Computer Algebra
Independent Integration Tests

Summer 2024

4-Trig-functions/4-Miscellaneous/256-4.2

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May 18, 2024 Compiled on May 18, 2024 at 1:58am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**619**]. This is test number [256].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	98.71 (611)	1.29 (8)
Maple	85.30 (528)	14.70 (91)
Fricas	85.30 (528)	14.70 (91)
Rubi	73.67 (456)	26.33 (163)
Giac	72.86 (451)	27.14 (168)
Maxima	69.31 (429)	30.69 (190)
Mupad	64.14 (397)	35.86 (222)
Sympy	34.25 (212)	65.75 (407)
Reduce	30.53 (189)	69.47 (430)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

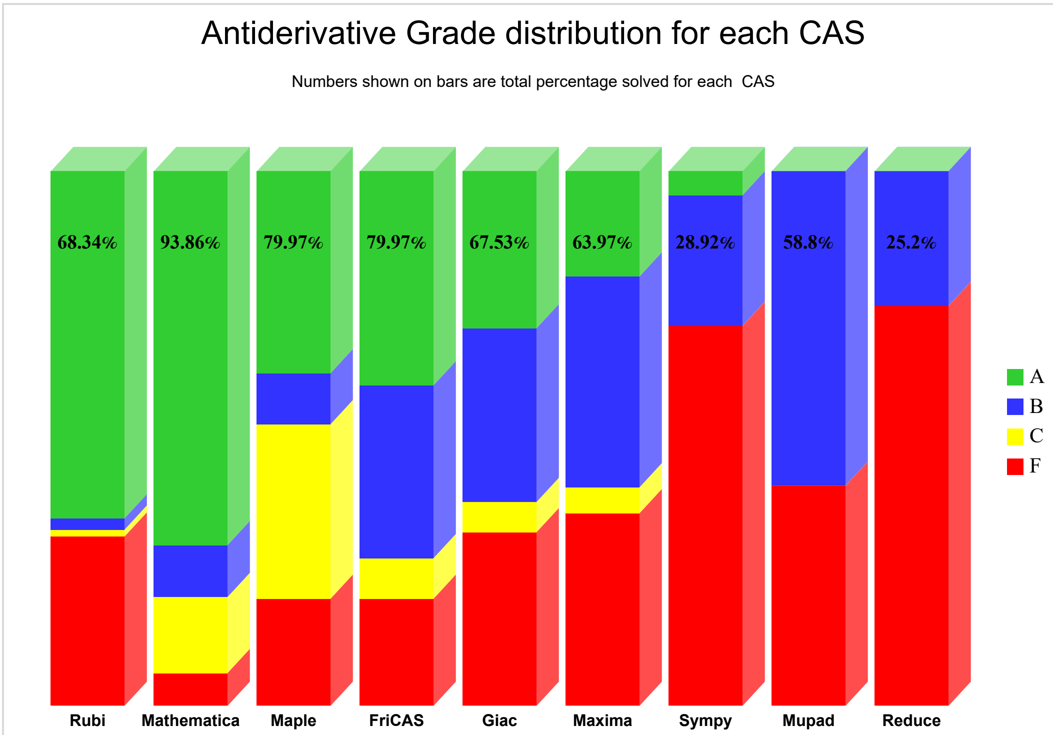
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

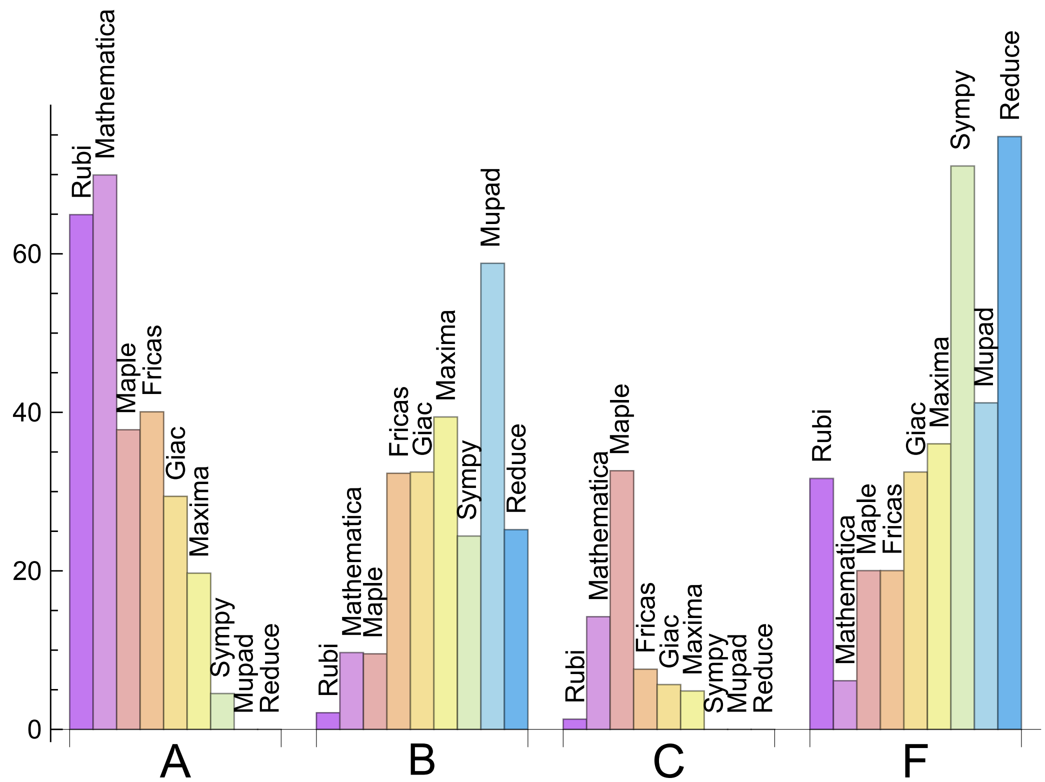
System	% A grade	% B grade	% C grade	% F grade
Mathematica	69.952	9.693	14.216	6.139
Rubi	64.943	2.100	1.292	31.664
Fricas	40.065	32.310	7.593	20.032
Maple	37.803	9.532	32.633	20.032
Giac	29.402	32.472	5.654	32.472
Maxima	19.709	39.418	4.847	36.026
Sympy	4.523	24.394	0.000	71.082
Mupad	0.000	58.805	0.000	41.195
Reduce	0.000	25.202	0.000	74.798

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	8	62.50	37.50	0.00
Fricas	91	90.11	0.00	9.89
Maple	91	84.62	15.38	0.00
Rubi	163	100.00	0.00	0.00
Giac	168	91.07	7.74	1.19
Maxima	190	96.32	2.63	1.05
Mupad	222	0.00	100.00	0.00
Sympy	407	33.42	61.67	4.91
Reduce	430	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.09
Rubi	0.33
Maxima	2.07
Mathematica	2.10
Reduce	3.18
Giac	3.71
Sympy	10.49
Maple	11.45
Mupad	14.44

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	76.19	1.25	48.00	1.00
Fricas	128.32	34.60	84.50	1.68
Mathematica	168.32	26.73	64.00	1.00
Reduce	230.59	9.27	78.00	1.49
Mupad	823.91	571.01	60.00	1.27
Sympy	2001.17	65.65	174.00	3.56
Giac	7902.74	5309.63	89.00	1.85
Maxima	32262.11	30592.01	163.00	6.55
Maple	3868820.05	51334.60	84.00	1.50

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

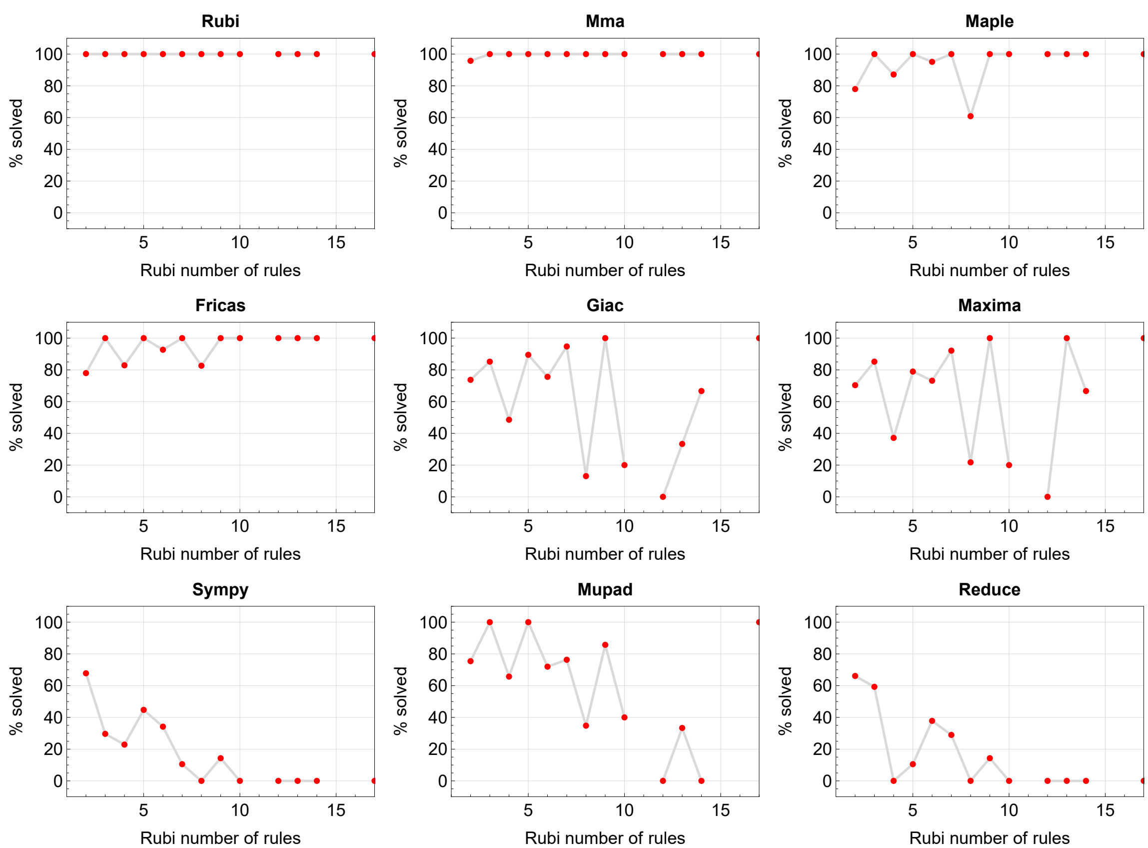


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

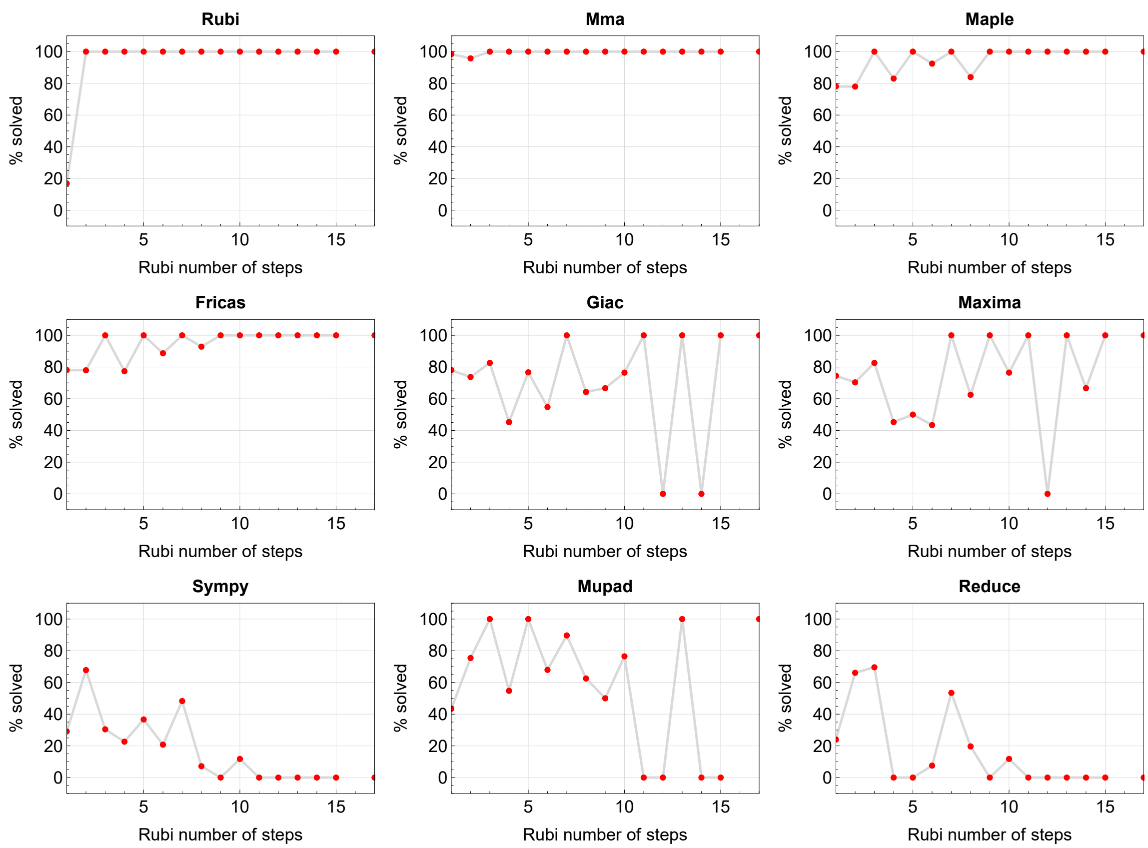


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

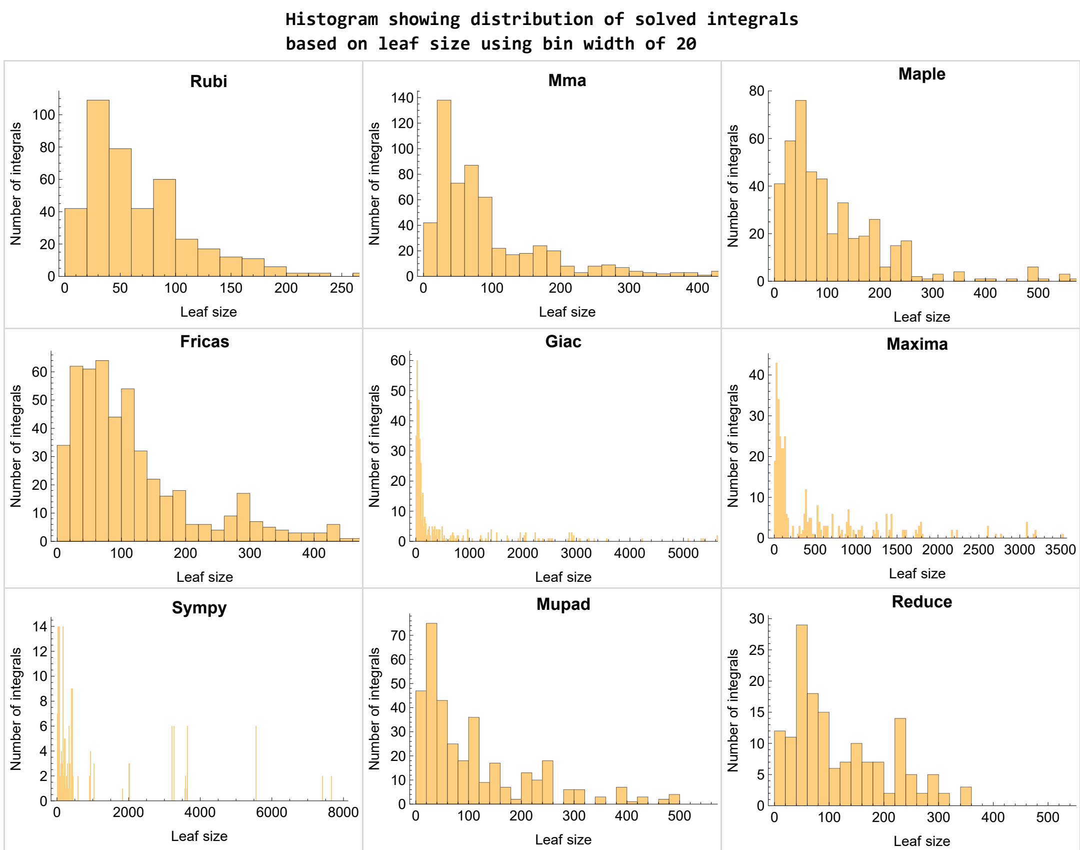


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

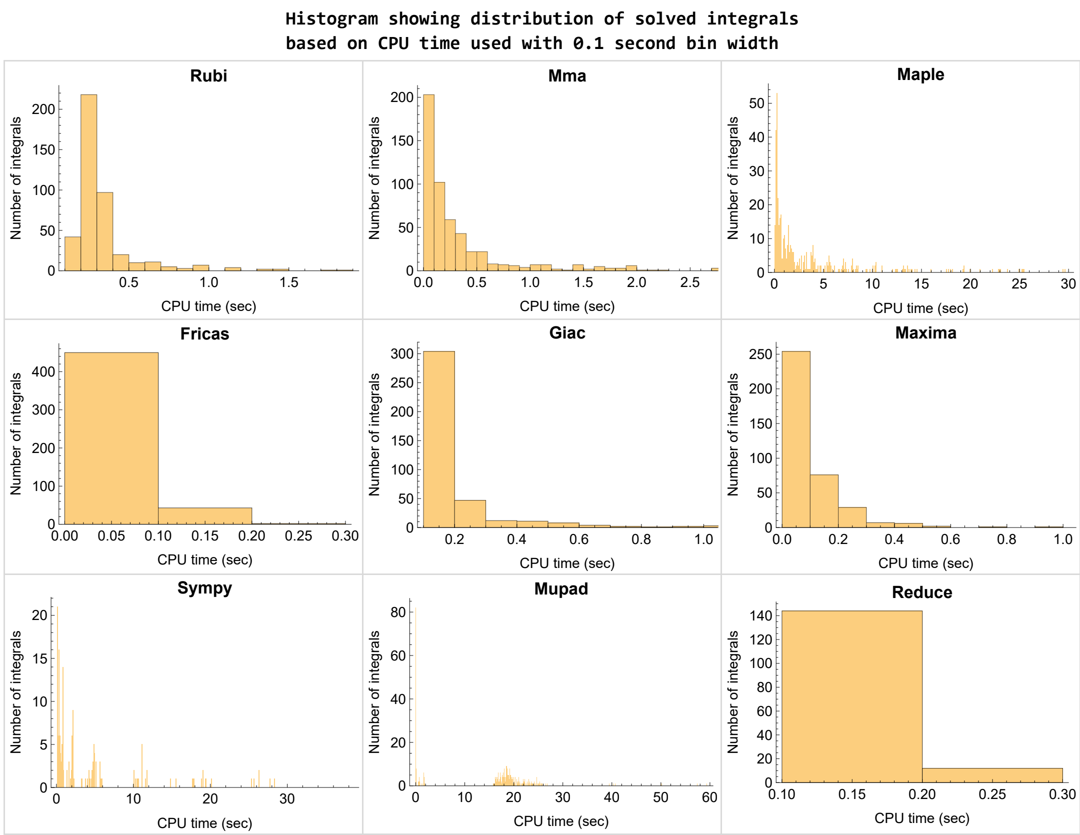


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

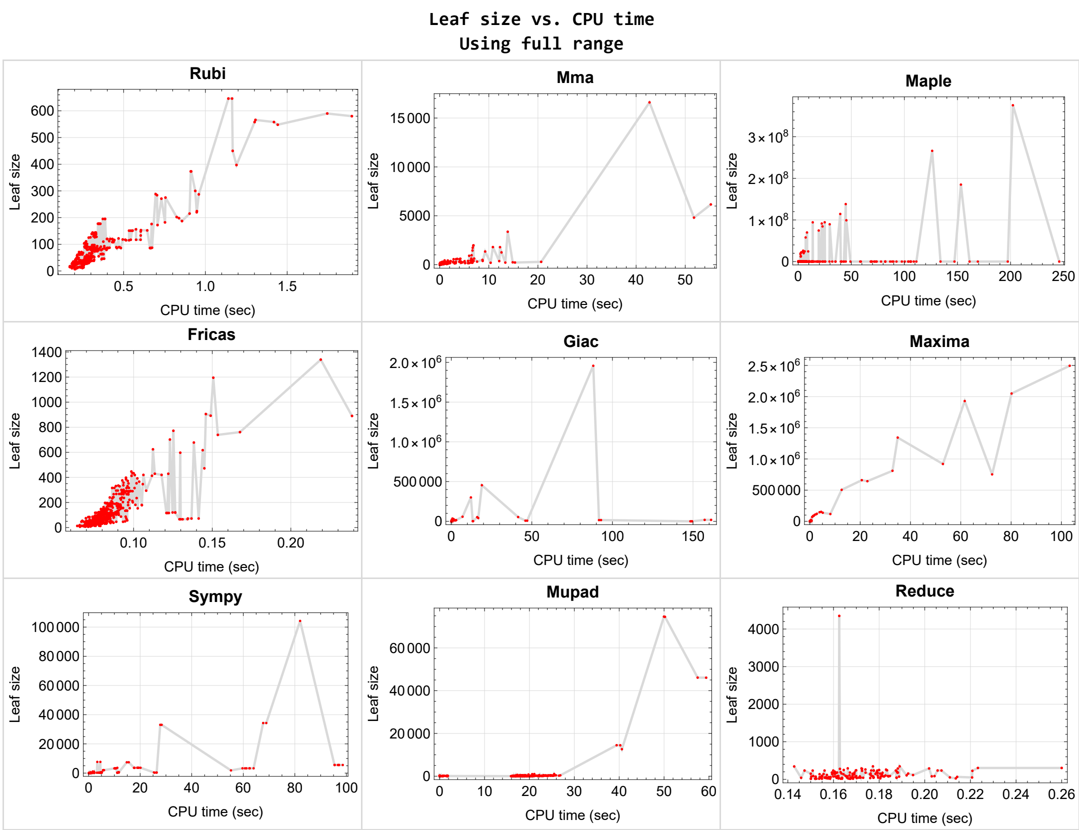


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{130, 131, 132, 133, 134, 135, 139, 140, 144, 145, 206, 207, 208, 212, 213, 214, 218, 219, 220, 224, 225, 229, 230, 347, 348, 349, 350, 351, 352, 356, 357, 361, 362}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {137, 222, 354, 432, 465, 467, 559}

Mathematica {34, 36, 49, 51, 109, 138, 143, 184, 222, 226, 227, 228, 323, 324, 326, 353, 354, 359, 360, 534, 535, 537, 596, 598}

Maple {108, 252, 325, 326, 483, 484, 485, 486, 487, 493, 494, 495, 496, 500, 508, 509, 571, 572, 573, 574, 579, 580, 581, 582, 583, 586, 587, 588, 589, 603, 604, 608, 609, 613, 614, 618, 619}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

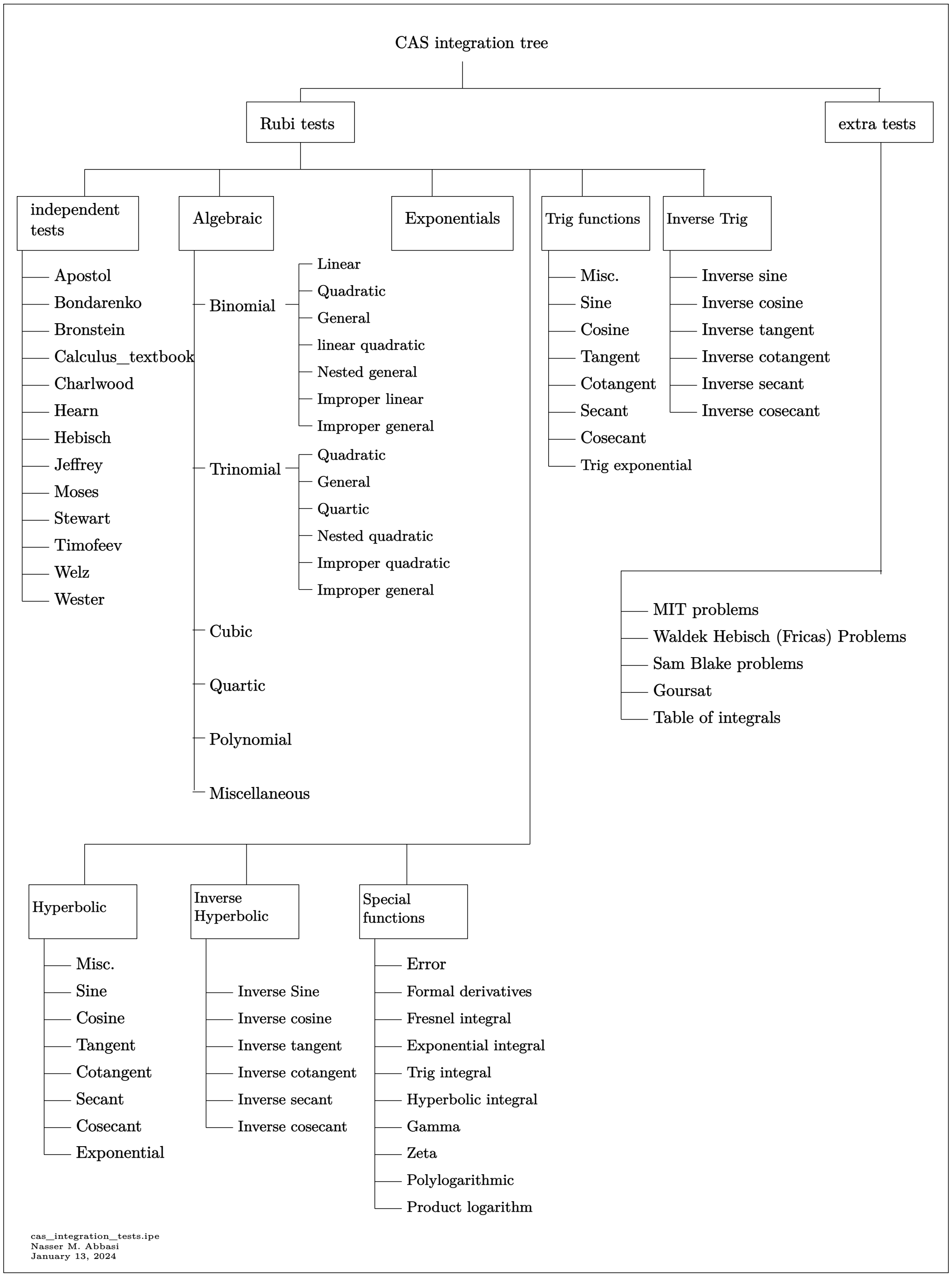
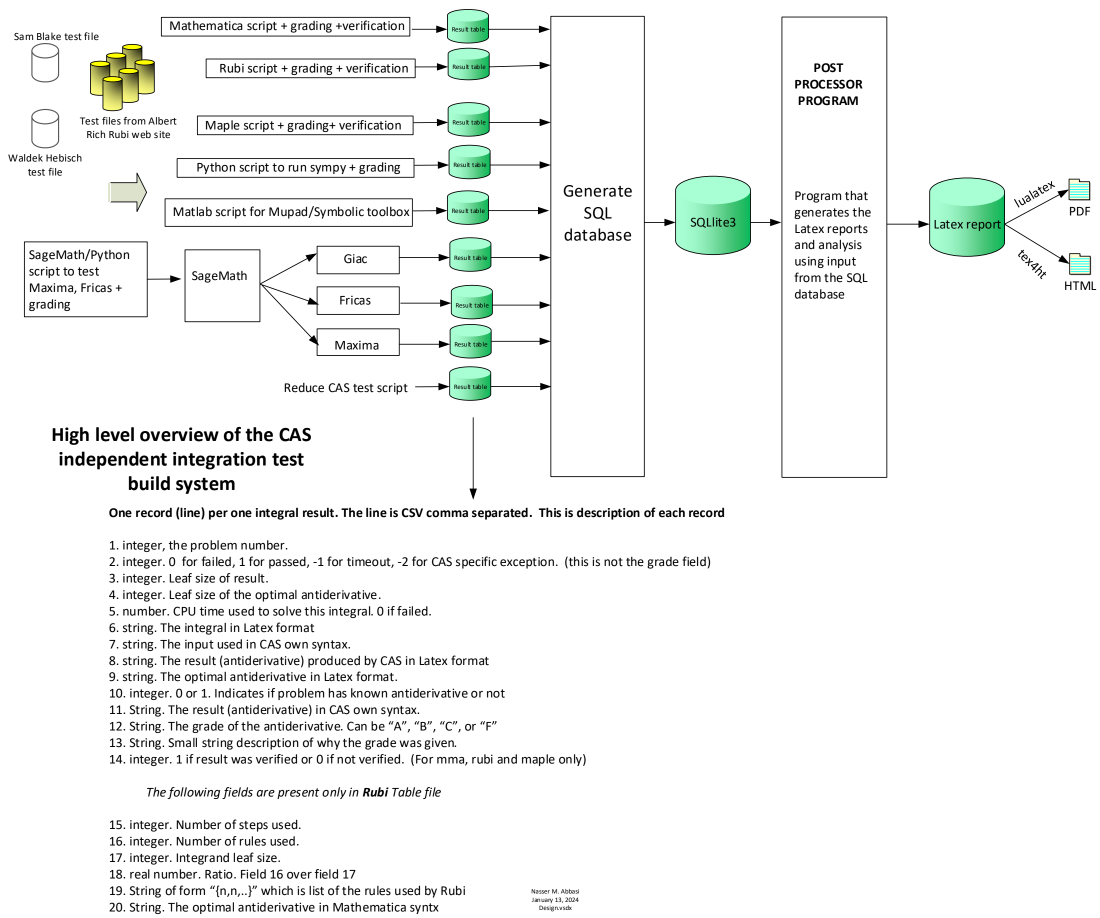


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 96, 101, 110, 111, 112, 116, 117, 118, 123, 124, 125, 136, 137, 138, 141, 142, 143, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 161, 162, 163, 169, 175, 181, 185, 186, 187, 191, 192, 193, 197, 198, 199, 221, 223, 226, 227, 228, 231, 232, 233, 234, 235, 236, 256, 257, 261, 262, 263, 264, 277, 278, 279, 280, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 311, 312, 313, 318, 327, 328, 329, 333, 334, 335, 340, 341, 342, 353, 355, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 387, 388, 389, 390, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 610, 611, 612 }

B grade { 23, 53, 222, 354, 600, 601, 602, 605, 606, 607, 615, 616, 617 }

C grade { 603, 604, 608, 609, 613, 614, 618, 619 }

F normal fail { 90, 91, 92, 93, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 158, 159, 160, 164, 165, 166, 167, 168, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 182, 183, 184, 188, 189, 190, 194, 195, 196, 200, 201, 202, 203, 204, 205, 209, 210, 211, 215, 216, 217, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 260, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 307, 308, 309, 310, 314, 315, 316, 317, 319, 320, 321, 322, 323, 324, 325, 326, 330, 331, 332, 336, 337, 338, 339, 343, 344, 345, 346, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 35, 37, 38, 39, 41, 42, 43, 44, 45, 46, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 110, 111, 112, 113, 115, 116, 117, 118, 119, 121, 123, 124, 125, 126, 128, 138, 143, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 168, 170, 171, 181, 185, 186, 187, 188, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 202, 203, 204, 205, 209, 210, 211, 217, 221, 222, 223, 226, 227, 228, 231, 236, 239, 241, 243, 249, 250, 256, 258, 259, 260, 261, 263, 264, 265, 267, 268, 271, 272, 275, 276, 277, 279, 280, 281, 283, 284, 285, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 300, 301, 302, 303, 304, 305, 306, 309, 310, 311, 312, 313, 314, 318, 319, 320, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 343, 344, 345, 359, 360, 363, 368, 369, 370, 371, 373, 374, 375, 377, 378, 381, 382, 385, 386, 389, 390, 393, 394, 395, 398, 399, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 472, 473, 474, 475, 476, 477, 478, 480, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 536, 538, 539, 540, 541, 542, 543, 544, 545, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 583, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619 }

B grade { 19, 20, 32, 40, 47, 48, 50, 52, 53, 56, 68, 91, 92, 114, 120, 122, 127, 129, 158, 159, 174, 180, 189, 195, 201, 215, 216, 238, 257, 269, 270, 273, 274, 286, 287, 290, 307, 308, 339, 346, 353, 354, 355, 379, 380, 383, 384, 396, 397, 400, 421, 450, 452, 454, 471, 482, 547, 549, 567, 569 }

C grade { 34, 36, 49, 51, 54, 66, 82, 104, 105, 106, 107, 108, 109, 150, 165, 166, 167, 169, 172, 173, 175, 176, 177, 178, 179, 182, 183, 184, 232, 233, 234, 235, 237, 240, 242, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 262, 266, 278, 282, 299, 315, 316, 317, 321, 322, 323, 324, 325, 326, 364, 365, 366, 367, 372, 376, 387, 388, 391, 392, 420, 422, 442, 451, 453, 479, 481, 518, 520, 534, 535, 537, 546, 548, 568, 582, 584, 596, 598 }

F normal fail { 136, 137, 141, 142, 358 }

F(-1) timedout fail { 132, 208, 220 }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 27, 32, 33, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 53, 57, 58, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 87, 88, 89, 93, 94, 95, 96, 98, 99, 100, 110, 111, 112, 116, 117, 118, 123, 124, 125, 146, 147, 148, 151, 153, 155, 156, 157, 161, 162, 163, 170, 181, 185, 186, 187, 191, 192, 193, 197, 198, 199, 263, 267, 272, 276, 288, 292, 293, 294, 295, 296, 297, 300, 304, 305, 306, 310, 311, 312, 313, 318, 327, 328, 329, 333, 334, 335, 340, 341, 342, 373, 377, 382, 386, 389, 393, 398, 402, 407, 408, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428, 429, 430, 432, 434, 435, 436, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 465, 467, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 516, 517, 518, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 555, 556, 557, 559, 561, 562, 563, 565, 566, 567, 568, 569, 599 }

B grade { 52, 101, 102, 171, 319, 410, 427, 433, 437, 483, 484, 485, 486, 487, 493, 494, 495, 496, 497, 498, 500, 508, 509, 515, 519, 520, 521, 522, 523, 554, 560, 564, 571, 572, 573, 574, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 600, 601, 602, 605, 606, 607, 610, 611, 612, 615, 616, 617 }

C grade { 9, 24, 25, 26, 28, 29, 30, 31, 34, 49, 50, 51, 54, 55, 56, 59, 60, 65, 81, 82, 83, 84, 85, 86, 90, 91, 92, 97, 103, 104, 105, 106, 107, 108, 109, 149, 150, 152, 154, 158, 159, 160, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 261, 262, 264, 265, 266, 268, 269, 270, 271, 273, 274, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 289, 290, 291, 298, 299, 301, 302, 303, 307, 308, 309, 314, 315, 316, 317, 320, 321, 322, 323, 324, 325, 326, 363, 364, 365, 366, 367, 368, 371, 372, 374, 375, 376, 378, 379, 380, 381, 383, 384, 385, 387, 388, 390, 391, 392, 394, 395, 396, 397, 399, 400, 401, 403, 404, 405, 406, 409, 431, 464, 466, 468, 488, 489, 503, 504, 507, 510, 511, 512, 513, 514, 525, 526, 527, 528, 529, 530, 531, 532, 558, 575, 576, 590, 591, 594, 595, 603, 604, 608, 609, 613, 614, 618, 619 }

F normal fail { 19, 20, 21, 40, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 136, 137, 138, 141, 142, 143, 188, 189, 190, 194, 195, 196, 200, 201, 202, 203, 204, 205, 209, 210, 211, 215, 216, 217, 221, 222, 223, 226, 227, 228, 256, 257, 258, 259, 260, 330, 331, 332, 336, 337, 338, 339, 343, 344, 345, 346, 353, 354, 355, 358, 359, 360, 369, 370, 533, 534, 535, 536, 537, 596, 597, 598 }

F(-1) timedout fail { 490, 491, 492, 499, 501, 502, 505, 506, 524, 570, 577, 578, 592, 593 }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 7, 11, 12, 13, 14, 15, 16, 17, 18, 23, 24, 30, 33, 35, 37, 38, 39, 41, 42, 43, 44, 45, 46, 49, 50, 58, 65, 67, 70, 71, 73, 75, 77, 78, 79, 81, 82, 83, 85, 87, 88, 89, 91, 92, 93, 94, 95, 98, 99, 100, 110, 111, 112, 116, 117, 118, 123, 124, 125, 146, 147, 149, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 167, 169, 174, 180, 181, 185, 186, 187, 191, 192, 193, 197, 198, 199, 243, 261, 263, 264, 270, 271, 272, 277, 278, 279, 281, 286, 287, 288, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 315, 327, 328, 329, 333, 334, 335, 340, 341, 342, 371, 372, 373, 374, 380, 381, 382, 387, 389, 391, 396, 397, 398, 411, 412, 413, 414, 415, 416, 417, 422, 423, 424, 425, 426, 427, 428, 429, 431, 433, 434, 435, 436, 438, 439, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 455, 456, 457, 458, 459, 460, 461, 462, 464, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 487, 488, 489, 490, 503, 504, 505, 506, 511, 512, 513, 514, 525, 526, 530, 531, 532, 538, 539, 540, 541, 542, 543, 544, 550, 551, 552, 553, 554, 555, 556, 558, 560, 561, 562, 563, 564, 565, 566, 574, 575, 576, 577, 591, 592, 593, 599 }

B grade { 6, 8, 9, 10, 22, 25, 26, 27, 28, 29, 31, 32, 34, 36, 47, 48, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 68, 69, 72, 74, 76, 80, 84, 86, 90, 96, 97, 101, 102, 103, 148, 150, 158, 168, 170, 171, 175, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 245, 249, 250, 253, 262, 265, 266, 267, 268, 269, 273, 274, 275, 276, 280, 282, 283, 284, 285, 289, 290, 291, 292, 293, 297, 307, 313, 314, 318, 319, 320, 363, 364, 365, 366, 367, 368, 375, 376, 377, 378, 379, 383, 384, 385, 386, 388, 390, 392, 393, 394, 395, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 430, 432, 437, 440, 451, 452, 453, 454, 463, 465, 467, 479, 480, 481, 482, 483, 484, 485, 486, 499, 500, 501, 502, 507, 508, 509, 510, 527, 528, 529, 545, 546, 547, 548, 549, 557, 559, 567, 568, 569, 570, 571, 572, 573, 586, 587, 588, 589, 590, 594, 595, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619 }

C grade { 104, 105, 106, 107, 108, 109, 165, 166, 172, 173, 176, 177, 178, 179, 182, 183, 184, 237, 242, 244, 246, 247, 248, 251, 252, 254, 255, 316, 317, 321, 322, 323, 324, 325, 326, 496, 497, 498, 519, 520, 521, 522, 523, 524, 583, 584, 585 }

F normal fail { 19, 20, 21, 40, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 141, 142, 143, 188, 189, 190, 194, 195, 196, 200, 201, 202, 203, 204, 205, 209, 210, 211, 215, 216, 217, 226, 227, 228, 256, 257, 258, 259, 260, 330, 331, 332, 336, 337, 338, 339, 343, 344, 345, 346, 358, 359, 360, 369, 370, 491, 492, 493, 494, 495, 515, 516, 517, 518, 533, 534, 535, 536, 537, 578, 579, 580, 581, 582, 596, 597, 598 }

F(-1) timedout fail { }

F(-2) exception fail { 136, 137, 138, 221, 222, 223, 353, 354, 355 }

Maxima

A grade { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 16, 17, 18, 37, 38, 39, 41, 42, 43, 44, 45, 46, 62, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 87, 88, 89, 94, 95, 96, 112, 146, 147, 148, 155, 156, 157, 161, 162, 163, 164, 187, 239, 243, 293, 294, 295, 296, 297, 304, 305, 306, 311, 312, 313, 329, 411, 412, 413, 414, 415, 416, 417, 423, 424, 425, 426, 427, 433, 434, 435, 436, 443, 444, 445, 446, 447, 448, 449, 450, 455, 456, 457, 458, 459, 460, 461, 469, 470, 471, 472, 473, 474, 476, 538, 539, 540, 541, 542, 543, 544, 550, 551, 552, 553, 554, 560, 561, 562, 563, 599 }

B grade { 6, 7, 8, 22, 27, 28, 29, 32, 33, 47, 52, 53, 54, 57, 58, 63, 64, 65, 66, 81, 82, 83, 84, 85, 86, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 110, 111, 116, 117, 118, 123, 124, 125, 149, 150, 151, 152, 153, 154, 158, 159, 160, 167, 168, 169, 170, 171, 174, 175, 180, 181, 185, 186, 191, 192, 193, 197, 198, 199, 231, 232, 233, 234, 235, 236, 240, 245, 249, 250, 253, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 314, 315, 318, 319, 320, 327, 328, 333, 334, 335, 340, 341, 342, 363, 364, 365, 366, 367, 368, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 428, 429, 430, 431, 432, 437, 438, 439, 440, 441, 442, 451, 452, 453, 454, 462, 463, 464, 465, 466, 467, 468, 475, 477, 478, 479, 480, 481, 482, 545, 546, 547, 548, 549, 555, 556, 557, 558, 559, 564, 565, 566, 567, 568, 569, 600, 601, 602, 605, 606, 607, 610, 611, 612, 615, 616, 617 }

C grade { 104, 105, 106, 165, 166, 172, 173, 176, 177, 178, 179, 182, 183, 184, 237, 242, 244, 246, 247, 248, 251, 252, 254, 255, 316, 317, 321, 322, 323, 324 }

F normal fail { 9, 10, 19, 20, 21, 23, 24, 25, 26, 30, 31, 34, 35, 36, 40, 48, 49, 50, 51, 55, 56, 59, 60, 61, 67, 68, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 136, 137, 138, 141, 142, 143, 188, 189, 190, 194, 195, 196, 200, 201, 202, 203, 204, 205, 209, 210, 211, 215, 216, 217, 221, 222, 223, 226, 227, 228, 256, 257, 258, 259, 260, 330, 331, 332, 336, 337, 338, 339, 343, 344, 345, 346, 353, 354, 355, 358, 359, 360, 369, 370, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 603, 604, 608, 609, 613, 614, 618, 619 }

F(-1) timedout fail { 107, 108, 109, 325, 326 }

F(-2) exception fail { 238, 241 }

Giac

A grade { 1, 2, 3, 4, 5, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 30, 33, 35, 37, 38, 39, 41, 42, 43, 44, 45, 46, 58, 61, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 87, 88, 89, 94, 95, 96, 110, 111, 112, 116, 117, 118, 123, 124, 125, 146, 147, 148, 155, 156, 157, 161, 162, 163, 185, 186, 187, 191, 192, 193, 197, 198, 199, 293, 294, 295, 296, 297, 304, 305, 306, 311, 312, 313, 327, 328, 329, 333, 334, 335, 340, 341, 342, 411, 412, 413, 414, 415, 416, 417, 420, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 440, 442, 444, 446, 448, 450, 451, 453, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 470, 472, 474, 476, 478, 479, 481, 538, 539, 540, 541, 542, 543, 544, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 599 }

B grade { 6, 8, 10, 23, 27, 28, 29, 31, 32, 34, 36, 52, 53, 54, 55, 56, 57, 59, 60, 62, 63, 64, 66, 68, 81, 82, 83, 84, 85, 86, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 149, 150, 151, 152, 153, 154, 158, 159, 160, 164, 167, 168, 169, 170, 171, 174, 175, 180, 181, 234, 235, 236, 238, 239, 240, 241, 243, 245, 249, 250, 253, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 314, 315, 318, 319, 320, 366, 367, 368, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 418, 419, 421, 439, 441, 443, 445, 447, 449, 452, 454, 469, 471, 473, 475, 477, 480, 482, 487, 488, 489, 503, 545, 574, 575, 576, 590, 600, 601, 602, 605, 606, 607, 610, 611, 612, 615, 616, 617 }

C grade { 104, 105, 106, 107, 108, 109, 165, 166, 172, 173, 176, 177, 178, 179, 182, 183, 184, 237, 242, 244, 246, 247, 248, 251, 252, 254, 255, 316, 317, 321, 322, 323, 324, 325, 326 }

F normal fail { 19, 20, 21, 22, 24, 25, 26, 40, 47, 48, 49, 50, 51, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 136, 137, 138, 141, 142, 143, 188, 189, 190, 194, 195, 196, 200, 201, 202, 203, 204, 205, 209, 210, 211, 215, 216, 217, 221, 222, 223, 226, 227, 228, 231, 232, 233, 256, 257, 258, 259, 260, 330, 331, 332, 336, 337, 338, 339, 343, 344, 345, 346, 353, 354, 355, 358, 359, 360, 363, 364, 365, 369, 370, 483, 484, 485, 486, 491, 492, 493, 494, 496, 497, 498, 499, 502, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 570, 571, 572, 573, 579, 580, 581, 582, 583, 584, 585, 586, 588, 589, 594, 595, 596, 597, 598, 603, 604, 608, 609, 613, 614, 618, 619 }

F(-1) timedout fail { 490, 495, 501, 504, 505, 506, 515, 525, 577, 578, 591, 592, 593 }

F(-2) exception fail { 500, 587 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 101, 110, 111, 112, 116, 117, 118, 123, 124, 125, 146, 147, 148, 149, 150, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 167, 168, 169, 170, 173, 174, 175, 185, 186, 187, 191, 192, 193, 197, 198, 199, 232, 233, 234, 235, 238, 239, 240, 241, 243, 244, 245, 246, 247, 252, 255, 261, 262, 265, 266, 269, 270, 273, 274, 277, 278, 281, 282, 285, 286, 289, 290, 293, 294, 295, 296, 297, 298, 299, 304, 305, 306, 307, 308, 311, 312, 313, 314, 315, 318, 327, 328, 329, 333, 334, 335, 340, 341, 342, 364, 365, 366, 367, 371, 372, 375, 376, 379, 380, 383, 384, 387, 388, 391, 392, 395, 396, 399, 400, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 487, 488, 489, 490, 503, 504, 505, 506, 511, 512, 513, 514, 529, 530, 531, 532, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 574, 575, 576, 577, 590, 591, 592, 593, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619 }

C grade { }

F normal fail { }

F(-1) timedout fail { 19, 20, 21, 40, 83, 84, 85, 86, 92, 93, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 136, 137, 138, 141, 142, 143, 151, 152, 153, 154, 160, 166, 171, 172, 176, 177, 178, 179, 180, 181, 182, 183, 184, 188, 189, 190, 194, 195, 196, 200, 201, 202, 203, 204, 205, 209, 210, 211, 215, 216, 217, 221, 222, 223, 226, 227, 228, 231, 236, 237, 242, 248, 249, 250, 251, 253, 254, 256, 257, 258, 259, 260, 263, 264, 267, 268, 271, 272, 275, 276, 279, 280, 283, 284, 287, 288, 291, 292, 300, 301, 302, 303, 309, 310, 316, 317, 319, 320, 321, 322, 323, 324, 325, 326, 330, 331, 332, 336, 337, 338, 339, 343, 344, 345, 346, 353, 354, 355, 358, 359, 360, 363, 368, 369, 370, 373, 374, 377, 378, 381, 382, 385, 386, 389, 390, 393, 394, 397, 398, 401, 402, 483, 484, 485, 486, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 507, 508, 509, 510, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 533, 534, 535, 536, 537, 570, 571, 572, 573, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 594, 595, 596, 597, 598 }

