right)e^{(2\,a)}-48i\,(bx+d)e^{(2\,a)}\sin\left(2\,d\right)+i\left(e^{(6i\,bx+4i\,d)}+e^{(-6i\,bx-4i\,d)}\right)e^{(2\,a)}-6i\left(e^{(4i\,bx+2}\right.}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*sin(b*x+d)^4,x, algorithm="giac")
```

output
```
-1/192*(48*(b*x + d)*cos(2*d)*e^(2*a) - 48*I*(b*x + d)*e^(2*a)*sin(2*d) +
I*(e^(6*I*b*x + 4*I*d) + e^(-6*I*b*x - 4*I*d))*e^(2*a) - 6*I*(e^(4*I*b*x +
 2*I*d) + e^(-4*I*b*x - 2*I*d))*e^(2*a) - 3*I*(e^(2*I*b*x + 4*I*d) + e^(-2
*I*b*x - 4*I*d))*e^(2*a) + 18*I*(e^(2*I*b*x) + e^(-2*I*b*x))*e^(2*a) - 36*
e^(2*a)*sin(2*b*x) - 6*e^(2*a)*sin(2*b*x + 4*d) - 12*e^(2*a)*sin(-4*b*x -
2*d) + 2*e^(2*a)*sin(-6*b*x - 4*d))/b
```

Mupad [B] (verification not implemented)

Time = 15.68 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.11

$$\int e^{2(a+ibx)}\sin^4(d+bx)\,dx = -\frac{x\,\mathrm{e}^{2\,a}\,\left(\cos\left(2\,d\right)-\sin\left(2\,d\right)\,1\mathrm{i}\right)}{4} - \frac{\mathrm{e}^{2\,a}\,\left(\cos\left(2\,b\,x\right)+\sin\left(2\,b\,x\right)\,1\mathrm{i}\right)\,3\mathrm{i}}{16\,b} + \frac{\mathrm{e}^{2\,a}\,\left(\cos\left(2\,b\,x\right)-\sin\left(2\,b\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(4\,d\right)-\sin\left(4\,d\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{32\,b} + \frac{\mathrm{e}^{2\,a}\,\left(\cos\left(4\,b\,x\right)+\sin\left(4\,b\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(2\,d\right)+\sin\left(2\,d\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{16\,b} - \frac{\mathrm{e}^{2\,a}\,\left(\cos\left(6\,b\,x\right)+\sin\left(6\,b\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(4\,d\right)+\sin\left(4\,d\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{96\,b}$$

input
```
int(exp(2*a + b*x*2i)*sin(d + b*x)^4,x)
```

output
```
(exp(2*a)*(cos(2*b*x) - sin(2*b*x)*1i)*(cos(4*d) - sin(4*d)*1i)*1i)/(32*b)
 - (exp(2*a)*(cos(2*b*x) + sin(2*b*x)*1i)*3i)/(16*b) - (x*exp(2*a)*(cos(2*
d) - sin(2*d)*1i))/4 + (exp(2*a)*(cos(4*b*x) + sin(4*b*x)*1i)*(cos(2*d) +
sin(2*d)*1i)*1i)/(16*b) - (exp(2*a)*(cos(6*b*x) + sin(6*b*x)*1i)*(cos(4*d)
 + sin(4*d)*1i)*1i)/(96*b)
```

Reduce [F]

$$\int e^{2(a+ibx)}\sin^4(d+bx)\,dx = e^{2a}\left(\int e^{2bix}\sin\left(bx+d\right)^4dx\right)$$

input
```
int(exp(2*a+2*I*b*x)*sin(b*x+d)^4,x)
```

output
```
e**(2*a)*int(e**(2*b*i*x)*sin(b*x + d)**4,x)
```

3.9 $\int e^{\frac{5}{3}(a+ibx)} \sin(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int e^{\frac{5}{3}(a+ibx)} \sin(d+bx)\,dx = \frac{3e^{\frac{5}{3}(a-id)+\frac{2}{3}i(d+bx)}}{4b} - \frac{3e^{\frac{5}{3}(a-id)+\frac{8}{3}i(d+bx)}}{16b}$$

output
```
3/4*exp(5/3*a-5/3*I*d+2/3*I*(b*x+d))/b-3/16*exp(5/3*a-5/3*I*d+8/3*I*(b*x+d
))/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int e^{\frac{5}{3}(a+ibx)} \sin(d+bx)\,dx = -\frac{3e^{\frac{5a}{3}+\frac{2ibx}{3}}\left((-4+e^{2ibx})\cos(d)+i(4+e^{2ibx})\sin(d)\right)}{16b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Sin[d + b*x],x]
```

output
```
(-3*E^((5*a)/3 + ((2*I)/3)*b*x)*((-4 + E^((2*I)*b*x))*Cos[d] + I*(4 + E^((
2*I)*b*x))*Sin[d]))/(16*b)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\sin(bx+d)\,dx$$

$$\Big\downarrow \text{4932}$$

$$\frac{9e^{\frac{5}{3}(a+ibx)}\cos(bx+d)}{16b}-\frac{15ie^{\frac{5}{3}(a+ibx)}\sin(bx+d)}{16b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Sin[d + b*x],x]
```

output
```
(9*E^((5*(a + I*b*x))/3)*Cos[d + b*x])/(16*b) - (((15*I)/16)*E^((5*(a + I*
b*x))/3)*Sin[d + b*x])/b
```

Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

method	result	size
parallelrisch	$-\frac{3(5i\sin(bx+d)-3\cos(bx+d))\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}}{16b}$	34
default	$\frac{9\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\cos(bx+d)}{16b}-\frac{15i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\sin(bx+d)}{16b}$	45
orering	$-\frac{15i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\sin(bx+d)}{8b}+\frac{\frac{15ib\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\sin(bx+d)}{16}+\frac{9\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}b\cos(bx+d)}{16}}{b^2}$	68
norman	$\frac{\frac{9\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}}{16b}-\frac{9\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2}{16b}-\frac{15i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{8b}}{1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2}$	84

input
```
int(exp(5/3*a+5/3*I*b*x)*sin(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-3/16*(5*I*sin(b*x+d)-3*cos(b*x+d))*exp(5/3*a+5/3*I*b*x)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.49

$$\int e^{\frac{5}{3}(a+ibx)}\sin(d+bx)\,dx = -\frac{3\left(e^{\left(\frac{8}{3}i\,bx+\frac{5}{3}\,a+i\,d\right)}-4\,e^{\left(\frac{2}{3}i\,bx+\frac{5}{3}\,a-i\,d\right)}\right)}{16\,b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d),x, algorithm="fricas")
```

output
```
-3/16*(e^(8/3*I*b*x + 5/3*a + I*d) - 4*e^(2/3*I*b*x + 5/3*a - I*d))/b
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.46

$$\int e^{\frac{5}{3}(a+ibx)}\sin(d+bx)\,dx = \begin{cases} \frac{\left(-12be^{\frac{5a}{3}}e^{2id}e^{\frac{8ibx}{3}}+48be^{\frac{5a}{3}}e^{\frac{2ibx}{3}}\right)e^{-id}}{64b^2} & \text{for}\ b^2e^{id}\neq 0 \\ \frac{x\left(-ie^{\frac{5a}{3}}e^{2id}+ie^{\frac{5a}{3}}\right)e^{-id}}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d),x)
```

output
```
Piecewise(((-12*b*exp(5*a/3)*exp(2*I*d)*exp(8*I*b*x/3) + 48*b*exp(5*a/3)*e
xp(2*I*b*x/3))*exp(-I*d)/(64*b**2), Ne(b**2*exp(I*d), 0)), (x*(-I*exp(5*a/
3)*exp(2*I*d) + I*exp(5*a/3))*exp(-I*d)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.51

$$\int e^{\frac{5}{3}(a+ibx)}\sin(d+bx)\,dx = \frac{3\,(3\,b\cos{(bx+d)} - 5i\,b\sin{(bx+d)})e^{\left(\frac{5}{3}\,i\,bx+\frac{5}{3}\,a\right)}}{16\,b^2}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d),x, algorithm="maxima")
```

output
```
3/16*(3*b*cos(b*x + d) - 5*I*b*sin(b*x + d))*e^(5/3*I*b*x + 5/3*a)/b^2
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(35) = 70.

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.31

$$\int e^{\frac{5}{3}(a+ibx)}\sin(d+bx)\,dx =$$
$$-\frac{3\left(\left(e^{\left(\frac{8}{3}\,i\,bx+i\,d\right)} - e^{\left(-\frac{8}{3}\,i\,bx-i\,d\right)}\right)e^{\left(\frac{5}{3}\,a\right)} - 4\left(e^{\left(\frac{2}{3}\,i\,bx-i\,d\right)} - e^{\left(-\frac{2}{3}\,i\,bx+i\,d\right)}\right)e^{\left(\frac{5}{3}\,a\right)} + 2\,\cos\left(\frac{8}{3}\,bx+d\right)e^{\left(\frac{5}{3}\,a\right)} - 8\,\mathrm{c}}{32\,b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d),x, algorithm="giac")
```

output
```
-3/32*((e^(8/3*I*b*x + I*d) - e^(-8/3*I*b*x - I*d))*e^(5/3*a) - 4*(e^(2/3*
I*b*x - I*d) - e^(-2/3*I*b*x + I*d))*e^(5/3*a) + 2*cos(8/3*b*x + d)*e^(5/3
*a) - 8*cos(-2/3*b*x + d)*e^(5/3*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63

$$\int e^{\frac{5}{3}(a+ibx)} \sin(d+bx) \, dx = \frac{3\,\mathrm{e}^{\frac{5\,a}{3}}\,\left(\cos\left(\frac{5\,b\,x}{3}\right)+\sin\left(\frac{5\,b\,x}{3}\right)\,1{}\mathrm{i}\right)\,\left(3\,\cos\left(d+b\,x\right)-\sin\left(d+b\,x\right)\,5{}\mathrm{i}\right)}{16\,b}$$

input

```
int(exp((5*a)/3 + (b*x*5i)/3)*sin(d + b*x),x)
```

output

```
(3*exp((5*a)/3)*(cos((5*b*x)/3) + sin((5*b*x)/3)*1i)*(3*cos(d + b*x) - sin
(d + b*x)*5i))/(16*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int e^{\frac{5}{3}(a+ibx)} \sin(d+bx) \, dx = \frac{3e^{\frac{5bix}{3}+\frac{5a}{3}} (3\cos(bx+d) - 5\sin(bx+d)\, i)}{16b}$$

input

```
int(exp(5/3*a+5/3*I*b*x)*sin(b*x+d),x)
```

output

```
(3*e**((5*a + 5*b*i*x)/3)*(3*cos(b*x + d) - 5*sin(b*x + d)*i))/(16*b)
```

3.10 $\int e^{\frac{5}{3}(a+ibx)}\sin^2(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int e^{\frac{5}{3}(a+ibx)}\sin^2(d+bx)\,dx = -\frac{3ie^{\frac{5}{3}(a-id)-\frac{1}{3}i(d+bx)}}{4b}-\frac{3ie^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{10b}+\frac{3ie^{\frac{5}{3}(a-id)+\frac{11}{3}i(d+bx)}}{44b}$$

output
```
-3/4*I*exp(5/3*a-5/3*I*d-1/3*I*(b*x+d))/b-3/10*I*exp(5/3*a-5/3*I*d+5/3*I*(
b*x+d))/b+3/44*I*exp(5/3*a-5/3*I*d+11/3*I*(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int e^{\frac{5}{3}(a+ibx)}\sin^2(d+bx)\,dx = \frac{3ie^{\frac{5a}{3}-\frac{ibx}{3}}\left(-22e^{2ibx}+5\left(-11+e^{4ibx}\right)\cos(2d)+5i\left(11+e^{4ibx}\right)\sin(2d)\right)}{220b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Sin[d + b*x]^2,x]
```

output
```
(((3*I)/220)*E^((5*a)/3 - (I/3)*b*x)*(-22*E^((2*I)*b*x) + 5*(-11 + E^((4*I
)*b*x))*Cos[2*d] + (5*I)*(11 + E^((4*I)*b*x))*Sin[2*d]))/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4934, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)} \sin^2(bx+d) \, dx$$

$$\Big\downarrow \text{4934}$$

$$\frac{18}{11} \int e^{\frac{5}{3}(a+ibx)} dx + \frac{15ie^{\frac{5}{3}(a+ibx)} \sin^2(bx+d)}{11b} - \frac{18e^{\frac{5}{3}(a+ibx)} \sin(bx+d) \cos(bx+d)}{11b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{15ie^{\frac{5}{3}(a+ibx)} \sin^2(bx+d)}{11b} - \frac{18e^{\frac{5}{3}(a+ibx)} \sin(bx+d) \cos(bx+d)}{11b} - \frac{54ie^{\frac{5}{3}(a+ibx)}}{55b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Sin[d + b*x]^2,x]
```

output
```
(((-54*I)/55)*E^((5*(a + I*b*x))/3))/b - (18*E^((5*(a + I*b*x))/3)*Cos[d +
 b*x]*Sin[d + b*x])/(11*b) + (((15*I)/11)*E^((5*(a + I*b*x))/3)*Sin[d + b*
x]^2)/b
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.41

method	result
parallelrisch	$-\frac{3\left(25 i \cos \left(2 b x +2 d \right)+11 i +30 \sin \left(2 b x +2 d \right)\right) {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}}}{110 b}$
default	$-\frac{3 i {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}}}{10 b}-\frac{15 i {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \cos \left(2 b x +2 d \right)}{22 b}-\frac{9 \,{\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \sin \left(2 b x +2 d \right)}{11 b}$
norman	$\frac{-\frac{36 \,{\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)}{11 b}+\frac{36 \,{\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{3}}{11 b}-\frac{54 i {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}}}{55 b}+\frac{192 i {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{2}}{55 b}-\frac{54 i {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \tan \left(\frac{b x}{2}+\frac{d}{2}\right.}{55 b}}{\left(1+\tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{2}\right)^{2}}$
orering	$\frac{117 i {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \sin \left(b x +d \right)^{2}}{55 b}-\frac{27 \left(\frac{5 i b \,{\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \sin \left(b x +d \right)^{2}}{3}+2 \,{\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \sin \left(b x +d \right) b \cos \left(b x +d \right)\right)}{11 b^{2}}-\frac{27 i \left(-\frac{43 b^{2} {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \mathrm{s}}{9}\right.}{}$

input
```
int(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/110*(25*I*cos(2*b*x+2*d)+11*I+30*sin(2*b*x+2*d))*exp(5/3*a+5/3*I*b*x)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int e^{\frac{5}{3}(a+ibx)}\sin^2(d+bx)\,dx = -\frac{3\left(-5i\,e^{\left(4i\,bx+\frac{5}{3}\,a+\frac{7}{3}\,i\,d\right)}+22i\,e^{\left(2i\,bx+\frac{5}{3}\,a+\frac{1}{3}\,i\,d\right)}+55i\,e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,i\,d\right)}\right)e^{\left(-\frac{1}{3}\,i\,bx-\frac{1}{3}\,i\,d\right)}}{220\,b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^2,x, algorithm="fricas")
```

output
```
-3/220*(-5*I*e^(4*I*b*x + 5/3*a + 7/3*I*d) + 22*I*e^(2*I*b*x + 5/3*a + 1/3
*I*d) + 55*I*e^(5/3*a - 5/3*I*d))*e^(-1/3*I*b*x - 1/3*I*d)/b
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.44

$$\int e^{\frac{5}{3}(a+ibx)}\sin^2(d+bx)\,dx = \begin{cases} \frac{\left(120ib^2e^{\frac{5a}{3}}e^{4id}e^{\frac{11ibx}{3}}-528ib^2e^{\frac{5a}{3}}e^{2id}e^{\frac{5ibx}{3}}-1320ib^2e^{\frac{5a}{3}}e^{-\frac{ibx}{3}}\right)e^{-2id}}{1760b^3} & \text{for } b^3e^{2id}\neq 0 \\ \frac{x\left(-e^{\frac{5a}{3}}e^{4id}+2e^{\frac{5a}{3}}e^{2id}-e^{\frac{5a}{3}}\right)e^{-2id}}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)**2,x)
```

output
```
Piecewise(((120*I*b**2*exp(5*a/3)*exp(4*I*d)*exp(11*I*b*x/3) - 528*I*b**2*
exp(5*a/3)*exp(2*I*d)*exp(5*I*b*x/3) - 1320*I*b**2*exp(5*a/3)*exp(-I*b*x/3
))*exp(-2*I*d)/(1760*b**3), Ne(b**3*exp(2*I*d), 0)), (x*(-exp(5*a/3)*exp(4
*I*d) + 2*exp(5*a/3)*exp(2*I*d) - exp(5*a/3))*exp(-2*I*d)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int e^{\frac{5}{3}(a+ibx)}\sin^2(d+bx)\,dx$$
$$=\frac{3\left(-22i\cos\left(\frac{5}{3}bx\right)e^{\left(\frac{5}{3}a\right)}+5i\cos\left(\frac{11}{3}bx+2d\right)e^{\left(\frac{5}{3}a\right)}-55i\cos\left(\frac{1}{3}bx+2d\right)e^{\left(\frac{5}{3}a\right)}+22e^{\left(\frac{5}{3}a\right)}\sin\left(\frac{5}{3}bx\right)-\right.}{220b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^2,x, algorithm="maxima")
```

output
```
3/220*(-22*I*cos(5/3*b*x)*e^(5/3*a) + 5*I*cos(11/3*b*x + 2*d)*e^(5/3*a) -
55*I*cos(1/3*b*x + 2*d)*e^(5/3*a) + 22*e^(5/3*a)*sin(5/3*b*x) - 5*e^(5/3*a
)*sin(11/3*b*x + 2*d) - 55*e^(5/3*a)*sin(1/3*b*x + 2*d))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(49) = 98$.

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int e^{\frac{5}{3}(a+ibx)}\sin^2(d+bx)\,dx =$$
$$-\frac{3\left(-5i\left(e^{\left(\frac{11}{3}ibx+2id\right)}+e^{\left(-\frac{11}{3}ibx-2id\right)}\right)e^{\left(\frac{5}{3}a\right)}+55i\left(e^{\left(\frac{1}{3}ibx+2id\right)}+e^{\left(-\frac{1}{3}ibx-2id\right)}\right)e^{\left(\frac{5}{3}a\right)}+22i\left(e^{\left(\frac{5}{3}ibx\right)}+e^{\left(\right.}\right.\right.}{440b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^2,x, algorithm="giac")
```

output
```
-3/440*(-5*I*(e^(11/3*I*b*x + 2*I*d) + e^(-11/3*I*b*x - 2*I*d))*e^(5/3*a)
+ 55*I*(e^(1/3*I*b*x + 2*I*d) + e^(-1/3*I*b*x - 2*I*d))*e^(5/3*a) + 22*I*(
e^(5/3*I*b*x) + e^(-5/3*I*b*x))*e^(5/3*a) - 44*e^(5/3*a)*sin(5/3*b*x) + 10
*e^(5/3*a)*sin(11/3*b*x + 2*d) + 110*e^(5/3*a)*sin(1/3*b*x + 2*d))/b
```

Mupad [B] (verification not implemented)

Time = 15.50 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.40

$$\int e^{\frac{5}{3}(a+ibx)}\sin^2(d+bx)\,dx = -\frac{3\,\mathrm{e}^{\frac{5\,a}{3}+\frac{b\,x\,5\mathrm{i}}{3}}\,\left(\cos\left(2\,d+2\,b\,x\right)\,25\mathrm{i}+30\,\sin\left(2\,d+2\,b\,x\right)+11\mathrm{i}\right)}{110\,b}$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)*sin(d + b*x)^2,x)
```

output
```
-(3*exp((5*a)/3 + (b*x*5i)/3)*(cos(2*d + 2*b*x)*25i + 30*sin(2*d + 2*b*x)
+ 11i))/(110*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.44

$$\int e^{\frac{5}{3}(a+ibx)}\sin^2(d+bx)\,dx = \frac{3e^{\frac{5bix}{3}+\frac{5a}{3}}\left(-30\cos\left(bx+d\right)\sin\left(bx+d\right)+25\sin\left(bx+d\right)^2 i-18i\right)}{55b}$$

input
```
int(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^2,x)
```

output
```
(3*e**((5*a + 5*b*i*x)/3)*( - 30*cos(b*x + d)*sin(b*x + d) + 25*sin(b*x +
d)**2*i - 18*i))/(55*b)
```

3.11 $\int e^{\frac{5}{3}(a+ibx)}\sin^3(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int e^{\frac{5}{3}(a+ibx)}\sin^3(d+bx)\,dx = \frac{9e^{\frac{5}{3}(a-id)+\frac{2}{3}i(d+bx)}}{16b} + \frac{3e^{\frac{5}{3}(a-id)-\frac{4}{3}i(d+bx)}}{32b} - \frac{9e^{\frac{5}{3}(a-id)+\frac{8}{3}i(d+bx)}}{64b} + \frac{3e^{\frac{5}{3}(a-id)+\frac{14}{3}i(d+bx)}}{112b}$$

output
```
9/16*exp(5/3*a-5/3*I*d+2/3*I*(b*x+d))/b+3/32*exp(5/3*a-5/3*I*d-4/3*I*(b*x+
d))/b-9/64*exp(5/3*a-5/3*I*d+8/3*I*(b*x+d))/b+3/112*exp(5/3*a-5/3*I*d+14/3
*I*(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int e^{\frac{5}{3}(a+ibx)}\sin^3(d+bx)\,dx = \frac{3e^{\frac{5a}{3}-\frac{4ibx}{3}}\left(-21e^{2ibx}\left(-4+e^{2ibx}\right)\cos(d)+2\left(7+2e^{6ibx}\right)\cos(3d)-21ie^{2ibx}\left(4+e^{2ibx}\right)\sin(d)+2i\left(-7+2e^{6\right.}\right.}{448b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Sin[d + b*x]^3,x]
```

output
```
(3*E^((5*a)/3 - ((4*I)/3)*b*x)*(-21*E^((2*I)*b*x)*(-4 + E^((2*I)*b*x))*Cos
[d] + 2*(7 + 2*E^((6*I)*b*x))*Cos[3*d] - (21*I)*E^((2*I)*b*x)*(4 + E^((2*I
)*b*x))*Sin[d] + (2*I)*(-7 + 2*E^((6*I)*b*x))*Sin[3*d]))/(448*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4934, 4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\sin^3(bx+d)\,dx$$

$$\Big\downarrow \text{4934}$$

$$\frac{27}{28}\int e^{\frac{5}{3}(a+ibx)}\sin(d+bx)dx+\frac{15ie^{\frac{5}{3}(a+ibx)}\sin^3(bx+d)}{56b}-\frac{27e^{\frac{5}{3}(a+ibx)}\sin^2(bx+d)\cos(bx+d)}{56b}$$

$$\Big\downarrow \text{4932}$$

$$\frac{15ie^{\frac{5}{3}(a+ibx)}\sin^3(bx+d)}{56b}-\frac{27e^{\frac{5}{3}(a+ibx)}\sin^2(bx+d)\cos(bx+d)}{56b}+\frac{27}{28}\left(\frac{9e^{\frac{5}{3}(a+ibx)}\cos(bx+d)}{16b}-\frac{15ie^{\frac{5}{3}(a+ibx)}\sin(bx+d)}{16b}\right)$$

input
```
Int[E^((5*(a + I*b*x))/3)*Sin[d + b*x]^3,x]
```

output
```
(-27*E^((5*(a + I*b*x))/3)*Cos[d + b*x]*Sin[d + b*x]^2)/(56*b) + (((15*I)/
56)*E^((5*(a + I*b*x))/3)*Sin[d + b*x]^3)/b + (27*((9*E^((5*(a + I*b*x))/3
)*Cos[d + b*x])/(16*b) - (((15*I)/16)*E^((5*(a + I*b*x))/3)*Sin[d + b*x])/
b))/28
```

Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.44

method	result
parallelrisch	$-\frac{3\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \left(105 i \sin \left(bx +d \right)+10 i \sin \left(3bx +3d \right)-63 \cos \left(bx +d \right)-18 \cos \left(3bx +3d \right)\right)}{448 b}$
default	$\frac{27\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \cos \left(3bx +3d \right)}{224 b}-\frac{15 i \mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \sin \left(3bx +3d \right)}{224 b}+\frac{27\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \cos \left(bx +d \right)}{64 b}-\frac{45 i \mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \sin \left(bx +d \right)}{64 b}$
norman	$\frac{\frac{243\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}}{448 b}-\frac{621\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \tan \left(\frac{bx}{2}+\frac{d}{2}\right)^{2}}{448 b}+\frac{621\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \tan \left(\frac{bx}{2}+\frac{d}{2}\right)^{4}}{448 b}-\frac{243\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \tan \left(\frac{bx}{2}+\frac{d}{2}\right)^{6}}{448 b}-\frac{405 i \mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \tan \left(\frac{bx}{2}+\right.}{224 b}}{\left(1+\tan \left(\frac{bx}{2}+\frac{d}{2}\right)^{2}\right)^{3}}$
orering	$-\frac{75 i \mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \sin \left(bx +d \right)^{3}}{56 b}-\frac{135 \left(\frac{5 i b\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \sin \left(bx +d \right)^{3}}{3}+3\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \sin \left(bx +d \right)^{2} b \cos \left(bx +d \right)\right)}{224 b^{2}}-\frac{135 i \left(-\frac{52 b^{2} \mathrm{e}^{\frac{5a}{3}+}}{}\right.}{}$

input
```
int(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/448*exp(5/3*a+5/3*I*b*x)/b*(105*I*sin(b*x+d)+10*I*sin(3*b*x+3*d)-63*cos
(b*x+d)-18*cos(3*b*x+3*d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.52

$$\int e^{\frac{5}{3}(a+ibx)}\sin^3(d+bx)\,dx = \frac{3\left(4\,e^{\left(6i\,bx+\frac{5}{3}\,a+\frac{13}{3}\,i\,d\right)}-21\,e^{\left(4i\,bx+\frac{5}{3}\,a+\frac{7}{3}\,i\,d\right)}+84\,e^{\left(2i\,bx+\frac{5}{3}\,a+\frac{1}{3}\,i\,d\right)}+14\,e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,i\,d\right)}\right)e^{\left(-\frac{4}{3}\,i\,bx-\frac{4}{3}\,i\,d\right)}}{448\,b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^3,x, algorithm="fricas")
```

output
```
3/448*(4*e^(6*I*b*x + 5/3*a + 13/3*I*d) - 21*e^(4*I*b*x + 5/3*a + 7/3*I*d)
 + 84*e^(2*I*b*x + 5/3*a + 1/3*I*d) + 14*e^(5/3*a - 5/3*I*d))*e^(-4/3*I*b*
x - 4/3*I*d)/b
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.56

$$\int e^{\frac{5}{3}(a+ibx)}\sin^3(d+bx)\,dx = \begin{cases} \frac{\left(98304b^3e^{\frac{5a}{3}}e^{7id}e^{\frac{14ibx}{3}}-516096b^3e^{\frac{5a}{3}}e^{5id}e^{\frac{8ibx}{3}}+2064384b^3e^{\frac{5a}{3}}e^{3id}e^{\frac{2ibx}{3}}+344064b^3e^{\frac{5a}{3}}e^{id}e^{-\frac{4ibx}{3}}\right)e^{-4id}}{3670016b^4} & \text{for } b^4e^{4id}\neq 0 \\ \frac{x\left(ie^{\frac{5a}{3}}e^{6id}-3ie^{\frac{5a}{3}}e^{4id}+3ie^{\frac{5a}{3}}e^{2id}-ie^{\frac{5a}{3}}\right)e^{-3id}}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)**3,x)
```

output
```
Piecewise(((98304*b**3*exp(5*a/3)*exp(7*I*d)*exp(14*I*b*x/3) - 516096*b**3
*exp(5*a/3)*exp(5*I*d)*exp(8*I*b*x/3) + 2064384*b**3*exp(5*a/3)*exp(3*I*d)
*exp(2*I*b*x/3) + 344064*b**3*exp(5*a/3)*exp(I*d)*exp(-4*I*b*x/3))*exp(-4*
I*d)/(3670016*b**4), Ne(b**4*exp(4*I*d), 0)), (x*(I*exp(5*a/3)*exp(6*I*d)
- 3*I*exp(5*a/3)*exp(4*I*d) + 3*I*exp(5*a/3)*exp(2*I*d) - I*exp(5*a/3))*ex
p(-3*I*d)/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int e^{\frac{5}{3}(a+ibx)}\sin^3(d+bx)\,dx$$
$$= \frac{3\left(4\cos\left(\frac{14}{3}bx+3d\right)e^{\left(\frac{5}{3}a\right)}-21\cos\left(\frac{8}{3}bx+d\right)e^{\left(\frac{5}{3}a\right)}+14\cos\left(\frac{4}{3}bx+3d\right)e^{\left(\frac{5}{3}a\right)}+84\cos\left(\frac{2}{3}bx-d\right)e^{\left(\frac{5}{3}\right.}\right.}{}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^3,x, algorithm="maxima")
```

output
```
3/448*(4*cos(14/3*b*x + 3*d)*e^(5/3*a) - 21*cos(8/3*b*x + d)*e^(5/3*a) + 1
4*cos(4/3*b*x + 3*d)*e^(5/3*a) + 84*cos(2/3*b*x - d)*e^(5/3*a) + 4*I*e^(5/
3*a)*sin(14/3*b*x + 3*d) - 21*I*e^(5/3*a)*sin(8/3*b*x + d) - 14*I*e^(5/3*a
)*sin(4/3*b*x + 3*d) + 84*I*e^(5/3*a)*sin(2/3*b*x - d))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(69) = 138.

Time = 0.17 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.32

$$\int e^{\frac{5}{3}(a+ibx)}\sin^3(d+bx)\,dx$$
$$= \frac{3\left(4\left(e^{\left(\frac{14}{3}ibx+3id\right)}-e^{\left(-\frac{14}{3}ibx-3id\right)}\right)e^{\left(\frac{5}{3}a\right)}-21\left(e^{\left(\frac{8}{3}ibx+id\right)}-e^{\left(-\frac{8}{3}ibx-id\right)}\right)e^{\left(\frac{5}{3}a\right)}-14\left(e^{\left(\frac{4}{3}ibx+3id\right)}-e^{\left(-\frac{4}{3}ib\right.}\right.\right.}{}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^3,x, algorithm="giac")
```

output
```
3/896*(4*(e^(14/3*I*b*x + 3*I*d) - e^(-14/3*I*b*x - 3*I*d))*e^(5/3*a) - 21
*(e^(8/3*I*b*x + I*d) - e^(-8/3*I*b*x - I*d))*e^(5/3*a) - 14*(e^(4/3*I*b*x
 + 3*I*d) - e^(-4/3*I*b*x - 3*I*d))*e^(5/3*a) + 84*(e^(2/3*I*b*x - I*d) -
e^(-2/3*I*b*x + I*d))*e^(5/3*a) + 8*cos(14/3*b*x + 3*d)*e^(5/3*a) - 42*cos
(8/3*b*x + d)*e^(5/3*a) + 28*cos(4/3*b*x + 3*d)*e^(5/3*a) + 168*cos(-2/3*b
*x + d)*e^(5/3*a))/b
```

Mupad [B] (verification not implemented)

Time = 15.95 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.03

$$\int e^{\frac{5}{3}(a+ibx)}\sin^3(d+bx)\,dx = \frac{3\,\mathrm{e}^{\frac{5\,a}{3}}\,\left(\cos\left(\frac{4\,b\,x}{3}\right)-\sin\left(\frac{4\,b\,x}{3}\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(3\,d\right)-\sin\left(3\,d\right)\,1{}\mathrm{i}\right)}{32\,b} + \frac{3\,\mathrm{e}^{\frac{5\,a}{3}}\,\left(\cos\left(\frac{14\,b\,x}{3}\right)+\sin\left(\frac{14\,b\,x}{3}\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(3\,d\right)+\sin\left(3\,d\right)\,1{}\mathrm{i}\right)}{112\,b} + \frac{9\,\mathrm{e}^{\frac{5\,a}{3}}\,\left(\cos\left(\frac{2\,b\,x}{3}\right)+\sin\left(\frac{2\,b\,x}{3}\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(d\right)-\sin\left(d\right)\,1{}\mathrm{i}\right)}{16\,b} - \frac{9\,\mathrm{e}^{\frac{5\,a}{3}}\,\left(\cos\left(\frac{8\,b\,x}{3}\right)+\sin\left(\frac{8\,b\,x}{3}\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(d\right)+\sin\left(d\right)\,1{}\mathrm{i}\right)}{64\,b}$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)*sin(d + b*x)^3,x)
```

output
```
(3*exp((5*a)/3)*(cos((4*b*x)/3) - sin((4*b*x)/3)*1i)*(cos(3*d) - sin(3*d)*
1i))/(32*b) + (3*exp((5*a)/3)*(cos((14*b*x)/3) + sin((14*b*x)/3)*1i)*(cos(
3*d) + sin(3*d)*1i))/(112*b) + (9*exp((5*a)/3)*(cos((2*b*x)/3) + sin((2*b*
x)/3)*1i)*(cos(d) - sin(d)*1i))/(16*b) - (9*exp((5*a)/3)*(cos((8*b*x)/3) +
 sin((8*b*x)/3)*1i)*(cos(d) + sin(d)*1i))/(64*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.47

$$\int e^{\frac{5}{3}(a+ibx)}\sin^3(d+bx)\,dx = \frac{3e^{\frac{5bix}{3}+\frac{5a}{3}}\left(-72\cos\left(bx+d\right)\sin\left(bx+d\right)^2+81\cos\left(bx+d\right)+40\sin\left(bx+d\right)^3i-135\sin\left(bx+d\right)i\right)}{448b}$$

input
```
int(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^3,x)
```

output
```
(3*e**((5*a + 5*b*i*x)/3)*( - 72*cos(b*x + d)*sin(b*x + d)**2 + 81*cos(b*x
 + d) + 40*sin(b*x + d)**3*i - 135*sin(b*x + d)*i))/(448*b)
```

3.12 $\int e^{\frac{5}{3}(a+ibx)}\sin^4(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 171

$$\int e^{\frac{5}{3}(a+ibx)}\sin^4(d+bx)\,dx=-\frac{3ie^{\frac{5}{3}(a-id)-\frac{1}{3}i(d+bx)}}{4b}-\frac{9ie^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{40b}+\frac{3ie^{\frac{5}{3}(a-id)-\frac{7}{3}i(d+bx)}}{112b}+\frac{3ie^{\frac{5}{3}(a-id)+\frac{11}{3}i(d+bx)}}{44b}-\frac{3ie^{\frac{5}{3}(a-id)+\frac{17}{3}i(d+bx)}}{272b}$$

output
```
-3/4*I*exp(5/3*a-5/3*I*d-1/3*I*(b*x+d))/b-9/40*I*exp(5/3*a-5/3*I*d+5/3*I*(
b*x+d))/b+3/112*I*exp(5/3*a-5/3*I*d-7/3*I*(b*x+d))/b+3/44*I*exp(5/3*a-5/3*
I*d+11/3*I*(b*x+d))/b-3/272*I*exp(5/3*a-5/3*I*d+17/3*I*(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.78

$$\int e^{\frac{5}{3}(a+ibx)}\sin^4(d+bx)\,dx=$$
$$-\frac{3ie^{\frac{5a}{3}-\frac{7ibx}{3}}\left(7854e^{4ibx}-2380e^{2ibx}\left(-11+e^{4ibx}\right)\cos(2d)+55\left(-17+7e^{8ibx}\right)\cos(4d)-26180ie^{2ibx}\sin(2d\ldots}{104720b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Sin[d + b*x]^4,x]
```

output
```
(((-3*I)/104720)*E^((5*a)/3 - ((7*I)/3)*b*x)*(7854*E^((4*I)*b*x) - 2380*E^
((2*I)*b*x)*(-11 + E^((4*I)*b*x))*Cos[2*d] + 55*(-17 + 7*E^((8*I)*b*x))*Co
s[4*d] - (26180*I)*E^((2*I)*b*x)*Sin[2*d] - (2380*I)*E^((6*I)*b*x)*Sin[2*d
] + (935*I)*Sin[4*d] + (385*I)*E^((8*I)*b*x)*Sin[4*d]))/b
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4934, 4934, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\sin^4(bx+d)\,dx$$

$$\Big\downarrow \text{4934}$$

$$\frac{108}{119}\int e^{\frac{5}{3}(a+ibx)}\sin^2(d+bx)dx+\frac{15ie^{\frac{5}{3}(a+ibx)}\sin^4(bx+d)}{119b}-\frac{36e^{\frac{5}{3}(a+ibx)}\sin^3(bx+d)\cos(bx+d)}{119b}$$

$$\Big\downarrow \text{4934}$$

$$\frac{108}{119}\left(\frac{18}{11}\int e^{\frac{5}{3}(a+ibx)}dx+\frac{15ie^{\frac{5}{3}(a+ibx)}\sin^2(bx+d)}{11b}-\frac{18e^{\frac{5}{3}(a+ibx)}\sin(bx+d)\cos(bx+d)}{11b}\right)+\frac{15ie^{\frac{5}{3}(a+ibx)}\sin^4(bx+d)}{119b}-\frac{36e^{\frac{5}{3}(a+ibx)}\sin^3(bx+d)\cos(bx+d)}{119b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{15ie^{\frac{5}{3}(a+ibx)}\sin^4(bx+d)}{119b}-\frac{36e^{\frac{5}{3}(a+ibx)}\sin^3(bx+d)\cos(bx+d)}{119b}+\frac{108}{119}\left(\frac{15ie^{\frac{5}{3}(a+ibx)}\sin^2(bx+d)}{11b}-\frac{18e^{\frac{5}{3}(a+ibx)}\sin(bx+d)\cos(bx+d)}{11b}-\frac{54ie^{\frac{5}{3}(a+ibx)}}{55b}\right)$$

input
```
Int[E^((5*(a + I*b*x))/3)*Sin[d + b*x]^4,x]
```

output
```
(-36*E^((5*(a + I*b*x))/3)*Cos[d + b*x]*Sin[d + b*x]^3)/(119*b) + (((15*I)
/119)*E^((5*(a + I*b*x))/3)*Sin[d + b*x]^4)/b + (108*((((-54*I)/55)*E^((5*
(a + I*b*x))/3))/b - (18*E^((5*(a + I*b*x))/3)*Cos[d + b*x]*Sin[d + b*x])/
(11*b) + (((15*I)/11)*E^((5*(a + I*b*x))/3)*Sin[d + b*x]^2)/b))/119
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.38

method	result
parallelrisch	$\frac{3(275i\cos(4bx+4d)-11900i\cos(2bx+2d)-3927i+660\sin(4bx+4d)-14280\sin(2bx+2d))\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}}{52360b}$
default	$-\frac{9i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}}{40b}+\frac{15i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\cos(4bx+4d)}{952b}+\frac{9\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\sin(4bx+4d)}{238b}-\frac{15i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\cos(2bx+2d)}{22b}-\frac{9\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\,\mathrm{s}}{11}$
norman	$\frac{-\frac{3888\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{1309b}-\frac{1008\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^3}{187b}+\frac{1008\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^5}{187b}+\frac{3888\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^7}{1309b}-\frac{5832}{}}{\left(1+\tan\right.}$
orering	$\frac{15573i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\sin(bx+d)^4}{6545b}-\frac{2610\left(\frac{5ib\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\sin(bx+d)^4}{3}+4\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\sin(bx+d)^3b\cos(bx+d)\right)}{1309b^2}+\frac{54i\left(-\frac{61b^2\mathrm{e}^{\frac{5a}{3}+}}{}\right.}{}$

input
```
int(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
3/52360*(275*I*cos(4*b*x+4*d)-11900*I*cos(2*b*x+2*d)-3927*I+660*sin(4*b*x+
4*d)-14280*sin(2*b*x+2*d))*exp(5/3*a+5/3*I*b*x)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.47

$$\int e^{\frac{5}{3}(a+ibx)}\sin^4(d+bx)\,dx = -\frac{3\left(385i\,e^{\left(8i\,bx+\frac{5}{3}\,a+\frac{19}{3}\,i\,d\right)}-2380i\,e^{\left(6i\,bx+\frac{5}{3}\,a+\frac{13}{3}\,i\,d\right)}+7854i\,e^{\left(4i\,bx+\frac{5}{3}\,a+\frac{7}{3}\,i\,d\right)}+26180i\,e^{\left(2i\,bx+\frac{5}{3}\,a+\frac{1}{3}\,i\,d\right)}-935i}{104720\,b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^4,x, algorithm="fricas")
```

output
```
-3/104720*(385*I*e^(8*I*b*x + 5/3*a + 19/3*I*d) - 2380*I*e^(6*I*b*x + 5/3*
a + 13/3*I*d) + 7854*I*e^(4*I*b*x + 5/3*a + 7/3*I*d) + 26180*I*e^(2*I*b*x
+ 5/3*a + 1/3*I*d) - 935*I*e^(5/3*a - 5/3*I*d))*e^(-7/3*I*b*x - 7/3*I*d)/b
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.44

$$\int e^{\frac{5}{3}(a+ibx)}\sin^4(d+bx)\,dx$$

$$= \begin{cases} \frac{\left(-2365440ib^4e^{\frac{5a}{3}}e^{10id}e^{\frac{17ibx}{3}}+14622720ib^4e^{\frac{5a}{3}}e^{8id}e^{\frac{11ibx}{3}}-48254976ib^4e^{\frac{5a}{3}}e^{6id}e^{\frac{5ibx}{3}}-160849920ib^4e^{\frac{5a}{3}}e^{4id}e^{-\frac{ibx}{3}}+5744640ib^4e^{\frac{5a}{3}}e^{2id}e}{214466560b^5} \\ \frac{x\left(e^{\frac{5a}{3}}e^{8id}-4e^{\frac{5a}{3}}e^{6id}+6e^{\frac{5a}{3}}e^{4id}-4e^{\frac{5a}{3}}e^{2id}+e^{\frac{5a}{3}}\right)e^{-4id}}{16} \end{cases}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)**4,x)
```

output
```
Piecewise(((-2365440*I*b**4*exp(5*a/3)*exp(10*I*d)*exp(17*I*b*x/3) + 14622
720*I*b**4*exp(5*a/3)*exp(8*I*d)*exp(11*I*b*x/3) - 48254976*I*b**4*exp(5*a
/3)*exp(6*I*d)*exp(5*I*b*x/3) - 160849920*I*b**4*exp(5*a/3)*exp(4*I*d)*exp
(-I*b*x/3) + 5744640*I*b**4*exp(5*a/3)*exp(2*I*d)*exp(-7*I*b*x/3))*exp(-6*
I*d)/(214466560*b**5), Ne(b**5*exp(6*I*d), 0)), (x*(exp(5*a/3)*exp(8*I*d)
- 4*exp(5*a/3)*exp(6*I*d) + 6*exp(5*a/3)*exp(4*I*d) - 4*exp(5*a/3)*exp(2*I
*d) + exp(5*a/3))*exp(-4*I*d)/16, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\int e^{\frac{5}{3}(a+ibx)}\sin^4(d+bx)\,dx =$$

$$-\frac{3\left(7854i\cos\left(\frac{5}{3}bx\right)e^{\left(\frac{5}{3}a\right)}+385i\cos\left(\frac{17}{3}bx+4d\right)e^{\left(\frac{5}{3}a\right)}-2380i\cos\left(\frac{11}{3}bx+2d\right)e^{\left(\frac{5}{3}a\right)}-935i\cos\left(\frac{7}{3}b\right.\right.}{}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^4,x, algorithm="maxima")
```

output
```
-3/104720*(7854*I*cos(5/3*b*x)*e^(5/3*a) + 385*I*cos(17/3*b*x + 4*d)*e^(5/
3*a) - 2380*I*cos(11/3*b*x + 2*d)*e^(5/3*a) - 935*I*cos(7/3*b*x + 4*d)*e^(
5/3*a) + 26180*I*cos(1/3*b*x + 2*d)*e^(5/3*a) - 7854*e^(5/3*a)*sin(5/3*b*x
) - 385*e^(5/3*a)*sin(17/3*b*x + 4*d) + 2380*e^(5/3*a)*sin(11/3*b*x + 2*d)
 - 935*e^(5/3*a)*sin(7/3*b*x + 4*d) + 26180*e^(5/3*a)*sin(1/3*b*x + 2*d))/
b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(83) = 166.

Time = 0.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.13

$$\int e^{\frac{5}{3}(a+ibx)}\sin^4(d+bx)\,dx =$$

$$-\frac{3\left(385i\left(e^{\left(\frac{17}{3}ibx+4id\right)}+e^{\left(-\frac{17}{3}ibx-4id\right)}\right)e^{\left(\frac{5}{3}a\right)}-2380i\left(e^{\left(\frac{11}{3}ibx+2id\right)}+e^{\left(-\frac{11}{3}ibx-2id\right)}\right)e^{\left(\frac{5}{3}a\right)}-935i\left(e^{\left(\frac{7}{3}ibx\right.}\right.\right.}{}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^4,x, algorithm="giac")
```

output
```
-3/209440*(385*I*(e^(17/3*I*b*x + 4*I*d) + e^(-17/3*I*b*x - 4*I*d))*e^(5/3
*a) - 2380*I*(e^(11/3*I*b*x + 2*I*d) + e^(-11/3*I*b*x - 2*I*d))*e^(5/3*a)
- 935*I*(e^(7/3*I*b*x + 4*I*d) + e^(-7/3*I*b*x - 4*I*d))*e^(5/3*a) + 26180
*I*(e^(1/3*I*b*x + 2*I*d) + e^(-1/3*I*b*x - 2*I*d))*e^(5/3*a) + 7854*I*(e^
(5/3*I*b*x) + e^(-5/3*I*b*x))*e^(5/3*a) - 15708*e^(5/3*a)*sin(5/3*b*x) - 7
70*e^(5/3*a)*sin(17/3*b*x + 4*d) + 4760*e^(5/3*a)*sin(11/3*b*x + 2*d) - 18
70*e^(5/3*a)*sin(7/3*b*x + 4*d) + 52360*e^(5/3*a)*sin(1/3*b*x + 2*d))/b
```

Mupad [B] (verification not implemented)

Time = 15.81 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int e^{\frac{5}{3}(a+ibx)}\sin^4(d+bx)\,dx = -\frac{e^{\frac{5a}{3}}\left(\cos\left(\frac{5bx}{3}\right)+\sin\left(\frac{5bx}{3}\right)1i\right)9i}{40b} - \frac{e^{\frac{5a}{3}}\left(\cos\left(\frac{bx}{3}\right)-\sin\left(\frac{bx}{3}\right)1i\right)\left(\cos(2d)-\sin(2d)1i\right)3i}{4b} + \frac{e^{\frac{5a}{3}}\left(\cos\left(\frac{7bx}{3}\right)-\sin\left(\frac{7bx}{3}\right)1i\right)\left(\cos(4d)-\sin(4d)1i\right)3i}{112b} + \frac{e^{\frac{5a}{3}}\left(\cos\left(\frac{11bx}{3}\right)+\sin\left(\frac{11bx}{3}\right)1i\right)\left(\cos(2d)+\sin(2d)1i\right)3i}{44b} - \frac{e^{\frac{5a}{3}}\left(\cos\left(\frac{17bx}{3}\right)+\sin\left(\frac{17bx}{3}\right)1i\right)\left(\cos(4d)+\sin(4d)1i\right)3i}{272b}$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)*sin(d + b*x)^4,x)
```

output
```
(exp((5*a)/3)*(cos((7*b*x)/3) - sin((7*b*x)/3)*1i)*(cos(4*d) - sin(4*d)*1i
)*3i)/(112*b) - (exp((5*a)/3)*(cos((b*x)/3) - sin((b*x)/3)*1i)*(cos(2*d) -
 sin(2*d)*1i)*3i)/(4*b) - (exp((5*a)/3)*(cos((5*b*x)/3) + sin((5*b*x)/3)*1
i)*9i)/(40*b) + (exp((5*a)/3)*(cos((11*b*x)/3) + sin((11*b*x)/3)*1i)*(cos(
2*d) + sin(2*d)*1i)*3i)/(44*b) - (exp((5*a)/3)*(cos((17*b*x)/3) + sin((17*
b*x)/3)*1i)*(cos(4*d) + sin(4*d)*1i)*3i)/(272*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\sin^4(d+bx)\,dx=\int e^{\frac{5bix}{3}+\frac{5a}{3}}\sin{(bx+d)}^4\,dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*sin(b*x+d)^4,x)
```

output
```
int(e**((5*a + 5*b*i*x)/3)*sin(b*x + d)**4,x)
```

3.13 $\int F^{c(a+bx)} \sin(d+ex) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int F^{c(a+bx)} \sin(d+ex) \, dx = -\frac{F^{c(a+bx)}(e\cos(d+ex) - bc\log(F)\sin(d+ex))}{e^2+b^2c^2\log^2(F)}$$

output
```
-F^(c*(b*x+a))*(e*cos(e*x+d)-b*c*ln(F)*sin(e*x+d))/(e^2+b^2*c^2*ln(F)^2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int F^{c(a+bx)} \sin(d+ex) \, dx = \frac{F^{c(a+bx)}(-e\cos(d+ex) + bc\log(F)\sin(d+ex))}{e^2+b^2c^2\log^2(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sin[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(-(e*Cos[d + e*x]) + b*c*Log[F]*Sin[d + e*x]))/(e^2 + b^2
*c^2*Log[F]^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \ 4932$$

$$\frac{bc\log(F)\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}-\frac{e\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}$$

input
```
Int[F^(c*(a + b*x))*Sin[d + e*x],x]
```

output
```
-((e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)) + (b*c*F^(c*(
a + b*x))*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{F^{c(bx+a)}(bc\ln(F)\sin(ex+d)-e\cos(ex+d))}{e^2+b^2c^2\ln(F)^2}$	49
risch	$-\frac{e\,F^{c(bx+a)}\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{cb\ln(F)F^{c(bx+a)}\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}$	74
orering	$\frac{2cb\ln(F)F^{c(bx+a)}\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}-\frac{F^{c(bx+a)}bc\ln(F)\sin(ex+d)+F^{c(bx+a)}e\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}$	97
norman	$\frac{\frac{e\,\mathrm{e}^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}-\frac{e\,\mathrm{e}^{c(bx+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2bc\ln(F)\mathrm{e}^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$	130

input
```
int(F^(c*(b*x+a))*sin(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
F^(c*(b*x+a))*(b*c*ln(F)*sin(e*x+d)-e*cos(e*x+d))/(e^2+b^2*c^2*ln(F)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\sin(d+ex)\,dx=\frac{(bc\log{(F)}\sin{(ex+d)}-e\cos{(ex+d)})F^{bcx+ac}}{b^2c^2\log{(F)}^2+e^2}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="fricas")
```

output
```
(b*c*log(F)*sin(e*x + d) - e*cos(e*x + d))*F^(b*c*x + a*c)/(b^2*c^2*log(F)
^2 + e^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 286, normalized size of antiderivative = 5.84

$$\int F^{c(a+bx)}\sin(d+ex)\,dx$$
$$= \begin{cases} x\sin(d) & \text{for } F=1\wedge e=0\\ F^{ac}x\sin(d) & \text{for } b=0\wedge e=0\\ x\sin(d) & \text{for } c=0\wedge e=0\\ -\frac{F^{ac+bcx}x\sin(ibcx\log(F)-d)}{2}+\frac{iF^{ac+bcx}x\cos(ibcx\log(F)-d)}{2}-\frac{iF^{ac+bcx}\cos(ibcx\log(F)-d)}{2bc\log(F)} & \text{for } e=-ibc\log(F)\\ \frac{F^{ac+bcx}x\sin(ibcx\log(F)+d)}{2}-\frac{iF^{ac+bcx}x\cos(ibcx\log(F)+d)}{2}+\frac{iF^{ac+bcx}\cos(ibcx\log(F)+d)}{2bc\log(F)} & \text{for } e=ibc\log(F)\\ \frac{F^{ac+bcx}bc\log(F)\sin(d+ex)}{b^2c^2\log(F)^2+e^2}-\frac{F^{ac+bcx}e\cos(d+ex)}{b^2c^2\log(F)^2+e^2} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*sin(e*x+d),x)
```

output
```
Piecewise((x*sin(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sin(d), Eq(b, 0) &
Eq(e, 0)), (x*sin(d), Eq(c, 0) & Eq(e, 0)), (-F**(a*c + b*c*x)*x*sin(I*b*c
*x*log(F) - d)/2 + I*F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F) - d)/2 - I*F**(
a*c + b*c*x)*cos(I*b*c*x*log(F) - d)/(2*b*c*log(F)), Eq(e, -I*b*c*log(F)))
, (F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F) + d)/2 - I*F**(a*c + b*c*x)*x*cos
(I*b*c*x*log(F) + d)/2 + I*F**(a*c + b*c*x)*cos(I*b*c*x*log(F) + d)/(2*b*c
*log(F)), Eq(e, I*b*c*log(F))), (F**(a*c + b*c*x)*b*c*log(F)*sin(d + e*x)/
(b**2*c**2*log(F)**2 + e**2) - F**(a*c + b*c*x)*e*cos(d + e*x)/(b**2*c**2*
log(F)**2 + e**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(48) = 96.

Time = 0.04 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.96

$$\int F^{c(a+bx)}\sin(d+ex)\,dx =$$
$$-\frac{(F^{ac}bc\log(F)\sin(d)+F^{ac}e\cos(d))F^{bcx}\cos(ex+2d)-(F^{ac}bc\log(F)\sin(d)-F^{ac}e\cos(d))F^{bcx}\,\text{co}}{2\left(b^2c^2\cos(d)^2\log(F)^2+b^2c\right.}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="maxima")
```

output
```
-1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2
*d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) -
(F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) -
(F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))/(b^2*c^
2*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^
2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 634, normalized size of antiderivative = 12.94

$$\int F^{c(a+bx)}\sin(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="giac")
```

output
```
(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn
(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi
*b*c + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 2*e)*cos(1/2*pi*b*c*x*sgn(F) -
1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(ab
s(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F))) - (2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1
/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*
c*sgn(F) - pi*b*c - 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c - 2*e)*cos(1/2*pi*b*
c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b
^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs
(F)) + a*c*log(abs(F))) - (-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x +
1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I
*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I
*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*c
*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) +
a*c*log(abs(F))) - (I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi
*a*c*sgn(F) - 1/2*I*pi*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c
+ 4*b*c*log(abs(F)) - 4*I*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*
x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(F)
+ 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*l...
```

Mupad [B] (verification not implemented)

Time = 15.64 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\sin(d+ex)\,dx = -\frac{F^{a\,c+b\,c\,x}\,(e\,\cos{(d+e\,x)}-b\,c\,\sin{(d+e\,x)}\,\ln{(F)})}{b^2\,c^2\ln{(F)}^2+e^2}$$

input
```
int(F^(c*(a + b*x))*sin(d + e*x),x)
```

output
```
-(F^(a*c + b*c*x)*(e*cos(d + e*x) - b*c*sin(d + e*x)*log(F)))/(e^2 + b^2*c
^2*log(F)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\sin(d+ex)\,dx = \frac{f^{bcx+ac}(-\cos{(ex+d)}\,e+\log(f)\sin{(ex+d)}\,bc)}{\log{(f)}^2\,b^2c^2+e^2}$$

input
```
int(F^(c*(b*x+a))*sin(e*x+d),x)
```

output
```
(f**(a*c + b*c*x)*( - cos(d + e*x)*e + log(f)*sin(d + e*x)*b*c))/(log(f)**
2*b**2*c**2 + e**2)
```

3.14 $\int F^{c(a+bx)} \sin^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 102

$$\int F^{c(a+bx)} \sin^2(d+ex)\,dx = \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)} - \frac{F^{c(a+bx)}\sin(d+ex)(2e\cos(d+ex)-bc\log(F)\sin(d+ex))}{4e^2+b^2c^2\log^2(F)}$$

output
```
2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)-F^(c*(b*x+a))*sin(e*
x+d)*(2*e*cos(e*x+d)-b*c*ln(F)*sin(e*x+d))/(4*e^2+b^2*c^2*ln(F)^2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)} \sin^2(d+ex)\,dx = \frac{F^{c(a+bx)}\left(4e^2+b^2c^2\log^2(F)-b^2c^2\cos(2(d+ex))\log^2(F)-2bce\log(F)\sin(2(d+ex))\right)}{8bce^2\log(F)+2b^3c^3\log^3(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sin[d + e*x]^2,x]
```

output
```
(F^(c*(a + b*x))*(4*e^2 + b^2*c^2*Log[F]^2 - b^2*c^2*Cos[2*(d + e*x)]*Log[
F]^2 - 2*b*c*e*Log[F]*Sin[2*(d + e*x)]))/(8*b*c*e^2*Log[F] + 2*b^3*c^3*Log
[F]^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4934, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4934}$$

$$\frac{2e^2\int F^{c(a+bx)}dx}{b^2c^2\log^2(F)+4e^2}+\frac{bc\log(F)\sin^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}-\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}$$

$$\Big\downarrow \text{2624}$$

$$\frac{bc\log(F)\sin^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}-\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}$$

input
```
Int[F^(c*(a + b*x))*Sin[d + e*x]^2,x]
```

output
```
(2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F]^2)) - (2*e*F^(
c*(a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*Log[F]^2) + (b*c*
F^(c*(a + b*x))*Log[F]*Sin[d + e*x]^2)/(4*e^2 + b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

method	result
parallelrisch	$-\frac{F^{c(bx+a)}\left(\frac{b^{2}c^{2}\ln(F)^{2}\cos(2ex+2d)}{2}-\frac{b^{2}c^{2}\ln(F)^{2}}{2}+e\sin(2ex+2d)bc\ln(F)-2e^{2}\right)}{bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}$
risch	$\frac{F^{c(bx+a)}}{2bc\ln(F)}-\frac{F^{c(bx+a)}bc\ln(F)\cos(2ex+2d)}{2\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}-\frac{e\,F^{c(bx+a)}\sin(2ex+2d)}{4e^{2}+b^{2}c^{2}\ln(F)^{2}}$
orering	$\frac{\left(3b^{2}c^{2}\ln(F)^{2}+4e^{2}\right)F^{c(bx+a)}\sin(ex+d)^{2}}{bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}-\frac{3\left(F^{c(bx+a)}bc\ln(F)\sin(ex+d)^{2}+2F^{c(bx+a)}\sin(ex+d)e\cos(ex+d)\right)}{4e^{2}+b^{2}c^{2}\ln(F)^{2}}+\frac{F^{c(b}\cdots}{\cdots}$
norman	$\frac{-\frac{4e\,\mathrm{e}^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{4e^{2}+b^{2}c^{2}\ln(F)^{2}}+\frac{4e\,\mathrm{e}^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{4e^{2}+b^{2}c^{2}\ln(F)^{2}}+\frac{2e^{2}\mathrm{e}^{c(bx+a)\ln(F)}}{bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}+\frac{2e^{2}\mathrm{e}^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{4}}{bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}+\frac{4\cdots}{\cdots}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}\right)^{2}}$

input
```
int(F^(c*(b*x+a))*sin(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-F^(c*(b*x+a))*(1/2*b^2*c^2*ln(F)^2*cos(2*e*x+2*d)-1/2*b^2*c^2*ln(F)^2+e*s
in(2*e*x+2*d)*b*c*ln(F)-2*e^2)/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

$$\int F^{c(a+bx)} \sin^2(d+ex)\,dx = -\frac{\left(2\,bce\cos\left(ex+d\right)\log\left(F\right)\sin\left(ex+d\right)+\left(b^2c^2\cos\left(ex+d\right)^2-b^2c^2\right)\log\left(F\right)^2-2\,e^2\right)F^{bcx+ac}}{b^3c^3\log\left(F\right)^3+4\,bce^2\log\left(F\right)}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="fricas")
```

output
```
-(2*b*c*e*cos(e*x + d)*log(F)*sin(e*x + d) + (b^2*c^2*cos(e*x + d)^2 - b^2
*c^2)*log(F)^2 - 2*e^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + 4*b*c*e^2*log(
F))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 740, normalized size of antiderivative = 7.25

$$\int F^{c(a+bx)} \sin^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*sin(e*x+d)**2,x)
```

output
```
Piecewise((x*sin(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (x*sin
(d + e*x)**2/2 + x*cos(d + e*x)**2/2 - sin(d + e*x)*cos(d + e*x)/(2*e), Eq
(F, 1)), (F**(a*c)*(x*sin(d + e*x)**2/2 + x*cos(d + e*x)**2/2 - sin(d + e*
x)*cos(d + e*x)/(2*e)), Eq(b, 0)), (x*sin(d + e*x)**2/2 + x*cos(d + e*x)**
2/2 - sin(d + e*x)*cos(d + e*x)/(2*e), Eq(c, 0)), (F**(a*c + b*c*x)*x*sin(
I*b*c*x*log(F)/2 - d)**2/4 - I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 - d
)*cos(I*b*c*x*log(F)/2 - d)/2 - F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 -
d)**2/4 + F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 - d)**2/(b*c*log(F)) - I*F
**(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 - d)*cos(I*b*c*x*log(F)/2 - d)/(2*b*c
*log(F)), Eq(e, -I*b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/
2 + d)**2/4 - I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 + d)*cos(I*b*c*x*l
og(F)/2 + d)/2 - F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 + d)**2/4 + F**(a
*c + b*c*x)*sin(I*b*c*x*log(F)/2 + d)**2/(b*c*log(F)) - I*F**(a*c + b*c*x)
*sin(I*b*c*x*log(F)/2 + d)*cos(I*b*c*x*log(F)/2 + d)/(2*b*c*log(F)), Eq(e,
 I*b*c*log(F)/2)), (F**(a*c + b*c*x)*b**2*c**2*log(F)**2*sin(d + e*x)**2/(
b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) - 2*F**(a*c + b*c*x)*b*c*e*log(F)
*sin(d + e*x)*cos(d + e*x)/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2*F
**(a*c + b*c*x)*e**2*sin(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log
(F)) + 2*F**(a*c + b*c*x)*e**2*cos(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*
c*e**2*log(F)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(100) = 200$.

Time = 0.05 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.49

$$\int F^{c(a+bx)} \sin^2(d+ex)\,dx = -\frac{\left(F^{ac}b^2c^2\cos(2\,d)\log(F)^2 + 2\,F^{ac}bce\log(F)\sin(2\,d)\right)F^{bcx}\cos(2\,ex) + \left(F^{ac}b^2c^2\cos(2\,d)\log(F)^2 - 2\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d)
)*F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*
c*e*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^
2*sin(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a
*c)*b^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)
*sin(2*e*x + 4*d) - 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c
^2*log(F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*
F^(b*c*x))/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2 + 4*
(b*c*cos(2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 915, normalized size of antiderivative = 8.97

$$\int F^{c(a+bx)} \sin^2(d+ex) \, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1
/2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sg
n(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*sin(1/2*pi*b*c*x*
sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)/(4*b
^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(abs
(F)) + a*c*log(abs(F))) - 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*
x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)*log(abs(F))/(4*b^2*c^2*l
og(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c
 - 4*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*p
i*a*c - 2*e*x - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c -
4*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*cos(-1/2*pi*b*c*
x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b
^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b
*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a
*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(a
bs(F)) + a*c*log(abs(F))) + I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*
x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(4*I*pi*b*c*sgn(
F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*I*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F
) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)
/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c...
```

Mupad [B] (verification not implemented)

Time = 16.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx = \frac{F^{a\,c+b\,c\,x}\left(2\,e^2+\frac{b^2\,c^2\,\ln(F)^2}{2}-\frac{b^2\,c^2\,\ln(F)^2\,\cos(2\,d+2\,e\,x)}{2}-b\,c\,e\,\ln\left(F\right)\,\sin\left(2\,d+2\,e\,x\right)\right)}{b\,c\,\ln\left(F\right)\,\left(b^2\,c^2\,\ln\left(F\right)^2+4\,e^2\right)}$$

input
```
int(F^(c*(a + b*x))*sin(d + e*x)^2,x)
```

output
```
(F^(a*c + b*c*x)*(2*e^2 + (b^2*c^2*log(F)^2)/2 - (b^2*c^2*log(F)^2*cos(2*d
 + 2*e*x))/2 - b*c*e*log(F)*sin(2*d + 2*e*x)))/(b*c*log(F)*(4*e^2 + b^2*c^
2*log(F)^2))
```

Reduce [F]

$$\int F^{c(a+bx)} \sin^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx} \sin{(ex+d)}^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sin(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sin(d + e*x)**2,x)
```

3.15 $\int F^{c(a+bx)}\sin^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 130

$$\begin{aligned}\int F^{c(a+bx)}\sin^3(d+ex)\,dx &= -\frac{6e^2F^{c(a+bx)}(e\cos(d+ex)-bc\log(F)\sin(d+ex))}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}\\ &\quad-\frac{F^{c(a+bx)}\sin^2(d+ex)(3e\cos(d+ex)-bc\log(F)\sin(d+ex))}{9e^2+b^2c^2\log^2(F)}\end{aligned}$$

output
```
-6*e^2*F^(c*(b*x+a))*(e*cos(e*x+d)-b*c*ln(F)*sin(e*x+d))/(9*e^4+10*b^2*c^2
*e^2*ln(F)^2+b^4*c^4*ln(F)^4)-F^(c*(b*x+a))*sin(e*x+d)^2*(3*e*cos(e*x+d)-b
*c*ln(F)*sin(e*x+d))/(9*e^2+b^2*c^2*ln(F)^2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.18

$$\begin{aligned}\int F^{c(a+bx)}\sin^3(d+ex)\,dx\\ = \frac{F^{c(a+bx)}\left(-3e\cos(d+ex)\left(9e^2+b^2c^2\log^2(F)\right)+3\cos(3(d+ex))\left(e^3+b^2c^2e\log^2(F)\right)-2bc\log(F)\left(-1\right.\right.}{4\left(9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(\right.}\end{aligned}$$

input
```
Integrate[F^(c*(a + b*x))*Sin[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(-3*e*Cos[d + e*x]*(9*e^2 + b^2*c^2*Log[F]^2) + 3*Cos[3*(
d + e*x)]*(e^3 + b^2*c^2*e*Log[F]^2) - 2*b*c*Log[F]*(-13*e^2 - b^2*c^2*Log
[F]^2 + Cos[2*(d + e*x)]*(e^2 + b^2*c^2*Log[F]^2))*Sin[d + e*x]))/(4*(9*e^
4 + 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4934, 4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4934}$$

$$\frac{6e^2\int F^{c(a+bx)}\sin(d+ex)dx}{b^2c^2\log^2(F)+9e^2}+\frac{bc\log(F)\sin^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}-\frac{3e\sin^2(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}$$

$$\Big\downarrow \text{4932}$$

$$\frac{bc\log(F)\sin^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}-\frac{3e\sin^2(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{6e^2\left(\frac{bc\log(F)\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}-\frac{e\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}\right)}{b^2c^2\log^2(F)+9e^2}$$

input
```
Int[F^(c*(a + b*x))*Sin[d + e*x]^3,x]
```

output
```
(-3*e*F^(c*(a + b*x))*Cos[d + e*x]*Sin[d + e*x]^2)/(9*e^2 + b^2*c^2*Log[F]
^2) + (b*c*F^(c*(a + b*x))*Log[F]*Sin[d + e*x]^3)/(9*e^2 + b^2*c^2*Log[F]^
2) + (6*e^2*(-((e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2))
+ (b*c*F^(c*(a + b*x))*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)))/(9*
e^2 + b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

method	result
parallelrisch	$\frac{3\left(\left(b^2c^2\ln(F)^2e+e^3\right)\cos(3ex+3d)-\frac{bc\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)\sin(3ex+3d)}{3}+\left(9e^2+b^2c^2\ln(F)^2\right)\left(bc\ln(F)\sin(ex+d)-e\cos(ex\right.\right.}{4b^4c^4\ln(F)^4+40b^2c^2e^2\ln(F)^2+36e^4}$
risch	$-\frac{3e\,F^{c(bx+a)}\cos(ex+d)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{3cb\ln(F)F^{c(bx+a)}\sin(ex+d)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{3e\,F^{c(bx+a)}\cos(3ex+3d)}{4\left(9e^2+b^2c^2\ln(F)^2\right)}-\frac{cb\ln(F)F^{c(bx+a)}\sin(3ex+3d)}{4\left(9e^2+b^2c^2\ln(F)^2\right)}$
norman	$\frac{-\frac{6e^3e^{c(bx+a)\ln(F)}}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+\frac{6e^3e^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{6e\left(2b^2c^2\ln(F)^2+3e^2\right)e^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+\frac{6e\left(2\right.}{}}{}$
orering	$\frac{4\ln(F)bc\left(b^2c^2\ln(F)^2+5e^2\right)F^{c(bx+a)}\sin(ex+d)^3}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{2\left(3b^2c^2\ln(F)^2+5e^2\right)\left(F^{c(bx+a)}bc\ln(F)\sin(ex+d)^3+3F^{c(bx+a)}\sin(\right.}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}$
default	$\frac{F^{ac}\left(\frac{-\frac{4e\,e^{bcx\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{4e\,e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}+\frac{8bc\ln(F)e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}+\frac{4e\left(b^2c^2\ln(F)^2+3e^2\right)e^{bcx\ln(F)}}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{4e\left(11b\right.}{}\right.}{}$

input
```
int(F^(c*(b*x+a))*sin(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
3*((b^2*c^2*ln(F)^2*e+e^3)*cos(3*e*x+3*d)-1/3*b*c*ln(F)*(e^2+b^2*c^2*ln(F)
^2)*sin(3*e*x+3*d)+(9*e^2+b^2*c^2*ln(F)^2)*(b*c*ln(F)*sin(e*x+d)-e*cos(e*x
+d)))*F^(c*(b*x+a))/(4*b^4*c^4*ln(F)^4+40*b^2*c^2*e^2*ln(F)^2+36*e^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.32

$$\int F^{c(a+bx)}\sin^3(d+ex)\,dx$$
$$=\frac{\left(3\,e^3\cos\left(ex+d\right)^3-9\,e^3\cos\left(ex+d\right)+3\left(b^2c^2e\cos\left(ex+d\right)^3-b^2c^2e\cos\left(ex+d\right)\right)\log\left(F\right)^2-\left(\left(b^3c^3\cos\right.\right.\right.}{b^4c^4\log\left(F\right)^4+10\,b^2c^2e^2\log}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="fricas")
```

output
```
(3*e^3*cos(e*x + d)^3 - 9*e^3*cos(e*x + d) + 3*(b^2*c^2*e*cos(e*x + d)^3 -
 b^2*c^2*e*cos(e*x + d))*log(F)^2 - ((b^3*c^3*cos(e*x + d)^2 - b^3*c^3)*lo
g(F)^3 + (b*c*e^2*cos(e*x + d)^2 - 7*b*c*e^2)*log(F))*sin(e*x + d))*F^(b*c
*x + a*c)/(b^4*c^4*log(F)^4 + 10*b^2*c^2*e^2*log(F)^2 + 9*e^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.56 (sec) , antiderivative size = 1773, normalized size of antiderivative = 13.64

$$\int F^{c(a+bx)} \sin^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*sin(e*x+d)**3,x)
```

output
```
Piecewise((x*sin(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sin(d)**3, Eq(b,
 0) & Eq(e, 0)), (x*sin(d)**3, Eq(c, 0) & Eq(e, 0)), (-3*F**(a*c + b*c*x)*
x*sin(I*b*c*x*log(F) - d)**3/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)
 - d)**2*cos(I*b*c*x*log(F) - d)/8 - 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(
F) - d)*cos(I*b*c*x*log(F) - d)**2/8 + 3*I*F**(a*c + b*c*x)*x*cos(I*b*c*x*
log(F) - d)**3/8 + 3*F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)**3/(8*b*c*lo
g(F)) - I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)**2*cos(I*b*c*x*log(F) -
 d)/(b*c*log(F)) + F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)*cos(I*b*c*x*lo
g(F) - d)**2/(4*b*c*log(F)) - 5*I*F**(a*c + b*c*x)*cos(I*b*c*x*log(F) - d)
**3/(8*b*c*log(F)), Eq(e, -I*b*c*log(F))), (-F**(a*c + b*c*x)*x*sin(I*b*c*
x*log(F)/3 - d)**3/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 - d)**2
*cos(I*b*c*x*log(F)/3 - d)/8 + 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 -
 d)*cos(I*b*c*x*log(F)/3 - d)**2/8 - I*F**(a*c + b*c*x)*x*cos(I*b*c*x*log(
F)/3 - d)**3/8 + F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)**3/(8*b*c*log(
F)) - 3*I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)**2*cos(I*b*c*x*log(F)
/3 - d)/(b*c*log(F)) - 15*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)*cos(I
*b*c*x*log(F)/3 - d)**2/(4*b*c*log(F)) + 11*I*F**(a*c + b*c*x)*cos(I*b*c*x
*log(F)/3 - d)**3/(8*b*c*log(F)), Eq(e, -I*b*c*log(F)/3)), (F**(a*c + b*c*
x)*x*sin(I*b*c*x*log(F)/3 + d)**3/8 - 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*l
og(F)/3 + d)**2*cos(I*b*c*x*log(F)/3 + d)/8 - 3*F**(a*c + b*c*x)*x*sin(...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(128) = 256$.

Time = 0.08 (sec) , antiderivative size = 813, normalized size of antiderivative = 6.25

$$\int F^{c(a+bx)} \sin^3(d+ex) \, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/8*((F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) - 3*F^(a*c)*b^2*c^2*e*cos(3*d)*lo
g(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(3*d) - 3*F^(a*c)*e^3*cos(3*d))*F^(b*c*
x)*cos(3*e*x) - (F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + 3*F^(a*c)*b^2*c^2*e*c
os(3*d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 3*F^(a*c)*e^3*cos(3*d
))*F^(b*c*x)*cos(3*e*x + 6*d) + 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + F^(
a*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 9*F
^(a*c)*e^3*cos(3*d))*F^(b*c*x)*cos(e*x + 4*d) - 3*(F^(a*c)*b^3*c^3*log(F)^
3*sin(3*d) - F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F
)*sin(3*d) - 9*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*cos(e*x - 2*d) + (F^(a*c)*b
^3*c^3*cos(3*d)*log(F)^3 + 3*F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*d) + F^(a*c)
*b*c*e^2*cos(3*d)*log(F) + 3*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*sin(3*e*x) +
(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - 3*F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*d)
 + F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 3*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*sin
(3*e*x + 6*d) - 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - F^(a*c)*b^2*c^2*e*l
og(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 9*F^(a*c)*e^3*sin(3
*d))*F^(b*c*x)*sin(e*x + 4*d) - 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 + F^(
a*c)*b^2*c^2*e*log(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F) + 9*F
^(a*c)*e^3*sin(3*d))*F^(b*c*x)*sin(e*x - 2*d))/(b^4*c^4*cos(3*d)^2*log(F)^
4 + b^4*c^4*log(F)^4*sin(3*d)^2 + 9*(cos(3*d)^2 + sin(3*d)^2)*e^4 + 10*(b^
2*c^2*cos(3*d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(3*d)^2)*e^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 1275, normalized size of antiderivative = 9.81

$$\int F^{c(a+bx)} \sin^3(d+ex) \, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="giac")
```

output
```
-1/4*(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*
c*sgn(F) - 1/2*pi*a*c + 3*e*x + 3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sg
n(F) - pi*b*c + 6*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 6*e)*cos(1/2*pi*b*c*x*
sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 3*e*x + 3*d)/(4*b
^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 6*e)^2))*e^(b*c*x*log(abs
(F)) + a*c*log(abs(F))) + 3/4*(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) -
 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(a
bs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 2
*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*
c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))
*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 3/4*(2*b*c*log(abs(F))*sin(1/2*
pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)
/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2) - (pi*b*c*sg
n(F) - pi*b*c - 2*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*s
gn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) -
pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*(2*b*c*log
(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*
pi*a*c - 3*e*x - 3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c -
 6*e)^2) - (pi*b*c*sgn(F) - pi*b*c - 6*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi
*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 3*e*x - 3*d)/(4*b^2*c^2*log(a...
```

Mupad [B] (verification not implemented)

Time = 16.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.46

$$\int F^{c(a+bx)}\sin^3(d+ex)\,dx = -\frac{3\,F^{c\,(a+b\,x)}\,\left(\cos\left(e\,x\right)-\sin\left(e\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(d\right)-\sin\left(d\right)\,1\mathrm{i}\right)}{8\,\left(e+b\,c\,\ln\left(F\right)\,1\mathrm{i}\right)} + \frac{F^{c\,(a+b\,x)}\,\left(\cos\left(3\,e\,x\right)+\sin\left(3\,e\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,d\right)+\sin\left(3\,d\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,\left(b\,c\,\ln\left(F\right)+e\,3\mathrm{i}\right)} + \frac{F^{c\,(a+b\,x)}\,\left(\cos\left(3\,e\,x\right)-\sin\left(3\,e\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,d\right)-\sin\left(3\,d\right)\,1\mathrm{i}\right)}{8\,\left(3\,e+b\,c\,\ln\left(F\right)\,1\mathrm{i}\right)} - \frac{F^{c\,(a+b\,x)}\,\left(\cos\left(e\,x\right)+\sin\left(e\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(d\right)+\sin\left(d\right)\,1\mathrm{i}\right)\,3\mathrm{i}}{8\,\left(b\,c\,\ln\left(F\right)+e\,1\mathrm{i}\right)}$$

input
```
int(F^(c*(a + b*x))*sin(d + e*x)^3,x)
```

output
```
(F^(c*(a + b*x))*(cos(3*e*x) + sin(3*e*x)*1i)*(cos(3*d) + sin(3*d)*1i)*1i)
/(8*(e*3i + b*c*log(F))) - (3*F^(c*(a + b*x))*(cos(e*x) - sin(e*x)*1i)*(co
s(d) - sin(d)*1i))/(8*(e + b*c*log(F)*1i)) + (F^(c*(a + b*x))*(cos(3*e*x)
- sin(3*e*x)*1i)*(cos(3*d) - sin(3*d)*1i))/(8*(3*e + b*c*log(F)*1i)) - (F^
(c*(a + b*x))*(cos(e*x) + sin(e*x)*1i)*(cos(d) + sin(d)*1i)*3i)/(8*(e*1i +
 b*c*log(F)))
```

Reduce [F]

$$\int F^{c(a+bx)}\sin^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\sin\left(ex+d\right)^3dx\right)$$

input
```
int(F^(c*(b*x+a))*sin(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sin(d + e*x)**3,x)
```

3.16 $\int F^{c(a+bx)}\sin^4(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 195

$$\int F^{c(a+bx)}\sin^4(d+ex)\,dx = \frac{24e^4F^{c(a+bx)}}{bc\log(F)\left(64e^4+20b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)\right)} - \frac{12e^2F^{c(a+bx)}\sin(d+ex)(2e\cos(d+ex)-bc\log(F)\sin(d+ex))}{64e^4+20b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} - \frac{F^{c(a+bx)}\sin^3(d+ex)(4e\cos(d+ex)-bc\log(F)\sin(d+ex))}{16e^2+b^2c^2\log^2(F)}$$

output
```
24*e^4*F^(c*(b*x+a))/b/c/ln(F)/(64*e^4+20*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F
)^4)-12*e^2*F^(c*(b*x+a))*sin(e*x+d)*(2*e*cos(e*x+d)-b*c*ln(F)*sin(e*x+d))
/(64*e^4+20*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^4)-F^(c*(b*x+a))*sin(e*x+d)^
3*(4*e*cos(e*x+d)-b*c*ln(F)*sin(e*x+d))/(16*e^2+b^2*c^2*ln(F)^2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\sin^4(d+ex)\,dx=\frac{1}{8}F^{c(a+bx)}\left(\frac{3}{bc\log(F)}-\frac{4\cos(2ex)(bc\cos(2d)\log(F)+2e\sin(2d))}{4e^2+b^2c^2\log^2(F)}+\frac{\cos(4ex)(bc\cos(4d)\log(F)+4e\sin(4d))}{16e^2+b^2c^2\log^2(F)}-\frac{4(2e\cos(2d)-bc\log(F)\sin(2d))\sin(2ex)}{4e^2+b^2c^2\log^2(F)}+\frac{(4e\cos(4d)-bc\log(F)\sin(4d))\sin(4ex)}{16e^2+b^2c^2\log^2(F)}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Sin[d + e*x]^4,x]
```

output
```
(F^(c*(a + b*x))*(3/(b*c*Log[F]) - (4*Cos[2*e*x]*(b*c*Cos[2*d]*Log[F] + 2*
e*Sin[2*d]))/(4*e^2 + b^2*c^2*Log[F]^2) + (Cos[4*e*x]*(b*c*Cos[4*d]*Log[F]
 + 4*e*Sin[4*d]))/(16*e^2 + b^2*c^2*Log[F]^2) - (4*(2*e*Cos[2*d] - b*c*Log
[F]*Sin[2*d])*Sin[2*e*x])/(4*e^2 + b^2*c^2*Log[F]^2) + ((4*e*Cos[4*d] - b*
c*Log[F]*Sin[4*d])*Sin[4*e*x])/(16*e^2 + b^2*c^2*Log[F]^2)))/8
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4934, 4934, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4934}$$

$$\frac{12e^2 \int F^{c(a+bx)} \sin^2(d+ex)dx}{b^2c^2 \log^2(F)+16e^2}+\frac{bc \log(F) \sin^4(d+ex)F^{c(a+bx)}}{b^2c^2 \log^2(F)+16e^2}-\frac{4e \sin^3(d+ex) \cos(d+ex)F^{c(a+bx)}}{b^2c^2 \log^2(F)+16e^2}$$

$$\Big\downarrow \text{4934}$$

$$\frac{12e^2 \left(\frac{2e^2 \int F^{c(a+bx)}dx}{b^2c^2 \log^2(F)+4e^2}+\frac{bc \log(F) \sin^2(d+ex)F^{c(a+bx)}}{b^2c^2 \log^2(F)+4e^2}-\frac{2e \sin(d+ex) \cos(d+ex)F^{c(a+bx)}}{b^2c^2 \log^2(F)+4e^2}\right)}{b^2c^2 \log^2(F)+16e^2}+\frac{bc \log(F) \sin^4(d+ex)F^{c(a+bx)}}{b^2c^2 \log^2(F)+16e^2}-\frac{4e \sin^3(d+ex) \cos(d+ex)F^{c(a+bx)}}{b^2c^2 \log^2(F)+16e^2}$$

$$\Big\downarrow \text{2624}$$

$$\frac{bc \log(F) \sin^4(d+ex)F^{c(a+bx)}}{b^2c^2 \log^2(F)+16e^2}-\frac{4e \sin^3(d+ex) \cos(d+ex)F^{c(a+bx)}}{b^2c^2 \log^2(F)+16e^2}+\frac{12e^2 \left(\frac{bc \log(F) \sin^2(d+ex)F^{c(a+bx)}}{b^2c^2 \log^2(F)+4e^2}-\frac{2e \sin(d+ex) \cos(d+ex)F^{c(a+bx)}}{b^2c^2 \log^2(F)+4e^2}+\frac{2e^2F^{c(a+bx)}}{bc \log(F) \left(b^2c^2 \log^2(F)+4e^2\right)}\right)}{b^2c^2 \log^2(F)+16e^2}$$

input
```
Int[F^(c*(a + b*x))*Sin[d + e*x]^4,x]
```

output
```
(-4*e*F^(c*(a + b*x))*Cos[d + e*x]*Sin[d + e*x]^3)/(16*e^2 + b^2*c^2*Log[F
]^2) + (b*c*F^(c*(a + b*x))*Log[F]*Sin[d + e*x]^4)/(16*e^2 + b^2*c^2*Log[F
]^2) + (12*e^2*((2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F
]^2)) - (2*e*F^(c*(a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*L
og[F]^2) + (b*c*F^(c*(a + b*x))*Log[F]*Sin[d + e*x]^2)/(4*e^2 + b^2*c^2*Lo
g[F]^2)))/(16*e^2 + b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.97

method	result
risch	$\frac{3F^{c(bx+a)}}{8bc\ln(F)}+\frac{F^{c(bx+a)}bc\ln(F)\cos(4ex+4d)}{8b^2c^2\ln(F)^2+128e^2}+\frac{e\,F^{c(bx+a)}\sin(4ex+4d)}{2b^2c^2\ln(F)^2+32e^2}-\frac{F^{c(bx+a)}bc\ln(F)\cos(2ex+2d)}{2\left(4e^2+b^2c^2\ln(F)^2\right)}-\frac{e\,F^{c(bx+a}}{4e^2+b}$
parallelrisch	$\frac{\left(\left(-4b^4c^4\ln(F)^4-64b^2c^2e^2\ln(F)^2\right)\cos(2ex+2d)+b^2c^2\ln(F)^2\left(4e^2+b^2c^2\ln(F)^2\right)\cos(4ex+4d)+\left(-8\ln(F)^3b^3c^3e-128\ln(\right.\right.}{8\ln(F)bc\left(4e^2+b^2c^2\ln(F)^2\right)\left(16e^2+\right.}$
default	$\frac{F^{ac}\left(\frac{2F^{bcx}}{bc\ln(F)}+\frac{-\frac{16e\,\mathrm{e}^{bcx\ln(F)}\tan(ex+d)}{4e^2+b^2c^2\ln(F)^2}-\frac{4bc\ln(F)\mathrm{e}^{bcx\ln(F)}}{4e^2+b^2c^2\ln(F)^2}+\frac{4bc\ln(F)\mathrm{e}^{bcx\ln(F)}\tan(ex+d)^2}{4e^2+b^2c^2\ln(F)^2}}{1+\tan(ex+d)^2}+\frac{\frac{16e\,\mathrm{e}^{bcx\ln(F)}\tan(ex+d)}{16e^2+b^2c^2\ln(F)^2}-\frac{16e\,\mathrm{e}^{bc}}{16}}{}\right.}{}$
norman	$\frac{-\frac{48e^3\mathrm{e}^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{64e^4+20b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+\frac{48e^3\mathrm{e}^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^7}{64e^4+20b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{16\left(2b^2c^2\ln(F)^2+11e^2\right)e\,\mathrm{e}^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{64e^4+20b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+}{}$
orering	$\frac{\left(5b^4c^4\ln(F)^4+60b^2c^2e^2\ln(F)^2+64e^4\right)F^{c(bx+a)}\sin(ex+d)^4}{\ln(F)bc\left(64e^4+20b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4\right)}-\frac{10\left(b^2c^2\ln(F)^2+6e^2\right)\left(F^{c(bx+a)}bc\ln(F)\sin(ex+d)^4+4F^{c}\right.}{64e^4+20b^2c^2e^2\ln(F)^2+b^4c^4\ln}$

input
```
int(F^(c*(b*x+a))*sin(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
3/8*F^(c*(b*x+a))/b/c/ln(F)+1/8/(16*e^2+b^2*c^2*ln(F)^2)*F^(c*(b*x+a))*b*c
*ln(F)*cos(4*e*x+4*d)+1/2/(16*e^2+b^2*c^2*ln(F)^2)*e*F^(c*(b*x+a))*sin(4*e
*x+4*d)-1/2/(4*e^2+b^2*c^2*ln(F)^2)*F^(c*(b*x+a))*b*c*ln(F)*cos(2*e*x+2*d)
-e*F^(c*(b*x+a))/(4*e^2+b^2*c^2*ln(F)^2)*sin(2*e*x+2*d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.19

$$\int F^{c(a+bx)} \sin^4(d+ex)\,dx = \frac{\left(\left(b^4c^4\cos(ex+d)^4 - 2\,b^4c^4\cos(ex+d)^2 + b^4c^4\right)\log(F)^4 + 24\,e^4 + 4\left(b^2c^2e^2\cos(ex+d)^4 - 5\,b^2c^2e^2\cos\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^4,x, algorithm="fricas")
```

output
```
((b^4*c^4*cos(e*x + d)^4 - 2*b^4*c^4*cos(e*x + d)^2 + b^4*c^4)*log(F)^4 +
24*e^4 + 4*(b^2*c^2*e^2*cos(e*x + d)^4 - 5*b^2*c^2*e^2*cos(e*x + d)^2 + 4*
b^2*c^2*e^2)*log(F)^2 + 4*((b^3*c^3*e*cos(e*x + d)^3 - b^3*c^3*e*cos(e*x +
 d))*log(F)^3 + 2*(2*b*c*e^3*cos(e*x + d)^3 - 5*b*c*e^3*cos(e*x + d))*log(
F))*sin(e*x + d))*F^(b*c*x + a*c)/(b^5*c^5*log(F)^5 + 20*b^3*c^3*e^2*log(F
)^3 + 64*b*c*e^4*log(F))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.63 (sec) , antiderivative size = 2538, normalized size of antiderivative = 13.02

$$\int F^{c(a+bx)} \sin^4(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*sin(e*x+d)**4,x)
```

output
```
Piecewise((x*sin(d)**4, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (3*x*s
in(d + e*x)**4/8 + 3*x*sin(d + e*x)**2*cos(d + e*x)**2/4 + 3*x*cos(d + e*x
)**4/8 - 5*sin(d + e*x)**3*cos(d + e*x)/(8*e) - 3*sin(d + e*x)*cos(d + e*x
)**3/(8*e), Eq(F, 1)), (F**(a*c)*(3*x*sin(d + e*x)**4/8 + 3*x*sin(d + e*x)
**2*cos(d + e*x)**2/4 + 3*x*cos(d + e*x)**4/8 - 5*sin(d + e*x)**3*cos(d +
e*x)/(8*e) - 3*sin(d + e*x)*cos(d + e*x)**3/(8*e)), Eq(b, 0)), (3*x*sin(d
+ e*x)**4/8 + 3*x*sin(d + e*x)**2*cos(d + e*x)**2/4 + 3*x*cos(d + e*x)**4/
8 - 5*sin(d + e*x)**3*cos(d + e*x)/(8*e) - 3*sin(d + e*x)*cos(d + e*x)**3/
(8*e), Eq(c, 0)), (F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 - d)**4/4 - I*F
**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 - d)**3*cos(I*b*c*x*log(F)/2 - d)/2
 - I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 - d)*cos(I*b*c*x*log(F)/2 - d
)**3/2 - F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 - d)**4/4 + F**(a*c + b*c
*x)*sin(I*b*c*x*log(F)/2 - d)**4/(24*b*c*log(F)) + 17*I*F**(a*c + b*c*x)*s
in(I*b*c*x*log(F)/2 - d)**3*cos(I*b*c*x*log(F)/2 - d)/(12*b*c*log(F)) + F*
*(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 - d)**2*cos(I*b*c*x*log(F)/2 - d)**2/(
b*c*log(F)) + 3*I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 - d)*cos(I*b*c*x*l
og(F)/2 - d)**3/(4*b*c*log(F)) + 5*F**(a*c + b*c*x)*cos(I*b*c*x*log(F)/2 -
 d)**4/(8*b*c*log(F)), Eq(e, -I*b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sin(I*
b*c*x*log(F)/4 - d)**4/16 - I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/4 - d)
**3*cos(I*b*c*x*log(F)/4 - d)/4 - 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(192) = 384$.

Time = 0.09 (sec) , antiderivative size = 1046, normalized size of antiderivative = 5.36

$$\int F^{c(a+bx)} \sin^4(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^4,x, algorithm="maxima")
```

output
```
1/16*((F^(a*c)*b^4*c^4*cos(4*d)*log(F)^4 + 4*F^(a*c)*b^3*c^3*e*log(F)^3*si
n(4*d) + 4*F^(a*c)*b^2*c^2*e^2*cos(4*d)*log(F)^2 + 16*F^(a*c)*b*c*e^3*log(
F)*sin(4*d))*F^(b*c*x)*cos(4*e*x) + (F^(a*c)*b^4*c^4*cos(4*d)*log(F)^4 - 4
*F^(a*c)*b^3*c^3*e*log(F)^3*sin(4*d) + 4*F^(a*c)*b^2*c^2*e^2*cos(4*d)*log(
F)^2 - 16*F^(a*c)*b*c*e^3*log(F)*sin(4*d))*F^(b*c*x)*cos(4*e*x + 8*d) - 4*
(F^(a*c)*b^4*c^4*cos(4*d)*log(F)^4 - 2*F^(a*c)*b^3*c^3*e*log(F)^3*sin(4*d)
 + 16*F^(a*c)*b^2*c^2*e^2*cos(4*d)*log(F)^2 - 32*F^(a*c)*b*c*e^3*log(F)*si
n(4*d))*F^(b*c*x)*cos(2*e*x + 6*d) - 4*(F^(a*c)*b^4*c^4*cos(4*d)*log(F)^4
+ 2*F^(a*c)*b^3*c^3*e*log(F)^3*sin(4*d) + 16*F^(a*c)*b^2*c^2*e^2*cos(4*d)*
log(F)^2 + 32*F^(a*c)*b*c*e^3*log(F)*sin(4*d))*F^(b*c*x)*cos(2*e*x - 2*d)
- (F^(a*c)*b^4*c^4*log(F)^4*sin(4*d) - 4*F^(a*c)*b^3*c^3*e*cos(4*d)*log(F)
^3 + 4*F^(a*c)*b^2*c^2*e^2*log(F)^2*sin(4*d) - 16*F^(a*c)*b*c*e^3*cos(4*d)
*log(F))*F^(b*c*x)*sin(4*e*x) + (F^(a*c)*b^4*c^4*log(F)^4*sin(4*d) + 4*F^(
a*c)*b^3*c^3*e*cos(4*d)*log(F)^3 + 4*F^(a*c)*b^2*c^2*e^2*log(F)^2*sin(4*d)
 + 16*F^(a*c)*b*c*e^3*cos(4*d)*log(F))*F^(b*c*x)*sin(4*e*x + 8*d) - 4*(F^(
a*c)*b^4*c^4*log(F)^4*sin(4*d) + 2*F^(a*c)*b^3*c^3*e*cos(4*d)*log(F)^3 + 1
6*F^(a*c)*b^2*c^2*e^2*log(F)^2*sin(4*d) + 32*F^(a*c)*b*c*e^3*cos(4*d)*log(
F))*F^(b*c*x)*sin(2*e*x + 6*d) + 4*(F^(a*c)*b^4*c^4*log(F)^4*sin(4*d) - 2*
F^(a*c)*b^3*c^3*e*cos(4*d)*log(F)^3 + 16*F^(a*c)*b^2*c^2*e^2*log(F)^2*sin(
4*d) - 32*F^(a*c)*b*c*e^3*cos(4*d)*log(F))*F^(b*c*x)*sin(2*e*x - 2*d) +...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 1554, normalized size of antiderivative = 7.97

$$\int F^{c(a+bx)} \sin^4(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^4,x, algorithm="giac")
```

output
```
1/8*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/
2*pi*a*c + 4*e*x + 4*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn
(F) - pi*b*c + 8*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 8*e)*sin(1/2*pi*b*c*x*s
gn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 4*e*x + 4*d)/(4*b^
2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 8*e)^2))*e^(b*c*x*log(abs(
F)) + a*c*log(abs(F))) - 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x
 + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*lo
g(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c
+ 4*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi
*a*c + 2*e*x + 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4
*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*(2*b*c*cos(1/2*pi*b*
c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)*
log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) +
 (pi*b*c*sgn(F) - pi*b*c - 4*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1
/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (p
i*b*c*sgn(F) - pi*b*c - 4*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) +
 1/8*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1
/2*pi*a*c - 4*e*x - 4*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sg
n(F) - pi*b*c - 8*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 8*e)*sin(1/2*pi*b*c*x*
sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 4*e*x - 4*d)/(...
```

Mupad [B] (verification not implemented)

Time = 16.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.14

$$\int F^{c(a+bx)}\sin^4(d+ex)\,dx = \frac{F^{c\,(a+b\,x)}\,(\cos(2\,e\,x)-\sin(2\,e\,x)\,1\mathrm{i})\,(\cos(2\,d)-\sin(2\,d)\,1\mathrm{i})}{4\,(-b\,c\,\ln(F)+e\,2\mathrm{i})} - \frac{F^{c\,(a+b\,x)}\,(\cos(4\,e\,x)-\sin(4\,e\,x)\,1\mathrm{i})\,(\cos(4\,d)-\sin(4\,d)\,1\mathrm{i})}{16\,(-b\,c\,\ln(F)+e\,4\mathrm{i})} + \frac{3\,F^{c\,(a+b\,x)}}{8\,b\,c\,\ln(F)} + \frac{F^{c\,(a+b\,x)}\,(\cos(2\,e\,x)+\sin(2\,e\,x)\,1\mathrm{i})\,(\cos(2\,d)+\sin(2\,d)\,1\mathrm{i})\,1\mathrm{i}}{4\,(2\,e-b\,c\,\ln(F)\,1\mathrm{i})} - \frac{F^{c\,(a+b\,x)}\,(\cos(4\,e\,x)+\sin(4\,e\,x)\,1\mathrm{i})\,(\cos(4\,d)+\sin(4\,d)\,1\mathrm{i})\,1\mathrm{i}}{16\,(4\,e-b\,c\,\ln(F)\,1\mathrm{i})}$$

input
```
int(F^(c*(a + b*x))*sin(d + e*x)^4,x)
```

output
```
(F^(c*(a + b*x))*(cos(2*e*x) - sin(2*e*x)*1i)*(cos(2*d) - sin(2*d)*1i))/(4
*(e*2i - b*c*log(F))) + (F^(c*(a + b*x))*(cos(2*e*x) + sin(2*e*x)*1i)*(cos
(2*d) + sin(2*d)*1i)*1i)/(4*(2*e - b*c*log(F)*1i)) - (F^(c*(a + b*x))*(cos
(4*e*x) - sin(4*e*x)*1i)*(cos(4*d) - sin(4*d)*1i))/(16*(e*4i - b*c*log(F))
) - (F^(c*(a + b*x))*(cos(4*e*x) + sin(4*e*x)*1i)*(cos(4*d) + sin(4*d)*1i)
*1i)/(16*(4*e - b*c*log(F)*1i)) + (3*F^(c*(a + b*x)))/(8*b*c*log(F))
```

Reduce [F]

$$\int F^{c(a+bx)}\sin^4(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\sin(ex+d)^4\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sin(e*x+d)^4,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sin(d + e*x)**4,x)
```

3.17 $\int e^{a+ibx}\sin^n(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int e^{a+ibx}\sin^n(a+bx)\,dx = -\frac{ie^{a+ibx}\left(1-e^{2i(a+bx)}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1-n}{2},-n,\frac{3-n}{2},e^{2i(a+bx)}\right)\sin^n(a+bx)}{b(1-n)}$$

output
```
-I*exp(a+I*b*x)*hypergeom([-n, 1/2-1/2*n],[3/2-1/2*n],exp(2*I*(b*x+a)))*si
n(b*x+a)^n/b/((1-exp(2*I*(b*x+a)))^n)/(1-n)
```

Mathematica [A] (verified)

Time = 6.88 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.24

$$\int e^{a+ibx}\sin^n(a+bx)\,dx = \frac{ie^{a+ibx}\left(2-2e^{2i(a+bx)}\right)^{-n}\left(-ie^{-i(a+bx)}\left(-1+e^{2i(a+bx)}\right)\right)^n\text{Hypergeometric2F1}\left(\frac{1-n}{2},-n,\frac{3-n}{2},e^{2i(a+bx)}\right)}{b(-1+n)}$$

input
```
Integrate[E^(a + I*b*x)*Sin[a + b*x]^n,x]
```

output
```
(I*E^(a + I*b*x)*(((-I)*(-1 + E^((2*I)*(a + b*x))))/E^(I*(a + b*x)))^n*Hyp
ergeometric2F1[(1 - n)/2, -n, (3 - n)/2, E^((2*I)*(a + b*x))])/(b*(2 - 2*E
^((2*I)*(a + b*x)))^n*(-1 + n))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {4940, 2683, 2682, 2681}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\sin^n(a+bx)\,dx$$

$$\Big\downarrow \text{4940}$$

$$e^{in(a+bx)}\left(-1+e^{2i(a+bx)}\right)^{-n}\sin^n(a+bx)\int e^{a+ibx-in(a+bx)}\left(-1+e^{2i(a+bx)}\right)^n dx$$

$$\Big\downarrow \text{2683}$$

$$e^{in(a+bx)}\left(-1+e^{2i(a+bx)}\right)^{-n}\sin^n(a+bx)\int e^{a(1-in)+ib(1-n)x}\left(-1+e^{2i(a+bx)}\right)^n dx$$

$$\Big\downarrow \text{2682}$$

$$e^{in(a+bx)}\left(1-e^{2i(a+bx)}\right)^{-n}\sin^n(a+bx)\int e^{a(1-in)+ib(1-n)x}\left(1-e^{2i(a+bx)}\right)^n dx$$

$$\Big\downarrow \text{2681}$$

$$-\frac{i\left(1-e^{2i(a+bx)}\right)^{-n}\exp(in(a+bx)+a(1-in)+ib(1-n)x)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},-n,\frac{3-n}{2},e^{2i(a+bx)}\right)\sin^n}{b(1-n)}$$

input
```
Int[E^(a + I*b*x)*Sin[a + b*x]^n,x]
```

output
```
((-I)*E^(a*(1 - I*n) + I*b*(1 - n)*x + I*n*(a + b*x))*Hypergeometric2F1[(1
 - n)/2, -n, (3 - n)/2, E^((2*I)*(a + b*x))]*Sin[a + b*x]^n)/(b*(1 - E^((2
*I)*(a + b*x)))^n*(1 - n))
```

Defintions of rubi rules used

rule 2681
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

rule 2682
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p   Int[G^(h*(f + g*x))*(1 + (b/a)*F^(e*(c + d*x)))^p, x], x
] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] &&  !(ILtQ[p, 0] || GtQ[a,
 0])
```

rule 2683
```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

rule 4940
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[E^(I*n*(d + e*x))*(Sin[d + e*x]^n/(-1 + E^(2*I*(d + e*x)))^n)
Int[F^(c*(a + b*x))*((-1 + E^(2*I*(d + e*x)))^n/E^(I*n*(d + e*x))), x], x]
/; FreeQ[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int e^{ibx+a} \sin (bx+a)^n \, dx$$

input
```
int(exp(a+I*b*x)*sin(b*x+a)^n,x)
```

output
```
int(exp(a+I*b*x)*sin(b*x+a)^n,x)
```

Fricas [F]

$$\int e^{a+ibx}\sin^n(a+bx)\,dx=\int \sin\left(bx+a\right)^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral((1/2*(-I*e^(2*I*b*x + 2*I*a) + I)*e^(-I*b*x - I*a))^n*e^(I*b*x +
a), x)
```

Sympy [F]

$$\int e^{a+ibx}\sin^n(a+bx)\,dx=e^a\int e^{ibx}\sin^n\left(a+bx\right)dx$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+a)**n,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*sin(a + b*x)**n, x)
```

Maxima [F]

$$\int e^{a+ibx}\sin^n(a+bx)\,dx=\int \sin\left(bx+a\right)^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^n*e^(I*b*x + a), x)
```

Giac [F]

$$\int e^{a+ibx}\sin^n(a+bx)\,dx=\int \sin(bx+a)^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*sin(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^n*e^(I*b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\sin^n(a+bx)\,dx=\int \mathrm{e}^{a+b\,x\,1\mathrm{i}}\,\sin(a+b\,x)^n\,dx$$

input
```
int(exp(a + b*x*1i)*sin(a + b*x)^n,x)
```

output
```
int(exp(a + b*x*1i)*sin(a + b*x)^n, x)
```

Reduce [F]

$$\int e^{a+ibx}\sin^n(a+bx)\,dx=\frac{e^a i\left(-e^{bix}\sin(bx+a)^n+\left(\int\frac{e^{bix}\sin(bx+a)^n\cos(bx+a)}{\sin(bx+a)}dx\right)bn\right)}{b}$$

input
```
int(exp(a+I*b*x)*sin(b*x+a)^n,x)
```

output
```
(e**a*i*( - e**(b*i*x)*sin(a + b*x)**n + int((e**(b*i*x)*sin(a + b*x)**n*c
os(a + b*x))/sin(a + b*x),x)*b*n))/b
```

3.18 $\int F^{c(a+bx)}(f\sin(d+ex))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int F^{c(a+bx)}(f\sin(d+ex))^n\,dx = -\frac{\left(1-e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(-n,\frac{-en-ibc\log(F)}{2e},\frac{1}{2}\left(2-n-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)(f\,\text{si}}{ien-bc\log(F)}$$

output
```
-F^(c*(b*x+a))*hypergeom([-n, 1/2*(-e*n-I*b*c*ln(F))/e],[1-1/2*n-1/2*I*b*c
*ln(F)/e],exp(2*I*(e*x+d)))*(f*sin(e*x+d))^n/((1-exp(2*I*(e*x+d)))^n)/(I*e
*n-b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}(f\sin(d+ex))^n\,dx = \frac{\left(1-e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(-n,-\frac{i(-ien+bc\log(F))}{2e},1-\frac{i(-ien+bc\log(F))}{2e},e^{2i(d+ex)}\right)(f\,\text{sin}}{-ien+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sin[d + e*x])^n,x]
```

output
```
(F^(c*(a + b*x))*Hypergeometric2F1[-n, ((-1/2*I)*((-I)*e*n + b*c*Log[F]))/
e, 1 - ((I/2)*((-I)*e*n + b*c*Log[F]))/e, E^((2*I)*(d + e*x))]*(f*Sin[d +
e*x])^n)/((1 - E^((2*I)*(d + e*x)))^n*((-I)*e*n + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7271, 4940, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\sin(d+ex))^n\,dx$$

$$\Big\downarrow \ 7271$$

$$\sin^{-n}(d+ex)(f\sin(d+ex))^n\int F^{c(a+bx)}\sin^n(d+ex)dx$$

$$\Big\downarrow \ 4940$$

$$e^{in(d+ex)}\left(-1+e^{2i(d+ex)}\right)^{-n}(f\sin(d+ex))^n\int e^{-in(d+ex)}\left(-1+e^{2i(d+ex)}\right)^nF^{c(a+bx)}dx$$

$$\Big\downarrow \ 2689$$

$$-\frac{\left(1-e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}(f\sin(d+ex))^n\,\text{Hypergeometric2F1}\left(-n,-\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(-n-\frac{ibc\log(F)}{e}+2\right),e^{2i}\right.}{-bc\log(F)+ien}$$

input
```
Int[F^(c*(a + b*x))*(f*Sin[d + e*x])^n,x]
```

output
```
-((F^(c*(a + b*x))*Hypergeometric2F1[-n, -1/2*(e*n + I*b*c*Log[F])/e, (2 -
 n - (I*b*c*Log[F])/e)/2, E^((2*I)*(d + e*x))]*(f*Sin[d + e*x])^n)/((1 - E
^((2*I)*(d + e*x)))^n*(I*e*n - b*c*Log[F])))
```

Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 4940
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[E^(I*n*(d + e*x))*(Sin[d + e*x]^n/(-1 + E^(2*I*(d + e*x)))^n)
Int[F^(c*(a + b*x))*((-1 + E^(2*I*(d + e*x)))^n/E^(I*n*(d + e*x))), x], x]
/; FreeQ[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}(f \sin (ex+d))^n \, dx$$

input
```
int(F^(c*(b*x+a))*(f*sin(e*x+d))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*sin(e*x+d))^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\sin(d+ex))^n\,dx=\int(f\sin(ex+d))^nF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sin(e*x+d))^n,x, algorithm="fricas")
```

output
```
integral((f*sin(e*x + d))^n*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\sin(d+ex))^n\,dx=\int F^{c(a+bx)}(f\sin(d+ex))^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sin(e*x+d))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sin(d + e*x))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\sin(d+ex))^n\,dx=\int(f\sin(ex+d))^nF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sin(e*x+d))^n,x, algorithm="maxima")
```

output
```
integrate((f*sin(e*x + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\sin(d+ex))^n\,dx=\int (f\sin(ex+d))^nF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sin(e*x+d))^n,x, algorithm="giac")
```

output
```
integrate((f*sin(e*x + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\sin(d+ex))^n\,dx=\int F^{c\,(a+b\,x)}\,(f\,\sin(d+e\,x))^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*sin(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*sin(d + e*x))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\sin(d+ex))^n\,dx=\frac{f^{ac+n}\left(f^{bcx}\sin(ex+d)^n-\left(\int\frac{f^{bcx}\sin(ex+d)^n\cos(ex+d)}{\sin(ex+d)}dx\right)en\right)}{\log(f)\,bc}$$

input
```
int(F^(c*(b*x+a))*(f*sin(e*x+d))^n,x)
```

output
```
(f**(a*c + n)*(f**(b*c*x)*sin(d + e*x)**n - int((f**(b*c*x)*sin(d + e*x)**
n*cos(d + e*x))/sin(d + e*x),x)*e*n))/(log(f)*b*c)
```

3.19 $\int F^{c(a+bx)}\left(f\sin\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx$

Optimal result

Integrand size = 31, antiderivative size = 96

$$\int F^{c(a+bx)}\left(f\sin\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx = -\frac{F^{c(a+bx)}(2+n)\left(i\cos\left(d+\frac{ibcx\log(F)}{2+n}\right)-\sin\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)\left(f\sin\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^{1+n}}{bcf(1+n)\log(F)}$$

output
```
-F^(c*(b*x+a))*(2+n)*(I*cos(d+I*b*c*x*ln(F)/(2+n))-sin(d+I*b*c*x*ln(F)/(2+
n)))*(f*sin(d+I*b*c*x*ln(F)/(2+n)))^(1+n)/b/c/f/(1+n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int F^{c(a+bx)}\left(f\sin\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx = \frac{F^{c(a+bx)}\left(1-e^{2id}F^{-\frac{2bcx}{2+n}}\right)(2+n)\left(f\sin\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n}{2bc(1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sin[d + (I*b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
(F^(c*(a + b*x))*(1 - E^((2*I)*d)/F^((2*b*c*x)/(2 + n)))*(2 + n)*(f*Sin[d
+ (I*b*c*x*Log[F])/(2 + n)])^n)/(2*b*c*(1 + n)*Log[F])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.69, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {7271, 4936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\sin\left(d+\frac{ibcx\log(F)}{n+2}\right)\right)^n dx$$

$$\Big\downarrow \text{7271}$$

$$\sin^{-n}\left(d+\frac{ibcx\log(F)}{n+2}\right)\left(f\sin\left(d+\frac{ibcx\log(F)}{n+2}\right)\right)^n\int F^{c(a+bx)}\sin^n\left(d+\frac{ibcx\log(F)}{n+2}\right)dx$$

$$\Big\downarrow \text{4936}$$

$$\sin^{-n}\left(d+\frac{ibcx\log(F)}{n+2}\right)\left(f\sin\left(d+\frac{ibcx\log(F)}{n+2}\right)\right)^n\left(\frac{(n+2)F^{c(a+bx)}\sin^{n+2}\left(d+\frac{ibcx\log(F)}{n+2}\right)}{bc(n+1)\log(F)}-\frac{i(n+2)F^{c(a}}{}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Sin[d + (I*b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
((f*Sin[d + (I*b*c*x*Log[F])/(2 + n)])^n*(((-I)*F^(c*(a + b*x))*(2 + n)*Co
s[d + (I*b*c*x*Log[F])/(2 + n)]*Sin[d + (I*b*c*x*Log[F])/(2 + n)]^(1 + n))
/(b*c*(1 + n)*Log[F]) + (F^(c*(a + b*x))*(2 + n)*Sin[d + (I*b*c*x*Log[F])/
(2 + n)]^(2 + n))/(b*c*(1 + n)*Log[F])))/Sin[d + (I*b*c*x*Log[F])/(2 + n)]
^n
```

Defintions of rubi rules used

rule 4936
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^(n + 2)/(e^2*(n + 1)
*(n + 2))), x] + Simp[F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n + 1)/(e
*(n + 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[e^2*(n + 2)^2 + b^
2*c^2*Log[F]^2, 0] && NeQ[n, -1] && NeQ[n, -2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(f\sin\left(d+\frac{ibcx\ln(F)}{n+2}\right)\right)^n dx$$

input
```
int(F^(c*(b*x+a))*(f*sin(d+I*b*c*x*ln(F)/(n+2)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*sin(d+I*b*c*x*ln(F)/(n+2)))^n,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16

$$\int F^{c(a+bx)}\left(f\sin\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n dx = -\frac{\left((n+2)e^{\left(-\frac{2\,(bcx\log(F)-i\,dn-2i\,d)}{n+2}\right)}-n-2\right)\left(\frac{1}{2}\left(-i\,fe^{\left(-\frac{2\,(bcx\log(F)-i\,dn-2i\,d)}{n+2}\right)}+i\,f\right)e^{\left(\frac{bcx\log(F)-i\,dn-2i\,d}{n+2}\right)}\right)^n F}{2\,(bcn+bc)\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(f*sin(d+I*b*c*x*log(F)/(2+n)))^n,x, algorithm="fr
icas")
```

output
```
-1/2*((n + 2)*e^(-2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) - n - 2)*(1/2*
(-I*f*e^(-2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) + I*f)*e^((b*c*x*log(F
) - I*d*n - 2*I*d)/(n + 2)))^n*F^(b*c*x + a*c)/((b*c*n + b*c)*log(F))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\sin\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n dx=\int F^{c(a+bx)}\left(f\sin\left(\frac{ibcx\log(F)}{n+2}+d\right)\right)^n dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sin(d+I*b*c*x*ln(F)/(2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sin(I*b*c*x*log(F)/(n + 2) + d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\sin\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n dx=\int\left(f\sin\left(\frac{i\,bcx\log(F)}{n+2}+d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sin(d+I*b*c*x*log(F)/(2+n)))^n,x, algorithm="ma
xima")
```

output
```
integrate((f*sin(I*b*c*x*log(F)/(n + 2) + d))^n*F^((b*x + a)*c), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(88) = 176$.

Time = 0.61 (sec) , antiderivative size = 550, normalized size of antiderivative = 5.73

$$\int F^{c(a+bx)}\left(f\sin\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n dx=\text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(f*sin(d+I*b*c*x*log(F)/(2+n)))^n,x, algorithm="gi
ac")
```

output
```
1/2*(F^(a*c)*n*e^((2*b*c*x*log(F) + I*d*n^2 - n^2*log(2) + n^2*log(I*f*e^(
2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) - I*f) + 2*I*d*n - 2*n*log(2) +
2*n*log(I*f*e^(2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) - I*f))/(n + 2) +
 2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) - F^(a*c)*n*e^((2*b*c*x*log(F)
+ I*d*n^2 - n^2*log(2) + n^2*log(I*f*e^(2*(b*c*x*log(F) - I*d*n - 2*I*d)/(
n + 2)) - I*f) + 2*I*d*n - 2*n*log(2) + 2*n*log(I*f*e^(2*(b*c*x*log(F) - I
*d*n - 2*I*d)/(n + 2)) - I*f))/(n + 2)) + 2*F^(a*c)*e^((2*b*c*x*log(F) + I
*d*n^2 - n^2*log(2) + n^2*log(I*f*e^(2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n +
 2)) - I*f) + 2*I*d*n - 2*n*log(2) + 2*n*log(I*f*e^(2*(b*c*x*log(F) - I*d*
n - 2*I*d)/(n + 2)) - I*f))/(n + 2) + 2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n
+ 2)) - 2*F^(a*c)*e^((2*b*c*x*log(F) + I*d*n^2 - n^2*log(2) + n^2*log(I*f*
e^(2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) - I*f) + 2*I*d*n - 2*n*log(2)
 + 2*n*log(I*f*e^(2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) - I*f))/(n + 2
)))/(b*c*n*e^(2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2))*log(F) + b*c*e^(2*
(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2))*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\sin\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx = \int F^{c\,(a+b\,x)}\,\left(f\,\sin\left(d+\frac{b\,c\,x\,\ln\left(F\right)\,1{}\mathrm{i}}{n+2}\right)\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*sin(d + (b*c*x*log(F)*1i)/(n + 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*sin(d + (b*c*x*log(F)*1i)/(n + 2)))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\left(f\sin\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n dx$$
$$= \frac{f^{ac+n}\left(f^{bcx}\sin\left(\frac{\log(f)bcix+dn+2d}{n+2}\right)^n n+2f^{bcx}\sin\left(\frac{\log(f)bcix+dn+2d}{n+2}\right)^n-\left(\int\frac{f^{bcx}\sin\left(\frac{\log(f)bcix+dn+2d}{n+2}\right)^n\cos\left(\frac{\log(f)bcix+dn+2d}{n+2}\right)}{\sin\left(\frac{\log(f)bcix+dn+2d}{n+2}\right)}\right.}{\log(f)\,bc\,(n+2)}$$

input
```
int(F^(c*(b*x+a))*(f*sin(d+I*b*c*x*log(F)/(2+n)))^n,x)
```

output
```
(f**(a*c + n)*(f**(b*c*x)*sin((log(f)*b*c*i*x + d*n + 2*d)/(n + 2))**n*n +
 2*f**(b*c*x)*sin((log(f)*b*c*i*x + d*n + 2*d)/(n + 2))**n - int((f**(b*c*
x)*sin((log(f)*b*c*i*x + d*n + 2*d)/(n + 2))**n*cos((log(f)*b*c*i*x + d*n
+ 2*d)/(n + 2)))/sin((log(f)*b*c*i*x + d*n + 2*d)/(n + 2)),x)*log(f)*b*c*i
*n))/(log(f)*b*c*(n + 2))
```

3.20 $\int F^{c(a+bx)}\left(f\sin\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n dx$

Optimal result

Integrand size = 31, antiderivative size = 93

$$\int F^{c(a+bx)}\left(f\sin\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n dx = \frac{F^{c(a+bx)}(2+n)\left(f\sin\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^{1+n}\left(i\cos\left(d-\frac{ibcx\log(F)}{2+n}\right)+\sin\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)}{bcf(1+n)\log(F)}$$

output
```
F^(c*(b*x+a))*(2+n)*(-f*sin(-d+I*b*c*x*ln(F)/(2+n)))^(1+n)*(I*cos(-d+I*b*c
*x*ln(F)/(2+n))-sin(-d+I*b*c*x*ln(F)/(2+n)))/b/c/f/(1+n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}\left(f\sin\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n dx = \frac{e^{-2id}F^{c\left(a+\frac{bnx}{2+n}\right)}\left(-1+e^{2id}F^{\frac{2bcx}{2+n}}\right)(2+n)\left(f\sin\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n}{2bc(1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sin[d - (I*b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
(F^(c*(a + (b*n*x)/(2 + n)))*(-1 + E^((2*I)*d)*F^((2*b*c*x)/(2 + n)))*(2 +
 n)*(f*Sin[d - (I*b*c*x*Log[F])/(2 + n)])^n)/(2*b*c*E^((2*I)*d)*(1 + n)*Lo
g[F])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {7271, 4936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\sin\left(d-\frac{ibcx\log(F)}{n+2}\right)\right)^n dx$$

$$\Big\downarrow \text{7271}$$

$$\sin^{-n}\left(d-\frac{ibcx\log(F)}{n+2}\right)\left(f\sin\left(d-\frac{ibcx\log(F)}{n+2}\right)\right)^n\int F^{c(a+bx)}\sin^n\left(d-\frac{ibcx\log(F)}{n+2}\right)dx$$

$$\Big\downarrow \text{4936}$$

$$\sin^{-n}\left(d-\frac{ibcx\log(F)}{n+2}\right)\left(f\sin\left(d-\frac{ibcx\log(F)}{n+2}\right)\right)^n\left(\frac{(n+2)F^{c(a+bx)}\sin^{n+2}\left(d-\frac{ibcx\log(F)}{n+2}\right)}{bc(n+1)\log(F)}+\frac{i(n+2)F^{c(a}}{}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Sin[d - (I*b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
((f*Sin[d - (I*b*c*x*Log[F])/(2 + n)])^n*((I*F^(c*(a + b*x))*(2 + n)*Cos[d
 - (I*b*c*x*Log[F])/(2 + n)]*Sin[d - (I*b*c*x*Log[F])/(2 + n)]^(1 + n))/(b
*c*(1 + n)*Log[F]) + (F^(c*(a + b*x))*(2 + n)*Sin[d - (I*b*c*x*Log[F])/(2
+ n)]^(2 + n))/(b*c*(1 + n)*Log[F])))/Sin[d - (I*b*c*x*Log[F])/(2 + n)]^n
```

Defintions of rubi rules used

rule 4936
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^(n + 2)/(e^2*(n + 1)
*(n + 2))), x] + Simp[F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n + 1)/(e
*(n + 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[e^2*(n + 2)^2 + b^
2*c^2*Log[F]^2, 0] && NeQ[n, -1] && NeQ[n, -2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(-f\sin\left(-d+\frac{ibcx\ln(F)}{n+2}\right)\right)^n dx$$

input
```
int(F^(c*(b*x+a))*(-f*sin(-d+I*b*c*x*ln(F)/(n+2)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(-f*sin(-d+I*b*c*x*ln(F)/(n+2)))^n,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int F^{c(a+bx)}\left(f\sin\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n dx = -\frac{\left((n+2)e^{\left(-\frac{2\,(bcx\log(F)+i\,dn+2i\,d)}{n+2}\right)}-n-2\right)\left(\frac{1}{2}\left(i\,fe^{\left(-\frac{2\,(bcx\log(F)+i\,dn+2i\,d)}{n+2}\right)}-i\,f\right)e^{\left(\frac{bcx\log(F)+i\,dn+2i\,d}{n+2}\right)}\right)^n F^{bcx}}{2\,(bcn+bc)\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(-f*sin(-d+I*b*c*x*log(F)/(2+n)))^n,x, algorithm="
fricas")
```

output
```
-1/2*((n + 2)*e^(-2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) - n - 2)*(1/2*
(I*f*e^(-2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) - I*f)*e^((b*c*x*log(F)
 + I*d*n + 2*I*d)/(n + 2)))^n*F^(b*c*x + a*c)/((b*c*n + b*c)*log(F))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\sin\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx$$
$$=\int F^{c(a+bx)}\left(-f\sin\left(\frac{ibcx\log(F)}{n+2}-d\right)\right)^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(-f*sin(-d+I*b*c*x*ln(F)/(2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(-f*sin(I*b*c*x*log(F)/(n + 2) - d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\sin\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx$$
$$=\int\left(-f\sin\left(\frac{i\,bcx\log(F)}{n+2}-d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(-f*sin(-d+I*b*c*x*log(F)/(2+n)))^n,x, algorithm="
maxima")
```

output
```
integrate((-f*sin(I*b*c*x*log(F)/(n + 2) - d))^n*F^((b*x + a)*c), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(94) = 188$.

Time = 0.58 (sec) , antiderivative size = 550, normalized size of antiderivative = 5.91

$$\int F^{c(a+bx)}\left(f\sin\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(-f*sin(-d+I*b*c*x*log(F)/(2+n)))^n,x, algorithm="
giac")
```

output
```
1/2*(F^(a*c)*n*e^((2*b*c*x*log(F) - I*d*n^2 - n^2*log(2) + n^2*log(-I*f*e^
(2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) + I*f) - 2*I*d*n - 2*n*log(2) +
 2*n*log(-I*f*e^(2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) + I*f))/(n + 2)
 + 2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) - F^(a*c)*n*e^((2*b*c*x*log(F
) - I*d*n^2 - n^2*log(2) + n^2*log(-I*f*e^(2*(b*c*x*log(F) + I*d*n + 2*I*d
)/(n + 2)) + I*f) - 2*I*d*n - 2*n*log(2) + 2*n*log(-I*f*e^(2*(b*c*x*log(F)
 + I*d*n + 2*I*d)/(n + 2)) + I*f))/(n + 2)) + 2*F^(a*c)*e^((2*b*c*x*log(F)
 - I*d*n^2 - n^2*log(2) + n^2*log(-I*f*e^(2*(b*c*x*log(F) + I*d*n + 2*I*d)
/(n + 2)) + I*f) - 2*I*d*n - 2*n*log(2) + 2*n*log(-I*f*e^(2*(b*c*x*log(F)
+ I*d*n + 2*I*d)/(n + 2)) + I*f))/(n + 2) + 2*(b*c*x*log(F) + I*d*n + 2*I*
d)/(n + 2)) - 2*F^(a*c)*e^((2*b*c*x*log(F) - I*d*n^2 - n^2*log(2) + n^2*lo
g(-I*f*e^(2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) + I*f) - 2*I*d*n - 2*n
*log(2) + 2*n*log(-I*f*e^(2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) + I*f)
)/(n + 2)))/(b*c*n*e^(2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2))*log(F) + b
*c*e^(2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2))*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\sin\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n dx = \int F^{c\,(a+b\,x)}\,{\left(f\,\sin\left(d-\frac{b\,c\,x\,\ln\left(F\right)\,1{}\mathrm{i}}{n+2}\right)\right)}^n \,d x$$

input
```
int(F^(c*(a + b*x))*(f*sin(d - (b*c*x*log(F)*1i)/(n + 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*sin(d - (b*c*x*log(F)*1i)/(n + 2)))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\left(f\sin\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n dx$$
$$=\frac{f^{ac+n}(-1)^n\left(f^{bcx}\sin\left(\frac{\log(f)bcix-dn-2d}{n+2}\right)^n n+2f^{bcx}\sin\left(\frac{\log(f)bcix-dn-2d}{n+2}\right)^n-\left(\int\frac{f^{bcx}\sin\left(\frac{\log(f)bcix-dn-2d}{n+2}\right)^n\cos\left(\right.}{\sin\left(\frac{\log(f)bcix-dn-}{n+2}\right.}\right.\right.}{\log(f)\,bc\,(n+2)}$$

input
```
int(F^(c*(b*x+a))*(-f*sin(-d+I*b*c*x*log(F)/(2+n)))^n,x)
```

output
```
(f**(a*c + n)*( - 1)**n*(f**(b*c*x)*sin((log(f)*b*c*i*x - d*n - 2*d)/(n +
2))**n*n + 2*f**(b*c*x)*sin((log(f)*b*c*i*x - d*n - 2*d)/(n + 2))**n - int
((f**(b*c*x)*sin((log(f)*b*c*i*x - d*n - 2*d)/(n + 2))**n*cos((log(f)*b*c*
i*x - d*n - 2*d)/(n + 2)))/sin((log(f)*b*c*i*x - d*n - 2*d)/(n + 2)),x)*lo
g(f)*b*c*i*n))/(log(f)*b*c*(n + 2))
```

3.21 $\int e^{a+ibx}\cos(d+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int e^{a+ibx}\cos(d+bx)\,dx=-\frac{ie^{a-id+2i(d+bx)}}{4b}+\frac{1}{2}e^{a-id}x$$

output
```
-1/4*I*exp(a-I*d+2*I*(b*x+d))/b+1/2*exp(a-I*d)*x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int e^{a+ibx}\cos(d+bx)\,dx=\frac{e^a\left(\left(-ie^{2ibx}+2bx\right)\cos(d)+\left(e^{2ibx}-2ibx\right)\sin(d)\right)}{4b}$$

input
```
Integrate[E^(a + I*b*x)*Cos[d + b*x],x]
```

output
```
(E^a*(((-I)*E^((2*I)*b*x) + 2*b*x)*Cos[d] + (E^((2*I)*b*x) - (2*I)*b*x)*Si
n[d]))/(4*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\cos(bx+d)\,dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{2}e^{a+2ibx+id}+\frac{1}{2}e^{a-id}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}xe^{a-id}-\frac{ie^{a+2ibx+id}}{4b}$$

input
```
Int[E^(a + I*b*x)*Cos[d + b*x],x]
```

output
```
((-1/4*I)*E^(a + I*d + (2*I)*b*x))/b + (E^(a - I*d)*x)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(32) = 64.