F(-2) exception fail { }

Sympy

A grade { 3, 4, 5, 7, 11, 12, 13, 14, 15, 16, 17, 18, 41, 42, 43, 44, 45, 46, 65, 77, 147, 155, 157, 162, 296, 304, 306, 312 }

B grade { 1, 2, 6, 8, 27, 37, 38, 39, 52, 62, 63, 64, 66, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 87, 88, 89, 90, 94, 95, 96, 97, 110, 111, 112, 116, 117, 118, 123, 124, 125, 146, 148, 149, 150, 156, 158, 161, 163, 164, 167, 168, 169, 175, 185, 186, 187, 191, 192, 193, 197, 198, 199, 261, 262, 265, 266, 269, 273, 277, 278, 281, 282, 285, 289, 293, 294, 295, 297, 298, 299, 305, 307, 311, 313, 314, 327, 328, 329, 333, 334, 335, 340, 341, 342, 371, 372, 375, 376, 379, 383, 387, 388, 391, 392, 395, 399, 403, 404, 405, 406, 410, 411, 412, 413, 414, 415, 416, 417, 423, 424, 425, 426, 427, 433, 434, 435, 436, 437, 449, 450, 538, 539, 540, 541, 542, 543, 544, 550, 551, 552, 553, 554, 560, 561, 562, 563, 564, 599 }

C grade { }

F normal fail { 9, 10, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 67, 68, 101, 102, 103, 104, 105, 106, 107, 108, 113, 119, 126, 137, 138, 170, 171, 172, 176, 177, 178, 182, 183, 184, 188, 194, 200, 203, 204, 205, 222, 223, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 245, 246, 247, 249, 250, 253, 254, 256, 257, 258, 259, 260, 318, 319, 320, 321, 322, 323, 324, 325, 330, 336, 343, 354, 355, 363, 364, 365, 366, 367, 368, 369, 370, 407, 408, 409, 451, 452, 453, 454, 463, 464, 465, 466, 467, 479, 480, 481, 482, 533, 534, 535, 536, 537, 596, 597, 598 }

F(-1) timedout fail { 83, 84, 85, 86, 91, 92, 93, 98, 99, 100, 109, 114, 115, 120, 121, 122, 127, 128, 129, 136, 141, 142, 143, 152, 153, 154, 160, 166, 173, 174, 179, 180, 181, 189, 190, 195, 196, 201, 202, 209, 210, 211, 215, 216, 217, 221, 226, 227, 228, 244, 248, 251, 252, 255, 264, 268, 271, 272, 275, 276, 279, 280, 283, 284, 286, 287, 288, 290, 291, 292, 301, 302, 303, 309, 310, 316, 317, 326, 331, 332, 337, 338, 339, 344, 345, 346, 353, 358, 359, 360, 374, 378, 381, 382, 385, 386, 389, 390, 393, 394, 396, 397, 398, 400, 401, 402, 419, 420, 421, 422, 429, 430, 431, 432, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 455, 456, 457, 458, 459, 460, 461, 462, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 546, 547, 548, 549, 556, 557, 558, 559, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619 }

F(-2) exception fail { 151, 159, 165, 263, 267, 270, 274, 300, 308, 315, 373, 377, 380, 384, 418, 428, 438, 545, 555, 565 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 13, 14, 15, 16, 17, 18, 37, 38, 39, 41, 42, 43, 44, 45, 46, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 85, 87, 88, 89, 93, 94, 95, 96, 110, 111, 112, 116, 117, 118, 124, 125, 146, 147, 148, 151, 153, 155, 156, 157, 162, 181, 185, 186, 187, 191, 192, 193, 198, 199, 263, 267, 272, 276, 279, 283, 288, 292, 293, 294, 295, 296, 297, 300, 302, 304, 305, 306, 310, 311, 312, 313, 327, 328, 329, 333, 334, 335, 341, 342, 373, 377, 382, 386, 389, 393, 398, 402, 411, 412, 413, 414, 415, 416, 417, 423, 424, 425, 426, 427, 430, 432, 433, 434, 435, 436, 437, 538, 539, 540, 541, 542, 543, 544, 550, 551, 552, 553, 554, 557, 559, 560, 561, 562, 563, 564, 599, 600, 601, 602, 605, 606, 607, 610, 611, 612, 615, 616, 617 }

C grade { }

F normal fail { 6, 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 81, 82, 84, 86, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 119, 120, 121, 122, 123, 126, 127, 128, 129, 136, 137, 138, 141, 142, 143, 149, 150, 152, 154, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 188, 189, 190, 194, 195, 196, 197, 200, 201, 202, 203, 204, 205, 209, 210, 211, 215, 216, 217, 221, 222, 223, 226, 227, 228, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254,

255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 268, 269, 270, 271, 273, 274, 275, 277, 278, 280, 281, 282, 284, 285, 286, 287, 289, 290, 291, 298, 299, 301, 303, 307, 308, 309, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 330, 331, 332, 336, 337, 338, 339, 340, 343, 344, 345, 346, 353, 354, 355, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 374, 375, 376, 378, 379, 380, 381, 383, 384, 385, 387, 388, 390, 391, 392, 394, 395, 396, 397, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 428, 429, 431, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 545, 546, 547, 548, 549, 555, 556, 558, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 603, 604, 608, 609, 613, 614, 618, 619
}

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	34	105	34	34	35
N.S.	1	1.00	1.00	0.91	0.89	0.97	3.00	0.97	0.97	1.00
time (sec)	N/A	0.192	0.056	0.322	0.024	0.069	0.308	0.101	0.176	0.520

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	28	26	78	29	25	64
N.S.	1	1.00	0.71	0.80	0.80	0.74	2.23	0.83	0.71	1.83
time (sec)	N/A	0.204	0.038	0.281	0.025	0.071	0.295	0.113	0.214	16.399

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	10	20	6	17	6
N.S.	1	1.00	1.00	0.80	0.73	0.67	1.33	0.40	1.13	0.40
time (sec)	N/A	0.175	0.004	0.181	0.023	0.066	0.134	0.105	0.152	0.031

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	20	13	17	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	1.18	0.76	1.00	0.76
time (sec)	N/A	0.173	0.005	0.209	0.033	0.068	0.132	0.149	0.155	0.030

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	18	20	13	17	13
N.S.	1	1.00	1.00	0.82	0.76	1.06	1.18	0.76	1.00	0.76
time (sec)	N/A	0.183	0.005	0.220	0.027	0.070	0.144	0.136	0.150	0.031

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	9	35	17	15	17	9	5
N.S.	1	1.00	1.00	1.29	5.00	2.43	2.14	2.43	1.29	0.71
time (sec)	N/A	0.201	0.001	0.070	0.105	0.074	0.428	0.112	0.153	16.410

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	15	15	14	125	58	76	31	9	17
N.S.	1	0.33	0.33	0.31	2.78	1.29	1.69	0.69	0.20	0.38
time (sec)	N/A	0.198	0.023	0.118	0.137	0.076	0.824	0.162	0.158	16.885

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	171	50	294	48	9	27
N.S.	1	1.00	1.00	1.08	6.58	1.92	11.31	1.85	0.35	1.04
time (sec)	N/A	0.203	0.019	0.131	0.131	0.077	3.782	0.148	0.154	16.601

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	87	84	42	0	201	0	105	9	217
N.S.	1	1.09	1.05	0.52	0.00	2.51	0.00	1.31	0.11	2.71
time (sec)	N/A	0.272	0.101	0.173	0.000	0.097	0.000	0.187	0.149	16.558

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	38	30	47	0	68	0	68	9	35
N.S.	1	1.06	0.83	1.31	0.00	1.89	0.00	1.89	0.25	0.97
time (sec)	N/A	0.257	0.026	0.171	0.000	0.085	0.000	0.144	0.174	16.466

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	16	6	7	6	8	5	6	9	6
N.S.	1	2.00	0.75	0.88	0.75	1.00	0.62	0.75	1.12	0.75
time (sec)	N/A	0.199	0.014	0.069	0.026	0.073	0.407	0.121	0.226	16.676

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	11	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	1.38	0.75
time (sec)	N/A	0.207	0.002	0.079	0.025	0.066	1.316	0.111	0.161	0.027

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	32	33	51	34	33	96
N.S.	1	1.00	1.00	0.91	0.91	0.94	1.46	0.97	0.94	2.74
time (sec)	N/A	0.197	0.042	0.292	0.025	0.070	0.284	0.118	0.157	16.886

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	28	27	25	44	29	26	57
N.S.	1	1.00	0.74	0.80	0.77	0.71	1.26	0.83	0.74	1.63
time (sec)	N/A	0.203	0.042	0.283	0.031	0.074	0.298	0.105	0.181	16.227

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	28	28	24	37	29	24	55
N.S.	1	1.00	0.69	0.80	0.80	0.69	1.06	0.83	0.69	1.57
time (sec)	N/A	0.196	0.035	0.262	0.024	0.071	0.304	0.123	0.158	0.118

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	9	20	11	17	9
N.S.	1	1.00	1.00	0.80	0.73	0.60	1.33	0.73	1.13	0.60
time (sec)	N/A	0.173	0.009	0.184	0.025	0.070	0.138	0.107	0.154	0.026

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	20	13	17	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	1.18	0.76	1.00	0.76
time (sec)	N/A	0.173	0.004	0.187	0.032	0.066	0.135	0.103	0.152	0.029

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	17	20	13	17	17
N.S.	1	1.00	1.00	0.82	0.76	1.00	1.18	0.76	1.00	1.00
time (sec)	N/A	0.171	0.021	0.202	0.031	0.072	0.138	0.125	0.150	0.030

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	125	218	0	0	0	0	0	11	0
N.S.	1	1.23	2.14	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.286	0.146	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	105	200	0	0	0	0	0	9	0
N.S.	1	1.24	2.35	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.262	0.116	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	111	165	0	0	0	0	0	9	0
N.S.	1	1.26	1.88	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.270	0.103	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	27	20	18	141	38	0	0	9	17
N.S.	1	1.35	1.00	0.90	7.05	1.90	0.00	0.00	0.45	0.85
time (sec)	N/A	0.196	0.012	0.104	0.116	0.079	0.000	0.000	0.155	18.093

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	57	21	38	0	39	0	364	9	26
N.S.	1	2.38	0.88	1.58	0.00	1.62	0.00	15.17	0.38	1.08
time (sec)	N/A	0.284	0.021	0.166	0.000	0.080	0.000	0.218	0.171	17.111

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	80	69	55	0	101	0	0	9	103
N.S.	1	1.04	0.90	0.71	0.00	1.31	0.00	0.00	0.12	1.34
time (sec)	N/A	0.297	0.055	0.218	0.000	0.091	0.000	0.000	0.150	16.920

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	112	100	236	0	136	0	0	9	107
N.S.	1	1.30	1.16	2.74	0.00	1.58	0.00	0.00	0.10	1.24
time (sec)	N/A	0.412	0.115	0.188	0.000	0.095	0.000	0.000	0.171	17.447

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	84	103	0	134	0	0	9	131
N.S.	1	1.04	0.94	1.16	0.00	1.51	0.00	0.00	0.10	1.47
time (sec)	N/A	0.382	0.095	0.200	0.000	0.095	0.000	0.000	0.156	18.570

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	14	10	12	37	19	19	19	9	10
N.S.	1	1.40	1.00	1.20	3.70	1.90	1.90	1.90	0.90	1.00
time (sec)	N/A	0.191	0.008	0.100	0.107	0.075	0.398	0.112	0.165	17.527

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	66	127	36	0	34	9	16
N.S.	1	1.00	1.00	3.30	6.35	1.80	0.00	1.70	0.45	0.80
time (sec)	N/A	0.196	0.013	0.125	0.130	0.077	0.000	0.142	0.179	17.663

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	30	28	88	173	52	0	50	9	29
N.S.	1	1.07	1.00	3.14	6.18	1.86	0.00	1.79	0.32	1.04
time (sec)	N/A	0.263	0.024	0.160	0.129	0.086	0.000	0.121	0.177	18.470

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	82	76	55	0	107	0	111	9	119
N.S.	1	1.01	0.94	0.68	0.00	1.32	0.00	1.37	0.11	1.47
time (sec)	N/A	0.371	0.169	0.184	0.000	0.090	0.000	0.223	0.223	17.278

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	38	124	0	70	0	70	9	37
N.S.	1	1.11	1.00	3.26	0.00	1.84	0.00	1.84	0.24	0.97
time (sec)	N/A	0.284	0.042	0.200	0.000	0.083	0.000	0.140	0.160	18.533

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	35	13	129	33	0	49	13	12
N.S.	1	1.00	2.33	0.87	8.60	2.20	0.00	3.27	0.87	0.80
time (sec)	N/A	0.187	0.057	0.260	0.111	0.075	0.000	0.128	0.191	0.137

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	27	17	18	81	19	0	24	13	15
N.S.	1	1.29	0.81	0.86	3.86	0.90	0.00	1.14	0.62	0.71
time (sec)	N/A	0.201	0.007	0.275	0.105	0.072	0.000	0.113	0.171	0.131

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	B	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	4814	47	0	121	0	133	13	112
N.S.	1	1.00	67.80	0.66	0.00	1.70	0.00	1.87	0.18	1.58
time (sec)	N/A	0.242	51.792	0.313	0.000	0.085	0.000	0.124	0.161	17.629

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	80	57	43	0	72	0	67	13	47
N.S.	1	1.13	0.80	0.61	0.00	1.01	0.00	0.94	0.18	0.66
time (sec)	N/A	0.311	0.088	0.311	0.000	0.080	0.000	0.118	0.167	17.837

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	627	80	0	153	0	182	13	118
N.S.	1	1.00	7.38	0.94	0.00	1.80	0.00	2.14	0.15	1.39
time (sec)	N/A	0.293	6.429	0.363	0.000	0.103	0.000	0.146	0.161	0.119

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	34	75	34	34	108
N.S.	1	1.00	1.00	0.91	0.89	0.97	2.14	0.97	0.97	3.09
time (sec)	N/A	0.207	0.039	0.378	0.030	0.071	0.322	0.105	0.170	17.176

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	28	25	56	29	25	58
N.S.	1	1.00	0.71	0.80	0.80	0.71	1.60	0.83	0.71	1.66
time (sec)	N/A	0.203	0.038	0.351	0.033	0.070	0.312	0.128	0.166	16.762

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	28	25	56	29	25	58
N.S.	1	1.00	0.71	0.80	0.80	0.71	1.60	0.83	0.71	1.66
time (sec)	N/A	0.201	0.022	0.324	0.029	0.072	0.272	0.138	0.162	0.143

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	92	179	0	0	0	0	0	9	0
N.S.	1	1.21	2.36	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.265	0.138	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	6	22	6	17	6
N.S.	1	1.00	1.00	0.80	0.73	0.40	1.47	0.40	1.13	0.40
time (sec)	N/A	0.178	0.004	0.213	0.026	0.071	0.131	0.132	0.163	0.019

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	22	13	17	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	1.29	0.76	1.00	0.76
time (sec)	N/A	0.174	0.004	0.225	0.024	0.079	0.134	0.103	0.169	0.028

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	22	13	17	14
N.S.	1	1.00	1.00	0.82	0.76	0.76	1.29	0.76	1.00	0.82
time (sec)	N/A	0.174	0.005	0.250	0.025	0.068	0.160	0.128	0.164	0.027

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	12	20	11	17	9
N.S.	1	1.00	1.00	0.80	0.73	0.80	1.33	0.73	1.13	0.60
time (sec)	N/A	0.171	0.004	0.202	0.024	0.071	0.128	0.112	0.167	0.027

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	7	20	13	17	7
N.S.	1	1.00	1.00	0.82	0.76	0.41	1.18	0.76	1.00	0.41
time (sec)	N/A	0.171	0.004	0.223	0.036	0.069	0.148	0.135	0.158	0.021

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	18	20	13	17	13
N.S.	1	1.00	1.00	0.82	0.76	1.06	1.18	0.76	1.00	0.76
time (sec)	N/A	0.175	0.007	0.244	0.024	0.071	0.144	0.106	0.167	0.026

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	27	45	18	133	38	0	0	9	42
N.S.	1	1.35	2.25	0.90	6.65	1.90	0.00	0.00	0.45	2.10
time (sec)	N/A	0.200	0.061	0.151	0.118	0.079	0.000	0.000	0.176	16.779

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	48	19	0	38	0	0	9	42
N.S.	1	1.00	2.29	0.90	0.00	1.81	0.00	0.00	0.43	2.00
time (sec)	N/A	0.193	0.046	0.207	0.000	0.080	0.000	0.000	0.170	16.955

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	80	6161	54	0	101	0	0	9	295
N.S.	1	1.04	80.01	0.70	0.00	1.31	0.00	0.00	0.12	3.83
time (sec)	N/A	0.276	55.221	0.231	0.000	0.085	0.000	0.000	0.222	16.881

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	84	215	54	0	109	0	0	9	407
N.S.	1	1.02	2.62	0.66	0.00	1.33	0.00	0.00	0.11	4.96
time (sec)	N/A	0.312	0.485	0.196	0.000	0.087	0.000	0.000	0.196	17.015

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	628	99	0	134	0	0	9	787
N.S.	1	1.04	7.06	1.11	0.00	1.51	0.00	0.00	0.10	8.84
time (sec)	N/A	0.339	7.008	0.203	0.000	0.096	0.000	0.000	0.167	19.788

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	14	25	20	37	21	19	19	9	20
N.S.	1	1.40	2.50	2.00	3.70	2.10	1.90	1.90	0.90	2.00
time (sec)	N/A	0.191	0.021	0.141	0.027	0.074	0.430	0.104	0.161	17.298

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	55	47	36	131	39	0	39	9	39
N.S.	1	2.75	2.35	1.80	6.55	1.95	0.00	1.95	0.45	1.95
time (sec)	N/A	0.284	0.027	0.225	0.110	0.082	0.000	0.135	0.156	16.890

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	30	73	81	165	53	0	50	9	67
N.S.	1	1.07	2.61	2.89	5.89	1.89	0.00	1.79	0.32	2.39
time (sec)	N/A	0.248	0.060	0.160	0.122	0.081	0.000	0.121	0.171	16.464

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	110	133	224	0	137	0	117	9	611
N.S.	1	1.31	1.58	2.67	0.00	1.63	0.00	1.39	0.11	7.27
time (sec)	N/A	0.376	0.120	0.213	0.000	0.091	0.000	0.126	0.159	17.319

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	87	113	0	71	0	70	9	86
N.S.	1	1.11	2.29	2.97	0.00	1.87	0.00	1.84	0.24	2.26
time (sec)	N/A	0.271	0.082	0.209	0.000	0.095	0.000	0.143	0.160	16.443

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	13	137	33	0	31	53	12
N.S.	1	1.00	1.00	0.87	9.13	2.20	0.00	2.07	3.53	0.80
time (sec)	N/A	0.188	0.009	0.197	0.115	0.076	0.000	0.142	0.165	16.263

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	15	15	13	76	53	0	31	11	16
N.S.	1	0.34	0.34	0.30	1.73	1.20	0.00	0.70	0.25	0.36
time (sec)	N/A	0.199	0.019	0.260	0.123	0.073	0.000	0.128	0.183	16.835

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	67	46	0	121	0	99	61	95
N.S.	1	1.00	0.94	0.65	0.00	1.70	0.00	1.39	0.86	1.34
time (sec)	N/A	0.228	0.075	0.259	0.000	0.090	0.000	0.208	0.202	16.544

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	89	84	44	0	201	0	105	11	217
N.S.	1	1.22	1.15	0.60	0.00	2.75	0.00	1.44	0.15	2.97
time (sec)	N/A	0.294	0.088	0.275	0.000	0.099	0.000	0.208	0.165	16.328

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	80	0	154	0	132	61	118
N.S.	1	1.00	0.95	0.94	0.00	1.81	0.00	1.55	0.72	1.39
time (sec)	N/A	0.272	0.061	0.299	0.000	0.098	0.000	0.189	0.167	0.118

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	14	19	21	20	21	57	10
N.S.	1	1.00	1.00	1.40	1.90	2.10	2.00	2.10	5.70	1.00
time (sec)	N/A	0.184	0.006	0.237	0.029	0.078	0.523	0.108	0.162	15.953

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	18	14	18	129	25	427	25	55	12
N.S.	1	1.29	1.00	1.29	9.21	1.79	30.50	1.79	3.93	0.86
time (sec)	N/A	0.200	0.007	0.235	0.126	0.076	1.899	0.131	0.162	0.026

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	9	35	19	15	17	9	5
N.S.	1	1.00	1.00	1.29	5.00	2.71	2.14	2.43	1.29	0.71
time (sec)	N/A	0.203	0.001	0.137	0.032	0.074	0.450	0.135	0.182	0.029

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	27	21	27	129	19	17	24	9	17
N.S.	1	1.29	1.00	1.29	6.14	0.90	0.81	1.14	0.43	0.81
time (sec)	N/A	0.195	0.007	0.184	0.114	0.074	0.603	0.107	0.166	15.885

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	66	28	163	52	248	48	9	55
N.S.	1	1.00	2.54	1.08	6.27	2.00	9.54	1.85	0.35	2.12
time (sec)	N/A	0.208	0.045	0.218	0.120	0.076	3.222	0.109	0.163	16.085

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	80	57	80	0	72	0	67	9	51
N.S.	1	1.13	0.80	1.13	0.00	1.01	0.00	0.94	0.13	0.72
time (sec)	N/A	0.282	0.066	0.226	0.000	0.082	0.000	0.131	0.228	0.515

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	37	83	47	0	70	0	68	9	74
N.S.	1	1.03	2.31	1.31	0.00	1.94	0.00	1.89	0.25	2.06
time (sec)	N/A	0.249	0.059	0.263	0.000	0.084	0.000	0.116	0.173	16.116

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	57	63	25	42	57
N.S.	1	1.00	1.00	0.79	0.76	1.73	1.91	0.76	1.27	1.73
time (sec)	N/A	0.209	0.060	1.402	0.034	0.096	0.595	0.140	0.169	0.094

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	39	71	25	50	78
N.S.	1	1.00	1.00	0.79	0.76	1.18	2.15	0.76	1.52	2.36
time (sec)	N/A	0.217	0.064	1.612	0.030	0.073	0.629	0.109	0.166	16.291

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	32	48	19	33	25
N.S.	1	1.00	1.00	0.80	0.76	1.28	1.92	0.76	1.32	1.00
time (sec)	N/A	0.213	0.010	0.666	0.024	0.070	0.273	0.121	0.162	0.091

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	42	42	17	27	17
N.S.	1	1.00	1.00	0.78	0.74	1.83	1.83	0.74	1.17	0.74
time (sec)	N/A	0.202	0.002	0.662	0.027	0.075	0.298	0.108	0.158	16.329

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	49	65	49	42	150
N.S.	1	1.00	1.00	0.79	0.76	1.48	1.97	1.48	1.27	4.55
time (sec)	N/A	0.215	0.010	1.219	0.025	0.095	0.585	0.124	0.177	17.250

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	23	67	70	23	50	198
N.S.	1	1.00	1.00	0.77	0.74	2.16	2.26	0.74	1.61	6.39
time (sec)	N/A	0.207	0.059	1.559	0.030	0.134	0.586	0.128	0.154	18.629

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	31	25	228	31	4350	25
N.S.	1	1.00	1.00	0.78	0.76	0.61	5.56	0.76	106.10	0.61
time (sec)	N/A	0.229	0.029	2.518	0.029	0.075	1.355	0.108	0.163	18.085

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	50	58	23	50	36
N.S.	1	1.00	0.96	0.89	0.85	1.85	2.15	0.85	1.85	1.33
time (sec)	N/A	0.194	0.037	0.345	0.025	0.071	0.177	0.107	0.221	17.370

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	44	61	23	57	36
N.S.	1	1.00	0.96	0.89	0.85	1.63	2.26	0.85	2.11	1.33
time (sec)	N/A	0.200	0.028	0.288	0.029	0.072	0.182	0.105	0.154	16.661

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	54	50	81	170	53	103	52
N.S.	1	1.00	0.82	0.89	0.82	1.33	2.79	0.87	1.69	0.85
time (sec)	N/A	0.222	0.068	1.381	0.039	0.076	0.967	0.137	0.183	19.807

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	43	37	58	75	42	77	39
N.S.	1	1.00	0.79	0.90	0.77	1.21	1.56	0.88	1.60	0.81
time (sec)	N/A	0.221	0.064	0.684	0.035	0.072	0.431	0.113	0.160	18.695

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	50	58	23	50	36
N.S.	1	1.00	0.96	0.89	0.85	1.85	2.15	0.85	1.85	1.33
time (sec)	N/A	0.198	0.005	0.000	0.028	0.071	0.189	0.148	0.155	0.006

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	26	68	108	31	333	236	15	111
N.S.	1	1.08	1.00	2.62	4.15	1.19	12.81	9.08	0.58	4.27
time (sec)	N/A	0.236	0.106	0.501	0.051	0.077	4.375	0.122	0.152	17.875

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	90	115	454	71	3264	349	17	252
N.S.	1	1.00	2.50	3.19	12.61	1.97	90.67	9.69	0.47	7.00
time (sec)	N/A	0.294	0.078	0.998	0.061	0.080	64.002	0.145	0.151	22.266

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	63	399	47	0	145	42	0
N.S.	1	1.00	0.90	1.62	10.23	1.21	0.00	3.72	1.08	0.00
time (sec)	N/A	0.300	0.139	1.602	0.046	0.067	0.000	0.135	0.173	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	63	67	189	1773	141	0	2221	264	0
N.S.	1	0.94	1.00	2.82	26.46	2.10	0.00	33.15	3.94	0.00
time (sec)	N/A	0.373	0.374	4.411	0.095	0.083	0.000	0.159	0.162	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	52	58	97	1076	75	0	301	80	0
N.S.	1	0.87	0.97	1.62	17.93	1.25	0.00	5.02	1.33	0.00
time (sec)	N/A	0.321	0.274	5.578	0.049	0.070	0.000	0.150	0.203	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	89	79	257	3879	197	0	8035	366	0
N.S.	1	0.95	0.84	2.73	41.27	2.10	0.00	85.48	3.89	0.00
time (sec)	N/A	0.471	0.878	19.345	0.212	0.082	0.000	0.194	0.170	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	79	92	77	118	175	91	141	77
N.S.	1	1.00	0.77	0.89	0.75	1.15	1.70	0.88	1.37	0.75
time (sec)	N/A	0.268	0.205	3.818	0.037	0.076	2.121	0.172	0.156	1.837

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	54	61	57	111	204	60	182	57
N.S.	1	1.00	0.82	0.92	0.86	1.68	3.09	0.91	2.76	0.86
time (sec)	N/A	0.245	0.080	1.383	0.038	0.081	0.924	0.114	0.159	18.142

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	44	37	81	75	43	78	39
N.S.	1	1.00	0.80	0.90	0.76	1.65	1.53	0.88	1.59	0.80
time (sec)	N/A	0.225	0.035	0.664	0.034	0.076	0.395	0.121	0.184	17.911

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	52	138	118	91	3215	688	17	223
N.S.	1	0.00	1.41	3.73	3.19	2.46	86.89	18.59	0.46	6.03
time (sec)	N/A	0.000	0.110	0.651	0.057	0.084	10.433	0.149	0.159	18.443

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	A	F(-1)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	181	173	711	86	0	1402	19	148
N.S.	1	0.00	3.69	3.53	14.51	1.76	0.00	28.61	0.39	3.02
time (sec)	N/A	0.000	0.443	1.304	0.066	0.081	0.000	0.163	0.168	17.978

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	247	243	1595	148	0	2867	19	0
N.S.	1	0.00	2.78	2.73	17.92	1.66	0.00	32.21	0.21	0.00
time (sec)	N/A	0.000	2.853	2.723	0.097	0.079	0.000	0.180	0.156	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	0	80	58	900	84	0	653	66	0
N.S.	1	0.00	1.48	1.07	16.67	1.56	0.00	12.09	1.22	0.00
time (sec)	N/A	0.000	0.248	3.210	0.046	0.072	0.000	0.163	0.214	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	102	114	105	193	405	113	290	104
N.S.	1	1.00	0.81	0.90	0.83	1.53	3.21	0.90	2.30	0.83
time (sec)	N/A	0.324	0.195	8.096	0.042	0.089	4.872	0.171	0.202	21.932

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	77	90	77	149	175	89	177	77
N.S.	1	1.00	0.76	0.89	0.76	1.48	1.73	0.88	1.75	0.76
time (sec)	N/A	0.275	0.181	3.813	0.039	0.082	2.160	0.117	0.160	1.638

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	56	50	128	172	55	89	52
N.S.	1	1.00	0.83	0.89	0.79	2.03	2.73	0.87	1.41	0.83
time (sec)	N/A	0.229	0.046	1.517	0.038	0.082	0.878	0.128	0.155	18.751

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	72	161	162	115	33056	2488	17	169
N.S.	1	0.00	1.14	2.56	2.57	1.83	524.70	39.49	0.27	2.68
time (sec)	N/A	0.000	0.296	0.859	0.062	0.087	28.337	0.159	0.160	17.831

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	A	F(-1)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	0	133	232	1066	160	0	2919	19	390
N.S.	1	0.00	0.38	0.66	3.02	0.45	0.00	8.27	0.05	1.10
time (sec)	N/A	0.000	0.136	1.902	0.082	0.090	0.000	0.186	0.219	24.169

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	0	294	196	1738	173	0	6427	19	0
N.S.	1	0.00	0.94	0.63	5.55	0.55	0.00	20.53	0.06	0.00
time (sec)	N/A	0.000	0.996	3.971	0.103	0.084	0.000	0.255	0.241	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	627	0	105	350	3532	232	0	16081	19	0
N.S.	1	0.00	0.17	0.56	5.63	0.37	0.00	25.65	0.03	0.00
time (sec)	N/A	0.000	0.468	6.575	0.194	0.086	0.000	0.307	0.235	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	28	79	564	110	0	396	15	249
N.S.	1	1.11	0.78	2.19	15.67	3.06	0.00	11.00	0.42	6.92
time (sec)	N/A	0.259	0.207	0.509	0.061	0.082	0.000	0.170	0.235	22.108

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	B	B	F	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	73	300	19853	180	0	1917	17	0
N.S.	1	0.00	1.20	4.92	325.46	2.95	0.00	31.43	0.28	0.00
time (sec)	N/A	0.000	0.331	1.016	0.370	0.105	0.000	0.404	0.300	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	F	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	0	155	215	105891	203	0	7053	17	0
N.S.	1	0.00	1.87	2.59	1275.80	2.45	0.00	84.98	0.20	0.00
time (sec)	N/A	0.000	0.666	1.794	1.970	0.093	0.000	0.474	0.236	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupac
grade	N/A	F	C	C	C	C	F	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	404	261	146269	358	0	3043	19	0
N.S.	1	0.00	404.00	261.00	146269.00	358.00	0.00	3043.00	19.00	0.00
time (sec)	N/A	0.000	6.395	1.066	3.876	0.094	0.000	0.272	0.252	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupa
grade	N/A	F	C	C	C	C	F	C	F	F(-1
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBI
size	1	0	382	770	810260	678	0	10037	19	0
N.S.	1	0.00	382.00	770.00	810260.00	678.00	0.00	10037.00	19.00	0.00
time (sec)	N/A	0.000	8.747	2.251	32.838	0.138	0.000	2.316	0.264	0.00

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mu
grade	N/A	F	C	C	C	C	F	C	F	F(-
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TE
size	1	0	328	627	2049662	624	0	27104	19	0
N.S.	1	0.00	328.00	627.00	2049662.00	624.00	0.00	27104.00	19.00	0.0
time (sec)	N/A	0.000	4.283	3.851	80.185	0.112	0.000	1.489	0.398	0.0

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F(-1)	C	F	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	347	749	0	772	0	13206	19	0
N.S.	1	0.00	347.00	749.00	0.00	772.00	0.00	13206.00	19.00	0.00
time (sec)	N/A	0.000	4.088	2.727	0.000	0.125	0.000	0.598	0.374	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mup
grade	N/A	F	C	C	F(-1)	C	F	C	F	F(-
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TB
size	1	0	221	1740	0	1339	0	41036	19	0
N.S.	1	0.00	221.00	1740.00	0.00	1339.00	0.00	41036.00	19.00	0.0
time (sec)	N/A	0.000	15.325	4.269	0.000	0.219	0.000	16.751	1.858	0.00

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mu
grade	N/A	F	C	C	F(-1)	C	F(-1)	C	F	F(
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	T
size	1	0	1966	1222	0	1195	0	57794	19	(
N.S.	1	0.00	1966.00	1222.00	0.00	1195.00	0.00	57794.00	19.00	0.
time (sec)	N/A	0.000	6.835	7.971	0.000	0.151	0.000	7.000	7.772	0.0

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	86	84	916	122	921	84	145	311
N.S.	1	1.00	0.95	0.92	10.07	1.34	10.12	0.92	1.59	3.42
time (sec)	N/A	0.281	0.418	4.377	0.112	0.081	2.003	0.193	0.161	18.525

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	414	71	408	56	73	98
N.S.	1	1.00	1.11	0.92	6.68	1.15	6.58	0.90	1.18	1.58
time (sec)	N/A	0.248	0.508	1.742	0.060	0.074	0.719	0.112	0.152	17.279

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	40	42	153	40	42	84
N.S.	1	1.00	1.00	0.93	0.93	0.98	3.56	0.93	0.98	1.95
time (sec)	N/A	0.211	0.147	0.503	0.037	0.070	0.318	0.299	0.146	17.473

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	0	124	0	0	0	0	0	15	0
N.S.	1	0.00	0.90	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.658	0.000	0.000	0.000	0.000	0.000	0.146	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	0	273	0	0	0	0	0	17	0
N.S.	1	0.00	2.08	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	4.319	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	0	170	0	0	0	0	0	17	0
N.S.	1	0.00	1.21	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	1.923	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1362	192	2004	129	263	469
N.S.	1	1.00	1.10	0.92	9.46	1.33	13.92	0.90	1.83	3.26
time (sec)	N/A	0.327	1.155	13.016	0.101	0.083	5.714	0.408	0.174	18.593

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	106	89	620	118	1027	80	118	177
N.S.	1	1.00	1.20	1.01	7.05	1.34	11.67	0.91	1.34	2.01
time (sec)	N/A	0.269	0.533	5.046	0.065	0.079	1.615	0.109	0.157	17.221

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	80	63	371	69	410	61	79	105
N.S.	1	1.00	1.18	0.93	5.46	1.01	6.03	0.90	1.16	1.54
time (sec)	N/A	0.240	0.213	1.745	0.053	0.077	0.733	0.134	0.172	17.351

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	0	160	0	0	0	0	0	17	0
N.S.	1	0.00	1.07	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.861	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	0	290	0	0	0	0	0	19	0
N.S.	1	0.00	2.07	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.000	3.298	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	0	206	0	0	0	0	0	19	0
N.S.	1	0.00	1.24	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	1.871	0.000	0.000	0.000	0.000	0.000	57.661	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	0	563	0	0	0	0	0	19	0
N.S.	1	0.00	3.68	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	2.820	0.000	0.000	0.000	0.000	0.000	75.207	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	177	190	2612	291	3580	181	19	997
N.S.	1	1.00	0.91	0.97	13.39	1.49	18.36	0.93	0.10	5.11
time (sec)	N/A	0.385	1.164	43.133	0.207	0.098	17.813	0.142	200.021	21.093

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	1360	189	2030	124	258	437
N.S.	1	1.00	1.11	0.92	9.86	1.37	14.71	0.90	1.87	3.17
time (sec)	N/A	0.332	1.153	13.195	0.101	0.088	5.808	0.124	0.183	19.158

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	91	90	789	115	933	89	145	494
N.S.	1	1.00	0.94	0.93	8.13	1.19	9.62	0.92	1.49	5.09
time (sec)	N/A	0.268	0.341	3.942	0.083	0.079	2.054	0.124	0.178	17.848

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	0	261	0	0	0	0	0	17	0
N.S.	1	0.00	0.90	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	1.144	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	0	570	0	0	0	0	0	19	0
N.S.	1	0.00	2.09	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	4.025	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	0	310	0	0	0	0	0	19	0
N.S.	1	0.00	1.10	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	1.840	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	265	0	913	0	0	0	0	0	19	0
N.S.	1	0.00	3.45	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	5.992	0.000	0.000	0.000	0.000	0.000	200.017	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	13	15	15	14	15	15	19
N.S.	1	1.00	1.15	1.00	1.15	1.15	1.08	1.15	1.15	1.46
time (sec)	N/A	0.213	14.843	0.030	0.503	0.071	0.694	0.245	0.183	16.976

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	2521	17	15	17	17	19
N.S.	1	1.00	1.13	1.00	168.07	1.13	1.00	1.13	1.13	1.27
time (sec)	N/A	0.220	14.462	0.033	1.745	0.075	1.317	0.168	0.147	16.434

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	0	15	9659	17	15	17	17	19
N.S.	1	1.00	0.00	1.00	643.93	1.13	1.00	1.13	1.13	1.27
time (sec)	N/A	0.224	0.000	0.033	10.967	0.076	3.128	10.884	0.160	16.496

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	4492	19	17	19	19	19
N.S.	1	1.00	1.12	1.00	264.24	1.12	1.00	1.12	1.12	1.12
time (sec)	N/A	0.255	14.371	0.038	3.392	0.072	3.252	0.249	0.159	16.428

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	15005	19	17	19	19	19
N.S.	1	1.00	1.12	1.00	882.65	1.12	1.00	1.12	1.12	1.12
time (sec)	N/A	0.229	78.086	0.040	18.562	0.075	10.874	56.001	152.214	16.234

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	36042	19	17	19	19	19
N.S.	1	1.00	1.12	1.00	2120.12	1.12	1.00	1.12	1.12	1.12
time (sec)	N/A	0.238	25.571	0.040	55.406	0.084	26.368	0.940	200.020	16.672

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	477	590	0	0	0	0	0	0	18	0
N.S.	1	1.24	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.745	0.000	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	263	450	0	0	0	0	0	0	18	0
N.S.	1	1.71	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.167	0.000	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	300	195	0	0	0	0	0	16	0
N.S.	1	1.28	0.83	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.938	10.365	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	15	17	17	17	17	16	19
N.S.	1	1.00	1.12	0.88	1.00	1.00	1.00	1.00	0.94	1.12
time (sec)	N/A	0.226	12.953	0.044	1.016	0.071	0.831	0.134	0.161	17.121

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	19	19	19	19	18	19
N.S.	1	1.00	1.11	0.89	1.00	1.00	1.00	1.00	0.95	1.00
time (sec)	N/A	0.254	20.324	0.050	1.514	0.076	2.088	0.150	0.170	16.428

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	473	580	0	0	0	0	0	0	19	0
N.S.	1	1.23	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.896	0.000	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	397	0	0	0	0	0	0	19	0
N.S.	1	1.26	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.190	0.000	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	287	182	0	0	0	0	0	17	0
N.S.	1	1.25	0.79	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.960	1.497	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	15	17	17	19
N.S.	1	1.00	1.13	1.00	1.13	1.13	1.00	1.13	1.13	1.27
time (sec)	N/A	0.226	1.598	0.040	0.743	0.076	2.599	0.154	0.162	16.631

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	19	19	17	19	19	19
N.S.	1	1.00	1.12	1.00	1.12	1.12	1.00	1.12	1.12	1.12
time (sec)	N/A	0.247	1.845	0.042	0.865	0.079	10.204	0.185	0.156	17.315

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	54	50	64	172	53	91	52
N.S.	1	1.00	0.82	0.89	0.82	1.05	2.82	0.87	1.49	0.85
time (sec)	N/A	0.232	0.107	1.327	0.037	0.075	0.851	0.117	0.155	19.235

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	43	37	44	73	42	79	37
N.S.	1	1.00	0.79	0.90	0.77	0.92	1.52	0.88	1.65	0.77
time (sec)	N/A	0.215	0.064	0.648	0.034	0.073	0.368	0.115	0.161	18.362

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	49	58	23	51	36
N.S.	1	1.00	0.96	0.89	0.85	1.81	2.15	0.85	1.89	1.33
time (sec)	N/A	0.187	0.027	0.260	0.029	0.074	0.179	0.113	0.157	16.209

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	69	73	31	435	158	25	112
N.S.	1	1.00	1.00	2.56	2.70	1.15	16.11	5.85	0.93	4.15
time (sec)	N/A	0.219	0.102	0.599	0.047	0.083	4.827	0.118	0.159	16.608

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	88	115	387	69	5545	248	17	254
N.S.	1	1.00	2.59	3.38	11.38	2.03	163.09	7.29	0.50	7.47
time (sec)	N/A	0.269	0.074	1.517	0.177	0.080	95.375	0.146	0.173	21.604

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-2)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	34	40	391	42	0	174	45	0
N.S.	1	1.00	0.89	1.05	10.29	1.11	0.00	4.58	1.18	0.00
time (sec)	N/A	0.290	0.126	1.991	0.039	0.071	0.000	0.129	0.171	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	63	64	191	1424	94	0	495	17	0
N.S.	1	0.94	0.96	2.85	21.25	1.40	0.00	7.39	0.25	0.00
time (sec)	N/A	0.372	0.316	10.041	0.215	0.085	0.000	0.138	0.178	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	54	48	76	1074	53	0	327	115	0
N.S.	1	0.92	0.81	1.29	18.20	0.90	0.00	5.54	1.95	0.00
time (sec)	N/A	0.315	0.244	7.281	0.049	0.074	0.000	0.150	0.195	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	89	78	259	3096	107	0	756	17	0
N.S.	1	0.95	0.83	2.76	32.94	1.14	0.00	8.04	0.18	0.00
time (sec)	N/A	0.475	0.694	59.020	0.312	0.087	0.000	0.147	0.177	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	79	92	77	91	173	91	134	77
N.S.	1	1.00	0.77	0.89	0.75	0.88	1.68	0.88	1.30	0.75
time (sec)	N/A	0.267	0.147	3.730	0.043	0.079	2.121	0.147	0.168	1.753

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	54	61	57	94	204	60	182	55
N.S.	1	1.00	0.79	0.90	0.84	1.38	3.00	0.88	2.68	0.81
time (sec)	N/A	0.252	0.093	1.355	0.042	0.079	0.865	0.120	0.167	19.470

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	44	37	67	73	43	78	37
N.S.	1	1.00	0.80	0.90	0.76	1.37	1.49	0.88	1.59	0.76
time (sec)	N/A	0.222	0.054	0.658	0.037	0.076	0.378	0.135	0.162	17.696

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	129	146	140	85	3645	961	17	217
N.S.	1	0.00	3.58	4.06	3.89	2.36	101.25	26.69	0.47	6.03
time (sec)	N/A	0.000	0.108	1.096	0.192	0.085	18.888	0.144	0.177	18.793

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	A	F(-2)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	0	177	167	534	82	0	760	802	146
N.S.	1	0.00	3.54	3.34	10.68	1.64	0.00	15.20	16.04	2.92
time (sec)	N/A	0.000	0.421	3.068	0.062	0.081	0.000	0.133	0.216	18.607

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	0	165	250	1263	113	0	1502	19	0
N.S.	1	0.00	1.88	2.84	14.35	1.28	0.00	17.07	0.22	0.00
time (sec)	N/A	0.000	1.958	7.967	0.224	0.086	0.000	0.172	0.174	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	102	114	106	156	405	113	19	104
N.S.	1	1.00	0.80	0.89	0.83	1.22	3.16	0.88	0.15	0.81
time (sec)	N/A	0.334	0.183	7.020	0.057	0.092	5.124	0.128	0.227	22.658

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	77	90	77	119	173	89	205	77
N.S.	1	1.00	0.76	0.89	0.76	1.18	1.71	0.88	2.03	0.76
time (sec)	N/A	0.270	0.107	3.688	0.039	0.083	2.153	0.134	0.188	1.670

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	56	52	107	172	55	17	51
N.S.	1	1.00	0.83	0.89	0.83	1.70	2.73	0.87	0.27	0.81
time (sec)	N/A	0.233	0.047	1.302	0.041	0.086	0.883	0.120	0.179	19.768

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	A	A	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	0	73	159	111	109	34286	2306	17	164
N.S.	1	0.00	1.14	2.48	1.73	1.70	535.72	36.03	0.27	2.56
time (sec)	N/A	0.000	0.235	2.317	0.051	0.086	67.709	0.164	0.190	17.938

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F(-2)	C	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	131	235	871	149	0	2956	19	382
N.S.	1	0.00	131.00	235.00	871.00	149.00	0.00	2956.00	19.00	382.00
time (sec)	N/A	0.000	0.134	6.655	0.220	0.092	0.000	0.185	0.163	24.926

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F(-1)	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	284	193	1403	123	0	4224	19	0
N.S.	1	0.00	284.00	193.00	1403.00	123.00	0.00	4224.00	19.00	0.00
time (sec)	N/A	0.000	0.753	17.540	0.090	0.089	0.000	0.188	0.166	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	B	A	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	117	163	155	107	33056	3325	17	165
N.S.	1	0.00	1.86	2.59	2.46	1.70	524.70	52.78	0.27	2.62
time (sec)	N/A	0.000	0.299	1.447	0.065	0.087	27.788	0.206	0.161	1.121

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	50	137	118	87	3215	527	17	223
N.S.	1	0.00	1.39	3.81	3.28	2.42	89.31	14.64	0.47	6.19
time (sec)	N/A	0.000	0.097	0.811	0.059	0.095	10.252	0.282	0.165	19.363

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	29	58	70	106	30	333	482	15	115
N.S.	1	1.07	2.15	2.59	3.93	1.11	12.33	17.85	0.56	4.26
time (sec)	N/A	0.228	0.113	0.493	0.057	0.075	4.259	0.154	0.162	19.383

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	F	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	28	53	430	111	0	391	50	311
N.S.	1	0.00	0.78	1.47	11.94	3.08	0.00	10.86	1.39	8.64
time (sec)	N/A	0.000	0.151	0.487	0.056	0.083	0.000	0.148	0.168	23.770

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	B	B	F	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	0	72	282	14127	294	0	2933	17	0
N.S.	1	0.00	1.20	4.70	235.45	4.90	0.00	48.88	0.28	0.00
time (sec)	N/A	0.000	0.278	0.984	0.352	0.108	0.000	0.375	0.163	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	151	158	85407	392	0	6981	17	0
N.S.	1	0.00	151.00	158.00	85407.00	392.00	0.00	6981.00	17.00	0.00
time (sec)	N/A	0.000	0.562	1.453	1.261	0.102	0.000	0.365	0.164	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F(-1)	C	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	134	231	1042	159	0	5372	19	393
N.S.	1	0.00	134.00	231.00	1042.00	159.00	0.00	5372.00	19.00	393.00
time (sec)	N/A	0.000	0.142	3.703	0.085	0.087	0.000	0.588	0.172	25.916

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	A	F(-1)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	0	181	175	711	83	0	2042	19	149
N.S.	1	0.00	3.55	3.43	13.94	1.63	0.00	40.04	0.37	2.92
time (sec)	N/A	0.000	0.404	1.913	0.065	0.085	0.000	0.197	0.165	18.823

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	90	115	450	71	3264	893	17	252
N.S.	1	1.00	2.57	3.29	12.86	2.03	93.26	25.51	0.49	7.20
time (sec)	N/A	0.291	0.065	1.086	0.059	0.080	61.217	0.249	0.159	23.198

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	97	357	17016	178	0	2468	158	0
N.S.	1	0.00	97.00	357.00	17016.00	178.00	0.00	2468.00	158.00	0.00
time (sec)	N/A	0.000	0.296	1.169	0.406	0.105	0.000	0.232	0.190	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	64	176	114453	330	0	3227	19	0
N.S.	1	0.00	64.00	176.00	114453.00	330.00	0.00	3227.00	19.00	0.00
time (sec)	N/A	0.000	0.864	1.792	2.550	0.096	0.000	0.238	0.158	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupa
grade	N/A	F	C	C	C	C	F	C	F	F(-1
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBI
size	1	0	491	858	660194	739	0	14520	19	0
N.S.	1	0.00	491.00	858.00	660194.00	739.00	0.00	14520.00	19.00	0.0
time (sec)	N/A	0.000	6.600	4.130	20.636	0.154	0.000	3.028	0.157	0.00

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F(-1)	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	437	198	1738	165	0	5339	19	0
N.S.	1	0.00	437.00	198.00	1738.00	165.00	0.00	5339.00	19.00	0.00
time (sec)	N/A	0.000	1.000	7.099	0.102	0.090	0.000	0.280	0.155	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	247	243	1603	146	0	5628	19	0
N.S.	1	0.00	2.84	2.79	18.43	1.68	0.00	64.69	0.22	0.00
time (sec)	N/A	0.000	2.806	4.079	0.098	0.084	0.000	0.438	0.157	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	36	395	45	0	327	41	0
N.S.	1	1.00	0.92	0.95	10.39	1.18	0.00	8.61	1.08	0.00
time (sec)	N/A	0.318	0.160	1.457	0.041	0.070	0.000	0.151	0.157	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	165	207	85585	189	0	2551	91	0
N.S.	1	0.00	165.00	207.00	85585.00	189.00	0.00	2551.00	91.00	0.00
time (sec)	N/A	0.000	0.636	2.127	1.377	0.090	0.000	0.215	0.163	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupa
grade	N/A	F	C	C	C	C	F	C	F	F(-1
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TB
size	1	0	381	934	643576	617	0	13498	19	0
N.S.	1	0.00	381.00	934.00	643576.00	617.00	0.00	13498.00	19.00	0.0
time (sec)	N/A	0.000	6.610	5.505	22.864	0.144	0.000	0.768	0.180	0.00

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mu
grade	N/A	F	C	C	C	C	F	C	F	F(
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	T
size	1	0	1451	483	2494919	702	0	13903	19	
N.S.	1	0.00	1451.00	483.00	2494919.00	702.00	0.00	13903.00	19.00	0
time (sec)	N/A	0.000	6.673	8.553	103.291	0.123	0.000	0.515	0.170	0.