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

method	result	size
orering	$-\frac{(-2bx+i)\mathrm{e}^{ibx+a}\cos(bx+d)}{2b}+\frac{ix\left(ib\,\mathrm{e}^{ibx+a}\cos(bx+d)-\mathrm{e}^{ibx+a}b\sin(bx+d)\right)}{2b}$	71
parallelrisch	$-\frac{\mathrm{e}^{ibx+a}\left(\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}xb+2i\tan\left(\frac{bx}{2}+\frac{d}{2}\right)xb-bx-2\tan\left(\frac{bx}{2}+\frac{d}{2}\right)\right)}{2b\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}\right)}$	73
norman	$\frac{\frac{\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{b}+\frac{x\,\mathrm{e}^{ibx+a}}{2}-\frac{x\,\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}}{2}-ix\,\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}}$	93

input
```
int(exp(a+I*b*x)*cos(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*b*x+I)/b*exp(a+I*b*x)*cos(b*x+d)+1/2*I/b*x*(I*b*exp(a+I*b*x)*cos(
b*x+d)-exp(a+I*b*x)*b*sin(b*x+d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int e^{a+ibx}\cos(d+bx)\,dx=\frac{2\,bxe^{(a-i\,d)}-i\,e^{(2i\,bx+a+i\,d)}}{4\,b}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d),x, algorithm="fricas")
```

output
```
1/4*(2*b*x*e^(a - I*d) - I*e^(2*I*b*x + a + I*d))/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.57

$$\int e^{a+ibx}\cos(d+bx)\,dx=\frac{xe^ae^{-id}}{2}+\begin{cases}-\frac{ie^ae^{id}e^{2ibx}}{4b} & \text{for } b\neq 0\\ x\left(\frac{(e^ae^{2id}+e^a)e^{-id}}{2}-\frac{e^ae^{-id}}{2}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d),x)
```

output
```
x*exp(a)*exp(-I*d)/2 + Piecewise((-I*exp(a)*exp(I*d)*exp(2*I*b*x)/(4*b), N
e(b, 0)), (x*((exp(a)*exp(2*I*d) + exp(a))*exp(-I*d)/2 - exp(a)*exp(-I*d)/
2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

$$\int e^{a+ibx}\cos(d+bx)\,dx=\frac{1}{2}\,xe^{(a-i\,d)}-\frac{i\,e^{(2i\,bx+a+i\,d)}}{4\,b}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d),x, algorithm="maxima")
```

output
```
1/2*x*e^(a - I*d) - 1/4*I*e^(2*I*b*x + a + I*d)/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(25) = 50.

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.52

$$\int e^{a+ibx}\cos(d+bx)\,dx$$
$$=\frac{4\,(bx+d)\cos\left(d\right)e^{a}-4i\,(bx+d)e^{a}\sin\left(d\right)-i\left(e^{(2i\,bx+i\,d)}+e^{(-2i\,bx-i\,d)}\right)e^{a}-2\,e^{a}\sin\left(-2\,bx-d\right)}{8\,b}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d),x, algorithm="giac")
```

output
```
1/8*(4*(b*x + d)*cos(d)*e^a - 4*I*(b*x + d)*e^a*sin(d) - I*(e^(2*I*b*x + I
*d) + e^(-2*I*b*x - I*d))*e^a - 2*e^a*sin(-2*b*x - d))/b
```

Mupad [B] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int e^{a+ibx}\cos(d+bx)\,dx = \frac{\mathrm{e}^{a}\,\left(2\,x\,\cos\left(d\right)-x\,\sin\left(d\right)\,2\mathrm{i}\right)}{4} - \frac{\mathrm{e}^{a}\,\left(-\sin\left(d+2\,b\,x\right)+\cos\left(d+2\,b\,x\right)\,1\mathrm{i}\right)}{4\,b}$$

input
```
int(cos(d + b*x)*exp(a + b*x*1i),x)
```

output
```
(exp(a)*(2*x*cos(d) - x*sin(d)*2i))/4 - (exp(a)*(cos(d + 2*b*x)*1i - sin(d
 + 2*b*x)))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int e^{a+ibx}\cos(d+bx)\,dx = \frac{e^{bix+a}(\cos\left(bx+d\right)bx-\cos\left(bx+d\right)i-\sin\left(bx+d\right)bix)}{2b}$$

input
```
int(exp(a+I*b*x)*cos(b*x+d),x)
```

output
```
(e**(a + b*i*x)*(cos(b*x + d)*b*x - cos(b*x + d)*i - sin(b*x + d)*b*i*x))/
(2*b)
```

3.22 $\int e^{a+ibx}\cos^2(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int e^{a+ibx}\cos^2(d+bx)\,dx = \frac{ie^{a-id-i(d+bx)}}{4b} - \frac{ie^{a-id+i(d+bx)}}{2b} - \frac{ie^{a-id+3i(d+bx)}}{12b}$$

output
```
1/4*I*exp(a-I*d-I*(b*x+d))/b-1/2*I*exp(a-I*d+I*(b*x+d))/b-1/12*I*exp(a-I*d
+3*I*(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int e^{a+ibx}\cos^2(d+bx)\,dx = \frac{e^{a-ibx}\left(-6ie^{2ibx} - i\left(-3+e^{4ibx}\right)\cos(2d) + \left(3+e^{4ibx}\right)\sin(2d)\right)}{12b}$$

input
```
Integrate[E^(a + I*b*x)*Cos[d + b*x]^2,x]
```

output
```
(E^(a - I*b*x)*((-6*I)*E^((2*I)*b*x) - I*(-3 + E^((4*I)*b*x))*Cos[2*d] + (
3 + E^((4*I)*b*x))*Sin[2*d]))/(12*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4935, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\cos^2(bx+d)\,dx$$

$$\Big\downarrow \text{4935}$$

$$\frac{2}{3}\int e^{a+ibx}dx+\frac{ie^{a+ibx}\cos^2(bx+d)}{3b}+\frac{2e^{a+ibx}\sin(bx+d)\cos(bx+d)}{3b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{ie^{a+ibx}\cos^2(bx+d)}{3b}+\frac{2e^{a+ibx}\sin(bx+d)\cos(bx+d)}{3b}-\frac{2ie^{a+ibx}}{3b}$$

input
```
Int[E^(a + I*b*x)*Cos[d + b*x]^2,x]
```

output
```
(((-2*I)/3)*E^(a + I*b*x))/b + ((I/3)*E^(a + I*b*x)*Cos[d + b*x]^2)/b + (2
*E^(a + I*b*x)*Cos[d + b*x]*Sin[d + b*x])/(3*b)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

method	result
default	$-\frac{i e^{ibx+a}}{2b}+\frac{i e^{ibx+a}\cos(2bx+2d)}{6b}+\frac{e^{ibx+a}\sin(2bx+2d)}{3b}$
parallelrisch	$-\frac{2\,e^{ibx+a}\left(3i\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}+3\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}+i-\tan\left(\frac{bx}{2}+\frac{d}{2}\right)\right)}{3b\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}\right)^{2}}$
norman	$\frac{\frac{8\,e^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{3b}-\frac{2i e^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}}{b}+\frac{i e^{ibx+a}}{3b}-\frac{i e^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{4}}{b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}\right)^{2}}$
orering	$-\frac{i e^{ibx+a}\cos(bx+d)^{2}}{3b}-\frac{ib\,e^{ibx+a}\cos(bx+d)^{2}-2\,e^{ibx+a}\cos(bx+d)b\sin(bx+d)}{b^{2}}-\frac{i\left(-3b^{2}e^{ibx+a}\cos(bx+d)^{2}-4ib^{2}e^{ibx+a}\right.}{}$

input
```
int(exp(a+I*b*x)*cos(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b*exp(a+I*b*x)+1/6*I/b*exp(a+I*b*x)*cos(2*b*x+2*d)+1/3/b*exp(a+I*b*
x)*sin(2*b*x+2*d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int e^{a+ibx}\cos^2(d+bx)\,dx=\frac{\left(-i\,e^{(4i\,bx+a+3i\,d)}-6i\,e^{(2i\,bx+a+i\,d)}+3i\,e^{(a-i\,d)}\right)e^{(-i\,bx-i\,d)}}{12\,b}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/12*(-I*e^(4*I*b*x + a + 3*I*d) - 6*I*e^(2*I*b*x + a + I*d) + 3*I*e^(a -
I*d))*e^(-I*b*x - I*d)/b
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(56) = 112$.

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.48

$$\int e^{a+ibx}\cos^2(d+bx)\,dx = \begin{cases} \frac{(-8ib^2e^ae^{4id}e^{3ibx}-48ib^2e^ae^{2id}e^{ibx}+24ib^2e^ae^{-ibx})e^{-2id}}{96b^3} & \text{for } b^3e^{2id}\neq 0 \\ \frac{x(e^ae^{4id}+2e^ae^{2id}+e^a)e^{-2id}}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d)**2,x)
```

output
```
Piecewise(((-8*I*b**2*exp(a)*exp(4*I*d)*exp(3*I*b*x) - 48*I*b**2*exp(a)*ex
p(2*I*d)*exp(I*b*x) + 24*I*b**2*exp(a)*exp(-I*b*x))*exp(-2*I*d)/(96*b**3),
 Ne(b**3*exp(2*I*d), 0)), (x*(exp(a)*exp(4*I*d) + 2*exp(a)*exp(2*I*d) + ex
p(a))*exp(-2*I*d)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int e^{a+ibx}\cos^2(d+bx)\,dx$$
$$= \frac{-i\cos(3bx+2d)e^a+3i\cos(bx+2d)e^a-6i\cos(bx)e^a+e^a\sin(3bx+2d)+3e^a\sin(bx+2d)+6}{12b}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d)^2,x, algorithm="maxima")
```

output
```
1/12*(-I*cos(3*b*x + 2*d)*e^a + 3*I*cos(b*x + 2*d)*e^a - 6*I*cos(b*x)*e^a
+ e^a*sin(3*b*x + 2*d) + 3*e^a*sin(b*x + 2*d) + 6*e^a*sin(b*x))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(43) = 86$.

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int e^{a+ibx}\cos^2(d+bx)\,dx =$$
$$-\frac{i\left(e^{(3i\,bx+2i\,d)}+e^{(-3i\,bx-2i\,d)}\right)e^{a}-3i\left(e^{(i\,bx+2i\,d)}+e^{(-i\,bx-2i\,d)}\right)e^{a}+6i\left(e^{(i\,bx)}+e^{(-i\,bx)}\right)e^{a}-6\,e^{a}\sin\left(bx+\right.}{24\,b}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d)^2,x, algorithm="giac")
```

output
```
-1/24*(I*(e^(3*I*b*x + 2*I*d) + e^(-3*I*b*x - 2*I*d))*e^a - 3*I*(e^(I*b*x
+ 2*I*d) + e^(-I*b*x - 2*I*d))*e^a + 6*I*(e^(I*b*x) + e^(-I*b*x))*e^a - 6*
e^a*sin(b*x + 2*d) - 12*e^a*sin(b*x) + 2*e^a*sin(-3*b*x - 2*d))/b
```

Mupad [B] (verification not implemented)

Time = 15.96 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.48

$$\int e^{a+ibx}\cos^2(d+bx)\,dx = \frac{\mathrm{e}^{a+b\,x\,1\mathrm{i}}\,\left(\cos\left(2\,d+2\,b\,x\right)\,1\mathrm{i}+2\,\sin\left(2\,d+2\,b\,x\right)-3\mathrm{i}\right)}{6\,b}$$

input
```
int(cos(d + b*x)^2*exp(a + b*x*1i),x)
```

output
```
(exp(a + b*x*1i)*(cos(2*d + 2*b*x)*1i + 2*sin(2*d + 2*b*x) - 3i))/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

$$\int e^{a+ibx}\cos^2(d+bx)\,dx$$
$$=\frac{e^{bix+a}\left(2\cos\left(bx+d\right)\sin\left(bx+d\right)+2\cos\left(bx+d\right)i-\sin\left(bx+d\right)^{2}i+2\sin\left(bx+d\right)-i\right)}{3b}$$

input
```
int(exp(a+I*b*x)*cos(b*x+d)^2,x)
```

output
```
(e**(a + b*i*x)*(2*cos(b*x + d)*sin(b*x + d) + 2*cos(b*x + d)*i - sin(b*x
+ d)**2*i + 2*sin(b*x + d) - i))/(3*b)
```

3.23 $\int e^{a+ibx}\cos^3(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 96

$$\int e^{a+ibx}\cos^3(d+bx)\,dx=\frac{ie^{a-id-2i(d+bx)}}{16b}-\frac{3ie^{a-id+2i(d+bx)}}{16b}-\frac{ie^{a-id+4i(d+bx)}}{32b}+\frac{3}{8}e^{a-id}x$$

output
```
1/16*I*exp(a-I*d-2*I*(b*x+d))/b-3/16*I*exp(a-I*d+2*I*(b*x+d))/b-1/32*I*exp
(a-I*d+4*I*(b*x+d))/b+3/8*exp(a-I*d)*x
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int e^{a+ibx}\cos^3(d+bx)\,dx$$
$$=\frac{e^{a-2ibx}\left(6e^{2ibx}\left(-ie^{2ibx}+2bx\right)\cos(d)-i\left(-2+e^{6ibx}\right)\cos(3d)+6e^{4ibx}\sin(d)-12ibe^{2ibx}x\sin(d)+2\sin(3\ldots}{32b}$$

input
```
Integrate[E^(a + I*b*x)*Cos[d + b*x]^3,x]
```

output
```
(E^(a - (2*I)*b*x)*(6*E^((2*I)*b*x)*((-I)*E^((2*I)*b*x) + 2*b*x)*Cos[d] -
I*(-2 + E^((6*I)*b*x))*Cos[3*d] + 6*E^((4*I)*b*x)*Sin[d] - (12*I)*b*E^((2*
I)*b*x)*x*Sin[d] + 2*Sin[3*d] + E^((6*I)*b*x)*Sin[3*d]))/(32*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4935, 4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\cos^3(bx+d)\,dx$$

$$\Big\downarrow \text{4935}$$

$$\frac{3}{4}\int e^{a+ibx}\cos(d+bx)dx+\frac{ie^{a+ibx}\cos^3(bx+d)}{8b}+\frac{3e^{a+ibx}\sin(bx+d)\cos^2(bx+d)}{8b}$$

$$\Big\downarrow \text{4976}$$

$$\frac{3}{4}\int\left(\frac{1}{2}e^{a-id}+\frac{1}{2}e^{a+id+2ibx}\right)dx+\frac{ie^{a+ibx}\cos^3(bx+d)}{8b}+\frac{3e^{a+ibx}\sin(bx+d)\cos^2(bx+d)}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4}\left(\frac{1}{2}xe^{a-id}-\frac{ie^{a+2ibx+id}}{4b}\right)+\frac{ie^{a+ibx}\cos^3(bx+d)}{8b}+\frac{3e^{a+ibx}\sin(bx+d)\cos^2(bx+d)}{8b}$$

input
```
Int[E^(a + I*b*x)*Cos[d + b*x]^3,x]
```

output
```
(3*(((-1/4*I)*E^(a + I*d + (2*I)*b*x))/b + (E^(a - I*d)*x)/2))/4 + ((I/8)*
E^(a + I*b*x)*Cos[d + b*x]^3)/b + (3*E^(a + I*b*x)*Cos[d + b*x]^2*Sin[d +
b*x])/(8*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.79

method	result
parallelrisch	$-\frac{\left(12 i x b \sin \left(b x+d\right)-12 b x \cos \left(b x+d\right)+i \cos \left(b x+d\right)-i \cos \left(3 b x+3 d\right)-11 \sin \left(b x+d\right)-3 \sin \left(3 b x+3 d\right)\right) {\mathrm e}^{i b x+a}}{32 b}$
norman	$\frac{3 x \,{\mathrm e}^{i b x+a}}{8}+\frac{3 x \,{\mathrm e}^{i b x+a} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{2}}{8}-\frac{3 x \,{\mathrm e}^{i b x+a} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{4}}{8}-\frac{3 x \,{\mathrm e}^{i b x+a} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{6}}{8}-\frac{i {\mathrm e}^{i b x+a} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{2}}{2 b}+\frac{i {\mathrm e}^{i b x+a} \tan \left(\frac{b x}{2}+ \right.}{2 b}$
orering	$-\frac{\left(-4 b x+i\right) {\mathrm e}^{i b x+a} \cos \left(b x+d\right)^{3}}{4 b}+\frac{i \left(b x+i\right) \left(i b \,{\mathrm e}^{i b x+a} \cos \left(b x+d\right)^{3}-3 \,{\mathrm e}^{i b x+a} \cos \left(b x+d\right)^{2} b \sin \left(b x+d\right)\right)}{4 b^{2}}-\frac{\left(-4 b x+i\right) \left(-4 \cos (\right.}{}$

input
```
int(exp(a+I*b*x)*cos(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(12*I*x*b*sin(b*x+d)-12*b*x*cos(b*x+d)+I*cos(b*x+d)-I*cos(3*b*x+3*d)
-11*sin(b*x+d)-3*sin(3*b*x+3*d))*exp(a+I*b*x)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.64

$$\int e^{a+ibx}\cos^3(d+bx)\,dx = \frac{\left(12\,bxe^{(2i\,bx+a+i\,d)} - i\,e^{(6i\,bx+a+5i\,d)} - 6i\,e^{(4i\,bx+a+3i\,d)} + 2i\,e^{(a-i\,d)}\right)e^{(-2i\,bx-2i\,d)}}{32\,b}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/32*(12*b*x*e^(2*I*b*x + a + I*d) - I*e^(6*I*b*x + a + 5*I*d) - 6*I*e^(4*
I*b*x + a + 3*I*d) + 2*I*e^(a - I*d))*e^(-2*I*b*x - 2*I*d)/b
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.69

$$\int e^{a+ibx}\cos^3(d+bx)\,dx = \frac{3xe^ae^{-id}}{8} + \begin{cases} \frac{(-256ib^2e^ae^{6id}e^{4ibx}-1536ib^2e^ae^{4id}e^{2ibx}+512ib^2e^ae^{-2ibx})e^{-3id}}{8192b^3} & \text{for } b^3e^{3id}\neq 0 \\ x\left(\frac{(e^ae^{6id}+3e^ae^{4id}+3e^ae^{2id}+e^a)e^{-3id}}{8}-\frac{3e^ae^{-id}}{8}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d)**3,x)
```

output
```
3*x*exp(a)*exp(-I*d)/8 + Piecewise(((-256*I*b**2*exp(a)*exp(6*I*d)*exp(4*I
*b*x) - 1536*I*b**2*exp(a)*exp(4*I*d)*exp(2*I*b*x) + 512*I*b**2*exp(a)*exp
(-2*I*b*x))*exp(-3*I*d)/(8192*b**3), Ne(b**3*exp(3*I*d), 0)), (x*((exp(a)*
exp(6*I*d) + 3*exp(a)*exp(4*I*d) + 3*exp(a)*exp(2*I*d) + exp(a))*exp(-3*I*
d)/8 - 3*exp(a)*exp(-I*d)/8), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int e^{a+ibx}\cos^3(d+bx)\,dx = \frac{12\,(b\cos{(d)}\,e^{a} - i\,be^{a}\sin{(d)})x - i\,\cos{(4\,bx+3\,d)}\,e^{a} + 2i\,\cos{(2\,bx+3\,d)}\,e^{a} - 6i\,\cos{(2\,bx+d)}\,e^{a} + e^{a}\,\mathrm{s}}{32\,b}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d)^3,x, algorithm="maxima")
```

output
```
1/32*(12*(b*cos(d)*e^a - I*b*e^a*sin(d))*x - I*cos(4*b*x + 3*d)*e^a + 2*I*
cos(2*b*x + 3*d)*e^a - 6*I*cos(2*b*x + d)*e^a + e^a*sin(4*b*x + 3*d) + 2*e
^a*sin(2*b*x + 3*d) + 6*e^a*sin(2*b*x + d))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(55) = 110.

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.42

$$\int e^{a+ibx}\cos^3(d+bx)\,dx = \frac{24\,(bx+d)\cos{(d)}\,e^{a} - 24i\,(bx+d)e^{a}\sin{(d)} - i\,\left(e^{(4i\,bx+3i\,d)} + e^{(-4i\,bx-3i\,d)}\right)e^{a} + 2i\,\left(e^{(2i\,bx+3i\,d)} + e^{(-2i\,bx-}\right.}{}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d)^3,x, algorithm="giac")
```

output
```
1/64*(24*(b*x + d)*cos(d)*e^a - 24*I*(b*x + d)*e^a*sin(d) - I*(e^(4*I*b*x
+ 3*I*d) + e^(-4*I*b*x - 3*I*d))*e^a + 2*I*(e^(2*I*b*x + 3*I*d) + e^(-2*I*
b*x - 3*I*d))*e^a - 6*I*(e^(2*I*b*x + I*d) + e^(-2*I*b*x - I*d))*e^a + 4*e
^a*sin(2*b*x + 3*d) - 12*e^a*sin(-2*b*x - d) - 2*e^a*sin(-4*b*x - 3*d))/b
```

Mupad [B] (verification not implemented)

Time = 16.85 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int e^{a+ibx}\cos^3(d+bx)\,dx = \frac{3\,x\,\mathrm{e}^{a}\,(\cos(d)-\sin(d)\ 1\mathrm{i})}{8} + \frac{\mathrm{e}^{a}\,(\cos(2\,b\,x)-\sin(2\,b\,x)\ 1\mathrm{i})\ (\cos(3\,d)-\sin(3\,d)\ 1\mathrm{i})\ 1\mathrm{i}}{16\,b} - \frac{\mathrm{e}^{a}\,(\cos(4\,b\,x)+\sin(4\,b\,x)\ 1\mathrm{i})\ (\cos(3\,d)+\sin(3\,d)\ 1\mathrm{i})\ 1\mathrm{i}}{32\,b} - \frac{\mathrm{e}^{a}\,(\cos(2\,b\,x)+\sin(2\,b\,x)\ 1\mathrm{i})\ (\cos(d)+\sin(d)\ 1\mathrm{i})\ 3\mathrm{i}}{16\,b}$$

input
```
int(cos(d + b*x)^3*exp(a + b*x*1i),x)
```

output
```
(3*x*exp(a)*(cos(d) - sin(d)*1i))/8 + (exp(a)*(cos(2*b*x) - sin(2*b*x)*1i)
*(cos(3*d) - sin(3*d)*1i)*1i)/(16*b) - (exp(a)*(cos(4*b*x) + sin(4*b*x)*1i
)*(cos(3*d) + sin(3*d)*1i)*1i)/(32*b) - (exp(a)*(cos(2*b*x) + sin(2*b*x)*1
i)*(cos(d) + sin(d)*1i)*3i)/(16*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.74

$$\int e^{a+ibx}\cos^3(d+bx)\,dx = \frac{e^{bix+a}\left(-\cos(bx+d)\sin(bx+d)^2 i+3\cos(bx+d)\,bx-5\cos(bx+d)\,i-3\sin(bx+d)^3-3\sin(bx+d)\right.}{8b}$$

input
```
int(exp(a+I*b*x)*cos(b*x+d)^3,x)
```

output
```
(e**(a + b*i*x)*( - cos(b*x + d)*sin(b*x + d)**2*i + 3*cos(b*x + d)*b*x -
5*cos(b*x + d)*i - 3*sin(b*x + d)**3 - 3*sin(b*x + d)*b*i*x))/(8*b)
```

3.24 $\int e^{a+ibx}\cos^4(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\int e^{a+ibx}\cos^4(d+bx)\,dx=\frac{ie^{a-id-i(d+bx)}}{4b}-\frac{3ie^{a-id+i(d+bx)}}{8b}+\frac{ie^{a-id-3i(d+bx)}}{48b}-\frac{ie^{a-id+3i(d+bx)}}{12b}-\frac{ie^{a-id+5i(d+bx)}}{80b}$$

output
```
1/4*I*exp(a-I*d-I*(b*x+d))/b-3/8*I*exp(a-I*d+I*(b*x+d))/b+1/48*I*exp(a-I*d
-3*I*(b*x+d))/b-1/12*I*exp(a-I*d+3*I*(b*x+d))/b-1/80*I*exp(a-I*d+5*I*(b*x+
d))/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int e^{a+ibx}\cos^4(d+bx)\,dx=\frac{e^{a-3ibx}\left(-90ie^{4ibx}-20ie^{2ibx}\left(-3+e^{4ibx}\right)\cos(2d)-i\left(-5+3e^{8ibx}\right)\cos(4d)+60e^{2ibx}\sin(2d)+20e^{6ibx}\sin(\right.}{240b}$$

input
```
Integrate[E^(a + I*b*x)*Cos[d + b*x]^4,x]
```

output
```
(E^(a - (3*I)*b*x)*((-90*I)*E^((4*I)*b*x) - (20*I)*E^((2*I)*b*x)*(-3 + E^(
(4*I)*b*x))*Cos[2*d] - I*(-5 + 3*E^((8*I)*b*x))*Cos[4*d] + 60*E^((2*I)*b*x
)*Sin[2*d] + 20*E^((6*I)*b*x)*Sin[2*d] + 5*Sin[4*d] + 3*E^((8*I)*b*x)*Sin[
4*d]))/(240*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4935, 4935, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\cos^4(bx+d)\,dx$$

$$\Big\downarrow \text{4935}$$

$$\frac{4}{5}\int e^{a+ibx}\cos^2(d+bx)dx+\frac{ie^{a+ibx}\cos^4(bx+d)}{15b}+\frac{4e^{a+ibx}\sin(bx+d)\cos^3(bx+d)}{15b}$$

$$\Big\downarrow \text{4935}$$

$$\frac{4}{5}\left(\frac{2}{3}\int e^{a+ibx}dx+\frac{ie^{a+ibx}\cos^2(bx+d)}{3b}+\frac{2e^{a+ibx}\sin(bx+d)\cos(bx+d)}{3b}\right)+\frac{ie^{a+ibx}\cos^4(bx+d)}{15b}+\frac{4e^{a+ibx}\sin(bx+d)\cos^3(bx+d)}{15b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{ie^{a+ibx}\cos^4(bx+d)}{15b}+\frac{4e^{a+ibx}\sin(bx+d)\cos^3(bx+d)}{15b}+\frac{4}{5}\left(\frac{ie^{a+ibx}\cos^2(bx+d)}{3b}+\frac{2e^{a+ibx}\sin(bx+d)\cos(bx+d)}{3b}-\frac{2ie^{a+ibx}}{3b}\right)$$

input
```
Int[E^(a + I*b*x)*Cos[d + b*x]^4,x]
```

output
```
((I/15)*E^(a + I*b*x)*Cos[d + b*x]^4)/b + (4*E^(a + I*b*x)*Cos[d + b*x]^3*
Sin[d + b*x])/(15*b) + (4*((((-2*I)/3)*E^(a + I*b*x))/b + ((I/3)*E^(a + I*
b*x)*Cos[d + b*x]^2)/b + (2*E^(a + I*b*x)*Cos[d + b*x]*Sin[d + b*x])/(3*b)
))/5
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

method	result
default	$-\frac{3i\mathrm{e}^{ibx+a}}{8b}+\frac{i\mathrm{e}^{ibx+a}\cos(4bx+4d)}{120b}+\frac{\mathrm{e}^{ibx+a}\sin(4bx+4d)}{30b}+\frac{i\mathrm{e}^{ibx+a}\cos(2bx+2d)}{6b}+\frac{\mathrm{e}^{ibx+a}\sin(2bx+2d)}{3b}$
parallelrisch	$-\frac{2\,\mathrm{e}^{ibx+a}\left(15\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{7}+15i\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{6}+5\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{5}+25i\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{4}+13\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}+21i\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}-\right.}{15b\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}\right)^{4}}$
norman	$\frac{-\frac{2i\mathrm{e}^{ibx+a}}{5b}-\frac{2i\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{6}}{b}-\frac{14i\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}}{5b}-\frac{10i\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{4}}{3b}-\frac{2\,\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{7}}{b}+\frac{6\,\mathrm{e}^{ibx+a}\tan\left(\frac{bx}{2}\right.}{5b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}\right)^{4}}$
orering	$-\frac{i\mathrm{e}^{ibx+a}\cos(bx+d)^{4}}{5b}-\frac{10\left(ib\,\mathrm{e}^{ibx+a}\cos(bx+d)^{4}-4\,\mathrm{e}^{ibx+a}\cos(bx+d)^{3}b\sin(bx+d)\right)}{9b^{2}}-\frac{2i\left(-5b^{2}\mathrm{e}^{ibx+a}\cos(bx+d)^{4}-8ib^{2}\right.}{}$

input
```
int(exp(a+I*b*x)*cos(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-3/8*I/b*exp(a+I*b*x)+1/120*I/b*exp(a+I*b*x)*cos(4*b*x+4*d)+1/30/b*exp(a+I
*b*x)*sin(4*b*x+4*d)+1/6*I/b*exp(a+I*b*x)*cos(2*b*x+2*d)+1/3/b*exp(a+I*b*x
)*sin(2*b*x+2*d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.52

$$\int e^{a+ibx}\cos^4(d+bx)\,dx = \frac{\left(-3i\,e^{(8i\,bx+a+7i\,d)}-20i\,e^{(6i\,bx+a+5i\,d)}-90i\,e^{(4i\,bx+a+3i\,d)}+60i\,e^{(2i\,bx+a+i\,d)}+5i\,e^{(a-i\,d)}\right)e^{(-3i\,bx-3i\,d)}}{240\,b}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d)^4,x, algorithm="fricas")
```

output
```
1/240*(-3*I*e^(8*I*b*x + a + 7*I*d) - 20*I*e^(6*I*b*x + a + 5*I*d) - 90*I*
e^(4*I*b*x + a + 3*I*d) + 60*I*e^(2*I*b*x + a + I*d) + 5*I*e^(a - I*d))*e^
(-3*I*b*x - 3*I*d)/b
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(99) = 198.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.49

$$\int e^{a+ibx}\cos^4(d+bx)\,dx = \begin{cases} \frac{\left(-18432ib^4e^ae^{10id}e^{5ibx}-122880ib^4e^ae^{8id}e^{3ibx}-552960ib^4e^ae^{6id}e^{ibx}+368640ib^4e^ae^{4id}e^{-ibx}+30720ib^4e^ae^{2id}e^{-3ibx}\right)e^{-6id}}{1474560b^5} & \text{for } b^5e^{6id} \\ \frac{x\left(e^ae^{8id}+4e^ae^{6id}+6e^ae^{4id}+4e^ae^{2id}+e^a\right)e^{-4id}}{16} & \text{otherwis} \end{cases}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d)**4,x)
```

output
```
Piecewise(((-18432*I*b**4*exp(a)*exp(10*I*d)*exp(5*I*b*x) - 122880*I*b**4*
exp(a)*exp(8*I*d)*exp(3*I*b*x) - 552960*I*b**4*exp(a)*exp(6*I*d)*exp(I*b*x
) + 368640*I*b**4*exp(a)*exp(4*I*d)*exp(-I*b*x) + 30720*I*b**4*exp(a)*exp(
2*I*d)*exp(-3*I*b*x))*exp(-6*I*d)/(1474560*b**5), Ne(b**5*exp(6*I*d), 0)),
 (x*(exp(a)*exp(8*I*d) + 4*exp(a)*exp(6*I*d) + 6*exp(a)*exp(4*I*d) + 4*exp
(a)*exp(2*I*d) + exp(a))*exp(-4*I*d)/16, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int e^{a+ibx}\cos^4(d+bx)\,dx$$
$$= \frac{-3i\,\cos\left(5\,bx+4\,d\right)e^a+5i\,\cos\left(3\,bx+4\,d\right)e^a-20i\,\cos\left(3\,bx+2\,d\right)e^a+60i\,\cos\left(bx+2\,d\right)e^a-90i\,\cos\left(\right.}{}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d)^4,x, algorithm="maxima")
```

output
```
1/240*(-3*I*cos(5*b*x + 4*d)*e^a + 5*I*cos(3*b*x + 4*d)*e^a - 20*I*cos(3*b
*x + 2*d)*e^a + 60*I*cos(b*x + 2*d)*e^a - 90*I*cos(b*x)*e^a + 3*e^a*sin(5*
b*x + 4*d) + 5*e^a*sin(3*b*x + 4*d) + 20*e^a*sin(3*b*x + 2*d) + 60*e^a*sin
(b*x + 2*d) + 90*e^a*sin(b*x))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(73) = 146.

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int e^{a+ibx}\cos^4(d+bx)\,dx =$$
$$-\frac{3i\left(e^{(5i\,bx+4i\,d)}+e^{(-5i\,bx-4i\,d)}\right)e^a-5i\left(e^{(3i\,bx+4i\,d)}+e^{(-3i\,bx-4i\,d)}\right)e^a+20i\left(e^{(3i\,bx+2i\,d)}+e^{(-3i\,bx-2i\,d)}\right)e^a-}{}$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+d)^4,x, algorithm="giac")
```

output
```
-1/480*(3*I*(e^(5*I*b*x + 4*I*d) + e^(-5*I*b*x - 4*I*d))*e^a - 5*I*(e^(3*I
*b*x + 4*I*d) + e^(-3*I*b*x - 4*I*d))*e^a + 20*I*(e^(3*I*b*x + 2*I*d) + e^
(-3*I*b*x - 2*I*d))*e^a - 60*I*(e^(I*b*x + 2*I*d) + e^(-I*b*x - 2*I*d))*e^
a + 90*I*(e^(I*b*x) + e^(-I*b*x))*e^a - 10*e^a*sin(3*b*x + 4*d) - 120*e^a*
sin(b*x + 2*d) - 180*e^a*sin(b*x) + 40*e^a*sin(-3*b*x - 2*d) + 6*e^a*sin(-
5*b*x - 4*d))/b
```

Mupad [B] (verification not implemented)

Time = 16.98 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.14

$$\int e^{a+ibx}\cos^4(d+bx)\,dx = -\frac{e^a\left(\cos(b\,x)+\sin(b\,x)\ 1\mathrm{i}\right)\,3\mathrm{i}}{8\,b} + \frac{e^a\left(\cos(b\,x)-\sin(b\,x)\ 1\mathrm{i}\right)\left(\cos(2\,d)-\sin(2\,d)\ 1\mathrm{i}\right)\,1\mathrm{i}}{4\,b} - \frac{e^a\left(\cos(3\,b\,x)+\sin(3\,b\,x)\ 1\mathrm{i}\right)\left(\cos(2\,d)+\sin(2\,d)\ 1\mathrm{i}\right)\,1\mathrm{i}}{12\,b} + \frac{e^a\left(\cos(3\,b\,x)-\sin(3\,b\,x)\ 1\mathrm{i}\right)\left(\cos(4\,d)-\sin(4\,d)\ 1\mathrm{i}\right)\,1\mathrm{i}}{48\,b} - \frac{e^a\left(\cos(5\,b\,x)+\sin(5\,b\,x)\ 1\mathrm{i}\right)\left(\cos(4\,d)+\sin(4\,d)\ 1\mathrm{i}\right)\,1\mathrm{i}}{80\,b}$$

input
```
int(cos(d + b*x)^4*exp(a + b*x*1i),x)
```

output
```
(exp(a)*(cos(b*x) - sin(b*x)*1i)*(cos(2*d) - sin(2*d)*1i)*1i)/(4*b) - (exp
(a)*(cos(b*x) + sin(b*x)*1i)*3i)/(8*b) - (exp(a)*(cos(3*b*x) + sin(3*b*x)*
1i)*(cos(2*d) + sin(2*d)*1i)*1i)/(12*b) + (exp(a)*(cos(3*b*x) - sin(3*b*x)
*1i)*(cos(4*d) - sin(4*d)*1i)*1i)/(48*b) - (exp(a)*(cos(5*b*x) + sin(5*b*x
)*1i)*(cos(4*d) + sin(4*d)*1i)*1i)/(80*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62

$$\int e^{a+ibx}\cos^4(d+bx)\,dx$$
$$=\frac{e^{bix+a}\left(-4\cos\left(bx+d\right)\sin\left(bx+d\right)^{3}+12\cos\left(bx+d\right)\sin\left(bx+d\right)+12\cos\left(bx+d\right)i+\sin\left(bx+d\right)^{4}i-6\right.}{15b}$$

input
```
int(exp(a+I*b*x)*cos(b*x+d)^4,x)
```

output
```
(e**(a + b*i*x)*( - 4*cos(b*x + d)*sin(b*x + d)**3 + 12*cos(b*x + d)*sin(b
*x + d) + 12*cos(b*x + d)*i + sin(b*x + d)**4*i - 6*sin(b*x + d)**2*i + 12
*sin(b*x + d) - 3*i))/(15*b)
```

3.25 $\int e^{2(a+ibx)} \cos(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int e^{2(a+ibx)} \cos(d+bx)\,dx = -\frac{ie^{2(a-id)+i(d+bx)}}{2b} - \frac{ie^{2(a-id)+3i(d+bx)}}{6b}$$

output
```
-1/2*I*exp(2*a-2*I*d+I*(b*x+d))/b-1/6*I*exp(2*a-2*I*d+3*I*(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int e^{2(a+ibx)} \cos(d+bx)\,dx = \frac{e^{2a+ibx}\left(-i\left(3+e^{2ibx}\right)\cos(d)+\left(-3+e^{2ibx}\right)\sin(d)\right)}{6b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Cos[d + b*x],x]
```

output
```
(E^(2*a + I*b*x)*((-I)*(3 + E^((2*I)*b*x))*Cos[d] + (-3 + E^((2*I)*b*x))*S
in[d]))/(6*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\cos(bx+d)\,dx$$

$$\Big\downarrow \ 4933$$

$$-\frac{e^{2(a+ibx)}\sin(bx+d)}{3b}-\frac{2ie^{2(a+ibx)}\cos(bx+d)}{3b}$$

input
```
Int[E^(2*(a + I*b*x))*Cos[d + b*x],x]
```

output
```
(((-2*I)/3)*E^(2*(a + I*b*x))*Cos[d + b*x])/b - (E^(2*(a + I*b*x))*Sin[d +
 b*x])/(3*b)
```

Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

method	result	size
parallelrisch	$-\frac{e^{2ibx+2a}(2i\cos(bx+d)+\sin(bx+d))}{3b}$	32
default	$-\frac{2ie^{2ibx+2a}\cos(bx+d)}{3b}-\frac{e^{2ibx+2a}\sin(bx+d)}{3b}$	45
orering	$-\frac{4ie^{2ibx+2a}\cos(bx+d)}{3b}+\frac{2ib\,e^{2ibx+2a}\cos(bx+d)-e^{2ibx+2a}b\sin(bx+d)}{3b^2}$	69
norman	$\frac{-\frac{2\,e^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{3b}-\frac{2ie^{2ibx+2a}}{3b}+\frac{2ie^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2}{3b}}{1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2}$	85

input
```
int(exp(2*a+2*I*b*x)*cos(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/3*exp(2*a+2*I*b*x)*(2*I*cos(b*x+d)+sin(b*x+d))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int e^{2(a+ibx)}\cos(d+bx)\,dx=\frac{-i\,e^{(3i\,bx+2\,a+i\,d)}-3i\,e^{(i\,bx+2\,a-i\,d)}}{6\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d),x, algorithm="fricas")
```

output
```
1/6*(-I*e^(3*I*b*x + 2*a + I*d) - 3*I*e^(I*b*x + 2*a - I*d))/b
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int e^{2(a+ibx)}\cos(d+bx)\,dx=\begin{cases}\frac{(-2ibe^{2a}e^{2id}e^{3ibx}-6ibe^{2a}e^{ibx})e^{-id}}{12b^2} & \text{for } b^2e^{id}\neq 0\\ \frac{x(e^{2a}e^{2id}+e^{2a})e^{-id}}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d),x)
```

output
```
Piecewise(((-2*I*b*exp(2*a)*exp(2*I*d)*exp(3*I*b*x) - 6*I*b*exp(2*a)*exp(I
*b*x))*exp(-I*d)/(12*b**2), Ne(b**2*exp(I*d), 0)), (x*(exp(2*a)*exp(2*I*d)
 + exp(2*a))*exp(-I*d)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

$$\int e^{2(a+ibx)}\cos(d+bx)\,dx = -\frac{(2i\,b\cos{(bx+d)}+b\sin{(bx+d)})e^{(2i\,bx+2\,a)}}{3\,b^2}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d),x, algorithm="maxima")
```

output
```
-1/3*(2*I*b*cos(b*x + d) + b*sin(b*x + d))*e^(2*I*b*x + 2*a)/b^2
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(35) = 70.

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.38

$$\int e^{2(a+ibx)}\cos(d+bx)\,dx = -\frac{i\left(e^{(3i\,bx+i\,d)}+e^{(-3i\,bx-i\,d)}\right)e^{(2\,a)}+3i\left(e^{(i\,bx-i\,d)}+e^{(-i\,bx+i\,d)}\right)e^{(2\,a)}+6\,e^{(2\,a)}\sin{(-bx+d)}+2\,e^{(2\,a)}\sin{(-}}{12\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d),x, algorithm="giac")
```

output
```
-1/12*(I*(e^(3*I*b*x + I*d) + e^(-3*I*b*x - I*d))*e^(2*a) + 3*I*(e^(I*b*x
- I*d) + e^(-I*b*x + I*d))*e^(2*a) + 6*e^(2*a)*sin(-b*x + d) + 2*e^(2*a)*s
in(-3*b*x - d))/b
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.51

$$\int e^{2(a+ibx)}\cos(d+bx)\,dx=-\frac{\mathrm{e}^{2\,a+b\,x\,2\mathrm{i}}\,\left(\sin\left(d+b\,x\right)+\cos\left(d+b\,x\right)\,2\mathrm{i}\right)}{3\,b}$$

input
```
int(cos(d + b*x)*exp(2*a + b*x*2i),x)
```

output
```
-(exp(2*a + b*x*2i)*(cos(d + b*x)*2i + sin(d + b*x)))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int e^{2(a+ibx)}\cos(d+bx)\,dx=\frac{e^{2bix+2a}\left(-2\cos\left(bx+d\right)i-\sin\left(bx+d\right)\right)}{3b}$$

input
```
int(exp(2*a+2*I*b*x)*cos(b*x+d),x)
```

output
```
(e**(2*a + 2*b*i*x)*( - 2*cos(b*x + d)*i - sin(b*x + d)))/(3*b)
```

3.26 $\int e^{2(a+ibx)} \cos^2(d+bx)\, dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int e^{2(a+ibx)} \cos^2(d+bx)\, dx = -\frac{ie^{2(a-id)+2i(d+bx)}}{4b} - \frac{ie^{2(a-id)+4i(d+bx)}}{16b} + \frac{1}{4}e^{2a-2id}x$$

output
```
-1/4*I*exp(2*a-2*I*d+2*I*(b*x+d))/b-1/16*I*exp(2*a-2*I*d+4*I*(b*x+d))/b+1/
4*exp(2*a-2*I*d)*x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int e^{2(a+ibx)} \cos^2(d+bx)\, dx = \frac{e^{2a}\left(-4ie^{2ibx} + \left(-ie^{4ibx} + 4bx\right)\cos(2d) + \left(e^{4ibx} - 4ibx\right)\sin(2d)\right)}{16b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Cos[d + b*x]^2,x]
```

output
```
(E^(2*a)*((-4*I)*E^((2*I)*b*x) + ((-I)*E^((4*I)*b*x) + 4*b*x)*Cos[2*d] + (
E^((4*I)*b*x) - (4*I)*b*x)*Sin[2*d]))/(16*b)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4967, 4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\cos^2(bx+d)\,dx$$

$$\Big\downarrow \text{4967}$$

$$\int e^{2a+2ibx}\cos^2(bx+d)dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{4}e^{2a+4ibx+2id}+\frac{1}{2}e^{2a+2ibx}+\frac{1}{4}e^{2(a-id)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ie^{2(a+id)+4ibx}}{16b}-\frac{ie^{2a+2ibx}}{4b}+\frac{1}{4}xe^{2a-2id}$$

input
```
Int[E^(2*(a + I*b*x))*Cos[d + b*x]^2,x]
```

output
```
((-1/4*I)*E^(2*a + (2*I)*b*x))/b - ((I/16)*E^(2*(a + I*d) + (4*I)*b*x))/b
+ (E^(2*a - (2*I)*d)*x)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4967
```
Int[(F_)^((c_.)*(u_))*(G_)[v_]^(n_.), x_Symbol] :> Int[F^(c*ExpandToSum[u,
x])*G[ExpandToSum[v, x]]^n, x] /; FreeQ[{F, c, n}, x] && TrigQ[G] && Linear
Q[{u, v}, x] && !LinearMatchQ[{u, v}, x]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

method	result
parallelrisch	$-\frac{\left(\left(-bx-i\right)\cos\left(2bx+2d\right)+ibx\sin\left(2bx+2d\right)+i-\frac{3\sin\left(2bx+2d\right)}{2}\right)e^{2ibx+2a}}{4b}$
orering	$-\frac{(-4bx+3i)e^{2ibx+2a}\cos(bx+d)^2}{4b}-\frac{i(-6bx+i)\left(2ib\,e^{2ibx+2a}\cos(bx+d)^2-2\,e^{2ibx+2a}\cos(bx+d)b\sin(bx+d)\right)}{8b^2}-\frac{x\left(-6\,\mathrm{co}\right.}{}$
norman	$\frac{ix\,e^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^3+\frac{x\,e^{2ibx+2a}}{4}-\frac{3x\,e^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2}{2}+\frac{x\,e^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^4}{4}-\frac{2ie^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2}{b}-ix\,e^{2ib}}{\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2\right)^2}$

input
```
int(exp(2*a+2*I*b*x)*cos(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*((-b*x-I)*cos(2*b*x+2*d)+I*b*x*sin(2*b*x+2*d)+I-3/2*sin(2*b*x+2*d))*e
xp(2*a+2*I*b*x)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.56

$$\int e^{2(a+ibx)}\cos^2(d+bx)\,dx=\frac{4\,bxe^{(2\,a-2i\,d)}-i\,e^{(4i\,bx+2\,a+2i\,d)}-4i\,e^{(2i\,bx+2\,a)}}{16\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/16*(4*b*x*e^(2*a - 2*I*d) - I*e^(4*I*b*x + 2*a + 2*I*d) - 4*I*e^(2*I*b*x
 + 2*a))/b
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.57

$$\int e^{2(a+ibx)}\cos^2(d+bx)\,dx = \frac{xe^{2a}e^{-2id}}{4} + \begin{cases} \frac{-4ibe^{2a}e^{2id}e^{4ibx}-16ibe^{2a}e^{2ibx}}{64b^2} & \text{for } b^2 \neq 0 \\ x\left(\frac{\left(e^{2a}e^{4id}+2e^{2a}e^{2id}+e^{2a}\right)e^{-2id}}{4} - \frac{e^{2a}e^{-2id}}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d)**2,x)
```

output
```
x*exp(2*a)*exp(-2*I*d)/4 + Piecewise(((-4*I*b*exp(2*a)*exp(2*I*d)*exp(4*I*
b*x) - 16*I*b*exp(2*a)*exp(2*I*b*x))/(64*b**2), Ne(b**2, 0)), (x*((exp(2*a
)*exp(4*I*d) + 2*exp(2*a)*exp(2*I*d) + exp(2*a))*exp(-2*I*d)/4 - exp(2*a)*
exp(-2*I*d)/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int e^{2(a+ibx)}\cos^2(d+bx)\,dx = \frac{4\left(b\cos(2d)e^{(2a)} - i\,be^{(2a)}\sin(2d)\right)x - 4i\,\cos(2bx)e^{(2a)} - i\,\cos(4bx+2d)e^{(2a)} + 4e^{(2a)}\sin(2bx) +}{16b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d)^2,x, algorithm="maxima")
```

output
```
1/16*(4*(b*cos(2*d)*e^(2*a) - I*b*e^(2*a)*sin(2*d))*x - 4*I*cos(2*b*x)*e^(
2*a) - I*cos(4*b*x + 2*d)*e^(2*a) + 4*e^(2*a)*sin(2*b*x) + e^(2*a)*sin(4*b
*x + 2*d))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(43) = 86$.

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int e^{2(a+ibx)}\cos^2(d+bx)\,dx$$
$$= \frac{8\,(bx+d)\cos(2\,d)\,e^{(2\,a)} - 8i\,(bx+d)e^{(2\,a)}\sin(2\,d) - i\left(e^{(4i\,bx+2i\,d)} + e^{(-4i\,bx-2i\,d)}\right)e^{(2\,a)} - 4i\left(e^{(2i\,bx)} + e^{(-}\right.}{32\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d)^2,x, algorithm="giac")
```

output
```
1/32*(8*(b*x + d)*cos(2*d)*e^(2*a) - 8*I*(b*x + d)*e^(2*a)*sin(2*d) - I*(e
^(4*I*b*x + 2*I*d) + e^(-4*I*b*x - 2*I*d))*e^(2*a) - 4*I*(e^(2*I*b*x) + e^
(-2*I*b*x))*e^(2*a) + 8*e^(2*a)*sin(2*b*x) - 2*e^(2*a)*sin(-4*b*x - 2*d))/
b
```

Mupad [B] (verification not implemented)

Time = 15.70 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int e^{2(a+ibx)}\cos^2(d+bx)\,dx = \frac{x\,\mathrm{e}^{2\,a-d\,2\mathrm{i}}}{4} - \frac{\mathrm{e}^{2\,a+d\,2\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}}{16\,b} - \frac{\mathrm{e}^{2\,a+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{4\,b}$$

input
```
int(cos(d + b*x)^2*exp(2*a + b*x*2i),x)
```

output
```
(x*exp(2*a - d*2i))/4 - (exp(2*a + d*2i + b*x*4i)*1i)/(16*b) - (exp(2*a +
b*x*2i)*1i)/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int e^{2(a+ibx)}\cos^2(d+bx)\,dx$$
$$= \frac{e^{2bix+2a}\left(-4\cos(bx+d)\sin(bx+d)\,bix-4\sin(bx+d)^2\,bx+2\sin(bx+d)^2\,i+2bx-3i\right)}{8b}$$

input
```
int(exp(2*a+2*I*b*x)*cos(b*x+d)^2,x)
```

output
```
(e**(2*a + 2*b*i*x)*( - 4*cos(b*x + d)*sin(b*x + d)*b*i*x - 4*sin(b*x + d)
**2*b*x + 2*sin(b*x + d)**2*i + 2*b*x - 3*i))/(8*b)
```

3.27 $\int e^{2(a+ibx)}\cos^3(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int e^{2(a+ibx)}\cos^3(d+bx)\,dx=\frac{ie^{2(a-id)-i(d+bx)}}{8b}-\frac{3ie^{2(a-id)+i(d+bx)}}{8b}-\frac{ie^{2(a-id)+3i(d+bx)}}{8b}-\frac{ie^{2(a-id)+5i(d+bx)}}{40b}$$

output
```
1/8*I*exp(2*a-2*I*d-I*(b*x+d))/b-3/8*I*exp(2*a-2*I*d+I*(b*x+d))/b-1/8*I*ex
p(2*a-2*I*d+3*I*(b*x+d))/b-1/40*I*exp(2*a-2*I*d+5*I*(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int e^{2(a+ibx)}\cos^3(d+bx)\,dx=\frac{e^{2a-ibx}\left(-5ie^{2ibx}\left(3+e^{2ibx}\right)\cos(d)-i\left(-5+e^{6ibx}\right)\cos(3d)-15e^{2ibx}\sin(d)+5e^{4ibx}\sin(d)+5\sin(3d)+e}{40b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Cos[d + b*x]^3,x]
```

output
```
(E^(2*a - I*b*x)*((-5*I)*E^((2*I)*b*x)*(3 + E^((2*I)*b*x))*Cos[d] - I*(-5
+ E^((6*I)*b*x))*Cos[3*d] - 15*E^((2*I)*b*x)*Sin[d] + 5*E^((4*I)*b*x)*Sin[
d] + 5*Sin[3*d] + E^((6*I)*b*x)*Sin[3*d]))/(40*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4935, 4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)} \cos^3(bx+d) \, dx$$

$$\Big\downarrow \text{4935}$$

$$\frac{6}{5} \int e^{2(a+ibx)} \cos(d+bx) dx + \frac{2ie^{2(a+ibx)} \cos^3(bx+d)}{5b} + \frac{3e^{2(a+ibx)} \sin(bx+d) \cos^2(bx+d)}{5b}$$

$$\Big\downarrow \text{4933}$$

$$\frac{2ie^{2(a+ibx)} \cos^3(bx+d)}{5b} + \frac{3e^{2(a+ibx)} \sin(bx+d) \cos^2(bx+d)}{5b} + \frac{6}{5} \left(-\frac{e^{2(a+ibx)} \sin(bx+d)}{3b} - \frac{2ie^{2(a+ibx)} \cos(bx+d)}{3b}\right)$$

input
```
Int[E^(2*(a + I*b*x))*Cos[d + b*x]^3,x]
```

output
```
(((2*I)/5)*E^(2*(a + I*b*x))*Cos[d + b*x]^3)/b + (3*E^(2*(a + I*b*x))*Cos[
d + b*x]^2*Sin[d + b*x])/(5*b) + (6*((((-2*I)/3)*E^(2*(a + I*b*x))*Cos[d +
 b*x])/b - (E^(2*(a + I*b*x))*Sin[d + b*x])/(3*b)))/5
```

Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

method	result
default	$\frac{i\,\mathrm{e}^{2ibx+2a}\cos(3bx+3d)}{10b}+\frac{3\,\mathrm{e}^{2ibx+2a}\sin(3bx+3d)}{20b}-\frac{i\,\mathrm{e}^{2ibx+2a}\cos(bx+d)}{2b}-\frac{\mathrm{e}^{2ibx+2a}\sin(bx+d)}{4b}$
parallelrisch	$\frac{4\tan\left(\frac{bx}{2}+\frac{d}{2}\right)\mathrm{e}^{2ibx+2a}\left(i\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{5}-\frac{3\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{4}}{2}-5i\tan\left(\frac{bx}{2}+\frac{d}{2}\right)-5\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}+\frac{5}{2}\right)}{5b\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}\right)^{3}}$
norman	$\frac{-\frac{4\,\mathrm{e}^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}}{b}-\frac{4i\,\mathrm{e}^{2ibx+2a}}{5b}+\frac{4i\,\mathrm{e}^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{4}}{b}+\frac{2\,\mathrm{e}^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{5}}{b}-\frac{6\,\mathrm{e}^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{5b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}\right)^{3}}$
orering	$-\frac{8i\,\mathrm{e}^{2ibx+2a}\cos(bx+d)^{3}}{15b}-\frac{14\left(2ib\,\mathrm{e}^{2ibx+2a}\cos(bx+d)^{3}-3\,\mathrm{e}^{2ibx+2a}\cos(bx+d)^{2}b\sin(bx+d)\right)}{15b^{2}}-\frac{8i\left(-7b^{2}\mathrm{e}^{2ibx+2a}\cos(bx\right.}{}$

input
```
int(exp(2*a+2*I*b*x)*cos(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*I/b*exp(2*a+2*I*b*x)*cos(3*b*x+3*d)+3/20/b*exp(2*a+2*I*b*x)*sin(3*b*x
+3*d)-1/2*I/b*exp(2*a+2*I*b*x)*cos(b*x+d)-1/4/b*exp(2*a+2*I*b*x)*sin(b*x+d
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.53

$$\int e^{2(a+ibx)}\cos^3(d+bx)\,dx = \frac{\left(-i\,e^{(6i\,bx+2\,a+4i\,d)}-5i\,e^{(4i\,bx+2\,a+2i\,d)}-15i\,e^{(2i\,bx+2\,a)}+5i\,e^{(2\,a-2i\,d)}\right)e^{(-i\,bx-i\,d)}}{40\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/40*(-I*e^(6*I*b*x + 2*a + 4*I*d) - 5*I*e^(4*I*b*x + 2*a + 2*I*d) - 15*I*
e^(2*I*b*x + 2*a) + 5*I*e^(2*a - 2*I*d))*e^(-I*b*x - I*d)/b
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.46

$$\int e^{2(a+ibx)}\cos^3(d+bx)\,dx = \begin{cases} \frac{\left(-512ib^3e^{2a}e^{7id}e^{5ibx}-2560ib^3e^{2a}e^{5id}e^{3ibx}-7680ib^3e^{2a}e^{3id}e^{ibx}+2560ib^3e^{2a}e^{id}e^{-ibx}\right)e^{-4id}}{20480b^4} & \text{for } b^4e^{4id}\neq 0 \\ \frac{x\left(e^{2a}e^{6id}+3e^{2a}e^{4id}+3e^{2a}e^{2id}+e^{2a}\right)e^{-3id}}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d)**3,x)
```

output
```
Piecewise(((-512*I*b**3*exp(2*a)*exp(7*I*d)*exp(5*I*b*x) - 2560*I*b**3*exp
(2*a)*exp(5*I*d)*exp(3*I*b*x) - 7680*I*b**3*exp(2*a)*exp(3*I*d)*exp(I*b*x)
 + 2560*I*b**3*exp(2*a)*exp(I*d)*exp(-I*b*x))*exp(-4*I*d)/(20480*b**4), Ne
(b**4*exp(4*I*d), 0)), (x*(exp(2*a)*exp(6*I*d) + 3*exp(2*a)*exp(4*I*d) + 3
*exp(2*a)*exp(2*I*d) + exp(2*a))*exp(-3*I*d)/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.97

$$\int e^{2(a+ibx)} \cos^3(d+bx)\,dx$$
$$= \frac{-i\,\cos\left(5\,bx+3\,d\right)e^{(2\,a)}-5i\,\cos\left(3\,bx+d\right)e^{(2\,a)}+5i\,\cos\left(bx+3\,d\right)e^{(2\,a)}-15i\,\cos\left(bx-d\right)e^{(2\,a)}+e^{(2\,a)}}{40\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d)^3,x, algorithm="maxima")
```

output
```
1/40*(-I*cos(5*b*x + 3*d)*e^(2*a) - 5*I*cos(3*b*x + d)*e^(2*a) + 5*I*cos(b
*x + 3*d)*e^(2*a) - 15*I*cos(b*x - d)*e^(2*a) + e^(2*a)*sin(5*b*x + 3*d) +
 5*e^(2*a)*sin(3*b*x + d) + 5*e^(2*a)*sin(b*x + 3*d) + 15*e^(2*a)*sin(b*x
- d))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(69) = 138.

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.35

$$\int e^{2(a+ibx)} \cos^3(d+bx)\,dx =$$
$$-\frac{i\left(e^{(5i\,bx+3i\,d)}+e^{(-5i\,bx-3i\,d)}\right)e^{(2\,a)}+5i\left(e^{(3i\,bx+i\,d)}+e^{(-3i\,bx-i\,d)}\right)e^{(2\,a)}-5i\left(e^{(i\,bx+3i\,d)}+e^{(-i\,bx-3i\,d)}\right)e^{(2\,a)}}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/80*(I*(e^(5*I*b*x + 3*I*d) + e^(-5*I*b*x - 3*I*d))*e^(2*a) + 5*I*(e^(3*
I*b*x + I*d) + e^(-3*I*b*x - I*d))*e^(2*a) - 5*I*(e^(I*b*x + 3*I*d) + e^(-
I*b*x - 3*I*d))*e^(2*a) + 15*I*(e^(I*b*x - I*d) + e^(-I*b*x + I*d))*e^(2*a
) - 10*e^(2*a)*sin(b*x + 3*d) + 30*e^(2*a)*sin(-b*x + d) + 10*e^(2*a)*sin(
-3*b*x - d) + 2*e^(2*a)*sin(-5*b*x - 3*d))/b
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10

$$\int e^{2(a+ibx)}\cos^3(d+bx)\,dx = \frac{e^{2a}\left(\cos(bx)-\sin(bx)\,1i\right)\left(\cos(3d)-\sin(3d)\,1i\right)1i}{8b} - \frac{e^{2a}\left(\cos(5bx)+\sin(5bx)\,1i\right)\left(\cos(3d)+\sin(3d)\,1i\right)1i}{40b} - \frac{e^{2a}\left(\cos(bx)+\sin(bx)\,1i\right)\left(\cos(d)-\sin(d)\,1i\right)3i}{8b} - \frac{e^{2a}\left(\cos(3bx)+\sin(3bx)\,1i\right)\left(\cos(d)+\sin(d)\,1i\right)1i}{8b}$$

input
```
int(cos(d + b*x)^3*exp(2*a + b*x*2i),x)
```

output
```
(exp(2*a)*(cos(b*x) - sin(b*x)*1i)*(cos(3*d) - sin(3*d)*1i)*1i)/(8*b) - (e
xp(2*a)*(cos(5*b*x) + sin(5*b*x)*1i)*(cos(3*d) + sin(3*d)*1i)*1i)/(40*b) -
 (exp(2*a)*(cos(b*x) + sin(b*x)*1i)*(cos(d) - sin(d)*1i)*3i)/(8*b) - (exp(
2*a)*(cos(3*b*x) + sin(3*b*x)*1i)*(cos(d) + sin(d)*1i)*1i)/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int e^{2(a+ibx)}\cos^3(d+bx)\,dx = \frac{e^{2bix+2a}\left(-4\cos(bx+d)\sin(bx+d)^2 i - 2\cos(bx+d)\sin(bx+d) - 4\cos(bx+d)\,i - 6\sin(bx+d)^3 + \right.}{10b}$$

input
```
int(exp(2*a+2*I*b*x)*cos(b*x+d)^3,x)
```

output
```
(e**(2*a + 2*b*i*x)*( - 4*cos(b*x + d)*sin(b*x + d)**2*i - 2*cos(b*x + d)*
sin(b*x + d) - 4*cos(b*x + d)*i - 6*sin(b*x + d)**3 + 2*sin(b*x + d)**2*i
+ 2*sin(b*x + d) - i))/(10*b)
```

3.28 $\int e^{2(a+ibx)}\cos^4(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\int e^{2(a+ibx)}\cos^4(d+bx)\,dx=\frac{ie^{2(a-id)-2i(d+bx)}}{32b}-\frac{3ie^{2(a-id)+2i(d+bx)}}{16b}-\frac{ie^{2(a-id)+4i(d+bx)}}{16b}-\frac{ie^{2(a-id)+6i(d+bx)}}{96b}+\frac{1}{4}e^{2a-2id}x$$

output
```
1/32*I*exp(2*a-2*I*d-2*I*(b*x+d))/b-3/16*I*exp(2*a-2*I*d+2*I*(b*x+d))/b-1/
16*I*exp(2*a-2*I*d+4*I*(b*x+d))/b-1/96*I*exp(2*a-2*I*d+6*I*(b*x+d))/b+1/4*
exp(2*a-2*I*d)*x
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int e^{2(a+ibx)}\cos^4(d+bx)\,dx$$
$$=\frac{e^{2a-2ibx}\left(-18ie^{4ibx}+6e^{2ibx}\left(-ie^{4ibx}+4bx\right)\cos(2d)-i\left(-3+e^{8ibx}\right)\cos(4d)+6e^{6ibx}\sin(2d)-24ibe^{2ibx}x\,s\right.}{96b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Cos[d + b*x]^4,x]
```

output
```
(E^(2*a - (2*I)*b*x)*((-18*I)*E^((4*I)*b*x) + 6*E^((2*I)*b*x)*((-I)*E^((4*
I)*b*x) + 4*b*x)*Cos[2*d] - I*(-3 + E^((8*I)*b*x))*Cos[4*d] + 6*E^((6*I)*b
*x)*Sin[2*d] - (24*I)*b*E^((2*I)*b*x)*x*Sin[2*d] + 3*Sin[4*d] + E^((8*I)*b
*x)*Sin[4*d]))/(96*b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4935, 4967, 4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\cos^4(bx+d)\,dx$$

$$\Big\downarrow \; 4935$$

$$\int e^{2(a+ibx)}\cos^2(d+bx)dx+\frac{ie^{2(a+ibx)}\cos^4(bx+d)}{6b}+\frac{e^{2(a+ibx)}\sin(bx+d)\cos^3(bx+d)}{3b}$$

$$\Big\downarrow \; 4967$$

$$\int e^{2a+2ibx}\cos^2(d+bx)dx+\frac{ie^{2(a+ibx)}\cos^4(bx+d)}{6b}+\frac{e^{2(a+ibx)}\sin(bx+d)\cos^3(bx+d)}{3b}$$

$$\Big\downarrow \; 4976$$

$$\int\left(\frac{1}{4}e^{2(a-id)}+\frac{1}{2}e^{2a+2ibx}+\frac{1}{4}e^{2a+2id+4ibx}\right)dx+\frac{ie^{2(a+ibx)}\cos^4(bx+d)}{6b}+\frac{e^{2(a+ibx)}\sin(bx+d)\cos^3(bx+d)}{3b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{ie^{2(a+id)+4ibx}}{16b}+\frac{ie^{2(a+ibx)}\cos^4(bx+d)}{6b}+\frac{e^{2(a+ibx)}\sin(bx+d)\cos^3(bx+d)}{3b}-\frac{ie^{2a+2ibx}}{4b}+\frac{1}{4}xe^{2a-2id}$$

input
```
Int[E^(2*(a + I*b*x))*Cos[d + b*x]^4,x]
```

output
```
((-1/4*I)*E^(2*a + (2*I)*b*x))/b - ((I/16)*E^(2*(a + I*d) + (4*I)*b*x))/b
+ (E^(2*a - (2*I)*d)*x)/4 + ((I/6)*E^(2*(a + I*b*x))*Cos[d + b*x]^4)/b + (
E^(2*(a + I*b*x))*Cos[d + b*x]^3*Sin[d + b*x])/(3*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

rule 4967
```
Int[(F_)^((c_.)*(u_))*(G_)[v_]^(n_.), x_Symbol] :> Int[F^(c*ExpandToSum[u,
x])*G[ExpandToSum[v, x]]^n, x] /; FreeQ[{F, c, n}, x] && TrigQ[G] && Linear
Q[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.67

method	result
parallelrisch	$-\frac{{\mathrm e}^{2 i b x +2 a} \left(12 i b x \sin \left(2 b x +2 d \right)-12 b x \cos \left(2 b x +2 d \right)-i \cos \left(4 b x +4 d \right)-8 i \cos \left(2 b x +2 d \right)+9 i -2 \sin \left(4 b x +4 d \right)-14 \sin \left(2 b x +2 d \right)\right)}{48 b}$
norman	$\frac{-i x \,{\mathrm e}^{2 i b x +2 a} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{3}-\frac{2 i {\mathrm e}^{2 i b x +2 a} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{6}}{b}+\frac{x \,{\mathrm e}^{2 i b x +2 a}}{4}-x \,{\mathrm e}^{2 i b x +2 a} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{2}-\frac{5 x \,{\mathrm e}^{2 i b x +2 a} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{4}}{2}-x \,{\mathrm e}^{2}}{}$
orering	$-\frac{\left(-12 b x +5 i \right) {\mathrm e}^{2 i b x +2 a} \cos \left(b x +d \right)^{4}}{12 b}+\frac{5 i \left(2 b x +i \right) \left(2 i b \,{\mathrm e}^{2 i b x +2 a} \cos \left(b x +d \right)^{4}-4 \,{\mathrm e}^{2 i b x +2 a} \cos \left(b x +d \right)^{3} b \sin \left(b x +d \right)\right)}{24 b^{2}}-\frac{5 \left(-2 b\right.}{}$

input
```
int(exp(2*a+2*I*b*x)*cos(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48*exp(2*a+2*I*b*x)*(12*I*b*x*sin(2*b*x+2*d)-12*b*x*cos(2*b*x+2*d)-I*co
s(4*b*x+4*d)-8*I*cos(2*b*x+2*d)+9*I-2*sin(4*b*x+4*d)-14*sin(2*b*x+2*d))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.58

$$\int e^{2(a+ibx)}\cos^4(d+bx)\,dx = \frac{\left(24\,bxe^{(2i\,bx+2\,a)} - i\,e^{(8i\,bx+2\,a+6i\,d)} - 6i\,e^{(6i\,bx+2\,a+4i\,d)} - 18i\,e^{(4i\,bx+2\,a+2i\,d)} + 3i\,e^{(2\,a-2i\,d)}\right)e^{(-2i\,bx-2i\,d)}}{96\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d)^4,x, algorithm="fricas")
```

output
```
1/96*(24*b*x*e^(2*I*b*x + 2*a) - I*e^(8*I*b*x + 2*a + 6*I*d) - 6*I*e^(6*I*
b*x + 2*a + 4*I*d) - 18*I*e^(4*I*b*x + 2*a + 2*I*d) + 3*I*e^(2*a - 2*I*d))
*e^(-2*I*b*x - 2*I*d)/b
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.59

$$\int e^{2(a+ibx)}\cos^4(d+bx)\,dx = \frac{xe^{2a}e^{-2id}}{4} + \begin{cases} \frac{\left(-8192ib^3e^{2a}e^{8id}e^{6ibx} - 49152ib^3e^{2a}e^{6id}e^{4ibx} - 147456ib^3e^{2a}e^{4id}e^{2ibx} + 24576ib^3e^{2a}e^{-2ibx}\right)e^{-4id}}{786432b^4} & \text{for}\ b^4e^{4id} \neq 0 \\ x\left(\frac{\left(e^{2a}e^{8id} + 4e^{2a}e^{6id} + 6e^{2a}e^{4id} + 4e^{2a}e^{2id} + e^{2a}\right)e^{-4id}}{16} - \frac{e^{2a}e^{-2id}}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d)**4,x)
```

output
```
x*exp(2*a)*exp(-2*I*d)/4 + Piecewise(((-8192*I*b**3*exp(2*a)*exp(8*I*d)*ex
p(6*I*b*x) - 49152*I*b**3*exp(2*a)*exp(6*I*d)*exp(4*I*b*x) - 147456*I*b**3
*exp(2*a)*exp(4*I*d)*exp(2*I*b*x) + 24576*I*b**3*exp(2*a)*exp(-2*I*b*x))*e
xp(-4*I*d)/(786432*b**4), Ne(b**4*exp(4*I*d), 0)), (x*((exp(2*a)*exp(8*I*d
) + 4*exp(2*a)*exp(6*I*d) + 6*exp(2*a)*exp(4*I*d) + 4*exp(2*a)*exp(2*I*d)
+ exp(2*a))*exp(-4*I*d)/16 - exp(2*a)*exp(-2*I*d)/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04

$$\int e^{2(a+ibx)}\cos^4(d+bx)\,dx$$
$$= \frac{24\left(b\cos\left(2\,d\right)e^{\left(2\,a\right)} - i\,be^{\left(2\,a\right)}\sin\left(2\,d\right)\right)x - 18i\,\cos\left(2\,bx\right)e^{\left(2\,a\right)} - i\,\cos\left(6\,bx + 4\,d\right)e^{\left(2\,a\right)} - 6i\,\cos\left(4\,bx + 2\right.}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d)^4,x, algorithm="maxima")
```

output
```
1/96*(24*(b*cos(2*d)*e^(2*a) - I*b*e^(2*a)*sin(2*d))*x - 18*I*cos(2*b*x)*e
^(2*a) - I*cos(6*b*x + 4*d)*e^(2*a) - 6*I*cos(4*b*x + 2*d)*e^(2*a) + 3*I*c
os(2*b*x + 4*d)*e^(2*a) + 18*e^(2*a)*sin(2*b*x) + e^(2*a)*sin(6*b*x + 4*d)
 + 6*e^(2*a)*sin(4*b*x + 2*d) + 3*e^(2*a)*sin(2*b*x + 4*d))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(77) = 154.

Time = 0.13 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.34

$$\int e^{2(a+ibx)}\cos^4(d+bx)\,dx$$
$$= \frac{48\left(bx+d\right)\cos\left(2\,d\right)e^{\left(2\,a\right)} - 48i\left(bx+d\right)e^{\left(2\,a\right)}\sin\left(2\,d\right) - i\left(e^{\left(6i\,bx+4i\,d\right)} + e^{\left(-6i\,bx-4i\,d\right)}\right)e^{\left(2\,a\right)} - 6i\left(e^{\left(4i\,bx+2i\,d\right)}\right.}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*cos(b*x+d)^4,x, algorithm="giac")
```

output
```
1/192*(48*(b*x + d)*cos(2*d)*e^(2*a) - 48*I*(b*x + d)*e^(2*a)*sin(2*d) - I
*(e^(6*I*b*x + 4*I*d) + e^(-6*I*b*x - 4*I*d))*e^(2*a) - 6*I*(e^(4*I*b*x +
2*I*d) + e^(-4*I*b*x - 2*I*d))*e^(2*a) + 3*I*(e^(2*I*b*x + 4*I*d) + e^(-2*
I*b*x - 4*I*d))*e^(2*a) - 18*I*(e^(2*I*b*x) + e^(-2*I*b*x))*e^(2*a) + 36*e
^(2*a)*sin(2*b*x) + 6*e^(2*a)*sin(2*b*x + 4*d) - 12*e^(2*a)*sin(-4*b*x - 2
*d) - 2*e^(2*a)*sin(-6*b*x - 4*d))/b
```

Mupad [B] (verification not implemented)

Time = 15.60 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.11

$$\int e^{2(a+ibx)}\cos^4(d+bx)\,dx = \frac{x\,e^{2\,a}\,\left(\cos\left(2\,d\right)-\sin\left(2\,d\right)\,1\mathrm{i}\right)}{4} - \frac{e^{2\,a}\,\left(\cos\left(2\,b\,x\right)+\sin\left(2\,b\,x\right)\,1\mathrm{i}\right)\,3\mathrm{i}}{16\,b} + \frac{e^{2\,a}\,\left(\cos\left(2\,b\,x\right)-\sin\left(2\,b\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(4\,d\right)-\sin\left(4\,d\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{32\,b} - \frac{e^{2\,a}\,\left(\cos\left(4\,b\,x\right)+\sin\left(4\,b\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(2\,d\right)+\sin\left(2\,d\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{16\,b} - \frac{e^{2\,a}\,\left(\cos\left(6\,b\,x\right)+\sin\left(6\,b\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(4\,d\right)+\sin\left(4\,d\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{96\,b}$$

input
```
int(cos(d + b*x)^4*exp(2*a + b*x*2i),x)
```

output
```
(x*exp(2*a)*(cos(2*d) - sin(2*d)*1i))/4 - (exp(2*a)*(cos(2*b*x) + sin(2*b*
x)*1i)*3i)/(16*b) + (exp(2*a)*(cos(2*b*x) - sin(2*b*x)*1i)*(cos(4*d) - sin
(4*d)*1i)*1i)/(32*b) - (exp(2*a)*(cos(4*b*x) + sin(4*b*x)*1i)*(cos(2*d) +
sin(2*d)*1i)*1i)/(16*b) - (exp(2*a)*(cos(6*b*x) + sin(6*b*x)*1i)*(cos(4*d)
 + sin(4*d)*1i)*1i)/(96*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.66

$$\int e^{2(a+ibx)}\cos^4(d+bx)\,dx$$
$$= \frac{e^{2bix+2a}\left(-8\cos\left(bx+d\right)\sin\left(bx+d\right)^3-12\cos\left(bx+d\right)\sin\left(bx+d\right)bix+4\sin\left(bx+d\right)^4i-12\sin\left(bx+\right.\right.}{24b}$$

input
```
int(exp(2*a+2*I*b*x)*cos(b*x+d)^4,x)
```

output
```
(e**(2*a + 2*b*i*x)*( - 8*cos(b*x + d)*sin(b*x + d)**3 - 12*cos(b*x + d)*s
in(b*x + d)*b*i*x + 4*sin(b*x + d)**4*i - 12*sin(b*x + d)**2*b*x + 6*sin(b
*x + d)**2*i + 6*b*x - 9*i))/(24*b)
```

3.29 $\int e^{\frac{5}{3}(a+ibx)}\cos(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int e^{\frac{5}{3}(a+ibx)}\cos(a+bx)\,dx=-\frac{3ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a+\frac{2}{3}i(a+bx)}}{4b}-\frac{3ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a+\frac{8}{3}i(a+bx)}}{16b}$$

output
```
-3/4*I*exp((5/3-5/3*I)*a+2/3*I*(b*x+a))/b-3/16*I*exp((5/3-5/3*I)*a+8/3*I*(
b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int e^{\frac{5}{3}(a+ibx)}\cos(a+bx)\,dx=\frac{3e^{\frac{5a}{3}+\frac{2ibx}{3}}\left(-i\left(4+e^{2ibx}\right)\cos(a)+\left(-4+e^{2ibx}\right)\sin(a)\right)}{16b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Cos[a + b*x],x]
```

output
```
(3*E^((5*a)/3 + ((2*I)/3)*b*x)*((-I)*(4 + E^((2*I)*b*x))*Cos[a] + (-4 + E^
((2*I)*b*x))*Sin[a]))/(16*b)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4933}$$

$$-\frac{9e^{\frac{5}{3}(a+ibx)}\sin(a+bx)}{16b}-\frac{15ie^{\frac{5}{3}(a+ibx)}\cos(a+bx)}{16b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Cos[a + b*x],x]
```

output
```
(((-15*I)/16)*E^((5*(a + I*b*x))/3)*Cos[a + b*x])/b - (9*E^((5*(a + I*b*x)
)/3)*Sin[a + b*x])/(16*b)
```

Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

method	result	size
parallelrisch	$-\frac{3\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\left(5i\cos\left(bx+a\right)+3\sin\left(bx+a\right)\right)}{16b}$	34
default	$-\frac{15i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\cos\left(bx+a\right)}{16b}-\frac{9\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\sin\left(bx+a\right)}{16b}$	45
orering	$-\frac{15i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\cos\left(bx+a\right)}{8b}+\frac{\frac{15ib\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\cos\left(bx+a\right)}{16}-\frac{9\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}b\sin\left(bx+a\right)}{16}}{b^{2}}$	69
norman	$\frac{-\frac{9\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{15i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}}{16b}+\frac{15i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}{16b}}{1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}$	85

input
```
int(exp(5/3*a+5/3*I*b*x)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-3/16*exp(5/3*a+5/3*I*b*x)*(5*I*cos(b*x+a)+3*sin(b*x+a))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.43

$$\int e^{\frac{5}{3}(a+ibx)}\cos(a+bx)\,dx=-\frac{3\left(i\,e^{\left(\frac{8}{3}\,i\,bx+\left(i+\frac{5}{3}\right)\,a\right)}+4i\,e^{\left(\frac{2}{3}\,i\,bx-\left(i-\frac{5}{3}\right)\,a\right)}\right)}{16\,b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a),x, algorithm="fricas")
```

output
```
-3/16*(I*e^(8/3*I*b*x + (I + 5/3)*a) + 4*I*e^(2/3*I*b*x - (I - 5/3)*a))/b
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int e^{\frac{5}{3}(a+ibx)}\cos(a+bx)\,dx=\begin{cases}\frac{\left(-12ibe^{\frac{5a}{3}}e^{2ia}e^{\frac{8ibx}{3}}-48ibe^{\frac{5a}{3}}e^{\frac{2ibx}{3}}\right)e^{-ia}}{64b^{2}} & \text{for } b^{2}e^{ia}\neq 0\\ \frac{x\left(e^{\frac{5a}{3}}e^{2ia}+e^{\frac{5a}{3}}\right)e^{-ia}}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a),x)
```

output
```
Piecewise(((-12*I*b*exp(5*a/3)*exp(2*I*a)*exp(8*I*b*x/3) - 48*I*b*exp(5*a/
3)*exp(2*I*b*x/3))*exp(-I*a)/(64*b**2), Ne(b**2*exp(I*a), 0)), (x*(exp(5*a
/3)*exp(2*I*a) + exp(5*a/3))*exp(-I*a)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.51

$$\int e^{\frac{5}{3}(a+ibx)}\cos(a+bx)\,dx = \frac{3\left(-5i\,b\cos\left(bx+a\right)-3\,b\sin\left(bx+a\right)\right)e^{\left(\frac{5}{3}i\,bx+\frac{5}{3}\,a\right)}}{16\,b^2}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a),x, algorithm="maxima")
```

output
```
3/16*(-5*I*b*cos(b*x + a) - 3*b*sin(b*x + a))*e^(5/3*I*b*x + 5/3*a)/b^2
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(29) = 58.

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int e^{\frac{5}{3}(a+ibx)}\cos(a+bx)\,dx =$$
$$-\frac{3\left(i\left(e^{\left(\frac{8}{3}i\,bx+i\,a\right)}+e^{\left(-\frac{8}{3}i\,bx-i\,a\right)}\right)e^{\left(\frac{5}{3}\,a\right)}+4i\left(e^{\left(\frac{2}{3}i\,bx-i\,a\right)}+e^{\left(-\frac{2}{3}i\,bx+i\,a\right)}\right)e^{\left(\frac{5}{3}\,a\right)}-2\,e^{\left(\frac{5}{3}\,a\right)}\sin\left(\frac{8}{3}\,bx+a\right)+8}{32\,b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a),x, algorithm="giac")
```

output
```
-3/32*(I*(e^(8/3*I*b*x + I*a) + e^(-8/3*I*b*x - I*a))*e^(5/3*a) + 4*I*(e^(
2/3*I*b*x - I*a) + e^(-2/3*I*b*x + I*a))*e^(5/3*a) - 2*e^(5/3*a)*sin(8/3*b
*x + a) + 8*e^(5/3*a)*sin(-2/3*b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int e^{\frac{5}{3}(a+ibx)}\cos(a+bx)\,dx$$
$$= -\frac{\mathrm{e}^{\frac{5\,a}{3}}\,\left(\cos\left(\frac{5\,b\,x}{3}\right)+\sin\left(\frac{5\,b\,x}{3}\right)\,1\mathrm{i}\right)\,\left(5\,\cos\left(a+b\,x\right)-\sin\left(a+b\,x\right)\,3\mathrm{i}\right)\,3\mathrm{i}}{16\,b}$$

input
```
int(cos(a + b*x)*exp((5*a)/3 + (b*x*5i)/3),x)
```

output
```
-(exp((5*a)/3)*(cos((5*b*x)/3) + sin((5*b*x)/3)*1i)*(5*cos(a + b*x) - sin(
a + b*x)*3i)*3i)/(16*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int e^{\frac{5}{3}(a+ibx)}\cos(a+bx)\,dx = \frac{3e^{\frac{5bix}{3}+\frac{5a}{3}}\left(-5\cos\left(bx+a\right)i-3\sin\left(bx+a\right)\right)}{16b}$$

input
```
int(exp(5/3*a+5/3*I*b*x)*cos(b*x+a),x)
```

output
```
(3*e**((5*a + 5*b*i*x)/3)*( - 5*cos(a + b*x)*i - 3*sin(a + b*x)))/(16*b)
```

3.30 $\int e^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int e^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)\,dx=\frac{3ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a-\frac{1}{3}i(a+bx)}}{4b}-\frac{3ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a+\frac{5}{3}i(a+bx)}}{10b}-\frac{3ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a+\frac{11}{3}i(a+bx)}}{44b}$$

output
```
3/4*I*exp((5/3-5/3*I)*a-1/3*I*(b*x+a))/b-3/10*I*exp((5/3-5/3*I)*a+5/3*I*(b
*x+a))/b-3/44*I*exp((5/3-5/3*I)*a+11/3*I*(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$\int e^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)\,dx$$
$$=-\frac{3ie^{\frac{5a}{3}-\frac{ibx}{3}}\left(22e^{2ibx}+5\left(-11+e^{4ibx}\right)\cos(2a)+5i\left(11+e^{4ibx}\right)\sin(2a)\right)}{220b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Cos[a + b*x]^2,x]
```

output
```
(((-3*I)/220)*E^((5*a)/3 - (I/3)*b*x)*(22*E^((2*I)*b*x) + 5*(-11 + E^((4*I
)*b*x))*Cos[2*a] + (5*I)*(11 + E^((4*I)*b*x))*Sin[2*a]))/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4935, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4935}$$

$$\frac{18}{11}\int e^{\frac{5}{3}(a+ibx)}dx+\frac{15ie^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)}{11b}+\frac{18e^{\frac{5}{3}(a+ibx)}\sin(a+bx)\cos(a+bx)}{11b}$$

$$\Big\downarrow \text{2624}$$

$$-\frac{54ie^{\frac{5}{3}(a+ibx)}}{55b}+\frac{15ie^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)}{11b}+\frac{18e^{\frac{5}{3}(a+ibx)}\sin(a+bx)\cos(a+bx)}{11b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Cos[a + b*x]^2,x]
```

output
```
(((-54*I)/55)*E^((5*(a + I*b*x))/3))/b + (((15*I)/11)*E^((5*(a + I*b*x))/3
)*Cos[a + b*x]^2)/b + (18*E^((5*(a + I*b*x))/3)*Cos[a + b*x]*Sin[a + b*x])
/(11*b)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.43

method	result
parallelrisch	$\frac{3\,{\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \left(25 i \cos \left(2 b x +2 a \right)-11 i +30 \sin \left(2 b x +2 a \right)\right)}{110 b}$
default	$-\frac{3 i {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}}}{10 b}+\frac{15 i {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \cos \left(2 b x +2 a \right)}{22 b}+\frac{9\,{\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \sin \left(2 b x +2 a \right)}{11 b}$
norman	$\frac{\frac{36\,{\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \tan \left(\frac{b x}{2}+\frac{a}{2}\right)}{11 b}-\frac{36\,{\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}}{11 b}+\frac{21 i {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}}}{55 b}-\frac{258 i {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}}{55 b}+\frac{21 i {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{4}}{55 b}}{\left(1+\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}\right)^{2}}$
orering	$\frac{117 i {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \cos \left(b x +a \right)^{2}}{55 b}-\frac{27 \left(\frac{5 i b\,{\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \cos \left(b x +a \right)^{2}}{3}-2\,{\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}} \cos \left(b x +a \right) b \sin \left(b x +a \right)\right)}{11 b^{2}}-\frac{27 i \left(-\frac{43 b^{2} {\mathrm e}^{\frac{5 a}{3}+\frac{5 i b x}{3}}}{9}\right.}{}$

input
```
int(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/110*exp(5/3*a+5/3*I*b*x)*(25*I*cos(2*b*x+2*a)-11*I+30*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.44

$$\int e^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)\,dx$$
$$= -\frac{3\left(5i\,e^{\left(4i\,bx+\left(\frac{7}{3}i+\frac{5}{3}\right)a\right)}+22i\,e^{\left(2i\,bx+\left(\frac{1}{3}i+\frac{5}{3}\right)a\right)}-55i\,e^{\left(-\left(\frac{5}{3}i-\frac{5}{3}\right)a\right)}\right)e^{\left(-\frac{1}{3}i\,bx-\frac{1}{3}i\,a\right)}}{220\,b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
-3/220*(5*I*e^(4*I*b*x + (7/3*I + 5/3)*a) + 22*I*e^(2*I*b*x + (1/3*I + 5/3
)*a) - 55*I*e^(-(5/3*I - 5/3)*a))*e^(-1/3*I*b*x - 1/3*I*a)/b
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.53

$$\int e^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)\,dx$$
$$= \begin{cases} \frac{\left(-120ib^2e^{\frac{5a}{3}}e^{4ia}e^{\frac{11ibx}{3}}-528ib^2e^{\frac{5a}{3}}e^{2ia}e^{\frac{5ibx}{3}}+1320ib^2e^{\frac{5a}{3}}e^{-\frac{ibx}{3}}\right)e^{-2ia}}{1760b^3} & \text{for } b^3e^{2ia}\neq 0 \\ \frac{x\left(e^{\frac{5a}{3}}e^{4ia}+2e^{\frac{5a}{3}}e^{2ia}+e^{\frac{5a}{3}}\right)e^{-2ia}}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)**2,x)
```

output
```
Piecewise(((-120*I*b**2*exp(5*a/3)*exp(4*I*a)*exp(11*I*b*x/3) - 528*I*b**2
*exp(5*a/3)*exp(2*I*a)*exp(5*I*b*x/3) + 1320*I*b**2*exp(5*a/3)*exp(-I*b*x/
3))*exp(-2*I*a)/(1760*b**3), Ne(b**3*exp(2*I*a), 0)), (x*(exp(5*a/3)*exp(4
*I*a) + 2*exp(5*a/3)*exp(2*I*a) + exp(5*a/3))*exp(-2*I*a)/4, True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(43) = 86.

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int e^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)\,dx = -\frac{3\left(22i\cos\left(\frac{5}{3}\,bx\right)e^{\left(\frac{5}{3}\,a\right)}+5i\cos\left(\frac{11}{3}\,bx+2\,a\right)e^{\left(\frac{5}{3}\,a\right)}-55i\cos\left(\frac{1}{3}\,bx+2\,a\right)e^{\left(\frac{5}{3}\,a\right)}-22\,e^{\left(\frac{5}{3}\,a\right)}\sin\left(\frac{5}{3}\,bx\right)-\right.}{220\,b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
-3/220*(22*I*cos(5/3*b*x)*e^(5/3*a) + 5*I*cos(11/3*b*x + 2*a)*e^(5/3*a) -
55*I*cos(1/3*b*x + 2*a)*e^(5/3*a) - 22*e^(5/3*a)*sin(5/3*b*x) - 5*e^(5/3*a
)*sin(11/3*b*x + 2*a) - 55*e^(5/3*a)*sin(1/3*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(43) = 86.

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

$$\int e^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)\,dx = -\frac{3\left(5i\left(e^{\left(\frac{11}{3}\,i\,bx+2i\,a\right)}+e^{\left(-\frac{11}{3}\,i\,bx-2i\,a\right)}\right)e^{\left(\frac{5}{3}\,a\right)}-55i\left(e^{\left(\frac{1}{3}\,i\,bx+2i\,a\right)}+e^{\left(-\frac{1}{3}\,i\,bx-2i\,a\right)}\right)e^{\left(\frac{5}{3}\,a\right)}+22i\left(e^{\left(\frac{5}{3}\,i\,bx\right)}+e^{\left(-\right.}\right.\right.}{440\,b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
-3/440*(5*I*(e^(11/3*I*b*x + 2*I*a) + e^(-11/3*I*b*x - 2*I*a))*e^(5/3*a) -
 55*I*(e^(1/3*I*b*x + 2*I*a) + e^(-1/3*I*b*x - 2*I*a))*e^(5/3*a) + 22*I*(e
^(5/3*I*b*x) + e^(-5/3*I*b*x))*e^(5/3*a) - 44*e^(5/3*a)*sin(5/3*b*x) - 10*
e^(5/3*a)*sin(11/3*b*x + 2*a) - 110*e^(5/3*a)*sin(1/3*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 15.67 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

$$\int e^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)\,dx = \frac{3\,\mathrm{e}^{\frac{5\,a}{3}+\frac{b\,x\,5\mathrm{i}}{3}}\,\left(\cos\left(2\,a+2\,b\,x\right)\,25\mathrm{i}+30\,\sin\left(2\,a+2\,b\,x\right)-11\mathrm{i}\right)}{110\,b}$$

input
```
int(cos(a + b*x)^2*exp((5*a)/3 + (b*x*5i)/3),x)
```

output
```
(3*exp((5*a)/3 + (b*x*5i)/3)*(cos(2*a + 2*b*x)*25i + 30*sin(2*a + 2*b*x) -
 11i))/(110*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

$$\int e^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)\,dx = \frac{3e^{\frac{5bix}{3}+\frac{5a}{3}}\left(30\cos\left(bx+a\right)\sin\left(bx+a\right)-25\sin\left(bx+a\right)^{2}i+7i\right)}{55b}$$

input
```
int(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^2,x)
```

output
```
(3*e**((5*a + 5*b*i*x)/3)*(30*cos(a + b*x)*sin(a + b*x) - 25*sin(a + b*x)*
*2*i + 7*i))/(55*b)
```

3.31 $\int e^{\frac{5}{3}(a+ibx)} \cos^3(a+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int e^{\frac{5}{3}(a+ibx)} \cos^3(a+bx)\,dx = -\frac{9ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a+\frac{2}{3}i(a+bx)}}{16b}+\frac{3ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a-\frac{4}{3}i(a+bx)}}{32b}-\frac{9ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a+\frac{8}{3}i(a+bx)}}{64b}-\frac{3ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a+\frac{14}{3}i(a+bx)}}{112b}$$

output
```
-9/16*I*exp((5/3-5/3*I)*a+2/3*I*(b*x+a))/b+3/32*I*exp((5/3-5/3*I)*a-4/3*I*
(b*x+a))/b-9/64*I*exp((5/3-5/3*I)*a+8/3*I*(b*x+a))/b-3/112*I*exp((5/3-5/3*
I)*a+14/3*I*(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.87

$$\int e^{\frac{5}{3}(a+ibx)} \cos^3(a+bx)\,dx = \frac{3e^{\frac{5a}{3}-\frac{4ibx}{3}}\left(-21ie^{2ibx}\left(4+e^{2ibx}\right)\cos(a)-2i\left(-7+2e^{6ibx}\right)\cos(3a)-84e^{2ibx}\sin(a)+21e^{4ibx}\sin(a)+14\sin}{448b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Cos[a + b*x]^3,x]
```

output
```
(3*E^((5*a)/3 - ((4*I)/3)*b*x)*((-21*I)*E^((2*I)*b*x)*(4 + E^((2*I)*b*x))*
Cos[a] - (2*I)*(-7 + 2*E^((6*I)*b*x))*Cos[3*a] - 84*E^((2*I)*b*x)*Sin[a] +
 21*E^((4*I)*b*x)*Sin[a] + 14*Sin[3*a] + 4*E^((6*I)*b*x)*Sin[3*a]))/(448*b
)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4935, 4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4935}$$

$$\frac{27}{28}\int e^{\frac{5}{3}(a+ibx)}\cos(a+bx)dx+\frac{15ie^{\frac{5}{3}(a+ibx)}\cos^3(a+bx)}{56b}+\frac{27e^{\frac{5}{3}(a+ibx)}\sin(a+bx)\cos^2(a+bx)}{56b}$$

$$\Big\downarrow \text{4933}$$

$$\frac{15ie^{\frac{5}{3}(a+ibx)}\cos^3(a+bx)}{56b}+\frac{27e^{\frac{5}{3}(a+ibx)}\sin(a+bx)\cos^2(a+bx)}{56b}+\frac{27}{28}\left(-\frac{9e^{\frac{5}{3}(a+ibx)}\sin(a+bx)}{16b}-\frac{15ie^{\frac{5}{3}(a+ibx)}\cos(a+bx)}{16b}\right)$$

input
```
Int[E^((5*(a + I*b*x))/3)*Cos[a + b*x]^3,x]
```

output
```
(((15*I)/56)*E^((5*(a + I*b*x))/3)*Cos[a + b*x]^3)/b + (27*E^((5*(a + I*b*
x))/3)*Cos[a + b*x]^2*Sin[a + b*x])/(56*b) + (27*((((-15*I)/16)*E^((5*(a +
 I*b*x))/3)*Cos[a + b*x])/b - (9*E^((5*(a + I*b*x))/3)*Sin[a + b*x])/(16*b
)))/28
```

Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.44

method	result
parallelrisch	$-\frac{3\,{\mathrm e}^{\frac{5a}{3}+\frac{5ibx}{3}} \left(105 i \cos \left(bx+a\right)-10 i \cos \left(3bx+3a\right)+63 \sin \left(bx+a\right)-18 \sin \left(3bx+3a\right)\right)}{448 b}$
default	$\frac{15 i {\mathrm e}^{\frac{5a}{3}+\frac{5ibx}{3}} \cos \left(3bx+3a\right)}{224 b}+\frac{27\,{\mathrm e}^{\frac{5a}{3}+\frac{5ibx}{3}} \sin \left(3bx+3a\right)}{224 b}-\frac{45 i {\mathrm e}^{\frac{5a}{3}+\frac{5ibx}{3}} \cos \left(bx+a\right)}{64 b}-\frac{27\,{\mathrm e}^{\frac{5a}{3}+\frac{5ibx}{3}} \sin \left(bx+a\right)}{64 b}$
norman	$\frac{-\frac{27\,{\mathrm e}^{\frac{5a}{3}+\frac{5ibx}{3}} \tan \left(\frac{bx}{2}+\frac{a}{2}\right)}{224 b}-\frac{459\,{\mathrm e}^{\frac{5a}{3}+\frac{5ibx}{3}} \tan \left(\frac{bx}{2}+\frac{a}{2}\right)^{3}}{112 b}-\frac{27\,{\mathrm e}^{\frac{5a}{3}+\frac{5ibx}{3}} \tan \left(\frac{bx}{2}+\frac{a}{2}\right)^{5}}{224 b}-\frac{285 i {\mathrm e}^{\frac{5a}{3}+\frac{5ibx}{3}}}{448 b}-\frac{765 i {\mathrm e}^{\frac{5a}{3}+\frac{5ibx}{3}} \tan \left(\frac{bx}{2}+\right.}{448 b}}{\left(1+\tan \left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)^{3}}$
orering	$-\frac{75 i {\mathrm e}^{\frac{5a}{3}+\frac{5ibx}{3}} \cos \left(bx+a\right)^{3}}{56 b}-\frac{135 \left(\frac{5 i b\,{\mathrm e}^{\frac{5a}{3}+\frac{5ibx}{3}} \cos \left(bx+a\right)^{3}}{3}-3\,{\mathrm e}^{\frac{5a}{3}+\frac{5ibx}{3}} \cos \left(bx+a\right)^{2} b \sin \left(bx+a\right)\right)}{224 b^{2}}-\frac{135 i \left(-\frac{52 b^{2} {\mathrm e}^{\frac{5a}{3}+}\right.}{}$

input
```
int(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/448*exp(5/3*a+5/3*I*b*x)*(105*I*cos(b*x+a)-10*I*cos(3*b*x+3*a)+63*sin(b
*x+a)-18*sin(3*b*x+3*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.42

$$\int e^{\frac{5}{3}(a+ibx)}\cos^3(a+bx)\,dx = -\frac{3\left(4i\,e^{\left(6i\,bx+\left(\frac{13}{3}i+\frac{5}{3}\right)a\right)}+21i\,e^{\left(4i\,bx+\left(\frac{7}{3}i+\frac{5}{3}\right)a\right)}+84i\,e^{\left(2i\,bx+\left(\frac{1}{3}i+\frac{5}{3}\right)a\right)}-14i\,e^{\left(-\left(\frac{5}{3}i-\frac{5}{3}\right)a\right)}\right)e^{\left(-\frac{4}{3}i\,bx-\frac{4}{3}i\,a\right)}}{448\,b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^3,x, algorithm="fricas")
```

output
```
-3/448*(4*I*e^(6*I*b*x + (13/3*I + 5/3)*a) + 21*I*e^(4*I*b*x + (7/3*I + 5/
3)*a) + 84*I*e^(2*I*b*x + (1/3*I + 5/3)*a) - 14*I*e^(-(5/3*I - 5/3)*a))*e^
(-4/3*I*b*x - 4/3*I*a)/b
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.56

$$\int e^{\frac{5}{3}(a+ibx)}\cos^3(a+bx)\,dx$$
$$= \begin{cases} \frac{\left(-98304ib^3e^{\frac{5a}{3}}e^{7ia}e^{\frac{14ibx}{3}}-516096ib^3e^{\frac{5a}{3}}e^{5ia}e^{\frac{8ibx}{3}}-2064384ib^3e^{\frac{5a}{3}}e^{3ia}e^{\frac{2ibx}{3}}+344064ib^3e^{\frac{5a}{3}}e^{ia}e^{-\frac{4ibx}{3}}\right)e^{-4ia}}{3670016b^4} & \text{for } b^4e^{4ia}\neq 0 \\ \frac{x\left(e^{\frac{5a}{3}}e^{6ia}+3e^{\frac{5a}{3}}e^{4ia}+3e^{\frac{5a}{3}}e^{2ia}+e^{\frac{5a}{3}}\right)e^{-3ia}}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)**3,x)
```

output
```
Piecewise(((-98304*I*b**3*exp(5*a/3)*exp(7*I*a)*exp(14*I*b*x/3) - 516096*I
*b**3*exp(5*a/3)*exp(5*I*a)*exp(8*I*b*x/3) - 2064384*I*b**3*exp(5*a/3)*exp
(3*I*a)*exp(2*I*b*x/3) + 344064*I*b**3*exp(5*a/3)*exp(I*a)*exp(-4*I*b*x/3)
)*exp(-4*I*a)/(3670016*b**4), Ne(b**4*exp(4*I*a), 0)), (x*(exp(5*a/3)*exp(
6*I*a) + 3*exp(5*a/3)*exp(4*I*a) + 3*exp(5*a/3)*exp(2*I*a) + exp(5*a/3))*e
xp(-3*I*a)/8, True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(57) = 114.

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int e^{\frac{5}{3}(a+ibx)}\cos^3(a+bx)\,dx = -\frac{3\left(4i\cos\left(\frac{14}{3}bx+3a\right)e^{\left(\frac{5}{3}a\right)}+21i\cos\left(\frac{8}{3}bx+a\right)e^{\left(\frac{5}{3}a\right)}-14i\cos\left(\frac{4}{3}bx+3a\right)e^{\left(\frac{5}{3}a\right)}+84i\cos\left(\frac{2}{3}bx-\cdots\right.}{\cdots}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^3,x, algorithm="maxima")
```

output
```
-3/448*(4*I*cos(14/3*b*x + 3*a)*e^(5/3*a) + 21*I*cos(8/3*b*x + a)*e^(5/3*a
) - 14*I*cos(4/3*b*x + 3*a)*e^(5/3*a) + 84*I*cos(2/3*b*x - a)*e^(5/3*a) -
4*e^(5/3*a)*sin(14/3*b*x + 3*a) - 21*e^(5/3*a)*sin(8/3*b*x + a) - 14*e^(5/
3*a)*sin(4/3*b*x + 3*a) - 84*e^(5/3*a)*sin(2/3*b*x - a))/b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.26

$$\int e^{\frac{5}{3}(a+ibx)}\cos^3(a+bx)\,dx = -\frac{3\left(4i\left(e^{\left(\frac{14}{3}ibx+3ia\right)}+e^{\left(-\frac{14}{3}ibx-3ia\right)}\right)e^{\left(\frac{5}{3}a\right)}+21i\left(e^{\left(\frac{8}{3}ibx+ia\right)}+e^{\left(-\frac{8}{3}ibx-ia\right)}\right)e^{\left(\frac{5}{3}a\right)}-14i\left(e^{\left(\frac{4}{3}ibx+3ia\right)}+e^{\cdots}\right.\right.}{\cdots}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^3,x, algorithm="giac")
```

output
```
-3/896*(4*I*(e^(14/3*I*b*x + 3*I*a) + e^(-14/3*I*b*x - 3*I*a))*e^(5/3*a) +
 21*I*(e^(8/3*I*b*x + I*a) + e^(-8/3*I*b*x - I*a))*e^(5/3*a) - 14*I*(e^(4/
3*I*b*x + 3*I*a) + e^(-4/3*I*b*x - 3*I*a))*e^(5/3*a) + 84*I*(e^(2/3*I*b*x
- I*a) + e^(-2/3*I*b*x + I*a))*e^(5/3*a) - 8*e^(5/3*a)*sin(14/3*b*x + 3*a)
 - 42*e^(5/3*a)*sin(8/3*b*x + a) - 28*e^(5/3*a)*sin(4/3*b*x + 3*a) + 168*e
^(5/3*a)*sin(-2/3*b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 15.62 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

$$\int e^{\frac{5}{3}(a+ibx)}\cos^3(a+bx)\,dx = -\frac{e^{a\,\left(\frac{5}{3}-\mathrm{i}\right)+\frac{b\,x\,2\mathrm{i}}{3}}\,9\mathrm{i}}{16\,b}+\frac{e^{a\,\left(\frac{5}{3}-3\mathrm{i}\right)-\frac{b\,x\,4\mathrm{i}}{3}}\,3\mathrm{i}}{32\,b}-\frac{e^{a\,\left(\frac{5}{3}+1\mathrm{i}\right)+\frac{b\,x\,8\mathrm{i}}{3}}\,9\mathrm{i}}{64\,b}-\frac{e^{a\,\left(\frac{5}{3}+3\mathrm{i}\right)+\frac{b\,x\,14\mathrm{i}}{3}}\,3\mathrm{i}}{112\,b}$$

input
```
int(cos(a + b*x)^3*exp((5*a)/3 + (b*x*5i)/3),x)
```

output
```
(exp(a*(5/3 - 3i) - (b*x*4i)/3)*3i)/(32*b) - (exp(a*(5/3 - 1i) + (b*x*2i)/
3)*9i)/(16*b) - (exp(a*(5/3 + 1i) + (b*x*8i)/3)*9i)/(64*b) - (exp(a*(5/3 +
 3i) + (b*x*14i)/3)*3i)/(112*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.47

$$\int e^{\frac{5}{3}(a+ibx)}\cos^3(a+bx)\,dx = \frac{3e^{\frac{5bix}{3}+\frac{5a}{3}}\left(-40\cos\left(bx+a\right)\sin\left(bx+a\right)^2 i-95\cos\left(bx+a\right) i-72\sin\left(bx+a\right)^3-9\sin\left(bx+a\right)\right)}{448b}$$

input
```
int(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^3,x)
```

output
```
(3*e**((5*a + 5*b*i*x)/3)*( - 40*cos(a + b*x)*sin(a + b*x)**2*i - 95*cos(a
 + b*x)*i - 72*sin(a + b*x)**3 - 9*sin(a + b*x)))/(448*b)
```

3.32 $\int e^{\frac{5}{3}(a+ibx)}\cos^4(a+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 161

$$\int e^{\frac{5}{3}(a+ibx)}\cos^4(a+bx)\,dx=\frac{3ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a-\frac{1}{3}i(a+bx)}}{4b}-\frac{9ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a+\frac{5}{3}i(a+bx)}}{40b}+\frac{3ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a-\frac{7}{3}i(a+bx)}}{112b}-\frac{3ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a+\frac{11}{3}i(a+bx)}}{44b}-\frac{3ie^{\left(\frac{5}{3}-\frac{5i}{3}\right)a+\frac{17}{3}i(a+bx)}}{272b}$$

output
```
3/4*I*exp((5/3-5/3*I)*a-1/3*I*(b*x+a))/b-9/40*I*exp((5/3-5/3*I)*a+5/3*I*(b
*x+a))/b+3/112*I*exp((5/3-5/3*I)*a-7/3*I*(b*x+a))/b-3/44*I*exp((5/3-5/3*I)
*a+11/3*I*(b*x+a))/b-3/272*I*exp((5/3-5/3*I)*a+17/3*I*(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

$$\int e^{\frac{5}{3}(a+ibx)}\cos^4(a+bx)\,dx=-\frac{3ie^{\frac{5a}{3}-\frac{7ibx}{3}}\left(7854e^{4ibx}+2380e^{2ibx}\left(-11+e^{4ibx}\right)\cos(2a)+55\left(-17+7e^{8ibx}\right)\cos(4a)+26180ie^{2ibx}\sin(2a\right.}{104720b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Cos[a + b*x]^4,x]
```

output
```
(((-3*I)/104720)*E^((5*a)/3 - ((7*I)/3)*b*x)*(7854*E^((4*I)*b*x) + 2380*E^
((2*I)*b*x)*(-11 + E^((4*I)*b*x))*Cos[2*a] + 55*(-17 + 7*E^((8*I)*b*x))*Co
s[4*a] + (26180*I)*E^((2*I)*b*x)*Sin[2*a] + (2380*I)*E^((6*I)*b*x)*Sin[2*a
] + (935*I)*Sin[4*a] + (385*I)*E^((8*I)*b*x)*Sin[4*a]))/b
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4935, 4935, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\cos^4(a+bx)\,dx$$

$$\Big\downarrow \text{4935}$$

$$\frac{108}{119}\int e^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)dx+\frac{15ie^{\frac{5}{3}(a+ibx)}\cos^4(a+bx)}{119b}+\frac{36e^{\frac{5}{3}(a+ibx)}\sin(a+bx)\cos^3(a+bx)}{119b}$$

$$\Big\downarrow \text{4935}$$

$$\frac{108}{119}\left(\frac{18}{11}\int e^{\frac{5}{3}(a+ibx)}dx+\frac{15ie^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)}{11b}+\frac{18e^{\frac{5}{3}(a+ibx)}\sin(a+bx)\cos(a+bx)}{11b}\right)+\frac{15ie^{\frac{5}{3}(a+ibx)}\cos^4(a+bx)}{119b}+\frac{36e^{\frac{5}{3}(a+ibx)}\sin(a+bx)\cos^3(a+bx)}{119b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{15ie^{\frac{5}{3}(a+ibx)}\cos^4(a+bx)}{119b}+\frac{36e^{\frac{5}{3}(a+ibx)}\sin(a+bx)\cos^3(a+bx)}{119b}+\frac{108}{119}\left(-\frac{54ie^{\frac{5}{3}(a+ibx)}}{55b}+\frac{15ie^{\frac{5}{3}(a+ibx)}\cos^2(a+bx)}{11b}+\frac{18e^{\frac{5}{3}(a+ibx)}\sin(a+bx)\cos(a+bx)}{11b}\right)$$

input
```
Int[E^((5*(a + I*b*x))/3)*Cos[a + b*x]^4,x]
```

output
```
(((15*I)/119)*E^((5*(a + I*b*x))/3)*Cos[a + b*x]^4)/b + (36*E^((5*(a + I*b
*x))/3)*Cos[a + b*x]^3*Sin[a + b*x])/(119*b) + (108*((((-54*I)/55)*E^((5*(
a + I*b*x))/3))/b + (((15*I)/11)*E^((5*(a + I*b*x))/3)*Cos[a + b*x]^2)/b +
 (18*E^((5*(a + I*b*x))/3)*Cos[a + b*x]*Sin[a + b*x])/(11*b)))/119
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.40

method	result
parallelrisch	$\frac{3\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\left(275i\cos(4bx+4a)+11900i\cos(2bx+2a)-3927i+660\sin(4bx+4a)+14280\sin(2bx+2a)\right)}{52360b}$
default	$-\frac{9i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}}{40b}+\frac{15i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\cos(4bx+4a)}{952b}+\frac{9\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\sin(4bx+4a)}{238b}+\frac{15i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\cos(2bx+2a)}{22b}+\frac{9\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\,\mathrm{s}}{11}$
norman	$\frac{\frac{4680\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{1309b}+\frac{216\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{187b}-\frac{216\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{187b}-\frac{4680\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{1309b}+\frac{3093i\mathrm{e}^{\frac{5a}{3}}}{654}}{\left(1+\tan\right.}$
orering	$\frac{15573i\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\cos(bx+a)^4}{6545b}-\frac{2610\left(\frac{5ib\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\cos(bx+a)^4}{3}-4\,\mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}}\cos(bx+a)^3b\sin(bx+a)\right)}{1309b^2}+\frac{54i\left(-\frac{61b^2\mathrm{e}^{\frac{5a}{3}+}}{}\right.}{}$

input
```
int(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
3/52360*exp(5/3*a+5/3*I*b*x)*(275*I*cos(4*b*x+4*a)+11900*I*cos(2*b*x+2*a)-
3927*I+660*sin(4*b*x+4*a)+14280*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.40

$$\int e^{\frac{5}{3}(a+ibx)}\cos^4(a+bx)\,dx = -\frac{3\left(385i\,e^{\left(8i\,bx+\left(\frac{19}{3}i+\frac{5}{3}\right)a\right)}+2380i\,e^{\left(6i\,bx+\left(\frac{13}{3}i+\frac{5}{3}\right)a\right)}+7854i\,e^{\left(4i\,bx+\left(\frac{7}{3}i+\frac{5}{3}\right)a\right)}-26180i\,e^{\left(2i\,bx+\left(\frac{1}{3}i+\frac{5}{3}\right)a\right)}-935\right.}{104720\,b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^4,x, algorithm="fricas")
```

output
```
-3/104720*(385*I*e^(8*I*b*x + (19/3*I + 5/3)*a) + 2380*I*e^(6*I*b*x + (13/
3*I + 5/3)*a) + 7854*I*e^(4*I*b*x + (7/3*I + 5/3)*a) - 26180*I*e^(2*I*b*x
+ (1/3*I + 5/3)*a) - 935*I*e^(-(5/3*I - 5/3)*a))*e^(-7/3*I*b*x - 7/3*I*a)/
b
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.53

$$\int e^{\frac{5}{3}(a+ibx)}\cos^4(a+bx)\,dx = \begin{cases} \frac{\left(-2365440ib^4e^{\frac{5a}{3}}e^{10ia}e^{\frac{17ibx}{3}}-14622720ib^4e^{\frac{5a}{3}}e^{8ia}e^{\frac{11ibx}{3}}-48254976ib^4e^{\frac{5a}{3}}e^{6ia}e^{\frac{5ibx}{3}}+160849920ib^4e^{\frac{5a}{3}}e^{4ia}e^{-\frac{ibx}{3}}+5744640ib^4e^{\frac{5a}{3}}e^{2ia}e\right.}{214466560b^5} \\ \frac{x\left(e^{\frac{5a}{3}}e^{8ia}+4e^{\frac{5a}{3}}e^{6ia}+6e^{\frac{5a}{3}}e^{4ia}+4e^{\frac{5a}{3}}e^{2ia}+e^{\frac{5a}{3}}\right)e^{-4ia}}{16} \end{cases}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)**4,x)
```

output
```
Piecewise(((-2365440*I*b**4*exp(5*a/3)*exp(10*I*a)*exp(17*I*b*x/3) - 14622
720*I*b**4*exp(5*a/3)*exp(8*I*a)*exp(11*I*b*x/3) - 48254976*I*b**4*exp(5*a
/3)*exp(6*I*a)*exp(5*I*b*x/3) + 160849920*I*b**4*exp(5*a/3)*exp(4*I*a)*exp
(-I*b*x/3) + 5744640*I*b**4*exp(5*a/3)*exp(2*I*a)*exp(-7*I*b*x/3))*exp(-6*
I*a)/(214466560*b**5), Ne(b**5*exp(6*I*a), 0)), (x*(exp(5*a/3)*exp(8*I*a)
+ 4*exp(5*a/3)*exp(6*I*a) + 6*exp(5*a/3)*exp(4*I*a) + 4*exp(5*a/3)*exp(2*I
*a) + exp(5*a/3))*exp(-4*I*a)/16, True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(71) = 142.

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\int e^{\frac{5}{3}(a+ibx)} \cos^4(a+bx)\,dx =$$

$$-\frac{3\left(7854i\,\cos\left(\frac{5}{3}\,bx\right)e^{\left(\frac{5}{3}\,a\right)}+385i\,\cos\left(\frac{17}{3}\,bx+4\,a\right)e^{\left(\frac{5}{3}\,a\right)}+2380i\,\cos\left(\frac{11}{3}\,bx+2\,a\right)e^{\left(\frac{5}{3}\,a\right)}-935i\,\cos\left(\frac{7}{3}\,b\right.\right.}{}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^4,x, algorithm="maxima")
```

output
```
-3/104720*(7854*I*cos(5/3*b*x)*e^(5/3*a) + 385*I*cos(17/3*b*x + 4*a)*e^(5/
3*a) + 2380*I*cos(11/3*b*x + 2*a)*e^(5/3*a) - 935*I*cos(7/3*b*x + 4*a)*e^(
5/3*a) - 26180*I*cos(1/3*b*x + 2*a)*e^(5/3*a) - 7854*e^(5/3*a)*sin(5/3*b*x
) - 385*e^(5/3*a)*sin(17/3*b*x + 4*a) - 2380*e^(5/3*a)*sin(11/3*b*x + 2*a)
 - 935*e^(5/3*a)*sin(7/3*b*x + 4*a) - 26180*e^(5/3*a)*sin(1/3*b*x + 2*a))/
b
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(71) = 142$.

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.20

$$\int e^{\frac{5}{3}(a+ibx)}\cos^4(a+bx)\,dx =$$
$$-\frac{3\left(385i\left(e^{\left(\frac{17}{3}i\,bx+4i\,a\right)}+e^{\left(-\frac{17}{3}i\,bx-4i\,a\right)}\right)e^{\left(\frac{5}{3}a\right)}+2380i\left(e^{\left(\frac{11}{3}i\,bx+2i\,a\right)}+e^{\left(-\frac{11}{3}i\,bx-2i\,a\right)}\right)e^{\left(\frac{5}{3}a\right)}-935i\left(e^{\left(\frac{7}{3}i\,bx\right.}\right.\right.}{}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^4,x, algorithm="giac")
```

output
```
-3/209440*(385*I*(e^(17/3*I*b*x + 4*I*a) + e^(-17/3*I*b*x - 4*I*a))*e^(5/3
*a) + 2380*I*(e^(11/3*I*b*x + 2*I*a) + e^(-11/3*I*b*x - 2*I*a))*e^(5/3*a)
- 935*I*(e^(7/3*I*b*x + 4*I*a) + e^(-7/3*I*b*x - 4*I*a))*e^(5/3*a) - 26180
*I*(e^(1/3*I*b*x + 2*I*a) + e^(-1/3*I*b*x - 2*I*a))*e^(5/3*a) + 7854*I*(e^
(5/3*I*b*x) + e^(-5/3*I*b*x))*e^(5/3*a) - 15708*e^(5/3*a)*sin(5/3*b*x) - 7
70*e^(5/3*a)*sin(17/3*b*x + 4*a) - 4760*e^(5/3*a)*sin(11/3*b*x + 2*a) - 18
70*e^(5/3*a)*sin(7/3*b*x + 4*a) - 52360*e^(5/3*a)*sin(1/3*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 15.89 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.55

$$\int e^{\frac{5}{3}(a+ibx)}\cos^4(a+bx)\,dx = -\frac{e^{\frac{5a}{3}+\frac{b\,x\,5\mathrm{i}}{3}}\,9\mathrm{i}}{40\,b}+\frac{e^{a\,\left(\frac{5}{3}-2\mathrm{i}\right)-\frac{b\,x\,1\mathrm{i}}{3}}\,3\mathrm{i}}{4\,b}+\frac{e^{a\,\left(\frac{5}{3}-4\mathrm{i}\right)-\frac{b\,x\,7\mathrm{i}}{3}}\,3\mathrm{i}}{112\,b} - \frac{e^{a\,\left(\frac{5}{3}+2\mathrm{i}\right)+\frac{b\,x\,11\mathrm{i}}{3}}\,3\mathrm{i}}{44\,b}-\frac{e^{a\,\left(\frac{5}{3}+4\mathrm{i}\right)+\frac{b\,x\,17\mathrm{i}}{3}}\,3\mathrm{i}}{272\,b}$$

input
```
int(cos(a + b*x)^4*exp((5*a)/3 + (b*x*5i)/3),x)
```

output
```
(exp(a*(5/3 - 2i) - (b*x*1i)/3)*3i)/(4*b) - (exp((5*a)/3 + (b*x*5i)/3)*9i)
/(40*b) + (exp(a*(5/3 - 4i) - (b*x*7i)/3)*3i)/(112*b) - (exp(a*(5/3 + 2i)
+ (b*x*11i)/3)*3i)/(44*b) - (exp(a*(5/3 + 4i) + (b*x*17i)/3)*3i)/(272*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.45

$$\int e^{\frac{5}{3}(a+ibx)}\cos^4(a+bx)\,dx$$
$$=\frac{3e^{\frac{5bix}{3}+\frac{5a}{3}}\left(-660\cos(bx+a)\sin(bx+a)^3+3900\cos(bx+a)\sin(bx+a)+275\sin(bx+a)^4 i-3250\sin\right.}{6545b}$$

input
```
int(exp(5/3*a+5/3*I*b*x)*cos(b*x+a)^4,x)
```

output
```
(3*e**((5*a + 5*b*i*x)/3)*( - 660*cos(a + b*x)*sin(a + b*x)**3 + 3900*cos(
a + b*x)*sin(a + b*x) + 275*sin(a + b*x)**4*i - 3250*sin(a + b*x)**2*i + 1
031*i))/(6545*b)
```

3.33 $\int F^{c(a+bx)} \cos(d + ex) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 47

$$\int F^{c(a+bx)} \cos(d+ex) \, dx = \frac{F^{c(a+bx)}(bc\cos(d+ex)\log(F)+e\sin(d+ex))}{e^2+b^2c^2\log^2(F)}$$

output
```
F^(c*(b*x+a))*(b*c*cos(e*x+d)*ln(F)+e*sin(e*x+d))/(e^2+b^2*c^2*ln(F)^2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)} \cos(d+ex) \, dx = \frac{F^{c(a+bx)}(bc\cos(d+ex)\log(F)+e\sin(d+ex))}{e^2+b^2c^2\log^2(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Cos[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(b*c*Cos[d + e*x]*Log[F] + e*Sin[d + e*x]))/(e^2 + b^2*c^
2*Log[F]^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4933}$$

$$\frac{e\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}+\frac{bc\log(F)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}$$

input
```
Int[F^(c*(a + b*x))*Cos[d + e*x],x]
```

output
```
(b*c*F^(c*(a + b*x))*Cos[d + e*x]*Log[F])/(e^2 + b^2*c^2*Log[F]^2) + (e*F^
(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
parallelrisch	$\frac{F^{c(bx+a)}(bc\cos(ex+d)\ln(F)+e\sin(ex+d))}{e^2+b^2c^2\ln(F)^2}$	48
risch	$\frac{F^{c(bx+a)}bc\ln(F)\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{e\,F^{c(bx+a)}\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}$	73
orering	$\frac{2F^{c(bx+a)}bc\ln(F)\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}-\frac{F^{c(bx+a)}bc\ln(F)\cos(ex+d)-F^{c(bx+a)}e\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}$	98
norman	$\frac{\frac{bc\ln(F)e^{c(bx+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2e\,e^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}-\frac{bc\ln(F)e^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$	133

input
```
int(F^(c*(b*x+a))*cos(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
F^(c*(b*x+a))*(b*c*cos(e*x+d)*ln(F)+e*sin(e*x+d))/(e^2+b^2*c^2*ln(F)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\cos(d+ex)\,dx=\frac{(bc\cos(ex+d)\log(F)+e\sin(ex+d))F^{bcx+ac}}{b^2c^2\log(F)^2+e^2}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="fricas")
```

output
```
(b*c*cos(e*x + d)*log(F) + e*sin(e*x + d))*F^(b*c*x + a*c)/(b^2*c^2*log(F)
^2 + e^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 347, normalized size of antiderivative = 7.38

$$\int F^{c(a+bx)}\cos(d+ex)\,dx$$
$$= \begin{cases} x\cos(d) & \text{fo} \\ F^{ac}x\cos(d) & \text{fo} \\ x\cos(d) & \text{fo} \\ \frac{iF^{ac+bcx}x\sin(ibcx\log(F)-d)}{2}+\frac{F^{ac+bcx}x\cos(ibcx\log(F)-d)}{2}-\frac{iF^{ac+bcx}\sin(ibcx\log(F)-d)}{bc\log(F)}-\frac{F^{ac+bcx}\cos(ibcx\log(F)-d)}{2bc\log(F)} & \text{fo} \\ \frac{iF^{ac+bcx}x\sin(ibcx\log(F)+d)}{2}+\frac{F^{ac+bcx}x\cos(ibcx\log(F)+d)}{2}-\frac{iF^{ac+bcx}\sin(ibcx\log(F)+d)}{bc\log(F)}-\frac{F^{ac+bcx}\cos(ibcx\log(F)+d)}{2bc\log(F)} & \text{fo} \\ \frac{F^{ac+bcx}bc\log(F)\cos(d+ex)}{b^2c^2\log(F)^2+e^2}+\frac{F^{ac+bcx}e\sin(d+ex)}{b^2c^2\log(F)^2+e^2} & \text{ot} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*cos(e*x+d),x)
```

output
```
Piecewise((x*cos(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cos(d), Eq(b, 0) &
Eq(e, 0)), (x*cos(d), Eq(c, 0) & Eq(e, 0)), (I*F**(a*c + b*c*x)*x*sin(I*b*
c*x*log(F) - d)/2 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F) - d)/2 - I*F**(a
*c + b*c*x)*sin(I*b*c*x*log(F) - d)/(b*c*log(F)) - F**(a*c + b*c*x)*cos(I*
b*c*x*log(F) - d)/(2*b*c*log(F)), Eq(e, -I*b*c*log(F))), (I*F**(a*c + b*c*
x)*x*sin(I*b*c*x*log(F) + d)/2 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F) + d
)/2 - I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F) + d)/(b*c*log(F)) - F**(a*c +
b*c*x)*cos(I*b*c*x*log(F) + d)/(2*b*c*log(F)), Eq(e, I*b*c*log(F))), (F**(
a*c + b*c*x)*b*c*log(F)*cos(d + e*x)/(b**2*c**2*log(F)**2 + e**2) + F**(a*
c + b*c*x)*e*sin(d + e*x)/(b**2*c**2*log(F)**2 + e**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(47) = 94$.