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	87	84	912	106	918	84	191	297
N.S.	1	1.00	0.96	0.92	10.02	1.16	10.09	0.92	2.10	3.26
time (sec)	N/A	0.271	0.352	3.967	0.093	0.082	2.043	0.133	0.163	18.095

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	71	57	414	66	405	56	86	97
N.S.	1	1.00	1.15	0.92	6.68	1.06	6.53	0.90	1.39	1.56
time (sec)	N/A	0.244	0.518	1.738	0.056	0.076	0.740	0.117	0.181	17.153

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	40	42	155	40	43	85
N.S.	1	1.00	1.00	0.93	0.93	0.98	3.60	0.93	1.00	1.98
time (sec)	N/A	0.212	0.108	0.480	0.036	0.069	0.330	0.124	0.159	17.325

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	0	124	0	0	0	0	0	25	0
N.S.	1	0.00	0.90	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.000	0.376	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	0	289	0	0	0	0	0	1081	0
N.S.	1	0.00	2.14	0.00	0.00	0.00	0.00	0.00	8.01	0.00
time (sec)	N/A	0.000	3.979	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	0	170	0	0	0	0	0	0	0
N.S.	1	0.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	1.751	0.000	0.000	0.000	0.000	0.000	0.501	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1362	174	2004	129	279	495
N.S.	1	1.00	1.10	0.92	9.46	1.21	13.92	0.90	1.94	3.44
time (sec)	N/A	0.330	1.152	13.135	0.186	0.084	5.681	0.136	0.163	18.986

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	108	89	620	108	1027	80	135	177
N.S.	1	1.00	1.23	1.01	7.05	1.23	11.67	0.91	1.53	2.01
time (sec)	N/A	0.265	0.495	5.194	0.080	0.079	1.613	0.108	0.169	17.873

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	76	63	371	70	410	61	70	105
N.S.	1	1.00	1.12	0.93	5.46	1.03	6.03	0.90	1.03	1.54
time (sec)	N/A	0.235	0.530	1.747	0.079	0.075	0.768	0.130	0.170	17.785

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	0	192	0	0	0	0	0	19	0
N.S.	1	0.00	1.22	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.509	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	0	295	0	0	0	0	0	19	0
N.S.	1	0.00	2.05	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	2.744	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	0	237	0	0	0	0	0	19	0
N.S.	1	0.00	1.36	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	4.643	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	176	190	2612	264	3580	181	19	951
N.S.	1	1.00	0.90	0.97	13.39	1.35	18.36	0.93	0.10	4.88
time (sec)	N/A	0.387	1.100	40.977	0.445	0.097	17.759	0.116	200.022	20.623

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	1360	179	2020	124	293	438
N.S.	1	1.00	1.11	0.92	9.86	1.30	14.64	0.90	2.12	3.17
time (sec)	N/A	0.321	1.278	12.079	0.152	0.085	5.992	0.107	0.166	18.377

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	90	785	116	933	89	156	471
N.S.	1	1.00	0.93	0.93	8.09	1.20	9.62	0.92	1.61	4.86
time (sec)	N/A	0.273	0.312	3.998	0.118	0.080	2.091	0.110	0.163	18.123

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	0	267	0	0	0	0	0	19	0
N.S.	1	0.00	0.92	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.703	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	0	593	0	0	0	0	0	19	0
N.S.	1	0.00	2.11	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	3.558	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	0	313	0	0	0	0	0	19	0
N.S.	1	0.00	1.11	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	1.779	0.000	0.000	0.000	0.000	0.000	10.075	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	282	0	290	0	0	0	0	0	17	0
N.S.	1	0.00	1.03	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	20.659	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	0	213	0	0	0	0	0	17	0
N.S.	1	0.00	1.43	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	13.360	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	0	125	0	0	0	0	0	15	0
N.S.	1	0.00	0.94	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.822	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	13	15	15	14	15	15	19
N.S.	1	1.00	1.15	1.00	1.15	1.15	1.08	1.15	1.15	1.46
time (sec)	N/A	0.206	16.959	0.036	0.646	0.070	0.804	0.301	0.156	17.307

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	15	17	17	19
N.S.	1	1.00	1.13	1.00	1.13	1.13	1.00	1.13	1.13	1.27
time (sec)	N/A	0.231	13.910	0.056	1.210	0.070	1.668	0.680	0.165	17.462

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	0	15	17	17	15	17	17	19
N.S.	1	1.00	0.00	1.00	1.13	1.13	1.00	1.13	1.13	1.27
time (sec)	N/A	0.228	0.000	0.036	4.682	0.073	4.482	1.238	0.165	16.759

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	0	372	0	0	0	0	0	19	0
N.S.	1	0.00	1.32	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	6.907	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	0	152	0	0	0	0	0	19	0
N.S.	1	0.00	1.09	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.000	3.235	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	0	158	0	0	0	0	0	17	0
N.S.	1	0.00	1.17	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	1.685	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	2521	17	15	17	17	19
N.S.	1	1.00	1.13	1.00	168.07	1.13	1.00	1.13	1.13	1.27
time (sec)	N/A	0.246	20.688	0.052	3.995	0.076	1.699	0.161	0.153	16.708

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	4492	19	17	19	19	19
N.S.	1	1.00	1.12	1.00	264.24	1.12	1.00	1.12	1.12	1.12
time (sec)	N/A	0.267	16.461	0.038	7.734	0.077	4.743	0.236	0.162	16.760

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	7340	19	17	19	19	19
N.S.	1	1.00	1.12	1.00	431.76	1.12	1.00	1.12	1.12	1.12
time (sec)	N/A	0.241	118.288	0.042	16.827	0.079	14.520	0.362	0.169	16.805

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	275	0	1795	0	0	0	0	0	19	0
N.S.	1	0.00	6.53	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	12.274	0.000	0.000	0.000	0.000	0.000	200.015	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	0	365	0	0	0	0	0	19	0
N.S.	1	0.00	2.20	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	12.116	0.000	0.000	0.000	0.000	0.000	185.997	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	0	190	0	0	0	0	0	17	0
N.S.	1	0.00	1.39	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	3.263	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	9655	17	15	17	17	19
N.S.	1	1.00	1.13	1.00	643.67	1.13	1.00	1.13	1.13	1.27
time (sec)	N/A	0.226	42.719	0.036	21.531	0.078	4.417	14.963	0.162	16.471

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	15013	19	17	19	19	19
N.S.	1	1.00	1.12	1.00	883.12	1.12	1.00	1.12	1.12	1.12
time (sec)	N/A	0.234	17.655	0.039	42.397	0.079	14.269	48.284	11.892	16.542

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	0	17	22516	19	17	19	19	19
N.S.	1	1.00	0.00	1.00	1324.47	1.12	1.00	1.12	1.12	1.12
time (sec)	N/A	0.226	0.000	0.043	76.459	0.083	43.176	115.625	200.024	17.445

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	477	558	368	0	0	0	0	0	18	0
N.S.	1	1.17	0.77	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.301	2.230	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	258	646	359	0	0	0	0	0	18	0
N.S.	1	2.50	1.39	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.141	5.068	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	284	207	0	0	0	0	0	16	0
N.S.	1	1.21	0.88	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.704	0.497	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	15	17	17	17	17	16	19
N.S.	1	1.00	1.12	0.88	1.00	1.00	1.00	1.00	0.94	1.12
time (sec)	N/A	0.222	16.755	0.043	0.409	0.070	0.783	0.135	0.146	16.960

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	19	19	19	19	18	19
N.S.	1	1.00	1.11	0.89	1.00	1.00	1.00	1.00	0.95	1.00
time (sec)	N/A	0.239	26.172	0.050	0.576	0.075	2.098	0.140	0.147	16.674

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	473	548	329	0	0	0	0	0	19	0
N.S.	1	1.16	0.70	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.442	1.574	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	373	242	0	0	0	0	0	19	0
N.S.	1	1.19	0.77	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.913	0.817	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	271	181	0	0	0	0	0	17	0
N.S.	1	1.18	0.79	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.731	1.446	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	15	17	17	19
N.S.	1	1.00	1.13	1.00	1.13	1.13	1.00	1.13	1.13	1.27
time (sec)	N/A	0.223	1.614	0.039	0.455	0.076	2.821	0.141	0.181	16.628

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	19	19	17	19	19	19
N.S.	1	1.00	1.12	1.00	1.12	1.12	1.00	1.12	1.12	1.12
time (sec)	N/A	0.237	1.813	0.045	0.566	0.076	10.433	0.160	0.175	16.234

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	86	70	186	1027	376	0	0	59	0
N.S.	1	1.19	0.97	2.58	14.26	5.22	0.00	0.00	0.82	0.00
time (sec)	N/A	0.668	0.267	0.235	0.193	0.093	0.000	0.000	0.151	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	109	143	520	315	0	0	17	294
N.S.	1	1.00	2.48	3.25	11.82	7.16	0.00	0.00	0.39	6.68
time (sec)	N/A	0.365	0.073	0.158	0.191	0.101	0.000	0.000	0.155	21.281

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	94	99	131	188	0	0	15	227
N.S.	1	1.00	3.24	3.41	4.52	6.48	0.00	0.00	0.52	7.83
time (sec)	N/A	0.241	0.038	0.115	0.177	0.084	0.000	0.000	0.167	21.116

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	93	95	105	197	0	226	15	233
N.S.	1	1.00	3.21	3.28	3.62	6.79	0.00	7.79	0.52	8.03
time (sec)	N/A	0.239	0.044	0.137	0.055	0.086	0.000	0.147	0.178	20.980

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	111	143	612	316	0	577	17	290
N.S.	1	1.00	2.41	3.11	13.30	6.87	0.00	12.54	0.37	6.30
time (sec)	N/A	0.378	0.080	0.221	0.062	0.088	0.000	0.173	0.158	23.088

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	87	71	184	1254	372	0	870	62	0
N.S.	1	1.18	0.96	2.49	16.95	5.03	0.00	11.76	0.84	0.00
time (sec)	N/A	0.674	0.261	0.326	0.083	0.096	0.000	0.187	0.163	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mup
grade	N/A	F	C	C	C	C	F	C	F	F(-
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TB
size	1	0	239	226	1755	412	0	1957357	19	0
N.S.	1	0.00	239.00	226.00	1755.00	412.00	0.00	1957357.00	19.00	0.0
time (sec)	N/A	0.000	1.624	0.400	0.121	0.099	0.000	88.120	0.158	0.00

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	F(-2)	B	F	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	189	200	0	300	0	300537	802	205
N.S.	1	0.00	2.22	2.35	0.00	3.53	0.00	3535.73	9.44	2.41
time (sec)	N/A	0.000	1.426	0.305	0.000	0.092	0.000	12.193	0.222	1.690

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	A	B	F	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	45	139	90	105	0	11424	17	131
N.S.	1	0.00	0.92	2.84	1.84	2.14	0.00	233.14	0.35	2.67
time (sec)	N/A	0.000	0.155	0.266	0.048	0.081	0.000	0.644	0.146	0.858

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	B	B	F	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	0	91	143	125	106	0	1224	17	132
N.S.	1	0.00	1.90	2.98	2.60	2.21	0.00	25.50	0.35	2.75
time (sec)	N/A	0.000	0.212	0.327	0.068	0.084	0.000	0.167	0.149	17.721

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F(-2)	B	F	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	0	190	206	0	301	0	1401	19	208
N.S.	1	0.00	1.46	1.58	0.00	2.32	0.00	10.78	0.15	1.60
time (sec)	N/A	0.000	1.431	0.513	0.000	0.094	0.000	0.200	0.159	18.506

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	422	230	2160	419	0	2856	19	0
N.S.	1	0.00	422.00	230.00	2160.00	419.00	0.00	2856.00	19.00	0.00
time (sec)	N/A	0.000	6.281	0.770	0.131	0.101	0.000	0.204	0.183	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	A	A	F	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	0	103	243	137	173	0	454260	17	12617
N.S.	1	0.00	0.97	2.29	1.29	1.63	0.00	4285.47	0.16	119.03
time (sec)	N/A	0.000	0.561	1.152	0.063	0.089	0.000	19.004	0.160	40.603

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F(-1)	C	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	80	558	14864	413	0	2953	17	14459
N.S.	1	0.00	80.00	558.00	14864.00	413.00	0.00	2953.00	17.00	14459.00
time (sec)	N/A	0.000	0.521	0.210	0.513	0.112	0.000	0.690	0.175	40.244

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	B	B	F	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	121	242	615	232	0	404	15	46071
N.S.	1	0.00	1.81	3.61	9.18	3.46	0.00	6.03	0.22	687.63
time (sec)	N/A	0.000	0.100	0.143	0.175	0.092	0.000	0.222	0.165	59.377

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F	C	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	122	252	527	236	0	378	15	46101
N.S.	1	0.00	122.00	252.00	527.00	236.00	0.00	378.00	15.00	46101.00
time (sec)	N/A	0.000	0.108	0.128	0.067	0.094	0.000	0.149	0.171	57.480

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F	C	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	82	550	20524	433	0	1206	17	14480
N.S.	1	0.00	82.00	550.00	20524.00	433.00	0.00	1206.00	17.00	14480.00
time (sec)	N/A	0.000	0.537	0.162	0.533	0.099	0.000	0.219	0.159	39.433

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F(-1)	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	74	384	120050	352	0	3075	19	0
N.S.	1	0.00	74.00	384.00	120050.00	352.00	0.00	3075.00	19.00	0.00
time (sec)	N/A	0.000	1.423	0.258	2.734	0.097	0.000	0.480	0.149	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	F	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	0	76	174	11681	134	0	1312	17	0
N.S.	1	0.00	1.31	3.00	201.40	2.31	0.00	22.62	0.29	0.00
time (sec)	N/A	0.000	0.448	0.171	0.210	0.086	0.000	0.200	0.149	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	F	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	0	76	180	14405	136	0	1208	17	0
N.S.	1	0.00	1.36	3.21	257.23	2.43	0.00	21.57	0.30	0.00
time (sec)	N/A	0.000	0.448	0.119	0.281	0.086	0.000	0.170	0.161	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F(-1)	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	434	402	151939	353	0	2538	19	0
N.S.	1	0.00	434.00	402.00	151939.00	353.00	0.00	2538.00	19.00	0.00
time (sec)	N/A	0.000	6.431	0.137	4.474	0.102	0.000	0.211	0.172	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mu
grade	N/A	F	C	C	C	C	F(-1)	C	F	I
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TI
size	1	0	185	1267	752466	892	0	13648	19	74
N.S.	1	0.00	185.00	1267.00	752466.00	892.00	0.00	13648.00	19.00	7453
time (sec)	N/A	0.000	1.929	1.591	72.448	0.149	0.000	1.156	0.148	50.

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	B	B	F	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	0	322	742	116810	420	0	6012	17	0
N.S.	1	0.00	3.13	7.20	1134.08	4.08	0.00	58.37	0.17	0.00
time (sec)	N/A	0.000	6.756	0.475	8.138	0.118	0.000	0.428	0.169	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	198	762	137271	429	0	3565	17	0
N.S.	1	0.00	198.00	762.00	137271.00	429.00	0.00	3565.00	17.00	0.00
time (sec)	N/A	0.000	6.659	0.204	5.125	0.122	0.000	0.341	0.148	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mup
grade	N/A	F	C	C	C	C	F(-1)	C	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TB
size	1	0	186	1263	919799	905	0	7181	19	746
N.S.	1	0.00	186.00	1263.00	919799.00	905.00	0.00	7181.00	19.00	74632
time (sec)	N/A	0.000	4.337	0.273	52.887	0.146	0.000	0.820	0.209	49.9

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	143	116	0	0	0	0	0	15	0
N.S.	1	1.23	1.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.322	1.132	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	139	260	0	0	0	0	0	15	0
N.S.	1	1.24	2.32	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.305	1.547	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	178	0	0	0	0	0	19	0
N.S.	1	0.00	1.06	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	1.657	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	0	156	0	0	0	0	0	17	0
N.S.	1	0.00	1.10	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	5.767	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	0	269	0	0	0	0	0	17	0
N.S.	1	0.00	0.92	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	1.210	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	69	73	31	435	158	25	112
N.S.	1	1.00	1.00	2.56	2.70	1.15	16.11	5.85	0.93	4.15
time (sec)	N/A	0.220	0.079	0.001	0.040	0.082	4.839	0.136	0.151	0.002

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	88	115	387	69	5545	248	17	254
N.S.	1	1.00	2.59	3.38	11.38	2.03	163.09	7.29	0.50	7.47
time (sec)	N/A	0.264	0.048	0.099	0.184	0.083	98.582	0.138	0.157	0.002

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-2)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	34	40	391	42	0	174	45	0
N.S.	1	1.00	0.89	1.05	10.29	1.11	0.00	4.58	1.18	0.00
time (sec)	N/A	0.289	0.093	1.313	0.042	0.071	0.000	0.151	0.156	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	63	64	191	1424	94	0	495	17	0
N.S.	1	0.94	0.96	2.85	21.25	1.40	0.00	7.39	0.25	0.00
time (sec)	N/A	0.360	0.293	0.090	0.220	0.085	0.000	0.145	0.180	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	24	59	70	52	435	164	27	112
N.S.	1	0.00	1.00	2.46	2.92	2.17	18.12	6.83	1.12	4.67
time (sec)	N/A	0.000	0.107	0.618	0.051	0.081	4.937	0.130	0.156	0.777

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	84	108	365	175	5545	249	19	254
N.S.	1	0.00	2.55	3.27	11.06	5.30	168.03	7.55	0.58	7.70
time (sec)	N/A	0.000	0.080	1.499	0.184	0.084	96.679	0.128	0.154	22.734

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	F(-2)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	34	42	425	126	0	192	51	0
N.S.	1	0.00	0.92	1.14	11.49	3.41	0.00	5.19	1.38	0.00
time (sec)	N/A	0.000	0.120	2.088	0.043	0.080	0.000	0.127	0.156	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	0	61	181	1423	447	0	500	19	0
N.S.	1	0.00	0.94	2.78	21.89	6.88	0.00	7.69	0.29	0.00
time (sec)	N/A	0.000	0.304	10.352	0.214	0.099	0.000	0.156	0.177	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	129	146	140	85	3645	961	17	217
N.S.	1	0.00	3.58	4.06	3.89	2.36	101.25	26.69	0.47	6.03
time (sec)	N/A	0.000	0.081	0.118	0.192	0.084	19.004	0.149	0.151	0.002

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	A	F(-2)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	0	177	167	534	82	0	760	802	146
N.S.	1	0.00	3.54	3.34	10.68	1.64	0.00	15.20	16.04	2.92
time (sec)	N/A	0.000	0.349	0.294	0.064	0.080	0.000	0.144	0.202	0.002

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	0	165	250	1263	113	0	1502	19	0
N.S.	1	0.00	1.88	2.84	14.35	1.28	0.00	17.07	0.22	0.00
time (sec)	N/A	0.000	1.902	1.476	0.235	0.085	0.000	0.186	0.167	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	0	80	56	894	73	0	718	77	0
N.S.	1	0.00	0.95	0.67	10.64	0.87	0.00	8.55	0.92	0.00
time (sec)	N/A	0.000	0.236	5.697	0.044	0.076	0.000	0.137	0.166	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	76	136	106	117	3645	961	19	217
N.S.	1	0.00	2.24	4.00	3.12	3.44	107.21	28.26	0.56	6.38
time (sec)	N/A	0.000	0.145	1.085	0.185	0.086	20.154	0.159	0.197	1.661

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	B	F(-2)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	0	182	159	542	184	0	790	826	146
N.S.	1	0.00	3.87	3.38	11.53	3.91	0.00	16.81	17.57	3.11
time (sec)	N/A	0.000	0.427	3.056	0.056	0.092	0.000	0.152	0.219	18.149

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	0	168	240	1217	335	0	1502	21	0
N.S.	1	0.00	2.00	2.86	14.49	3.99	0.00	17.88	0.25	0.00
time (sec)	N/A	0.000	2.117	8.319	0.206	0.090	0.000	0.169	0.163	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	0	83	62	927	222	0	786	89	0
N.S.	1	0.00	1.04	0.78	11.59	2.78	0.00	9.82	1.11	0.00
time (sec)	N/A	0.000	0.233	5.482	0.051	0.092	0.000	0.150	0.184	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	26	68	108	31	333	236	15	111
N.S.	1	1.08	1.00	2.62	4.15	1.19	12.81	9.08	0.58	4.27
time (sec)	N/A	0.233	0.073	0.446	0.053	0.078	4.261	0.135	0.154	0.002

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	90	115	454	71	3264	349	17	252
N.S.	1	1.00	2.50	3.19	12.61	1.97	90.67	9.69	0.47	7.00
time (sec)	N/A	0.283	0.051	0.001	0.056	0.076	62.544	0.147	0.155	0.002

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	63	399	47	0	145	42	0
N.S.	1	1.00	0.90	1.62	10.23	1.21	0.00	3.72	1.08	0.00
time (sec)	N/A	0.296	0.116	0.000	0.039	0.068	0.000	0.133	0.168	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	63	67	189	1773	141	0	2221	264	0
N.S.	1	0.94	1.00	2.82	26.46	2.10	0.00	33.15	3.94	0.00
time (sec)	N/A	0.377	0.357	0.000	0.090	0.081	0.000	0.155	0.169	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	A	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	26	60	100	52	337	241	19	114
N.S.	1	0.00	1.04	2.40	4.00	2.08	13.48	9.64	0.76	4.56
time (sec)	N/A	0.000	0.099	0.452	0.047	0.080	4.744	0.134	0.186	0.712

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	85	108	461	169	3264	347	19	252
N.S.	1	0.00	2.58	3.27	13.97	5.12	98.91	10.52	0.58	7.64
time (sec)	N/A	0.000	0.081	0.970	0.055	0.085	62.341	0.143	0.161	22.001

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	34	60	433	121	0	155	46	0
N.S.	1	0.00	0.92	1.62	11.70	3.27	0.00	4.19	1.24	0.00
time (sec)	N/A	0.000	0.138	1.618	0.041	0.078	0.000	0.150	0.180	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	0	60	179	1798	398	0	2220	296	0
N.S.	1	0.00	0.92	2.75	27.66	6.12	0.00	34.15	4.55	0.00
time (sec)	N/A	0.000	0.312	4.421	0.093	0.095	0.000	0.184	0.159	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	52	138	118	91	3215	688	17	223
N.S.	1	0.00	1.41	3.73	3.19	2.46	86.89	18.59	0.46	6.03
time (sec)	N/A	0.000	0.076	0.494	0.057	0.084	10.029	0.136	0.181	0.002

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	A	F(-1)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	181	173	711	86	0	1402	19	148
N.S.	1	0.00	3.69	3.53	14.51	1.76	0.00	28.61	0.39	3.02
time (sec)	N/A	0.000	0.350	0.000	0.065	0.079	0.000	0.165	0.162	0.002

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	247	243	1595	148	0	2867	19	0
N.S.	1	0.00	2.78	2.73	17.92	1.66	0.00	32.21	0.21	0.00
time (sec)	N/A	0.000	2.740	0.000	0.091	0.080	0.000	0.177	0.160	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	80	58	900	84	0	653	66	0
N.S.	1	0.00	0.94	0.68	10.59	0.99	0.00	7.68	0.78	0.00
time (sec)	N/A	0.000	0.187	0.000	0.045	0.071	0.000	0.135	0.166	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	48	127	116	118	3216	684	21	223
N.S.	1	0.00	1.50	3.97	3.62	3.69	100.50	21.38	0.66	6.97
time (sec)	N/A	0.000	0.100	0.646	0.053	0.085	10.609	0.133	0.159	1.677

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	B	F(-1)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	0	186	165	718	192	0	1410	21	148
N.S.	1	0.00	4.23	3.75	16.32	4.36	0.00	32.05	0.48	3.36
time (sec)	N/A	0.000	0.433	1.171	0.066	0.089	0.000	0.177	0.178	18.599

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	162	233	1571	331	0	2867	23	0
N.S.	1	0.00	1.91	2.74	18.48	3.89	0.00	33.73	0.27	0.00
time (sec)	N/A	0.000	5.170	2.703	0.096	0.089	0.000	0.171	0.163	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	81	62	937	205	0	705	76	0
N.S.	1	0.00	1.03	0.78	11.86	2.59	0.00	8.92	0.96	0.00
time (sec)	N/A	0.000	0.241	3.256	0.047	0.086	0.000	0.155	0.161	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	50	58	23	49	36
N.S.	1	1.00	0.96	0.89	0.85	1.85	2.15	0.85	1.81	1.33
time (sec)	N/A	0.194	0.023	0.329	0.029	0.071	0.178	0.124	0.157	17.273

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	44	58	23	55	36
N.S.	1	1.00	0.96	0.89	0.85	1.63	2.15	0.85	2.04	1.33
time (sec)	N/A	0.189	0.021	0.279	0.030	0.073	0.167	0.140	0.170	16.825

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	54	50	71	172	53	103	51
N.S.	1	1.00	0.82	0.89	0.82	1.16	2.82	0.87	1.69	0.84
time (sec)	N/A	0.218	0.056	1.329	0.041	0.076	0.832	0.136	0.179	18.856

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	43	37	53	73	42	74	37
N.S.	1	1.00	0.79	0.90	0.77	1.10	1.52	0.88	1.54	0.77
time (sec)	N/A	0.213	0.060	0.744	0.039	0.075	0.376	0.119	0.155	0.647

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	50	58	23	49	36
N.S.	1	1.00	0.96	0.89	0.85	1.85	2.15	0.85	1.81	1.33
time (sec)	N/A	0.184	0.001	0.000	0.028	0.072	0.168	0.138	0.178	0.006

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	66	74	31	435	440	102	109
N.S.	1	1.00	1.00	2.54	2.85	1.19	16.73	16.92	3.92	4.19
time (sec)	N/A	0.216	0.084	0.582	0.045	0.080	4.633	0.183	0.155	17.079

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	89	119	391	69	5545	1341	122	246
N.S.	1	1.00	2.54	3.40	11.17	1.97	158.43	38.31	3.49	7.03
time (sec)	N/A	0.268	0.069	1.468	0.178	0.078	97.094	0.209	0.165	21.928

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-2)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	26	382	40	0	315	46	0
N.S.	1	1.00	0.92	0.68	10.05	1.05	0.00	8.29	1.21	0.00
time (sec)	N/A	0.297	0.141	1.823	0.044	0.072	0.000	0.171	0.188	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	63	64	192	1420	94	0	12158	615	0
N.S.	1	0.94	0.96	2.87	21.19	1.40	0.00	181.46	9.18	0.00
time (sec)	N/A	0.365	0.331	9.389	0.202	0.081	0.000	0.997	0.215	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	54	49	95	1056	60	0	5419	94	0
N.S.	1	0.92	0.83	1.61	17.90	1.02	0.00	91.85	1.59	0.00
time (sec)	N/A	0.312	0.267	7.230	0.048	0.074	0.000	0.284	0.168	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	89	78	260	3092	113	0	52256	1269	0
N.S.	1	0.95	0.83	2.77	32.89	1.20	0.00	555.91	13.50	0.00
time (sec)	N/A	0.455	0.510	68.342	0.286	0.088	0.000	16.098	0.310	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	79	92	77	91	173	91	134	77
N.S.	1	1.00	0.77	0.89	0.75	0.88	1.68	0.88	1.30	0.75
time (sec)	N/A	0.259	0.118	3.832	0.042	0.077	2.100	0.122	0.181	1.660

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	54	61	57	94	204	60	182	53
N.S.	1	1.00	0.82	0.92	0.86	1.42	3.09	0.91	2.76	0.80
time (sec)	N/A	0.241	0.066	1.285	0.042	0.078	0.836	0.133	0.169	19.156

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	44	37	67	73	43	75	37
N.S.	1	1.00	0.80	0.90	0.76	1.37	1.49	0.88	1.53	0.76
time (sec)	N/A	0.212	0.036	0.659	0.037	0.075	0.360	0.142	0.150	0.682

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	127	145	140	89	3645	1708	17	217
N.S.	1	0.00	3.63	4.14	4.00	2.54	104.14	48.80	0.49	6.20
time (sec)	N/A	0.000	0.106	1.037	0.194	0.084	19.023	0.224	0.162	19.141

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	A	F(-2)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	0	177	165	534	85	0	1954	802	145
N.S.	1	0.00	3.69	3.44	11.12	1.77	0.00	40.71	16.71	3.02
time (sec)	N/A	0.000	0.405	2.761	0.056	0.082	0.000	0.209	0.212	19.271

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	0	165	250	1251	115	0	6958	19	0
N.S.	1	0.00	1.88	2.84	14.22	1.31	0.00	79.07	0.22	0.00
time (sec)	N/A	0.000	1.901	6.937	0.214	0.090	0.000	0.444	0.202	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	0	80	56	886	74	0	429	77	0
N.S.	1	0.00	0.96	0.67	10.67	0.89	0.00	5.17	0.93	0.00
time (sec)	N/A	0.000	0.224	5.393	0.046	0.076	0.000	0.189	0.181	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	102	114	105	172	406	113	290	104
N.S.	1	1.00	0.81	0.90	0.83	1.37	3.22	0.90	2.30	0.83
time (sec)	N/A	0.297	0.167	7.123	0.045	0.092	4.973	0.139	0.172	21.022

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	77	90	77	138	173	89	205	77
N.S.	1	1.00	0.76	0.89	0.76	1.37	1.71	0.88	2.03	0.76
time (sec)	N/A	0.276	0.126	3.792	0.045	0.083	2.231	0.130	0.161	1.460

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	56	50	117	172	55	89	51
N.S.	1	1.00	0.83	0.89	0.79	1.86	2.73	0.87	1.41	0.81
time (sec)	N/A	0.228	0.044	1.299	0.037	0.089	0.884	0.118	0.151	19.420

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	74	157	115	115	34286	2907	17	168
N.S.	1	0.00	1.17	2.49	1.83	1.83	544.22	46.14	0.27	2.67
time (sec)	N/A	0.000	0.252	2.234	0.048	0.089	68.924	0.185	0.182	18.702

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	B	A	F(-2)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	131	238	883	156	0	6607	19	399
N.S.	1	0.00	1.68	3.05	11.32	2.00	0.00	84.71	0.24	5.12
time (sec)	N/A	0.000	0.132	6.825	0.218	0.090	0.000	0.636	0.156	24.667

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F(-1)	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	291	192	1401	132	0	5096	0	0
N.S.	1	0.00	291.00	192.00	1401.00	132.00	0.00	5096.00	0.00	0.00
time (sec)	N/A	0.000	0.605	15.971	0.086	0.086	0.000	0.299	0.299	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F(-1)	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	105	357	2778	165	0	22836	0	0
N.S.	1	0.00	105.00	357.00	2778.00	165.00	0.00	22836.00	0.00	0.00
time (sec)	N/A	0.000	0.553	40.118	0.307	0.092	0.000	1.073	0.424	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	54	349	107	0	171	119	249
N.S.	1	1.00	0.78	1.50	9.69	2.97	0.00	4.75	3.31	6.92
time (sec)	N/A	0.238	0.155	0.576	0.049	0.085	0.000	0.174	0.161	25.029

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	B	B	F	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	96	356	12393	181	0	2905	147	0
N.S.	1	0.00	1.57	5.84	203.16	2.97	0.00	47.62	2.41	0.00
time (sec)	N/A	0.000	0.324	1.508	0.343	0.101	0.000	0.363	0.162	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	F	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	0	153	211	69828	180	0	2285	605	0
N.S.	1	0.00	1.82	2.51	831.29	2.14	0.00	27.20	7.20	0.00
time (sec)	N/A	0.000	0.683	3.365	0.900	0.091	0.000	0.235	0.198	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F	C	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	87	180	91340	315	0	2375	137	0
N.S.	1	0.00	87.00	180.00	91340.00	315.00	0.00	2375.00	137.00	0.00
time (sec)	N/A	0.000	1.350	3.211	1.600	0.095	0.000	0.199	0.166	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupa
grade	N/A	F	C	C	C	C	F	C	F	F(-1
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBI
size	1	0	491	943	504922	472	0	15002	0	0
N.S.	1	0.00	491.00	943.00	504922.00	472.00	0.00	15002.00	0.00	0.00
time (sec)	N/A	0.000	8.709	7.395	12.635	0.145	0.000	2.871	0.573	0.00

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mu
grade	N/A	F	C	C	C	C	F	C	F	F(
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TE
size	1	0	966	506	1342628	429	0	13226	19	0
N.S.	1	0.00	966.00	506.00	1342628.00	429.00	0.00	13226.00	19.00	0.0
time (sec)	N/A	0.000	6.488	18.169	34.918	0.113	0.000	0.427	200.021	0.0

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mu
grade	N/A	F	C	C	C	C	F	C	F	F(
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	T
size	1	0	1733	492	1929031	597	0	10230	0	
N.S.	1	0.00	1733.00	492.00	1929031.00	597.00	0.00	10230.00	0.00	0
time (sec)	N/A	0.000	6.720	17.778	61.564	0.130	0.000	0.363	1.152	0.

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupa
grade	N/A	F	C	C	F(-1)	C	F	C	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	243	1761	0	890	0	53926	0	0
N.S.	1	0.00	243.00	1761.00	0.00	890.00	0.00	53926.00	0.00	0.00
time (sec)	N/A	0.000	14.864	35.207	0.000	0.239	0.000	41.484	1.105	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mup
grade	N/A	F	C	C	F(-1)	C	F(-1)	C	F	F(-
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TB
size	1	0	1972	1201	0	761	0	34934	0	0
N.S.	1	0.00	1972.00	1201.00	0.00	761.00	0.00	34934.00	0.00	0.0
time (sec)	N/A	0.000	6.877	68.422	0.000	0.168	0.000	0.718	3.811	0.00

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	85	84	914	109	918	84	179	313
N.S.	1	1.00	0.93	0.92	10.04	1.20	10.09	0.92	1.97	3.44
time (sec)	N/A	0.261	0.336	3.945	0.088	0.083	1.944	0.139	0.188	21.035

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	416	63	405	56	78	98
N.S.	1	1.00	1.11	0.92	6.71	1.02	6.53	0.90	1.26	1.58
time (sec)	N/A	0.232	0.487	1.643	0.059	0.074	0.700	0.114	0.182	19.031

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	40	42	153	40	42	84
N.S.	1	1.00	1.00	0.93	0.93	0.98	3.56	0.93	0.98	1.95
time (sec)	N/A	0.205	0.117	0.597	0.037	0.071	0.303	0.120	0.188	19.569

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	0	132	0	0	0	0	0	108	0
N.S.	1	0.00	0.93	0.00	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.000	0.486	0.000	0.000	0.000	0.000	0.000	0.421	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	0	165	0	0	0	0	0	0	0
N.S.	1	0.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	1.006	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	0	173	0	0	0	0	0	0	0
N.S.	1	0.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	2.033	0.000	0.000	0.000	0.000	0.000	0.311	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1362	163	2004	129	303	495
N.S.	1	1.00	1.10	0.92	9.46	1.13	13.92	0.90	2.10	3.44
time (sec)	N/A	0.310	1.096	12.384	0.107	0.081	5.605	0.120	0.260	21.038

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	105	89	620	105	1027	80	154	177
N.S.	1	1.00	1.19	1.01	7.05	1.19	11.67	0.91	1.75	2.01
time (sec)	N/A	0.263	0.528	4.631	0.062	0.078	1.644	0.123	0.161	20.066

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	74	63	371	64	410	61	80	104
N.S.	1	1.00	1.09	0.93	5.46	0.94	6.03	0.90	1.18	1.53
time (sec)	N/A	0.236	0.514	1.687	0.056	0.072	0.751	0.114	0.177	19.609

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	0	155	0	0	0	0	0	17	0
N.S.	1	0.00	0.99	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	1.024	0.000	0.000	0.000	0.000	0.000	0.328	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	0	158	0	0	0	0	0	19	0
N.S.	1	0.00	1.10	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	1.460	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	0	205	0	0	0	0	0	19	0
N.S.	1	0.00	1.18	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	6.369	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	0	429	0	0	0	0	0	19	0
N.S.	1	0.00	2.73	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	1.643	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	176	190	2614	240	3577	181	19	999
N.S.	1	1.00	0.90	0.97	13.41	1.23	18.34	0.93	0.10	5.12
time (sec)	N/A	0.373	1.079	44.812	0.209	0.093	17.604	0.128	200.027	25.515

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	1360	154	2020	124	303	437
N.S.	1	1.00	1.11	0.92	9.86	1.12	14.64	0.90	2.20	3.17
time (sec)	N/A	0.315	1.144	13.523	0.103	0.083	5.675	0.119	0.223	23.108

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	90	787	101	937	89	179	495
N.S.	1	1.00	0.93	0.93	8.11	1.04	9.66	0.92	1.85	5.10
time (sec)	N/A	0.270	0.322	3.744	0.083	0.077	1.975	0.133	0.174	22.199

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	298	0	314	0	0	0	0	0	17	0
N.S.	1	0.00	1.05	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	7.603	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	289	0	385	0	0	0	0	0	19	0
N.S.	1	0.00	1.33	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	5.764	0.000	0.000	0.000	0.000	0.000	0.275	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	0	300	0	0	0	0	0	19	0
N.S.	1	0.00	1.03	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	1.774	0.000	0.000	0.000	0.000	0.000	132.082	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	0	1347	0	0	0	0	0	19	0
N.S.	1	0.00	4.79	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	9.249	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	13	15	15	14	15	130	19
N.S.	1	1.00	1.15	1.00	1.15	1.15	1.08	1.15	10.00	1.46
time (sec)	N/A	0.202	9.617	0.059	0.262	0.067	0.861	0.304	0.177	21.701

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	1488	17	15	17	17	19
N.S.	1	1.00	1.13	1.00	99.20	1.13	1.00	1.13	1.13	1.27
time (sec)	N/A	0.217	15.135	0.089	0.876	0.072	1.955	0.187	0.181	22.011

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	6267	17	15	17	17	19
N.S.	1	1.00	1.13	1.00	417.80	1.13	1.00	1.13	1.13	1.27
time (sec)	N/A	0.215	10.434	0.040	5.449	0.080	4.936	16.997	0.175	21.232

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	2684	19	17	19	144	19
N.S.	1	1.00	1.12	1.00	157.88	1.12	1.00	1.12	8.47	1.12
time (sec)	N/A	0.265	21.160	0.043	1.719	0.077	5.199	0.251	0.221	20.046

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	10269	19	17	19	7625	19
N.S.	1	1.00	1.12	1.00	604.06	1.12	1.00	1.12	448.53	1.12
time (sec)	N/A	0.231	13.545	0.046	12.409	0.081	15.620	40.988	0.837	20.303

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	24945	19	17	19	12086	19
N.S.	1	1.00	1.12	1.00	1467.35	1.12	1.00	1.12	710.94	1.12
time (sec)	N/A	0.246	11.442	0.059	28.056	0.090	40.306	0.942	5.447	21.387

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	485	566	16601	0	0	0	0	0	18	0
N.S.	1	1.17	34.23	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.306	42.731	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	258	646	3372	0	0	0	0	0	18	0
N.S.	1	2.50	13.07	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.163	13.847	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	288	614	0	0	0	0	0	16	0
N.S.	1	1.21	2.57	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.696	4.199	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	15	17	17	17	17	16	19
N.S.	1	1.00	1.12	0.88	1.00	1.00	1.00	1.00	0.94	1.12
time (sec)	N/A	0.221	10.680	0.053	0.365	0.072	0.880	0.138	0.163	19.734

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	19	19	19	19	18	19
N.S.	1	1.00	1.11	0.89	1.00	1.00	1.00	1.00	0.95	1.00
time (sec)	N/A	0.250	20.189	0.055	0.471	0.073	2.541	0.141	0.166	19.289

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	481	558	0	0	0	0	0	0	19	0
N.S.	1	1.16	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.419	0.000	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	373	241	0	0	0	0	0	19	0
N.S.	1	1.19	0.77	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.911	0.735	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	275	184	0	0	0	0	0	17	0
N.S.	1	1.18	0.79	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.756	1.433	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	15	17	17	19
N.S.	1	1.00	1.13	1.00	1.13	1.13	1.00	1.13	1.13	1.27
time (sec)	N/A	0.229	0.886	0.053	0.378	0.077	3.178	0.170	0.183	19.744

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	19	19	17	19	19	19
N.S.	1	1.00	1.12	1.00	1.12	1.12	1.00	1.12	1.12	1.12
time (sec)	N/A	0.245	0.939	0.052	0.516	0.079	12.301	0.168	0.166	19.173

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	86	70	181	1027	366	0	0	355	0
N.S.	1	1.19	0.97	2.51	14.26	5.08	0.00	0.00	4.93	0.00
time (sec)	N/A	0.664	0.264	0.333	0.181	0.104	0.000	0.000	0.183	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	111	149	526	316	0	0	109	285
N.S.	1	1.00	2.41	3.24	11.43	6.87	0.00	0.00	2.37	6.20
time (sec)	N/A	0.368	0.076	0.220	0.180	0.086	0.000	0.000	0.191	25.111

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	93	97	131	196	0	0	15	237
N.S.	1	1.00	3.10	3.23	4.37	6.53	0.00	0.00	0.50	7.90
time (sec)	N/A	0.253	0.037	0.151	0.161	0.083	0.000	0.000	0.185	23.441

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	94	93	105	190	0	234	15	231
N.S.	1	1.00	3.24	3.21	3.62	6.55	0.00	8.07	0.52	7.97
time (sec)	N/A	0.241	0.039	0.179	0.047	0.086	0.000	0.141	0.191	23.859

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	112	145	613	316	0	627	17	289
N.S.	1	1.00	2.43	3.15	13.33	6.87	0.00	13.63	0.37	6.28
time (sec)	N/A	0.381	0.078	0.255	0.060	0.097	0.000	0.161	0.211	23.536

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	86	71	179	1254	385	0	963	193	0
N.S.	1	1.18	0.97	2.45	17.18	5.27	0.00	13.19	2.64	0.00
time (sec)	N/A	0.661	0.246	0.360	0.070	0.095	0.000	0.212	0.187	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	134	114	0	0	0	0	0	15	0
N.S.	1	1.21	1.03	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.325	1.063	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	130	108	0	0	0	0	0	15	0
N.S.	1	1.21	1.01	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.316	1.023	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	66	74	31	435	440	102	109
N.S.	1	1.00	1.00	2.54	2.85	1.19	16.73	16.92	3.92	4.19
time (sec)	N/A	0.217	0.065	0.677	0.044	0.080	4.803	0.156	0.196	0.003

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	89	119	391	69	5545	1341	122	246
N.S.	1	1.00	2.54	3.40	11.17	1.97	158.43	38.31	3.49	7.03
time (sec)	N/A	0.265	0.038	1.706	0.172	0.085	97.141	0.215	0.185	0.002

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-2)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	26	382	40	0	315	46	0
N.S.	1	1.00	0.92	0.68	10.05	1.05	0.00	8.29	1.21	0.00
time (sec)	N/A	0.294	0.113	1.876	0.040	0.070	0.000	0.157	0.172	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	63	64	192	1420	94	0	12158	615	0
N.S.	1	0.94	0.96	2.87	21.19	1.40	0.00	181.46	9.18	0.00
time (sec)	N/A	0.365	0.310	10.329	0.208	0.087	0.000	1.036	0.241	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	23	56	69	51	435	439	102	109
N.S.	1	0.00	1.00	2.43	3.00	2.22	18.91	19.09	4.43	4.74
time (sec)	N/A	0.000	0.090	0.652	0.043	0.083	4.610	0.173	0.175	0.728

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	159	112	368	169	5545	1342	144	246
N.S.	1	0.00	4.68	3.29	10.82	4.97	163.09	39.47	4.24	7.24
time (sec)	N/A	0.000	0.075	1.691	0.172	0.084	95.498	0.216	0.187	26.593

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	F(-2)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	33	28	419	122	0	313	52	0
N.S.	1	0.00	0.89	0.76	11.32	3.30	0.00	8.46	1.41	0.00
time (sec)	N/A	0.000	0.139	1.927	0.056	0.078	0.000	0.151	0.211	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	0	61	182	1427	433	0	12160	719	0
N.S.	1	0.00	0.94	2.80	21.95	6.66	0.00	187.08	11.06	0.00
time (sec)	N/A	0.000	0.359	10.378	0.202	0.100	0.000	0.951	0.227	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	127	145	140	89	3645	1708	17	217
N.S.	1	0.00	3.63	4.14	4.00	2.54	104.14	48.80	0.49	6.20
time (sec)	N/A	0.000	0.076	1.076	0.193	0.084	19.364	0.226	0.164	0.003

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	A	F(-2)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	0	177	165	534	85	0	1954	802	145
N.S.	1	0.00	3.69	3.44	11.12	1.77	0.00	40.71	16.71	3.02
time (sec)	N/A	0.000	0.345	3.064	0.055	0.084	0.000	0.215	0.220	0.003

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	0	165	250	1251	115	0	6958	19	0
N.S.	1	0.00	1.88	2.84	14.22	1.31	0.00	79.07	0.22	0.00
time (sec)	N/A	0.000	1.882	7.959	0.211	0.085	0.000	0.450	0.179	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	0	80	56	886	74	0	429	77	0
N.S.	1	0.00	0.96	0.67	10.67	0.89	0.00	5.17	0.93	0.00
time (sec)	N/A	0.000	0.170	5.343	0.044	0.077	0.000	0.192	0.182	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	74	135	106	121	3645	1725	19	217
N.S.	1	0.00	2.24	4.09	3.21	3.67	110.45	52.27	0.58	6.58
time (sec)	N/A	0.000	0.152	1.128	0.175	0.086	19.426	0.238	0.179	1.778

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	B	F(-2)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	0	182	157	542	191	0	1950	826	145
N.S.	1	0.00	4.04	3.49	12.04	4.24	0.00	43.33	18.36	3.22
time (sec)	N/A	0.000	0.421	3.290	0.055	0.087	0.000	0.188	0.243	22.083

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	0	168	240	1217	337	0	6975	21	0
N.S.	1	0.00	1.95	2.79	14.15	3.92	0.00	81.10	0.24	0.00
time (sec)	N/A	0.000	1.916	7.950	0.213	0.093	0.000	0.444	0.191	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	81	55	919	221	0	427	89	0
N.S.	1	0.00	1.00	0.68	11.35	2.73	0.00	5.27	1.10	0.00
time (sec)	N/A	0.000	0.232	5.335	0.045	0.097	0.000	0.164	0.185	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	29	58	70	106	30	333	482	15	115
N.S.	1	1.07	2.15	2.59	3.93	1.11	12.33	17.85	0.56	4.26
time (sec)	N/A	0.226	0.106	0.555	0.046	0.079	4.029	0.164	0.169	0.841

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	90	115	450	71	3264	893	17	252
N.S.	1	1.00	2.57	3.29	12.86	2.03	93.26	25.51	0.49	7.20
time (sec)	N/A	0.270	0.045	1.211	0.058	0.079	60.656	0.237	0.201	24.737

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	36	395	45	0	327	41	0
N.S.	1	1.00	0.92	0.95	10.39	1.18	0.00	8.61	1.08	0.00
time (sec)	N/A	0.294	0.120	1.525	0.039	0.070	0.000	0.156	0.164	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	65	190	1777	125	0	11407	259	0
N.S.	1	1.07	1.08	3.17	29.62	2.08	0.00	190.12	4.32	0.00
time (sec)	N/A	0.369	0.355	5.492	0.099	0.082	0.000	1.116	0.198	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	B	A	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	55	61	102	52	337	484	19	114
N.S.	1	0.00	2.29	2.54	4.25	2.17	14.04	20.17	0.79	4.75
time (sec)	N/A	0.000	0.128	0.626	0.045	0.080	4.795	0.170	0.179	0.765

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	158	108	457	171	3264	982	19	252
N.S.	1	0.00	4.94	3.38	14.28	5.34	102.00	30.69	0.59	7.88
time (sec)	N/A	0.000	0.076	1.406	0.060	0.086	59.682	0.192	0.177	24.679

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	34	38	432	123	0	327	47	0
N.S.	1	0.00	0.94	1.06	12.00	3.42	0.00	9.08	1.31	0.00
time (sec)	N/A	0.000	0.151	1.802	0.040	0.078	0.000	0.155	0.168	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	0	60	180	1794	420	0	11496	287	0
N.S.	1	0.00	1.00	3.00	29.90	7.00	0.00	191.60	4.78	0.00
time (sec)	N/A	0.000	0.330	5.688	0.094	0.106	0.000	1.065	0.170	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	50	137	118	87	3215	527	17	223
N.S.	1	0.00	1.39	3.81	3.28	2.42	89.31	14.64	0.47	6.19
time (sec)	N/A	0.000	0.070	0.935	0.056	0.083	10.019	0.256	0.157	1.778

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	A	F(-1)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	0	181	175	711	83	0	2042	19	149
N.S.	1	0.00	3.55	3.43	13.94	1.63	0.00	40.04	0.37	2.92
time (sec)	N/A	0.000	0.345	1.974	0.065	0.085	0.000	0.197	0.161	20.995

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	A	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	247	243	1603	146	0	5628	19	0
N.S.	1	0.00	2.84	2.79	18.43	1.68	0.00	64.69	0.22	0.00
time (sec)	N/A	0.000	2.729	4.136	0.089	0.084	0.000	0.439	0.175	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	0	82	55	908	86	0	441	72	0
N.S.	1	0.00	0.98	0.65	10.81	1.02	0.00	5.25	0.86	0.00
time (sec)	N/A	0.000	0.229	3.818	0.045	0.075	0.000	0.184	0.189	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	B	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	52	128	116	114	3216	1043	21	223
N.S.	1	0.00	1.58	3.88	3.52	3.45	97.45	31.61	0.64	6.76
time (sec)	N/A	0.000	0.108	0.928	0.059	0.084	10.578	0.213	0.172	1.731

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	C	B	B	F(-1)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	0	194	167	718	182	0	2046	21	149
N.S.	1	0.00	4.22	3.63	15.61	3.96	0.00	44.48	0.46	3.24
time (sec)	N/A	0.000	0.437	1.947	0.069	0.091	0.000	0.199	0.176	20.726

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	B	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	164	233	1571	329	0	6142	23	0
N.S.	1	0.00	1.93	2.74	18.48	3.87	0.00	72.26	0.27	0.00
time (sec)	N/A	0.000	5.968	4.145	0.102	0.091	0.000	0.391	0.176	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	0	81	58	937	210	0	441	84	0
N.S.	1	0.00	1.01	0.72	11.71	2.62	0.00	5.51	1.05	0.00
time (sec)	N/A	0.000	0.245	3.780	0.047	0.086	0.000	0.188	0.170	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	47	31	173	371	145	7672	81	15	207
N.S.	1	1.21	0.79	4.44	9.51	3.72	196.72	2.08	0.38	5.31
time (sec)	N/A	0.294	0.369	0.124	0.063	0.083	3.372	0.154	0.165	21.781

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	41	28	145	290	145	7679	81	19	196
N.S.	1	1.21	0.82	4.26	8.53	4.26	225.85	2.38	0.56	5.76
time (sec)	N/A	0.296	0.368	0.130	0.057	0.083	4.564	0.155	0.177	21.416

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	51	31	177	549	118	7404	348	15	207
N.S.	1	1.31	0.79	4.54	14.08	3.03	189.85	8.92	0.38	5.31
time (sec)	N/A	0.287	0.355	0.141	0.082	0.081	14.823	0.146	0.170	21.230

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	44	30	149	432	118	7417	345	19	200
N.S.	1	1.29	0.88	4.38	12.71	3.47	218.15	10.15	0.56	5.88
time (sec)	N/A	0.297	0.358	0.144	0.070	0.083	15.583	0.157	0.176	21.849

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	54	349	107	0	171	119	249
N.S.	1	1.00	0.78	1.50	9.69	2.97	0.00	4.75	3.31	6.92
time (sec)	N/A	0.231	0.136	0.564	0.140	0.087	0.000	0.152	0.173	0.003

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	53	322	93	0	169	119	249
N.S.	1	1.00	0.79	1.61	9.76	2.82	0.00	5.12	3.61	7.55
time (sec)	N/A	0.223	0.161	0.576	0.054	0.084	0.000	0.155	0.170	25.219

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	28	92	564	110	0	396	15	249
N.S.	1	1.11	0.78	2.56	15.67	3.06	0.00	11.00	0.42	6.92
time (sec)	N/A	0.238	0.151	0.503	0.089	0.081	0.000	0.161	0.177	25.212

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	37	29	80	536	96	1824	397	19	249
N.S.	1	1.12	0.88	2.42	16.24	2.91	55.27	12.03	0.58	7.55
time (sec)	N/A	0.239	0.158	0.548	0.089	0.086	55.224	0.159	0.181	25.406

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	54	47	92	91	83	269	46	181	45
N.S.	1	0.89	0.77	1.51	1.49	1.36	4.41	0.75	2.97	0.74
time (sec)	N/A	0.290	0.327	25.234	0.060	0.084	26.386	0.129	0.188	0.089

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	54	47	82	80	46	235	80	154	46
N.S.	1	0.89	0.77	1.34	1.31	0.75	3.85	1.31	2.52	0.75
time (sec)	N/A	0.291	0.234	14.594	0.050	0.085	11.174	0.137	0.193	18.044

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	37	70	69	63	197	36	134	36
N.S.	1	0.91	0.80	1.52	1.50	1.37	4.28	0.78	2.91	0.78
time (sec)	N/A	0.289	0.160	8.281	0.057	0.081	4.986	0.126	0.176	0.079

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	37	60	58	36	163	58	107	36
N.S.	1	0.91	0.80	1.30	1.26	0.78	3.54	1.26	2.33	0.78
time (sec)	N/A	0.281	0.093	4.067	0.045	0.075	2.187	0.128	0.192	18.516

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	27	55	47	41	126	26	86	26
N.S.	1	0.97	0.87	1.77	1.52	1.32	4.06	0.84	2.77	0.84
time (sec)	N/A	0.277	0.058	2.927	0.050	0.072	0.828	0.227	0.204	18.160

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	27	41	36	26	92	36	59	26
N.S.	1	0.97	0.87	1.32	1.16	0.84	2.97	1.16	1.90	0.84
time (sec)	N/A	0.275	0.041	1.431	0.050	0.073	0.400	0.151	0.159	0.053

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	15	26	26	21	51	13	39	44
N.S.	1	1.00	0.50	0.87	0.87	0.70	1.70	0.43	1.30	1.47
time (sec)	N/A	0.192	0.018	0.491	0.024	0.069	0.168	0.140	0.161	18.476

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	20	115	28	0	28	18	12
N.S.	1	1.00	1.00	1.43	8.21	2.00	0.00	2.00	1.29	0.86
time (sec)	N/A	0.224	0.006	0.153	0.184	0.075	0.000	0.116	0.172	18.321

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	23	50	31	236	52	0	52	20	26
N.S.	1	0.82	1.79	1.11	8.43	1.86	0.00	1.86	0.71	0.93
time (sec)	N/A	0.249	0.136	0.389	0.048	0.076	0.000	0.156	0.176	18.461

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	54	29	51	808	85	0	63	20	48
N.S.	1	1.15	0.62	1.09	17.19	1.81	0.00	1.34	0.43	1.02
time (sec)	N/A	0.273	0.016	0.748	0.239	0.076	0.000	0.190	0.175	0.070

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	66	205	71	2174	112	0	160	297	60
N.S.	1	1.06	3.31	1.15	35.06	1.81	0.00	2.58	4.79	0.97
time (sec)	N/A	0.300	0.432	1.813	0.154	0.081	0.000	0.142	0.195	18.280

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	97	31	87	3088	140	0	85	374	79
N.S.	1	1.17	0.37	1.05	37.20	1.69	0.00	1.02	4.51	0.95
time (sec)	N/A	0.315	0.022	3.978	0.406	0.085	0.000	0.125	0.184	18.409

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	42	68	74	72	46	593	36	250	46
N.S.	1	0.95	1.55	1.68	1.64	1.05	13.48	0.82	5.68	1.05
time (sec)	N/A	0.296	0.235	13.217	0.046	0.081	11.576	0.131	0.180	18.061

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	86	62	66	65	67	434	68	227	110
N.S.	1	1.13	0.82	0.87	0.86	0.88	5.71	0.89	2.99	1.45
time (sec)	N/A	0.398	0.145	6.924	0.111	0.080	5.105	0.133	0.221	19.260

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	30	48	52	50	36	359	26	172	33
N.S.	1	1.03	1.66	1.79	1.72	1.24	12.38	0.90	5.93	1.14
time (sec)	N/A	0.284	0.079	3.567	0.045	0.076	2.169	0.251	0.179	0.052

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	54	40	42	41	46	231	44	149	43
N.S.	1	1.10	0.82	0.86	0.84	0.94	4.71	0.90	3.04	0.88
time (sec)	N/A	0.315	0.044	2.359	0.045	0.072	0.852	0.156	0.168	18.586

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	28	26	24	131	13	96	13
N.S.	1	1.00	1.00	1.87	1.73	1.60	8.73	0.87	6.40	0.87
time (sec)	N/A	0.240	0.004	0.816	0.072	0.073	0.377	0.147	0.165	0.041

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	55	14	0	18	20	12
N.S.	1	1.00	1.00	0.93	3.93	1.00	0.00	1.29	1.43	0.86
time (sec)	N/A	0.234	0.010	0.237	0.055	0.075	0.000	0.116	0.178	17.838

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	53	19	0	11	22	11
N.S.	1	1.00	1.00	0.92	4.08	1.46	0.00	0.85	1.69	0.85
time (sec)	N/A	0.249	0.007	0.414	0.047	0.069	0.000	0.157	0.171	17.965

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	27	43	24	641	56	0	41	78	35
N.S.	1	0.90	1.43	0.80	21.37	1.87	0.00	1.37	2.60	1.17
time (sec)	N/A	0.275	0.016	0.507	0.068	0.078	0.000	0.191	0.166	0.093

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	36	48	46	308	43	0	32	22	33
N.S.	1	0.86	1.14	1.10	7.33	1.02	0.00	0.76	0.52	0.79
time (sec)	N/A	0.286	0.205	2.524	0.057	0.069	0.000	0.146	0.193	18.885

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	49	56	62	3164	112	0	74	156	74
N.S.	1	0.82	0.93	1.03	52.73	1.87	0.00	1.23	2.60	1.23
time (sec)	N/A	0.304	0.159	1.697	0.220	0.081	0.000	0.121	0.176	0.171

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	37	97	80	73	447	36	345	36
N.S.	1	0.91	0.80	2.11	1.74	1.59	9.72	0.78	7.50	0.78
time (sec)	N/A	0.293	0.237	22.870	0.056	0.078	26.380	0.151	0.189	18.046

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	54	47	71	69	46	366	69	288	46
N.S.	1	0.89	0.77	1.16	1.13	0.75	6.00	1.13	4.72	0.75
time (sec)	N/A	0.297	0.141	12.531	0.093	0.080	11.703	0.155	0.188	16.742

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	27	55	47	53	284	26	241	26
N.S.	1	0.97	0.87	1.77	1.52	1.71	9.16	0.84	7.77	0.84
time (sec)	N/A	0.287	0.089	5.535	0.102	0.075	4.916	0.265	0.183	0.060

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	37	55	47	36	202	47	184	36
N.S.	1	0.91	0.80	1.20	1.02	0.78	4.39	1.02	4.00	0.78
time (sec)	N/A	0.289	0.067	2.799	0.101	0.072	2.166	0.171	0.179	17.709

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	34	31	117	13	137	13
N.S.	1	1.00	1.00	2.73	2.27	2.07	7.80	0.87	9.13	0.87
time (sec)	N/A	0.248	0.006	1.678	0.102	0.072	0.827	0.114	0.159	0.044

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	23	27	29	124	36	0	36	20	23
N.S.	1	0.82	0.96	1.04	4.43	1.29	0.00	1.29	0.71	0.82
time (sec)	N/A	0.246	0.011	0.357	0.362	0.079	0.000	0.122	0.173	17.679

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	83	13	0	28	22	13
N.S.	1	1.00	1.00	1.08	6.38	1.00	0.00	2.15	1.69	1.00
time (sec)	N/A	0.245	0.009	0.950	0.069	0.066	0.000	0.170	0.173	0.032

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	38	37	480	61	0	48	22	36
N.S.	1	1.12	1.12	1.09	14.12	1.79	0.00	1.41	0.65	1.06
time (sec)	N/A	0.306	0.010	1.472	0.261	0.079	0.000	0.203	0.182	0.067

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	35	61	41	987	67	0	98	22	37
N.S.	1	0.81	1.42	0.95	22.95	1.56	0.00	2.28	0.51	0.86
time (sec)	N/A	0.280	0.080	3.706	0.153	0.084	0.000	0.146	0.169	0.073

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	85	29	69	1805	95	0	73	22	67
N.S.	1	1.25	0.43	1.01	26.54	1.40	0.00	1.07	0.32	0.99
time (sec)	N/A	0.301	0.027	8.436	0.441	0.080	0.000	0.125	0.189	18.447

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	54	119	47	91	46	0	270	20	46
N.S.	1	0.89	1.95	0.77	1.49	0.75	0.00	4.43	0.33	0.75
time (sec)	N/A	0.292	0.264	13.576	0.096	0.089	0.000	0.172	0.238	18.718

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	54	61	47	80	73	0	46	20	45
N.S.	1	0.89	1.00	0.77	1.31	1.20	0.00	0.75	0.33	0.74
time (sec)	N/A	0.297	0.028	5.691	0.114	0.080	0.000	0.140	0.200	0.066

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	89	37	69	36	0	204	20	36
N.S.	1	0.91	1.93	0.80	1.50	0.78	0.00	4.43	0.43	0.78
time (sec)	N/A	0.284	0.157	4.087	0.102	0.076	0.000	0.134	0.212	17.711

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	46	37	58	53	0	36	20	36
N.S.	1	0.91	1.00	0.80	1.26	1.15	0.00	0.78	0.43	0.78
time (sec)	N/A	0.281	0.018	1.431	0.057	0.073	0.000	0.121	0.185	17.621

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	59	27	47	26	0	138	20	26
N.S.	1	0.97	1.90	0.87	1.52	0.84	0.00	4.45	0.65	0.84
time (sec)	N/A	0.281	0.109	0.984	0.103	0.074	0.000	0.128	0.173	0.054

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	31	40	36	33	0	26	20	26
N.S.	1	0.97	1.00	1.29	1.16	1.06	0.00	0.84	0.65	0.84
time (sec)	N/A	0.285	0.015	0.387	0.081	0.073	0.000	0.296	0.171	18.081

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	13	104225	52	20	13
N.S.	1	1.00	1.00	0.93	1.53	0.87	6948.33	3.47	1.33	0.87
time (sec)	N/A	0.252	0.005	0.246	0.048	0.070	82.037	0.148	0.166	0.040

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	12	11	11	3636	11	18	11
N.S.	1	1.00	2.09	1.09	1.00	1.00	330.55	1.00	1.64	1.00
time (sec)	N/A	0.217	0.004	0.117	0.044	0.068	11.150	0.114	0.158	0.024

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	23	29	31	233	50	0	38	18	26
N.S.	1	0.82	1.04	1.11	8.32	1.79	0.00	1.36	0.64	0.93
time (sec)	N/A	0.255	0.012	0.147	0.193	0.077	0.000	0.118	0.182	19.035

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	54	143	53	974	96	0	137	20	49
N.S.	1	1.15	3.04	1.13	20.72	2.04	0.00	2.91	0.43	1.04
time (sec)	N/A	0.283	0.284	0.198	0.067	0.077	0.000	0.154	0.194	0.074

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	66	31	69	1780	130	0	72	20	61
N.S.	1	1.06	0.50	1.11	28.71	2.10	0.00	1.16	0.32	0.98
time (sec)	N/A	0.302	0.013	0.237	0.274	0.080	0.000	0.244	0.192	0.094

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	97	268	89	3846	148	0	206	20	78
N.S.	1	1.17	3.23	1.07	46.34	1.78	0.00	2.48	0.24	0.94
time (sec)	N/A	0.322	0.450	0.322	0.208	0.082	0.000	0.142	0.194	18.960

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	187	85	103	87	87	0	95	22	149
N.S.	1	1.21	0.55	0.66	0.56	0.56	0.00	0.61	0.14	0.96
time (sec)	N/A	0.857	0.622	29.478	0.112	0.092	0.000	0.176	5.884	20.585

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	42	59	47	72	36	0	46	22	35
N.S.	1	0.95	1.34	1.07	1.64	0.82	0.00	1.05	0.50	0.80
time (sec)	N/A	0.297	0.033	17.971	0.095	0.079	0.000	0.145	1.891	19.559

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	133	62	75	65	66	0	75	22	109
N.S.	1	1.20	0.56	0.68	0.59	0.59	0.00	0.68	0.20	0.98
time (sec)	N/A	0.604	0.126	10.249	0.107	0.082	0.000	0.157	1.662	20.169

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	30	48	27	50	26	0	36	22	25
N.S.	1	1.03	1.66	0.93	1.72	0.90	0.00	1.24	0.76	0.86
time (sec)	N/A	0.281	0.078	5.506	0.049	0.073	0.000	0.132	0.324	18.623

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	79	40	45	43	47	0	55	22	65
N.S.	1	1.32	0.67	0.75	0.72	0.78	0.00	0.92	0.37	1.08
time (sec)	N/A	0.395	0.063	2.023	0.051	0.073	0.000	0.134	0.315	19.282

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	26	13	0	13	22	13
N.S.	1	1.00	1.00	1.08	2.00	1.00	0.00	1.00	1.69	1.00
time (sec)	N/A	0.255	0.004	0.939	0.066	0.073	0.000	0.298	0.185	18.222

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	27	20	18	18	22	0	29	22	17
N.S.	1	1.29	0.95	0.86	0.86	1.05	0.00	1.38	1.05	0.81
time (sec)	N/A	0.244	0.019	0.408	0.095	0.070	0.000	0.155	0.184	18.045

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	14	12	13	81	14	0	13	20	13
N.S.	1	1.17	1.00	1.08	6.75	1.17	0.00	1.08	1.67	1.08
time (sec)	N/A	0.228	0.008	0.302	0.101	0.078	0.000	0.114	0.172	18.459

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	27	44	24	656	65	0	37	20	36
N.S.	1	0.90	1.47	0.80	21.87	2.17	0.00	1.23	0.67	1.20
time (sec)	N/A	0.265	0.019	0.178	0.075	0.081	0.000	0.122	0.166	0.082

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	34	48	46	308	54	0	35	22	37
N.S.	1	0.81	1.14	1.10	7.33	1.29	0.00	0.83	0.52	0.88
time (sec)	N/A	0.282	0.197	0.260	0.146	0.070	0.000	0.162	0.181	18.936

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	47	54	62	3188	138	0	74	22	82
N.S.	1	0.78	0.90	1.03	53.13	2.30	0.00	1.23	0.37	1.37
time (sec)	N/A	0.293	0.236	0.332	0.239	0.082	0.000	0.199	0.196	0.175

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	60	90	68	1227	86	0	56	22	55
N.S.	1	0.83	1.25	0.94	17.04	1.19	0.00	0.78	0.31	0.76
time (sec)	N/A	0.302	0.139	0.457	0.125	0.074	0.000	0.137	0.317	18.795

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	73	76	98	7650	194	0	94	22	114
N.S.	1	0.81	0.84	1.09	85.00	2.16	0.00	1.04	0.24	1.27
time (sec)	N/A	0.321	0.283	0.646	0.722	0.085	0.000	0.122	0.317	18.390

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	80	132	90	2710	118	0	76	22	83
N.S.	1	0.78	1.29	0.88	26.57	1.16	0.00	0.75	0.22	0.81
time (sec)	N/A	0.314	0.268	0.825	0.252	0.076	0.000	0.157	4.158	18.488

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	54	119	47	91	46	0	314	22	46
N.S.	1	0.89	1.95	0.77	1.49	0.75	0.00	5.15	0.36	0.75
time (sec)	N/A	0.315	0.933	106.031	0.116	0.093	0.000	0.228	111.881	18.531

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	66	76	57	91	83	0	56	22	55
N.S.	1	0.87	1.00	0.75	1.20	1.09	0.00	0.74	0.29	0.72
time (sec)	N/A	0.307	0.069	69.330	0.060	0.087	0.000	0.189	200.016	0.060

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	104	37	80	36	0	248	22	36
N.S.	1	0.91	2.26	0.80	1.74	0.78	0.00	5.39	0.48	0.78
time (sec)	N/A	0.285	0.541	42.891	0.086	0.083	0.000	0.203	24.166	0.084

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	54	61	47	69	63	0	46	22	45
N.S.	1	0.89	1.00	0.77	1.13	1.03	0.00	0.75	0.36	0.74
time (sec)	N/A	0.306	0.031	25.444	0.109	0.075	0.000	0.171	72.024	0.066

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	27	27	47	26	0	182	22	26
N.S.	1	0.97	0.87	0.87	1.52	0.84	0.00	5.87	0.71	0.84
time (sec)	N/A	0.283	0.095	14.333	0.052	0.073	0.000	0.161	4.168	19.070

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	37	37	47	43	0	36	22	36
N.S.	1	0.91	0.80	0.80	1.02	0.93	0.00	0.78	0.48	0.78
time (sec)	N/A	0.290	0.067	7.862	0.161	0.076	0.000	0.162	10.180	18.847

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	34	13	0	74	22	13
N.S.	1	1.00	1.00	0.93	2.27	0.87	0.00	4.93	1.47	0.87
time (sec)	N/A	0.253	0.005	3.023	0.120	0.072	0.000	0.146	0.280	18.603

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	28	22	23	21	0	22	22	24
N.S.	1	0.96	1.04	0.81	0.85	0.78	0.00	0.81	0.81	0.89
time (sec)	N/A	0.256	0.008	1.426	0.092	0.072	0.000	0.322	0.608	0.046

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	21	44	29	92	38	0	54	22	22
N.S.	1	0.88	1.83	1.21	3.83	1.58	0.00	2.25	0.92	0.92
time (sec)	N/A	0.256	0.030	1.087	0.124	0.087	0.000	0.172	0.162	18.632

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	84	13	0	13	20	13
N.S.	1	1.00	1.00	1.27	7.64	1.18	0.00	1.18	1.82	1.18
time (sec)	N/A	0.248	0.008	0.658	0.100	0.064	0.000	0.129	0.170	18.989

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	35	31	41	834	94	0	52	20	38
N.S.	1	0.81	0.72	0.95	19.40	2.19	0.00	1.21	0.47	0.88
time (sec)	N/A	0.283	0.016	0.217	0.376	0.075	0.000	0.121	0.165	0.067

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	85	129	71	2237	132	0	160	22	66
N.S.	1	1.25	1.90	1.04	32.90	1.94	0.00	2.35	0.32	0.97
time (sec)	N/A	0.300	3.021	0.327	0.248	0.081	0.000	0.170	0.223	0.093

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	78	31	87	3095	166	0	82	22	71
N.S.	1	1.01	0.40	1.13	40.19	2.16	0.00	1.06	0.29	0.92
time (sec)	N/A	0.304	0.029	0.418	0.491	0.083	0.000	0.197	0.533	19.179

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	128	278	107	4268	194	0	268	22	100
N.S.	1	1.23	2.67	1.03	41.04	1.87	0.00	2.58	0.21	0.96
time (sec)	N/A	0.333	0.737	0.610	0.417	0.084	0.000	0.162	5.663	20.233

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	136	151	98	266160190	0	291	0	0	30	0
N.S.	1	1.11	0.72	1957060.22	0.00	2.14	0.00	0.00	0.22	0.00
time (sec)	N/A	0.551	0.288	126.185	0.000	0.094	0.000	0.000	0.159	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	110	120	86	94440518	0	280	0	0	28	0
N.S.	1	1.09	0.78	858550.16	0.00	2.55	0.00	0.00	0.25	0.00
time (sec)	N/A	0.398	0.194	24.987	0.000	0.093	0.000	0.000	0.160	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	89	72	20170856	0	266	0	0	19	0
N.S.	1	1.06	0.86	240129.24	0.00	3.17	0.00	0.00	0.23	0.00
time (sec)	N/A	0.297	0.109	5.187	0.000	0.092	0.000	0.000	0.174	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	50	980083	0	240	0	0	30	0
N.S.	1	1.00	0.86	16897.98	0.00	4.14	0.00	0.00	0.52	0.00
time (sec)	N/A	0.216	0.091	0.627	0.000	0.085	0.000	0.000	0.159	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupa
grade	N/A	A	A	B	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	94317036	0	39	0	2029	30	34
N.S.	1	1.00	0.96	4100740.70	0.00	1.70	0.00	88.22	1.30	1.48
time (sec)	N/A	0.197	0.012	13.717	0.000	0.078	0.000	13.573	0.184	18.66

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	597	0	69	0	7875	30	108
N.S.	1	1.00	0.81	11.26	0.00	1.30	0.00	148.58	0.57	2.04
time (sec)	N/A	0.283	0.194	69.211	0.000	0.080	0.000	47.104	0.155	22.354

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	84	52	308	0	88	0	18022	30	131
N.S.	1	1.06	0.66	3.90	0.00	1.11	0.00	228.13	0.38	1.66
time (sec)	N/A	0.384	0.279	97.645	0.000	0.083	0.000	157.235	0.225	22.279

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	115	67	0	0	113	0	0	30	351
N.S.	1	1.10	0.64	0.00	0.00	1.08	0.00	0.00	0.29	3.34
time (sec)	N/A	0.515	0.328	0.000	0.000	0.087	0.000	0.000	0.153	24.468

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	108	96	0	0	0	0	0	32	0
N.S.	1	1.10	0.98	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.431	1.651	180.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	66	0	0	0	0	0	32	0
N.S.	1	1.07	0.96	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.329	0.262	180.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	76	84470649	0	0	0	0	30	0
N.S.	1	1.07	1.10	1224212.30	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.327	0.298	22.921	0.000	0.000	0.000	0.000	0.155	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	26159629	0	0	0	0	21	0
N.S.	1	1.00	0.85	653990.72	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.252	0.142	4.657	0.000	0.000	0.000	0.000	0.159	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	75	11960757	0	0	0	0	32	0
N.S.	1	1.00	1.88	299018.92	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.254	0.242	2.413	0.000	0.000	0.000	0.000	0.189	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	22356405	0	156	0	0	32	0
N.S.	1	1.00	0.91	496809.00	0.00	3.47	0.00	0.00	0.71	0.00
time (sec)	N/A	0.266	0.169	2.974	0.000	0.093	0.000	0.000	0.146	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	83	123	0	94	0	0	32	0
N.S.	1	1.00	1.73	2.56	0.00	1.96	0.00	0.00	0.67	0.00
time (sec)	N/A	0.261	0.237	19.258	0.000	0.087	0.000	0.000	0.148	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	78	66	227	0	212	0	0	32	0
N.S.	1	1.01	0.86	2.95	0.00	2.75	0.00	0.00	0.42	0.00
time (sec)	N/A	0.337	0.762	147.190	0.000	0.099	0.000	0.000	0.150	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	151	98	0	0	290	0	0	30	0
N.S.	1	1.11	0.72	0.00	0.00	2.13	0.00	0.00	0.22	0.00
time (sec)	N/A	0.533	0.342	180.000	0.000	0.099	0.000	0.000	0.178	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	110	120	86	89676612	0	281	0	0	21	0
N.S.	1	1.09	0.78	815241.93	0.00	2.55	0.00	0.00	0.19	0.00
time (sec)	N/A	0.418	0.232	29.678	0.000	0.093	0.000	0.000	0.157	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	89	74	0	0	268	0	0	32	0
N.S.	1	1.06	0.88	0.00	0.00	3.19	0.00	0.00	0.38	0.00
time (sec)	N/A	0.313	0.183	180.000	0.000	0.089	0.000	0.000	0.154	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	86	72	0	0	296	0	0	32	0
N.S.	1	1.06	0.89	0.00	0.00	3.65	0.00	0.00	0.40	0.00
time (sec)	N/A	0.403	0.172	180.000	0.000	0.093	0.000	0.000	0.154	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	727	0	48	0	15648	32	85
N.S.	1	1.00	0.96	25.96	0.00	1.71	0.00	558.86	1.14	3.04
time (sec)	N/A	0.196	0.171	35.099	0.000	0.078	0.000	92.915	0.170	20.782

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	35	4684	0	55	0	0	32	88
N.S.	1	1.00	0.64	85.16	0.00	1.00	0.00	0.00	0.58	1.60
time (sec)	N/A	0.292	0.276	161.408	0.000	0.083	0.000	0.000	0.159	23.006

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	86	55	0	0	79	0	0	32	300
N.S.	1	1.06	0.68	0.00	0.00	0.98	0.00	0.00	0.40	3.70
time (sec)	N/A	0.399	0.399	0.000	0.000	0.087	0.000	0.000	0.163	24.641

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	117	62	0	0	98	0	0	32	383
N.S.	1	1.09	0.58	0.00	0.00	0.92	0.00	0.00	0.30	3.58
time (sec)	N/A	0.510	0.578	0.000	0.000	0.086	0.000	0.000	0.171	26.883

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	153	98	973	0	290	0	0	30	0
N.S.	1	1.12	0.72	7.15	0.00	2.13	0.00	0.00	0.22	0.00
time (sec)	N/A	0.606	0.358	10.902	0.000	0.095	0.000	0.000	0.181	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	110	122	86	99228795	0	281	0	0	30	0
N.S.	1	1.11	0.78	902079.95	0.00	2.55	0.00	0.00	0.27	0.00
time (sec)	N/A	0.469	0.207	45.377	0.000	0.093	0.000	0.000	0.160	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	81	91	70	24409652	0	266	0	0	28	0
N.S.	1	1.12	0.86	301353.73	0.00	3.28	0.00	0.00	0.35	0.00
time (sec)	N/A	0.361	0.129	9.441	0.000	0.087	0.000	0.000	0.156	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	60	52	157	0	242	0	0	19	0
N.S.	1	1.13	0.98	2.96	0.00	4.57	0.00	0.00	0.36	0.00
time (sec)	N/A	0.275	0.083	4.737	0.000	0.087	0.000	0.000	0.146	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	308	0	39	0	0	30	24
N.S.	1	1.00	0.96	12.83	0.00	1.62	0.00	0.00	1.25	1.00
time (sec)	N/A	0.196	0.091	3.970	0.000	0.077	0.000	0.000	0.148	19.498

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	55	43	194	0	74	0	0	30	103
N.S.	1	1.04	0.81	3.66	0.00	1.40	0.00	0.00	0.57	1.94
time (sec)	N/A	0.347	0.145	8.520	0.000	0.079	0.000	0.000	0.148	22.225

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	86	52	481	0	103	0	0	30	136
N.S.	1	1.09	0.66	6.09	0.00	1.30	0.00	0.00	0.38	1.72
time (sec)	N/A	0.458	0.167	81.954	0.000	0.083	0.000	0.000	0.145	22.911

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	117	67	222	0	118	0	0	30	350
N.S.	1	1.11	0.64	2.11	0.00	1.12	0.00	0.00	0.29	3.33
time (sec)	N/A	0.576	0.208	111.306	0.000	0.086	0.000	0.000	0.153	23.512

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	116	66	204	0	0	0	0	32	0
N.S.	1	1.09	0.62	1.92	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.434	0.562	49.227	0.000	0.000	0.000	0.000	0.152	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	116	76	139	0	0	0	0	32	0
N.S.	1	1.09	0.72	1.31	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.443	0.327	32.059	0.000	0.000	0.000	0.000	0.150	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	82	34	137	0	0	0	0	32	0
N.S.	1	1.09	0.45	1.83	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.329	0.184	23.092	0.000	0.000	0.000	0.000	0.144	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	77	52	111	0	0	0	0	30	0
N.S.	1	1.10	0.74	1.59	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.342	0.262	7.271	0.000	0.000	0.000	0.000	0.146	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	37	176	0	155	0	0	21	0
N.S.	1	1.00	0.84	4.00	0.00	3.52	0.00	0.00	0.48	0.00
time (sec)	N/A	0.262	0.169	3.427	0.000	0.093	0.000	0.000	0.141	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	62	123	0	103	0	0	32	0
N.S.	1	1.00	1.29	2.56	0.00	2.15	0.00	0.00	0.67	0.00
time (sec)	N/A	0.274	0.223	6.164	0.000	0.086	0.000	0.000	0.140	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	78	64	227	0	266	0	0	32	0
N.S.	1	1.01	0.83	2.95	0.00	3.45	0.00	0.00	0.42	0.00
time (sec)	N/A	0.347	0.548	13.924	0.000	0.096	0.000	0.000	0.155	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	82	66	154	0	179	0	0	32	0
N.S.	1	1.06	0.86	2.00	0.00	2.32	0.00	0.00	0.42	0.00
time (sec)	N/A	0.342	0.508	58.417	0.000	0.097	0.000	0.000	0.149	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	112	85	240	0	346	0	0	32	0
N.S.	1	1.06	0.80	2.26	0.00	3.26	0.00	0.00	0.30	0.00
time (sec)	N/A	0.439	0.770	245.864	0.000	0.106	0.000	0.000	0.148	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	116	86	0	0	235	0	0	32	0
N.S.	1	1.09	0.81	0.00	0.00	2.22	0.00	0.00	0.30	0.00
time (sec)	N/A	0.441	0.529	0.000	0.000	0.102	0.000	0.000	0.149	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	215	100	441	0	291	0	0	32	0
N.S.	1	1.13	0.53	2.32	0.00	1.53	0.00	0.00	0.17	0.00
time (sec)	N/A	0.903	0.594	108.867	0.000	0.098	0.000	0.000	0.165	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	182	84	973	0	280	0	0	32	0
N.S.	1	1.11	0.51	5.93	0.00	1.71	0.00	0.00	0.20	0.00
time (sec)	N/A	0.753	0.381	100.343	0.000	0.097	0.000	0.000	0.164	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	148	70	243	0	268	0	0	32	0
N.S.	1	1.17	0.55	1.91	0.00	2.11	0.00	0.00	0.25	0.00
time (sec)	N/A	0.605	0.207	103.651	0.000	0.092	0.000	0.000	0.163	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	118	68	542	0	295	0	0	30	0
N.S.	1	1.13	0.65	5.21	0.00	2.84	0.00	0.00	0.29	0.00
time (sec)	N/A	0.494	0.172	9.771	0.000	0.089	0.000	0.000	0.175	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	192	0	53	0	0	21	95
N.S.	1	1.00	0.96	6.86	0.00	1.89	0.00	0.00	0.75	3.39
time (sec)	N/A	0.200	0.123	7.563	0.000	0.079	0.000	0.000	0.178	20.607

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	35	482	0	76	0	0	32	93
N.S.	1	1.00	0.64	8.76	0.00	1.38	0.00	0.00	0.58	1.69
time (sec)	N/A	0.291	0.154	10.217	0.000	0.081	0.000	0.000	0.155	24.107

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	86	55	222	0	104	0	0	32	302
N.S.	1	1.06	0.68	2.74	0.00	1.28	0.00	0.00	0.40	3.73
time (sec)	N/A	0.483	0.185	72.240	0.000	0.083	0.000	0.000	0.154	24.241

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	117	62	560	0	131	0	0	32	383
N.S.	1	1.09	0.58	5.23	0.00	1.22	0.00	0.00	0.30	3.58
time (sec)	N/A	0.604	0.191	197.367	0.000	0.086	0.000	0.000	0.153	25.696

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	0	0	0	0	0	22	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.339	0.347	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	278	0	0	0	0	0	20	0
N.S.	1	1.00	3.39	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.304	0.905	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	254	0	0	0	0	0	20	0
N.S.	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.307	0.840	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	61	0	0	0	0	0	22	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.325	0.341	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	1257	0	0	0	0	0	22	0
N.S.	1	1.00	14.79	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.329	12.578	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	54	47	93	91	46	270	46	182	46
N.S.	1	0.89	0.77	1.52	1.49	0.75	4.43	0.75	2.98	0.75
time (sec)	N/A	0.282	0.302	25.362	0.041	0.086	25.624	0.143	0.162	0.069

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	54	47	81	80	73	233	80	153	45
N.S.	1	0.89	0.77	1.33	1.31	1.20	3.82	1.31	2.51	0.74
time (sec)	N/A	0.283	0.208	14.013	0.041	0.082	11.118	0.135	0.180	0.064

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	37	71	69	36	199	36	135	36
N.S.	1	0.91	0.80	1.54	1.50	0.78	4.33	0.78	2.93	0.78
time (sec)	N/A	0.277	0.168	7.852	0.036	0.078	4.789	0.133	0.164	0.093

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	37	69	58	53	162	58	106	36
N.S.	1	0.91	0.80	1.50	1.26	1.15	3.52	1.26	2.30	0.78
time (sec)	N/A	0.276	0.083	4.131	0.036	0.080	2.040	0.143	0.167	20.228

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	27	55	47	26	128	26	88	26
N.S.	1	0.97	0.87	1.77	1.52	0.84	4.13	0.84	2.84	0.84
time (sec)	N/A	0.268	0.050	3.003	0.034	0.077	0.809	0.304	0.162	0.053

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	27	37	36	33	90	36	59	26
N.S.	1	0.97	0.87	1.19	1.16	1.06	2.90	1.16	1.90	0.84
time (sec)	N/A	0.273	0.040	1.483	0.038	0.074	0.371	0.151	0.158	0.050

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	15	27	26	13	53	13	40	43
N.S.	1	1.00	0.50	0.90	0.87	0.43	1.77	0.43	1.33	1.43
time (sec)	N/A	0.186	0.004	0.541	0.034	0.070	0.166	0.116	0.168	19.663

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	22	84	30	0	28	18	12
N.S.	1	1.00	1.00	1.57	6.00	2.14	0.00	2.00	1.29	0.86
time (sec)	N/A	0.216	0.003	0.240	0.039	0.074	0.000	0.114	0.188	19.138

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	23	29	31	233	50	0	38	20	26
N.S.	1	0.82	1.04	1.11	8.32	1.79	0.00	1.36	0.71	0.93
time (sec)	N/A	0.252	0.017	0.536	0.147	0.077	0.000	0.154	0.147	19.324

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	54	143	53	974	96	0	63	239	49
N.S.	1	1.15	3.04	1.13	20.72	2.04	0.00	1.34	5.09	1.04
time (sec)	N/A	0.263	0.281	1.186	0.051	0.076	0.000	0.202	0.152	0.070

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	66	31	69	1780	130	0	72	302	61
N.S.	1	1.06	0.50	1.11	28.71	2.10	0.00	1.16	4.87	0.98
time (sec)	N/A	0.290	0.014	2.650	0.200	0.079	0.000	0.128	0.160	0.091

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	97	268	89	3846	148	0	85	368	78
N.S.	1	1.17	3.23	1.07	46.34	1.78	0.00	1.02	4.43	0.94
time (sec)	N/A	0.307	0.457	5.776	0.145	0.081	0.000	0.119	0.181	19.213

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	42	68	74	72	36	597	36	250	35
N.S.	1	0.95	1.55	1.68	1.64	0.82	13.57	0.82	5.68	0.80
time (sec)	N/A	0.291	0.266	13.348	0.037	0.077	11.772	0.143	0.164	19.353

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	86	62	66	65	66	434	68	227	109
N.S.	1	1.13	0.82	0.87	0.86	0.87	5.71	0.89	2.99	1.43
time (sec)	N/A	0.382	0.092	7.012	0.036	0.080	5.158	0.155	0.173	20.839

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	30	48	52	50	26	362	26	173	26
N.S.	1	1.07	1.71	1.86	1.79	0.93	12.93	0.93	6.18	0.93
time (sec)	N/A	0.280	0.056	3.915	0.034	0.074	2.199	0.325	0.161	19.355

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	54	40	44	43	47	231	46	150	43
N.S.	1	1.10	0.82	0.90	0.88	0.96	4.71	0.94	3.06	0.88
time (sec)	N/A	0.313	0.047	2.594	0.034	0.075	0.815	0.163	0.150	19.700

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	26	13	133	13	96	13
N.S.	1	1.00	1.00	2.00	1.73	0.87	8.87	0.87	6.40	0.87
time (sec)	N/A	0.237	0.003	0.959	0.032	0.071	0.375	0.113	0.177	0.048

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	16	14	13	82	14	0	13	20	14
N.S.	1	1.14	1.00	0.93	5.86	1.00	0.00	0.93	1.43	1.00
time (sec)	N/A	0.234	0.004	0.457	0.057	0.076	0.000	0.123	0.158	19.136

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	53	19	0	13	22	11
N.S.	1	1.00	1.00	0.92	4.08	1.46	0.00	1.00	1.69	0.85
time (sec)	N/A	0.237	0.007	1.113	0.034	0.066	0.000	0.180	0.162	19.162

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	27	43	24	656	65	0	41	89	36
N.S.	1	0.90	1.43	0.80	21.87	2.17	0.00	1.37	2.97	1.20
time (sec)	N/A	0.264	0.016	1.023	0.056	0.081	0.000	0.196	0.162	0.077

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	34	48	46	308	54	0	35	22	37
N.S.	1	0.81	1.14	1.10	7.33	1.29	0.00	0.83	0.52	0.88
time (sec)	N/A	0.292	0.209	5.729	0.052	0.067	0.000	0.139	0.191	19.986

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	47	54	62	3188	138	0	74	167	82
N.S.	1	0.78	0.90	1.03	53.13	2.30	0.00	1.23	2.78	1.37
time (sec)	N/A	0.292	0.239	3.284	0.110	0.081	0.000	0.143	0.160	20.276

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	37	97	80	36	447	36	346	36
N.S.	1	0.91	0.80	2.11	1.74	0.78	9.72	0.78	7.52	0.78
time (sec)	N/A	0.285	0.266	23.766	0.042	0.085	25.343	0.172	0.177	19.425

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	54	47	70	69	63	366	69	281	45
N.S.	1	0.89	0.77	1.15	1.13	1.03	6.00	1.13	4.61	0.74
time (sec)	N/A	0.291	0.140	12.490	0.046	0.078	11.163	0.151	0.172	0.064

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	27	49	47	26	284	26	242	26
N.S.	1	0.97	0.87	1.58	1.52	0.84	9.16	0.84	7.81	0.84
time (sec)	N/A	0.279	0.065	5.965	0.039	0.075	4.849	0.373	0.170	0.055

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	37	55	47	43	202	47	177	36
N.S.	1	0.91	0.80	1.20	1.02	0.93	4.39	1.02	3.85	0.78
time (sec)	N/A	0.292	0.038	2.993	0.038	0.074	2.077	0.189	0.181	0.062

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	34	13	117	13	138	13
N.S.	1	1.00	1.00	2.73	2.27	0.87	7.80	0.87	9.20	0.87
time (sec)	N/A	0.247	0.006	1.808	0.038	0.071	0.801	0.131	0.154	20.545

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	23	46	29	92	38	0	36	20	22
N.S.	1	0.82	1.64	1.04	3.29	1.36	0.00	1.29	0.71	0.79
time (sec)	N/A	0.255	0.010	1.051	0.041	0.081	0.000	0.145	0.155	0.059

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	84	13	0	13	22	13
N.S.	1	1.00	1.00	1.08	6.46	1.00	0.00	1.00	1.69	1.00
time (sec)	N/A	0.249	0.006	2.393	0.045	0.067	0.000	0.165	0.159	0.031

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	79	39	558	72	0	48	22	36
N.S.	1	1.12	2.32	1.15	16.41	2.12	0.00	1.41	0.65	1.06
time (sec)	N/A	0.303	0.008	5.500	0.045	0.076	0.000	0.214	0.189	0.071

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	35	31	41	834	94	0	52	22	38
N.S.	1	0.81	0.72	0.95	19.40	2.19	0.00	1.21	0.51	0.88
time (sec)	N/A	0.283	0.015	10.947	0.167	0.083	0.000	0.169	0.160	0.034