Time = 0.04 (sec) , antiderivative size = 192, normalized size of antiderivative = 4.09

$$\int F^{c(a+bx)}\cos(d+ex)\,dx$$
$$= \frac{(F^{ac}bc\cos(d)\log(F) - F^{ac}e\sin(d))F^{bcx}\cos(ex+2d) + (F^{ac}bc\cos(d)\log(F) + F^{ac}e\sin(d))F^{bcx}\cos(}{2\left(b^2c^2\cos(d)^2\log(F)^2 + b^2c^2\log\right.}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="maxima")
```

output
```
1/2*((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x + 2*
d) + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x) + (
F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x + 2*d) - (
F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x))/(b^2*c^2
*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2
)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 631, normalized size of antiderivative = 13.43

$$\int F^{c(a+bx)}\cos(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="giac")
```

output
```
(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi
*a*c + e*x + d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi
*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*e)*sin(1/2*pi*b*c*x*sgn(F) -
1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(ab
s(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F))) + (2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sg
n(F) - 1/2*pi*a*c - e*x - d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*
c*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 2*e)*sin(1/2*pi*b*
c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b
^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs
(F)) + a*c*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x +
 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*
I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*
I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*
c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) +
 a*c*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I
*pi*a*c*sgn(F) - 1/2*I*pi*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b
*c + 4*b*c*log(abs(F)) - 4*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b
*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(
F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*...
```

Mupad [B] (verification not implemented)

Time = 15.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\cos(d+ex)\,dx = \frac{F^{a\,c+b\,c\,x}\,\left(e\,\sin\left(d+e\,x\right)+b\,c\,\cos\left(d+e\,x\right)\,\ln\left(F\right)\right)}{b^2\,c^2\,{\ln\left(F\right)}^2+e^2}$$

input
```
int(F^(c*(a + b*x))*cos(d + e*x),x)
```

output
```
(F^(a*c + b*c*x)*(e*sin(d + e*x) + b*c*cos(d + e*x)*log(F)))/(e^2 + b^2*c^
2*log(F)^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\cos(d+ex)\,dx = \frac{f^{bcx+ac}(\cos\left(ex+d\right)\log(f)\,bc+\sin\left(ex+d\right)e)}{\log\left(f\right)^2 b^2c^2+e^2}$$

input
```
int(F^(c*(b*x+a))*cos(e*x+d),x)
```

output
```
(f**(a*c + b*c*x)*(cos(d + e*x)*log(f)*b*c + sin(d + e*x)*e))/(log(f)**2*b
**2*c**2 + e**2)
```

3.34 $\int F^{c(a+bx)} \cos^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int F^{c(a+bx)} \cos^2(d+ex)\,dx = \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)} + \frac{F^{c(a+bx)}\cos(d+ex)(bc\cos(d+ex)\log(F)+2e\sin(d+ex))}{4e^2+b^2c^2\log^2(F)}$$

output
```
2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+F^(c*(b*x+a))*cos(e*
x+d)*(b*c*cos(e*x+d)*ln(F)+2*e*sin(e*x+d))/(4*e^2+b^2*c^2*ln(F)^2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85

$$\int F^{c(a+bx)} \cos^2(d+ex)\,dx = \frac{F^{c(a+bx)}\left(4e^2+b^2c^2\log^2(F)+b^2c^2\cos(2(d+ex))\log^2(F)+2bce\log(F)\sin(2(d+ex))\right)}{8bce^2\log(F)+2b^3c^3\log^3(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Cos[d + e*x]^2,x]
```

output
```
(F^(c*(a + b*x))*(4*e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cos[2*(d + e*x)]*Log[
F]^2 + 2*b*c*e*Log[F]*Sin[2*(d + e*x)]))/(8*b*c*e^2*Log[F] + 2*b^3*c^3*Log
[F]^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4935, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4935}$$

$$\frac{2e^2\int F^{c(a+bx)}dx}{b^2c^2\log^2(F)+4e^2}+\frac{bc\log(F)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}$$

$$\Big\downarrow \text{2624}$$

$$\frac{bc\log(F)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}$$

input
```
Int[F^(c*(a + b*x))*Cos[d + e*x]^2,x]
```

output
```
(2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F]^2)) + (b*c*F^(
c*(a + b*x))*Cos[d + e*x]^2*Log[F])/(4*e^2 + b^2*c^2*Log[F]^2) + (2*e*F^(c
*(a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

method	result
parallelrisch	$\frac{F^{c(bx+a)}\left(b^{2}c^{2}\ln(F)^{2}\cos(2ex+2d)+b^{2}c^{2}\ln(F)^{2}+2e\sin(2ex+2d)bc\ln(F)+4e^{2}\right)}{2bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}$
risch	$\frac{F^{c(bx+a)}}{2bc\ln(F)}+\frac{F^{c(bx+a)}bc\ln(F)\cos(2ex+2d)}{2b^{2}c^{2}\ln(F)^{2}+8e^{2}}+\frac{e\,F^{c(bx+a)}\sin(2ex+2d)}{4e^{2}+b^{2}c^{2}\ln(F)^{2}}$
orering	$\frac{\left(3b^{2}c^{2}\ln(F)^{2}+4e^{2}\right)F^{c(bx+a)}\cos(ex+d)^{2}}{bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}-\frac{3\left(F^{c(bx+a)}bc\ln(F)\cos(ex+d)^{2}-2F^{c(bx+a)}\sin(ex+d)e\cos(ex+d)\right)}{4e^{2}+b^{2}c^{2}\ln(F)^{2}}+ F^{c(b$
norman	$\frac{\frac{\left(b^{2}c^{2}\ln(F)^{2}+2e^{2}\right)e^{c(bx+a)\ln(F)}}{bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}+\frac{\left(b^{2}c^{2}\ln(F)^{2}+2e^{2}\right)e^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{4}}{bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}+\frac{4e\,e^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{4e^{2}+b^{2}c^{2}\ln(F)^{2}}-\frac{4e\,e^{c(bx+a)\ln(}}{4e^{2}+b^{2}}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}\right)^{2}}$

input
```
int(F^(c*(b*x+a))*cos(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*F^(c*(b*x+a))*(b^2*c^2*ln(F)^2*cos(2*e*x+2*d)+b^2*c^2*ln(F)^2+2*e*sin(
2*e*x+2*d)*b*c*ln(F)+4*e^2)/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx = \frac{\left(b^2c^2\cos\left(ex+d\right)^2\log\left(F\right)^2+2\,bce\cos\left(ex+d\right)\log\left(F\right)\sin\left(ex+d\right)+2\,e^2\right)F^{bcx+ac}}{b^3c^3\log\left(F\right)^3+4\,bce^2\log\left(F\right)}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="fricas")
```

output
```
(b^2*c^2*cos(e*x + d)^2*log(F)^2 + 2*b*c*e*cos(e*x + d)*log(F)*sin(e*x + d
) + 2*e^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + 4*b*c*e^2*log(F))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 743, normalized size of antiderivative = 7.43

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cos(e*x+d)**2,x)
```

output
```
Piecewise((x*cos(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (x*sin
(d + e*x)**2/2 + x*cos(d + e*x)**2/2 + sin(d + e*x)*cos(d + e*x)/(2*e), Eq
(F, 1)), (F**(a*c)*(x*sin(d + e*x)**2/2 + x*cos(d + e*x)**2/2 + sin(d + e*
x)*cos(d + e*x)/(2*e)), Eq(b, 0)), (x*sin(d + e*x)**2/2 + x*cos(d + e*x)**
2/2 + sin(d + e*x)*cos(d + e*x)/(2*e), Eq(c, 0)), (-F**(a*c + b*c*x)*x*sin
(I*b*c*x*log(F)/2 - d)**2/4 + I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 -
d)*cos(I*b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 -
 d)**2/4 + F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 - d)**2/(b*c*log(F)) - 3*
I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 - d)*cos(I*b*c*x*log(F)/2 - d)/(2*
b*c*log(F)), Eq(e, -I*b*c*log(F)/2)), (-F**(a*c + b*c*x)*x*sin(I*b*c*x*log
(F)/2 + d)**2/4 + I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 + d)*cos(I*b*c
*x*log(F)/2 + d)/2 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 + d)**2/4 + F
**(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 + d)**2/(b*c*log(F)) - 3*I*F**(a*c +
b*c*x)*sin(I*b*c*x*log(F)/2 + d)*cos(I*b*c*x*log(F)/2 + d)/(2*b*c*log(F)),
 Eq(e, I*b*c*log(F)/2)), (F**(a*c + b*c*x)*b**2*c**2*log(F)**2*cos(d + e*x
)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2*F**(a*c + b*c*x)*b*c*e*
log(F)*sin(d + e*x)*cos(d + e*x)/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F))
 + 2*F**(a*c + b*c*x)*e**2*sin(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e*
*2*log(F)) + 2*F**(a*c + b*c*x)*e**2*cos(d + e*x)**2/(b**3*c**3*log(F)**3
+ 4*b*c*e**2*log(F)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(100) = 200$.

Time = 0.05 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.56

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx$$
$$= \frac{\left(F^{ac}b^2c^2\cos(2\,d)\log(F)^2 + 2\,F^{ac}bce\log(F)\sin(2\,d)\right)F^{bcx}\cos(2\,ex) + \left(F^{ac}b^2c^2\cos(2\,d)\log(F)^2 - 2\,F^{\cdots}\right.}{\cdots}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))
*F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c
*e*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2
*sin(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a*
c)*b^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*
sin(2*e*x + 4*d) + 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c^
2*log(F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*F
^(b*c*x))/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2 + 4*(
b*c*cos(2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 915, normalized size of antiderivative = 9.15

$$\int F^{c(a+bx)} \cos^2(d+ex) \, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="giac")
```

output
```
1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/
2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn
(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*sin(1/2*pi*b*c*x*s
gn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)/(4*b^
2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(abs(
F)) + a*c*log(abs(F))) + 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x
 + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)*log(abs(F))/(4*b^2*c^2*lo
g(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c
- 4*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi
*a*c - 2*e*x - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4
*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*cos(-1/2*pi*b*c*x
*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^
2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*
c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*
c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(ab
s(F)) + a*c*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x
+ 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(4*I*pi*b*c*sgn(F)
 - 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F)
+ 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(
-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x...
```

Mupad [B] (verification not implemented)

Time = 16.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int F^{c(a+bx)} \cos^2(d+ex)\,dx = \frac{2\,F^{a\,c+b\,c\,x}\,e^2+F^{a\,c+b\,c\,x}\,b^2\,c^2\cos\left(d+e\,x\right)^2\ln\left(F\right)^2+2\,F^{a\,c+b\,c\,x}\,b\,c\,e\,\cos\left(d+e\,x\right)\,\sin\left(d+e\,x\right)\,\ln\left(F\right)}{b^3\,c^3\ln\left(F\right)^3+4\,b\,c\,e^2\,\ln\left(F\right)}$$

input
```
int(F^(c*(a + b*x))*cos(d + e*x)^2,x)
```

output
```
(2*F^(a*c + b*c*x)*e^2 + F^(a*c + b*c*x)*b^2*c^2*cos(d + e*x)^2*log(F)^2 +
 2*F^(a*c + b*c*x)*b*c*e*cos(d + e*x)*sin(d + e*x)*log(F))/(b^3*c^3*log(F)
^3 + 4*b*c*e^2*log(F))
```

Reduce [F]

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cos(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*cos(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cos(d + e*x)**2,x)
```

3.35 $\int F^{c(a+bx)}\cos^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 127

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \frac{6e^2F^{c(a+bx)}(bc\cos(d+ex)\log(F)+e\sin(d+ex))}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{F^{c(a+bx)}\cos^2(d+ex)(bc\cos(d+ex)\log(F)+3e\sin(d+ex))}{9e^2+b^2c^2\log^2(F)}$$

output
```
6*e^2*F^(c*(b*x+a))*(b*c*cos(e*x+d)*ln(F)+e*sin(e*x+d))/(9*e^4+10*b^2*c^2*
e^2*ln(F)^2+b^4*c^4*ln(F)^4)+F^(c*(b*x+a))*cos(e*x+d)^2*(b*c*cos(e*x+d)*ln
(F)+3*e*sin(e*x+d))/(9*e^2+b^2*c^2*ln(F)^2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.22

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(bc\cos(3(d+ex))\log(F)\left(e^2+b^2c^2\log^2(F)\right)+3bc\cos(d+ex)\log(F)\left(9e^2+b^2c^2\log^2(F)\right)+6e\cdots\right)}{4\left(9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(\cdots\right)}$$

input
```
Integrate[F^(c*(a + b*x))*Cos[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(b*c*Cos[3*(d + e*x)]*Log[F]*(e^2 + b^2*c^2*Log[F]^2) + 3
*b*c*Cos[d + e*x]*Log[F]*(9*e^2 + b^2*c^2*Log[F]^2) + 6*e*(5*e^2 + b^2*c^2
*Log[F]^2 + Cos[2*(d + e*x)]*(e^2 + b^2*c^2*Log[F]^2))*Sin[d + e*x]))/(4*(
9*e^4 + 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4935, 4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4935}$$

$$\frac{6e^2\int F^{c(a+bx)}\cos(d+ex)dx}{b^2c^2\log^2(F)+9e^2}+\frac{bc\log(F)\cos^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{3e\sin(d+ex)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}$$

$$\Big\downarrow \text{4933}$$

$$\frac{bc\log(F)\cos^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{3e\sin(d+ex)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{6e^2\left(\frac{e\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}+\frac{bc\log(F)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}\right)}{b^2c^2\log^2(F)+9e^2}$$

input
```
Int[F^(c*(a + b*x))*Cos[d + e*x]^3,x]
```

output
```
(b*c*F^(c*(a + b*x))*Cos[d + e*x]^3*Log[F])/(9*e^2 + b^2*c^2*Log[F]^2) + (
3*e*F^(c*(a + b*x))*Cos[d + e*x]^2*Sin[d + e*x])/(9*e^2 + b^2*c^2*Log[F]^2
) + (6*e^2*((b*c*F^(c*(a + b*x))*Cos[d + e*x]*Log[F])/(e^2 + b^2*c^2*Log[F
]^2) + (e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)))/(9*e^2
+ b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13

method	result
parallelrisch	$\frac{\left(bc\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)\cos(3ex+3d)+\left(3b^2c^2\ln(F)^2e+3e^3\right)\sin(3ex+3d)+3\left(9e^2+b^2c^2\ln(F)^2\right)(bc\cos(ex+d)\ln(F)+e\ldots}{4b^4c^4\ln(F)^4+40b^2c^2e^2\ln(F)^2+36e^4}$
risch	$\frac{3F^{c(bx+a)}bc\ln(F)\cos(ex+d)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{3e\,F^{c(bx+a)}\sin(ex+d)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{F^{c(bx+a)}bc\ln(F)\cos(3ex+3d)}{4b^2c^2\ln(F)^2+36e^2}+\frac{3e\,F^{c(bx+a)}\sin(3ex+3d)}{4\left(9e^2+b^2c^2\ln(F)^2\right)}$
orering	$\frac{4\ln(F)bc\left(b^2c^2\ln(F)^2+5e^2\right)F^{c(bx+a)}\cos(ex+d)^3}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{2\left(3b^2c^2\ln(F)^2+5e^2\right)\left(F^{c(bx+a)}bc\ln(F)\cos(ex+d)^3-3F^{c(bx+a)}\cos\ldots}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}$
norman	$\frac{\frac{\ln(F)bc\left(b^2c^2\ln(F)^2+7e^2\right)e^{c(bx+a)\ln(F)}}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{12e\left(b^2c^2\ln(F)^2-e^2\right)e^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+\frac{6e\left(b^2c^2\ln(F)^2+3e^2\right)e^{c(bx+a)\ln(F)}\ldots}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(\ldots}}{\ldots}$

input
```
int(F^(c*(b*x+a))*cos(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
(b*c*ln(F)*(e^2+b^2*c^2*ln(F)^2)*cos(3*e*x+3*d)+(3*b^2*c^2*ln(F)^2*e+3*e^3
)*sin(3*e*x+3*d)+3*(9*e^2+b^2*c^2*ln(F)^2)*(b*c*cos(e*x+d)*ln(F)+e*sin(e*x
+d)))*F^(c*(b*x+a))/(4*b^4*c^4*ln(F)^4+40*b^2*c^2*e^2*ln(F)^2+36*e^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \frac{\left(b^3c^3\cos\left(ex+d\right)^3\log\left(F\right)^3+\left(bce^2\cos\left(ex+d\right)^3+6\,bce^2\cos\left(ex+d\right)\right)\log\left(F\right)+3\left(b^2c^2e\cos\left(ex+d\right)^2\log\right.}{b^4c^4\log\left(F\right)^4+10\,b^2c^2e^2\log\left(F\right)^2+9\,e^4}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="fricas")
```

output
```
(b^3*c^3*cos(e*x + d)^3*log(F)^3 + (b*c*e^2*cos(e*x + d)^3 + 6*b*c*e^2*cos
(e*x + d))*log(F) + 3*(b^2*c^2*e*cos(e*x + d)^2*log(F)^2 + e^3*cos(e*x + d
)^2 + 2*e^3)*sin(e*x + d))*F^(b*c*x + a*c)/(b^4*c^4*log(F)^4 + 10*b^2*c^2*
e^2*log(F)^2 + 9*e^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.96 (sec) , antiderivative size = 1579, normalized size of antiderivative = 12.43

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cos(e*x+d)**3,x)
```

output
```
Piecewise((x*cos(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cos(d)**3, Eq(b,
 0) & Eq(e, 0)), (x*cos(d)**3, Eq(c, 0) & Eq(e, 0)), (3*I*F**(a*c + b*c*x)
*x*sin(I*b*c*x*log(F) - d)**3/8 + 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)
- d)**2*cos(I*b*c*x*log(F) - d)/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log
(F) - d)*cos(I*b*c*x*log(F) - d)**2/8 + 3*F**(a*c + b*c*x)*x*cos(I*b*c*x*l
og(F) - d)**3/8 - 3*I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)**3/(8*b*c*l
og(F)) - 3*I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)*cos(I*b*c*x*log(F) -
 d)**2/(4*b*c*log(F)) - F**(a*c + b*c*x)*cos(I*b*c*x*log(F) - d)**3/(8*b*c
*log(F)), Eq(e, -I*b*c*log(F))), (-I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)
/3 - d)**3/8 - 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 - d)**2*cos(I*b*c
*x*log(F)/3 - d)/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 - d)*cos(
I*b*c*x*log(F)/3 - d)**2/8 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/3 - d)*
*3/8 + I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)) + 3*
I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)*cos(I*b*c*x*log(F)/3 - d)**2/
(4*b*c*log(F)) + 9*F**(a*c + b*c*x)*cos(I*b*c*x*log(F)/3 - d)**3/(8*b*c*lo
g(F)), Eq(e, -I*b*c*log(F)/3)), (-I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/
3 + d)**3/8 - 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 + d)**2*cos(I*b*c*
x*log(F)/3 + d)/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 + d)*cos(I
*b*c*x*log(F)/3 + d)**2/8 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/3 + d)**
3/8 + I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 + d)**3/(8*b*c*log(F)) + ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(127) = 254$.

Time = 0.07 (sec) , antiderivative size = 813, normalized size of antiderivative = 6.40

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/8*((F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 + 3*F^(a*c)*b^2*c^2*e*log(F)^2*sin
(3*d) + F^(a*c)*b*c*e^2*cos(3*d)*log(F) + 3*F^(a*c)*e^3*sin(3*d))*F^(b*c*x
)*cos(3*e*x) + (F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - 3*F^(a*c)*b^2*c^2*e*lo
g(F)^2*sin(3*d) + F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 3*F^(a*c)*e^3*sin(3*d)
)*F^(b*c*x)*cos(3*e*x + 6*d) + 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - F^(a
*c)*b^2*c^2*e*log(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 9*F^
(a*c)*e^3*sin(3*d))*F^(b*c*x)*cos(e*x + 4*d) + 3*(F^(a*c)*b^3*c^3*cos(3*d)
*log(F)^3 + F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*
d)*log(F) + 9*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*cos(e*x - 2*d) - (F^(a*c)*b^
3*c^3*log(F)^3*sin(3*d) - 3*F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + F^(a*c)*
b*c*e^2*log(F)*sin(3*d) - 3*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*sin(3*e*x) + (
F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + 3*F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^2
+ F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 3*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*sin(
3*e*x + 6*d) + 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + F^(a*c)*b^2*c^2*e*co
s(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 9*F^(a*c)*e^3*cos(3*
d))*F^(b*c*x)*sin(e*x + 4*d) - 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) - F^(a
*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*d) - 9*F^
(a*c)*e^3*cos(3*d))*F^(b*c*x)*sin(e*x - 2*d))/(b^4*c^4*cos(3*d)^2*log(F)^4
 + b^4*c^4*log(F)^4*sin(3*d)^2 + 9*(cos(3*d)^2 + sin(3*d)^2)*e^4 + 10*(b^2
*c^2*cos(3*d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(3*d)^2)*e^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1271, normalized size of antiderivative = 10.01

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="giac")
```

output
```
1/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/
2*pi*a*c + 3*e*x + 3*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn
(F) - pi*b*c + 6*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 6*e)*sin(1/2*pi*b*c*x*s
gn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 3*e*x + 3*d)/(4*b^
2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 6*e)^2))*e^(b*c*x*log(abs(
F)) + a*c*log(abs(F))) + 3/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x
 + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)*log(abs(F))/(4*b^2*c^2*log(ab
s(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*
e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c
 + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*
e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)*log(abs(F))/
(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*sgn
(F) - pi*b*c - 2*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sg
n(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - p
i*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*(2*b*c*cos(
1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 3*e*
x - 3*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c -
6*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 6*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*
b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 3*e*x - 3*d)/(4*b^2*c^2*log(ab...
```

Mupad [B] (verification not implemented)

Time = 16.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.50

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = -\frac{F^{c\,(a+b\,x)}\,\left(\cos\left(e\,x\right)+\sin\left(e\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(d\right)+\sin\left(d\right)\,1\mathrm{i}\right)\,3\mathrm{i}}{8\,\left(e-b\,c\,\ln\left(F\right)\,1\mathrm{i}\right)} - \frac{F^{c\,(a+b\,x)}\,\left(\cos\left(3\,e\,x\right)-\sin\left(3\,e\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,d\right)-\sin\left(3\,d\right)\,1\mathrm{i}\right)}{8\,\left(-b\,c\,\ln\left(F\right)+e\,3\mathrm{i}\right)} - \frac{F^{c\,(a+b\,x)}\,\left(\cos\left(3\,e\,x\right)+\sin\left(3\,e\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,d\right)+\sin\left(3\,d\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,\left(3\,e-b\,c\,\ln\left(F\right)\,1\mathrm{i}\right)} - \frac{3\,F^{c\,(a+b\,x)}\,\left(\cos\left(e\,x\right)-\sin\left(e\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(d\right)-\sin\left(d\right)\,1\mathrm{i}\right)}{8\,\left(-b\,c\,\ln\left(F\right)+e\,1\mathrm{i}\right)}$$

input
```
int(F^(c*(a + b*x))*cos(d + e*x)^3,x)
```

output
```
- (F^(c*(a + b*x))*(cos(e*x) + sin(e*x)*1i)*(cos(d) + sin(d)*1i)*3i)/(8*(e
 - b*c*log(F)*1i)) - (F^(c*(a + b*x))*(cos(3*e*x) - sin(3*e*x)*1i)*(cos(3*
d) - sin(3*d)*1i))/(8*(e*3i - b*c*log(F))) - (F^(c*(a + b*x))*(cos(3*e*x)
+ sin(3*e*x)*1i)*(cos(3*d) + sin(3*d)*1i)*1i)/(8*(3*e - b*c*log(F)*1i)) -
(3*F^(c*(a + b*x))*(cos(e*x) - sin(e*x)*1i)*(cos(d) - sin(d)*1i))/(8*(e*1i
 - b*c*log(F)))
```

Reduce [F]

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cos(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*cos(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cos(d + e*x)**3,x)
```

3.36 $\int F^{c(a+bx)}\cos^4(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 192

$$\int F^{c(a+bx)}\cos^4(d+ex)\,dx = \frac{24e^4F^{c(a+bx)}}{bc\log(F)\left(64e^4+20b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)\right)} + \frac{12e^2F^{c(a+bx)}\cos(d+ex)(bc\cos(d+ex)\log(F)+2e\sin(d+ex))}{64e^4+20b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{F^{c(a+bx)}\cos^3(d+ex)(bc\cos(d+ex)\log(F)+4e\sin(d+ex))}{16e^2+b^2c^2\log^2(F)}$$

output
```
24*e^4*F^(c*(b*x+a))/b/c/ln(F)/(64*e^4+20*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F
)^4)+12*e^2*F^(c*(b*x+a))*cos(e*x+d)*(b*c*cos(e*x+d)*ln(F)+2*e*sin(e*x+d))
/(64*e^4+20*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^4)+F^(c*(b*x+a))*cos(e*x+d)^
3*(b*c*cos(e*x+d)*ln(F)+4*e*sin(e*x+d))/(16*e^2+b^2*c^2*ln(F)^2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.21

$$\int F^{c(a+bx)}\cos^4(d+ex)\,dx$$
$$= \frac{F^{c(a+bx)}\left(192e^4+60b^2c^2e^2\log^2(F)+3b^4c^4\log^4(F)+\cos(4(d+ex))\left(4b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)\right)+\cdots\right.}{\cdots}$$

input
```
Integrate[F^(c*(a + b*x))*Cos[d + e*x]^4,x]
```

output
```
(F^(c*(a + b*x))*(192*e^4 + 60*b^2*c^2*e^2*Log[F]^2 + 3*b^4*c^4*Log[F]^4 +
 Cos[4*(d + e*x)]*(4*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) + 4*Cos[2*(d
 + e*x)]*(16*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) + 128*b*c*e^3*Log[F]
*Sin[2*(d + e*x)] + 8*b^3*c^3*e*Log[F]^3*Sin[2*(d + e*x)] + 16*b*c*e^3*Log
[F]*Sin[4*(d + e*x)] + 4*b^3*c^3*e*Log[F]^3*Sin[4*(d + e*x)]))/(8*(64*b*c*
e^4*Log[F] + 20*b^3*c^3*e^2*Log[F]^3 + b^5*c^5*Log[F]^5))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4935, 4935, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{4935}$$
$$\frac{12e^2\int F^{c(a+bx)}\cos^2(d+ex)dx}{b^2c^2\log^2(F)+16e^2}+\frac{bc\log(F)\cos^4(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+16e^2}+\frac{4e\sin(d+ex)\cos^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+16e^2}$$
$$\Big\downarrow \text{4935}$$

$$\frac{12e^2\left(\frac{2e^2\int F^{c(a+bx)}dx}{b^2c^2\log^2(F)+4e^2}+\frac{bc\log(F)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}\right)}{b^2c^2\log^2(F)+16e^2}+\frac{bc\log(F)\cos^4(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+16e^2}+\frac{4e\sin(d+ex)\cos^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+16e^2}$$

$$\Big\downarrow \text{2624}$$

$$\frac{bc\log(F)\cos^4(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+16e^2}+\frac{4e\sin(d+ex)\cos^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+16e^2}+\frac{12e^2\left(\frac{bc\log(F)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)(b^2c^2\log^2(F)+4e^2)}\right)}{b^2c^2\log^2(F)+16e^2}$$

input
```
Int[F^(c*(a + b*x))*Cos[d + e*x]^4,x]
```

output
```
(b*c*F^(c*(a + b*x))*Cos[d + e*x]^4*Log[F])/(16*e^2 + b^2*c^2*Log[F]^2) +
(4*e*F^(c*(a + b*x))*Cos[d + e*x]^3*Sin[d + e*x])/(16*e^2 + b^2*c^2*Log[F]
^2) + (12*e^2*((2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F]
^2)) + (b*c*F^(c*(a + b*x))*Cos[d + e*x]^2*Log[F])/(4*e^2 + b^2*c^2*Log[F]
^2) + (2*e*F^(c*(a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*Log
[F]^2)))/(16*e^2 + b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.98

method	result
risch	$\frac{3F^{c(bx+a)}}{8bc\ln(F)}+\frac{F^{c(bx+a)}bc\ln(F)\cos(4ex+4d)}{8b^2c^2\ln(F)^2+128e^2}+\frac{e\,F^{c(bx+a)}\sin(4ex+4d)}{2b^2c^2\ln(F)^2+32e^2}+\frac{F^{c(bx+a)}bc\ln(F)\cos(2ex+2d)}{2b^2c^2\ln(F)^2+8e^2}+\frac{e\,F^{c(bx+a}}{4e^2+b}$
parallelrisch	$\frac{\left(4\left(b^4c^4\ln(F)^4+16b^2c^2e^2\ln(F)^2\right)\cos(2ex+2d)+b^2c^2\ln(F)^2\left(4e^2+b^2c^2\ln(F)^2\right)\cos(4ex+4d)+8\left(\ln(F)^3b^3c^3e+16\ln(F)bc\right.\right.}{8\ln(F)bc\left(4e^2+b^2c^2\ln(F)^2\right)\left(16e^2+b^2\right.}$
default	$\frac{F^{ac}\left(\frac{2F^{bcx}}{bc\ln(F)}+\frac{\frac{16e\,e^{bcx\ln(F)}\tan(ex+d)}{4e^2+b^2c^2\ln(F)^2}+\frac{4bc\ln(F)e^{bcx\ln(F)}}{4e^2+b^2c^2\ln(F)^2}-\frac{4bc\ln(F)e^{bcx\ln(F)}\tan(ex+d)^2}{4e^2+b^2c^2\ln(F)^2}}{1+\tan(ex+d)^2}+\frac{\frac{16e\,e^{bcx\ln(F)}\tan(ex+d)}{16e^2+b^2c^2\ln(F)^2}-\frac{16e\,e^{bcx}}{16e^2}\right.}{}$
norman	$\frac{\frac{\left(b^4c^4\ln(F)^4+16b^2c^2e^2\ln(F)^2+24e^4\right)e^{c(bx+a)\ln(F)}}{cb\ln(F)\left(64e^4+20b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4\right)}+\frac{\left(b^4c^4\ln(F)^4+16b^2c^2e^2\ln(F)^2+24e^4\right)e^{c(bx+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^8}{cb\ln(F)\left(64e^4+20b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4\right)}-\frac{24e\left(b^2c^2\right.}{64}}{}$
orering	$\frac{\left(5b^4c^4\ln(F)^4+60b^2c^2e^2\ln(F)^2+64e^4\right)F^{c(bx+a)}\cos(ex+d)^4}{cb\ln(F)\left(64e^4+20b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4\right)}-\frac{10\left(b^2c^2\ln(F)^2+6e^2\right)\left(F^{c(bx+a)}bc\ln(F)\cos(ex+d)^4-4F^{c}\right.}{64e^4+20b^2c^2e^2\ln(F)^2+b^4c^4\ln}$

input
```
int(F^(c*(b*x+a))*cos(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
3/8*F^(c*(b*x+a))/b/c/ln(F)+1/8/(16*e^2+b^2*c^2*ln(F)^2)*F^(c*(b*x+a))*b*c
*ln(F)*cos(4*e*x+4*d)+1/2/(16*e^2+b^2*c^2*ln(F)^2)*e*F^(c*(b*x+a))*sin(4*e
*x+4*d)+1/2/(4*e^2+b^2*c^2*ln(F)^2)*F^(c*(b*x+a))*b*c*ln(F)*cos(2*e*x+2*d)
+e*F^(c*(b*x+a))/(4*e^2+b^2*c^2*ln(F)^2)*sin(2*e*x+2*d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}\cos^4(d+ex)\,dx$$
$$=\frac{\left(b^4c^4\cos\left(ex+d\right)^4\log\left(F\right)^4+24\,e^4+4\left(b^2c^2e^2\cos\left(ex+d\right)^4+3\,b^2c^2e^2\cos\left(ex+d\right)^2\right)\log\left(F\right)^2+4\left(b^3c^3e\right.\right.}{b^5c^5\log\left(F\right)^5+20\,b^3c^3e^2\log}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^4,x, algorithm="fricas")
```

output
```
(b^4*c^4*cos(e*x + d)^4*log(F)^4 + 24*e^4 + 4*(b^2*c^2*e^2*cos(e*x + d)^4
+ 3*b^2*c^2*e^2*cos(e*x + d)^2)*log(F)^2 + 4*(b^3*c^3*e*cos(e*x + d)^3*log
(F)^3 + 2*(2*b*c*e^3*cos(e*x + d)^3 + 3*b*c*e^3*cos(e*x + d))*log(F))*sin(
e*x + d))*F^(b*c*x + a*c)/(b^5*c^5*log(F)^5 + 20*b^3*c^3*e^2*log(F)^3 + 64
*b*c*e^4*log(F))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.70 (sec) , antiderivative size = 2538, normalized size of antiderivative = 13.22

$$\int F^{c(a+bx)} \cos^4(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cos(e*x+d)**4,x)
```

output
```
Piecewise((x*cos(d)**4, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (3*x*s
in(d + e*x)**4/8 + 3*x*sin(d + e*x)**2*cos(d + e*x)**2/4 + 3*x*cos(d + e*x
)**4/8 + 3*sin(d + e*x)**3*cos(d + e*x)/(8*e) + 5*sin(d + e*x)*cos(d + e*x
)**3/(8*e), Eq(F, 1)), (F**(a*c)*(3*x*sin(d + e*x)**4/8 + 3*x*sin(d + e*x)
**2*cos(d + e*x)**2/4 + 3*x*cos(d + e*x)**4/8 + 3*sin(d + e*x)**3*cos(d +
e*x)/(8*e) + 5*sin(d + e*x)*cos(d + e*x)**3/(8*e)), Eq(b, 0)), (3*x*sin(d
+ e*x)**4/8 + 3*x*sin(d + e*x)**2*cos(d + e*x)**2/4 + 3*x*cos(d + e*x)**4/
8 + 3*sin(d + e*x)**3*cos(d + e*x)/(8*e) + 5*sin(d + e*x)*cos(d + e*x)**3/
(8*e), Eq(c, 0)), (-F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 - d)**4/4 + I*
F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 - d)**3*cos(I*b*c*x*log(F)/2 - d)/
2 + I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 - d)*cos(I*b*c*x*log(F)/2 -
d)**3/2 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 - d)**4/4 + 13*F**(a*c +
 b*c*x)*sin(I*b*c*x*log(F)/2 - d)**4/(24*b*c*log(F)) - 7*I*F**(a*c + b*c*x
)*sin(I*b*c*x*log(F)/2 - d)**3*cos(I*b*c*x*log(F)/2 - d)/(12*b*c*log(F)) +
 F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 - d)**2*cos(I*b*c*x*log(F)/2 - d)**
2/(b*c*log(F)) - 5*I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 - d)*cos(I*b*c*
x*log(F)/2 - d)**3/(4*b*c*log(F)) + F**(a*c + b*c*x)*cos(I*b*c*x*log(F)/2
- d)**4/(8*b*c*log(F)), Eq(e, -I*b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sin(I
*b*c*x*log(F)/4 - d)**4/16 - I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/4 - d
)**3*cos(I*b*c*x*log(F)/4 - d)/4 - 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(192) = 384$.

Time = 0.09 (sec) , antiderivative size = 1046, normalized size of antiderivative = 5.45

$$\int F^{c(a+bx)}\cos^4(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^4,x, algorithm="maxima")
```

output
```
1/16*((F^(a*c)*b^4*c^4*cos(4*d)*log(F)^4 + 4*F^(a*c)*b^3*c^3*e*log(F)^3*si
n(4*d) + 4*F^(a*c)*b^2*c^2*e^2*cos(4*d)*log(F)^2 + 16*F^(a*c)*b*c*e^3*log(
F)*sin(4*d))*F^(b*c*x)*cos(4*e*x) + (F^(a*c)*b^4*c^4*cos(4*d)*log(F)^4 - 4
*F^(a*c)*b^3*c^3*e*log(F)^3*sin(4*d) + 4*F^(a*c)*b^2*c^2*e^2*cos(4*d)*log(
F)^2 - 16*F^(a*c)*b*c*e^3*log(F)*sin(4*d))*F^(b*c*x)*cos(4*e*x + 8*d) + 4*
(F^(a*c)*b^4*c^4*cos(4*d)*log(F)^4 - 2*F^(a*c)*b^3*c^3*e*log(F)^3*sin(4*d)
 + 16*F^(a*c)*b^2*c^2*e^2*cos(4*d)*log(F)^2 - 32*F^(a*c)*b*c*e^3*log(F)*si
n(4*d))*F^(b*c*x)*cos(2*e*x + 6*d) + 4*(F^(a*c)*b^4*c^4*cos(4*d)*log(F)^4
+ 2*F^(a*c)*b^3*c^3*e*log(F)^3*sin(4*d) + 16*F^(a*c)*b^2*c^2*e^2*cos(4*d)*
log(F)^2 + 32*F^(a*c)*b*c*e^3*log(F)*sin(4*d))*F^(b*c*x)*cos(2*e*x - 2*d)
- (F^(a*c)*b^4*c^4*log(F)^4*sin(4*d) - 4*F^(a*c)*b^3*c^3*e*cos(4*d)*log(F)
^3 + 4*F^(a*c)*b^2*c^2*e^2*log(F)^2*sin(4*d) - 16*F^(a*c)*b*c*e^3*cos(4*d)
*log(F))*F^(b*c*x)*sin(4*e*x) + (F^(a*c)*b^4*c^4*log(F)^4*sin(4*d) + 4*F^(
a*c)*b^3*c^3*e*cos(4*d)*log(F)^3 + 4*F^(a*c)*b^2*c^2*e^2*log(F)^2*sin(4*d)
 + 16*F^(a*c)*b*c*e^3*cos(4*d)*log(F))*F^(b*c*x)*sin(4*e*x + 8*d) + 4*(F^(
a*c)*b^4*c^4*log(F)^4*sin(4*d) + 2*F^(a*c)*b^3*c^3*e*cos(4*d)*log(F)^3 + 1
6*F^(a*c)*b^2*c^2*e^2*log(F)^2*sin(4*d) + 32*F^(a*c)*b*c*e^3*cos(4*d)*log(
F))*F^(b*c*x)*sin(2*e*x + 6*d) - 4*(F^(a*c)*b^4*c^4*log(F)^4*sin(4*d) - 2*
F^(a*c)*b^3*c^3*e*cos(4*d)*log(F)^3 + 16*F^(a*c)*b^2*c^2*e^2*log(F)^2*sin(
4*d) - 32*F^(a*c)*b*c*e^3*cos(4*d)*log(F))*F^(b*c*x)*sin(2*e*x - 2*d) +...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 1554, normalized size of antiderivative = 8.09

$$\int F^{c(a+bx)}\cos^4(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^4,x, algorithm="giac")
```

output
```
1/8*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/
2*pi*a*c + 4*e*x + 4*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn
(F) - pi*b*c + 8*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 8*e)*sin(1/2*pi*b*c*x*s
gn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 4*e*x + 4*d)/(4*b^
2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 8*e)^2))*e^(b*c*x*log(abs(
F)) + a*c*log(abs(F))) + 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x
 + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*lo
g(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c
+ 4*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi
*a*c + 2*e*x + 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4
*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/2*(2*b*c*cos(1/2*pi*b*
c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)*
log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) +
 (pi*b*c*sgn(F) - pi*b*c - 4*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1
/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (p
i*b*c*sgn(F) - pi*b*c - 4*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) +
 1/8*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1
/2*pi*a*c - 4*e*x - 4*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sg
n(F) - pi*b*c - 8*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 8*e)*sin(1/2*pi*b*c*x*
sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 4*e*x - 4*d)/(...
```

Mupad [B] (verification not implemented)

Time = 16.39 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.17

$$\int F^{c(a+bx)}\cos^4(d+ex)\,dx$$
$$= \frac{3F^{c(a+bx)}}{8bc\ln(F)} - \frac{F^{c(a+bx)}(\cos(4ex)-\sin(4ex)\,1\mathrm{i})(\cos(4d)-\sin(4d)\,1\mathrm{i})}{16(-bc\ln(F)+e\,4\mathrm{i})}$$
$$-\frac{F^{c(a+bx)}(\cos(2ex)-\sin(2ex)\,1\mathrm{i})(\cos(2d)-\sin(2d)\,1\mathrm{i})}{4(-bc\ln(F)+e\,2\mathrm{i})}$$
$$-\frac{F^{c(a+bx)}(\cos(2ex)+\sin(2ex)\,1\mathrm{i})(\cos(2d)+\sin(2d)\,1\mathrm{i})\,1\mathrm{i}}{4(2e-bc\ln(F)\,1\mathrm{i})}$$
$$-\frac{F^{c(a+bx)}(\cos(4ex)+\sin(4ex)\,1\mathrm{i})(\cos(4d)+\sin(4d)\,1\mathrm{i})\,1\mathrm{i}}{16(4e-bc\ln(F)\,1\mathrm{i})}$$

input
```
int(F^(c*(a + b*x))*cos(d + e*x)^4,x)
```

output
```
(3*F^(c*(a + b*x)))/(8*b*c*log(F)) - (F^(c*(a + b*x))*(cos(2*e*x) + sin(2*
e*x)*1i)*(cos(2*d) + sin(2*d)*1i)*1i)/(4*(2*e - b*c*log(F)*1i)) - (F^(c*(a
 + b*x))*(cos(4*e*x) - sin(4*e*x)*1i)*(cos(4*d) - sin(4*d)*1i))/(16*(e*4i
- b*c*log(F))) - (F^(c*(a + b*x))*(cos(4*e*x) + sin(4*e*x)*1i)*(cos(4*d) +
 sin(4*d)*1i)*1i)/(16*(4*e - b*c*log(F)*1i)) - (F^(c*(a + b*x))*(cos(2*e*x
) - sin(2*e*x)*1i)*(cos(2*d) - sin(2*d)*1i))/(4*(e*2i - b*c*log(F)))
```

Reduce [F]

$$\int F^{c(a+bx)}\cos^4(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cos(ex+d)^4\,dx\right)$$

input
```
int(F^(c*(b*x+a))*cos(e*x+d)^4,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cos(d + e*x)**4,x)
```

3.37 $\int e^{a+ibx} \cos^n(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int e^{a+ibx} \cos^n(a+bx)\,dx = -\frac{ie^{a+ibx}\left(1+e^{2i(a+bx)}\right)^{-n} \cos^n(a+bx) \operatorname{Hypergeometric2F1}\left(\frac{1-n}{2},-n,\frac{3-n}{2},-e^{2i(a+bx)}\right)}{b(1-n)}$$

output
```
-I*exp(a+I*b*x)*cos(b*x+a)^n*hypergeom([-n, 1/2-1/2*n],[3/2-1/2*n],-exp(2*
I*(b*x+a)))/b/((1+exp(2*I*(b*x+a)))^n)/(1-n)
```

Mathematica [A] (verified)

Time = 6.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int e^{a+ibx} \cos^n(a+bx)\,dx = \frac{i2^{-n}e^{a+ibx}\left(1+e^{2i(a+bx)}\right)^{-n} \left(e^{-i(a+bx)}\left(1+e^{2i(a+bx)}\right)\right)^n \operatorname{Hypergeometric2F1}\left(\frac{1-n}{2},-n,\frac{3-n}{2},-e^{2i(a+bx)}\right)}{b(-1+n)}$$

input
```
Integrate[E^(a + I*b*x)*Cos[a + b*x]^n,x]
```

output
```
(I*E^(a + I*b*x)*((1 + E^((2*I)*(a + b*x)))/E^(I*(a + b*x)))^n*Hypergeomet
ric2F1[(1 - n)/2, -n, (3 - n)/2, -E^((2*I)*(a + b*x))])/(2^n*b*(1 + E^((2*
I)*(a + b*x)))^n*(-1 + n))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4941, 2683, 2681}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\cos^n(a+bx)\,dx$$

$$\Big\downarrow \; 4941$$

$$e^{in(a+bx)}\left(1+e^{2i(a+bx)}\right)^{-n}\cos^n(a+bx)\int e^{a+ibx-in(a+bx)}\left(1+e^{2i(a+bx)}\right)^n dx$$

$$\Big\downarrow \; 2683$$

$$e^{in(a+bx)}\left(1+e^{2i(a+bx)}\right)^{-n}\cos^n(a+bx)\int e^{a(1-in)+ib(1-n)x}\left(1+e^{2i(a+bx)}\right)^n dx$$

$$\Big\downarrow \; 2681$$

$$-\frac{i\left(1+e^{2i(a+bx)}\right)^{-n}\exp(in(a+bx)+a(1-in)+ib(1-n)x)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},-n,\frac{3-n}{2},-e^{2i(a+bx)}\right)\text{co}}{b(1-n)}$$

input
```
Int[E^(a + I*b*x)*Cos[a + b*x]^n,x]
```

output
```
((-I)*E^(a*(1 - I*n) + I*b*(1 - n)*x + I*n*(a + b*x))*Cos[a + b*x]^n*Hyper
geometric2F1[(1 - n)/2, -n, (3 - n)/2, -E^((2*I)*(a + b*x))])/(b*(1 + E^((
2*I)*(a + b*x)))^n*(1 - n))
```

Defintions of rubi rules used

rule 2681
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

rule 2683
```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] && !LinearMatchQ[{u, v}, x]
```

rule 4941
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[E^(I*n*(d + e*x))*(Cos[d + e*x]^n/(1 + E^(2*I*(d + e*x)))^n)   I
nt[F^(c*(a + b*x))*((1 + E^(2*I*(d + e*x)))^n/E^(I*n*(d + e*x))), x], x] /;
 FreeQ[{F, a, b, c, d, e, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int e^{ibx+a} \cos (bx+a)^n \, dx$$

input
```
int(exp(a+I*b*x)*cos(b*x+a)^n,x)
```

output
```
int(exp(a+I*b*x)*cos(b*x+a)^n,x)
```

Fricas [F]

$$\int e^{a+ibx} \cos^n(a+bx) \, dx = \int \cos (bx+a)^n e^{(i\,bx+a)} \, dx$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral((1/2*(e^(2*I*b*x + 2*I*a) + 1)*e^(-I*b*x - I*a))^n*e^(I*b*x + a),
 x)
```

Sympy [F]

$$\int e^{a+ibx} \cos^n(a+bx)\,dx = e^a \int e^{ibx} \cos^n(a+bx)\,dx$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+a)**n,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*cos(a + b*x)**n, x)
```

Maxima [F]

$$\int e^{a+ibx} \cos^n(a+bx)\,dx = \int \cos(bx+a)^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^n*e^(I*b*x + a), x)
```

Giac [F]

$$\int e^{a+ibx} \cos^n(a+bx)\,dx = \int \cos(bx+a)^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*cos(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^n*e^(I*b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx} \cos^n(a+bx) \, dx = \int \cos(a+b\,x)^n \, \mathrm{e}^{a+b\,x\,1\mathrm{i}} \, dx$$

input
```
int(cos(a + b*x)^n*exp(a + b*x*1i),x)
```

output
```
int(cos(a + b*x)^n*exp(a + b*x*1i), x)
```

Reduce [F]

$$\int e^{a+ibx} \cos^n(a+bx) \, dx = -\frac{e^{a} i \left(e^{bix} \cos(bx+a)^{n} + \left(\int \frac{e^{bix} \cos(bx+a)^{n} \sin(bx+a)}{\cos(bx+a)} dx\right) bn\right)}{b}$$

input
```
int(exp(a+I*b*x)*cos(b*x+a)^n,x)
```

output
```
( - e**a*i*(e**(b*i*x)*cos(a + b*x)**n + int((e**(b*i*x)*cos(a + b*x)**n*s
in(a + b*x))/cos(a + b*x),x)*b*n))/b
```

3.38 $\int F^{c(a+bx)}(f\cos(d+ex))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int F^{c(a+bx)}(f\cos(d+ex))^n\,dx = -\frac{\left(1+e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}(f\cos(d+ex))^n\,\text{Hypergeometric2F1}\left(-n,\frac{-en-ibc\log(F)}{2e},\frac{1}{2}\left(2-n-\frac{ibc\log(F)}{e}\right.\right.}{ien-bc\log(F)}$$

output
```
-F^(c*(b*x+a))*(f*cos(e*x+d))^n*hypergeom([-n, 1/2*(-e*n-I*b*c*ln(F))/e],[
1-1/2*n-1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/((1+exp(2*I*(e*x+d)))^n)/(I*
e*n-b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}(f\cos(d+ex))^n\,dx = \frac{\left(1+e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}(f\cos(d+ex))^n\,\text{Hypergeometric2F1}\left(-n,-\frac{i(-ien+bc\log(F))}{2e},1-\frac{i(-ien+bc\log(F))}{2e}\right.}{-ien+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Cos[d + e*x])^n,x]
```

output
```
(F^(c*(a + b*x))*(f*Cos[d + e*x])^n*Hypergeometric2F1[-n, ((-1/2*I)*((-I)*
e*n + b*c*Log[F]))/e, 1 - ((I/2)*((-I)*e*n + b*c*Log[F]))/e, -E^((2*I)*(d
+ e*x))])/((1 + E^((2*I)*(d + e*x)))^n*((-I)*e*n + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7271, 4941, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\cos(d+ex))^n\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\cos^{-n}(d+ex)(f\cos(d+ex))^n\int F^{c(a+bx)}\cos^n(d+ex)dx$$

$$\Big\downarrow \text{ 4941}$$

$$e^{in(d+ex)}\left(1+e^{2i(d+ex)}\right)^{-n}(f\cos(d+ex))^n\int e^{-in(d+ex)}\left(1+e^{2i(d+ex)}\right)^nF^{c(a+bx)}dx$$

$$\Big\downarrow \text{ 2689}$$

$$-\frac{\left(1+e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}(f\cos(d+ex))^n\,\text{Hypergeometric2F1}\left(-n,-\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(-n-\frac{ibc\log(F)}{e}+2\right),-e\right.}{-bc\log(F)+ien}$$

input
```
Int[F^(c*(a + b*x))*(f*Cos[d + e*x])^n,x]
```

output
```
-((F^(c*(a + b*x))*(f*Cos[d + e*x])^n*Hypergeometric2F1[-n, -1/2*(e*n + I*
b*c*Log[F])/e, (2 - n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))])/((1 +
E^((2*I)*(d + e*x)))^n*(I*e*n - b*c*Log[F])))
```

Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 4941
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[E^(I*n*(d + e*x))*(Cos[d + e*x]^n/(1 + E^(2*I*(d + e*x)))^n)   I
nt[F^(c*(a + b*x))*((1 + E^(2*I*(d + e*x)))^n/E^(I*n*(d + e*x))), x], x] /;
 FreeQ[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}(f\cos(ex+d))^n\,dx$$

input
```
int(F^(c*(b*x+a))*(f*cos(e*x+d))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*cos(e*x+d))^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\cos(d+ex))^n\,dx=\int (f\cos(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cos(e*x+d))^n,x, algorithm="fricas")
```

output
```
integral((f*cos(e*x + d))^n*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\cos(d+ex))^n\,dx=\int F^{c(a+bx)}(f\cos(d+ex))^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*cos(e*x+d))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*cos(d + e*x))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\cos(d+ex))^n\,dx=\int (f\cos(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cos(e*x+d))^n,x, algorithm="maxima")
```

output
```
integrate((f*cos(e*x + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\cos(d+ex))^n\,dx=\int (f\cos(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cos(e*x+d))^n,x, algorithm="giac")
```

output
```
integrate((f*cos(e*x + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\cos(d+ex))^n\,dx=\int F^{c\,(a+b\,x)}\,\left(f\,\cos\left(d+e\,x\right)\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*cos(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*cos(d + e*x))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\cos(d+ex))^n\,dx=\frac{f^{ac+n}\left(f^{bcx}\cos\left(ex+d\right)^n+\left(\int\frac{f^{bcx}\cos(ex+d)^n\sin(ex+d)}{\cos(ex+d)}dx\right)en\right)}{\log\left(f\right)bc}$$

input
```
int(F^(c*(b*x+a))*(f*cos(e*x+d))^n,x)
```

output
```
(f**(a*c + n)*(f**(b*c*x)*cos(d + e*x)**n + int((f**(b*c*x)*cos(d + e*x)**
n*sin(d + e*x))/cos(d + e*x),x)*e*n))/(log(f)*b*c)
```

3.39 $\int F^{c(a+bx)}\left(f\cos\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx$

Optimal result

Integrand size = 31, antiderivative size = 93

$$\int F^{c(a+bx)}\left(f\cos\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx = \frac{F^{c(a+bx)}(2+n)\left(f\cos\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^{1+n}\left(\cos\left(d+\frac{ibcx\log(F)}{2+n}\right)+i\sin\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)}{bcf(1+n)\log(F)}$$

output
```
F^(c*(b*x+a))*(2+n)*(f*cos(d+I*b*c*x*ln(F)/(2+n)))^(1+n)*(cos(d+I*b*c*x*ln
(F)/(2+n))+I*sin(d+I*b*c*x*ln(F)/(2+n)))/b/c/f/(1+n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

$$\int F^{c(a+bx)}\left(f\cos\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx = \frac{F^{c(a+bx)}\left(1+e^{2id}F^{-\frac{2bcx}{2+n}}\right)(2+n)\left(f\cos\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n}{2bc(1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Cos[d + (I*b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
(F^(c*(a + b*x))*(1 + E^((2*I)*d)/F^((2*b*c*x)/(2 + n)))*(2 + n)*(f*Cos[d
+ (I*b*c*x*Log[F])/(2 + n)])^n)/(2*b*c*(1 + n)*Log[F])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {7271, 4937}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\cos\left(d+\frac{ibcx\log(F)}{n+2}\right)\right)^n dx$$

$$\Big\downarrow \text{7271}$$

$$\cos^{-n}\left(d+\frac{ibcx\log(F)}{n+2}\right)\left(f\cos\left(d+\frac{ibcx\log(F)}{n+2}\right)\right)^n\int F^{c(a+bx)}\cos^n\left(d+\frac{ibcx\log(F)}{n+2}\right)dx$$

$$\Big\downarrow \text{4937}$$

$$\cos^{-n}\left(d+\frac{ibcx\log(F)}{n+2}\right)\left(f\cos\left(d+\frac{ibcx\log(F)}{n+2}\right)\right)^n\left(\frac{(n+2)F^{c(a+bx)}\cos^{n+2}\left(d+\frac{ibcx\log(F)}{n+2}\right)}{bc(n+1)\log(F)}+\frac{i(n+2)F^{c(}}{}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Cos[d + (I*b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
((f*Cos[d + (I*b*c*x*Log[F])/(2 + n)])^n*((F^(c*(a + b*x))*(2 + n)*Cos[d +
 (I*b*c*x*Log[F])/(2 + n)]^(2 + n))/(b*c*(1 + n)*Log[F]) + (I*F^(c*(a + b*
x))*(2 + n)*Cos[d + (I*b*c*x*Log[F])/(2 + n)]^(1 + n)*Sin[d + (I*b*c*x*Log
[F])/(2 + n)])/(b*c*(1 + n)*Log[F])))/Cos[d + (I*b*c*x*Log[F])/(2 + n)]^n
```

Defintions of rubi rules used

rule 4937
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^(n + 2)/(e^2*(n + 1)
*(n + 2))), x] - Simp[F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(n + 1)/(e
*(n + 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[e^2*(n + 2)^2 + b^
2*c^2*Log[F]^2, 0] && NeQ[n, -1] && NeQ[n, -2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(f\cos\left(d+\frac{ibcx\ln(F)}{n+2}\right)\right)^{n}dx$$

input
```
int(F^(c*(b*x+a))*(f*cos(d+I*b*c*x*ln(F)/(n+2)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*cos(d+I*b*c*x*ln(F)/(n+2)))^n,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int F^{c(a+bx)}\left(f\cos\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^{n}dx = \frac{\left((n+2)e^{\left(-\frac{2\,(bcx\log(F)-i\,dn-2i\,d)}{n+2}\right)}+n+2\right)\left(\frac{1}{2}\left(fe^{\left(-\frac{2\,(bcx\log(F)-i\,dn-2i\,d)}{n+2}\right)}+f\right)e^{\left(\frac{bcx\log(F)-i\,dn-2i\,d}{n+2}\right)}\right)^{n}F^{bcx+ac}}{2\,(bcn+bc)\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(f*cos(d+I*b*c*x*log(F)/(2+n)))^n,x, algorithm="fr
icas")
```

output
```
1/2*((n + 2)*e^(-2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) + n + 2)*(1/2*(
f*e^(-2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) + f)*e^((b*c*x*log(F) - I*
d*n - 2*I*d)/(n + 2)))^n*F^(b*c*x + a*c)/((b*c*n + b*c)*log(F))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\cos\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx=\int F^{c(a+bx)}\left(f\cos\left(\frac{ibcx\log\left(F\right)}{n+2}+d\right)\right)^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*cos(d+I*b*c*x*ln(F)/(2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*cos(I*b*c*x*log(F)/(n + 2) + d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\cos\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx$$
$$=\int\left(f\cos\left(\frac{i\,bcx\log\left(F\right)}{n+2}+d\right)\right)^nF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cos(d+I*b*c*x*log(F)/(2+n)))^n,x, algorithm="ma
xima")
```

output
```
integrate((f*cos(I*b*c*x*log(F)/(n + 2) + d))^n*F^((b*x + a)*c), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(85) = 170$.

Time = 0.66 (sec) , antiderivative size = 525, normalized size of antiderivative = 5.65

$$\int F^{c(a+bx)}\left(f\cos\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(f*cos(d+I*b*c*x*log(F)/(2+n)))^n,x, algorithm="gi
ac")
```

output
```
1/2*(F^(a*c)*n*e^((2*b*c*x*log(F) + I*d*n^2 - n^2*log(2) + n^2*log(f*e^(2*
(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) + f) + 2*I*d*n - 2*n*log(2) + 2*n*
log(f*e^(2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) + f))/(n + 2) + 2*(b*c*
x*log(F) - I*d*n - 2*I*d)/(n + 2)) + F^(a*c)*n*e^((2*b*c*x*log(F) + I*d*n^
2 - n^2*log(2) + n^2*log(f*e^(2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) +
f) + 2*I*d*n - 2*n*log(2) + 2*n*log(f*e^(2*(b*c*x*log(F) - I*d*n - 2*I*d)/
(n + 2)) + f))/(n + 2)) + 2*F^(a*c)*e^((2*b*c*x*log(F) + I*d*n^2 - n^2*log
(2) + n^2*log(f*e^(2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) + f) + 2*I*d*
n - 2*n*log(2) + 2*n*log(f*e^(2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) +
f))/(n + 2) + 2*(b*c*x*log(F) - I*d*n - 2*I*d)/(n + 2)) + 2*F^(a*c)*e^((2*
b*c*x*log(F) + I*d*n^2 - n^2*log(2) + n^2*log(f*e^(2*(b*c*x*log(F) - I*d*n
 - 2*I*d)/(n + 2)) + f) + 2*I*d*n - 2*n*log(2) + 2*n*log(f*e^(2*(b*c*x*log
(F) - I*d*n - 2*I*d)/(n + 2)) + f))/(n + 2)))/(b*c*n*e^(2*(b*c*x*log(F) -
I*d*n - 2*I*d)/(n + 2))*log(F) + b*c*e^(2*(b*c*x*log(F) - I*d*n - 2*I*d)/(
n + 2))*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\cos\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n dx = \int F^{c\,(a+b\,x)}\,\left(f\,\cos\left(d+\frac{b\,c\,x\,\ln\left(F\right)\,1{}\mathrm{i}}{n+2}\right)\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*cos(d + (b*c*x*log(F)*1i)/(n + 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*cos(d + (b*c*x*log(F)*1i)/(n + 2)))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\left(f\cos\left(d+\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx$$
$$=\frac{f^{ac+n}\left(f^{bcx}\cos\left(\frac{\log(f)bcix+dn+2d}{n+2}\right)^n n+2f^{bcx}\cos\left(\frac{\log(f)bcix+dn+2d}{n+2}\right)^n+\left(\int\frac{f^{bcx}\cos\left(\frac{\log(f)bcix+dn+2d}{n+2}\right)^n\sin\left(\frac{\log(f)bci}{n}\right.}{\cos\left(\frac{\log(f)bcix+dn+2d}{n+2}\right)}\right.\right.}{\log(f)\,bc\,(n+2)}$$

input
```
int(F^(c*(b*x+a))*(f*cos(d+I*b*c*x*log(F)/(2+n)))^n,x)
```

output
```
(f**(a*c + n)*(f**(b*c*x)*cos((log(f)*b*c*i*x + d*n + 2*d)/(n + 2))**n*n +
 2*f**(b*c*x)*cos((log(f)*b*c*i*x + d*n + 2*d)/(n + 2))**n + int((f**(b*c*
x)*cos((log(f)*b*c*i*x + d*n + 2*d)/(n + 2))**n*sin((log(f)*b*c*i*x + d*n
+ 2*d)/(n + 2)))/cos((log(f)*b*c*i*x + d*n + 2*d)/(n + 2)),x)*log(f)*b*c*i
*n))/(log(f)*b*c*(n + 2))
```

3.40 $\int F^{c(a+bx)}\left(f\cos\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx$

Optimal result

Integrand size = 31, antiderivative size = 93

$$\int F^{c(a+bx)}\left(f\cos\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx =\frac{F^{c(a+bx)}(2+n)\left(f\cos\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^{1+n}\left(\cos\left(d-\frac{ibcx\log(F)}{2+n}\right)-i\sin\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)}{bcf(1+n)\log(F)}$$

output
```
F^(c*(b*x+a))*(2+n)*(f*cos(-d+I*b*c*x*ln(F)/(2+n)))^(1+n)*(cos(-d+I*b*c*x*
ln(F)/(2+n))+I*sin(-d+I*b*c*x*ln(F)/(2+n)))/b/c/f/(1+n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}\left(f\cos\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx =\frac{e^{-2id}F^{c\left(a+\frac{bnx}{2+n}\right)}\left(1+e^{2id}F^{\frac{2bcx}{2+n}}\right)(2+n)\left(f\cos\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n}{2bc(1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Cos[d - (I*b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
(F^(c*(a + (b*n*x)/(2 + n)))*(1 + E^((2*I)*d)*F^((2*b*c*x)/(2 + n)))*(2 +
n)*(f*Cos[d - (I*b*c*x*Log[F])/(2 + n)])^n)/(2*b*c*E^((2*I)*d)*(1 + n)*Log
[F])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {7271, 4937}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\cos\left(d-\frac{ibcx\log(F)}{n+2}\right)\right)^n dx$$

$$\Big\downarrow \text{ 7271}$$

$$\cos^{-n}\left(d-\frac{ibcx\log(F)}{n+2}\right)\left(f\cos\left(d-\frac{ibcx\log(F)}{n+2}\right)\right)^n\int F^{c(a+bx)}\cos^n\left(d-\frac{ibcx\log(F)}{n+2}\right)dx$$

$$\Big\downarrow \text{ 4937}$$

$$\cos^{-n}\left(d-\frac{ibcx\log(F)}{n+2}\right)\left(f\cos\left(d-\frac{ibcx\log(F)}{n+2}\right)\right)^n\left(\frac{(n+2)F^{c(a+bx)}\cos^{n+2}\left(d-\frac{ibcx\log(F)}{n+2}\right)}{bc(n+1)\log(F)}-\frac{i(n+2)F^{c(}}{}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Cos[d - (I*b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
((f*Cos[d - (I*b*c*x*Log[F])/(2 + n)])^n*((F^(c*(a + b*x))*(2 + n)*Cos[d -
 (I*b*c*x*Log[F])/(2 + n)]^(2 + n))/(b*c*(1 + n)*Log[F]) - (I*F^(c*(a + b*
x))*(2 + n)*Cos[d - (I*b*c*x*Log[F])/(2 + n)]^(1 + n)*Sin[d - (I*b*c*x*Log
[F])/(2 + n)])/(b*c*(1 + n)*Log[F])))/Cos[d - (I*b*c*x*Log[F])/(2 + n)]^n
```

Defintions of rubi rules used

rule 4937
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^(n + 2)/(e^2*(n + 1)
*(n + 2))), x] - Simp[F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(n + 1)/(e
*(n + 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[e^2*(n + 2)^2 + b^
2*c^2*Log[F]^2, 0] && NeQ[n, -1] && NeQ[n, -2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(f\cos\left(-d+\frac{ibcx\ln(F)}{n+2}\right)\right)^n dx$$

input
```
int(F^(c*(b*x+a))*(f*cos(-d+I*b*c*x*ln(F)/(n+2)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*cos(-d+I*b*c*x*ln(F)/(n+2)))^n,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int F^{c(a+bx)}\left(f\cos\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n dx = \frac{\left((n+2)e^{\left(-\frac{2(bcx\log(F)+i\,dn+2i\,d)}{n+2}\right)}+n+2\right)\left(\frac{1}{2}\left(fe^{\left(-\frac{2(bcx\log(F)+i\,dn+2i\,d)}{n+2}\right)}+f\right)e^{\left(\frac{bcx\log(F)+i\,dn+2i\,d}{n+2}\right)}\right)^n F^{bcx+ac}}{2\,(bcn+bc)\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(f*cos(-d+I*b*c*x*log(F)/(2+n)))^n,x, algorithm="f
ricas")
```

output
```
1/2*((n + 2)*e^(-2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) + n + 2)*(1/2*(
f*e^(-2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) + f)*e^((b*c*x*log(F) + I*
d*n + 2*I*d)/(n + 2)))^n*F^(b*c*x + a*c)/((b*c*n + b*c)*log(F))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\cos\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx$$
$$=\int F^{c(a+bx)}\left(f\cos\left(\frac{ibcx\log\left(F\right)}{n+2}-d\right)\right)^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*cos(-d+I*b*c*x*ln(F)/(2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*cos(I*b*c*x*log(F)/(n + 2) - d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\cos\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx$$
$$=\int\left(f\cos\left(\frac{i\,bcx\log\left(F\right)}{n+2}-d\right)\right)^nF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cos(-d+I*b*c*x*log(F)/(2+n)))^n,x, algorithm="m
axima")
```

output
```
integrate((f*cos(I*b*c*x*log(F)/(n + 2) - d))^n*F^((b*x + a)*c), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(91) = 182$.

Time = 0.76 (sec) , antiderivative size = 525, normalized size of antiderivative = 5.65

$$\int F^{c(a+bx)}\left(f\cos\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(f*cos(-d+I*b*c*x*log(F)/(2+n)))^n,x, algorithm="g
iac")
```

output
```
1/2*(F^(a*c)*n*e^((2*b*c*x*log(F) - I*d*n^2 - n^2*log(2) + n^2*log(f*e^(2*
(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) + f) - 2*I*d*n - 2*n*log(2) + 2*n*
log(f*e^(2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) + f))/(n + 2) + 2*(b*c*
x*log(F) + I*d*n + 2*I*d)/(n + 2)) + F^(a*c)*n*e^((2*b*c*x*log(F) - I*d*n^
2 - n^2*log(2) + n^2*log(f*e^(2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) +
f) - 2*I*d*n - 2*n*log(2) + 2*n*log(f*e^(2*(b*c*x*log(F) + I*d*n + 2*I*d)/
(n + 2)) + f))/(n + 2)) + 2*F^(a*c)*e^((2*b*c*x*log(F) - I*d*n^2 - n^2*log
(2) + n^2*log(f*e^(2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) + f) - 2*I*d*
n - 2*n*log(2) + 2*n*log(f*e^(2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) +
f))/(n + 2) + 2*(b*c*x*log(F) + I*d*n + 2*I*d)/(n + 2)) + 2*F^(a*c)*e^((2*
b*c*x*log(F) - I*d*n^2 - n^2*log(2) + n^2*log(f*e^(2*(b*c*x*log(F) + I*d*n
 + 2*I*d)/(n + 2)) + f) - 2*I*d*n - 2*n*log(2) + 2*n*log(f*e^(2*(b*c*x*log
(F) + I*d*n + 2*I*d)/(n + 2)) + f))/(n + 2)))/(b*c*n*e^(2*(b*c*x*log(F) +
I*d*n + 2*I*d)/(n + 2))*log(F) + b*c*e^(2*(b*c*x*log(F) + I*d*n + 2*I*d)/(
n + 2))*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\cos\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n\,dx = \int F^{c\,(a+b\,x)}\,\left(f\,\cos\left(d-\frac{b\,c\,x\,\ln\left(F\right)\,1{}\mathrm{i}}{n+2}\right)\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*cos(d - (b*c*x*log(F)*1i)/(n + 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*cos(d - (b*c*x*log(F)*1i)/(n + 2)))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\left(f\cos\left(d-\frac{ibcx\log(F)}{2+n}\right)\right)^n dx = \frac{f^{ac+n}\left(f^{bcx}\cos\left(\frac{\log(f)bcix-dn-2d}{n+2}\right)^n n+2f^{bcx}\cos\left(\frac{\log(f)bcix-dn-2d}{n+2}\right)^n+\left(\int\frac{f^{bcx}\cos\left(\frac{\log(f)bcix-dn-2d}{n+2}\right)^n\sin\left(\frac{\log(f)bci}{n}\right.}{\cos\left(\frac{\log(f)bcix-dn-2d}{n+2}\right)}\right.}{\log(f)\,bc\,(n+2)}$$

input
```
int(F^(c*(b*x+a))*(f*cos(-d+I*b*c*x*log(F)/(2+n)))^n,x)
```

output
```
(f**(a*c + n)*(f**(b*c*x)*cos((log(f)*b*c*i*x - d*n - 2*d)/(n + 2))**n*n +
 2*f**(b*c*x)*cos((log(f)*b*c*i*x - d*n - 2*d)/(n + 2))**n + int((f**(b*c*
x)*cos((log(f)*b*c*i*x - d*n - 2*d)/(n + 2))**n*sin((log(f)*b*c*i*x - d*n
- 2*d)/(n + 2)))/cos((log(f)*b*c*i*x - d*n - 2*d)/(n + 2)),x)*log(f)*b*c*i
*n))/(log(f)*b*c*(n + 2))
```

3.41 $\int e^{a+ibx}\tan(d+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int e^{a+ibx}\tan(d+bx)\,dx=-\frac{e^{a-id+i(d+bx)}}{b}+\frac{2e^{a-id}\arctan\left(e^{i(d+bx)}\right)}{b}$$

output
```
-exp(a-I*d+I*(b*x+d))/b+2*exp(a-I*d)*arctan(exp(I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int e^{a+ibx}\tan(d+bx)\,dx=-\frac{e^a\left(e^{ibx}-2\arctan\left(e^{ibx}(\cos(d)+i\sin(d))\right)(\cos(d)-i\sin(d))\right)}{b}$$

input
```
Integrate[E^(a + I*b*x)*Tan[d + b*x],x]
```

output
```
-((E^a*(E^(I*b*x) - 2*ArcTan[E^(I*b*x)*(Cos[d] + I*Sin[d])]*(Cos[d] - I*Si
n[d])))/b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\tan(bx+d)\,dx$$

$$\Big\downarrow \text{4942}$$

$$i\int\left(\frac{2e^{a+ibx}}{1+e^{2i(d+bx)}}-e^{a+ibx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{ie^{a+ibx}}{b}-\frac{2ie^{a-id}\arctan\left(e^{ibx+id}\right)}{b}\right)$$

input
```
Int[E^(a + I*b*x)*Tan[d + b*x],x]
```

output
```
I*((I*E^(a + I*b*x))/b - ((2*I)*E^(a - I*d)*ArcTan[E^(I*d + I*b*x)])/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{e^a e^{ibx}}{b}+\frac{2 e^a e^{-id} \arctan\left(e^{i(bx+d)}\right)}{b}$	37

input
```
int(exp(a+I*b*x)*tan(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-exp(a)*exp(I*b*x)/b+2*exp(a)/b*exp(-I*d)*arctan(exp(I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int e^{a+ibx}\tan(d+bx)\,dx = \frac{i\,e^{(a-i\,d)}\log\left(e^{(i\,bx+i\,d)}+i\right)-i\,e^{(a-i\,d)}\log\left(e^{(i\,bx+i\,d)}-i\right)-e^{(i\,bx+a)}}{b}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d),x, algorithm="fricas")
```

output
```
(I*e^(a - I*d)*log(e^(I*b*x + I*d) + I) - I*e^(a - I*d)*log(e^(I*b*x + I*d
) - I) - e^(I*b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int e^{a+ibx}\tan(d+bx)\,dx = \begin{cases}-\frac{e^a e^{ibx}}{b} & \text{for } b\neq 0\\ -ixe^a & \text{otherwise}\end{cases} + \frac{e^a e^{-id}\operatorname{RootSum}\left(z^2+1,\left(i\mapsto i\log\left(ie^{-id}+e^{ibx}\right)\right)\right)}{b}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d),x)
```

output
```
Piecewise((-exp(a)*exp(I*b*x)/b, Ne(b, 0)), (-I*x*exp(a), True)) + exp(a)*
exp(-I*d)*RootSum(_z**2 + 1, Lambda(_i, _i*log(_i*exp(-I*d) + exp(I*b*x)))
)/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(34) = 68$.

Time = 0.15 (sec) , antiderivative size = 329, normalized size of antiderivative = 6.58

$$\int e^{a+ibx}\tan(d+bx)\,dx =$$

$$-\frac{2\left(\cos\left(d\right)e^{a}-i\,e^{a}\sin\left(d\right)\right)\arctan\left(\frac{2\left(\cos(bx+2\,d)\cos(d)+\sin(bx+2\,d)\sin(d)\right)}{\cos(bx+2\,d)^{2}+\cos(d)^{2}+2\,\cos(d)\sin(bx+2\,d)+\sin(bx+2\,d)^{2}-2\,\cos(bx+2\,d)\sin(d)+\sin(d)^{2}},\right.}{}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d),x, algorithm="maxima")
```

output
```
-1/2*(2*(cos(d)*e^a - I*e^a*sin(d))*arctan2(2*(cos(b*x + 2*d)*cos(d) + sin
(b*x + 2*d)*sin(d))/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d)
 + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2), (cos(b*x + 2*d)
^2 - cos(d)^2 + sin(b*x + 2*d)^2 - sin(d)^2)/(cos(b*x + 2*d)^2 + cos(d)^2
+ 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + s
in(d)^2)) + 2*cos(b*x)*e^a + (I*cos(d)*e^a + e^a*sin(d))*log((cos(b*x + 2*
d)^2 + cos(d)^2 - 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 + 2*cos(b*x +
 2*d)*sin(d) + sin(d)^2)/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x +
 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2)) + 2*I*e^a*
sin(b*x))/b
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int e^{a+ibx}\tan(d+bx)\,dx = \frac{i\,e^{(a-i\,d)}\log\left(e^{(i\,bx+i\,d)}+i\right)-i\,e^{(a-i\,d)}\log\left(e^{(i\,bx+i\,d)}-i\right)-e^{(i\,bx+a)}}{b}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d),x, algorithm="giac")
```

output
```
(I*e^(a - I*d)*log(e^(I*b*x + I*d) + I) - I*e^(a - I*d)*log(e^(I*b*x + I*d
) - I) - e^(I*b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 17.73 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.78

$$\int e^{a+ibx}\tan(d+bx)\,dx = -\frac{e^{a+b\,x\,1i}}{b}+\frac{\sqrt{-e^{2\,a-d\,2i}}\,\ln\left(-e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}\,2i-e^{2\,a}\,e^{-d\,2i}\,\sqrt{-e^{2\,a}\,e^{-d\,2i}}\,2i\right)}{b} - \frac{\sqrt{-e^{2\,a-d\,2i}}\,\ln\left(-e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}\,2i+e^{2\,a}\,e^{-d\,2i}\,\sqrt{-e^{2\,a}\,e^{-d\,2i}}\,2i\right)}{b}$$

input
```
int(exp(a + b*x*1i)*tan(d + b*x),x)
```

output
```
((-exp(2*a - d*2i))^(1/2)*log(- exp(3*a)*exp(-d*2i)*exp(b*x*1i)*2i - exp(2
*a)*exp(-d*2i)*(-exp(2*a)*exp(-d*2i))^(1/2)*2i))/b - exp(a + b*x*1i)/b - (
(-exp(2*a - d*2i))^(1/2)*log(exp(2*a)*exp(-d*2i)*(-exp(2*a)*exp(-d*2i))^(1
/2)*2i - exp(3*a)*exp(-d*2i)*exp(b*x*1i)*2i))/b
```

Reduce [F]

$$\int e^{a+ibx}\tan(d+bx)\,dx = e^{a}\left(\int e^{bix}\tan(bx+d)\,dx\right)$$

input
```
int(exp(a+I*b*x)*tan(b*x+d),x)
```

output
```
e**a*int(e**(b*i*x)*tan(b*x + d),x)
```

3.42 $\int e^{a+ibx} \tan^2(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int e^{a+ibx}\tan^2(d+bx)\,dx=\frac{ie^{a-id+i(d+bx)}}{b}+\frac{2ie^{a-id+i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)}-\frac{2ie^{a-id}\arctan\left(e^{i(d+bx)}\right)}{b}$$

output
```
I*exp(a-I*d+I*(b*x+d))/b+2*I*exp(a-I*d+I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))-2
*I*exp(a-I*d)*arctan(exp(I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

$$\int e^{a+ibx}\tan^2(d+bx)\,dx$$
$$=\frac{e^a\left(-e^{ibx}\left(\left(3+e^{2ibx}\right)\cos(d)+i\left(-3+e^{2ibx}\right)\sin(d)\right)+2\arctan\left(e^{ibx}(\cos(d)+i\sin(d))\right)\left(e^{2ibx}+\cos^2(d)-\right.\right.}{b\left(i\left(1+e^{2ibx}\right)\cos(d)-\left(-1+e^{2ibx}\right)\sin(d)\right)}$$

input
```
Integrate[E^(a + I*b*x)*Tan[d + b*x]^2,x]
```

output
```
(E^a*(-(E^(I*b*x)*((3 + E^((2*I)*b*x))*Cos[d] + I*(-3 + E^((2*I)*b*x))*Sin
[d])) + 2*ArcTan[E^(I*b*x)*(Cos[d] + I*Sin[d])]*(E^((2*I)*b*x) + Cos[d]^2
- Sin[d]^2 - I*Sin[2*d])))/(b*(I*(1 + E^((2*I)*b*x))*Cos[d] - (-1 + E^((2*
I)*b*x))*Sin[d]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{4942, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\tan^2(bx+d)\,dx$$

$$\Big\downarrow \text{4942}$$

$$-\int \left(e^{a+ibx}-\frac{4e^{a+ibx}}{1+e^{2i(d+bx)}}+\frac{4e^{a+ibx}}{\left(1+e^{2i(d+bx)}\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2ie^{a-id}\arctan\left(e^{ibx+id}\right)}{b}+\frac{2ie^{a+ibx}}{b\left(1+e^{2i(bx+d)}\right)}+\frac{ie^{a+ibx}}{b}$$

input
```
Int[E^(a + I*b*x)*Tan[d + b*x]^2,x]
```

output
```
(I*E^(a + I*b*x))/b + ((2*I)*E^(a + I*b*x))/(b*(1 + E^((2*I)*(d + b*x))))
- ((2*I)*E^(a - I*d)*ArcTan[E^(I*d + I*b*x)])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{i e^{a} e^{i b x}}{b}+\frac{2 i e^{a} e^{i b x}}{b \left(1+e^{2 i \left(b x +d \right)}\right)}-\frac{2 i e^{a} e^{-i d} \arctan\left(e^{i \left(b x +d \right)}\right)}{b}$	66

input
```
int(exp(a+I*b*x)*tan(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
I/b*exp(a)*exp(I*b*x)+2*I*exp(a)*exp(I*b*x)/b/(1+exp(2*I*(b*x+d)))-2*I*exp
(a)/b*exp(-I*d)*arctan(exp(I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int e^{a+ibx}\tan^2(d+bx)\,dx$$
$$=\frac{\left(e^{(2i\,bx+a+i\,d)}+e^{(a-i\,d)}\right)\log\left(e^{(i\,bx+i\,d)}+i\right)-\left(e^{(2i\,bx+a+i\,d)}+e^{(a-i\,d)}\right)\log\left(e^{(i\,bx+i\,d)}-i\right)+i\,e^{(3i\,bx+a+2i\,d)}+}{be^{(2i\,bx+2i\,d)}+b}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d)^2,x, algorithm="fricas")
```

output
```
((e^(2*I*b*x + a + I*d) + e^(a - I*d))*log(e^(I*b*x + I*d) + I) - (e^(2*I*
b*x + a + I*d) + e^(a - I*d))*log(e^(I*b*x + I*d) - I) + I*e^(3*I*b*x + a
+ 2*I*d) + 3*I*e^(I*b*x + a))/(b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int e^{a+ibx}\tan^2(d+bx)\,dx = \begin{cases} \frac{ie^a e^{ibx}}{b} & \text{for } b \neq 0 \\ -xe^a & \text{otherwise} \end{cases} + \frac{2ie^a e^{ibx}}{be^{2id}e^{2ibx}+b} + \frac{\left(-\log\left(e^{ibx}-ie^{-id}\right)+\log\left(e^{ibx}+ie^{-id}\right)\right)e^a e^{-id}}{b}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d)**2,x)
```

output
```
Piecewise((I*exp(a)*exp(I*b*x)/b, Ne(b, 0)), (-x*exp(a), True)) + 2*I*exp(
a)*exp(I*b*x)/(b*exp(2*I*d)*exp(2*I*b*x) + b) + (-log(exp(I*b*x) - I*exp(-
I*d)) + log(exp(I*b*x) + I*exp(-I*d)))*exp(a)*exp(-I*d)/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(59) = 118.

Time = 0.16 (sec) , antiderivative size = 472, normalized size of antiderivative = 5.02

$$\int e^{a+ibx}\tan^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d)^2,x, algorithm="maxima")
```

output
```
(2*((cos(d)*e^a - I*e^a*sin(d))*cos(2*b*x + 2*d) + cos(d)*e^a - (-I*cos(d)
*e^a - e^a*sin(d))*sin(2*b*x + 2*d) - I*e^a*sin(d))*arctan2(2*(cos(b*x + 2
*d)*cos(d) + sin(b*x + 2*d)*sin(d))/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d
)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2),
 (cos(b*x + 2*d)^2 - cos(d)^2 + sin(b*x + 2*d)^2 - sin(d)^2)/(cos(b*x + 2*
d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x +
 2*d)*sin(d) + sin(d)^2)) + 2*cos(3*b*x + 2*d)*e^a + 6*cos(b*x)*e^a + ((I*
cos(d)*e^a + e^a*sin(d))*cos(2*b*x + 2*d) + I*cos(d)*e^a - (cos(d)*e^a - I
*e^a*sin(d))*sin(2*b*x + 2*d) + e^a*sin(d))*log((cos(b*x + 2*d)^2 + cos(d)
^2 - 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 + 2*cos(b*x + 2*d)*sin(d)
+ sin(d)^2)/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b
*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2)) + 2*I*e^a*sin(3*b*x + 2
*d) + 6*I*e^a*sin(b*x))/(-2*I*b*cos(2*b*x + 2*d) + 2*b*sin(2*b*x + 2*d) -
2*I*b)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(59) = 118$.

Time = 0.53 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.43

$$\int e^{a+ibx}\tan^2(d+bx)\,dx = -\frac{e^{(2i\,bx+a+i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+1\right)+e^{(a-i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+1\right)-e^{(2i\,bx+a+i\,d)}\log\left(-i\,e^{(i\,bx+i\,d)}+1\right)-e^{(a}}{b\left(e^{(2i\,bx+2i\,d)}+1\right)}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d)^2,x, algorithm="giac")
```

output
```
-(e^(2*I*b*x + a + I*d)*log(I*e^(I*b*x + I*d) + 1) + e^(a - I*d)*log(I*e^(
I*b*x + I*d) + 1) - e^(2*I*b*x + a + I*d)*log(-I*e^(I*b*x + I*d) + 1) - e^
(a - I*d)*log(-I*e^(I*b*x + I*d) + 1) - I*e^(3*I*b*x + a + 2*I*d) - 3*I*e^
(I*b*x + a))/(b*(e^(2*I*b*x + 2*I*d) + 1))
```

Mupad [B] (verification not implemented)

Time = 17.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.85

$$\int e^{a+ibx}\tan^2(d+bx)\,dx = \frac{e^{a+b\,x\,1i}\,1i}{b} + \frac{e^{3\,a-d\,2i+b\,x\,1i}\,2i}{b\,\left(e^{2\,a-d\,2i}+e^{2\,a+b\,x\,2i}\right)} + \frac{\sqrt{e^{2\,a-d\,2i}}\,\ln\left(-2\,e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}-e^{2\,a}\,e^{-d\,2i}\,\sqrt{e^{2\,a}\,e^{-d\,2i}}\,2i\right)}{b} - \frac{\sqrt{e^{2\,a-d\,2i}}\,\ln\left(-2\,e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}+e^{2\,a}\,e^{-d\,2i}\,\sqrt{e^{2\,a}\,e^{-d\,2i}}\,2i\right)}{b}$$

input
```
int(exp(a + b*x*1i)*tan(d + b*x)^2,x)
```

output
```
(exp(a + b*x*1i)*1i)/b + (exp(3*a - d*2i + b*x*1i)*2i)/(b*(exp(2*a - d*2i)
 + exp(2*a + b*x*2i))) + (exp(2*a - d*2i)^(1/2)*log(- 2*exp(3*a)*exp(-d*2i
)*exp(b*x*1i) - exp(2*a)*exp(-d*2i)*(exp(2*a)*exp(-d*2i))^(1/2)*2i))/b - (
exp(2*a - d*2i)^(1/2)*log(exp(2*a)*exp(-d*2i)*(exp(2*a)*exp(-d*2i))^(1/2)*
2i - 2*exp(3*a)*exp(-d*2i)*exp(b*x*1i)))/b
```

Reduce [F]

$$\int e^{a+ibx}\tan^2(d+bx)\,dx = e^a\left(\int e^{bix}\tan(bx+d)^2\,dx\right)$$

input
```
int(exp(a+I*b*x)*tan(b*x+d)^2,x)
```

output
```
e**a*int(e**(b*i*x)*tan(b*x + d)**2,x)
```

3.43 $\int e^{a+ibx} \tan^3(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 125

$$\int e^{a+ibx}\tan^3(d+bx)\,dx = \frac{e^{a-id+i(d+bx)}}{b}-\frac{2e^{a-id+i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)^2}+\frac{3e^{a-id+i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)}-\frac{3e^{a-id}\arctan\left(e^{i(d+bx)}\right)}{b}$$

output
```
exp(a-I*d+I*(b*x+d))/b-2*exp(a-I*d+I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^2+3*e
xp(a-I*d+I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))-3*exp(a-I*d)*arctan(exp(I*(b*x+
d)))/b
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int e^{a+ibx}\tan^3(d+bx)\,dx = \frac{e^a\left(-3\arctan\left(e^{ibx}(\cos(d)+i\sin(d))\right)(\cos(d)-i\sin(d))+\frac{e^{ibx}\left(5e^{2ibx}+\left(2+e^{4ibx}\right)\cos(2d)+i\left(-2+e^{4ibx}\right)\sin(2d)\right)}{\left(\left(1+e^{2ibx}\right)\cos(d)+i\left(-1+e^{2ibx}\right)\sin(d)\right)^2}\right)}{b}$$

input
```
Integrate[E^(a + I*b*x)*Tan[d + b*x]^3,x]
```

output
```
(E^a*(-3*ArcTan[E^(I*b*x)*(Cos[d] + I*Sin[d])]*(Cos[d] - I*Sin[d]) + (E^(I
*b*x)*(5*E^((2*I)*b*x) + (2 + E^((4*I)*b*x))*Cos[2*d] + I*(-2 + E^((4*I)*b
*x))*Sin[2*d]))/((1 + E^((2*I)*b*x))*Cos[d] + I*(-1 + E^((2*I)*b*x))*Sin[d
])^2))/b
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx} \tan^3(bx+d)\,dx$$

$$\Big\downarrow \text{4942}$$

$$-i\int \left(-e^{a+ibx}+\frac{6e^{a+ibx}}{1+e^{2i(d+bx)}}-\frac{12e^{a+ibx}}{\left(1+e^{2i(d+bx)}\right)^2}+\frac{8e^{a+ibx}}{\left(1+e^{2i(d+bx)}\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{3ie^{a-id}\arctan\left(e^{ibx+id}\right)}{b}+\frac{3ie^{a+ibx}}{b\left(1+e^{2i(bx+d)}\right)}-\frac{2ie^{a+ibx}}{b\left(1+e^{2i(bx+d)}\right)^2}+\frac{ie^{a+ibx}}{b}\right)$$

input
```
Int[E^(a + I*b*x)*Tan[d + b*x]^3,x]
```

output
```
(-I)*((I*E^(a + I*b*x))/b - ((2*I)*E^(a + I*b*x))/(b*(1 + E^((2*I)*(d + b*
x)))^2) + ((3*I)*E^(a + I*b*x))/(b*(1 + E^((2*I)*(d + b*x)))) - ((3*I)*E^(
a - I*d)*ArcTan[E^(I*d + I*b*x)])/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{e^{a} e^{i b x}}{b}+\frac{3 e^{a} e^{3 i b x} e^{2 i d}+e^{a} e^{i b x}}{b \left(1+e^{2 i (b x+d)}\right)^{2}}-\frac{3 e^{a} e^{-i d} \arctan \left(e^{i (b x+d)}\right)}{b}$	78

input
```
int(exp(a+I*b*x)*tan(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
exp(a)*exp(I*b*x)/b+1/b/(1+exp(2*I*(b*x+d)))^2*(3*exp(a)*exp(3*I*b*x)*exp(
2*I*d)+exp(a)*exp(I*b*x))-3*exp(a)/b*exp(-I*d)*arctan(exp(I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.26

$$\int e^{a+ibx} \tan^3(d+bx)\,dx =$$

$$-\frac{3 \left(i e^{(4 i bx+a+3 i d)}+2 i e^{(2 i bx+a+i d)}+i e^{(a-i d)}\right) \log \left(e^{(i bx+i d)}+i\right)+3 \left(-i e^{(4 i bx+a+3 i d)}-2 i e^{(2 i bx+a+i d)}-\right.}{2 \left(b e^{(4 i bx+4 i d)}+2 b e^{(2 i bx+2 i d)}-\right.}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d)^3,x, algorithm="fricas")
```

output
```
-1/2*(3*(I*e^(4*I*b*x + a + 3*I*d) + 2*I*e^(2*I*b*x + a + I*d) + I*e^(a -
I*d))*log(e^(I*b*x + I*d) + I) + 3*(-I*e^(4*I*b*x + a + 3*I*d) - 2*I*e^(2*
I*b*x + a + I*d) - I*e^(a - I*d))*log(e^(I*b*x + I*d) - I) - 2*e^(5*I*b*x
+ a + 4*I*d) - 10*e^(3*I*b*x + a + 2*I*d) - 4*e^(I*b*x + a))/(b*e^(4*I*b*x
 + 4*I*d) + 2*b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int e^{a+ibx}\tan^3(d+bx)\,dx = \frac{3e^ae^{2id}e^{3ibx}+e^ae^{ibx}}{be^{4id}e^{4ibx}+2be^{2id}e^{2ibx}+b} + \begin{cases} \frac{e^ae^{ibx}}{b} & \text{for } b\neq 0 \\ ixe^a & \text{otherwise} \end{cases} + \frac{3e^ae^{-id}\operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(-2ie^{-id}+e^{ibx}\right)\right)\right)}{b}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d)**3,x)
```

output
```
(3*exp(a)*exp(2*I*d)*exp(3*I*b*x) + exp(a)*exp(I*b*x))/(b*exp(4*I*d)*exp(4
*I*b*x) + 2*b*exp(2*I*d)*exp(2*I*b*x) + b) + Piecewise((exp(a)*exp(I*b*x)/
b, Ne(b, 0)), (I*x*exp(a), True)) + 3*exp(a)*exp(-I*d)*RootSum(4*_z**2 + 1
, Lambda(_i, _i*log(-2*_i*exp(-I*d) + exp(I*b*x))))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(83) = 166.

Time = 0.17 (sec) , antiderivative size = 617, normalized size of antiderivative = 4.94

$$\int e^{a+ibx}\tan^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d)^3,x, algorithm="maxima")
```

output
```
-(6*((I*cos(d)*e^a + e^a*sin(d))*cos(4*b*x + 4*d) + 2*(I*cos(d)*e^a + e^a*
sin(d))*cos(2*b*x + 2*d) + I*cos(d)*e^a - (cos(d)*e^a - I*e^a*sin(d))*sin(
4*b*x + 4*d) - 2*(cos(d)*e^a - I*e^a*sin(d))*sin(2*b*x + 2*d) + e^a*sin(d)
)*arctan2(2*(cos(b*x + 2*d)*cos(d) + sin(b*x + 2*d)*sin(d))/(cos(b*x + 2*d
)^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x +
2*d)*sin(d) + sin(d)^2), (cos(b*x + 2*d)^2 - cos(d)^2 + sin(b*x + 2*d)^2 -
 sin(d)^2)/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b*
x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2)) + 4*I*cos(5*b*x + 4*d)*e
^a + 20*I*cos(3*b*x + 2*d)*e^a + 8*I*cos(b*x)*e^a - 3*((cos(d)*e^a - I*e^a
*sin(d))*cos(4*b*x + 4*d) + 2*(cos(d)*e^a - I*e^a*sin(d))*cos(2*b*x + 2*d)
 + cos(d)*e^a - (-I*cos(d)*e^a - e^a*sin(d))*sin(4*b*x + 4*d) - 2*(-I*cos(
d)*e^a - e^a*sin(d))*sin(2*b*x + 2*d) - I*e^a*sin(d))*log((cos(b*x + 2*d)^
2 + cos(d)^2 - 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 + 2*cos(b*x + 2*
d)*sin(d) + sin(d)^2)/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*
d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2)) - 4*e^a*sin(5
*b*x + 4*d) - 20*e^a*sin(3*b*x + 2*d) - 8*e^a*sin(b*x))/(-4*I*b*cos(4*b*x
+ 4*d) - 8*I*b*cos(2*b*x + 2*d) + 4*b*sin(4*b*x + 4*d) + 8*b*sin(2*b*x + 2
*d) - 4*I*b)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(83) = 166.

Time = 0.94 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.59

$$\int e^{a+ibx}\tan^3(d+bx)\,dx = \frac{-3i\,e^{(4i\,bx+a+3i\,d)}\log\left(e^{(i\,bx+i\,d)}+i\right)-6i\,e^{(2i\,bx+a+i\,d)}\log\left(e^{(i\,bx+i\,d)}+i\right)-3i\,e^{(a-i\,d)}\log\left(e^{(i\,bx+i\,d)}+i\right)+3i}{}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d)^3,x, algorithm="giac")
```

output
```
1/2*(-3*I*e^(4*I*b*x + a + 3*I*d)*log(e^(I*b*x + I*d) + I) - 6*I*e^(2*I*b*
x + a + I*d)*log(e^(I*b*x + I*d) + I) - 3*I*e^(a - I*d)*log(e^(I*b*x + I*d
) + I) + 3*I*e^(4*I*b*x + a + 3*I*d)*log(e^(I*b*x + I*d) - I) + 6*I*e^(2*I
*b*x + a + I*d)*log(e^(I*b*x + I*d) - I) + 3*I*e^(a - I*d)*log(e^(I*b*x +
I*d) - I) + 2*e^(5*I*b*x + a + 4*I*d) + 10*e^(3*I*b*x + a + 2*I*d) + 4*e^(
I*b*x + a))/(b*(e^(4*I*b*x + 4*I*d) + 2*e^(2*I*b*x + 2*I*d) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\tan^3(d+bx)\,dx = \int \mathrm{e}^{a+b\,x\,1\mathrm{i}}\,\tan(d+b\,x)^3\,dx$$

input
```
int(exp(a + b*x*1i)*tan(d + b*x)^3,x)
```

output
```
int(exp(a + b*x*1i)*tan(d + b*x)^3, x)
```

Reduce [F]

$$\int e^{a+ibx}\tan^3(d+bx)\,dx = e^a\left(\int e^{bix}\tan(bx+d)^3\,dx\right)$$

input
```
int(exp(a+I*b*x)*tan(b*x+d)^3,x)
```

output
```
e**a*int(e**(b*i*x)*tan(b*x + d)**3,x)
```

3.44 $\int e^{a+ibx} \tan^4(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 178

$$\int e^{a+ibx}\tan^4(d+bx)\,dx = -\frac{ie^{a-id+i(d+bx)}}{b}-\frac{8ie^{a-id+i(d+bx)}}{3b\left(1+e^{2i(d+bx)}\right)^3}+\frac{14ie^{a-id+i(d+bx)}}{3b\left(1+e^{2i(d+bx)}\right)^2}-\frac{5ie^{a-id+i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)}+\frac{3ie^{a-id}\arctan\left(e^{i(d+bx)}\right)}{b}$$

output
```
-I*exp(a-I*d+I*(b*x+d))/b-8/3*I*exp(a-I*d+I*(b*x+d))/b/(1+exp(2*I*(b*x+d))
)^3+14/3*I*exp(a-I*d+I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^2-5*I*exp(a-I*d+I*(
b*x+d))/b/(1+exp(2*I*(b*x+d)))+3*I*exp(a-I*d)*arctan(exp(I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.30

$$\int e^{a+ibx}\tan^4(d+bx)\,dx = \frac{e^a\left(-3ie^{ibx}+9i\arctan\left(e^{ibx}(\cos(d)+i\sin(d))\right)\cos(d)+9\arctan\left(e^{ibx}(\cos(d)+i\sin(d))\right)\sin(d)+\frac{}{i(1+e^2}\right.}{3b}$$

input
```
Integrate[E^(a + I*b*x)*Tan[d + b*x]^4,x]
```

output
```
(E^a*((-3*I)*E^(I*b*x) + (9*I)*ArcTan[E^(I*b*x)*(Cos[d] + I*Sin[d])]*Cos[d
] + 9*ArcTan[E^(I*b*x)*(Cos[d] + I*Sin[d])]*Sin[d] + (15*E^(I*b*x)*(Cos[d]
 - I*Sin[d]))/(I*(1 + E^((2*I)*b*x))*Cos[d] - (-1 + E^((2*I)*b*x))*Sin[d])
 + (8*E^(I*b*x)*(Cos[d] - I*Sin[d])^3)/((-I)*(1 + E^((2*I)*b*x))*Cos[d] +
(-1 + E^((2*I)*b*x))*Sin[d])^3 + (14*E^(I*b*x)*(I*Cos[2*d] + Sin[2*d]))/((
1 + E^((2*I)*b*x))*Cos[d] + I*(-1 + E^((2*I)*b*x))*Sin[d])^2))/(3*b)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx} \tan^4(bx+d)\,dx$$

$$\Big\downarrow \text{4942}$$

$$\int \left(-\frac{8e^{a+ibx}}{1+e^{2i(bx+d)}}+\frac{24e^{a+ibx}}{\left(1+e^{2i(bx+d)}\right)^2}-\frac{32e^{a+ibx}}{\left(1+e^{2i(bx+d)}\right)^3}+\frac{16e^{a+ibx}}{\left(1+e^{2i(bx+d)}\right)^4}+e^{a+ibx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3ie^{a-id}\arctan\left(e^{ibx+id}\right)}{b}-\frac{5ie^{a+ibx}}{b\left(1+e^{2i(bx+d)}\right)}+\frac{14ie^{a+ibx}}{3b\left(1+e^{2i(bx+d)}\right)^2}-\frac{8ie^{a+ibx}}{3b\left(1+e^{2i(bx+d)}\right)^3}-\frac{ie^{a+ibx}}{b}$$

input
```
Int[E^(a + I*b*x)*Tan[d + b*x]^4,x]
```

output
```
((-I)*E^(a + I*b*x))/b - (((8*I)/3)*E^(a + I*b*x))/(b*(1 + E^((2*I)*(d + b
*x)))^3) + (((14*I)/3)*E^(a + I*b*x))/(b*(1 + E^((2*I)*(d + b*x)))^2) - ((
5*I)*E^(a + I*b*x))/(b*(1 + E^((2*I)*(d + b*x)))) + ((3*I)*E^(a - I*d)*Arc
Tan[E^(I*d + I*b*x)])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.56

method	result	size
risch	$-\frac{i e^{a} e^{i b x}}{b}-\frac{i \left(15 e^{a} e^{5 i b x} e^{4 i d}+16 e^{a} e^{3 i b x} e^{2 i d}+9 e^{a} e^{i b x}\right)}{3 b \left(1+e^{2 i (b x+d)}\right)^{3}}+\frac{3 i e^{a} e^{-i d} \arctan\left(e^{i (b x+d)}\right)}{b}$	99

input
```
int(exp(a+I*b*x)*tan(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-I/b*exp(a)*exp(I*b*x)-1/3*I/b/(1+exp(2*I*(b*x+d)))^3*(15*exp(a)*exp(5*I*b
*x)*exp(4*I*d)+16*exp(a)*exp(3*I*b*x)*exp(2*I*d)+9*exp(a)*exp(I*b*x))+3*I*
exp(a)/b*exp(-I*d)*arctan(exp(I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.11

$$\int e^{a+ibx}\tan^4(d+bx)\,dx = -\frac{9\left(e^{(6i\,bx+a+5i\,d)}+3\,e^{(4i\,bx+a+3i\,d)}+3\,e^{(2i\,bx+a+i\,d)}+e^{(a-i\,d)}\right)\log\left(e^{(i\,bx+i\,d)}+i\right)-9\left(e^{(6i\,bx+a+5i\,d)}+3\,e^{(4i\right.}{6\left(be^{(6i\,bx+6i\,d)}\right.}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d)^4,x, algorithm="fricas")
```

output
```
-1/6*(9*(e^(6*I*b*x + a + 5*I*d) + 3*e^(4*I*b*x + a + 3*I*d) + 3*e^(2*I*b*
x + a + I*d) + e^(a - I*d))*log(e^(I*b*x + I*d) + I) - 9*(e^(6*I*b*x + a +
 5*I*d) + 3*e^(4*I*b*x + a + 3*I*d) + 3*e^(2*I*b*x + a + I*d) + e^(a - I*d
))*log(e^(I*b*x + I*d) - I) + 6*I*e^(7*I*b*x + a + 6*I*d) + 48*I*e^(5*I*b*
x + a + 4*I*d) + 50*I*e^(3*I*b*x + a + 2*I*d) + 24*I*e^(I*b*x + a))/(b*e^(
6*I*b*x + 6*I*d) + 3*b*e^(4*I*b*x + 4*I*d) + 3*b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.03

$$\int e^{a+ibx}\tan^4(d+bx)\,dx = \frac{-15ie^ae^{4id}e^{5ibx}-16ie^ae^{2id}e^{3ibx}-9ie^ae^{ibx}}{3be^{6id}e^{6ibx}+9be^{4id}e^{4ibx}+9be^{2id}e^{2ibx}+3b} + \begin{cases} -\frac{ie^ae^{ibx}}{b} & \text{for } b\neq 0 \\ xe^a & \text{otherwise} \end{cases} + \frac{3\left(\frac{\log\left(e^{ibx}-ie^{-id}\right)}{2}-\frac{\log\left(e^{ibx}+ie^{-id}\right)}{2}\right)e^ae^{-id}}{b}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d)**4,x)
```

output
```
(-15*I*exp(a)*exp(4*I*d)*exp(5*I*b*x) - 16*I*exp(a)*exp(2*I*d)*exp(3*I*b*x
) - 9*I*exp(a)*exp(I*b*x))/(3*b*exp(6*I*d)*exp(6*I*b*x) + 9*b*exp(4*I*d)*e
xp(4*I*b*x) + 9*b*exp(2*I*d)*exp(2*I*b*x) + 3*b) + Piecewise((-I*exp(a)*ex
p(I*b*x)/b, Ne(b, 0)), (x*exp(a), True)) + 3*(log(exp(I*b*x) - I*exp(-I*d)
)/2 - log(exp(I*b*x) + I*exp(-I*d))/2)*exp(a)*exp(-I*d)/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 763 vs. 2(109) = 218.

Time = 0.17 (sec) , antiderivative size = 763, normalized size of antiderivative = 4.29

$$\int e^{a+ibx}\tan^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d)^4,x, algorithm="maxima")
```

output
```
-(18*((cos(d)*e^a - I*e^a*sin(d))*cos(6*b*x + 6*d) + 3*(cos(d)*e^a - I*e^a
*sin(d))*cos(4*b*x + 4*d) + 3*(cos(d)*e^a - I*e^a*sin(d))*cos(2*b*x + 2*d)
 + cos(d)*e^a - (-I*cos(d)*e^a - e^a*sin(d))*sin(6*b*x + 6*d) - 3*(-I*cos(
d)*e^a - e^a*sin(d))*sin(4*b*x + 4*d) - 3*(-I*cos(d)*e^a - e^a*sin(d))*sin
(2*b*x + 2*d) - I*e^a*sin(d))*arctan2(2*(cos(b*x + 2*d)*cos(d) + sin(b*x +
 2*d)*sin(d))/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin
(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2), (cos(b*x + 2*d)^2 - c
os(d)^2 + sin(b*x + 2*d)^2 - sin(d)^2)/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*co
s(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^
2)) + 12*cos(7*b*x + 6*d)*e^a + 96*cos(5*b*x + 4*d)*e^a + 100*cos(3*b*x +
2*d)*e^a + 48*cos(b*x)*e^a - 9*((-I*cos(d)*e^a - e^a*sin(d))*cos(6*b*x + 6
*d) + 3*(-I*cos(d)*e^a - e^a*sin(d))*cos(4*b*x + 4*d) + 3*(-I*cos(d)*e^a -
 e^a*sin(d))*cos(2*b*x + 2*d) - I*cos(d)*e^a + (cos(d)*e^a - I*e^a*sin(d))
*sin(6*b*x + 6*d) + 3*(cos(d)*e^a - I*e^a*sin(d))*sin(4*b*x + 4*d) + 3*(co
s(d)*e^a - I*e^a*sin(d))*sin(2*b*x + 2*d) - e^a*sin(d))*log((cos(b*x + 2*d
)^2 + cos(d)^2 - 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 + 2*cos(b*x +
2*d)*sin(d) + sin(d)^2)/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x +
2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2)) + 12*I*e^a*
sin(7*b*x + 6*d) + 96*I*e^a*sin(5*b*x + 4*d) + 100*I*e^a*sin(3*b*x + 2*d)
+ 48*I*e^a*sin(b*x))/(-12*I*b*cos(6*b*x + 6*d) - 36*I*b*cos(4*b*x + 4*d...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(109) = 218$.

Time = 1.43 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.73

$$\int e^{a+ibx} \tan^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+d)^4,x, algorithm="giac")
```

output
```
1/96*(147*e^(6*I*b*x + a + 5*I*d)*log(I*e^(I*b*x + I*d) + 1) + 441*e^(4*I*
b*x + a + 3*I*d)*log(I*e^(I*b*x + I*d) + 1) + 441*e^(2*I*b*x + a + I*d)*lo
g(I*e^(I*b*x + I*d) + 1) + 147*e^(a - I*d)*log(I*e^(I*b*x + I*d) + 1) + 3*
e^(6*I*b*x + a + 5*I*d)*log(I*e^(I*b*x + I*d) - 1) + 9*e^(4*I*b*x + a + 3*
I*d)*log(I*e^(I*b*x + I*d) - 1) + 9*e^(2*I*b*x + a + I*d)*log(I*e^(I*b*x +
 I*d) - 1) + 3*e^(a - I*d)*log(I*e^(I*b*x + I*d) - 1) - 147*e^(6*I*b*x + a
 + 5*I*d)*log(-I*e^(I*b*x + I*d) + 1) - 441*e^(4*I*b*x + a + 3*I*d)*log(-I
*e^(I*b*x + I*d) + 1) - 441*e^(2*I*b*x + a + I*d)*log(-I*e^(I*b*x + I*d) +
 1) - 147*e^(a - I*d)*log(-I*e^(I*b*x + I*d) + 1) - 3*e^(6*I*b*x + a + 5*I
*d)*log(-I*e^(I*b*x + I*d) - 1) - 9*e^(4*I*b*x + a + 3*I*d)*log(-I*e^(I*b*
x + I*d) - 1) - 9*e^(2*I*b*x + a + I*d)*log(-I*e^(I*b*x + I*d) - 1) - 3*e^
(a - I*d)*log(-I*e^(I*b*x + I*d) - 1) - 96*I*e^(7*I*b*x + a + 6*I*d) - 768
*I*e^(5*I*b*x + a + 4*I*d) - 800*I*e^(3*I*b*x + a + 2*I*d) - 384*I*e^(I*b*
x + a))/(b*(e^(6*I*b*x + 6*I*d) + 3*e^(4*I*b*x + 4*I*d) + 3*e^(2*I*b*x + 2
*I*d) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\tan^4(d+bx)\,dx=\int \mathrm{e}^{a+b\,x\,1\mathrm{i}}\,\tan(d+b\,x)^4\,dx$$

input
```
int(exp(a + b*x*1i)*tan(d + b*x)^4,x)
```

output
```
int(exp(a + b*x*1i)*tan(d + b*x)^4, x)
```

Reduce [F]

$$\int e^{a+ibx}\tan^4(d+bx)\,dx=e^a\left(\int e^{bix}\tan(bx+d)^4\,dx\right)$$

input
```
int(exp(a+I*b*x)*tan(b*x+d)^4,x)
```

output
```
e**a*int(e**(b*i*x)*tan(b*x + d)**4,x)
```

3.45 $\int e^{2(a+ibx)}\tan(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int e^{2(a+ibx)}\tan(d+bx)\,dx=-\frac{e^{2(a-id)+2i(d+bx)}}{2b}+\frac{e^{2a-2id}\log\left(1+e^{2i(d+bx)}\right)}{b}$$

output
```
-1/2*exp(2*a-2*I*d+2*I*(b*x+d))/b+exp(2*a-2*I*d)*ln(1+exp(2*I*(b*x+d)))/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 139 vs. 2(58) = 116.

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.40

$$\int e^{2(a+ibx)}\tan(d+bx)\,dx=$$
$$-\frac{e^{2a}(\cos(d)-i\sin(d))\left(\cos(d)\left(e^{2ibx}-\log\left(1+e^{4ibx}+2e^{2ibx}\cos(2d)\right)\right)+\arctan\left(\frac{\left(-1+e^{2ibx}\right)\tan(d)}{1+e^{2ibx}}\right)(-2\ldots}{2b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Tan[d + b*x],x]
```

output
```
-1/2*(E^(2*a)*(Cos[d] - I*Sin[d])*(Cos[d]*(E^((2*I)*b*x) - Log[1 + E^((4*I
)*b*x) + 2*E^((2*I)*b*x)*Cos[2*d]]) + ArcTan[((-1 + E^((2*I)*b*x))*Tan[d])
/(1 + E^((2*I)*b*x))]*((-2*I)*Cos[d] - 2*Sin[d]) + I*(E^((2*I)*b*x) + Log[
1 + E^((4*I)*b*x) + 2*E^((2*I)*b*x)*Cos[2*d]])*Sin[d]))/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\tan(bx+d)\,dx$$
$$\Big\downarrow \text{4942}$$
$$i\int \left(\frac{2e^{2(a+ibx)}}{1+e^{2i(d+bx)}}-e^{2(a+ibx)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$i\left(\frac{ie^{2(a+ibx)}}{2b}-\frac{ie^{2a-2id}\log\left(1+e^{2i(bx+d)}\right)}{b}\right)$$

input
```
Int[E^(2*(a + I*b*x))*Tan[d + b*x],x]
```

output
```
I*(((I/2)*E^(2*(a + I*b*x)))/b - (I*E^(2*a - (2*I)*d)*Log[1 + E^((2*I)*(d
+ b*x))])/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{e^{2a} e^{2ibx}}{2b}+\frac{e^{2a} e^{-2id} \ln\left(1+e^{2i(bx+d)}\right)}{b}$	42

input
```
int(exp(2*a+2*I*b*x)*tan(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(2*a)*exp(2*I*b*x)/b+exp(2*a)/b*exp(-2*I*d)*ln(1+exp(2*I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int e^{2(a+ibx)} \tan(d+bx)\,dx = \frac{2\,e^{(2\,a-2i\,d)} \log\left(e^{(2i\,bx+2i\,d)}+1\right)-e^{(2i\,bx+2\,a)}}{2\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d),x, algorithm="fricas")
```

output
```
1/2*(2*e^(2*a - 2*I*d)*log(e^(2*I*b*x + 2*I*d) + 1) - e^(2*I*b*x + 2*a))/b
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int e^{2(a+ibx)}\tan(d+bx)\,dx=\begin{cases}-\frac{e^{2a}e^{2ibx}}{2b} & \text{for } b\neq 0\\ -ixe^{2a} & \text{otherwise}\end{cases}+\frac{e^{2a}e^{-2id}\log\left(e^{2ibx}+e^{-2id}\right)}{b}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d),x)
```

output
```
Piecewise((-exp(2*a)*exp(2*I*b*x)/(2*b), Ne(b, 0)), (-I*x*exp(2*a), True))
 + exp(2*a)*exp(-2*I*d)*log(exp(2*I*b*x) + exp(-2*I*d))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(39) = 78.

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.48

$$\int e^{2(a+ibx)}\tan(d+bx)\,dx$$
$$=\frac{2\left(i\cos(2d)e^{(2a)}+e^{(2a)}\sin(2d)\right)\arctan(\sin(2bx)-\sin(2d),\cos(2bx)+\cos(2d))-\cos(2bx)e^{(2a)}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d),x, algorithm="maxima")
```

output
```
1/2*(2*(I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*arctan2(sin(2*b*x) - sin(2*
d), cos(2*b*x) + cos(2*d)) - cos(2*b*x)*e^(2*a) + (cos(2*d)*e^(2*a) - I*e^
(2*a)*sin(2*d))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*d) + cos(2*d)^2 + si
n(2*b*x)^2 - 2*sin(2*b*x)*sin(2*d) + sin(2*d)^2) - I*e^(2*a)*sin(2*b*x))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int e^{2(a+ibx)}\tan(d+bx)\,dx = \frac{2\,e^{(2\,a-2i\,d)}\log\left(e^{(2i\,bx+2i\,d)}+1\right)-e^{(2i\,bx+2\,a)}}{2\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d),x, algorithm="giac")
```

output
```
1/2*(2*e^(2*a - 2*I*d)*log(e^(2*I*b*x + 2*I*d) + 1) - e^(2*I*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int e^{2(a+ibx)}\tan(d+bx)\,dx = -\frac{e^{2\,a}\,e^{b\,x\,2i}}{2\,b}+\frac{e^{2\,a}\,e^{-d\,2i}\,\ln\left(e^{2\,a}\,e^{b\,x\,2i}+e^{2\,a}\,e^{-d\,2i}\right)}{b}$$

input
```
int(exp(2*a + b*x*2i)*tan(d + b*x),x)
```

output
```
(exp(2*a)*exp(-d*2i)*log(exp(2*a)*exp(b*x*2i) + exp(2*a)*exp(-d*2i)))/b -
(exp(2*a)*exp(b*x*2i))/(2*b)
```

Reduce [F]

$$\int e^{2(a+ibx)}\tan(d+bx)\,dx = e^{2a}\left(\int e^{2bix}\tan\left(bx+d\right)dx\right)$$

input
```
int(exp(2*a+2*I*b*x)*tan(b*x+d),x)
```

output
```
e**(2*a)*int(e**(2*b*i*x)*tan(b*x + d),x)
```

3.46 $\int e^{2(a+ibx)} \tan^2(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int e^{2(a+ibx)} \tan^2(d+bx)\,dx = \frac{ie^{2(a-id)+2i(d+bx)}}{2b} - \frac{2ie^{2a-2id}}{b\left(1+e^{2i(d+bx)}\right)} - \frac{2ie^{2a-2id}\log\left(1+e^{2i(d+bx)}\right)}{b}$$

output
```
1/2*I*exp(2*a-2*I*d+2*I*(b*x+d))/b-2*I*exp(2*a-2*I*d)/b/(1+exp(2*I*(b*x+d)
))-2*I*exp(2*a-2*I*d)*ln(1+exp(2*I*(b*x+d)))/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 201 vs. 2(96) = 192.

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.09

$$\int e^{2(a+ibx)} \tan^2(d+bx)\,dx = \frac{e^{2a}\left(ie^{2ibx} + 4\arctan\left(\frac{\left(-1+e^{2ibx}\right)\tan(d)}{1+e^{2ibx}}\right)\cos(2d) - 2i\cos(2d)\log\left(1+e^{4ibx}+2e^{2ibx}\cos(2d)\right) + \frac{4(\cos(}{i\left(1+e^{2ibx}\right)\cos(}\right.}{2b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Tan[d + b*x]^2,x]
```

output
```
(E^(2*a)*(I*E^((2*I)*b*x) + 4*ArcTan[((-1 + E^((2*I)*b*x))*Tan[d])/(1 + E^
((2*I)*b*x))]*Cos[2*d] - (2*I)*Cos[2*d]*Log[1 + E^((4*I)*b*x) + 2*E^((2*I)
*b*x)*Cos[2*d]] + (4*(Cos[d] - I*Sin[d])^3)/(I*(1 + E^((2*I)*b*x))*Cos[d]
- (-1 + E^((2*I)*b*x))*Sin[d]) - (4*I)*ArcTan[((-1 + E^((2*I)*b*x))*Tan[d]
)/(1 + E^((2*I)*b*x))]*Sin[2*d] - 2*Log[1 + E^((4*I)*b*x) + 2*E^((2*I)*b*x
)*Cos[2*d]]*Sin[2*d]))/(2*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\tan^2(bx+d)\,dx$$

$$\Big\downarrow \; 4942$$

$$-\int \left(e^{2(a+ibx)}-\frac{4e^{2(a+ibx)}}{1+e^{2i(d+bx)}}+\frac{4e^{2(a+ibx)}}{\left(1+e^{2i(d+bx)}\right)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2ie^{2a-2id}}{b\left(1+e^{2i(bx+d)}\right)}-\frac{2ie^{2a-2id}\log\left(1+e^{2i(bx+d)}\right)}{b}+\frac{ie^{2(a+ibx)}}{2b}$$

input
```
Int[E^(2*(a + I*b*x))*Tan[d + b*x]^2,x]
```

output
```
((I/2)*E^(2*(a + I*b*x)))/b - ((2*I)*E^(2*a - (2*I)*d))/(b*(1 + E^((2*I)*(
d + b*x)))) - ((2*I)*E^(2*a - (2*I)*d)*Log[1 + E^((2*I)*(d + b*x))])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{i \mathrm{e}^{2 a} \mathrm{e}^{2 i b x}}{2 b}+\frac{2 i \mathrm{e}^{2 a} \mathrm{e}^{2 i b x}}{b \left(1+\mathrm{e}^{2 i \left(b x +d \right)}\right)}-\frac{2 i \mathrm{e}^{2 a} \mathrm{e}^{-2 i d} \ln \left(1+\mathrm{e}^{2 i \left(b x +d \right)}\right)}{b}$	74

input
```
int(exp(2*a+2*I*b*x)*tan(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b*exp(2*a)*exp(2*I*b*x)+2*I*exp(2*a)*exp(2*I*b*x)/b/(1+exp(2*I*(b*x+
d)))-2*I*exp(2*a)/b*exp(-2*I*d)*ln(1+exp(2*I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

$$\int e^{2(a+ibx)}\tan^2(d+bx)\,dx = -\frac{4\left(i\,e^{(2i\,bx+2\,a)}+i\,e^{(2\,a-2i\,d)}\right)\log\left(e^{(2i\,bx+2i\,d)}+1\right)-i\,e^{(4i\,bx+2\,a+2i\,d)}-i\,e^{(2i\,bx+2\,a)}+4i\,e^{(2\,a-2i\,d)}}{2\left(be^{(2i\,bx+2i\,d)}+b\right)}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d)^2,x, algorithm="fricas")
```

output
```
-1/2*(4*(I*e^(2*I*b*x + 2*a) + I*e^(2*a - 2*I*d))*log(e^(2*I*b*x + 2*I*d)
+ 1) - I*e^(4*I*b*x + 2*a + 2*I*d) - I*e^(2*I*b*x + 2*a) + 4*I*e^(2*a - 2*
I*d))/(b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

$$\int e^{2(a+ibx)}\tan^2(d+bx)\,dx = \begin{cases} \frac{ie^{2a}e^{2ibx}}{2b} & \text{for } b \neq 0 \\ -xe^{2a} & \text{otherwise} \end{cases} - \frac{2ie^{2a}}{be^{4id}e^{2ibx} + be^{2id}} - \frac{2ie^{2a}e^{-2id}\log\left(e^{2ibx} + e^{-2id}\right)}{b}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d)**2,x)
```

output
```
Piecewise((I*exp(2*a)*exp(2*I*b*x)/(2*b), Ne(b, 0)), (-x*exp(2*a), True))
- 2*I*exp(2*a)/(b*exp(4*I*d)*exp(2*I*b*x) + b*exp(2*I*d)) - 2*I*exp(2*a)*e
xp(-2*I*d)*log(exp(2*I*b*x) + exp(-2*I*d))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(66) = 132.

Time = 0.05 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.70

$$\int e^{2(a+ibx)}\tan^2(d+bx)\,dx = \frac{4\left(-i\cos(2d)^2e^{(2a)} - ie^{(2a)}\sin(2d)^2 + \left(-i\cos(2d)e^{(2a)} - e^{(2a)}\sin(2d)\right)\cos(2bx+4d) + \left(\cos(2d)\right.\right.}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d)^2,x, algorithm="maxima")
```

output
```
(4*(-I*cos(2*d)^2*e^(2*a) - I*e^(2*a)*sin(2*d)^2 + (-I*cos(2*d)*e^(2*a) -
e^(2*a)*sin(2*d))*cos(2*b*x + 4*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d
))*sin(2*b*x + 4*d))*arctan2(sin(2*b*x) - sin(2*d), cos(2*b*x) + cos(2*d))
 + cos(4*b*x + 4*d)*e^(2*a) + cos(2*b*x + 2*d)*e^(2*a) - 2*(cos(2*d)^2*e^(
2*a) + e^(2*a)*sin(2*d)^2 + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(2*
b*x + 4*d) - (-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*sin(2*b*x + 4*d))*lo
g(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*d) + cos(2*d)^2 + sin(2*b*x)^2 - 2*sin
(2*b*x)*sin(2*d) + sin(2*d)^2) + I*e^(2*a)*sin(4*b*x + 4*d) + I*e^(2*a)*si
n(2*b*x + 2*d) - 4*e^(2*a))/(-2*I*b*cos(2*b*x + 4*d) - 2*I*b*cos(2*d) + 2*
b*sin(2*b*x + 4*d) + 2*b*sin(2*d))
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03

$$\int e^{2(a+ibx)}\tan^2(d+bx)\,dx = \frac{-4i\,e^{(2i\,bx+2\,a)}\log\left(e^{(2i\,bx+2i\,d)}+1\right)-4i\,e^{(2\,a-2i\,d)}\log\left(e^{(2i\,bx+2i\,d)}+1\right)+i\,e^{(4i\,bx+2\,a+2i\,d)}+i\,e^{(2i\,bx+2\,a)}-4i}{2\,b(e^{(2i\,bx+2i\,d)}+1)}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d)^2,x, algorithm="giac")
```

output
```
1/2*(-4*I*e^(2*I*b*x + 2*a)*log(e^(2*I*b*x + 2*I*d) + 1) - 4*I*e^(2*a - 2*
I*d)*log(e^(2*I*b*x + 2*I*d) + 1) + I*e^(4*I*b*x + 2*a + 2*I*d) + I*e^(2*I
*b*x + 2*a) - 4*I*e^(2*a - 2*I*d))/(b*(e^(2*I*b*x + 2*I*d) + 1))
```

Mupad [B] (verification not implemented)

Time = 16.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int e^{2(a+ibx)}\tan^2(d+bx)\,dx = \frac{\mathrm{e}^{2\,a+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2\,b}-\frac{\mathrm{e}^{2\,a-d\,2\mathrm{i}}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+\mathrm{e}^{2\,a}\,\mathrm{e}^{-d\,2\mathrm{i}}\right)\,2\mathrm{i}}{b} - \frac{\mathrm{e}^{4\,a-d\,4\mathrm{i}}\,2\mathrm{i}}{b\,\left(\mathrm{e}^{2\,a-d\,2\mathrm{i}}+\mathrm{e}^{2\,a+b\,x\,2\mathrm{i}}\right)}$$

input
```
int(exp(2*a + b*x*2i)*tan(d + b*x)^2,x)
```

output
```
(exp(2*a + b*x*2i)*1i)/(2*b) - (exp(2*a - d*2i)*log(exp(2*a)*exp(b*x*2i) +
 exp(2*a)*exp(-d*2i))*2i)/b - (exp(4*a - d*4i)*2i)/(b*(exp(2*a - d*2i) + e
xp(2*a + b*x*2i)))
```

Reduce [F]

$$\int e^{2(a+ibx)}\tan^2(d+bx)\,dx = e^{2a}\left(\int e^{2bix}\tan{(bx+d)}^2\,dx\right)$$

input
```
int(exp(2*a+2*I*b*x)*tan(b*x+d)^2,x)
```

output
```
e**(2*a)*int(e**(2*b*i*x)*tan(b*x + d)**2,x)
```

3.47 $\int e^{2(a+ibx)} \tan^3(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int e^{2(a+ibx)} \tan^3(d+bx)\,dx = \frac{e^{2(a-id)+2i(d+bx)}}{2b} + \frac{2e^{2a-2id}}{b\left(1+e^{2i(d+bx)}\right)^2} - \frac{6e^{2a-2id}}{b\left(1+e^{2i(d+bx)}\right)} - \frac{3e^{2a-2id}\log\left(1+e^{2i(d+bx)}\right)}{b}$$

output
```
1/2*exp(2*a-2*I*d+2*I*(b*x+d))/b+2*exp(2*a-2*I*d)/b/(1+exp(2*I*(b*x+d)))^2
-6*exp(2*a-2*I*d)/b/(1+exp(2*I*(b*x+d)))-3*exp(2*a-2*I*d)*ln(1+exp(2*I*(b*
x+d)))/b
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.99

$$\int e^{2(a+ibx)} \tan^3(d+bx)\,dx$$
$$= \frac{e^{2a}\left(e^{2ibx} - 6i\arctan\left(\frac{(-1+e^{2ibx})\tan(d)}{1+e^{2ibx}}\right)\cos(2d) - 3\cos(2d)\log\left(1+e^{4ibx}+2e^{2ibx}\cos(2d)\right) + \frac{4(\cos}{((1+e^{2ibx})\cos(}\right.}{}$$

input
```
Integrate[E^(2*(a + I*b*x))*Tan[d + b*x]^3,x]
```

output
```
(E^(2*a)*(E^((2*I)*b*x) - (6*I)*ArcTan[((-1 + E^((2*I)*b*x))*Tan[d])/(1 +
E^((2*I)*b*x))]*Cos[2*d] - 3*Cos[2*d]*Log[1 + E^((4*I)*b*x) + 2*E^((2*I)*b
*x)*Cos[2*d]] + (4*(Cos[d] - I*Sin[d])^4)/((1 + E^((2*I)*b*x))*Cos[d] + I*
(-1 + E^((2*I)*b*x))*Sin[d])^2 - (12*(Cos[d] - I*Sin[d])^3)/((1 + E^((2*I)
*b*x))*Cos[d] + I*(-1 + E^((2*I)*b*x))*Sin[d]) - 6*ArcTan[((-1 + E^((2*I)*
b*x))*Tan[d])/(1 + E^((2*I)*b*x))]*Sin[2*d] + (3*I)*Log[1 + E^((4*I)*b*x)
+ 2*E^((2*I)*b*x)*Cos[2*d]]*Sin[2*d]))/(2*b)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\tan^3(bx+d)\,dx$$

$$\Big\downarrow \text{4942}$$

$$-i\int\left(-e^{2(a+ibx)}+\frac{6e^{2(a+ibx)}}{1+e^{2i(d+bx)}}-\frac{12e^{2(a+ibx)}}{\left(1+e^{2i(d+bx)}\right)^2}+\frac{8e^{2(a+ibx)}}{\left(1+e^{2i(d+bx)}\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{6ie^{2a-2id}}{b\left(1+e^{2i(bx+d)}\right)}+\frac{2ie^{2a-2id}}{b\left(1+e^{2i(bx+d)}\right)^2}-\frac{3ie^{2a-2id}\log\left(1+e^{2i(bx+d)}\right)}{b}+\frac{ie^{2(a+ibx)}}{2b}\right)$$

input
```
Int[E^(2*(a + I*b*x))*Tan[d + b*x]^3,x]
```

output
```
(-I)*(((I/2)*E^(2*(a + I*b*x)))/b + ((2*I)*E^(2*a - (2*I)*d))/(b*(1 + E^((
2*I)*(d + b*x)))^2) - ((6*I)*E^(2*a - (2*I)*d))/(b*(1 + E^((2*I)*(d + b*x)
))) - ((3*I)*E^(2*a - (2*I)*d)*Log[1 + E^((2*I)*(d + b*x))])/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{e^{2a} e^{2ibx}}{2b}+\frac{4 e^{4ibx} e^{2id} e^{2a}+2 e^{2a} e^{2ibx}}{b\left(1+e^{2i(bx+d)}\right)^{2}}-\frac{3 e^{2a} e^{-2id} \ln\left(1+e^{2i(bx+d)}\right)}{b}$	90

input
```
int(exp(2*a+2*I*b*x)*tan(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*a)*exp(2*I*b*x)/b+2/b/(1+exp(2*I*(b*x+d)))^2*(2*exp(4*I*b*x)*exp
(2*I*d)*exp(2*a)+exp(2*a)*exp(2*I*b*x))-3*exp(2*a)/b*exp(-2*I*d)*ln(1+exp(
2*I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03

$$\int e^{2(a+ibx)} \tan^3(d+bx)\,dx = -\frac{6\left(e^{(4i\,bx+2\,a+2i\,d)}+2\,e^{(2i\,bx+2\,a)}+e^{(2\,a-2i\,d)}\right)\log\left(e^{(2i\,bx+2i\,d)}+1\right)-e^{(6i\,bx+2\,a+4i\,d)}-2\,e^{(4i\,bx+2\,a+2i\,d)}+11}{2\left(be^{(4i\,bx+4i\,d)}+2\,be^{(2i\,bx+2i\,d)}+b\right)}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d)^3,x, algorithm="fricas")
```

output
```
-1/2*(6*(e^(4*I*b*x + 2*a + 2*I*d) + 2*e^(2*I*b*x + 2*a) + e^(2*a - 2*I*d)
)*log(e^(2*I*b*x + 2*I*d) + 1) - e^(6*I*b*x + 2*a + 4*I*d) - 2*e^(4*I*b*x
+ 2*a + 2*I*d) + 11*e^(2*I*b*x + 2*a) + 8*e^(2*a - 2*I*d))/(b*e^(4*I*b*x +
 4*I*d) + 2*b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10

$$\int e^{2(a+ibx)}\tan^3(d+bx)\,dx = \frac{-6e^{2a}e^{2id}e^{2ibx}-4e^{2a}}{be^{6id}e^{4ibx}+2be^{4id}e^{2ibx}+be^{2id}} + \begin{cases} \frac{e^{2a}e^{2ibx}}{2b} & \text{for } b\neq 0 \\ ixe^{2a} & \text{otherwise} \end{cases} - \frac{3e^{2a}e^{-2id}\log\left(e^{2ibx}+e^{-2id}\right)}{b}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d)**3,x)
```

output
```
(-6*exp(2*a)*exp(2*I*d)*exp(2*I*b*x) - 4*exp(2*a))/(b*exp(6*I*d)*exp(4*I*b
*x) + 2*b*exp(4*I*d)*exp(2*I*b*x) + b*exp(2*I*d)) + Piecewise((exp(2*a)*ex
p(2*I*b*x)/(2*b), Ne(b, 0)), (I*x*exp(2*a), True)) - 3*exp(2*a)*exp(-2*I*d
)*log(exp(2*I*b*x) + exp(-2*I*d))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(92) = 184.

Time = 0.06 (sec) , antiderivative size = 535, normalized size of antiderivative = 4.42

$$\int e^{2(a+ibx)}\tan^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d)^3,x, algorithm="maxima")
```

output
```
-(6*(cos(2*d)^2*e^(2*a) + e^(2*a)*sin(2*d)^2 + (cos(2*d)*e^(2*a) - I*e^(2*
a)*sin(2*d))*cos(4*b*x + 6*d) + 2*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*
cos(2*b*x + 4*d) + (I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*sin(4*b*x + 6*d
) + 2*(I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*sin(2*b*x + 4*d))*arctan2(si
n(2*b*x) - sin(2*d), cos(2*b*x) + cos(2*d)) + I*cos(6*b*x + 6*d)*e^(2*a) +
 2*I*cos(4*b*x + 4*d)*e^(2*a) - 11*I*cos(2*b*x + 2*d)*e^(2*a) + 3*(-I*cos(
2*d)^2*e^(2*a) - I*e^(2*a)*sin(2*d)^2 + (-I*cos(2*d)*e^(2*a) - e^(2*a)*sin
(2*d))*cos(4*b*x + 6*d) + 2*(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*cos(2
*b*x + 4*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(4*b*x + 6*d) + 2
*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(2*b*x + 4*d))*log(cos(2*b*x)^
2 + 2*cos(2*b*x)*cos(2*d) + cos(2*d)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2
*d) + sin(2*d)^2) - e^(2*a)*sin(6*b*x + 6*d) - 2*e^(2*a)*sin(4*b*x + 4*d)
+ 11*e^(2*a)*sin(2*b*x + 2*d) - 8*I*e^(2*a))/(-2*I*b*cos(4*b*x + 6*d) - 4*
I*b*cos(2*b*x + 4*d) - 2*I*b*cos(2*d) + 2*b*sin(4*b*x + 6*d) + 4*b*sin(2*b
*x + 4*d) + 2*b*sin(2*d))
```

Giac [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

$$\int e^{2(a+ibx)}\tan^3(d+bx)\,dx = -\frac{6\,e^{(4i\,bx+2\,a+2i\,d)}\log\left(e^{(2i\,bx+2i\,d)}+1\right)+12\,e^{(2i\,bx+2\,a)}\log\left(e^{(2i\,bx+2i\,d)}+1\right)+6\,e^{(2\,a-2i\,d)}\log\left(e^{(2i\,bx+2i\,d)}+1\right.}{2\,b(e^{(4i\,bx+4i\,d)}+2\,e^{(2i\,bx+2i\,d)}+1)}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/2*(6*e^(4*I*b*x + 2*a + 2*I*d)*log(e^(2*I*b*x + 2*I*d) + 1) + 12*e^(2*I
*b*x + 2*a)*log(e^(2*I*b*x + 2*I*d) + 1) + 6*e^(2*a - 2*I*d)*log(e^(2*I*b*
x + 2*I*d) + 1) - e^(6*I*b*x + 2*a + 4*I*d) - 2*e^(4*I*b*x + 2*a + 2*I*d)
+ 11*e^(2*I*b*x + 2*a) + 8*e^(2*a - 2*I*d))/(b*(e^(4*I*b*x + 4*I*d) + 2*e^
(2*I*b*x + 2*I*d) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+ibx)}\tan^3(d+bx)\,dx = \int e^{2\,a+b\,x\,2\mathrm{i}}\tan(d+b\,x)^3\,dx$$

input
```
int(exp(2*a + b*x*2i)*tan(d + b*x)^3,x)
```

output
```
int(exp(2*a + b*x*2i)*tan(d + b*x)^3, x)
```

Reduce [F]

$$\int e^{2(a+ibx)}\tan^3(d+bx)\,dx = e^{2a}\left(\int e^{2bix}\tan(bx+d)^3\,dx\right)$$

input
```
int(exp(2*a+2*I*b*x)*tan(b*x+d)^3,x)
```

output
```
e**(2*a)*int(e**(2*b*i*x)*tan(b*x + d)**3,x)
```

3.48 $\int e^{2(a+ibx)} \tan^4(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 164

$$\int e^{2(a+ibx)} \tan^4(d+bx)\,dx = -\frac{ie^{2(a-id)+2i(d+bx)}}{2b} + \frac{8ie^{2a-2id}}{3b\left(1+e^{2i(d+bx)}\right)^3} - \frac{8ie^{2a-2id}}{b\left(1+e^{2i(d+bx)}\right)^2} + \frac{12ie^{2a-2id}}{b\left(1+e^{2i(d+bx)}\right)} + \frac{4ie^{2a-2id}\log\left(1+e^{2i(d+bx)}\right)}{b}$$

output

```
-1/2*I*exp(2*a-2*I*d+2*I*(b*x+d))/b+8/3*I*exp(2*a-2*I*d)/b/(1+exp(2*I*(b*x
+d)))^3-8*I*exp(2*a-2*I*d)/b/(1+exp(2*I*(b*x+d)))^2+12*I*exp(2*a-2*I*d)/b/
(1+exp(2*I*(b*x+d)))+4*I*exp(2*a-2*I*d)*ln(1+exp(2*I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.91

$$\int e^{2(a+ibx)}\tan^4(d+bx)\,dx = \frac{1}{6}e^{2a}\left(-\frac{3ie^{2ibx}}{b}-\frac{24\arctan\left(\frac{(-1+e^{2ibx})\tan(d)}{1+e^{2ibx}}\right)\cos(2d)}{b}+\frac{12i\cos(2d)\log\left(1+e^{4ibx}+2e^{2ibx}\cos(2d)\right)}{b}+\frac{16(\cos(d)-i\sin(d))^5}{b\left(i\left(1+e^{2ibx}\right)\cos(d)-\left(-1+e^{2ibx}\right)\sin(d)\right)^3}-\frac{48i(\cos(d)-i\sin(d))^4}{b\left(\left(1+e^{2ibx}\right)\cos(d)+i\left(-1+e^{2ibx}\right)\sin(d)\right)^2}+\frac{72(\cos(d)-i\sin(d))^3}{-ib\left(1+e^{2ibx}\right)\cos(d)+b\left(-1+e^{2ibx}\right)\sin(d)}+\frac{24i\arctan\left(\frac{(-1+e^{2ibx})\tan(d)}{1+e^{2ibx}}\right)\sin(2d)}{b}+\frac{12\log\left(1+e^{4ibx}+2e^{2ibx}\cos(2d)\right)\sin(2d)}{b}\right)$$

input
```
Integrate[E^(2*(a + I*b*x))*Tan[d + b*x]^4,x]
```

output
```
(E^(2*a)*(((-3*I)*E^((2*I)*b*x))/b - (24*ArcTan[((-1 + E^((2*I)*b*x))*Tan[
d])/(1 + E^((2*I)*b*x))]*Cos[2*d])/b + ((12*I)*Cos[2*d]*Log[1 + E^((4*I)*b
*x) + 2*E^((2*I)*b*x)*Cos[2*d]])/b + (16*(Cos[d] - I*Sin[d])^5)/(b*(I*(1 +
 E^((2*I)*b*x))*Cos[d] - (-1 + E^((2*I)*b*x))*Sin[d])^3) - ((48*I)*(Cos[d]
 - I*Sin[d])^4)/(b*((1 + E^((2*I)*b*x))*Cos[d] + I*(-1 + E^((2*I)*b*x))*Si
n[d])^2) + (72*(Cos[d] - I*Sin[d])^3)/((-I)*b*(1 + E^((2*I)*b*x))*Cos[d] +
 b*(-1 + E^((2*I)*b*x))*Sin[d]) + ((24*I)*ArcTan[((-1 + E^((2*I)*b*x))*Tan
[d])/(1 + E^((2*I)*b*x))]*Sin[2*d])/b + (12*Log[1 + E^((4*I)*b*x) + 2*E^((
2*I)*b*x)*Cos[2*d]]*Sin[2*d])/b))/6
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\tan^4(bx+d)\,dx$$

$$\Big\downarrow \text{4942}$$

$$\int \left(-\frac{8e^{2(a+ibx)}}{1+e^{2i(bx+d)}}+\frac{24e^{2(a+ibx)}}{\left(1+e^{2i(bx+d)}\right)^2}-\frac{32e^{2(a+ibx)}}{\left(1+e^{2i(bx+d)}\right)^3}+\frac{16e^{2(a+ibx)}}{\left(1+e^{2i(bx+d)}\right)^4}+e^{2(a+ibx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{12ie^{2a-2id}}{b\left(1+e^{2i(bx+d)}\right)}-\frac{8ie^{2a-2id}}{b\left(1+e^{2i(bx+d)}\right)^2}+\frac{8ie^{2a-2id}}{3b\left(1+e^{2i(bx+d)}\right)^3}+\frac{4ie^{2a-2id}\log\left(1+e^{2i(bx+d)}\right)}{b}-\frac{ie^{2(a+ibx)}}{2b}$$

input
```
Int[E^(2*(a + I*b*x))*Tan[d + b*x]^4,x]
```

output
```
((-1/2*I)*E^(2*(a + I*b*x)))/b + (((8*I)/3)*E^(2*a - (2*I)*d))/(b*(1 + E^(
(2*I)*(d + b*x)))^3) - ((8*I)*E^(2*a - (2*I)*d))/(b*(1 + E^((2*I)*(d + b*x
)))^2) + ((12*I)*E^(2*a - (2*I)*d))/(b*(1 + E^((2*I)*(d + b*x)))) + ((4*I)
*E^(2*a - (2*I)*d)*Log[1 + E^((2*I)*(d + b*x))])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{i e^{2a} e^{2ibx}}{2b}-\frac{4i\left(5\,e^{6ibx} e^{4id} e^{2a}+6\,e^{4ibx} e^{2id} e^{2a}+3\,e^{2a} e^{2ibx}\right)}{3b\left(1+e^{2i(bx+d)}\right)^{3}}+\frac{4i e^{2a} e^{-2id}\ln\left(1+e^{2i(bx+d)}\right)}{b}$	111

input
```
int(exp(2*a+2*I*b*x)*tan(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b*exp(2*a)*exp(2*I*b*x)-4/3*I/b/(1+exp(2*I*(b*x+d)))^3*(5*exp(6*I*b
*x)*exp(4*I*d)*exp(2*a)+6*exp(4*I*b*x)*exp(2*I*d)*exp(2*a)+3*exp(2*a)*exp(
2*I*b*x))+4*I*exp(2*a)/b*exp(-2*I*d)*ln(1+exp(2*I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.03

$$\int e^{2(a+ibx)}\tan^4(d+bx)\,dx = -\frac{24\left(-i\,e^{(6i\,bx+2\,a+4i\,d)}-3i\,e^{(4i\,bx+2\,a+2i\,d)}-3i\,e^{(2i\,bx+2\,a)}-i\,e^{(2\,a-2i\,d)}\right)\log\left(e^{(2i\,bx+2i\,d)}+1\right)+3i\,e^{(8i\,bx+2\,a}}{6\left(be^{(6i\,bx+6i\,d)}+3\,be^{(4i\,bx+4i\,d)}+3\,be^{(2i\,b}}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d)^4,x, algorithm="fricas")
```

output
```
-1/6*(24*(-I*e^(6*I*b*x + 2*a + 4*I*d) - 3*I*e^(4*I*b*x + 2*a + 2*I*d) - 3
*I*e^(2*I*b*x + 2*a) - I*e^(2*a - 2*I*d))*log(e^(2*I*b*x + 2*I*d) + 1) + 3
*I*e^(8*I*b*x + 2*a + 6*I*d) + 9*I*e^(6*I*b*x + 2*a + 4*I*d) - 63*I*e^(4*I
*b*x + 2*a + 2*I*d) - 93*I*e^(2*I*b*x + 2*a) - 40*I*e^(2*a - 2*I*d))/(b*e^
(6*I*b*x + 6*I*d) + 3*b*e^(4*I*b*x + 4*I*d) + 3*b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.12

$$\int e^{2(a+ibx)}\tan^4(d+bx)\,dx = \frac{36ie^{2a}e^{4id}e^{4ibx}+48ie^{2a}e^{2id}e^{2ibx}+20ie^{2a}}{3be^{8id}e^{6ibx}+9be^{6id}e^{4ibx}+9be^{4id}e^{2ibx}+3be^{2id}} + \begin{cases} -\frac{ie^{2a}e^{2ibx}}{2b} & \text{for } b\neq 0 \\ xe^{2a} & \text{otherwise} \end{cases} + \frac{4ie^{2a}e^{-2id}\log\left(e^{2ibx}+e^{-2id}\right)}{b}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d)**4,x)
```

output
```
(36*I*exp(2*a)*exp(4*I*d)*exp(4*I*b*x) + 48*I*exp(2*a)*exp(2*I*d)*exp(2*I*
b*x) + 20*I*exp(2*a))/(3*b*exp(8*I*d)*exp(6*I*b*x) + 9*b*exp(6*I*d)*exp(4*
I*b*x) + 9*b*exp(4*I*d)*exp(2*I*b*x) + 3*b*exp(2*I*d)) + Piecewise((-I*exp
(2*a)*exp(2*I*b*x)/(2*b), Ne(b, 0)), (x*exp(2*a), True)) + 4*I*exp(2*a)*ex
p(-2*I*d)*log(exp(2*I*b*x) + exp(-2*I*d))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 718 vs. 2(118) = 236.

Time = 0.09 (sec) , antiderivative size = 718, normalized size of antiderivative = 4.38

$$\int e^{2(a+ibx)}\tan^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d)^4,x, algorithm="maxima")
```

output
```
-(24*(-I*cos(2*d)^2*e^(2*a) - I*e^(2*a)*sin(2*d)^2 + (-I*cos(2*d)*e^(2*a)
- e^(2*a)*sin(2*d))*cos(6*b*x + 8*d) + 3*(-I*cos(2*d)*e^(2*a) - e^(2*a)*si
n(2*d))*cos(4*b*x + 6*d) + 3*(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*cos(
2*b*x + 4*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(6*b*x + 8*d) +
3*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(4*b*x + 6*d) + 3*(cos(2*d)*e
^(2*a) - I*e^(2*a)*sin(2*d))*sin(2*b*x + 4*d))*arctan2(sin(2*b*x) - sin(2*
d), cos(2*b*x) + cos(2*d)) + 3*cos(8*b*x + 8*d)*e^(2*a) + 9*cos(6*b*x + 6*
d)*e^(2*a) - 63*cos(4*b*x + 4*d)*e^(2*a) - 93*cos(2*b*x + 2*d)*e^(2*a) - 1
2*(cos(2*d)^2*e^(2*a) + e^(2*a)*sin(2*d)^2 + (cos(2*d)*e^(2*a) - I*e^(2*a)
*sin(2*d))*cos(6*b*x + 8*d) + 3*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*co
s(4*b*x + 6*d) + 3*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(2*b*x + 4*d
) - (-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*sin(6*b*x + 8*d) - 3*(-I*cos(
2*d)*e^(2*a) - e^(2*a)*sin(2*d))*sin(4*b*x + 6*d) - 3*(-I*cos(2*d)*e^(2*a)
 - e^(2*a)*sin(2*d))*sin(2*b*x + 4*d))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos
(2*d) + cos(2*d)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*d) + sin(2*d)^2) +
3*I*e^(2*a)*sin(8*b*x + 8*d) + 9*I*e^(2*a)*sin(6*b*x + 6*d) - 63*I*e^(2*a)
*sin(4*b*x + 4*d) - 93*I*e^(2*a)*sin(2*b*x + 2*d) - 40*e^(2*a))/(-6*I*b*co
s(6*b*x + 8*d) - 18*I*b*cos(4*b*x + 6*d) - 18*I*b*cos(2*b*x + 4*d) - 6*I*b
*cos(2*d) + 6*b*sin(6*b*x + 8*d) + 18*b*sin(4*b*x + 6*d) + 18*b*sin(2*b*x
+ 4*d) + 6*b*sin(2*d))
```

Giac [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.23

$$\int e^{2(a+ibx)}\tan^4(d+bx)\,dx$$
$$= \frac{24i\,e^{(6i\,bx+2\,a+4i\,d)}\log\left(e^{(2i\,bx+2i\,d)}+1\right)+72i\,e^{(4i\,bx+2\,a+2i\,d)}\log\left(e^{(2i\,bx+2i\,d)}+1\right)+72i\,e^{(2i\,bx+2\,a)}\log\left(e^{(2i\,bx+}\right.}{6\,b(e^{(6i\,bx+}}$$

input
```
integrate(exp(2*a+2*I*b*x)*tan(b*x+d)^4,x, algorithm="giac")
```

output
```
1/6*(24*I*e^(6*I*b*x + 2*a + 4*I*d)*log(e^(2*I*b*x + 2*I*d) + 1) + 72*I*e^
(4*I*b*x + 2*a + 2*I*d)*log(e^(2*I*b*x + 2*I*d) + 1) + 72*I*e^(2*I*b*x + 2
*a)*log(e^(2*I*b*x + 2*I*d) + 1) + 24*I*e^(2*a - 2*I*d)*log(e^(2*I*b*x + 2
*I*d) + 1) - 3*I*e^(8*I*b*x + 2*a + 6*I*d) - 9*I*e^(6*I*b*x + 2*a + 4*I*d)
 + 63*I*e^(4*I*b*x + 2*a + 2*I*d) + 93*I*e^(2*I*b*x + 2*a) + 40*I*e^(2*a -
 2*I*d))/(b*(e^(6*I*b*x + 6*I*d) + 3*e^(4*I*b*x + 4*I*d) + 3*e^(2*I*b*x +
2*I*d) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+ibx)} \tan^4(d+bx)\,dx = \int e^{2\,a+b\,x\,2\mathrm{i}} \tan(d+b\,x)^4\,dx$$

input
```
int(exp(2*a + b*x*2i)*tan(d + b*x)^4,x)
```

output
```
int(exp(2*a + b*x*2i)*tan(d + b*x)^4, x)
```

Reduce [F]

$$\int e^{2(a+ibx)} \tan^4(d+bx)\,dx = e^{2a}\left(\int e^{2bix} \tan(bx+d)^4\,dx\right)$$

input
```
int(exp(2*a+2*I*b*x)*tan(b*x+d)^4,x)
```

output
```
e**(2*a)*int(e**(2*b*i*x)*tan(b*x + d)**4,x)
```

3.49 $\int e^{\frac{5}{3}(a+ibx)}\tan(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 204

$$\int e^{\frac{5}{3}(a+ibx)}\tan(d+bx)\,dx = -\frac{3e^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{5b}+\frac{2e^{\frac{5}{3}(a-id)}\arctan\left(e^{\frac{1}{3}i(d+bx)}\right)}{b} - \frac{e^{\frac{5}{3}(a-id)}\arctan\left(\sqrt{3}-2e^{\frac{1}{3}i(d+bx)}\right)}{b} + \frac{e^{\frac{5}{3}(a-id)}\arctan\left(\sqrt{3}+2e^{\frac{1}{3}i(d+bx)}\right)}{b} - \frac{\sqrt{3}e^{\frac{5}{3}(a-id)}\operatorname{arctanh}\left(\frac{\sqrt{3}e^{\frac{1}{3}i(d+bx)}}{1+e^{\frac{2}{3}i(d+bx)}}\right)}{b}$$

output
```
-3/5*exp(5/3*a-5/3*I*d+5/3*I*(b*x+d))/b+2*exp(5/3*a-5/3*I*d)*arctan(exp(1/
3*I*(b*x+d)))/b+exp(5/3*a-5/3*I*d)*arctan(-3^(1/2)+2*exp(1/3*I*(b*x+d)))/b
+exp(5/3*a-5/3*I*d)*arctan(3^(1/2)+2*exp(1/3*I*(b*x+d)))/b-3^(1/2)*exp(5/3
*a-5/3*I*d)*arctanh(3^(1/2)*exp(1/3*I*(b*x+d))/(1+exp(2/3*I*(b*x+d))))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.49

$$\int e^{\frac{5}{3}(a+ibx)}\tan(d+bx)\,dx = -\frac{e^{5a/3}\left(9e^{\frac{5ibx}{3}}+5\text{RootSum}\left[\cos(d)-i\sin(d)+\cos(d)\#1^6+i\sin(d)\#1^6\&,\frac{bx+3i\log\left(e^{\frac{ibx}{3}}-\#1\right)}{\#1}\&\right](i\cos$$

$$15b$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Tan[d + b*x],x]
```

output
```
-1/15*(E^((5*a)/3)*(9*E^(((5*I)/3)*b*x) + 5*RootSum[Cos[d] - I*Sin[d] + Co
s[d]*#1^6 + I*Sin[d]*#1^6 & , (b*x + (3*I)*Log[E^((I/3)*b*x) - #1])/#1 & ]
*(I*Cos[2*d] + Sin[2*d])))/b
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.75, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\tan(bx+d)\,dx$$

$$\Big\downarrow \text{4942}$$

$$i\int\left(\frac{2e^{\frac{5}{3}(a+ibx)}}{1+e^{2i(d+bx)}}-e^{\frac{5}{3}(a+ibx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{2ie^{\frac{5}{3}(a-id)}\arctan\left(e^{\frac{1}{3}(a+ibx)+\frac{1}{3}(-a+id)}\right)}{b}+\frac{ie^{\frac{5}{3}(a-id)}\arctan\left(e^{-a/3}\left(\sqrt{3}e^{a/3}-2e^{\frac{1}{3}(a+ibx)+\frac{id}{3}}\right)\right)}{b}-\frac{ie^{\frac{5}{3}(a-id)}\text{arc}\right.$$

input
```
Int[E^((5*(a + I*b*x))/3)*Tan[d + b*x],x]
```

output
```
I*((((3*I)/5)*E^((5*(a + I*b*x))/3))/b - ((2*I)*E^((5*(a - I*d))/3)*ArcTan
[E^((-a + I*d)/3 + (a + I*b*x)/3)])/b + (I*E^((5*(a - I*d))/3)*ArcTan[(Sqr
t[3]*E^(a/3) - 2*E^((I/3)*d + (a + I*b*x)/3))/E^(a/3)])/b - (I*E^((5*(a -
I*d))/3)*ArcTan[(Sqrt[3]*E^(a/3) + 2*E^((I/3)*d + (a + I*b*x)/3))/E^(a/3)]
)/b - ((I/2)*Sqrt[3]*E^((5*(a - I*d))/3)*Log[1 - Sqrt[3]*E^((I/3)*d + (I/3
)*b*x) + E^(((2*I)/3)*d + ((2*I)/3)*b*x)])/b + ((I/2)*Sqrt[3]*E^((5*(a - I
*d))/3)*Log[1 + Sqrt[3]*E^((I/3)*d + (I/3)*b*x) + E^(((2*I)/3)*d + ((2*I)/
3)*b*x)])/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [F]

$$\int e^{\frac{5a}{3}+\frac{5ibx}{3}}\tan(bx+d)\,dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*tan(b*x+d),x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*tan(b*x+d),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(139) = 278$.

Time = 0.09 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.97

$$\int e^{\frac{5}{3}(a+ibx)}\tan(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*tan(b*x+d),x, algorithm="fricas")
```

output
```
-1/10*(5*(sqrt(3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) - I*e^(5/3*a - 5/3*I*
d))*log(1/2*(sqrt(3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) + 2*e^(1/3*I*b*x +
 5/3*a - 4/3*I*d) + I*e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) + 5*(sqrt
(3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) + I*e^(5/3*a - 5/3*I*d))*log(1/2*(s
qrt(3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) + 2*e^(1/3*I*b*x + 5/3*a - 4/3*I
*d) - I*e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 5*(sqrt(3)*b*sqrt(b^(
-2))*e^(5/3*a - 5/3*I*d) - I*e^(5/3*a - 5/3*I*d))*log(-1/2*(sqrt(3)*b*sqrt
(b^(-2))*e^(5/3*a - 5/3*I*d) - 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) + I*e^(5/
3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 5*(sqrt(3)*b*sqrt(b^(-2))*e^(5/3*a
 - 5/3*I*d) + I*e^(5/3*a - 5/3*I*d))*log(-1/2*(sqrt(3)*b*sqrt(b^(-2))*e^(5
/3*a - 5/3*I*d) - 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) - I*e^(5/3*a - 5/3*I*d
))*e^(-5/3*a + 5/3*I*d)) - 10*I*e^(5/3*a - 5/3*I*d)*log(e^(1/3*I*b*x + 1/3
*I*d) + I) + 10*I*e^(5/3*a - 5/3*I*d)*log(e^(1/3*I*b*x + 1/3*I*d) - I) + 6
*e^(5/3*I*b*x + 5/3*a))/b
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.39

$$\int e^{\frac{5}{3}(a+ibx)}\tan(d+bx)\,dx = \begin{cases} -\frac{3e^{\frac{5a}{3}}e^{\frac{5ibx}{3}}}{5b} & \text{for } b \neq 0 \\ -ixe^{\frac{5a}{3}} & \text{otherwise} \end{cases} + \frac{\operatorname{RootSum}\left(z^6e^{10id} + e^{10a}, \left(i \mapsto i\log\left(i^5e^{-\frac{25a}{3}}e^{8id} + e^{\frac{ibx}{3}}\right)\right)\right)}{b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*tan(b*x+d),x)
```

output
```
Piecewise((-3*exp(5*a/3)*exp(5*I*b*x/3)/(5*b), Ne(b, 0)), (-I*x*exp(5*a/3)
, True)) + RootSum(_z**6*exp(10*I*d) + exp(10*a), Lambda(_i, _i*log(_i**5*
exp(-25*a/3)*exp(8*I*d) + exp(I*b*x/3))))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7602 vs. $2(139) = 278$.

Time = 80.97 (sec) , antiderivative size = 7602, normalized size of antiderivative = 37.26

$$\int e^{\frac{5}{3}(a+ibx)}\tan(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*tan(b*x+d),x, algorithm="maxima")
```

output
```
-1/20*(10*(I*sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) + sqrt
(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))))*arctan2(sqrt(3)*cos(1/
6*arctan2(sin(2*d), cos(2*d)))*sin(1/3*b*x) + sqrt(3)*cos(1/3*b*x)*sin(1/6
*arctan2(sin(2*d), cos(2*d))) + cos(1/3*arctan2(sin(2*d), cos(2*d)))*sin(2
/3*b*x) + cos(2/3*b*x)*sin(1/3*arctan2(sin(2*d), cos(2*d))), sqrt(3)*cos(1
/3*b*x)*cos(1/6*arctan2(sin(2*d), cos(2*d))) - sqrt(3)*sin(1/3*b*x)*sin(1/
6*arctan2(sin(2*d), cos(2*d))) + cos(2/3*b*x)*cos(1/3*arctan2(sin(2*d), co
s(2*d))) - sin(2/3*b*x)*sin(1/3*arctan2(sin(2*d), cos(2*d))) + 1) + 10*(-I
*sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - sqrt(3)*e^(5/3*a
)*sin(5/6*arctan2(sin(2*d), cos(2*d))))*arctan2(-sqrt(3)*cos(1/6*arctan2(s
in(2*d), cos(2*d)))*sin(1/3*b*x) - sqrt(3)*cos(1/3*b*x)*sin(1/6*arctan2(si
n(2*d), cos(2*d))) + cos(1/3*arctan2(sin(2*d), cos(2*d)))*sin(2/3*b*x) + c
os(2/3*b*x)*sin(1/3*arctan2(sin(2*d), cos(2*d))), -sqrt(3)*cos(1/3*b*x)*co
s(1/6*arctan2(sin(2*d), cos(2*d))) + sqrt(3)*sin(1/3*b*x)*sin(1/6*arctan2(
sin(2*d), cos(2*d))) + cos(2/3*b*x)*cos(1/3*arctan2(sin(2*d), cos(2*d))) -
 sin(2/3*b*x)*sin(1/3*arctan2(sin(2*d), cos(2*d))) + 1) + 10*(((-I*cos(2*d
)*e^(5/3*a) - e^(5/3*a)*sin(2*d))*cos(1/3*arctan2(sin(2*d), cos(2*d))) + (
cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*sin(1/3*arctan2(sin(2*d), cos(2
*d))))*cos(1/2*pi + 1/6*arctan2(sin(2*d), cos(2*d))) - ((cos(2*d)*e^(5/3*a
) - I*e^(5/3*a)*sin(2*d))*cos(1/3*arctan2(sin(2*d), cos(2*d))) - (-I*co...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3205 vs. $2(139) = 278$.

Time = 26.99 (sec) , antiderivative size = 3205, normalized size of antiderivative = 15.71

$$\int e^{\frac{5}{3}(a+ibx)}\tan(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*tan(b*x+d),x, algorithm="giac")
```

output
```
1/2*(2*(5*sqrt(3)*cos(1/12*pi*sgn(cos(2*d))*sgn(sin(2*d)) + 1/6*pi*floor(2
*d/pi + 1/2) - 1/12*pi*sgn(sin(2*d)) - 1/3*d)^4*e^(125/3*a)*sin(1/12*pi*sg
n(cos(2*d))*sgn(sin(2*d)) + 1/6*pi*floor(2*d/pi + 1/2) - 1/12*pi*sgn(sin(2
*d)) - 1/3*d)/(cos(2*d)^2 + sin(2*d)^2)^(5/12) - 10*sqrt(3)*cos(1/12*pi*sg
n(cos(2*d))*sgn(sin(2*d)) + 1/6*pi*floor(2*d/pi + 1/2) - 1/12*pi*sgn(sin(2
*d)) - 1/3*d)^2*e^(125/3*a)*sin(1/12*pi*sgn(cos(2*d))*sgn(sin(2*d)) + 1/6*
pi*floor(2*d/pi + 1/2) - 1/12*pi*sgn(sin(2*d)) - 1/3*d)^3/(cos(2*d)^2 + si
n(2*d)^2)^(5/12) + sqrt(3)*e^(125/3*a)*sin(1/12*pi*sgn(cos(2*d))*sgn(sin(2
*d)) + 1/6*pi*floor(2*d/pi + 1/2) - 1/12*pi*sgn(sin(2*d)) - 1/3*d)^5/(cos(
2*d)^2 + sin(2*d)^2)^(5/12) + cos(1/12*pi*sgn(cos(2*d))*sgn(sin(2*d)) + 1/
6*pi*floor(2*d/pi + 1/2) - 1/12*pi*sgn(sin(2*d)) - 1/3*d)^5*e^(125/3*a)/(c
os(2*d)^2 + sin(2*d)^2)^(5/12) - 10*cos(1/12*pi*sgn(cos(2*d))*sgn(sin(2*d)
) + 1/6*pi*floor(2*d/pi + 1/2) - 1/12*pi*sgn(sin(2*d)) - 1/3*d)^3*e^(125/3
*a)*sin(1/12*pi*sgn(cos(2*d))*sgn(sin(2*d)) + 1/6*pi*floor(2*d/pi + 1/2) -
 1/12*pi*sgn(sin(2*d)) - 1/3*d)^2/(cos(2*d)^2 + sin(2*d)^2)^(5/12) + 5*cos
(1/12*pi*sgn(cos(2*d))*sgn(sin(2*d)) + 1/6*pi*floor(2*d/pi + 1/2) - 1/12*p
i*sgn(sin(2*d)) - 1/3*d)*e^(125/3*a)*sin(1/12*pi*sgn(cos(2*d))*sgn(sin(2*d
)) + 1/6*pi*floor(2*d/pi + 1/2) - 1/12*pi*sgn(sin(2*d)) - 1/3*d)^4/(cos(2*
d)^2 + sin(2*d)^2)^(5/12))*arctan((sqrt(3)*e^(25/3*a - 1/3*I*d) + 2*e^(1/3
*I*b*x + 25/3*a))*e^(-25/3*a + 1/3*I*d))*e^(-40*a) - 2*(5*sqrt(3)*cos(1...
```

Mupad [B] (verification not implemented)

Time = 18.56 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.29

$$\int e^{\frac{5}{3}(a+ibx)} \tan(d+bx)\,dx = \text{Too large to display}$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)*tan(d + b*x),x)
```

output
```
((-exp(10*a - d*10i))^(1/6)*log(4*exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i
)/3)*(-exp(10*a)*exp(-d*10i))^(1/6) - 4*exp(6*a)*exp(-d*6i)))/b - ((-exp(1
0*a - d*10i))^(1/6)*log(- 4*exp(6*a)*exp(-d*6i) - 4*exp(a/3)*exp(4*a)*exp(
-d*4i)*exp((b*x*1i)/3)*(-exp(10*a)*exp(-d*10i))^(1/6)))/b - (3*exp((5*a)/3
 + (b*x*5i)/3))/(5*b) - (log(- 4*exp(6*a)*exp(-d*6i) - 4*exp(a/3)*exp(4*a)
*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 - 1/2)*(-exp(10*a)*exp(-d*10i)
)^(1/6))*(-exp(10*a - d*10i))^(1/6)*((3^(1/2)*1i)/2 - 1/2))/b + (log(4*exp
(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 - 1/2)*(-exp(10*
a)*exp(-d*10i))^(1/6) - 4*exp(6*a)*exp(-d*6i))*(-exp(10*a - d*10i))^(1/6)*
((3^(1/2)*1i)/2 - 1/2))/b - (log(- 4*exp(6*a)*exp(-d*6i) - 4*exp(a/3)*exp(
4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 + 1/2)*(-exp(10*a)*exp(-d*
10i))^(1/6))*(-exp(10*a - d*10i))^(1/6)*((3^(1/2)*1i)/2 + 1/2))/b + (log(4
*exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 + 1/2)*(-exp
(10*a)*exp(-d*10i))^(1/6) - 4*exp(6*a)*exp(-d*6i))*(-exp(10*a - d*10i))^(1
/6)*((3^(1/2)*1i)/2 + 1/2))/b
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\tan(d+bx)\,dx=\int e^{\frac{5bix}{3}+\frac{5a}{3}}\tan\left(bx+d\right)dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*tan(b*x+d),x)
```

output
```
int(e**((5*a + 5*b*i*x)/3)*tan(b*x + d),x)
```

3.50 $\int e^{\frac{5}{3}(a+ibx)}\tan^2(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 268

$$\int e^{\frac{5}{3}(a+ibx)}\tan^2(d+bx)\,dx = \frac{3ie^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{5b} + \frac{2ie^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)} - \frac{10ie^{\frac{5}{3}(a-id)}\arctan\left(e^{\frac{1}{3}i(d+bx)}\right)}{3b} + \frac{5ie^{\frac{5}{3}(a-id)}\arctan\left(\sqrt{3}-2e^{\frac{1}{3}i(d+bx)}\right)}{3b} - \frac{5ie^{\frac{5}{3}(a-id)}\arctan\left(\sqrt{3}+2e^{\frac{1}{3}i(d+bx)}\right)}{3b} + \frac{5ie^{\frac{5}{3}(a-id)}\text{arctanh}\left(\frac{\sqrt{3}e^{\frac{1}{3}i(d+bx)}}{1+e^{\frac{2}{3}i(d+bx)}}\right)}{\sqrt{3}b}$$

output
```
3/5*I*exp(5/3*a-5/3*I*d+5/3*I*(b*x+d))/b+2*I*exp(5/3*a-5/3*I*d+5/3*I*(b*x+
d))/b/(1+exp(2*I*(b*x+d)))-10/3*I*exp(5/3*a-5/3*I*d)*arctan(exp(1/3*I*(b*x
+d)))/b-5/3*I*exp(5/3*a-5/3*I*d)*arctan(-3^(1/2)+2*exp(1/3*I*(b*x+d)))/b-5
/3*I*exp(5/3*a-5/3*I*d)*arctan(3^(1/2)+2*exp(1/3*I*(b*x+d)))/b+5/3*I*exp(5
/3*a-5/3*I*d)*arctanh(3^(1/2)*exp(1/3*I*(b*x+d))/(1+exp(2/3*I*(b*x+d))))*3
^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.59

$$\int e^{\frac{5}{3}(a+ibx)}\tan^2(d+bx)\,dx = \frac{1}{45}e^{5a/3}\left(\frac{27ie^{\frac{5ibx}{3}}}{b} - \frac{25\text{RootSum}\left[\cos(d)-i\sin(d)+\cos(d)\#1^6+i\sin(d)\#1^6\&,\frac{bx+3i\log\left(e^{\frac{ibx}{3}}-\#1\right)}{\#1}\&\right](\cos(d)-i\sin(d))^2}{b} + \frac{90e^{\frac{5ibx}{3}}(\cos(d)-i\sin(d))}{-ib\left(1+e^{2ibx}\right)\cos(d)+b\left(-1+e^{2ibx}\right)\sin(d)}\right)$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Tan[d + b*x]^2,x]
```

output
```
(E^((5*a)/3)*(((27*I)*E^(((5*I)/3)*b*x))/b - (25*RootSum[Cos[d] - I*Sin[d]
 + Cos[d]*#1^6 + I*Sin[d]*#1^6 & , (b*x + (3*I)*Log[E^((I/3)*b*x) - #1])/#
1 & ]*(Cos[d] - I*Sin[d])^2)/b + (90*E^(((5*I)/3)*b*x)*(Cos[d] - I*Sin[d])
)/((-I)*b*(1 + E^((2*I)*b*x))*Cos[d] + b*(-1 + E^((2*I)*b*x))*Sin[d])))/45
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)} \tan^2(bx+d)\,dx$$

$$\Big\downarrow \textbf{4942}$$

$$-\int \left(e^{\frac{5}{3}(a+ibx)} - \frac{4e^{\frac{5}{3}(a+ibx)}}{1+e^{2i(d+bx)}} + \frac{4e^{\frac{5}{3}(a+ibx)}}{\left(1+e^{2i(d+bx)}\right)^2}\right)dx$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{10ie^{\frac{5}{3}(a-id)} \arctan\left(e^{\frac{1}{3}(a+ibx)+\frac{1}{3}(-a+id)}\right)}{3b} + \frac{5ie^{\frac{5}{3}(a-id)} \arctan\left(e^{-a/3}\left(\sqrt{3}e^{a/3} - 2e^{\frac{1}{3}(a+ibx)+\frac{id}{3}}\right)\right)}{3b} - \frac{5ie^{\frac{5}{3}(a-id)} \arctan\left(e^{-a/3}\left(\sqrt{3}e^{a/3} + 2e^{\frac{1}{3}(a+ibx)+\frac{id}{3}}\right)\right)}{3b} + \frac{2ie^{\frac{5}{3}(a+ibx)}}{b\left(1+e^{2i(bx+d)}\right)} - \frac{i\sqrt{3}e^{\frac{5}{3}(a-id)} \log\left(-\sqrt{3}e^{\frac{ibx}{3}+\frac{id}{3}} + e^{\frac{2ibx}{3}+\frac{2id}{3}} + 1\right)}{b} + \frac{ie^{\frac{5}{3}(a-id)} \log\left(-\sqrt{3}e^{\frac{ibx}{3}+\frac{id}{3}} + e^{\frac{2ibx}{3}+\frac{2id}{3}} + 1\right)}{2\sqrt{3}b} + \frac{i\sqrt{3}e^{\frac{5}{3}(a-id)} \log\left(\sqrt{3}e^{\frac{ibx}{3}+\frac{id}{3}} + e^{\frac{2ibx}{3}+\frac{2id}{3}} + 1\right)}{b} - \frac{ie^{\frac{5}{3}(a-id)} \log\left(\sqrt{3}e^{\frac{ibx}{3}+\frac{id}{3}} + e^{\frac{2ibx}{3}+\frac{2id}{3}} + 1\right)}{2\sqrt{3}b} + \frac{3ie^{\frac{5}{3}(a+ibx)}}{5b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Tan[d + b*x]^2,x]
```

output
```
(((3*I)/5)*E^((5*(a + I*b*x))/3))/b + ((2*I)*E^((5*(a + I*b*x))/3))/(b*(1
+ E^((2*I)*(d + b*x)))) - (((10*I)/3)*E^((5*(a - I*d))/3)*ArcTan[E^((-a +
I*d)/3 + (a + I*b*x)/3)])/b + (((5*I)/3)*E^((5*(a - I*d))/3)*ArcTan[(Sqrt[
3]*E^(a/3) - 2*E^((I/3)*d + (a + I*b*x)/3))/E^(a/3)])/b - (((5*I)/3)*E^((5
*(a - I*d))/3)*ArcTan[(Sqrt[3]*E^(a/3) + 2*E^((I/3)*d + (a + I*b*x)/3))/E^
(a/3)])/b + ((I/2)*E^((5*(a - I*d))/3)*Log[1 - Sqrt[3]*E^((I/3)*d + (I/3)*
b*x) + E^(((2*I)/3)*d + ((2*I)/3)*b*x)])/(Sqrt[3]*b) - (I*Sqrt[3]*E^((5*(a
 - I*d))/3)*Log[1 - Sqrt[3]*E^((I/3)*d + (I/3)*b*x) + E^(((2*I)/3)*d + ((2
*I)/3)*b*x)])/b - ((I/2)*E^((5*(a - I*d))/3)*Log[1 + Sqrt[3]*E^((I/3)*d +
(I/3)*b*x) + E^(((2*I)/3)*d + ((2*I)/3)*b*x)])/(Sqrt[3]*b) + (I*Sqrt[3]*E^
((5*(a - I*d))/3)*Log[1 + Sqrt[3]*E^((I/3)*d + (I/3)*b*x) + E^(((2*I)/3)*d
 + ((2*I)/3)*b*x)])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [F]

$$\int \mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \tan \left(bx+d\right)^{2}dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)^2,x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(168) = 336.

Time = 0.09 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.14

$$\int e^{\frac{5}{3}(a+ibx)} \tan^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/30*(25*(3*sqrt(1/3)*(b*e^(2*I*b*x + 2*I*d) + b)*sqrt(-e^(10/3*a - 10/3*I
*d)/b^2) - e^(2*I*b*x + 5/3*a + 1/3*I*d) - e^(5/3*a - 5/3*I*d))*log(-1/2*(
3*I*sqrt(1/3)*b*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) - 2*e^(1/3*I*b*x + 5/3*a
- 4/3*I*d) + I*e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) + 25*(3*sqrt(1/3
)*(b*e^(2*I*b*x + 2*I*d) + b)*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) + e^(2*I*b*
x + 5/3*a + 1/3*I*d) + e^(5/3*a - 5/3*I*d))*log(-1/2*(3*I*sqrt(1/3)*b*sqrt
(-e^(10/3*a - 10/3*I*d)/b^2) - 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) - I*e^(5/
3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 25*(3*sqrt(1/3)*(b*e^(2*I*b*x + 2*
I*d) + b)*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) + e^(2*I*b*x + 5/3*a + 1/3*I*d)
 + e^(5/3*a - 5/3*I*d))*log(-1/2*(-3*I*sqrt(1/3)*b*sqrt(-e^(10/3*a - 10/3*
I*d)/b^2) - 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) + I*e^(5/3*a - 5/3*I*d))*e^(
-5/3*a + 5/3*I*d)) - 25*(3*sqrt(1/3)*(b*e^(2*I*b*x + 2*I*d) + b)*sqrt(-e^(
10/3*a - 10/3*I*d)/b^2) - e^(2*I*b*x + 5/3*a + 1/3*I*d) - e^(5/3*a - 5/3*I
*d))*log(-1/2*(-3*I*sqrt(1/3)*b*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) - 2*e^(1/
3*I*b*x + 5/3*a - 4/3*I*d) - I*e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d))
+ 50*(e^(2*I*b*x + 5/3*a + 1/3*I*d) + e^(5/3*a - 5/3*I*d))*log(e^(1/3*I*b*
x + 1/3*I*d) + I) - 50*(e^(2*I*b*x + 5/3*a + 1/3*I*d) + e^(5/3*a - 5/3*I*d
))*log(e^(1/3*I*b*x + 1/3*I*d) - I) + 18*I*e^(11/3*I*b*x + 5/3*a + 2*I*d)
+ 78*I*e^(5/3*I*b*x + 5/3*a))/(b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.66

$$\int e^{\frac{5}{3}(a+ibx)}\tan^2(d+bx)\,dx = \begin{cases} \frac{3ie^{\frac{5a}{3}}e^{\frac{5ibx}{3}}}{5b} & \text{for}\ b \neq 0 \\ -xe^{\frac{5a}{3}} & \text{otherwise} \end{cases} + \frac{2ie^{\frac{5a}{3}}e^{\frac{5ibx}{3}}}{be^{2id}e^{2ibx}+b} + \frac{\operatorname{RootSum}\left(27z^3e^{5id}-125e^{5a},\left(i\mapsto i\log\left(\frac{243i^5ie^{-\frac{25a}{3}}e^{8id}}{3125}+e^{\frac{ibx}{3}}\right)\right)\right)+\operatorname{RootSum}\left(27z^3e^{5id}+125e^{5a},\left(i\right.\right.}{b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)**2,x)
```

output
```
Piecewise((3*I*exp(5*a/3)*exp(5*I*b*x/3)/(5*b), Ne(b, 0)), (-x*exp(5*a/3),
 True)) + 2*I*exp(5*a/3)*exp(5*I*b*x/3)/(b*exp(2*I*d)*exp(2*I*b*x) + b) +
(RootSum(27*_z**3*exp(5*I*d) - 125*exp(5*a), Lambda(_i, _i*log(243*_i**5*I
*exp(-25*a/3)*exp(8*I*d)/3125 + exp(I*b*x/3)))) + RootSum(27*_z**3*exp(5*I
*d) + 125*exp(5*a), Lambda(_i, _i*log(243*_i**5*I*exp(-25*a/3)*exp(8*I*d)/
3125 + exp(I*b*x/3)))))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 9984 vs. $2(168) = 336$.

Time = 82.58 (sec) , antiderivative size = 9984, normalized size of antiderivative = 37.25

$$\int e^{\frac{5}{3}(a+ibx)}\tan^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)^2,x, algorithm="maxima")
```

output
```
-60*(50*(-I*sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - sqrt(
3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))) + (-I*sqrt(3)*cos(5/6*ar
ctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(s
in(2*d), cos(2*d))))*cos(2*b*x + 2*d) + (sqrt(3)*cos(5/6*arctan2(sin(2*d),
 cos(2*d)))*e^(5/3*a) - I*sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(
2*d))))*sin(2*b*x + 2*d))*arctan2(sqrt(3)*cos(1/6*arctan2(sin(2*d), cos(2*
d)))*sin(1/3*b*x) + sqrt(3)*cos(1/3*b*x)*sin(1/6*arctan2(sin(2*d), cos(2*d
))) + cos(1/3*arctan2(sin(2*d), cos(2*d)))*sin(2/3*b*x) + cos(2/3*b*x)*sin
(1/3*arctan2(sin(2*d), cos(2*d))), sqrt(3)*cos(1/3*b*x)*cos(1/6*arctan2(si
n(2*d), cos(2*d))) - sqrt(3)*sin(1/3*b*x)*sin(1/6*arctan2(sin(2*d), cos(2*
d))) + cos(2/3*b*x)*cos(1/3*arctan2(sin(2*d), cos(2*d))) - sin(2/3*b*x)*si
n(1/3*arctan2(sin(2*d), cos(2*d))) + 1) + 50*(I*sqrt(3)*cos(5/6*arctan2(si
n(2*d), cos(2*d)))*e^(5/3*a) + sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d),
 cos(2*d))) + (I*sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) +
sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))))*cos(2*b*x + 2*d) -
 (sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - I*sqrt(3)*e^(5/
3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))))*sin(2*b*x + 2*d))*arctan2(-sqrt
(3)*cos(1/6*arctan2(sin(2*d), cos(2*d)))*sin(1/3*b*x) - sqrt(3)*cos(1/3*b*
x)*sin(1/6*arctan2(sin(2*d), cos(2*d))) + cos(1/3*arctan2(sin(2*d), cos(2*
d)))*sin(2/3*b*x) + cos(2/3*b*x)*sin(1/3*arctan2(sin(2*d), cos(2*d))), ...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4151 vs. $2(168) = 336$.

Time = 0.98 (sec) , antiderivative size = 4151, normalized size of antiderivative = 15.49

$$\int e^{\frac{5}{3}(a+ibx)}\tan^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)^2,x, algorithm="giac")
```

output
```
1/6*I*(10*(5*sqrt(3)*cos(2*d)*cos(1/3*d)^4*sin(1/3*d) - 10*sqrt(3)*cos(2*d
)*cos(1/3*d)^2*sin(1/3*d)^3 + sqrt(3)*cos(2*d)*sin(1/3*d)^5 - cos(2*d)*cos
(1/3*d)^5 + 10*cos(2*d)*cos(1/3*d)^3*sin(1/3*d)^2 - 5*cos(2*d)*cos(1/3*d)*
sin(1/3*d)^4)*arctan((sqrt(3)*e^(-1/3*I*d) + 2*e^(1/3*I*b*x))*e^(1/3*I*d))
/(cos(2*d)^2*cos(1/3*d)^10 + cos(1/3*d)^10*sin(2*d)^2 + 5*cos(2*d)^2*cos(1
/3*d)^8*sin(1/3*d)^2 + 5*cos(1/3*d)^8*sin(2*d)^2*sin(1/3*d)^2 + 10*cos(2*d
)^2*cos(1/3*d)^6*sin(1/3*d)^4 + 10*cos(1/3*d)^6*sin(2*d)^2*sin(1/3*d)^4 +
10*cos(2*d)^2*cos(1/3*d)^4*sin(1/3*d)^6 + 10*cos(1/3*d)^4*sin(2*d)^2*sin(1
/3*d)^6 + 5*cos(2*d)^2*cos(1/3*d)^2*sin(1/3*d)^8 + 5*cos(1/3*d)^2*sin(2*d)
^2*sin(1/3*d)^8 + cos(2*d)^2*sin(1/3*d)^10 + sin(2*d)^2*sin(1/3*d)^10) - 1
0*I*(5*sqrt(3)*cos(1/3*d)^4*sin(2*d)*sin(1/3*d) - 10*sqrt(3)*cos(1/3*d)^2*
sin(2*d)*sin(1/3*d)^3 + sqrt(3)*sin(2*d)*sin(1/3*d)^5 - cos(1/3*d)^5*sin(2
*d) + 10*cos(1/3*d)^3*sin(2*d)*sin(1/3*d)^2 - 5*cos(1/3*d)*sin(2*d)*sin(1/
3*d)^4)*arctan((sqrt(3)*e^(-1/3*I*d) + 2*e^(1/3*I*b*x))*e^(1/3*I*d))/(cos(
2*d)^2*cos(1/3*d)^10 + cos(1/3*d)^10*sin(2*d)^2 + 5*cos(2*d)^2*cos(1/3*d)^
8*sin(1/3*d)^2 + 5*cos(1/3*d)^8*sin(2*d)^2*sin(1/3*d)^2 + 10*cos(2*d)^2*co
s(1/3*d)^6*sin(1/3*d)^4 + 10*cos(1/3*d)^6*sin(2*d)^2*sin(1/3*d)^4 + 10*cos
(2*d)^2*cos(1/3*d)^4*sin(1/3*d)^6 + 10*cos(1/3*d)^4*sin(2*d)^2*sin(1/3*d)^
6 + 5*cos(2*d)^2*cos(1/3*d)^2*sin(1/3*d)^8 + 5*cos(1/3*d)^2*sin(2*d)^2*sin
(1/3*d)^8 + cos(2*d)^2*sin(1/3*d)^10 + sin(2*d)^2*sin(1/3*d)^10) - 10*(...
```

Mupad [B] (verification not implemented)

Time = 21.11 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.87

$$\int e^{\frac{5}{3}(a+ibx)} \tan^2(d+bx)\,dx = \text{Too large to display}$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)*tan(d + b*x)^2,x)
```

output
```
(exp((5*a)/3 + (b*x*5i)/3)*3i)/(5*b) + (exp((11*a)/3 - d*2i + (b*x*5i)/3)*
2i)/(b*(exp(2*a - d*2i) + exp(2*a + b*x*2i))) + (5*exp(10*a - d*10i)^(1/6)
*log((100*exp(6*a)*exp(-d*6i))/9 - (exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*
1i)/3)*(exp(10*a)*exp(-d*10i))^(1/6)*100i)/9))/(3*b) - (5*exp(10*a - d*10i
)^(1/6)*log((100*exp(6*a)*exp(-d*6i))/9 + (exp(a/3)*exp(4*a)*exp(-d*4i)*ex
p((b*x*1i)/3)*(exp(10*a)*exp(-d*10i))^(1/6)*100i)/9))/(3*b) + (5*log((100*
exp(6*a)*exp(-d*6i))/9 - (exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3
^(1/2)*1i)/2 - 1/2)*(exp(10*a)*exp(-d*10i))^(1/6)*100i)/9)*exp(10*a - d*10
i)^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(3*b) - (5*log((100*exp(6*a)*exp(-d*6i))/
9 + (exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 - 1/2)*(
exp(10*a)*exp(-d*10i))^(1/6)*100i)/9)*exp(10*a - d*10i)^(1/6)*((3^(1/2)*1i
)/2 - 1/2))/(3*b) + (5*log((100*exp(6*a)*exp(-d*6i))/9 - (exp(a/3)*exp(4*a
)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 + 1/2)*(exp(10*a)*exp(-d*10i)
)^(1/6)*100i)/9)*exp(10*a - d*10i)^(1/6)*((3^(1/2)*1i)/2 + 1/2))/(3*b) - (
5*log((100*exp(6*a)*exp(-d*6i))/9 + (exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x
*1i)/3)*((3^(1/2)*1i)/2 + 1/2)*(exp(10*a)*exp(-d*10i))^(1/6)*100i)/9)*exp(
10*a - d*10i)^(1/6)*((3^(1/2)*1i)/2 + 1/2))/(3*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\tan^2(d+bx)\,dx=\int e^{\frac{5bix}{3}+\frac{5a}{3}}\tan\left(bx+d\right)^2dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)^2,x)
```

output
```
int(e**((5*a + 5*b*i*x)/3)*tan(b*x + d)**2,x)
```

3.51 $\int e^{\frac{5}{3}(a+ibx)}\tan^3(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 305

$$\begin{aligned}\int e^{\frac{5}{3}(a+ibx)}\tan^3(d+bx)\,dx &= \frac{3e^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{5b}-\frac{2e^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)^2}\\&\quad+\frac{11e^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{3b\left(1+e^{2i(d+bx)}\right)}-\frac{43e^{\frac{5}{3}(a-id)}\arctan\left(e^{\frac{1}{3}i(d+bx)}\right)}{9b}\\&\quad+\frac{43e^{\frac{5}{3}(a-id)}\arctan\left(\sqrt{3}-2e^{\frac{1}{3}i(d+bx)}\right)}{18b}\\&\quad-\frac{43e^{\frac{5}{3}(a-id)}\arctan\left(\sqrt{3}+2e^{\frac{1}{3}i(d+bx)}\right)}{18b}\\&\quad+\frac{43e^{\frac{5}{3}(a-id)}\operatorname{arctanh}\left(\frac{\sqrt{3}e^{\frac{1}{3}i(d+bx)}}{1+e^{\frac{2}{3}i(d+bx)}}\right)}{6\sqrt{3}b}\end{aligned}$$

output
```
3/5*exp(5/3*a-5/3*I*d+5/3*I*(b*x+d))/b-2*exp(5/3*a-5/3*I*d+5/3*I*(b*x+d))/
b/(1+exp(2*I*(b*x+d)))^2+11/3*exp(5/3*a-5/3*I*d+5/3*I*(b*x+d))/b/(1+exp(2*
I*(b*x+d)))-43/9*exp(5/3*a-5/3*I*d)*arctan(exp(1/3*I*(b*x+d)))/b-43/18*exp
(5/3*a-5/3*I*d)*arctan(-3^(1/2)+2*exp(1/3*I*(b*x+d)))/b-43/18*exp(5/3*a-5/
3*I*d)*arctan(3^(1/2)+2*exp(1/3*I*(b*x+d)))/b+43/18*3^(1/2)*exp(5/3*a-5/3*
I*d)*arctanh(3^(1/2)*exp(1/3*I*(b*x+d))/(1+exp(2/3*I*(b*x+d))))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.59

$$\int e^{\frac{5}{3}(a+ibx)}\tan^3(d+bx)\,dx$$
$$=\frac{e^{5a/3}\left(215\text{RootSum}\left[\cos(d)-i\sin(d)+\cos(d)\#1^6+i\sin(d)\#1^6\&,\frac{bx+3i\log\left(e^{\frac{ibx}{3}}-\#1\right)}{\#1}\&\right](i\cos(2d)+\text{si}\right.}{270b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Tan[d + b*x]^3,x]
```

output
```
(E^((5*a)/3)*(215*RootSum[Cos[d] - I*Sin[d] + Cos[d]*#1^6 + I*Sin[d]*#1^6
& , (b*x + (3*I)*Log[E^((I/3)*b*x) - #1])/#1 & ]*(I*Cos[2*d] + Sin[2*d]) +
 (18*E^(((5*I)/3)*b*x)*(73*E^((2*I)*b*x) + (34 + 9*E^((4*I)*b*x))*Cos[2*d]
 + I*(-34 + 9*E^((4*I)*b*x))*Sin[2*d]))/((1 + E^((2*I)*b*x))*Cos[d] + I*(-
1 + E^((2*I)*b*x))*Sin[d])^2))/(270*b)
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.89, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\tan^3(bx+d)\,dx$$
$$\Big\downarrow \text{4942}$$
$$-i\int\left(-e^{\frac{5}{3}(a+ibx)}+\frac{6e^{\frac{5}{3}(a+ibx)}}{1+e^{2i(d+bx)}}-\frac{12e^{\frac{5}{3}(a+ibx)}}{\left(1+e^{2i(d+bx)}\right)^2}+\frac{8e^{\frac{5}{3}(a+ibx)}}{\left(1+e^{2i(d+bx)}\right)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{43ie^{\frac{5}{3}(a-id)}\arctan\left(e^{\frac{1}{3}(a+ibx)+\frac{1}{3}(-a+id)}\right)}{9b}+\frac{43ie^{\frac{5}{3}(a-id)}\arctan\left(e^{-a/3}\left(\sqrt{3}e^{a/3}-2e^{\frac{1}{3}(a+ibx)+\frac{id}{3}}\right)\right)}{18b}-\frac{43ie^{\frac{5}{3}(}}{}\right.$$

input
```
Int[E^((5*(a + I*b*x))/3)*Tan[d + b*x]^3,x]
```

output
```
(-I)*((((3*I)/5)*E^((5*(a + I*b*x))/3))/b - ((2*I)*E^((5*(a + I*b*x))/3))/
(b*(1 + E^((2*I)*(d + b*x)))^2) + (((11*I)/3)*E^((5*(a + I*b*x))/3))/(b*(1
 + E^((2*I)*(d + b*x)))) - (((43*I)/9)*E^((5*(a - I*d))/3)*ArcTan[E^((-a +
 I*d)/3 + (a + I*b*x)/3)])/b + (((43*I)/18)*E^((5*(a - I*d))/3)*ArcTan[(Sq
rt[3]*E^(a/3) - 2*E^((I/3)*d + (a + I*b*x)/3))/E^(a/3)])/b - (((43*I)/18)*
E^((5*(a - I*d))/3)*ArcTan[(Sqrt[3]*E^(a/3) + 2*E^((I/3)*d + (a + I*b*x)/3
))/E^(a/3)])/b - (((7*I)/12)*E^((5*(a - I*d))/3)*Log[1 - Sqrt[3]*E^((I/3)*
d + (I/3)*b*x) + E^(((2*I)/3)*d + ((2*I)/3)*b*x)])/(Sqrt[3]*b) - (I*Sqrt[3
]*E^((5*(a - I*d))/3)*Log[1 - Sqrt[3]*E^((I/3)*d + (I/3)*b*x) + E^(((2*I)/
3)*d + ((2*I)/3)*b*x)])/b + (((7*I)/12)*E^((5*(a - I*d))/3)*Log[1 + Sqrt[3
]*E^((I/3)*d + (I/3)*b*x) + E^(((2*I)/3)*d + ((2*I)/3)*b*x)])/(Sqrt[3]*b)
+ (I*Sqrt[3]*E^((5*(a - I*d))/3)*Log[1 + Sqrt[3]*E^((I/3)*d + (I/3)*b*x) +
 E^(((2*I)/3)*d + ((2*I)/3)*b*x)])/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [F]

$$\int e^{\frac{5a}{3}+\frac{5ibx}{3}} \tan\left(bx+d\right)^{3} dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)^3,x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(195) = 390$.

Time = 0.10 (sec) , antiderivative size = 731, normalized size of antiderivative = 2.40

$$\int e^{\frac{5}{3}(a+ibx)} \tan^3(d+bx)\, dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/180*(215*(3*sqrt(1/3)*(b*e^(4*I*b*x + 4*I*d) + 2*b*e^(2*I*b*x + 2*I*d) +
 b)*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) - I*e^(4*I*b*x + 5/3*a + 7/3*I*d) - 2
*I*e^(2*I*b*x + 5/3*a + 1/3*I*d) - I*e^(5/3*a - 5/3*I*d))*log(1/2*(3*sqrt(
1/3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) + 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d
) + I*e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) + 215*(3*sqrt(1/3)*(b*e^(
4*I*b*x + 4*I*d) + 2*b*e^(2*I*b*x + 2*I*d) + b)*sqrt(b^(-2))*e^(5/3*a - 5/
3*I*d) + I*e^(4*I*b*x + 5/3*a + 7/3*I*d) + 2*I*e^(2*I*b*x + 5/3*a + 1/3*I*
d) + I*e^(5/3*a - 5/3*I*d))*log(1/2*(3*sqrt(1/3)*b*sqrt(b^(-2))*e^(5/3*a -
 5/3*I*d) + 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) - I*e^(5/3*a - 5/3*I*d))*e^(
-5/3*a + 5/3*I*d)) - 215*(3*sqrt(1/3)*(b*e^(4*I*b*x + 4*I*d) + 2*b*e^(2*I*
b*x + 2*I*d) + b)*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) - I*e^(4*I*b*x + 5/3*a
+ 7/3*I*d) - 2*I*e^(2*I*b*x + 5/3*a + 1/3*I*d) - I*e^(5/3*a - 5/3*I*d))*lo
g(-1/2*(3*sqrt(1/3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) - 2*e^(1/3*I*b*x +
5/3*a - 4/3*I*d) + I*e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 215*(3*s
qrt(1/3)*(b*e^(4*I*b*x + 4*I*d) + 2*b*e^(2*I*b*x + 2*I*d) + b)*sqrt(b^(-2)
)*e^(5/3*a - 5/3*I*d) + I*e^(4*I*b*x + 5/3*a + 7/3*I*d) + 2*I*e^(2*I*b*x +
 5/3*a + 1/3*I*d) + I*e^(5/3*a - 5/3*I*d))*log(-1/2*(3*sqrt(1/3)*b*sqrt(b^
(-2))*e^(5/3*a - 5/3*I*d) - 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) - I*e^(5/3*a
 - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 430*(I*e^(4*I*b*x + 5/3*a + 7/3*I*d)
+ 2*I*e^(2*I*b*x + 5/3*a + 1/3*I*d) + I*e^(5/3*a - 5/3*I*d))*log(e^(1/3...
```

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.55

$$\int e^{\frac{5}{3}(a+ibx)}\tan^3(d+bx)\,dx = \frac{11e^{\frac{5a}{3}}e^{2id}e^{\frac{11ibx}{3}}+5e^{\frac{5a}{3}}e^{\frac{5ibx}{3}}}{3be^{4id}e^{4ibx}+6be^{2id}e^{2ibx}+3b} + \begin{cases} \frac{3e^{\frac{5a}{3}}e^{\frac{5ibx}{3}}}{5b} & \text{for}\ b\neq 0 \\ ixe^{\frac{5a}{3}} & \text{otherwise} \end{cases} + \frac{\operatorname{RootSum}\left(34012224z^6e^{10id}+6321363049e^{10a},\left(i\mapsto i\log\left(-\frac{1889568i^5e^{-\frac{25a}{3}}e^{8id}}{147008443}+e^{\frac{ibx}{3}}\right)\right)\right)}{b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)**3,x)
```

output
```
(11*exp(5*a/3)*exp(2*I*d)*exp(11*I*b*x/3) + 5*exp(5*a/3)*exp(5*I*b*x/3))/(
3*b*exp(4*I*d)*exp(4*I*b*x) + 6*b*exp(2*I*d)*exp(2*I*b*x) + 3*b) + Piecewi
se((3*exp(5*a/3)*exp(5*I*b*x/3)/(5*b), Ne(b, 0)), (I*x*exp(5*a/3), True))
+ RootSum(34012224*_z**6*exp(10*I*d) + 6321363049*exp(10*a), Lambda(_i, _i
*log(-1889568*_i**5*exp(-25*a/3)*exp(8*I*d)/147008443 + exp(I*b*x/3))))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 12429 vs. $2(195) = 390$.

Time = 81.65 (sec) , antiderivative size = 12429, normalized size of antiderivative = 40.75

$$\int e^{\frac{5}{3}(a+ibx)} \tan^3(d+bx) \, dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)^3,x, algorithm="maxima")
```

output
```
360*(430*(sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - I*sqrt(
3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))) + (sqrt(3)*cos(5/6*arcta
n2(sin(2*d), cos(2*d)))*e^(5/3*a) - I*sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(si
n(2*d), cos(2*d))))*cos(4*b*x + 4*d) + 2*(sqrt(3)*cos(5/6*arctan2(sin(2*d)
, cos(2*d)))*e^(5/3*a) - I*sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos
(2*d))))*cos(2*b*x + 2*d) + (I*sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d))
)*e^(5/3*a) + sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))))*sin(
4*b*x + 4*d) + 2*(I*sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a)
 + sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))))*sin(2*b*x + 2*d
))*arctan2(sqrt(3)*cos(1/6*arctan2(sin(2*d), cos(2*d)))*sin(1/3*b*x) + sqr
t(3)*cos(1/3*b*x)*sin(1/6*arctan2(sin(2*d), cos(2*d))) + cos(1/3*arctan2(s
in(2*d), cos(2*d)))*sin(2/3*b*x) + cos(2/3*b*x)*sin(1/3*arctan2(sin(2*d),
cos(2*d))), sqrt(3)*cos(1/3*b*x)*cos(1/6*arctan2(sin(2*d), cos(2*d))) - sq
rt(3)*sin(1/3*b*x)*sin(1/6*arctan2(sin(2*d), cos(2*d))) + cos(2/3*b*x)*cos
(1/3*arctan2(sin(2*d), cos(2*d))) - sin(2/3*b*x)*sin(1/3*arctan2(sin(2*d),
 cos(2*d))) + 1) - 430*(sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/
3*a) - I*sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))) + (sqrt(3)
*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - I*sqrt(3)*e^(5/3*a)*sin(
5/6*arctan2(sin(2*d), cos(2*d))))*cos(4*b*x + 4*d) + 2*(sqrt(3)*cos(5/6*ar
ctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - I*sqrt(3)*e^(5/3*a)*sin(5/6*arct...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1290 vs. $2(195) = 390$.

Time = 1.60 (sec) , antiderivative size = 1290, normalized size of antiderivative = 4.23

$$\int e^{\frac{5}{3}(a+ibx)} \tan^3(d+bx)\,dx = \text{Too large to display}$$

input

```
integrate(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)^3,x, algorithm="giac")
```

output

```
1/36*I*(-86*I*(5*sqrt(3)*cos(1/3*d)^4*e^(5/3*a)*sin(1/3*d) - 10*sqrt(3)*co
s(1/3*d)^2*e^(5/3*a)*sin(1/3*d)^3 + sqrt(3)*e^(5/3*a)*sin(1/3*d)^5 - cos(1
/3*d)^5*e^(5/3*a) + 10*cos(1/3*d)^3*e^(5/3*a)*sin(1/3*d)^2 - 5*cos(1/3*d)*
e^(5/3*a)*sin(1/3*d)^4)*arctan((sqrt(3)*e^(-1/3*I*d) + 2*e^(1/3*I*b*x))*e^
(1/3*I*d))/(cos(1/3*d)^10 + 5*cos(1/3*d)^8*sin(1/3*d)^2 + 10*cos(1/3*d)^6*
sin(1/3*d)^4 + 10*cos(1/3*d)^4*sin(1/3*d)^6 + 5*cos(1/3*d)^2*sin(1/3*d)^8
+ sin(1/3*d)^10) + 86*I*(5*sqrt(3)*cos(1/3*d)^4*e^(5/3*a)*sin(1/3*d) - 10*
sqrt(3)*cos(1/3*d)^2*e^(5/3*a)*sin(1/3*d)^3 + sqrt(3)*e^(5/3*a)*sin(1/3*d)
^5 + cos(1/3*d)^5*e^(5/3*a) - 10*cos(1/3*d)^3*e^(5/3*a)*sin(1/3*d)^2 + 5*c
os(1/3*d)*e^(5/3*a)*sin(1/3*d)^4)*arctan(-(sqrt(3)*e^(-1/3*I*d) - 2*e^(1/3
*I*b*x))*e^(1/3*I*d))/(cos(1/3*d)^10 + 5*cos(1/3*d)^8*sin(1/3*d)^2 + 10*co
s(1/3*d)^6*sin(1/3*d)^4 + 10*cos(1/3*d)^4*sin(1/3*d)^6 + 5*cos(1/3*d)^2*si
n(1/3*d)^8 + sin(1/3*d)^10) + 172*I*(cos(1/3*d)^5*e^(5/3*a) - 10*cos(1/3*d
)^3*e^(5/3*a)*sin(1/3*d)^2 + 5*cos(1/3*d)*e^(5/3*a)*sin(1/3*d)^4)*arctan(e
^(1/3*I*b*x + 1/3*I*d))/(cos(1/3*d)^10 + 5*cos(1/3*d)^8*sin(1/3*d)^2 + 10*
cos(1/3*d)^6*sin(1/3*d)^4 + 10*cos(1/3*d)^4*sin(1/3*d)^6 + 5*cos(1/3*d)^2*
sin(1/3*d)^8 + sin(1/3*d)^10) - 43*I*(sqrt(3)*cos(1/3*d)^5*e^(5/3*a) - 10*
sqrt(3)*cos(1/3*d)^3*e^(5/3*a)*sin(1/3*d)^2 + 5*sqrt(3)*cos(1/3*d)*e^(5/3*
a)*sin(1/3*d)^4 + 5*cos(1/3*d)^4*e^(5/3*a)*sin(1/3*d) - 10*cos(1/3*d)^2*e^
(5/3*a)*sin(1/3*d)^3 + e^(5/3*a)*sin(1/3*d)^5)*log(sqrt(3)*e^(1/3*I*b*x...
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{3}(a+ibx)}\tan^3(d+bx)\,dx=\int e^{\frac{5\,a}{3}+\frac{b\,x\,5\mathrm{i}}{3}}\tan(d+b\,x)^3\,dx$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)*tan(d + b*x)^3,x)
```

output
```
int(exp((5*a)/3 + (b*x*5i)/3)*tan(d + b*x)^3, x)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\tan^3(d+bx)\,dx=\int e^{\frac{5bix}{3}+\frac{5a}{3}}\tan(bx+d)^3\,dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*tan(b*x+d)^3,x)
```

output
```
int(e**((5*a + 5*b*i*x)/3)*tan(b*x + d)**3,x)
```

3.52 $\int F^{c(a+bx)}\tan(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int F^{c(a+bx)}\tan(d+ex)\,dx = -\frac{iF^{c(a+bx)}}{bc\log(F)}+\frac{2iF^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)}$$

output
```
-I*F^(c*(b*x+a))/b/c/ln(F)+2*I*F^(c*(b*x+a))*hypergeom([1, -1/2*I*b*c*ln(F
)/e],[1-1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int F^{c(a+bx)}\tan(d+ex)\,dx = \frac{iF^{c(a+bx)}\left(-1+2\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)\right)}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Tan[d + e*x],x]
```

output
```
(I*F^(c*(a + b*x))*(-1 + 2*Hypergeometric2F1[1, ((-1/2*I)*b*c*Log[F])/e, 1
 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]))/(b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4942}$$

$$i\int \left(\frac{2F^{c(a+bx)}}{1+e^{2i(d+ex)}}-F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{F^{c(a+bx)}}{bc\log(F)}+\frac{2F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)}\right)$$

input
```
Int[F^(c*(a + b*x))*Tan[d + e*x],x]
```

output
```
I*(-(F^(c*(a + b*x))/(b*c*Log[F])) + (2*F^(c*(a + b*x))*Hypergeometric2F1[
1, ((-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))
])/(b*c*Log[F]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \tan(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*tan(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*tan(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)} \tan(d+ex)\,dx = \int F^{(bx+a)c} \tan(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*tan(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*tan(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\tan(d+ex)\,dx = \int F^{c(a+bx)}\tan{(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*tan(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*tan(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\tan(d+ex)\,dx = \int F^{(bx+a)c}\tan{(ex+d)}\,dx$$

input
```
integrate(F^(c*(b*x+a))*tan(e*x+d),x, algorithm="maxima")
```

output
```
2*(F^(b*c*x)*F^(a*c)*b*c*log(F)*sin(2*e*x + 2*d) - 2*F^(b*c*x)*F^(a*c)*e*c
os(2*e*x + 2*d) - 2*F^(b*c*x)*F^(a*c)*e + 2*(F^(a*c)*b^2*c^2*e*log(F)^2 +
4*F^(a*c)*e^3 + (F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*c)*e^3)*cos(2*e*x + 2
*d)^2 + (F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*c)*e^3)*sin(2*e*x + 2*d)^2 +
2*(F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*c)*e^3)*cos(2*e*x + 2*d))*integrate
((F^(b*c*x)*b*c*cos(4*e*x + 4*d)*log(F) + 2*F^(b*c*x)*b*c*cos(2*e*x + 2*d)
*log(F) + F^(b*c*x)*b*c*log(F) + 2*F^(b*c*x)*e*sin(4*e*x + 4*d) + 4*F^(b*c
*x)*e*sin(2*e*x + 2*d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + 4*e^2)*cos
(4*e*x + 4*d)^2 + 4*(b^2*c^2*log(F)^2 + 4*e^2)*cos(2*e*x + 2*d)^2 + (b^2*c
^2*log(F)^2 + 4*e^2)*sin(4*e*x + 4*d)^2 + 4*(b^2*c^2*log(F)^2 + 4*e^2)*sin
(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(b^2*c^2*log(F)^2 + 4*e^2)*sin(2*e*x +
2*d)^2 + 4*e^2 + 2*(b^2*c^2*log(F)^2 + 4*e^2 + 2*(b^2*c^2*log(F)^2 + 4*e^2
)*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) + 4*(b^2*c^2*log(F)^2 + 4*e^2)*cos(2*
e*x + 2*d)), x))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + 4*e^2)*cos(2*e*x
+ 2*d)^2 + (b^2*c^2*log(F)^2 + 4*e^2)*sin(2*e*x + 2*d)^2 + 4*e^2 + 2*(b^2*
c^2*log(F)^2 + 4*e^2)*cos(2*e*x + 2*d))
```

Giac [F]

$$\int F^{c(a+bx)}\tan(d+ex)\,dx=\int F^{(bx+a)c}\tan\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*tan(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*tan(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\tan(d+ex)\,dx=\int F^{c\,(a+b\,x)}\tan(d+e\,x)\,dx$$

input
```
int(F^(c*(a + b*x))*tan(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))*tan(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\tan(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\tan\left(ex+d\right)dx\right)$$

input
```
int(F^(c*(b*x+a))*tan(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*tan(d + e*x),x)
```

3.53 $\int F^{c(a+bx)} \tan^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 112

$$\int F^{c(a+bx)} \tan^2(d+ex)\,dx = \frac{2iF^{c(a+bx)}}{e\left(1+e^{2i(d+ex)}\right)} - \frac{2iF^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{e} - \frac{F^{c(a+bx)}}{bc\log(F)}$$

output
```
2*I*F^(c*(b*x+a))/e/(1+exp(2*I*(e*x+d)))-2*I*F^(c*(b*x+a))*hypergeom([1, -
1/2*I*b*c*ln(F)/e],[1-1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/e-F^(c*(b*x+a)
)/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int F^{c(a+bx)}\tan^2(d+ex)\,dx = F^{c(a+bx)}\left(\frac{2i}{e+ee^{2id}}-\frac{2i\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{e}-\frac{1}{bc\log(F)}+\frac{\sec(d)\sec(d+ex)\sin(ex)}{e}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Tan[d + e*x]^2,x]
```

output
```
F^(c*(a + b*x))*((2*I)/(e + e*E^((2*I)*d)) - ((2*I)*Hypergeometric2F1[1, (
(-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/e
 - 1/(b*c*Log[F]) + (Sec[d]*Sec[d + e*x]*Sin[e*x])/e)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(d+ex)F^{c(a+bx)}\,dx$$

$$\big\downarrow \text{ 4942}$$

$$-\int \left(-\frac{4F^{c(a+bx)}}{1+e^{2i(d+ex)}}+\frac{4F^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2}+F^{c(a+bx)}\right)dx$$

$$\big\downarrow \text{ 2009}$$

$$\frac{4F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)}-\frac{4F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)}-\frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Tan[d + e*x]^2,x]
```

output
```
-(F^(c*(a + b*x))/(b*c*Log[F])) + (4*F^(c*(a + b*x))*Hypergeometric2F1[1,
((-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/
(b*c*Log[F]) - (4*F^(c*(a + b*x))*Hypergeometric2F1[2, ((-1/2*I)*b*c*Log[F
])/e, 1 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\tan(ex+d)^2\,dx$$

input
```
int(F^(c*(b*x+a))*tan(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*tan(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\tan^2(d+ex)\,dx = \int F^{(bx+a)c}\tan{(ex+d)}^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*tan(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\tan^2(d+ex)\,dx = \int F^{c(a+bx)}\tan^2{(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*tan(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*tan(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\tan^2(d+ex)\,dx = \int F^{(bx+a)c}\tan{(ex+d)}^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="maxima")
```

output
```
-((F^(a*c)*b^4*c^4*log(F)^4 + 20*F^(a*c)*b^2*c^2*e^2*log(F)^2 + 64*F^(a*c)
*e^4)*F^(b*c*x)*cos(4*e*x + 4*d)^2 - 4*(F^(a*c)*b^4*c^4*log(F)^4 + 12*F^(a
*c)*b^2*c^2*e^2*log(F)^2 - 64*F^(a*c)*e^4)*F^(b*c*x)*cos(2*e*x + 2*d)^2 +
(F^(a*c)*b^4*c^4*log(F)^4 + 20*F^(a*c)*b^2*c^2*e^2*log(F)^2 + 64*F^(a*c)*e
^4)*F^(b*c*x)*sin(4*e*x + 4*d)^2 - 4*(F^(a*c)*b^4*c^4*log(F)^4 + 12*F^(a*c
)*b^2*c^2*e^2*log(F)^2 - 64*F^(a*c)*e^4)*F^(b*c*x)*sin(2*e*x + 2*d)^2 - 16
*(11*F^(a*c)*b^2*c^2*e^2*log(F)^2 - 16*F^(a*c)*e^4)*F^(b*c*x)*cos(2*e*x +
2*d) + 8*(5*F^(a*c)*b^3*c^3*e*log(F)^3 - 16*F^(a*c)*b*c*e^3*log(F))*F^(b*c
*x)*sin(2*e*x + 2*d) + (F^(a*c)*b^4*c^4*log(F)^4 - 76*F^(a*c)*b^2*c^2*e^2*
log(F)^2 + 64*F^(a*c)*e^4)*F^(b*c*x) + 2*(8*(F^(a*c)*b^2*c^2*e^2*log(F)^2
+ 16*F^(a*c)*e^4)*F^(b*c*x)*cos(2*e*x + 2*d) + 4*(F^(a*c)*b^3*c^3*e*log(F)
^3 + 16*F^(a*c)*b*c*e^3*log(F))*F^(b*c*x)*sin(2*e*x + 2*d) + (F^(a*c)*b^4*
c^4*log(F)^4 - 28*F^(a*c)*b^2*c^2*e^2*log(F)^2 + 64*F^(a*c)*e^4)*F^(b*c*x)
)*cos(4*e*x + 4*d) + 16*(F^(a*c)*b^6*c^6*e*log(F)^6 + 20*F^(a*c)*b^4*c^4*e
^3*log(F)^4 + 64*F^(a*c)*b^2*c^2*e^5*log(F)^2 + (F^(a*c)*b^6*c^6*e*log(F)^
6 + 20*F^(a*c)*b^4*c^4*e^3*log(F)^4 + 64*F^(a*c)*b^2*c^2*e^5*log(F)^2)*cos
(4*e*x + 4*d)^2 + 4*(F^(a*c)*b^6*c^6*e*log(F)^6 + 20*F^(a*c)*b^4*c^4*e^3*l
og(F)^4 + 64*F^(a*c)*b^2*c^2*e^5*log(F)^2)*cos(2*e*x + 2*d)^2 + (F^(a*c)*b
^6*c^6*e*log(F)^6 + 20*F^(a*c)*b^4*c^4*e^3*log(F)^4 + 64*F^(a*c)*b^2*c^2*e
^5*log(F)^2)*sin(4*e*x + 4*d)^2 + 4*(F^(a*c)*b^6*c^6*e*log(F)^6 + 20*F^...
```

Giac [F]

$$\int F^{c(a+bx)}\tan^2(d+ex)\,dx=\int F^{(bx+a)c}\tan\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*tan(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\tan^2(d+ex)\,dx=\int F^{c\,(a+b\,x)}\,\tan(d+e\,x)^2\,dx$$

input
```
int(F^(c*(a + b*x))*tan(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))*tan(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\tan^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\tan\left(ex+d\right)^2dx\right)$$

input
```
int(F^(c*(b*x+a))*tan(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*tan(d + e*x)**2,x)
```

3.54 $\int F^{c(a+bx)} \tan^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 171

$$\int F^{c(a+bx)} \tan^3(d+ex)\,dx = -\frac{2F^{c(a+bx)}}{e\left(1+e^{2i(d+ex)}\right)^2}+\frac{iF^{c(a+bx)}}{bc\log(F)}+\frac{F^{c(a+bx)}(2e-ibc\log(F))}{e^2\left(1+e^{2i(d+ex)}\right)} - iF^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)\left(\frac{2}{bc\log(F)}-\frac{bc\log(F)}{e^2}\right)$$

output
```
-2*F^(c*(b*x+a))/e/(1+exp(2*I*(e*x+d)))^2+I*F^(c*(b*x+a))/b/c/ln(F)+F^(c*(
b*x+a))*(2*e-I*b*c*ln(F))/e^2/(1+exp(2*I*(e*x+d)))-I*F^(c*(b*x+a))*hyperge
om([1, -1/2*I*b*c*ln(F)/e],[1-1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))*(2/b/c
/ln(F)-b*c*ln(F)/e^2)
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\tan^3(d+ex)\,dx$$
$$= \frac{F^{c(a+bx)}\left(2ie^2 - ib^2c^2\log^2(F) - 2i\,\text{Hypergeometric2F1}\left(1, -\frac{ibc\log(F)}{2e}, 1 - \frac{ibc\log(F)}{2e}, -\cos(2(d+ex)) - i\right.\right.}{}$$

input
```
Integrate[F^(c*(a + b*x))*Tan[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*((2*I)*e^2 - I*b^2*c^2*Log[F]^2 - (2*I)*Hypergeometric2F1
[1, ((-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, -Cos[2*(d + e*x)] -
 I*Sin[2*(d + e*x)]]*(2*e^2 - b^2*c^2*Log[F]^2) + b*c*e*Log[F]*Sec[d + e*x
]^2 - b^2*c^2*Log[F]^2*Sec[d]*Sec[d + e*x]*Sin[e*x] - b^2*c^2*Log[F]^2*Tan
[d]))/(2*b*c*e^2*Log[F])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{4942}$$
$$-i\int \left(\frac{6F^{c(a+bx)}}{1+e^{2i(d+ex)}} - \frac{12F^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2} + \frac{8F^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^3} - F^{c(a+bx)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{6F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)}-\frac{12F^{c(a+bx)}\text{Hypergeometric2F1}\left(2\right.}{bc}\right.$$

input
```
Int[F^(c*(a + b*x))*Tan[d + e*x]^3,x]
```

output
```
(-I)*(-(F^(c*(a + b*x))/(b*c*Log[F])) + (6*F^(c*(a + b*x))*Hypergeometric2
F1[1, ((-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*
x))])/(b*c*Log[F]) - (12*F^(c*(a + b*x))*Hypergeometric2F1[2, ((-1/2*I)*b*
c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/(b*c*Log[F])
 + (8*F^(c*(a + b*x))*Hypergeometric2F1[3, ((-1/2*I)*b*c*Log[F])/e, 1 - ((
I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/(b*c*Log[F]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\tan(ex+d)^3\,dx$$

input
```
int(F^(c*(b*x+a))*tan(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*tan(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \tan^3(d+ex)\, dx = \int F^{(bx+a)c} \tan{(ex+d)}^3 \, dx$$

input
```
integrate(F^(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*tan(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \tan^3(d+ex)\, dx = \int F^{c(a+bx)} \tan^3{(d+ex)}\, dx$$

input
```
integrate(F**(c*(b*x+a))*tan(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*tan(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \tan^3(d+ex)\, dx = \int F^{(bx+a)c} \tan{(ex+d)}^3 \, dx$$

input
```
integrate(F^(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="maxima")
```

output
```
2*(18*(F^(a*c)*b^4*c^4*e*log(F)^4 + 52*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 576*
F^(a*c)*e^5)*F^(b*c*x)*cos(4*e*x + 4*d)^2 - 54*(F^(a*c)*b^4*c^4*e*log(F)^4
 + 28*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 288*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x
+ 2*d)^2 + 18*(F^(a*c)*b^4*c^4*e*log(F)^4 + 52*F^(a*c)*b^2*c^2*e^3*log(F)^
2 + 576*F^(a*c)*e^5)*F^(b*c*x)*sin(4*e*x + 4*d)^2 - 54*(F^(a*c)*b^4*c^4*e*
log(F)^4 + 28*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 288*F^(a*c)*e^5)*F^(b*c*x)*si
n(2*e*x + 2*d)^2 + 18*(3*F^(a*c)*b^4*c^4*e*log(F)^4 - 212*F^(a*c)*b^2*c^2*
e^3*log(F)^2 + 640*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) - 3*(F^(a*c)*b^
5*c^5*log(F)^5 - 268*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 1216*F^(a*c)*b*c*e^4*l
og(F))*F^(b*c*x)*sin(2*e*x + 2*d) + 24*(F^(a*c)*b^4*c^4*e*log(F)^4 - 46*F^
(a*c)*b^2*c^2*e^3*log(F)^2 + 88*F^(a*c)*e^5)*F^(b*c*x) + 3*(2*(F^(a*c)*b^4
*c^4*e*log(F)^4 + 52*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 576*F^(a*c)*e^5)*F^(b*
c*x)*cos(4*e*x + 4*d) - 6*(F^(a*c)*b^4*c^4*e*log(F)^4 + 28*F^(a*c)*b^2*c^2
*e^3*log(F)^2 - 288*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) + (F^(a*c)*b^5
*c^5*log(F)^5 + 52*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 576*F^(a*c)*b*c*e^4*log(
F))*F^(b*c*x)*sin(4*e*x + 4*d) + 36*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 36*F^(
a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(2*e*x + 2*d) + 8*(F^(a*c)*b^4*c^4*e*log
(F)^4 - 46*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 88*F^(a*c)*e^5)*F^(b*c*x))*cos(6
*e*x + 6*d) - 3*(12*(F^(a*c)*b^4*c^4*e*log(F)^4 + 16*F^(a*c)*b^2*c^2*e^3*l
og(F)^2 - 720*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) + 3*(F^(a*c)*b^5*...
```

Giac [F]

$$\int F^{c(a+bx)}\tan^3(d+ex)\,dx=\int F^{(bx+a)c}\tan(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*tan(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\tan^3(d+ex)\,dx=\int F^{c\,(a+b\,x)}\,\tan(d+e\,x)^3\,dx$$

input
```
int(F^(c*(a + b*x))*tan(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))*tan(d + e*x)^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\tan^3(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\tan(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*tan(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*tan(d + e*x)**3,x)
```

3.55 $\int F^{c(a+bx)} \tan^4(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 235

$$\int F^{c(a+bx)} \tan^4(d+ex)\,dx = -\frac{8iF^{c(a+bx)}}{3e\left(1+e^{2i(d+ex)}\right)^3}+\frac{F^{c(a+bx)}}{bc\log(F)}+\frac{2F^{c(a+bx)}(6ie+bc\log(F))}{3e^2\left(1+e^{2i(d+ex)}\right)^2}+\frac{iF^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)\left(8e^2-b^2c^2\log^2(F)\right)}{3e^3}-\frac{F^{c(a+bx)}\left(12ie^2+2bce\log(F)-ib^2c^2\log^2(F)\right)}{3e^3\left(1+e^{2i(d+ex)}\right)}$$

output
```
-8/3*I*F^(c*(b*x+a))/e/(1+exp(2*I*(e*x+d)))^3+F^(c*(b*x+a))/b/c/ln(F)+2/3*
F^(c*(b*x+a))*(6*I*e+b*c*ln(F))/e^2/(1+exp(2*I*(e*x+d)))^2+1/3*I*F^(c*(b*x
+a))*hypergeom([1, -1/2*I*b*c*ln(F)/e],[1-1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x
+d)))*(8*e^2-b^2*c^2*ln(F)^2)/e^3-1/3*F^(c*(b*x+a))*(12*I*e^2+2*b*c*e*ln(F
)-I*b^2*c^2*ln(F)^2)/e^3/(1+exp(2*I*(e*x+d)))
```

Mathematica [A] (verified)

Time = 6.93 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\tan^4(d+ex)\,dx$$
$$=\frac{F^{ac+bcx}}{bc\log(F)}+\frac{F^{ac+bcx}\sec(d)\sec^2(d+ex)(-bc\cos(d)\log(F)+2e\sin(d))}{6e^2}$$
$$-\frac{F^{ac+bcx}\left(8e^2-b^2c^2\log^2(F)\right)\sec(d)\sec(d+ex)\sin(ex)}{6e^3}$$
$$+\frac{F^{ac+bcx}\sec(d)\sec^3(d+ex)\sin(ex)}{3e}$$
$$+\frac{iF^{c(a+bx)}\left(8e^2-b^2c^2\log^2(F)\right)\left(-1+2\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-\cos(2(d+ex\right.\right.}{6e^3}$$

input
```
Integrate[F^(c*(a + b*x))*Tan[d + e*x]^4,x]
```

output
```
F^(a*c + b*c*x)/(b*c*Log[F]) + (F^(a*c + b*c*x)*Sec[d]*Sec[d + e*x]^2*(-(b
*c*Cos[d]*Log[F]) + 2*e*Sin[d]))/(6*e^2) - (F^(a*c + b*c*x)*(8*e^2 - b^2*c
^2*Log[F]^2)*Sec[d]*Sec[d + e*x]*Sin[e*x])/(6*e^3) + (F^(a*c + b*c*x)*Sec[
d]*Sec[d + e*x]^3*Sin[e*x])/(3*e) + ((I/6)*F^(c*(a + b*x))*(8*e^2 - b^2*c^
2*Log[F]^2)*(-1 + 2*Hypergeometric2F1[1, ((-1/2*I)*b*c*Log[F])/e, 1 - ((I/
2)*b*c*Log[F])/e, -Cos[2*(d + e*x)] - I*Sin[2*(d + e*x)]] + I*Tan[d]))/e^3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{4942}$$

$$\int \left(-\frac{8F^{c(a+bx)}}{1+e^{2i(d+ex)}} + \frac{24F^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2} - \frac{32F^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^3} + \frac{16F^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^4} + F^{c(a+bx)} \right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)} + \frac{24F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)} - \frac{32F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)} + \frac{16F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(4,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)} + \frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Tan[d + e*x]^4,x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (8*F^(c*(a + b*x))*Hypergeometric2F1[1, ((-
1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/(b*
c*Log[F]) + (24*F^(c*(a + b*x))*Hypergeometric2F1[2, ((-1/2*I)*b*c*Log[F])
/e, 1 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/(b*c*Log[F]) - (32*F^
(c*(a + b*x))*Hypergeometric2F1[3, ((-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c
*Log[F])/e, -E^((2*I)*(d + e*x))])/(b*c*Log[F]) + (16*F^(c*(a + b*x))*Hype
rgeometric2F1[4, ((-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, -E^((2
*I)*(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \tan(ex+d)^4 \, dx$$

input
```
int(F^(c*(b*x+a))*tan(e*x+d)^4,x)
```

output
```
int(F^(c*(b*x+a))*tan(e*x+d)^4,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \tan^4(d+ex) \, dx = \int F^{(bx+a)c} \tan(ex+d)^4 \, dx$$

input
```
integrate(F^(c*(b*x+a))*tan(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*tan(e*x + d)^4, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \tan^4(d+ex) \, dx = \int F^{c(a+bx)} \tan^4(d+ex) \, dx$$

input
```
integrate(F**(c*(b*x+a))*tan(e*x+d)**4,x)
```

output
```
Integral(F**(c*(a + b*x))*tan(d + e*x)**4, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\tan^4(d+ex)\,dx = \int F^{(bx+a)c}\tan\left(ex+d\right)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*tan(e*x+d)^4,x, algorithm="maxima")
```

output
```
((F^(a*c)*b^8*c^8*log(F)^8 + 120*F^(a*c)*b^6*c^6*e^2*log(F)^6 + 4368*F^(a*
c)*b^4*c^4*e^4*log(F)^4 + 52480*F^(a*c)*b^2*c^2*e^6*log(F)^2 + 147456*F^(a
*c)*e^8)*F^(b*c*x)*cos(8*e*x + 8*d)^2 - 16*(F^(a*c)*b^8*c^8*log(F)^8 + 112
*F^(a*c)*b^6*c^6*e^2*log(F)^6 + 3440*F^(a*c)*b^4*c^4*e^4*log(F)^4 + 21248*
F^(a*c)*b^2*c^2*e^6*log(F)^2 - 147456*F^(a*c)*e^8)*F^(b*c*x)*cos(6*e*x + 6
*d)^2 + 36*(F^(a*c)*b^8*c^8*log(F)^8 + 56*F^(a*c)*b^6*c^6*e^2*log(F)^6 - 2
032*F^(a*c)*b^4*c^4*e^4*log(F)^4 - 94976*F^(a*c)*b^2*c^2*e^6*log(F)^2 + 14
7456*F^(a*c)*e^8)*F^(b*c*x)*cos(4*e*x + 4*d)^2 - 16*(F^(a*c)*b^8*c^8*log(F
)^8 - 208*F^(a*c)*b^6*c^6*e^2*log(F)^6 - 14480*F^(a*c)*b^4*c^4*e^4*log(F)^
4 + 185088*F^(a*c)*b^2*c^2*e^6*log(F)^2 - 147456*F^(a*c)*e^8)*F^(b*c*x)*co
s(2*e*x + 2*d)^2 + (F^(a*c)*b^8*c^8*log(F)^8 + 120*F^(a*c)*b^6*c^6*e^2*log
(F)^6 + 4368*F^(a*c)*b^4*c^4*e^4*log(F)^4 + 52480*F^(a*c)*b^2*c^2*e^6*log(
F)^2 + 147456*F^(a*c)*e^8)*F^(b*c*x)*sin(8*e*x + 8*d)^2 - 16*(F^(a*c)*b^8*
c^8*log(F)^8 + 112*F^(a*c)*b^6*c^6*e^2*log(F)^6 + 3440*F^(a*c)*b^4*c^4*e^4
*log(F)^4 + 21248*F^(a*c)*b^2*c^2*e^6*log(F)^2 - 147456*F^(a*c)*e^8)*F^(b*
c*x)*sin(6*e*x + 6*d)^2 + 36*(F^(a*c)*b^8*c^8*log(F)^8 + 56*F^(a*c)*b^6*c^
6*e^2*log(F)^6 - 2032*F^(a*c)*b^4*c^4*e^4*log(F)^4 - 94976*F^(a*c)*b^2*c^2
*e^6*log(F)^2 + 147456*F^(a*c)*e^8)*F^(b*c*x)*sin(4*e*x + 4*d)^2 - 16*(F^(
a*c)*b^8*c^8*log(F)^8 - 208*F^(a*c)*b^6*c^6*e^2*log(F)^6 - 14480*F^(a*c)*b
^4*c^4*e^4*log(F)^4 + 185088*F^(a*c)*b^2*c^2*e^6*log(F)^2 - 147456*F^(a...
```

Giac [F]

$$\int F^{c(a+bx)}\tan^4(d+ex)\,dx = \int F^{(bx+a)c}\tan\left(ex+d\right)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*tan(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*tan(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\tan^4(d+ex)\,dx=\int F^{c\,(a+b\,x)}\,\tan(d+e\,x)^4\,dx$$

input
```
int(F^(c*(a + b*x))*tan(d + e*x)^4,x)
```

output
```
int(F^(c*(a + b*x))*tan(d + e*x)^4, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\tan^4(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\tan\left(ex+d\right)^4dx\right)$$

input
```
int(F^(c*(b*x+a))*tan(e*x+d)^4,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*tan(d + e*x)**4,x)
```

3.56 $\int e^{a+ibx} \tan^n(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int e^{a+ibx} \tan^n(a+bx)\,dx = -\frac{ie^{a+ibx}\left(1-e^{2i(a+bx)}\right)^{-n}\left(1+e^{2i(a+bx)}\right)^{n}\operatorname{AppellF1}\left(\frac{1}{2},-n,n,\frac{3}{2},e^{2i(a+bx)},-e^{2i(a+bx)}\right)\tan^n(a+bx)}{b}$$

output
```
-I*exp(a+I*b*x)*(1+exp(2*I*(b*x+a)))^n*AppellF1(1/2,n,-n,3/2,-exp(2*I*(b*x
+a)),exp(2*I*(b*x+a)))*tan(b*x+a)^n/b/((1-exp(2*I*(b*x+a)))^n)
```

Mathematica [F]

$$\int e^{a+ibx} \tan^n(a+bx)\,dx = \int e^{a+ibx} \tan^n(a+bx)\,dx$$

input
```
Integrate[E^(a + I*b*x)*Tan[a + b*x]^n,x]
```

output
```
Integrate[E^(a + I*b*x)*Tan[a + b*x]^n, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\tan^n(a+bx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int e^{a+ibx}\tan^n(a+bx)dx$$

input
```
Int[E^(a + I*b*x)*Tan[a + b*x]^n,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \mathrm{e}^{ibx+a}\tan\left(bx+a\right)^n dx$$

input
```
int(exp(a+I*b*x)*tan(b*x+a)^n,x)
```

output
```
int(exp(a+I*b*x)*tan(b*x+a)^n,x)
```

Fricas [F]

$$\int e^{a+ibx}\tan^n(a+bx)\,dx = \int \tan\left(bx+a\right)^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral(((-I*e^(2*I*b*x + 2*I*a) + I)/(e^(2*I*b*x + 2*I*a) + 1))^n*e^(I*b
*x + a), x)
```

Sympy [F]

$$\int e^{a+ibx}\tan^n(a+bx)\,dx = e^a\int e^{ibx}\tan^n\left(a+bx\right)dx$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+a)**n,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*tan(a + b*x)**n, x)
```

Maxima [F]

$$\int e^{a+ibx}\tan^n(a+bx)\,dx = \int \tan\left(bx+a\right)^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(tan(b*x + a)^n*e^(I*b*x + a), x)
```

Giac [F]

$$\int e^{a+ibx}\tan^n(a+bx)\,dx=\int \tan{(bx+a)}^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*tan(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(tan(b*x + a)^n*e^(I*b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\tan^n(a+bx)\,dx=\int e^{a+b\,x\,1\mathrm{i}}\,\tan(a+b\,x)^n\,dx$$

input
```
int(exp(a + b*x*1i)*tan(a + b*x)^n,x)
```

output
```
int(exp(a + b*x*1i)*tan(a + b*x)^n, x)
```

Reduce [F]

$$\int e^{a+ibx}\tan^n(a+bx)\,dx = \frac{e^a i\left(-e^{bix}\tan{(bx+a)}^n+\left(\int\frac{e^{bix}\tan(bx+a)^n}{\tan(bx+a)}dx\right)bn+\left(\int e^{bix}\tan{(bx+a)}^n\tan{(bx+a)}\,dx\right)bn\right)}{b}$$

input
```
int(exp(a+I*b*x)*tan(b*x+a)^n,x)
```

output
```
(e**a*i*( - e**(b*i*x)*tan(a + b*x)**n + int((e**(b*i*x)*tan(a + b*x)**n)/
tan(a + b*x),x)*b*n + int(e**(b*i*x)*tan(a + b*x)**n*tan(a + b*x),x)*b*n))
/b
```

3.57 $\int F^{c(a+bx)}(f\tan(d+ex))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\int F^{c(a+bx)}(f\tan(d+ex))^n\,dx = \frac{i2^{-1-n}\left(e^{2i(d+ex)}\right)^{\frac{ibc\log(F)}{2e}}\left(1-e^{2i(d+ex)}\right)\left(1+e^{2i(d+ex)}\right)^n F^{c(a+bx)}\,\mathrm{AppellF1}\left(1+n,1+\frac{ibc\log(F)}{2e},n,2+n,1\right.}{e(1+n)}$$

output
```
I*2^(-1-n)*exp(2*I*(e*x+d))^(1/2*I*b*c*ln(F)/e)*(1-exp(2*I*(e*x+d)))*(1+ex
p(2*I*(e*x+d)))^n*F^(c*(b*x+a))*AppellF1(1+n,1+1/2*I*b*c*ln(F)/e,n,2+n,1-e
xp(2*I*(e*x+d)),1/2-1/2*exp(2*I*(e*x+d)))*(f*tan(e*x+d))^n/e/(1+n)
```

Mathematica [F]

$$\int F^{c(a+bx)}(f\tan(d+ex))^n\,dx = \int F^{c(a+bx)}(f\tan(d+ex))^n\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*Tan[d + e*x])^n,x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*Tan[d + e*x])^n, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\tan(d+ex))^n\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\tan^{-n}(d+ex)(f\tan(d+ex))^n\int F^{c(a+bx)}\tan^n(d+ex)dx$$

$$\Big\downarrow \text{ 4967}$$

$$\tan^{-n}(d+ex)(f\tan(d+ex))^n\int F^{ac+bxc}\tan^n(d+ex)dx$$

$$\Big\downarrow \text{ 7299}$$

$$\tan^{-n}(d+ex)(f\tan(d+ex))^n\int F^{ac+bxc}\tan^n(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(f*Tan[d + e*x])^n,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4967
```
Int[(F_)^((c_.)*(u_))*(G_)[v_]^(n_.), x_Symbol] :> Int[F^(c*ExpandToSum[u,
x])*G[ExpandToSum[v, x]]^n, x] /; FreeQ[{F, c, n}, x] && TrigQ[G] && Linear
Q[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int F^{c(bx+a)}(f\tan(ex+d))^n\,dx$$

input
```
int(F^(c*(b*x+a))*(f*tan(e*x+d))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*tan(e*x+d))^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\tan(d+ex))^n\,dx = \int (f\tan(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*tan(e*x+d))^n,x, algorithm="fricas")
```

output
```
integral((f*tan(e*x + d))^n*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\tan(d+ex))^n\,dx = \int F^{c(a+bx)}(f\tan(d+ex))^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*tan(e*x+d))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*tan(d + e*x))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\tan(d+ex))^n\,dx = \int (f\tan(ex+d))^n\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*tan(e*x+d))^n,x, algorithm="maxima")
```

output
```
integrate((f*tan(e*x + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\tan(d+ex))^n\,dx = \int (f\tan(ex+d))^n\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*tan(e*x+d))^n,x, algorithm="giac")
```

output
```
integrate((f*tan(e*x + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\tan(d+ex))^n\,dx = \int F^{c\,(a+b\,x)}\,(f\tan(d+e\,x))^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*tan(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*tan(d + e*x))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\tan(d+ex))^n\,dx = f^{ac+n}\left(\int f^{bcx}\tan\left(ex+d\right)^n dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*tan(e*x+d))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*tan(d + e*x)**n,x)
```

3.58 $\int e^{a+ibx}\cot(d+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int e^{a+ibx}\cot(d+bx)\,dx=\frac{e^{a-id+i(d+bx)}}{b}-\frac{2e^{a-id}\text{arctanh}\left(e^{i(d+bx)}\right)}{b}$$

output
```
exp(a-I*d+I*(b*x+d))/b-2*exp(a-I*d)*arctanh(exp(I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int e^{a+ibx}\cot(d+bx)\,dx=\frac{e^a\left(e^{ibx}-2\text{arctanh}\left(e^{ibx}(\cos(d)+i\sin(d))\right)(\cos(d)-i\sin(d))\right)}{b}$$

input
```
Integrate[E^(a + I*b*x)*Cot[d + b*x],x]
```

output
```
(E^a*(E^(I*b*x) - 2*ArcTanh[E^(I*b*x)*(Cos[d] + I*Sin[d])]*(Cos[d] - I*Sin
[d])))/b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\cot(bx+d)\,dx$$

$$\Big\downarrow \text{4943}$$

$$-i\int\left(\frac{2e^{a+ibx}}{1-e^{2i(d+bx)}}-e^{a+ibx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{ie^{a+ibx}}{b}-\frac{2ie^{a-id}\text{arctanh}\left(e^{ibx+id}\right)}{b}\right)$$

input
```
Int[E^(a + I*b*x)*Cot[d + b*x],x]
```

output
```
(-I)*((I*E^(a + I*b*x))/b - ((2*I)*E^(a - I*d)*ArcTanh[E^(I*d + I*b*x)])/b
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{e^{a} e^{i b x}}{b}-\frac{2 e^{a} e^{-i d} \operatorname{arctanh}\left(e^{i(b x+d)}\right)}{b}$	36

input
```
int(exp(a+I*b*x)*cot(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
exp(a)*exp(I*b*x)/b-2*exp(a)/b*exp(-I*d)*arctanh(exp(I*(b*x+d)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(33) = 66.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.78

$$\int e^{a+ibx}\cot(d+bx)\,dx = -\frac{b\sqrt{\frac{1}{b^2}}e^{(a-i\,d)}\log\left(b\sqrt{\frac{1}{b^2}}e^{(a-i\,d)}+e^{(i\,bx+a)}\right)-b\sqrt{\frac{1}{b^2}}e^{(a-i\,d)}\log\left(-b\sqrt{\frac{1}{b^2}}e^{(a-i\,d)}+e^{(i\,bx+a)}\right)-e^{(i\,bx+a)}}{b}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d),x, algorithm="fricas")
```

output
```
-(b*sqrt(b^(-2))*e^(a - I*d)*log(b*sqrt(b^(-2))*e^(a - I*d) + e^(I*b*x + a
)) - b*sqrt(b^(-2))*e^(a - I*d)*log(-b*sqrt(b^(-2))*e^(a - I*d) + e^(I*b*x
 + a)) - e^(I*b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int e^{a+ibx}\cot(d+bx)\,dx = \begin{cases} \frac{e^a e^{ibx}}{b} & \text{for } b \neq 0 \\ ixe^a & \text{otherwise} \end{cases} + \frac{\left(\log\left(e^{ibx}-e^{-id}\right)-\log\left(e^{ibx}+e^{-id}\right)\right)e^a e^{-id}}{b}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d),x)
```

output
```
Piecewise((exp(a)*exp(I*b*x)/b, Ne(b, 0)), (I*x*exp(a), True)) + (log(exp(
I*b*x) - exp(-I*d)) - log(exp(I*b*x) + exp(-I*d)))*exp(a)*exp(-I*d)/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(33) = 66.

Time = 0.04 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.84

$$\int e^{a+ibx}\cot(d+bx)\,dx$$
$$= \frac{2\left(i\cos(d)e^a+e^a\sin(d)\right)\arctan\left(\sin(bx)+\sin(d),\cos(bx)-\cos(d)\right)+2\left(-i\cos(d)e^a-e^a\sin(d)\right)\text{ar}}{}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d),x, algorithm="maxima")
```

output
```
1/2*(2*(I*cos(d)*e^a + e^a*sin(d))*arctan2(sin(b*x) + sin(d), cos(b*x) - c
os(d)) + 2*(-I*cos(d)*e^a - e^a*sin(d))*arctan2(sin(b*x) - sin(d), cos(b*x
) + cos(d)) + 2*cos(b*x)*e^a - (cos(d)*e^a - I*e^a*sin(d))*log(cos(b*x)^2
+ 2*cos(b*x)*cos(d) + cos(d)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(d) + sin(d)^2
) + (cos(d)*e^a - I*e^a*sin(d))*log(cos(b*x)^2 - 2*cos(b*x)*cos(d) + cos(d
)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(d) + sin(d)^2) + 2*I*e^a*sin(b*x))/b
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int e^{a+ibx}\cot(d+bx)\,dx = -\frac{e^{(a-i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+i\right)-e^{(a-i\,d)}\log\left(-i\,e^{(i\,bx+i\,d)}+i\right)-e^{(i\,bx+a)}}{b}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d),x, algorithm="giac")
```

output
```
-(e^(a - I*d)*log(I*e^(I*b*x + I*d) + I) - e^(a - I*d)*log(-I*e^(I*b*x + I
*d) + I) - e^(I*b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 16.63 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.69

$$\int e^{a+ibx}\cot(d+bx)\,dx = \frac{e^{a+b\,x\,1i}}{b} - \frac{\sqrt{e^{2\,a-d\,2i}}\,\ln\left(-e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}\,2i-e^{2\,a}\,e^{-d\,2i}\,\sqrt{e^{2\,a}\,e^{-d\,2i}}\,2i\right)}{b} + \frac{\sqrt{e^{2\,a-d\,2i}}\,\ln\left(-e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}\,2i+e^{2\,a}\,e^{-d\,2i}\,\sqrt{e^{2\,a}\,e^{-d\,2i}}\,2i\right)}{b}$$

input
```
int(cot(d + b*x)*exp(a + b*x*1i),x)
```

output
```
exp(a + b*x*1i)/b - (exp(2*a - d*2i)^(1/2)*log(- exp(3*a)*exp(-d*2i)*exp(b
*x*1i)*2i - exp(2*a)*exp(-d*2i)*(exp(2*a)*exp(-d*2i))^(1/2)*2i))/b + (exp(
2*a - d*2i)^(1/2)*log(exp(2*a)*exp(-d*2i)*(exp(2*a)*exp(-d*2i))^(1/2)*2i -
 exp(3*a)*exp(-d*2i)*exp(b*x*1i)*2i))/b
```

Reduce [F]

$$\int e^{a+ibx}\cot(d+bx)\,dx = e^a\left(\int e^{bix}\cot(bx+d)\,dx\right)$$

input
```
int(exp(a+I*b*x)*cot(b*x+d),x)
```

output
```
e**a*int(e**(b*i*x)*cot(b*x + d),x)
```

3.59 $\int e^{a+ibx} \cot^2(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 96

$$\int e^{a+ibx} \cot^2(d+bx)\,dx = \frac{ie^{a-id+i(d+bx)}}{b} + \frac{2ie^{a-id+i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)} - \frac{2ie^{a-id}\text{arctanh}\left(e^{i(d+bx)}\right)}{b}$$

output
```
I*exp(a-I*d+I*(b*x+d))/b+2*I*exp(a-I*d+I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))-2
*I*exp(a-I*d)*arctanh(exp(I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.33

$$\int e^{a+ibx} \cot^2(d+bx)\,dx$$
$$= \frac{e^a\left(-e^{ibx}\left(\left(-3+e^{2ibx}\right)\cos(d)+i\left(3+e^{2ibx}\right)\sin(d)\right)+2\text{arctanh}\left(e^{ibx}(\cos(d)+i\sin(d))\right)\left(e^{2ibx}-\cos^2(d)-\right.\right.}{ib\left(-1+e^{2ibx}\right)\cos(d)-b\left(1+e^{2ibx}\right)\sin(d)}$$

input
```
Integrate[E^(a + I*b*x)*Cot[d + b*x]^2,x]
```

output
```
(E^a*(-(E^(I*b*x)*((-3 + E^((2*I)*b*x))*Cos[d] + I*(3 + E^((2*I)*b*x))*Sin
[d])) + 2*ArcTanh[E^(I*b*x)*(Cos[d] + I*Sin[d])]*(E^((2*I)*b*x) - Cos[d]^2
 + Sin[d]^2 + I*Sin[2*d]))/(I*b*(-1 + E^((2*I)*b*x))*Cos[d] - b*(1 + E^((
2*I)*b*x))*Sin[d])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\cot^2(bx+d)\,dx$$
$$\Big\downarrow \text{4943}$$
$$-\int \left(e^{a+ibx}-\frac{4e^{a+ibx}}{1-e^{2i(d+bx)}}+\frac{4e^{a+ibx}}{\left(1-e^{2i(d+bx)}\right)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2ie^{a-id}\text{arctanh}\left(e^{ibx+id}\right)}{b}+\frac{2ie^{a+ibx}}{b\left(1-e^{2i(bx+d)}\right)}+\frac{ie^{a+ibx}}{b}$$

input
```
Int[E^(a + I*b*x)*Cot[d + b*x]^2,x]
```

output
```
(I*E^(a + I*b*x))/b + ((2*I)*E^(a + I*b*x))/(b*(1 - E^((2*I)*(d + b*x))))
- ((2*I)*E^(a - I*d)*ArcTanh[E^(I*d + I*b*x)])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{i e^{a} e^{i b x}}{b}-\frac{2 i e^{a} e^{i b x}}{b \left(-1+e^{2 i (b x+d)}\right)}-\frac{2 i e^{a} e^{-i d} \operatorname{arctanh}\left(e^{i (b x+d)}\right)}{b}$	66

input
```
int(exp(a+I*b*x)*cot(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
I/b*exp(a)*exp(I*b*x)-2*I*exp(a)*exp(I*b*x)/b/(-1+exp(2*I*(b*x+d)))-2*I*ex
p(a)/b*exp(-I*d)*arctanh(exp(I*(b*x+d)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(59) = 118.

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.47

$$\int e^{a+ibx}\cot^2(d+bx)\,dx$$
$$= \frac{\left(-i\,be^{(2i\,bx+2i\,d)}+i\,b\right)\sqrt{\frac{1}{b^2}}e^{(a-i\,d)}\log\left(b\sqrt{\frac{1}{b^2}}e^{(a-i\,d)}+e^{(i\,bx+a)}\right)+\left(i\,be^{(2i\,bx+2i\,d)}-i\,b\right)\sqrt{\frac{1}{b^2}}e^{(a-i\,d)}\log\left(-b\sqrt{\right.}}{be^{(2i\,bx+2i\,d)}-b}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d)^2,x, algorithm="fricas")
```

output
```
((-I*b*e^(2*I*b*x + 2*I*d) + I*b)*sqrt(b^(-2))*e^(a - I*d)*log(b*sqrt(b^(-
2))*e^(a - I*d) + e^(I*b*x + a)) + (I*b*e^(2*I*b*x + 2*I*d) - I*b)*sqrt(b^
(-2))*e^(a - I*d)*log(-b*sqrt(b^(-2))*e^(a - I*d) + e^(I*b*x + a)) + I*e^(
3*I*b*x + a + 2*I*d) - 3*I*e^(I*b*x + a))/(b*e^(2*I*b*x + 2*I*d) - b)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int e^{a+ibx}\cot^2(d+bx)\,dx = \begin{cases} \frac{ie^ae^{ibx}}{b} & \text{for } b\neq 0 \\ -xe^a & \text{otherwise} \end{cases} - \frac{2ie^ae^{ibx}}{be^{2id}e^{2ibx}-b} + \frac{e^ae^{-id}\,\text{RootSum}\left(z^2+1,\left(i\mapsto i\log\left(iie^{-id}+e^{ibx}\right)\right)\right)}{b}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d)**2,x)
```

output
```
Piecewise((I*exp(a)*exp(I*b*x)/b, Ne(b, 0)), (-x*exp(a), True)) - 2*I*exp(
a)*exp(I*b*x)/(b*exp(2*I*d)*exp(2*I*b*x) - b) + exp(a)*exp(-I*d)*RootSum(_
z**2 + 1, Lambda(_i, _i*log(_i*I*exp(-I*d) + exp(I*b*x))))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(59) = 118.

Time = 0.05 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.41

$$\int e^{a+ibx}\cot^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d)^2,x, algorithm="maxima")
```

output
```
-(2*((-I*cos(d)*e^a - e^a*sin(d))*cos(2*b*x + 2*d) + I*cos(d)*e^a + (cos(d
)*e^a - I*e^a*sin(d))*sin(2*b*x + 2*d) + e^a*sin(d))*arctan2(sin(b*x) + si
n(d), cos(b*x) - cos(d)) + 2*((I*cos(d)*e^a + e^a*sin(d))*cos(2*b*x + 2*d)
 - I*cos(d)*e^a - (cos(d)*e^a - I*e^a*sin(d))*sin(2*b*x + 2*d) - e^a*sin(d
))*arctan2(sin(b*x) - sin(d), cos(b*x) + cos(d)) - 2*cos(3*b*x + 2*d)*e^a
+ 6*cos(b*x)*e^a + ((cos(d)*e^a - I*e^a*sin(d))*cos(2*b*x + 2*d) - cos(d)*
e^a - (-I*cos(d)*e^a - e^a*sin(d))*sin(2*b*x + 2*d) + I*e^a*sin(d))*log(co
s(b*x)^2 + 2*cos(b*x)*cos(d) + cos(d)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(d) +
 sin(d)^2) - ((cos(d)*e^a - I*e^a*sin(d))*cos(2*b*x + 2*d) - cos(d)*e^a +
(I*cos(d)*e^a + e^a*sin(d))*sin(2*b*x + 2*d) + I*e^a*sin(d))*log(cos(b*x)^
2 - 2*cos(b*x)*cos(d) + cos(d)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(d) + sin(d)
^2) - 2*I*e^a*sin(3*b*x + 2*d) + 6*I*e^a*sin(b*x))/(-2*I*b*cos(2*b*x + 2*d
) + 2*b*sin(2*b*x + 2*d) + 2*I*b)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(59) = 118$.

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.32

$$\int e^{a+ibx}\cot^2(d+bx)\,dx = \frac{-i\,e^{(2i\,bx+a+i\,d)}\log\left(e^{(i\,bx+i\,d)}+1\right)+i\,e^{(a-i\,d)}\log\left(e^{(i\,bx+i\,d)}+1\right)+i\,e^{(2i\,bx+a+i\,d)}\log\left(e^{(i\,bx+i\,d)}-1\right)-i\,e^{(a-}}{b(e^{(2i\,bx+2i\,d)}-1)}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d)^2,x, algorithm="giac")
```

output
```
(-I*e^(2*I*b*x + a + I*d)*log(e^(I*b*x + I*d) + 1) + I*e^(a - I*d)*log(e^(
I*b*x + I*d) + 1) + I*e^(2*I*b*x + a + I*d)*log(e^(I*b*x + I*d) - 1) - I*e
^(a - I*d)*log(e^(I*b*x + I*d) - 1) + I*e^(3*I*b*x + a + 2*I*d) - 3*I*e^(I
*b*x + a))/(b*(e^(2*I*b*x + 2*I*d) - 1))
```

Mupad [B] (verification not implemented)

Time = 16.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.90

$$\int e^{a+ibx}\cot^2(d+bx)\,dx=\frac{e^{a+b\,x\,1i}\,1i}{b}+\frac{e^{3\,a-d\,2i+b\,x\,1i}\,2i}{b\,\left(e^{2\,a-d\,2i}-e^{2\,a+b\,x\,2i}\right)}-\frac{\sqrt{-e^{2\,a-d\,2i}}\,\ln\left(2\,e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}-e^{2\,a}\,e^{-d\,2i}\,\sqrt{-e^{2\,a}\,e^{-d\,2i}}\,2i\right)}{b}+\frac{\sqrt{-e^{2\,a-d\,2i}}\,\ln\left(2\,e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}+e^{2\,a}\,e^{-d\,2i}\,\sqrt{-e^{2\,a}\,e^{-d\,2i}}\,2i\right)}{b}$$

input
```
int(cot(d + b*x)^2*exp(a + b*x*1i),x)
```

output
```
(exp(a + b*x*1i)*1i)/b + (exp(3*a - d*2i + b*x*1i)*2i)/(b*(exp(2*a - d*2i)
 - exp(2*a + b*x*2i))) - ((-exp(2*a - d*2i))^(1/2)*log(2*exp(3*a)*exp(-d*2
i)*exp(b*x*1i) - exp(2*a)*exp(-d*2i)*(-exp(2*a)*exp(-d*2i))^(1/2)*2i))/b +
 ((-exp(2*a - d*2i))^(1/2)*log(2*exp(3*a)*exp(-d*2i)*exp(b*x*1i) + exp(2*a
)*exp(-d*2i)*(-exp(2*a)*exp(-d*2i))^(1/2)*2i))/b
```

Reduce [F]

$$\int e^{a+ibx}\cot^2(d+bx)\,dx=e^a\left(\int e^{bix}\cot\left(bx+d\right)^2dx\right)$$

input
```
int(exp(a+I*b*x)*cot(b*x+d)^2,x)
```

output
```
e**a*int(e**(b*i*x)*cot(b*x + d)**2,x)
```

3.60 $\int e^{a+ibx} \cot^3(d+bx)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int e^{a+ibx} \cot^3(d+bx)\, dx = -\frac{e^{a-id+i(d+bx)}}{b} + \frac{2e^{a-id+i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)^2} - \frac{3e^{a-id+i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)} + \frac{3e^{a-id}\text{arctanh}\left(e^{i(d+bx)}\right)}{b}$$

output
```
-exp(a-I*d+I*(b*x+d))/b+2*exp(a-I*d+I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^2-3*
exp(a-I*d+I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))+3*exp(a-I*d)*arctanh(exp(I*(b*
x+d)))/b
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

$$\int e^{a+ibx} \cot^3(d+bx)\, dx = \frac{e^a\left(3\text{arctanh}\left(e^{ibx}(\cos(d)+i\sin(d))\right)(\cos(d)-i\sin(d)) - \frac{e^{ibx}\left(-5e^{2ibx}+\left(2+e^{4ibx}\right)\cos(2d)+i\left(-2+e^{4ibx}\right)\sin(2d)\right)}{\left(\left(-1+e^{2ibx}\right)\cos(d)+i\left(1+e^{2ibx}\right)\sin(d)\right)^2}\right)}{b}$$

input
```
Integrate[E^(a + I*b*x)*Cot[d + b*x]^3,x]
```

output
```
(E^a*(3*ArcTanh[E^(I*b*x)*(Cos[d] + I*Sin[d])]*(Cos[d] - I*Sin[d]) - (E^(I
*b*x)*(-5*E^((2*I)*b*x) + (2 + E^((4*I)*b*x))*Cos[2*d] + I*(-2 + E^((4*I)*
b*x))*Sin[2*d]))/((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[
d])^2))/b
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\cot^3(bx+d)\,dx$$

$$\Big\downarrow \text{4943}$$

$$i\int \left(-e^{a+ibx}+\frac{6e^{a+ibx}}{1-e^{2i(d+bx)}}-\frac{12e^{a+ibx}}{\left(1-e^{2i(d+bx)}\right)^2}+\frac{8e^{a+ibx}}{\left(1-e^{2i(d+bx)}\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{3ie^{a-id}\text{arctanh}\left(e^{ibx+id}\right)}{b}+\frac{3ie^{a+ibx}}{b\left(1-e^{2i(bx+d)}\right)}-\frac{2ie^{a+ibx}}{b\left(1-e^{2i(bx+d)}\right)^2}+\frac{ie^{a+ibx}}{b}\right)$$

input
```
Int[E^(a + I*b*x)*Cot[d + b*x]^3,x]
```

output
```
I*((I*E^(a + I*b*x))/b - ((2*I)*E^(a + I*b*x))/(b*(1 - E^((2*I)*(d + b*x))
)^2) + ((3*I)*E^(a + I*b*x))/(b*(1 - E^((2*I)*(d + b*x)))) - ((3*I)*E^(a -
 I*d)*ArcTanh[E^(I*d + I*b*x)])/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{e^{a} e^{i b x}}{b}+\frac{3 e^{a} e^{3 i b x} e^{2 i d}-e^{a} e^{i b x}}{b\left(-1+e^{2 i (b x+d)}\right)^{2}}+\frac{3 e^{a} e^{-i d} \operatorname{arctanh}\left(e^{i (b x+d)}\right)}{b}$	80

input
```
int(exp(a+I*b*x)*cot(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-exp(a)*exp(I*b*x)/b+1/b/(-1+exp(2*I*(b*x+d)))^2*(3*exp(a)*exp(3*I*b*x)*ex
p(2*I*d)-exp(a)*exp(I*b*x))+3*exp(a)/b*exp(-I*d)*arctanh(exp(I*(b*x+d)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(84) = 168.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.42

$$\int e^{a+ibx}\cot^3(d+bx)\,dx$$
$$= \frac{3\left(be^{(4i\,bx+4i\,d)}-2\,be^{(2i\,bx+2i\,d)}+b\right)\sqrt{\frac{1}{b^2}}e^{(a-i\,d)}\log\left(b\sqrt{\frac{1}{b^2}}e^{(a-i\,d)}+e^{(i\,bx+a)}\right)-3\left(be^{(4i\,bx+4i\,d)}-2\,be^{(2i\,bx+2i}\right.}{2\left(be^{(4i\,bx+4i\,d)}-2\,be^{(2}\right.}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(3*(b*e^(4*I*b*x + 4*I*d) - 2*b*e^(2*I*b*x + 2*I*d) + b)*sqrt(b^(-2))*
e^(a - I*d)*log(b*sqrt(b^(-2))*e^(a - I*d) + e^(I*b*x + a)) - 3*(b*e^(4*I*
b*x + 4*I*d) - 2*b*e^(2*I*b*x + 2*I*d) + b)*sqrt(b^(-2))*e^(a - I*d)*log(-
b*sqrt(b^(-2))*e^(a - I*d) + e^(I*b*x + a)) - 2*e^(5*I*b*x + a + 4*I*d) +
10*e^(3*I*b*x + a + 2*I*d) - 4*e^(I*b*x + a))/(b*e^(4*I*b*x + 4*I*d) - 2*b
*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$\int e^{a+ibx}\cot^3(d+bx)\,dx = \frac{3e^ae^{2id}e^{3ibx}-e^ae^{ibx}}{be^{4id}e^{4ibx}-2be^{2id}e^{2ibx}+b} + \begin{cases} -\frac{e^ae^{ibx}}{b} & \text{for } b\neq 0 \\ -ixe^a & \text{otherwise} \end{cases} + \frac{3\left(-\frac{\log\left(e^{ibx}-e^{-id}\right)}{2}+\frac{\log\left(e^{ibx}+e^{-id}\right)}{2}\right)e^ae^{-id}}{b}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d)**3,x)
```

output
```
(3*exp(a)*exp(2*I*d)*exp(3*I*b*x) - exp(a)*exp(I*b*x))/(b*exp(4*I*d)*exp(4
*I*b*x) - 2*b*exp(2*I*d)*exp(2*I*b*x) + b) + Piecewise((-exp(a)*exp(I*b*x)
/b, Ne(b, 0)), (-I*x*exp(a), True)) + 3*(-log(exp(I*b*x) - exp(-I*d))/2 +
log(exp(I*b*x) + exp(-I*d))/2)*exp(a)*exp(-I*d)/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 658 vs. 2(84) = 168.

Time = 0.06 (sec) , antiderivative size = 658, normalized size of antiderivative = 5.06

$$\int e^{a+ibx}\cot^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d)^3,x, algorithm="maxima")
```

output
```
-(6*((cos(d)*e^a - I*e^a*sin(d))*cos(4*b*x + 4*d) - 2*(cos(d)*e^a - I*e^a*
sin(d))*cos(2*b*x + 2*d) + cos(d)*e^a + (I*cos(d)*e^a + e^a*sin(d))*sin(4*
b*x + 4*d) + 2*(-I*cos(d)*e^a - e^a*sin(d))*sin(2*b*x + 2*d) - I*e^a*sin(d
))*arctan2(sin(b*x) + sin(d), cos(b*x) - cos(d)) - 6*((cos(d)*e^a - I*e^a*
sin(d))*cos(4*b*x + 4*d) - 2*(cos(d)*e^a - I*e^a*sin(d))*cos(2*b*x + 2*d)
+ cos(d)*e^a - (-I*cos(d)*e^a - e^a*sin(d))*sin(4*b*x + 4*d) - 2*(I*cos(d)
*e^a + e^a*sin(d))*sin(2*b*x + 2*d) - I*e^a*sin(d))*arctan2(sin(b*x) - sin
(d), cos(b*x) + cos(d)) - 4*I*cos(5*b*x + 4*d)*e^a + 20*I*cos(3*b*x + 2*d)
*e^a - 8*I*cos(b*x)*e^a + 3*((I*cos(d)*e^a + e^a*sin(d))*cos(4*b*x + 4*d)
+ 2*(-I*cos(d)*e^a - e^a*sin(d))*cos(2*b*x + 2*d) + I*cos(d)*e^a - (cos(d)
*e^a - I*e^a*sin(d))*sin(4*b*x + 4*d) + 2*(cos(d)*e^a - I*e^a*sin(d))*sin(
2*b*x + 2*d) + e^a*sin(d))*log(cos(b*x)^2 + 2*cos(b*x)*cos(d) + cos(d)^2 +
 sin(b*x)^2 - 2*sin(b*x)*sin(d) + sin(d)^2) + 3*((-I*cos(d)*e^a - e^a*sin(
d))*cos(4*b*x + 4*d) + 2*(I*cos(d)*e^a + e^a*sin(d))*cos(2*b*x + 2*d) - I*
cos(d)*e^a + (cos(d)*e^a - I*e^a*sin(d))*sin(4*b*x + 4*d) - 2*(cos(d)*e^a
- I*e^a*sin(d))*sin(2*b*x + 2*d) - e^a*sin(d))*log(cos(b*x)^2 - 2*cos(b*x)
*cos(d) + cos(d)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(d) + sin(d)^2) + 4*e^a*si
n(5*b*x + 4*d) - 20*e^a*sin(3*b*x + 2*d) + 8*e^a*sin(b*x))/(-4*I*b*cos(4*b
*x + 4*d) + 8*I*b*cos(2*b*x + 2*d) + 4*b*sin(4*b*x + 4*d) - 8*b*sin(2*b*x
+ 2*d) - 4*I*b)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(84) = 168$.

Time = 0.20 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.62

$$\int e^{a+ibx}\cot^3(d+bx)\,dx = \frac{3\,e^{(4i\,bx+a+3i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+i\right)-6\,e^{(2i\,bx+a+i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+i\right)+3\,e^{(a-i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+i\right)-3\,e}{}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d)^3,x, algorithm="giac")
```

output
```
1/2*(3*e^(4*I*b*x + a + 3*I*d)*log(I*e^(I*b*x + I*d) + I) - 6*e^(2*I*b*x +
 a + I*d)*log(I*e^(I*b*x + I*d) + I) + 3*e^(a - I*d)*log(I*e^(I*b*x + I*d)
 + I) - 3*e^(4*I*b*x + a + 3*I*d)*log(-I*e^(I*b*x + I*d) + I) + 6*e^(2*I*b
*x + a + I*d)*log(-I*e^(I*b*x + I*d) + I) - 3*e^(a - I*d)*log(-I*e^(I*b*x
+ I*d) + I) - 2*e^(5*I*b*x + a + 4*I*d) + 10*e^(3*I*b*x + a + 2*I*d) - 4*e
^(I*b*x + a))/(b*(e^(4*I*b*x + 4*I*d) - 2*e^(2*I*b*x + 2*I*d) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\cot^3(d+bx)\,dx=\int \cot(d+b\,x)^3\,\mathrm{e}^{a+b\,x\,1\mathrm{i}}\,dx$$

input
```
int(cot(d + b*x)^3*exp(a + b*x*1i),x)
```

output
```
int(cot(d + b*x)^3*exp(a + b*x*1i), x)
```

Reduce [F]

$$\int e^{a+ibx}\cot^3(d+bx)\,dx=e^{a}\left(\int e^{bix}\cot\left(bx+d\right)^{3}dx\right)$$

input
```
int(exp(a+I*b*x)*cot(b*x+d)^3,x)
```

output
```
e**a*int(e**(b*i*x)*cot(b*x + d)**3,x)
```

3.61 $\int e^{a+ibx} \cot^4(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 184

$$\int e^{a+ibx}\cot^4(d+bx)\,dx=-\frac{ie^{a-id+i(d+bx)}}{b}-\frac{8ie^{a-id+i(d+bx)}}{3b\left(1-e^{2i(d+bx)}\right)^3}+\frac{14ie^{a-id+i(d+bx)}}{3b\left(1-e^{2i(d+bx)}\right)^2}-\frac{5ie^{a-id+i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)}+\frac{3ie^{a-id}\text{arctanh}\left(e^{i(d+bx)}\right)}{b}$$

output
```
-I*exp(a-I*d+I*(b*x+d))/b-8/3*I*exp(a-I*d+I*(b*x+d))/b/(1-exp(2*I*(b*x+d))
)^3+14/3*I*exp(a-I*d+I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^2-5*I*exp(a-I*d+I*(
b*x+d))/b/(1-exp(2*I*(b*x+d)))+3*I*exp(a-I*d)*arctanh(exp(I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int e^{a+ibx}\cot^4(d+bx)\,dx =\frac{e^a\left(9\text{arctanh}\left(e^{ibx}(\cos(d)+i\sin(d))\right)(i\cos(d)+\sin(d))+\frac{e^{ibx}\left(\left(25e^{2ibx}-24e^{4ibx}\right)\cos(d)+3\left(-4+e^{6ibx}\right)\cos(3d)-ie^{2ibx}(2}{\left(-i\left(-1+e^{2ibx}\right)\cos(d)+\left(1+e^{2ib}\right.}\right.}{3b}$$

input
```
Integrate[E^(a + I*b*x)*Cot[d + b*x]^4,x]
```

output
```
(E^a*(9*ArcTanh[E^(I*b*x)*(Cos[d] + I*Sin[d])]*(I*Cos[d] + Sin[d]) + (E^(I
*b*x)*((25*E^((2*I)*b*x) - 24*E^((4*I)*b*x))*Cos[d] + 3*(-4 + E^((6*I)*b*x
))*Cos[3*d] - I*E^((2*I)*b*x)*(25 + 24*E^((2*I)*b*x))*Sin[d] + (3*I)*(4 +
E^((6*I)*b*x))*Sin[3*d]))/((-I)*(-1 + E^((2*I)*b*x))*Cos[d] + (1 + E^((2*I
)*b*x))*Sin[d])^3))/(3*b)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\cot^4(bx+d)\,dx$$

$$\Big\downarrow \text{4943}$$

$$\int \left(-\frac{8e^{a+ibx}}{1-e^{2i(bx+d)}}+\frac{24e^{a+ibx}}{\left(1-e^{2i(bx+d)}\right)^2}-\frac{32e^{a+ibx}}{\left(1-e^{2i(bx+d)}\right)^3}+\frac{16e^{a+ibx}}{\left(1-e^{2i(bx+d)}\right)^4}+e^{a+ibx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3ie^{a-id}\text{arctanh}\left(e^{ibx+id}\right)}{b}-\frac{5ie^{a+ibx}}{b\left(1-e^{2i(bx+d)}\right)}+\frac{14ie^{a+ibx}}{3b\left(1-e^{2i(bx+d)}\right)^2}-\frac{8ie^{a+ibx}}{3b\left(1-e^{2i(bx+d)}\right)^3}-\frac{ie^{a+ibx}}{b}$$

input
```
Int[E^(a + I*b*x)*Cot[d + b*x]^4,x]
```

output
```
((-I)*E^(a + I*b*x))/b - (((8*I)/3)*E^(a + I*b*x))/(b*(1 - E^((2*I)*(d + b
*x)))^3) + (((14*I)/3)*E^(a + I*b*x))/(b*(1 - E^((2*I)*(d + b*x)))^2) - ((
5*I)*E^(a + I*b*x))/(b*(1 - E^((2*I)*(d + b*x)))) + ((3*I)*E^(a - I*d)*Arc
Tanh[E^(I*d + I*b*x)])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.54

method	result	size
risch	$-\frac{i e^{a} e^{i b x}}{b}+\frac{i\left(15\,e^{a} e^{5 i b x} e^{4 i d}-16\,e^{a} e^{3 i b x} e^{2 i d}+9\,e^{a} e^{i b x}\right)}{3 b\left(-1+e^{2 i (b x+d)}\right)^{3}}+\frac{3 i e^{a} e^{-i d} \operatorname{arctanh}\left(e^{i (b x+d)}\right)}{b}$	99

input
```
int(exp(a+I*b*x)*cot(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-I/b*exp(a)*exp(I*b*x)+1/3*I/b/(-1+exp(2*I*(b*x+d)))^3*(15*exp(a)*exp(5*I*
b*x)*exp(4*I*d)-16*exp(a)*exp(3*I*b*x)*exp(2*I*d)+9*exp(a)*exp(I*b*x))+3*I
*exp(a)/b*exp(-I*d)*arctanh(exp(I*(b*x+d)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(109) = 218.

Time = 0.08 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.30

$$\int e^{a+ibx}\cot^4(d+bx)\,dx =$$

$$-\frac{9\left(-i\,be^{(6i\,bx+6i\,d)}+3i\,be^{(4i\,bx+4i\,d)}-3i\,be^{(2i\,bx+2i\,d)}+i\,b\right)\sqrt{\frac{1}{b^2}}e^{(a-i\,d)}\log\left(b\sqrt{\frac{1}{b^2}}e^{(a-i\,d)}+e^{(i\,bx+a)}\right)+9\,(\ldots}{}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d)^4,x, algorithm="fricas")
```

output
```
-1/6*(9*(-I*b*e^(6*I*b*x + 6*I*d) + 3*I*b*e^(4*I*b*x + 4*I*d) - 3*I*b*e^(2
*I*b*x + 2*I*d) + I*b)*sqrt(b^(-2))*e^(a - I*d)*log(b*sqrt(b^(-2))*e^(a -
I*d) + e^(I*b*x + a)) + 9*(I*b*e^(6*I*b*x + 6*I*d) - 3*I*b*e^(4*I*b*x + 4*
I*d) + 3*I*b*e^(2*I*b*x + 2*I*d) - I*b)*sqrt(b^(-2))*e^(a - I*d)*log(-b*sq
rt(b^(-2))*e^(a - I*d) + e^(I*b*x + a)) + 6*I*e^(7*I*b*x + a + 6*I*d) - 48
*I*e^(5*I*b*x + a + 4*I*d) + 50*I*e^(3*I*b*x + a + 2*I*d) - 24*I*e^(I*b*x
+ a))/(b*e^(6*I*b*x + 6*I*d) - 3*b*e^(4*I*b*x + 4*I*d) + 3*b*e^(2*I*b*x +
2*I*d) - b)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.95

$$\int e^{a+ibx}\cot^4(d+bx)\,dx = \frac{15ie^ae^{4id}e^{5ibx} - 16ie^ae^{2id}e^{3ibx} + 9ie^ae^{ibx}}{3be^{6id}e^{6ibx} - 9be^{4id}e^{4ibx} + 9be^{2id}e^{2ibx} - 3b} + \begin{cases} -\frac{ie^ae^{ibx}}{b} & \text{for } b \neq 0 \\ xe^a & \text{otherwise} \end{cases} + \frac{3e^ae^{-id}\operatorname{RootSum}\left(4z^2+1, \left(i \mapsto i\log\left(-2iie^{-id}+e^{ibx}\right)\right)\right)}{b}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d)**4,x)
```

output
```
(15*I*exp(a)*exp(4*I*d)*exp(5*I*b*x) - 16*I*exp(a)*exp(2*I*d)*exp(3*I*b*x)
 + 9*I*exp(a)*exp(I*b*x))/(3*b*exp(6*I*d)*exp(6*I*b*x) - 9*b*exp(4*I*d)*ex
p(4*I*b*x) + 9*b*exp(2*I*d)*exp(2*I*b*x) - 3*b) + Piecewise((-I*exp(a)*exp
(I*b*x)/b, Ne(b, 0)), (x*exp(a), True)) + 3*exp(a)*exp(-I*d)*RootSum(4*_z*
*2 + 1, Lambda(_i, _i*log(-2*_i*I*exp(-I*d) + exp(I*b*x))))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(109) = 218$.

Time = 0.09 (sec) , antiderivative size = 895, normalized size of antiderivative = 4.86

$$\int e^{a+ibx}\cot^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d)^4,x, algorithm="maxima")
```

output
```
(18*((-I*cos(d)*e^a - e^a*sin(d))*cos(6*b*x + 6*d) + 3*(I*cos(d)*e^a + e^a
*sin(d))*cos(4*b*x + 4*d) + 3*(-I*cos(d)*e^a - e^a*sin(d))*cos(2*b*x + 2*d
) + I*cos(d)*e^a + (cos(d)*e^a - I*e^a*sin(d))*sin(6*b*x + 6*d) - 3*(cos(d
)*e^a - I*e^a*sin(d))*sin(4*b*x + 4*d) + 3*(cos(d)*e^a - I*e^a*sin(d))*sin
(2*b*x + 2*d) + e^a*sin(d))*arctan2(sin(b*x) + sin(d), cos(b*x) - cos(d))
+ 18*((I*cos(d)*e^a + e^a*sin(d))*cos(6*b*x + 6*d) + 3*(-I*cos(d)*e^a - e^
a*sin(d))*cos(4*b*x + 4*d) + 3*(I*cos(d)*e^a + e^a*sin(d))*cos(2*b*x + 2*d
) - I*cos(d)*e^a - (cos(d)*e^a - I*e^a*sin(d))*sin(6*b*x + 6*d) + 3*(cos(d
)*e^a - I*e^a*sin(d))*sin(4*b*x + 4*d) - 3*(cos(d)*e^a - I*e^a*sin(d))*sin
(2*b*x + 2*d) - e^a*sin(d))*arctan2(sin(b*x) - sin(d), cos(b*x) + cos(d))
- 12*cos(7*b*x + 6*d)*e^a + 96*cos(5*b*x + 4*d)*e^a - 100*cos(3*b*x + 2*d)
*e^a + 48*cos(b*x)*e^a + 9*((cos(d)*e^a - I*e^a*sin(d))*cos(6*b*x + 6*d) -
 3*(cos(d)*e^a - I*e^a*sin(d))*cos(4*b*x + 4*d) + 3*(cos(d)*e^a - I*e^a*si
n(d))*cos(2*b*x + 2*d) - cos(d)*e^a + (I*cos(d)*e^a + e^a*sin(d))*sin(6*b*
x + 6*d) + 3*(-I*cos(d)*e^a - e^a*sin(d))*sin(4*b*x + 4*d) + 3*(I*cos(d)*e
^a + e^a*sin(d))*sin(2*b*x + 2*d) + I*e^a*sin(d))*log(cos(b*x)^2 + 2*cos(b
*x)*cos(d) + cos(d)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(d) + sin(d)^2) - 9*((c
os(d)*e^a - I*e^a*sin(d))*cos(6*b*x + 6*d) - 3*(cos(d)*e^a - I*e^a*sin(d))
*cos(4*b*x + 4*d) + 3*(cos(d)*e^a - I*e^a*sin(d))*cos(2*b*x + 2*d) - cos(d
)*e^a - (-I*cos(d)*e^a - e^a*sin(d))*sin(6*b*x + 6*d) - 3*(I*cos(d)*e^a...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(109) = 218.

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.47

$$\int e^{a+ibx}\cot^4(d+bx)\,dx$$
$$= \frac{9i\,e^{(6i\,bx+a+5i\,d)}\log\left(e^{(i\,bx+i\,d)}+1\right)-27i\,e^{(4i\,bx+a+3i\,d)}\log\left(e^{(i\,bx+i\,d)}+1\right)+27i\,e^{(2i\,bx+a+i\,d)}\log\left(e^{(i\,bx+i\,d)}+\right.}{}$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+d)^4,x, algorithm="giac")
```

output
```
1/6*(9*I*e^(6*I*b*x + a + 5*I*d)*log(e^(I*b*x + I*d) + 1) - 27*I*e^(4*I*b*
x + a + 3*I*d)*log(e^(I*b*x + I*d) + 1) + 27*I*e^(2*I*b*x + a + I*d)*log(e
^(I*b*x + I*d) + 1) - 9*I*e^(a - I*d)*log(e^(I*b*x + I*d) + 1) - 9*I*e^(6*
I*b*x + a + 5*I*d)*log(e^(I*b*x + I*d) - 1) + 27*I*e^(4*I*b*x + a + 3*I*d)
*log(e^(I*b*x + I*d) - 1) - 27*I*e^(2*I*b*x + a + I*d)*log(e^(I*b*x + I*d)
 - 1) + 9*I*e^(a - I*d)*log(e^(I*b*x + I*d) - 1) - 6*I*e^(7*I*b*x + a + 6*
I*d) + 48*I*e^(5*I*b*x + a + 4*I*d) - 50*I*e^(3*I*b*x + a + 2*I*d) + 24*I*
e^(I*b*x + a))/(b*(e^(6*I*b*x + 6*I*d) - 3*e^(4*I*b*x + 4*I*d) + 3*e^(2*I*
b*x + 2*I*d) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\cot^4(d+bx)\,dx=\int \cot(d+b\,x)^4\,\mathrm{e}^{a+b\,x\,1\mathrm{i}}\,dx$$

input
```
int(cot(d + b*x)^4*exp(a + b*x*1i),x)
```

output
```
int(cot(d + b*x)^4*exp(a + b*x*1i), x)
```

Reduce [F]

$$\int e^{a+ibx}\cot^4(d+bx)\,dx = e^a\left(\int e^{bix}\cot\left(bx+d\right)^4 dx\right)$$

input

```
int(exp(a+I*b*x)*cot(b*x+d)^4,x)
```

output

```
e**a*int(e**(b*i*x)*cot(b*x + d)**4,x)
```

3.62 $\int e^{2(a+ibx)} \cot(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 60

$$\int e^{2(a+ibx)} \cot(d+bx)\,dx = \frac{e^{2(a-id)+2i(d+bx)}}{2b} + \frac{e^{2a-2id} \log\left(1-e^{2i(d+bx)}\right)}{b}$$

output
```
1/2*exp(2*a-2*I*d+2*I*(b*x+d))/b+exp(2*a-2*I*d)*ln(1-exp(2*I*(b*x+d)))/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 138 vs. 2(60) = 120.