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	85	195	71	2237	132	0	73	22	66
N.S.	1	1.25	2.87	1.04	32.90	1.94	0.00	1.07	0.32	0.97
time (sec)	N/A	0.302	0.527	20.965	0.098	0.080	0.000	0.137	0.162	19.973

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	151	98	0	0	290	0	0	30	0
N.S.	1	1.11	0.72	0.00	0.00	2.13	0.00	0.00	0.22	0.00
time (sec)	N/A	0.543	0.343	180.000	0.000	0.097	0.000	0.000	0.162	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	110	120	86	138159568	0	281	0	0	28	0
N.S.	1	1.09	0.78	1255996.07	0.00	2.55	0.00	0.00	0.25	0.00
time (sec)	N/A	0.401	0.196	44.791	0.000	0.094	0.000	0.000	0.181	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	89	70	24398096	0	266	0	0	19	0
N.S.	1	1.06	0.83	290453.52	0.00	3.17	0.00	0.00	0.23	0.00
time (sec)	N/A	0.300	0.120	6.143	0.000	0.094	0.000	0.000	0.168	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	52	18464991	0	242	0	0	30	0
N.S.	1	1.00	0.90	318361.91	0.00	4.17	0.00	0.00	0.52	0.00
time (sec)	N/A	0.216	0.090	2.068	0.000	0.087	0.000	0.000	0.162	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupa
grade	N/A	A	A	B	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBI
size	24	24	23	57930080	0	39	0	2029	30	24
N.S.	1	1.00	0.96	2413753.33	0.00	1.62	0.00	84.54	1.25	1.00
time (sec)	N/A	0.195	0.014	7.040	0.000	0.082	0.000	13.306	0.157	19.91

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	194	0	74	0	7875	30	104
N.S.	1	1.00	0.81	3.66	0.00	1.40	0.00	148.58	0.57	1.96
time (sec)	N/A	0.289	0.148	71.531	0.000	0.077	0.000	46.024	0.192	22.772

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	84	52	481	0	103	0	18022	30	136
N.S.	1	1.06	0.66	6.09	0.00	1.30	0.00	228.13	0.38	1.72
time (sec)	N/A	0.397	0.240	82.128	0.000	0.089	0.000	161.206	0.145	23.082

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	115	67	0	0	118	0	0	30	350
N.S.	1	1.10	0.64	0.00	0.00	1.12	0.00	0.00	0.29	3.33
time (sec)	N/A	0.533	0.320	0.000	0.000	0.088	0.000	0.000	0.147	23.219

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	108	96	0	0	0	0	0	32	0
N.S.	1	1.10	0.98	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.438	0.387	180.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	66	185010647	0	0	0	0	32	0
N.S.	1	1.07	0.96	2681313.72	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.340	0.231	153.257	0.000	0.000	0.000	0.000	0.191	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	76	91497799	0	0	0	0	30	0
N.S.	1	1.07	1.10	1326055.06	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.335	0.307	22.220	0.000	0.000	0.000	0.000	0.143	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	24543726	0	0	0	0	21	0
N.S.	1	1.00	0.85	613593.15	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.266	0.130	4.773	0.000	0.000	0.000	0.000	0.139	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	54	12988217	0	0	0	0	32	0
N.S.	1	1.00	1.35	324705.42	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.263	0.467	2.525	0.000	0.000	0.000	0.000	0.156	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	20591474	0	156	0	0	32	0
N.S.	1	1.00	0.85	447640.74	0.00	3.39	0.00	0.00	0.70	0.00
time (sec)	N/A	0.272	0.218	2.449	0.000	0.090	0.000	0.000	0.156	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	62	123	0	103	0	0	32	0
N.S.	1	1.00	1.29	2.56	0.00	2.15	0.00	0.00	0.67	0.00
time (sec)	N/A	0.268	0.529	40.874	0.000	0.087	0.000	0.000	0.163	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	78	64	227	0	266	0	0	32	0
N.S.	1	1.01	0.83	2.95	0.00	3.45	0.00	0.00	0.42	0.00
time (sec)	N/A	0.363	0.686	134.033	0.000	0.096	0.000	0.000	0.158	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	136	151	99	375719658	0	291	0	0	30	0
N.S.	1	1.11	0.73	2762644.54	0.00	2.14	0.00	0.00	0.22	0.00
time (sec)	N/A	0.554	0.350	202.213	0.000	0.096	0.000	0.000	0.163	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	110	120	84	114426295	0	280	0	0	21	0
N.S.	1	1.09	0.76	1040239.04	0.00	2.55	0.00	0.00	0.19	0.00
time (sec)	N/A	0.431	0.211	39.623	0.000	0.095	0.000	0.000	0.170	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	89	73	74647250	0	268	0	0	32	0
N.S.	1	1.06	0.87	888657.74	0.00	3.19	0.00	0.00	0.38	0.00
time (sec)	N/A	0.323	0.188	19.345	0.000	0.091	0.000	0.000	0.157	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	82	87	70	69948460	0	295	0	0	32	0
N.S.	1	1.06	0.85	853030.00	0.00	3.60	0.00	0.00	0.39	0.00
time (sec)	N/A	0.414	0.171	8.301	0.000	0.090	0.000	0.000	0.140	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	192	0	53	0	15292	32	94
N.S.	1	1.00	0.96	6.86	0.00	1.89	0.00	546.14	1.14	3.36
time (sec)	N/A	0.210	0.224	19.984	0.000	0.079	0.000	91.672	0.136	21.923

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	35	482	0	76	0	0	32	93
N.S.	1	1.00	0.64	8.76	0.00	1.38	0.00	0.00	0.58	1.69
time (sec)	N/A	0.304	0.345	169.191	0.000	0.082	0.000	0.000	0.136	23.981

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	86	55	0	0	104	0	0	32	302
N.S.	1	1.06	0.68	0.00	0.00	1.28	0.00	0.00	0.40	3.73
time (sec)	N/A	0.405	0.477	0.000	0.000	0.082	0.000	0.000	0.162	23.685

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	117	62	0	0	131	0	0	32	383
N.S.	1	1.09	0.58	0.00	0.00	1.22	0.00	0.00	0.30	3.58
time (sec)	N/A	0.544	0.793	0.000	0.000	0.090	0.000	0.000	0.137	24.679

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	98	0	137	0	0	16	0
N.S.	1	1.00	0.94	3.16	0.00	4.42	0.00	0.00	0.52	0.00
time (sec)	N/A	0.191	0.025	0.342	0.000	0.082	0.000	0.000	0.137	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	33	25	99	0	137	0	0	10	0
N.S.	1	1.32	1.00	3.96	0.00	5.48	0.00	0.00	0.40	0.00
time (sec)	N/A	0.252	0.021	0.295	0.000	0.081	0.000	0.000	0.134	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	1801	0	0	0	0	0	22	0
N.S.	1	1.00	21.19	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.353	10.830	0.000	0.000	0.000	0.000	0.000	0.137	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	63	0	0	0	0	0	22	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.336	0.308	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	567	0	0	0	0	0	20	0
N.S.	1	1.00	6.83	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.325	3.316	0.000	0.000	0.000	0.000	0.000	0.137	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	37	60	58	36	318	36	343	36
N.S.	1	0.91	0.80	1.30	1.26	0.78	6.91	0.78	7.46	0.78
time (sec)	N/A	0.319	0.109	10.093	0.047	0.076	11.167	0.144	0.143	0.081

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	177	23	127	125	116	0	66	234	144
N.S.	1	7.70	1.00	5.52	5.43	5.04	0.00	2.87	10.17	6.26
time (sec)	N/A	0.343	0.076	106.256	0.046	0.121	0.000	0.301	0.147	20.027

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	177	23	127	125	116	0	66	234	144
N.S.	1	7.70	1.00	5.52	5.43	5.04	0.00	2.87	10.17	6.26
time (sec)	N/A	0.357	0.130	88.504	0.050	0.122	0.000	0.272	0.151	0.002

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	177	23	127	125	116	0	66	234	144
N.S.	1	7.70	1.00	5.52	5.43	5.04	0.00	2.87	10.17	6.26
time (sec)	N/A	0.346	0.130	92.561	0.048	0.121	0.000	0.273	0.179	0.002

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	172	35	964	0	77	0	0	31	41
N.S.	1	5.06	1.03	28.35	0.00	2.26	0.00	0.00	0.91	1.21
time (sec)	N/A	0.707	0.019	1.114	0.000	0.075	0.000	0.000	0.154	19.936

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	220	35	964	0	77	0	0	31	41
N.S.	1	6.47	1.03	28.35	0.00	2.26	0.00	0.00	0.91	1.21
time (sec)	N/A	0.947	0.003	0.707	0.000	0.078	0.000	0.000	0.174	19.627

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	177	23	128	125	120	0	125	235	127
N.S.	1	7.70	1.00	5.57	5.43	5.22	0.00	5.43	10.22	5.52
time (sec)	N/A	0.357	0.073	103.742	0.046	0.126	0.000	0.633	0.156	20.581

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	177	23	128	125	120	0	125	235	127
N.S.	1	7.70	1.00	5.57	5.43	5.22	0.00	5.43	10.22	5.52
time (sec)	N/A	0.354	0.130	86.636	0.042	0.126	0.000	0.598	0.160	0.002

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	177	23	128	125	120	0	125	235	127
N.S.	1	7.70	1.00	5.57	5.43	5.22	0.00	5.43	10.22	5.52
time (sec)	N/A	0.350	0.130	78.020	0.049	0.125	0.000	0.599	0.187	0.002

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	176	35	967	0	78	0	0	31	41
N.S.	1	5.18	1.03	28.44	0.00	2.29	0.00	0.00	0.91	1.21
time (sec)	N/A	0.671	0.018	1.074	0.000	0.078	0.000	0.000	0.171	18.918

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	224	35	967	0	78	0	0	31	41
N.S.	1	6.59	1.03	28.44	0.00	2.29	0.00	0.00	0.91	1.21
time (sec)	N/A	0.948	0.005	0.987	0.000	0.077	0.000	0.000	0.168	18.471

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	129	125	66	0	66	236	78
N.S.	1	1.00	1.00	5.61	5.43	2.87	0.00	2.87	10.26	3.39
time (sec)	N/A	0.202	0.065	105.510	0.044	0.130	0.000	0.532	0.170	18.672

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	129	125	66	0	66	236	78
N.S.	1	1.00	1.00	5.61	5.43	2.87	0.00	2.87	10.26	3.39
time (sec)	N/A	0.208	0.130	81.971	0.040	0.129	0.000	0.569	0.206	0.002

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	129	125	66	0	66	236	78
N.S.	1	1.00	1.00	5.61	5.43	2.87	0.00	2.87	10.26	3.39
time (sec)	N/A	0.203	0.126	73.385	0.040	0.131	0.000	0.529	0.177	0.002

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	35	152	36	792	0	76	0	0	31	39
N.S.	1	4.34	1.03	22.63	0.00	2.17	0.00	0.00	0.89	1.11
time (sec)	N/A	0.644	0.016	1.898	0.000	0.082	0.000	0.000	0.180	18.866

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	35	198	36	792	0	76	0	0	31	39
N.S.	1	5.66	1.03	22.63	0.00	2.17	0.00	0.00	0.89	1.11
time (sec)	N/A	0.838	0.003	1.417	0.000	0.079	0.000	0.000	0.177	20.262

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	177	23	126	125	72	0	125	235	102
N.S.	1	7.70	1.00	5.48	5.43	3.13	0.00	5.43	10.22	4.43
time (sec)	N/A	0.351	0.066	105.716	0.047	0.134	0.000	149.323	0.207	19.798

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	177	23	126	125	72	0	125	235	102
N.S.	1	7.70	1.00	5.48	5.43	3.13	0.00	5.43	10.22	4.43
time (sec)	N/A	0.346	0.127	89.672	0.045	0.137	0.000	149.376	0.178	0.003

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	177	23	126	125	72	0	125	235	102
N.S.	1	7.70	1.00	5.48	5.43	3.13	0.00	5.43	10.22	4.43
time (sec)	N/A	0.347	0.129	73.946	0.056	0.141	0.000	148.369	0.178	0.003

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	156	35	795	0	76	0	0	31	39
N.S.	1	4.59	1.03	23.38	0.00	2.24	0.00	0.00	0.91	1.15
time (sec)	N/A	0.576	0.016	3.150	0.000	0.080	0.000	0.000	0.178	19.142

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	202	35	795	0	76	0	0	31	39
N.S.	1	5.94	1.03	23.38	0.00	2.24	0.00	0.00	0.91	1.15
time (sec)	N/A	0.824	0.003	3.289	0.000	0.077	0.000	0.000	0.211	19.517

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [23] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	9	0.222
2	A	2	2	1.00	7	0.286
3	A	2	2	1.00	7	0.286
4	A	2	2	1.00	7	0.286
5	A	2	2	1.00	7	0.286
6	A	4	4	1.00	7	0.571
7	A	4	3	0.33	7	0.429
8	A	5	4	1.00	7	0.571
9	A	5	4	1.09	7	0.571
10	A	6	5	1.06	7	0.714
11	A	5	4	2.00	7	0.571
12	A	4	4	1.00	9	0.444
13	A	2	2	1.00	9	0.222
14	A	2	2	1.00	7	0.286
15	A	2	2	1.00	7	0.286
16	A	2	2	1.00	7	0.286
17	A	2	2	1.00	7	0.286
18	A	2	2	1.00	7	0.286
19	A	2	2	1.23	9	0.222
20	A	2	2	1.24	7	0.286
21	A	2	2	1.26	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
22	A	6	5	1.35	7	0.714
23	B	8	7	2.38	7	1.000
24	A	8	7	1.04	7	1.000
25	A	5	4	1.30	7	0.571
26	A	6	5	1.04	7	0.714
27	A	6	5	1.40	7	0.714
28	A	5	4	1.00	7	0.571
29	A	6	5	1.07	7	0.714
30	A	5	4	1.01	7	0.571
31	A	6	5	1.11	7	0.714
32	A	4	3	1.00	7	0.429
33	A	8	7	1.29	7	1.000
34	A	5	4	1.00	7	0.571
35	A	6	5	1.13	7	0.714
36	A	5	4	1.00	7	0.571
37	A	2	2	1.00	9	0.222
38	A	2	2	1.00	7	0.286
39	A	2	2	1.00	7	0.286
40	A	2	2	1.21	7	0.286
41	A	2	2	1.00	7	0.286
42	A	2	2	1.00	7	0.286
43	A	2	2	1.00	7	0.286
44	A	2	2	1.00	7	0.286
45	A	2	2	1.00	7	0.286
46	A	2	2	1.00	7	0.286
47	A	6	5	1.35	7	0.714
48	A	5	4	1.00	7	0.571
49	A	8	7	1.04	7	1.000
50	A	5	4	1.02	7	0.571
51	A	6	5	1.04	7	0.714
52	A	6	5	1.40	7	0.714
53	B	8	7	2.75	7	1.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	6	5	1.07	7	0.714
55	A	5	4	1.31	7	0.571
56	A	6	5	1.11	7	0.714
57	A	4	3	1.00	7	0.429
58	A	4	3	0.34	7	0.429
59	A	5	4	1.00	7	0.571
60	A	5	4	1.22	7	0.571
61	A	5	4	1.00	7	0.571
62	A	5	4	1.00	7	0.571
63	A	5	4	1.29	9	0.444
64	A	4	4	1.00	7	0.571
65	A	7	6	1.29	7	0.857
66	A	5	4	1.00	7	0.571
67	A	6	5	1.13	7	0.714
68	A	6	5	1.03	7	0.714
69	A	3	3	1.00	9	0.333
70	A	3	3	1.00	11	0.273
71	A	3	3	1.00	11	0.273
72	A	3	3	1.00	9	0.333
73	A	3	3	1.00	9	0.333
74	A	3	3	1.00	11	0.273
75	A	3	3	1.00	13	0.231
76	A	2	2	1.00	13	0.154
77	A	2	2	1.00	14	0.143
78	A	2	2	1.00	15	0.133
79	A	2	2	1.00	15	0.133
80	A	2	2	1.00	13	0.154
81	A	5	5	1.08	13	0.385
82	A	7	6	1.00	15	0.400
83	A	8	7	1.00	15	0.467
84	A	9	8	0.94	15	0.533
85	A	8	7	0.87	15	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	11	10	0.95	15	0.667
87	A	2	2	1.00	17	0.118
88	A	2	2	1.00	17	0.118
89	A	2	2	1.00	15	0.133
90	F	0	0	N/A	0.000	N/A
91	F	0	0	N/A	0.000	N/A
92	F	0	0	N/A	0.000	N/A
93	F	0	0	N/A	0.000	N/A
94	A	2	2	1.00	17	0.118
95	A	2	2	1.00	17	0.118
96	A	2	2	1.00	15	0.133
97	F	0	0	N/A	0.000	N/A
98	F	0	0	N/A	0.000	N/A
99	F	0	0	N/A	0.000	N/A
100	F	0	0	N/A	0.000	N/A
101	A	4	4	1.11	13	0.308
102	F	0	0	N/A	0.000	N/A
103	F	0	0	N/A	0.000	N/A
104	F	0	0	N/A	0.000	N/A
105	F	0	0	N/A	0.000	N/A
106	F	0	0	N/A	0.000	N/A
107	F	0	0	N/A	0.000	N/A
108	F	0	0	N/A	0.000	N/A
109	F	0	0	N/A	0.000	N/A
110	A	2	2	1.00	15	0.133
111	A	2	2	1.00	15	0.133
112	A	2	2	1.00	13	0.154
113	F	0	0	N/A	0.000	N/A
114	F	0	0	N/A	0.000	N/A
115	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
116	A	2	2	1.00	17	0.118
117	A	2	2	1.00	17	0.118
118	A	2	2	1.00	15	0.133
119	F	0	0	N/A	0.000	N/A
120	F	0	0	N/A	0.000	N/A
121	F	0	0	N/A	0.000	N/A
122	F	0	0	N/A	0.000	N/A
123	A	2	2	1.00	17	0.118
124	A	2	2	1.00	17	0.118
125	A	2	2	1.00	15	0.133
126	F	0	0	N/A	0.000	N/A
127	F	0	0	N/A	0.000	N/A
128	F	0	0	N/A	0.000	N/A
129	F	0	0	N/A	0.000	N/A
130	N/A	1	0	1.00	13	0.000
131	N/A	1	0	1.00	15	0.000
132	N/A	1	0	1.00	15	0.000
133	N/A	1	0	1.00	17	0.000
134	N/A	1	0	1.00	17	0.000
135	N/A	1	0	1.00	17	0.000
136	A	2	2	1.24	19	0.105
137	A	2	2	1.71	19	0.105
138	A	2	2	1.28	17	0.118
139	N/A	1	0	1.00	17	0.000
140	N/A	1	0	1.00	19	0.000
141	A	2	2	1.23	17	0.118
142	A	2	2	1.26	17	0.118
143	A	2	2	1.25	15	0.133
144	N/A	1	0	1.00	15	0.000
145	N/A	1	0	1.00	17	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
146	A	2	2	1.00	15	0.133
147	A	2	2	1.00	15	0.133
148	A	2	2	1.00	13	0.154
149	A	4	4	1.00	13	0.308
150	A	6	5	1.00	15	0.333
151	A	7	6	1.00	15	0.400
152	A	8	7	0.94	15	0.467
153	A	7	6	0.92	15	0.400
154	A	10	9	0.95	15	0.600
155	A	2	2	1.00	17	0.118
156	A	2	2	1.00	17	0.118
157	A	2	2	1.00	15	0.133
158	F	0	0	N/A	0.000	N/A
159	F	0	0	N/A	0.000	N/A
160	F	0	0	N/A	0.000	N/A
161	A	2	2	1.00	17	0.118
162	A	2	2	1.00	17	0.118
163	A	2	2	1.00	15	0.133
164	F	0	0	N/A	0.000	N/A
165	F	0	0	N/A	0.000	N/A
166	F	0	0	N/A	0.000	N/A
167	F	0	0	N/A	0.000	N/A
168	F	0	0	N/A	0.000	N/A
169	A	5	5	1.07	13	0.385
170	F	0	0	N/A	0.000	N/A
171	F	0	0	N/A	0.000	N/A
172	F	0	0	N/A	0.000	N/A
173	F	0	0	N/A	0.000	N/A
174	F	0	0	N/A	0.000	N/A
175	A	7	6	1.00	15	0.400
176	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
177	F	0	0	N/A	0.000	N/A
178	F	0	0	N/A	0.000	N/A
179	F	0	0	N/A	0.000	N/A
180	F	0	0	N/A	0.000	N/A
181	A	8	7	1.00	15	0.467
182	F	0	0	N/A	0.000	N/A
183	F	0	0	N/A	0.000	N/A
184	F	0	0	N/A	0.000	N/A
185	A	2	2	1.00	15	0.133
186	A	2	2	1.00	15	0.133
187	A	2	2	1.00	13	0.154
188	F	0	0	N/A	0.000	N/A
189	F	0	0	N/A	0.000	N/A
190	F	0	0	N/A	0.000	N/A
191	A	2	2	1.00	17	0.118
192	A	2	2	1.00	17	0.118
193	A	2	2	1.00	15	0.133
194	F	0	0	N/A	0.000	N/A
195	F	0	0	N/A	0.000	N/A
196	F	0	0	N/A	0.000	N/A
197	A	2	2	1.00	17	0.118
198	A	2	2	1.00	17	0.118
199	A	2	2	1.00	15	0.133
200	F	0	0	N/A	0.000	N/A
201	F	0	0	N/A	0.000	N/A
202	F	0	0	N/A	0.000	N/A
203	F	0	0	N/A	0.000	N/A
204	F	0	0	N/A	0.000	N/A
205	F	0	0	N/A	0.000	N/A
206	N/A	1	0	1.00	13	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
207	N/A	1	0	1.00	15	0.000
208	N/A	1	0	1.00	15	0.000
209	F	0	0	N/A	0.000	N/A
210	F	0	0	N/A	0.000	N/A
211	F	0	0	N/A	0.000	N/A
212	N/A	1	0	1.00	15	0.000
213	N/A	1	0	1.00	17	0.000
214	N/A	1	0	1.00	17	0.000
215	F	0	0	N/A	0.000	N/A
216	F	0	0	N/A	0.000	N/A
217	F	0	0	N/A	0.000	N/A
218	N/A	1	0	1.00	15	0.000
219	N/A	1	0	1.00	17	0.000
220	N/A	1	0	1.00	17	0.000
221	A	2	2	1.17	19	0.105
222	B	2	2	2.50	19	0.105
223	A	2	2	1.21	17	0.118
224	N/A	1	0	1.00	17	0.000
225	N/A	1	0	1.00	19	0.000
226	A	2	2	1.16	17	0.118
227	A	2	2	1.19	17	0.118
228	A	2	2	1.18	15	0.133
229	N/A	1	0	1.00	15	0.000
230	N/A	1	0	1.00	17	0.000
231	A	14	13	1.19	15	0.867
232	A	9	8	1.00	15	0.533
233	A	4	4	1.00	13	0.308
234	A	4	4	1.00	13	0.308
235	A	10	9	1.00	15	0.600
236	A	15	14	1.18	15	0.933

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
237	F	0	0	N/A	0.000	N/A
238	F	0	0	N/A	0.000	N/A
239	F	0	0	N/A	0.000	N/A
240	F	0	0	N/A	0.000	N/A
241	F	0	0	N/A	0.000	N/A
242	F	0	0	N/A	0.000	N/A
243	F	0	0	N/A	0.000	N/A
244	F	0	0	N/A	0.000	N/A
245	F	0	0	N/A	0.000	N/A
246	F	0	0	N/A	0.000	N/A
247	F	0	0	N/A	0.000	N/A
248	F	0	0	N/A	0.000	N/A
249	F	0	0	N/A	0.000	N/A
250	F	0	0	N/A	0.000	N/A
251	F	0	0	N/A	0.000	N/A
252	F	0	0	N/A	0.000	N/A
253	F	0	0	N/A	0.000	N/A
254	F	0	0	N/A	0.000	N/A
255	F	0	0	N/A	0.000	N/A
256	A	2	2	1.23	13	0.154
257	A	2	2	1.24	13	0.154
258	F	0	0	N/A	0.000	N/A
259	F	0	0	N/A	0.000	N/A
260	F	0	0	N/A	0.000	N/A
261	A	4	4	1.00	13	0.308
262	A	6	5	1.00	15	0.333
263	A	7	6	1.00	15	0.400
264	A	8	7	0.94	15	0.467
265	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
266	F	0	0	N/A	0.000	N/A
267	F	0	0	N/A	0.000	N/A
268	F	0	0	N/A	0.000	N/A
269	F	0	0	N/A	0.000	N/A
270	F	0	0	N/A	0.000	N/A
271	F	0	0	N/A	0.000	N/A
272	F	0	0	N/A	0.000	N/A
273	F	0	0	N/A	0.000	N/A
274	F	0	0	N/A	0.000	N/A
275	F	0	0	N/A	0.000	N/A
276	F	0	0	N/A	0.000	N/A
277	A	5	5	1.08	13	0.385
278	A	7	6	1.00	15	0.400
279	A	8	7	1.00	15	0.467
280	A	9	8	0.94	15	0.533
281	F	0	0	N/A	0.000	N/A
282	F	0	0	N/A	0.000	N/A
283	F	0	0	N/A	0.000	N/A
284	F	0	0	N/A	0.000	N/A
285	F	0	0	N/A	0.000	N/A
286	F	0	0	N/A	0.000	N/A
287	F	0	0	N/A	0.000	N/A
288	F	0	0	N/A	0.000	N/A
289	F	0	0	N/A	0.000	N/A
290	F	0	0	N/A	0.000	N/A
291	F	0	0	N/A	0.000	N/A
292	F	0	0	N/A	0.000	N/A
293	A	2	2	1.00	13	0.154
294	A	2	2	1.00	14	0.143
295	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
296	A	2	2	1.00	15	0.133
297	A	2	2	1.00	13	0.154
298	A	4	4	1.00	13	0.308
299	A	6	5	1.00	15	0.333
300	A	7	6	1.00	15	0.400
301	A	8	7	0.94	15	0.467
302	A	7	6	0.92	15	0.400
303	A	10	9	0.95	15	0.600
304	A	2	2	1.00	17	0.118
305	A	2	2	1.00	17	0.118
306	A	2	2	1.00	15	0.133
307	F	0	0	N/A	0.000	N/A
308	F	0	0	N/A	0.000	N/A
309	F	0	0	N/A	0.000	N/A
310	F	0	0	N/A	0.000	N/A
311	A	2	2	1.00	17	0.118
312	A	2	2	1.00	17	0.118
313	A	2	2	1.00	15	0.133
314	F	0	0	N/A	0.000	N/A
315	F	0	0	N/A	0.000	N/A
316	F	0	0	N/A	0.000	N/A
317	F	0	0	N/A	0.000	N/A
318	A	3	3	1.00	13	0.231
319	F	0	0	N/A	0.000	N/A
320	F	0	0	N/A	0.000	N/A
321	F	0	0	N/A	0.000	N/A
322	F	0	0	N/A	0.000	N/A
323	F	0	0	N/A	0.000	N/A
324	F	0	0	N/A	0.000	N/A
325	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
326	F	0	0	N/A	0.000	N/A
327	A	2	2	1.00	15	0.133
328	A	2	2	1.00	15	0.133
329	A	2	2	1.00	13	0.154
330	F	0	0	N/A	0.000	N/A
331	F	0	0	N/A	0.000	N/A
332	F	0	0	N/A	0.000	N/A
333	A	2	2	1.00	17	0.118
334	A	2	2	1.00	17	0.118
335	A	2	2	1.00	15	0.133
336	F	0	0	N/A	0.000	N/A
337	F	0	0	N/A	0.000	N/A
338	F	0	0	N/A	0.000	N/A
339	F	0	0	N/A	0.000	N/A
340	A	2	2	1.00	17	0.118
341	A	2	2	1.00	17	0.118
342	A	2	2	1.00	15	0.133
343	F	0	0	N/A	0.000	N/A
344	F	0	0	N/A	0.000	N/A
345	F	0	0	N/A	0.000	N/A
346	F	0	0	N/A	0.000	N/A
347	N/A	1	0	1.00	13	0.000
348	N/A	1	0	1.00	15	0.000
349	N/A	1	0	1.00	15	0.000
350	N/A	1	0	1.00	17	0.000
351	N/A	1	0	1.00	17	0.000
352	N/A	1	0	1.00	17	0.000
353	A	2	2	1.17	19	0.105
354	B	2	2	2.50	19	0.105
355	A	2	2	1.21	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
356	N/A	1	0	1.00	17	0.000
357	N/A	1	0	1.00	19	0.000
358	A	2	2	1.16	17	0.118
359	A	2	2	1.19	17	0.118
360	A	2	2	1.18	15	0.133
361	N/A	1	0	1.00	15	0.000
362	N/A	1	0	1.00	17	0.000
363	A	14	13	1.19	15	0.867
364	A	9	8	1.00	15	0.533
365	A	4	4	1.00	13	0.308
366	A	4	4	1.00	13	0.308
367	A	10	9	1.00	15	0.600
368	A	15	14	1.18	15	0.933
369	A	2	2	1.21	13	0.154
370	A	2	2	1.21	13	0.154
371	A	4	4	1.00	13	0.308
372	A	6	5	1.00	15	0.333
373	A	7	6	1.00	15	0.400
374	A	8	7	0.94	15	0.467
375	F	0	0	N/A	0.000	N/A
376	F	0	0	N/A	0.000	N/A
377	F	0	0	N/A	0.000	N/A
378	F	0	0	N/A	0.000	N/A
379	F	0	0	N/A	0.000	N/A
380	F	0	0	N/A	0.000	N/A
381	F	0	0	N/A	0.000	N/A
382	F	0	0	N/A	0.000	N/A
383	F	0	0	N/A	0.000	N/A
384	F	0	0	N/A	0.000	N/A
385	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
386	F	0	0	N/A	0.000	N/A
387	A	5	5	1.07	13	0.385
388	A	7	6	1.00	15	0.400
389	A	8	7	1.00	15	0.467
390	A	9	8	1.07	15	0.533
391	F	0	0	N/A	0.000	N/A
392	F	0	0	N/A	0.000	N/A
393	F	0	0	N/A	0.000	N/A
394	F	0	0	N/A	0.000	N/A
395	F	0	0	N/A	0.000	N/A
396	F	0	0	N/A	0.000	N/A
397	F	0	0	N/A	0.000	N/A
398	F	0	0	N/A	0.000	N/A
399	F	0	0	N/A	0.000	N/A
400	F	0	0	N/A	0.000	N/A
401	F	0	0	N/A	0.000	N/A
402	F	0	0	N/A	0.000	N/A
403	A	4	4	1.21	13	0.308
404	A	4	4	1.21	14	0.286
405	A	5	5	1.31	13	0.385
406	A	5	5	1.29	14	0.357
407	A	3	3	1.00	13	0.231
408	A	3	3	1.00	14	0.214
409	A	4	4	1.11	13	0.308
410	A	4	4	1.12	14	0.286
411	A	7	6	0.89	18	0.333
412	A	7	6	0.89	18	0.333
413	A	7	6	0.91	18	0.333
414	A	7	6	0.91	18	0.333
415	A	7	6	0.97	18	0.333
416	A	7	6	0.97	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
417	A	2	2	1.00	16	0.125
418	A	4	4	1.00	16	0.250
419	A	8	7	0.82	18	0.389
420	A	8	7	1.15	18	0.389
421	A	8	7	1.06	18	0.389
422	A	10	9	1.17	18	0.500
423	A	8	7	0.95	20	0.350
424	A	10	9	1.13	20	0.450
425	A	7	6	1.03	20	0.300
426	A	8	7	1.10	20	0.350
427	A	6	5	1.00	18	0.278
428	A	4	4	1.00	18	0.222
429	A	6	5	1.00	20	0.250
430	A	7	6	0.90	20	0.300
431	A	7	6	0.86	20	0.300
432	A	8	7	0.82	20	0.350
433	A	7	6	0.91	20	0.300
434	A	7	6	0.89	20	0.300
435	A	7	6	0.97	20	0.300
436	A	7	6	0.91	20	0.300
437	A	6	5	1.00	18	0.278
438	A	7	6	0.82	18	0.333
439	A	6	5	1.00	20	0.250
440	A	6	6	1.12	20	0.300
441	A	8	7	0.81	20	0.350
442	A	10	9	1.25	20	0.450
443	A	7	6	0.89	18	0.333
444	A	7	6	0.89	18	0.333
445	A	7	6	0.91	18	0.333
446	A	7	6	0.91	18	0.333
447	A	7	6	0.97	18	0.333
448	A	7	6	0.97	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
449	A	6	5	1.00	18	0.278
450	A	4	4	1.00	16	0.250
451	A	8	7	0.82	16	0.438
452	A	8	7	1.15	18	0.389
453	A	8	7	1.06	18	0.389
454	A	10	9	1.17	18	0.500
455	A	17	17	1.21	20	0.850
456	A	8	7	0.95	20	0.350
457	A	13	13	1.20	20	0.650
458	A	7	6	1.03	20	0.300
459	A	9	9	1.32	20	0.450
460	A	6	5	1.00	20	0.250
461	A	5	5	1.29	20	0.250
462	A	5	5	1.17	18	0.278
463	A	7	6	0.90	18	0.333
464	A	7	6	0.81	20	0.300
465	A	8	7	0.78	20	0.350
466	A	7	6	0.83	20	0.300
467	A	8	7	0.81	20	0.350
468	A	7	6	0.78	20	0.300
469	A	7	6	0.89	20	0.300
470	A	7	6	0.87	20	0.300
471	A	7	6	0.91	20	0.300
472	A	7	6	0.89	20	0.300
473	A	7	6	0.97	20	0.300
474	A	7	6	0.91	20	0.300
475	A	6	5	1.00	20	0.250
476	A	6	5	0.96	20	0.250
477	A	8	7	0.88	20	0.350
478	A	7	6	1.00	18	0.333
479	A	8	7	0.81	18	0.389
480	A	10	9	1.25	20	0.450

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	8	7	1.01	20	0.350
482	A	10	9	1.23	20	0.450
483	A	8	8	1.11	20	0.400
484	A	6	6	1.09	20	0.300
485	A	4	4	1.06	20	0.200
486	A	2	2	1.00	20	0.100
487	A	2	2	1.00	20	0.100
488	A	4	4	1.00	20	0.200
489	A	6	6	1.06	20	0.300
490	A	8	8	1.10	20	0.400
491	A	8	8	1.10	22	0.364
492	A	6	6	1.07	22	0.273
493	A	6	6	1.07	22	0.273
494	A	4	4	1.00	22	0.182
495	A	4	4	1.00	22	0.182
496	A	4	4	1.00	22	0.182
497	A	4	4	1.00	22	0.182
498	A	6	6	1.01	22	0.273
499	A	8	8	1.11	22	0.364
500	A	6	6	1.09	22	0.273
501	A	4	4	1.06	22	0.182
502	A	6	6	1.06	22	0.273
503	A	2	2	1.00	22	0.091
504	A	4	4	1.00	22	0.182
505	A	6	6	1.06	22	0.273
506	A	8	8	1.09	22	0.364
507	A	10	10	1.12	20	0.500
508	A	8	8	1.11	20	0.400
509	A	6	6	1.12	20	0.300
510	A	4	4	1.13	20	0.200
511	A	2	2	1.00	20	0.100
512	A	6	6	1.04	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
513	A	8	8	1.09	20	0.400
514	A	10	10	1.11	20	0.500
515	A	8	8	1.09	22	0.364
516	A	8	8	1.09	22	0.364
517	A	6	6	1.09	22	0.273
518	A	6	6	1.10	22	0.273
519	A	4	4	1.00	22	0.182
520	A	4	4	1.00	22	0.182
521	A	6	6	1.01	22	0.273
522	A	6	6	1.06	22	0.273
523	A	8	8	1.06	22	0.364
524	A	8	8	1.09	22	0.364
525	A	14	14	1.13	22	0.636
526	A	12	12	1.11	22	0.545
527	A	10	10	1.17	22	0.455
528	A	8	8	1.13	22	0.364
529	A	2	2	1.00	22	0.091
530	A	4	4	1.00	22	0.182
531	A	8	8	1.06	22	0.364
532	A	10	10	1.09	22	0.455
533	A	4	4	1.00	20	0.200
534	A	4	4	1.00	18	0.222
535	A	4	4	1.00	18	0.222
536	A	4	4	1.00	20	0.200
537	A	4	4	1.00	20	0.200
538	A	7	6	0.89	18	0.333
539	A	7	6	0.89	18	0.333
540	A	7	6	0.91	18	0.333
541	A	7	6	0.91	18	0.333
542	A	7	6	0.97	18	0.333
543	A	7	6	0.97	18	0.333
544	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
545	A	4	4	1.00	16	0.250
546	A	8	7	0.82	18	0.389
547	A	8	7	1.15	18	0.389
548	A	8	7	1.06	18	0.389
549	A	10	9	1.17	18	0.500
550	A	8	7	0.95	20	0.350
551	A	10	9	1.13	20	0.450
552	A	7	6	1.07	20	0.300
553	A	8	7	1.10	20	0.350
554	A	6	5	1.00	18	0.278
555	A	5	5	1.14	18	0.278
556	A	6	5	1.00	20	0.250
557	A	7	6	0.90	20	0.300
558	A	7	6	0.81	20	0.300
559	A	8	7	0.78	20	0.350
560	A	7	6	0.91	20	0.300
561	A	7	6	0.89	20	0.300
562	A	7	6	0.97	20	0.300
563	A	7	6	0.91	20	0.300
564	A	6	5	1.00	18	0.278
565	A	8	7	0.82	18	0.389
566	A	7	6	1.00	20	0.300
567	A	6	6	1.12	20	0.300
568	A	8	7	0.81	20	0.350
569	A	10	9	1.25	20	0.450
570	A	8	8	1.11	20	0.400
571	A	6	6	1.09	20	0.300
572	A	4	4	1.06	20	0.200
573	A	2	2	1.00	20	0.100
574	A	2	2	1.00	20	0.100
575	A	4	4	1.00	20	0.200
576	A	6	6	1.06	20	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
577	A	8	8	1.10	20	0.400
578	A	8	8	1.10	22	0.364
579	A	6	6	1.07	22	0.273
580	A	6	6	1.07	22	0.273
581	A	4	4	1.00	22	0.182
582	A	4	4	1.00	22	0.182
583	A	4	4	1.00	22	0.182
584	A	4	4	1.00	22	0.182
585	A	6	6	1.01	22	0.273
586	A	8	8	1.11	22	0.364
587	A	6	6	1.09	22	0.273
588	A	4	4	1.06	22	0.182
589	A	6	6	1.06	22	0.273
590	A	2	2	1.00	22	0.091
591	A	4	4	1.00	22	0.182
592	A	6	6	1.06	22	0.273
593	A	8	8	1.09	22	0.364
594	A	2	2	1.00	11	0.182
595	A	4	4	1.32	11	0.364
596	A	4	4	1.00	20	0.200
597	A	4	4	1.00	20	0.200
598	A	4	4	1.00	18	0.222
599	A	7	6	0.91	28	0.214
600	B	3	3	7.70	17	0.176
601	B	3	3	7.70	20	0.150
602	B	3	3	7.70	20	0.150
603	C	5	4	5.06	21	0.190
604	C	3	3	6.47	24	0.125
605	B	3	3	7.70	17	0.176
606	B	3	3	7.70	20	0.150
607	B	3	3	7.70	20	0.150
608	C	5	4	5.18	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
609	C	3	3	6.59	24	0.125
610	A	2	2	1.00	17	0.118
611	A	2	2	1.00	20	0.100
612	A	2	2	1.00	20	0.100
613	C	5	4	4.34	21	0.190
614	C	3	3	5.66	24	0.125
615	B	3	3	7.70	17	0.176
616	B	3	3	7.70	20	0.150
617	B	3	3	7.70	20	0.150
618	C	5	4	4.59	21	0.190
619	C	3	3	5.94	24	0.125

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sin(mx)\sin(nx)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int \sin(mx)\sin(nx)\,dx = \frac{\sin((m-n)x)}{2(m-n)} - \frac{\sin((m+n)x)}{2(m+n)}$$

output
```
sin((m-n)*x)/(2*m-2*n)-sin((m+n)*x)/(2*m+2*n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \sin(mx)\sin(nx)\,dx = \frac{\sin((m-n)x)}{2(m-n)} - \frac{\sin((m+n)x)}{2(m+n)}$$

input
```
Integrate[Sin[m*x]*Sin[n*x],x]
```

output
```
Sin[(m - n)*x]/(2*(m - n)) - Sin[(m + n)*x]/(2*(m + n))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(mx)\sin(nx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(\frac{1}{2}\cos(x(m-n))-\frac{1}{2}\cos(x(m+n))\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(x(m-n))}{2(m-n)}-\frac{\sin(x(m+n))}{2(m+n)}$$

input
```
Int[Sin[m*x]*Sin[n*x],x]
```

output
```
Sin[(m - n)*x]/(2*(m - n)) - Sin[(m + n)*x]/(2*(m + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result
default	$\frac{\sin((m-n)x)}{2m-2n}-\frac{\sin((m+n)x)}{2(m+n)}$
risch	$\frac{\sin((m-n)x)}{2m-2n}-\frac{\sin((m+n)x)}{2(m+n)}$
parallelrisch	$\frac{(m+n)\sin((m-n)x)-\sin((m+n)x)(m-n)}{2m^2-2n^2}$
norman	$\frac{-\frac{2m\tan\left(\frac{nx}{2}\right)}{m^2-n^2}+\frac{2n\tan\left(\frac{mx}{2}\right)}{m^2-n^2}+\frac{2m\tan\left(\frac{mx}{2}\right)^2\tan\left(\frac{nx}{2}\right)}{m^2-n^2}-\frac{2n\tan\left(\frac{mx}{2}\right)\tan\left(\frac{nx}{2}\right)^2}{m^2-n^2}}{\left(1+\tan\left(\frac{mx}{2}\right)^2\right)\left(1+\tan\left(\frac{nx}{2}\right)^2\right)}$
orering	$-\frac{2(m^2+n^2)(m\cos(mx)\sin(nx)+\sin(mx)n\cos(nx))}{m^4-2m^2n^2+n^4}-\frac{-m^3\cos(mx)\sin(nx)-3m^2\sin(mx)n\cos(nx)-3m\cos(mx)n^2\sin}{m^4-2m^2n^2+n^4}$

input
```
int(sin(m*x)*sin(n*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/(m-n)*sin((m-n)*x)-1/2/(m+n)*sin((m+n)*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \sin(mx)\sin(nx)\,dx = \frac{n\cos(nx)\sin(mx)-m\cos(mx)\sin(nx)}{m^2-n^2}$$

input
```
integrate(sin(m*x)*sin(n*x),x, algorithm="fricas")
```

output
```
(n*cos(n*x)*sin(m*x) - m*cos(m*x)*sin(n*x))/(m^2 - n^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(26) = 52$.

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int \sin(mx)\sin(nx)\,dx = \begin{cases} 0 & \text{for}\ m = 0 \wedge n = 0 \\ -\frac{x\sin^2(nx)}{2} - \frac{x\cos^2(nx)}{2} + \frac{\sin(nx)\cos(nx)}{2n} & \text{for}\ m = -n \\ \frac{x\sin^2(nx)}{2} + \frac{x\cos^2(nx)}{2} - \frac{\sin(nx)\cos(nx)}{2n} & \text{for}\ m = n \\ -\frac{m\sin(nx)\cos(mx)}{m^2-n^2} + \frac{n\sin(mx)\cos(nx)}{m^2-n^2} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(m*x)*sin(n*x),x)
```

output
```
Piecewise((0, Eq(m, 0) & Eq(n, 0)), (-x*sin(n*x)**2/2 - x*cos(n*x)**2/2 +
sin(n*x)*cos(n*x)/(2*n), Eq(m, -n)), (x*sin(n*x)**2/2 + x*cos(n*x)**2/2 -
sin(n*x)*cos(n*x)/(2*n), Eq(m, n)), (-m*sin(n*x)*cos(m*x)/(m**2 - n**2) +
n*sin(m*x)*cos(n*x)/(m**2 - n**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \sin(mx)\sin(nx)\,dx = -\frac{\sin((m+n)x)}{2(m+n)} + \frac{\sin((m-n)x)}{2(m-n)}$$

input
```
integrate(sin(m*x)*sin(n*x),x, algorithm="maxima")
```

output
```
-1/2*sin((m + n)*x)/(m + n) + 1/2*sin((m - n)*x)/(m - n)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \sin(mx)\sin(nx)\,dx = -\frac{\sin(mx+nx)}{2(m+n)} + \frac{\sin(mx-nx)}{2(m-n)}$$

input
```
integrate(sin(m*x)*sin(n*x),x, algorithm="giac")
```

output
```
-1/2*sin(m*x + n*x)/(m + n) + 1/2*sin(m*x - n*x)/(m - n)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \sin(mx)\sin(nx)\,dx = -\frac{m\,\cos(m\,x)\,\sin(n\,x) - n\,\cos(n\,x)\,\sin(m\,x)}{m^2-n^2}$$

input
```
int(sin(m*x)*sin(n*x),x)
```

output
```
-(m*cos(m*x)*sin(n*x) - n*cos(n*x)*sin(m*x))/(m^2 - n^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \sin(mx)\sin(nx)\,dx = \frac{-\cos(mx)\sin(nx)\,m + \cos(nx)\sin(mx)\,n}{m^2-n^2}$$

input
```
int(sin(m*x)*sin(n*x),x)
```

output
```
( - cos(m*x)*sin(n*x)*m + cos(n*x)*sin(m*x)*n)/(m**2 - n**2)
```

3.2 $\int \sin(x)\sin(mx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \sin(x)\sin(mx)\,dx = \frac{\sin((1-m)x)}{2(1-m)} - \frac{\sin((1+m)x)}{2(1+m)}$$

output
```
sin((1-m)*x)/(2-2*m)-sin((1+m)*x)/(2+2*m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \sin(x)\sin(mx)\,dx = \frac{-m\cos(mx)\sin(x)+\cos(x)\sin(mx)}{-1+m^2}$$

input
```
Integrate[Sin[x]*Sin[m*x],x]
```

output
```
(-(m*Cos[m*x]*Sin[x]) + Cos[x]*Sin[m*x])/(-1 + m^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sin(mx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(\frac{1}{2}\cos((1-m)x)-\frac{1}{2}\cos((m+1)x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin((1-m)x)}{2(1-m)}-\frac{\sin((m+1)x)}{2(m+1)}$$

input
```
Int[Sin[x]*Sin[m*x],x]
```

output
```
Sin[(1 - m)*x]/(2*(1 - m)) - Sin[(1 + m)*x]/(2*(1 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result
default	$\frac{\sin(x(-1+m))}{-2+2m}-\frac{\sin((1+m)x)}{2(1+m)}$
parallelrisch	$\frac{(1-m)\sin((1+m)x)+(1+m)\sin(x(-1+m))}{2m^{2}-2}$
risch	$\frac{\sin(x(-1+m))}{-2+2m}+\frac{\sin((1+m)x)}{2(-1+m)(1+m)}-\frac{\sin((1+m)x)m}{2(-1+m)(1+m)}$
orering	$-\frac{2(m^{2}+1)(\cos(x)\sin(mx)+\sin(x)m\cos(mx))}{m^{4}-2m^{2}+1}-\frac{-\cos(x)\sin(mx)-3\sin(x)m\cos(mx)-3\cos(x)m^{2}\sin(mx)-\sin(x)m^{3}\,\mathrm{c}}{m^{4}-2m^{2}+1}$
norman	$\frac{\frac{2\tan\left(\frac{mx}{2}\right)}{m^{2}-1}-\frac{2m\tan\left(\frac{x}{2}\right)}{m^{2}-1}-\frac{2\tan\left(\frac{x}{2}\right)^{2}\tan\left(\frac{mx}{2}\right)}{m^{2}-1}+\frac{2m\tan\left(\frac{x}{2}\right)\tan\left(\frac{mx}{2}\right)^{2}}{m^{2}-1}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan\left(\frac{mx}{2}\right)^{2}\right)}$

input
```
int(sin(x)*sin(m*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-1+m)*sin(x*(-1+m))-1/2/(1+m)*sin((1+m)*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \sin(x)\sin(mx)\,dx = -\frac{m\cos(mx)\sin(x)-\cos(x)\sin(mx)}{m^{2}-1}$$

input
```
integrate(sin(x)*sin(m*x),x, algorithm="fricas")
```

output
```
-(m*cos(m*x)*sin(x) - cos(x)*sin(m*x))/(m^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(22) = 44$.

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.23

$$\int \sin(x)\sin(mx)\,dx = \begin{cases} -\frac{x\sin^2(x)}{2} - \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} & \text{for } m = -1 \\ \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} - \frac{\sin(x)\cos(x)}{2} & \text{for } m = 1 \\ -\frac{m\sin(x)\cos(mx)}{m^2-1} + \frac{\sin(mx)\cos(x)}{m^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(x)*sin(m*x),x)
```

output
```
Piecewise((-x*sin(x)**2/2 - x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, -1)), (
x*sin(x)**2/2 + x*cos(x)**2/2 - sin(x)*cos(x)/2, Eq(m, 1)), (-m*sin(x)*cos
(m*x)/(m**2 - 1) + sin(m*x)*cos(x)/(m**2 - 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \sin(x)\sin(mx)\,dx = -\frac{\sin((m+1)x)}{2(m+1)} - \frac{\sin(-(m-1)x)}{2(m-1)}$$

input
```
integrate(sin(x)*sin(m*x),x, algorithm="maxima")
```

output
```
-1/2*sin((m + 1)*x)/(m + 1) - 1/2*sin(-(m - 1)*x)/(m - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \sin(x)\sin(mx)\,dx = -\frac{\sin(mx+x)}{2(m+1)} + \frac{\sin(mx-x)}{2(m-1)}$$

input
```
integrate(sin(x)*sin(m*x),x, algorithm="giac")
```

output
```
-1/2*sin(m*x + x)/(m + 1) + 1/2*sin(m*x - x)/(m - 1)
```

Mupad [B] (verification not implemented)