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.30

$$\int e^{2(a+ibx)} \cot(d+bx)\,dx$$
$$= \frac{e^{2a}(\cos(d)-i\sin(d))\left(\cos(d)\left(e^{2ibx}+\log\left(1+e^{4ibx}-2e^{2ibx}\cos(2d)\right)\right)+i\left(e^{2ibx}-\log\left(1+e^{4ibx}-2e^{2ibx}\right.\right.\right.}{2b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Cot[d + b*x],x]
```

output
```
(E^(2*a)*(Cos[d] - I*Sin[d])*(Cos[d]*(E^((2*I)*b*x) + Log[1 + E^((4*I)*b*x
) - 2*E^((2*I)*b*x)*Cos[2*d]]) + I*(E^((2*I)*b*x) - Log[1 + E^((4*I)*b*x)
- 2*E^((2*I)*b*x)*Cos[2*d]])*Sin[d] + 2*ArcTan[((1 + E^((2*I)*b*x))*Tan[d]
)/(-1 + E^((2*I)*b*x))]*(I*Cos[d] + Sin[d])))/(2*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)} \cot(bx+d)\,dx$$

$$\Big\downarrow \text{4943}$$

$$-i\int \left(\frac{2e^{2(a+ibx)}}{1-e^{2i(d+bx)}}-e^{2(a+ibx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{ie^{2a-2id}\log\left(1-e^{2i(bx+d)}\right)}{b}+\frac{ie^{2(a+ibx)}}{2b}\right)$$

input
```
Int[E^(2*(a + I*b*x))*Cot[d + b*x],x]
```

output
```
(-I)*(((I/2)*E^(2*(a + I*b*x)))/b + (I*E^(2*a - (2*I)*d)*Log[1 - E^((2*I)*
(d + b*x))])/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{e^{2a} e^{2ibx}}{2b} + \frac{e^{2a} e^{-2id} \ln\left(-1+e^{2i(bx+d)}\right)}{b}$	42

input
```
int(exp(2*a+2*I*b*x)*cot(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*a)*exp(2*I*b*x)/b+exp(2*a)/b*exp(-2*I*d)*ln(-1+exp(2*I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

$$\int e^{2(a+ibx)} \cot(d+bx)\, dx = \frac{2\, e^{(2\, a-2i\, d)} \log\left(e^{(2i\, bx+2i\, d)} - 1\right) + e^{(2i\, bx+2\, a)}}{2\, b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d),x, algorithm="fricas")
```

output
```
1/2*(2*e^(2*a - 2*I*d)*log(e^(2*I*b*x + 2*I*d) - 1) + e^(2*I*b*x + 2*a))/b
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int e^{2(a+ibx)}\cot(d+bx)\,dx = \begin{cases} \frac{e^{2a}e^{2ibx}}{2b} & \text{for } b\neq 0 \\ ixe^{2a} & \text{otherwise} \end{cases} + \frac{e^{2a}e^{-2id}\log\left(e^{2ibx}-e^{-2id}\right)}{b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d),x)
```

output
```
Piecewise((exp(2*a)*exp(2*I*b*x)/(2*b), Ne(b, 0)), (I*x*exp(2*a), True)) +
 exp(2*a)*exp(-2*I*d)*log(exp(2*I*b*x) - exp(-2*I*d))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.72

$$\int e^{2(a+ibx)}\cot(d+bx)\,dx$$
$$= \frac{2\left(i\cos(2d)e^{(2a)}+e^{(2a)}\sin(2d)\right)\arctan\left(\sin(bx)+\sin(d),\cos(bx)-\cos(d)\right)+2\left(i\cos(2d)e^{(2a)}+e\right.}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d),x, algorithm="maxima")
```

output
```
1/2*(2*(I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*arctan2(sin(b*x) + sin(d),
cos(b*x) - cos(d)) + 2*(I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*arctan2(sin
(b*x) - sin(d), cos(b*x) + cos(d)) + cos(2*b*x)*e^(2*a) + (cos(2*d)*e^(2*a
) - I*e^(2*a)*sin(2*d))*log(cos(b*x)^2 + 2*cos(b*x)*cos(d) + cos(d)^2 + si
n(b*x)^2 - 2*sin(b*x)*sin(d) + sin(d)^2) + (cos(2*d)*e^(2*a) - I*e^(2*a)*s
in(2*d))*log(cos(b*x)^2 - 2*cos(b*x)*cos(d) + cos(d)^2 + sin(b*x)^2 + 2*si
n(b*x)*sin(d) + sin(d)^2) + I*e^(2*a)*sin(2*b*x))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

$$\int e^{2(a+ibx)}\cot(d+bx)\,dx = \frac{2\,e^{(2\,a-2i\,d)}\log\left(e^{(2i\,bx+2i\,d)}-1\right)+e^{(2i\,bx+2\,a)}}{2\,b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d),x, algorithm="giac")
```

output
```
1/2*(2*e^(2*a - 2*I*d)*log(e^(2*I*b*x + 2*I*d) - 1) + e^(2*I*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int e^{2(a+ibx)}\cot(d+bx)\,dx = \frac{e^{2\,a}\,e^{b\,x\,2i}}{2\,b}+\frac{e^{2\,a}\,e^{-d\,2i}\,\ln\left(e^{2\,a}\,e^{b\,x\,2i}-e^{2\,a}\,e^{-d\,2i}\right)}{b}$$

input
```
int(cot(d + b*x)*exp(2*a + b*x*2i),x)
```

output
```
(exp(2*a)*exp(b*x*2i))/(2*b) + (exp(2*a)*exp(-d*2i)*log(exp(2*a)*exp(b*x*2
i) - exp(2*a)*exp(-d*2i)))/b
```

Reduce [F]

$$\int e^{2(a+ibx)}\cot(d+bx)\,dx = e^{2a}\left(\int e^{2bix}\cot\left(bx+d\right)dx\right)$$

input
```
int(exp(2*a+2*I*b*x)*cot(b*x+d),x)
```

output
```
e**(2*a)*int(e**(2*b*i*x)*cot(b*x + d),x)
```

3.63 $\int e^{2(a+ibx)} \cot^2(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int e^{2(a+ibx)} \cot^2(d+bx)\,dx = \frac{ie^{2(a-id)+2i(d+bx)}}{2b} + \frac{2ie^{2a-2id}}{b\left(1-e^{2i(d+bx)}\right)} + \frac{2ie^{2a-2id}\log\left(1-e^{2i(d+bx)}\right)}{b}$$

output

```
1/2*I*exp(2*a-2*I*d+2*I*(b*x+d))/b+2*I*exp(2*a-2*I*d)/b/(1-exp(2*I*(b*x+d)
))+2*I*exp(2*a-2*I*d)*ln(1-exp(2*I*(b*x+d)))/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 215 vs. 2(100) = 200.

Time = 0.31 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.15

$$\int e^{2(a+ibx)}\cot^2(d+bx)\,dx = \frac{1}{2}e^{2a}\left(\frac{ie^{2ibx}}{b} - \frac{4\arctan\left(\frac{(1+e^{2ibx})\tan(d)}{-1+e^{2ibx}}\right)\cos(2d)}{b} + \frac{2i\cos(2d)\log\left(1+e^{4ibx}-2e^{2ibx}\cos(2d)\right)}{b} + \frac{4(\cos(d)-i\sin(d))^3}{ib\left(-1+e^{2ibx}\right)\cos(d)-b\left(1+e^{2ibx}\right)\sin(d)} + \frac{4i\arctan\left(\frac{(1+e^{2ibx})\tan(d)}{-1+e^{2ibx}}\right)\sin(2d)}{b} + \frac{2\log\left(1+e^{4ibx}-2e^{2ibx}\cos(2d)\right)\sin(2d)}{b}\right)$$

input
```
Integrate[E^(2*(a + I*b*x))*Cot[d + b*x]^2,x]
```

output
```
(E^(2*a)*((I*E^((2*I)*b*x))/b - (4*ArcTan[((1 + E^((2*I)*b*x))*Tan[d])/(-1
 + E^((2*I)*b*x))]*Cos[2*d])/b + ((2*I)*Cos[2*d]*Log[1 + E^((4*I)*b*x) - 2
*E^((2*I)*b*x)*Cos[2*d]])/b + (4*(Cos[d] - I*Sin[d])^3)/(I*b*(-1 + E^((2*I
)*b*x))*Cos[d] - b*(1 + E^((2*I)*b*x))*Sin[d]) + ((4*I)*ArcTan[((1 + E^((2
*I)*b*x))*Tan[d])/(-1 + E^((2*I)*b*x))]*Sin[2*d])/b + (2*Log[1 + E^((4*I)*
b*x) - 2*E^((2*I)*b*x)*Cos[2*d]]*Sin[2*d])/b))/2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\cot^2(bx+d)\,dx$$

$$\Big\downarrow \; 4943$$

$$-\int \left(e^{2(a+ibx)}-\frac{4e^{2(a+ibx)}}{1-e^{2i(d+bx)}}+\frac{4e^{2(a+ibx)}}{\left(1-e^{2i(d+bx)}\right)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2ie^{2a-2id}}{b\left(1-e^{2i(bx+d)}\right)}+\frac{2ie^{2a-2id}\log\left(1-e^{2i(bx+d)}\right)}{b}+\frac{ie^{2(a+ibx)}}{2b}$$

input
```
Int[E^(2*(a + I*b*x))*Cot[d + b*x]^2,x]
```

output
```
((I/2)*E^(2*(a + I*b*x)))/b + ((2*I)*E^(2*a - (2*I)*d))/(b*(1 - E^((2*I)*(
d + b*x)))) + ((2*I)*E^(2*a - (2*I)*d)*Log[1 - E^((2*I)*(d + b*x))])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{i e^{2a} e^{2ibx}}{2b}-\frac{2i e^{2a} e^{2ibx}}{b\left(-1+e^{2i(bx+d)}\right)}+\frac{2i e^{2a} e^{-2id}\ln\left(-1+e^{2i(bx+d)}\right)}{b}$	74

input
```
int(exp(2*a+2*I*b*x)*cot(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b*exp(2*a)*exp(2*I*b*x)-2*I*exp(2*a)*exp(2*I*b*x)/b/(-1+exp(2*I*(b*x
+d)))+2*I*exp(2*a)/b*exp(-2*I*d)*ln(-1+exp(2*I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int e^{2(a+ibx)}\cot^2(d+bx)\,dx = -\frac{4\left(-i\,e^{(2i\,bx+2\,a)}+i\,e^{(2\,a-2i\,d)}\right)\log\left(e^{(2i\,bx+2i\,d)}-1\right)-i\,e^{(4i\,bx+2\,a+2i\,d)}+i\,e^{(2i\,bx+2\,a)}+4i\,e^{(2\,a-2i\,d)}}{2\left(be^{(2i\,bx+2i\,d)}-b\right)}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d)^2,x, algorithm="fricas")
```

output
```
-1/2*(4*(-I*e^(2*I*b*x + 2*a) + I*e^(2*a - 2*I*d))*log(e^(2*I*b*x + 2*I*d)
 - 1) - I*e^(4*I*b*x + 2*a + 2*I*d) + I*e^(2*I*b*x + 2*a) + 4*I*e^(2*a - 2
*I*d))/(b*e^(2*I*b*x + 2*I*d) - b)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int e^{2(a+ibx)}\cot^2(d+bx)\,dx = \begin{cases} \frac{ie^{2a}e^{2ibx}}{2b} & \text{for } b\neq 0 \\ -xe^{2a} & \text{otherwise} \end{cases} - \frac{2ie^{2a}}{be^{4id}e^{2ibx}-be^{2id}} + \frac{2ie^{2a}e^{-2id}\log\left(e^{2ibx}-e^{-2id}\right)}{b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d)**2,x)
```

output
```
Piecewise((I*exp(2*a)*exp(2*I*b*x)/(2*b), Ne(b, 0)), (-x*exp(2*a), True))
- 2*I*exp(2*a)/(b*exp(4*I*d)*exp(2*I*b*x) - b*exp(2*I*d)) + 2*I*exp(2*a)*e
xp(-2*I*d)*log(exp(2*I*b*x) - exp(-2*I*d))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(68) = 136$.

Time = 0.06 (sec) , antiderivative size = 569, normalized size of antiderivative = 5.69

$$\int e^{2(a+ibx)}\cot^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d)^2,x, algorithm="maxima")
```

output
```
-(4*(I*cos(2*d)^2*e^(2*a) + I*e^(2*a)*sin(2*d)^2 + (-I*cos(2*d)*e^(2*a) -
e^(2*a)*sin(2*d))*cos(2*b*x + 4*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d
))*sin(2*b*x + 4*d))*arctan2(sin(b*x) + sin(d), cos(b*x) - cos(d)) + 4*(I*
cos(2*d)^2*e^(2*a) + I*e^(2*a)*sin(2*d)^2 + (-I*cos(2*d)*e^(2*a) - e^(2*a)
*sin(2*d))*cos(2*b*x + 4*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(
2*b*x + 4*d))*arctan2(sin(b*x) - sin(d), cos(b*x) + cos(d)) - cos(4*b*x +
4*d)*e^(2*a) + cos(2*b*x + 2*d)*e^(2*a) + 2*(cos(2*d)^2*e^(2*a) + e^(2*a)*
sin(2*d)^2 - (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(2*b*x + 4*d) + (-
I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*sin(2*b*x + 4*d))*log(cos(b*x)^2 +
2*cos(b*x)*cos(d) + cos(d)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(d) + sin(d)^2)
+ 2*(cos(2*d)^2*e^(2*a) + e^(2*a)*sin(2*d)^2 - (cos(2*d)*e^(2*a) - I*e^(2*
a)*sin(2*d))*cos(2*b*x + 4*d) + (-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*s
in(2*b*x + 4*d))*log(cos(b*x)^2 - 2*cos(b*x)*cos(d) + cos(d)^2 + sin(b*x)^
2 + 2*sin(b*x)*sin(d) + sin(d)^2) - I*e^(2*a)*sin(4*b*x + 4*d) + I*e^(2*a)
*sin(2*b*x + 2*d) + 4*e^(2*a))/(-2*I*b*cos(2*b*x + 4*d) + 2*I*b*cos(2*d) +
 2*b*sin(2*b*x + 4*d) - 2*b*sin(2*d))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int e^{2(a+ibx)}\cot^2(d+bx)\,dx = \frac{4i\,e^{(2i\,bx+2\,a)}\log\left(e^{(2i\,bx+2i\,d)}-1\right)-4i\,e^{(2\,a-2i\,d)}\log\left(e^{(2i\,bx+2i\,d)}-1\right)+i\,e^{(4i\,bx+2\,a+2i\,d)}-i\,e^{(2i\,bx+2\,a)}-4i\,e^{(}}{2\,b(e^{(2i\,bx+2i\,d)}-1)}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d)^2,x, algorithm="giac")
```

output
```
1/2*(4*I*e^(2*I*b*x + 2*a)*log(e^(2*I*b*x + 2*I*d) - 1) - 4*I*e^(2*a - 2*I
*d)*log(e^(2*I*b*x + 2*I*d) - 1) + I*e^(4*I*b*x + 2*a + 2*I*d) - I*e^(2*I*
b*x + 2*a) - 4*I*e^(2*a - 2*I*d))/(b*(e^(2*I*b*x + 2*I*d) - 1))
```

Mupad [B] (verification not implemented)

Time = 16.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int e^{2(a+ibx)}\cot^2(d+bx)\,dx = \frac{e^{2\,a+b\,x\,2i}\,1i}{2\,b}+\frac{e^{2\,a-d\,2i}\,\ln\left(e^{2\,a}\,e^{b\,x\,2i}-e^{2\,a}\,e^{-d\,2i}\right)\,2i}{b}+\frac{e^{4\,a-d\,4i}\,2i}{b\,\left(e^{2\,a-d\,2i}-e^{2\,a+b\,x\,2i}\right)}$$

input
```
int(cot(d + b*x)^2*exp(2*a + b*x*2i),x)
```

output
```
(exp(2*a + b*x*2i)*1i)/(2*b) + (exp(2*a - d*2i)*log(exp(2*a)*exp(b*x*2i) -
 exp(2*a)*exp(-d*2i))*2i)/b + (exp(4*a - d*4i)*2i)/(b*(exp(2*a - d*2i) - e
xp(2*a + b*x*2i)))
```

Reduce [F]

$$\int e^{2(a+ibx)}\cot^2(d+bx)\,dx = e^{2a}\left(\int e^{2bix}\cot(bx+d)^2\,dx\right)$$

input
```
int(exp(2*a+2*I*b*x)*cot(b*x+d)^2,x)
```

output
```
e**(2*a)*int(e**(2*b*i*x)*cot(b*x + d)**2,x)
```

3.64 $\int e^{2(a+ibx)} \cot^3(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 127

$$\int e^{2(a+ibx)} \cot^3(d+bx)\,dx = -\frac{e^{2(a-id)+2i(d+bx)}}{2b} + \frac{2e^{2a-2id}}{b\left(1-e^{2i(d+bx)}\right)^2} - \frac{6e^{2a-2id}}{b\left(1-e^{2i(d+bx)}\right)} - \frac{3e^{2a-2id}\log\left(1-e^{2i(d+bx)}\right)}{b}$$

output
```
-1/2*exp(2*a-2*I*d+2*I*(b*x+d))/b+2*exp(2*a-2*I*d)/b/(1-exp(2*I*(b*x+d)))^
2-6*exp(2*a-2*I*d)/b/(1-exp(2*I*(b*x+d)))-3*exp(2*a-2*I*d)*ln(1-exp(2*I*(b
*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.91

$$\int e^{2(a+ibx)} \cot^3(d+bx)\,dx$$

$$= \frac{e^{2a}\left(-e^{2ibx} - 6i\arctan\left(\frac{(1+e^{2ibx})\tan(d)}{-1+e^{2ibx}}\right)\cos(2d) - 3\cos(2d)\log\left(1+e^{4ibx}-2e^{2ibx}\cos(2d)\right) + \frac{4(\cos \cdots}{((-1+e^{2ibx})\cos \cdots}\right.}{\cdots}$$

input
```
Integrate[E^(2*(a + I*b*x))*Cot[d + b*x]^3,x]
```

output
```
(E^(2*a)*(-E^((2*I)*b*x) - (6*I)*ArcTan[((1 + E^((2*I)*b*x))*Tan[d])/(-1 +
 E^((2*I)*b*x))]*Cos[2*d] - 3*Cos[2*d]*Log[1 + E^((4*I)*b*x) - 2*E^((2*I)*
b*x)*Cos[2*d]] + (4*(Cos[d] - I*Sin[d])^4)/((-1 + E^((2*I)*b*x))*Cos[d] +
I*(1 + E^((2*I)*b*x))*Sin[d])^2 + (12*(Cos[d] - I*Sin[d])^3)/((-1 + E^((2*
I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d]) - 6*ArcTan[((1 + E^((2*I)*
b*x))*Tan[d])/(-1 + E^((2*I)*b*x))]*Sin[2*d] + (3*I)*Log[1 + E^((4*I)*b*x)
 - 2*E^((2*I)*b*x)*Cos[2*d]]*Sin[2*d]))/(2*b)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\cot^3(bx+d)\,dx$$

$$\Big\downarrow \text{4943}$$

$$i\int\left(-e^{2(a+ibx)}+\frac{6e^{2(a+ibx)}}{1-e^{2i(d+bx)}}-\frac{12e^{2(a+ibx)}}{\left(1-e^{2i(d+bx)}\right)^2}+\frac{8e^{2(a+ibx)}}{\left(1-e^{2i(d+bx)}\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{6ie^{2a-2id}}{b\left(1-e^{2i(bx+d)}\right)}-\frac{2ie^{2a-2id}}{b\left(1-e^{2i(bx+d)}\right)^2}+\frac{3ie^{2a-2id}\log\left(1-e^{2i(bx+d)}\right)}{b}+\frac{ie^{2(a+ibx)}}{2b}\right)$$

input
```
Int[E^(2*(a + I*b*x))*Cot[d + b*x]^3,x]
```

output
```
I*(((I/2)*E^(2*(a + I*b*x)))/b - ((2*I)*E^(2*a - (2*I)*d))/(b*(1 - E^((2*I
)*(d + b*x)))^2) + ((6*I)*E^(2*a - (2*I)*d))/(b*(1 - E^((2*I)*(d + b*x))))
 + ((3*I)*E^(2*a - (2*I)*d)*Log[1 - E^((2*I)*(d + b*x))])/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{e^{2a} e^{2ibx}}{2b}+\frac{4\,e^{4ibx} e^{2id} e^{2a}-2\,e^{2a} e^{2ibx}}{b\left(-1+e^{2i(bx+d)}\right)^{2}}-\frac{3\,e^{2a} e^{-2id} \ln\left(-1+e^{2i(bx+d)}\right)}{b}$	91

input
```
int(exp(2*a+2*I*b*x)*cot(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(2*a)*exp(2*I*b*x)/b+2/b/(-1+exp(2*I*(b*x+d)))^2*(2*exp(4*I*b*x)*e
xp(2*I*d)*exp(2*a)-exp(2*a)*exp(2*I*b*x))-3*exp(2*a)/b*exp(-2*I*d)*ln(-1+e
xp(2*I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.97

$$\int e^{2(a+ibx)}\cot^3(d+bx)\,dx = -\frac{6\left(e^{(4i\,bx+2\,a+2i\,d)}-2\,e^{(2i\,bx+2\,a)}+e^{(2\,a-2i\,d)}\right)\log\left(e^{(2i\,bx+2i\,d)}-1\right)+e^{(6i\,bx+2\,a+4i\,d)}-2\,e^{(4i\,bx+2\,a+2i\,d)}-11}{2\left(be^{(4i\,bx+4i\,d)}-2\,be^{(2i\,bx+2i\,d)}+b\right)}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d)^3,x, algorithm="fricas")
```

output
```
-1/2*(6*(e^(4*I*b*x + 2*a + 2*I*d) - 2*e^(2*I*b*x + 2*a) + e^(2*a - 2*I*d)
)*log(e^(2*I*b*x + 2*I*d) - 1) + e^(6*I*b*x + 2*a + 4*I*d) - 2*e^(4*I*b*x
+ 2*a + 2*I*d) - 11*e^(2*I*b*x + 2*a) + 8*e^(2*a - 2*I*d))/(b*e^(4*I*b*x +
 4*I*d) - 2*b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\int e^{2(a+ibx)}\cot^3(d+bx)\,dx = \frac{6e^{2a}e^{2id}e^{2ibx}-4e^{2a}}{be^{6id}e^{4ibx}-2be^{4id}e^{2ibx}+be^{2id}} + \begin{cases} -\frac{e^{2a}e^{2ibx}}{2b} & \text{for } b\neq 0 \\ -ixe^{2a} & \text{otherwise} \end{cases} - \frac{3e^{2a}e^{-2id}\log\left(e^{2ibx}-e^{-2id}\right)}{b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d)**3,x)
```

output
```
(6*exp(2*a)*exp(2*I*d)*exp(2*I*b*x) - 4*exp(2*a))/(b*exp(6*I*d)*exp(4*I*b*
x) - 2*b*exp(4*I*d)*exp(2*I*b*x) + b*exp(2*I*d)) + Piecewise((-exp(2*a)*ex
p(2*I*b*x)/(2*b), Ne(b, 0)), (-I*x*exp(2*a), True)) - 3*exp(2*a)*exp(-2*I*
d)*log(exp(2*I*b*x) - exp(-2*I*d))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 869 vs. 2(94) = 188.

Time = 0.07 (sec) , antiderivative size = 869, normalized size of antiderivative = 6.84

$$\int e^{2(a+ibx)}\cot^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d)^3,x, algorithm="maxima")
```

output
```
-(6*(cos(2*d)^2*e^(2*a) + e^(2*a)*sin(2*d)^2 + (cos(2*d)*e^(2*a) - I*e^(2*
a)*sin(2*d))*cos(4*b*x + 6*d) - 2*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*
cos(2*b*x + 4*d) + (I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*sin(4*b*x + 6*d
) + 2*(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*sin(2*b*x + 4*d))*arctan2(s
in(b*x) + sin(d), cos(b*x) - cos(d)) + 6*(cos(2*d)^2*e^(2*a) + e^(2*a)*sin
(2*d)^2 + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(4*b*x + 6*d) - 2*(co
s(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(2*b*x + 4*d) + (I*cos(2*d)*e^(2*a
) + e^(2*a)*sin(2*d))*sin(4*b*x + 6*d) + 2*(-I*cos(2*d)*e^(2*a) - e^(2*a)*
sin(2*d))*sin(2*b*x + 4*d))*arctan2(sin(b*x) - sin(d), cos(b*x) + cos(d))
- I*cos(6*b*x + 6*d)*e^(2*a) + 2*I*cos(4*b*x + 4*d)*e^(2*a) + 11*I*cos(2*b
*x + 2*d)*e^(2*a) + 3*(-I*cos(2*d)^2*e^(2*a) - I*e^(2*a)*sin(2*d)^2 + (-I*
cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*cos(4*b*x + 6*d) + 2*(I*cos(2*d)*e^(2
*a) + e^(2*a)*sin(2*d))*cos(2*b*x + 4*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*s
in(2*d))*sin(4*b*x + 6*d) - 2*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(
2*b*x + 4*d))*log(cos(b*x)^2 + 2*cos(b*x)*cos(d) + cos(d)^2 + sin(b*x)^2 -
 2*sin(b*x)*sin(d) + sin(d)^2) + 3*(-I*cos(2*d)^2*e^(2*a) - I*e^(2*a)*sin(
2*d)^2 + (-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*cos(4*b*x + 6*d) + 2*(I*
cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*cos(2*b*x + 4*d) + (cos(2*d)*e^(2*a)
- I*e^(2*a)*sin(2*d))*sin(4*b*x + 6*d) - 2*(cos(2*d)*e^(2*a) - I*e^(2*a)*s
in(2*d))*sin(2*b*x + 4*d))*log(cos(b*x)^2 - 2*cos(b*x)*cos(d) + cos(d)^...
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17

$$\int e^{2(a+ibx)}\cot^3(d+bx)\,dx =$$

$$-\frac{6\,e^{(4i\,bx+2\,a+2i\,d)}\log\left(e^{(2i\,bx+2i\,d)}-1\right)-12\,e^{(2i\,bx+2\,a)}\log\left(e^{(2i\,bx+2i\,d)}-1\right)+6\,e^{(2\,a-2i\,d)}\log\left(e^{(2i\,bx+2i\,d)}-\right.}{2\,b(e^{(4i\,bx+4i\,d)}-2\,e^{(2i\,bx+2i\,d)}+1)}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/2*(6*e^(4*I*b*x + 2*a + 2*I*d)*log(e^(2*I*b*x + 2*I*d) - 1) - 12*e^(2*I
*b*x + 2*a)*log(e^(2*I*b*x + 2*I*d) - 1) + 6*e^(2*a - 2*I*d)*log(e^(2*I*b*
x + 2*I*d) - 1) + e^(6*I*b*x + 2*a + 4*I*d) - 2*e^(4*I*b*x + 2*a + 2*I*d)
- 11*e^(2*I*b*x + 2*a) + 8*e^(2*a - 2*I*d))/(b*(e^(4*I*b*x + 4*I*d) - 2*e^
(2*I*b*x + 2*I*d) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+ibx)}\cot^3(d+bx)\,dx = \int \cot(d+b\,x)^3\,\mathrm{e}^{2\,a+b\,x\,2\mathrm{i}}\,dx$$

input
```
int(cot(d + b*x)^3*exp(2*a + b*x*2i),x)
```

output
```
int(cot(d + b*x)^3*exp(2*a + b*x*2i), x)
```

Reduce [F]

$$\int e^{2(a+ibx)}\cot^3(d+bx)\,dx = e^{2a}\left(\int e^{2bix}\cot(bx+d)^3\,dx\right)$$

input
```
int(exp(2*a+2*I*b*x)*cot(b*x+d)^3,x)
```

output
```
e**(2*a)*int(e**(2*b*i*x)*cot(b*x + d)**3,x)
```

3.65 $\int e^{2(a+ibx)} \cot^4(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 172

$$\int e^{2(a+ibx)} \cot^4(d+bx)\,dx = -\frac{ie^{2(a-id)+2i(d+bx)}}{2b} - \frac{8ie^{2a-2id}}{3b\left(1-e^{2i(d+bx)}\right)^3} + \frac{8ie^{2a-2id}}{b\left(1-e^{2i(d+bx)}\right)^2} - \frac{12ie^{2a-2id}}{b\left(1-e^{2i(d+bx)}\right)} - \frac{4ie^{2a-2id}\log\left(1-e^{2i(d+bx)}\right)}{b}$$

output
```
-1/2*I*exp(2*a-2*I*d+2*I*(b*x+d))/b-8/3*I*exp(2*a-2*I*d)/b/(1-exp(2*I*(b*x
+d)))^3+8*I*exp(2*a-2*I*d)/b/(1-exp(2*I*(b*x+d)))^2-12*I*exp(2*a-2*I*d)/b/
(1-exp(2*I*(b*x+d)))-4*I*exp(2*a-2*I*d)*ln(1-exp(2*I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.71

$$\int e^{2(a+ibx)} \cot^4(d+bx)\,dx$$
$$= \frac{e^{2a}\left(-3ie^{2ibx} + 24\arctan\left(\frac{(1+e^{2ibx})\tan(d)}{-1+e^{2ibx}}\right)\cos(2d) - 12i\cos(2d)\log\left(1+e^{4ibx}-2e^{2ibx}\cos(2d)\right) + \frac{}{((-1+e^{2i}}\right.}{}$$

input
```
Integrate[E^(2*(a + I*b*x))*Cot[d + b*x]^4,x]
```

output
```
(E^(2*a)*((-3*I)*E^((2*I)*b*x) + 24*ArcTan[((1 + E^((2*I)*b*x))*Tan[d])/(-
1 + E^((2*I)*b*x))]*Cos[2*d] - (12*I)*Cos[2*d]*Log[1 + E^((4*I)*b*x) - 2*E
^((2*I)*b*x)*Cos[2*d]] + (16*(I*Cos[d] + Sin[d])^5)/((-1 + E^((2*I)*b*x))*
Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d])^3 - (24*I)*ArcTan[((1 + E^((2*I)*b*
x))*Tan[d])/(-1 + E^((2*I)*b*x))]*Sin[2*d] - 12*Log[1 + E^((4*I)*b*x) - 2*
E^((2*I)*b*x)*Cos[2*d]]*Sin[2*d] + (72*(I*Cos[3*d] + Sin[3*d]))/((-1 + E^(
(2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d]) + (48*(I*Cos[4*d] + Sin
[4*d]))/((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d])^2))/(
6*b)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\cot^4(bx+d)\,dx$$

$$\Big\downarrow \text{4943}$$

$$\int \left(-\frac{8e^{2(a+ibx)}}{1-e^{2i(bx+d)}}+\frac{24e^{2(a+ibx)}}{\left(1-e^{2i(bx+d)}\right)^2}-\frac{32e^{2(a+ibx)}}{\left(1-e^{2i(bx+d)}\right)^3}+\frac{16e^{2(a+ibx)}}{\left(1-e^{2i(bx+d)}\right)^4}+e^{2(a+ibx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{12ie^{2a-2id}}{b\left(1-e^{2i(bx+d)}\right)}+\frac{8ie^{2a-2id}}{b\left(1-e^{2i(bx+d)}\right)^2}-\frac{8ie^{2a-2id}}{3b\left(1-e^{2i(bx+d)}\right)^3}-\frac{4ie^{2a-2id}\log\left(1-e^{2i(bx+d)}\right)}{b}-\frac{ie^{2(a+ibx)}}{2b}$$

input
```
Int[E^(2*(a + I*b*x))*Cot[d + b*x]^4,x]
```

output
```
((-1/2*I)*E^(2*(a + I*b*x)))/b - (((8*I)/3)*E^(2*a - (2*I)*d))/(b*(1 - E^(
(2*I)*(d + b*x)))^3) + ((8*I)*E^(2*a - (2*I)*d))/(b*(1 - E^((2*I)*(d + b*x
)))^2) - ((12*I)*E^(2*a - (2*I)*d))/(b*(1 - E^((2*I)*(d + b*x)))) - ((4*I)
*E^(2*a - (2*I)*d)*Log[1 - E^((2*I)*(d + b*x))])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{i e^{2a} e^{2ibx}}{2b}+\frac{4i\left(5\,e^{6ibx} e^{4id} e^{2a}-6\,e^{4ibx} e^{2id} e^{2a}+3\,e^{2a} e^{2ibx}\right)}{3b\left(-1+e^{2i(bx+d)}\right)^{3}}-\frac{4i e^{2a} e^{-2id} \ln\left(-1+e^{2i(bx+d)}\right)}{b}$	111

input
```
int(exp(2*a+2*I*b*x)*cot(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b*exp(2*a)*exp(2*I*b*x)+4/3*I/b/(-1+exp(2*I*(b*x+d)))^3*(5*exp(6*I*
b*x)*exp(4*I*d)*exp(2*a)-6*exp(4*I*b*x)*exp(2*I*d)*exp(2*a)+3*exp(2*a)*exp
(2*I*b*x))-4*I*exp(2*a)/b*exp(-2*I*d)*ln(-1+exp(2*I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99

$$\int e^{2(a+ibx)}\cot^4(d+bx)\,dx = -\frac{24\left(i\,e^{(6i\,bx+2\,a+4i\,d)}-3i\,e^{(4i\,bx+2\,a+2i\,d)}+3i\,e^{(2i\,bx+2\,a)}-i\,e^{(2\,a-2i\,d)}\right)\log\left(e^{(2i\,bx+2i\,d)}-1\right)+3i\,e^{(8i\,bx+2\,a+6}}{6\left(be^{(6i\,bx+6i\,d)}-3\,be^{(4i\,bx+4i\,d)}+3\,be^{(2i\,bx}\right.}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d)^4,x, algorithm="fricas")
```

output
```
-1/6*(24*(I*e^(6*I*b*x + 2*a + 4*I*d) - 3*I*e^(4*I*b*x + 2*a + 2*I*d) + 3*
I*e^(2*I*b*x + 2*a) - I*e^(2*a - 2*I*d))*log(e^(2*I*b*x + 2*I*d) - 1) + 3*
I*e^(8*I*b*x + 2*a + 6*I*d) - 9*I*e^(6*I*b*x + 2*a + 4*I*d) - 63*I*e^(4*I*
b*x + 2*a + 2*I*d) + 93*I*e^(2*I*b*x + 2*a) - 40*I*e^(2*a - 2*I*d))/(b*e^(
6*I*b*x + 6*I*d) - 3*b*e^(4*I*b*x + 4*I*d) + 3*b*e^(2*I*b*x + 2*I*d) - b)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.07

$$\int e^{2(a+ibx)}\cot^4(d+bx)\,dx = \frac{36ie^{2a}e^{4id}e^{4ibx}-48ie^{2a}e^{2id}e^{2ibx}+20ie^{2a}}{3be^{8id}e^{6ibx}-9be^{6id}e^{4ibx}+9be^{4id}e^{2ibx}-3be^{2id}} + \begin{cases} -\frac{ie^{2a}e^{2ibx}}{2b} & \text{for } b\neq 0 \\ xe^{2a} & \text{otherwise} \end{cases} - \frac{4ie^{2a}e^{-2id}\log\left(e^{2ibx}-e^{-2id}\right)}{b}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d)**4,x)
```

output
```
(36*I*exp(2*a)*exp(4*I*d)*exp(4*I*b*x) - 48*I*exp(2*a)*exp(2*I*d)*exp(2*I*
b*x) + 20*I*exp(2*a))/(3*b*exp(8*I*d)*exp(6*I*b*x) - 9*b*exp(6*I*d)*exp(4*
I*b*x) + 9*b*exp(4*I*d)*exp(2*I*b*x) - 3*b*exp(2*I*d)) + Piecewise((-I*exp
(2*a)*exp(2*I*b*x)/(2*b), Ne(b, 0)), (x*exp(2*a), True)) - 4*I*exp(2*a)*ex
p(-2*I*d)*log(exp(2*I*b*x) - exp(-2*I*d))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1177 vs. $2(120) = 240$.

Time = 0.11 (sec) , antiderivative size = 1177, normalized size of antiderivative = 6.84

$$\int e^{2(a+ibx)} \cot^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d)^4,x, algorithm="maxima")
```

output
```
(24*(I*cos(2*d)^2*e^(2*a) + I*e^(2*a)*sin(2*d)^2 + (-I*cos(2*d)*e^(2*a) -
e^(2*a)*sin(2*d))*cos(6*b*x + 8*d) + 3*(I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2
*d))*cos(4*b*x + 6*d) + 3*(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*cos(2*b
*x + 4*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(6*b*x + 8*d) - 3*(
cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(4*b*x + 6*d) + 3*(cos(2*d)*e^(2
*a) - I*e^(2*a)*sin(2*d))*sin(2*b*x + 4*d))*arctan2(sin(b*x) + sin(d), cos
(b*x) - cos(d)) + 24*(I*cos(2*d)^2*e^(2*a) + I*e^(2*a)*sin(2*d)^2 + (-I*co
s(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*cos(6*b*x + 8*d) + 3*(I*cos(2*d)*e^(2*a
) + e^(2*a)*sin(2*d))*cos(4*b*x + 6*d) + 3*(-I*cos(2*d)*e^(2*a) - e^(2*a)*
sin(2*d))*cos(2*b*x + 4*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(6
*b*x + 8*d) - 3*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(4*b*x + 6*d) +
 3*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(2*b*x + 4*d))*arctan2(sin(b
*x) - sin(d), cos(b*x) + cos(d)) - 3*cos(8*b*x + 8*d)*e^(2*a) + 9*cos(6*b*
x + 6*d)*e^(2*a) + 63*cos(4*b*x + 4*d)*e^(2*a) - 93*cos(2*b*x + 2*d)*e^(2*
a) + 12*(cos(2*d)^2*e^(2*a) + e^(2*a)*sin(2*d)^2 - (cos(2*d)*e^(2*a) - I*e
^(2*a)*sin(2*d))*cos(6*b*x + 8*d) + 3*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*
d))*cos(4*b*x + 6*d) - 3*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(2*b*x
 + 4*d) + (-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*sin(6*b*x + 8*d) + 3*(I
*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*sin(4*b*x + 6*d) + 3*(-I*cos(2*d)*e^
(2*a) - e^(2*a)*sin(2*d))*sin(2*b*x + 4*d))*log(cos(b*x)^2 + 2*cos(b*x)...
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.17

$$\int e^{2(a+ibx)}\cot^4(d+bx)\,dx = \frac{-24i\,e^{(6i\,bx+2\,a+4i\,d)}\log\left(e^{(2i\,bx+2i\,d)}-1\right)+72i\,e^{(4i\,bx+2\,a+2i\,d)}\log\left(e^{(2i\,bx+2i\,d)}-1\right)-72i\,e^{(2i\,bx+2\,a)}\log\left(e^{(2i\,b}\right.}{6\,b\left(e^{(6i\,bx}\right.}$$

input
```
integrate(exp(2*a+2*I*b*x)*cot(b*x+d)^4,x, algorithm="giac")
```

output
```
1/6*(-24*I*e^(6*I*b*x + 2*a + 4*I*d)*log(e^(2*I*b*x + 2*I*d) - 1) + 72*I*e
^(4*I*b*x + 2*a + 2*I*d)*log(e^(2*I*b*x + 2*I*d) - 1) - 72*I*e^(2*I*b*x +
2*a)*log(e^(2*I*b*x + 2*I*d) - 1) + 24*I*e^(2*a - 2*I*d)*log(e^(2*I*b*x +
2*I*d) - 1) - 3*I*e^(8*I*b*x + 2*a + 6*I*d) + 9*I*e^(6*I*b*x + 2*a + 4*I*d
) + 63*I*e^(4*I*b*x + 2*a + 2*I*d) - 93*I*e^(2*I*b*x + 2*a) + 40*I*e^(2*a
- 2*I*d))/(b*(e^(6*I*b*x + 6*I*d) - 3*e^(4*I*b*x + 4*I*d) + 3*e^(2*I*b*x +
 2*I*d) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+ibx)}\cot^4(d+bx)\,dx = \int \cot(d+b\,x)^4\,\mathrm{e}^{2\,a+b\,x\,2\mathrm{i}}\,dx$$

input
```
int(cot(d + b*x)^4*exp(2*a + b*x*2i),x)
```

output
```
int(cot(d + b*x)^4*exp(2*a + b*x*2i), x)
```

Reduce [F]

$$\int e^{2(a+ibx)}\cot^4(d+bx)\,dx = e^{2a}\left(\int e^{2bix}\cot\left(bx+d\right)^4dx\right)$$

input
```
int(exp(2*a+2*I*b*x)*cot(b*x+d)^4,x)
```

output
```
e**(2*a)*int(e**(2*b*i*x)*cot(b*x + d)**4,x)
```

3.66 $\int e^{\frac{5}{3}(a+ibx)} \cot(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 208

$$\int e^{\frac{5}{3}(a+ibx)} \cot(d+bx)\,dx = \frac{3e^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{5b} - \frac{\sqrt{3}e^{\frac{5}{3}(a-id)} \arctan\left(\frac{1-2e^{\frac{1}{3}i(d+bx)}}{\sqrt{3}}\right)}{b} + \frac{\sqrt{3}e^{\frac{5}{3}(a-id)} \arctan\left(\frac{1+2e^{\frac{1}{3}i(d+bx)}}{\sqrt{3}}\right)}{b} - \frac{2e^{\frac{5}{3}(a-id)}\text{arctanh}\left(e^{\frac{1}{3}i(d+bx)}\right)}{b} - \frac{e^{\frac{5}{3}(a-id)}\text{arctanh}\left(\frac{e^{\frac{1}{3}i(d+bx)}}{1+e^{\frac{2}{3}i(d+bx)}}\right)}{b}$$

output
```
3/5*exp(5/3*a-5/3*I*d+5/3*I*(b*x+d))/b-3^(1/2)*exp(5/3*a-5/3*I*d)*arctan(1
/3*(1-2*exp(1/3*I*(b*x+d)))*3^(1/2))/b+3^(1/2)*exp(5/3*a-5/3*I*d)*arctan(1
/3*(1+2*exp(1/3*I*(b*x+d)))*3^(1/2))/b-2*exp(5/3*a-5/3*I*d)*arctanh(exp(1/
3*I*(b*x+d)))/b-exp(5/3*a-5/3*I*d)*arctanh(exp(1/3*I*(b*x+d))/(1+exp(2/3*I
*(b*x+d))))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.50

$$\int e^{\frac{5}{3}(a+ibx)} \cot(d+bx)\,dx = \frac{e^{5a/3}\left(9e^{\frac{5ibx}{3}} + \text{RootSum}\left[-\cos(d)+i\sin(d)+\cos(d)\#1^6+i\sin(d)\#1^6\&,\frac{bx+3i\log\left(e^{\frac{ibx}{3}}-\#1\right)}{\#1}\&\right](-5i\,\text{co}\right.}{15b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Cot[d + b*x],x]
```

output
```
(E^((5*a)/3)*(9*E^(((5*I)/3)*b*x) + RootSum[-Cos[d] + I*Sin[d] + Cos[d]*#1
^6 + I*Sin[d]*#1^6 & , (b*x + (3*I)*Log[E^((I/3)*b*x) - #1])/#1 & ]*((-5*I
)*Cos[2*d] - 5*Sin[2*d])))/(15*b)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.61, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)} \cot(bx+d)\,dx$$

$$\Big\downarrow \text{ 4943}$$

$$-i\int \left(\frac{2e^{\frac{5}{3}(a+ibx)}}{1-e^{2i(d+bx)}} - e^{\frac{5}{3}(a+ibx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-i\left(-\frac{i\sqrt{3}e^{\frac{5}{3}(a-id)}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2e^{\frac{1}{3}(a+ibx)+\frac{1}{3}(-a+id)}}{\sqrt{3}}\right)}{b}+\frac{i\sqrt{3}e^{\frac{5}{3}(a-id)}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2e^{\frac{1}{3}(a+ibx)+\frac{1}{3}(-a+id)}}{\sqrt{3}}\right)}{b}-\frac{2ie^{\frac{5}{3}(a}}{}\right.$$

input
```
Int[E^((5*(a + I*b*x))/3)*Cot[d + b*x],x]
```

output
```
(-I)*((((3*I)/5)*E^((5*(a + I*b*x))/3))/b - (I*Sqrt[3]*E^((5*(a - I*d))/3)
*ArcTan[1/Sqrt[3] - (2*E^((-a + I*d)/3 + (a + I*b*x)/3))/Sqrt[3]])/b + (I*
Sqrt[3]*E^((5*(a - I*d))/3)*ArcTan[1/Sqrt[3] + (2*E^((-a + I*d)/3 + (a + I
*b*x)/3))/Sqrt[3]])/b - ((2*I)*E^((5*(a - I*d))/3)*ArcTanh[E^((-a + I*d)/3
 + (a + I*b*x)/3)])/b + ((I/2)*E^((5*(a - I*d))/3)*Log[1 - E^((I/3)*d + (I
/3)*b*x) + E^(((2*I)/3)*d + ((2*I)/3)*b*x)])/b - ((I/2)*E^((5*(a - I*d))/3
)*Log[1 + E^((I/3)*d + (I/3)*b*x) + E^(((2*I)/3)*d + ((2*I)/3)*b*x)])/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [F]

$$\int e^{\frac{5a}{3}+\frac{5ibx}{3}} \cot(bx+d)\, dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*cot(b*x+d),x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*cot(b*x+d),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(144) = 288.

Time = 0.09 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.72

$$\int e^{\frac{5}{3}(a+ibx)} \cot(d+bx)\,dx =$$

$$-\frac{10\,b\frac{1}{b^6}^{\frac{1}{6}}e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,i\,d\right)}\log\left(b^5\frac{1}{b^6}^{\frac{5}{6}}e^{\left(\frac{1}{3}\,a-\frac{1}{3}\,i\,d\right)}+e^{\left(\frac{1}{3}\,i\,bx+\frac{1}{3}\,a\right)}\right)-10\,b\frac{1}{b^6}^{\frac{1}{6}}e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,i\,d\right)}\log\left(-b^5\frac{1}{b^6}^{\frac{5}{6}}e^{\left(\frac{1}{3}\,a-\frac{1}{3}\,i\,d\right)}+e^{\left(\frac{1}{3}\,i\,bx\right.}\right.}{}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cot(b*x+d),x, algorithm="fricas")
```

output
```
-1/10*(10*b*(b^(-6))^(1/6)*e^(5/3*a - 5/3*I*d)*log(b^5*(b^(-6))^(5/6)*e^(1
/3*a - 1/3*I*d) + e^(1/3*I*b*x + 1/3*a)) - 10*b*(b^(-6))^(1/6)*e^(5/3*a -
5/3*I*d)*log(-b^5*(b^(-6))^(5/6)*e^(1/3*a - 1/3*I*d) + e^(1/3*I*b*x + 1/3*
a)) + 5*(-I*sqrt(3)*b + b)*(b^(-6))^(1/6)*e^(5/3*a - 5/3*I*d)*log(1/2*(I*s
qrt(3)*b^5 + b^5)*(b^(-6))^(5/6)*e^(1/3*a - 1/3*I*d) + e^(1/3*I*b*x + 1/3*
a)) + 5*(-I*sqrt(3)*b - b)*(b^(-6))^(1/6)*e^(5/3*a - 5/3*I*d)*log(1/2*(I*s
qrt(3)*b^5 - b^5)*(b^(-6))^(5/6)*e^(1/3*a - 1/3*I*d) + e^(1/3*I*b*x + 1/3*
a)) + 5*(I*sqrt(3)*b + b)*(b^(-6))^(1/6)*e^(5/3*a - 5/3*I*d)*log(1/2*(-I*s
qrt(3)*b^5 + b^5)*(b^(-6))^(5/6)*e^(1/3*a - 1/3*I*d) + e^(1/3*I*b*x + 1/3*
a)) + 5*(I*sqrt(3)*b - b)*(b^(-6))^(1/6)*e^(5/3*a - 5/3*I*d)*log(1/2*(-I*s
qrt(3)*b^5 - b^5)*(b^(-6))^(5/6)*e^(1/3*a - 1/3*I*d) + e^(1/3*I*b*x + 1/3*
a)) - 6*e^(5/3*I*b*x + 5/3*a))/b
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

$$\int e^{\frac{5}{3}(a+ibx)} \cot(d+bx)\,dx = \begin{cases} \frac{3e^{\frac{5a}{3}}e^{\frac{5ibx}{3}}}{5b} & \text{for } b \neq 0 \\ ixe^{\frac{5a}{3}} & \text{otherwise} \end{cases}$$

$$+\frac{\operatorname{RootSum}\left(z^3e^{5id}-e^{5a},\left(i\mapsto i\log\left(-i^5e^{-\frac{25a}{3}}e^{8id}+e^{\frac{ibx}{3}}\right)\right)\right)+\operatorname{RootSum}\left(z^3e^{5id}+e^{5a},\left(i\mapsto i\log\left(-i\right.\right.\right.}{b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cot(b*x+d),x)
```

output
```
Piecewise((3*exp(5*a/3)*exp(5*I*b*x/3)/(5*b), Ne(b, 0)), (I*x*exp(5*a/3),
True)) + (RootSum(_z**3*exp(5*I*d) - exp(5*a), Lambda(_i, _i*log(-_i**5*ex
p(-25*a/3)*exp(8*I*d) + exp(I*b*x/3)))) + RootSum(_z**3*exp(5*I*d) + exp(5
*a), Lambda(_i, _i*log(-_i**5*exp(-25*a/3)*exp(8*I*d) + exp(I*b*x/3)))))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3081 vs. $2(144) = 288$.

Time = 0.43 (sec) , antiderivative size = 3081, normalized size of antiderivative = 14.81

$$\int e^{\frac{5}{3}(a+ibx)}\cot(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cot(b*x+d),x, algorithm="maxima")
```

output
```
-1/20*(10*((-I*cos(2*d)*e^(5/3*a) - e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*
arctan2(sin(d), cos(d))) + (cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*sin
(-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*arctan2(cos(1/3*b*x)*sin(-1/3*pi
+ 1/3*arctan2(sin(d), cos(d))) - cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d))
)*sin(1/3*b*x) - sin(-2/3*pi + 2/3*arctan2(sin(d), cos(d))) + sin(2/3*b*x)
, -cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))*cos(1/3*b*x) - sin(-1/3*pi +
 1/3*arctan2(sin(d), cos(d)))*sin(1/3*b*x) + cos(-2/3*pi + 2/3*arctan2(sin
(d), cos(d))) + cos(2/3*b*x)) + 10*((-I*cos(2*d)*e^(5/3*a) - e^(5/3*a)*sin
(2*d))*cos(1/3*arctan2(sin(d), cos(d))) + (cos(2*d)*e^(5/3*a) - I*e^(5/3*a
)*sin(2*d))*sin(1/3*arctan2(sin(d), cos(d))))*arctan2(-cos(1/3*arctan2(sin
(d), cos(d)))*sin(1/3*b*x) + cos(1/3*b*x)*sin(1/3*arctan2(sin(d), cos(d)))
 + sin(2/3*b*x) - sin(2/3*arctan2(sin(d), cos(d))), -cos(1/3*b*x)*cos(1/3*
arctan2(sin(d), cos(d))) - sin(1/3*b*x)*sin(1/3*arctan2(sin(d), cos(d))) +
 cos(2/3*b*x) + cos(2/3*arctan2(sin(d), cos(d)))) - 10*((sqrt(3)*cos(2*d)*
e^(5/3*a) - I*sqrt(3)*e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d)
, cos(d))) - (-I*sqrt(3)*cos(2*d)*e^(5/3*a) - sqrt(3)*e^(5/3*a)*sin(2*d))*
sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*arctan2(1/2*(sqrt(3)*cos(-1/3*
pi + 1/3*arctan2(sin(d), cos(d)))^2 - 2*sqrt(3)*cos(-1/3*pi + 1/3*arctan2(
sin(d), cos(d)))*cos(1/3*b*x) + sqrt(3)*sin(-1/3*pi + 1/3*arctan2(sin(d),
cos(d)))^2 + 2*sqrt(3)*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))*sin(1...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49619 vs. 2(144) = 288.

Time = 50.19 (sec) , antiderivative size = 49619, normalized size of antiderivative = 238.55

$$\int e^{\frac{5}{3}(a+ibx)}\cot(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cot(b*x+d),x, algorithm="giac")
```

output
```
-1/4*(2*(-e^(10*a - 7*I*d))^(2/3)*e^(-5*a + 3*I*d)*log(-(-e^(10*a - 7*I*d)
)^(1/3) + e^(1/3*I*b*x + 10/3*a - 2*I*d)) - 2*e^(5/3*a - 5/3*I*d)*log(e^(1
/3*I*b*x + 10/3*a - 2*I*d) - e^(10/3*a - 7/3*I*d)) - 2*((cos(2*d)^6*cos(d)
^10 + 3*cos(2*d)^4*cos(d)^10*sin(2*d)^2 + 3*cos(2*d)^2*cos(d)^10*sin(2*d)^
4 + cos(d)^10*sin(2*d)^6 + 5*cos(2*d)^6*cos(d)^8*sin(d)^2 + 15*cos(2*d)^4*
cos(d)^8*sin(2*d)^2*sin(d)^2 + 15*cos(2*d)^2*cos(d)^8*sin(2*d)^4*sin(d)^2
+ 5*cos(d)^8*sin(2*d)^6*sin(d)^2 + 10*cos(2*d)^6*cos(d)^6*sin(d)^4 + 30*co
s(2*d)^4*cos(d)^6*sin(2*d)^2*sin(d)^4 + 30*cos(2*d)^2*cos(d)^6*sin(2*d)^4*
sin(d)^4 + 10*cos(d)^6*sin(2*d)^6*sin(d)^4 + 10*cos(2*d)^6*cos(d)^4*sin(d)
^6 + 30*cos(2*d)^4*cos(d)^4*sin(2*d)^2*sin(d)^6 + 30*cos(2*d)^2*cos(d)^4*s
in(2*d)^4*sin(d)^6 + 10*cos(d)^4*sin(2*d)^6*sin(d)^6 + 5*cos(2*d)^6*cos(d)
^2*sin(d)^8 + 15*cos(2*d)^4*cos(d)^2*sin(2*d)^2*sin(d)^8 + 15*cos(2*d)^2*c
os(d)^2*sin(2*d)^4*sin(d)^8 + 5*cos(d)^2*sin(2*d)^6*sin(d)^8 + cos(2*d)^6*
sin(d)^10 + 3*cos(2*d)^4*sin(2*d)^2*sin(d)^10 + 3*cos(2*d)^2*sin(2*d)^4*si
n(d)^10 + sin(2*d)^6*sin(d)^10)^(1/3)*(sqrt(3)*cos(2*d)^2*cos(d)*e^(5/3*a)
 - sqrt(3)*cos(d)*e^(5/3*a)*sin(2*d)^2 + 2*sqrt(3)*cos(2*d)*e^(5/3*a)*sin(
2*d)*sin(d))*cos(-1/6*pi*sgn(cos(2*d)^3*cos(d) - 3*cos(2*d)*cos(d)*sin(2*d
)^2 - 3*cos(2*d)^2*sin(2*d)*sin(d) + sin(2*d)^3*sin(d))*sgn(-3*cos(2*d)^2*
cos(d)*sin(2*d) + cos(d)*sin(2*d)^3 - cos(2*d)^3*sin(d) + 3*cos(2*d)*sin(2
*d)^2*sin(d)) + 1/6*pi*sgn(-3*cos(2*d)^2*cos(d)*sin(2*d) + cos(d)*sin(2...
```

Mupad [B] (verification not implemented)

Time = 19.83 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.16

$$\int e^{\frac{5}{3}(a+ibx)}\cot(d+bx)\,dx = \text{Too large to display}$$

input
```
int(cot(d + b*x)*exp((5*a)/3 + (b*x*5i)/3),x)
```

output
```
(3*exp((5*a)/3 + (b*x*5i)/3))/(5*b) + (exp(10*a - d*10i)^(1/6)*log(4*exp(6
*a)*exp(-d*6i) - 4*exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*(exp(10*a)
*exp(-d*10i))^(1/6)))/b - (exp(10*a - d*10i)^(1/6)*log(4*exp(6*a)*exp(-d*6
i) + 4*exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*(exp(10*a)*exp(-d*10i)
)^(1/6)))/b + (log(4*exp(6*a)*exp(-d*6i) - 4*exp(a/3)*exp(4*a)*exp(-d*4i)*
exp((b*x*1i)/3)*((3^(1/2)*1i)/2 - 1/2)*(exp(10*a)*exp(-d*10i))^(1/6))*exp(
10*a - d*10i)^(1/6)*((3^(1/2)*1i)/2 - 1/2))/b - (log(4*exp(6*a)*exp(-d*6i)
 + 4*exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 - 1/2)*(
exp(10*a)*exp(-d*10i))^(1/6))*exp(10*a - d*10i)^(1/6)*((3^(1/2)*1i)/2 - 1/
2))/b + (log(4*exp(6*a)*exp(-d*6i) - 4*exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b
*x*1i)/3)*((3^(1/2)*1i)/2 + 1/2)*(exp(10*a)*exp(-d*10i))^(1/6))*exp(10*a -
 d*10i)^(1/6)*((3^(1/2)*1i)/2 + 1/2))/b - (log(4*exp(6*a)*exp(-d*6i) + 4*e
xp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 + 1/2)*(exp(10
*a)*exp(-d*10i))^(1/6))*exp(10*a - d*10i)^(1/6)*((3^(1/2)*1i)/2 + 1/2))/b
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\cot(d+bx)\,dx = \int e^{\frac{5bix}{3}+\frac{5a}{3}}\cot(bx+d)\,dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*cot(b*x+d),x)
```

output
```
int(e**((5*a + 5*b*i*x)/3)*cot(b*x + d),x)
```

3.67 $\int e^{\frac{5}{3}(a+ibx)} \cot^2(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 272

$$\int e^{\frac{5}{3}(a+ibx)} \cot^2(d+bx)\,dx = \frac{3ie^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{5b} + \frac{2ie^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)} - \frac{5ie^{\frac{5}{3}(a-id)}\arctan\left(\frac{1-2e^{\frac{1}{3}i(d+bx)}}{\sqrt{3}}\right)}{\sqrt{3}b} + \frac{5ie^{\frac{5}{3}(a-id)}\arctan\left(\frac{1+2e^{\frac{1}{3}i(d+bx)}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{10ie^{\frac{5}{3}(a-id)}\operatorname{arctanh}\left(e^{\frac{1}{3}i(d+bx)}\right)}{3b} - \frac{5ie^{\frac{5}{3}(a-id)}\operatorname{arctanh}\left(\frac{e^{\frac{1}{3}i(d+bx)}}{1+e^{\frac{2}{3}i(d+bx)}}\right)}{3b}$$

output
```
3/5*I*exp(5/3*a-5/3*I*d+5/3*I*(b*x+d))/b+2*I*exp(5/3*a-5/3*I*d+5/3*I*(b*x+
d))/b/(1-exp(2*I*(b*x+d)))-5/3*I*exp(5/3*a-5/3*I*d)*arctan(1/3*(1-2*exp(1/
3*I*(b*x+d)))*3^(1/2))*3^(1/2)/b+5/3*I*exp(5/3*a-5/3*I*d)*arctan(1/3*(1+2*
exp(1/3*I*(b*x+d)))*3^(1/2))*3^(1/2)/b-10/3*I*exp(5/3*a-5/3*I*d)*arctanh(e
xp(1/3*I*(b*x+d)))/b-5/3*I*exp(5/3*a-5/3*I*d)*arctanh(exp(1/3*I*(b*x+d))/(
1+exp(2/3*I*(b*x+d))))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.59

$$\int e^{\frac{5}{3}(a+ibx)}\cot^2(d+bx)\,dx = \frac{1}{45}e^{5a/3}\left(\frac{27ie^{\frac{5ibx}{3}}}{b} + \frac{25\text{RootSum}\left[-\cos(d)+i\sin(d)+\cos(d)\#1^6+i\sin(d)\#1^6\&,\frac{bx+3i\log\left(e^{\frac{ibx}{3}}-\#1\right)}{\#1}\&\right](\cos(d)-i\sin(d)}{b} + \frac{90e^{\frac{5ibx}{3}}(\cos(d)-i\sin(d))}{ib\left(-1+e^{2ibx}\right)\cos(d)-b\left(1+e^{2ibx}\right)\sin(d)}\right)$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Cot[d + b*x]^2,x]
```

output
```
(E^((5*a)/3)*(((27*I)*E^(((5*I)/3)*b*x))/b + (25*RootSum[-Cos[d] + I*Sin[d
] + Cos[d]*#1^6 + I*Sin[d]*#1^6 & , (b*x + (3*I)*Log[E^((I/3)*b*x) - #1])/
#1 & ]*(Cos[d] - I*Sin[d])^2)/b + (90*E^(((5*I)/3)*b*x)*(Cos[d] - I*Sin[d]
))/(I*b*(-1 + E^((2*I)*b*x))*Cos[d] - b*(1 + E^((2*I)*b*x))*Sin[d])))/45
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\cot^2(bx+d)\,dx$$

$$\Big\downarrow \text{4943}$$

$$-\int \left(e^{\frac{5}{3}(a+ibx)}-\frac{4e^{\frac{5}{3}(a+ibx)}}{1-e^{2i(d+bx)}}+\frac{4e^{\frac{5}{3}(a+ibx)}}{\left(1-e^{2i(d+bx)}\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2i\sqrt{3}e^{\frac{5}{3}(a-id)}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2e^{\frac{1}{3}(a+ibx)+\frac{1}{3}(-a+id)}}{\sqrt{3}}\right)}{b}+\frac{ie^{\frac{5}{3}(a-id)}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2e^{\frac{1}{3}(a+ibx)+\frac{1}{3}(-a+id)}}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{2i\sqrt{3}e^{\frac{5}{3}(a-id)}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2e^{\frac{1}{3}(a+ibx)+\frac{1}{3}(-a+id)}}{\sqrt{3}}\right)}{b}-\frac{ie^{\frac{5}{3}(a-id)}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2e^{\frac{1}{3}(a+ibx)+\frac{1}{3}(-a+id)}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{10ie^{\frac{5}{3}(a-id)}\text{arctanh}\left(e^{\frac{1}{3}(a+ibx)+\frac{1}{3}(-a+id)}\right)}{3b}+\frac{2ie^{\frac{5}{3}(a+ibx)}}{b\left(1-e^{2i(bx+d)}\right)}+\frac{5ie^{\frac{5}{3}(a-id)}\log\left(-e^{\frac{ibx}{3}+\frac{id}{3}}+e^{\frac{2ibx}{3}+\frac{2id}{3}}+1\right)}{6b}-\frac{5ie^{\frac{5}{3}(a-id)}\log\left(e^{\frac{ibx}{3}+\frac{id}{3}}+e^{\frac{2ibx}{3}+\frac{2id}{3}}+1\right)}{6b}+\frac{3ie^{\frac{5}{3}(a+ibx)}}{5b}$$

input

```
Int[E^((5*(a + I*b*x))/3)*Cot[d + b*x]^2,x]
```

output

```
(((3*I)/5)*E^((5*(a + I*b*x))/3))/b + ((2*I)*E^((5*(a + I*b*x))/3))/(b*(1
 - E^((2*I)*(d + b*x)))) + (I*E^((5*(a - I*d))/3)*ArcTan[1/Sqrt[3] - (2*E^(
(-a + I*d)/3 + (a + I*b*x)/3))/Sqrt[3]])/(Sqrt[3]*b) - ((2*I)*Sqrt[3]*E^((
5*(a - I*d))/3)*ArcTan[1/Sqrt[3] - (2*E^((-a + I*d)/3 + (a + I*b*x)/3))/Sq
rt[3]])/b - (I*E^((5*(a - I*d))/3)*ArcTan[1/Sqrt[3] + (2*E^((-a + I*d)/3 +
 (a + I*b*x)/3))/Sqrt[3]])/(Sqrt[3]*b) + ((2*I)*Sqrt[3]*E^((5*(a - I*d))/3
)*ArcTan[1/Sqrt[3] + (2*E^((-a + I*d)/3 + (a + I*b*x)/3))/Sqrt[3]])/b - ((
(10*I)/3)*E^((5*(a - I*d))/3)*ArcTanh[E^((-a + I*d)/3 + (a + I*b*x)/3)])/b
 + (((5*I)/6)*E^((5*(a - I*d))/3)*Log[1 - E^((I/3)*d + (I/3)*b*x) + E^(((2
*I)/3)*d + ((2*I)/3)*b*x)])/b - (((5*I)/6)*E^((5*(a - I*d))/3)*Log[1 + E^(
(I/3)*d + (I/3)*b*x) + E^(((2*I)/3)*d + ((2*I)/3)*b*x)])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [F]

$$\int \mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \cot \left(bx+d\right)^{2} dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)^2,x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(172) = 344$.

Time $= 0.09$ (sec) , antiderivative size $= 563$, normalized size of antiderivative $= 2.07$

$$\int e^{\frac{5}{3}(a+ibx)} \cot^2(d+bx)\, dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)^2,x, algorithm="fricas")
```

output

```
1/30*(50*(b*e^(2*I*b*x + 2*I*d) - b)*(-e^(10*a - 10*I*d)/b^6)^(1/6)*log(I*
b^5*(-e^(10*a - 10*I*d)/b^6)^(5/6)*e^(-8*a + 8*I*d) + e^(1/3*I*b*x + 1/3*a
)) - 50*(b*e^(2*I*b*x + 2*I*d) - b)*(-e^(10*a - 10*I*d)/b^6)^(1/6)*log(-I*
b^5*(-e^(10*a - 10*I*d)/b^6)^(5/6)*e^(-8*a + 8*I*d) + e^(1/3*I*b*x + 1/3*a
)) - 25*((-I*sqrt(3)*b - b)*e^(2*I*b*x + 2*I*d) + I*sqrt(3)*b + b)*(-e^(10
*a - 10*I*d)/b^6)^(1/6)*log(1/2*(sqrt(3)*b^5 + I*b^5)*(-e^(10*a - 10*I*d)/
b^6)^(5/6)*e^(-8*a + 8*I*d) + e^(1/3*I*b*x + 1/3*a)) - 25*((I*sqrt(3)*b +
b)*e^(2*I*b*x + 2*I*d) - I*sqrt(3)*b - b)*(-e^(10*a - 10*I*d)/b^6)^(1/6)*l
og(-1/2*(sqrt(3)*b^5 + I*b^5)*(-e^(10*a - 10*I*d)/b^6)^(5/6)*e^(-8*a + 8*I
*d) + e^(1/3*I*b*x + 1/3*a)) - 25*((-I*sqrt(3)*b + b)*e^(2*I*b*x + 2*I*d)
+ I*sqrt(3)*b - b)*(-e^(10*a - 10*I*d)/b^6)^(1/6)*log(1/2*(sqrt(3)*b^5 - I
*b^5)*(-e^(10*a - 10*I*d)/b^6)^(5/6)*e^(-8*a + 8*I*d) + e^(1/3*I*b*x + 1/3
*a)) - 25*((I*sqrt(3)*b - b)*e^(2*I*b*x + 2*I*d) - I*sqrt(3)*b + b)*(-e^(1
0*a - 10*I*d)/b^6)^(1/6)*log(-1/2*(sqrt(3)*b^5 - I*b^5)*(-e^(10*a - 10*I*d
)/b^6)^(5/6)*e^(-8*a + 8*I*d) + e^(1/3*I*b*x + 1/3*a)) + 18*I*e^(11/3*I*b*
x + 5/3*a + 2*I*d) - 78*I*e^(5/3*I*b*x + 5/3*a))/(b*e^(2*I*b*x + 2*I*d) -
b)
```

Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.46

$$\int e^{\frac{5}{3}(a+ibx)}\cot^2(d+bx)\,dx$$
$$=\begin{cases}\frac{3ie^{\frac{5a}{3}}e^{\frac{5ibx}{3}}}{5b} & \text{for } b\neq 0\\ -xe^{\frac{5a}{3}} & \text{otherwise}\end{cases}-\frac{2ie^{\frac{5a}{3}}e^{\frac{5ibx}{3}}}{be^{2id}e^{2ibx}-b}$$
$$+\frac{\operatorname{RootSum}\left(729z^6e^{10id}+15625e^{10a},\left(i\mapsto i\log\left(\frac{243i^5ie^{-\frac{25a}{3}}e^{8id}}{3125}+e^{\frac{ibx}{3}}\right)\right)\right)}{b}$$

input

```
integrate(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)**2,x)
```

output

```
Piecewise((3*I*exp(5*a/3)*exp(5*I*b*x/3)/(5*b), Ne(b, 0)), (-x*exp(5*a/3),
 True)) - 2*I*exp(5*a/3)*exp(5*I*b*x/3)/(b*exp(2*I*d)*exp(2*I*b*x) - b) +
RootSum(729*_z**6*exp(10*I*d) + 15625*exp(10*a), Lambda(_i, _i*log(243*_i*
*5*I*exp(-25*a/3)*exp(8*I*d)/3125 + exp(I*b*x/3))))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5024 vs. $2(172) = 344$.

Time = 0.51 (sec) , antiderivative size = 5024, normalized size of antiderivative = 18.47

$$\int e^{\frac{5}{3}(a+ibx)} \cot^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)^2,x, algorithm="maxima")
```

output
```
-60*(50*((I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arc
tan2(sin(d), cos(d))) + ((-I*cos(2*d)*e^(5/3*a) - e^(5/3*a)*sin(2*d))*cos(
-1/3*pi + 1/3*arctan2(sin(d), cos(d))) + (cos(2*d)*e^(5/3*a) - I*e^(5/3*a)
*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*cos(2*b*x + 2*d) -
(cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(
d), cos(d))) + ((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos(-1/3*pi +
1/3*arctan2(sin(d), cos(d))) + (I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))
*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*sin(2*b*x + 2*d))*arctan2(cos
(1/3*b*x)*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) - cos(-1/3*pi + 1/3*a
rctan2(sin(d), cos(d)))*sin(1/3*b*x) - sin(-2/3*pi + 2/3*arctan2(sin(d), c
os(d))) + sin(2/3*b*x), -cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))*cos(1/
3*b*x) - sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))*sin(1/3*b*x) + cos(-2/
3*pi + 2/3*arctan2(sin(d), cos(d))) + cos(2/3*b*x)) + 50*(((-I*cos(2*d)*e^
(5/3*a) - e^(5/3*a)*sin(2*d))*cos(1/3*arctan2(sin(d), cos(d))) + (cos(2*d)
*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*sin(1/3*arctan2(sin(d), cos(d))))*cos(2
*b*x + 2*d) + (I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))*cos(1/3*arctan2(
sin(d), cos(d))) + ((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos(1/3*ar
ctan2(sin(d), cos(d))) + (I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))*sin(1
/3*arctan2(sin(d), cos(d))))*sin(2*b*x + 2*d) - (cos(2*d)*e^(5/3*a) - I*e^
(5/3*a)*sin(2*d))*sin(1/3*arctan2(sin(d), cos(d))))*arctan2(-cos(1/3*ar...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2700 vs. 2(172) = 344.

Time = 4.12 (sec) , antiderivative size = 2700, normalized size of antiderivative = 9.93

$$\int e^{\frac{5}{3}(a+ibx)} \cot^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)^2,x, algorithm="giac")
```

output
```
-1/6*I*(10*(-e^(-I*d))^(2/3)*e^(-3*I*d)*log(-(-e^(-I*d))^(1/3) + e^(1/3*I*
b*x)) - 10*e^(-11/3*I*d)*log(e^(1/3*I*b*x) - e^(-1/3*I*d)) - 10*(sqrt(3)*c
os(1/3*d)^2*cos(d)^3 - sqrt(3)*cos(d)^3*sin(1/3*d)^2 - 3*sqrt(3)*cos(1/3*d
)^2*cos(d)*sin(d)^2 + 3*sqrt(3)*cos(d)*sin(1/3*d)^2*sin(d)^2 - 2*cos(1/3*d
)*cos(d)^3*sin(1/3*d) + 6*cos(1/3*d)*cos(d)*sin(1/3*d)*sin(d)^2)*arctan(1/
3*sqrt(3)*(2*e^(1/3*I*b*x) + e^(-1/3*I*d))*e^(1/3*I*d))/(cos(1/3*d)^4*cos(
d)^6 + 2*cos(1/3*d)^2*cos(d)^6*sin(1/3*d)^2 + cos(d)^6*sin(1/3*d)^4 + 3*co
s(1/3*d)^4*cos(d)^4*sin(d)^2 + 6*cos(1/3*d)^2*cos(d)^4*sin(1/3*d)^2*sin(d)
^2 + 3*cos(d)^4*sin(1/3*d)^4*sin(d)^2 + 3*cos(1/3*d)^4*cos(d)^2*sin(d)^4 +
 6*cos(1/3*d)^2*cos(d)^2*sin(1/3*d)^2*sin(d)^4 + 3*cos(d)^2*sin(1/3*d)^4*s
in(d)^4 + cos(1/3*d)^4*sin(d)^6 + 2*cos(1/3*d)^2*sin(1/3*d)^2*sin(d)^6 + s
in(1/3*d)^4*sin(d)^6) + 10*I*(3*sqrt(3)*cos(1/3*d)^2*cos(d)^2*sin(d) - 3*s
qrt(3)*cos(d)^2*sin(1/3*d)^2*sin(d) - sqrt(3)*cos(1/3*d)^2*sin(d)^3 + sqrt
(3)*sin(1/3*d)^2*sin(d)^3 - 6*cos(1/3*d)*cos(d)^2*sin(1/3*d)*sin(d) + 2*co
s(1/3*d)*sin(1/3*d)*sin(d)^3)*arctan(1/3*sqrt(3)*(2*e^(1/3*I*b*x) + e^(-1/
3*I*d))*e^(1/3*I*d))/(cos(1/3*d)^4*cos(d)^6 + 2*cos(1/3*d)^2*cos(d)^6*sin(
1/3*d)^2 + cos(d)^6*sin(1/3*d)^4 + 3*cos(1/3*d)^4*cos(d)^4*sin(d)^2 + 6*co
s(1/3*d)^2*cos(d)^4*sin(1/3*d)^2*sin(d)^2 + 3*cos(d)^4*sin(1/3*d)^4*sin(d)
^2 + 3*cos(1/3*d)^4*cos(d)^2*sin(d)^4 + 6*cos(1/3*d)^2*cos(d)^2*sin(1/3*d)
^2*sin(d)^4 + 3*cos(d)^2*sin(1/3*d)^4*sin(d)^4 + cos(1/3*d)^4*sin(d)^6 ...
```

Mupad [B] (verification not implemented)

Time = 20.95 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.92

$$\int e^{\frac{5}{3}(a+ibx)}\cot^2(d+bx)\,dx = \text{Too large to display}$$

input
```
int(cot(d + b*x)^2*exp((5*a)/3 + (b*x*5i)/3),x)
```

output
```
(exp((5*a)/3 + (b*x*5i)/3)*3i)/(5*b) + (5*(-exp(10*a - d*10i))^(1/6)*log(-
 (100*exp(6*a)*exp(-d*6i))/9 - (exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/
3)*(-exp(10*a)*exp(-d*10i))^(1/6)*100i)/9))/(3*b) - (5*(-exp(10*a - d*10i)
)^(1/6)*log((exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*(-exp(10*a)*exp(
-d*10i))^(1/6)*100i)/9 - (100*exp(6*a)*exp(-d*6i))/9))/(3*b) + (exp((11*a)
/3 - d*2i + (b*x*5i)/3)*2i)/(b*(exp(2*a - d*2i) - exp(2*a + b*x*2i))) + (5
*log(- (100*exp(6*a)*exp(-d*6i))/9 - (exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*
x*1i)/3)*((3^(1/2)*1i)/2 - 1/2)*(-exp(10*a)*exp(-d*10i))^(1/6)*100i)/9)*(-
exp(10*a - d*10i))^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(3*b) - (5*log((exp(a/3)*
exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 - 1/2)*(-exp(10*a)*exp
(-d*10i))^(1/6)*100i)/9 - (100*exp(6*a)*exp(-d*6i))/9)*(-exp(10*a - d*10i)
)^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(3*b) + (5*log(- (100*exp(6*a)*exp(-d*6i))
/9 - (exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 + 1/2)*
(-exp(10*a)*exp(-d*10i))^(1/6)*100i)/9)*(-exp(10*a - d*10i))^(1/6)*((3^(1/
2)*1i)/2 + 1/2))/(3*b) - (5*log((exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)
/3)*((3^(1/2)*1i)/2 + 1/2)*(-exp(10*a)*exp(-d*10i))^(1/6)*100i)/9 - (100*e
xp(6*a)*exp(-d*6i))/9)*(-exp(10*a - d*10i))^(1/6)*((3^(1/2)*1i)/2 + 1/2))/
(3*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\cot^2(d+bx)\,dx = \int e^{\frac{5bix}{3}+\frac{5a}{3}}\cot(bx+d)^2\,dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)^2,x)
```

output
```
int(e**((5*a + 5*b*i*x)/3)*cot(b*x + d)**2,x)
```

3.68 $\int e^{\frac{5}{3}(a+ibx)}\cot^3(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 313

$$\begin{aligned}\int e^{\frac{5}{3}(a+ibx)}\cot^3(d+bx)\,dx = & -\frac{3e^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{5b}+\frac{2e^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)^2}\\ & -\frac{11e^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{3b\left(1-e^{2i(d+bx)}\right)}+\frac{43e^{\frac{5}{3}(a-id)}\arctan\left(\frac{1-2e^{\frac{1}{3}i(d+bx)}}{\sqrt{3}}\right)}{6\sqrt{3}b}\\ & -\frac{43e^{\frac{5}{3}(a-id)}\arctan\left(\frac{1+2e^{\frac{1}{3}i(d+bx)}}{\sqrt{3}}\right)}{6\sqrt{3}b}\\ & +\frac{43e^{\frac{5}{3}(a-id)}\text{arctanh}\left(e^{\frac{1}{3}i(d+bx)}\right)}{9b}\\ & +\frac{43e^{\frac{5}{3}(a-id)}\text{arctanh}\left(\frac{e^{\frac{1}{3}i(d+bx)}}{1+e^{\frac{2}{3}i(d+bx)}}\right)}{18b}\end{aligned}$$

output
```
-3/5*exp(5/3*a-5/3*I*d+5/3*I*(b*x+d))/b+2*exp(5/3*a-5/3*I*d+5/3*I*(b*x+d))
/b/(1-exp(2*I*(b*x+d)))^2-11/3*exp(5/3*a-5/3*I*d+5/3*I*(b*x+d))/b/(1-exp(2
*I*(b*x+d)))+43/18*3^(1/2)*exp(5/3*a-5/3*I*d)*arctan(1/3*(1-2*exp(1/3*I*(b
*x+d)))*3^(1/2))/b-43/18*3^(1/2)*exp(5/3*a-5/3*I*d)*arctan(1/3*(1+2*exp(1/
3*I*(b*x+d)))*3^(1/2))/b+43/9*exp(5/3*a-5/3*I*d)*arctanh(exp(1/3*I*(b*x+d)
))/b+43/18*exp(5/3*a-5/3*I*d)*arctanh(exp(1/3*I*(b*x+d))/(1+exp(2/3*I*(b*x
+d))))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.58

$$\int e^{\frac{5}{3}(a+ibx)}\cot^3(d+bx)\,dx$$
$$= \frac{e^{5a/3}\left(215\text{RootSum}\left[-\cos(d)+i\sin(d)+\cos(d)\#1^6+i\sin(d)\#1^6\&,\frac{bx+3i\log\left(e^{\frac{ibx}{3}}-\#1\right)}{\#1}\&\right](i\cos(2d)+\right.}{270b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Cot[d + b*x]^3,x]
```

output
```
(E^((5*a)/3)*(215*RootSum[-Cos[d] + I*Sin[d] + Cos[d]*#1^6 + I*Sin[d]*#1^6
 & , (b*x + (3*I)*Log[E^((I/3)*b*x) - #1])/#1 & ]*(I*Cos[2*d] + Sin[2*d])
- (18*E^(((5*I)/3)*b*x)*(-73*E^((2*I)*b*x) + (34 + 9*E^((4*I)*b*x))*Cos[2*
d] + I*(-34 + 9*E^((4*I)*b*x))*Sin[2*d]))/((-1 + E^((2*I)*b*x))*Cos[d] + I
*(1 + E^((2*I)*b*x))*Sin[d])^2))/(270*b)
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\cot^3(bx+d)\,dx$$
$$\Big\downarrow \text{ 4943}$$
$$i\int\left(-e^{\frac{5}{3}(a+ibx)}+\frac{6e^{\frac{5}{3}(a+ibx)}}{1-e^{2i(d+bx)}}-\frac{12e^{\frac{5}{3}(a+ibx)}}{\left(1-e^{2i(d+bx)}\right)^2}+\frac{8e^{\frac{5}{3}(a+ibx)}}{\left(1-e^{2i(d+bx)}\right)^3}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$i\left(-\frac{2i\sqrt{3}e^{\frac{5}{3}(a-id)}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2e^{\frac{1}{3}(a+ibx)+\frac{1}{3}(-a+id)}}{\sqrt{3}}\right)}{b}-\frac{7ie^{\frac{5}{3}(a-id)}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2e^{\frac{1}{3}(a+ibx)+\frac{1}{3}(-a+id)}}{\sqrt{3}}\right)}{6\sqrt{3}b}+\frac{2i\sqrt{3}e^{\frac{5}{3}(a}}{}\right.$$

input
```
Int[E^((5*(a + I*b*x))/3)*Cot[d + b*x]^3,x]
```

output
```
I*((((3*I)/5)*E^((5*(a + I*b*x))/3))/b - ((2*I)*E^((5*(a + I*b*x))/3))/(b*
(1 - E^((2*I)*(d + b*x)))^2) + (((11*I)/3)*E^((5*(a + I*b*x))/3))/(b*(1 -
E^((2*I)*(d + b*x)))) - (((7*I)/6)*E^((5*(a - I*d))/3)*ArcTan[1/Sqrt[3] -
(2*E^((-a + I*d)/3 + (a + I*b*x)/3))/Sqrt[3]])/(Sqrt[3]*b) - ((2*I)*Sqrt[3
]*E^((5*(a - I*d))/3)*ArcTan[1/Sqrt[3] - (2*E^((-a + I*d)/3 + (a + I*b*x)/
3))/Sqrt[3]])/b + (((7*I)/6)*E^((5*(a - I*d))/3)*ArcTan[1/Sqrt[3] + (2*E^(
(-a + I*d)/3 + (a + I*b*x)/3))/Sqrt[3]])/(Sqrt[3]*b) + ((2*I)*Sqrt[3]*E^((
5*(a - I*d))/3)*ArcTan[1/Sqrt[3] + (2*E^((-a + I*d)/3 + (a + I*b*x)/3))/Sq
rt[3]])/b - (((43*I)/9)*E^((5*(a - I*d))/3)*ArcTanh[E^((-a + I*d)/3 + (a +
 I*b*x)/3)])/b + (((43*I)/36)*E^((5*(a - I*d))/3)*Log[1 - E^((I/3)*d + (I/
3)*b*x) + E^(((2*I)/3)*d + ((2*I)/3)*b*x)])/b - (((43*I)/36)*E^((5*(a - I*
d))/3)*Log[1 + E^((I/3)*d + (I/3)*b*x) + E^(((2*I)/3)*d + ((2*I)/3)*b*x)])
/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [F]

$$\int e^{\frac{5a}{3}+\frac{5ibx}{3}} \cot \left(bx+d\right)^{3} dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)^3,x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(199) = 398.

Time = 0.10 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.96

$$\int e^{\frac{5}{3}(a+ibx)} \cot^3(d+bx)\, dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/180*(430*(b*e^(4*I*b*x + 4*I*d) - 2*b*e^(2*I*b*x + 2*I*d) + b)*(b^(-6))^
(1/6)*e^(5/3*a - 5/3*I*d)*log(b^5*(b^(-6))^(5/6)*e^(1/3*a - 1/3*I*d) + e^(
1/3*I*b*x + 1/3*a)) - 430*(b*e^(4*I*b*x + 4*I*d) - 2*b*e^(2*I*b*x + 2*I*d)
 + b)*(b^(-6))^(1/6)*e^(5/3*a - 5/3*I*d)*log(-b^5*(b^(-6))^(5/6)*e^(1/3*a
- 1/3*I*d) + e^(1/3*I*b*x + 1/3*a)) - 215*((-I*sqrt(3)*b + b)*e^(4*I*b*x +
 4*I*d) + 2*(I*sqrt(3)*b - b)*e^(2*I*b*x + 2*I*d) - I*sqrt(3)*b + b)*(b^(-
6))^(1/6)*e^(5/3*a - 5/3*I*d)*log(-1/2*(I*sqrt(3)*b^5 + b^5)*(b^(-6))^(5/6
)*e^(1/3*a - 1/3*I*d) + e^(1/3*I*b*x + 1/3*a)) - 215*((-I*sqrt(3)*b - b)*e
^(4*I*b*x + 4*I*d) + 2*(I*sqrt(3)*b + b)*e^(2*I*b*x + 2*I*d) - I*sqrt(3)*b
 - b)*(b^(-6))^(1/6)*e^(5/3*a - 5/3*I*d)*log(-1/2*(I*sqrt(3)*b^5 - b^5)*(b
^(-6))^(5/6)*e^(1/3*a - 1/3*I*d) + e^(1/3*I*b*x + 1/3*a)) - 215*((I*sqrt(3
)*b + b)*e^(4*I*b*x + 4*I*d) + 2*(-I*sqrt(3)*b - b)*e^(2*I*b*x + 2*I*d) +
I*sqrt(3)*b + b)*(b^(-6))^(1/6)*e^(5/3*a - 5/3*I*d)*log(-1/2*(-I*sqrt(3)*b
^5 + b^5)*(b^(-6))^(5/6)*e^(1/3*a - 1/3*I*d) + e^(1/3*I*b*x + 1/3*a)) - 21
5*((I*sqrt(3)*b - b)*e^(4*I*b*x + 4*I*d) + 2*(-I*sqrt(3)*b + b)*e^(2*I*b*x
 + 2*I*d) + I*sqrt(3)*b - b)*(b^(-6))^(1/6)*e^(5/3*a - 5/3*I*d)*log(-1/2*(
-I*sqrt(3)*b^5 - b^5)*(b^(-6))^(5/6)*e^(1/3*a - 1/3*I*d) + e^(1/3*I*b*x +
1/3*a)) - 108*e^(17/3*I*b*x + 5/3*a + 4*I*d) + 876*e^(11/3*I*b*x + 5/3*a +
 2*I*d) - 408*e^(5/3*I*b*x + 5/3*a))/(b*e^(4*I*b*x + 4*I*d) - 2*b*e^(2*I*b
*x + 2*I*d) + b)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.71

$$\int e^{\frac{5}{3}(a+ibx)}\cot^3(d+bx)\,dx = \frac{11e^{\frac{5a}{3}}e^{2id}e^{\frac{11ibx}{3}}-5e^{\frac{5a}{3}}e^{\frac{5ibx}{3}}}{3be^{4id}e^{4ibx}-6be^{2id}e^{2ibx}+3b}+\begin{cases}-\frac{3e^{\frac{5a}{3}}e^{\frac{5ibx}{3}}}{5b} & \text{for } b\neq 0\\ -ixe^{\frac{5a}{3}} & \text{otherwise}\end{cases}$$
$$+\frac{\operatorname{RootSum}\left(5832z^3e^{5id}-79507e^{5a},\left(i\mapsto i\log\left(\frac{1889568i^5e^{-\frac{25a}{3}}e^{8id}}{147008443}+e^{\frac{ibx}{3}}\right)\right)\right)+\operatorname{RootSum}\left(5832z^3e^{5id}+\right.}{b}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)**3,x)
```

output
```
(11*exp(5*a/3)*exp(2*I*d)*exp(11*I*b*x/3) - 5*exp(5*a/3)*exp(5*I*b*x/3))/(
3*b*exp(4*I*d)*exp(4*I*b*x) - 6*b*exp(2*I*d)*exp(2*I*b*x) + 3*b) + Piecewi
se((-3*exp(5*a/3)*exp(5*I*b*x/3)/(5*b), Ne(b, 0)), (-I*x*exp(5*a/3), True)
) + (RootSum(5832*_z**3*exp(5*I*d) - 79507*exp(5*a), Lambda(_i, _i*log(188
9568*_i**5*exp(-25*a/3)*exp(8*I*d)/147008443 + exp(I*b*x/3)))) + RootSum(5
832*_z**3*exp(5*I*d) + 79507*exp(5*a), Lambda(_i, _i*log(1889568*_i**5*exp
(-25*a/3)*exp(8*I*d)/147008443 + exp(I*b*x/3)))))/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6962 vs. $2(199) = 398$.

Time = 0.68 (sec) , antiderivative size = 6962, normalized size of antiderivative = 22.24

$$\int e^{\frac{5}{3}(a+ibx)}\cot^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)^3,x, algorithm="maxima")
```

output
```
-360*(430*((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*a
rctan2(sin(d), cos(d))) + ((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos
(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) - (-I*cos(2*d)*e^(5/3*a) - e^(5/3*
a)*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*cos(4*b*x + 4*d)
- 2*((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arctan2
(sin(d), cos(d))) + (I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))*sin(-1/3*p
i + 1/3*arctan2(sin(d), cos(d))))*cos(2*b*x + 2*d) - (-I*cos(2*d)*e^(5/3*a
) - e^(5/3*a)*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) - ((-I*
cos(2*d)*e^(5/3*a) - e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d),
 cos(d))) + (cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*sin(-1/3*pi + 1/3*
arctan2(sin(d), cos(d))))*sin(4*b*x + 4*d) - 2*((I*cos(2*d)*e^(5/3*a) + e^
(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) - (cos(2*d)*e
^(5/3*a) - I*e^(5/3*a)*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))
))*sin(2*b*x + 2*d))*arctan2(cos(1/3*b*x)*sin(-1/3*pi + 1/3*arctan2(sin(d)
, cos(d))) - cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))*sin(1/3*b*x) - sin
(-2/3*pi + 2/3*arctan2(sin(d), cos(d))) + sin(2/3*b*x), -cos(-1/3*pi + 1/3
*arctan2(sin(d), cos(d)))*cos(1/3*b*x) - sin(-1/3*pi + 1/3*arctan2(sin(d),
 cos(d)))*sin(1/3*b*x) + cos(-2/3*pi + 2/3*arctan2(sin(d), cos(d))) + cos(
2/3*b*x)) + 430*(((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos(1/3*arct
an2(sin(d), cos(d))) - (-I*cos(2*d)*e^(5/3*a) - e^(5/3*a)*sin(2*d))*sin...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5078 vs. $2(199) = 398$.

Time = 7.41 (sec) , antiderivative size = 5078, normalized size of antiderivative = 16.22

$$\int e^{\frac{5}{3}(a+ibx)}\cot^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)^3,x, algorithm="giac")
```

output
```
1/36*I*(-86*I*(-e^(-I*d))^(2/3)*e^(5/3*a - I*d)*log(-(-e^(-I*d))^(1/3) + e
^(1/3*I*b*x)) + 86*I*e^(5/3*a - 5/3*I*d)*log(e^(1/3*I*b*x) - e^(-1/3*I*d))
 - 86*(sqrt(3)*cos(1/3*d)^2*cos(d)^4*e^(5/3*a)*sin(3*d) - sqrt(3)*cos(d)^4
*e^(5/3*a)*sin(3*d)*sin(1/3*d)^2 - 4*sqrt(3)*cos(3*d)*cos(1/3*d)^2*cos(d)^
3*e^(5/3*a)*sin(d) + 4*sqrt(3)*cos(3*d)*cos(d)^3*e^(5/3*a)*sin(1/3*d)^2*si
n(d) - 6*sqrt(3)*cos(1/3*d)^2*cos(d)^2*e^(5/3*a)*sin(3*d)*sin(d)^2 + 6*sqr
t(3)*cos(d)^2*e^(5/3*a)*sin(3*d)*sin(1/3*d)^2*sin(d)^2 + 4*sqrt(3)*cos(3*d
)*cos(1/3*d)^2*cos(d)*e^(5/3*a)*sin(d)^3 - 4*sqrt(3)*cos(3*d)*cos(d)*e^(5/
3*a)*sin(1/3*d)^2*sin(d)^3 + sqrt(3)*cos(1/3*d)^2*e^(5/3*a)*sin(3*d)*sin(d
)^4 - sqrt(3)*e^(5/3*a)*sin(3*d)*sin(1/3*d)^2*sin(d)^4 - 2*cos(1/3*d)*cos(
d)^4*e^(5/3*a)*sin(3*d)*sin(1/3*d) + 8*cos(3*d)*cos(1/3*d)*cos(d)^3*e^(5/3
*a)*sin(1/3*d)*sin(d) + 12*cos(1/3*d)*cos(d)^2*e^(5/3*a)*sin(3*d)*sin(1/3*
d)*sin(d)^2 - 8*cos(3*d)*cos(1/3*d)*cos(d)*e^(5/3*a)*sin(1/3*d)*sin(d)^3 -
 2*cos(1/3*d)*e^(5/3*a)*sin(3*d)*sin(1/3*d)*sin(d)^4)*arctan(1/3*sqrt(3)*(
2*e^(1/3*I*b*x) + e^(-1/3*I*d))*e^(1/3*I*d))/(cos(1/3*d)^4*cos(d)^8 + 2*co
s(1/3*d)^2*cos(d)^8*sin(1/3*d)^2 + cos(d)^8*sin(1/3*d)^4 + 4*cos(1/3*d)^4*
cos(d)^6*sin(d)^2 + 8*cos(1/3*d)^2*cos(d)^6*sin(1/3*d)^2*sin(d)^2 + 4*cos(
d)^6*sin(1/3*d)^4*sin(d)^2 + 6*cos(1/3*d)^4*cos(d)^4*sin(d)^4 + 12*cos(1/3
*d)^2*cos(d)^4*sin(1/3*d)^2*sin(d)^4 + 6*cos(d)^4*sin(1/3*d)^4*sin(d)^4 +
4*cos(1/3*d)^4*cos(d)^2*sin(d)^6 + 8*cos(1/3*d)^2*cos(d)^2*sin(1/3*d)^2...
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{3}(a+ibx)}\cot^3(d+bx)\,dx=\int \cot(d+b\,x)^3\,e^{\frac{5\,a}{3}+\frac{b\,x\,5\mathrm{i}}{3}}\,dx$$

input
```
int(cot(d + b*x)^3*exp((5*a)/3 + (b*x*5i)/3),x)
```

output
```
int(cot(d + b*x)^3*exp((5*a)/3 + (b*x*5i)/3), x)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\cot^3(d+bx)\,dx=\int e^{\frac{5bix}{3}+\frac{5a}{3}}\cot\left(bx+d\right)^3dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*cot(b*x+d)^3,x)
```

output
```
int(e**((5*a + 5*b*i*x)/3)*cot(b*x + d)**3,x)
```

3.69 $\int F^{c(a+bx)} \cot(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int F^{c(a+bx)} \cot(d+ex)\,dx = \frac{iF^{c(a+bx)}}{bc\log(F)} - \frac{2iF^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1, -\frac{ibc\log(F)}{2e}, 1-\frac{ibc\log(F)}{2e}, e^{2i(d+ex)}\right)}{bc\log(F)}$$

output
```
I*F^(c*(b*x+a))/b/c/ln(F)-2*I*F^(c*(b*x+a))*hypergeom([1, -1/2*I*b*c*ln(F)
/e],[1-1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int F^{c(a+bx)} \cot(d+ex)\,dx = -\frac{iF^{c(a+bx)}\left(-1+2\,\text{Hypergeometric2F1}\left(1, -\frac{ibc\log(F)}{2e}, 1-\frac{ibc\log(F)}{2e}, e^{2i(d+ex)}\right)\right)}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Cot[d + e*x],x]
```

output
```
((-I)*F^(c*(a + b*x))*(-1 + 2*Hypergeometric2F1[1, ((-1/2*I)*b*c*Log[F])/e
, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))]))/(b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4943}$$

$$-i\int \left(\frac{2F^{c(a+bx)}}{1-e^{2i(d+ex)}}-F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{F^{c(a+bx)}}{bc\log(F)}+\frac{2F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{bc\log(F)}\right)$$

input
```
Int[F^(c*(a + b*x))*Cot[d + e*x],x]
```

output
```
(-I)*(-(F^(c*(a + b*x))/(b*c*Log[F])) + (2*F^(c*(a + b*x))*Hypergeometric2
F1[1, ((-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x
))])/(b*c*Log[F]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \cot(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*cot(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*cot(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)} \cot(d+ex)\,dx = \int F^{(bx+a)c} \cot(ex+d)\;dx$$

input
```
integrate(F^(c*(b*x+a))*cot(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*cot(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)} \cot(d+ex)\,dx = \int F^{c(a+bx)} \cot\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*cot(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*cot(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)} \cot(d+ex)\,dx = \int F^{(bx+a)c} \cot\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*cot(e*x+d),x, algorithm="maxima")
```

output
```
2*(F^(b*c*x)*F^(a*c)*b*c*log(F)*sin(2*e*x + 2*d) - 2*F^(b*c*x)*F^(a*c)*e*c
os(2*e*x + 2*d) + 2*F^(b*c*x)*F^(a*c)*e - 2*(F^(a*c)*b^2*c^2*e*log(F)^2 +
4*F^(a*c)*e^3 + (F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*c)*e^3)*cos(2*e*x + 2
*d)^2 + (F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*c)*e^3)*sin(2*e*x + 2*d)^2 -
2*(F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*c)*e^3)*cos(2*e*x + 2*d))*integrate
((F^(b*c*x)*b*c*cos(4*e*x + 4*d)*log(F) - 2*F^(b*c*x)*b*c*cos(2*e*x + 2*d)
*log(F) + F^(b*c*x)*b*c*log(F) + 2*F^(b*c*x)*e*sin(4*e*x + 4*d) - 4*F^(b*c
*x)*e*sin(2*e*x + 2*d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + 4*e^2)*cos
(4*e*x + 4*d)^2 + 4*(b^2*c^2*log(F)^2 + 4*e^2)*cos(2*e*x + 2*d)^2 + (b^2*c
^2*log(F)^2 + 4*e^2)*sin(4*e*x + 4*d)^2 - 4*(b^2*c^2*log(F)^2 + 4*e^2)*sin
(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(b^2*c^2*log(F)^2 + 4*e^2)*sin(2*e*x +
2*d)^2 + 4*e^2 + 2*(b^2*c^2*log(F)^2 + 4*e^2 - 2*(b^2*c^2*log(F)^2 + 4*e^2
)*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) - 4*(b^2*c^2*log(F)^2 + 4*e^2)*cos(2*
e*x + 2*d)), x))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + 4*e^2)*cos(2*e*x
+ 2*d)^2 + (b^2*c^2*log(F)^2 + 4*e^2)*sin(2*e*x + 2*d)^2 + 4*e^2 - 2*(b^2*
c^2*log(F)^2 + 4*e^2)*cos(2*e*x + 2*d))
```

Giac [F]

$$\int F^{c(a+bx)}\cot(d+ex)\,dx = \int F^{(bx+a)c}\cot(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*cot(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*cot(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\cot(d+ex)\,dx = \int F^{c\,(a+b\,x)}\cot(d+e\,x)\,dx$$

input
```
int(F^(c*(a + b*x))*cot(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))*cot(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\cot(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cot(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*cot(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cot(d + e*x),x)
```

3.70 $\int F^{c(a+bx)} \cot^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 112

$$\int F^{c(a+bx)}\cot^2(d+ex)\,dx = \frac{2iF^{c(a+bx)}}{e\left(1-e^{2i(d+ex)}\right)} - \frac{2iF^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{e} - \frac{F^{c(a+bx)}}{bc\log(F)}$$

output
```
2*I*F^(c*(b*x+a))/e/(1-exp(2*I*(e*x+d)))-2*I*F^(c*(b*x+a))*hypergeom([1, -
1/2*I*b*c*ln(F)/e],[1-1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))/e-F^(c*(b*x+a))
/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}\cot^2(d+ex)\,dx$$
$$= F^{c(a+bx)}\left(\frac{2i}{e-ee^{2id}}-\frac{2i\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{e}-\frac{1}{bc\log(F)}+\frac{\csc(d)\csc(d+ex)\sin(ex)}{e}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Cot[d + e*x]^2,x]
```

output
```
F^(c*(a + b*x))*((2*I)/(e - e*E^((2*I)*d)) - ((2*I)*Hypergeometric2F1[1, (
(-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])/e
- 1/(b*c*Log[F]) + (Csc[d]*Csc[d + e*x]*Sin[e*x])/e)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{4943}$$
$$-\int \left(-\frac{4F^{c(a+bx)}}{1-e^{2i(d+ex)}}+\frac{4F^{c(a+bx)}}{\left(1-e^{2i(d+ex)}\right)^2}+F^{c(a+bx)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{4F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{bc\log(F)}-\frac{4F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{bc\log(F)}-\frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Cot[d + e*x]^2,x]
```

output
```
-(F^(c*(a + b*x))/(b*c*Log[F])) + (4*F^(c*(a + b*x))*Hypergeometric2F1[1,
((-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])/(
b*c*Log[F]) - (4*F^(c*(a + b*x))*Hypergeometric2F1[2, ((-1/2*I)*b*c*Log[F]
)/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \cot(ex+d)^2\,dx$$

input
```
int(F^(c*(b*x+a))*cot(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*cot(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \cot^2(d+ex)\,dx = \int F^{(bx+a)c} \cot{(ex+d)}^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*cot(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \cot^2(d+ex)\,dx = \int F^{c(a+bx)} \cot^2{(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*cot(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*cot(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \cot^2(d+ex)\,dx = \int F^{(bx+a)c} \cot{(ex+d)}^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="maxima")
```

output
```
-((F^(a*c)*b^4*c^4*log(F)^4 + 20*F^(a*c)*b^2*c^2*e^2*log(F)^2 + 64*F^(a*c)
*e^4)*F^(b*c*x)*cos(4*e*x + 4*d)^2 - 4*(F^(a*c)*b^4*c^4*log(F)^4 + 12*F^(a
*c)*b^2*c^2*e^2*log(F)^2 - 64*F^(a*c)*e^4)*F^(b*c*x)*cos(2*e*x + 2*d)^2 +
(F^(a*c)*b^4*c^4*log(F)^4 + 20*F^(a*c)*b^2*c^2*e^2*log(F)^2 + 64*F^(a*c)*e
^4)*F^(b*c*x)*sin(4*e*x + 4*d)^2 - 4*(F^(a*c)*b^4*c^4*log(F)^4 + 12*F^(a*c
)*b^2*c^2*e^2*log(F)^2 - 64*F^(a*c)*e^4)*F^(b*c*x)*sin(2*e*x + 2*d)^2 + 16
*(11*F^(a*c)*b^2*c^2*e^2*log(F)^2 - 16*F^(a*c)*e^4)*F^(b*c*x)*cos(2*e*x +
2*d) - 8*(5*F^(a*c)*b^3*c^3*e*log(F)^3 - 16*F^(a*c)*b*c*e^3*log(F))*F^(b*c
*x)*sin(2*e*x + 2*d) + (F^(a*c)*b^4*c^4*log(F)^4 - 76*F^(a*c)*b^2*c^2*e^2*
log(F)^2 + 64*F^(a*c)*e^4)*F^(b*c*x) - 2*(8*(F^(a*c)*b^2*c^2*e^2*log(F)^2
+ 16*F^(a*c)*e^4)*F^(b*c*x)*cos(2*e*x + 2*d) + 4*(F^(a*c)*b^3*c^3*e*log(F)
^3 + 16*F^(a*c)*b*c*e^3*log(F))*F^(b*c*x)*sin(2*e*x + 2*d) - (F^(a*c)*b^4*
c^4*log(F)^4 - 28*F^(a*c)*b^2*c^2*e^2*log(F)^2 + 64*F^(a*c)*e^4)*F^(b*c*x)
)*cos(4*e*x + 4*d) - 16*(F^(a*c)*b^6*c^6*e*log(F)^6 + 20*F^(a*c)*b^4*c^4*e
^3*log(F)^4 + 64*F^(a*c)*b^2*c^2*e^5*log(F)^2 + (F^(a*c)*b^6*c^6*e*log(F)^
6 + 20*F^(a*c)*b^4*c^4*e^3*log(F)^4 + 64*F^(a*c)*b^2*c^2*e^5*log(F)^2)*cos
(4*e*x + 4*d)^2 + 4*(F^(a*c)*b^6*c^6*e*log(F)^6 + 20*F^(a*c)*b^4*c^4*e^3*l
og(F)^4 + 64*F^(a*c)*b^2*c^2*e^5*log(F)^2)*cos(2*e*x + 2*d)^2 + (F^(a*c)*b
^6*c^6*e*log(F)^6 + 20*F^(a*c)*b^4*c^4*e^3*log(F)^4 + 64*F^(a*c)*b^2*c^2*e
^5*log(F)^2)*sin(4*e*x + 4*d)^2 - 4*(F^(a*c)*b^6*c^6*e*log(F)^6 + 20*F^...
```

Giac [F]

$$\int F^{c(a+bx)} \cot^2(d+ex)\,dx = \int F^{(bx+a)c} \cot\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*cot(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\cot^2(d+ex)\,dx=\int F^{c\,(a+b\,x)}\cot(d+e\,x)^2\,dx$$

input
```
int(F^(c*(a + b*x))*cot(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))*cot(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\cot^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\cot\left(ex+d\right)^2dx\right)$$

input
```
int(F^(c*(b*x+a))*cot(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cot(d + e*x)**2,x)
```

3.71 $\int F^{c(a+bx)} \cot^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 174

$$\int F^{c(a+bx)}\cot^3(d+ex)\,dx = \frac{2F^{c(a+bx)}}{e\left(1-e^{2i(d+ex)}\right)^2} - \frac{iF^{c(a+bx)}}{bc\log(F)} - \frac{F^{c(a+bx)}(2e-ibc\log(F))}{e^2\left(1-e^{2i(d+ex)}\right)} + iF^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)\left(\frac{2}{bc\log(F)}-\frac{bc\log(F)}{e^2}\right)$$

output

```
2*F^(c*(b*x+a))/e/(1-exp(2*I*(e*x+d)))^2-I*F^(c*(b*x+a))/b/c/ln(F)-F^(c*(b
*x+a))*(2*e-I*b*c*ln(F))/e^2/(1-exp(2*I*(e*x+d)))+I*F^(c*(b*x+a))*hypergeo
m([1, -1/2*I*b*c*ln(F)/e],[1-1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))*(2/b/c/l
n(F)-b*c*ln(F)/e^2)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

$$\int F^{c(a+bx)}\cot^3(d+ex)\,dx$$
$$= \frac{F^{c(a+bx)}\left(-2ie^2-bce\csc^2(d+ex)\log(F)+ib^2c^2\log^2(F)-b^2c^2\cot(d)\log^2(F)+2i\,\text{Hypergeometric2F1}\right.}{}$$

input
```
Integrate[F^(c*(a + b*x))*Cot[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*((-2*I)*e^2 - b*c*e*Csc[d + e*x]^2*Log[F] + I*b^2*c^2*Log
[F]^2 - b^2*c^2*Cot[d]*Log[F]^2 + (2*I)*Hypergeometric2F1[1, ((-1/2*I)*b*c
*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, Cos[2*(d + e*x)] + I*Sin[2*(d + e*x)
]]*(2*e^2 - b^2*c^2*Log[F]^2) + b^2*c^2*Csc[d]*Csc[d + e*x]*Log[F]^2*Sin[e
*x]))/(2*b*c*e^2*Log[F])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{4943}$$
$$i\int \left(\frac{6F^{c(a+bx)}}{1-e^{2i(d+ex)}}-\frac{12F^{c(a+bx)}}{\left(1-e^{2i(d+ex)}\right)^2}+\frac{8F^{c(a+bx)}}{\left(1-e^{2i(d+ex)}\right)^3}-F^{c(a+bx)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$i\left(\frac{6F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{bc\log(F)}-\frac{12F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,-\right.}{bc\log(\right.}$$

input
```
Int[F^(c*(a + b*x))*Cot[d + e*x]^3,x]
```

output
```
I*(-(F^(c*(a + b*x))/(b*c*Log[F])) + (6*F^(c*(a + b*x))*Hypergeometric2F1[
1, ((-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))]
)/(b*c*Log[F]) - (12*F^(c*(a + b*x))*Hypergeometric2F1[2, ((-1/2*I)*b*c*Lo
g[F])/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])/(b*c*Log[F]) + (8
*F^(c*(a + b*x))*Hypergeometric2F1[3, ((-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*
b*c*Log[F])/e, E^((2*I)*(d + e*x))])/(b*c*Log[F]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\cot(ex+d)^3\,dx$$

input
```
int(F^(c*(b*x+a))*cot(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*cot(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\cot^3(d+ex)\,dx=\int F^{(bx+a)c}\cot\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*cot(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\cot^3(d+ex)\,dx=\int F^{c(a+bx)}\cot^3\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*cot(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*cot(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\cot^3(d+ex)\,dx=\int F^{(bx+a)c}\cot\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="maxima")
```

output
```
-2*(18*(F^(a*c)*b^4*c^4*e*log(F)^4 + 52*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 576
*F^(a*c)*e^5)*F^(b*c*x)*cos(4*e*x + 4*d)^2 - 54*(F^(a*c)*b^4*c^4*e*log(F)^
4 + 28*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 288*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x
 + 2*d)^2 + 18*(F^(a*c)*b^4*c^4*e*log(F)^4 + 52*F^(a*c)*b^2*c^2*e^3*log(F)
^2 + 576*F^(a*c)*e^5)*F^(b*c*x)*sin(4*e*x + 4*d)^2 - 54*(F^(a*c)*b^4*c^4*e
*log(F)^4 + 28*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 288*F^(a*c)*e^5)*F^(b*c*x)*s
in(2*e*x + 2*d)^2 - 18*(3*F^(a*c)*b^4*c^4*e*log(F)^4 - 212*F^(a*c)*b^2*c^2
*e^3*log(F)^2 + 640*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) + 3*(F^(a*c)*b
^5*c^5*log(F)^5 - 268*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 1216*F^(a*c)*b*c*e^4*
log(F))*F^(b*c*x)*sin(2*e*x + 2*d) + 24*(F^(a*c)*b^4*c^4*e*log(F)^4 - 46*F
^(a*c)*b^2*c^2*e^3*log(F)^2 + 88*F^(a*c)*e^5)*F^(b*c*x) - 3*(2*(F^(a*c)*b^
4*c^4*e*log(F)^4 + 52*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 576*F^(a*c)*e^5)*F^(b
*c*x)*cos(4*e*x + 4*d) + 6*(F^(a*c)*b^4*c^4*e*log(F)^4 + 28*F^(a*c)*b^2*c^
2*e^3*log(F)^2 - 288*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) + (F^(a*c)*b^
5*c^5*log(F)^5 + 52*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 576*F^(a*c)*b*c*e^4*log
(F))*F^(b*c*x)*sin(4*e*x + 4*d) - 36*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 36*F^
(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(2*e*x + 2*d) + 8*(F^(a*c)*b^4*c^4*e*lo
g(F)^4 - 46*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 88*F^(a*c)*e^5)*F^(b*c*x))*cos(
6*e*x + 6*d) + 3*(12*(F^(a*c)*b^4*c^4*e*log(F)^4 + 16*F^(a*c)*b^2*c^2*e^3*
log(F)^2 - 720*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) + 3*(F^(a*c)*b^5...
```

Giac [F]

$$\int F^{c(a+bx)}\cot^3(d+ex)\,dx=\int F^{(bx+a)c}\cot(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*cot(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\cot^3(d+ex)\,dx = \int F^{c\,(a+b\,x)}\cot(d+e\,x)^3\,dx$$

input
```
int(F^(c*(a + b*x))*cot(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))*cot(d + e*x)^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\cot^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cot(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*cot(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cot(d + e*x)**3,x)
```

3.72 $\int F^{c(a+bx)}\cot^4(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 239

$$\int F^{c(a+bx)}\cot^4(d+ex)\,dx = -\frac{8iF^{c(a+bx)}}{3e\left(1-e^{2i(d+ex)}\right)^3}+\frac{F^{c(a+bx)}}{bc\log(F)}+\frac{2F^{c(a+bx)}(6ie+bc\log(F))}{3e^2\left(1-e^{2i(d+ex)}\right)^2}+\frac{iF^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)\left(8e^2-b^2c^2\log^2(F)\right)}{3e^3}-\frac{F^{c(a+bx)}\left(12ie^2+2bce\log(F)-ib^2c^2\log^2(F)\right)}{3e^3\left(1-e^{2i(d+ex)}\right)}$$

output
```
-8/3*I*F^(c*(b*x+a))/e/(1-exp(2*I*(e*x+d)))^3+F^(c*(b*x+a))/b/c/ln(F)+2/3*
F^(c*(b*x+a))*(6*I*e+b*c*ln(F))/e^2/(1-exp(2*I*(e*x+d)))^2+1/3*I*F^(c*(b*x
+a))*hypergeom([1, -1/2*I*b*c*ln(F)/e],[1-1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+
d)))*(8*e^2-b^2*c^2*ln(F)^2)/e^3-1/3*F^(c*(b*x+a))*(12*I*e^2+2*b*c*e*ln(F)
-I*b^2*c^2*ln(F)^2)/e^3/(1-exp(2*I*(e*x+d)))
```

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.80

$$\int F^{c(a+bx)}\cot^4(d+ex)\,dx = \frac{1}{6}F^{c(a+bx)}\left(\frac{6}{bc\log(F)}-\frac{\csc^2(d+ex)(2e\cot(d)+bc\log(F))}{e^2}-\frac{2i\left(1+\left(-1+e^{2id}\right)\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)\right)\left(-8e^2+b^2c^2\log^2(F)\right)}{e^3\left(-1+e^{2id}\right)}+\frac{2\csc(d)\csc^3(d+ex)\sin(ex)}{e}-\frac{\csc(d)\csc(d+ex)\left(8e^2-b^2c^2\log^2(F)\right)\sin(ex)}{e^3}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Cot[d + e*x]^4,x]
```

output
```
(F^(c*(a + b*x))*(6/(b*c*Log[F]) - (Csc[d + e*x]^2*(2*e*Cot[d] + b*c*Log[F
]))/e^2 - ((2*I)*(1 + (-1 + E^((2*I)*d))*Hypergeometric2F1[1, ((-1/2*I)*b*
c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])*(-8*e^2 + b^2
*c^2*Log[F]^2))/(e^3*(-1 + E^((2*I)*d))) + (2*Csc[d]*Csc[d + e*x]^3*Sin[e*
x])/e - (Csc[d]*Csc[d + e*x]*(8*e^2 - b^2*c^2*Log[F]^2)*Sin[e*x])/e^3))/6
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4943}$$

$$\int \left(-\frac{8F^{c(a+bx)}}{1-e^{2i(d+ex)}}+\frac{24F^{c(a+bx)}}{\left(1-e^{2i(d+ex)}\right)^2}-\frac{32F^{c(a+bx)}}{\left(1-e^{2i(d+ex)}\right)^3}+\frac{16F^{c(a+bx)}}{\left(1-e^{2i(d+ex)}\right)^4}+F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{bc\log(F)}+\frac{24F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{bc\log(F)}-\frac{32F^{c(a+bx)}\text{Hypergeometric2F1}\left(3,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{bc\log(F)}+\frac{16F^{c(a+bx)}\text{Hypergeometric2F1}\left(4,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{bc\log(F)}+\frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Cot[d + e*x]^4,x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (8*F^(c*(a + b*x))*Hypergeometric2F1[1, ((-
1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])/(b*c
*Log[F]) + (24*F^(c*(a + b*x))*Hypergeometric2F1[2, ((-1/2*I)*b*c*Log[F])/
e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])/(b*c*Log[F]) - (32*F^(c
*(a + b*x))*Hypergeometric2F1[3, ((-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*L
og[F])/e, E^((2*I)*(d + e*x))])/(b*c*Log[F]) + (16*F^(c*(a + b*x))*Hyperge
ometric2F1[4, ((-1/2*I)*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*
(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \cot(ex+d)^4 \, dx$$

input
```
int(F^(c*(b*x+a))*cot(e*x+d)^4,x)
```

output
```
int(F^(c*(b*x+a))*cot(e*x+d)^4,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \cot^4(d+ex) \, dx = \int F^{(bx+a)c} \cot(ex+d)^4 \, dx$$

input
```
integrate(F^(c*(b*x+a))*cot(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*cot(e*x + d)^4, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \cot^4(d+ex) \, dx = \int F^{c(a+bx)} \cot^4(d+ex) \, dx$$

input
```
integrate(F**(c*(b*x+a))*cot(e*x+d)**4,x)
```

output
```
Integral(F**(c*(a + b*x))*cot(d + e*x)**4, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\cot^4(d+ex)\,dx = \int F^{(bx+a)c}\cot(ex+d)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*cot(e*x+d)^4,x, algorithm="maxima")
```

output
```
((F^(a*c)*b^8*c^8*log(F)^8 + 120*F^(a*c)*b^6*c^6*e^2*log(F)^6 + 4368*F^(a*
c)*b^4*c^4*e^4*log(F)^4 + 52480*F^(a*c)*b^2*c^2*e^6*log(F)^2 + 147456*F^(a
*c)*e^8)*F^(b*c*x)*cos(8*e*x + 8*d)^2 - 16*(F^(a*c)*b^8*c^8*log(F)^8 + 112
*F^(a*c)*b^6*c^6*e^2*log(F)^6 + 3440*F^(a*c)*b^4*c^4*e^4*log(F)^4 + 21248*
F^(a*c)*b^2*c^2*e^6*log(F)^2 - 147456*F^(a*c)*e^8)*F^(b*c*x)*cos(6*e*x + 6
*d)^2 + 36*(F^(a*c)*b^8*c^8*log(F)^8 + 56*F^(a*c)*b^6*c^6*e^2*log(F)^6 - 2
032*F^(a*c)*b^4*c^4*e^4*log(F)^4 - 94976*F^(a*c)*b^2*c^2*e^6*log(F)^2 + 14
7456*F^(a*c)*e^8)*F^(b*c*x)*cos(4*e*x + 4*d)^2 - 16*(F^(a*c)*b^8*c^8*log(F
)^8 - 208*F^(a*c)*b^6*c^6*e^2*log(F)^6 - 14480*F^(a*c)*b^4*c^4*e^4*log(F)^
4 + 185088*F^(a*c)*b^2*c^2*e^6*log(F)^2 - 147456*F^(a*c)*e^8)*F^(b*c*x)*co
s(2*e*x + 2*d)^2 + (F^(a*c)*b^8*c^8*log(F)^8 + 120*F^(a*c)*b^6*c^6*e^2*log
(F)^6 + 4368*F^(a*c)*b^4*c^4*e^4*log(F)^4 + 52480*F^(a*c)*b^2*c^2*e^6*log(
F)^2 + 147456*F^(a*c)*e^8)*F^(b*c*x)*sin(8*e*x + 8*d)^2 - 16*(F^(a*c)*b^8*
c^8*log(F)^8 + 112*F^(a*c)*b^6*c^6*e^2*log(F)^6 + 3440*F^(a*c)*b^4*c^4*e^4
*log(F)^4 + 21248*F^(a*c)*b^2*c^2*e^6*log(F)^2 - 147456*F^(a*c)*e^8)*F^(b*
c*x)*sin(6*e*x + 6*d)^2 + 36*(F^(a*c)*b^8*c^8*log(F)^8 + 56*F^(a*c)*b^6*c^
6*e^2*log(F)^6 - 2032*F^(a*c)*b^4*c^4*e^4*log(F)^4 - 94976*F^(a*c)*b^2*c^2
*e^6*log(F)^2 + 147456*F^(a*c)*e^8)*F^(b*c*x)*sin(4*e*x + 4*d)^2 - 16*(F^(
a*c)*b^8*c^8*log(F)^8 - 208*F^(a*c)*b^6*c^6*e^2*log(F)^6 - 14480*F^(a*c)*b
^4*c^4*e^4*log(F)^4 + 185088*F^(a*c)*b^2*c^2*e^6*log(F)^2 - 147456*F^(a...
```

Giac [F]

$$\int F^{c(a+bx)}\cot^4(d+ex)\,dx = \int F^{(bx+a)c}\cot(ex+d)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*cot(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*cot(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \cot^4(d+ex)\,dx = \int F^{c\,(a+b\,x)} \cot(d+e\,x)^4\,dx$$

input
```
int(F^(c*(a + b*x))*cot(d + e*x)^4,x)
```

output
```
int(F^(c*(a + b*x))*cot(d + e*x)^4, x)
```

Reduce [F]

$$\int F^{c(a+bx)} \cot^4(d+ex)\,dx = f^{ac}\left(\int f^{bcx} \cot(ex+d)^4\,dx\right)$$

input
```
int(F^(c*(b*x+a))*cot(e*x+d)^4,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cot(d + e*x)**4,x)
```

3.73 $\int e^{a+ibx}\cot^n(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int e^{a+ibx}\cot^n(a+bx)\,dx = -\frac{ie^{a+ibx}\left(1-e^{2i(a+bx)}\right)^n\left(1+e^{2i(a+bx)}\right)^{-n}\operatorname{AppellF1}\left(\frac{1}{2},n,-n,\frac{3}{2},e^{2i(a+bx)},-e^{2i(a+bx)}\right)\cot^n(a+bx)}{b}$$

output
```
-I*exp(a+I*b*x)*(1-exp(2*I*(b*x+a)))^n*AppellF1(1/2,-n,n,3/2,-exp(2*I*(b*x
+a)),exp(2*I*(b*x+a)))*cot(b*x+a)^n/b/((1+exp(2*I*(b*x+a)))^n)
```

Mathematica [F]

$$\int e^{a+ibx}\cot^n(a+bx)\,dx = \int e^{a+ibx}\cot^n(a+bx)\,dx$$

input
```
Integrate[E^(a + I*b*x)*Cot[a + b*x]^n,x]
```

output
```
Integrate[E^(a + I*b*x)*Cot[a + b*x]^n, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\cot^n(a+bx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int e^{a+ibx}\cot^n(a+bx)dx$$

input
```
Int[E^(a + I*b*x)*Cot[a + b*x]^n,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int e^{ibx+a}\cot\left(bx+a\right)^n dx$$

input
```
int(exp(a+I*b*x)*cot(b*x+a)^n,x)
```

output
```
int(exp(a+I*b*x)*cot(b*x+a)^n,x)
```

Fricas [F]

$$\int e^{a+ibx} \cot^n(a+bx)\,dx = \int \cot(bx+a)^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral(((I*e^(2*I*b*x + 2*I*a) + I)/(e^(2*I*b*x + 2*I*a) - 1))^n*e^(I*b*
x + a), x)
```

Sympy [F]

$$\int e^{a+ibx} \cot^n(a+bx)\,dx = e^a \int e^{ibx} \cot^n(a+bx)\,dx$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+a)**n,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*cot(a + b*x)**n, x)
```

Maxima [F]

$$\int e^{a+ibx} \cot^n(a+bx)\,dx = \int \cot(bx+a)^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(cot(b*x + a)^n*e^(I*b*x + a), x)
```

Giac [F]

$$\int e^{a+ibx}\cot^n(a+bx)\,dx=\int \cot{(bx+a)}^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*cot(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)^n*e^(I*b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\cot^n(a+bx)\,dx=\int \cot(a+b\,x)^n\,\mathrm{e}^{a+b\,x\,1\mathrm{i}}\,dx$$

input
```
int(cot(a + b*x)^n*exp(a + b*x*1i),x)
```

output
```
int(cot(a + b*x)^n*exp(a + b*x*1i), x)
```

Reduce [F]

$$\int e^{a+ibx}\cot^n(a+bx)\,dx=e^a\left(\int e^{bix}\cot{(bx+a)}^n\,dx\right)$$

input
```
int(exp(a+I*b*x)*cot(b*x+a)^n,x)
```

output
```
e**a*int(e**(b*i*x)*cot(a + b*x)**n,x)
```

3.74 $\int F^{c(a+bx)}(f\cot(d+ex))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 158

$$\int F^{c(a+bx)}(f\cot(d+ex))^n\,dx = \frac{i2^{-1+n}\left(e^{2i(d+ex)}\right)^{\frac{ibc\log(F)}{2e}}\left(1-e^{2i(d+ex)}\right)\left(1+e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\operatorname{AppellF1}\left(1-n,1+\frac{ibc\log(F)}{2e},-n,2-\right.}{e(1-n)}$$

output
```
I*2^(-1+n)*exp(2*I*(e*x+d))^(1/2*I*b*c*ln(F)/e)*(1-exp(2*I*(e*x+d)))*F^(c*
(b*x+a))*AppellF1(1-n,1+1/2*I*b*c*ln(F)/e,-n,2-n,1-exp(2*I*(e*x+d)),1/2-1/
2*exp(2*I*(e*x+d)))*(f*cot(e*x+d))^n/e/((1+exp(2*I*(e*x+d)))^n)/(1-n)
```

Mathematica [F]

$$\int F^{c(a+bx)}(f\cot(d+ex))^n\,dx = \int F^{c(a+bx)}(f\cot(d+ex))^n\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*Cot[d + e*x])^n,x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*Cot[d + e*x])^n, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\cot(d+ex))^n\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cot^{-n}(d+ex)(f\cot(d+ex))^n\int F^{c(a+bx)}\cot^n(d+ex)dx$$

$$\Big\downarrow \text{4967}$$

$$\cot^{-n}(d+ex)(f\cot(d+ex))^n\int F^{ac+bxc}\cot^n(d+ex)dx$$

$$\Big\downarrow \text{7299}$$

$$\cot^{-n}(d+ex)(f\cot(d+ex))^n\int F^{ac+bxc}\cot^n(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(f*Cot[d + e*x])^n,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4967
```
Int[(F_)^((c_.)*(u_))*(G_)[v_]^(n_.), x_Symbol] :> Int[F^(c*ExpandToSum[u,
x])*G[ExpandToSum[v, x]]^n, x] /; FreeQ[{F, c, n}, x] && TrigQ[G] && Linear
Q[{u, v}, x] && !LinearMatchQ[{u, v}, x]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int F^{c(bx+a)}(f\cot(ex+d))^n\,dx$$

input
```
int(F^(c*(b*x+a))*(f*cot(e*x+d))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*cot(e*x+d))^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\cot(d+ex))^n\,dx = \int (f\cot(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cot(e*x+d))^n,x, algorithm="fricas")
```

output
```
integral((f*cot(e*x + d))^n*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\cot(d+ex))^n\,dx = \int F^{c(a+bx)}(f\cot(d+ex))^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*cot(e*x+d))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*cot(d + e*x))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\cot(d+ex))^n\,dx = \int (f\cot(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cot(e*x+d))^n,x, algorithm="maxima")
```

output
```
integrate((f*cot(e*x + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\cot(d+ex))^n\,dx = \int (f\cot(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cot(e*x+d))^n,x, algorithm="giac")
```

output
```
integrate((f*cot(e*x + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\cot(d+ex))^n\,dx = \int F^{c\,(a+b\,x)}\,(f\cot(d+e\,x))^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*cot(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*cot(d + e*x))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\cot(d+ex))^n\,dx = f^{ac+n}\left(\int f^{bcx}\cot\left(ex+d\right)^n dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*cot(e*x+d))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*cot(d + e*x)**n,x)
```

3.75 $\int e^{a+ibx}\sec(d+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int e^{a+ibx}\sec(d+bx)\,dx=-\frac{ie^{a-id}\log\left(1+e^{2i(d+bx)}\right)}{b}$$

output
```
-I*exp(a-I*d)*ln(1+exp(2*I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int e^{a+ibx}\sec(d+bx)\,dx=-\frac{ie^{a-id}\log\left(1+e^{2i(d+bx)}\right)}{b}$$

input
```
Integrate[E^(a + I*b*x)*Sec[d + b*x],x]
```

output
```
((-I)*E^(a - I*d)*Log[1 + E^((2*I)*(d + b*x))])/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx} \sec(bx+d)\,dx$$

$$\Big\downarrow \; 4951$$

$$-\frac{ie^{a-id}\log\left(1+e^{2i(bx+d)}\right)}{b}$$

input
```
Int[E^(a + I*b*x)*Sec[d + b*x],x]
```

output
```
((-I)*E^(a - I*d)*Log[1 + E^((2*I)*(d + b*x))])/b
```

Defintions of rubi rules used

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
risch	$-\frac{\mathrm{i}\,\mathrm{e}^{-id+a}\ln\left(1+\mathrm{e}^{2i(bx+d)}\right)}{b}$	26

input
```
int(exp(a+I*b*x)*sec(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-I*exp(a-I*d)*ln(1+exp(2*I*(b*x+d)))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int e^{a+ibx}\sec(d+bx)\,dx=-\frac{i\,e^{(a-i\,d)}\log\left(e^{(2i\,bx+2i\,d)}+1\right)}{b}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d),x, algorithm="fricas")
```

output
```
-I*e^(a - I*d)*log(e^(2*I*b*x + 2*I*d) + 1)/b
```

Sympy [F]

$$\int e^{a+ibx}\sec(d+bx)\,dx=e^{a}\int e^{ibx}\sec\left(bx+d\right)dx$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d),x)
```

output
```
exp(a)*Integral(exp(I*b*x)*sec(b*x + d), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(23) = 46.

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.57

$$\int e^{a+ibx}\sec(d+bx)\,dx$$
$$=\frac{2\left(\cos\left(d\right)e^{a}-i\,e^{a}\sin\left(d\right)\right)\arctan\left(\sin\left(2\,bx\right)-\sin\left(2\,d\right),\cos\left(2\,bx\right)+\cos\left(2\,d\right)\right)-\left(i\,\cos\left(d\right)e^{a}+e^{a}\sin\left(d\right)\right)}{2\,b}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d),x, algorithm="maxima")
```

output
```
1/2*(2*(cos(d)*e^a - I*e^a*sin(d))*arctan2(sin(2*b*x) - sin(2*d), cos(2*b*
x) + cos(2*d)) - (I*cos(d)*e^a + e^a*sin(d))*log(cos(2*b*x)^2 + 2*cos(2*b*
x)*cos(2*d) + cos(2*d)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*d) + sin(2*d)
^2))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int e^{a+ibx}\sec(d+bx)\,dx = -\frac{i\,e^{(a-i\,d)}\log\left(e^{(2i\,bx+2i\,d)}+1\right)}{b}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d),x, algorithm="giac")
```

output
```
-I*e^(a - I*d)*log(e^(2*I*b*x + 2*I*d) + 1)/b
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int e^{a+ibx}\sec(d+bx)\,dx = -\frac{e^{a-d\,1i}\,\ln\left(e^{2\,a}\,e^{b\,x\,2i}+e^{2\,a}\,e^{-d\,2i}\right)\,1i}{b}$$

input
```
int(exp(a + b*x*1i)/cos(d + b*x),x)
```

output
```
-(exp(a - d*1i)*log(exp(2*a)*exp(b*x*2i) + exp(2*a)*exp(-d*2i))*1i)/b
```

Reduce [F]

$$\int e^{a+ibx}\sec(d+bx)\,dx$$

$$= \frac{e^a\left(-3e^{bix}\cos\left(bx+d\right)\sec\left(bx+d\right)\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2 i+3e^{bix}\cos\left(bx+d\right)\sec\left(bx+d\right)i+2e^{bix}\cos\left(bx+d\right)\right.}{}$$

input
```
int(exp(a+I*b*x)*sec(b*x+d),x)
```

output
```
(e**a*( - 3*e**(b*i*x)*cos(b*x + d)*sec(b*x + d)*tan((b*x + d)/2)**2*i + 3
*e**(b*i*x)*cos(b*x + d)*sec(b*x + d)*i + 2*e**(b*i*x)*cos(b*x + d)*tan((b
*x + d)/2)**2*i - 2*e**(b*i*x)*cos(b*x + d)*tan((b*x + d)/2) - 4*e**(b*i*x
)*cos(b*x + d)*i - 6*cos(b*x + d)*int(e**(b*i*x)/(tan((b*x + d)/2)**2 - 1)
,x)*tan((b*x + d)/2)**2*b + 6*cos(b*x + d)*int(e**(b*i*x)/(tan((b*x + d)/2
)**2 - 1),x)*b - e**(b*i*x)*sin(b*x + d)*tan((b*x + d)/2)**2 + e**(b*i*x)*
sin(b*x + d) + 2*e**(b*i*x)*tan((b*x + d)/2)**2*i - 2*e**(b*i*x)*i))/(3*co
s(b*x + d)*b*(tan((b*x + d)/2)**2 - 1))
```

3.76 $\int e^{a+ibx}\sec^2(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int e^{a+ibx}\sec^2(d+bx)\,dx=\frac{2ie^{a-id+i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)}-\frac{2ie^{a-id}\arctan\left(e^{i(d+bx)}\right)}{b}$$

output
```
2*I*exp(a-I*d+I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))-2*I*exp(a-I*d)*arctan(exp(
I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int e^{a+ibx}\sec^2(d+bx)\,dx=\frac{2ie^a\left(\frac{e^{ibx}}{1+e^{2i(d+bx)}}-e^{-id}\arctan\left(e^{i(d+bx)}\right)\right)}{b}$$

input
```
Integrate[E^(a + I*b*x)*Sec[d + b*x]^2,x]
```

output
```
((2*I)*E^a*(E^(I*b*x)/(1 + E^((2*I)*(d + b*x))) - ArcTan[E^(I*(d + b*x))]/
E^(I*d)))/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\sec^2(bx+d)\,dx$$

$$\Big\downarrow \text{ 4951}$$

$$\frac{2ie^{a-2i(bx+d)+ibx}\left(\frac{e^{2i(bx+d)}}{1+e^{2i(bx+d)}}-e^{i(bx+d)}\arctan\left(e^{i(bx+d)}\right)\right)}{b}$$

input
```
Int[E^(a + I*b*x)*Sec[d + b*x]^2,x]
```

output
```
((2*I)*E^(a + I*b*x - (2*I)*(d + b*x))*(E^((2*I)*(d + b*x))/(1 + E^((2*I)*
(d + b*x))) - E^(I*(d + b*x))*ArcTan[E^(I*(d + b*x))]))/b
```

Defintions of rubi rules used

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{2i\mathrm{e}^{a}\mathrm{e}^{ibx}}{b\left(1+\mathrm{e}^{2i(bx+d)}\right)}-\frac{2i\mathrm{e}^{a}\mathrm{e}^{-id}\arctan\left(\mathrm{e}^{i(bx+d)}\right)}{b}$	52

input
```
int(exp(a+I*b*x)*sec(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
2*I*exp(a)*exp(I*b*x)/b/(1+exp(2*I*(b*x+d)))-2*I*exp(a)*exp(-I*d)*arctan(e
xp(I*(b*x+d)))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int e^{a+ibx}\sec^2(d+bx)\,dx = \frac{\left(e^{(2i\,bx+a+i\,d)}+e^{(a-i\,d)}\right)\log\left(e^{(i\,bx+i\,d)}+i\right)-\left(e^{(2i\,bx+a+i\,d)}+e^{(a-i\,d)}\right)\log\left(e^{(i\,bx+i\,d)}-i\right)+2i\,e^{(i\,bx+a)}}{be^{(2i\,bx+2i\,d)}+b}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)^2,x, algorithm="fricas")
```

output
```
((e^(2*I*b*x + a + I*d) + e^(a - I*d))*log(e^(I*b*x + I*d) + I) - (e^(2*I*
b*x + a + I*d) + e^(a - I*d))*log(e^(I*b*x + I*d) - I) + 2*I*e^(I*b*x + a)
)/(b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [F]

$$\int e^{a+ibx}\sec^2(d+bx)\,dx = e^a\int e^{ibx}\sec^2(bx+d)\,dx$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*sec(b*x + d)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(47) = 94$.