Time = 16.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int \sin(x)\sin(mx)\,dx = \begin{cases} \frac{x}{2} - \frac{\sin(2x)}{4} & \text{if } m = 1 \\ \frac{\sin(2x)}{4} - \frac{x}{2} & \text{if } m = -1 \\ \frac{\sin(x(m-1))}{2m-2} - \frac{\sin(x(m+1))}{2m+2} & \text{if } m \neq -1 \wedge m \neq 1 \end{cases}$$

input
```
int(sin(m*x)*sin(x),x)
```

output
```
piecewise(m == 1, x/2 - sin(2*x)/4, m == -1, - x/2 + sin(2*x)/4, m ~= -1 &
 m ~= 1, sin(x*(m - 1))/(2*m - 2) - sin(x*(m + 1))/(2*m + 2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \sin(x)\sin(mx)\,dx = \frac{-\cos(mx)\sin(x)m + \cos(x)\sin(mx)}{m^2-1}$$

input
```
int(sin(x)*sin(m*x),x)
```

output
```
( - cos(m*x)*sin(x)*m + cos(x)*sin(m*x))/(m**2 - 1)
```

3.3 $\int \sin(x)\sin(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \sin(x)\sin(2x)\,dx = \frac{\sin(x)}{2} - \frac{1}{6}\sin(3x)$$

output
```
1/2*sin(x)-1/6*sin(3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sin(x)\sin(2x)\,dx = \frac{\sin(x)}{2} - \frac{1}{6}\sin(3x)$$

input
```
Integrate[Sin[x]*Sin[2*x],x]
```

output
```
Sin[x]/2 - Sin[3*x]/6
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sin(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\sin(2x)dx$$

$$\Big\downarrow \text{4770}$$

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

input
```
Int[Sin[x]*Sin[2*x],x]
```

output
```
Sin[x]/2 - Sin[3*x]/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sin(x)}{2}-\frac{\sin(3x)}{6}$	12
risch	$\frac{\sin(x)}{2}-\frac{\sin(3x)}{6}$	12
parallelrisch	$\frac{\sin(x)}{2}-\frac{\sin(3x)}{6}$	12
orering	$\frac{\cos(x)\sin(2x)}{3}-\frac{2\cos(2x)\sin(x)}{3}$	18
norman	$\frac{-\frac{2\tan(x)\tan\left(\frac{x}{2}\right)^{2}}{3}+\frac{4\tan(x)^{2}\tan\left(\frac{x}{2}\right)}{3}+\frac{2\tan(x)}{3}-\frac{4\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(\tan(x)^{2}+1\right)}$	51

input
```
int(sin(x)*sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)-1/6*sin(3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \sin(x)\sin(2x)\,dx = -\frac{2}{3}\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)$$

input
```
integrate(sin(x)*sin(2*x),x, algorithm="fricas")
```

output
```
-2/3*(cos(x)^2 - 1)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \sin(x)\sin(2x)\,dx = -\frac{2\sin{(x)}\cos{(2x)}}{3} + \frac{\sin{(2x)}\cos{(x)}}{3}$$

input
```
integrate(sin(x)*sin(2*x),x)
```

output
```
-2*sin(x)*cos(2*x)/3 + sin(2*x)*cos(x)/3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \sin(x)\sin(2x)\,dx = -\frac{1}{6}\sin{(3x)} + \frac{1}{2}\sin{(x)}$$

input
```
integrate(sin(x)*sin(2*x),x, algorithm="maxima")
```

output
```
-1/6*sin(3*x) + 1/2*sin(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \sin(x)\sin(2x)\,dx = \frac{2}{3}\sin{(x)}^3$$

input
```
integrate(sin(x)*sin(2*x),x, algorithm="giac")
```

output
```
2/3*sin(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \sin(x)\sin(2x)\,dx = \frac{2\sin(x)^3}{3}$$

input
```
int(sin(2*x)*sin(x),x)
```

output
```
(2*sin(x)^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \sin(x)\sin(2x)\,dx = -\frac{2\cos(2x)\sin(x)}{3} + \frac{\cos(x)\sin(2x)}{3}$$

input
```
int(sin(x)*sin(2*x),x)
```

output
```
( - 2*cos(2*x)*sin(x) + cos(x)*sin(2*x))/3
```

3.4 $\int \sin(x)\sin(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \sin(x)\sin(3x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{8}\sin(4x)$$

output
```
1/4*sin(2*x)-1/8*sin(4*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sin(x)\sin(3x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{8}\sin(4x)$$

input
```
Integrate[Sin[x]*Sin[3*x],x]
```

output
```
Sin[2*x]/4 - Sin[4*x]/8
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sin(3x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(x)\sin(3x)dx$$

$$\Big\downarrow \ 4770$$

$$\frac{1}{4}\sin(2x) - \frac{1}{8}\sin(4x)$$

input
```
Int[Sin[x]*Sin[3*x],x]
```

output
```
Sin[2*x]/4 - Sin[4*x]/8
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sin(2x)}{4}-\frac{\sin(4x)}{8}$	14
risch	$\frac{\sin(2x)}{4}-\frac{\sin(4x)}{8}$	14
parallelrisch	$\frac{\sin(2x)}{4}-\frac{\sin(4x)}{8}$	14
orering	$\frac{\sin(3x)\cos(x)}{8}-\frac{3\cos(3x)\sin(x)}{8}$	18
norman	$\frac{\frac{3\tan\left(\frac{x}{2}\right)\tan\left(\frac{3x}{2}\right)^{2}}{4}-\frac{\tan\left(\frac{x}{2}\right)^{2}\tan\left(\frac{3x}{2}\right)}{4}-\frac{3\tan\left(\frac{x}{2}\right)}{4}+\frac{\tan\left(\frac{3x}{2}\right)}{4}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan\left(\frac{3x}{2}\right)^{2}\right)}$	59

input
```
int(sin(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(2*x)-1/8*sin(4*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(3x)\,dx = -\left(\cos\left(x\right)^{3}-\cos\left(x\right)\right)\sin\left(x\right)$$

input
```
integrate(sin(x)*sin(3*x),x, algorithm="fricas")
```

output
```
-(cos(x)^3 - cos(x))*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sin(x)\sin(3x)\,dx = -\frac{3\sin(x)\cos(3x)}{8} + \frac{\sin(3x)\cos(x)}{8}$$

input
```
integrate(sin(x)*sin(3*x),x)
```

output
```
-3*sin(x)*cos(3*x)/8 + sin(3*x)*cos(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(3x)\,dx = -\frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(sin(x)*sin(3*x),x, algorithm="maxima")
```

output
```
-1/8*sin(4*x) + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(3x)\,dx = -\frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(sin(x)*sin(3*x),x, algorithm="giac")
```

output
```
-1/8*sin(4*x) + 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(3x)\,dx = \frac{\sin(2\,x)}{4} - \frac{\sin(4\,x)}{8}$$

input
```
int(sin(3*x)*sin(x),x)
```

output
```
sin(2*x)/4 - sin(4*x)/8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sin(x)\sin(3x)\,dx = -\frac{3\cos(3x)\sin(x)}{8} + \frac{\cos(x)\sin(3x)}{8}$$

input
```
int(sin(x)*sin(3*x),x)
```

output
```
( - 3*cos(3*x)*sin(x) + cos(x)*sin(3*x))/8
```

3.5 $\int \sin(x)\sin(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \sin(x)\sin(4x)\,dx = \frac{1}{6}\sin(3x) - \frac{1}{10}\sin(5x)$$

output
```
1/6*sin(3*x)-1/10*sin(5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sin(x)\sin(4x)\,dx = \frac{1}{6}\sin(3x) - \frac{1}{10}\sin(5x)$$

input
```
Integrate[Sin[x]*Sin[4*x],x]
```

output
```
Sin[3*x]/6 - Sin[5*x]/10
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sin(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\sin(4x)dx$$

$$\Big\downarrow \text{4770}$$

$$\frac{1}{6}\sin(3x)-\frac{1}{10}\sin(5x)$$

input
```
Int[Sin[x]*Sin[4*x],x]
```

output
```
Sin[3*x]/6 - Sin[5*x]/10
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sin(3x)}{6}-\frac{\sin(5x)}{10}$	14
risch	$\frac{\sin(3x)}{6}-\frac{\sin(5x)}{10}$	14
parallelrisch	$\frac{\sin(3x)}{6}-\frac{\sin(5x)}{10}$	14
orering	$\frac{\cos(x)\sin(4x)}{15}-\frac{4\sin(x)\cos(4x)}{15}$	18
norman	$\frac{-\frac{2\tan(2x)\tan\left(\frac{x}{2}\right)^{2}}{15}+\frac{8\tan(2x)^{2}\tan\left(\frac{x}{2}\right)}{15}+\frac{2\tan(2x)}{15}-\frac{8\tan\left(\frac{x}{2}\right)}{15}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan(2x)^{2}\right)}$	59

input
```
int(sin(x)*sin(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*sin(3*x)-1/10*sin(5*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \sin(x)\sin(4x)\,dx = -\frac{4}{15}\left(6\,\cos\left(x\right)^{4}-7\,\cos\left(x\right)^{2}+1\right)\sin\left(x\right)$$

input
```
integrate(sin(x)*sin(4*x),x, algorithm="fricas")
```

output
```
-4/15*(6*cos(x)^4 - 7*cos(x)^2 + 1)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sin(x)\sin(4x)\,dx = -\frac{4\sin(x)\cos(4x)}{15} + \frac{\sin(4x)\cos(x)}{15}$$

input
```
integrate(sin(x)*sin(4*x),x)
```

output
```
-4*sin(x)*cos(4*x)/15 + sin(4*x)*cos(x)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(4x)\,dx = -\frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

input
```
integrate(sin(x)*sin(4*x),x, algorithm="maxima")
```

output
```
-1/10*sin(5*x) + 1/6*sin(3*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(4x)\,dx = -\frac{8}{5}\sin(x)^5 + \frac{4}{3}\sin(x)^3$$

input
```
integrate(sin(x)*sin(4*x),x, algorithm="giac")
```

output
```
-8/5*sin(x)^5 + 4/3*sin(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(4x)\,dx = \frac{\sin(3\,x)}{6} - \frac{\sin(5\,x)}{10}$$

input
```
int(sin(4*x)*sin(x),x)
```

output
```
sin(3*x)/6 - sin(5*x)/10
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sin(x)\sin(4x)\,dx = -\frac{4\cos(4x)\sin(x)}{15} + \frac{\cos(x)\sin(4x)}{15}$$

input
```
int(sin(x)*sin(4*x),x)
```

output
```
( - 4*cos(4*x)*sin(x) + cos(x)*sin(4*x))/15
```

3.6 $\int \csc(2x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \csc(2x)\sin(x)\,dx = \frac{1}{2}\text{arctanh}(\sin(x))$$

output
```
1/2*arctanh(sin(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \csc(2x)\sin(x)\,dx = \frac{1}{2}\coth^{-1}(\sin(x))$$

input
```
Integrate[Csc[2*x]*Sin[x],x]
```

output
```
ArcCoth[Sin[x]]/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4776, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\csc(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\sin(2x)}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{\int \sec(x)dx}{2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\text{arctanh}(\sin(x))$$

input
```
Int[Csc[2*x]*Sin[x],x]
```

output
```
ArcTanh[Sin[x]]/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{\ln(\sec(x)+\tan(x))}{2}$	9
risch	$-\frac{\ln(e^{ix}-i)}{2}+\frac{\ln(e^{ix}+i)}{2}$	24

input
```
int(csc(2*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(sec(x)+tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \csc(2x)\sin(x)\,dx = \frac{1}{4}\log(\sin(x)+1) - \frac{1}{4}\log(-\sin(x)+1)$$

input
```
integrate(csc(2*x)*sin(x),x, algorithm="fricas")
```

output
```
1/4*log(sin(x) + 1) - 1/4*log(-sin(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.14

$$\int \csc(2x)\sin(x)\,dx = -\frac{\log(\sin(x)-1)}{4} + \frac{\log(\sin(x)+1)}{4}$$

input
```
integrate(csc(2*x)*sin(x),x)
```

output
```
-log(sin(x) - 1)/4 + log(sin(x) + 1)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(5) = 10$.

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 5.00

$$\int \csc(2x)\sin(x)\,dx = \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) - \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right)$$

input
```
integrate(csc(2*x)*sin(x),x, algorithm="maxima")
```

output
```
1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*log(cos(x)^2 + sin(x)^2
- 2*sin(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \csc(2x)\sin(x)\,dx = \frac{1}{4}\log\left(\sin\left(x\right)+1\right) - \frac{1}{4}\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate(csc(2*x)*sin(x),x, algorithm="giac")
```

output
```
1/4*log(sin(x) + 1) - 1/4*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 16.41 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \csc(2x)\sin(x)\,dx = \frac{\operatorname{atanh}(\sin\left(x\right))}{2}$$

input
```
int(sin(x)/sin(2*x),x)
```

output
```
atanh(sin(x))/2
```

Reduce [F]

$$\int \csc(2x)\sin(x)\,dx = \int \csc\left(2x\right)\sin\left(x\right)dx$$

input
```
int(csc(2*x)*sin(x),x)
```

output
```
int(csc(2*x)*sin(x),x)
```

3.7 $\int \csc(3x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 45

$$\int \csc(3x)\sin(x)\,dx = -\frac{\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{2\sqrt{3}}+\frac{\log\left(\sqrt{3}\cos(x)+\sin(x)\right)}{2\sqrt{3}}$$

output
```
-1/6*ln(3^(1/2)*cos(x)-sin(x))*3^(1/2)+1/6*ln(3^(1/2)*cos(x)+sin(x))*3^(1/
2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.33

$$\int \csc(3x)\sin(x)\,dx = \frac{\text{arctanh}\left(\frac{\tan(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[Csc[3*x]*Sin[x],x]
```

output
```
ArcTanh[Tan[x]/Sqrt[3]]/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.33, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4889, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\csc(3x)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(x)}{\sin(3x)}dx$$
$$\Big\downarrow \text{ 4889}$$
$$\int \frac{1}{3-\tan^2(x)}d\tan(x)$$
$$\Big\downarrow \text{ 219}$$
$$\frac{\text{arctanh}\left(\frac{\tan(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[Csc[3*x]*Sin[x],x]
```

output
```
ArcTanh[Tan[x]/Sqrt[3]]/Sqrt[3]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.31

method	result	size
default	$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\tan(x)\sqrt{3}}{3}\right)}{3}$	14
risch	$\frac{\sqrt{3}\,\ln\left(\mathrm{e}^{2ix}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}-\frac{\sqrt{3}\,\ln\left(\mathrm{e}^{2ix}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}$	40

input
```
int(csc(3*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arctanh(1/3*tan(x)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \csc(3x)\sin(x)\,dx$$
$$=\frac{1}{12}\sqrt{3}\log\left(-\frac{8\cos(x)^4-16\cos(x)^2-4\left(2\sqrt{3}\cos(x)^3+\sqrt{3}\cos(x)\right)\sin(x)-1}{16\cos(x)^4-8\cos(x)^2+1}\right)$$

input
```
integrate(csc(3*x)*sin(x),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(-(8*cos(x)^4 - 16*cos(x)^2 - 4*(2*sqrt(3)*cos(x)^3 + sqrt
(3)*cos(x))*sin(x) - 1)/(16*cos(x)^4 - 8*cos(x)^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int \csc(3x)\sin(x)\,dx = \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)}{6} - \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)-\frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)+\frac{\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)}{6}$$

input
```
integrate(csc(3*x)*sin(x),x)
```

output
```
sqrt(3)*log(tan(x/2) - sqrt(3))/6 - sqrt(3)*log(tan(x/2) - sqrt(3)/3)/6 +
sqrt(3)*log(tan(x/2) + sqrt(3)/3)/6 - sqrt(3)*log(tan(x/2) + sqrt(3))/6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(33) = 66$.

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.78

$$\int \csc(3x)\sin(x)\,dx = -\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right) + \frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)-\frac{4}{3}\cos(x)+\frac{4}{3}\right) + \frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2-\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right) - \frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2-\frac{4}{3}\sqrt{3}\sin(x)-\frac{4}{3}\cos(x)+\frac{4}{3}\right)$$

input
```
integrate(csc(3*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*c
os(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*
sin(x) - 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2
- 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) - 1/12*sqrt(3)*log(4/3*cos(x)^2 +
 4/3*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \csc(3x)\sin(x)\,dx = -\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\tan\left(x\right)\right|}{\left|2\sqrt{3}+2\tan\left(x\right)\right|}\right)$$

input
```
integrate(csc(3*x)*sin(x),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(x))/abs(2*sqrt(3) + 2*tan(x)))
```

Mupad [B] (verification not implemented)

Time = 16.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.38

$$\int \csc(3x)\sin(x)\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\sin(x)}{3\cos(x)}\right)}{3}$$

input
```
int(sin(x)/sin(3*x),x)
```

output
```
(3^(1/2)*atanh((3^(1/2)*sin(x))/(3*cos(x))))/3
```

Reduce [F]

$$\int \csc(3x)\sin(x)\,dx = \int \csc(3x)\sin(x)\,dx$$

input
```
int(csc(3*x)*sin(x),x)
```

output
```
int(csc(3*x)*sin(x),x)
```

3.8 $\int \csc(4x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \csc(4x)\sin(x)\,dx=-\frac{1}{4}\text{arctanh}(\sin(x))+\frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

output
```
-1/4*arctanh(sin(x))+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc(4x)\sin(x)\,dx=-\frac{1}{4}\text{arctanh}(\sin(x))+\frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

input
```
Integrate[Csc[4*x]*Sin[x],x]
```

output
```
-1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4878, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\csc(4x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)}{\sin(4x)}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{1}{8\sin^4(x)-12\sin^2(x)+4}d\sin(x)$$

$$\Big\downarrow \text{ 1406}$$

$$2\int \frac{1}{8\sin^2(x)-8}d\sin(x)-2\int \frac{1}{8\sin^2(x)-4}d\sin(x)$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\text{arctanh}(\sin(x))$$

input
```
Int[Csc[4*x]*Sin[x],x]
```

output
```
-1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{\ln(1+\sin(x))}{8}+\frac{\ln(\sin(x)-1)}{8}+\frac{\operatorname{arctanh}\left(\sqrt{2}\,\sin(x)\right)\sqrt{2}}{4}$	28
risch	$-\frac{\ln(\mathrm{e}^{ix}+i)}{4}+\frac{\ln(\mathrm{e}^{ix}-i)}{4}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{8}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{8}$	72

input
```
int(csc(4*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(1+sin(x))+1/8*ln(sin(x)-1)+1/4*arctanh(2^(1/2)*sin(x))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \csc(4x)\sin(x)\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{2\,\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\,\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

input
```
integrate(csc(4*x)*sin(x),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(22) = 44$.

Time = 3.78 (sec) , antiderivative size = 294, normalized size of antiderivative = 11.31

$$\int \csc(4x)\sin(x)\,dx = \frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808} + \frac{39202\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808} - \frac{39202\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808} - \frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808} + \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{27720\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808} - \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808} - \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808}$$

input
```
integrate(csc(4*x)*sin(x),x)
```

output
```
27720*sqrt(2)*log(tan(x/2) - 1)/(110880*sqrt(2) + 156808) + 39202*log(tan(
x/2) - 1)/(110880*sqrt(2) + 156808) - 39202*log(tan(x/2) + 1)/(110880*sqrt
(2) + 156808) - 27720*sqrt(2)*log(tan(x/2) + 1)/(110880*sqrt(2) + 156808)
+ 27720*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt
(2)*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(
x/2) + 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2)
 + 1 + sqrt(2))/(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - s
qrt(2) - 1)/(110880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) - 1)/
(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(110
880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2)
+ 156808)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(18) = 36$.

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.58

$$\int \csc(4x)\sin(x)\,dx = \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right) - \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right) + \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right) - \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right) - \frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right) + \frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)$$

input
```
integrate(csc(4*x)*sin(x),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) -
2*sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(
2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)
^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^
2 + 2*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \csc(4x)\sin(x)\,dx = -\frac{1}{8}\sqrt{2}\log\left(\frac{|-2\sqrt{2}+4\sin(x)|}{|2\sqrt{2}+4\sin(x)|}\right) - \frac{1}{8}\log(\sin(x)+1) + \frac{1}{8}\log(-\sin(x)+1)$$

input
```
integrate(csc(4*x)*sin(x),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 16.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \csc(4x)\sin(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin\left(x\right)\right)}{4} - \frac{\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

input
```
int(sin(x)/sin(4*x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*sin(x)))/4 - atanh(sin(x/2)/cos(x/2))/2
```

Reduce [F]

$$\int \csc(4x)\sin(x)\,dx = \int \csc\left(4x\right)\sin\left(x\right)dx$$

input
```
int(csc(4*x)*sin(x),x)
```

output
```
int(csc(4*x)*sin(x),x)
```

3.9 $\int \csc(5x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 80

$$\int \csc(5x)\sin(x)\,dx = -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}\tan(x)\right) + \sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\operatorname{arctanh}\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}\tan(x)\right)$$

output
```
-1/10*(10+2*5^(1/2))^(1/2)*arctanh(1/5*(25-10*5^(1/2))^(1/2)*tan(x))+2^(1/
2)/(25+5*5^(1/2))^(1/2)*arctanh(1/5*(25+10*5^(1/2))^(1/2)*tan(x))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

$$\int \csc(5x)\sin(x)\,dx = \frac{\sqrt{5+\sqrt{5}}\operatorname{arctanh}\left(\frac{\left(-3+\sqrt{5}\right)\tan(x)}{\sqrt{10-2\sqrt{5}}}\right) + \sqrt{5-\sqrt{5}}\operatorname{arctanh}\left(\frac{\left(3+\sqrt{5}\right)\tan(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

input
```
Integrate[Csc[5*x]*Sin[x],x]
```

output
```
(Sqrt[5 + Sqrt[5]]*ArcTanh[((-3 + Sqrt[5])*Tan[x])/Sqrt[10 - 2*Sqrt[5]]] +
 Sqrt[5 - Sqrt[5]]*ArcTanh[((3 + Sqrt[5])*Tan[x])/Sqrt[2*(5 + Sqrt[5])]])/
(5*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4889, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\csc(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\sin(5x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tan^2(x)+1}{\tan^4(x)-10\tan^2(x)+5}d\tan(x)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\tan^2(x)-2\sqrt{5}-5}d\tan(x)+\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{\tan^2(x)+2\sqrt{5}-5}d\tan(x)$$

$$\Big\downarrow \text{220}$$

$$-\frac{\left(5-3\sqrt{5}\right)\text{arctanh}\left(\frac{\tan(x)}{\sqrt{5-2\sqrt{5}}}\right)}{10\sqrt{5-2\sqrt{5}}}-\frac{\left(5+3\sqrt{5}\right)\text{arctanh}\left(\frac{\tan(x)}{\sqrt{5+2\sqrt{5}}}\right)}{10\sqrt{5+2\sqrt{5}}}$$

input
```
Int[Csc[5*x]*Sin[x],x]
```

output
```
-1/10*((5 - 3*Sqrt[5])*ArcTanh[Tan[x]/Sqrt[5 - 2*Sqrt[5]]])/Sqrt[5 - 2*Sqr
t[5]] - ((5 + 3*Sqrt[5])*ArcTanh[Tan[x]/Sqrt[5 + 2*Sqrt[5]]])/(10*Sqrt[5 +
 2*Sqrt[5]])
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

method	result	size
risch	$\sum_{\_R=\mathrm{RootOf}(2000\_Z^4-100\_Z^2+1)} \_R\ln\left(\mathrm{e}^{2ix}-500i\_R^3+50\_R^2+15i\_R-1\right)$	42
default	$-\frac{\sqrt{5}\left(\sqrt{5}-3\right)\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{5-2\sqrt{5}}}\right)}{10\sqrt{5-2\sqrt{5}}}-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{5+2\sqrt{5}}}\right)}{10\sqrt{5+2\sqrt{5}}}$	66

input

```
int(csc(5*x)*sin(x),x,method=_RETURNVERBOSE)
```

output

```
sum(_R*ln(exp(2*I*x)-500*I*_R^3+50*_R^2+15*I*_R-1),_R=RootOf(2000*_Z^4-100
*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(53) = 106$.

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.51

$$\int \csc(5x)\sin(x)\,dx$$
$$= -\frac{1}{20}\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(2\left(\sqrt{5}-1\right)\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\cos(x)\sin(x)+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}+3\right)+\frac{1}{20}\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(-2\left(\sqrt{5}-1\right)\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\cos(x)\sin(x)+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}+3\right)-\frac{1}{20}\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(2\left(\sqrt{5}+1\right)\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\cos(x)\sin(x)+2\left(\sqrt{5}-1\right)\cos(x)^2-\sqrt{5}-3\right)+\frac{1}{20}\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(-2\left(\sqrt{5}+1\right)\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\cos(x)\sin(x)+2\left(\sqrt{5}-1\right)\cos(x)^2-\sqrt{5}-3\right)$$

input
```
integrate(csc(5*x)*sin(x),x, algorithm="fricas")
```

output
```
-1/20*sqrt(1/2*sqrt(5) + 5/2)*log(2*(sqrt(5) - 1)*sqrt(1/2*sqrt(5) + 5/2)*
cos(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) + 3) + 1/20*sqrt(1/2*sq
rt(5) + 5/2)*log(-2*(sqrt(5) - 1)*sqrt(1/2*sqrt(5) + 5/2)*cos(x)*sin(x) +
2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) + 3) - 1/20*sqrt(-1/2*sqrt(5) + 5/2)*lo
g(2*(sqrt(5) + 1)*sqrt(-1/2*sqrt(5) + 5/2)*cos(x)*sin(x) + 2*(sqrt(5) - 1)
*cos(x)^2 - sqrt(5) - 3) + 1/20*sqrt(-1/2*sqrt(5) + 5/2)*log(-2*(sqrt(5) +
 1)*sqrt(-1/2*sqrt(5) + 5/2)*cos(x)*sin(x) + 2*(sqrt(5) - 1)*cos(x)^2 - sq
rt(5) - 3)
```

Sympy [F]

$$\int \csc(5x)\sin(x)\,dx = \int \sin(x)\csc(5x)\,dx$$

input
```
integrate(csc(5*x)*sin(x),x)
```

output
```
Integral(sin(x)*csc(5*x), x)
```

Maxima [F]

$$\int \csc(5x)\sin(x)\,dx = \int \csc(5x)\sin(x)\,dx$$

input
```
integrate(csc(5*x)*sin(x),x, algorithm="maxima")
```

output
```
integrate(csc(5*x)*sin(x), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31

$$\int \csc(5x)\sin(x)\,dx = -\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|\sqrt{2\sqrt{5}+5}+\tan(x)\right|\right) + \frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|-\sqrt{2\sqrt{5}+5}+\tan(x)\right|\right) + \frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|\sqrt{-2\sqrt{5}+5}+\tan(x)\right|\right) - \frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|-\sqrt{-2\sqrt{5}+5}+\tan(x)\right|\right)$$

input
```
integrate(csc(5*x)*sin(x),x, algorithm="giac")
```

output
```
-1/20*sqrt(2*sqrt(5) + 10)*log(abs(sqrt(2*sqrt(5) + 5) + tan(x))) + 1/20*s
qrt(2*sqrt(5) + 10)*log(abs(-sqrt(2*sqrt(5) + 5) + tan(x))) + 1/20*sqrt(-2
*sqrt(5) + 10)*log(abs(sqrt(-2*sqrt(5) + 5) + tan(x))) - 1/20*sqrt(-2*sqrt
(5) + 10)*log(abs(-sqrt(-2*sqrt(5) + 5) + tan(x)))
```

Mupad [B] (verification not implemented)

Time = 16.56 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.71

$$\int \csc(5x)\sin(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(-\frac{34359738368\sqrt{2}\tan\left(\frac{x}{2}\right)\sqrt{\sqrt{5}+5}}{1953125\left(\frac{90194313216\sqrt{5}}{1953125}-\frac{90194313216\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{1953125}-\frac{201863462912\tan\left(\frac{x}{2}\right)^2}{1953125}+\frac{201863462912}{1953125}\right)}-\frac{77\cdots}{9765625\left(\frac{90194313216\sqrt{5}}{1953125}-\frac{9\cdots}{\cdots}\right.}\cdots\right.}{10} - \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{77309411328\sqrt{2}\sqrt{5}\tan\left(\frac{x}{2}\right)\sqrt{5-\sqrt{5}}}{9765625\left(\frac{90194313216\sqrt{5}}{1953125}-\frac{90194313216\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{1953125}+\frac{201863462912\tan\left(\frac{x}{2}\right)^2}{1953125}-\frac{201863462912}{1953125}\right)}-\frac{\cdots}{1953125\left(\frac{90194313216\sqrt{5}}{1953125}-\frac{9\cdots}{\cdots}\right.}\cdots\right.}{10}$$

input
```
int(sin(x)/sin(5*x),x)
```

output
```
(2^(1/2)*atanh(- (34359738368*2^(1/2)*tan(x/2)*(5^(1/2) + 5)^(1/2))/(19531
25*((90194313216*5^(1/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)/19531
25 - (201863462912*tan(x/2)^2)/1953125 + 201863462912/1953125)) - (7730941
1328*2^(1/2)*5^(1/2)*tan(x/2)*(5^(1/2) + 5)^(1/2))/(9765625*((90194313216*
5^(1/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)/1953125 - (20186346291
2*tan(x/2)^2)/1953125 + 201863462912/1953125)))*(5^(1/2) + 5)^(1/2))/10 -
(2^(1/2)*atanh((77309411328*2^(1/2)*5^(1/2)*tan(x/2)*(5 - 5^(1/2))^(1/2))/
(9765625*((90194313216*5^(1/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)
/1953125 + (201863462912*tan(x/2)^2)/1953125 - 201863462912/1953125)) - (3
4359738368*2^(1/2)*tan(x/2)*(5 - 5^(1/2))^(1/2))/(1953125*((90194313216*5^
(1/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)/1953125 + (201863462912*
tan(x/2)^2)/1953125 - 201863462912/1953125)))*(5 - 5^(1/2))^(1/2))/10
```

Reduce [F]

$$\int \csc(5x)\sin(x)\,dx = \int \csc(5x)\sin(x)\,dx$$

input
```
int(csc(5*x)*sin(x),x)
```

output
```
int(csc(5*x)*sin(x),x)
```

3.10 $\int \csc(6x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 36

$$\int \csc(6x)\sin(x)\,dx = \frac{1}{6}\text{arctanh}(\sin(x)) + \frac{1}{6}\text{arctanh}(2\sin(x)) - \frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
1/6*arctanh(sin(x))+1/6*arctanh(2*sin(x))-1/6*arctanh(2/3*sin(x)*3^(1/2))*
3^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \csc(6x)\sin(x)\,dx = \frac{1}{6}\left(\text{arctanh}(\sin(x)) + \text{arctanh}(2\sin(x)) - \sqrt{3}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)\right)$$

input
```
Integrate[Csc[6*x]*Sin[x],x]
```

output
```
(ArcTanh[Sin[x]] + ArcTanh[2*Sin[x]] - Sqrt[3]*ArcTanh[(2*Sin[x])/Sqrt[3]]
)/6
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4878, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\csc(6x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(x)}{\sin(6x)}dx$$

$$\Big\downarrow \ 4878$$

$$\int \frac{1}{2\left(-16\sin^6(x)+32\sin^4(x)-19\sin^2(x)+3\right)}d\sin(x)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\int \frac{1}{-16\sin^6(x)+32\sin^4(x)-19\sin^2(x)+3}d\sin(x)$$

$$\Big\downarrow \ 2460$$

$$\frac{1}{2}\int \left(\frac{2}{4\sin^2(x)-3}-\frac{2}{3\left(4\sin^2(x)-1\right)}-\frac{1}{3\left(\sin^2(x)-1\right)}\right)d\sin(x)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{1}{3}\text{arctanh}(\sin(x))+\frac{1}{3}\text{arctanh}(2\sin(x))-\frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

input
```
Int[Csc[6*x]*Sin[x],x]
```

output
```
(ArcTanh[Sin[x]]/3 + ArcTanh[2*Sin[x]]/3 - ArcTanh[(2*Sin[x])/Sqrt[3]]/Sqr
t[3])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

method	result
default	$\frac{\ln(1+\sin(x))}{12}-\frac{\ln(2\sin(x)-1)}{12}+\frac{\ln(2\sin(x)+1)}{12}-\frac{\ln(\sin(x)-1)}{12}-\frac{\operatorname{arctanh}\left(\frac{2\sin(x)\sqrt{3}}{3}\right)\sqrt{3}}{6}$
risch	$-\frac{\ln(\mathrm{e}^{ix}-i)}{6}+\frac{\ln(\mathrm{e}^{ix}+i)}{6}+\frac{\ln(i\mathrm{e}^{ix}+\mathrm{e}^{2ix}-1)}{12}+\frac{\sqrt{3}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{3}\,\mathrm{e}^{ix}-1\right)}{12}-\frac{\sqrt{3}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{3}\,\mathrm{e}^{ix}-1\right)}{12}-\frac{\ln(-i\mathrm{e}^{ix}+\mathrm{e}^{2ix}-1}{12}$

input
```
int(csc(6*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(1+sin(x))-1/12*ln(2*sin(x)-1)+1/12*ln(2*sin(x)+1)-1/12*ln(sin(x)-1
)-1/6*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(26) = 52.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.89

$$\int \csc(6x)\sin(x)\,dx = \frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\sin(x)-7}{4\cos(x)^2-1}\right) + \frac{1}{12}\log(2\sin(x)+1) + \frac{1}{12}\log(\sin(x)+1) - \frac{1}{12}\log(-\sin(x)+1) - \frac{1}{12}\log(-2\sin(x)+1)$$

input
```
integrate(csc(6*x)*sin(x),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) +
1/12*log(2*sin(x) + 1) + 1/12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) - 1/
12*log(-2*sin(x) + 1)
```

Sympy [F]

$$\int \csc(6x)\sin(x)\,dx = \int \sin(x)\csc(6x)\,dx$$

input
```
integrate(csc(6*x)*sin(x),x)
```

output
```
Integral(sin(x)*csc(6*x), x)
```

Maxima [F]

$$\int \csc(6x)\sin(x)\,dx = \int \csc(6x)\sin(x)\,dx$$

input
```
integrate(csc(6*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*c
os(x) + 4/3) - 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*
sin(x) - 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2
- 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 +
 4/3*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) + integrate(-1/6*((
cos(3*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - cos(2*x)*cos(x) +
(sin(3*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)*sin(x) + cos(x
))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*s
in(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/12*log(cos(x)^2 +
sin(x)^2 + 2*sin(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.89

$$\int \csc(6x)\sin(x)\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\sin(x)\right|}{\left|4\sqrt{3}+8\sin(x)\right|}\right) + \frac{1}{12}\log(\sin(x)+1) - \frac{1}{12}\log(-\sin(x)+1) + \frac{1}{12}\log(|2\sin(x)+1|) - \frac{1}{12}\log(|2\sin(x)-1|)$$

input
```
integrate(csc(6*x)*sin(x),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) + 1
/12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) + 1/12*log(abs(2*sin(x) + 1))
- 1/12*log(abs(2*sin(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 16.47 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \csc(6x)\sin(x)\,dx=\frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3}+\frac{\operatorname{atanh}(2\,\sin{(x)})}{6}-\frac{\sqrt{3}\operatorname{atanh}\left(\frac{2\sqrt{3}\,\sin(x)}{3}\right)}{6}$$

input
```
int(sin(x)/sin(6*x),x)
```

output
```
atanh(sin(x/2)/cos(x/2))/3 + atanh(2*sin(x))/6 - (3^(1/2)*atanh((2*3^(1/2)
*sin(x))/3))/6
```

Reduce [F]

$$\int \csc(6x)\sin(x)\,dx=\int \csc{(6x)}\sin{(x)}\,dx$$

input
```
int(csc(6*x)*sin(x),x)
```

output
```
int(csc(6*x)*sin(x),x)
```

3.11 $\int \csc(x)\sin(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \csc(x)\sin(3x)\,dx = x + 2\cos(x)\sin(x)$$

output
```
x+2*cos(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(x)\sin(3x)\,dx = x + \sin(2x)$$

input
```
Integrate[Csc[x]*Sin[3*x],x]
```

output
```
x + Sin[2*x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4889, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x)\csc(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(3x)}{\sin(x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{3-\tan^2(x)}{\left(\tan^2(x)+1\right)^2}d\tan(x)$$

$$\Big\downarrow \text{298}$$

$$\int \frac{1}{\tan^2(x)+1}d\tan(x)+\frac{2\tan(x)}{\tan^2(x)+1}$$

$$\Big\downarrow \text{216}$$

$$\arctan(\tan(x))+\frac{2\tan(x)}{\tan^2(x)+1}$$

input
```
Int[Csc[x]*Sin[3*x],x]
```

output
```
ArcTan[Tan[x]] + (2*Tan[x])/(1 + Tan[x]^2)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$x+\sin(2x)$	7
risch	$x+\sin(2x)$	7

input
```
int(csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
x+sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \csc(x)\sin(3x)\,dx = 2\cos(x)\sin(x) + x$$

input
```
integrate(csc(x)*sin(3*x),x, algorithm="fricas")
```

output
```
2*cos(x)*sin(x) + x
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \csc(x)\sin(3x)\,dx = x + \sin(2x)$$

input
```
integrate(csc(x)*sin(3*x),x)
```

output
```
x + sin(2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(x)\sin(3x)\,dx = x + \sin(2\,x)$$

input
```
integrate(csc(x)*sin(3*x),x, algorithm="maxima")
```

output
```
x + sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(x)\sin(3x)\,dx = x + \sin(2x)$$

input
```
integrate(csc(x)*sin(3*x),x, algorithm="giac")
```

output
```
x + sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 16.68 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(x)\sin(3x)\,dx = x + \sin(2x)$$

input
```
int(sin(3*x)/sin(x),x)
```

output
```
x + sin(2*x)
```

Reduce [F]

$$\int \csc(x)\sin(3x)\,dx = \int \csc(x)\sin(3x)\,dx$$

input
```
int(csc(x)*sin(3*x),x)
```

output
```
int(csc(x)*sin(3*x),x)
```

3.12 $\int \csc(3x)\sin(6x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \csc(3x)\sin(6x)\,dx = \frac{2}{3}\sin(3x)$$

output
```
2/3*sin(3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \csc(3x)\sin(6x)\,dx = \frac{2}{3}\sin(3x)$$

input
```
Integrate[Csc[3*x]*Sin[6*x],x]
```

output
```
(2*Sin[3*x])/3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4776, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(6x)\csc(3x)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(6x)}{\sin(3x)}dx$$
$$\Big\downarrow \text{ 4776}$$
$$2\int \cos(3x)dx$$
$$\Big\downarrow \text{ 3042}$$
$$2\int \sin\left(3x+\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \text{ 3117}$$
$$\frac{2}{3}\sin(3x)$$

input
```
Int[Csc[3*x]*Sin[6*x],x]
```

output
```
(2*Sin[3*x])/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{2\sin(3x)}{3}$	7
derivativedivides	$\frac{2}{3\csc(3x)}$	9
default	$\frac{2}{3\csc(3x)}$	9

input
```
int(csc(3*x)*sin(6*x),x,method=_RETURNVERBOSE)
```

output
```
2/3*sin(3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(3x)\sin(6x)\,dx = \frac{2}{3}\sin(3x)$$

input
```
integrate(csc(3*x)*sin(6*x),x, algorithm="fricas")
```

output
```
2/3*sin(3*x)
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \csc(3x)\sin(6x)\,dx = \frac{2\sin(3x)}{3}$$

input
```
integrate(csc(3*x)*sin(6*x),x)
```

output
```
2*sin(3*x)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(3x)\sin(6x)\,dx = \frac{2}{3}\sin(3x)$$

input
```
integrate(csc(3*x)*sin(6*x),x, algorithm="maxima")
```

output
```
2/3*sin(3*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(3x)\sin(6x)\,dx = \frac{2}{3}\sin(3x)$$

input
```
integrate(csc(3*x)*sin(6*x),x, algorithm="giac")
```

output
```
2/3*sin(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(3x)\sin(6x)\,dx = \frac{2\sin(3x)}{3}$$

input
```
int(sin(6*x)/sin(3*x),x)
```

output
```
(2*sin(3*x))/3
```

Reduce [F]

$$\int \csc(3x)\sin(6x)\,dx = \int \csc(3x)\sin(6x)\,dx$$

input
```
int(csc(3*x)*sin(6*x),x)
```

output
```
int(csc(3*x)*sin(6*x),x)
```

3.13 $\int \cos(mx)\sin(nx)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int \cos(mx)\sin(nx)\,dx = \frac{\cos((m-n)x)}{2(m-n)} - \frac{\cos((m+n)x)}{2(m+n)}$$

output
```
cos((m-n)*x)/(2*m-2*n)-cos((m+n)*x)/(2*m+2*n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \cos(mx)\sin(nx)\,dx = \frac{\cos((m-n)x)}{2(m-n)} - \frac{\cos((m+n)x)}{2(m+n)}$$

input
```
Integrate[Cos[m*x]*Sin[n*x],x]
```

output
```
Cos[(m - n)*x]/(2*(m - n)) - Cos[(m + n)*x]/(2*(m + n))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(mx)\sin(nx)\,dx$$

$$\Big\downarrow \; 5085$$

$$\int \left(\frac{1}{2}\sin(x(m+n))-\frac{1}{2}\sin(x(m-n))\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\cos(x(m-n))}{2(m-n)}-\frac{\cos(x(m+n))}{2(m+n)}$$

input
```
Int[Cos[m*x]*Sin[n*x],x]
```

output
```
Cos[(m - n)*x]/(2*(m - n)) - Cos[(m + n)*x]/(2*(m + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result
default	$\frac{\cos((m-n)x)}{2m-2n}-\frac{\cos((m+n)x)}{2(m+n)}$
parallelrisch	$\frac{(m+n)\cos((m-n)x)+(-m+n)\cos((m+n)x)-2n}{2m^2-2n^2}$
risch	$\frac{\cos((m-n)x)m}{2(m+n)(m-n)}+\frac{\cos((m-n)x)n}{2(m+n)(m-n)}-\frac{\cos((m+n)x)m}{2(m+n)(m-n)}+\frac{\cos((m+n)x)n}{2(m+n)(m-n)}$
norman	$\frac{-\frac{2n\tan\left(\frac{mx}{2}\right)^2}{m^2-n^2}-\frac{2n\tan\left(\frac{nx}{2}\right)^2}{m^2-n^2}+\frac{4m\tan\left(\frac{mx}{2}\right)\tan\left(\frac{nx}{2}\right)}{m^2-n^2}}{\left(1+\tan\left(\frac{mx}{2}\right)^2\right)\left(1+\tan\left(\frac{nx}{2}\right)^2\right)}$
orering	$-\frac{2(m^2+n^2)(-\sin(mx)\sin(nx)m+\cos(mx)\cos(nx)n)}{m^4-2m^2n^2+n^4}-\frac{m^3\sin(mx)\sin(nx)-3m^2\cos(mx)\cos(nx)n+3m\sin(mx)n^2 \sin}{m^4-2m^2n^2+n^4}$

input
```
int(cos(m*x)*sin(n*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*cos((m-n)*x)/(m-n)-1/2*cos((m+n)*x)/(m+n)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \cos(mx)\sin(nx)\,dx=\frac{n\cos(mx)\cos(nx)+m\sin(mx)\sin(nx)}{m^2-n^2}$$

input
```
integrate(cos(m*x)*sin(n*x),x, algorithm="fricas")
```

output
```
(n*cos(m*x)*cos(n*x) + m*sin(m*x)*sin(n*x))/(m^2 - n^2)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int \cos(mx)\sin(nx)\,dx = \begin{cases} 0 & \text{for } m = 0 \wedge n = 0 \\ \frac{\sin^2(nx)}{2n} & \text{for } m = -n \vee m = n \\ \frac{m\sin(mx)\sin(nx)}{m^2-n^2} + \frac{n\cos(mx)\cos(nx)}{m^2-n^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(m*x)*sin(n*x),x)
```

output
```
Piecewise((0, Eq(m, 0) & Eq(n, 0)), (sin(n*x)**2/(2*n), Eq(m, n) | Eq(m, -
n)), (m*sin(m*x)*sin(n*x)/(m**2 - n**2) + n*cos(m*x)*cos(n*x)/(m**2 - n**2
), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \cos(mx)\sin(nx)\,dx = -\frac{\cos((m+n)x)}{2(m+n)} + \frac{\cos(-(m-n)x)}{2(m-n)}$$

input
```
integrate(cos(m*x)*sin(n*x),x, algorithm="maxima")
```

output
```
-1/2*cos((m + n)*x)/(m + n) + 1/2*cos(-(m - n)*x)/(m - n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \cos(mx)\sin(nx)\,dx = -\frac{\cos(mx+nx)}{2(m+n)} + \frac{\cos(mx-nx)}{2(m-n)}$$

input
```
integrate(cos(m*x)*sin(n*x),x, algorithm="giac")
```

output
```
-1/2*cos(m*x + n*x)/(m + n) + 1/2*cos(m*x - n*x)/(m - n)
```

Mupad [B] (verification not implemented)

Time = 16.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.74

$$\int \cos(mx)\sin(nx)\,dx$$
$$= \begin{cases} -\frac{\cos(2\,n\,x)}{4\,n} & \text{if } (m = n \vee m + n = 0) \wedge n \neq 0 \\ \frac{\cos(x\,(m-n))}{2\,m-2\,n} - \frac{\cos(x\,(m+n))}{2\,m+2\,n} & \text{if } (m = 0 \wedge n = 0) \vee (m \neq n \wedge ((m \neq n \wedge m + n \neq 0) \vee n = 0) \wedge m + n \end{cases}$$

input
```
int(cos(m*x)*sin(n*x),x)
```

output
```
piecewise((m == n | m + n == 0) & n ~= 0, -cos(2*n*x)/(4*n), m == 0 & n ==
 0 | m ~= n & (m ~= n & m + n ~= 0 | n == 0) & m + n ~= 0, - cos(x*(m + n)
)/(2*m + 2*n) + cos(x*(m - n))/(2*m - 2*n))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \cos(mx)\sin(nx)\,dx = \frac{\cos(mx)\cos(nx)\,n + \sin(mx)\sin(nx)\,m}{m^2 - n^2}$$

input
```
int(cos(m*x)*sin(n*x),x)
```

output
```
(cos(m*x)*cos(n*x)*n + sin(m*x)*sin(n*x)*m)/(m**2 - n**2)
```

3.14 $\int \cos(x)\sin(nx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \cos(x)\sin(nx)\,dx = \frac{\cos((1-n)x)}{2(1-n)} - \frac{\cos((1+n)x)}{2(1+n)}$$

output
```
cos((1-n)*x)/(2-2*n)-cos((1+n)*x)/(2+2*n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \cos(x)\sin(nx)\,dx = \frac{n\cos(x)\cos(nx)+\sin(x)\sin(nx)}{1-n^2}$$

input
```
Integrate[Cos[x]*Sin[n*x],x]
```

output
```
(n*Cos[x]*Cos[n*x] + Sin[x]*Sin[n*x])/(1 - n^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sin(nx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{1}{2}\sin((n+1)x)-\frac{1}{2}\sin((1-n)x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos((1-n)x)}{2(1-n)}-\frac{\cos((n+1)x)}{2(n+1)}$$

input
```
Int[Cos[x]*Sin[n*x],x]
```

output
```
Cos[(1 - n)*x]/(2*(1 - n)) - Cos[(1 + n)*x]/(2*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result
default	$-\frac{\cos(x(-1+n))}{2(-1+n)}-\frac{\cos((1+n)x)}{2(1+n)}$
risch	$-\frac{\cos(x(-1+n))}{2(-1+n)}-\frac{\cos((1+n)x)}{2(1+n)}$
parallelrisch	$\frac{(-1-n)\cos(x(-1+n))+(1-n)\cos((1+n)x)+2n}{2n^2-2}$
norman	$\frac{\frac{2n\tan\left(\frac{x}{2}\right)^2}{n^2-1}+\frac{2n\tan\left(\frac{nx}{2}\right)^2}{n^2-1}-\frac{4\tan\left(\frac{x}{2}\right)\tan\left(\frac{nx}{2}\right)}{n^2-1}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan\left(\frac{nx}{2}\right)^2\right)}$
orering	$-\frac{2(n^2+1)(-\sin(x)\sin(nx)+\cos(x)n\cos(nx))}{n^4-2n^2+1}-\frac{\sin(x)\sin(nx)-3\cos(x)n\cos(nx)+3\sin(x)n^2\sin(nx)-\cos(x)n^3\cos(nx)}{n^4-2n^2+1}$

input
```
int(cos(x)*sin(n*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-1+n)*cos(x*(-1+n))-1/2/(1+n)*cos((1+n)*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cos(x)\sin(nx)\,dx=-\frac{n\cos(nx)\cos(x)+\sin(nx)\sin(x)}{n^2-1}$$

input
```
integrate(cos(x)*sin(n*x),x, algorithm="fricas")
```

output
```
-(n*cos(n*x)*cos(x) + sin(n*x)*sin(x))/(n^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \cos(x)\sin(nx)\,dx = \begin{cases} -\frac{\sin^2(x)}{2} & \text{for } n=-1 \\ \frac{\sin^2(x)}{2} & \text{for } n=1 \\ -\frac{n\cos(x)\cos(nx)}{n^2-1} - \frac{\sin(x)\sin(nx)}{n^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(x)*sin(n*x),x)
```

output
```
Piecewise((-sin(x)**2/2, Eq(n, -1)), (sin(x)**2/2, Eq(n, 1)), (-n*cos(x)*c
os(n*x)/(n**2 - 1) - sin(x)*sin(n*x)/(n**2 - 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \cos(x)\sin(nx)\,dx = -\frac{\cos((n+1)x)}{2(n+1)} - \frac{\cos((n-1)x)}{2(n-1)}$$

input
```
integrate(cos(x)*sin(n*x),x, algorithm="maxima")
```

output
```
-1/2*cos((n + 1)*x)/(n + 1) - 1/2*cos((n - 1)*x)/(n - 1)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \cos(x)\sin(nx)\,dx = -\frac{\cos(nx+x)}{2(n+1)} - \frac{\cos(nx-x)}{2(n-1)}$$

input
```
integrate(cos(x)*sin(n*x),x, algorithm="giac")
```

output
```
-1/2*cos(n*x + x)/(n + 1) - 1/2*cos(n*x - x)/(n - 1)
```

Mupad [B] (verification not implemented)

Time = 16.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \cos(x)\sin(nx)\,dx = \begin{cases} \frac{\sin(x)^2}{2} & \text{if } n = 1 \\ \frac{\cos(x)^2}{2} & \text{if } n = -1 \\ -\frac{\cos(x\,(n-1))}{2\,n-2} - \frac{\cos(x\,(n+1))}{2\,n+2} & \text{if } n \neq -1 \land n \neq 1 \end{cases}$$

input
```
int(sin(n*x)*cos(x),x)
```

output
```
piecewise(n == 1, sin(x)^2/2, n == -1, cos(x)^2/2, n ~= -1 & n ~= 1, - cos
(x*(n - 1))/(2*n - 2) - cos(x*(n + 1))/(2*n + 2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \cos(x)\sin(nx)\,dx = \frac{-\cos(nx)\cos(x)\,n - \sin(nx)\sin(x)}{n^2 - 1}$$

input
```
int(cos(x)*sin(n*x),x)
```

output
```
( - (cos(n*x)*cos(x)*n + sin(n*x)*sin(x)))/(n**2 - 1)
```

3.15 $\int \cos(mx)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \cos(mx)\sin(x)\,dx = -\frac{\cos((1-m)x)}{2(1-m)} - \frac{\cos((1+m)x)}{2(1+m)}$$

output
```
-1/2*cos((1-m)*x)/(1-m)-cos((1+m)*x)/(2+2*m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \cos(mx)\sin(x)\,dx = \frac{\cos(x)\cos(mx) + m\sin(x)\sin(mx)}{-1+m^2}$$

input
```
Integrate[Cos[m*x]*Sin[x],x]
```

output
```
(Cos[x]*Cos[m*x] + m*Sin[x]*Sin[m*x])/(-1 + m^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(mx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{1}{2}\sin((1-m)x)+\frac{1}{2}\sin((m+1)x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((m+1)x)}{2(m+1)}$$

input
```
Int[Cos[m*x]*Sin[x],x]
```

output
```
-1/2*Cos[(1 - m)*x]/(1 - m) - Cos[(1 + m)*x]/(2*(1 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result
default	$\frac{\cos(x(-1+m))}{-2+2m}-\frac{\cos((1+m)x)}{2(1+m)}$
parallelrisch	$\frac{(1+m)\cos(x(-1+m))-2+(1-m)\cos((1+m)x)}{2m^2-2}$
risch	$\frac{\cos(x(-1+m))}{-2+2m}+\frac{\cos((1+m)x)}{2(1+m)(-1+m)}-\frac{\cos((1+m)x)m}{2(1+m)(-1+m)}$
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)^2}{m^2-1}-\frac{2\tan\left(\frac{mx}{2}\right)^2}{m^2-1}+\frac{4m\tan\left(\frac{x}{2}\right)\tan\left(\frac{mx}{2}\right)}{m^2-1}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan\left(\frac{mx}{2}\right)^2\right)}$
orering	$-\frac{2(m^2+1)(-\sin(x)\sin(mx)m+\cos(x)\cos(mx))}{m^4-2m^2+1}-\frac{3\sin(x)\sin(mx)m-3\cos(x)m^2\cos(mx)+\sin(x)m^3\sin(mx)-\cos(x)\,c}{m^4-2m^2+1}$

input
```
int(cos(m*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(x*(-1+m))/(-1+m)-1/2*cos((1+m)*x)/(1+m)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \cos(mx)\sin(x)\,dx=\frac{m\sin(mx)\sin(x)+\cos(mx)\cos(x)}{m^2-1}$$

input
```
integrate(cos(m*x)*sin(x),x, algorithm="fricas")
```

output
```
(m*sin(m*x)*sin(x) + cos(m*x)*cos(x))/(m^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \cos(mx)\sin(x)\,dx = \begin{cases} \frac{\sin^2(x)}{2} & \text{for } m = -1 \vee m = 1 \\ \frac{m\sin(x)\sin(mx)}{m^2-1} + \frac{\cos(x)\cos(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(m*x)*sin(x),x)
```

output
```
Piecewise((sin(x)**2/2, Eq(m, -1) | Eq(m, 1)), (m*sin(x)*sin(m*x)/(m**2 -
1) + cos(x)*cos(m*x)/(m**2 - 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \cos(mx)\sin(x)\,dx = -\frac{\cos((m+1)x)}{2(m+1)} + \frac{\cos(-(m-1)x)}{2(m-1)}$$

input
```
integrate(cos(m*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/2*cos((m + 1)*x)/(m + 1) + 1/2*cos(-(m - 1)*x)/(m - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \cos(mx)\sin(x)\,dx = -\frac{\cos(mx+x)}{2(m+1)} + \frac{\cos(mx-x)}{2(m-1)}$$

input
```
integrate(cos(m*x)*sin(x),x, algorithm="giac")
```

output
```
-1/2*cos(m*x + x)/(m + 1) + 1/2*cos(m*x - x)/(m - 1)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int \cos(mx)\sin(x)\,dx = \begin{cases} \frac{\sin(x)^2}{2} & \text{if } m = -1 \lor m = 1 \\ \frac{\cos(x\,(m-1))}{2\,m-2} - \frac{\cos(x\,(m+1))}{2\,m+2} & \text{if } m \neq -1 \land m \neq 1 \end{cases}$$

input
```
int(cos(m*x)*sin(x),x)
```

output
```
piecewise(m == -1 | m == 1, sin(x)^2/2, m ~= -1 & m ~= 1, cos(x*(m - 1))/(
2*m - 2) - cos(x*(m + 1))/(2*m + 2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \cos(mx)\sin(x)\,dx = \frac{\cos(mx)\cos(x) + \sin(mx)\sin(x)\,m}{m^2 - 1}$$

input
```
int(cos(m*x)*sin(x),x)
```

output
```
(cos(m*x)*cos(x) + sin(m*x)*sin(x)*m)/(m**2 - 1)
```

3.16 $\int \cos(2x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cos(2x)\sin(x)\,dx = \frac{\cos(x)}{2} - \frac{1}{6}\cos(3x)$$

output
```
1/2*cos(x)-1/6*cos(3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(2x)\sin(x)\,dx = \frac{\cos(x)}{2} - \frac{1}{6}\cos(3x)$$

input
```
Integrate[Cos[2*x]*Sin[x],x]
```

output
```
Cos[x]/2 - Cos[3*x]/6
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\cos(2x)dx$$

$$\Big\downarrow \text{4772}$$

$$\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

input
```
Int[Cos[2*x]*Sin[x],x]
```

output
```
Cos[x]/2 - Cos[3*x]/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$	12
risch	$\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$	12
parallelrisch	$\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}-\frac{1}{3}$	13
orering	$\frac{2\sin(x)\sin(2x)}{3}+\frac{\cos(x)\cos(2x)}{3}$	18
norman	$\frac{-\frac{2\tan(x)^{2}}{3}-\frac{2\tan\left(\frac{x}{2}\right)^{2}}{3}+\frac{8\tan\left(\frac{x}{2}\right)\tan(x)}{3}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(\tan(x)^{2}+1\right)}$	43

input
```
int(cos(2*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(x)-1/6*cos(3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \cos(2x)\sin(x)\,dx=-\frac{2}{3}\,\cos\left(x\right)^{3}+\cos\left(x\right)$$

input
```
integrate(cos(2*x)*sin(x),x, algorithm="fricas")
```

output
```
-2/3*cos(x)^3 + cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(2x)\sin(x)\,dx = \frac{2\sin(x)\sin(2x)}{3} + \frac{\cos(x)\cos(2x)}{3}$$

input
```
integrate(cos(2*x)*sin(x),x)
```

output
```
2*sin(x)*sin(2*x)/3 + cos(x)*cos(2*x)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(2x)\sin(x)\,dx = -\frac{1}{6}\cos(3x) + \frac{1}{2}\cos(x)$$

input
```
integrate(cos(2*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/6*cos(3*x) + 1/2*cos(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(2x)\sin(x)\,dx = -\frac{1}{6}\cos(3x) + \frac{1}{2}\cos(x)$$

input
```
integrate(cos(2*x)*sin(x),x, algorithm="giac")
```

output
```
-1/6*cos(3*x) + 1/2*cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \cos(2x)\sin(x)\,dx = \cos(x) - \frac{2\cos(x)^3}{3}$$

input
```
int(cos(2*x)*sin(x),x)
```

output
```
cos(x) - (2*cos(x)^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(2x)\sin(x)\,dx = \frac{\cos(2x)\cos(x)}{3} + \frac{2\sin(2x)\sin(x)}{3}$$

input
```
int(cos(2*x)*sin(x),x)
```

output
```
(cos(2*x)*cos(x) + 2*sin(2*x)*sin(x))/3
```

3.17 $\int \cos(3x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(3x)\sin(x)\,dx = \frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x)$$

output
```
1/4*cos(2*x)-1/8*cos(4*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(3x)\sin(x)\,dx = \frac{\cos^2(x)}{2} - \frac{1}{8}\cos(4x)$$

input
```
Integrate[Cos[3*x]*Sin[x],x]
```

output
```
Cos[x]^2/2 - Cos[4*x]/8
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\cos(3x)dx$$

$$\Big\downarrow \text{4772}$$

$$\frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x)$$

input
```
Int[Cos[3*x]*Sin[x],x]
```

output
```
Cos[2*x]/4 - Cos[4*x]/8
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$	14
risch	$\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$	14
parallelrisch	$-\frac{\cos(4x)}{8}-\frac{1}{8}+\frac{\cos(2x)}{4}$	15
orering	$\frac{3\sin(x)\sin(3x)}{8}+\frac{\cos(x)\cos(3x)}{8}$	18
norman	$\frac{-\frac{\tan\left(\frac{x}{2}\right)^{2}}{4}-\frac{\tan\left(\frac{3x}{2}\right)^{2}}{4}+\frac{3\tan\left(\frac{x}{2}\right)\tan\left(\frac{3x}{2}\right)}{2}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan\left(\frac{3x}{2}\right)^{2}\right)}$	49

input
```
int(cos(3*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/4*cos(2*x)-1/8*cos(4*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(3x)\sin(x)\,dx = -\cos\left(x\right)^{4} + \frac{3}{2}\cos\left(x\right)^{2}$$

input
```
integrate(cos(3*x)*sin(x),x, algorithm="fricas")
```

output
```
-cos(x)^4 + 3/2*cos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cos(3x)\sin(x)\,dx = \frac{3\sin(x)\sin(3x)}{8} + \frac{\cos(x)\cos(3x)}{8}$$

input
```
integrate(cos(3*x)*sin(x),x)
```

output
```
3*sin(x)*sin(3*x)/8 + cos(x)*cos(3*x)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(3x)\sin(x)\,dx = -\frac{1}{8}\cos(4x) + \frac{1}{4}\cos(2x)$$

input
```
integrate(cos(3*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/8*cos(4*x) + 1/4*cos(2*x)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(3x)\sin(x)\,dx = -\sin(x)^4 + \frac{1}{2}\sin(x)^2$$

input
```
integrate(cos(3*x)*sin(x),x, algorithm="giac")
```

output
```
-sin(x)^4 + 1/2*sin(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(3x)\sin(x)\,dx = \frac{3\cos(x)^2}{2} - \cos(x)^4$$

input
```
int(cos(3*x)*sin(x),x)
```

output
```
(3*cos(x)^2)/2 - cos(x)^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(3x)\sin(x)\,dx = \frac{\cos(3x)\cos(x)}{8} + \frac{3\sin(3x)\sin(x)}{8}$$

input
```
int(cos(3*x)*sin(x),x)
```

output
```
(cos(3*x)*cos(x) + 3*sin(3*x)*sin(x))/8
```

3.18 $\int \cos(4x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(4x)\sin(x)\,dx=\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

output
```
1/6*cos(3*x)-1/10*cos(5*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sin(x)\,dx=\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

input
```
Integrate[Cos[4*x]*Sin[x],x]
```

output
```
Cos[3*x]/6 - Cos[5*x]/10
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\cos(4x)dx$$

$$\Big\downarrow \text{4772}$$

$$\frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

input
```
Int[Cos[4*x]*Sin[x],x]
```

output
```
Cos[3*x]/6 - Cos[5*x]/10
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}$	14
risch	$\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}$	14
parallelrisch	$\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}-\frac{1}{15}$	15
orering	$\frac{\cos(x)\cos(4x)}{15}+\frac{4\sin(x)\sin(4x)}{15}$	18
norman	$\frac{-\frac{2\tan(2x)^2}{15}-\frac{2\tan\left(\frac{x}{2}\right)^2}{15}+\frac{16\tan\left(\frac{x}{2}\right)\tan(2x)}{15}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan(2x)^2\right)}$	49

input
```
int(sin(x)*cos(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*cos(3*x)-1/10*cos(5*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sin(x)\,dx = -\frac{8}{5}\cos(x)^5+\frac{8}{3}\cos(x)^3-\cos(x)$$

input
```
integrate(cos(4*x)*sin(x),x, algorithm="fricas")
```

output
```
-8/5*cos(x)^5 + 8/3*cos(x)^3 - cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cos(4x)\sin(x)\,dx = \frac{4\sin{(x)}\sin{(4x)}}{15} + \frac{\cos{(x)}\cos{(4x)}}{15}$$

input
```
integrate(cos(4*x)*sin(x),x)
```

output
```
4*sin(x)*sin(4*x)/15 + cos(x)*cos(4*x)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(4x)\sin(x)\,dx = -\frac{1}{10}\cos{(5x)} + \frac{1}{6}\cos{(3x)}$$

input
```
integrate(cos(4*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/10*cos(5*x) + 1/6*cos(3*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(4x)\sin(x)\,dx = -\frac{1}{10}\cos{(5x)} + \frac{1}{6}\cos{(3x)}$$

input
```
integrate(cos(4*x)*sin(x),x, algorithm="giac")
```

output
```
-1/10*cos(5*x) + 1/6*cos(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sin(x)\,dx = -\frac{8\cos(x)^5}{5} + \frac{8\cos(x)^3}{3} - \cos(x)$$

input
```
int(cos(4*x)*sin(x),x)
```

output
```
(8*cos(x)^3)/3 - cos(x) - (8*cos(x)^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sin(x)\,dx = \frac{\cos(4x)\cos(x)}{15} + \frac{4\sin(4x)\sin(x)}{15}$$

input
```
int(cos(4*x)*sin(x),x)
```

output
```
(cos(4*x)*cos(x) + 4*sin(4*x)*sin(x))/15
```

3.19 $\int \sin(mx)\tan(nx)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 102

$$\int \sin(mx)\tan(nx)\,dx = \frac{i\cos(mx)}{m} - \frac{ie^{-imx}\,\text{Hypergeometric2F1}\left(1,-\frac{m}{2n},1-\frac{m}{2n},-e^{2inx}\right)}{m} - \frac{ie^{imx}\,\text{Hypergeometric2F1}\left(1,\frac{m}{2n},1+\frac{m}{2n},-e^{2inx}\right)}{m}$$

output
```
I*cos(m*x)/m-I*hypergeom([1, -1/2*m/n],[1-1/2*m/n],-exp(2*I*n*x))/exp(I*m*
x)/m-I*exp(I*m*x)*hypergeom([1, 1/2*m/n],[1+1/2*m/n],-exp(2*I*n*x))/m
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 218 vs. 2(102) = 204.

Time = 0.15 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.14

$$\int \sin(mx)\tan(nx)\,dx$$
$$= ie^{-2imx}\left(e^{i(m+2n)x}m(m+2n)\,\text{Hypergeometric2F1}\left(1,1-\frac{m}{2n},2-\frac{m}{2n},-e^{2inx}\right)+(m-2n)\left(e^{i(3m+2n)x}m\,\text{H}\right.\right.$$

input
```
Integrate[Sin[m*x]*Tan[n*x],x]
```

output
```
((I/2)*(E^(I*(m + 2*n)*x)*m*(m + 2*n)*Hypergeometric2F1[1, 1 - m/(2*n), 2
- m/(2*n), -E^((2*I)*n*x)] + (m - 2*n)*(E^(I*(3*m + 2*n)*x)*m*Hypergeometr
ic2F1[1, 1 + m/(2*n), 2 + m/(2*n), -E^((2*I)*n*x)] - E^(I*m*x)*(m + 2*n)*(
Hypergeometric2F1[1, -1/2*m/n, 1 - m/(2*n), -E^((2*I)*n*x)] + E^((2*I)*m*x
)*Hypergeometric2F1[1, m/(2*n), 1 + m/(2*n), -E^((2*I)*n*x)]))))/(E^((2*I)
*m*x)*(m^3 - 4*m*n^2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5068, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(mx)\tan(nx)\,dx$$

$$\Big\downarrow \text{5068}$$

$$\int \left(-\frac{e^{-imx}}{1+e^{2inx}}+\frac{e^{imx}}{1+e^{2inx}}+\frac{1}{2}e^{-imx}-\frac{1}{2}e^{imx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ie^{-imx}\text{Hypergeometric2F1}\left(1,-\frac{m}{2n},1-\frac{m}{2n},-e^{2inx}\right)}{m}-\frac{ie^{imx}\text{Hypergeometric2F1}\left(1,\frac{m}{2n},\frac{m}{2n}+1,-e^{2inx}\right)}{m}+\frac{ie^{-imx}}{2m}+\frac{ie^{imx}}{2m}$$

input
```
Int[Sin[m*x]*Tan[n*x],x]
```

output
```
(I/2)/(E^(I*m*x)*m) + ((I/2)*E^(I*m*x))/m - (I*Hypergeometric2F1[1, -1/2*m
/n, 1 - m/(2*n), -E^((2*I)*n*x)])/(E^(I*m*x)*m) - (I*E^(I*m*x)*Hypergeomet
ric2F1[1, m/(2*n), 1 + m/(2*n), -E^((2*I)*n*x)])/m
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5068
```
Int[Sin[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(E^
(I*(a + b*x))*2) - E^(I*(a + b*x))/2 - 1/(E^(I*(a + b*x))*(1 + E^(2*I*(c +
d*x)))) + E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

Maple [F]

$$\int \sin(mx)\tan(nx)\,dx$$

input
```
int(sin(m*x)*tan(n*x),x)
```

output
```
int(sin(m*x)*tan(n*x),x)
```

Fricas [F]

$$\int \sin(mx)\tan(nx)\,dx = \int \sin(mx)\tan(nx)\,dx$$

input
```
integrate(sin(m*x)*tan(n*x),x, algorithm="fricas")
```

output
```
integral(sin(m*x)*tan(n*x), x)
```

Sympy [F]

$$\int \sin(mx)\tan(nx)\,dx = \int \sin(mx)\tan(nx)\,dx$$

input
```
integrate(sin(m*x)*tan(n*x),x)
```

output
```
Integral(sin(m*x)*tan(n*x), x)
```

Maxima [F]

$$\int \sin(mx)\tan(nx)\,dx = \int \sin(mx)\tan(nx)\,dx$$

input
```
integrate(sin(m*x)*tan(n*x),x, algorithm="maxima")
```

output
```
integrate(sin(m*x)*tan(n*x), x)
```

Giac [F]

$$\int \sin(mx)\tan(nx)\,dx = \int \sin(mx)\tan(nx)\,dx$$

input
```
integrate(sin(m*x)*tan(n*x),x, algorithm="giac")
```

output
```
integrate(sin(m*x)*tan(n*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(mx)\tan(nx)\,dx = \int \sin\left(m\,x\right)\,\tan\left(n\,x\right)\,dx$$

input
```
int(sin(m*x)*tan(n*x),x)
```

output
```
int(sin(m*x)*tan(n*x), x)
```

Reduce [F]

$$\int \sin(mx)\tan(nx)\,dx = \int \sin\left(mx\right)\tan\left(nx\right)dx$$

input
```
int(sin(m*x)*tan(n*x),x)
```

output
```
int(sin(m*x)*tan(n*x),x)
```

3.20 $\int \sin(x)\tan(nx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 85

$$\int \sin(x)\tan(nx)\,dx = i\cos(x) - ie^{-ix}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-e^{2inx}\right) - ie^{ix}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-e^{2inx}\right)$$

output
```
I*cos(x)-I*hypergeom([1, -1/2/n],[1-1/2/n],-exp(2*I*n*x))/exp(I*x)-I*exp(I
*x)*hypergeom([1, 1/2/n],[1+1/2/n],-exp(2*I*n*x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 200 vs. 2(85) = 170.

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.35

$$\int \sin(x)\tan(nx)\,dx = -\frac{ie^{-2ix}\left(e^{i(x+2nx)}(1+2n)\,\text{Hypergeometric2F1}\left(1,1-\frac{1}{2n},2-\frac{1}{2n},-e^{2inx}\right)+(-1+2n)\left(-e^{i(3+2n)x}\,\text{Hype}\right.\right.}{}$$

input
```
Integrate[Sin[x]*Tan[n*x],x]
```

output
```
((-1/2*I)*(E^(I*(x + 2*n*x))*(1 + 2*n)*Hypergeometric2F1[1, 1 - 1/(2*n), 2
 - 1/(2*n), -E^((2*I)*n*x)] + (-1 + 2*n)*(-(E^(I*(3 + 2*n)*x)*Hypergeometr
ic2F1[1, 1 + 1/(2*n), 2 + 1/(2*n), -E^((2*I)*n*x)]) + E^(I*x)*(1 + 2*n)*(H
ypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), -E^((2*I)*n*x)] + E^((2*I)*x)*H
ypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -E^((2*I)*n*x)]))))/(E^((2*I)*x)
*(-1 + 4*n^2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5068, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan(nx)\,dx$$

$$\Big\downarrow \text{5068}$$

$$\int \left(-\frac{e^{-ix}}{1+e^{2inx}}+\frac{e^{ix}}{1+e^{2inx}}+\frac{e^{-ix}}{2}-\frac{e^{ix}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-ie^{-ix}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-e^{2inx}\right)-ie^{ix}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-e^{2inx}\right)+\frac{1}{2}ie^{-ix}+\frac{1}{2}ie^{ix}$$

input
```
Int[Sin[x]*Tan[n*x],x]
```

output
```
(I/2)/E^(I*x) + (I/2)*E^(I*x) - (I*Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2
*n), -E^((2*I)*n*x)])/E^(I*x) - I*E^(I*x)*Hypergeometric2F1[1, 1/(2*n), (2
 + n^(-1))/2, -E^((2*I)*n*x)]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5068
```
Int[Sin[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(E^
(I*(a + b*x))*2) - E^(I*(a + b*x))/2 - 1/(E^(I*(a + b*x))*(1 + E^(2*I*(c +
d*x)))) + E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

Maple [F]

$$\int \sin(x)\tan(nx)\,dx$$

input
```
int(sin(x)*tan(n*x),x)
```

output
```
int(sin(x)*tan(n*x),x)
```

Fricas [F]

$$\int \sin(x)\tan(nx)\,dx = \int \sin(x)\tan(nx)\,dx$$

input
```
integrate(sin(x)*tan(n*x),x, algorithm="fricas")
```

output
```
integral(sin(x)*tan(n*x), x)
```

Sympy [F]

$$\int \sin(x)\tan(nx)\,dx = \int \sin{\left(x \right)} \tan{\left(n x \right)}\, dx$$

input
```
integrate(sin(x)*tan(n*x),x)
```

output
```
Integral(sin(x)*tan(n*x), x)
```

Maxima [F]

$$\int \sin(x)\tan(nx)\,dx = \int \sin\left(x\right) \tan\left(n x\right) \,{d x}$$

input
```
integrate(sin(x)*tan(n*x),x, algorithm="maxima")
```

output
```
integrate(sin(x)*tan(n*x), x)
```

Giac [F]

$$\int \sin(x)\tan(nx)\,dx = \int \sin\left(x\right) \tan\left(n x\right) \,{d x}$$

input
```
integrate(sin(x)*tan(n*x),x, algorithm="giac")
```

output
```
integrate(sin(x)*tan(n*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(x)\tan(nx)\,dx = \int \tan(n\,x)\,\sin\left(x\right)\,dx$$

input
```
int(tan(n*x)*sin(x),x)
```

output
```
int(tan(n*x)*sin(x), x)
```

Reduce [F]

$$\int \sin(x)\tan(nx)\,dx = \int \sin\left(x\right)\tan\left(nx\right)dx$$

input
```
int(sin(x)*tan(n*x),x)
```

output
```
int(sin(x)*tan(n*x),x)
```

3.21 $\int \sin(mx)\tan(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 88

$$\int \sin(mx)\tan(x)\,dx = \frac{i\cos(mx)}{m} - \frac{ie^{-imx}\,\text{Hypergeometric2F1}\left(1,-\frac{m}{2},1-\frac{m}{2},-e^{2ix}\right)}{m} - \frac{ie^{imx}\,\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},-e^{2ix}\right)}{m}$$

output
```
I*cos(m*x)/m-I*hypergeom([1, -1/2*m],[1-1/2*m],-exp(2*I*x))/exp(I*m*x)/m-I
*exp(I*m*x)*hypergeom([1, 1/2*m],[1+1/2*m],-exp(2*I*x))/m
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.88

$$\int \sin(mx)\tan(x)\,dx = \frac{ie^{-imx}\left(e^{2ix}m(2+m)\,\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},2-\frac{m}{2},-e^{2ix}\right)+(-2+m)\left(e^{2i(1+m)x}m\,\text{Hypergeom}\right.\right.}{}$$

input
```
Integrate[Sin[m*x]*Tan[x],x]
```

output
```
((I/2)*(E^((2*I)*x)*m*(2 + m)*Hypergeometric2F1[1, 1 - m/2, 2 - m/2, -E^((
2*I)*x)] + (-2 + m)*(E^((2*I)*(1 + m)*x)*m*Hypergeometric2F1[1, 1 + m/2, 2
 + m/2, -E^((2*I)*x)] - (2 + m)*(Hypergeometric2F1[1, -1/2*m, 1 - m/2, -E^
((2*I)*x)] + E^((2*I)*m*x)*Hypergeometric2F1[1, m/2, 1 + m/2, -E^((2*I)*x)
]))))/(E^(I*m*x)*m*(-4 + m^2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5068, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sin(mx)\,dx$$

$$\Big\downarrow \text{5068}$$

$$\int \left(\frac{1}{2}e^{-imx}-\frac{1}{2}e^{imx}-\frac{e^{-imx}}{1+e^{2ix}}+\frac{e^{imx}}{1+e^{2ix}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ie^{-imx}\,\text{Hypergeometric2F1}\left(1,-\frac{m}{2},1-\frac{m}{2},-e^{2ix}\right)}{m}-\frac{ie^{imx}\,\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},-e^{2ix}\right)}{m}+\frac{ie^{-imx}}{2m}+\frac{ie^{imx}}{2m}$$

input
```
Int[Sin[m*x]*Tan[x],x]
```

output
```
(I/2)/(E^(I*m*x)*m) + ((I/2)*E^(I*m*x))/m - (I*Hypergeometric2F1[1, -1/2*m
, 1 - m/2, -E^((2*I)*x)])/(E^(I*m*x)*m) - (I*E^(I*m*x)*Hypergeometric2F1[1
, m/2, (2 + m)/2, -E^((2*I)*x)])/m
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5068
```
Int[Sin[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(E^
(I*(a + b*x))*2) - E^(I*(a + b*x))/2 - 1/(E^(I*(a + b*x))*(1 + E^(2*I*(c +
d*x)))) + E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

Maple [F]

$$\int \sin(mx)\tan(x)\,dx$$

input
```
int(sin(m*x)*tan(x),x)
```

output
```
int(sin(m*x)*tan(x),x)
```

Fricas [F]

$$\int \sin(mx)\tan(x)\,dx = \int \sin(mx)\tan(x)\,\,dx$$

input
```
integrate(sin(m*x)*tan(x),x, algorithm="fricas")
```

output
```
integral(sin(m*x)*tan(x), x)
```

Sympy [F]

$$\int \sin(mx)\tan(x)\,dx = \int \sin(mx)\tan(x)\,dx$$

input
```
integrate(sin(m*x)*tan(x),x)
```

output
```
Integral(sin(m*x)*tan(x), x)
```

Maxima [F]

$$\int \sin(mx)\tan(x)\,dx = \int \sin(mx)\tan(x)\,dx$$

input
```
integrate(sin(m*x)*tan(x),x, algorithm="maxima")
```

output
```
1/2*((m^2 - 4)*cos((m + 2)*x)^3*sin(2*x) - (m^2 + 4*m + 4)*cos(m*x)^3*sin(
2*x) - (m^2 + (m^2 - 4)*cos(2*x) - 4)*sin((m + 2)*x)^3 + (m^2 + (m^2 + 4*m
 + 4)*cos(2*x) + 4*m + 4)*sin(m*x)^3 + ((m^2 - 4)*cos((m - 2)*x)*sin(2*x)
+ (m^2 - 4*m - 12)*cos(m*x)*sin(2*x) + (m^2 + (m^2 - 4)*cos(2*x) - 4)*sin(
(m - 2)*x) + (m^2 + (m^2 + 4*m + 4)*cos(2*x) + 4*m + 4)*sin(m*x))*cos((m +
 2)*x)^2 + ((m^2 - 4)*cos((m - 2)*x)*sin(2*x) + (m^2 + (m^2 - 4)*cos(2*x)
- 4)*sin((m - 2)*x))*cos(m*x)^2 + ((m^2 - 4)*cos((m + 2)*x)*sin(2*x) + (m^
2 - 4)*cos((m - 2)*x)*sin(2*x) - (m^2 + 4*m + 4)*cos(m*x)*sin(2*x) + (m^2
+ (m^2 - 4)*cos(2*x) - 4)*sin((m - 2)*x) - (m^2 + (m^2 - 4*m - 12)*cos(2*x
) - 4*m - 12)*sin(m*x))*sin((m + 2)*x)^2 + ((m^2 - 4)*cos((m - 2)*x)*sin(2
*x) - (m^2 + 4*m + 4)*cos(m*x)*sin(2*x) + (m^2 + (m^2 - 4)*cos(2*x) - 4)*s
in((m - 2)*x))*sin(m*x)^2 - ((m^2 + 8*m + 12)*cos(m*x)^2*sin(2*x) - (m^2 -
 4)*sin(m*x)^2*sin(2*x) - 2*(m^2 + (m^2 + 4*m + 4)*cos(2*x) + 4*m + 4)*cos
(m*x)*sin(m*x) - 2*((m^2 - 4)*cos((m - 2)*x)*sin(2*x) + (m^2 + (m^2 - 4)*c
os(2*x) - 4)*sin((m - 2)*x))*cos(m*x))*cos((m + 2)*x) - 4*((m^3 + (m^3 - 4
*m)*cos(2*x)^2 + (m^3 - 4*m)*sin(2*x)^2 + 2*(m^3 - 4*m)*cos(2*x) - 4*m)*co
s((m + 2)*x)^2 + 2*(m^3 + (m^3 - 4*m)*cos(2*x)^2 + (m^3 - 4*m)*sin(2*x)^2
+ 2*(m^3 - 4*m)*cos(2*x) - 4*m)*cos((m + 2)*x)*cos(m*x) + (m^3 + (m^3 - 4*
m)*cos(2*x)^2 + (m^3 - 4*m)*sin(2*x)^2 + 2*(m^3 - 4*m)*cos(2*x) - 4*m)*cos
(m*x)^2 + (m^3 + (m^3 - 4*m)*cos(2*x)^2 + (m^3 - 4*m)*sin(2*x)^2 + 2*(m...
```

Giac [F]

$$\int \sin(mx)\tan(x)\,dx = \int \sin(mx)\tan(x)\,dx$$

input
```
integrate(sin(m*x)*tan(x),x, algorithm="giac")
```

output
```
integrate(sin(m*x)*tan(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(mx)\tan(x)\,dx = \int \sin(m\,x)\,\tan(x)\,dx$$

input
```
int(sin(m*x)*tan(x),x)
```

output
```
int(sin(m*x)*tan(x), x)
```

Reduce [F]

$$\int \sin(mx)\tan(x)\,dx = \int \sin(mx)\tan(x)\,dx$$

input
```
int(sin(m*x)*tan(x),x)
```

output
```
int(sin(m*x)*tan(x),x)
```

3.22 $\int \sin(x)\tan(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 20

$$\int \sin(x)\tan(2x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}} - \sin(x)$$

output
```
1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)-sin(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sin(x)\tan(2x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}} - \sin(x)$$

input
```
Integrate[Sin[x]*Tan[2*x],x]
```

output
```
ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] - Sin[x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4878, 27, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan(2x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)\tan(2x)dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{2\sin^2(x)}{1-2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 27}$$

$$2\int \frac{\sin^2(x)}{1-2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 262}$$

$$2\left(\frac{1}{2}\int \frac{1}{1-2\sin^2(x)}d\sin(x)-\frac{\sin(x)}{2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$2\left(\frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{\sin(x)}{2}\right)$$

input
```
Int[Sin[x]*Tan[2*x],x]
```

output
```
2*(ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) - Sin[x]/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\operatorname{arctanh}\left(\sqrt{2}\,\sin(x)\right)\sqrt{2}}{2}-\sin\left(x\right)$	18
risch	$\frac{i e^{i x}}{2}-\frac{i e^{-i x}}{2}-\frac{\sqrt{2}\,\ln\left(e^{2 i x}-i\sqrt{2}\,e^{i x}-1\right)}{4}+\frac{\sqrt{2}\,\ln\left(e^{2 i x}+i\sqrt{2}\,e^{i x}-1\right)}{4}$	66

input
```
int(sin(x)*tan(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(2^(1/2)*sin(x))*2^(1/2)-sin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \sin(x)\tan(2x)\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \sin(x)$$

input
```
integrate(sin(x)*tan(2*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - s
in(x)
```

Sympy [F]

$$\int \sin(x)\tan(2x)\,dx = \int \sin(x)\tan(2x)\,dx$$

input
```
integrate(sin(x)*tan(2*x),x)
```

output
```
Integral(sin(x)*tan(2*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 7.05

$$\int \sin(x)\tan(2x)\,dx = \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) + \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) - \sin(x)$$

input
```
integrate(sin(x)*tan(2*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - sin(x)
```

Giac [F]

$$\int \sin(x)\tan(2x)\,dx = \int \sin(x)\tan(2x)\,dx$$

input
```
integrate(sin(x)*tan(2*x),x, algorithm="giac")
```

output
```
integrate(sin(x)*tan(2*x), x)
```

Mupad [B] (verification not implemented)

Time = 18.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sin(x)\tan(2x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin\left(x\right)\right)}{2} - \sin\left(x\right)$$

input
```
int(tan(2*x)*sin(x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*sin(x)))/2 - sin(x)
```

Reduce [F]

$$\int \sin(x)\tan(2x)\,dx = \int \sin\left(x\right)\tan\left(2x\right)dx$$

input
```
int(sin(x)*tan(2*x),x)
```

output
```
int(sin(x)*tan(2*x),x)
```

3.23 $\int \sin(x)\tan(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 24

$$\int \sin(x)\tan(3x)\,dx = \frac{1}{3}\text{arctanh}\left(\frac{3\sin(x)}{1+2\sin^2(x)}\right) - \sin(x)$$

output
```
1/3*arctanh(3*sin(x)/(1+2*sin(x)^2))-sin(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \sin(x)\tan(3x)\,dx = \frac{1}{3}\text{arctanh}(\sin(x)) + \frac{1}{3}\text{arctanh}(2\sin(x)) - \sin(x)$$

input
```
Integrate[Sin[x]*Tan[3*x],x]
```

output
```
ArcTanh[Sin[x]]/3 + ArcTanh[2*Sin[x]]/3 - Sin[x]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(24) = 48.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4878, 1602, 27, 1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\tan(3x)dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{\sin^2(x)\left(3-4\sin^2(x)\right)}{4\sin^4(x)-5\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{1602}$$

$$-\frac{1}{4}\int -\frac{4\left(1-2\sin^2(x)\right)}{4\sin^4(x)-5\sin^2(x)+1}d\sin(x)-\sin(x)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1-2\sin^2(x)}{4\sin^4(x)-5\sin^2(x)+1}d\sin(x)-\sin(x)$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{4}\int \frac{1}{\sin^2(x)-\frac{\sin(x)}{2}-\frac{1}{2}}d\sin(x)-\frac{1}{4}\int \frac{1}{\sin^2(x)+\frac{\sin(x)}{2}-\frac{1}{2}}d\sin(x)-\sin(x)$$

$$\Big\downarrow \text{1081}$$

$$-\frac{1}{4}\int \left(-\frac{2}{3(\sin(x)+1)}-\frac{4}{3(1-2\sin(x))}\right)d\sin(x)-\frac{1}{4}\int \left(-\frac{4}{3(2\sin(x)+1)}-\frac{2}{3(1-\sin(x))}\right)d\sin(x)-\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$-\sin(x)+\frac{1}{4}\left(\frac{2}{3}\log(\sin(x)+1)-\frac{2}{3}\log(1-2\sin(x))\right)+\frac{1}{4}\left(\frac{2}{3}\log(2\sin(x)+1)-\frac{2}{3}\log(1-\sin(x))\right)$$

input
```
Int[Sin[x]*Tan[3*x],x]
```

output
```
((-2*Log[1 - 2*Sin[x]])/3 + (2*Log[1 + Sin[x]])/3)/4 + ((-2*Log[1 - Sin[x]
])/3 + (2*Log[1 + 2*Sin[x]])/3)/4 - Sin[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

method	result	size
default	$\frac{\ln(1+\sin(x))}{6}-\frac{\ln(\sin(x)-1)}{6}+\frac{\ln(2\sin(x)+1)}{6}-\frac{\ln(2\sin(x)-1)}{6}-\sin(x)$	38
risch	$\frac{i e^{ix}}{2}-\frac{i e^{-ix}}{2}+\frac{\ln(e^{ix}+i)}{3}-\frac{\ln(e^{ix}-i)}{3}+\frac{\ln(i e^{ix}+e^{2ix}-1)}{6}-\frac{\ln(-i e^{ix}+e^{2ix}-1)}{6}$	76

input
```
int(sin(x)*tan(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(1+sin(x))-1/6*ln(sin(x)-1)+1/6*ln(2*sin(x)+1)-1/6*ln(2*sin(x)-1)-si
n(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \sin(x)\tan(3x)\,dx = \frac{1}{6}\log\left(2\sin(x)+1\right)+\frac{1}{6}\log\left(\sin(x)+1\right) - \frac{1}{6}\log\left(-\sin(x)+1\right)-\frac{1}{6}\log\left(-2\sin(x)+1\right)-\sin(x)$$

input
```
integrate(sin(x)*tan(3*x),x, algorithm="fricas")
```

output
```
1/6*log(2*sin(x) + 1) + 1/6*log(sin(x) + 1) - 1/6*log(-sin(x) + 1) - 1/6*l
og(-2*sin(x) + 1) - sin(x)
```

Sympy [F]

$$\int \sin(x)\tan(3x)\,dx = \int \sin{(x)}\tan{(3x)}\,dx$$

input
```
integrate(sin(x)*tan(3*x),x)
```

output
```
Integral(sin(x)*tan(3*x), x)
```

Maxima [F]

$$\int \sin(x)\tan(3x)\,dx = \int \sin\left(x\right)\tan\left(3\,x\right)\,dx$$

input
```
integrate(sin(x)*tan(3*x),x, algorithm="maxima")
```

output
```
integrate(-1/3*((cos(3*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - c
os(2*x)*cos(x) + (sin(3*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*
x)*sin(x) + cos(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 -
 sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/6
*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/6*log(cos(x)^2 + sin(x)^2 - 2
*sin(x) + 1) - sin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(22) = 44$.