Time = 0.16 (sec) , antiderivative size = 446, normalized size of antiderivative = 6.46

$$\int e^{a+ibx}\sec^2(d+bx)\,dx$$
$$= \frac{2\left((\cos(d)e^a - ie^a\sin(d))\cos(2bx+2d) + \cos(d)e^a - (-i\cos(d)e^a - e^a\sin(d))\sin(2bx+2d) - ie\right.}{}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)^2,x, algorithm="maxima")
```

output
```
(2*((cos(d)*e^a - I*e^a*sin(d))*cos(2*b*x + 2*d) + cos(d)*e^a - (-I*cos(d)
*e^a - e^a*sin(d))*sin(2*b*x + 2*d) - I*e^a*sin(d))*arctan2(2*(cos(b*x + 2
*d)*cos(d) + sin(b*x + 2*d)*sin(d))/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d
)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2),
 (cos(b*x + 2*d)^2 - cos(d)^2 + sin(b*x + 2*d)^2 - sin(d)^2)/(cos(b*x + 2*
d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x +
 2*d)*sin(d) + sin(d)^2)) + 4*cos(b*x)*e^a + ((I*cos(d)*e^a + e^a*sin(d))*
cos(2*b*x + 2*d) + I*cos(d)*e^a - (cos(d)*e^a - I*e^a*sin(d))*sin(2*b*x +
2*d) + e^a*sin(d))*log((cos(b*x + 2*d)^2 + cos(d)^2 - 2*cos(d)*sin(b*x + 2
*d) + sin(b*x + 2*d)^2 + 2*cos(b*x + 2*d)*sin(d) + sin(d)^2)/(cos(b*x + 2*
d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x +
 2*d)*sin(d) + sin(d)^2)) + 4*I*e^a*sin(b*x))/(-2*I*b*cos(2*b*x + 2*d) + 2
*b*sin(2*b*x + 2*d) - 2*I*b)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(47) = 94.

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

$$\int e^{a+ibx}\sec^2(d+bx)\,dx = -\frac{e^{(2i\,bx+a+2i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+1\right)+e^{a}\log\left(i\,e^{(i\,bx+i\,d)}+1\right)-e^{(2i\,bx+a+2i\,d)}\log\left(-i\,e^{(i\,bx+i\,d)}+1\right)-e^{a}\log}{b\left(e^{(2i\,bx+3i\,d)}+e^{(i\,d)}\right)}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)^2,x, algorithm="giac")
```

output
```
-(e^(2*I*b*x + a + 2*I*d)*log(I*e^(I*b*x + I*d) + 1) + e^a*log(I*e^(I*b*x
+ I*d) + 1) - e^(2*I*b*x + a + 2*I*d)*log(-I*e^(I*b*x + I*d) + 1) - e^a*lo
g(-I*e^(I*b*x + I*d) + 1) - 2*I*e^(I*b*x + a + I*d))/(b*(e^(2*I*b*x + 3*I*
d) + e^(I*d)))
```

Mupad [B] (verification not implemented)

Time = 17.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.38

$$\int e^{a+ibx}\sec^2(d+bx)\,dx = \frac{\ln\left(-2\,e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}-e^{2\,a}\,e^{-d\,2i}\,\sqrt{e^{2\,a}\,e^{-d\,2i}}\,2i\right)\,\sqrt{e^{2\,a}\,e^{-d\,2i}}}{b} - \frac{\ln\left(-2\,e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}+e^{2\,a}\,e^{-d\,2i}\,\sqrt{e^{2\,a}\,e^{-d\,2i}}\,2i\right)\,\sqrt{e^{2\,a}\,e^{-d\,2i}}}{b} + \frac{e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}\,2i}{b\,e^{2\,a}\,e^{-d\,2i}+b\,e^{2\,a}\,e^{b\,x\,2i}}$$

input
```
int(exp(a + b*x*1i)/cos(d + b*x)^2,x)
```

output
```
(log(- 2*exp(3*a)*exp(-d*2i)*exp(b*x*1i) - exp(2*a)*exp(-d*2i)*(exp(2*a)*e
xp(-d*2i))^(1/2)*2i)*(exp(2*a)*exp(-d*2i))^(1/2))/b - (log(exp(2*a)*exp(-d
*2i)*(exp(2*a)*exp(-d*2i))^(1/2)*2i - 2*exp(3*a)*exp(-d*2i)*exp(b*x*1i))*(
exp(2*a)*exp(-d*2i))^(1/2))/b + (exp(3*a)*exp(-d*2i)*exp(b*x*1i)*2i)/(b*ex
p(2*a)*exp(-d*2i) + b*exp(2*a)*exp(b*x*2i))
```

Reduce [F]

$$\int e^{a+ibx}\sec^2(d+bx)\,dx = \text{too large to display}$$

input
```
int(exp(a+I*b*x)*sec(b*x+d)^2,x)
```

output
```
(e**a*( - 6*e**(b*i*x)*cos(b*x + d)*sin(b*x + d)*tan((b*x + d)/2)**4 + 12*
e**(b*i*x)*cos(b*x + d)*sin(b*x + d)*tan((b*x + d)/2)**2 - 6*e**(b*i*x)*co
s(b*x + d)*sin(b*x + d) - 14*e**(b*i*x)*cos(b*x + d)*tan((b*x + d)/2)**4*i
 + 28*e**(b*i*x)*cos(b*x + d)*tan((b*x + d)/2)**2*i - 14*e**(b*i*x)*cos(b*
x + d)*i - 15*e**(b*i*x)*sec(b*x + d)**2*sin(b*x + d)**2*tan((b*x + d)/2)*
*4*i + 30*e**(b*i*x)*sec(b*x + d)**2*sin(b*x + d)**2*tan((b*x + d)/2)**2*i
 - 15*e**(b*i*x)*sec(b*x + d)**2*sin(b*x + d)**2*i + 15*e**(b*i*x)*sec(b*x
 + d)**2*tan((b*x + d)/2)**4*i - 30*e**(b*i*x)*sec(b*x + d)**2*tan((b*x +
d)/2)**2*i + 15*e**(b*i*x)*sec(b*x + d)**2*i - 4*e**(b*i*x)*sin(b*x + d)**
2*tan((b*x + d)/2)**4*i + 16*e**(b*i*x)*sin(b*x + d)**2*tan((b*x + d)/2)**
3 + 24*e**(b*i*x)*sin(b*x + d)**2*tan((b*x + d)/2)**2*i - 48*e**(b*i*x)*si
n(b*x + d)**2*tan((b*x + d)/2) + 48*e**(b*i*x)*sin(b*x + d)**2*i + 2*e**(b
*i*x)*sin(b*x + d)*tan((b*x + d)/2)**4 - 4*e**(b*i*x)*sin(b*x + d)*tan((b*
x + d)/2)**2 + 2*e**(b*i*x)*sin(b*x + d) - 14*e**(b*i*x)*tan((b*x + d)/2)*
*4*i - 16*e**(b*i*x)*tan((b*x + d)/2)**3 + 12*e**(b*i*x)*tan((b*x + d)/2)*
*2*i + 48*e**(b*i*x)*tan((b*x + d)/2) - 66*e**(b*i*x)*i - 100*int(e**(b*i*
x)/(tan((b*x + d)/2)**6 - 3*tan((b*x + d)/2)**4 + 3*tan((b*x + d)/2)**2 -
1),x)*sin(b*x + d)**2*tan((b*x + d)/2)**4*b + 200*int(e**(b*i*x)/(tan((b*x
 + d)/2)**6 - 3*tan((b*x + d)/2)**4 + 3*tan((b*x + d)/2)**2 - 1),x)*sin(b*
x + d)**2*tan((b*x + d)/2)**2*b - 100*int(e**(b*i*x)/(tan((b*x + d)/2)*...
```

3.77 $\int e^{a+ibx} \sec^3(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 40

$$\int e^{a+ibx} \sec^3(d+bx)\,dx = -\frac{2ie^{a-id+4i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)^2}$$

output
```
-2*I*exp(a-I*d+4*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int e^{a+ibx} \sec^3(d+bx)\,dx = -\frac{2ie^{a+3id+4ibx}}{b\left(1+e^{2i(d+bx)}\right)^2}$$

input
```
Integrate[E^(a + I*b*x)*Sec[d + b*x]^3,x]
```

output
```
((-2*I)*E^(a + (3*I)*d + (4*I)*b*x))/(b*(1 + E^((2*I)*(d + b*x)))^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{4946\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx} \sec^3(bx+d) \, dx$$

$$\Big\downarrow \text{4946}$$

$$\frac{e^{a+ibx} \tan(bx+d) \sec(bx+d)}{2b} - \frac{ie^{a+ibx} \sec(bx+d)}{2b}$$

input
```
Int[E^(a + I*b*x)*Sec[d + b*x]^3,x]
```

output
```
((-1/2*I)*E^(a + I*b*x)*Sec[d + b*x])/b + (E^(a + I*b*x)*Sec[d + b*x]*Tan[
d + b*x])/(2*b)
```

Defintions of rubi rules used

rule 4946
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e
*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[b^2*c^2*Log[F]^2 +
 e^2*(n - 2)^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$-\frac{e^{ibx+a}\sec(bx+d)(i-\tan(bx+d))}{2b}$	31
risch	$-\frac{2i e^{4ibx+3id+a}}{\left(1+e^{2i(bx+d)}\right)^2 b}$	32
norman	$\frac{\frac{e^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{b}+\frac{e^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^3}{b}-\frac{i e^{ibx+a}}{2b}+\frac{i e^{ibx+a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^4}{2b}}{\left(\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2-1\right)^2}$	101

input
```
int(exp(a+I*b*x)*sec(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(a+I*b*x)*sec(b*x+d)*(I-tan(b*x+d))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int e^{a+ibx}\sec^3(d+bx)\,dx=-\frac{2\left(-2i\,e^{(2i\,bx+a+i\,d)}-i\,e^{(a-i\,d)}\right)}{be^{(4i\,bx+4i\,d)}+2\,be^{(2i\,bx+2i\,d)}+b}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)^3,x, algorithm="fricas")
```

output
```
-2*(-2*I*e^(2*I*b*x + a + I*d) - I*e^(a - I*d))/(b*e^(4*I*b*x + 4*I*d) + 2
*b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [F]

$$\int e^{a+ibx}\sec^3(d+bx)\,dx = e^a\int e^{ibx}\sec^3(bx+d)\,dx$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)**3,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*sec(b*x + d)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(28) = 56.

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int e^{a+ibx}\sec^3(d+bx)\,dx = \frac{2\,(2\,\cos(2\,bx+2\,d)\,e^a+2i\,e^a\sin(2\,bx+2\,d)+e^a)}{-i\,b\cos(4\,bx+5\,d)-2i\,b\cos(2\,bx+3\,d)-i\,b\cos(d)+b\sin(4\,bx+5\,d)+2\,b\sin(2\,bx+3\,d)+b\sin(d)}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)^3,x, algorithm="maxima")
```

output
```
2*(2*cos(2*b*x + 2*d)*e^a + 2*I*e^a*sin(2*b*x + 2*d) + e^a)/(-I*b*cos(4*b*
x + 5*d) - 2*I*b*cos(2*b*x + 3*d) - I*b*cos(d) + b*sin(4*b*x + 5*d) + 2*b*
sin(2*b*x + 3*d) + b*sin(d))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int e^{a+ibx}\sec^3(d+bx)\,dx = -\frac{2\left(-2i\,e^{(2i\,bx+a+2i\,d)}-i\,e^{a}\right)}{b(e^{(4i\,bx+5i\,d)}+2\,e^{(2i\,bx+3i\,d)}+e^{(i\,d)})}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)^3,x, algorithm="giac")
```

output
```
-2*(-2*I*e^(2*I*b*x + a + 2*I*d) - I*e^a)/(b*(e^(4*I*b*x + 5*I*d) + 2*e^(2
*I*b*x + 3*I*d) + e^(I*d)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\sec^3(d+bx)\,dx = \int \frac{e^{a+b\,x\,1i}}{\cos\left(d+b\,x\right)^3}\,dx$$

input
```
int(exp(a + b*x*1i)/cos(d + b*x)^3,x)
```

output
```
int(exp(a + b*x*1i)/cos(d + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.60

$$\int e^{a+ibx}\sec^3(d+bx)\,dx$$
$$= \frac{e^{bix+a}\left(-2\cos\left(bx+d\right)\sec\left(bx+d\right)^3\sin\left(bx+d\right)^2 i+2\cos\left(bx+d\right)\sec\left(bx+d\right)^3 i-\cos\left(bx+d\right)\sin\left(bx+\right.\right.}{2\cos\left(bx+d\right)b\left(\sin\left(bx+d\right)^2-1\right)}$$

input
```
int(exp(a+I*b*x)*sec(b*x+d)^3,x)
```

output
```
(e**(a + b*i*x)*( - 2*cos(b*x + d)*sec(b*x + d)**3*sin(b*x + d)**2*i + 2*c
os(b*x + d)*sec(b*x + d)**3*i - cos(b*x + d)*sin(b*x + d) - sin(b*x + d)**
2*i - i))/(2*cos(b*x + d)*b*(sin(b*x + d)**2 - 1))
```

3.78 $\int e^{a+ibx} \sec^4(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 151

$$\int e^{a+ibx}\sec^4(d+bx)\,dx=\frac{8ie^{a-id+3i(d+bx)}}{3b\left(1+e^{2i(d+bx)}\right)^3}+\frac{2ie^{a-id+i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)^2}-\frac{ie^{a-id+i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)}-\frac{ie^{a-id}\arctan\left(e^{i(d+bx)}\right)}{b}$$

output
```
8/3*I*exp(a-I*d+3*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^3+2*I*exp(a-I*d+I*(b*x
+d))/b/(1+exp(2*I*(b*x+d)))^2-I*exp(a-I*d+I*(b*x+d))/b/(1+exp(2*I*(b*x+d))
)-I*exp(a-I*d)*arctan(exp(I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.57

$$\int e^{a+ibx}\sec^4(d+bx)\,dx=-\frac{ie^a\left(\frac{e^{ibx}\left(-3-8e^{2i(d+bx)}+3e^{4i(d+bx)}\right)}{\left(1+e^{2i(d+bx)}\right)^3}+3e^{-id}\arctan\left(e^{i(d+bx)}\right)\right)}{3b}$$

input
```
Integrate[E^(a + I*b*x)*Sec[d + b*x]^4,x]
```

output
```
((-1/3*I)*E^a*((E^(I*b*x)*(-3 - 8*E^((2*I)*(d + b*x)) + 3*E^((4*I)*(d + b*
x))))/(1 + E^((2*I)*(d + b*x)))^3 + (3*ArcTan[E^(I*(d + b*x))])/E^(I*d)))/
b
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4948, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\sec^4(bx+d)\,dx$$

$$\Big\downarrow \text{4948}$$

$$\frac{1}{2}\int e^{a+ibx}\sec^2(d+bx)dx-\frac{ie^{a+ibx}\sec^2(bx+d)}{6b}+\frac{e^{a+ibx}\tan(bx+d)\sec^2(bx+d)}{3b}$$

$$\Big\downarrow \text{4951}$$

$$\frac{ie^{a-2i(bx+d)+ibx}\left(\frac{e^{2i(bx+d)}}{1+e^{2i(bx+d)}}-e^{i(bx+d)}\arctan\left(e^{i(bx+d)}\right)\right)}{b}-\frac{ie^{a+ibx}\sec^2(bx+d)}{6b}+\frac{e^{a+ibx}\tan(bx+d)\sec^2(bx+d)}{3b}$$

input
```
Int[E^(a + I*b*x)*Sec[d + b*x]^4,x]
```

output
```
(I*E^(a + I*b*x - (2*I)*(d + b*x))*(E^((2*I)*(d + b*x))/(1 + E^((2*I)*(d +
 b*x))) - E^(I*(d + b*x))*ArcTan[E^(I*(d + b*x))]))/b - ((I/6)*E^(a + I*b*
x)*Sec[d + b*x]^2)/b + (E^(a + I*b*x)*Sec[d + b*x]^2*Tan[d + b*x])/(3*b)
```

Defintions of rubi rules used

rule 4948
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(
e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2))   Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b,
 c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && N
eQ[n, 2]
```

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.56

method	result	size
risch	$-\frac{i\left(3\,\mathrm{e}^{a}\mathrm{e}^{5ibx}\mathrm{e}^{4id}-8\,\mathrm{e}^{a}\mathrm{e}^{3ibx}\mathrm{e}^{2id}-3\,\mathrm{e}^{a}\mathrm{e}^{ibx}\right)}{3b\left(1+\mathrm{e}^{2i(bx+d)}\right)^{3}}-\frac{i\mathrm{e}^{a}\mathrm{e}^{-id}\arctan\left(\mathrm{e}^{i(bx+d)}\right)}{b}$	85

input
```
int(exp(a+I*b*x)*sec(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/b/(1+exp(2*I*(b*x+d)))^3*(3*exp(a)*exp(5*I*b*x)*exp(4*I*d)-8*exp(a)
*exp(3*I*b*x)*exp(2*I*d)-3*exp(a)*exp(I*b*x))-I*exp(a)/b*exp(-I*d)*arctan(
exp(I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.23

$$\int e^{a+ibx}\sec^4(d+bx)\,dx$$
$$= \frac{3\left(e^{(6i\,bx+a+5i\,d)}+3\,e^{(4i\,bx+a+3i\,d)}+3\,e^{(2i\,bx+a+i\,d)}+e^{(a-i\,d)}\right)\log\left(e^{(i\,bx+i\,d)}+i\right)-3\left(e^{(6i\,bx+a+5i\,d)}+3\,e^{(4i\,bx-}\right.}{6\left(be^{(6i\,bx+6i\,d)}+3\,be^{(4i\,bx+4i}\right.}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)^4,x, algorithm="fricas")
```

output
```
1/6*(3*(e^(6*I*b*x + a + 5*I*d) + 3*e^(4*I*b*x + a + 3*I*d) + 3*e^(2*I*b*x
 + a + I*d) + e^(a - I*d))*log(e^(I*b*x + I*d) + I) - 3*(e^(6*I*b*x + a +
5*I*d) + 3*e^(4*I*b*x + a + 3*I*d) + 3*e^(2*I*b*x + a + I*d) + e^(a - I*d)
)*log(e^(I*b*x + I*d) - I) - 6*I*e^(5*I*b*x + a + 4*I*d) + 16*I*e^(3*I*b*x
 + a + 2*I*d) + 6*I*e^(I*b*x + a))/(b*e^(6*I*b*x + 6*I*d) + 3*b*e^(4*I*b*x
 + 4*I*d) + 3*b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [F]

$$\int e^{a+ibx}\sec^4(d+bx)\,dx = e^a\int e^{ibx}\sec^4(bx+d)\,dx$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)**4,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*sec(b*x + d)**4, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(100) = 200$.

Time = 0.19 (sec) , antiderivative size = 736, normalized size of antiderivative = 4.87

$$\int e^{a+ibx}\sec^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)^4,x, algorithm="maxima")
```

output
```
(6*((cos(d)*e^a - I*e^a*sin(d))*cos(6*b*x + 6*d) + 3*(cos(d)*e^a - I*e^a*s
in(d))*cos(4*b*x + 4*d) + 3*(cos(d)*e^a - I*e^a*sin(d))*cos(2*b*x + 2*d) +
 cos(d)*e^a - (-I*cos(d)*e^a - e^a*sin(d))*sin(6*b*x + 6*d) - 3*(-I*cos(d)
*e^a - e^a*sin(d))*sin(4*b*x + 4*d) - 3*(-I*cos(d)*e^a - e^a*sin(d))*sin(2
*b*x + 2*d) - I*e^a*sin(d))*arctan2(2*(cos(b*x + 2*d)*cos(d) + sin(b*x + 2
*d)*sin(d))/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b
*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2), (cos(b*x + 2*d)^2 - cos
(d)^2 + sin(b*x + 2*d)^2 - sin(d)^2)/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(
d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2)
) - 12*cos(5*b*x + 4*d)*e^a + 32*cos(3*b*x + 2*d)*e^a + 12*cos(b*x)*e^a -
3*((-I*cos(d)*e^a - e^a*sin(d))*cos(6*b*x + 6*d) + 3*(-I*cos(d)*e^a - e^a*
sin(d))*cos(4*b*x + 4*d) + 3*(-I*cos(d)*e^a - e^a*sin(d))*cos(2*b*x + 2*d)
 - I*cos(d)*e^a + (cos(d)*e^a - I*e^a*sin(d))*sin(6*b*x + 6*d) + 3*(cos(d)
*e^a - I*e^a*sin(d))*sin(4*b*x + 4*d) + 3*(cos(d)*e^a - I*e^a*sin(d))*sin(
2*b*x + 2*d) - e^a*sin(d))*log((cos(b*x + 2*d)^2 + cos(d)^2 - 2*cos(d)*sin
(b*x + 2*d) + sin(b*x + 2*d)^2 + 2*cos(b*x + 2*d)*sin(d) + sin(d)^2)/(cos(
b*x + 2*d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*c
os(b*x + 2*d)*sin(d) + sin(d)^2)) - 12*I*e^a*sin(5*b*x + 4*d) + 32*I*e^a*s
in(3*b*x + 2*d) + 12*I*e^a*sin(b*x))/(-12*I*b*cos(6*b*x + 6*d) - 36*I*b*co
s(4*b*x + 4*d) - 36*I*b*cos(2*b*x + 2*d) + 12*b*sin(6*b*x + 6*d) + 36*b...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(100) = 200$.

Time = 0.15 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.80

$$\int e^{a+ibx}\sec^4(d+bx)\,dx = -\frac{3\,e^{(6i\,bx+a+6i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+1\right)+9\,e^{(4i\,bx+a+4i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+1\right)+9\,e^{(2i\,bx+a+2i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}\right.}{}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)^4,x, algorithm="giac")
```

output
```
-1/6*(3*e^(6*I*b*x + a + 6*I*d)*log(I*e^(I*b*x + I*d) + 1) + 9*e^(4*I*b*x
+ a + 4*I*d)*log(I*e^(I*b*x + I*d) + 1) + 9*e^(2*I*b*x + a + 2*I*d)*log(I*
e^(I*b*x + I*d) + 1) + 3*e^a*log(I*e^(I*b*x + I*d) + 1) - 3*e^(6*I*b*x + a
 + 6*I*d)*log(-I*e^(I*b*x + I*d) + 1) - 9*e^(4*I*b*x + a + 4*I*d)*log(-I*e
^(I*b*x + I*d) + 1) - 9*e^(2*I*b*x + a + 2*I*d)*log(-I*e^(I*b*x + I*d) + 1
) - 3*e^a*log(-I*e^(I*b*x + I*d) + 1) + 6*I*e^(5*I*b*x + a + 5*I*d) - 16*I
*e^(3*I*b*x + a + 3*I*d) - 6*I*e^(I*b*x + a + I*d))/(b*(e^(6*I*b*x + 7*I*d
) + 3*e^(4*I*b*x + 5*I*d) + 3*e^(2*I*b*x + 3*I*d) + e^(I*d)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\sec^4(d+bx)\,dx = \int \frac{e^{a+b\,x\,1i}}{\cos\left(d+b\,x\right)^4}\,dx$$

input
```
int(exp(a + b*x*1i)/cos(d + b*x)^4,x)
```

output
```
int(exp(a + b*x*1i)/cos(d + b*x)^4, x)
```

Reduce [F]

$$\int e^{a+ibx}\sec^4(d+bx)\,dx = \text{too large to display}$$

input
```
int(exp(a+I*b*x)*sec(b*x+d)^4,x)
```

output
```
(e**a*( - 20*e**(b*i*x)*cos(b*x + d)*sin(b*x + d)**3*tan((b*x + d)/2)**8 +
 80*e**(b*i*x)*cos(b*x + d)*sin(b*x + d)**3*tan((b*x + d)/2)**6 - 120*e**(
b*i*x)*cos(b*x + d)*sin(b*x + d)**3*tan((b*x + d)/2)**4 + 80*e**(b*i*x)*co
s(b*x + d)*sin(b*x + d)**3*tan((b*x + d)/2)**2 - 20*e**(b*i*x)*cos(b*x + d
)*sin(b*x + d)**3 - 152*e**(b*i*x)*cos(b*x + d)*sin(b*x + d)**2*tan((b*x +
 d)/2)**8*i + 608*e**(b*i*x)*cos(b*x + d)*sin(b*x + d)**2*tan((b*x + d)/2)
**6*i - 912*e**(b*i*x)*cos(b*x + d)*sin(b*x + d)**2*tan((b*x + d)/2)**4*i
+ 608*e**(b*i*x)*cos(b*x + d)*sin(b*x + d)**2*tan((b*x + d)/2)**2*i - 152*
e**(b*i*x)*cos(b*x + d)*sin(b*x + d)**2*i + 100*e**(b*i*x)*cos(b*x + d)*si
n(b*x + d)*tan((b*x + d)/2)**8 - 400*e**(b*i*x)*cos(b*x + d)*sin(b*x + d)*
tan((b*x + d)/2)**6 + 600*e**(b*i*x)*cos(b*x + d)*sin(b*x + d)*tan((b*x +
d)/2)**4 - 400*e**(b*i*x)*cos(b*x + d)*sin(b*x + d)*tan((b*x + d)/2)**2 +
100*e**(b*i*x)*cos(b*x + d)*sin(b*x + d) + 180*e**(b*i*x)*cos(b*x + d)*tan
((b*x + d)/2)**8*i - 720*e**(b*i*x)*cos(b*x + d)*tan((b*x + d)/2)**6*i + 1
080*e**(b*i*x)*cos(b*x + d)*tan((b*x + d)/2)**4*i - 720*e**(b*i*x)*cos(b*x
 + d)*tan((b*x + d)/2)**2*i + 180*e**(b*i*x)*cos(b*x + d)*i - 315*e**(b*i*
x)*sec(b*x + d)**4*sin(b*x + d)**4*tan((b*x + d)/2)**8*i + 1260*e**(b*i*x)
*sec(b*x + d)**4*sin(b*x + d)**4*tan((b*x + d)/2)**6*i - 1890*e**(b*i*x)*s
ec(b*x + d)**4*sin(b*x + d)**4*tan((b*x + d)/2)**4*i + 1260*e**(b*i*x)*sec
(b*x + d)**4*sin(b*x + d)**4*tan((b*x + d)/2)**2*i - 315*e**(b*i*x)*sec...
```

3.79 $\int e^{a+ibx} \sec^5(d+bx) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 96

$$\int e^{a+ibx} \sec^5(d+bx) \, dx = \frac{4ie^{a-id}}{b\left(1+e^{2i(d+bx)}\right)^4} - \frac{32ie^{a-id}}{3b\left(1+e^{2i(d+bx)}\right)^3} + \frac{8ie^{a-id}}{b\left(1+e^{2i(d+bx)}\right)^2}$$

output
```
4*I*exp(a-I*d)/b/(1+exp(2*I*(b*x+d)))^4-32/3*I*exp(a-I*d)/b/(1+exp(2*I*(b*
x+d)))^3+8*I*exp(a-I*d)/b/(1+exp(2*I*(b*x+d)))^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.54

$$\int e^{a+ibx} \sec^5(d+bx) \, dx = -\frac{4ie^{a+5id+6ibx}\left(4+e^{2i(d+bx)}\right)}{3b\left(1+e^{2i(d+bx)}\right)^4}$$

input
```
Integrate[E^(a + I*b*x)*Sec[d + b*x]^5,x]
```

output
```
(((-4*I)/3)*E^(a + (5*I)*d + (6*I)*b*x)*(4 + E^((2*I)*(d + b*x))))/(b*(1 +
 E^((2*I)*(d + b*x)))^4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4948, 4946}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx} \sec^5(bx+d) \, dx$$
$$\Big\downarrow \text{4948}$$
$$\frac{2}{3} \int e^{a+ibx} \sec^3(d+bx)dx - \frac{ie^{a+ibx} \sec^3(bx+d)}{12b} + \frac{e^{a+ibx} \tan(bx+d) \sec^3(bx+d)}{4b}$$
$$\Big\downarrow \text{4946}$$
$$-\frac{ie^{a+ibx} \sec^3(bx+d)}{12b} + \frac{e^{a+ibx} \tan(bx+d) \sec^3(bx+d)}{4b} + \frac{2}{3} \left(\frac{e^{a+ibx} \tan(bx+d) \sec(bx+d)}{2b} - \frac{ie^{a+ibx} \sec(bx+d)}{2b} \right)$$

input
```
Int[E^(a + I*b*x)*Sec[d + b*x]^5,x]
```

output
```
((-1/12*I)*E^(a + I*b*x)*Sec[d + b*x]^3)/b + (E^(a + I*b*x)*Sec[d + b*x]^3
*Tan[d + b*x])/(4*b) + (2*(((-1/2*I)*E^(a + I*b*x)*Sec[d + b*x])/b + (E^(a
 + I*b*x)*Sec[d + b*x]*Tan[d + b*x])/(2*b)))/3
```

Defintions of rubi rules used

rule 4946
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e
*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[b^2*c^2*Log[F]^2 +
 e^2*(n - 2)^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

rule 4948
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(
e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2))   Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b,
 c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && N
eQ[n, 2]
```

Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.52

method	result
risch	$-\frac{4 i \left(\mathrm{e}^{8 i b x} \mathrm{e}^{7 i d} \mathrm{e}^{a}+4 \,\mathrm{e}^{6 i b x} \mathrm{e}^{5 i d} \mathrm{e}^{a}\right)}{3 \left(1+\mathrm{e}^{2 i \left(b x +d \right)}\right)^{4} b}$
parallelrisch	$\frac{\mathrm{e}^{i b x +a} \left(5 i \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{8}-6 i \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{6}+14 \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{7}+10 \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{5}+6 i \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{2}+10 \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{3}-5 i+14\right.}{12 b \left(\tan \left(\frac{b x}{2}+\frac{d}{2}\right)-1\right)^{4} \left(\tan \left(\frac{b x}{2}+\frac{d}{2}\right)+1\right)^{4}}$

input
```
int(exp(a+I*b*x)*sec(b*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-4/3*I/(1+exp(2*I*(b*x+d)))^4/b*(exp(8*I*b*x)*exp(7*I*d)*exp(a)+4*exp(6*I*
b*x)*exp(5*I*d)*exp(a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90

$$\int e^{a+ibx} \sec^5(d+bx)\,dx$$
$$= -\frac{4 \left(-6 i \, e^{\left(4 i \, b x + a + 3 i \, d\right)} - 4 i \, e^{\left(2 i \, b x + a + i \, d\right)} - i \, e^{\left(a - i \, d\right)}\right)}{3 \left(b e^{\left(8 i \, b x + 8 i \, d\right)} + 4 \, b e^{\left(6 i \, b x + 6 i \, d\right)} + 6 \, b e^{\left(4 i \, b x + 4 i \, d\right)} + 4 \, b e^{\left(2 i \, b x + 2 i \, d\right)} + b\right)}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)^5,x, algorithm="fricas")
```

output
```
-4/3*(-6*I*e^(4*I*b*x + a + 3*I*d) - 4*I*e^(2*I*b*x + a + I*d) - I*e^(a -
I*d))/(b*e^(8*I*b*x + 8*I*d) + 4*b*e^(6*I*b*x + 6*I*d) + 6*b*e^(4*I*b*x +
4*I*d) + 4*b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [F]

$$\int e^{a+ibx}\sec^5(d+bx)\,dx = e^a\int e^{ibx}\sec^5(bx+d)\,dx$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)**5,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*sec(b*x + d)**5, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(73) = 146.

Time = 0.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.73

$$\int e^{a+ibx}\sec^5(d+bx)\,dx$$
$$= \frac{4\,(6\,\cos(4\,bx+4\,d)\,e^a+4\,\cos(2\,bx+2\,d)\,e^a+6i\,e}{-3i\,b\cos(8\,bx+9\,d)-12i\,b\cos(6\,bx+7\,d)-18i\,b\cos(4\,bx+5\,d)-12i\,b\cos(2\,bx+3\,d)-3i\,b\cos(d)}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)^5,x, algorithm="maxima")
```

output
```
4*(6*cos(4*b*x + 4*d)*e^a + 4*cos(2*b*x + 2*d)*e^a + 6*I*e^a*sin(4*b*x + 4
*d) + 4*I*e^a*sin(2*b*x + 2*d) + e^a)/(-3*I*b*cos(8*b*x + 9*d) - 12*I*b*co
s(6*b*x + 7*d) - 18*I*b*cos(4*b*x + 5*d) - 12*I*b*cos(2*b*x + 3*d) - 3*I*b
*cos(d) + 3*b*sin(8*b*x + 9*d) + 12*b*sin(6*b*x + 7*d) + 18*b*sin(4*b*x +
5*d) + 12*b*sin(2*b*x + 3*d) + 3*b*sin(d))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int e^{a+ibx}\sec^5(d+bx)\,dx = -\frac{4\left(-6i\,e^{(4i\,bx+a+4i\,d)}-4i\,e^{(2i\,bx+a+2i\,d)}-i\,e^{a}\right)}{3\,b(e^{(8i\,bx+9i\,d)}+4\,e^{(6i\,bx+7i\,d)}+6\,e^{(4i\,bx+5i\,d)}+4\,e^{(2i\,bx+3i\,d)}+e^{(i\,d)})}$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+d)^5,x, algorithm="giac")
```

output
```
-4/3*(-6*I*e^(4*I*b*x + a + 4*I*d) - 4*I*e^(2*I*b*x + a + 2*I*d) - I*e^a)/
(b*(e^(8*I*b*x + 9*I*d) + 4*e^(6*I*b*x + 7*I*d) + 6*e^(4*I*b*x + 5*I*d) +
4*e^(2*I*b*x + 3*I*d) + e^(I*d)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\sec^5(d+bx)\,dx = \int \frac{e^{a+b\,x\,1i}}{\cos\left(d+b\,x\right)^5}\,dx$$

input
```
int(exp(a + b*x*1i)/cos(d + b*x)^5,x)
```

output
```
int(exp(a + b*x*1i)/cos(d + b*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.73

$$\int e^{a+ibx}\sec^5(d+bx)\,dx = \frac{e^{bix+a}\left(-12\cos\left(bx+d\right)\sec\left(bx+d\right)^5\sin\left(bx+d\right)^4 i+24\cos\left(bx+d\right)\sec\left(bx+d\right)^5\sin\left(bx+d\right)^2 i-12\,\mathrm{co}\right.}{12\cos\left(bx+d\right)b}$$

input
```
int(exp(a+I*b*x)*sec(b*x+d)^5,x)
```

output
```
(e**(a + b*i*x)*( - 12*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**4*i + 24
*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**2*i - 12*cos(b*x + d)*sec(b*x
+ d)**5*i - 4*cos(b*x + d)*sin(b*x + d)**3 + 7*cos(b*x + d)*sin(b*x + d) -
 4*sin(b*x + d)**4*i + 9*sin(b*x + d)**2*i + 7*i))/(12*cos(b*x + d)*b*(sin
(b*x + d)**4 - 2*sin(b*x + d)**2 + 1))
```

3.80 $\int e^{2(a+ibx)}\sec(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int e^{2(a+ibx)}\sec(d+bx)\,dx=-\frac{2ie^{2(a-id)+i(d+bx)}}{b}+\frac{2ie^{2a-2id}\arctan\left(e^{i(d+bx)}\right)}{b}$$

output
```
-2*I*exp(2*a-2*I*d+I*(b*x+d))/b+2*I*exp(2*a-2*I*d)*arctan(exp(I*(b*x+d)))/
b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int e^{2(a+ibx)}\sec(d+bx)\,dx=-\frac{2ie^{2a-2id}\left(e^{i(d+bx)}-\arctan\left(e^{i(d+bx)}\right)\right)}{b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Sec[d + b*x],x]
```

output
```
((-2*I)*E^(2*a - (2*I)*d)*(E^(I*(d + b*x)) - ArcTan[E^(I*(d + b*x))]))/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\sec(bx+d)\,dx$$

$$\Big\downarrow \text{4951}$$

$$-\frac{2ie^{2(a+ibx)-3i(bx+d)}\left(e^{2i(bx+d)}-e^{i(bx+d)}\arctan\left(e^{i(bx+d)}\right)\right)}{b}$$

input
```
Int[E^(2*(a + I*b*x))*Sec[d + b*x],x]
```

output
```
((-2*I)*E^(2*(a + I*b*x) - (3*I)*(d + b*x))*(E^((2*I)*(d + b*x)) - E^(I*(d
 + b*x))*ArcTan[E^(I*(d + b*x))]))/b
```

Defintions of rubi rules used

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{2i\mathrm{e}^{2a}\mathrm{e}^{-id}\mathrm{e}^{ibx}}{b}+\frac{2i\mathrm{e}^{2a}\mathrm{e}^{-2id}\arctan\left(\mathrm{e}^{i(bx+d)}\right)}{b}$	48

input
```
int(exp(2*a+2*I*b*x)*sec(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-2*I/b*exp(2*a)*exp(-I*d)*exp(I*b*x)+2*I/b*exp(2*a)*exp(-2*I*d)*arctan(exp
(I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int e^{2(a+ibx)}\sec(d+bx)\,dx = -\frac{e^{(2\,a-2i\,d)}\log\left(e^{(i\,bx+i\,d)}+i\right)-e^{(2\,a-2i\,d)}\log\left(e^{(i\,bx+i\,d)}-i\right)+2i\,e^{(i\,bx+2\,a-i\,d)}}{b}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d),x, algorithm="fricas")
```

output
```
-(e^(2*a - 2*I*d)*log(e^(I*b*x + I*d) + I) - e^(2*a - 2*I*d)*log(e^(I*b*x
+ I*d) - I) + 2*I*e^(I*b*x + 2*a - I*d))/b
```

Sympy [F]

$$\int e^{2(a+ibx)}\sec(d+bx)\,dx = e^{2a}\int e^{2ibx}\sec{(bx+d)}\,dx$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*I*b*x)*sec(b*x + d), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(41) = 82.

Time = 0.16 (sec) , antiderivative size = 358, normalized size of antiderivative = 6.07

$$\int e^{2(a+ibx)}\sec(d+bx)\,dx$$
$$= \frac{2\left(-i\cos(2d)e^{(2a)} - e^{(2a)}\sin(2d)\right)\arctan\left(\frac{2(\cos(bx+2d)\cos(d)+\sin(bx+2d)\sin(d))}{\cos(bx+2d)^2+\cos(d)^2+2\cos(d)\sin(bx+2d)+\sin(bx+2d)^2-2\cos(bx+2d)\sin(d}\right.}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d),x, algorithm="maxima")
```

output
```
1/2*(2*(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*arctan2(2*(cos(b*x + 2*d)*
cos(d) + sin(b*x + 2*d)*sin(d))/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d)*si
n(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2), (co
s(b*x + 2*d)^2 - cos(d)^2 + sin(b*x + 2*d)^2 - sin(d)^2)/(cos(b*x + 2*d)^2
 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d
)*sin(d) + sin(d)^2)) - 4*I*cos(b*x - d)*e^(2*a) + (cos(2*d)*e^(2*a) - I*e
^(2*a)*sin(2*d))*log((cos(b*x + 2*d)^2 + cos(d)^2 - 2*cos(d)*sin(b*x + 2*d
) + sin(b*x + 2*d)^2 + 2*cos(b*x + 2*d)*sin(d) + sin(d)^2)/(cos(b*x + 2*d)
^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2
*d)*sin(d) + sin(d)^2)) + 4*e^(2*a)*sin(b*x - d))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int e^{2(a+ibx)}\sec(d+bx)\,dx$$
$$= -\frac{e^{(2a-2id)}\log\left(ie^{(ibx+id)}-1\right) - e^{(2a-2id)}\log\left(-ie^{(ibx+id)}-1\right) + 2ie^{(ibx+2a-id)}}{b}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d),x, algorithm="giac")
```

output
```
-(e^(2*a - 2*I*d)*log(I*e^(I*b*x + I*d) - 1) - e^(2*a - 2*I*d)*log(-I*e^(I
*b*x + I*d) - 1) + 2*I*e^(I*b*x + 2*a - I*d))/b
```

Mupad [B] (verification not implemented)

Time = 18.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.34

$$\int e^{2(a+ibx)}\sec(d+bx)\,dx = -\frac{\mathrm{e}^{2\,a-d\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,2\mathrm{i}}{b} + \frac{\sqrt{\mathrm{e}^{4\,a-d\,4\mathrm{i}}}\,\ln\left(2\,\mathrm{e}^{4\,a}\,\mathrm{e}^{-d\,3\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}-\mathrm{e}^{2\,a}\,\mathrm{e}^{-d\,2\mathrm{i}}\,\sqrt{\mathrm{e}^{4\,a}\,\mathrm{e}^{-d\,4\mathrm{i}}}\,2\mathrm{i}\right)}{b} - \frac{\sqrt{\mathrm{e}^{4\,a-d\,4\mathrm{i}}}\,\ln\left(2\,\mathrm{e}^{4\,a}\,\mathrm{e}^{-d\,3\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}+\mathrm{e}^{2\,a}\,\mathrm{e}^{-d\,2\mathrm{i}}\,\sqrt{\mathrm{e}^{4\,a}\,\mathrm{e}^{-d\,4\mathrm{i}}}\,2\mathrm{i}\right)}{b}$$

input
```
int(exp(2*a + b*x*2i)/cos(d + b*x),x)
```

output
```
(exp(4*a - d*4i)^(1/2)*log(2*exp(4*a)*exp(-d*3i)*exp(b*x*1i) - exp(2*a)*ex
p(-d*2i)*(exp(4*a)*exp(-d*4i))^(1/2)*2i))/b - (exp(2*a - d*1i + b*x*1i)*2i
)/b - (exp(4*a - d*4i)^(1/2)*log(2*exp(4*a)*exp(-d*3i)*exp(b*x*1i) + exp(2
*a)*exp(-d*2i)*(exp(4*a)*exp(-d*4i))^(1/2)*2i))/b
```

Reduce [F]

$$\int e^{2(a+ibx)}\sec(d+bx)\,dx = \frac{e^{2a}\left(-3e^{2bix}\cos(bx+d)\sec(bx+d)\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^2 i+3e^{2bix}\cos(bx+d)\sec(bx+d)\,i+e^{2bix}\cos(bx+d\right.}{}$$

input
```
int(exp(2*a+2*I*b*x)*sec(b*x+d),x)
```

output

```
(e**(2*a)*( - 3*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d)*tan((b*x + d)/2)**2
*i + 3*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d)*i + e**(2*b*i*x)*cos(b*x + d
)*tan((b*x + d)/2)**2*i - 2*e**(2*b*i*x)*cos(b*x + d)*tan((b*x + d)/2) - 5
*e**(2*b*i*x)*cos(b*x + d)*i - 12*cos(b*x + d)*int(e**(2*b*i*x)/(tan((b*x
+ d)/2)**2 - 1),x)*tan((b*x + d)/2)**2*b + 12*cos(b*x + d)*int(e**(2*b*i*x
)/(tan((b*x + d)/2)**2 - 1),x)*b - e**(2*b*i*x)*sin(b*x + d)*tan((b*x + d)
/2)**2 + e**(2*b*i*x)*sin(b*x + d) + e**(2*b*i*x)*tan((b*x + d)/2)**2*i -
e**(2*b*i*x)*i))/(6*cos(b*x + d)*b*(tan((b*x + d)/2)**2 - 1))
```

3.81 $\int e^{2(a+ibx)} \sec^2(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int e^{2(a+ibx)} \sec^2(d+bx)\,dx = -\frac{2ie^{2a-2id}}{b\left(1+e^{2i(d+bx)}\right)} - \frac{2ie^{2a-2id}\log\left(1+e^{2i(d+bx)}\right)}{b}$$

output
```
-2*I*exp(2*a-2*I*d)/b/(1+exp(2*I*(b*x+d)))-2*I*exp(2*a-2*I*d)*ln(1+exp(2*I
*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int e^{2(a+ibx)} \sec^2(d+bx)\,dx = -\frac{2ie^{2a-2id}\left(-e^{2i(d+bx)} + \left(1+e^{2i(d+bx)}\right)\log\left(1+e^{2i(d+bx)}\right)\right)}{b\left(1+e^{2i(d+bx)}\right)}$$

input
```
Integrate[E^(2*(a + I*b*x))*Sec[d + b*x]^2,x]
```

output
```
((-2*I)*E^(2*a - (2*I)*d)*(-E^((2*I)*(d + b*x)) + (1 + E^((2*I)*(d + b*x))
)*Log[1 + E^((2*I)*(d + b*x))]))/(b*(1 + E^((2*I)*(d + b*x))))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.73, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\sec^2(bx+d)\,dx$$

$$\Big\downarrow \; 4951$$

$$\frac{2ie^{2(a+ibx)+4i(bx+d)}\left(e^{-4i(bx+d)}\left(1+e^{2i(bx+d)}\right)^2-e^{-6i(bx+d)}\left(1+e^{2i(bx+d)}\right)^3\log\left(1+e^{2i(bx+d)}\right)\right)}{b\left(1+e^{2i(bx+d)}\right)^3}$$

input
```
Int[E^(2*(a + I*b*x))*Sec[d + b*x]^2,x]
```

output
```
((2*I)*E^(2*(a + I*b*x) + (4*I)*(d + b*x))*((1 + E^((2*I)*(d + b*x)))^2/E^
((4*I)*(d + b*x)) - ((1 + E^((2*I)*(d + b*x)))^3*Log[1 + E^((2*I)*(d + b*x
))])/E^((6*I)*(d + b*x))))/(b*(1 + E^((2*I)*(d + b*x)))^3)
```

Defintions of rubi rules used

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{2i\mathrm{e}^{2a}\mathrm{e}^{2ibx}}{b(1+\mathrm{e}^{2i(bx+d)})}-\frac{2i\mathrm{e}^{2a}\mathrm{e}^{-2id}\ln(1+\mathrm{e}^{2i(bx+d)})}{b}$	58

input
```
int(exp(2*a+2*I*b*x)*sec(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
2*I*exp(2*a)*exp(2*I*b*x)/b/(1+exp(2*I*(b*x+d)))-2*I*exp(2*a)*exp(-2*I*d)*
ln(1+exp(2*I*(b*x+d)))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int e^{2(a+ibx)}\sec^2(d+bx)\,dx = -\frac{2\left(\left(i\,e^{(2i\,bx+2\,a)}+i\,e^{(2\,a-2i\,d)}\right)\log\left(e^{(2i\,bx+2i\,d)}+1\right)+i\,e^{(2\,a-2i\,d)}\right)}{be^{(2i\,bx+2i\,d)}+b}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)^2,x, algorithm="fricas")
```

output
```
-2*((I*e^(2*I*b*x + 2*a) + I*e^(2*a - 2*I*d))*log(e^(2*I*b*x + 2*I*d) + 1)
 + I*e^(2*a - 2*I*d))/(b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [F]

$$\int e^{2(a+ibx)}\sec^2(d+bx)\,dx = e^{2a}\int e^{2ibx}\sec^2(bx+d)\,dx$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*I*b*x)*sec(b*x + d)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(52) = 104$.

Time = 0.04 (sec) , antiderivative size = 293, normalized size of antiderivative = 4.44

$$\int e^{2(a+ibx)}\sec^2(d+bx)\,dx$$
$$= \frac{2\left(-i\cos(2d)^2e^{(2a)} - ie^{(2a)}\sin(2d)^2 + \left(-i\cos(2d)e^{(2a)} - e^{(2a)}\sin(2d)\right)\cos(2bx+4d) + \left(\cos(2d)\right.\right.}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)^2,x, algorithm="maxima")
```

output
```
(2*(-I*cos(2*d)^2*e^(2*a) - I*e^(2*a)*sin(2*d)^2 + (-I*cos(2*d)*e^(2*a) -
e^(2*a)*sin(2*d))*cos(2*b*x + 4*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d
))*sin(2*b*x + 4*d))*arctan2(sin(2*b*x) - sin(2*d), cos(2*b*x) + cos(2*d))
 - (cos(2*d)^2*e^(2*a) + e^(2*a)*sin(2*d)^2 + (cos(2*d)*e^(2*a) - I*e^(2*a
)*sin(2*d))*cos(2*b*x + 4*d) + (I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*sin
(2*b*x + 4*d))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*d) + cos(2*d)^2 + sin
(2*b*x)^2 - 2*sin(2*b*x)*sin(2*d) + sin(2*d)^2) - 2*e^(2*a))/(-I*b*cos(2*b
*x + 4*d) - I*b*cos(2*d) + b*sin(2*b*x + 4*d) + b*sin(2*d))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int e^{2(a+ibx)}\sec^2(d+bx)\,dx$$
$$= -\frac{2\left(ie^{(2i\,bx+2\,a)}\log\left(e^{(2i\,bx+2i\,d)}+1\right) + ie^{(2\,a-2i\,d)}\log\left(e^{(2i\,bx+2i\,d)}+1\right) + ie^{(2\,a-2i\,d)}\right)}{b(e^{(2i\,bx+2i\,d)}+1)}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)^2,x, algorithm="giac")
```

output
```
-2*(I*e^(2*I*b*x + 2*a)*log(e^(2*I*b*x + 2*I*d) + 1) + I*e^(2*a - 2*I*d)*l
og(e^(2*I*b*x + 2*I*d) + 1) + I*e^(2*a - 2*I*d))/(b*(e^(2*I*b*x + 2*I*d) +
 1))
```

Mupad [B] (verification not implemented)

Time = 16.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int e^{2(a+ibx)}\sec^2(d+bx)\,dx = -\frac{\mathrm{e}^{2\,a-d\,2\mathrm{i}}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+\mathrm{e}^{2\,a}\,\mathrm{e}^{-d\,2\mathrm{i}}\right)\,2\mathrm{i}}{b} - \frac{\mathrm{e}^{4\,a-d\,4\mathrm{i}}\,2\mathrm{i}}{b\,\left(\mathrm{e}^{2\,a-d\,2\mathrm{i}}+\mathrm{e}^{2\,a+b\,x\,2\mathrm{i}}\right)}$$

input
```
int(exp(2*a + b*x*2i)/cos(d + b*x)^2,x)
```

output
```
- (exp(2*a - d*2i)*log(exp(2*a)*exp(b*x*2i) + exp(2*a)*exp(-d*2i))*2i)/b -
 (exp(4*a - d*4i)*2i)/(b*(exp(2*a - d*2i) + exp(2*a + b*x*2i)))
```

Reduce [F]

$$\int e^{2(a+ibx)}\sec^2(d+bx)\,dx = \text{too large to display}$$

input
```
int(exp(2*a+2*I*b*x)*sec(b*x+d)^2,x)
```

output
```
(e**(2*a)*( - 42*e**(2*b*i*x)*cos(b*x + d)*sin(b*x + d)*tan((b*x + d)/2)**
4 + 84*e**(2*b*i*x)*cos(b*x + d)*sin(b*x + d)*tan((b*x + d)/2)**2 - 42*e**
(2*b*i*x)*cos(b*x + d)*sin(b*x + d) - 392*e**(2*b*i*x)*cos(b*x + d)*tan((b
*x + d)/2)**4*i + 784*e**(2*b*i*x)*cos(b*x + d)*tan((b*x + d)/2)**2*i - 39
2*e**(2*b*i*x)*cos(b*x + d)*i - 315*e**(2*b*i*x)*sec(b*x + d)**2*sin(b*x +
 d)**2*tan((b*x + d)/2)**4*i + 630*e**(2*b*i*x)*sec(b*x + d)**2*sin(b*x +
d)**2*tan((b*x + d)/2)**2*i - 315*e**(2*b*i*x)*sec(b*x + d)**2*sin(b*x + d
)**2*i + 315*e**(2*b*i*x)*sec(b*x + d)**2*tan((b*x + d)/2)**4*i - 630*e**(
2*b*i*x)*sec(b*x + d)**2*tan((b*x + d)/2)**2*i + 315*e**(2*b*i*x)*sec(b*x
+ d)**2*i + 80*e**(2*b*i*x)*sin(b*x + d)**2*tan((b*x + d)/2)**4*i + 128*e*
*(2*b*i*x)*sin(b*x + d)**2*tan((b*x + d)/2)**3 + 96*e**(2*b*i*x)*sin(b*x +
 d)**2*tan((b*x + d)/2)**2*i - 640*e**(2*b*i*x)*sin(b*x + d)**2*tan((b*x +
 d)/2) + 556*e**(2*b*i*x)*sin(b*x + d)**2*i + 112*e**(2*b*i*x)*sin(b*x + d
)*tan((b*x + d)/2)**4 - 224*e**(2*b*i*x)*sin(b*x + d)*tan((b*x + d)/2)**2
+ 112*e**(2*b*i*x)*sin(b*x + d) - 437*e**(2*b*i*x)*tan((b*x + d)/2)**4*i -
 128*e**(2*b*i*x)*tan((b*x + d)/2)**3 + 618*e**(2*b*i*x)*tan((b*x + d)/2)*
*2*i + 640*e**(2*b*i*x)*tan((b*x + d)/2) - 913*e**(2*b*i*x)*i - 3000*int(e
**(2*b*i*x)/(tan((b*x + d)/2)**6 - 3*tan((b*x + d)/2)**4 + 3*tan((b*x + d)
/2)**2 - 1),x)*sin(b*x + d)**2*tan((b*x + d)/2)**4*b + 6000*int(e**(2*b*i*
x)/(tan((b*x + d)/2)**6 - 3*tan((b*x + d)/2)**4 + 3*tan((b*x + d)/2)**2...
```

3.82 $\int e^{2(a+ibx)}\sec^3(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 117

$$\int e^{2(a+ibx)}\sec^3(d+bx)\,dx=\frac{2ie^{2(a-id)+3i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)^2}+\frac{3ie^{2(a-id)+i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)}-\frac{3ie^{2a-2id}\arctan\left(e^{i(d+bx)}\right)}{b}$$

output
```
2*I*exp(2*a-2*I*d+3*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^2+3*I*exp(2*a-2*I*d+
I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))-3*I*exp(2*a-2*I*d)*arctan(exp(I*(b*x+d))
)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int e^{2(a+ibx)}\sec^3(d+bx)\,dx=\frac{e^{2a-2id}\left(\frac{2ie^{i(d+bx)}\left(3+5e^{2i(d+bx)}\right)}{\left(1+e^{2i(d+bx)}\right)^2}-6i\arctan\left(e^{i(d+bx)}\right)\right)}{2b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Sec[d + b*x]^3,x]
```

output
```
(E^(2*a - (2*I)*d)*(((2*I)*E^(I*(d + b*x))*(3 + 5*E^((2*I)*(d + b*x))))/(1
 + E^((2*I)*(d + b*x)))^2 - (6*I)*ArcTan[E^(I*(d + b*x))]))/(2*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4948, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\sec^3(bx+d)\,dx$$

$$\Big\downarrow \text{4948}$$

$$-\frac{3}{2}\int e^{2(a+ibx)}\sec(d+bx)dx-\frac{ie^{2(a+ibx)}\sec(bx+d)}{b}+\frac{e^{2(a+ibx)}\tan(bx+d)\sec(bx+d)}{2b}$$

$$\Big\downarrow \text{4951}$$

$$\frac{3ie^{2(a+ibx)-3i(bx+d)}\left(e^{2i(bx+d)}-e^{i(bx+d)}\arctan\left(e^{i(bx+d)}\right)\right)}{b}-\frac{ie^{2(a+ibx)}\sec(bx+d)}{b}+\frac{e^{2(a+ibx)}\tan(bx+d)\sec(bx+d)}{2b}$$

input
```
Int[E^(2*(a + I*b*x))*Sec[d + b*x]^3,x]
```

output
```
((3*I)*E^(2*(a + I*b*x) - (3*I)*(d + b*x))*(E^((2*I)*(d + b*x)) - E^(I*(d
+ b*x))*ArcTan[E^(I*(d + b*x))]))/b - (I*E^(2*(a + I*b*x))*Sec[d + b*x])/b
 + (E^(2*(a + I*b*x))*Sec[d + b*x]*Tan[d + b*x])/(2*b)
```

Defintions of rubi rules used

rule 4948
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(
e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2))   Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b,
 c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && N
eQ[n, 2]
```

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

method	result	size
risch	$-\frac{i\left(3\,\mathrm{e}^{5ibx}\mathrm{e}^{3id}\mathrm{e}^{2a}+\mathrm{e}^{3ibx}\mathrm{e}^{id}\mathrm{e}^{2a}\right)}{\left(1+\mathrm{e}^{2i(bx+d)}\right)^{2}b}+\frac{3i\mathrm{e}^{2a}\mathrm{e}^{-id}\mathrm{e}^{ibx}}{b}-\frac{3i\mathrm{e}^{2a}\mathrm{e}^{-2id}\arctan\left(\mathrm{e}^{i(bx+d)}\right)}{b}$	101

input
```
int(exp(2*a+2*I*b*x)*sec(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-I/(1+exp(2*I*(b*x+d)))^2/b*(3*exp(5*I*b*x)*exp(3*I*d)*exp(2*a)+exp(3*I*b*
x)*exp(I*d)*exp(2*a))+3*I/b*exp(2*a)*exp(I*b*x)*exp(-I*d)-3*I/b*exp(2*a)*e
xp(-2*I*d)*arctan(exp(I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.28

$$\int e^{2(a+ibx)}\sec^3(d+bx)\,dx$$
$$= \frac{3\left(e^{(4i\,bx+2\,a+2i\,d)}+2\,e^{(2i\,bx+2\,a)}+e^{(2\,a-2i\,d)}\right)\log\left(e^{(i\,bx+i\,d)}+i\right)-3\left(e^{(4i\,bx+2\,a+2i\,d)}+2\,e^{(2i\,bx+2\,a)}+e^{(2\,a-2i\,d}\right.}{2\left(be^{(4i\,bx+4i\,d)}+2\,be^{(2i\,bx+2i\,d)}+b\right)}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(3*(e^(4*I*b*x + 2*a + 2*I*d) + 2*e^(2*I*b*x + 2*a) + e^(2*a - 2*I*d))
*log(e^(I*b*x + I*d) + I) - 3*(e^(4*I*b*x + 2*a + 2*I*d) + 2*e^(2*I*b*x +
2*a) + e^(2*a - 2*I*d))*log(e^(I*b*x + I*d) - I) + 10*I*e^(3*I*b*x + 2*a +
 I*d) + 6*I*e^(I*b*x + 2*a - I*d))/(b*e^(4*I*b*x + 4*I*d) + 2*b*e^(2*I*b*x
 + 2*I*d) + b)
```

Sympy [F]

$$\int e^{2(a+ibx)}\sec^3(d+bx)\,dx = e^{2a}\int e^{2ibx}\sec^3\left(bx+d\right)dx$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*I*b*x)*sec(b*x + d)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 735 vs. 2(84) = 168.

Time = 0.16 (sec) , antiderivative size = 735, normalized size of antiderivative = 6.28

$$\int e^{2(a+ibx)}\sec^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)^3,x, algorithm="maxima")
```

output
```
(6*((cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(4*b*x + 5*d) + 2*(cos(2*d)
*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(2*b*x + 3*d) + (cos(d)*e^(2*a) + I*e^(2
*a)*sin(d))*cos(2*d) - (-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*sin(4*b*x
+ 5*d) - 2*(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*sin(2*b*x + 3*d) - (I*
cos(d)*e^(2*a) - e^(2*a)*sin(d))*sin(2*d))*arctan2(2*(cos(b*x + 2*d)*cos(d
) + sin(b*x + 2*d)*sin(d))/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x
 + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2), (cos(b*x
 + 2*d)^2 - cos(d)^2 + sin(b*x + 2*d)^2 - sin(d)^2)/(cos(b*x + 2*d)^2 + co
s(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin
(d) + sin(d)^2)) + 20*cos(3*b*x + 2*d)*e^(2*a) + 12*cos(b*x)*e^(2*a) - 3*(
(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*cos(4*b*x + 5*d) + 2*(-I*cos(2*d)
*e^(2*a) - e^(2*a)*sin(2*d))*cos(2*b*x + 3*d) + (-I*cos(d)*e^(2*a) + e^(2*
a)*sin(d))*cos(2*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(4*b*x +
5*d) + 2*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(2*b*x + 3*d) - (cos(d
)*e^(2*a) + I*e^(2*a)*sin(d))*sin(2*d))*log((cos(b*x + 2*d)^2 + cos(d)^2 -
 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 + 2*cos(b*x + 2*d)*sin(d) + si
n(d)^2)/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b*x +
 2*d)^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2)) + 20*I*e^(2*a)*sin(3*b*x +
2*d) + 12*I*e^(2*a)*sin(b*x))/(-4*I*b*cos(4*b*x + 5*d) - 8*I*b*cos(2*b*x +
 3*d) - 4*I*b*cos(d) + 4*b*sin(4*b*x + 5*d) + 8*b*sin(2*b*x + 3*d) + 4*...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(84) = 168.

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.81

$$\int e^{2(a+ibx)}\sec^3(d+bx)\,dx$$
$$= \frac{3\,e^{(4i\,bx+2\,a+2i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}-1\right)+6\,e^{(2i\,bx+2\,a)}\log\left(i\,e^{(i\,bx+i\,d)}-1\right)+3\,e^{(2\,a-2i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}-1\right)-3}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)^3,x, algorithm="giac")
```

output
```
1/2*(3*e^(4*I*b*x + 2*a + 2*I*d)*log(I*e^(I*b*x + I*d) - 1) + 6*e^(2*I*b*x
 + 2*a)*log(I*e^(I*b*x + I*d) - 1) + 3*e^(2*a - 2*I*d)*log(I*e^(I*b*x + I*
d) - 1) - 3*e^(4*I*b*x + 2*a + 2*I*d)*log(-I*e^(I*b*x + I*d) - 1) - 6*e^(2
*I*b*x + 2*a)*log(-I*e^(I*b*x + I*d) - 1) - 3*e^(2*a - 2*I*d)*log(-I*e^(I*
b*x + I*d) - 1) + 10*I*e^(3*I*b*x + 2*a + I*d) + 6*I*e^(I*b*x + 2*a - I*d)
)/(b*(e^(4*I*b*x + 4*I*d) + 2*e^(2*I*b*x + 2*I*d) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+ibx)}\sec^3(d+bx)\,dx=\int \frac{\mathrm{e}^{2\,a+b\,x\,2\mathrm{i}}}{{\cos\left(d+b\,x\right)}^3}\,dx$$

input
```
int(exp(2*a + b*x*2i)/cos(d + b*x)^3,x)
```

output
```
int(exp(2*a + b*x*2i)/cos(d + b*x)^3, x)
```

Reduce [F]

$$\int e^{2(a+ibx)}\sec^3(d+bx)\,dx=\text{too large to display}$$

input
```
int(exp(2*a+2*I*b*x)*sec(b*x+d)^3,x)
```

output
```
(e**(2*a)*( - 70*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d)**3*sin(b*x + d)**2
*tan((b*x + d)/2)**6*i + 210*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d)**3*sin
(b*x + d)**2*tan((b*x + d)/2)**4*i - 210*e**(2*b*i*x)*cos(b*x + d)*sec(b*x
 + d)**3*sin(b*x + d)**2*tan((b*x + d)/2)**2*i + 70*e**(2*b*i*x)*cos(b*x +
 d)*sec(b*x + d)**3*sin(b*x + d)**2*i + 70*e**(2*b*i*x)*cos(b*x + d)*sec(b
*x + d)**3*tan((b*x + d)/2)**6*i - 210*e**(2*b*i*x)*cos(b*x + d)*sec(b*x +
 d)**3*tan((b*x + d)/2)**4*i + 210*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d)*
*3*tan((b*x + d)/2)**2*i - 70*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d)**3*i
+ 6*e**(2*b*i*x)*cos(b*x + d)*sin(b*x + d)**2*tan((b*x + d)/2)**6*i + 6*e*
*(2*b*i*x)*cos(b*x + d)*sin(b*x + d)**2*tan((b*x + d)/2)**5 - 6*e**(2*b*i*
x)*cos(b*x + d)*sin(b*x + d)**2*tan((b*x + d)/2)**4*i - 36*e**(2*b*i*x)*co
s(b*x + d)*sin(b*x + d)**2*tan((b*x + d)/2)**3 - 42*e**(2*b*i*x)*cos(b*x +
 d)*sin(b*x + d)**2*tan((b*x + d)/2)**2*i + 102*e**(2*b*i*x)*cos(b*x + d)*
sin(b*x + d)**2*tan((b*x + d)/2) + 138*e**(2*b*i*x)*cos(b*x + d)*sin(b*x +
 d)**2*i - 52*e**(2*b*i*x)*cos(b*x + d)*sin(b*x + d)*tan((b*x + d)/2)**6 +
 156*e**(2*b*i*x)*cos(b*x + d)*sin(b*x + d)*tan((b*x + d)/2)**4 - 156*e**(
2*b*i*x)*cos(b*x + d)*sin(b*x + d)*tan((b*x + d)/2)**2 + 52*e**(2*b*i*x)*c
os(b*x + d)*sin(b*x + d) - 65*e**(2*b*i*x)*cos(b*x + d)*tan((b*x + d)/2)**
6*i - 6*e**(2*b*i*x)*cos(b*x + d)*tan((b*x + d)/2)**5 + 183*e**(2*b*i*x)*c
os(b*x + d)*tan((b*x + d)/2)**4*i + 36*e**(2*b*i*x)*cos(b*x + d)*tan((b...
```

3.83 $\int e^{2(a+ibx)} \sec^4(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int e^{2(a+ibx)} \sec^4(d+bx)\,dx = -\frac{8ie^{2(a-id)+6i(d+bx)}}{3b\left(1+e^{2i(d+bx)}\right)^3}$$

output
```
-8/3*I*exp(2*a-2*I*d+6*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int e^{2(a+ibx)} \sec^4(d+bx)\,dx = -\frac{8ie^{2a+4id+6ibx}}{3b\left(1+e^{2i(d+bx)}\right)^3}$$

input
```
Integrate[E^(2*(a + I*b*x))*Sec[d + b*x]^4,x]
```

output
```
(((-8*I)/3)*E^(2*a + (4*I)*d + (6*I)*b*x))/(b*(1 + E^((2*I)*(d + b*x)))^3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4946}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)} \sec^4(bx+d) \, dx$$

$$\Big\downarrow \text{4946}$$

$$\frac{e^{2(a+ibx)} \tan(bx+d) \sec^2(bx+d)}{3b} - \frac{ie^{2(a+ibx)} \sec^2(bx+d)}{3b}$$

input
```
Int[E^(2*(a + I*b*x))*Sec[d + b*x]^4,x]
```

output
```
((-1/3*I)*E^(2*(a + I*b*x))*Sec[d + b*x]^2)/b + (E^(2*(a + I*b*x))*Sec[d +
 b*x]^2*Tan[d + b*x])/(3*b)
```

Defintions of rubi rules used

rule 4946
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e
*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[b^2*c^2*Log[F]^2 +
 e^2*(n - 2)^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{8 i \mathrm{e}^{6 i b x +4 i d +2 a}}{3 \left(1+\mathrm{e}^{2 i \left(b x +d \right)}\right)^{3} b}$
parallelrisch	$-\frac{4 \,\mathrm{e}^{2 i b x +2 a} \left(i \cos \left(b x +d \right)-\sin \left(b x +d \right)\right)}{3 b \left(\cos \left(3 b x +3 d \right)+3 \cos \left(b x +d \right)\right)}$
norman	$\frac{-\frac{2 \,\mathrm{e}^{2 i b x +2 a} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)}{3 b}-\frac{4 \,\mathrm{e}^{2 i b x +2 a} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{3}}{3 b}-\frac{2 \,\mathrm{e}^{2 i b x +2 a} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{5}}{3 b}+\frac{i \mathrm{e}^{2 i b x +2 a}}{3 b}+\frac{i \mathrm{e}^{2 i b x +2 a} \tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{2}}{3 b}-\frac{i \mathrm{e}^{2 i b x +2 a} \mathrm{t}}{3}}{\left(\tan \left(\frac{b x}{2}+\frac{d}{2}\right)^{2}-1\right)^{3}}$

input
```
int(exp(2*a+2*I*b*x)*sec(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-8/3*I/(1+exp(2*I*(b*x+d)))^3/b*exp(6*I*b*x+4*I*d+2*a)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(30) = 60.

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.71

$$\int e^{2(a+ibx)} \sec^4(d+bx)\,dx = -\frac{8\left(-3i\,e^{(4i\,bx+2\,a+2i\,d)} - 3i\,e^{(2i\,bx+2\,a)} - i\,e^{(2\,a-2i\,d)}\right)}{3\left(be^{(6i\,bx+6i\,d)} + 3\,be^{(4i\,bx+4i\,d)} + 3\,be^{(2i\,bx+2i\,d)} + b\right)}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)^4,x, algorithm="fricas")
```

output
```
-8/3*(-3*I*e^(4*I*b*x + 2*a + 2*I*d) - 3*I*e^(2*I*b*x + 2*a) - I*e^(2*a -
2*I*d))/(b*e^(6*I*b*x + 6*I*d) + 3*b*e^(4*I*b*x + 4*I*d) + 3*b*e^(2*I*b*x
+ 2*I*d) + b)
```

Sympy [F]

$$\int e^{2(a+ibx)}\sec^4(d+bx)\,dx = e^{2a}\int e^{2ibx}\sec^4(bx+d)\,dx$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)**4,x)
```

output
```
exp(2*a)*Integral(exp(2*I*b*x)*sec(b*x + d)**4, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(30) = 60.

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.47

$$\int e^{2(a+ibx)}\sec^4(d+bx)\,dx$$
$$= \frac{8\left(3\cos\left(4\,bx+4\,d\right)e^{(2\,a)}+3\cos\left(2\,bx+2\,d\right)e^{(2\,a)}+3i\,e^{(2\,a)}\sin\left(4\,bx+4\,d\right)+3i\,\cdots\right.}{-3i\,b\cos\left(6\,bx+8\,d\right)-9i\,b\cos\left(4\,bx+6\,d\right)-9i\,b\cos\left(2\,bx+4\,d\right)-3i\,b\cos\left(2\,d\right)+3\,b\sin\left(6\,bx+8\,d\right)+9\cdots}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)^4,x, algorithm="maxima")
```

output
```
8*(3*cos(4*b*x + 4*d)*e^(2*a) + 3*cos(2*b*x + 2*d)*e^(2*a) + 3*I*e^(2*a)*s
in(4*b*x + 4*d) + 3*I*e^(2*a)*sin(2*b*x + 2*d) + e^(2*a))/(-3*I*b*cos(6*b*
x + 8*d) - 9*I*b*cos(4*b*x + 6*d) - 9*I*b*cos(2*b*x + 4*d) - 3*I*b*cos(2*d
) + 3*b*sin(6*b*x + 8*d) + 9*b*sin(4*b*x + 6*d) + 9*b*sin(2*b*x + 4*d) + 3
*b*sin(2*d))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int e^{2(a+ibx)}\sec^4(d+bx)\,dx = -\frac{8\left(-3i\,e^{(4i\,bx+2\,a+2i\,d)} - 3i\,e^{(2i\,bx+2\,a)} - i\,e^{(2\,a-2i\,d)}\right)}{3\,b(e^{(6i\,bx+6i\,d)} + 3\,e^{(4i\,bx+4i\,d)} + 3\,e^{(2i\,bx+2i\,d)} + 1)}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)^4,x, algorithm="giac")
```

output
```
-8/3*(-3*I*e^(4*I*b*x + 2*a + 2*I*d) - 3*I*e^(2*I*b*x + 2*a) - I*e^(2*a -
2*I*d))/(b*(e^(6*I*b*x + 6*I*d) + 3*e^(4*I*b*x + 4*I*d) + 3*e^(2*I*b*x + 2
*I*d) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+ibx)}\sec^4(d+bx)\,dx = \int \frac{e^{2\,a+b\,x\,2i}}{\cos\left(d+b\,x\right)^4}\,dx$$

input
```
int(exp(2*a + b*x*2i)/cos(d + b*x)^4,x)
```

output
```
int(exp(2*a + b*x*2i)/cos(d + b*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.53

$$\int e^{2(a+ibx)}\sec^4(d+bx)\,dx = \frac{e^{2bix+2a}\left(2\cos\left(bx+d\right)\sin\left(bx+d\right) - 3\sec\left(bx+d\right)^4\sin\left(bx+d\right)^4 i + 6\sec\left(bx+d\right)^4\sin\left(bx+d\right)^2 i - 3\sec\right.}{6b\left(\sin\left(bx+d\right)^4 - 2\sin\left(bx+d\right)^2 + 1\right)}$$

input
```
int(exp(2*a+2*I*b*x)*sec(b*x+d)^4,x)
```

output
```
(e**(2*a + 2*b*i*x)*(2*cos(b*x + d)*sin(b*x + d) - 3*sec(b*x + d)**4*sin(b
*x + d)**4*i + 6*sec(b*x + d)**4*sin(b*x + d)**2*i - 3*sec(b*x + d)**4*i +
 2*sin(b*x + d)**2*i + i))/(6*b*(sin(b*x + d)**4 - 2*sin(b*x + d)**2 + 1))
```

3.84 $\int e^{2(a+ibx)}\sec^5(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 211

$$\int e^{2(a+ibx)}\sec^5(d+bx)\,dx=\frac{4ie^{2(a-id)+5i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)^4}+\frac{10ie^{2(a-id)+3i(d+bx)}}{3b\left(1+e^{2i(d+bx)}\right)^3}+\frac{5ie^{2(a-id)+i(d+bx)}}{2b\left(1+e^{2i(d+bx)}\right)^2}-\frac{5ie^{2(a-id)+i(d+bx)}}{4b\left(1+e^{2i(d+bx)}\right)}-\frac{5ie^{2a-2id}\arctan\left(e^{i(d+bx)}\right)}{4b}$$

output
```
4*I*exp(2*a-2*I*d+5*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^4+10/3*I*exp(2*a-2*I
*d+3*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^3+5/2*I*exp(2*a-2*I*d+I*(b*x+d))/b/
(1+exp(2*I*(b*x+d)))^2-5/4*I*exp(2*a-2*I*d+I*(b*x+d))/b/(1+exp(2*I*(b*x+d)
))-5/4*I*exp(2*a-2*I*d)*arctan(exp(I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.50

$$\int e^{2(a+ibx)}\sec^5(d+bx)\,dx=\frac{e^{2a-2id}\left(-\frac{2ie^{i(d+bx)}\left(-15-55e^{2i(d+bx)}-73e^{4i(d+bx)}+15e^{6i(d+bx)}\right)}{\left(1+e^{2i(d+bx)}\right)^4}-30i\arctan\left(e^{i(d+bx)}\right)\right)}{24b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Sec[d + b*x]^5,x]
```

output
```
(E^(2*a - (2*I)*d)*(((-2*I)*E^(I*(d + b*x))*(-15 - 55*E^((2*I)*(d + b*x))
- 73*E^((4*I)*(d + b*x)) + 15*E^((6*I)*(d + b*x))))/(1 + E^((2*I)*(d + b*x
)))^4 - (30*I)*ArcTan[E^(I*(d + b*x))]))/(24*b)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4948, 4948, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)} \sec^5(bx+d) \, dx$$

$$\Big\downarrow \text{4948}$$

$$\frac{5}{12} \int e^{2(a+ibx)} \sec^3(d+bx)dx - \frac{ie^{2(a+ibx)} \sec^3(bx+d)}{6b} + \frac{e^{2(a+ibx)} \tan(bx+d) \sec^3(bx+d)}{4b}$$

$$\Big\downarrow \text{4948}$$

$$\frac{5}{12} \left(-\frac{3}{2} \int e^{2(a+ibx)} \sec(d+bx)dx - \frac{ie^{2(a+ibx)} \sec(bx+d)}{b} + \frac{e^{2(a+ibx)} \tan(bx+d) \sec(bx+d)}{2b}\right) - \frac{ie^{2(a+ibx)} \sec^3(bx+d)}{6b} + \frac{e^{2(a+ibx)} \tan(bx+d) \sec^3(bx+d)}{4b}$$

$$\Big\downarrow \text{4951}$$

$$\frac{5}{12} \left(\frac{3ie^{2(a+ibx)-3i(bx+d)} \left(e^{2i(bx+d)} - e^{i(bx+d)} \arctan\left(e^{i(bx+d)}\right)\right)}{b} - \frac{ie^{2(a+ibx)} \sec(bx+d)}{b} + \frac{e^{2(a+ibx)} \tan(bx+d) \text{se}}{2b} \right. - \frac{ie^{2(a+ibx)} \sec^3(bx+d)}{6b} + \frac{e^{2(a+ibx)} \tan(bx+d) \sec^3(bx+d)}{4b}$$

input
```
Int[E^(2*(a + I*b*x))*Sec[d + b*x]^5,x]
```

output
```
((-1/6*I)*E^(2*(a + I*b*x))*Sec[d + b*x]^3)/b + (E^(2*(a + I*b*x))*Sec[d +
 b*x]^3*Tan[d + b*x])/(4*b) + (5*(((3*I)*E^(2*(a + I*b*x) - (3*I)*(d + b*x
))*(E^((2*I)*(d + b*x)) - E^(I*(d + b*x))*ArcTan[E^(I*(d + b*x))]))/b - (I
*E^(2*(a + I*b*x))*Sec[d + b*x])/b + (E^(2*(a + I*b*x))*Sec[d + b*x]*Tan[d
 + b*x])/(2*b)))/12
```

Defintions of rubi rules used

rule 4948
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(
e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2))   Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b,
 c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && N
eQ[n, 2]
```

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 4.79 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{i\left(15\,\mathrm{e}^{9ibx}\mathrm{e}^{7id}\mathrm{e}^{2a}+75\,\mathrm{e}^{7ibx}\mathrm{e}^{5id}\mathrm{e}^{2a}+17\,\mathrm{e}^{5ibx}\mathrm{e}^{3id}\mathrm{e}^{2a}+5\,\mathrm{e}^{3ibx}\mathrm{e}^{id}\mathrm{e}^{2a}\right)}{12\left(1+\mathrm{e}^{2i(bx+d)}\right)^{4}b}-\frac{5i\mathrm{e}^{2a}\mathrm{e}^{-2id}\arctan\left(\mathrm{e}^{i(bx+d)}\right)}{4b}+\frac{5i\mathrm{e}^{2a}\mathrm{e}^{-id}\mathrm{e}^{ibx}}{4b}$	136

input
```
int(exp(2*a+2*I*b*x)*sec(b*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*I/(1+exp(2*I*(b*x+d)))^4/b*(15*exp(9*I*b*x)*exp(7*I*d)*exp(2*a)+75*e
xp(7*I*b*x)*exp(5*I*d)*exp(2*a)+17*exp(5*I*b*x)*exp(3*I*d)*exp(2*a)+5*exp(
3*I*b*x)*exp(I*d)*exp(2*a))-5/4*I/b*exp(2*a)*exp(-2*I*d)*arctan(exp(I*(b*x
+d)))+5/4*I/b*exp(2*a)*exp(I*b*x)*exp(-I*d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.22

$$\int e^{2(a+ibx)}\sec^5(d+bx)\,dx$$
$$= \frac{15\left(e^{(8i\,bx+2\,a+6i\,d)}+4\,e^{(6i\,bx+2\,a+4i\,d)}+6\,e^{(4i\,bx+2\,a+2i\,d)}+4\,e^{(2i\,bx+2\,a)}+e^{(2\,a-2i\,d)}\right)\log\left(e^{(i\,bx+i\,d)}+i\right)-15\,(}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)^5,x, algorithm="fricas")
```

output
```
1/24*(15*(e^(8*I*b*x + 2*a + 6*I*d) + 4*e^(6*I*b*x + 2*a + 4*I*d) + 6*e^(4
*I*b*x + 2*a + 2*I*d) + 4*e^(2*I*b*x + 2*a) + e^(2*a - 2*I*d))*log(e^(I*b*
x + I*d) + I) - 15*(e^(8*I*b*x + 2*a + 6*I*d) + 4*e^(6*I*b*x + 2*a + 4*I*d
) + 6*e^(4*I*b*x + 2*a + 2*I*d) + 4*e^(2*I*b*x + 2*a) + e^(2*a - 2*I*d))*l
og(e^(I*b*x + I*d) - I) - 30*I*e^(7*I*b*x + 2*a + 5*I*d) + 146*I*e^(5*I*b*
x + 2*a + 3*I*d) + 110*I*e^(3*I*b*x + 2*a + I*d) + 30*I*e^(I*b*x + 2*a - I
*d))/(b*e^(8*I*b*x + 8*I*d) + 4*b*e^(6*I*b*x + 6*I*d) + 6*b*e^(4*I*b*x + 4
*I*d) + 4*b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [F]

$$\int e^{2(a+ibx)}\sec^5(d+bx)\,dx = e^{2a}\int e^{2ibx}\sec^5(bx+d)\,dx$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)**5,x)
```

output
```
exp(2*a)*Integral(exp(2*I*b*x)*sec(b*x + d)**5, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1095 vs. $2(144) = 288$.

Time = 0.20 (sec) , antiderivative size = 1095, normalized size of antiderivative = 5.19

$$\int e^{2(a+ibx)} \sec^5(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)^5,x, algorithm="maxima")
```

output
```
(30*((cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(8*b*x + 9*d) + 4*(cos(2*d
)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(6*b*x + 7*d) + 6*(cos(2*d)*e^(2*a) - I
*e^(2*a)*sin(2*d))*cos(4*b*x + 5*d) + 4*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(
2*d))*cos(2*b*x + 3*d) + (cos(d)*e^(2*a) + I*e^(2*a)*sin(d))*cos(2*d) - (-
I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*sin(8*b*x + 9*d) - 4*(-I*cos(2*d)*e
^(2*a) - e^(2*a)*sin(2*d))*sin(6*b*x + 7*d) - 6*(-I*cos(2*d)*e^(2*a) - e^(
2*a)*sin(2*d))*sin(4*b*x + 5*d) - 4*(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d
))*sin(2*b*x + 3*d) - (I*cos(d)*e^(2*a) - e^(2*a)*sin(d))*sin(2*d))*arctan
2(2*(cos(b*x + 2*d)*cos(d) + sin(b*x + 2*d)*sin(d))/(cos(b*x + 2*d)^2 + co
s(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)^2 - 2*cos(b*x + 2*d)*sin
(d) + sin(d)^2), (cos(b*x + 2*d)^2 - cos(d)^2 + sin(b*x + 2*d)^2 - sin(d)^
2)/(cos(b*x + 2*d)^2 + cos(d)^2 + 2*cos(d)*sin(b*x + 2*d) + sin(b*x + 2*d)
^2 - 2*cos(b*x + 2*d)*sin(d) + sin(d)^2)) - 60*cos(7*b*x + 6*d)*e^(2*a) +
292*cos(5*b*x + 4*d)*e^(2*a) + 220*cos(3*b*x + 2*d)*e^(2*a) + 60*cos(b*x)*
e^(2*a) - 15*((-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*cos(8*b*x + 9*d) +
4*(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*cos(6*b*x + 7*d) + 6*(-I*cos(2*
d)*e^(2*a) - e^(2*a)*sin(2*d))*cos(4*b*x + 5*d) + 4*(-I*cos(2*d)*e^(2*a) -
 e^(2*a)*sin(2*d))*cos(2*b*x + 3*d) + (-I*cos(d)*e^(2*a) + e^(2*a)*sin(d))
*cos(2*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(8*b*x + 9*d) + 4*(
cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(6*b*x + 7*d) + 6*(cos(2*d)*e...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(144) = 288.

Time = 0.15 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.77

$$\int e^{2(a+ibx)}\sec^5(d+bx)\,dx =$$
$$-\frac{15\,e^{(8i\,bx+2\,a+6i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+1\right)+60\,e^{(6i\,bx+2\,a+4i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+1\right)+90\,e^{(4i\,bx+2\,a+2i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+1\right)}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*sec(b*x+d)^5,x, algorithm="giac")
```

output
```
-1/24*(15*e^(8*I*b*x + 2*a + 6*I*d)*log(I*e^(I*b*x + I*d) + 1) + 60*e^(6*I
*b*x + 2*a + 4*I*d)*log(I*e^(I*b*x + I*d) + 1) + 90*e^(4*I*b*x + 2*a + 2*I
*d)*log(I*e^(I*b*x + I*d) + 1) + 60*e^(2*I*b*x + 2*a)*log(I*e^(I*b*x + I*d
) + 1) + 15*e^(2*a - 2*I*d)*log(I*e^(I*b*x + I*d) + 1) - 15*e^(8*I*b*x + 2
*a + 6*I*d)*log(-I*e^(I*b*x + I*d) + 1) - 60*e^(6*I*b*x + 2*a + 4*I*d)*log
(-I*e^(I*b*x + I*d) + 1) - 90*e^(4*I*b*x + 2*a + 2*I*d)*log(-I*e^(I*b*x +
I*d) + 1) - 60*e^(2*I*b*x + 2*a)*log(-I*e^(I*b*x + I*d) + 1) - 15*e^(2*a -
 2*I*d)*log(-I*e^(I*b*x + I*d) + 1) + 30*I*e^(7*I*b*x + 2*a + 5*I*d) - 146
*I*e^(5*I*b*x + 2*a + 3*I*d) - 110*I*e^(3*I*b*x + 2*a + I*d) - 30*I*e^(I*b
*x + 2*a - I*d))/(b*(e^(8*I*b*x + 8*I*d) + 4*e^(6*I*b*x + 6*I*d) + 6*e^(4*
I*b*x + 4*I*d) + 4*e^(2*I*b*x + 2*I*d) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+ibx)}\sec^5(d+bx)\,dx = \int \frac{e^{2\,a+b\,x\,2i}}{\cos\left(d+b\,x\right)^5}\,dx$$

input
```
int(exp(2*a + b*x*2i)/cos(d + b*x)^5,x)
```

output
```
int(exp(2*a + b*x*2i)/cos(d + b*x)^5, x)
```

Reduce [F]

$$\int e^{2(a+ibx)}\sec^5(d+bx)\,dx = \text{too large to display}$$

input
```
int(exp(2*a+2*I*b*x)*sec(b*x+d)^5,x)
```

output
```
(e**(2*a)*( - 2079*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)*
*4*tan((b*x + d)/2)**10*i + 10395*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d)**
5*sin(b*x + d)**4*tan((b*x + d)/2)**8*i - 20790*e**(2*b*i*x)*cos(b*x + d)*
sec(b*x + d)**5*sin(b*x + d)**4*tan((b*x + d)/2)**6*i + 20790*e**(2*b*i*x)
*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**4*tan((b*x + d)/2)**4*i - 1039
5*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**4*tan((b*x + d)/
2)**2*i + 2079*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**4*i
 + 4158*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**2*tan((b*x
 + d)/2)**10*i - 20790*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d)**5*sin(b*x +
 d)**2*tan((b*x + d)/2)**8*i + 41580*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d
)**5*sin(b*x + d)**2*tan((b*x + d)/2)**6*i - 41580*e**(2*b*i*x)*cos(b*x +
d)*sec(b*x + d)**5*sin(b*x + d)**2*tan((b*x + d)/2)**4*i + 20790*e**(2*b*i
*x)*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**2*tan((b*x + d)/2)**2*i - 4
158*e**(2*b*i*x)*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**2*i - 2079*e**
(2*b*i*x)*cos(b*x + d)*sec(b*x + d)**5*tan((b*x + d)/2)**10*i + 10395*e**(
2*b*i*x)*cos(b*x + d)*sec(b*x + d)**5*tan((b*x + d)/2)**8*i - 20790*e**(2*
b*i*x)*cos(b*x + d)*sec(b*x + d)**5*tan((b*x + d)/2)**6*i + 20790*e**(2*b*
i*x)*cos(b*x + d)*sec(b*x + d)**5*tan((b*x + d)/2)**4*i - 10395*e**(2*b*i*
x)*cos(b*x + d)*sec(b*x + d)**5*tan((b*x + d)/2)**2*i + 2079*e**(2*b*i*x)*
cos(b*x + d)*sec(b*x + d)**5*i + 45*e**(2*b*i*x)*cos(b*x + d)*sin(b*x +...
```

3.85 $\int e^{\frac{5}{3}(a+ibx)}\sec(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 171

$$\int e^{\frac{5}{3}(a+ibx)}\sec(d+bx)\,dx = -\frac{3ie^{\frac{5}{3}(a-id)+\frac{2}{3}i(d+bx)}}{b} - \frac{i\sqrt{3}e^{\frac{5}{3}(a-id)}\arctan\left(\frac{1-2e^{\frac{2}{3}i(d+bx)}}{\sqrt{3}}\right)}{b} + \frac{ie^{\frac{5}{3}(a-id)}\log\left(1+e^{\frac{2}{3}i(d+bx)}\right)}{b} - \frac{ie^{\frac{5}{3}(a-id)}\log\left(1-e^{\frac{2}{3}i(d+bx)}+e^{\frac{4}{3}i(d+bx)}\right)}{2b}$$

output
```
-3*I*exp(5/3*a-5/3*I*d+2/3*I*(b*x+d))/b-I*3^(1/2)*exp(5/3*a-5/3*I*d)*arcta
n(1/3*(1-2*exp(2/3*I*(b*x+d)))*3^(1/2))/b+I*exp(5/3*a-5/3*I*d)*ln(1+exp(2/
3*I*(b*x+d)))/b-1/2*I*exp(5/3*a-5/3*I*d)*ln(1-exp(2/3*I*(b*x+d))+exp(4/3*I
*(b*x+d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.30

$$\int e^{\frac{5}{3}(a+ibx)} \sec(d+bx)\, dx = -\frac{3ie^{\frac{5a}{3}+id+\frac{8ibx}{3}} \text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},-e^{2i(d+bx)}\right)}{4b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Sec[d + b*x],x]
```

output
```
(((-3*I)/4)*E^((5*a)/3 + I*d + ((8*I)/3)*b*x)*Hypergeometric2F1[1, 4/3, 7/
3, -E^((2*I)*(d + b*x))])/b
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.32, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)} \sec(bx+d)\, dx$$

$$\Big\downarrow \text{ 4951}$$

$$-\frac{3ie^{\frac{5}{3}(a+ibx)+i(bx+d)} \text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},-e^{2i(d+bx)}\right)}{4b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Sec[d + b*x],x]
```

output
```
(((-3*I)/4)*E^((5*(a + I*b*x))/3 + I*(d + b*x))*Hypergeometric2F1[1, 4/3,
7/3, -E^((2*I)*(d + b*x))])/b
```

Defintions of rubi rules used

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int e^{\frac{5a}{3}+\frac{5ibx}{3}} \sec (bx+d)\, dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d),x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.25

$$\int e^{\frac{5}{3}(a+ibx)} \sec(d+bx)\, dx =$$
$$-\frac{\left(\sqrt{3}b\sqrt{\frac{1}{b^2}}e^{\left(\frac{5}{3}a-\frac{5}{3}id\right)}+i\,e^{\left(\frac{5}{3}a-\frac{5}{3}id\right)}\right)\log\left(\frac{1}{2}\left(i\sqrt{3}b\sqrt{\frac{1}{b^2}}e^{\left(\frac{5}{3}a-\frac{5}{3}id\right)}+2\,e^{\left(\frac{2}{3}ibx+\frac{5}{3}a-id\right)}-e^{\left(\frac{5}{3}a-\frac{5}{3}id\right)}\right)e^{\left(-\frac{5}{3}a+\right.}\right.}{}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d),x, algorithm="fricas")
```

output
```
-1/2*((sqrt(3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) + I*e^(5/3*a - 5/3*I*d))
*log(1/2*(I*sqrt(3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) + 2*e^(2/3*I*b*x +
5/3*a - I*d) - e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - (sqrt(3)*b*sqr
t(b^(-2))*e^(5/3*a - 5/3*I*d) - I*e^(5/3*a - 5/3*I*d))*log(1/2*(-I*sqrt(3)
*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) + 2*e^(2/3*I*b*x + 5/3*a - I*d) - e^(5
/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 2*I*e^(5/3*a - 5/3*I*d)*log(e^(2/
3*I*b*x + 2/3*I*d) + 1) + 6*I*e^(2/3*I*b*x + 5/3*a - I*d))/b
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+ibx)} \sec(d+bx)\,dx = e^{\frac{5a}{3}} \int e^{\frac{5ibx}{3}} \sec(bx+d)\,dx$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d),x)
```

output
```
exp(5*a/3)*Integral(exp(5*I*b*x/3)*sec(b*x + d), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6613 vs. $2(114) = 228$.

Time = 82.65 (sec) , antiderivative size = 6613, normalized size of antiderivative = 38.67

$$\int e^{\frac{5}{3}(a+ibx)} \sec(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d),x, algorithm="maxima")
```

output
```
1/4*(2*((cos(d)*e^(5/3*a) - I*e^(5/3*a)*sin(d))*cos(1/3*arctan2(sin(2*d),
cos(2*d))) - (I*cos(d)*e^(5/3*a) + e^(5/3*a)*sin(d))*sin(1/3*arctan2(sin(2
*d), cos(2*d))))*arctan2(sqrt(3)*cos(1/6*arctan2(sin(2*d), cos(2*d)))*sin(
1/3*b*x) + sqrt(3)*cos(1/3*b*x)*sin(1/6*arctan2(sin(2*d), cos(2*d))) + cos
(1/3*arctan2(sin(2*d), cos(2*d)))*sin(2/3*b*x) + cos(2/3*b*x)*sin(1/3*arct
an2(sin(2*d), cos(2*d))), sqrt(3)*cos(1/3*b*x)*cos(1/6*arctan2(sin(2*d), c
os(2*d))) - sqrt(3)*sin(1/3*b*x)*sin(1/6*arctan2(sin(2*d), cos(2*d))) + co
s(2/3*b*x)*cos(1/3*arctan2(sin(2*d), cos(2*d))) - sin(2/3*b*x)*sin(1/3*arc
tan2(sin(2*d), cos(2*d))) + 1) + 2*((cos(d)*e^(5/3*a) - I*e^(5/3*a)*sin(d)
)*cos(1/3*arctan2(sin(2*d), cos(2*d))) - (I*cos(d)*e^(5/3*a) + e^(5/3*a)*s
in(d))*sin(1/3*arctan2(sin(2*d), cos(2*d))))*arctan2(-sqrt(3)*cos(1/6*arct
an2(sin(2*d), cos(2*d)))*sin(1/3*b*x) - sqrt(3)*cos(1/3*b*x)*sin(1/6*arcta
n2(sin(2*d), cos(2*d))) + cos(1/3*arctan2(sin(2*d), cos(2*d)))*sin(2/3*b*x
) + cos(2/3*b*x)*sin(1/3*arctan2(sin(2*d), cos(2*d))), -sqrt(3)*cos(1/3*b*
x)*cos(1/6*arctan2(sin(2*d), cos(2*d))) + sqrt(3)*sin(1/3*b*x)*sin(1/6*arc
tan2(sin(2*d), cos(2*d))) + cos(2/3*b*x)*cos(1/3*arctan2(sin(2*d), cos(2*d
))) - sin(2/3*b*x)*sin(1/3*arctan2(sin(2*d), cos(2*d))) + 1) - 4*((cos(d)*
e^(5/3*a) - I*e^(5/3*a)*sin(d))*cos(1/3*arctan2(sin(2*d), cos(2*d))) + (-I
*cos(d)*e^(5/3*a) - e^(5/3*a)*sin(d))*sin(1/3*arctan2(sin(2*d), cos(2*d)))
)*arctan2(cos(1/3*arctan2(sin(2*d), cos(2*d)))*sin(2/3*b*x) + cos(2/3*b...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2578 vs. 2(114) = 228.

Time = 1.69 (sec) , antiderivative size = 2578, normalized size of antiderivative = 15.08

$$\int e^{\frac{5}{3}(a+ibx)}\sec(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d),x, algorithm="giac")
```

output
```
1/2*I*(2*e^(-20/3*I*d)*log(e^(2/3*I*b*x) + e^(-2/3*I*d)) - 2*((sqrt(3)*cos
(d)^6 - 15*sqrt(3)*cos(d)^4*sin(d)^2 + 15*sqrt(3)*cos(d)^2*sin(d)^4 - sqrt
(3)*sin(d)^6)*(cos(d)^4 + 2*cos(d)^2*sin(d)^2 + sin(d)^4)^(1/3)*cos(-2/3*p
i*floor(d/pi - 2*floor(1/2*d/pi + 1/2) + 1/2) - 4/3*pi*floor(1/2*d/pi + 1/
2) + 2/3*d) + (cos(d)^6 - 15*cos(d)^4*sin(d)^2 + 15*cos(d)^2*sin(d)^4 - si
n(d)^6)*(cos(d)^4 + 2*cos(d)^2*sin(d)^2 + sin(d)^4)^(1/3)*sin(-2/3*pi*floo
r(d/pi - 2*floor(1/2*d/pi + 1/2) + 1/2) - 4/3*pi*floor(1/2*d/pi + 1/2) + 2
/3*d))*arctan(1/3*sqrt(3)*(2*e^(2/3*I*b*x) - e^(-2/3*I*d))*e^(2/3*I*d))/((
cos(d)^14 + 7*cos(d)^12*sin(d)^2 + 21*cos(d)^10*sin(d)^4 + 35*cos(d)^8*sin
(d)^6 + 35*cos(d)^6*sin(d)^8 + 21*cos(d)^4*sin(d)^10 + 7*cos(d)^2*sin(d)^1
2 + sin(d)^14)*cos(-2/3*pi*floor(d/pi - 2*floor(1/2*d/pi + 1/2) + 1/2) - 4
/3*pi*floor(1/2*d/pi + 1/2) + 2/3*d)^2 + (cos(d)^14 + 7*cos(d)^12*sin(d)^2
 + 21*cos(d)^10*sin(d)^4 + 35*cos(d)^8*sin(d)^6 + 35*cos(d)^6*sin(d)^8 + 2
1*cos(d)^4*sin(d)^10 + 7*cos(d)^2*sin(d)^12 + sin(d)^14)*sin(-2/3*pi*floor
(d/pi - 2*floor(1/2*d/pi + 1/2) + 1/2) - 4/3*pi*floor(1/2*d/pi + 1/2) + 2/
3*d)^2) + 4*I*((3*sqrt(3)*cos(d)^5*sin(d) - 10*sqrt(3)*cos(d)^3*sin(d)^3 +
 3*sqrt(3)*cos(d)*sin(d)^5)*(cos(d)^4 + 2*cos(d)^2*sin(d)^2 + sin(d)^4)^(1
/3)*cos(-2/3*pi*floor(d/pi - 2*floor(1/2*d/pi + 1/2) + 1/2) - 4/3*pi*floor
(1/2*d/pi + 1/2) + 2/3*d) + (3*cos(d)^5*sin(d) - 10*cos(d)^3*sin(d)^3 + 3*
cos(d)*sin(d)^5)*(cos(d)^4 + 2*cos(d)^2*sin(d)^2 + sin(d)^4)^(1/3)*sin(...
```

Mupad [B] (verification not implemented)

Time = 19.70 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.54

$$\int e^{\frac{5}{3}(a+ibx)}\sec(d+bx)\,dx = -\frac{e^{\frac{5a}{3}-d\,1i+\frac{b\,x\,2i}{3}}\,3i}{b} + \frac{(1i)^{1/3}\left(-e^{5a-d\,5i}\right)^{1/3}\ln\left(-(1i)^{1/3}\,e^{2a}\,e^{-d\,2i}\left(-e^{5a}\,e^{-d\,5i}\right)^{1/3}2i+2\,e^{3a}\,e^{\frac{2a}{3}}\,e^{-d\,3i}\,e^{\frac{b\,x\,2i}{3}}\right)}{b} + \frac{(1i)^{1/3}\left(-e^{5a-d\,5i}\right)^{1/3}\ln\left(2\,e^{3a}\,e^{\frac{2a}{3}}\,e^{-d\,3i}\,e^{\frac{b\,x\,2i}{3}}-(1i)^{1/3}\,e^{2a}\,e^{-d\,2i}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-e^{5a}\,e^{-d\,5i}\right)^{1/3}2i\right)\left(-\frac{1}{2}\right.}{b} - \frac{(1i)^{1/3}\left(-e^{5a-d\,5i}\right)^{1/3}\ln\left(2\,e^{3a}\,e^{\frac{2a}{3}}\,e^{-d\,3i}\,e^{\frac{b\,x\,2i}{3}}+(1i)^{1/3}\,e^{2a}\,e^{-d\,2i}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-e^{5a}\,e^{-d\,5i}\right)^{1/3}2i\right)\left(\frac{1}{2}+\right.}{b}$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)/cos(d + b*x),x)
```

output
```
(1i^(1/3)*(-exp(5*a - d*5i))^(1/3)*log(2*exp(3*a)*exp((2*a)/3)*exp(-d*3i)*
exp((b*x*2i)/3) - 1i^(1/3)*exp(2*a)*exp(-d*2i)*(-exp(5*a)*exp(-d*5i))^(1/3
)*2i))/b - (exp((5*a)/3 - d*1i + (b*x*2i)/3)*3i)/b + (1i^(1/3)*(-exp(5*a -
 d*5i))^(1/3)*log(2*exp(3*a)*exp((2*a)/3)*exp(-d*3i)*exp((b*x*2i)/3) - 1i^
(1/3)*exp(2*a)*exp(-d*2i)*((3^(1/2)*1i)/2 - 1/2)*(-exp(5*a)*exp(-d*5i))^(1
/3)*2i)*((3^(1/2)*1i)/2 - 1/2))/b - (1i^(1/3)*(-exp(5*a - d*5i))^(1/3)*log
(2*exp(3*a)*exp((2*a)/3)*exp(-d*3i)*exp((b*x*2i)/3) + 1i^(1/3)*exp(2*a)*ex
p(-d*2i)*((3^(1/2)*1i)/2 + 1/2)*(-exp(5*a)*exp(-d*5i))^(1/3)*2i)*((3^(1/2)
*1i)/2 + 1/2))/b
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\sec(d+bx)\,dx = \frac{-3e^{\frac{5bix}{3}+\frac{5a}{3}}\sec\left(bx+d\right)\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}i+3e^{\frac{5bix}{3}+\frac{5a}{3}}\sec\left(bx+d\right)i-6e^{\frac{5bix}{3}+\frac{5a}{3}}i-10\left(\int\frac{e^{\frac{5bix}{3}+\frac{5a}{3}}}{\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}-1}dx\right)\mathrm{t}}{5b\left(\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}-1\right)}$$

input
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d),x)
```

output
```
( - 3*e**((5*a + 5*b*i*x)/3)*sec(b*x + d)*tan((b*x + d)/2)**2*i + 3*e**((5
*a + 5*b*i*x)/3)*sec(b*x + d)*i - 6*e**((5*a + 5*b*i*x)/3)*i - 10*int(e**(
(5*a + 5*b*i*x)/3)/(tan((b*x + d)/2)**2 - 1),x)*tan((b*x + d)/2)**2*b + 10
*int(e**((5*a + 5*b*i*x)/3)/(tan((b*x + d)/2)**2 - 1),x)*b)/(5*b*(tan((b*x
 + d)/2)**2 - 1))
```

3.86 $\int e^{\frac{5}{3}(a+ibx)}\sec^2(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 234

$$\int e^{\frac{5}{3}(a+ibx)}\sec^2(d+bx)\,dx=\frac{2ie^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)}-\frac{10ie^{\frac{5}{3}(a-id)}\arctan\left(e^{\frac{1}{3}i(d+bx)}\right)}{3b}+\frac{5ie^{\frac{5}{3}(a-id)}\arctan\left(\sqrt{3}-2e^{\frac{1}{3}i(d+bx)}\right)}{3b}-\frac{5ie^{\frac{5}{3}(a-id)}\arctan\left(\sqrt{3}+2e^{\frac{1}{3}i(d+bx)}\right)}{3b}+\frac{5ie^{\frac{5}{3}(a-id)}\operatorname{arctanh}\left(\frac{\sqrt{3}e^{\frac{1}{3}i(d+bx)}}{1+e^{\frac{2}{3}i(d+bx)}}\right)}{\sqrt{3}b}$$

output
```
2*I*exp(5/3*a-5/3*I*d+5/3*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))-10/3*I*exp(5/3
*a-5/3*I*d)*arctan(exp(1/3*I*(b*x+d)))/b-5/3*I*exp(5/3*a-5/3*I*d)*arctan(-
3^(1/2)+2*exp(1/3*I*(b*x+d)))/b-5/3*I*exp(5/3*a-5/3*I*d)*arctan(3^(1/2)+2*
exp(1/3*I*(b*x+d)))/b+5/3*I*exp(5/3*a-5/3*I*d)*arctanh(3^(1/2)*exp(1/3*I*(
b*x+d))/(1+exp(2/3*I*(b*x+d))))*3^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.22

$$\int e^{\frac{5}{3}(a+ibx)}\sec^2(d+bx)\,dx=-\frac{12ie^{\frac{5a}{3}+2id+\frac{11ibx}{3}}\text{Hypergeometric2F1}\left(\frac{11}{6},2,\frac{17}{6},-e^{2i(d+bx)}\right)}{11b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Sec[d + b*x]^2,x]
```

output
```
(((-12*I)/11)*E^((5*a)/3 + (2*I)*d + ((11*I)/3)*b*x)*Hypergeometric2F1[11/
6, 2, 17/6, -E^((2*I)*(d + b*x))])/b
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.23, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\sec^2(bx+d)\,dx$$

$$\Big\downarrow \text{4951}$$

$$-\frac{12ie^{\frac{5}{3}(a+ibx)+2i(bx+d)}\text{Hypergeometric2F1}\left(\frac{11}{6},2,\frac{17}{6},-e^{2i(d+bx)}\right)}{11b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Sec[d + b*x]^2,x]
```

output
```
(((-12*I)/11)*E^((5*(a + I*b*x))/3 + (2*I)*(d + b*x))*Hypergeometric2F1[11
/6, 2, 17/6, -E^((2*I)*(d + b*x))])/b
```

Defintions of rubi rules used

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int \mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \sec (bx+d)^{2} dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^2,x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(154) = 308.

Time = 0.09 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.39

$$\int e^{\frac{5}{3}(a+ibx)} \sec^2(d+bx) \, dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/6*(5*(3*sqrt(1/3)*(b*e^(2*I*b*x + 2*I*d) + b)*sqrt(-e^(10/3*a - 10/3*I*d
)/b^2) - e^(2*I*b*x + 5/3*a + 1/3*I*d) - e^(5/3*a - 5/3*I*d))*log(-1/2*(3*
I*sqrt(1/3)*b*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) - 2*e^(1/3*I*b*x + 5/3*a -
4/3*I*d) + I*e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) + 5*(3*sqrt(1/3)*(
b*e^(2*I*b*x + 2*I*d) + b)*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) + e^(2*I*b*x +
 5/3*a + 1/3*I*d) + e^(5/3*a - 5/3*I*d))*log(-1/2*(3*I*sqrt(1/3)*b*sqrt(-e
^(10/3*a - 10/3*I*d)/b^2) - 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) - I*e^(5/3*a
 - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 5*(3*sqrt(1/3)*(b*e^(2*I*b*x + 2*I*d)
 + b)*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) + e^(2*I*b*x + 5/3*a + 1/3*I*d) + e
^(5/3*a - 5/3*I*d))*log(-1/2*(-3*I*sqrt(1/3)*b*sqrt(-e^(10/3*a - 10/3*I*d)
/b^2) - 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) + I*e^(5/3*a - 5/3*I*d))*e^(-5/3
*a + 5/3*I*d)) - 5*(3*sqrt(1/3)*(b*e^(2*I*b*x + 2*I*d) + b)*sqrt(-e^(10/3*
a - 10/3*I*d)/b^2) - e^(2*I*b*x + 5/3*a + 1/3*I*d) - e^(5/3*a - 5/3*I*d))*
log(-1/2*(-3*I*sqrt(1/3)*b*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) - 2*e^(1/3*I*b
*x + 5/3*a - 4/3*I*d) - I*e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) + 10*
(e^(2*I*b*x + 5/3*a + 1/3*I*d) + e^(5/3*a - 5/3*I*d))*log(e^(1/3*I*b*x + 1
/3*I*d) + I) - 10*(e^(2*I*b*x + 5/3*a + 1/3*I*d) + e^(5/3*a - 5/3*I*d))*lo
g(e^(1/3*I*b*x + 1/3*I*d) - I) + 12*I*e^(5/3*I*b*x + 5/3*a))/(b*e^(2*I*b*x
 + 2*I*d) + b)
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+ibx)}\sec^2(d+bx)\,dx=e^{\frac{5a}{3}}\int e^{\frac{5ibx}{3}}\sec^2{(bx+d)}\,dx$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)**2,x)
```

output
```
exp(5*a/3)*Integral(exp(5*I*b*x/3)*sec(b*x + d)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 9954 vs. 2(154) = 308.

Time = 82.28 (sec) , antiderivative size = 9954, normalized size of antiderivative = 42.54

$$\int e^{\frac{5}{3}(a+ibx)}\sec^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^2,x, algorithm="maxima")
```

output
```
-12*(10*(-I*sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - sqrt(
3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))) + (-I*sqrt(3)*cos(5/6*ar
ctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(s
in(2*d), cos(2*d))))*cos(2*b*x + 2*d) + (sqrt(3)*cos(5/6*arctan2(sin(2*d),
 cos(2*d)))*e^(5/3*a) - I*sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(
2*d))))*sin(2*b*x + 2*d))*arctan2(sqrt(3)*cos(1/6*arctan2(sin(2*d), cos(2*
d)))*sin(1/3*b*x) + sqrt(3)*cos(1/3*b*x)*sin(1/6*arctan2(sin(2*d), cos(2*d
))) + cos(1/3*arctan2(sin(2*d), cos(2*d)))*sin(2/3*b*x) + cos(2/3*b*x)*sin
(1/3*arctan2(sin(2*d), cos(2*d))), sqrt(3)*cos(1/3*b*x)*cos(1/6*arctan2(si
n(2*d), cos(2*d))) - sqrt(3)*sin(1/3*b*x)*sin(1/6*arctan2(sin(2*d), cos(2*
d))) + cos(2/3*b*x)*cos(1/3*arctan2(sin(2*d), cos(2*d))) - sin(2/3*b*x)*si
n(1/3*arctan2(sin(2*d), cos(2*d))) + 1) + 10*(I*sqrt(3)*cos(5/6*arctan2(si
n(2*d), cos(2*d)))*e^(5/3*a) + sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d),
 cos(2*d))) + (I*sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) +
sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))))*cos(2*b*x + 2*d) -
 (sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - I*sqrt(3)*e^(5/
3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))))*sin(2*b*x + 2*d))*arctan2(-sqrt
(3)*cos(1/6*arctan2(sin(2*d), cos(2*d)))*sin(1/3*b*x) - sqrt(3)*cos(1/3*b*
x)*sin(1/6*arctan2(sin(2*d), cos(2*d))) + cos(1/3*arctan2(sin(2*d), cos(2*
d)))*sin(2/3*b*x) + cos(2/3*b*x)*sin(1/3*arctan2(sin(2*d), cos(2*d))), ...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(154) = 308$.

Time = 0.31 (sec) , antiderivative size = 880, normalized size of antiderivative = 3.76

$$\int e^{\frac{5}{3}(a+ibx)} \sec^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^2,x, algorithm="giac")
```

output
```
1/6*I*(20*I*(-e^(I*d))^(1/3)*arctan((sqrt(3)*e^(-1/3*I*d) + 2*e^(1/3*I*b*x
))*e^(1/3*I*d))*cos(d)*sin(d)/(cos(d)^4*e^(2*I*d) + 2*cos(d)^2*e^(2*I*d)*s
in(d)^2 + e^(2*I*d)*sin(d)^4) + 20*I*(-e^(I*d))^(1/3)*arctan(-(sqrt(3)*e^(
-1/3*I*d) - 2*e^(1/3*I*b*x))*e^(1/3*I*d))*cos(d)*sin(d)/(cos(d)^4*e^(2*I*d
) + 2*cos(d)^2*e^(2*I*d)*sin(d)^2 + e^(2*I*d)*sin(d)^4) + 40*I*(-e^(I*d))^
(1/3)*arctan(e^(1/3*I*b*x + 1/3*I*d))*cos(d)*sin(d)/(cos(d)^4*e^(2*I*d) +
2*cos(d)^2*e^(2*I*d)*sin(d)^2 + e^(2*I*d)*sin(d)^4) - 30*I*(-e^(I*d))^(1/3
)*cos(d)*log(sqrt(3)*e^(1/3*I*b*x - 1/3*I*d) + e^(2/3*I*b*x) + e^(-2/3*I*d
))*sin(d)/(sqrt(3)*cos(d)^4*e^(2*I*d) + 2*sqrt(3)*cos(d)^2*e^(2*I*d)*sin(d
)^2 + sqrt(3)*e^(2*I*d)*sin(d)^4) + 30*I*(-e^(I*d))^(1/3)*cos(d)*log(-sqrt
(3)*e^(1/3*I*b*x - 1/3*I*d) + e^(2/3*I*b*x) + e^(-2/3*I*d))*sin(d)/(sqrt(3
)*cos(d)^4*e^(2*I*d) + 2*sqrt(3)*cos(d)^2*e^(2*I*d)*sin(d)^2 + sqrt(3)*e^(
2*I*d)*sin(d)^4) - 10*((-e^(I*d))^(1/3)*cos(d)^2 - (-e^(I*d))^(1/3)*sin(d)
^2)*arctan((sqrt(3)*e^(-1/3*I*d) + 2*e^(1/3*I*b*x))*e^(1/3*I*d))/(cos(d)^4
*e^(2*I*d) + 2*cos(d)^2*e^(2*I*d)*sin(d)^2 + e^(2*I*d)*sin(d)^4) - 10*((-e
^(I*d))^(1/3)*cos(d)^2 - (-e^(I*d))^(1/3)*sin(d)^2)*arctan(-(sqrt(3)*e^(-1
/3*I*d) - 2*e^(1/3*I*b*x))*e^(1/3*I*d))/(cos(d)^4*e^(2*I*d) + 2*cos(d)^2*e
^(2*I*d)*sin(d)^2 + e^(2*I*d)*sin(d)^4) - 20*((-e^(I*d))^(1/3)*cos(d)^2 -
(-e^(I*d))^(1/3)*sin(d)^2)*arctan(e^(1/3*I*b*x + 1/3*I*d))/(cos(d)^4*e^(2*
I*d) + 2*cos(d)^2*e^(2*I*d)*sin(d)^2 + e^(2*I*d)*sin(d)^4) + 15*((-e^(I...
```

Mupad [B] (verification not implemented)

Time = 20.83 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.07

$$\int e^{\frac{5}{3}(a+ibx)}\sec^2(d+bx)\,dx = \text{Too large to display}$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)/cos(d + b*x)^2,x)
```

output
```
(exp((11*a)/3 - d*2i + (b*x*5i)/3)*2i)/(b*(exp(2*a - d*2i) + exp(2*a + b*x
*2i))) + (5*exp(10*a - d*10i)^(1/6)*log((100*exp(6*a)*exp(-d*6i))/9 - (exp
(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*(exp(10*a)*exp(-d*10i))^(1/6)*10
0i)/9))/(3*b) - (5*exp(10*a - d*10i)^(1/6)*log((100*exp(6*a)*exp(-d*6i))/9
 + (exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*(exp(10*a)*exp(-d*10i))^(
1/6)*100i)/9))/(3*b) + (5*log((100*exp(6*a)*exp(-d*6i))/9 - (exp(a/3)*exp(
4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 - 1/2)*(exp(10*a)*exp(-d*1
0i))^(1/6)*100i)/9)*exp(10*a - d*10i)^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(3*b)
- (5*log((100*exp(6*a)*exp(-d*6i))/9 + (exp(a/3)*exp(4*a)*exp(-d*4i)*exp((
b*x*1i)/3)*((3^(1/2)*1i)/2 - 1/2)*(exp(10*a)*exp(-d*10i))^(1/6)*100i)/9)*e
xp(10*a - d*10i)^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(3*b) + (5*log((100*exp(6*a
)*exp(-d*6i))/9 - (exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*
1i)/2 + 1/2)*(exp(10*a)*exp(-d*10i))^(1/6)*100i)/9)*exp(10*a - d*10i)^(1/6
)*((3^(1/2)*1i)/2 + 1/2))/(3*b) - (5*log((100*exp(6*a)*exp(-d*6i))/9 + (ex
p(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 + 1/2)*(exp(10*
a)*exp(-d*10i))^(1/6)*100i)/9)*exp(10*a - d*10i)^(1/6)*((3^(1/2)*1i)/2 + 1
/2))/(3*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\sec^2(d+bx)\,dx = \text{too large to display}$$

input
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^2,x)
```

output
```
( - 60987150*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)*tan((b*x + d
)/2)**4 + 121974300*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)*tan((
b*x + d)/2)**2 - 60987150*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)
 - 337034250*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*tan((b*x + d)/2)**4*i + 6
74068500*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*tan((b*x + d)/2)**2*i - 33703
4250*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*i - 274656165*e**((5*a + 5*b*i*x)
/3)*sec(b*x + d)**2*sin(b*x + d)**2*tan((b*x + d)/2)**4*i + 549312330*e**(
(5*a + 5*b*i*x)/3)*sec(b*x + d)**2*sin(b*x + d)**2*tan((b*x + d)/2)**2*i -
 274656165*e**((5*a + 5*b*i*x)/3)*sec(b*x + d)**2*sin(b*x + d)**2*i + 2746
56165*e**((5*a + 5*b*i*x)/3)*sec(b*x + d)**2*tan((b*x + d)/2)**4*i - 54931
2330*e**((5*a + 5*b*i*x)/3)*sec(b*x + d)**2*tan((b*x + d)/2)**2*i + 274656
165*e**((5*a + 5*b*i*x)/3)*sec(b*x + d)**2*i + 50368500*e**((5*a + 5*b*i*x
)/3)*sin(b*x + d)**2*tan((b*x + d)/2)**4*i + 153090000*e**((5*a + 5*b*i*x)
/3)*sin(b*x + d)**2*tan((b*x + d)/2)**3 + 154413000*e**((5*a + 5*b*i*x)/3)
*sin(b*x + d)**2*tan((b*x + d)/2)**2*i - 640710000*e**((5*a + 5*b*i*x)/3)*
sin(b*x + d)**2*tan((b*x + d)/2) + 477974256*e**((5*a + 5*b*i*x)/3)*sin(b*
x + d)**2*i + 80246250*e**((5*a + 5*b*i*x)/3)*sin(b*x + d)*tan((b*x + d)/2
)**4 - 160492500*e**((5*a + 5*b*i*x)/3)*sin(b*x + d)*tan((b*x + d)/2)**2 +
 80246250*e**((5*a + 5*b*i*x)/3)*sin(b*x + d) - 375847290*e**((5*a + 5*b*i
*x)/3)*tan((b*x + d)/2)**4*i - 153090000*e**((5*a + 5*b*i*x)/3)*tan((b*...
```

3.87 $\int e^{\frac{5}{3}(a+ibx)} \sec^3(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 239

$$\int e^{\frac{5}{3}(a+ibx)} \sec^3(d+bx)\,dx = \frac{2ie^{\frac{5}{3}(a-id)+\frac{8}{3}i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)^2} + \frac{8ie^{\frac{5}{3}(a-id)+\frac{2}{3}i(d+bx)}}{3b\left(1+e^{2i(d+bx)}\right)} + \frac{8ie^{\frac{5}{3}(a-id)}\arctan\left(\frac{1-2e^{\frac{2}{3}i(d+bx)}}{\sqrt{3}}\right)}{3\sqrt{3}b} - \frac{8ie^{\frac{5}{3}(a-id)}\log\left(1+e^{\frac{2}{3}i(d+bx)}\right)}{9b} + \frac{4ie^{\frac{5}{3}(a-id)}\log\left(1-e^{\frac{2}{3}i(d+bx)}+e^{\frac{4}{3}i(d+bx)}\right)}{9b}$$

output
```
2*I*exp(5/3*a-5/3*I*d+8/3*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^2+8/3*I*exp(5/
3*a-5/3*I*d+2/3*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))+8/9*I*exp(5/3*a-5/3*I*d)
*arctan(1/3*(1-2*exp(2/3*I*(b*x+d)))*3^(1/2))*3^(1/2)/b-8/9*I*exp(5/3*a-5/
3*I*d)*ln(1+exp(2/3*I*(b*x+d)))/b+4/9*I*exp(5/3*a-5/3*I*d)*ln(1-exp(2/3*I*
(b*x+d))+exp(4/3*I*(b*x+d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.32

$$\int e^{\frac{5}{3}(a+ibx)}\sec^3(d+bx)\,dx$$
$$=\frac{e^{\frac{5}{3}(a+ibx)}\left(4ie^{i(d+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},-e^{2i(d+bx)}\right)+\sec(d+bx)(-5i+3\tan(d+bx))\right)}{6b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Sec[d + b*x]^3,x]
```

output
```
(E^((5*(a + I*b*x))/3)*((4*I)*E^(I*(d + b*x))*Hypergeometric2F1[1, 4/3, 7/
3, -E^((2*I)*(d + b*x))] + Sec[d + b*x]*(-5*I + 3*Tan[d + b*x])))/(6*b)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.49, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4948, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\sec^3(bx+d)\,dx$$
$$\Big\downarrow \text{ 4948}$$
$$-\frac{8}{9}\int e^{\frac{5}{3}(a+ibx)}\sec(d+bx)dx-\frac{5ie^{\frac{5}{3}(a+ibx)}\sec(bx+d)}{6b}+\frac{e^{\frac{5}{3}(a+ibx)}\tan(bx+d)\sec(bx+d)}{2b}$$
$$\Big\downarrow \text{ 4951}$$
$$\frac{2ie^{\frac{5}{3}(a+ibx)+i(bx+d)}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},-e^{2i(d+bx)}\right)}{3b}-\frac{5ie^{\frac{5}{3}(a+ibx)}\sec(bx+d)}{6b}+\frac{e^{\frac{5}{3}(a+ibx)}\tan(bx+d)\sec(bx+d)}{2b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Sec[d + b*x]^3,x]
```

output
```
(((2*I)/3)*E^((5*(a + I*b*x))/3 + I*(d + b*x))*Hypergeometric2F1[1, 4/3, 7
/3, -E^((2*I)*(d + b*x))])/b - (((5*I)/6)*E^((5*(a + I*b*x))/3)*Sec[d + b*
x])/b + (E^((5*(a + I*b*x))/3)*Sec[d + b*x]*Tan[d + b*x])/(2*b)
```

Defintions of rubi rules used

rule 4948
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(
e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2))   Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b,
 c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && N
eQ[n, 2]
```

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int \mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \sec (bx+d)^3 \, dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^3,x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(157) = 314.

Time = 0.08 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.61

$$\int e^{\frac{5}{3}(a+ibx)}\sec^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^3,x, algorithm="fricas")
```

output
```
-2/9*(2*(3*sqrt(1/3)*(b*e^(4*I*b*x + 4*I*d) + 2*b*e^(2*I*b*x + 2*I*d) + b)
*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) - I*e^(4*I*b*x + 5/3*a + 7/3*I*d) - 2*I*
e^(2*I*b*x + 5/3*a + 1/3*I*d) - I*e^(5/3*a - 5/3*I*d))*log(-1/2*(3*I*sqrt(
1/3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) - 2*e^(2/3*I*b*x + 5/3*a - I*d) +
e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 2*(3*sqrt(1/3)*(b*e^(4*I*b*x
+ 4*I*d) + 2*b*e^(2*I*b*x + 2*I*d) + b)*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) +
 I*e^(4*I*b*x + 5/3*a + 7/3*I*d) + 2*I*e^(2*I*b*x + 5/3*a + 1/3*I*d) + I*e
^(5/3*a - 5/3*I*d))*log(-1/2*(-3*I*sqrt(1/3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3
*I*d) - 2*e^(2/3*I*b*x + 5/3*a - I*d) + e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5
/3*I*d)) + 4*(I*e^(4*I*b*x + 5/3*a + 7/3*I*d) + 2*I*e^(2*I*b*x + 5/3*a + 1
/3*I*d) + I*e^(5/3*a - 5/3*I*d))*log(e^(2/3*I*b*x + 2/3*I*d) + 1) - 21*I*e
^(8/3*I*b*x + 5/3*a + I*d) - 12*I*e^(2/3*I*b*x + 5/3*a - I*d))/(b*e^(4*I*b
*x + 4*I*d) + 2*b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+ibx)}\sec^3(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5ibx}{3}}\sec^3\left(bx+d\right)dx$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)**3,x)
```

output
```
exp(5*a/3)*Integral(exp(5*I*b*x/3)*sec(b*x + d)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11528 vs. $2(157) = 314$.

Time = 82.52 (sec) , antiderivative size = 11528, normalized size of antiderivative = 48.23

$$\int e^{\frac{5}{3}(a+ibx)} \sec^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^3,x, algorithm="maxima")
```

output
```
18*(2*(((I*cos(d)*e^(5/3*a) + e^(5/3*a)*sin(d))*cos(1/3*arctan2(sin(2*d),
cos(2*d))) + (cos(d)*e^(5/3*a) - I*e^(5/3*a)*sin(d))*sin(1/3*arctan2(sin(2
*d), cos(2*d))))*cos(4*b*x + 5*d) + 2*((I*cos(d)*e^(5/3*a) + e^(5/3*a)*sin
(d))*cos(1/3*arctan2(sin(2*d), cos(2*d))) + (cos(d)*e^(5/3*a) - I*e^(5/3*a
)*sin(d))*sin(1/3*arctan2(sin(2*d), cos(2*d))))*cos(2*b*x + 3*d) + (I*cos(
d)^2*e^(5/3*a) + I*e^(5/3*a)*sin(d)^2)*cos(1/3*arctan2(sin(2*d), cos(2*d))
) - ((cos(d)*e^(5/3*a) - I*e^(5/3*a)*sin(d))*cos(1/3*arctan2(sin(2*d), cos
(2*d))) - (I*cos(d)*e^(5/3*a) + e^(5/3*a)*sin(d))*sin(1/3*arctan2(sin(2*d)
, cos(2*d))))*sin(4*b*x + 5*d) - 2*((cos(d)*e^(5/3*a) - I*e^(5/3*a)*sin(d)
)*cos(1/3*arctan2(sin(2*d), cos(2*d))) - (I*cos(d)*e^(5/3*a) + e^(5/3*a)*s
in(d))*sin(1/3*arctan2(sin(2*d), cos(2*d))))*sin(2*b*x + 3*d) + (cos(d)^2*
e^(5/3*a) + e^(5/3*a)*sin(d)^2)*sin(1/3*arctan2(sin(2*d), cos(2*d))))*arct
an2(sqrt(3)*cos(1/6*arctan2(sin(2*d), cos(2*d)))*sin(1/3*b*x) + sqrt(3)*co
s(1/3*b*x)*sin(1/6*arctan2(sin(2*d), cos(2*d))) + cos(1/3*arctan2(sin(2*d)
, cos(2*d)))*sin(2/3*b*x) + cos(2/3*b*x)*sin(1/3*arctan2(sin(2*d), cos(2*d
))), sqrt(3)*cos(1/3*b*x)*cos(1/6*arctan2(sin(2*d), cos(2*d))) - sqrt(3)*s
in(1/3*b*x)*sin(1/6*arctan2(sin(2*d), cos(2*d))) + cos(2/3*b*x)*cos(1/3*ar
ctan2(sin(2*d), cos(2*d))) - sin(2/3*b*x)*sin(1/3*arctan2(sin(2*d), cos(2*
d))) + 1) + 2*(((I*cos(d)*e^(5/3*a) + e^(5/3*a)*sin(d))*cos(1/3*arctan2(si
n(2*d), cos(2*d))) + (cos(d)*e^(5/3*a) - I*e^(5/3*a)*sin(d))*sin(1/3*ar...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(157) = 314$.

Time = 0.28 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.55

$$\int e^{\frac{5}{3}(a+ibx)}\sec^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^3,x, algorithm="giac")
```

output
```
-2/9*I*(4*e^(-14/3*I*d)*log(e^(2/3*I*b*x) + e^(-2/3*I*d)) + 12*(cos(d)^4*e
^(4/3*I*d) - 6*cos(d)^2*e^(4/3*I*d)*sin(d)^2 + e^(4/3*I*d)*sin(d)^4)*arcta
n(1/3*sqrt(3)*(2*e^(2/3*I*b*x) - e^(-2/3*I*d))*e^(2/3*I*d))/(sqrt(3)*cos(d
)^8*e^(2*I*d) + 4*sqrt(3)*cos(d)^6*e^(2*I*d)*sin(d)^2 + 6*sqrt(3)*cos(d)^4
*e^(2*I*d)*sin(d)^4 + 4*sqrt(3)*cos(d)^2*e^(2*I*d)*sin(d)^6 + sqrt(3)*e^(2
*I*d)*sin(d)^8) - 48*I*(cos(d)^3*e^(4/3*I*d)*sin(d) - cos(d)*e^(4/3*I*d)*s
in(d)^3)*arctan(1/3*sqrt(3)*(2*e^(2/3*I*b*x) - e^(-2/3*I*d))*e^(2/3*I*d))/
(sqrt(3)*cos(d)^8*e^(2*I*d) + 4*sqrt(3)*cos(d)^6*e^(2*I*d)*sin(d)^2 + 6*sq
rt(3)*cos(d)^4*e^(2*I*d)*sin(d)^4 + 4*sqrt(3)*cos(d)^2*e^(2*I*d)*sin(d)^6
+ sqrt(3)*e^(2*I*d)*sin(d)^8) + 8*I*(cos(d)^3*e^(4/3*I*d)*sin(d) - cos(d)*
e^(4/3*I*d)*sin(d)^3)*log(-e^(2/3*I*b*x - 2/3*I*d) + e^(4/3*I*b*x) + e^(-4
/3*I*d))/(cos(d)^8*e^(2*I*d) + 4*cos(d)^6*e^(2*I*d)*sin(d)^2 + 6*cos(d)^4*
e^(2*I*d)*sin(d)^4 + 4*cos(d)^2*e^(2*I*d)*sin(d)^6 + e^(2*I*d)*sin(d)^8) -
 2*(cos(d)^4 - 6*cos(d)^2*sin(d)^2 + sin(d)^4)*log(-e^(2/3*I*b*x - 2/3*I*d
) + e^(4/3*I*b*x) + e^(-4/3*I*d))/(cos(d)^8*e^(2/3*I*d) + 4*cos(d)^6*e^(2/
3*I*d)*sin(d)^2 + 6*cos(d)^4*e^(2/3*I*d)*sin(d)^4 + 4*cos(d)^2*e^(2/3*I*d)
*sin(d)^6 + e^(2/3*I*d)*sin(d)^8) - 3*(7*e^(8/3*I*b*x + 2*I*d) + 4*e^(2/3*
I*b*x))*e^(-4*I*d)/(e^(2*I*b*x + 2*I*d) + 1)^2)*e^(5/3*a + 3*I*d)/b - 4/3*
(-7*I*e^(8/3*I*b*x + 5/3*a + 2*I*d) - 4*I*e^(2/3*I*b*x + 5/3*a))/(b*(e^(4*
I*b*x + 5*I*d) + 2*e^(2*I*b*x + 3*I*d) + e^(I*d)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{3}(a+ibx)}\sec^3(d+bx)\,dx=\int \frac{\mathrm{e}^{\frac{5\,a}{3}+\frac{b\,x\,5\mathrm{i}}{3}}}{\cos\left(d+b\,x\right)^3}\,dx$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)/cos(d + b*x)^3,x)
```

output
```
int(exp((5*a)/3 + (b*x*5i)/3)/cos(d + b*x)^3, x)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\sec^3(d+bx)\,dx=\text{too large to display}$$

input
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^3,x)
```

output

```
( - 20337895914*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sec(b*x + d)**3*sin(b*
x + d)**2*tan((b*x + d)/2)**6*i + 61013687742*e**((5*a + 5*b*i*x)/3)*cos(b
*x + d)*sec(b*x + d)**3*sin(b*x + d)**2*tan((b*x + d)/2)**4*i - 6101368774
2*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sec(b*x + d)**3*sin(b*x + d)**2*tan(
(b*x + d)/2)**2*i + 20337895914*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sec(b*
x + d)**3*sin(b*x + d)**2*i + 20337895914*e**((5*a + 5*b*i*x)/3)*cos(b*x +
 d)*sec(b*x + d)**3*tan((b*x + d)/2)**6*i - 61013687742*e**((5*a + 5*b*i*x
)/3)*cos(b*x + d)*sec(b*x + d)**3*tan((b*x + d)/2)**4*i + 61013687742*e**(
(5*a + 5*b*i*x)/3)*cos(b*x + d)*sec(b*x + d)**3*tan((b*x + d)/2)**2*i - 20
337895914*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sec(b*x + d)**3*i + 73947427
2*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)**2*tan((b*x + d)/2)**6*
i + 1758348000*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)**2*tan((b*
x + d)/2)**5 + 712157184*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)*
*2*tan((b*x + d)/2)**4*i - 9117360000*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*
sin(b*x + d)**2*tan((b*x + d)/2)**3 - 11240537184*e**((5*a + 5*b*i*x)/3)*c
os(b*x + d)*sin(b*x + d)**2*tan((b*x + d)/2)**2*i + 22218861600*e**((5*a +
 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)**2*tan((b*x + d)/2) + 28174777728*e
**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)**2*i - 14640179355*e**((5*
a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)*tan((b*x + d)/2)**6 + 4392053806
5*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)*tan((b*x + d)/2)**4 ...
```

3.88 $\int e^{\frac{5}{3}(a+ibx)}\sec^4(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 336

$$\begin{aligned}\int e^{\frac{5}{3}(a+ibx)}\sec^4(d+bx)\,dx &= \frac{8ie^{\frac{5}{3}(a-id)+\frac{11}{3}i(d+bx)}}{3b\left(1+e^{2i(d+bx)}\right)^3}+\frac{22ie^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{9b\left(1+e^{2i(d+bx)}\right)^2}\\
&\quad-\frac{55ie^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{27b\left(1+e^{2i(d+bx)}\right)}-\frac{55e^{\frac{5}{3}(a-id)}\arctan\left(e^{\frac{1}{3}i(d+bx)}\right)}{81b}\\
&\quad+\frac{55ie^{\frac{5}{3}(a-id)}\arctan\left(\sqrt{3}-2e^{\frac{1}{3}i(d+bx)}\right)}{162b}\\
&\quad-\frac{55ie^{\frac{5}{3}(a-id)}\arctan\left(\sqrt{3}+2e^{\frac{1}{3}i(d+bx)}\right)}{162b}\\
&\quad+\frac{55ie^{\frac{5}{3}(a-id)}\operatorname{arctanh}\left(\frac{\sqrt{3}e^{\frac{1}{3}i(d+bx)}}{1+e^{\frac{2}{3}i(d+bx)}}\right)}{54\sqrt{3}b}\end{aligned}$$

output
```
8/3*I*exp(5/3*a-5/3*I*d+11/3*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^3+22/9*I*ex
p(5/3*a-5/3*I*d+5/3*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^2-55/27*I*exp(5/3*a-
5/3*I*d+5/3*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))-55/81*I*exp(5/3*a-5/3*I*d)*a
rctan(exp(1/3*I*(b*x+d)))/b-55/162*I*exp(5/3*a-5/3*I*d)*arctan(-3^(1/2)+2*
exp(1/3*I*(b*x+d)))/b-55/162*I*exp(5/3*a-5/3*I*d)*arctan(3^(1/2)+2*exp(1/3
*I*(b*x+d)))/b+55/162*I*exp(5/3*a-5/3*I*d)*arctanh(3^(1/2)*exp(1/3*I*(b*x+
d))/(1+exp(2/3*I*(b*x+d))))*3^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.24

$$\int e^{\frac{5}{3}(a+ibx)}\sec^4(d+bx)\,dx = \frac{e^{\frac{5}{3}(a+ibx)}\left(-4ie^{2i(d+bx)}\,\text{Hypergeometric2F1}\left(\frac{11}{6},2,\frac{17}{6},-e^{2i(d+bx)}\right)+\sec^2(d+bx)(-5i+6\tan(d+bx))\right)}{18b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Sec[d + b*x]^4,x]
```

output
```
(E^((5*(a + I*b*x))/3)*((-4*I)*E^((2*I)*(d + b*x))*Hypergeometric2F1[11/6,
 2, 17/6, -E^((2*I)*(d + b*x))] + Sec[d + b*x]^2*(-5*I + 6*Tan[d + b*x])))
/(18*b)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.36, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4948, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\sec^4(bx+d)\,dx$$

$$\Big\downarrow \text{4948}$$

$$\frac{11}{54}\int e^{\frac{5}{3}(a+ibx)}\sec^2(d+bx)dx-\frac{5ie^{\frac{5}{3}(a+ibx)}\sec^2(bx+d)}{18b}+\frac{e^{\frac{5}{3}(a+ibx)}\tan(bx+d)\sec^2(bx+d)}{3b}$$

$$\Big\downarrow \text{4951}$$

$$-\frac{2ie^{\frac{5}{3}(a+ibx)+2i(bx+d)}\text{Hypergeometric2F1}\left(\frac{11}{6},2,\frac{17}{6},-e^{2i(d+bx)}\right)}{9b}-\frac{5ie^{\frac{5}{3}(a+ibx)}\sec^2(bx+d)}{18b}+\frac{e^{\frac{5}{3}(a+ibx)}\tan(bx+d)\sec^2(bx+d)}{3b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Sec[d + b*x]^4,x]
```

output
```
(((-2*I)/9)*E^((5*(a + I*b*x))/3 + (2*I)*(d + b*x))*Hypergeometric2F1[11/6
, 2, 17/6, -E^((2*I)*(d + b*x))])/b - (((5*I)/18)*E^((5*(a + I*b*x))/3)*Se
c[d + b*x]^2)/b + (E^((5*(a + I*b*x))/3)*Sec[d + b*x]^2*Tan[d + b*x])/(3*b
)
```

Defintions of rubi rules used

rule 4948
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(
e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2))   Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b,
 c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && N
eQ[n, 2]
```

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int e^{\frac{5a}{3}+\frac{5ibx}{3}} \sec(bx+d)^4\, dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^4,x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^4,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 875 vs. 2(211) = 422.

Time = 0.11 (sec) , antiderivative size = 875, normalized size of antiderivative = 2.60

$$\int e^{\frac{5}{3}(a+ibx)}\sec^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^4,x, algorithm="fricas")
```

output
```
1/324*(55*(3*sqrt(1/3)*(b*e^(6*I*b*x + 6*I*d) + 3*b*e^(4*I*b*x + 4*I*d) +
3*b*e^(2*I*b*x + 2*I*d) + b)*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) - e^(6*I*b*x
 + 5/3*a + 13/3*I*d) - 3*e^(4*I*b*x + 5/3*a + 7/3*I*d) - 3*e^(2*I*b*x + 5/
3*a + 1/3*I*d) - e^(5/3*a - 5/3*I*d))*log(-1/2*(3*I*sqrt(1/3)*b*sqrt(-e^(1
0/3*a - 10/3*I*d)/b^2) - 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) + I*e^(5/3*a -
5/3*I*d))*e^(-5/3*a + 5/3*I*d)) + 55*(3*sqrt(1/3)*(b*e^(6*I*b*x + 6*I*d) +
 3*b*e^(4*I*b*x + 4*I*d) + 3*b*e^(2*I*b*x + 2*I*d) + b)*sqrt(-e^(10/3*a -
10/3*I*d)/b^2) + e^(6*I*b*x + 5/3*a + 13/3*I*d) + 3*e^(4*I*b*x + 5/3*a + 7
/3*I*d) + 3*e^(2*I*b*x + 5/3*a + 1/3*I*d) + e^(5/3*a - 5/3*I*d))*log(-1/2*
(3*I*sqrt(1/3)*b*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) - 2*e^(1/3*I*b*x + 5/3*a
 - 4/3*I*d) - I*e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 55*(3*sqrt(1/
3)*(b*e^(6*I*b*x + 6*I*d) + 3*b*e^(4*I*b*x + 4*I*d) + 3*b*e^(2*I*b*x + 2*I
*d) + b)*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) + e^(6*I*b*x + 5/3*a + 13/3*I*d)
 + 3*e^(4*I*b*x + 5/3*a + 7/3*I*d) + 3*e^(2*I*b*x + 5/3*a + 1/3*I*d) + e^(
5/3*a - 5/3*I*d))*log(-1/2*(-3*I*sqrt(1/3)*b*sqrt(-e^(10/3*a - 10/3*I*d)/b
^2) - 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) + I*e^(5/3*a - 5/3*I*d))*e^(-5/3*a
 + 5/3*I*d)) - 55*(3*sqrt(1/3)*(b*e^(6*I*b*x + 6*I*d) + 3*b*e^(4*I*b*x + 4
*I*d) + 3*b*e^(2*I*b*x + 2*I*d) + b)*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) - e^
(6*I*b*x + 5/3*a + 13/3*I*d) - 3*e^(4*I*b*x + 5/3*a + 7/3*I*d) - 3*e^(2*I*
b*x + 5/3*a + 1/3*I*d) - e^(5/3*a - 5/3*I*d))*log(-1/2*(-3*I*sqrt(1/3)*...
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+ibx)}\sec^4(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5ibx}{3}}\sec^4(bx+d)\,dx$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)**4,x)
```

output
```
exp(5*a/3)*Integral(exp(5*I*b*x/3)*sec(b*x + d)**4, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14770 vs. 2(211) = 422.

Time = 82.67 (sec) , antiderivative size = 14770, normalized size of antiderivative = 43.96

$$\int e^{\frac{5}{3}(a+ibx)}\sec^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^4,x, algorithm="maxima")
```

output
```
-648*(110*(-I*sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - sqr
t(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))) + (-I*sqrt(3)*cos(5/6*
arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - sqrt(3)*e^(5/3*a)*sin(5/6*arctan2
(sin(2*d), cos(2*d))))*cos(6*b*x + 6*d) + 3*(-I*sqrt(3)*cos(5/6*arctan2(si
n(2*d), cos(2*d)))*e^(5/3*a) - sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d),
 cos(2*d))))*cos(4*b*x + 4*d) + 3*(-I*sqrt(3)*cos(5/6*arctan2(sin(2*d), co
s(2*d)))*e^(5/3*a) - sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))
))*cos(2*b*x + 2*d) + (sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3
*a) - I*sqrt(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))))*sin(6*b*x
+ 6*d) + 3*(sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - I*sqr
t(3)*e^(5/3*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))))*sin(4*b*x + 4*d) + 3*
(sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) - I*sqrt(3)*e^(5/3
*a)*sin(5/6*arctan2(sin(2*d), cos(2*d))))*sin(2*b*x + 2*d))*arctan2(sqrt(3
)*cos(1/6*arctan2(sin(2*d), cos(2*d)))*sin(1/3*b*x) + sqrt(3)*cos(1/3*b*x)
*sin(1/6*arctan2(sin(2*d), cos(2*d))) + cos(1/3*arctan2(sin(2*d), cos(2*d)
))*sin(2/3*b*x) + cos(2/3*b*x)*sin(1/3*arctan2(sin(2*d), cos(2*d))), sqrt(
3)*cos(1/3*b*x)*cos(1/6*arctan2(sin(2*d), cos(2*d))) - sqrt(3)*sin(1/3*b*x
)*sin(1/6*arctan2(sin(2*d), cos(2*d))) + cos(2/3*b*x)*cos(1/3*arctan2(sin(
2*d), cos(2*d))) - sin(2/3*b*x)*sin(1/3*arctan2(sin(2*d), cos(2*d))) + 1)
+ 110*(I*sqrt(3)*cos(5/6*arctan2(sin(2*d), cos(2*d)))*e^(5/3*a) + sqrt(...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1418 vs. 2(211) = 422.

Time = 0.40 (sec) , antiderivative size = 1418, normalized size of antiderivative = 4.22

$$\int e^{\frac{5}{3}(a+ibx)}\sec^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^4,x, algorithm="giac")
```

output
```
-1/324*I*(110*((-e^(I*d))^(1/3)*cos(d)^4 - 6*(-e^(I*d))^(1/3)*cos(d)^2*sin
(d)^2 + (-e^(I*d))^(1/3)*sin(d)^4)*arctan((sqrt(3)*e^(-1/3*I*d) + 2*e^(1/3
*I*b*x))*e^(1/3*I*d))/(cos(d)^8*e^(2*I*d) + 4*cos(d)^6*e^(2*I*d)*sin(d)^2
+ 6*cos(d)^4*e^(2*I*d)*sin(d)^4 + 4*cos(d)^2*e^(2*I*d)*sin(d)^6 + e^(2*I*d
)*sin(d)^8) - 440*I*((-e^(I*d))^(1/3)*cos(d)^3*sin(d) - (-e^(I*d))^(1/3)*c
os(d)*sin(d)^3)*arctan((sqrt(3)*e^(-1/3*I*d) + 2*e^(1/3*I*b*x))*e^(1/3*I*d
))/(cos(d)^8*e^(2*I*d) + 4*cos(d)^6*e^(2*I*d)*sin(d)^2 + 6*cos(d)^4*e^(2*I
*d)*sin(d)^4 + 4*cos(d)^2*e^(2*I*d)*sin(d)^6 + e^(2*I*d)*sin(d)^8) + 110*(
(-e^(I*d))^(1/3)*cos(d)^4 - 6*(-e^(I*d))^(1/3)*cos(d)^2*sin(d)^2 + (-e^(I*
d))^(1/3)*sin(d)^4)*arctan(-(sqrt(3)*e^(-1/3*I*d) - 2*e^(1/3*I*b*x))*e^(1/
3*I*d))/(cos(d)^8*e^(2*I*d) + 4*cos(d)^6*e^(2*I*d)*sin(d)^2 + 6*cos(d)^4*e
^(2*I*d)*sin(d)^4 + 4*cos(d)^2*e^(2*I*d)*sin(d)^6 + e^(2*I*d)*sin(d)^8) -
440*I*((-e^(I*d))^(1/3)*cos(d)^3*sin(d) - (-e^(I*d))^(1/3)*cos(d)*sin(d)^3
)*arctan(-(sqrt(3)*e^(-1/3*I*d) - 2*e^(1/3*I*b*x))*e^(1/3*I*d))/(cos(d)^8*
e^(2*I*d) + 4*cos(d)^6*e^(2*I*d)*sin(d)^2 + 6*cos(d)^4*e^(2*I*d)*sin(d)^4
+ 4*cos(d)^2*e^(2*I*d)*sin(d)^6 + e^(2*I*d)*sin(d)^8) + 220*((-e^(I*d))^(1
/3)*cos(d)^4 - 6*(-e^(I*d))^(1/3)*cos(d)^2*sin(d)^2 + (-e^(I*d))^(1/3)*sin
(d)^4)*arctan(e^(1/3*I*b*x + 1/3*I*d))/(cos(d)^8*e^(2*I*d) + 4*cos(d)^6*e^
(2*I*d)*sin(d)^2 + 6*cos(d)^4*e^(2*I*d)*sin(d)^4 + 4*cos(d)^2*e^(2*I*d)*si
n(d)^6 + e^(2*I*d)*sin(d)^8) - 880*(I*(-e^(I*d))^(1/3)*cos(d)^3*sin(d) ...
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{3}(a+ibx)}\sec^4(d+bx)\,dx=\int \frac{\mathrm{e}^{\frac{5\,a}{3}+\frac{b\,x\,5\mathrm{i}}{3}}}{{\cos\left(d+b\,x\right)}^4}\,dx$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)/cos(d + b*x)^4,x)
```

output
```
int(exp((5*a)/3 + (b*x*5i)/3)/cos(d + b*x)^4, x)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)} \sec^4(d+bx)\,dx = \text{too large to display}$$

input
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^4,x)
```

output
```
(12571893556506900*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)**3*tan
((b*x + d)/2)**8 - 50287574226027600*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*s
in(b*x + d)**3*tan((b*x + d)/2)**6 + 75431361339041400*e**((5*a + 5*b*i*x)
/3)*cos(b*x + d)*sin(b*x + d)**3*tan((b*x + d)/2)**4 - 50287574226027600*e
**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)**3*tan((b*x + d)/2)**2 + 1
2571893556506900*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)**3 - 153
074995337979000*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)**2*tan((b
*x + d)/2)**8*i + 612299981351916000*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*s
in(b*x + d)**2*tan((b*x + d)/2)**6*i - 918449972027874000*e**((5*a + 5*b*i
*x)/3)*cos(b*x + d)*sin(b*x + d)**2*tan((b*x + d)/2)**4*i + 61229998135191
6000*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)**2*tan((b*x + d)/2)*
*2*i - 153074995337979000*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)
**2*i + 508084206288975900*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d
)*tan((b*x + d)/2)**8 - 2032336825155903600*e**((5*a + 5*b*i*x)/3)*cos(b*x
 + d)*sin(b*x + d)*tan((b*x + d)/2)**6 + 3048505237733855400*e**((5*a + 5*
b*i*x)/3)*cos(b*x + d)*sin(b*x + d)*tan((b*x + d)/2)**4 - 2032336825155903
600*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d)*tan((b*x + d)/2)**2 +
 508084206288975900*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sin(b*x + d) + 128
276132456884500*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*tan((b*x + d)/2)**8*i
- 513104529827538000*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*tan((b*x + d)/...
```

3.89 $\int e^{\frac{5}{3}(a+ibx)}\sec^5(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 337

$$\int e^{\frac{5}{3}(a+ibx)}\sec^5(d+bx)\,dx=\frac{4ie^{\frac{5}{3}(a-id)+\frac{14}{3}i(d+bx)}}{b\left(1+e^{2i(d+bx)}\right)^4}+\frac{28ie^{\frac{5}{3}(a-id)+\frac{8}{3}i(d+bx)}}{9b\left(1+e^{2i(d+bx)}\right)^3}+\frac{56ie^{\frac{5}{3}(a-id)+\frac{2}{3}i(d+bx)}}{27b\left(1+e^{2i(d+bx)}\right)^2}-\frac{56ie^{\frac{5}{3}(a-id)+\frac{2}{3}i(d+bx)}}{81b\left(1+e^{2i(d+bx)}\right)}+\frac{112ie^{\frac{5}{3}(a-id)}\arctan\left(\frac{1-2e^{\frac{2}{3}i(d+bx)}}{\sqrt{3}}\right)}{81\sqrt{3}b}-\frac{112ie^{\frac{5}{3}(a-id)}\log\left(1+e^{\frac{2}{3}i(d+bx)}\right)}{243b}+\frac{56ie^{\frac{5}{3}(a-id)}\log\left(1-e^{\frac{2}{3}i(d+bx)}+e^{\frac{4}{3}i(d+bx)}\right)}{243b}$$

output
```
4*I*exp(5/3*a-5/3*I*d+14/3*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^4+28/9*I*exp(
5/3*a-5/3*I*d+8/3*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^3+56/27*I*exp(5/3*a-5/
3*I*d+2/3*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))^2-56/81*I*exp(5/3*a-5/3*I*d+2/
3*I*(b*x+d))/b/(1+exp(2*I*(b*x+d)))+112/243*I*exp(5/3*a-5/3*I*d)*arctan(1/
3*(1-2*exp(2/3*I*(b*x+d)))*3^(1/2))*3^(1/2)/b-112/243*I*exp(5/3*a-5/3*I*d)
*ln(1+exp(2/3*I*(b*x+d)))/b+56/243*I*exp(5/3*a-5/3*I*d)*ln(1-exp(2/3*I*(b*
x+d))+exp(4/3*I*(b*x+d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.30

$$\int e^{\frac{5}{3}(a+ibx)}\sec^5(d+bx)\,dx$$
$$=\frac{e^{\frac{5}{3}(a+ibx)}\left(112ie^{i(d+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},-e^{2i(d+bx)}\right)+28\sec(d+bx)(-5i+3\tan(d+bx))+9\right.}{324b}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Sec[d + b*x]^5,x]
```

output
```
(E^((5*(a + I*b*x))/3)*((112*I)*E^(I*(d + b*x))*Hypergeometric2F1[1, 4/3,
7/3, -E^((2*I)*(d + b*x))] + 28*Sec[d + b*x]*(-5*I + 3*Tan[d + b*x]) + 9*S
ec[d + b*x]^3*(-5*I + 9*Tan[d + b*x])))/(324*b)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.56, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4948, 4948, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\sec^5(bx+d)\,dx$$
$$\Big\downarrow \text{ 4948}$$
$$\frac{14}{27}\int e^{\frac{5}{3}(a+ibx)}\sec^3(d+bx)dx-\frac{5ie^{\frac{5}{3}(a+ibx)}\sec^3(bx+d)}{36b}+\frac{e^{\frac{5}{3}(a+ibx)}\tan(bx+d)\sec^3(bx+d)}{4b}$$
$$\Big\downarrow \text{ 4948}$$

$$\frac{14}{27}\left(-\frac{8}{9}\int e^{\frac{5}{3}(a+ibx)}\sec(d+bx)dx-\frac{5ie^{\frac{5}{3}(a+ibx)}\sec(bx+d)}{6b}+\frac{e^{\frac{5}{3}(a+ibx)}\tan(bx+d)\sec(bx+d)}{2b}\right)-\frac{5ie^{\frac{5}{3}(a+ibx)}\sec^3(bx+d)}{36b}+\frac{e^{\frac{5}{3}(a+ibx)}\tan(bx+d)\sec^3(bx+d)}{4b}$$

$$\Big\downarrow \text{4951}$$

$$\frac{14}{27}\left(\frac{2ie^{\frac{5}{3}(a+ibx)+i(bx+d)}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},-e^{2i(d+bx)}\right)}{3b}-\frac{5ie^{\frac{5}{3}(a+ibx)}\sec(bx+d)}{6b}+\frac{e^{\frac{5}{3}(a+ibx)}\tan(bx+ \cdots}{2b}\right.$$
$$\frac{5ie^{\frac{5}{3}(a+ibx)}\sec^3(bx+d)}{36b}+\frac{e^{\frac{5}{3}(a+ibx)}\tan(bx+d)\sec^3(bx+d)}{4b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Sec[d + b*x]^5,x]
```

output
```
(((-5*I)/36)*E^((5*(a + I*b*x))/3)*Sec[d + b*x]^3)/b + (E^((5*(a + I*b*x))
/3)*Sec[d + b*x]^3*Tan[d + b*x])/(4*b) + (14*(((((2*I)/3)*E^((5*(a + I*b*x)
)/3 + I*(d + b*x))*Hypergeometric2F1[1, 4/3, 7/3, -E^((2*I)*(d + b*x))])/b
 - (((5*I)/6)*E^((5*(a + I*b*x))/3)*Sec[d + b*x])/b + (E^((5*(a + I*b*x))/
3)*Sec[d + b*x]*Tan[d + b*x])/(2*b)))/27
```

Defintions of rubi rules used

rule 4948
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(
e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2))   Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b,
 c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && N
eQ[n, 2]
```

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int e^{\frac{5a}{3}+\frac{5ibx}{3}} \sec (bx+d)^5 \, dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^5,x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^5,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 569 vs. 2(217) = 434.

Time = 0.09 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.69

$$\int e^{\frac{5}{3}(a+ibx)} \sec^5(d+bx) \, dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^5,x, algorithm="fricas")
```

output
```
-4/243*(14*(3*sqrt(1/3)*(b*e^(8*I*b*x + 8*I*d) + 4*b*e^(6*I*b*x + 6*I*d) +
 6*b*e^(4*I*b*x + 4*I*d) + 4*b*e^(2*I*b*x + 2*I*d) + b)*sqrt(b^(-2))*e^(5/
3*a - 5/3*I*d) - I*e^(8*I*b*x + 5/3*a + 19/3*I*d) - 4*I*e^(6*I*b*x + 5/3*a
 + 13/3*I*d) - 6*I*e^(4*I*b*x + 5/3*a + 7/3*I*d) - 4*I*e^(2*I*b*x + 5/3*a
+ 1/3*I*d) - I*e^(5/3*a - 5/3*I*d))*log(-1/2*(3*I*sqrt(1/3)*b*sqrt(b^(-2))
*e^(5/3*a - 5/3*I*d) - 2*e^(2/3*I*b*x + 5/3*a - I*d) + e^(5/3*a - 5/3*I*d)
)*e^(-5/3*a + 5/3*I*d)) - 14*(3*sqrt(1/3)*(b*e^(8*I*b*x + 8*I*d) + 4*b*e^(
6*I*b*x + 6*I*d) + 6*b*e^(4*I*b*x + 4*I*d) + 4*b*e^(2*I*b*x + 2*I*d) + b)*
sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) + I*e^(8*I*b*x + 5/3*a + 19/3*I*d) + 4*I*
e^(6*I*b*x + 5/3*a + 13/3*I*d) + 6*I*e^(4*I*b*x + 5/3*a + 7/3*I*d) + 4*I*e
^(2*I*b*x + 5/3*a + 1/3*I*d) + I*e^(5/3*a - 5/3*I*d))*log(-1/2*(-3*I*sqrt(
1/3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) - 2*e^(2/3*I*b*x + 5/3*a - I*d) +
e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) + 28*(I*e^(8*I*b*x + 5/3*a + 19
/3*I*d) + 4*I*e^(6*I*b*x + 5/3*a + 13/3*I*d) + 6*I*e^(4*I*b*x + 5/3*a + 7/
3*I*d) + 4*I*e^(2*I*b*x + 5/3*a + 1/3*I*d) + I*e^(5/3*a - 5/3*I*d))*log(e^
(2/3*I*b*x + 2/3*I*d) + 1) + 42*I*e^(20/3*I*b*x + 5/3*a + 5*I*d) - 432*I*e
^(14/3*I*b*x + 5/3*a + 3*I*d) - 315*I*e^(8/3*I*b*x + 5/3*a + I*d) - 84*I*e
^(2/3*I*b*x + 5/3*a - I*d))/(b*e^(8*I*b*x + 8*I*d) + 4*b*e^(6*I*b*x + 6*I*
d) + 6*b*e^(4*I*b*x + 4*I*d) + 4*b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+ibx)}\sec^5(d+bx)\,dx=e^{\frac{5a}{3}}\int e^{\frac{5ibx}{3}}\sec^5\left(bx+d\right)dx$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)**5,x)
```

output
```
exp(5*a/3)*Integral(exp(5*I*b*x/3)*sec(b*x + d)**5, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 16387 vs. 2(217) = 434.

Time = 87.29 (sec) , antiderivative size = 16387, normalized size of antiderivative = 48.63

$$\int e^{\frac{5}{3}(a+ibx)} \sec^5(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^5,x, algorithm="maxima")
```

output
```
972*(14*(((I*cos(d)*e^(5/3*a) + e^(5/3*a)*sin(d))*cos(1/3*arctan2(sin(2*d)
, cos(2*d))) + (cos(d)*e^(5/3*a) - I*e^(5/3*a)*sin(d))*sin(1/3*arctan2(sin
(2*d), cos(2*d))))*cos(8*b*x + 9*d) + 4*((I*cos(d)*e^(5/3*a) + e^(5/3*a)*s
in(d))*cos(1/3*arctan2(sin(2*d), cos(2*d))) + (cos(d)*e^(5/3*a) - I*e^(5/3
*a)*sin(d))*sin(1/3*arctan2(sin(2*d), cos(2*d))))*cos(6*b*x + 7*d) + 6*((I
*cos(d)*e^(5/3*a) + e^(5/3*a)*sin(d))*cos(1/3*arctan2(sin(2*d), cos(2*d)))
 + (cos(d)*e^(5/3*a) - I*e^(5/3*a)*sin(d))*sin(1/3*arctan2(sin(2*d), cos(2
*d))))*cos(4*b*x + 5*d) + 4*((I*cos(d)*e^(5/3*a) + e^(5/3*a)*sin(d))*cos(1
/3*arctan2(sin(2*d), cos(2*d))) + (cos(d)*e^(5/3*a) - I*e^(5/3*a)*sin(d))*
sin(1/3*arctan2(sin(2*d), cos(2*d))))*cos(2*b*x + 3*d) + (I*cos(d)^2*e^(5/
3*a) + I*e^(5/3*a)*sin(d)^2)*cos(1/3*arctan2(sin(2*d), cos(2*d))) - ((cos(
d)*e^(5/3*a) - I*e^(5/3*a)*sin(d))*cos(1/3*arctan2(sin(2*d), cos(2*d))) -
(I*cos(d)*e^(5/3*a) + e^(5/3*a)*sin(d))*sin(1/3*arctan2(sin(2*d), cos(2*d)
)))*sin(8*b*x + 9*d) - 4*((cos(d)*e^(5/3*a) - I*e^(5/3*a)*sin(d))*cos(1/3*
arctan2(sin(2*d), cos(2*d))) - (I*cos(d)*e^(5/3*a) + e^(5/3*a)*sin(d))*sin
(1/3*arctan2(sin(2*d), cos(2*d))))*sin(6*b*x + 7*d) - 6*((cos(d)*e^(5/3*a)
 - I*e^(5/3*a)*sin(d))*cos(1/3*arctan2(sin(2*d), cos(2*d))) - (I*cos(d)*e^
(5/3*a) + e^(5/3*a)*sin(d))*sin(1/3*arctan2(sin(2*d), cos(2*d))))*sin(4*b*
x + 5*d) - 4*((cos(d)*e^(5/3*a) - I*e^(5/3*a)*sin(d))*cos(1/3*arctan2(sin(
2*d), cos(2*d))) - (I*cos(d)*e^(5/3*a) + e^(5/3*a)*sin(d))*sin(1/3*arct...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(217) = 434$.

Time = 0.33 (sec) , antiderivative size = 880, normalized size of antiderivative = 2.61

$$\int e^{\frac{5}{3}(a+ibx)} \sec^5(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^5,x, algorithm="giac")
```

output
```
-4/243*I*(28*e^(-20/3*I*d)*log(e^(2/3*I*b*x) + e^(-2/3*I*d)) + 84*(cos(d)^
6*e^(4/3*I*d) - 15*cos(d)^4*e^(4/3*I*d)*sin(d)^2 + 15*cos(d)^2*e^(4/3*I*d)
*sin(d)^4 - e^(4/3*I*d)*sin(d)^6)*arctan(1/3*sqrt(3)*(2*e^(2/3*I*b*x) - e^
(-2/3*I*d))*e^(2/3*I*d))/(sqrt(3)*cos(d)^12*e^(2*I*d) + 6*sqrt(3)*cos(d)^1
0*e^(2*I*d)*sin(d)^2 + 15*sqrt(3)*cos(d)^8*e^(2*I*d)*sin(d)^4 + 20*sqrt(3)
*cos(d)^6*e^(2*I*d)*sin(d)^6 + 15*sqrt(3)*cos(d)^4*e^(2*I*d)*sin(d)^8 + 6*
sqrt(3)*cos(d)^2*e^(2*I*d)*sin(d)^10 + sqrt(3)*e^(2*I*d)*sin(d)^12) - 168*
I*(3*cos(d)^5*e^(4/3*I*d)*sin(d) - 10*cos(d)^3*e^(4/3*I*d)*sin(d)^3 + 3*co
s(d)*e^(4/3*I*d)*sin(d)^5)*arctan(1/3*sqrt(3)*(2*e^(2/3*I*b*x) - e^(-2/3*I
*d))*e^(2/3*I*d))/(sqrt(3)*cos(d)^12*e^(2*I*d) + 6*sqrt(3)*cos(d)^10*e^(2*
I*d)*sin(d)^2 + 15*sqrt(3)*cos(d)^8*e^(2*I*d)*sin(d)^4 + 20*sqrt(3)*cos(d)
^6*e^(2*I*d)*sin(d)^6 + 15*sqrt(3)*cos(d)^4*e^(2*I*d)*sin(d)^8 + 6*sqrt(3)
*cos(d)^2*e^(2*I*d)*sin(d)^10 + sqrt(3)*e^(2*I*d)*sin(d)^12) - 14*(cos(d)^
6*e^(4/3*I*d) - 15*cos(d)^4*e^(4/3*I*d)*sin(d)^2 + 15*cos(d)^2*e^(4/3*I*d)
*sin(d)^4 - e^(4/3*I*d)*sin(d)^6)*log(-e^(2/3*I*b*x - 2/3*I*d) + e^(4/3*I*
b*x) + e^(-4/3*I*d))/(cos(d)^12*e^(2*I*d) + 6*cos(d)^10*e^(2*I*d)*sin(d)^2
 + 15*cos(d)^8*e^(2*I*d)*sin(d)^4 + 20*cos(d)^6*e^(2*I*d)*sin(d)^6 + 15*co
s(d)^4*e^(2*I*d)*sin(d)^8 + 6*cos(d)^2*e^(2*I*d)*sin(d)^10 + e^(2*I*d)*sin
(d)^12) + 28*I*(3*cos(d)^5*e^(4/3*I*d)*sin(d) - 10*cos(d)^3*e^(4/3*I*d)*si
n(d)^3 + 3*cos(d)*e^(4/3*I*d)*sin(d)^5)*log(-e^(2/3*I*b*x - 2/3*I*d) + ...
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{3}(a+ibx)}\sec^5(d+bx)\,dx=\int \frac{\mathrm{e}^{\frac{5\,a}{3}+\frac{b\,x\,5\mathrm{i}}{3}}}{\cos\left(d+b\,x\right)^5}\,dx$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)/cos(d + b*x)^5,x)
```

output
```
int(exp((5*a)/3 + (b*x*5i)/3)/cos(d + b*x)^5, x)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\sec^5(d+bx)\,dx=\text{too large to display}$$

input
```
int(exp(5/3*a+5/3*I*b*x)*sec(b*x+d)^5,x)
```

output
```
( - 59051536552700868*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sec(b*x + d)**5*
sin(b*x + d)**4*tan((b*x + d)/2)**10*i + 295257682763504340*e**((5*a + 5*b
*i*x)/3)*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**4*tan((b*x + d)/2)**8*
i - 590515365527008680*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sec(b*x + d)**5
*sin(b*x + d)**4*tan((b*x + d)/2)**6*i + 590515365527008680*e**((5*a + 5*b
*i*x)/3)*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**4*tan((b*x + d)/2)**4*
i - 295257682763504340*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sec(b*x + d)**5
*sin(b*x + d)**4*tan((b*x + d)/2)**2*i + 59051536552700868*e**((5*a + 5*b*
i*x)/3)*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**4*i + 11810307310540173
6*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**2*tan(
(b*x + d)/2)**10*i - 590515365527008680*e**((5*a + 5*b*i*x)/3)*cos(b*x + d
)*sec(b*x + d)**5*sin(b*x + d)**2*tan((b*x + d)/2)**8*i + 1181030731054017
360*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**2*ta
n((b*x + d)/2)**6*i - 1181030731054017360*e**((5*a + 5*b*i*x)/3)*cos(b*x +
 d)*sec(b*x + d)**5*sin(b*x + d)**2*tan((b*x + d)/2)**4*i + 59051536552700
8680*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sec(b*x + d)**5*sin(b*x + d)**2*t
an((b*x + d)/2)**2*i - 118103073105401736*e**((5*a + 5*b*i*x)/3)*cos(b*x +
 d)*sec(b*x + d)**5*sin(b*x + d)**2*i - 59051536552700868*e**((5*a + 5*b*i
*x)/3)*cos(b*x + d)*sec(b*x + d)**5*tan((b*x + d)/2)**10*i + 2952576827635
04340*e**((5*a + 5*b*i*x)/3)*cos(b*x + d)*sec(b*x + d)**5*tan((b*x + d)...
```

3.90 $\int F^{c(a+bx)}\sec(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int F^{c(a+bx)}\sec(d+ex)\,dx = \frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{ie+bc\log(F)}$$

output
```
2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))/e],[3/2-1
/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/(I*e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\sec(d+ex)\,dx = \frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{ibc\log(F)}{2e},\frac{3}{2}-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{ie+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sec[d + e*x],x]
```

output
```
(2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, 1/2 - ((I/2)*b*c*L
og[F])/e, 3/2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/(I*e + b*c*Lo
g[F])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{4951\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4951}$$

$$\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{bc\log(F)+ie}$$

input
```
Int[F^(c*(a + b*x))*Sec[d + e*x],x]
```

output
```
(2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])
/(2*e), (3 - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))])/(I*e + b*c*Log[F]
)
```

Defintions of rubi rules used

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \sec(ex+d)\, dx$$

input
```
int(F^(c*(b*x+a))*sec(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*sec(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)} \sec(d+ex)\, dx = \int F^{(bx+a)c} \sec(ex+d)\, dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sec(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)} \sec(d+ex)\, dx = \int F^{c(a+bx)} \sec(d+ex)\, dx$$

input
```
integrate(F**(c*(b*x+a))*sec(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*sec(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\sec(d+ex)\,dx=\int F^{(bx+a)c}\sec(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="maxima")
```

output
```
2*(F^(b*c*x)*F^(a*c)*b*c*cos(e*x + d)*log(F) - F^(b*c*x)*F^(a*c)*e*sin(e*x
 + d) + (F^(b*c*x)*F^(a*c)*b*c*cos(e*x + d)*log(F) - F^(b*c*x)*F^(a*c)*e*s
in(e*x + d))*cos(2*e*x + 2*d) + 2*(F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^
3 + (F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*cos(2*e*x + 2*d)^2 + (F^(a*
c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*sin(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^2*c
^2*e*log(F)^2 + F^(a*c)*e^3)*cos(2*e*x + 2*d))*integrate((F^(b*c*x)*b*c*lo
g(F)*sin(e*x + d) + F^(b*c*x)*e*cos(e*x + d) + (F^(b*c*x)*b*c*log(F)*sin(e
*x + d) + F^(b*c*x)*e*cos(e*x + d))*cos(4*e*x + 4*d) + 2*(F^(b*c*x)*b*c*lo
g(F)*sin(e*x + d) + F^(b*c*x)*e*cos(e*x + d))*cos(2*e*x + 2*d) - (F^(b*c*x
)*b*c*cos(e*x + d)*log(F) - F^(b*c*x)*e*sin(e*x + d))*sin(4*e*x + 4*d) - 2
*(F^(b*c*x)*b*c*cos(e*x + d)*log(F) - F^(b*c*x)*e*sin(e*x + d))*sin(2*e*x
+ 2*d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^2)*cos(4*e*x + 4*d)^2 +
4*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)^2 + (b^2*c^2*log(F)^2 + e^2)*s
in(4*e*x + 4*d)^2 + 4*(b^2*c^2*log(F)^2 + e^2)*sin(4*e*x + 4*d)*sin(2*e*x
+ 2*d) + 4*(b^2*c^2*log(F)^2 + e^2)*sin(2*e*x + 2*d)^2 + e^2 + 2*(b^2*c^2*
log(F)^2 + e^2 + 2*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d))*cos(4*e*x +
4*d) + 4*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)), x) + (F^(b*c*x)*F^(a*
c)*b*c*log(F)*sin(e*x + d) + F^(b*c*x)*F^(a*c)*e*cos(e*x + d))*sin(2*e*x +
 2*d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)^2 + (
b^2*c^2*log(F)^2 + e^2)*sin(2*e*x + 2*d)^2 + e^2 + 2*(b^2*c^2*log(F)^2 ...
```

Giac [F]

$$\int F^{c(a+bx)}\sec(d+ex)\,dx=\int F^{(bx+a)c}\sec(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sec(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sec(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cos\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))/cos(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))/cos(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sec(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\sec\left(ex+d\right)dx\right)$$

input
```
int(F^(c*(b*x+a))*sec(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sec(d + e*x),x)
```

3.91 $\int F^{c(a+bx)} \sec^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 84

$$\int F^{c(a+bx)} \sec^2(d+ex)\,dx = \frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2-\frac{ibc\log(F)}{e}\right),\frac{1}{2}\left(4-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

output
```
4*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b*c*ln(F)/e],[2-1/2
*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/(2*I*e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int F^{c(a+bx)} \sec^2(d+ex)\,dx = \frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sec[d + e*x]^2,x]
```

output
```
(4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/((2*I)*e + b*
c*Log[F])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{4951\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 4951$$

$$\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)+2ie}$$

input
```
Int[F^(c*(a + b*x))*Sec[d + e*x]^2,x]
```

output
```
(4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/((2*I)*e + b*
c*Log[F])
```

Defintions of rubi rules used

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \sec (ex + d)^2 \, dx$$

input
```
int(F^(c*(b*x+a))*sec(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*sec(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \sec^2(d + ex) \, dx = \int F^{(bx+a)c} \sec (ex + d)^2 \, dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sec(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \sec^2(d + ex) \, dx = \int F^{c(a+bx)} \sec^2 (d + ex) \, dx$$

input
```
integrate(F**(c*(b*x+a))*sec(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*sec(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx = \int F^{(bx+a)c}\sec(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="maxima")
```

output
```
4*(24*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3 + 16*
F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^3*c^3*
log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 + (F^(a
*c)*b^3*c^3*log(F)^3 + 64*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*
d) - 2*(5*F^(a*c)*b^2*c^2*e*log(F)^2 - 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x
 + 2*d) + (24*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) + (F^(a*c)*b^3*c^3*log(F)^3
 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) - 2*(F^(a*c)*b^2*
c^2*e*log(F)^2 + 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x + 2*d))*cos(4*e*x + 4
*d) - 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64
*F^(a*c)*b*c*e^5*log(F) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3
*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(4*e*x + 4*d)^2 + 4*(F^(a*c)
*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5
*log(F))*cos(2*e*x + 2*d)^2 + (F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3
*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*sin(4*e*x + 4*d)^2 + 4*(F^(
a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c
*e^5*log(F))*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(F^(a*c)*b^5*c^5*e*log(
F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*sin(2*
e*x + 2*d)^2 + 2*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(
F)^3 + 64*F^(a*c)*b*c*e^5*log(F) + 2*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a
*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(2*e*x + 2*d))...
```

Giac [F]

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx = \int F^{(bx+a)c}\sec(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sec(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx=\int\frac{F^{c\,(a+b\,x)}}{\cos\left(d+e\,x\right)^2}\,dx$$

input
```
int(F^(c*(a + b*x))/cos(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))/cos(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\sec\left(ex+d\right)^2dx\right)$$

input
```
int(F^(c*(b*x+a))*sec(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sec(d + e*x)**2,x)
```

3.92 $\int F^{c(a+bx)} \sec^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 84

$$\int F^{c(a+bx)} \sec^3(d+ex)\,dx = \frac{8e^{3i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),\frac{1}{2}\left(5-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{3ie+bc\log(F)}$$

output
```
8*exp(3*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([3, 3/2-1/2*I*b*c*ln(F)/e],[5/2
-1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/(3*I*e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.33

$$\int F^{c(a+bx)} \sec^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(2e^{i(d+ex)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{3}{2}-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)(-ie+bc\log(F))+\sec(d\right.}{2e^2}$$

input
```
Integrate[F^(c*(a + b*x))*Sec[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(2*E^(I*(d + e*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F]
)/(2*e), 3/2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*((-I)*e + b*c*L
og[F]) + Sec[d + e*x]*(-(b*c*Log[F]) + e*Tan[d + e*x])))/(2*e^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4948, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4948}$$

$$\frac{1}{2}\left(\frac{b^2c^2\log^2(F)}{e^2}+1\right)\int F^{c(a+bx)}\sec(d+ex)dx-\frac{bc\log(F)\sec(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tan(d+ex)\sec(d+ex)F^{c(a+bx)}}{2e}$$

$$\Big\downarrow \text{4951}$$

$$\frac{e^{i(d+ex)}F^{c(a+bx)}\left(\frac{b^2c^2\log^2(F)}{e^2}+1\right)\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{bc\log(F)+ie}-\frac{bc\log(F)\sec(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tan(d+ex)\sec(d+ex)F^{c(a+bx)}}{2e}$$

input
```
Int[F^(c*(a + b*x))*Sec[d + e*x]^3,x]
```

output
```
(E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])/(
2*e), (3 - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))]*(1 + (b^2*c^2*Log[F]
^2)/e^2))/(I*e + b*c*Log[F]) - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d + e*x])/(
2*e^2) + (F^(c*(a + b*x))*Sec[d + e*x]*Tan[d + e*x])/(2*e)
```

Defintions of rubi rules used

rule 4948
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(
e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2))   Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b,
 c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && N
eQ[n, 2]
```

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \sec (ex+d)^3 \,dx$$

input
```
int(F^(c*(b*x+a))*sec(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*sec(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \sec^3(d+ex) \, dx = \int F^{(bx+a)c} \sec (ex+d)^3 \, dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sec(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\sec^3(d+ex)\,dx = \int F^{c(a+bx)}\sec^3\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*sec(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*sec(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\sec^3(d+ex)\,dx = \int F^{(bx+a)c}\sec\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="maxima")
```

output
```
8*(48*F^(b*c*x)*F^(a*c)*b*c*e^2*cos(e*x + d)*log(F) + 6*(F^(a*c)*b^2*c^2*e
*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d) + (48*F^(b*c*x)*F^(a*c)
*b*c*e^2*cos(e*x + d)*log(F) + (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*
e^2*log(F))*F^(b*c*x)*cos(3*e*x + 3*d) - 3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2
5*F^(a*c)*e^3)*F^(b*c*x)*sin(3*e*x + 3*d) + 6*(F^(a*c)*b^2*c^2*e*log(F)^2
- 15*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d))*cos(6*e*x + 6*d) + 3*(48*F^(b*c*
x)*F^(a*c)*b*c*e^2*cos(e*x + d)*log(F) + (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^
(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(3*e*x + 3*d) - 3*(F^(a*c)*b^2*c^2*e*lo
g(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*sin(3*e*x + 3*d) + 6*(F^(a*c)*b^2*c^2*e
*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d))*cos(4*e*x + 4*d) + (3*
(F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x
 + 2*d) + 9*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*sin(2*
e*x + 2*d) + (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c
*x))*cos(3*e*x + 3*d) + 18*(8*F^(b*c*x)*F^(a*c)*b*c*e^2*cos(e*x + d)*log(F
) + (F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d))*
cos(2*e*x + 2*d) - 6*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*
e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*
b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a
*c)*e^6*sin(d) + (F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*
log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b...
```

Giac [F]

$$\int F^{c(a+bx)}\sec^3(d+ex)\,dx = \int F^{(bx+a)c}\sec(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sec(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sec^3(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cos(d+e\,x)^3}\,dx$$

input
```
int(F^(c*(a + b*x))/cos(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))/cos(d + e*x)^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sec^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\sec(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sec(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sec(d + e*x)**3,x)
```

3.93 $\int e^{a+ibx}\sec^n(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int e^{a+ibx}\sec^n(a+bx)\,dx = -\frac{ie^{a+ibx}\left(1+e^{2i(a+bx)}\right)^n \text{Hypergeometric2F1}\left(n,\frac{1+n}{2},\frac{3+n}{2},-e^{2i(a+bx)}\right)\sec^n(a+bx)}{b(1+n)}$$

output
```
-I*exp(a+I*b*x)*(1+exp(2*I*(b*x+a)))^n*hypergeom([n, 1/2+1/2*n],[3/2+1/2*n
],-exp(2*I*(b*x+a)))*sec(b*x+a)^n/b/(1+n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int e^{a+ibx}\sec^n(a+bx)\,dx = -\frac{ie^{a+ibx}\left(1+e^{2i(a+bx)}\right) \text{Hypergeometric2F1}\left(1,\frac{3-n}{2},\frac{3+n}{2},-e^{2i(a+bx)}\right)\sec^n(a+bx)}{b(1+n)}$$

input
```
Integrate[E^(a + I*b*x)*Sec[a + b*x]^n,x]
```

output
```
((-I)*E^(a + I*b*x)*(1 + E^((2*I)*(a + b*x)))*Hypergeometric2F1[1, (3 - n)
/2, (3 + n)/2, -E^((2*I)*(a + b*x))]*Sec[a + b*x]^n)/(b*(1 + n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4954, 2681}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\sec^n(a+bx)\,dx$$

$$\Big\downarrow \text{4954}$$

$$e^{-in(a+bx)}\left(1+e^{2i(a+bx)}\right)^n\sec^n(a+bx)\int e^{a(in+1)+ib(n+1)x}\left(1+e^{2i(a+bx)}\right)^{-n}dx$$

$$\Big\downarrow \text{2681}$$

$$-\frac{i\left(1+e^{2i(a+bx)}\right)^n\exp(-in(a+bx)+a(1+in)+ib(n+1)x)\,\text{Hypergeometric2F1}\left(n,\frac{n+1}{2},\frac{n+3}{2},-e^{2i(a+bx)}\right)\sec}{b(n+1)}$$

input
```
Int[E^(a + I*b*x)*Sec[a + b*x]^n,x]
```

output
```
((-I)*E^(a*(1 + I*n) + I*b*(1 + n)*x - I*n*(a + b*x))*(1 + E^((2*I)*(a + b
*x)))^n*Hypergeometric2F1[n, (1 + n)/2, (3 + n)/2, -E^((2*I)*(a + b*x))]*S
ec[a + b*x]^n)/(b*(1 + n))
```

Defintions of rubi rules used

rule 2681
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

rule 4954
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[(1 + E^(2*I*(d + e*x)))^n*(Sec[d + e*x]^n/E^(I*n*(d + e*x)))
Int[SimplifyIntegrand[F^(c*(a + b*x))*(E^(I*n*(d + e*x))/(1 + E^(2*I*(d + e
*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \mathrm{e}^{ibx+a} \sec (bx+a)^n \, dx$$

input
```
int(exp(a+I*b*x)*sec(b*x+a)^n,x)
```

output
```
int(exp(a+I*b*x)*sec(b*x+a)^n,x)
```

Fricas [F]

$$\int e^{a+ibx} \sec^n(a+bx) \, dx = \int \sec (bx+a)^n e^{(i\,bx+a)} \, dx$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral((2*e^(I*b*x + I*a)/(e^(2*I*b*x + 2*I*a) + 1))^n*e^(I*b*x + a), x)
```

Sympy [F]

$$\int e^{a+ibx}\sec^n(a+bx)\,dx = e^a\int e^{ibx}\sec^n{(a+bx)}\,dx$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+a)**n,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*sec(a + b*x)**n, x)
```

Maxima [F]

$$\int e^{a+ibx}\sec^n(a+bx)\,dx = \int \sec{(bx+a)}^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^n*e^(I*b*x + a), x)
```

Giac [F]

$$\int e^{a+ibx}\sec^n(a+bx)\,dx = \int \sec{(bx+a)}^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*sec(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^n*e^(I*b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\sec^n(a+bx)\,dx=\int \mathrm{e}^{a+b\,x\,1\mathrm{i}}\left(\frac{1}{\cos\left(a+b\,x\right)}\right)^n\,dx$$

input
```
int(exp(a + b*x*1i)*(1/cos(a + b*x))^n,x)
```

output
```
int(exp(a + b*x*1i)*(1/cos(a + b*x))^n, x)
```

Reduce [F]

$$\int e^{a+ibx}\sec^n(a+bx)\,dx=e^a\left(\int e^{bix}\sec\left(bx+a\right)^n dx\right)$$

input
```
int(exp(a+I*b*x)*sec(b*x+a)^n,x)
```

output
```
e**a*int(e**(b*i*x)*sec(a + b*x)**n,x)
```

3.94 $\int F^{c(a+bx)}(f\sec(d+ex))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 99

$$\int F^{c(a+bx)}(f\sec(d+ex))^n\,dx$$
$$=\frac{\left(1+e^{2i(d+ex)}\right)^n F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(n,\frac{1}{2}\left(n-\frac{ibc\log(F)}{e}\right),\frac{1}{2}\left(2+n-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)(f\,\mathrm{s}}{ien+bc\log(F)}$$

output
```
(1+exp(2*I*(e*x+d)))^n*F^(c*(b*x+a))*hypergeom([n, 1/2*n-1/2*I*b*c*ln(F)/e
],[1+1/2*n-1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))*(f*sec(e*x+d))^n/(I*e*n+b
*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)}(f\sec(d+ex))^n\,dx =$$
$$-\frac{i\left(1+e^{2i(d+ex)}\right)^n F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(n,\frac{en-ibc\log(F)}{2e},\frac{1}{2}\left(2+n-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)(f\sec(}{en-ibc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sec[d + e*x])^n,x]
```

output
```
((-I)*(1 + E^((2*I)*(d + e*x)))^n*F^(c*(a + b*x))*Hypergeometric2F1[n, (e*
n - I*b*c*Log[F])/(2*e), (2 + n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x)
)]*(f*Sec[d + e*x])^n)/(e*n - I*b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7271, 4954, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\sec(d+ex))^n\,dx$$

$$\Big\downarrow \text{7271}$$

$$\sec^{-n}(d+ex)(f\sec(d+ex))^n\int F^{c(a+bx)}\sec^n(d+ex)dx$$

$$\Big\downarrow \text{4954}$$

$$e^{-in(d+ex)}\left(1+e^{2i(d+ex)}\right)^n(f\sec(d+ex))^n\int e^{idn+iexn}\left(1+e^{2i(d+ex)}\right)^{-n}F^{ac+bxc}dx$$

$$\Big\downarrow \text{2689}$$

$$\frac{e^{-in(d+ex)+idn+ienx}\left(1+e^{2i(d+ex)}\right)^n F^{ac+bcx}(f\sec(d+ex))^n\,\text{Hypergeometric2F1}\left(n,\frac{en-ibc\log(F)}{2e},\frac{1}{2}\left(n-\frac{ibc\log(F)}{e}\right.\right.}{bc\log(F)+ien}$$

input
```
Int[F^(c*(a + b*x))*(f*Sec[d + e*x])^n,x]
```

output
```
(E^(I*d*n + I*e*n*x - I*n*(d + e*x))*(1 + E^((2*I)*(d + e*x)))^n*F^(a*c +
b*c*x)*Hypergeometric2F1[n, (e*n - I*b*c*Log[F])/(2*e), (2 + n - (I*b*c*Lo
g[F])/e)/2, -E^((2*I)*(d + e*x))]*(f*Sec[d + e*x])^n)/(I*e*n + b*c*Log[F])
```

Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] && !IntegerQ[p]
```

rule 4954
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[(1 + E^(2*I*(d + e*x)))^n*(Sec[d + e*x]^n/E^(I*n*(d + e*x)))
Int[SimplifyIntegrand[F^(c*(a + b*x))*(E^(I*n*(d + e*x))/(1 + E^(2*I*(d + e
*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] && !IntegerQ[n]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}(f\sec(ex+d))^n\,dx$$

input
```
int(F^(c*(b*x+a))*(f*sec(e*x+d))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*sec(e*x+d))^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\sec(d+ex))^n\,dx = \int (f\sec(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sec(e*x+d))^n,x, algorithm="fricas")
```

output
```
integral((f*sec(e*x + d))^n*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\sec(d+ex))^n\,dx = \int F^{c(a+bx)}(f\sec(d+ex))^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sec(e*x+d))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sec(d + e*x))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\sec(d+ex))^n\,dx = \int (f\sec(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sec(e*x+d))^n,x, algorithm="maxima")
```

output
```
integrate((f*sec(e*x + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\sec(d+ex))^n\,dx=\int (f\sec(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sec(e*x+d))^n,x, algorithm="giac")
```

output
```
integrate((f*sec(e*x + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\sec(d+ex))^n\,dx=\int F^{c\,(a+b\,x)}\left(\frac{f}{\cos(d+e\,x)}\right)^n dx$$

input
```
int(F^(c*(a + b*x))*(f/cos(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f/cos(d + e*x))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\sec(d+ex))^n\,dx=f^{ac+n}\left(\int f^{bcx}\sec(ex+d)^n\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*sec(e*x+d))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*sec(d + e*x)**n,x)
```

3.95 $\int F^{c(a+bx)} \left(f \sec \left(d+\frac{ibcx \log(F)}{-2+n}\right)\right)^n \, dx$

Optimal result

Integrand size = 31, antiderivative size = 141

$$\begin{aligned}\int F^{c(a+bx)} \left(f \sec \left(d+\frac{ibcx \log(F)}{-2+n}\right)\right)^n \, dx &= \frac{f^2 F^{c(a+bx)}(2-n) \left(f \sec \left(d-\frac{ibcx \log(F)}{2-n}\right)\right)^{-2+n}}{bc(1-n)\log(F)} \\ &\quad - \frac{if F^{c(a+bx)}(2-n) \left(f \sec \left(d-\frac{ibcx \log(F)}{2-n}\right)\right)^{-1+n} \sin\left(d-\frac{ibcx \log(F)}{2-n}\right)}{bc(1-n)\log(F)}\end{aligned}$$

output
```
f^2*F^(c*(b*x+a))*(2-n)*(f*sec(-d+I*b*c*x*ln(F)/(2-n)))^(-2+n)/b/c/(1-n)/l
n(F)+I*f*F^(c*(b*x+a))*(2-n)*(f*sec(-d+I*b*c*x*ln(F)/(2-n)))^(-1+n)*sin(-d
+I*b*c*x*ln(F)/(2-n))/b/c/(1-n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.56

$$\int F^{c(a+bx)}\left(f\sec\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \frac{e^{-2id}F^{c(a+bx)}\left(e^{2id}+F^{\frac{2bcx}{-2+n}}\right)(-2+n)\left(f\sec\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n}{2bc(-1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sec[d + (I*b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
(F^(c*(a + b*x))*(E^((2*I)*d) + F^((2*b*c*x)/(-2 + n)))*(-2 + n)*(f*Sec[d
+ (I*b*c*x*Log[F])/(-2 + n)])^n)/(2*b*c*E^((2*I)*d)*(-1 + n)*Log[F])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {7271, 4946}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\sec\left(d+\frac{ibcx\log(F)}{n-2}\right)\right)^n dx$$

$$\Big\downarrow \text{7271}$$

$$\sec^{-n}\left(d-\frac{ibcx\log(F)}{2-n}\right)\left(f\sec\left(d-\frac{ibcx\log(F)}{2-n}\right)\right)^n\int F^{c(a+bx)}\sec^n\left(d-\frac{ibcx\log(F)}{2-n}\right)dx$$

$$\Big\downarrow \text{4946}$$

$$\sec^{-n}\left(d-\frac{ibcx\log(F)}{2-n}\right)\left(f\sec\left(d-\frac{ibcx\log(F)}{2-n}\right)\right)^n\left(\frac{(2-n)F^{c(a+bx)}\sec^{n-2}\left(d-\frac{ibcx\log(F)}{2-n}\right)}{bc(1-n)\log(F)}-\frac{i(2-n)F^{c(\ldots}}{\ldots}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Sec[d + (I*b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
((f*Sec[d - (I*b*c*x*Log[F])/(2 - n)])^n*((F^(c*(a + b*x))*(2 - n)*Sec[d -
 (I*b*c*x*Log[F])/(2 - n)]^(-2 + n))/(b*c*(1 - n)*Log[F]) - (I*F^(c*(a + b
*x))*(2 - n)*Sec[d - (I*b*c*x*Log[F])/(2 - n)]^(-1 + n)*Sin[d - (I*b*c*x*L
og[F])/(2 - n)])/(b*c*(1 - n)*Log[F])))/Sec[d - (I*b*c*x*Log[F])/(2 - n)]^
n
```

Defintions of rubi rules used

rule 4946
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e
*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[b^2*c^2*Log[F]^2 +
 e^2*(n - 2)^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(f\sec\left(d+\frac{ibcx\ln(F)}{-2+n}\right)\right)^{n}dx$$

input
```
int(F^(c*(b*x+a))*(f*sec(d+I*b*c*x*ln(F)/(-2+n)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*sec(d+I*b*c*x*ln(F)/(-2+n)))^n,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int F^{c(a+bx)}\left(f\sec\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n\,dx$$
$$=\frac{\left((n-2)e^{\left(-\frac{2\,(bcx\log(F)-i\,dn+2i\,d)}{n-2}\right)}+n-2\right)F^{bcx+ac}\left(\frac{2\,fe^{\left(-\frac{bcx\log(F)-i\,dn+2i\,d}{n-2}\right)}}{e^{\left(-\frac{2\,(bcx\log(F)-i\,dn+2i\,d)}{n-2}\right)}+1}\right)^n e^{\left(\frac{2\,(bcx\log(F)-i\,dn+2i\,d)}{n-2}\right)}}{2\,(bcn-bc)\log\left(F\right)}$$

input
```
integrate(F^(c*(b*x+a))*(f*sec(d+I*b*c*x*log(F)/(-2+n)))^n,x, algorithm="f
ricas")
```

output
```
1/2*((n - 2)*e^(-2*(b*c*x*log(F) - I*d*n + 2*I*d)/(n - 2)) + n - 2)*F^(b*c
*x + a*c)*(2*f*e^(-(b*c*x*log(F) - I*d*n + 2*I*d)/(n - 2))/(e^(-2*(b*c*x*l
og(F) - I*d*n + 2*I*d)/(n - 2)) + 1))^n*e^(2*(b*c*x*log(F) - I*d*n + 2*I*d
)/(n - 2))/((b*c*n - b*c)*log(F))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\sec\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n\,dx=\int F^{c(a+bx)}\left(f\sec\left(\frac{ibcx\log\left(F\right)}{n-2}+d\right)\right)^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sec(d+I*b*c*x*ln(F)/(-2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sec(I*b*c*x*log(F)/(n - 2) + d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\sec\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \int \left(f\sec\left(\frac{i\,bcx\log\left(F\right)}{n-2}+d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sec(d+I*b*c*x*log(F)/(-2+n)))^n,x, algorithm="m
axima")
```

output
```
integrate((f*sec(I*b*c*x*log(F)/(n - 2) + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\left(f\sec\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \int \left(f\sec\left(\frac{i\,bcx\log\left(F\right)}{n-2}+d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sec(d+I*b*c*x*log(F)/(-2+n)))^n,x, algorithm="g
iac")
```

output
```
integrate((f*sec(I*b*c*x*log(F)/(n - 2) + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\sec\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \int F^{c\,(a+b\,x)}\left(\frac{f}{\cos\left(d+\frac{b\,c\,x\,\ln(F)\,1i}{n-2}\right)}\right)^n dx$$

input
```
int(F^(c*(a + b*x))*(f/cos(d + (b*c*x*log(F)*1i)/(n - 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f/cos(d + (b*c*x*log(F)*1i)/(n - 2)))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\left(f\sec\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx$$
$$= f^{ac+n}\left(\int f^{bcx}\sec\left(\frac{\log(f)\,bcix+dn-2d}{n-2}\right)^n dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*sec(d+I*b*c*x*log(F)/(-2+n)))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*sec((log(f)*b*c*i*x + d*n - 2*d)/(n - 2))**n,x
)
```

3.96 $\int F^{c(a+bx)}\left(f\sec\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n\,dx$

Optimal result

Integrand size = 31, antiderivative size = 141

$$\int F^{c(a+bx)}\left(f\sec\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n\,dx = \frac{f^2F^{c(a+bx)}(2-n)\left(f\sec\left(d+\frac{ibcx\log(F)}{2-n}\right)\right)^{-2+n}}{bc(1-n)\log(F)} + \frac{ifF^{c(a+bx)}(2-n)\left(f\sec\left(d+\frac{ibcx\log(F)}{2-n}\right)\right)^{-1+n}\sin\left(d+\frac{ibcx\log(F)}{2-n}\right)}{bc(1-n)\log(F)}$$

output

```
f^2*F^(c*(b*x+a))*(2-n)*(f*sec(d+I*b*c*x*ln(F)/(2-n)))^(-2+n)/b/c/(1-n)/ln
(F)+I*f*F^(c*(b*x+a))*(2-n)*(f*sec(d+I*b*c*x*ln(F)/(2-n)))^(-1+n)*sin(d+I*
b*c*x*ln(F)/(2-n))/b/c/(1-n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

$$\int F^{c(a+bx)}\left(f\sec\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \frac{F^{c(a+bx)}\left(1+e^{2id}F^{\frac{2bcx}{-2+n}}\right)(-2+n)\left(f\sec\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n}{2bc(-1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sec[d - (I*b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
(F^(c*(a + b*x))*(1 + E^((2*I)*d)*F^((2*b*c*x)/(-2 + n)))*(-2 + n)*(f*Sec[
d - (I*b*c*x*Log[F])/(-2 + n)])^n)/(2*b*c*(-1 + n)*Log[F])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {7271, 4946}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\sec\left(d-\frac{ibcx\log(F)}{n-2}\right)\right)^n dx$$

$$\Big\downarrow \text{ 7271}$$

$$\sec^{-n}\left(d+\frac{ibcx\log(F)}{2-n}\right)\left(f\sec\left(d+\frac{ibcx\log(F)}{2-n}\right)\right)^n\int F^{c(a+bx)}\sec^n\left(d+\frac{ibcx\log(F)}{2-n}\right)dx$$

$$\Big\downarrow \text{ 4946}$$

$$\sec^{-n}\left(d+\frac{ibcx\log(F)}{2-n}\right)\left(f\sec\left(d+\frac{ibcx\log(F)}{2-n}\right)\right)^n\left(\frac{(2-n)F^{c(a+bx)}\sec^{n-2}\left(d+\frac{ibcx\log(F)}{2-n}\right)}{bc(1-n)\log(F)}+\frac{i(2-n)F^{c(}}{}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Sec[d - (I*b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
((f*Sec[d + (I*b*c*x*Log[F])/(2 - n)])^n*((F^(c*(a + b*x))*(2 - n)*Sec[d +
 (I*b*c*x*Log[F])/(2 - n)]^(-2 + n))/(b*c*(1 - n)*Log[F]) + (I*F^(c*(a + b
*x))*(2 - n)*Sec[d + (I*b*c*x*Log[F])/(2 - n)]^(-1 + n)*Sin[d + (I*b*c*x*L
og[F])/(2 - n)])/(b*c*(1 - n)*Log[F])))/Sec[d + (I*b*c*x*Log[F])/(2 - n)]^
n
```

Defintions of rubi rules used

rule 4946
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e
*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[b^2*c^2*Log[F]^2 +
 e^2*(n - 2)^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(f\sec\left(-d+\frac{ibcx\ln(F)}{-2+n}\right)\right)^n dx$$

input
```
int(F^(c*(b*x+a))*(f*sec(-d+I*b*c*x*ln(F)/(-2+n)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*sec(-d+I*b*c*x*ln(F)/(-2+n)))^n,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int F^{c(a+bx)}\left(f\sec\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \frac{\left((n-2)e^{\left(-\frac{2\,(bcx\log(F)+i\,dn-2i\,d)}{n-2}\right)}+n-2\right)F^{bcx+ac}\left(\frac{2\,fe^{\left(-\frac{bcx\log(F)+i\,dn-2i\,d}{n-2}\right)}}{e^{\left(-\frac{2\,(bcx\log(F)+i\,dn-2i\,d)}{n-2}\right)}+1}\right)^n e^{\left(\frac{2\,(bcx\log(F)+i\,dn-2i\,d)}{n-2}\right)}}{2\,(bcn-bc)\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(f*sec(-d+I*b*c*x*log(F)/(-2+n)))^n,x, algorithm="
fricas")
```

output
```
1/2*((n - 2)*e^(-2*(b*c*x*log(F) + I*d*n - 2*I*d)/(n - 2)) + n - 2)*F^(b*c
*x + a*c)*(2*f*e^(-(b*c*x*log(F) + I*d*n - 2*I*d)/(n - 2))/(e^(-2*(b*c*x*l
og(F) + I*d*n - 2*I*d)/(n - 2)) + 1))^n*e^(2*(b*c*x*log(F) + I*d*n - 2*I*d
)/(n - 2))/((b*c*n - b*c)*log(F))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\sec\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \int F^{c(a+bx)}\left(f\sec\left(\frac{ibcx\log(F)}{n-2}-d\right)\right)^n dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sec(-d+I*b*c*x*ln(F)/(-2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sec(I*b*c*x*log(F)/(n - 2) - d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\sec\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx$$
$$=\int\left(f\sec\left(\frac{i\,bcx\log(F)}{n-2}-d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sec(-d+I*b*c*x*log(F)/(-2+n)))^n,x, algorithm="
maxima")
```

output
```
integrate((f*sec(I*b*c*x*log(F)/(n - 2) - d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\left(f\sec\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx$$
$$=\int\left(f\sec\left(\frac{i\,bcx\log(F)}{n-2}-d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sec(-d+I*b*c*x*log(F)/(-2+n)))^n,x, algorithm="
giac")
```

output
```
integrate((f*sec(I*b*c*x*log(F)/(n - 2) - d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\sec\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx=\int F^{c\,(a+b\,x)}\left(\frac{f}{\cos\left(d-\frac{b\,c\,x\,\ln(F)\,1i}{n-2}\right)}\right)^n dx$$

input
```
int(F^(c*(a + b*x))*(f/cos(d - (b*c*x*log(F)*1i)/(n - 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f/cos(d - (b*c*x*log(F)*1i)/(n - 2)))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\left(f\sec\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx$$
$$=f^{ac+n}\left(\int f^{bcx}\sec\left(\frac{\log(f)\,bcix-dn+2d}{n-2}\right)^n dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*sec(-d+I*b*c*x*log(F)/(-2+n)))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*sec((log(f)*b*c*i*x - d*n + 2*d)/(n - 2))**n,x
)
```

3.97 $\int e^{a+ibx}\csc(d+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 29

$$\int e^{a+ibx}\csc(d+bx)\,dx=\frac{e^{a-id}\log\left(1-e^{2i(d+bx)}\right)}{b}$$

output
```
exp(a-I*d)*ln(1-exp(2*I*(b*x+d)))/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 75 vs. 2(29) = 58.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.59

$$\int e^{a+ibx}\csc(d+bx)\,dx = \frac{e^a\left(2i\arctan\left(\frac{(1+e^{2ibx})\tan(d)}{-1+e^{2ibx}}\right)+\log\left(1+e^{4ibx}-2e^{2ibx}\cos(2d)\right)\right)(\cos(d)-i\sin(d))}{2b}$$

input
```
Integrate[E^(a + I*b*x)*Csc[d + b*x],x]
```

output
```
(E^a*((2*I)*ArcTan[((1 + E^((2*I)*b*x))*Tan[d])/(-1 + E^((2*I)*b*x))] + Lo
g[1 + E^((4*I)*b*x) - 2*E^((2*I)*b*x)*Cos[2*d]])*(Cos[d] - I*Sin[d]))/(2*b
)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\csc(bx+d)\,dx$$

$$\Big\downarrow \text{4953}$$

$$\frac{e^{a-id}\log\left(1-e^{2i(bx+d)}\right)}{b}$$

input
```
Int[E^(a + I*b*x)*Csc[d + b*x],x]
```

output
```
(E^(a - I*d)*Log[1 - E^((2*I)*(d + b*x))])/b
```

Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{\ln\left(-1+\mathrm{e}^{2i(bx+d)}\right)\mathrm{e}^{-id+a}}{b}$	24

input
```
int(exp(a+I*b*x)*csc(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(-1+exp(2*I*(b*x+d)))/b*exp(a-I*d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int e^{a+ibx}\csc(d+bx)\,dx=\frac{e^{(a-i\,d)}\log\left(e^{(2i\,bx+2i\,d)}-1\right)}{b}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d),x, algorithm="fricas")
```

output
```
e^(a - I*d)*log(e^(2*I*b*x + 2*I*d) - 1)/b
```

Sympy [F]

$$\int e^{a+ibx}\csc(d+bx)\,dx=e^{a}\int e^{ibx}\csc\left(bx+d\right)dx$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d),x)
```

output
```
exp(a)*Integral(exp(I*b*x)*csc(b*x + d), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(24) = 48$.

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 5.86

$$\int e^{a+ibx}\csc(d+bx)\,dx$$
$$= \frac{2\left(i\cos\left(d\right)e^{a}+e^{a}\sin\left(d\right)\right)\arctan\left(\sin\left(bx\right)+\sin\left(d\right),\cos\left(bx\right)-\cos\left(d\right)\right)+2\left(i\cos\left(d\right)e^{a}+e^{a}\sin\left(d\right)\right)\mathrm{arct}}{}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d),x, algorithm="maxima")
```

output
```
1/2*(2*(I*cos(d)*e^a + e^a*sin(d))*arctan2(sin(b*x) + sin(d), cos(b*x) - c
os(d)) + 2*(I*cos(d)*e^a + e^a*sin(d))*arctan2(sin(b*x) - sin(d), cos(b*x)
 + cos(d)) + (cos(d)*e^a - I*e^a*sin(d))*log(cos(b*x)^2 + 2*cos(b*x)*cos(d
) + cos(d)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(d) + sin(d)^2) + (cos(d)*e^a -
I*e^a*sin(d))*log(cos(b*x)^2 - 2*cos(b*x)*cos(d) + cos(d)^2 + sin(b*x)^2 +
 2*sin(b*x)*sin(d) + sin(d)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int e^{a+ibx}\csc(d+bx)\,dx = \frac{e^{(a-i\,d)}\log\left(e^{(2i\,bx+2i\,d)}-1\right)}{b}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d),x, algorithm="giac")
```

output
```
e^(a - I*d)*log(e^(2*I*b*x + 2*I*d) - 1)/b
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int e^{a+ibx}\csc(d+bx)\,dx=\frac{\mathrm{e}^{a-d\,1\mathrm{i}}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-\mathrm{e}^{2\,a}\,\mathrm{e}^{-d\,2\mathrm{i}}\right)}{b}$$

input
```
int(exp(a + b*x*1i)/sin(d + b*x),x)
```

output
```
(exp(a - d*1i)*log(exp(2*a)*exp(b*x*2i) - exp(2*a)*exp(-d*2i)))/b
```

Reduce [F]

$$\int e^{a+ibx}\csc(d+bx)\,dx=e^{a}\left(\int e^{bix}\csc\left(bx+d\right)dx\right)$$

input
```
int(exp(a+I*b*x)*csc(b*x+d),x)
```

output
```
e**a*int(e**(b*i*x)*csc(b*x + d),x)
```

3.98 $\int e^{a+ibx}\csc^2(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 71

$$\int e^{a+ibx}\csc^2(d+bx)\,dx=\frac{2ie^{a-id+i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)}-\frac{2ie^{a-id}\text{arctanh}\left(e^{i(d+bx)}\right)}{b}$$

output
```
2*I*exp(a-I*d+I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))-2*I*exp(a-I*d)*arctanh(exp
(I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.52

$$\int e^{a+ibx}\csc^2(d+bx)\,dx$$
$$=\frac{2e^a(\cos(d)-i\sin(d))\left(e^{ibx}+\text{arctanh}\left(e^{ibx}(\cos(d)+i\sin(d))\right)\left(\left(-1+e^{2ibx}\right)\cos(d)+i\left(1+e^{2ibx}\right)\sin(d)\right)\right)}{ib\left(-1+e^{2ibx}\right)\cos(d)-b\left(1+e^{2ibx}\right)\sin(d)}$$

input
```
Integrate[E^(a + I*b*x)*Csc[d + b*x]^2,x]
```

output
```
(2*E^a*(Cos[d] - I*Sin[d])*(E^(I*b*x) + ArcTanh[E^(I*b*x)*(Cos[d] + I*Sin[
d])]*((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d])))/(I*b*(
-1 + E^((2*I)*b*x))*Cos[d] - b*(1 + E^((2*I)*b*x))*Sin[d])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\csc^2(bx+d)\,dx$$

$$\Big\downarrow \ 4953$$

$$\frac{2ie^{a-2i(bx+d)+ibx}\left(\frac{e^{2i(bx+d)}}{1-e^{2i(bx+d)}}-e^{i(bx+d)}\text{arctanh}\left(e^{i(bx+d)}\right)\right)}{b}$$

input
```
Int[E^(a + I*b*x)*Csc[d + b*x]^2,x]
```

output
```
((2*I)*E^(a + I*b*x - (2*I)*(d + b*x))*(E^((2*I)*(d + b*x))/(1 - E^((2*I)*
(d + b*x))) - E^(I*(d + b*x))*ArcTanh[E^(I*(d + b*x))]))/b
```

Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{2 i \,{\mathrm e}^{a} {\mathrm e}^{i b x}}{b \left(-1+{\mathrm e}^{2 i \left(b x +d \right)}\right)}-\frac{2 i \,{\mathrm e}^{a} {\mathrm e}^{-i d} \operatorname{arctanh}\left({\mathrm e}^{i \left(b x +d \right)}\right)}{b}$	52

input
```
int(exp(a+I*b*x)*csc(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*exp(a)*exp(I*b*x)/b/(-1+exp(2*I*(b*x+d)))-2*I*exp(a)*exp(-I*d)*arctan
h(exp(I*(b*x+d)))/b
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(47) = 94.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.35

$$\int e^{a+ibx}\csc^2(d+bx)\,dx$$
$$=\frac{\left(-i\,e^{(2i\,bx+a+i\,d)}+i\,e^{(a-i\,d)}\right)\log\left(e^{(i\,bx+i\,d)}+1\right)+\left(i\,e^{(2i\,bx+a+i\,d)}-i\,e^{(a-i\,d)}\right)\log\left(e^{(i\,bx+i\,d)}-1\right)-2i\,e^{(i\,bx+a)}}{be^{(2i\,bx+2i\,d)}-b}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)^2,x, algorithm="fricas")
```

output
```
((-I*e^(2*I*b*x + a + I*d) + I*e^(a - I*d))*log(e^(I*b*x + I*d) + 1) + (I*
e^(2*I*b*x + a + I*d) - I*e^(a - I*d))*log(e^(I*b*x + I*d) - 1) - 2*I*e^(I
*b*x + a))/(b*e^(2*I*b*x + 2*I*d) - b)
```

Sympy [F]

$$\int e^{a+ibx}\csc^2(d+bx)\,dx = e^a\int e^{ibx}\csc^2{(bx+d)}\,dx$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*csc(b*x + d)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(47) = 94$.

Time = 0.05 (sec) , antiderivative size = 397, normalized size of antiderivative = 5.59

$$\int e^{a+ibx}\csc^2(d+bx)\,dx = -\frac{2\left(\left(-i\cos{(d)}e^a - e^a\sin{(d)}\right)\cos{(2\,bx+2\,d)} + i\cos{(d)}e^a + \left(\cos{(d)}e^a - i\,e^a\sin{(d)}\right)\sin{(2\,bx+2\,d)} + \dots\right.}{\dots}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)^2,x, algorithm="maxima")
```

output
```
-(2*((-I*cos(d)*e^a - e^a*sin(d))*cos(2*b*x + 2*d) + I*cos(d)*e^a + (cos(d
)*e^a - I*e^a*sin(d))*sin(2*b*x + 2*d) + e^a*sin(d))*arctan2(sin(b*x) + si
n(d), cos(b*x) - cos(d)) + 2*((I*cos(d)*e^a + e^a*sin(d))*cos(2*b*x + 2*d)
 - I*cos(d)*e^a - (cos(d)*e^a - I*e^a*sin(d))*sin(2*b*x + 2*d) - e^a*sin(d
))*arctan2(sin(b*x) - sin(d), cos(b*x) + cos(d)) + 4*cos(b*x)*e^a + ((cos(
d)*e^a - I*e^a*sin(d))*cos(2*b*x + 2*d) - cos(d)*e^a - (-I*cos(d)*e^a - e^
a*sin(d))*sin(2*b*x + 2*d) + I*e^a*sin(d))*log(cos(b*x)^2 + 2*cos(b*x)*cos
(d) + cos(d)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(d) + sin(d)^2) - ((cos(d)*e^a
 - I*e^a*sin(d))*cos(2*b*x + 2*d) - cos(d)*e^a + (I*cos(d)*e^a + e^a*sin(d
))*sin(2*b*x + 2*d) + I*e^a*sin(d))*log(cos(b*x)^2 - 2*cos(b*x)*cos(d) + c
os(d)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(d) + sin(d)^2) + 4*I*e^a*sin(b*x))/(
-2*I*b*cos(2*b*x + 2*d) + 2*b*sin(2*b*x + 2*d) + 2*I*b)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(47) = 94.

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.62

$$\int e^{a+ibx}\csc^2(d+bx)\,dx = \frac{-i\,e^{(2i\,bx+a+2i\,d)}\log\left(e^{(i\,bx+i\,d)}+1\right)+i\,e^{a}\log\left(e^{(i\,bx+i\,d)}+1\right)+i\,e^{(2i\,bx+a+2i\,d)}\log\left(e^{(i\,bx+i\,d)}-1\right)-i\,e^{a}\log\left(\right.}{b\left(e^{(2i\,bx+3i\,d)}-e^{(i\,d)}\right)}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)^2,x, algorithm="giac")
```

output
```
(-I*e^(2*I*b*x + a + 2*I*d)*log(e^(I*b*x + I*d) + 1) + I*e^a*log(e^(I*b*x
+ I*d) + 1) + I*e^(2*I*b*x + a + 2*I*d)*log(e^(I*b*x + I*d) - 1) - I*e^a*l
og(e^(I*b*x + I*d) - 1) - 2*I*e^(I*b*x + a + I*d))/(b*(e^(2*I*b*x + 3*I*d)
 - e^(I*d)))
```

Mupad [B] (verification not implemented)

Time = 15.92 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.38

$$\int e^{a+ibx}\csc^2(d+bx)\,dx = -\frac{\ln\left(2\,e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}-e^{2\,a}\,e^{-d\,2i}\,\sqrt{-e^{2\,a}\,e^{-d\,2i}}\,2i\right)\sqrt{-e^{2\,a}\,e^{-d\,2i}}}{b} + \frac{\ln\left(2\,e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}+e^{2\,a}\,e^{-d\,2i}\,\sqrt{-e^{2\,a}\,e^{-d\,2i}}\,2i\right)\sqrt{-e^{2\,a}\,e^{-d\,2i}}}{b} + \frac{e^{3\,a}\,e^{-d\,2i}\,e^{b\,x\,1i}\,2i}{b\,e^{2\,a}\,e^{-d\,2i}-b\,e^{2\,a}\,e^{b\,x\,2i}}$$

input
```
int(exp(a + b*x*1i)/sin(d + b*x)^2,x)
```

output
```
(log(2*exp(3*a)*exp(-d*2i)*exp(b*x*1i) + exp(2*a)*exp(-d*2i)*(-exp(2*a)*ex
p(-d*2i))^(1/2)*2i)*(-exp(2*a)*exp(-d*2i))^(1/2))/b - (log(2*exp(3*a)*exp(
-d*2i)*exp(b*x*1i) - exp(2*a)*exp(-d*2i)*(-exp(2*a)*exp(-d*2i))^(1/2)*2i)*
(-exp(2*a)*exp(-d*2i))^(1/2))/b + (exp(3*a)*exp(-d*2i)*exp(b*x*1i)*2i)/(b*
exp(2*a)*exp(-d*2i) - b*exp(2*a)*exp(b*x*2i))
```

Reduce [F]

$$\int e^{a+ibx}\csc^2(d+bx)\,dx = e^a\left(\int e^{bix}\csc\left(bx+d\right)^2dx\right)$$

input
```
int(exp(a+I*b*x)*csc(b*x+d)^2,x)
```

output
```
e**a*int(e**(b*i*x)*csc(b*x + d)**2,x)
```

3.99 $\int e^{a+ibx} \csc^3(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 40

$$\int e^{a+ibx} \csc^3(d+bx)\,dx = \frac{2e^{a-id+4i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)^2}$$

output
```
2*exp(a-I*d+4*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.95

$$\int e^{a+ibx} \csc^3(d+bx)\,dx = \frac{2e^a(\cos(d)-i\sin(d))\left(2e^{2ibx}-\cos^2(d)+2i\cos(d)\sin(d)+\sin^2(d)\right)}{b\left((-1+e^{2ibx})\cos(d)+i\left(1+e^{2ibx}\right)\sin(d)\right)^2}$$

input
```
Integrate[E^(a + I*b*x)*Csc[d + b*x]^3,x]
```

output
```
(2*E^a*(Cos[d] - I*Sin[d])*(2*E^((2*I)*b*x) - Cos[d]^2 + (2*I)*Cos[d]*Sin[
d] + Sin[d]^2))/(b*((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Si
n[d])^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4947}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\csc^3(bx+d)\,dx$$

$$\Big\downarrow \text{ 4947}$$

$$\frac{e^{a+ibx}\cot(bx+d)\csc(bx+d)}{2b}-\frac{ie^{a+ibx}\csc(bx+d)}{2b}$$

input
```
Int[E^(a + I*b*x)*Csc[d + b*x]^3,x]
```

output
```
((-1/2*I)*E^(a + I*b*x)*Csc[d + b*x])/b + (E^(a + I*b*x)*Cot[d + b*x]*Csc[
d + b*x])/(2*b)
```

Defintions of rubi rules used

rule 4947
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/(e
*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[b^2*c^2*Log[F]^2 +
 e^2*(n - 2)^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{2\,\mathrm{e}^{4 i b x+3 i d+a}}{\left(-1+\mathrm{e}^{2 i (b x+d)}\right)^{2} b}$	31
parallelrisch	$-\frac{\left(\cot\left(\frac{b x}{2}+\frac{d}{2}\right)+2 i-\tan\left(\frac{b x}{2}+\frac{d}{2}\right)\right) \mathrm{e}^{i b x+a} \left(\cot\left(\frac{b x}{2}+\frac{d}{2}\right)+\tan\left(\frac{b x}{2}+\frac{d}{2}\right)\right)}{8 b}$	56
norman	$\frac{-\frac{\mathrm{e}^{i b x+a}}{8 b}+\frac{\mathrm{e}^{i b x+a} \tan\left(\frac{b x}{2}+\frac{d}{2}\right)^{4}}{8 b}-\frac{i \mathrm{e}^{i b x+a} \tan\left(\frac{b x}{2}+\frac{d}{2}\right)}{4 b}-\frac{i \mathrm{e}^{i b x+a} \tan\left(\frac{b x}{2}+\frac{d}{2}\right)^{3}}{4 b}}{\tan\left(\frac{b x}{2}+\frac{d}{2}\right)^{2}}$	99

input
```
int(exp(a+I*b*x)*csc(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
2/(-1+exp(2*I*(b*x+d)))^2/b*exp(4*I*b*x+3*I*d+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int e^{a+ibx}\csc^3(d+bx)\,dx = \frac{2\left(2\,e^{(2i\,bx+a+i\,d)}-e^{(a-i\,d)}\right)}{be^{(4i\,bx+4i\,d)}-2\,be^{(2i\,bx+2i\,d)}+b}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)^3,x, algorithm="fricas")
```

output
```
2*(2*e^(2*I*b*x + a + I*d) - e^(a - I*d))/(b*e^(4*I*b*x + 4*I*d) - 2*b*e^(
2*I*b*x + 2*I*d) + b)
```

Sympy [F]

$$\int e^{a+ibx}\csc^3(d+bx)\,dx = e^a\int e^{ibx}\csc^3(bx+d)\,dx$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)**3,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*csc(b*x + d)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(28) = 56.

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int e^{a+ibx}\csc^3(d+bx)\,dx = -\frac{2\,(2i\,\cos(2\,bx+2\,d)\,e^a - 2\,e^a\sin(2\,bx+2\,d) - i\,e^a)}{-i\,b\cos(4\,bx+5\,d) + 2i\,b\cos(2\,bx+3\,d) - i\,b\cos(d) + b\sin(4\,bx+5\,d) - 2\,b\sin(2\,bx+3\,d) + b\sin}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)^3,x, algorithm="maxima")
```

output
```
-2*(2*I*cos(2*b*x + 2*d)*e^a - 2*e^a*sin(2*b*x + 2*d) - I*e^a)/(-I*b*cos(4
*b*x + 5*d) + 2*I*b*cos(2*b*x + 3*d) - I*b*cos(d) + b*sin(4*b*x + 5*d) - 2
*b*sin(2*b*x + 3*d) + b*sin(d))
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int e^{a+ibx}\csc^3(d+bx)\,dx = \frac{2\left(2\,e^{(2i\,bx+a+2i\,d)}-e^{a}\right)}{b\left(e^{(4i\,bx+5i\,d)}-2\,e^{(2i\,bx+3i\,d)}+e^{(i\,d)}\right)}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)^3,x, algorithm="giac")
```

output
```
2*(2*e^(2*I*b*x + a + 2*I*d) - e^a)/(b*(e^(4*I*b*x + 5*I*d) - 2*e^(2*I*b*x
 + 3*I*d) + e^(I*d)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\csc^3(d+bx)\,dx = \int \frac{e^{a+b\,x\,1\mathrm{i}}}{\sin\left(d+b\,x\right)^3}\,dx$$

input
```
int(exp(a + b*x*1i)/sin(d + b*x)^3,x)
```

output
```
int(exp(a + b*x*1i)/sin(d + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int e^{a+ibx}\csc^3(d+bx)\,dx = -\frac{e^{bix+a}\left(\cos\left(bx+d\right)+\sin\left(bx+d\right)i\right)}{2\sin\left(bx+d\right)^2 b}$$

input
```
int(exp(a+I*b*x)*csc(b*x+d)^3,x)
```

output
```
( - e**(a + b*i*x)*(cos(b*x + d) + sin(b*x + d)*i))/(2*sin(b*x + d)**2*b)
```

3.100 $\int e^{a+ibx}\csc^4(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 157

$$\int e^{a+ibx}\csc^4(d+bx)\,dx = -\frac{8ie^{a-id+3i(d+bx)}}{3b\left(1-e^{2i(d+bx)}\right)^3}+\frac{2ie^{a-id+i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)^2} - \frac{ie^{a-id+i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)} - \frac{ie^{a-id}\operatorname{arctanh}\left(e^{i(d+bx)}\right)}{b}$$

output
```
-8/3*I*exp(a-I*d+3*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^3+2*I*exp(a-I*d+I*(b*
x+d))/b/(1-exp(2*I*(b*x+d)))^2-I*exp(a-I*d+I*(b*x+d))/b/(1-exp(2*I*(b*x+d)
))-I*exp(a-I*d)*arctanh(exp(I*(b*x+d)))/b
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87

$$\int e^{a+ibx}\csc^4(d+bx)\,dx = \frac{e^a\left(\operatorname{arctanh}\left(e^{ibx}(\cos(d)+i\sin(d))\right)(-3i\cos(d)-3\sin(d))+\frac{e^{ibx}(i\cos(d)+\sin(d))\left(8e^{2ibx}+3\left(-1+e^{4ibx}\right)\cos(2d)+3i(1-}{\left(\left(-1+e^{2ibx}\right)\cos(d)+i\left(1+e^{2ibx}\right)\sin(d)\right)^3}\right.}{3b}$$

input
```
Integrate[E^(a + I*b*x)*Csc[d + b*x]^4,x]
```

output
```
(E^a*(ArcTanh[E^(I*b*x)*(Cos[d] + I*Sin[d])]*((-3*I)*Cos[d] - 3*Sin[d]) +
(E^(I*b*x)*(I*Cos[d] + Sin[d])*(8*E^((2*I)*b*x) + 3*(-1 + E^((4*I)*b*x))*C
os[2*d] + (3*I)*(1 + E^((4*I)*b*x))*Sin[2*d]))/((-1 + E^((2*I)*b*x))*Cos[d
] + I*(1 + E^((2*I)*b*x))*Sin[d])^3))/(3*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.89, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4949, 4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\csc^4(bx+d)\,dx$$

$$\Big\downarrow \text{4949}$$

$$\frac{1}{2}\int e^{a+ibx}\csc^2(d+bx)dx-\frac{ie^{a+ibx}\csc^2(bx+d)}{6b}-\frac{e^{a+ibx}\cot(bx+d)\csc^2(bx+d)}{3b}$$

$$\Big\downarrow \text{4953}$$

$$\frac{ie^{a-2i(bx+d)+ibx}\left(\frac{e^{2i(bx+d)}}{1-e^{2i(bx+d)}}-e^{i(bx+d)}\text{arctanh}\left(e^{i(bx+d)}\right)\right)}{b}-\frac{ie^{a+ibx}\csc^2(bx+d)}{6b}-\frac{e^{a+ibx}\cot(bx+d)\csc^2(bx+d)}{3b}$$

input
```
Int[E^(a + I*b*x)*Csc[d + b*x]^4,x]
```

output
```
(I*E^(a + I*b*x - (2*I)*(d + b*x))*(E^((2*I)*(d + b*x))/(1 - E^((2*I)*(d +
 b*x))) - E^(I*(d + b*x))*ArcTanh[E^(I*(d + b*x))]))/b - ((I/6)*E^(a + I*b
*x)*Csc[d + b*x]^2)/b - (E^(a + I*b*x)*Cot[d + b*x]*Csc[d + b*x]^2)/(3*b)
```

Defintions of rubi rules used

rule 4949
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.54

method	result	size
risch	$\frac{i\left(3\,\mathrm{e}^{a}\mathrm{e}^{5ibx}\mathrm{e}^{4id}+8\,\mathrm{e}^{a}\mathrm{e}^{3ibx}\mathrm{e}^{2id}-3\,\mathrm{e}^{a}\mathrm{e}^{ibx}\right)}{3b\left(-1+\mathrm{e}^{2i(bx+d)}\right)^{3}}-\frac{i\mathrm{e}^{a}\mathrm{e}^{-id}\operatorname{arctanh}\left(\mathrm{e}^{i(bx+d)}\right)}{b}$	85

input
```
int(exp(a+I*b*x)*csc(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*I/b/(-1+exp(2*I*(b*x+d)))^3*(3*exp(a)*exp(5*I*b*x)*exp(4*I*d)+8*exp(a)
*exp(3*I*b*x)*exp(2*I*d)-3*exp(a)*exp(I*b*x))-I*exp(a)/b*exp(-I*d)*arctanh
(exp(I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.24

$$\int e^{a+ibx}\csc^4(d+bx)\,dx = -\frac{3\left(i\,e^{(6i\,bx+a+5i\,d)}-3i\,e^{(4i\,bx+a+3i\,d)}+3i\,e^{(2i\,bx+a+i\,d)}-i\,e^{(a-i\,d)}\right)\log\left(e^{(i\,bx+i\,d)}+1\right)+3\left(-i\,e^{(6i\,bx+a+5i\,d)}\cdots\right.}{6\left(be^{(6i\,bx+6i\,d)}-3\,b\cdots\right.}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)^4,x, algorithm="fricas")
```

output
```
-1/6*(3*(I*e^(6*I*b*x + a + 5*I*d) - 3*I*e^(4*I*b*x + a + 3*I*d) + 3*I*e^(
2*I*b*x + a + I*d) - I*e^(a - I*d))*log(e^(I*b*x + I*d) + 1) + 3*(-I*e^(6*
I*b*x + a + 5*I*d) + 3*I*e^(4*I*b*x + a + 3*I*d) - 3*I*e^(2*I*b*x + a + I*
d) + I*e^(a - I*d))*log(e^(I*b*x + I*d) - 1) - 6*I*e^(5*I*b*x + a + 4*I*d)
 - 16*I*e^(3*I*b*x + a + 2*I*d) + 6*I*e^(I*b*x + a))/(b*e^(6*I*b*x + 6*I*d
) - 3*b*e^(4*I*b*x + 4*I*d) + 3*b*e^(2*I*b*x + 2*I*d) - b)
```

Sympy [F]

$$\int e^{a+ibx}\csc^4(d+bx)\,dx = e^a\int e^{ibx}\csc^4{(bx+d)}\,dx$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)**4,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*csc(b*x + d)**4, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 870 vs. $2(100) = 200$.

Time = 0.08 (sec) , antiderivative size = 870, normalized size of antiderivative = 5.54

$$\int e^{a+ibx} \csc^4(d+bx)\, dx = \text{Too large to display}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)^4,x, algorithm="maxima")
```

output
```
-(6*((-I*cos(d)*e^a - e^a*sin(d))*cos(6*b*x + 6*d) + 3*(I*cos(d)*e^a + e^a
*sin(d))*cos(4*b*x + 4*d) + 3*(-I*cos(d)*e^a - e^a*sin(d))*cos(2*b*x + 2*d
) + I*cos(d)*e^a + (cos(d)*e^a - I*e^a*sin(d))*sin(6*b*x + 6*d) - 3*(cos(d
)*e^a - I*e^a*sin(d))*sin(4*b*x + 4*d) + 3*(cos(d)*e^a - I*e^a*sin(d))*sin
(2*b*x + 2*d) + e^a*sin(d))*arctan2(sin(b*x) + sin(d), cos(b*x) - cos(d))
+ 6*((I*cos(d)*e^a + e^a*sin(d))*cos(6*b*x + 6*d) + 3*(-I*cos(d)*e^a - e^a
*sin(d))*cos(4*b*x + 4*d) + 3*(I*cos(d)*e^a + e^a*sin(d))*cos(2*b*x + 2*d)
 - I*cos(d)*e^a - (cos(d)*e^a - I*e^a*sin(d))*sin(6*b*x + 6*d) + 3*(cos(d)
*e^a - I*e^a*sin(d))*sin(4*b*x + 4*d) - 3*(cos(d)*e^a - I*e^a*sin(d))*sin(
2*b*x + 2*d) - e^a*sin(d))*arctan2(sin(b*x) - sin(d), cos(b*x) + cos(d)) -
 12*cos(5*b*x + 4*d)*e^a - 32*cos(3*b*x + 2*d)*e^a + 12*cos(b*x)*e^a + 3*(
(cos(d)*e^a - I*e^a*sin(d))*cos(6*b*x + 6*d) - 3*(cos(d)*e^a - I*e^a*sin(d
))*cos(4*b*x + 4*d) + 3*(cos(d)*e^a - I*e^a*sin(d))*cos(2*b*x + 2*d) - cos
(d)*e^a + (I*cos(d)*e^a + e^a*sin(d))*sin(6*b*x + 6*d) + 3*(-I*cos(d)*e^a
- e^a*sin(d))*sin(4*b*x + 4*d) + 3*(I*cos(d)*e^a + e^a*sin(d))*sin(2*b*x +
 2*d) + I*e^a*sin(d))*log(cos(b*x)^2 + 2*cos(b*x)*cos(d) + cos(d)^2 + sin(
b*x)^2 - 2*sin(b*x)*sin(d) + sin(d)^2) - 3*((cos(d)*e^a - I*e^a*sin(d))*co
s(6*b*x + 6*d) - 3*(cos(d)*e^a - I*e^a*sin(d))*cos(4*b*x + 4*d) + 3*(cos(d
)*e^a - I*e^a*sin(d))*cos(2*b*x + 2*d) - cos(d)*e^a - (-I*cos(d)*e^a - e^a
*sin(d))*sin(6*b*x + 6*d) - 3*(I*cos(d)*e^a + e^a*sin(d))*sin(4*b*x + 4...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(100) = 200$.

Time = 0.18 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.64

$$\int e^{a+ibx} \csc^4(d+bx)\,dx$$
$$= \frac{-3i\,e^{(6i\,bx+a+6i\,d)}\log\left(e^{(i\,bx+i\,d)}+1\right)+9i\,e^{(4i\,bx+a+4i\,d)}\log\left(e^{(i\,bx+i\,d)}+1\right)-9i\,e^{(2i\,bx+a+2i\,d)}\log\left(e^{(i\,bx+i\,d)}+\right.}{}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)^4,x, algorithm="giac")
```

output
```
1/6*(-3*I*e^(6*I*b*x + a + 6*I*d)*log(e^(I*b*x + I*d) + 1) + 9*I*e^(4*I*b*
x + a + 4*I*d)*log(e^(I*b*x + I*d) + 1) - 9*I*e^(2*I*b*x + a + 2*I*d)*log(
e^(I*b*x + I*d) + 1) + 3*I*e^a*log(e^(I*b*x + I*d) + 1) + 3*I*e^(6*I*b*x +
 a + 6*I*d)*log(e^(I*b*x + I*d) - 1) - 9*I*e^(4*I*b*x + a + 4*I*d)*log(e^(
I*b*x + I*d) - 1) + 9*I*e^(2*I*b*x + a + 2*I*d)*log(e^(I*b*x + I*d) - 1) -
 3*I*e^a*log(e^(I*b*x + I*d) - 1) + 6*I*e^(5*I*b*x + a + 5*I*d) + 16*I*e^(
3*I*b*x + a + 3*I*d) - 6*I*e^(I*b*x + a + I*d))/(b*(e^(6*I*b*x + 7*I*d) -
3*e^(4*I*b*x + 5*I*d) + 3*e^(2*I*b*x + 3*I*d) - e^(I*d)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx} \csc^4(d+bx)\,dx = \int \frac{e^{a+b\,x\,1\mathrm{i}}}{\sin\left(d+b\,x\right)^4}\,dx$$

input
```
int(exp(a + b*x*1i)/sin(d + b*x)^4,x)
```

output
```
int(exp(a + b*x*1i)/sin(d + b*x)^4, x)
```

Reduce [F]

$$\int e^{a+ibx}\csc^4(d+bx)\,dx = e^a\left(\int e^{bix}\csc{(bx+d)}^4\,dx\right)$$

input
```
int(exp(a+I*b*x)*csc(b*x+d)^4,x)
```

output
```
e**a*int(e**(b*i*x)*csc(b*x + d)**4,x)
```

3.101 $\int e^{a+ibx}\csc^5(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 96

$$\int e^{a+ibx}\csc^5(d+bx)\,dx = -\frac{4e^{a-id}}{b\left(1-e^{2i(d+bx)}\right)^4}+\frac{32e^{a-id}}{3b\left(1-e^{2i(d+bx)}\right)^3}-\frac{8e^{a-id}}{b\left(1-e^{2i(d+bx)}\right)^2}$$

output
```
-4*exp(a-I*d)/b/(1-exp(2*I*(b*x+d)))^4+32/3*exp(a-I*d)/b/(1-exp(2*I*(b*x+d
)))^3-8*exp(a-I*d)/b/(1-exp(2*I*(b*x+d)))^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07

$$\int e^{a+ibx}\csc^5(d+bx)\,dx$$
$$= -\frac{4e^a\left(-4e^{2ibx}+\left(1+6e^{4ibx}\right)\cos(2d)+i\left(-1+6e^{4ibx}\right)\sin(2d)\right)(\cos(3d)-i\sin(3d))}{3b\left((-1+e^{2ibx})\cos(d)+i\left(1+e^{2ibx}\right)\sin(d)\right)^4}$$

input
```
Integrate[E^(a + I*b*x)*Csc[d + b*x]^5,x]
```

output
```
(-4*E^a*(-4*E^((2*I)*b*x) + (1 + 6*E^((4*I)*b*x))*Cos[2*d] + I*(-1 + 6*E^(
(4*I)*b*x))*Sin[2*d])*(Cos[3*d] - I*Sin[3*d]))/(3*b*((-1 + E^((2*I)*b*x))*
Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d])^4)
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4949, 4947}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\csc^5(bx+d)\,dx$$

$$\Big\downarrow \text{4949}$$

$$\frac{2}{3}\int e^{a+ibx}\csc^3(d+bx)dx-\frac{ie^{a+ibx}\csc^3(bx+d)}{12b}-\frac{e^{a+ibx}\cot(bx+d)\csc^3(bx+d)}{4b}$$

$$\Big\downarrow \text{4947}$$

$$-\frac{ie^{a+ibx}\csc^3(bx+d)}{12b}-\frac{e^{a+ibx}\cot(bx+d)\csc^3(bx+d)}{4b}+\frac{2}{3}\left(\frac{e^{a+ibx}\cot(bx+d)\csc(bx+d)}{2b}-\frac{ie^{a+ibx}\csc(bx+d)}{2b}\right)$$

input
```
Int[E^(a + I*b*x)*Csc[d + b*x]^5,x]
```

output
```
((-1/12*I)*E^(a + I*b*x)*Csc[d + b*x]^3)/b - (E^(a + I*b*x)*Cot[d + b*x]*C
sc[d + b*x]^3)/(4*b) + (2*(((-1/2*I)*E^(a + I*b*x)*Csc[d + b*x])/b + (E^(a
 + I*b*x)*Cot[d + b*x]*Csc[d + b*x])/(2*b)))/3
```

Defintions of rubi rules used

rule 4947
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/(e
*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[b^2*c^2*Log[F]^2 +
 e^2*(n - 2)^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

rule 4949
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{\frac{4\,{\rm e}^{8 i b x}{\rm e}^{7 i d}{\rm e}^{a}}{3}-\frac{16\,{\rm e}^{6 i b x}{\rm e}^{5 i d}{\rm e}^{a}}{3}}{\left(-1+{\rm e}^{2 i (b x+d)}\right)^{4} b}$	49
parallelrisch	$\frac{{\rm e}^{i b x+a} \csc\left(\frac{b x}{2}+\frac{d}{2}\right)^{4} \sec\left(\frac{b x}{2}+\frac{d}{2}\right)^{4} \left(i \sin(3 b x+3 d)-4 i \sin(b x+d)-4 \cos(b x+d)+\cos(3 b x+3 d)\right)}{192 b}$	75

input
```
int(exp(a+I*b*x)*csc(b*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
4/3/(-1+exp(2*I*(b*x+d)))^4/b*(exp(8*I*b*x)*exp(7*I*d)*exp(a)-4*exp(6*I*b*
x)*exp(5*I*d)*exp(a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int e^{a+ibx}\csc^5(d+bx)\,dx = -\frac{4\left(6\,e^{(4i\,bx+a+3i\,d)}-4\,e^{(2i\,bx+a+i\,d)}+e^{(a-i\,d)}\right)}{3\left(be^{(8i\,bx+8i\,d)}-4\,be^{(6i\,bx+6i\,d)}+6\,be^{(4i\,bx+4i\,d)}-4\,be^{(2i\,bx+2i\,d)}+b\right)}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)^5,x, algorithm="fricas")
```

output
```
-4/3*(6*e^(4*I*b*x + a + 3*I*d) - 4*e^(2*I*b*x + a + I*d) + e^(a - I*d))/(
b*e^(8*I*b*x + 8*I*d) - 4*b*e^(6*I*b*x + 6*I*d) + 6*b*e^(4*I*b*x + 4*I*d)
- 4*b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [F]

$$\int e^{a+ibx}\csc^5(d+bx)\,dx = e^a\int e^{ibx}\csc^5(bx+d)\,dx$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)**5,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*csc(b*x + d)**5, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(73) = 146.

Time = 0.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.75

$$\int e^{a+ibx}\csc^5(d+bx)\,dx = -\frac{4\left(-6i\,\cos(4\,bx+4\,d)\,e^a+4i\,\cos(2\,bx+2\,d)\,e^a+\right.}{-3i\,b\cos(8\,bx+9\,d)+12i\,b\cos(6\,bx+7\,d)-18i\,b\cos(4\,bx+5\,d)+12i\,b\cos(2\,bx+3\,d)-3i\,b\cos(}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)^5,x, algorithm="maxima")
```

output
```
-4*(-6*I*cos(4*b*x + 4*d)*e^a + 4*I*cos(2*b*x + 2*d)*e^a + 6*e^a*sin(4*b*x
 + 4*d) - 4*e^a*sin(2*b*x + 2*d) - I*e^a)/(-3*I*b*cos(8*b*x + 9*d) + 12*I*
b*cos(6*b*x + 7*d) - 18*I*b*cos(4*b*x + 5*d) + 12*I*b*cos(2*b*x + 3*d) - 3
*I*b*cos(d) + 3*b*sin(8*b*x + 9*d) - 12*b*sin(6*b*x + 7*d) + 18*b*sin(4*b*
x + 5*d) - 12*b*sin(2*b*x + 3*d) + 3*b*sin(d))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int e^{a+ibx}\csc^5(d+bx)\,dx = -\frac{4\left(6\,e^{(4i\,bx+a+4i\,d)}-4\,e^{(2i\,bx+a+2i\,d)}+e^{a}\right)}{3\,b\left(e^{(8i\,bx+9i\,d)}-4\,e^{(6i\,bx+7i\,d)}+6\,e^{(4i\,bx+5i\,d)}-4\,e^{(2i\,bx+3i\,d)}+e^{(i\,d)}\right)}$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+d)^5,x, algorithm="giac")
```

output
```
-4/3*(6*e^(4*I*b*x + a + 4*I*d) - 4*e^(2*I*b*x + a + 2*I*d) + e^a)/(b*(e^(
8*I*b*x + 9*I*d) - 4*e^(6*I*b*x + 7*I*d) + 6*e^(4*I*b*x + 5*I*d) - 4*e^(2*
I*b*x + 3*I*d) + e^(I*d)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\csc^5(d+bx)\,dx = \int \frac{e^{a+b\,x\,1i}}{\sin\left(d+b\,x\right)^5}\,dx$$

input
```
int(exp(a + b*x*1i)/sin(d + b*x)^5,x)
```

output
```
int(exp(a + b*x*1i)/sin(d + b*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int e^{a+ibx}\csc^5(d+bx)\,dx$$
$$=\frac{e^{bix+a}\left(-4\cos\left(bx+d\right)\sin\left(bx+d\right)^2-3\cos\left(bx+d\right)-4\sin\left(bx+d\right)^3i-\sin\left(bx+d\right)i\right)}{12\sin\left(bx+d\right)^4b}$$

input
```
int(exp(a+I*b*x)*csc(b*x+d)^5,x)
```

output
```
(e**(a + b*i*x)*( - 4*cos(b*x + d)*sin(b*x + d)**2 - 3*cos(b*x + d) - 4*si
n(b*x + d)**3*i - sin(b*x + d)*i))/(12*sin(b*x + d)**4*b)
```

3.102 $\int e^{2(a+ibx)}\csc(d+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int e^{2(a+ibx)}\csc(d+bx)\,dx = \frac{2e^{2(a-id)+i(d+bx)}}{b} - \frac{2e^{2a-2id}\operatorname{arctanh}\left(e^{i(d+bx)}\right)}{b}$$

output
```
2*exp(2*a-2*I*d+I*(b*x+d))/b-2*exp(2*a-2*I*d)*arctanh(exp(I*(b*x+d)))/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 249 vs. 2(55) = 110.

Time = 0.18 (sec) , antiderivative size = 249, normalized size of antiderivative = 4.53

$$\int e^{2(a+ibx)}\csc(d+bx)\,dx$$
$$= \frac{e^{2a}\left(4e^{ibx}\cos(d) + \cos(2d)\log\left(1+e^{2ibx}-2e^{ibx}\cos(d)\right) - \cos(2d)\log\left(1+e^{2ibx}+2e^{ibx}\cos(d)\right) - 4ie^{ibx}\,\mathrm{s}\right.}{}$$

input
```
Integrate[E^(2*(a + I*b*x))*Csc[d + b*x],x]
```

output
```
(E^(2*a)*(4*E^(I*b*x)*Cos[d] + Cos[2*d]*Log[1 + E^((2*I)*b*x) - 2*E^(I*b*x
)*Cos[d]] - Cos[2*d]*Log[1 + E^((2*I)*b*x) + 2*E^(I*b*x)*Cos[d]] - (4*I)*E
^(I*b*x)*Sin[d] + ArcTan[((-1 + E^(I*b*x))*Tan[d/2])/(1 + E^(I*b*x))]*((-2
*I)*Cos[2*d] - 2*Sin[2*d]) - I*Log[1 + E^((2*I)*b*x) - 2*E^(I*b*x)*Cos[d]]
*Sin[2*d] + I*Log[1 + E^((2*I)*b*x) + 2*E^(I*b*x)*Cos[d]]*Sin[2*d] + 2*Arc
Tan[((1 + E^(I*b*x))*Tan[d/2])/(-1 + E^(I*b*x))]*(I*Cos[2*d] + Sin[2*d])))
/(2*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{4953\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\csc(bx+d)\,dx$$

$$\Big\downarrow \; 4953$$

$$\frac{2e^{2(a+ibx)-3i(bx+d)}\left(e^{2i(bx+d)}-e^{i(bx+d)}\text{arctanh}\left(e^{i(bx+d)}\right)\right)}{b}$$

input
```
Int[E^(2*(a + I*b*x))*Csc[d + b*x],x]
```

output
```
(2*E^(2*(a + I*b*x) - (3*I)*(d + b*x))*(E^((2*I)*(d + b*x)) - E^(I*(d + b*
x))*ArcTanh[E^(I*(d + b*x))]))/b
```

Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{2\,\mathrm{e}^{2a}\mathrm{e}^{-id}\mathrm{e}^{ibx}}{b}-\frac{2\,\mathrm{e}^{2a}\mathrm{e}^{-2id}\operatorname{arctanh}\left(\mathrm{e}^{i(bx+d)}\right)}{b}$	46

input
```
int(exp(2*a+2*I*b*x)*csc(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
2/b*exp(2*a)*exp(-I*d)*exp(I*b*x)-2/b*exp(2*a)*exp(-2*I*d)*arctanh(exp(I*(
b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int e^{2(a+ibx)}\csc(d+bx)\,dx = -\frac{e^{(2\,a-2i\,d)}\log\left(e^{(i\,bx+i\,d)}+1\right)-e^{(2\,a-2i\,d)}\log\left(e^{(i\,bx+i\,d)}-1\right)-2\,e^{(i\,bx+2\,a-i\,d)}}{b}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d),x, algorithm="fricas")
```

output
```
-(e^(2*a - 2*I*d)*log(e^(I*b*x + I*d) + 1) - e^(2*a - 2*I*d)*log(e^(I*b*x
+ I*d) - 1) - 2*e^(I*b*x + 2*a - I*d))/b
```

Sympy [F]

$$\int e^{2(a+ibx)}\csc(d+bx)\,dx = e^{2a}\int e^{2ibx}\csc(bx+d)\,dx$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*I*b*x)*csc(b*x + d), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 232, normalized size of antiderivative = 4.22

$$\int e^{2(a+ibx)}\csc(d+bx)\,dx$$
$$= \frac{2\left(i\cos(2d)e^{(2a)}+e^{(2a)}\sin(2d)\right)\arctan(\sin(bx)+\sin(d),\cos(bx)-\cos(d))+2\left(-i\cos(2d)e^{(2a)}-\right.}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d),x, algorithm="maxima")
```

output
```
1/2*(2*(I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*arctan2(sin(b*x) + sin(d),
cos(b*x) - cos(d)) + 2*(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*arctan2(si
n(b*x) - sin(d), cos(b*x) + cos(d)) + 4*cos(b*x - d)*e^(2*a) - (cos(2*d)*e
^(2*a) - I*e^(2*a)*sin(2*d))*log(cos(b*x)^2 + 2*cos(b*x)*cos(d) + cos(d)^2
 + sin(b*x)^2 - 2*sin(b*x)*sin(d) + sin(d)^2) + (cos(2*d)*e^(2*a) - I*e^(2
*a)*sin(2*d))*log(cos(b*x)^2 - 2*cos(b*x)*cos(d) + cos(d)^2 + sin(b*x)^2 +
 2*sin(b*x)*sin(d) + sin(d)^2) + 4*I*e^(2*a)*sin(b*x - d))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int e^{2(a+ibx)}\csc(d+bx)\,dx = -\frac{e^{(2a-2id)}\log\left(ie^{(ibx+id)}+i\right)-e^{(2a-2id)}\log\left(-ie^{(ibx+id)}+i\right)-2e^{(ibx+2a-id)}}{b}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d),x, algorithm="giac")
```

output
```
-(e^(2*a - 2*I*d)*log(I*e^(I*b*x + I*d) + I) - e^(2*a - 2*I*d)*log(-I*e^(I
*b*x + I*d) + I) - 2*e^(I*b*x + 2*a - I*d))/b
```

Mupad [B] (verification not implemented)

Time = 17.42 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.53

$$\int e^{2(a+ibx)}\csc(d+bx)\,dx = \frac{2\,e^{2a-d\,1i+b\,x\,1i}}{b} - \frac{\sqrt{e^{4a-d\,4i}}\,\ln\left(-e^{4a}\,e^{-d\,3i}\,e^{b\,x\,1i}\,2i-e^{2a}\,e^{-d\,2i}\,\sqrt{e^{4a}\,e^{-d\,4i}}\,2i\right)}{b} + \frac{\sqrt{e^{4a-d\,4i}}\,\ln\left(-e^{4a}\,e^{-d\,3i}\,e^{b\,x\,1i}\,2i+e^{2a}\,e^{-d\,2i}\,\sqrt{e^{4a}\,e^{-d\,4i}}\,2i\right)}{b}$$

input
```
int(exp(2*a + b*x*2i)/sin(d + b*x),x)
```

output
```
(2*exp(2*a - d*1i + b*x*1i))/b - (exp(4*a - d*4i)^(1/2)*log(- exp(4*a)*exp
(-d*3i)*exp(b*x*1i)*2i - exp(2*a)*exp(-d*2i)*(exp(4*a)*exp(-d*4i))^(1/2)*2
i))/b + (exp(4*a - d*4i)^(1/2)*log(exp(2*a)*exp(-d*2i)*(exp(4*a)*exp(-d*4i
))^(1/2)*2i - exp(4*a)*exp(-d*3i)*exp(b*x*1i)*2i))/b
```

Reduce [F]

$$\int e^{2(a+ibx)}\csc(d+bx)\,dx = e^{2a}\left(\int e^{2bix}\csc(bx+d)\,dx\right)$$

input
```
int(exp(2*a+2*I*b*x)*csc(b*x+d),x)
```

output
```
e**(2*a)*int(e**(2*b*i*x)*csc(b*x + d),x)
```

3.103 $\int e^{2(a+ibx)}\csc^2(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int e^{2(a+ibx)}\csc^2(d+bx)\,dx=\frac{2ie^{2a-2id}}{b\left(1-e^{2i(d+bx)}\right)}+\frac{2ie^{2a-2id}\log\left(1-e^{2i(d+bx)}\right)}{b}$$

output
```
2*I*exp(2*a-2*I*d)/b/(1-exp(2*I*(b*x+d)))+2*I*exp(2*a-2*I*d)*ln(1-exp(2*I*
(b*x+d)))/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 183 vs. 2(70) = 140.

Time = 0.49 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.61

$$\int e^{2(a+ibx)}\csc^2(d+bx)\,dx$$
$$=\frac{e^{2a}\left(-2\arctan\left(\frac{(1+e^{2ibx})\tan(d)}{-1+e^{2ibx}}\right)(\cos(2d)-2i\cos(d)\sin(d))+\frac{(\cos(d)(-2+(-1+e^{2ibx})\log(1+e^{4ibx}-2e^{2ibx}\cos(2d)))+i}{(-1+e^{2ibx})}\right.}{b}$$

input
```
Integrate[E^(2*(a + I*b*x))*Csc[d + b*x]^2,x]
```

output
```
(E^(2*a)*(-2*ArcTan[((1 + E^((2*I)*b*x))*Tan[d])/(-1 + E^((2*I)*b*x))]*(Co
s[2*d] - (2*I)*Cos[d]*Sin[d]) + ((Cos[d]*(-2 + (-1 + E^((2*I)*b*x))*Log[1
+ E^((4*I)*b*x) - 2*E^((2*I)*b*x)*Cos[2*d]]) + I*(2 + (1 + E^((2*I)*b*x))*
Log[1 + E^((4*I)*b*x) - 2*E^((2*I)*b*x)*Cos[2*d]])*Sin[d])*(I*Cos[2*d] + S
in[2*d]))/((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d])))/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.73, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\csc^2(bx+d)\,dx$$

$$\Big\downarrow \text{4953}$$

$$\frac{2ie^{2(a+ibx)+4i(bx+d)}\left(e^{-4i(bx+d)}\left(1-e^{2i(bx+d)}\right)^2+e^{-6i(bx+d)}\left(1-e^{2i(bx+d)}\right)^3\log\left(1-e^{2i(bx+d)}\right)\right)}{b\left(1-e^{2i(bx+d)}\right)^3}$$

input
```
Int[E^(2*(a + I*b*x))*Csc[d + b*x]^2,x]
```

output
```
((2*I)*E^(2*(a + I*b*x) + (4*I)*(d + b*x))*((1 - E^((2*I)*(d + b*x)))^2/E^
((4*I)*(d + b*x)) + ((1 - E^((2*I)*(d + b*x)))^3*Log[1 - E^((2*I)*(d + b*x
))])/E^((6*I)*(d + b*x))))/(b*(1 - E^((2*I)*(d + b*x)))^3)
```

Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{2i e^{2a} e^{2ibx}}{b\left(-1+e^{2i(bx+d)}\right)}+\frac{2i e^{2a} e^{-2id} \ln\left(-1+e^{2i(bx+d)}\right)}{b}$	58

input
```
int(exp(2*a+2*I*b*x)*csc(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*exp(2*a)*exp(2*I*b*x)/b/(-1+exp(2*I*(b*x+d)))+2*I*exp(2*a)/b*exp(-2*I
*d)*ln(-1+exp(2*I*(b*x+d)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int e^{2(a+ibx)} \csc^2(d+bx)\,dx = -\frac{2\left(\left(-i\,e^{(2i\,bx+2\,a)}+i\,e^{(2\,a-2i\,d)}\right)\log\left(e^{(2i\,bx+2i\,d)}-1\right)+i\,e^{(2\,a-2i\,d)}\right)}{be^{(2i\,bx+2i\,d)}-b}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)^2,x, algorithm="fricas")
```

output
```
-2*((-I*e^(2*I*b*x + 2*a) + I*e^(2*a - 2*I*d))*log(e^(2*I*b*x + 2*I*d) - 1
) + I*e^(2*a - 2*I*d))/(b*e^(2*I*b*x + 2*I*d) - b)
```

Sympy [F]

$$\int e^{2(a+ibx)}\csc^2(d+bx)\,dx = e^{2a}\int e^{2ibx}\csc^2{(bx+d)}\,dx$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*I*b*x)*csc(b*x + d)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(54) = 108$.

Time = 0.05 (sec) , antiderivative size = 507, normalized size of antiderivative = 7.24

$$\int e^{2(a+ibx)}\csc^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)^2,x, algorithm="maxima")
```

output
```
-(2*(I*cos(2*d)^2*e^(2*a) + I*e^(2*a)*sin(2*d)^2 + (-I*cos(2*d)*e^(2*a) -
e^(2*a)*sin(2*d))*cos(2*b*x + 4*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d
))*sin(2*b*x + 4*d))*arctan2(sin(b*x) + sin(d), cos(b*x) - cos(d)) + 2*(I*
cos(2*d)^2*e^(2*a) + I*e^(2*a)*sin(2*d)^2 + (-I*cos(2*d)*e^(2*a) - e^(2*a)
*sin(2*d))*cos(2*b*x + 4*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(
2*b*x + 4*d))*arctan2(sin(b*x) - sin(d), cos(b*x) + cos(d)) + (cos(2*d)^2*
e^(2*a) + e^(2*a)*sin(2*d)^2 - (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos
(2*b*x + 4*d) - (I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*sin(2*b*x + 4*d))*
log(cos(b*x)^2 + 2*cos(b*x)*cos(d) + cos(d)^2 + sin(b*x)^2 - 2*sin(b*x)*si
n(d) + sin(d)^2) + (cos(2*d)^2*e^(2*a) + e^(2*a)*sin(2*d)^2 - (cos(2*d)*e^
(2*a) - I*e^(2*a)*sin(2*d))*cos(2*b*x + 4*d) - (I*cos(2*d)*e^(2*a) + e^(2*
a)*sin(2*d))*sin(2*b*x + 4*d))*log(cos(b*x)^2 - 2*cos(b*x)*cos(d) + cos(d)
^2 + sin(b*x)^2 + 2*sin(b*x)*sin(d) + sin(d)^2) + 2*e^(2*a))/(-I*b*cos(2*b
*x + 4*d) + I*b*cos(2*d) + b*sin(2*b*x + 4*d) - b*sin(2*d))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int e^{2(a+ibx)}\csc^2(d+bx)\,dx = -\frac{2\left(-i\,e^{(2i\,bx+2\,a)}\log\left(e^{(2i\,bx+2i\,d)}-1\right)+i\,e^{(2\,a-2i\,d)}\log\left(e^{(2i\,bx+2i\,d)}-1\right)+i\,e^{(2\,a-2i\,d)}\right)}{b(e^{(2i\,bx+2i\,d)}-1)}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)^2,x, algorithm="giac")
```

output
```
-2*(-I*e^(2*I*b*x + 2*a)*log(e^(2*I*b*x + 2*I*d) - 1) + I*e^(2*a - 2*I*d)*
log(e^(2*I*b*x + 2*I*d) - 1) + I*e^(2*a - 2*I*d))/(b*(e^(2*I*b*x + 2*I*d)
- 1))
```

Mupad [B] (verification not implemented)

Time = 15.63 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int e^{2(a+ibx)}\csc^2(d+bx)\,dx = \frac{e^{2\,a-d\,2i}\,\ln\left(e^{2\,a}\,e^{b\,x\,2i}-e^{2\,a}\,e^{-d\,2i}\right)\,2i}{b}+\frac{e^{4\,a-d\,4i}\,2i}{b\,\left(e^{2\,a-d\,2i}-e^{2\,a+b\,x\,2i}\right)}$$

input
```
int(exp(2*a + b*x*2i)/sin(d + b*x)^2,x)
```

output
```
(exp(2*a - d*2i)*log(exp(2*a)*exp(b*x*2i) - exp(2*a)*exp(-d*2i))*2i)/b + (
exp(4*a - d*4i)*2i)/(b*(exp(2*a - d*2i) - exp(2*a + b*x*2i)))
```

Reduce [F]

$$\int e^{2(a+ibx)}\csc^2(d+bx)\,dx = e^{2a}\left(\int e^{2bix}\csc\left(bx+d\right)^2 dx\right)$$

input
```
int(exp(2*a+2*I*b*x)*csc(b*x+d)^2,x)
```

output
```
e**(2*a)*int(e**(2*b*i*x)*csc(b*x + d)**2,x)
```

3.104 $\int e^{2(a+ibx)} \csc^3(d+bx) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int e^{2(a+ibx)} \csc^3(d+bx) \, dx = \frac{2e^{2(a-id)+3i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)^2} - \frac{3e^{2(a-id)+i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)} + \frac{3e^{2a-2id}\operatorname{arctanh}\left(e^{i(d+bx)}\right)}{b}$$

output
```
2*exp(2*a-2*I*d+3*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^2-3*exp(2*a-2*I*d+I*(b
*x+d))/b/(1-exp(2*I*(b*x+d)))+3*exp(2*a-2*I*d)*arctanh(exp(I*(b*x+d)))/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 387 vs. 2(115) = 230.

Time = 0.79 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.37

$$\int e^{2(a+ibx)} \csc^3(d+bx) \, dx = \frac{e^{2a}\left(6i\arctan\left(\frac{\left(-1+e^{ibx}\right)\tan\left(\frac{d}{2}\right)}{1+e^{ibx}}\right)\cos(2d) - 6i\arctan\left(\frac{\left(1+e^{ibx}\right)\tan\left(\frac{d}{2}\right)}{-1+e^{ibx}}\right)\cos(2d) - 3\cos(2d)\log\left(1+e^{2ibx} - \right.\right.}{}$$

input
```
Integrate[E^(2*(a + I*b*x))*Csc[d + b*x]^3,x]
```

output
```
(E^(2*a)*((6*I)*ArcTan[((-1 + E^(I*b*x))*Tan[d/2])/(1 + E^(I*b*x))]*Cos[2*
d] - (6*I)*ArcTan[((1 + E^(I*b*x))*Tan[d/2])/(-1 + E^(I*b*x))]*Cos[2*d] -
3*Cos[2*d]*Log[1 + E^((2*I)*b*x) - 2*E^(I*b*x)*Cos[d]] + 3*Cos[2*d]*Log[1
+ E^((2*I)*b*x) + 2*E^(I*b*x)*Cos[d]] + (8*E^(I*b*x)*(Cos[d] - I*Sin[d])^3
)/((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d])^2 + (20*E^(
I*b*x)*(Cos[d] - I*Sin[d])^2)/((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*
I)*b*x))*Sin[d]) + 6*ArcTan[((-1 + E^(I*b*x))*Tan[d/2])/(1 + E^(I*b*x))]*S
in[2*d] - 6*ArcTan[((1 + E^(I*b*x))*Tan[d/2])/(-1 + E^(I*b*x))]*Sin[2*d] +
 (3*I)*Log[1 + E^((2*I)*b*x) - 2*E^(I*b*x)*Cos[d]]*Sin[2*d] - (3*I)*Log[1
+ E^((2*I)*b*x) + 2*E^(I*b*x)*Cos[d]]*Sin[2*d]))/(4*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4949, 4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)} \csc^3(bx+d) \, dx$$

$$\Big\downarrow \text{4949}$$

$$-\frac{3}{2}\int e^{2(a+ibx)} \csc(d+bx)dx - \frac{ie^{2(a+ibx)} \csc(bx+d)}{b} - \frac{e^{2(a+ibx)} \cot(bx+d) \csc(bx+d)}{2b}$$

$$\Big\downarrow \text{4953}$$

$$-\frac{3e^{2(a+ibx)-3i(bx+d)}\left(e^{2i(bx+d)}-e^{i(bx+d)}\text{arctanh}\left(e^{i(bx+d)}\right)\right)}{b} - \frac{ie^{2(a+ibx)} \csc(bx+d)}{b} - \frac{e^{2(a+ibx)} \cot(bx+d) \csc(bx+d)}{2b}$$

input
```
Int[E^(2*(a + I*b*x))*Csc[d + b*x]^3,x]
```

output
```
(-3*E^(2*(a + I*b*x) - (3*I)*(d + b*x))*(E^((2*I)*(d + b*x)) - E^(I*(d + b
*x))*ArcTanh[E^(I*(d + b*x))]))/b - (I*E^(2*(a + I*b*x))*Csc[d + b*x])/b -
 (E^(2*(a + I*b*x))*Cot[d + b*x]*Csc[d + b*x])/(2*b)
```

Defintions of rubi rules used

rule 4949
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{3\,\mathrm{e}^{5 i b x}\mathrm{e}^{3 i d}\mathrm{e}^{2 a}-\mathrm{e}^{3 i b x}\mathrm{e}^{i d}\mathrm{e}^{2 a}}{\left(-1+\mathrm{e}^{2 i (b x+d)}\right)^{2} b}+\frac{3\,\mathrm{e}^{2 a}\mathrm{e}^{-2 i d}\operatorname{arctanh}\left(\mathrm{e}^{i (b x+d)}\right)}{b}-\frac{3\,\mathrm{e}^{2 a}\mathrm{e}^{-i d}\mathrm{e}^{i b x}}{b}$	98

input
```
int(exp(2*a+2*I*b*x)*csc(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(-1+exp(2*I*(b*x+d)))^2/b*(3*exp(5*I*b*x)*exp(3*I*d)*exp(2*a)-exp(3*I*b*
x)*exp(I*d)*exp(2*a))+3/b*exp(2*a)*exp(-2*I*d)*arctanh(exp(I*(b*x+d)))-3/b
*exp(2*a)*exp(-I*d)*exp(I*b*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.30

$$\int e^{2(a+ibx)}\csc^3(d+bx)\,dx = \frac{3\left(e^{(4i\,bx+2\,a+2i\,d)}-2\,e^{(2i\,bx+2\,a)}+e^{(2\,a-2i\,d)}\right)\log\left(e^{(i\,bx+i\,d)}+1\right)-3\left(e^{(4i\,bx+2\,a+2i\,d)}-2\,e^{(2i\,bx+2\,a)}+e^{(2\,a-2i\,d)}\ldots\right.}{2\left(be^{(4i\,bx+4i\,d)}-2\,be^{(2i\,bx+2i\,d)}+b\right)}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(3*(e^(4*I*b*x + 2*a + 2*I*d) - 2*e^(2*I*b*x + 2*a) + e^(2*a - 2*I*d))
*log(e^(I*b*x + I*d) + 1) - 3*(e^(4*I*b*x + 2*a + 2*I*d) - 2*e^(2*I*b*x +
2*a) + e^(2*a - 2*I*d))*log(e^(I*b*x + I*d) - 1) + 10*e^(3*I*b*x + 2*a + I
*d) - 6*e^(I*b*x + 2*a - I*d))/(b*e^(4*I*b*x + 4*I*d) - 2*b*e^(2*I*b*x + 2
*I*d) + b)
```

Sympy [F]

$$\int e^{2(a+ibx)}\csc^3(d+bx)\,dx = e^{2a}\int e^{2ibx}\csc^3\left(bx+d\right)dx$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*I*b*x)*csc(b*x + d)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 902 vs. 2(84) = 168.

Time = 0.07 (sec) , antiderivative size = 902, normalized size of antiderivative = 7.84

$$\int e^{2(a+ibx)}\csc^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)^3,x, algorithm="maxima")
```

output
```
-(6*((cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(4*b*x + 5*d) - 2*(cos(2*d
)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(2*b*x + 3*d) + (cos(d)*e^(2*a) + I*e^(
2*a)*sin(d))*cos(2*d) + (I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*sin(4*b*x
+ 5*d) + 2*(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*sin(2*b*x + 3*d) + (-I
*cos(d)*e^(2*a) + e^(2*a)*sin(d))*sin(2*d))*arctan2(sin(b*x) + sin(d), cos
(b*x) - cos(d)) - 6*((cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(4*b*x + 5
*d) - 2*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(2*b*x + 3*d) + (cos(d)
*e^(2*a) + I*e^(2*a)*sin(d))*cos(2*d) - (-I*cos(2*d)*e^(2*a) - e^(2*a)*sin
(2*d))*sin(4*b*x + 5*d) - 2*(I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*sin(2*
b*x + 3*d) - (I*cos(d)*e^(2*a) - e^(2*a)*sin(d))*sin(2*d))*arctan2(sin(b*x
) - sin(d), cos(b*x) + cos(d)) + 20*I*cos(3*b*x + 2*d)*e^(2*a) - 12*I*cos(
b*x)*e^(2*a) + 3*((I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*cos(4*b*x + 5*d)
 + 2*(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*cos(2*b*x + 3*d) + (I*cos(d)
*e^(2*a) - e^(2*a)*sin(d))*cos(2*d) - (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*
d))*sin(4*b*x + 5*d) + 2*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(2*b*x
 + 3*d) + (cos(d)*e^(2*a) + I*e^(2*a)*sin(d))*sin(2*d))*log(cos(b*x)^2 + 2
*cos(b*x)*cos(d) + cos(d)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(d) + sin(d)^2) +
 3*((-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*cos(4*b*x + 5*d) + 2*(I*cos(2
*d)*e^(2*a) + e^(2*a)*sin(2*d))*cos(2*b*x + 3*d) + (-I*cos(d)*e^(2*a) + e^
(2*a)*sin(d))*cos(2*d) + (cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*sin(4*...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(84) = 168$.

Time = 0.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.84

$$\int e^{2(a+ibx)}\csc^3(d+bx)\,dx$$
$$= \frac{3\,e^{(4i\,bx+2\,a+2i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+i\right)-6\,e^{(2i\,bx+2\,a)}\log\left(i\,e^{(i\,bx+i\,d)}+i\right)+3\,e^{(2\,a-2i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}+i\right)-3}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)^3,x, algorithm="giac")
```

output
```
1/2*(3*e^(4*I*b*x + 2*a + 2*I*d)*log(I*e^(I*b*x + I*d) + I) - 6*e^(2*I*b*x
 + 2*a)*log(I*e^(I*b*x + I*d) + I) + 3*e^(2*a - 2*I*d)*log(I*e^(I*b*x + I*
d) + I) - 3*e^(4*I*b*x + 2*a + 2*I*d)*log(-I*e^(I*b*x + I*d) + I) + 6*e^(2
*I*b*x + 2*a)*log(-I*e^(I*b*x + I*d) + I) - 3*e^(2*a - 2*I*d)*log(-I*e^(I*
b*x + I*d) + I) + 10*e^(3*I*b*x + 2*a + I*d) - 6*e^(I*b*x + 2*a - I*d))/(b
*(e^(4*I*b*x + 4*I*d) - 2*e^(2*I*b*x + 2*I*d) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+ibx)}\csc^3(d+bx)\,dx=\int \frac{e^{2\,a+b\,x\,2\mathrm{i}}}{\sin\left(d+b\,x\right)^3}\,dx$$

input
```
int(exp(2*a + b*x*2i)/sin(d + b*x)^3,x)
```

output
```
int(exp(2*a + b*x*2i)/sin(d + b*x)^3, x)
```

Reduce [F]

$$\int e^{2(a+ibx)}\csc^3(d+bx)\,dx=e^{2a}\left(\int e^{2bix}\csc\left(bx+d\right)^3dx\right)$$

input
```
int(exp(2*a+2*I*b*x)*csc(b*x+d)^3,x)
```

output
```
e**(2*a)*int(e**(2*b*i*x)*csc(b*x + d)**3,x)
```

3.105 $\int e^{2(a+ibx)}\csc^4(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int e^{2(a+ibx)}\csc^4(d+bx)\,dx=-\frac{8ie^{2(a-id)+6i(d+bx)}}{3b\left(1-e^{2i(d+bx)}\right)^3}$$

output
```
-8/3*I*exp(2*a-2*I*d+6*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 105 vs. 2(47) = 94.

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.23

$$\int e^{2(a+ibx)}\csc^4(d+bx)\,dx = \frac{8e^{2a}\left(-3e^{2ibx}+\left(1+3e^{4ibx}\right)\cos(2d)+i\left(-1+3e^{4ibx}\right)\sin(2d)\right)(i\cos(3d)+\sin(3d))}{3b\left(\left(-1+e^{2ibx}\right)\cos(d)+i\left(1+e^{2ibx}\right)\sin(d)\right)^3}$$

input
```
Integrate[E^(2*(a + I*b*x))*Csc[d + b*x]^4,x]
```

output
```
(8*E^(2*a)*(-3*E^((2*I)*b*x) + (1 + 3*E^((4*I)*b*x))*Cos[2*d] + I*(-1 + 3*
E^((4*I)*b*x))*Sin[2*d])*(I*Cos[3*d] + Sin[3*d]))/(3*b*((-1 + E^((2*I)*b*x
))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d])^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {4947}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)} \csc^4(bx+d) \, dx$$

$$\Big\downarrow \text{4947}$$

$$\frac{e^{2(a+ibx)} \cot(bx+d) \csc^2(bx+d)}{3b} - \frac{ie^{2(a+ibx)} \csc^2(bx+d)}{3b}$$

input
```
Int[E^(2*(a + I*b*x))*Csc[d + b*x]^4,x]
```

output
```
((-1/3*I)*E^(2*(a + I*b*x))*Csc[d + b*x]^2)/b + (E^(2*(a + I*b*x))*Cot[d +
 b*x]*Csc[d + b*x]^2)/(3*b)
```

Defintions of rubi rules used

rule 4947
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/(e
*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[b^2*c^2*Log[F]^2 +
 e^2*(n - 2)^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

method	result
risch	$\frac{8i\,\mathrm{e}^{6ibx+4id+2a}}{3\left(-1+\mathrm{e}^{2i(bx+d)}\right)^{3} b}$
parallelrisch	$-\frac{\mathrm{e}^{2ibx+2a}(i\sin(bx+d)+\cos(bx+d))\sec\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}\csc\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}}{24b}$
norman	$\frac{-\frac{\mathrm{e}^{2ibx+2a}}{24b}-\frac{\mathrm{e}^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}}{24b}+\frac{\mathrm{e}^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{4}}{24b}+\frac{\mathrm{e}^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{6}}{24b}-\frac{i\mathrm{e}^{2ibx+2a}\tan\left(\frac{bx}{2}+\frac{d}{2}\right)}{12b}-\frac{i\mathrm{e}^{2ibx+2a}\tan\left(\frac{b}{2}\right.}{6b}}{\tan\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}}$

input
```
int(exp(2*a+2*I*b*x)*csc(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
8/3*I/(-1+exp(2*I*(b*x+d)))^3/b*exp(6*I*b*x+4*I*d+2*a)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(30) = 60.

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int e^{2(a+ibx)}\csc^4(d+bx)\,dx = -\frac{8\left(-3i\,e^{(4i\,bx+2\,a+2i\,d)}+3i\,e^{(2i\,bx+2\,a)}-i\,e^{(2\,a-2i\,d)}\right)}{3\left(be^{(6i\,bx+6i\,d)}-3\,be^{(4i\,bx+4i\,d)}+3\,be^{(2i\,bx+2i\,d)}-b\right)}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)^4,x, algorithm="fricas")
```

output
```
-8/3*(-3*I*e^(4*I*b*x + 2*a + 2*I*d) + 3*I*e^(2*I*b*x + 2*a) - I*e^(2*a -
2*I*d))/(b*e^(6*I*b*x + 6*I*d) - 3*b*e^(4*I*b*x + 4*I*d) + 3*b*e^(2*I*b*x
+ 2*I*d) - b)
```

Sympy [F]

$$\int e^{2(a+ibx)}\csc^4(d+bx)\,dx = e^{2a}\int e^{2ibx}\csc^4(bx+d)\,dx$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)**4,x)
```

output
```
exp(2*a)*Integral(exp(2*I*b*x)*csc(b*x + d)**4, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(30) = 60$.

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.32

$$\int e^{2(a+ibx)}\csc^4(d+bx)\,dx = \frac{8\left(3\cos\left(4\,bx+4\,d\right)e^{(2\,a)}-3\cos\left(2\,bx+2\,d\right)e^{(2\,a)}+3i\,e^{(2\,a)}\sin\left(4\,bx+4\,d\right)-3i\right.}{-3i\,b\cos\left(6\,bx+8\,d\right)+9i\,b\cos\left(4\,bx+6\,d\right)-9i\,b\cos\left(2\,bx+4\,d\right)+3i\,b\cos\left(2\,d\right)+3\,b\sin\left(6\,bx+8\,d\right)-9}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)^4,x, algorithm="maxima")
```

output
```
8*(3*cos(4*b*x + 4*d)*e^(2*a) - 3*cos(2*b*x + 2*d)*e^(2*a) + 3*I*e^(2*a)*s
in(4*b*x + 4*d) - 3*I*e^(2*a)*sin(2*b*x + 2*d) + e^(2*a))/(-3*I*b*cos(6*b*
x + 8*d) + 9*I*b*cos(4*b*x + 6*d) - 9*I*b*cos(2*b*x + 4*d) + 3*I*b*cos(2*d
) + 3*b*sin(6*b*x + 8*d) - 9*b*sin(4*b*x + 6*d) + 9*b*sin(2*b*x + 4*d) - 3
*b*sin(2*d))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int e^{2(a+ibx)}\csc^4(d+bx)\,dx = -\frac{8\left(-3i\,e^{(4i\,bx+2\,a+2i\,d)}+3i\,e^{(2i\,bx+2\,a)}-i\,e^{(2\,a-2i\,d)}\right)}{3\,b(e^{(6i\,bx+6i\,d)}-3\,e^{(4i\,bx+4i\,d)}+3\,e^{(2i\,bx+2i\,d)}-1)}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)^4,x, algorithm="giac")
```

output
```
-8/3*(-3*I*e^(4*I*b*x + 2*a + 2*I*d) + 3*I*e^(2*I*b*x + 2*a) - I*e^(2*a -
2*I*d))/(b*(e^(6*I*b*x + 6*I*d) - 3*e^(4*I*b*x + 4*I*d) + 3*e^(2*I*b*x + 2
*I*d) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+ibx)}\csc^4(d+bx)\,dx = \int \frac{e^{2\,a+b\,x\,2i}}{\sin\left(d+b\,x\right)^4}\,dx$$

input
```
int(exp(2*a + b*x*2i)/sin(d + b*x)^4,x)
```

output
```
int(exp(2*a + b*x*2i)/sin(d + b*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int e^{2(a+ibx)}\csc^4(d+bx)\,dx = -\frac{e^{2bix+2a}(\cos(bx+d)+\sin(bx+d)\,i)}{3\sin(bx+d)^3\,b}$$

input
```
int(exp(2*a+2*I*b*x)*csc(b*x+d)^4,x)
```

output
```
( - e**(2*a + 2*b*i*x)*(cos(b*x + d) + sin(b*x + d)*i))/(3*sin(b*x + d)**3
*b)
```

3.106 $\int e^{2(a+ibx)}\csc^5(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 209

$$\int e^{2(a+ibx)}\csc^5(d+bx)\,dx=-\frac{4e^{2(a-id)+5i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)^4}+\frac{10e^{2(a-id)+3i(d+bx)}}{3b\left(1-e^{2i(d+bx)}\right)^3}-\frac{5e^{2(a-id)+i(d+bx)}}{2b\left(1-e^{2i(d+bx)}\right)^2}+\frac{5e^{2(a-id)+i(d+bx)}}{4b\left(1-e^{2i(d+bx)}\right)}+\frac{5e^{2a-2id}\text{arctanh}\left(e^{i(d+bx)}\right)}{4b}$$

output
```
-4*exp(2*a-2*I*d+5*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^4+10/3*exp(2*a-2*I*d+
3*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^3-5/2*exp(2*a-2*I*d+I*(b*x+d))/b/(1-ex
p(2*I*(b*x+d)))^2+5/4*exp(2*a-2*I*d+I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))+5/4*
exp(2*a-2*I*d)*arctanh(exp(I*(b*x+d)))/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 493 vs. 2(209) = 418.

Time = 0.97 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.36

$$\int e^{2(a+ibx)}\csc^5(d+bx)\,dx$$

$$=\frac{e^{2a}\left(30i\arctan\left(\frac{(-1+e^{ibx})\tan\left(\frac{d}{2}\right)}{1+e^{ibx}}\right)\cos(2d)-30i\arctan\left(\frac{(1+e^{ibx})\tan\left(\frac{d}{2}\right)}{-1+e^{ibx}}\right)\cos(2d)-15\cos(2d)\log\left(1+e^{2\cdots}\right.\right.}{}$$

input
```
Integrate[E^(2*(a + I*b*x))*Csc[d + b*x]^5,x]
```

output
```
(E^(2*a)*((30*I)*ArcTan[((-1 + E^(I*b*x))*Tan[d/2])/(1 + E^(I*b*x))]*Cos[2
*d] - (30*I)*ArcTan[((1 + E^(I*b*x))*Tan[d/2])/(-1 + E^(I*b*x))]*Cos[2*d]
- 15*Cos[2*d]*Log[1 + E^((2*I)*b*x) - 2*E^(I*b*x)*Cos[d]] + 15*Cos[2*d]*Lo
g[1 + E^((2*I)*b*x) + 2*E^(I*b*x)*Cos[d]] - (192*E^(I*b*x)*(Cos[d] - I*Sin
[d])^5)/((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d])^4 - (
544*E^(I*b*x)*(Cos[d] - I*Sin[d])^4)/((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 +
 E^((2*I)*b*x))*Sin[d])^3 - (472*E^(I*b*x)*(Cos[d] - I*Sin[d])^3)/((-1 + E
^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d])^2 - (60*E^(I*b*x)*(Co
s[d] - I*Sin[d])^2)/((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*S
in[d]) + 30*ArcTan[((-1 + E^(I*b*x))*Tan[d/2])/(1 + E^(I*b*x))]*Sin[2*d] -
 30*ArcTan[((1 + E^(I*b*x))*Tan[d/2])/(-1 + E^(I*b*x))]*Sin[2*d] + (15*I)*
Log[1 + E^((2*I)*b*x) - 2*E^(I*b*x)*Cos[d]]*Sin[2*d] - (15*I)*Log[1 + E^((
2*I)*b*x) + 2*E^(I*b*x)*Cos[d]]*Sin[2*d]))/(48*b)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4949, 4949, 4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+ibx)}\csc^5(bx+d)\,dx$$

$$\Big\downarrow \text{4949}$$

$$\frac{5}{12}\int e^{2(a+ibx)}\csc^3(d+bx)dx-\frac{ie^{2(a+ibx)}\csc^3(bx+d)}{6b}-\frac{e^{2(a+ibx)}\cot(bx+d)\csc^3(bx+d)}{4b}$$

$$\Big\downarrow \text{4949}$$

$$\frac{5}{12}\left(-\frac{3}{2}\int e^{2(a+ibx)}\csc(d+bx)dx-\frac{ie^{2(a+ibx)}\csc(bx+d)}{b}-\frac{e^{2(a+ibx)}\cot(bx+d)\csc(bx+d)}{2b}\right)-\frac{ie^{2(a+ibx)}\csc^3(bx+d)}{6b}-\frac{e^{2(a+ibx)}\cot(bx+d)\csc^3(bx+d)}{4b}$$

$$\Big\downarrow \text{4953}$$

$$\frac{5}{12}\left(-\frac{3e^{2(a+ibx)-3i(bx+d)}\left(e^{2i(bx+d)}-e^{i(bx+d)}\text{arctanh}\left(e^{i(bx+d)}\right)\right)}{b}-\frac{ie^{2(a+ibx)}\csc(bx+d)}{b}-\frac{e^{2(a+ibx)}\cot(bx+d)\,\text{c}}{2b}\right.$$
$$\frac{ie^{2(a+ibx)}\csc^3(bx+d)}{6b}-\frac{e^{2(a+ibx)}\cot(bx+d)\csc^3(bx+d)}{4b}$$

input
```
Int[E^(2*(a + I*b*x))*Csc[d + b*x]^5,x]
```

output
```
((-1/6*I)*E^(2*(a + I*b*x))*Csc[d + b*x]^3)/b - (E^(2*(a + I*b*x))*Cot[d +
 b*x]*Csc[d + b*x]^3)/(4*b) + (5*((-3*E^(2*(a + I*b*x) - (3*I)*(d + b*x))*
(E^((2*I)*(d + b*x)) - E^(I*(d + b*x))*ArcTanh[E^(I*(d + b*x))]))/b - (I*E
^(2*(a + I*b*x))*Csc[d + b*x])/b - (E^(2*(a + I*b*x))*Cot[d + b*x]*Csc[d +
 b*x])/(2*b)))/12
```

Defintions of rubi rules used

rule 4949
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{15\,\mathrm{e}^{9ibx}\mathrm{e}^{7id}\mathrm{e}^{2a}-75\,\mathrm{e}^{7ibx}\mathrm{e}^{5id}\mathrm{e}^{2a}+17\,\mathrm{e}^{5ibx}\mathrm{e}^{3id}\mathrm{e}^{2a}-5\,\mathrm{e}^{3ibx}\mathrm{e}^{id}\mathrm{e}^{2a}}{12\left(-1+\mathrm{e}^{2i(bx+d)}\right)^{4}b}+\frac{5\,\mathrm{e}^{2a}\mathrm{e}^{-2id}\,\mathrm{arctanh}\left(\mathrm{e}^{i(bx+d)}\right)}{4b}-\frac{5\,\mathrm{e}^{2a}\mathrm{e}^{-id}\mathrm{e}^{ibx}}{4b}$	133

input
```
int(exp(2*a+2*I*b*x)*csc(b*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
1/12/(-1+exp(2*I*(b*x+d)))^4/b*(15*exp(9*I*b*x)*exp(7*I*d)*exp(2*a)-75*exp
(7*I*b*x)*exp(5*I*d)*exp(2*a)+17*exp(5*I*b*x)*exp(3*I*d)*exp(2*a)-5*exp(3*
I*b*x)*exp(I*d)*exp(2*a))+5/4/b*exp(2*a)*exp(-2*I*d)*arctanh(exp(I*(b*x+d)
))-5/4/b*exp(2*a)*exp(-I*d)*exp(I*b*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.23

$$\int e^{2(a+ibx)}\csc^5(d+bx)\,dx = \frac{15\left(e^{(8i\,bx+2\,a+6i\,d)}-4\,e^{(6i\,bx+2\,a+4i\,d)}+6\,e^{(4i\,bx+2\,a+2i\,d)}-4\,e^{(2i\,bx+2\,a)}+e^{(2\,a-2i\,d)}\right)\log\left(e^{(i\,bx+i\,d)}+1\right)-15\,(}{}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)^5,x, algorithm="fricas")
```

output
```
1/24*(15*(e^(8*I*b*x + 2*a + 6*I*d) - 4*e^(6*I*b*x + 2*a + 4*I*d) + 6*e^(4
*I*b*x + 2*a + 2*I*d) - 4*e^(2*I*b*x + 2*a) + e^(2*a - 2*I*d))*log(e^(I*b*
x + I*d) + 1) - 15*(e^(8*I*b*x + 2*a + 6*I*d) - 4*e^(6*I*b*x + 2*a + 4*I*d
) + 6*e^(4*I*b*x + 2*a + 2*I*d) - 4*e^(2*I*b*x + 2*a) + e^(2*a - 2*I*d))*l
og(e^(I*b*x + I*d) - 1) - 30*e^(7*I*b*x + 2*a + 5*I*d) - 146*e^(5*I*b*x +
2*a + 3*I*d) + 110*e^(3*I*b*x + 2*a + I*d) - 30*e^(I*b*x + 2*a - I*d))/(b*
e^(8*I*b*x + 8*I*d) - 4*b*e^(6*I*b*x + 6*I*d) + 6*b*e^(4*I*b*x + 4*I*d) -
4*b*e^(2*I*b*x + 2*I*d) + b)
```

Sympy [F]

$$\int e^{2(a+ibx)}\csc^5(d+bx)\,dx = e^{2a}\int e^{2ibx}\csc^5(bx+d)\,dx$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)**5,x)
```

output
```
exp(2*a)*Integral(exp(2*I*b*x)*csc(b*x + d)**5, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1510 vs. $2(144) = 288$.

Time = 0.11 (sec) , antiderivative size = 1510, normalized size of antiderivative = 7.22

$$\int e^{2(a+ibx)}\csc^5(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)^5,x, algorithm="maxima")
```

output
```
-(30*((cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(8*b*x + 9*d) - 4*(cos(2*
d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(6*b*x + 7*d) + 6*(cos(2*d)*e^(2*a) -
I*e^(2*a)*sin(2*d))*cos(4*b*x + 5*d) - 4*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin
(2*d))*cos(2*b*x + 3*d) + (cos(d)*e^(2*a) + I*e^(2*a)*sin(d))*cos(2*d) + (
I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*sin(8*b*x + 9*d) + 4*(-I*cos(2*d)*e
^(2*a) - e^(2*a)*sin(2*d))*sin(6*b*x + 7*d) + 6*(I*cos(2*d)*e^(2*a) + e^(2
*a)*sin(2*d))*sin(4*b*x + 5*d) + 4*(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d)
)*sin(2*b*x + 3*d) + (-I*cos(d)*e^(2*a) + e^(2*a)*sin(d))*sin(2*d))*arctan
2(sin(b*x) + sin(d), cos(b*x) - cos(d)) - 30*((cos(2*d)*e^(2*a) - I*e^(2*a
)*sin(2*d))*cos(8*b*x + 9*d) - 4*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*c
os(6*b*x + 7*d) + 6*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(4*b*x + 5*
d) - 4*(cos(2*d)*e^(2*a) - I*e^(2*a)*sin(2*d))*cos(2*b*x + 3*d) + (cos(d)*
e^(2*a) + I*e^(2*a)*sin(d))*cos(2*d) - (-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(
2*d))*sin(8*b*x + 9*d) - 4*(I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*sin(6*b
*x + 7*d) - 6*(-I*cos(2*d)*e^(2*a) - e^(2*a)*sin(2*d))*sin(4*b*x + 5*d) -
4*(I*cos(2*d)*e^(2*a) + e^(2*a)*sin(2*d))*sin(2*b*x + 3*d) - (I*cos(d)*e^(
2*a) - e^(2*a)*sin(d))*sin(2*d))*arctan2(sin(b*x) - sin(d), cos(b*x) + cos
(d)) - 60*I*cos(7*b*x + 6*d)*e^(2*a) - 292*I*cos(5*b*x + 4*d)*e^(2*a) + 22
0*I*cos(3*b*x + 2*d)*e^(2*a) - 60*I*cos(b*x)*e^(2*a) + 15*((I*cos(2*d)*e^(
2*a) + e^(2*a)*sin(2*d))*cos(8*b*x + 9*d) + 4*(-I*cos(2*d)*e^(2*a) - e^...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(144) = 288.

Time = 0.15 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.79

$$\int e^{2(a+ibx)}\csc^5(d+bx)\,dx = -\frac{15\,e^{(8i\,bx+2\,a+6i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}-i\right)-60\,e^{(6i\,bx+2\,a+4i\,d)}\log\left(i\,e^{(i\,bx+i\,d)}-i\right)+90\,e^{(4i\,bx+2\,a+2i\,d)}\log\left(i\,e^{(i\,\ldots}\right.}{\ldots}$$

input
```
integrate(exp(2*a+2*I*b*x)*csc(b*x+d)^5,x, algorithm="giac")
```

output
```
-1/24*(15*e^(8*I*b*x + 2*a + 6*I*d)*log(I*e^(I*b*x + I*d) - I) - 60*e^(6*I
*b*x + 2*a + 4*I*d)*log(I*e^(I*b*x + I*d) - I) + 90*e^(4*I*b*x + 2*a + 2*I
*d)*log(I*e^(I*b*x + I*d) - I) - 60*e^(2*I*b*x + 2*a)*log(I*e^(I*b*x + I*d
) - I) + 15*e^(2*a - 2*I*d)*log(I*e^(I*b*x + I*d) - I) - 15*e^(8*I*b*x + 2
*a + 6*I*d)*log(-I*e^(I*b*x + I*d) - I) + 60*e^(6*I*b*x + 2*a + 4*I*d)*log
(-I*e^(I*b*x + I*d) - I) - 90*e^(4*I*b*x + 2*a + 2*I*d)*log(-I*e^(I*b*x +
I*d) - I) + 60*e^(2*I*b*x + 2*a)*log(-I*e^(I*b*x + I*d) - I) - 15*e^(2*a -
 2*I*d)*log(-I*e^(I*b*x + I*d) - I) + 30*e^(7*I*b*x + 2*a + 5*I*d) + 146*e
^(5*I*b*x + 2*a + 3*I*d) - 110*e^(3*I*b*x + 2*a + I*d) + 30*e^(I*b*x + 2*a
 - I*d))/(b*(e^(8*I*b*x + 8*I*d) - 4*e^(6*I*b*x + 6*I*d) + 6*e^(4*I*b*x +
4*I*d) - 4*e^(2*I*b*x + 2*I*d) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+ibx)}\csc^5(d+bx)\,dx=\int \frac{e^{2\,a+b\,x\,2i}}{\sin\left(d+b\,x\right)^5}\,dx$$

input
```
int(exp(2*a + b*x*2i)/sin(d + b*x)^5,x)
```

output
```
int(exp(2*a + b*x*2i)/sin(d + b*x)^5, x)
```

Reduce [F]

$$\int e^{2(a+ibx)}\csc^5(d+bx)\,dx=e^{2a}\left(\int e^{2bix}\csc\left(bx+d\right)^5dx\right)$$

input
```
int(exp(2*a+2*I*b*x)*csc(b*x+d)^5,x)
```

output
```
e**(2*a)*int(e**(2*b*i*x)*csc(b*x + d)**5,x)
```

3.107 $\int e^{\frac{5}{3}(a+ibx)}\csc(d+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 162

$$\int e^{\frac{5}{3}(a+ibx)}\csc(d+bx)\,dx=\frac{3e^{\frac{5}{3}(a-id)+\frac{2}{3}i(d+bx)}}{b}-\frac{\sqrt{3}e^{\frac{5}{3}(a-id)}\arctan\left(\frac{1+2e^{\frac{2}{3}i(d+bx)}}{\sqrt{3}}\right)}{b}+\frac{e^{\frac{5}{3}(a-id)}\log\left(1-e^{\frac{2}{3}i(d+bx)}\right)}{b}-\frac{e^{\frac{5}{3}(a-id)}\log\left(1+e^{\frac{2}{3}i(d+bx)}+e^{\frac{4}{3}i(d+bx)}\right)}{2b}$$

output
```
3*exp(5/3*a-5/3*I*d+2/3*I*(b*x+d))/b-3^(1/2)*exp(5/3*a-5/3*I*d)*arctan(1/3
*(1+2*exp(2/3*I*(b*x+d)))*3^(1/2))/b+exp(5/3*a-5/3*I*d)*ln(1-exp(2/3*I*(b*
x+d)))/b-1/2*exp(5/3*a-5/3*I*d)*ln(1+exp(2/3*I*(b*x+d))+exp(4/3*I*(b*x+d))
)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.32

$$\int e^{\frac{5}{3}(a+ibx)}\csc(d+bx)\,dx$$
$$= \frac{e^{5a/3}\left(\left(9e^{\frac{2ibx}{3}}+\text{RootSum}\left[-\cos\left(\frac{d}{2}\right)+i\sin\left(\frac{d}{2}\right)+\cos\left(\frac{d}{2}\right)\#1^3+i\sin\left(\frac{d}{2}\right)\#1^3\&,\frac{bx+3i\log\left(e^{\frac{ibx}{3}}-\#1\right)}{\#1}\&\right]\right(\right.\right.}{}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Csc[d + b*x],x]
```

output
```
(E^((5*a)/3)*((9*E^(((2*I)/3)*b*x) + RootSum[-Cos[d/2] + I*Sin[d/2] + Cos[
d/2]*#1^3 + I*Sin[d/2]*#1^3 & , (b*x + (3*I)*Log[E^((I/3)*b*x) - #1])/#1 &
 ]*((-I)*Cos[d] - Sin[d]))*(Cos[d] - I*Sin[d]) + RootSum[Cos[d/2] - I*Sin[
d/2] + Cos[d/2]*#1^3 + I*Sin[d/2]*#1^3 & , (b*x + (3*I)*Log[E^((I/3)*b*x)
- #1])/#1 & ]*(I*Cos[2*d] + Sin[2*d])))/(3*b)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.31, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\csc(bx+d)\,dx$$
$$\Big\downarrow \text{4953}$$
$$-\frac{3e^{\frac{5}{3}(a+ibx)+i(bx+d)}\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},e^{2i(d+bx)}\right)}{4b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Csc[d + b*x],x]
```

output
```
(-3*E^((5*(a + I*b*x))/3 + I*(d + b*x))*Hypergeometric2F1[1, 4/3, 7/3, E^(
(2*I)*(d + b*x))])/(4*b)
```

Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [F]

$$\int e^{\frac{5a}{3}+\frac{5ibx}{3}} \csc(bx+d)\, dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d),x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.33

$$\int e^{\frac{5}{3}(a+ibx)} \csc(d+bx)\, dx = -\frac{\left(\sqrt{3}b\sqrt{-\frac{e^{\left(\frac{10}{3}\,a-\frac{10}{3}\,i\,d\right)}}{b^2}}+e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,i\,d\right)}\right)\log\left(\frac{1}{2}\left(\sqrt{3}b\sqrt{-\frac{e^{\left(\frac{10}{3}\,a-\frac{10}{3}\,i\,d\right)}}{b^2}}+2\,e^{\left(\frac{2}{3}\,i\,bx+\frac{5}{3}\,a-i\,d\right)}+e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,i\,d\right)}\right)e^{\left(-\frac{5}{3}\right.}}{}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d),x, algorithm="fricas")
```

output
```
-1/2*((sqrt(3)*b*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) + e^(5/3*a - 5/3*I*d))*l
og(1/2*(sqrt(3)*b*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) + 2*e^(2/3*I*b*x + 5/3*
a - I*d) + e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - (sqrt(3)*b*sqrt(-e
^(10/3*a - 10/3*I*d)/b^2) - e^(5/3*a - 5/3*I*d))*log(-1/2*(sqrt(3)*b*sqrt(
-e^(10/3*a - 10/3*I*d)/b^2) - 2*e^(2/3*I*b*x + 5/3*a - I*d) - e^(5/3*a - 5
/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 2*e^(5/3*a - 5/3*I*d)*log(e^(2/3*I*b*x +
2/3*I*d) - 1) - 6*e^(2/3*I*b*x + 5/3*a - I*d))/b
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+ibx)} \csc(d+bx)\,dx = e^{\frac{5a}{3}} \int e^{\frac{5ibx}{3}} \csc(bx+d)\,dx$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d),x)
```

output
```
exp(5*a/3)*Integral(exp(5*I*b*x/3)*csc(b*x + d), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3086 vs. $2(113) = 226$.

Time = 0.43 (sec) , antiderivative size = 3086, normalized size of antiderivative = 19.05

$$\int e^{\frac{5}{3}(a+ibx)} \csc(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d),x, algorithm="maxima")
```

output
```
-1/4*(2*((-I*cos(2*d)*e^(5/3*a) - e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*ar
ctan2(sin(d), cos(d))) + (cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*sin(-
1/3*pi + 1/3*arctan2(sin(d), cos(d))))*arctan2(cos(1/3*b*x)*sin(-1/3*pi +
1/3*arctan2(sin(d), cos(d))) - cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))*
sin(1/3*b*x) - sin(-2/3*pi + 2/3*arctan2(sin(d), cos(d))) + sin(2/3*b*x),
-cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))*cos(1/3*b*x) - sin(-1/3*pi + 1
/3*arctan2(sin(d), cos(d)))*sin(1/3*b*x) + cos(-2/3*pi + 2/3*arctan2(sin(d
), cos(d))) + cos(2/3*b*x)) + 2*((I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d
))*cos(1/3*arctan2(sin(d), cos(d))) - (cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*si
n(2*d))*sin(1/3*arctan2(sin(d), cos(d))))*arctan2(-cos(1/3*arctan2(sin(d),
 cos(d)))*sin(1/3*b*x) + cos(1/3*b*x)*sin(1/3*arctan2(sin(d), cos(d))) + s
in(2/3*b*x) - sin(2/3*arctan2(sin(d), cos(d))), -cos(1/3*b*x)*cos(1/3*arct
an2(sin(d), cos(d))) - sin(1/3*b*x)*sin(1/3*arctan2(sin(d), cos(d))) + cos
(2/3*b*x) + cos(2/3*arctan2(sin(d), cos(d)))) - 2*((sqrt(3)*cos(2*d)*e^(5/
3*a) - I*sqrt(3)*e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d), cos
(d))) - (-I*sqrt(3)*cos(2*d)*e^(5/3*a) - sqrt(3)*e^(5/3*a)*sin(2*d))*sin(-
1/3*pi + 1/3*arctan2(sin(d), cos(d))))*arctan2(1/2*(sqrt(3)*cos(-1/3*pi +
1/3*arctan2(sin(d), cos(d)))^2 - 2*sqrt(3)*cos(-1/3*pi + 1/3*arctan2(sin(d
), cos(d)))*cos(1/3*b*x) + sqrt(3)*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d
)))^2 + 2*sqrt(3)*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))*sin(1/3*b*...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2572 vs. $2(113) = 226$.

Time = 1.78 (sec) , antiderivative size = 2572, normalized size of antiderivative = 15.88

$$\int e^{\frac{5}{3}(a+ibx)}\csc(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d),x, algorithm="giac")
```

output
```
1/2*(2*e^(-20/3*I*d)*log(e^(2/3*I*b*x) - e^(-2/3*I*d)) + 2*((sqrt(3)*cos(d
)^6 - 15*sqrt(3)*cos(d)^4*sin(d)^2 + 15*sqrt(3)*cos(d)^2*sin(d)^4 - sqrt(3
)*sin(d)^6)*(cos(d)^4 + 2*cos(d)^2*sin(d)^2 + sin(d)^4)^(1/3)*cos(-2/3*pi*
floor(d/pi - 2*floor(1/2*d/pi + 1/2) + 1/2) - 4/3*pi*floor(1/2*d/pi + 1/2)
 + 2/3*d) + (cos(d)^6 - 15*cos(d)^4*sin(d)^2 + 15*cos(d)^2*sin(d)^4 - sin(
d)^6)*(cos(d)^4 + 2*cos(d)^2*sin(d)^2 + sin(d)^4)^(1/3)*sin(-2/3*pi*floor(
d/pi - 2*floor(1/2*d/pi + 1/2) + 1/2) - 4/3*pi*floor(1/2*d/pi + 1/2) + 2/3
*d))*arctan(1/3*sqrt(3)*(2*e^(2/3*I*b*x) + e^(-2/3*I*d))*e^(2/3*I*d))/((co
s(d)^14 + 7*cos(d)^12*sin(d)^2 + 21*cos(d)^10*sin(d)^4 + 35*cos(d)^8*sin(d
)^6 + 35*cos(d)^6*sin(d)^8 + 21*cos(d)^4*sin(d)^10 + 7*cos(d)^2*sin(d)^12
+ sin(d)^14)*cos(-2/3*pi*floor(d/pi - 2*floor(1/2*d/pi + 1/2) + 1/2) - 4/3
*pi*floor(1/2*d/pi + 1/2) + 2/3*d)^2 + (cos(d)^14 + 7*cos(d)^12*sin(d)^2 +
 21*cos(d)^10*sin(d)^4 + 35*cos(d)^8*sin(d)^6 + 35*cos(d)^6*sin(d)^8 + 21*
cos(d)^4*sin(d)^10 + 7*cos(d)^2*sin(d)^12 + sin(d)^14)*sin(-2/3*pi*floor(d
/pi - 2*floor(1/2*d/pi + 1/2) + 1/2) - 4/3*pi*floor(1/2*d/pi + 1/2) + 2/3*
d)^2) - 4*I*((3*sqrt(3)*cos(d)^5*sin(d) - 10*sqrt(3)*cos(d)^3*sin(d)^3 + 3
*sqrt(3)*cos(d)*sin(d)^5)*(cos(d)^4 + 2*cos(d)^2*sin(d)^2 + sin(d)^4)^(1/3
)*cos(-2/3*pi*floor(d/pi - 2*floor(1/2*d/pi + 1/2) + 1/2) - 4/3*pi*floor(1
/2*d/pi + 1/2) + 2/3*d) + (3*cos(d)^5*sin(d) - 10*cos(d)^3*sin(d)^3 + 3*co
s(d)*sin(d)^5)*(cos(d)^4 + 2*cos(d)^2*sin(d)^2 + sin(d)^4)^(1/3)*sin(-2...
```

Mupad [B] (verification not implemented)

Time = 18.92 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.49

$$\int e^{\frac{5}{3}(a+ibx)}\csc(d+bx)\,dx$$
$$= \frac{3\,e^{\frac{5\,a}{3}-d\,1i+\frac{b\,x\,2i}{3}}}{b} + \frac{\left(e^{5\,a-d\,5i}\right)^{1/3}\,\ln\left(e^{2\,a}\,e^{-d\,2i}\,\left(e^{5\,a}\,e^{-d\,5i}\right)^{1/3}\,2i - e^{3\,a}\,e^{\frac{2\,a}{3}}\,e^{-d\,3i}\,e^{\frac{b\,x\,2i}{3}}\,2i\right)}{b}$$
$$+ \frac{\ln\left(-e^{3\,a}\,e^{\frac{2\,a}{3}}\,e^{-d\,3i}\,e^{\frac{b\,x\,2i}{3}}\,2i + e^{2\,a}\,e^{-d\,2i}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\left(e^{5\,a}\,e^{-d\,5i}\right)^{1/3}\,2i\right)\,\left(e^{5\,a-d\,5i}\right)^{1/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{b}$$
$$- \frac{\ln\left(-e^{3\,a}\,e^{\frac{2\,a}{3}}\,e^{-d\,3i}\,e^{\frac{b\,x\,2i}{3}}\,2i - e^{2\,a}\,e^{-d\,2i}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\left(e^{5\,a}\,e^{-d\,5i}\right)^{1/3}\,2i\right)\,\left(e^{5\,a-d\,5i}\right)^{1/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{b}$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)/sin(d + b*x),x)
```

output
```
(3*exp((5*a)/3 - d*1i + (b*x*2i)/3))/b + (exp(5*a - d*5i)^(1/3)*log(exp(2*
a)*exp(-d*2i)*(exp(5*a)*exp(-d*5i))^(1/3)*2i - exp(3*a)*exp((2*a)/3)*exp(-
d*3i)*exp((b*x*2i)/3)*2i))/b + (log(exp(2*a)*exp(-d*2i)*((3^(1/2)*1i)/2 -
1/2)*(exp(5*a)*exp(-d*5i))^(1/3)*2i - exp(3*a)*exp((2*a)/3)*exp(-d*3i)*exp
((b*x*2i)/3)*2i)*exp(5*a - d*5i)^(1/3)*((3^(1/2)*1i)/2 - 1/2))/b - (log(-
exp(3*a)*exp((2*a)/3)*exp(-d*3i)*exp((b*x*2i)/3)*2i - exp(2*a)*exp(-d*2i)*
((3^(1/2)*1i)/2 + 1/2)*(exp(5*a)*exp(-d*5i))^(1/3)*2i)*exp(5*a - d*5i)^(1/
3)*((3^(1/2)*1i)/2 + 1/2))/b
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\csc(d+bx)\,dx=\int e^{\frac{5bix}{3}+\frac{5a}{3}}\csc\left(bx+d\right)dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d),x)
```

output
```
int(e**((5*a + 5*b*i*x)/3)*csc(b*x + d),x)
```

3.108 $\int e^{\frac{5}{3}(a+ibx)}\csc^2(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 238

$$\int e^{\frac{5}{3}(a+ibx)}\csc^2(d+bx)\,dx = \frac{2ie^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)} - \frac{5ie^{\frac{5}{3}(a-id)}\arctan\left(\frac{1-2e^{\frac{1}{3}i(d+bx)}}{\sqrt{3}}\right)}{\sqrt{3}b} + \frac{5ie^{\frac{5}{3}(a-id)}\arctan\left(\frac{1+2e^{\frac{1}{3}i(d+bx)}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{10ie^{\frac{5}{3}(a-id)}\operatorname{arctanh}\left(e^{\frac{1}{3}i(d+bx)}\right)}{3b} - \frac{5ie^{\frac{5}{3}(a-id)}\operatorname{arctanh}\left(\frac{e^{\frac{1}{3}i(d+bx)}}{1+e^{\frac{2}{3}i(d+bx)}}\right)}{3b}$$

output
```
2*I*exp(5/3*a-5/3*I*d+5/3*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))-5/3*I*exp(5/3*
a-5/3*I*d)*arctan(1/3*(1-2*exp(1/3*I*(b*x+d)))*3^(1/2))*3^(1/2)/b+5/3*I*ex
p(5/3*a-5/3*I*d)*arctan(1/3*(1+2*exp(1/3*I*(b*x+d)))*3^(1/2))*3^(1/2)/b-10
/3*I*exp(5/3*a-5/3*I*d)*arctanh(exp(1/3*I*(b*x+d)))/b-5/3*I*exp(5/3*a-5/3*
I*d)*arctanh(exp(1/3*I*(b*x+d))/(1+exp(2/3*I*(b*x+d))))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.60

$$\int e^{\frac{5}{3}(a+ibx)}\csc^2(d+bx)\,dx = \frac{1}{9}e^{5a/3}(\cos(d) - i\sin(d))\left(\frac{5\text{RootSum}\left[-\cos(d)+i\sin(d)+\cos(d)\#1^6+i\sin(d)\#1^6\&,\frac{bx+3i\log\left(e^{\frac{ibx}{3}}-\#1\right)}{\#1}\&\right](\cos(d)}{b} + \frac{18e^{\frac{5ibx}{3}}}{ib\left(-1+e^{2ibx}\right)\cos(d)-b\left(1+e^{2ibx}\right)\sin(d)}\right)$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Csc[d + b*x]^2,x]
```

output
```
(E^((5*a)/3)*(Cos[d] - I*Sin[d])*((5*RootSum[-Cos[d] + I*Sin[d] + Cos[d]*#
1^6 + I*Sin[d]*#1^6 & , (b*x + (3*I)*Log[E^((I/3)*b*x) - #1])/#1 & ]*(Cos[
d] - I*Sin[d]))/b + (18*E^(((5*I)/3)*b*x))/(I*b*(-1 + E^((2*I)*b*x))*Cos[d
] - b*(1 + E^((2*I)*b*x))*Sin[d])))/9
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.22, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{4953\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\csc^2(bx+d)\,dx$$

$$\Big\downarrow \text{4953}$$

$$\frac{12ie^{\frac{5}{3}(a+ibx)+2i(bx+d)}\,\text{Hypergeometric2F1}\left(\frac{11}{6},2,\frac{17}{6},e^{2i(d+bx)}\right)}{11b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Csc[d + b*x]^2,x]
```

output
```
(((12*I)/11)*E^((5*(a + I*b*x))/3 + (2*I)*(d + b*x))*Hypergeometric2F1[11/
6, 2, 17/6, E^((2*I)*(d + b*x))])/b
```

Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [F]

$$\int e^{\frac{5a}{3}+\frac{5ibx}{3}}\csc\left(bx+d\right)^2dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^2,x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(158) = 316$.

Time = 0.09 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.37

$$\int e^{\frac{5}{3}(a+ibx)} \csc^2(d+bx) \, dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/6*(5*(3*sqrt(1/3)*(b*e^(2*I*b*x + 2*I*d) - b)*sqrt(b^(-2))*e^(5/3*a - 5/
3*I*d) + I*e^(2*I*b*x + 5/3*a + 1/3*I*d) - I*e^(5/3*a - 5/3*I*d))*log(-1/2
*(3*I*sqrt(1/3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) - 2*e^(1/3*I*b*x + 5/3*
a - 4/3*I*d) + e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) + 5*(3*sqrt(1/3)
*(b*e^(2*I*b*x + 2*I*d) - b)*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) - I*e^(2*I*b
*x + 5/3*a + 1/3*I*d) + I*e^(5/3*a - 5/3*I*d))*log(-1/2*(3*I*sqrt(1/3)*b*s
qrt(b^(-2))*e^(5/3*a - 5/3*I*d) - 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) - e^(5
/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 5*(3*sqrt(1/3)*(b*e^(2*I*b*x + 2*
I*d) - b)*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) - I*e^(2*I*b*x + 5/3*a + 1/3*I*
d) + I*e^(5/3*a - 5/3*I*d))*log(-1/2*(-3*I*sqrt(1/3)*b*sqrt(b^(-2))*e^(5/3
*a - 5/3*I*d) - 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) + e^(5/3*a - 5/3*I*d))*e
^(-5/3*a + 5/3*I*d)) - 5*(3*sqrt(1/3)*(b*e^(2*I*b*x + 2*I*d) - b)*sqrt(b^(
-2))*e^(5/3*a - 5/3*I*d) + I*e^(2*I*b*x + 5/3*a + 1/3*I*d) - I*e^(5/3*a -
5/3*I*d))*log(-1/2*(-3*I*sqrt(1/3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) - 2*
e^(1/3*I*b*x + 5/3*a - 4/3*I*d) - e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d
)) - 10*(I*e^(2*I*b*x + 5/3*a + 1/3*I*d) - I*e^(5/3*a - 5/3*I*d))*log(e^(1
/3*I*b*x + 1/3*I*d) + 1) - 10*(-I*e^(2*I*b*x + 5/3*a + 1/3*I*d) + I*e^(5/3
*a - 5/3*I*d))*log(e^(1/3*I*b*x + 1/3*I*d) - 1) - 12*I*e^(5/3*I*b*x + 5/3*
a))/(b*e^(2*I*b*x + 2*I*d) - b)
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+ibx)}\csc^2(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5ibx}{3}}\csc^2{(bx+d)}\,dx$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)**2,x)
```

output
```
exp(5*a/3)*Integral(exp(5*I*b*x/3)*csc(b*x + d)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4994 vs. 2(158) = 316.

Time = 0.49 (sec) , antiderivative size = 4994, normalized size of antiderivative = 20.98

$$\int e^{\frac{5}{3}(a+ibx)}\csc^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^2,x, algorithm="maxima")
```

output
```
-12*(10*((I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arc
tan2(sin(d), cos(d))) + ((-I*cos(2*d)*e^(5/3*a) - e^(5/3*a)*sin(2*d))*cos(
-1/3*pi + 1/3*arctan2(sin(d), cos(d))) + (cos(2*d)*e^(5/3*a) - I*e^(5/3*a)
*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*cos(2*b*x + 2*d) -
(cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(
d), cos(d))) + ((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos(-1/3*pi +
1/3*arctan2(sin(d), cos(d))) + (I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))
*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*sin(2*b*x + 2*d))*arctan2(cos
(1/3*b*x)*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) - cos(-1/3*pi + 1/3*a
rctan2(sin(d), cos(d)))*sin(1/3*b*x) - sin(-2/3*pi + 2/3*arctan2(sin(d), c
os(d))) + sin(2/3*b*x), -cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))*cos(1/
3*b*x) - sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))*sin(1/3*b*x) + cos(-2/
3*pi + 2/3*arctan2(sin(d), cos(d))) + cos(2/3*b*x)) + 10*(((-I*cos(2*d)*e^
(5/3*a) - e^(5/3*a)*sin(2*d))*cos(1/3*arctan2(sin(d), cos(d))) + (cos(2*d)
*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*sin(1/3*arctan2(sin(d), cos(d))))*cos(2
*b*x + 2*d) + (I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))*cos(1/3*arctan2(
sin(d), cos(d))) + ((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos(1/3*ar
ctan2(sin(d), cos(d))) + (I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))*sin(1
/3*arctan2(sin(d), cos(d))))*sin(2*b*x + 2*d) - (cos(2*d)*e^(5/3*a) - I*e^
(5/3*a)*sin(2*d))*sin(1/3*arctan2(sin(d), cos(d))))*arctan2(-cos(1/3*ar...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2675 vs. $2(158) = 316$.

Time = 4.16 (sec) , antiderivative size = 2675, normalized size of antiderivative = 11.24

$$\int e^{\frac{5}{3}(a+ibx)}\csc^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^2,x, algorithm="giac")
```

output
```
-1/6*I*(10*(-e^(-I*d))^(2/3)*e^(-3*I*d)*log(-(-e^(-I*d))^(1/3) + e^(1/3*I*
b*x)) - 10*e^(-11/3*I*d)*log(e^(1/3*I*b*x) - e^(-1/3*I*d)) - 10*(sqrt(3)*c
os(1/3*d)^2*cos(d)^3 - sqrt(3)*cos(d)^3*sin(1/3*d)^2 - 3*sqrt(3)*cos(1/3*d
)^2*cos(d)*sin(d)^2 + 3*sqrt(3)*cos(d)*sin(1/3*d)^2*sin(d)^2 - 2*cos(1/3*d
)*cos(d)^3*sin(1/3*d) + 6*cos(1/3*d)*cos(d)*sin(1/3*d)*sin(d)^2)*arctan(1/
3*sqrt(3)*(2*e^(1/3*I*b*x) + e^(-1/3*I*d))*e^(1/3*I*d))/(cos(1/3*d)^4*cos(
d)^6 + 2*cos(1/3*d)^2*cos(d)^6*sin(1/3*d)^2 + cos(d)^6*sin(1/3*d)^4 + 3*co
s(1/3*d)^4*cos(d)^4*sin(d)^2 + 6*cos(1/3*d)^2*cos(d)^4*sin(1/3*d)^2*sin(d)
^2 + 3*cos(d)^4*sin(1/3*d)^4*sin(d)^2 + 3*cos(1/3*d)^4*cos(d)^2*sin(d)^4 +
 6*cos(1/3*d)^2*cos(d)^2*sin(1/3*d)^2*sin(d)^4 + 3*cos(d)^2*sin(1/3*d)^4*s
in(d)^4 + cos(1/3*d)^4*sin(d)^6 + 2*cos(1/3*d)^2*sin(1/3*d)^2*sin(d)^6 + s
in(1/3*d)^4*sin(d)^6) + 10*I*(3*sqrt(3)*cos(1/3*d)^2*cos(d)^2*sin(d) - 3*s
qrt(3)*cos(d)^2*sin(1/3*d)^2*sin(d) - sqrt(3)*cos(1/3*d)^2*sin(d)^3 + sqrt
(3)*sin(1/3*d)^2*sin(d)^3 - 6*cos(1/3*d)*cos(d)^2*sin(1/3*d)*sin(d) + 2*co
s(1/3*d)*sin(1/3*d)*sin(d)^3)*arctan(1/3*sqrt(3)*(2*e^(1/3*I*b*x) + e^(-1/
3*I*d))*e^(1/3*I*d))/(cos(1/3*d)^4*cos(d)^6 + 2*cos(1/3*d)^2*cos(d)^6*sin(
1/3*d)^2 + cos(d)^6*sin(1/3*d)^4 + 3*cos(1/3*d)^4*cos(d)^4*sin(d)^2 + 6*co
s(1/3*d)^2*cos(d)^4*sin(1/3*d)^2*sin(d)^2 + 3*cos(d)^4*sin(1/3*d)^4*sin(d)
^2 + 3*cos(1/3*d)^4*cos(d)^2*sin(d)^4 + 6*cos(1/3*d)^2*cos(d)^2*sin(1/3*d)
^2*sin(d)^4 + 3*cos(d)^2*sin(1/3*d)^4*sin(d)^4 + cos(1/3*d)^4*sin(d)^6 ...
```

Mupad [B] (verification not implemented)

Time = 21.23 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.12

$$\int e^{\frac{5}{3}(a+ibx)}\csc^2(d+bx)\,dx = \text{Too large to display}$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)/sin(d + b*x)^2,x)
```

output
```
(5*(-exp(10*a - d*10i))^(1/6)*log(- (100*exp(6*a)*exp(-d*6i))/9 - (exp(a/3
)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*(-exp(10*a)*exp(-d*10i))^(1/6)*100i)
/9))/(3*b) - (5*(-exp(10*a - d*10i))^(1/6)*log((exp(a/3)*exp(4*a)*exp(-d*4
i)*exp((b*x*1i)/3)*(-exp(10*a)*exp(-d*10i))^(1/6)*100i)/9 - (100*exp(6*a)*
exp(-d*6i))/9))/(3*b) + (exp((11*a)/3 - d*2i + (b*x*5i)/3)*2i)/(b*(exp(2*a
 - d*2i) - exp(2*a + b*x*2i))) + (5*log(- (100*exp(6*a)*exp(-d*6i))/9 - (e
xp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 - 1/2)*(-exp(1
0*a)*exp(-d*10i))^(1/6)*100i)/9)*(-exp(10*a - d*10i))^(1/6)*((3^(1/2)*1i)/
2 - 1/2))/(3*b) - (5*log((exp(a/3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3
^(1/2)*1i)/2 - 1/2)*(-exp(10*a)*exp(-d*10i))^(1/6)*100i)/9 - (100*exp(6*a)
*exp(-d*6i))/9)*(-exp(10*a - d*10i))^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(3*b) +
 (5*log(- (100*exp(6*a)*exp(-d*6i))/9 - (exp(a/3)*exp(4*a)*exp(-d*4i)*exp(
(b*x*1i)/3)*((3^(1/2)*1i)/2 + 1/2)*(-exp(10*a)*exp(-d*10i))^(1/6)*100i)/9)
*(-exp(10*a - d*10i))^(1/6)*((3^(1/2)*1i)/2 + 1/2))/(3*b) - (5*log((exp(a/
3)*exp(4*a)*exp(-d*4i)*exp((b*x*1i)/3)*((3^(1/2)*1i)/2 + 1/2)*(-exp(10*a)*
exp(-d*10i))^(1/6)*100i)/9 - (100*exp(6*a)*exp(-d*6i))/9)*(-exp(10*a - d*1
0i))^(1/6)*((3^(1/2)*1i)/2 + 1/2))/(3*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\csc^2(d+bx)\,dx = \int e^{\frac{5bix}{3}+\frac{5a}{3}}\csc\left(bx+d\right)^2dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^2,x)
```

output
```
int(e**((5*a + 5*b*i*x)/3)*csc(b*x + d)**2,x)
```

3.109 $\int e^{\frac{5}{3}(a+ibx)} \csc^3(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 233

$$\int e^{\frac{5}{3}(a+ibx)} \csc^3(d+bx)\,dx = \frac{2e^{\frac{5}{3}(a-id)+\frac{8}{3}i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)^2} - \frac{8e^{\frac{5}{3}(a-id)+\frac{2}{3}i(d+bx)}}{3b\left(1-e^{2i(d+bx)}\right)} + \frac{8e^{\frac{5}{3}(a-id)}\arctan\left(\frac{1+2e^{\frac{2}{3}i(d+bx)}}{\sqrt{3}}\right)}{3\sqrt{3}b} - \frac{8e^{\frac{5}{3}(a-id)}\log\left(1-e^{\frac{2}{3}i(d+bx)}\right)}{9b} + \frac{4e^{\frac{5}{3}(a-id)}\log\left(1+e^{\frac{2}{3}i(d+bx)}+e^{\frac{4}{3}i(d+bx)}\right)}{9b}$$

output
```
2*exp(5/3*a-5/3*I*d+8/3*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^2-8/3*exp(5/3*a-
5/3*I*d+2/3*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))+8/9*3^(1/2)*exp(5/3*a-5/3*I*
d)*arctan(1/3*(1+2*exp(2/3*I*(b*x+d)))*3^(1/2))/b-8/9*exp(5/3*a-5/3*I*d)*l
n(1-exp(2/3*I*(b*x+d)))/b+4/9*exp(5/3*a-5/3*I*d)*ln(1+exp(2/3*I*(b*x+d))+e
xp(4/3*I*(b*x+d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.31

$$\int e^{\frac{5}{3}(a+ibx)}\csc^3(d+bx)\,dx$$
$$= \frac{2e^{5a/3}\left(-4i\text{RootSum}\left[\cos\left(\frac{d}{2}\right)-i\sin\left(\frac{d}{2}\right)+\cos\left(\frac{d}{2}\right)\#1^3+i\sin\left(\frac{d}{2}\right)\#1^3\&,\frac{bx+3i\log\left(e^{\frac{ibx}{3}}-\#1\right)}{\#1}\&\right](\cos(d)\ -}{}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Csc[d + b*x]^3,x]
```

output
```
(2*E^((5*a)/3)*((-4*I)*RootSum[Cos[d/2] - I*Sin[d/2] + Cos[d/2]*#1^3 + I*S
in[d/2]*#1^3 & , (b*x + (3*I)*Log[E^((I/3)*b*x) - #1])/#1 & ]*(Cos[d] - I*
Sin[d])^2 + (27*E^(((2*I)/3)*b*x)*(Cos[d] - I*Sin[d])^3)/((-1 + E^((2*I)*b
*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d])^2 + (63*E^(((2*I)/3)*b*x)*(Cos
[d] - I*Sin[d])^2)/((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Si
n[d]) + 4*RootSum[-Cos[d/2] + I*Sin[d/2] + Cos[d/2]*#1^3 + I*Sin[d/2]*#1^3
 & , (b*x + (3*I)*Log[E^((I/3)*b*x) - #1])/#1 & ]*(I*Cos[2*d] + Sin[2*d]))
)/(27*b)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.48, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4949, 4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\csc^3(bx+d)\,dx$$

$$\Big\downarrow \text{ 4949}$$

$$-\frac{8}{9}\int e^{\frac{5}{3}(a+ibx)}\csc(d+bx)dx-\frac{5ie^{\frac{5}{3}(a+ibx)}\csc(bx+d)}{6b}-\frac{e^{\frac{5}{3}(a+ibx)}\cot(bx+d)\csc(bx+d)}{2b}$$

$$\Big\downarrow \text{4953}$$

$$\frac{2e^{\frac{5}{3}(a+ibx)+i(bx+d)}\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},e^{2i(d+bx)}\right)}{3b}-\frac{5ie^{\frac{5}{3}(a+ibx)}\csc(bx+d)}{6b}-\frac{e^{\frac{5}{3}(a+ibx)}\cot(bx+d)\csc(bx+d)}{2b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Csc[d + b*x]^3,x]
```

output
```
(((-5*I)/6)*E^((5*(a + I*b*x))/3)*Csc[d + b*x])/b - (E^((5*(a + I*b*x))/3)
*Cot[d + b*x]*Csc[d + b*x])/(2*b) + (2*E^((5*(a + I*b*x))/3 + I*(d + b*x))
*Hypergeometric2F1[1, 4/3, 7/3, E^((2*I)*(d + b*x))])/(3*b)
```

Defintions of rubi rules used

rule 4949
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [F]

$$\int e^{\frac{5a}{3}+\frac{5ibx}{3}} \csc\left(bx+d\right)^{3} dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^3,x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(157) = 314$.

Time = 0.08 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.66

$$\int e^{\frac{5}{3}(a+ibx)} \csc^3(d+bx)\, dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^3,x, algorithm="fricas")
```

output
```
2/9*(2*(3*sqrt(1/3)*(b*e^(4*I*b*x + 4*I*d) - 2*b*e^(2*I*b*x + 2*I*d) + b)*
sqrt(-e^(10/3*a - 10/3*I*d)/b^2) + e^(4*I*b*x + 5/3*a + 7/3*I*d) - 2*e^(2*
I*b*x + 5/3*a + 1/3*I*d) + e^(5/3*a - 5/3*I*d))*log(1/2*(3*sqrt(1/3)*b*sqr
t(-e^(10/3*a - 10/3*I*d)/b^2) + 2*e^(2/3*I*b*x + 5/3*a - I*d) + e^(5/3*a -
 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 2*(3*sqrt(1/3)*(b*e^(4*I*b*x + 4*I*d) -
 2*b*e^(2*I*b*x + 2*I*d) + b)*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) - e^(4*I*b*
x + 5/3*a + 7/3*I*d) + 2*e^(2*I*b*x + 5/3*a + 1/3*I*d) - e^(5/3*a - 5/3*I*
d))*log(-1/2*(3*sqrt(1/3)*b*sqrt(-e^(10/3*a - 10/3*I*d)/b^2) - 2*e^(2/3*I*
b*x + 5/3*a - I*d) - e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 4*(e^(4*
I*b*x + 5/3*a + 7/3*I*d) - 2*e^(2*I*b*x + 5/3*a + 1/3*I*d) + e^(5/3*a - 5/
3*I*d))*log(e^(2/3*I*b*x + 2/3*I*d) - 1) + 21*e^(8/3*I*b*x + 5/3*a + I*d)
- 12*e^(2/3*I*b*x + 5/3*a - I*d))/(b*e^(4*I*b*x + 4*I*d) - 2*b*e^(2*I*b*x
+ 2*I*d) + b)
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+ibx)}\csc^3(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5ibx}{3}}\csc^3{(bx+d)}\,dx$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)**3,x)
```

output
```
exp(5*a/3)*Integral(exp(5*I*b*x/3)*csc(b*x + d)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7558 vs. $2(157) = 314$.

Time = 0.87 (sec) , antiderivative size = 7558, normalized size of antiderivative = 32.44

$$\int e^{\frac{5}{3}(a+ibx)}\csc^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^3,x, algorithm="maxima")
```

output
```
-18*(2*(((cos(d)*e^(5/3*a) + I*e^(5/3*a)*sin(d))*cos(2*d) - (I*cos(d)*e^(5
/3*a) - e^(5/3*a)*sin(d))*sin(2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d), cos(
d))) + ((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arct
an2(sin(d), cos(d))) - (-I*cos(2*d)*e^(5/3*a) - e^(5/3*a)*sin(2*d))*sin(-1
/3*pi + 1/3*arctan2(sin(d), cos(d))))*cos(4*b*x + 5*d) - 2*((cos(2*d)*e^(5
/3*a) - I*e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) +
 (I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin
(d), cos(d))))*cos(2*b*x + 3*d) - ((-I*cos(d)*e^(5/3*a) + e^(5/3*a)*sin(d)
)*cos(2*d) - (cos(d)*e^(5/3*a) + I*e^(5/3*a)*sin(d))*sin(2*d))*sin(-1/3*pi
 + 1/3*arctan2(sin(d), cos(d))) - ((-I*cos(2*d)*e^(5/3*a) - e^(5/3*a)*sin(
2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) + (cos(2*d)*e^(5/3*a) - I
*e^(5/3*a)*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*sin(4*b*x
 + 5*d) - 2*((I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3
*arctan2(sin(d), cos(d))) - (cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*si
n(-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*sin(2*b*x + 3*d))*arctan2(cos(1/
3*b*x)*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) - cos(-1/3*pi + 1/3*arct
an2(sin(d), cos(d)))*sin(1/3*b*x) - sin(-2/3*pi + 2/3*arctan2(sin(d), cos(
d))) + sin(2/3*b*x), -cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))*cos(1/3*b
*x) - sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))*sin(1/3*b*x) + cos(-2/3*p
i + 2/3*arctan2(sin(d), cos(d))) + cos(2/3*b*x)) - 2*(((cos(2*d)*e^(5/3...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(157) = 314$.

Time = 1.29 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.59

$$\int e^{\frac{5}{3}(a+ibx)}\csc^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^3,x, algorithm="giac")
```

output
```
-2/9*(4*e^(-14/3*I*d)*log(e^(2/3*I*b*x) - e^(-2/3*I*d)) - 12*(cos(d)^4*e^(
4/3*I*d) - 6*cos(d)^2*e^(4/3*I*d)*sin(d)^2 + e^(4/3*I*d)*sin(d)^4)*arctan(
1/3*sqrt(3)*(2*e^(2/3*I*b*x) + e^(-2/3*I*d))*e^(2/3*I*d))/(sqrt(3)*cos(d)^
8*e^(2*I*d) + 4*sqrt(3)*cos(d)^6*e^(2*I*d)*sin(d)^2 + 6*sqrt(3)*cos(d)^4*e
^(2*I*d)*sin(d)^4 + 4*sqrt(3)*cos(d)^2*e^(2*I*d)*sin(d)^6 + sqrt(3)*e^(2*I
*d)*sin(d)^8) + 48*I*(cos(d)^3*e^(4/3*I*d)*sin(d) - cos(d)*e^(4/3*I*d)*sin
(d)^3)*arctan(1/3*sqrt(3)*(2*e^(2/3*I*b*x) + e^(-2/3*I*d))*e^(2/3*I*d))/(s
qrt(3)*cos(d)^8*e^(2*I*d) + 4*sqrt(3)*cos(d)^6*e^(2*I*d)*sin(d)^2 + 6*sqrt
(3)*cos(d)^4*e^(2*I*d)*sin(d)^4 + 4*sqrt(3)*cos(d)^2*e^(2*I*d)*sin(d)^6 +
sqrt(3)*e^(2*I*d)*sin(d)^8) + 8*I*(cos(d)^3*e^(4/3*I*d)*sin(d) - cos(d)*e^
(4/3*I*d)*sin(d)^3)*log(e^(2/3*I*b*x - 2/3*I*d) + e^(4/3*I*b*x) + e^(-4/3*
I*d))/(cos(d)^8*e^(2*I*d) + 4*cos(d)^6*e^(2*I*d)*sin(d)^2 + 6*cos(d)^4*e^(
2*I*d)*sin(d)^4 + 4*cos(d)^2*e^(2*I*d)*sin(d)^6 + e^(2*I*d)*sin(d)^8) - 2*
(cos(d)^4 - 6*cos(d)^2*sin(d)^2 + sin(d)^4)*log(e^(2/3*I*b*x - 2/3*I*d) +
e^(4/3*I*b*x) + e^(-4/3*I*d))/(cos(d)^8*e^(2/3*I*d) + 4*cos(d)^6*e^(2/3*I*
d)*sin(d)^2 + 6*cos(d)^4*e^(2/3*I*d)*sin(d)^4 + 4*cos(d)^2*e^(2/3*I*d)*sin
(d)^6 + e^(2/3*I*d)*sin(d)^8) - 3*(7*e^(8/3*I*b*x + 2*I*d) - 4*e^(2/3*I*b*
x))*e^(-4*I*d)/(e^(2*I*b*x + 2*I*d) - 1)^2)*e^(5/3*a + 3*I*d)/b + 4/3*(7*e
^(8/3*I*b*x + 5/3*a + 2*I*d) - 4*e^(2/3*I*b*x + 5/3*a))/(b*(e^(4*I*b*x + 5
*I*d) - 2*e^(2*I*b*x + 3*I*d) + e^(I*d)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{3}(a+ibx)}\csc^3(d+bx)\,dx=\int \frac{\mathrm{e}^{\frac{5\,a}{3}+\frac{b\,x\,5\mathrm{i}}{3}}}{\sin\left(d+b\,x\right)^3}\,dx$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)/sin(d + b*x)^3,x)
```

output
```
int(exp((5*a)/3 + (b*x*5i)/3)/sin(d + b*x)^3, x)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\csc^3(d+bx)\,dx = \int e^{\frac{5bix}{3}+\frac{5a}{3}}\csc(bx+d)^3\,dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^3,x)
```

output
```
int(e**((5*a + 5*b*i*x)/3)*csc(b*x + d)**3,x)
```

3.110 $\int e^{\frac{5}{3}(a+ibx)} \csc^4(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 346

$$\begin{aligned}\int e^{\frac{5}{3}(a+ibx)} \csc^4(d+bx)\,dx &= -\frac{8ie^{\frac{5}{3}(a-id)+\frac{11}{3}i(d+bx)}}{3b\left(1-e^{2i(d+bx)}\right)^3}+\frac{22ie^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{9b\left(1-e^{2i(d+bx)}\right)^2}\\
&-\frac{55ie^{\frac{5}{3}(a-id)+\frac{5}{3}i(d+bx)}}{27b\left(1-e^{2i(d+bx)}\right)}-\frac{55ie^{\frac{5}{3}(a-id)}\arctan\left(\frac{1-2e^{\frac{1}{3}i(d+bx)}}{\sqrt{3}}\right)}{54\sqrt{3}b}\\
&+\frac{55ie^{\frac{5}{3}(a-id)}\arctan\left(\frac{1+2e^{\frac{1}{3}i(d+bx)}}{\sqrt{3}}\right)}{54\sqrt{3}b}\\
&-\frac{55ie^{\frac{5}{3}(a-id)}\operatorname{arctanh}\left(e^{\frac{1}{3}i(d+bx)}\right)}{81b}\\
&-\frac{55ie^{\frac{5}{3}(a-id)}\operatorname{arctanh}\left(\frac{e^{\frac{1}{3}i(d+bx)}}{1+e^{\frac{2}{3}i(d+bx)}}\right)}{162b}\end{aligned}$$

output
```
-8/3*I*exp(5/3*a-5/3*I*d+11/3*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^3+22/9*I*e
xp(5/3*a-5/3*I*d+5/3*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^2-55/27*I*exp(5/3*a
-5/3*I*d+5/3*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))-55/162*I*exp(5/3*a-5/3*I*d)
*arctan(1/3*(1-2*exp(1/3*I*(b*x+d)))*3^(1/2))*3^(1/2)/b+55/162*I*exp(5/3*a
-5/3*I*d)*arctan(1/3*(1+2*exp(1/3*I*(b*x+d)))*3^(1/2))*3^(1/2)/b-55/81*I*e
xp(5/3*a-5/3*I*d)*arctanh(exp(1/3*I*(b*x+d)))/b-55/162*I*exp(5/3*a-5/3*I*d
)*arctanh(exp(1/3*I*(b*x+d))/(1+exp(2/3*I*(b*x+d))))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.74

$$\int e^{\frac{5}{3}(a+ibx)}\csc^4(d+bx)\,dx$$

$$= \frac{e^{5a/3}\left(55\text{RootSum}\left[-\cos(d)+i\sin(d)+\cos(d)\#1^6+i\sin(d)\#1^6\&,\frac{bx+3i\log\left(e^{\frac{ibx}{3}}-\#1\right)}{\#1}\&\right](\cos(d)-i\sin\right.}{48}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Csc[d + b*x]^4,x]
```

output
```
(E^((5*a)/3)*(55*RootSum[-Cos[d] + I*Sin[d] + Cos[d]*#1^6 + I*Sin[d]*#1^6
& , (b*x + (3*I)*Log[E^((I/3)*b*x) - #1])/#1 & ]*(Cos[d] - I*Sin[d])^2 + (
990*E^(((5*I)/3)*b*x)*(I*Cos[d] + Sin[d]))/((-1 + E^((2*I)*b*x))*Cos[d] +
I*(1 + E^((2*I)*b*x))*Sin[d]) + (2484*E^(((5*I)/3)*b*x)*(I*Cos[2*d] + Sin[
2*d]))/((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d])^2 + (1
296*E^(((5*I)/3)*b*x)*(I*Cos[3*d] + Sin[3*d]))/((-1 + E^((2*I)*b*x))*Cos[d
] + I*(1 + E^((2*I)*b*x))*Sin[d])^3))/(486*b)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.34, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4949, 4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\csc^4(bx+d)\,dx$$

$$\Big\downarrow \text{ 4949}$$

$$\frac{11}{54}\int e^{\frac{5}{3}(a+ibx)}\csc^2(d+bx)dx-\frac{5ie^{\frac{5}{3}(a+ibx)}\csc^2(bx+d)}{18b}-\frac{e^{\frac{5}{3}(a+ibx)}\cot(bx+d)\csc^2(bx+d)}{3b}$$

$$\Big\downarrow \text{4953}$$

$$\frac{2ie^{\frac{5}{3}(a+ibx)+2i(bx+d)}\text{Hypergeometric2F1}\left(\frac{11}{6},2,\frac{17}{6},e^{2i(d+bx)}\right)}{9b}-\frac{5ie^{\frac{5}{3}(a+ibx)}\csc^2(bx+d)}{18b}-\frac{e^{\frac{5}{3}(a+ibx)}\cot(bx+d)\csc^2(bx+d)}{3b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Csc[d + b*x]^4,x]
```

output
```
(((-5*I)/18)*E^((5*(a + I*b*x))/3)*Csc[d + b*x]^2)/b - (E^((5*(a + I*b*x))
/3)*Cot[d + b*x]*Csc[d + b*x]^2)/(3*b) + (((2*I)/9)*E^((5*(a + I*b*x))/3 +
 (2*I)*(d + b*x))*Hypergeometric2F1[11/6, 2, 17/6, E^((2*I)*(d + b*x))])/b
```

Defintions of rubi rules used

rule 4949
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [F]

$$\int \mathrm{e}^{\frac{5a}{3}+\frac{5ibx}{3}} \csc \left(bx+d\right)^{4} dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^4,x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^4,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(215) = 430$.

Time = 0.10 (sec) , antiderivative size = 881, normalized size of antiderivative = 2.55

$$\int e^{\frac{5}{3}(a+ibx)} \csc^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^4,x, algorithm="fricas")
```

output
```
1/324*(55*(3*sqrt(1/3)*(b*e^(6*I*b*x + 6*I*d) - 3*b*e^(4*I*b*x + 4*I*d) +
3*b*e^(2*I*b*x + 2*I*d) - b)*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) + I*e^(6*I*b
*x + 5/3*a + 13/3*I*d) - 3*I*e^(4*I*b*x + 5/3*a + 7/3*I*d) + 3*I*e^(2*I*b*
x + 5/3*a + 1/3*I*d) - I*e^(5/3*a - 5/3*I*d))*log(-1/2*(3*I*sqrt(1/3)*b*sq
rt(b^(-2))*e^(5/3*a - 5/3*I*d) - 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) + e^(5/
3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) + 55*(3*sqrt(1/3)*(b*e^(6*I*b*x + 6*
I*d) - 3*b*e^(4*I*b*x + 4*I*d) + 3*b*e^(2*I*b*x + 2*I*d) - b)*sqrt(b^(-2))
*e^(5/3*a - 5/3*I*d) - I*e^(6*I*b*x + 5/3*a + 13/3*I*d) + 3*I*e^(4*I*b*x +
 5/3*a + 7/3*I*d) - 3*I*e^(2*I*b*x + 5/3*a + 1/3*I*d) + I*e^(5/3*a - 5/3*I
*d))*log(-1/2*(3*I*sqrt(1/3)*b*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) - 2*e^(1/3
*I*b*x + 5/3*a - 4/3*I*d) - e^(5/3*a - 5/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 5
5*(3*sqrt(1/3)*(b*e^(6*I*b*x + 6*I*d) - 3*b*e^(4*I*b*x + 4*I*d) + 3*b*e^(2
*I*b*x + 2*I*d) - b)*sqrt(b^(-2))*e^(5/3*a - 5/3*I*d) - I*e^(6*I*b*x + 5/3
*a + 13/3*I*d) + 3*I*e^(4*I*b*x + 5/3*a + 7/3*I*d) - 3*I*e^(2*I*b*x + 5/3*
a + 1/3*I*d) + I*e^(5/3*a - 5/3*I*d))*log(-1/2*(-3*I*sqrt(1/3)*b*sqrt(b^(-
2))*e^(5/3*a - 5/3*I*d) - 2*e^(1/3*I*b*x + 5/3*a - 4/3*I*d) + e^(5/3*a - 5
/3*I*d))*e^(-5/3*a + 5/3*I*d)) - 55*(3*sqrt(1/3)*(b*e^(6*I*b*x + 6*I*d) -
3*b*e^(4*I*b*x + 4*I*d) + 3*b*e^(2*I*b*x + 2*I*d) - b)*sqrt(b^(-2))*e^(5/3
*a - 5/3*I*d) + I*e^(6*I*b*x + 5/3*a + 13/3*I*d) - 3*I*e^(4*I*b*x + 5/3*a
+ 7/3*I*d) + 3*I*e^(2*I*b*x + 5/3*a + 1/3*I*d) - I*e^(5/3*a - 5/3*I*d))...
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+ibx)} \csc^4(d+bx)\,dx = e^{\frac{5a}{3}} \int e^{\frac{5ibx}{3}} \csc^4(bx+d)\,dx$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)**4,x)
```

output
```
exp(5*a/3)*Integral(exp(5*I*b*x/3)*csc(b*x + d)**4, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8894 vs. $2(215) = 430$.

Time = 1.05 (sec) , antiderivative size = 8894, normalized size of antiderivative = 25.71

$$\int e^{\frac{5}{3}(a+ibx)}\csc^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^4,x, algorithm="maxima")
```

output
```
-648*(110*((I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*a
rctan2(sin(d), cos(d))) + ((-I*cos(2*d)*e^(5/3*a) - e^(5/3*a)*sin(2*d))*co
s(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) + (cos(2*d)*e^(5/3*a) - I*e^(5/3*
a)*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*cos(6*b*x + 6*d)
+ 3*((I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arctan2
(sin(d), cos(d))) - (cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*sin(-1/3*p
i + 1/3*arctan2(sin(d), cos(d))))*cos(4*b*x + 4*d) + 3*((-I*cos(2*d)*e^(5/
3*a) - e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) + (c
os(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(d)
, cos(d))))*cos(2*b*x + 2*d) - (cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))
*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) + ((cos(2*d)*e^(5/3*a) - I*e^(
5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) + (I*cos(2*d)*
e^(5/3*a) + e^(5/3*a)*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))
)*sin(6*b*x + 6*d) - 3*((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos(-1
/3*pi + 1/3*arctan2(sin(d), cos(d))) - (-I*cos(2*d)*e^(5/3*a) - e^(5/3*a)*
sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*sin(4*b*x + 4*d) + 3
*((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arctan2(si
n(d), cos(d))) + (I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))*sin(-1/3*pi +
 1/3*arctan2(sin(d), cos(d))))*sin(2*b*x + 2*d))*arctan2(cos(1/3*b*x)*sin(
-1/3*pi + 1/3*arctan2(sin(d), cos(d))) - cos(-1/3*pi + 1/3*arctan2(sin(...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3636 vs. 2(215) = 430.

Time = 4.49 (sec) , antiderivative size = 3636, normalized size of antiderivative = 10.51

$$\int e^{\frac{5}{3}(a+ibx)}\csc^4(d+bx)\,dx = \text{Too large to display}$$

input

```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^4,x, algorithm="giac")
```

output

```
-1/324*I*(110*(-e^(-I*d))^(2/3)*e^(-5*I*d)*log(-(-e^(-I*d))^(1/3) + e^(1/3
*I*b*x)) - 110*e^(-17/3*I*d)*log(e^(1/3*I*b*x) - e^(-1/3*I*d)) - 110*(sqrt
(3)*cos(1/3*d)^2*cos(d)^5 - sqrt(3)*cos(d)^5*sin(1/3*d)^2 - 10*sqrt(3)*cos
(1/3*d)^2*cos(d)^3*sin(d)^2 + 10*sqrt(3)*cos(d)^3*sin(1/3*d)^2*sin(d)^2 +
5*sqrt(3)*cos(1/3*d)^2*cos(d)*sin(d)^4 - 5*sqrt(3)*cos(d)*sin(1/3*d)^2*sin
(d)^4 - 2*cos(1/3*d)*cos(d)^5*sin(1/3*d) + 20*cos(1/3*d)*cos(d)^3*sin(1/3*
d)*sin(d)^2 - 10*cos(1/3*d)*cos(d)*sin(1/3*d)*sin(d)^4)*arctan(1/3*sqrt(3)
*(2*e^(1/3*I*b*x) + e^(-1/3*I*d))*e^(1/3*I*d))/(cos(1/3*d)^4*cos(d)^10 + 2
*cos(1/3*d)^2*cos(d)^10*sin(1/3*d)^2 + cos(d)^10*sin(1/3*d)^4 + 5*cos(1/3*
d)^4*cos(d)^8*sin(d)^2 + 10*cos(1/3*d)^2*cos(d)^8*sin(1/3*d)^2*sin(d)^2 +
5*cos(d)^8*sin(1/3*d)^4*sin(d)^2 + 10*cos(1/3*d)^4*cos(d)^6*sin(d)^4 + 20*
cos(1/3*d)^2*cos(d)^6*sin(1/3*d)^2*sin(d)^4 + 10*cos(d)^6*sin(1/3*d)^4*sin
(d)^4 + 10*cos(1/3*d)^4*cos(d)^4*sin(d)^6 + 20*cos(1/3*d)^2*cos(d)^4*sin(1
/3*d)^2*sin(d)^6 + 10*cos(d)^4*sin(1/3*d)^4*sin(d)^6 + 5*cos(1/3*d)^4*cos(
d)^2*sin(d)^8 + 10*cos(1/3*d)^2*cos(d)^2*sin(1/3*d)^2*sin(d)^8 + 5*cos(d)^
2*sin(1/3*d)^4*sin(d)^8 + cos(1/3*d)^4*sin(d)^10 + 2*cos(1/3*d)^2*sin(1/3*
d)^2*sin(d)^10 + sin(1/3*d)^4*sin(d)^10) + 110*I*(5*sqrt(3)*cos(1/3*d)^2*c
os(d)^4*sin(d) - 5*sqrt(3)*cos(d)^4*sin(1/3*d)^2*sin(d) - 10*sqrt(3)*cos(1
/3*d)^2*cos(d)^2*sin(d)^3 + 10*sqrt(3)*cos(d)^2*sin(1/3*d)^2*sin(d)^3 + sq
rt(3)*cos(1/3*d)^2*sin(d)^5 - sqrt(3)*sin(1/3*d)^2*sin(d)^5 - 10*cos(1/...
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{3}(a+ibx)}\csc^4(d+bx)\,dx=\int \frac{\mathrm{e}^{\frac{5\,a}{3}+\frac{b\,x\,5\mathrm{i}}{3}}}{\sin\left(d+b\,x\right)^4}\,dx$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)/sin(d + b*x)^4,x)
```

output
```
int(exp((5*a)/3 + (b*x*5i)/3)/sin(d + b*x)^4, x)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\csc^4(d+bx)\,dx=\int e^{\frac{5bix}{3}+\frac{5a}{3}}\csc\left(bx+d\right)^4dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^4,x)
```

output
```
int(e**((5*a + 5*b*i*x)/3)*csc(b*x + d)**4,x)
```

3.111 $\int e^{\frac{5}{3}(a+ibx)} \csc^5(d+bx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 331

$$\begin{aligned}\int e^{\frac{5}{3}(a+ibx)}\csc^5(d+bx)\,dx =& -\frac{4e^{\frac{5}{3}(a-id)+\frac{14}{3}i(d+bx)}}{b\left(1-e^{2i(d+bx)}\right)^4}+\frac{28e^{\frac{5}{3}(a-id)+\frac{8}{3}i(d+bx)}}{9b\left(1-e^{2i(d+bx)}\right)^3}\\&-\frac{56e^{\frac{5}{3}(a-id)+\frac{2}{3}i(d+bx)}}{27b\left(1-e^{2i(d+bx)}\right)^2}+\frac{56e^{\frac{5}{3}(a-id)+\frac{2}{3}i(d+bx)}}{81b\left(1-e^{2i(d+bx)}\right)}\\&+\frac{112e^{\frac{5}{3}(a-id)}\arctan\left(\frac{1+2e^{\frac{2}{3}i(d+bx)}}{\sqrt{3}}\right)}{81\sqrt{3}b}\\&-\frac{112e^{\frac{5}{3}(a-id)}\log\left(1-e^{\frac{2}{3}i(d+bx)}\right)}{243b}\\&+\frac{56e^{\frac{5}{3}(a-id)}\log\left(1+e^{\frac{2}{3}i(d+bx)}+e^{\frac{4}{3}i(d+bx)}\right)}{243b}\end{aligned}$$

output

```
-4*exp(5/3*a-5/3*I*d+14/3*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^4+28/9*exp(5/3
*a-5/3*I*d+8/3*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^3-56/27*exp(5/3*a-5/3*I*d
+2/3*I*(b*x+d))/b/(1-exp(2*I*(b*x+d)))^2+56/81*exp(5/3*a-5/3*I*d+2/3*I*(b*
x+d))/b/(1-exp(2*I*(b*x+d)))+112/243*3^(1/2)*exp(5/3*a-5/3*I*d)*arctan(1/3
*(1+2*exp(2/3*I*(b*x+d)))*3^(1/2))/b-112/243*exp(5/3*a-5/3*I*d)*ln(1-exp(2
/3*I*(b*x+d)))/b+56/243*exp(5/3*a-5/3*I*d)*ln(1+exp(2/3*I*(b*x+d))+exp(4/3
*I*(b*x+d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.25

$$\int e^{\frac{5}{3}(a+ibx)}\csc^5(d+bx)\,dx$$
$$=\frac{4e^{5a/3}\left(-28i\text{RootSum}\left[\cos\left(\frac{d}{2}\right)-i\sin\left(\frac{d}{2}\right)+\cos\left(\frac{d}{2}\right)\#1^3+i\sin\left(\frac{d}{2}\right)\#1^3\&,\frac{bx+3i\log\left(e^{\frac{ibx}{3}}-\#1\right)}{\#1}\&\right](\cos(d)\right.}{}$$

input
```
Integrate[E^((5*(a + I*b*x))/3)*Csc[d + b*x]^5,x]
```

output
```
(4*E^((5*a)/3)*((-28*I)*RootSum[Cos[d/2] - I*Sin[d/2] + Cos[d/2]*#1^3 + I*
Sin[d/2]*#1^3 & , (b*x + (3*I)*Log[E^((I/3)*b*x) - #1])/#1 & ]*(Cos[d] - I
*Sin[d])^2 - (729*E^(((2*I)/3)*b*x)*(Cos[d] - I*Sin[d])^5)/((-1 + E^((2*I)
*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d])^4 - (2025*E^(((2*I)/3)*b*x)*
(Cos[d] - I*Sin[d])^4)/((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x)
)*Sin[d])^3 - (1674*E^(((2*I)/3)*b*x)*(Cos[d] - I*Sin[d])^3)/((-1 + E^((2*
I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x))*Sin[d])^2 - (126*E^(((2*I)/3)*b*x)
*(Cos[d] - I*Sin[d])^2)/((-1 + E^((2*I)*b*x))*Cos[d] + I*(1 + E^((2*I)*b*x
))*Sin[d]) + 28*RootSum[-Cos[d/2] + I*Sin[d/2] + Cos[d/2]*#1^3 + I*Sin[d/2
]*#1^3 & , (b*x + (3*I)*Log[E^((I/3)*b*x) - #1])/#1 & ]*(I*Cos[2*d] + Sin[
2*d])))/(729*b)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.56, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4949, 4949, 4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+ibx)}\csc^5(bx+d)\,dx$$

$$\Big\downarrow \text{4949}$$

$$\frac{14}{27}\int e^{\frac{5}{3}(a+ibx)}\csc^3(d+bx)dx-\frac{5ie^{\frac{5}{3}(a+ibx)}\csc^3(bx+d)}{36b}-\frac{e^{\frac{5}{3}(a+ibx)}\cot(bx+d)\csc^3(bx+d)}{4b}$$

$$\Big\downarrow \text{4949}$$

$$\frac{14}{27}\left(-\frac{8}{9}\int e^{\frac{5}{3}(a+ibx)}\csc(d+bx)dx-\frac{5ie^{\frac{5}{3}(a+ibx)}\csc(bx+d)}{6b}-\frac{e^{\frac{5}{3}(a+ibx)}\cot(bx+d)\csc(bx+d)}{2b}\right)-\frac{5ie^{\frac{5}{3}(a+ibx)}\csc^3(bx+d)}{36b}-\frac{e^{\frac{5}{3}(a+ibx)}\cot(bx+d)\csc^3(bx+d)}{4b}$$

$$\Big\downarrow \text{4953}$$

$$\frac{14}{27}\left(\frac{2e^{\frac{5}{3}(a+ibx)+i(bx+d)}\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},e^{2i(d+bx)}\right)}{3b}-\frac{5ie^{\frac{5}{3}(a+ibx)}\csc(bx+d)}{6b}-\frac{e^{\frac{5}{3}(a+ibx)}\cot(bx+d)\csc(bx+d)}{2b}\right)-\frac{5ie^{\frac{5}{3}(a+ibx)}\csc^3(bx+d)}{36b}-\frac{e^{\frac{5}{3}(a+ibx)}\cot(bx+d)\csc^3(bx+d)}{4b}$$

input
```
Int[E^((5*(a + I*b*x))/3)*Csc[d + b*x]^5,x]
```

output
```
(((-5*I)/36)*E^((5*(a + I*b*x))/3)*Csc[d + b*x]^3)/b - (E^((5*(a + I*b*x))
/3)*Cot[d + b*x]*Csc[d + b*x]^3)/(4*b) + (14*(((((-5*I)/6)*E^((5*(a + I*b*x
))/3)*Csc[d + b*x])/b - (E^((5*(a + I*b*x))/3)*Cot[d + b*x]*Csc[d + b*x])/
(2*b) + (2*E^((5*(a + I*b*x))/3 + I*(d + b*x))*Hypergeometric2F1[1, 4/3, 7
/3, E^((2*I)*(d + b*x))])/(3*b)))/27
```

Defintions of rubi rules used

rule 4949
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [F]

$$\int e^{\frac{5a}{3}+\frac{5ibx}{3}} \csc \left(bx+d\right)^{5} dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^5,x)
```

output
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^5,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(217) = 434.

Time = 0.09 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.73

$$\int e^{\frac{5}{3}(a+ibx)} \csc^5(d+bx)\, dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^5,x, algorithm="fricas")
```

output
```
4/243*(14*(3*sqrt(1/3)*(b*e^(8*I*b*x + 8*I*d) - 4*b*e^(6*I*b*x + 6*I*d) +
6*b*e^(4*I*b*x + 4*I*d) - 4*b*e^(2*I*b*x + 2*I*d) + b)*sqrt(-e^(10/3*a - 1
0/3*I*d)/b^2) + e^(8*I*b*x + 5/3*a + 19/3*I*d) - 4*e^(6*I*b*x + 5/3*a + 13
/3*I*d) + 6*e^(4*I*b*x + 5/3*a + 7/3*I*d) - 4*e^(2*I*b*x + 5/3*a + 1/3*I*d
) + e^(5/3*a - 5/3*I*d))*log(1/2*(3*sqrt(1/3)*b*sqrt(-e^(10/3*a - 10/3*I*d
)/b^2) + 2*e^(2/3*I*b*x + 5/3*a - I*d) + e^(5/3*a - 5/3*I*d))*e^(-5/3*a +
5/3*I*d)) - 14*(3*sqrt(1/3)*(b*e^(8*I*b*x + 8*I*d) - 4*b*e^(6*I*b*x + 6*I*
d) + 6*b*e^(4*I*b*x + 4*I*d) - 4*b*e^(2*I*b*x + 2*I*d) + b)*sqrt(-e^(10/3*
a - 10/3*I*d)/b^2) - e^(8*I*b*x + 5/3*a + 19/3*I*d) + 4*e^(6*I*b*x + 5/3*a
 + 13/3*I*d) - 6*e^(4*I*b*x + 5/3*a + 7/3*I*d) + 4*e^(2*I*b*x + 5/3*a + 1/
3*I*d) - e^(5/3*a - 5/3*I*d))*log(-1/2*(3*sqrt(1/3)*b*sqrt(-e^(10/3*a - 10
/3*I*d)/b^2) - 2*e^(2/3*I*b*x + 5/3*a - I*d) - e^(5/3*a - 5/3*I*d))*e^(-5/
3*a + 5/3*I*d)) - 28*(e^(8*I*b*x + 5/3*a + 19/3*I*d) - 4*e^(6*I*b*x + 5/3*
a + 13/3*I*d) + 6*e^(4*I*b*x + 5/3*a + 7/3*I*d) - 4*e^(2*I*b*x + 5/3*a + 1
/3*I*d) + e^(5/3*a - 5/3*I*d))*log(e^(2/3*I*b*x + 2/3*I*d) - 1) - 42*e^(20
/3*I*b*x + 5/3*a + 5*I*d) - 432*e^(14/3*I*b*x + 5/3*a + 3*I*d) + 315*e^(8/
3*I*b*x + 5/3*a + I*d) - 84*e^(2/3*I*b*x + 5/3*a - I*d))/(b*e^(8*I*b*x + 8
*I*d) - 4*b*e^(6*I*b*x + 6*I*d) + 6*b*e^(4*I*b*x + 4*I*d) - 4*b*e^(2*I*b*x
 + 2*I*d) + b)
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+ibx)} \csc^5(d+bx)\,dx = e^{\frac{5a}{3}} \int e^{\frac{5ibx}{3}} \csc^5(bx+d)\,dx$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)**5,x)
```

output
```
exp(5*a/3)*Integral(exp(5*I*b*x/3)*csc(b*x + d)**5, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11459 vs. 2(217) = 434.

Time = 2.55 (sec) , antiderivative size = 11459, normalized size of antiderivative = 34.62

$$\int e^{\frac{5}{3}(a+ibx)} \csc^5(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^5,x, algorithm="maxima")
```

output
```
-972*(14*(((cos(d)*e^(5/3*a) + I*e^(5/3*a)*sin(d))*cos(2*d) - (I*cos(d)*e^
(5/3*a) - e^(5/3*a)*sin(d))*sin(2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d), co
s(d))) + ((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*ar
ctan2(sin(d), cos(d))) - (-I*cos(2*d)*e^(5/3*a) - e^(5/3*a)*sin(2*d))*sin(
-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*cos(8*b*x + 9*d) - 4*((cos(2*d)*e^
(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))
 + (I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(s
in(d), cos(d))))*cos(6*b*x + 7*d) + 6*((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*s
in(2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) - (-I*cos(2*d)*e^(5/3*
a) - e^(5/3*a)*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*cos(4
*b*x + 5*d) - 4*((cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*cos(-1/3*pi +
 1/3*arctan2(sin(d), cos(d))) + (I*cos(2*d)*e^(5/3*a) + e^(5/3*a)*sin(2*d)
)*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))))*cos(2*b*x + 3*d) - ((-I*cos(
d)*e^(5/3*a) + e^(5/3*a)*sin(d))*cos(2*d) - (cos(d)*e^(5/3*a) + I*e^(5/3*a
)*sin(d))*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) - ((-I*cos(
2*d)*e^(5/3*a) - e^(5/3*a)*sin(2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d), cos
(d))) + (cos(2*d)*e^(5/3*a) - I*e^(5/3*a)*sin(2*d))*sin(-1/3*pi + 1/3*arct
an2(sin(d), cos(d))))*sin(8*b*x + 9*d) - 4*((I*cos(2*d)*e^(5/3*a) + e^(5/3
*a)*sin(2*d))*cos(-1/3*pi + 1/3*arctan2(sin(d), cos(d))) - (cos(2*d)*e^(5/
3*a) - I*e^(5/3*a)*sin(2*d))*sin(-1/3*pi + 1/3*arctan2(sin(d), cos(d)))...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 874 vs. 2(217) = 434.

Time = 1.29 (sec) , antiderivative size = 874, normalized size of antiderivative = 2.64

$$\int e^{\frac{5}{3}(a+ibx)} \csc^5(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^5,x, algorithm="giac")
```

output
```
-4/243*(28*e^(-20/3*I*d)*log(e^(2/3*I*b*x) - e^(-2/3*I*d)) - 84*(cos(d)^6*
e^(4/3*I*d) - 15*cos(d)^4*e^(4/3*I*d)*sin(d)^2 + 15*cos(d)^2*e^(4/3*I*d)*s
in(d)^4 - e^(4/3*I*d)*sin(d)^6)*arctan(1/3*sqrt(3)*(2*e^(2/3*I*b*x) + e^(-
2/3*I*d))*e^(2/3*I*d))/(sqrt(3)*cos(d)^12*e^(2*I*d) + 6*sqrt(3)*cos(d)^10*
e^(2*I*d)*sin(d)^2 + 15*sqrt(3)*cos(d)^8*e^(2*I*d)*sin(d)^4 + 20*sqrt(3)*c
os(d)^6*e^(2*I*d)*sin(d)^6 + 15*sqrt(3)*cos(d)^4*e^(2*I*d)*sin(d)^8 + 6*sq
rt(3)*cos(d)^2*e^(2*I*d)*sin(d)^10 + sqrt(3)*e^(2*I*d)*sin(d)^12) + 168*I*
(3*cos(d)^5*e^(4/3*I*d)*sin(d) - 10*cos(d)^3*e^(4/3*I*d)*sin(d)^3 + 3*cos(
d)*e^(4/3*I*d)*sin(d)^5)*arctan(1/3*sqrt(3)*(2*e^(2/3*I*b*x) + e^(-2/3*I*d
))*e^(2/3*I*d))/(sqrt(3)*cos(d)^12*e^(2*I*d) + 6*sqrt(3)*cos(d)^10*e^(2*I*
d)*sin(d)^2 + 15*sqrt(3)*cos(d)^8*e^(2*I*d)*sin(d)^4 + 20*sqrt(3)*cos(d)^6
*e^(2*I*d)*sin(d)^6 + 15*sqrt(3)*cos(d)^4*e^(2*I*d)*sin(d)^8 + 6*sqrt(3)*c
os(d)^2*e^(2*I*d)*sin(d)^10 + sqrt(3)*e^(2*I*d)*sin(d)^12) - 14*(cos(d)^6*
e^(4/3*I*d) - 15*cos(d)^4*e^(4/3*I*d)*sin(d)^2 + 15*cos(d)^2*e^(4/3*I*d)*s
in(d)^4 - e^(4/3*I*d)*sin(d)^6)*log(e^(2/3*I*b*x - 2/3*I*d) + e^(4/3*I*b*x
) + e^(-4/3*I*d))/(cos(d)^12*e^(2*I*d) + 6*cos(d)^10*e^(2*I*d)*sin(d)^2 +
15*cos(d)^8*e^(2*I*d)*sin(d)^4 + 20*cos(d)^6*e^(2*I*d)*sin(d)^6 + 15*cos(d
)^4*e^(2*I*d)*sin(d)^8 + 6*cos(d)^2*e^(2*I*d)*sin(d)^10 + e^(2*I*d)*sin(d)
^12) + 28*I*(3*cos(d)^5*e^(4/3*I*d)*sin(d) - 10*cos(d)^3*e^(4/3*I*d)*sin(d
)^3 + 3*cos(d)*e^(4/3*I*d)*sin(d)^5)*log(e^(2/3*I*b*x - 2/3*I*d) + e^(4...
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{3}(a+ibx)}\csc^5(d+bx)\,dx=\int \frac{\mathrm{e}^{\frac{5\,a}{3}+\frac{b\,x\,5\mathrm{i}}{3}}}{\sin\left(d+b\,x\right)^5}\,dx$$

input
```
int(exp((5*a)/3 + (b*x*5i)/3)/sin(d + b*x)^5,x)
```

output
```
int(exp((5*a)/3 + (b*x*5i)/3)/sin(d + b*x)^5, x)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+ibx)}\csc^5(d+bx)\,dx=\int e^{\frac{5bix}{3}+\frac{5a}{3}}\csc\left(bx+d\right)^5dx$$

input
```
int(exp(5/3*a+5/3*I*b*x)*csc(b*x+d)^5,x)
```

output
```
int(e**((5*a + 5*b*i*x)/3)*csc(b*x + d)**5,x)
```

3.112 $\int F^{c(a+bx)}\csc(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int F^{c(a+bx)}\csc(d+ex)\,dx = -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{e-ibc\log(F)}$$

output
```
-2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))/e],[3/2-
1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))/(e-I*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\int F^{c(a+bx)}\csc(d+ex)\,dx = \frac{iF^{c(a+bx)}\left(\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{e},1-\frac{ibc\log(F)}{e},-\cos(d+ex)-i\sin(d+ex)\right)-\text{Hypergeom}\right.}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Csc[d + e*x],x]
```

output
```
(I*F^(c*(a + b*x))*(Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c*L
og[F])/e, -Cos[d + e*x] - I*Sin[d + e*x]] - Hypergeometric2F1[1, ((-I)*b*c
*Log[F])/e, 1 - (I*b*c*Log[F])/e, Cos[d + e*x] + I*Sin[d + e*x]]))/(b*c*Lo
g[F])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{4953\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4953}$$

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{e-ibc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Csc[d + e*x],x]
```

output
```
(-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F]
)/(2*e), (3 - (I*b*c*Log[F])/e)/2, E^((2*I)*(d + e*x))])/(e - I*b*c*Log[F]
)
```

Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [F]

$$\int F^{c(bx+a)} \csc (ex+d)\, dx$$

input
```
int(F^(c*(b*x+a))*csc(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*csc(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)} \csc(d+ex)\, dx = \int F^{(bx+a)c} \csc (ex+d)\, dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csc(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)} \csc(d+ex)\, dx = \int F^{c(a+bx)} \csc (d+ex)\, dx$$

input
```
integrate(F**(c*(b*x+a))*csc(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*csc(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)} \csc(d+ex)\,dx = \int F^{(bx+a)c} \csc(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="maxima")
```

output
```
2*(F^(b*c*x)*F^(a*c)*b*c*log(F)*sin(e*x + d) + F^(b*c*x)*F^(a*c)*e*cos(e*x
 + d) - (F^(b*c*x)*F^(a*c)*b*c*log(F)*sin(e*x + d) + F^(b*c*x)*F^(a*c)*e*c
os(e*x + d))*cos(2*e*x + 2*d) - 2*(F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^
3 + (F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*cos(2*e*x + 2*d)^2 + (F^(a*
c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*sin(2*e*x + 2*d)^2 - 2*(F^(a*c)*b^2*c
^2*e*log(F)^2 + F^(a*c)*e^3)*cos(2*e*x + 2*d))*integrate((F^(b*c*x)*b*c*co
s(e*x + d)*log(F) - F^(b*c*x)*e*sin(e*x + d) + (F^(b*c*x)*b*c*cos(e*x + d)
*log(F) - F^(b*c*x)*e*sin(e*x + d))*cos(4*e*x + 4*d) - 2*(F^(b*c*x)*b*c*co
s(e*x + d)*log(F) - F^(b*c*x)*e*sin(e*x + d))*cos(2*e*x + 2*d) + (F^(b*c*x
)*b*c*log(F)*sin(e*x + d) + F^(b*c*x)*e*cos(e*x + d))*sin(4*e*x + 4*d) - 2
*(F^(b*c*x)*b*c*log(F)*sin(e*x + d) + F^(b*c*x)*e*cos(e*x + d))*sin(2*e*x
+ 2*d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^2)*cos(4*e*x + 4*d)^2 +
4*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)^2 + (b^2*c^2*log(F)^2 + e^2)*s
in(4*e*x + 4*d)^2 - 4*(b^2*c^2*log(F)^2 + e^2)*sin(4*e*x + 4*d)*sin(2*e*x
+ 2*d) + 4*(b^2*c^2*log(F)^2 + e^2)*sin(2*e*x + 2*d)^2 + e^2 + 2*(b^2*c^2*
log(F)^2 + e^2 - 2*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d))*cos(4*e*x +
4*d) - 4*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)), x) + (F^(b*c*x)*F^(a*
c)*b*c*cos(e*x + d)*log(F) - F^(b*c*x)*F^(a*c)*e*sin(e*x + d))*sin(2*e*x +
 2*d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)^2 + (
b^2*c^2*log(F)^2 + e^2)*sin(2*e*x + 2*d)^2 + e^2 - 2*(b^2*c^2*log(F)^2 ...
```

Giac [F]

$$\int F^{c(a+bx)} \csc(d+ex)\,dx = \int F^{(bx+a)c} \csc(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csc(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\csc(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))/sin(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))/sin(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\csc(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\csc\left(ex+d\right)dx\right)$$

input
```
int(F^(c*(b*x+a))*csc(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csc(d + e*x),x)
```

3.113 $\int F^{c(a+bx)} \csc^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 82

$$\int F^{c(a+bx)} \csc^2(d+ex)\,dx = -\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2-\frac{ibc\log(F)}{e}\right),\frac{1}{2}\left(4-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

output
```
-4*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b*c*ln(F)/e],[2-1/
2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))/(2*I*e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.23

$$\int F^{c(a+bx)} \csc^2(d+ex)\,dx = -\frac{2iF^{c(a+bx)}\left(\left(-1+e^{2id}\right)\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)+\csc(d+ex)\sin(d)\right.}{e\left(-1+e^{2id}\right)}$$

input
```
Integrate[F^(c*(a + b*x))*Csc[d + e*x]^2,x]
```

output
```
((-2*I)*F^(c*(a + b*x))*((-1 + E^((2*I)*d))*Hypergeometric2F1[1, ((-1/2*I)
*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))] + Csc[d + e
*x]*Sin[d]*(Cos[e*x] - I*Sin[e*x])))/(e*(-1 + E^((2*I)*d)))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 4953$$

$$-\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{bc\log(F)+2ie}$$

input
```
Int[F^(c*(a + b*x))*Csc[d + e*x]^2,x]
```

output
```
(-4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*
c*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])/((2*I)*e + b*
c*Log[F])
```

Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [F]

$$\int F^{c(bx+a)} \csc(ex+d)^2\,dx$$

input
```
int(F^(c*(b*x+a))*csc(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*csc(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx=\int F^{(bx+a)c}\csc(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csc(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx=\int F^{c(a+bx)}\csc^2(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*csc(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*csc(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \csc^2(d+ex)\,dx = \int F^{(bx+a)c} \csc(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="maxima")
```

output
```
4*(24*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3 + 16*
F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^3*c^3*
log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 - (F^(a
*c)*b^3*c^3*log(F)^3 + 64*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*
d) + 2*(5*F^(a*c)*b^2*c^2*e*log(F)^2 - 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x
 + 2*d) + (24*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) - (F^(a*c)*b^3*c^3*log(F)^3
 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 2*(F^(a*c)*b^2*
c^2*e*log(F)^2 + 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x + 2*d))*cos(4*e*x + 4
*d) + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64
*F^(a*c)*b*c*e^5*log(F) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3
*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(4*e*x + 4*d)^2 + 4*(F^(a*c)
*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5
*log(F))*cos(2*e*x + 2*d)^2 + (F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3
*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*sin(4*e*x + 4*d)^2 - 4*(F^(
a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c
*e^5*log(F))*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(F^(a*c)*b^5*c^5*e*log(
F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*sin(2*
e*x + 2*d)^2 + 2*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(
F)^3 + 64*F^(a*c)*b*c*e^5*log(F) - 2*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a
*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(2*e*x + 2*d))...
```

Giac [F]

$$\int F^{c(a+bx)} \csc^2(d+ex)\,dx = \int F^{(bx+a)c} \csc(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csc(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)^2}\,dx$$

input
```
int(F^(c*(a + b*x))/sin(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))/sin(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\csc\left(ex+d\right)^2dx\right)$$

input
```
int(F^(c*(b*x+a))*csc(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csc(d + e*x)**2,x)
```

3.114 $\int F^{c(a+bx)} \csc^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int F^{c(a+bx)} \csc^3(d+ex)\,dx = \frac{8e^{3i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),\frac{1}{2}\left(5-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{3e-ibc\log(F)}$$

output
```
8*exp(3*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([3, 3/2-1/2*I*b*c*ln(F)/e],[5/2
-1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))/(3*e-I*b*c*ln(F))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 334 vs. 2(83) = 166.

Time = 11.08 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.02

$$\int F^{c(a+bx)} \csc^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(-e\csc^2\left(\frac{1}{2}(d+ex)\right)-4bc\csc(d)\log(F)+\csc(d)\left(\frac{4e^2}{bc\log(F)}+4bc\log(F)\right)+e\sec^2\left(\frac{1}{2}(d+ex)\right)\cdots\right.}{\cdots}$$

input
```
Integrate[F^(c*(a + b*x))*Csc[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(-(e*Csc[(d + e*x)/2]^2) - 4*b*c*Csc[d]*Log[F] + Csc[d]*(
(4*e^2)/(b*c*Log[F]) + 4*b*c*Log[F]) + e*Sec[(d + e*x)/2]^2 - ((4*I)*(e^2
+ b^2*c^2*Log[F]^2)*(1 + Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*
b*c*Log[F])/e, Cos[d + e*x] + I*Sin[d + e*x]]*(-1 + Cos[d] + I*Sin[d])))/(
b*c*Log[F]*(-1 + Cos[d] + I*Sin[d])) - ((4*I)*(e^2 + b^2*c^2*Log[F]^2)*(1
- Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c*Log[F])/e, -Cos[d +
 e*x] - I*Sin[d + e*x]]*(1 + Cos[d] + I*Sin[d])))/(b*c*Log[F]*(1 + Cos[d]
+ I*Sin[d])) + 2*b*c*Csc[d/2]*Csc[(d + e*x)/2]*Log[F]*Sin[(e*x)/2] - 2*b*c
*Log[F]*Sec[d/2]*Sec[(d + e*x)/2]*Sin[(e*x)/2]))/(8*e^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4949, 4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4949}$$

$$\frac{1}{2}\left(\frac{b^2c^2\log^2(F)}{e^2}+1\right)\int F^{c(a+bx)}\csc(d+ex)dx-\frac{bc\log(F)\csc(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\cot(d+ex)\csc(d+ex)F^{c(a+bx)}}{2e}$$

$$\Big\downarrow \text{4953}$$

$$-\frac{e^{i(d+ex)}F^{c(a+bx)}\left(\frac{b^2c^2\log^2(F)}{e^2}+1\right)\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{e-ibc\log(F)}-\frac{bc\log(F)\csc(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\cot(d+ex)\csc(d+ex)F^{c(a+bx)}}{2e}$$

input
```
Int[F^(c*(a + b*x))*Csc[d + e*x]^3,x]
```

output
```
-1/2*(F^(c*(a + b*x))*Cot[d + e*x]*Csc[d + e*x])/e - (b*c*F^(c*(a + b*x))*
Csc[d + e*x]*Log[F])/(2*e^2) - (E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeome
tric2F1[1, (e - I*b*c*Log[F])/(2*e), (3 - (I*b*c*Log[F])/e)/2, E^((2*I)*(d
 + e*x))]*(1 + (b^2*c^2*Log[F]^2)/e^2))/(e - I*b*c*Log[F])
```

Defintions of rubi rules used

rule 4949
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Maple [F]

$$\int F^{c(bx+a)} \csc(ex+d)^3 \, dx$$

input
```
int(F^(c*(b*x+a))*csc(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*csc(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\csc^3(d+ex)\,dx = \int F^{(bx+a)c}\csc\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csc(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\csc^3(d+ex)\,dx = \int F^{c(a+bx)}\csc^3\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*csc(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*csc(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\csc^3(d+ex)\,dx = \int F^{(bx+a)c}\csc\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="maxima")
```

output
```
8*(48*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F)*sin(e*x + d) - 6*(F^(a*c)*b^2*c^2*e
*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*cos(e*x + d) - (48*F^(b*c*x)*F^(a*c)
*b*c*e^2*log(F)*sin(e*x + d) - 3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*
e^3)*F^(b*c*x)*cos(3*e*x + 3*d) - 6*(F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*
c)*e^3)*F^(b*c*x)*cos(e*x + d) - (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*
c*e^2*log(F))*F^(b*c*x)*sin(3*e*x + 3*d))*cos(6*e*x + 6*d) + 3*(48*F^(b*c*
x)*F^(a*c)*b*c*e^2*log(F)*sin(e*x + d) - 3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2
5*F^(a*c)*e^3)*F^(b*c*x)*cos(3*e*x + 3*d) - 6*(F^(a*c)*b^2*c^2*e*log(F)^2
- 15*F^(a*c)*e^3)*F^(b*c*x)*cos(e*x + d) - (F^(a*c)*b^3*c^3*log(F)^3 + 25*
F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(3*e*x + 3*d))*cos(4*e*x + 4*d) + 3*(
3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*cos(2*e*x + 2*d)
 - (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*
e*x + 2*d) - (F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x))*cos(
3*e*x + 3*d) - 18*(8*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F)*sin(e*x + d) - (F^(a
*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*cos(e*x + d))*cos(2*e*x
 + 2*d) - 6*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d
)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e
^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*c
os(d) + (F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*lo
g(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^...
```

Giac [F]

$$\int F^{c(a+bx)}\csc^3(d+ex)\,dx=\int F^{(bx+a)c}\csc(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csc(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\csc^3(d+ex)\,dx=\int\frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)^3}\,dx$$

input
```
int(F^(c*(a + b*x))/sin(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))/sin(d + e*x)^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\csc^3(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\csc\left(ex+d\right)^3dx\right)$$

input
```
int(F^(c*(b*x+a))*csc(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csc(d + e*x)**3,x)
```

3.115 $\int e^{a+ibx} \csc^n(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int e^{a+ibx} \csc^n(a+bx)\,dx = -\frac{ie^{a+ibx}\left(1-e^{2i(a+bx)}\right)^n \csc^n(a+bx) \text{Hypergeometric2F1}\left(n,\frac{1+n}{2},\frac{3+n}{2},e^{2i(a+bx)}\right)}{b(1+n)}$$

output
```
-I*exp(a+I*b*x)*(1-exp(2*I*(b*x+a)))^n*csc(b*x+a)^n*hypergeom([n, 1/2+1/2*
n],[3/2+1/2*n],exp(2*I*(b*x+a)))/b/(1+n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int e^{a+ibx} \csc^n(a+bx)\,dx = \frac{ie^{(1-2i)a-ibx}\left(-1+e^{2i(a+bx)}\right) \csc^n(a+bx) \text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{1+n}{2},e^{-2i(a+bx)}\right)}{b(-1+n)}$$

input
```
Integrate[E^(a + I*b*x)*Csc[a + b*x]^n,x]
```

output
```
(I*E^((1 - 2*I)*a - I*b*x)*(-1 + E^((2*I)*(a + b*x)))*Csc[a + b*x]^n*Hyper
geometric2F1[1, (1 - n)/2, (1 + n)/2, E^((-2*I)*(a + b*x))])/(b*(-1 + n))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4955, 2681}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+ibx}\csc^n(a+bx)\,dx$$

$$\Big\downarrow \ 4955$$

$$e^{in(a+bx)}\left(1-e^{-2i(a+bx)}\right)^n\csc^n(a+bx)\int e^{a(1-in)+ib(1-n)x}\left(1-e^{-2i(a+bx)}\right)^{-n}dx$$

$$\Big\downarrow \ 2681$$

$$-\frac{i\left(1-e^{-2i(a+bx)}\right)^n\exp(in(a+bx)+a(1-in)+ib(1-n)x)\,\text{Hypergeometric2F1}\left(\frac{n-1}{2},n,\frac{n+1}{2},e^{-2i(a+bx)}\right)\csc^n}{b(1-n)}$$

input
```
Int[E^(a + I*b*x)*Csc[a + b*x]^n,x]
```

output
```
((-I)*E^(a*(1 - I*n) + I*b*(1 - n)*x + I*n*(a + b*x))*(1 - E^((-2*I)*(a +
b*x)))^n*Csc[a + b*x]^n*Hypergeometric2F1[(-1 + n)/2, n, (1 + n)/2, E^((-2
*I)*(a + b*x))])/(b*(1 - n))
```

Defintions of rubi rules used

rule 2681
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

rule 4955
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(1 - E^(-2*I*(d + e*x)))^n*(Csc[d + e*x]^n/E^((-I)*n*(d + e*x))
)   Int[SimplifyIntegrand[F^(c*(a + b*x))*(1/(E^(I*n*(d + e*x))*(1 - E^(-2*
I*(d + e*x)))^n)), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ
[n]
```

Maple [F]

$$\int e^{ibx+a} \csc(bx+a)^n \, dx$$

input
```
int(exp(a+I*b*x)*csc(b*x+a)^n,x)
```

output
```
int(exp(a+I*b*x)*csc(b*x+a)^n,x)
```

Fricas [F]

$$\int e^{a+ibx} \csc^n(a+bx) \, dx = \int \csc(bx+a)^n e^{(i\,bx+a)} \, dx$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral((2*I*e^(I*b*x + I*a)/(e^(2*I*b*x + 2*I*a) - 1))^n*e^(I*b*x + a),
x)
```

Sympy [F]

$$\int e^{a+ibx}\csc^n(a+bx)\,dx=e^a\int e^{ibx}\csc^n(a+bx)\,dx$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+a)**n,x)
```

output
```
exp(a)*Integral(exp(I*b*x)*csc(a + b*x)**n, x)
```

Maxima [F]

$$\int e^{a+ibx}\csc^n(a+bx)\,dx=\int \csc(bx+a)^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^n*e^(I*b*x + a), x)
```

Giac [F]

$$\int e^{a+ibx}\csc^n(a+bx)\,dx=\int \csc(bx+a)^n e^{(i\,bx+a)}\,dx$$

input
```
integrate(exp(a+I*b*x)*csc(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^n*e^(I*b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+ibx}\csc^n(a+bx)\,dx=\int e^{a+b\,x\,1i}\left(\frac{1}{\sin\left(a+b\,x\right)}\right)^n\,dx$$

input
```
int(exp(a + b*x*1i)*(1/sin(a + b*x))^n,x)
```

output
```
int(exp(a + b*x*1i)*(1/sin(a + b*x))^n, x)
```

Reduce [F]

$$\int e^{a+ibx}\csc^n(a+bx)\,dx=e^a\left(\int e^{bix}\csc\left(bx+a\right)^n dx\right)$$

input
```
int(exp(a+I*b*x)*csc(b*x+a)^n,x)
```

output
```
e**a*int(e**(b*i*x)*csc(a + b*x)**n,x)
```

3.116 $\int F^{c(a+bx)}(f\csc(d+ex))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 99

$$\int F^{c(a+bx)}(f\csc(d+ex))^n\,dx = \frac{\left(1-e^{2i(d+ex)}\right)^n F^{c(a+bx)}(f\csc(d+ex))^n \operatorname{Hypergeometric2F1}\left(n,\frac{1}{2}\left(n-\frac{ibc\log(F)}{e}\right),\frac{1}{2}\left(2+n-\frac{ibc\log(F)}{e}\right)\right.}{ien+bc\log(F)}$$

output
```
(1-exp(2*I*(e*x+d)))^n*F^(c*(b*x+a))*(f*csc(e*x+d))^n*hypergeom([n, 1/2*n-
1/2*I*b*c*ln(F)/e],[1+1/2*n-1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))/(I*e*n+b*
c*ln(F))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)}(f\csc(d+ex))^n\,dx = \frac{i\left(1-e^{-2i(d+ex)}\right)^n F^{c(a+bx)}(f\csc(d+ex))^n \operatorname{Hypergeometric2F1}\left(n,\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(2+n+\frac{ibc\log(F)}{e}\right),e^{-2\cdots}\right.}{en+ibc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Csc[d + e*x])^n,x]
```

output
```
(I*(1 - E^((-2*I)*(d + e*x)))^n*F^(c*(a + b*x))*(f*Csc[d + e*x])^n*Hyperge
ometric2F1[n, (e*n + I*b*c*Log[F])/(2*e), (2 + n + (I*b*c*Log[F])/e)/2, E^
((-2*I)*(d + e*x))])/(e*n + I*b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7271, 4955, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\csc(d+ex))^n\,dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^{-n}(d+ex)(f\csc(d+ex))^n\int F^{c(a+bx)}\csc^n(d+ex)dx$$

$$\Big\downarrow \text{4955}$$

$$e^{in(d+ex)}\left(1-e^{-2i(d+ex)}\right)^n(f\csc(d+ex))^n\int e^{-idn-iexn}\left(1-e^{-2i(d+ex)}\right)^{-n}F^{ac+bxc}dx$$

$$\Big\downarrow \text{2689}$$

$$-\frac{e^{in(d+ex)-idn-ienx}\left(1-e^{-2i(d+ex)}\right)^n F^{ac+bcx}(f\csc(d+ex))^n\,\text{Hypergeometric2F1}\left(n,\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(n+\frac{ibc\log(\ldots}{e}\right.\right.}{-bc\log(F)+ien}$$

input
```
Int[F^(c*(a + b*x))*(f*Csc[d + e*x])^n,x]
```

output
```
-((E^((-I)*d*n - I*e*n*x + I*n*(d + e*x))*(1 - E^((-2*I)*(d + e*x)))^n*F^(
a*c + b*c*x)*(f*Csc[d + e*x])^n*Hypergeometric2F1[n, (e*n + I*b*c*Log[F])/
(2*e), (2 + n + (I*b*c*Log[F])/e)/2, E^((-2*I)*(d + e*x))])/(I*e*n - b*c*L
og[F]))
```

Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 4955
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(1 - E^(-2*I*(d + e*x)))^n*(Csc[d + e*x]^n/E^((-I)*n*(d + e*x))
)   Int[SimplifyIntegrand[F^(c*(a + b*x))*(1/(E^(I*n*(d + e*x))*(1 - E^(-2*
I*(d + e*x)))^n)), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ
[n]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}(f\csc(ex+d))^n\,dx$$

input
```
int(F^(c*(b*x+a))*(f*csc(e*x+d))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*csc(e*x+d))^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\csc(d+ex))^n\,dx=\int (f\csc(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csc(e*x+d))^n,x, algorithm="fricas")
```

output
```
integral((f*csc(e*x + d))^n*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\csc(d+ex))^n\,dx=\int F^{c(a+bx)}(f\csc(d+ex))^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*csc(e*x+d))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*csc(d + e*x))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\csc(d+ex))^n\,dx=\int (f\csc(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csc(e*x+d))^n,x, algorithm="maxima")
```

output
```
integrate((f*csc(e*x + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\csc(d+ex))^n\,dx = \int (f\csc(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csc(e*x+d))^n,x, algorithm="giac")
```

output
```
integrate((f*csc(e*x + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\csc(d+ex))^n\,dx = \int F^{c\,(a+b\,x)}\left(\frac{f}{\sin(d+e\,x)}\right)^n dx$$

input
```
int(F^(c*(a + b*x))*(f/sin(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f/sin(d + e*x))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\csc(d+ex))^n\,dx = f^{ac+n}\left(\int f^{bcx}\csc(ex+d)^n\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*csc(e*x+d))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*csc(d + e*x)**n,x)
```

3.117 $\int F^{c(a+bx)}\left(f\csc\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n\,dx$

Optimal result

Integrand size = 31, antiderivative size = 141

$$\int F^{c(a+bx)}\left(f\csc\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n\,dx = \frac{f^2F^{c(a+bx)}(2-n)\left(f\csc\left(d-\frac{ibcx\log(F)}{2-n}\right)\right)^{-2+n}}{bc(1-n)\log(F)} + \frac{ifF^{c(a+bx)}(2-n)\cos\left(d-\frac{ibcx\log(F)}{2-n}\right)\left(f\csc\left(d-\frac{ibcx\log(F)}{2-n}\right)\right)^{-1+n}}{bc(1-n)\log(F)}$$

output
```
f^2*F^(c*(b*x+a))*(2-n)*(-f*csc(-d+I*b*c*x*ln(F)/(2-n)))^(-2+n)/b/c/(1-n)/
ln(F)+I*f*F^(c*(b*x+a))*(2-n)*cos(-d+I*b*c*x*ln(F)/(2-n))*(-f*csc(-d+I*b*c
*x*ln(F)/(2-n)))^(-1+n)/b/c/(1-n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.57

$$\int F^{c(a+bx)}\left(f\csc\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \frac{e^{-2id}F^{c(a+bx)}\left(e^{2id}-F^{\frac{2bcx}{-2+n}}\right)(-2+n)\left(f\csc\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n}{2bc(-1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Csc[d + (I*b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
(F^(c*(a + b*x))*(E^((2*I)*d) - F^((2*b*c*x)/(-2 + n)))*(-2 + n)*(f*Csc[d
+ (I*b*c*x*Log[F])/(-2 + n)])^n)/(2*b*c*E^((2*I)*d)*(-1 + n)*Log[F])
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {7271, 4947}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\csc\left(d+\frac{ibcx\log(F)}{n-2}\right)\right)^n dx$$

$$\Big\downarrow \text{7271}$$

$$\csc^{-n}\left(d-\frac{ibcx\log(F)}{2-n}\right)\left(f\csc\left(d-\frac{ibcx\log(F)}{2-n}\right)\right)^n\int F^{c(a+bx)}\csc^n\left(d-\frac{ibcx\log(F)}{2-n}\right)dx$$

$$\Big\downarrow \text{4947}$$

$$\csc^{-n}\left(d-\frac{ibcx\log(F)}{2-n}\right)\left(f\csc\left(d-\frac{ibcx\log(F)}{2-n}\right)\right)^n\left(\frac{(2-n)F^{c(a+bx)}\csc^{n-2}\left(d-\frac{ibcx\log(F)}{2-n}\right)}{bc(1-n)\log(F)}-\frac{i(2-n)F^{c(}}{}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Csc[d + (I*b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
((f*Csc[d - (I*b*c*x*Log[F])/(2 - n)])^n*((F^(c*(a + b*x))*(2 - n)*Csc[d -
 (I*b*c*x*Log[F])/(2 - n)]^(-2 + n))/(b*c*(1 - n)*Log[F]) - (I*F^(c*(a + b
*x))*(2 - n)*Cos[d - (I*b*c*x*Log[F])/(2 - n)]*Csc[d - (I*b*c*x*Log[F])/(2
 - n)]^(-1 + n))/(b*c*(1 - n)*Log[F])))/Csc[d - (I*b*c*x*Log[F])/(2 - n)]^
n
```

Defintions of rubi rules used

rule 4947
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/(e
*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[b^2*c^2*Log[F]^2 +
 e^2*(n - 2)^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(f\csc\left(d+\frac{ibcx\ln(F)}{-2+n}\right)\right)^{n}dx$$

input
```
int(F^(c*(b*x+a))*(f*csc(d+I*b*c*x*ln(F)/(-2+n)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*csc(d+I*b*c*x*ln(F)/(-2+n)))^n,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}\left(f\csc\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx$$
$$=\frac{\left((n-2)e^{\left(-\frac{2\,(bcx\log(F)-i\,dn+2i\,d)}{n-2}\right)}-n+2\right)F^{bcx+ac}\left(\frac{2i\,fe^{\left(-\frac{bcx\log(F)-i\,dn+2i\,d}{n-2}\right)}}{e^{\left(-\frac{2\,(bcx\log(F)-i\,dn+2i\,d)}{n-2}\right)}-1}\right)^n e^{\left(\frac{2\,(bcx\log(F)-i\,dn+2i\,d)}{n-2}\right)}}{2\,(bcn-bc)\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(f*csc(d+I*b*c*x*log(F)/(-2+n)))^n,x, algorithm="f
ricas")
```

output
```
1/2*((n - 2)*e^(-2*(b*c*x*log(F) - I*d*n + 2*I*d)/(n - 2)) - n + 2)*F^(b*c
*x + a*c)*(2*I*f*e^(-(b*c*x*log(F) - I*d*n + 2*I*d)/(n - 2))/(e^(-2*(b*c*x
*log(F) - I*d*n + 2*I*d)/(n - 2)) - 1))^n*e^(2*(b*c*x*log(F) - I*d*n + 2*I
*d)/(n - 2))/((b*c*n - b*c)*log(F))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\csc\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx=\int F^{c(a+bx)}\left(f\csc\left(\frac{ibcx\log(F)}{n-2}+d\right)\right)^n dx$$

input
```
integrate(F**(c*(b*x+a))*(f*csc(d+I*b*c*x*ln(F)/(-2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*csc(I*b*c*x*log(F)/(n - 2) + d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\csc\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \int \left(f\csc\left(\frac{i\,bcx\log\left(F\right)}{n-2}+d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csc(d+I*b*c*x*log(F)/(-2+n)))^n,x, algorithm="m
axima")
```

output
```
integrate((f*csc(I*b*c*x*log(F)/(n - 2) + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\left(f\csc\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \int \left(f\csc\left(\frac{i\,bcx\log\left(F\right)}{n-2}+d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csc(d+I*b*c*x*log(F)/(-2+n)))^n,x, algorithm="g
iac")
```

output
```
integrate((f*csc(I*b*c*x*log(F)/(n - 2) + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\csc\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \int F^{c\,(a+b\,x)}\,{\left(\frac{f}{\sin\left(d+\frac{b\,c\,x\,\ln\left(F\right)\,1{}\mathrm{i}}{n-2}\right)}\right)}^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f/sin(d + (b*c*x*log(F)*1i)/(n - 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f/sin(d + (b*c*x*log(F)*1i)/(n - 2)))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\left(f\csc\left(d+\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx$$
$$=f^{ac+n}\left(\int f^{bcx}\csc\left(\frac{\log(f)\,bcix+dn-2d}{n-2}\right)^n dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*csc(d+I*b*c*x*log(F)/(-2+n)))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*csc((log(f)*b*c*i*x + d*n - 2*d)/(n - 2))**n,x
)
```

3.118 $\int F^{c(a+bx)}\left(f\csc\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n\,dx$

Optimal result

Integrand size = 31, antiderivative size = 141

$$\int F^{c(a+bx)}\left(f\csc\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n\,dx = \frac{f^2F^{c(a+bx)}(2-n)\left(f\csc\left(d+\frac{ibcx\log(F)}{2-n}\right)\right)^{-2+n}}{bc(1-n)\log(F)} - \frac{ifF^{c(a+bx)}(2-n)\cos\left(d+\frac{ibcx\log(F)}{2-n}\right)\left(f\csc\left(d+\frac{ibcx\log(F)}{2-n}\right)\right)^{-1+n}}{bc(1-n)\log(F)}$$

output
```
f^2*F^(c*(b*x+a))*(2-n)*(f*csc(d+I*b*c*x*ln(F)/(2-n)))^(-2+n)/b/c/(1-n)/ln
(F)-I*f*F^(c*(b*x+a))*(2-n)*cos(d+I*b*c*x*ln(F)/(2-n))*(f*csc(d+I*b*c*x*ln
(F)/(2-n)))^(-1+n)/b/c/(1-n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

$$\int F^{c(a+bx)}\left(f\csc\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = -\frac{F^{c(a+bx)}\left(-1+e^{2id}F^{\frac{2bcx}{-2+n}}\right)(-2+n)\left(f\csc\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n}{2bc(-1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Csc[d - (I*b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
-1/2*(F^(c*(a + b*x))*(-1 + E^((2*I)*d)*F^((2*b*c*x)/(-2 + n)))*(-2 + n)*(
f*Csc[d - (I*b*c*x*Log[F])/(-2 + n)])^n)/(b*c*(-1 + n)*Log[F])
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {7271, 4947}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\csc\left(d-\frac{ibcx\log(F)}{n-2}\right)\right)^n dx$$

$$\Big\downarrow \text{ 7271}$$

$$\csc^{-n}\left(d+\frac{ibcx\log(F)}{2-n}\right)\left(f\csc\left(d+\frac{ibcx\log(F)}{2-n}\right)\right)^n\int F^{c(a+bx)}\csc^n\left(d+\frac{ibcx\log(F)}{2-n}\right)dx$$

$$\Big\downarrow \text{ 4947}$$

$$\csc^{-n}\left(d+\frac{ibcx\log(F)}{2-n}\right)\left(f\csc\left(d+\frac{ibcx\log(F)}{2-n}\right)\right)^n\left(\frac{(2-n)F^{c(a+bx)}\csc^{n-2}\left(d+\frac{ibcx\log(F)}{2-n}\right)}{bc(1-n)\log(F)}+\frac{i(2-n)F^{c(\ldots}}{}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Csc[d - (I*b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
((f*Csc[d + (I*b*c*x*Log[F])/(2 - n)])^n*((F^(c*(a + b*x))*(2 - n)*Csc[d +
 (I*b*c*x*Log[F])/(2 - n)]^(-2 + n))/(b*c*(1 - n)*Log[F]) + (I*F^(c*(a + b
*x))*(2 - n)*Cos[d + (I*b*c*x*Log[F])/(2 - n)]*Csc[d + (I*b*c*x*Log[F])/(2
 - n)]^(-1 + n))/(b*c*(1 - n)*Log[F])))/Csc[d + (I*b*c*x*Log[F])/(2 - n)]^
n
```

Defintions of rubi rules used

rule 4947
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/(e
*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[b^2*c^2*Log[F]^2 +
 e^2*(n - 2)^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(-f\csc\left(-d+\frac{ibcx\ln(F)}{-2+n}\right)\right)^n dx$$

input
```
int(F^(c*(b*x+a))*(-f*csc(-d+I*b*c*x*ln(F)/(-2+n)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(-f*csc(-d+I*b*c*x*ln(F)/(-2+n)))^n,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}\left(f\csc\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \frac{\left((n-2)e^{\left(-\frac{2\,(bcx\log(F)+i\,dn-2i\,d)}{n-2}\right)}-n+2\right)F^{bcx+ac}\left(-\frac{2i\,fe^{\left(-\frac{bcx\log(F)+i\,dn-2i\,d}{n-2}\right)}}{e^{\left(-\frac{2\,(bcx\log(F)+i\,dn-2i\,d)}{n-2}\right)}-1}\right)^n e^{\left(\frac{2\,(bcx\log(F)+i\,dn-2i\,d)}{n-2}\right)}}{2\,(bcn-bc)\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(-f*csc(-d+I*b*c*x*log(F)/(-2+n)))^n,x, algorithm=
"fricas")
```

output
```
1/2*((n - 2)*e^(-2*(b*c*x*log(F) + I*d*n - 2*I*d)/(n - 2)) - n + 2)*F^(b*c
*x + a*c)*(-2*I*f*e^(-(b*c*x*log(F) + I*d*n - 2*I*d)/(n - 2))/(e^(-2*(b*c*
x*log(F) + I*d*n - 2*I*d)/(n - 2)) - 1))^n*e^(2*(b*c*x*log(F) + I*d*n - 2*
I*d)/(n - 2))/((b*c*n - b*c)*log(F))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\csc\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \int F^{c(a+bx)}\left(-f\csc\left(\frac{ibcx\log(F)}{n-2}-d\right)\right)^n dx$$

input
```
integrate(F**(c*(b*x+a))*(-f*csc(-d+I*b*c*x*ln(F)/(-2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(-f*csc(I*b*c*x*log(F)/(n - 2) - d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\csc\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \int\left(-f\csc\left(\frac{i\,bcx\log(F)}{n-2}-d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(-f*csc(-d+I*b*c*x*log(F)/(-2+n)))^n,x, algorithm=
"maxima")
```

output
```
integrate((-f*csc(I*b*c*x*log(F)/(n - 2) - d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\left(f\csc\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \int\left(-f\csc\left(\frac{i\,bcx\log(F)}{n-2}-d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(-f*csc(-d+I*b*c*x*log(F)/(-2+n)))^n,x, algorithm=
"giac")
```

output
```
integrate((-f*csc(I*b*c*x*log(F)/(n - 2) - d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\csc\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx = \int F^{c\,(a+b\,x)}\left(\frac{f}{\sin\left(d-\frac{b\,c\,x\,\ln(F)\,1\mathrm{i}}{n-2}\right)}\right)^n dx$$

input
```
int(F^(c*(a + b*x))*(f/sin(d - (b*c*x*log(F)*1i)/(n - 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f/sin(d - (b*c*x*log(F)*1i)/(n - 2)))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\left(f\csc\left(d-\frac{ibcx\log(F)}{-2+n}\right)\right)^n dx$$
$$= f^{ac+n}(-1)^n\left(\int f^{bcx}\csc\left(\frac{\log(f)\,bcix-dn+2d}{n-2}\right)^n dx\right)$$

input
```
int(F^(c*(b*x+a))*(-f*csc(-d+I*b*c*x*log(F)/(-2+n)))^n,x)
```

output
```
f**(a*c + n)*( - 1)**n*int(f**(b*c*x)*csc((log(f)*b*c*i*x - d*n + 2*d)/(n
- 2))**n,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
         fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
         gamma,loggamma,digamma,zeta,polylog,LambertW,
         elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file