Time = 0.22 (sec) , antiderivative size = 364, normalized size of antiderivative = 15.17

$$\int \sin(x)\tan(3x)\,dx = \text{Too large to display}$$

input
```
integrate(sin(x)*tan(3*x),x, algorithm="giac")
```

output
```
1/12*(log((tan(1/2*x)^4 + 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 + 8*tan(1/2*x)
+ 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - log((tan(1/2*x)^4
 - 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 - 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*
tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 2*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1
)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - 2*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x)
+ 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + log((tan(1/2*x)^4 + 8*tan(1/2*x)^3
 + 18*tan(1/2*x)^2 + 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)
) - log((tan(1/2*x)^4 - 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 - 8*tan(1/2*x) +
1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)) + 2*log(2*(tan(1/2*x)^2 + 2*tan(1/
2*x) + 1)/(tan(1/2*x)^2 + 1)) - 2*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/
(tan(1/2*x)^2 + 1)) - 24*tan(1/2*x))/(tan(1/2*x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 17.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \sin(x)\tan(3x)\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3} + \frac{\mathrm{atanh}(2\,\sin(x))}{3} - \sin(x)$$

input
```
int(tan(3*x)*sin(x),x)
```

output
```
(2*atanh(sin(x/2)/cos(x/2)))/3 + atanh(2*sin(x))/3 - sin(x)
```

Reduce [F]

$$\int \sin(x)\tan(3x)\,dx = \int \sin(x)\tan(3x)\,dx$$

input
```
int(sin(x)*tan(3*x),x)
```

output
```
int(sin(x)*tan(3*x),x)
```

3.24 $\int \sin(x)\tan(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 77

$$\int \sin(x)\tan(4x)\,dx=\frac{\operatorname{arctanh}\left(\sqrt{2\left(2-\sqrt{2}\right)}\sin(x)\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\operatorname{arctanh}\left(\sqrt{2\left(2+\sqrt{2}\right)}\sin(x)\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\sin(x)$$

output
```
1/2*arctanh((4-2*2^(1/2))^(1/2)*sin(x))/(4-2*2^(1/2))^(1/2)+1/2*arctanh((4
+2*2^(1/2))^(1/2)*sin(x))/(4+2*2^(1/2))^(1/2)-sin(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \sin(x)\tan(4x)\,dx=\frac{1}{4}\left(\sqrt{2-\sqrt{2}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\sqrt{2+\sqrt{2}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)-4\sin(x)\right)$$

input
```
Integrate[Sin[x]*Tan[4*x],x]
```

output
```
(Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]] + Sqrt[2 + Sqrt[2
]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]] - 4*Sin[x])/4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4878, 27, 1602, 27, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\tan(4x)dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{4\sin^2(x)\left(1-2\sin^2(x)\right)}{8\sin^4(x)-8\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{\sin^2(x)\left(1-2\sin^2(x)\right)}{8\sin^4(x)-8\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{1602}$$

$$4\left(-\frac{1}{8}\int -\frac{2\left(1-4\sin^2(x)\right)}{8\sin^4(x)-8\sin^2(x)+1}d\sin(x)-\frac{\sin(x)}{4}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{4}\int \frac{1-4\sin^2(x)}{8\sin^4(x)-8\sin^2(x)+1}d\sin(x)-\frac{\sin(x)}{4}\right)$$

$$\Big\downarrow \text{1480}$$

$$4\left(\frac{1}{4}\left(-\left(\left(2-\sqrt{2}\right)\int \frac{1}{8\sin^2(x)-2\left(2-\sqrt{2}\right)}d\sin(x)\right)-\left(2+\sqrt{2}\right)\int \frac{1}{8\sin^2(x)-2\left(2+\sqrt{2}\right)}d\sin(x)\right)-\frac{\sin(x)}{4}\right)$$

$$\Big\downarrow \text{220}$$

$$4\left(\frac{1}{4}\left(\frac{1}{4}\sqrt{2-\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)\right)-\frac{\sin(x)}{4}\right)$$

input
```
Int[Sin[x]*Tan[4*x],x]
```

output
```
4*(((Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2
+ Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]])/4)/4 - Sin[x]/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

method	result
risch	$\frac{i e^{ix}}{2}-\frac{i e^{-ix}}{2}-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(128\_Z^{4}-32\_Z^{2}+1\right)}\_R\ln\left(e^{2ix}-4i\_R e^{ix}-1\right)\right)}{2}$
default	$\frac{\sqrt{2}\,\sqrt{2+\sqrt{2}}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{4}+\frac{\left(\sqrt{2}-2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}-\sin(x)-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}$

input
```
int(sin(x)*tan(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*exp(I*x)-1/2*I*exp(-I*x)-1/2*sum(_R*ln(exp(2*I*x)-4*I*_R*exp(I*x)-1)
,_R=RootOf(128*_Z^4-32*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

$$\int \sin(x)\tan(4x)\,dx = \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}+2\,\sin\left(x\right)\right) - \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}-2\,\sin\left(x\right)\right) + \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}+2\,\sin\left(x\right)\right) - \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}-2\,\sin\left(x\right)\right) - \sin\left(x\right)$$

input
```
integrate(sin(x)*tan(4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + 2*sin(x)) - 1/8*sqrt(sqrt(2)
 + 2)*log(sqrt(sqrt(2) + 2) - 2*sin(x)) + 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(
-sqrt(2) + 2) + 2*sin(x)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)
- 2*sin(x)) - sin(x)
```

Sympy [F]

$$\int \sin(x)\tan(4x)\,dx = \int \sin\left(x\right)\tan\left(4x\right)\,dx$$

input
```
integrate(sin(x)*tan(4*x),x)
```

output
```
Integral(sin(x)*tan(4*x), x)
```

Maxima [F]

$$\int \sin(x)\tan(4x)\,dx = \int \sin\left(x\right)\tan\left(4\,x\right)\,dx$$

input
```
integrate(sin(x)*tan(4*x),x, algorithm="maxima")
```

output
```
integrate(((cos(7*x) + cos(x))*cos(8*x) + (sin(7*x) + sin(x))*sin(8*x) + c
os(7*x) + cos(x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1), x) - sin(x)
```

Giac [F]

$$\int \sin(x)\tan(4x)\,dx = \int \sin\left(x\right)\tan\left(4\,x\right)\,dx$$

input
```
integrate(sin(x)*tan(4*x),x, algorithm="giac")
```

output
```
integrate(sin(x)*tan(4*x), x)
```

Mupad [B] (verification not implemented)

Time = 16.92 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.34

$$\int \sin(x)\tan(4x)\,dx = \frac{\mathrm{atanh}\left(\frac{34\,\sin(x)\,\sqrt{\sqrt{2}+2}+24\,\sqrt{2}\,\sin(x)\,\sqrt{\sqrt{2}+2}}{41\,\sqrt{2}+58}\right)\,\sqrt{\sqrt{2}+2}}{4} - \sin\left(x\right) - \frac{\mathrm{atanh}\left(\frac{34\,\sin(x)\,\sqrt{2-\sqrt{2}}-24\,\sqrt{2}\,\sin(x)\,\sqrt{2-\sqrt{2}}}{41\,\sqrt{2}-58}\right)\,\sqrt{2-\sqrt{2}}}{4}$$

input
```
int(tan(4*x)*sin(x),x)
```

output
```
(atanh((34*sin(x)*(2^(1/2) + 2)^(1/2) + 24*2^(1/2)*sin(x)*(2^(1/2) + 2)^(1
/2))/(41*2^(1/2) + 58))*(2^(1/2) + 2)^(1/2))/4 - sin(x) - (atanh((34*sin(x
)*(2 - 2^(1/2))^(1/2) - 24*2^(1/2)*sin(x)*(2 - 2^(1/2))^(1/2))/(41*2^(1/2)
 - 58))*(2 - 2^(1/2))^(1/2))/4
```

Reduce [F]

$$\int \sin(x)\tan(4x)\,dx = \int \sin{(x)}\tan{(4x)}\,dx$$

input
```
int(sin(x)*tan(4*x),x)
```

output
```
int(sin(x)*tan(4*x),x)
```

3.25 $\int \sin(x)\tan(5x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 86

$$\int \sin(x)\tan(5x)\,dx = \frac{1}{5}\text{arctanh}(\sin(x)) + \frac{1}{5}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\text{arctanh}\left(\sqrt{2\left(3-\sqrt{5}\right)}\sin(x)\right) + \frac{1}{5}\sqrt{\frac{2}{3+\sqrt{5}}}\text{arctanh}\left(\sqrt{2\left(3+\sqrt{5}\right)}\sin(x)\right) - \sin(x)$$

output
```
1/5*arctanh(sin(x))+1/5*(1/2+1/2*5^(1/2))*arctanh((5^(1/2)-1)*sin(x))+1/5*
2^(1/2)/(3+5^(1/2))^(1/2)*arctanh((5^(1/2)+1)*sin(x))-sin(x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.16

$$\int \sin(x)\tan(5x)\,dx = \frac{1}{20}\left(4\text{arctanh}(\sin(x)) + \left(-1+\sqrt{5}\right)\log\left(1-\sqrt{5}-4\sin(x)\right) - \left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\sin(x)\right) - \left(-1+\sqrt{5}\right)\log\left(1-\sqrt{5}+4\sin(x)\right) + \left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\sin(x)\right) - 20\sin(x)\right)$$

input
```
Integrate[Sin[x]*Tan[5*x],x]
```

output
```
(4*ArcTanh[Sin[x]] + (-1 + Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]] - (1 + Sqr
t[5])*Log[1 + Sqrt[5] - 4*Sin[x]] - (-1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Sin
[x]] + (1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Sin[x]] - 20*Sin[x])/20
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4878, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan(5x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)\tan(5x)dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{\sin^2(x)\left(16\sin^4(x)-20\sin^2(x)+5\right)}{-16\sin^6(x)+28\sin^4(x)-13\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{ 2460}$$

$$\int \left(\frac{2(\sin(x)-1)}{5\left(4\sin^2(x)+2\sin(x)-1\right)}-\frac{1}{5\left(\sin^2(x)-1\right)}-\frac{2(\sin(x)+1)}{5\left(4\sin^2(x)-2\sin(x)-1\right)}-1\right)d\sin(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}\text{arctanh}(\sin(x))-\sin(x)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\sin(x)-\sqrt{5}+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\sin(x)+\sqrt{5}+1\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\sin(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(4\sin(x)+\sqrt{5}+1\right)$$

input
```
Int[Sin[x]*Tan[5*x],x]
```

output
```
ArcTanh[Sin[x]]/5 - ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]])/20 - ((1 +
 Sqrt[5])*Log[1 + Sqrt[5] - 4*Sin[x]])/20 + ((1 - Sqrt[5])*Log[1 - Sqrt[5]
 + 4*Sin[x]])/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Sin[x]])/20 - Sin[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.74

method	result
risch	$\frac{i\,\mathrm{e}^{ix}}{2}-\frac{i\,\mathrm{e}^{-ix}}{2}+\frac{\ln\left(\mathrm{e}^{ix}+i\right)}{5}-\frac{\ln\left(\mathrm{e}^{ix}-i\right)}{5}-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{i\left(\sqrt{5}+1\right)\mathrm{e}^{ix}}{2}-1\right)}{20}-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{i\left(\sqrt{5}+1\right)\mathrm{e}^{ix}}{2}-1\right)\sqrt{5}}{20}-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{i\left(\sqrt{5}-\right.}{2}\right.}{20}$

input
```
int(sin(x)*tan(5*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*exp(I*x)-1/2*I*exp(-I*x)+1/5*ln(exp(I*x)+I)-1/5*ln(exp(I*x)-I)-1/20*
ln(exp(2*I*x)-1/2*I*(5^(1/2)+1)*exp(I*x)-1)-1/20*ln(exp(2*I*x)-1/2*I*(5^(1
/2)+1)*exp(I*x)-1)*5^(1/2)-1/20*ln(exp(2*I*x)+1/2*I*(5^(1/2)-1)*exp(I*x)-1
)+1/20*ln(exp(2*I*x)+1/2*I*(5^(1/2)-1)*exp(I*x)-1)*5^(1/2)+1/20*ln(exp(2*I
*x)-1/2*I*(5^(1/2)-1)*exp(I*x)-1)-1/20*ln(exp(2*I*x)-1/2*I*(5^(1/2)-1)*exp
(I*x)-1)*5^(1/2)+1/20*ln(exp(2*I*x)+1/2*I*(5^(1/2)+1)*exp(I*x)-1)+1/20*ln(
exp(2*I*x)+1/2*I*(5^(1/2)+1)*exp(I*x)-1)*5^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(47) = 94$.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.58

$$\int \sin(x)\tan(5x)\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{8\cos(x)^2 - 4(\sqrt{5}-1)\sin(x) + \sqrt{5} - 11}{4\cos(x)^2 + 2\sin(x) - 3}\right) + \frac{1}{20}\sqrt{5}\log\left(-\frac{8\cos(x)^2 - 4(\sqrt{5}+1)\sin(x) - \sqrt{5} - 11}{4\cos(x)^2 - 2\sin(x) - 3}\right) - \frac{1}{20}\log\left(4\cos(x)^2 + 2\sin(x) - 3\right) + \frac{1}{20}\log\left(4\cos(x)^2 - 2\sin(x) - 3\right) + \frac{1}{10}\log(\sin(x) + 1) - \frac{1}{10}\log(-\sin(x) + 1) - \sin(x)$$

input
```
integrate(sin(x)*tan(5*x),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log((8*cos(x)^2 - 4*(sqrt(5) - 1)*sin(x) + sqrt(5) - 11)/(4*c
os(x)^2 + 2*sin(x) - 3)) + 1/20*sqrt(5)*log(-(8*cos(x)^2 - 4*(sqrt(5) + 1)
*sin(x) - sqrt(5) - 11)/(4*cos(x)^2 - 2*sin(x) - 3)) - 1/20*log(4*cos(x)^2
 + 2*sin(x) - 3) + 1/20*log(4*cos(x)^2 - 2*sin(x) - 3) + 1/10*log(sin(x) +
 1) - 1/10*log(-sin(x) + 1) - sin(x)
```

Sympy [F]

$$\int \sin(x)\tan(5x)\,dx = \int \sin{(x)}\tan{(5x)}\,dx$$

input
```
integrate(sin(x)*tan(5*x),x)
```

output
```
Integral(sin(x)*tan(5*x), x)
```

Maxima [F]

$$\int \sin(x)\tan(5x)\,dx = \int \sin{(x)}\tan{(5\,x)}\,dx$$

input
```
integrate(sin(x)*tan(5*x),x, algorithm="maxima")
```

output
```
integrate(-1/5*((3*cos(7*x) - cos(5*x) - cos(3*x) + 3*cos(x))*cos(8*x) - 3
*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(7*x) + (cos(5*x) + cos(3*x) - 3*
cos(x))*cos(6*x) - (cos(4*x) - cos(2*x) + 1)*cos(5*x) - (cos(3*x) - 3*cos(
x))*cos(4*x) + (cos(2*x) - 1)*cos(3*x) - 3*cos(2*x)*cos(x) + (3*sin(7*x) -
 sin(5*x) - sin(3*x) + 3*sin(x))*sin(8*x) - 3*(sin(6*x) - sin(4*x) + sin(2
*x))*sin(7*x) + (sin(5*x) + sin(3*x) - 3*sin(x))*sin(6*x) - (sin(4*x) - si
n(2*x))*sin(5*x) - (sin(3*x) - 3*sin(x))*sin(4*x) + sin(3*x)*sin(2*x) - 3*
sin(2*x)*sin(x) + 3*cos(x))/(2*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*
x) - cos(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(c
os(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) +
 sin(2*x))*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(
6*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1),
x) + 1/10*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/10*log(cos(x)^2 + si
n(x)^2 - 2*sin(x) + 1) - sin(x)
```

Giac [F]

$$\int \sin(x)\tan(5x)\,dx = \int \sin(x)\tan(5x)\,dx$$

input
```
integrate(sin(x)*tan(5*x),x, algorithm="giac")
```

output
```
integrate(sin(x)*tan(5*x), x)
```

Mupad [B] (verification not implemented)

Time = 17.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.24

$$\int \sin(x)\tan(5x)\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{5} + \frac{\mathrm{atan}\left(\frac{\sin(x)\,1042\mathrm{i}-\sqrt{5}\,\sin(x)\,466\mathrm{i}}{377\,\sqrt{5}-843}\right)\,1\mathrm{i}}{10} - \frac{\mathrm{atanh}\left(\sin(x)-\sqrt{5}\,\sin(x)\right)}{10} - \sin(x) - \frac{\sqrt{5}\,\mathrm{atanh}\left(\sin(x)-\sqrt{5}\,\sin(x)\right)}{10} - \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sin(x)\,1042\mathrm{i}-\sqrt{5}\,\sin(x)\,466\mathrm{i}}{377\,\sqrt{5}-843}\right)\,1\mathrm{i}}{10}$$

input
```
int(tan(5*x)*sin(x),x)
```

output
```
(atan((sin(x)*1042i - 5^(1/2)*sin(x)*466i)/(377*5^(1/2) - 843))*1i)/10 - a
tanh(sin(x) - 5^(1/2)*sin(x))/10 + (2*atanh(sin(x/2)/cos(x/2)))/5 - sin(x)
 - (5^(1/2)*atanh(sin(x) - 5^(1/2)*sin(x)))/10 - (5^(1/2)*atan((sin(x)*104
2i - 5^(1/2)*sin(x)*466i)/(377*5^(1/2) - 843))*1i)/10
```

Reduce [F]

$$\int \sin(x)\tan(5x)\,dx = \int \sin(x)\tan(5x)\,dx$$

input
```
int(sin(x)*tan(5*x),x)
```

output
```
int(sin(x)*tan(5*x),x)
```

3.26 $\int \sin(x)\tan(6x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 89

$$\int \sin(x)\tan(6x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}} + \frac{\operatorname{arctanh}\left(2\sqrt{2-\sqrt{3}}\sin(x)\right)}{6\sqrt{2-\sqrt{3}}} + \frac{\operatorname{arctanh}\left(2\sqrt{2+\sqrt{3}}\sin(x)\right)}{6\sqrt{2+\sqrt{3}}} - \sin(x)$$

output
```
1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*(1/2*6^(1/2)-1/2*2^(1/2)
)*sin(x))/(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*(1/2*6^(1/2)+1/2*2^(1/2)
)*sin(x))/(1/2*6^(1/2)+1/2*2^(1/2))-sin(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \sin(x)\tan(6x)\,dx = \frac{1}{6}\left(\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right) + \sqrt{2-\sqrt{3}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right) + \sqrt{2+\sqrt{3}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right) - 6\sin(x)\right)$$

input
```
Integrate[Sin[x]*Tan[6*x],x]
```

output
```
(Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]] + Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])/Sq
rt[2 - Sqrt[3]]] + Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]]
 - 6*Sin[x])/6
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4878, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\tan(6x)dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{2\sin^2(x)\left(16\sin^4(x)-16\sin^2(x)+3\right)}{-32\sin^6(x)+48\sin^4(x)-18\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\sin^2(x)\left(16\sin^4(x)-16\sin^2(x)+3\right)}{-32\sin^6(x)+48\sin^4(x)-18\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{2460}$$

$$2\int \left(\frac{1-8\sin^2(x)}{3\left(16\sin^4(x)-16\sin^2(x)+1\right)}-\frac{1}{6\left(2\sin^2(x)-1\right)}-\frac{1}{2}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{6\sqrt{2}}+\frac{1}{12}\sqrt{2-\sqrt{3}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{12}\sqrt{2+\sqrt{3}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)-\frac{\sin(x)}{2}\right)$$

input
```
Int[Sin[x]*Tan[6*x],x]
```

output
```
2*(ArcTanh[Sqrt[2]*Sin[x]]/(6*Sqrt[2]) + (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin
[x])/Sqrt[2 - Sqrt[3]]])/12 + (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2
 + Sqrt[3]]])/12 - Sin[x]/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

method	result
risch	$\frac{i e^{ix}}{2} - \frac{i e^{-ix}}{2} - \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1296\_Z^4-144\_Z^2+1\right)} \_R \ln\left(e^{2ix}-6i\_R e^{ix}-1\right)\right)}{2} - \frac{\sqrt{2}\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{12} + \frac{\sqrt{2}\ln\left(e^{2}\right.}{}$

input
```
int(sin(x)*tan(6*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*exp(I*x)-1/2*I*exp(-I*x)-1/2*sum(_R*ln(exp(2*I*x)-6*I*_R*exp(I*x)-1)
,_R=RootOf(1296*_Z^4-144*_Z^2+1))-1/12*2^(1/2)*ln(exp(2*I*x)-I*2^(1/2)*exp
(I*x)-1)+1/12*2^(1/2)*ln(exp(2*I*x)+I*2^(1/2)*exp(I*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(65) = 130.

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \sin(x)\tan(6x)\,dx = &\ \frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}+2\sin(x)\right)\\ &-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}-2\sin(x)\right)\\ &+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}+2\sin(x)\right)\\ &-\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}-2\sin(x)\right)\\ &+\frac{1}{12}\sqrt{2}\log\left(-\frac{2\cos(x)^2-2\sqrt{2}\sin(x)-3}{2\cos(x)^2-1}\right)-\sin(x)\end{aligned}$$

input
```
integrate(sin(x)*tan(6*x),x, algorithm="fricas")
```

output
```
1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) + 2*sin(x)) - 1/12*sqrt(sqrt(
3) + 2)*log(sqrt(sqrt(3) + 2) - 2*sin(x)) + 1/12*sqrt(-sqrt(3) + 2)*log(sq
rt(-sqrt(3) + 2) + 2*sin(x)) - 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) +
 2) - 2*sin(x)) + 1/12*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2
*cos(x)^2 - 1)) - sin(x)
```

Sympy [F]

$$\int \sin(x)\tan(6x)\,dx = \int \sin{(x)}\tan{(6x)}\,dx$$

input
```
integrate(sin(x)*tan(6*x),x)
```

output
```
Integral(sin(x)*tan(6*x), x)
```

Maxima [F]

$$\int \sin(x)\tan(6x)\,dx = \int \sin{(x)}\tan{(6\,x)}\;dx$$

input
```
integrate(sin(x)*tan(6*x),x, algorithm="maxima")
```

output
```
1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) -
2*sqrt(2)*sin(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(
2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)
^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + integrate(-1/3*((2*cos(7*x
) - cos(5*x) - cos(3*x) + 2*cos(x))*cos(8*x) - 2*(cos(4*x) - 1)*cos(7*x) +
 (cos(4*x) - 1)*cos(5*x) + (cos(3*x) - 2*cos(x))*cos(4*x) + (2*sin(7*x) -
sin(5*x) - sin(3*x) + 2*sin(x))*sin(8*x) + (sin(3*x) - 2*sin(x))*sin(4*x)
- 2*sin(7*x)*sin(4*x) + sin(5*x)*sin(4*x) - cos(3*x) + 2*cos(x))/(2*(cos(4
*x) - 1)*cos(8*x) - cos(8*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(
4*x) - sin(4*x)^2 + 2*cos(4*x) - 1), x) - sin(x)
```

Giac [F]

$$\int \sin(x)\tan(6x)\,dx = \int \sin(x)\tan(6x)\,dx$$

input
```
integrate(sin(x)*tan(6*x),x, algorithm="giac")
```

output
```
integrate(sin(x)*tan(6*x), x)
```

Mupad [B] (verification not implemented)

Time = 18.57 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

$$\int \sin(x)\tan(6x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x)\right)}{6} - \sin(x) - \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x)-\sqrt{6}\,\sin(x)\right)}{12} - \frac{\sqrt{6}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x)-\sqrt{6}\,\sin(x)\right)}{12} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sin(x)\,102818\mathrm{i}-\sqrt{6}\,\sin(x)\,59362\mathrm{i}}{40545\,\sqrt{2}\,\sqrt{6}-140452}\right)\,1\mathrm{i}}{12} - \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sin(x)\,102818\mathrm{i}-\sqrt{6}\,\sin(x)\,59362\mathrm{i}}{40545\,\sqrt{2}\,\sqrt{6}-140452}\right)\,1\mathrm{i}}{12}$$

input
```
int(tan(6*x)*sin(x),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*sin(x)*102818i - 6^(1/2)*sin(x)*59362i)/(40545*2^(1
/2)*6^(1/2) - 140452))*1i)/12 - sin(x) - (6^(1/2)*atan((2^(1/2)*sin(x)*102
818i - 6^(1/2)*sin(x)*59362i)/(40545*2^(1/2)*6^(1/2) - 140452))*1i)/12 + (
2^(1/2)*atanh(2^(1/2)*sin(x)))/6 - (2^(1/2)*atanh(2^(1/2)*sin(x) - 6^(1/2)
*sin(x)))/12 - (6^(1/2)*atanh(2^(1/2)*sin(x) - 6^(1/2)*sin(x)))/12
```

Reduce [F]

$$\int \sin(x)\tan(6x)\,dx = \int \sin(x)\tan(6x)\,dx$$

input
```
int(sin(x)*tan(6*x),x)
```

output
```
int(sin(x)*tan(6*x),x)
```

3.27 $\int \cot(2x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \cot(2x)\sin(x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(x)) + \sin(x)$$

output
```
-1/2*arctanh(sin(x))+sin(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cot(2x)\sin(x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(x)) + \sin(x)$$

input
```
Integrate[Cot[2*x]*Sin[x],x]
```

output
```
-1/2*ArcTanh[Sin[x]] + Sin[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4878, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cot(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(2x)}{\csc(x)}dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{1-2\sin^2(x)}{2\left(1-\sin^2(x)\right)}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1-2\sin^2(x)}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{299}$$

$$\frac{1}{2}\left(2\sin(x)-\int \frac{1}{1-\sin^2(x)}d\sin(x)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}(2\sin(x)-\operatorname{arctanh}(\sin(x)))$$

input
```
Int[Cot[2*x]*Sin[x],x]
```

output
```
(-ArcTanh[Sin[x]] + 2*Sin[x])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{\ln(\sec(x)+\tan(x))}{2}+\sin\left(x\right)$	12
risch	$-\frac{i e^{i x}}{2}+\frac{i e^{-i x}}{2}+\frac{\ln\left(e^{i x}-i\right)}{2}-\frac{\ln\left(e^{i x}+i\right)}{2}$	40

input
```
int(cot(2*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(sec(x)+tan(x))+sin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \cot(2x)\sin(x)\,dx = -\frac{1}{4}\log\left(\sin\left(x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(x\right)+1\right)+\sin\left(x\right)$$

input
```
integrate(cot(2*x)*sin(x),x, algorithm="fricas")
```

output
```
-1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1) + sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \cot(2x)\sin(x)\,dx = \frac{\log\left(\sin\left(x\right)-1\right)}{4}-\frac{\log\left(\sin\left(x\right)+1\right)}{4}+\sin\left(x\right)$$

input
```
integrate(cot(2*x)*sin(x),x)
```

output
```
log(sin(x) - 1)/4 - log(sin(x) + 1)/4 + sin(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(8) = 16$.

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.70

$$\int \cot(2x)\sin(x)\,dx = -\frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) + \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right) + \sin(x)$$

input
```
integrate(cot(2*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
 - 2*sin(x) + 1) + sin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \cot(2x)\sin(x)\,dx = -\frac{1}{4}\log(\sin(x) + 1) + \frac{1}{4}\log(-\sin(x) + 1) + \sin(x)$$

input
```
integrate(cot(2*x)*sin(x),x, algorithm="giac")
```

output
```
-1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1) + sin(x)
```

Mupad [B] (verification not implemented)

Time = 17.53 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cot(2x)\sin(x)\,dx = \sin(x) - \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

input
```
int(cot(2*x)*sin(x),x)
```

output
```
sin(x) - atanh(tan(x/2))
```

Reduce [F]

$$\int \cot(2x)\sin(x)\,dx = \int \cot(2x)\sin(x)\,dx$$

input
```
int(cot(2*x)*sin(x),x)
```

output
```
int(cot(2*x)*sin(x),x)
```

3.28 $\int \cot(3x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 20

$$\int \cot(3x)\sin(x)\,dx = -\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sin(x)$$

output
```
-1/3*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)+sin(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \cot(3x)\sin(x)\,dx = -\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sin(x)$$

input
```
Integrate[Cot[3*x]*Sin[x],x]
```

output
```
-(ArcTanh[(2*Sin[x])/Sqrt[3]]/Sqrt[3]) + Sin[x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4878, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cot(3x)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\cot(3x)}{\csc(x)}dx$$
$$\Big\downarrow \ 4878$$
$$\int \frac{1-4\sin^2(x)}{3-4\sin^2(x)}d\sin(x)$$
$$\Big\downarrow \ 299$$
$$\sin(x)-2\int \frac{1}{3-4\sin^2(x)}d\sin(x)$$
$$\Big\downarrow \ 219$$
$$\sin(x)-\frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[Cot[3*x]*Sin[x],x]
```

output
```
-(ArcTanh[(2*Sin[x])/Sqrt[3]]/Sqrt[3]) + Sin[x]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.30

method	result	size
risch	$-\frac{i e^{ix}}{2}+\frac{i e^{-ix}}{2}+\frac{\sqrt{3}\,\ln\left(e^{2ix}-i\sqrt{3}\,e^{ix}-1\right)}{6}-\frac{\sqrt{3}\,\ln\left(e^{2ix}+i\sqrt{3}\,e^{ix}-1\right)}{6}$	66

input
```
int(cot(3*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*exp(I*x)+1/2*I*exp(-I*x)+1/6*3^(1/2)*ln(exp(2*I*x)-I*3^(1/2)*exp(I*
x)-1)-1/6*3^(1/2)*ln(exp(2*I*x)+I*3^(1/2)*exp(I*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \cot(3x)\sin(x)\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{4\cos(x)^2 + 4\sqrt{3}\sin(x) - 7}{4\cos(x)^2 - 1}\right) + \sin(x)$$

input
```
integrate(cot(3*x)*sin(x),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) + s
in(x)
```

Sympy [F]

$$\int \cot(3x)\sin(x)\,dx = \int \sin(x)\cot(3x)\,dx$$

input
```
integrate(cot(3*x)*sin(x),x)
```

output
```
Integral(sin(x)*cot(3*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(16) = 32$.

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 6.35

$$\int \cot(3x)\sin(x)\,dx = -\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)-\frac{4}{3}\cos(x)+\frac{4}{3}\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2-\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2-\frac{4}{3}\sqrt{3}\sin(x)-\frac{4}{3}\cos(x)+\frac{4}{3}\right)+\sin(x)$$

input
```
integrate(cot(3*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*c
os(x) + 4/3) - 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*
sin(x) - 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2
- 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 +
 4/3*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) + sin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(16) = 32.

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \cot(3x)\sin(x)\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\sin(x)\right|}{\left|4\sqrt{3}+8\sin(x)\right|}\right)+\sin(x)$$

input
```
integrate(cot(3*x)*sin(x),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) + si
n(x)
```

Mupad [B] (verification not implemented)

Time = 17.66 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \cot(3x)\sin(x)\,dx = \sin(x) - \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,\sin(x)}{3}\right)}{3}$$

input
```
int(cot(3*x)*sin(x),x)
```

output
```
sin(x) - (3^(1/2)*atanh((2*3^(1/2)*sin(x))/3))/3
```

Reduce [F]

$$\int \cot(3x)\sin(x)\,dx = \int \cot(3x)\sin(x)\,dx$$

input
```
int(cot(3*x)*sin(x),x)
```

output
```
int(cot(3*x)*sin(x),x)
```

3.29 $\int \cot(4x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 28

$$\int \cot(4x)\sin(x)\,dx = -\frac{1}{4}\text{arctanh}(\sin(x)) - \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} + \sin(x)$$

output
```
-1/4*arctanh(sin(x))-1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)+sin(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \cot(4x)\sin(x)\,dx = -\frac{1}{4}\text{arctanh}(\sin(x)) - \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} + \sin(x)$$

input
```
Integrate[Cot[4*x]*Sin[x],x]
```

output
```
-1/4*ArcTanh[Sin[x]] - ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + Sin[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4878, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cot(4x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cot(4x)}{\csc(x)}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{8\sin^4(x)-8\sin^2(x)+1}{4\left(2\sin^4(x)-3\sin^2(x)+1\right)}d\sin(x)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\int \frac{8\sin^4(x)-8\sin^2(x)+1}{2\sin^4(x)-3\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{ 2205}$$

$$\frac{1}{4}\int \left(4-\frac{3-4\sin^2(x)}{2\sin^4(x)-3\sin^2(x)+1}\right)d\sin(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(-\text{arctanh}(\sin(x))-\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin(x)\right)+4\sin(x)\right)$$

input
```
Int[Cot[4*x]*Sin[x],x]
```

output
```
(-ArcTanh[Sin[x]] - Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]] + 4*Sin[x])/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.14

method	result	size
risch	$-\frac{i e^{i x}}{2}+\frac{i e^{-i x}}{2}-\frac{\ln \left(e^{i x}+i\right)}{4}+\frac{\ln \left(e^{i x}-i\right)}{4}+\frac{\sqrt{2}\, \ln \left(e^{2 i x}-i \sqrt{2}\, e^{i x}-1\right)}{8}-\frac{\sqrt{2}\, \ln \left(e^{2 i x}+i \sqrt{2}\, e^{i x}-1\right)}{8}$	88

input
```
int(cot(4*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*exp(I*x)+1/2*I*exp(-I*x)-1/4*ln(exp(I*x)+I)+1/4*ln(exp(I*x)-I)+1/8*
2^(1/2)*ln(exp(2*I*x)-I*2^(1/2)*exp(I*x)-1)-1/8*2^(1/2)*ln(exp(2*I*x)+I*2^
(1/2)*exp(I*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(20) = 40$.

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \cot(4x)\sin(x)\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2 + 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1) + \sin(x)$$

input
```
integrate(cot(4*x)*sin(x),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1) + sin(x)
```

Sympy [F]

$$\int \cot(4x)\sin(x)\,dx = \int \sin(x)\cot(4x)\,dx$$

input
```
integrate(cot(4*x)*sin(x),x)
```

output
```
Integral(sin(x)*cot(4*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(20) = 40$.

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 6.18

$$\int \cot(4x)\sin(x)\,dx = -\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) + \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) + \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) + \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right) + \sin(x)$$

input
```
integrate(cot(4*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*s
in(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) -
 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt
(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x
)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)
^2 + 2*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + sin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(20) = 40$.

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \cot(4x)\sin(x)\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\sin(x)\right|}{\left|2\sqrt{2} + 4\sin(x)\right|}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1) + \sin(x)$$

input
```
integrate(cot(4*x)*sin(x),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1) + sin(x)
```

Mupad [B] (verification not implemented)

Time = 18.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \cot(4x)\sin(x)\,dx = \sin(x) - \frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2} - \frac{\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\sin(x)\right)}{4}$$

input
```
int(cot(4*x)*sin(x),x)
```

output
```
sin(x) - atanh(sin(x/2)/cos(x/2))/2 - (2^(1/2)*atanh(2^(1/2)*sin(x)))/4
```

Reduce [F]

$$\int \cot(4x)\sin(x)\,dx = \int \cot(4x)\sin(x)\,dx$$

input
```
int(cot(4*x)*sin(x),x)
```

output
```
int(cot(4*x)*sin(x),x)
```

3.30 $\int \cot(5x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 81

$$\int \cot(5x)\sin(x)\,dx = -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{5}\left(5-\sqrt{5}\right)}\sin(x)\right) - \sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\operatorname{arctanh}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sin(x)\right)+\sin(x)$$

output
```
-1/10*(10+2*5^(1/2))^(1/2)*arctanh(1/5*(50-10*5^(1/2))^(1/2)*sin(x))-2^(1/
2)/(25+5*5^(1/2))^(1/2)*arctanh(1/5*(50+10*5^(1/2))^(1/2)*sin(x))+sin(x)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \cot(5x)\sin(x)\,dx = \frac{1}{10}\left(-\sqrt{10-2\sqrt{5}}\operatorname{arctanh}\left(\sqrt{2+\frac{2}{\sqrt{5}}}\sin(x)\right) - \sqrt{2\left(5+\sqrt{5}\right)}\operatorname{arctanh}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right)+10\sin(x)\right)$$

input
```
Integrate[Cot[5*x]*Sin[x],x]
```

output
```
(-(Sqrt[10 - 2*Sqrt[5]]*ArcTanh[Sqrt[2 + 2/Sqrt[5]]*Sin[x]]) - Sqrt[2*(5 +
 Sqrt[5])]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Sin[x]] + 10*Sin[x])/10
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4878, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cot(5x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cot(5x)}{\csc(x)}dx$$
$$\Big\downarrow \text{4878}$$
$$\int \frac{16\sin^4(x)-12\sin^2(x)+1}{16\sin^4(x)-20\sin^2(x)+5}d\sin(x)$$
$$\Big\downarrow \text{2205}$$
$$\int \left(1-\frac{4(1-2\sin^2(x))}{16\sin^4(x)-20\sin^2(x)+5}\right)d\sin(x)$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\text{arctanh}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sin(x)\right)+\sin(x)$$

input
```
Int[Cot[5*x]*Sin[x],x]
```

output
```
-1/5*(Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Sin[x]]) - (Sq
rt[(5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Sin[x]])/5 + Sin[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{i e^{i x}}{2}+\frac{i e^{-i x}}{2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(125\_Z^{4}-25\_Z^{2}+1\right)}\_R\ln\left(e^{2 i x}-5 i\_R\,e^{i x}-1\right)\right)}{2}$	55

input
```
int(cot(5*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*exp(I*x)+1/2*I*exp(-I*x)+1/2*sum(_R*ln(exp(2*I*x)-5*I*_R*exp(I*x)-1
),_R=RootOf(125*_Z^4-25*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.32

$$\int \cot(5x)\sin(x)\,dx = -\frac{1}{10}\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}+2\sin(x)\right) + \frac{1}{10}\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}-2\sin(x)\right) - \frac{1}{10}\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}+2\sin(x)\right) + \frac{1}{10}\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}-2\sin(x)\right) + \sin(x)$$

input
```
integrate(cot(5*x)*sin(x),x, algorithm="fricas")
```

output
```
-1/10*sqrt(1/2*sqrt(5) + 5/2)*log(sqrt(1/2*sqrt(5) + 5/2) + 2*sin(x)) + 1/
10*sqrt(1/2*sqrt(5) + 5/2)*log(sqrt(1/2*sqrt(5) + 5/2) - 2*sin(x)) - 1/10*
sqrt(-1/2*sqrt(5) + 5/2)*log(sqrt(-1/2*sqrt(5) + 5/2) + 2*sin(x)) + 1/10*s
qrt(-1/2*sqrt(5) + 5/2)*log(sqrt(-1/2*sqrt(5) + 5/2) - 2*sin(x)) + sin(x)
```

Sympy [F]

$$\int \cot(5x)\sin(x)\,dx = \int \sin(x)\cot(5x)\,dx$$

input
```
integrate(cot(5*x)*sin(x),x)
```

output
```
Integral(sin(x)*cot(5*x), x)
```

Maxima [F]

$$\int \cot(5x)\sin(x)\,dx = \int \cot(5x)\sin(x)\,dx$$

input
```
integrate(cot(5*x)*sin(x),x, algorithm="maxima")
```

output
```
-integrate(1/2*((cos(3*x) + cos(2*x) + cos(x))*cos(4*x) + (2*cos(2*x) + 2*
cos(x) + 1)*cos(3*x) + cos(3*x)^2 + (2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 +
 cos(x)^2 + (sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + 2*(sin(2*x) + sin(x)
)*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + cos(
x))/(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2
*x) + cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*
x)^2 + cos(x)^2 + 2*(sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 +
 2*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin
(x) + sin(x)^2 + 2*cos(x) + 1), x) - integrate(-1/2*((cos(3*x) - cos(2*x)
+ cos(x))*cos(4*x) + (2*cos(2*x) - 2*cos(x) + 1)*cos(3*x) - cos(3*x)^2 + (
2*cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + (sin(3*x) - sin(2*x) + si
n(x))*sin(4*x) + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2
+ 2*sin(2*x)*sin(x) - sin(x)^2 + cos(x))/(2*(cos(3*x) - cos(2*x) + cos(x)
- 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - cos(3*x)
^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) - sin(2
*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) - sin
(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x) +
sin(x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

$$\int \cot(5x)\sin(x)\,dx = -\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+5}+\sin(x)\right|\right) + \frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+5}+\sin(x)\right|\right) - \frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|\sqrt{-\frac{1}{8}\sqrt{5}+\frac{5}{8}}+\sin(x)\right|\right) + \frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|-\sqrt{-\frac{1}{8}\sqrt{5}+\frac{5}{8}}+\sin(x)\right|\right) + \sin(x)$$

input
```
integrate(cot(5*x)*sin(x),x, algorithm="giac")
```

output
```
-1/20*sqrt(2*sqrt(5) + 10)*log(abs(1/2*sqrt(1/2)*sqrt(sqrt(5) + 5) + sin(x
))) + 1/20*sqrt(2*sqrt(5) + 10)*log(abs(-1/2*sqrt(1/2)*sqrt(sqrt(5) + 5) +
 sin(x))) - 1/20*sqrt(-2*sqrt(5) + 10)*log(abs(sqrt(-1/8*sqrt(5) + 5/8) +
sin(x))) + 1/20*sqrt(-2*sqrt(5) + 10)*log(abs(-sqrt(-1/8*sqrt(5) + 5/8) +
sin(x))) + sin(x)
```

Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.47

$$\int \cot(5x)\sin(x)\,dx = \sin(x) - \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\frac{25\sqrt{2}\sin(x)\sqrt{\sqrt{5}+5}}{2}+\frac{11\sqrt{2}\sqrt{5}\sin(x)\sqrt{\sqrt{5}+5}}{2}}{20\sqrt{5}+45}\right)\sqrt{\sqrt{5}+5}}{10} + \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\frac{25\sqrt{2}\sin(x)\sqrt{5-\sqrt{5}}}{2}-\frac{11\sqrt{2}\sqrt{5}\sin(x)\sqrt{5-\sqrt{5}}}{2}}{20\sqrt{5}-45}\right)\sqrt{5-\sqrt{5}}}{10}$$

input
```
int(cot(5*x)*sin(x),x)
```

output
```
sin(x) - (2^(1/2)*atanh(((25*2^(1/2)*sin(x)*(5^(1/2) + 5)^(1/2))/2 + (11*2
^(1/2)*5^(1/2)*sin(x)*(5^(1/2) + 5)^(1/2))/2)/(20*5^(1/2) + 45))*(5^(1/2)
+ 5)^(1/2))/10 + (2^(1/2)*atanh(((25*2^(1/2)*sin(x)*(5 - 5^(1/2))^(1/2))/2
 - (11*2^(1/2)*5^(1/2)*sin(x)*(5 - 5^(1/2))^(1/2))/2)/(20*5^(1/2) - 45))*(
5 - 5^(1/2))^(1/2))/10
```

Reduce [F]

$$\int \cot(5x)\sin(x)\,dx = \int \cot(5x)\sin(x)\,dx$$

input
```
int(cot(5*x)*sin(x),x)
```

output
```
int(cot(5*x)*sin(x),x)
```

3.31 $\int \cot(6x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 38

$$\int \cot(6x)\sin(x)\,dx = -\frac{1}{6}\text{arctanh}(\sin(x)) - \frac{1}{6}\text{arctanh}(2\sin(x)) - \frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \sin(x)$$

output
```
-1/6*arctanh(sin(x))-1/6*arctanh(2*sin(x))-1/6*arctanh(2/3*sin(x)*3^(1/2))
*3^(1/2)+sin(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \cot(6x)\sin(x)\,dx = -\frac{1}{6}\text{arctanh}(\sin(x)) - \frac{1}{6}\text{arctanh}(2\sin(x)) - \frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \sin(x)$$

input
```
Integrate[Cot[6*x]*Sin[x],x]
```

output
```
-1/6*ArcTanh[Sin[x]] - ArcTanh[2*Sin[x]]/6 - ArcTanh[(2*Sin[x])/Sqrt[3]]/(
2*Sqrt[3]) + Sin[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4878, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cot(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(6x)}{\csc(x)}dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{-32\sin^6(x)+48\sin^4(x)-18\sin^2(x)+1}{2\left(-16\sin^6(x)+32\sin^4(x)-19\sin^2(x)+3\right)}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{-32\sin^6(x)+48\sin^4(x)-18\sin^2(x)+1}{-16\sin^6(x)+32\sin^4(x)-19\sin^2(x)+3}d\sin(x)$$

$$\Big\downarrow \text{2460}$$

$$\frac{1}{2}\int \left(\frac{2}{4\sin^2(x)-3}+\frac{2}{3\left(4\sin^2(x)-1\right)}+2+\frac{1}{3\left(\sin^2(x)-1\right)}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{3}\text{arctanh}(\sin(x))-\frac{1}{3}\text{arctanh}(2\sin(x))-\frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}+2\sin(x)\right)$$

input
```
Int[Cot[6*x]*Sin[x],x]
```

output
```
(-1/3*ArcTanh[Sin[x]] - ArcTanh[2*Sin[x]]/3 - ArcTanh[(2*Sin[x])/Sqrt[3]]/
Sqrt[3] + 2*Sin[x])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.26

method	result
risch	$-\frac{i e^{ix}}{2}+\frac{i e^{-ix}}{2}+\frac{\ln\left(e^{ix}-i\right)}{6}-\frac{\ln\left(e^{ix}+i\right)}{6}+\frac{\sqrt{3}\,\ln\left(e^{2ix}-i\sqrt{3}\,e^{ix}-1\right)}{12}-\frac{\sqrt{3}\,\ln\left(e^{2ix}+i\sqrt{3}\,e^{ix}-1\right)}{12}+\frac{\ln\left(-i e^{ix}+e^{2ix}-1\right)}{12}$

input
```
int(cot(6*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*exp(I*x)+1/2*I*exp(-I*x)+1/6*ln(exp(I*x)-I)-1/6*ln(exp(I*x)+I)+1/12
*3^(1/2)*ln(exp(2*I*x)-I*3^(1/2)*exp(I*x)-1)-1/12*3^(1/2)*ln(exp(2*I*x)+I*
3^(1/2)*exp(I*x)-1)+1/12*ln(-I*exp(I*x)+exp(2*I*x)-1)-1/12*ln(I*exp(I*x)+e
xp(2*I*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(28) = 56$.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\begin{aligned}\int \cot(6x)\sin(x)\,dx &= \frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\sin(x)-7}{4\cos(x)^2-1}\right)\\ &\quad-\frac{1}{12}\log(2\sin(x)+1)-\frac{1}{12}\log(\sin(x)+1)\\ &\quad+\frac{1}{12}\log(-\sin(x)+1)+\frac{1}{12}\log(-2\sin(x)+1)+\sin(x)\end{aligned}$$

input
```
integrate(cot(6*x)*sin(x),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) -
1/12*log(2*sin(x) + 1) - 1/12*log(sin(x) + 1) + 1/12*log(-sin(x) + 1) + 1/
12*log(-2*sin(x) + 1) + sin(x)
```

Sympy [F]

$$\int \cot(6x)\sin(x)\,dx = \int \sin(x)\cot(6x)\,dx$$

input
```
integrate(cot(6*x)*sin(x),x)
```

output
```
Integral(sin(x)*cot(6*x), x)
```

Maxima [F]

$$\int \cot(6x)\sin(x)\,dx = \int \cot(6x)\sin(x)\,dx$$

input
```
integrate(cot(6*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*c
os(x) + 4/3) - 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*
sin(x) - 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2
- 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 +
 4/3*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) - integrate(-1/6*((
cos(3*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - cos(2*x)*cos(x) +
(sin(3*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)*sin(x) + cos(x
))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*s
in(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 1/12*log(cos(x)^2 +
sin(x)^2 + 2*sin(x) + 1) + 1/12*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) +
sin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(28) = 56$.

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \cot(6x)\sin(x)\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\sin(x)\right|}{\left|4\sqrt{3}+8\sin(x)\right|}\right) - \frac{1}{12}\log(\sin(x)+1) + \frac{1}{12}\log(-\sin(x)+1) - \frac{1}{12}\log(|2\sin(x)+1|) + \frac{1}{12}\log(|2\sin(x)-1|) + \sin(x)$$

input
```
integrate(cot(6*x)*sin(x),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) - 1
/12*log(sin(x) + 1) + 1/12*log(-sin(x) + 1) - 1/12*log(abs(2*sin(x) + 1))
+ 1/12*log(abs(2*sin(x) - 1)) + sin(x)
```

Mupad [B] (verification not implemented)

Time = 18.53 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \cot(6x)\sin(x)\,dx = \sin(x) - \frac{\operatorname{atanh}(2\sin(x))}{6} - \frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3} - \frac{\sqrt{3}\operatorname{atanh}\left(\frac{2\sqrt{3}\sin(x)}{3}\right)}{6}$$

input
```
int(cot(6*x)*sin(x),x)
```

output
```
sin(x) - atanh(2*sin(x))/6 - atanh(sin(x/2)/cos(x/2))/3 - (3^(1/2)*atanh((
2*3^(1/2)*sin(x))/3))/6
```

Reduce [F]

$$\int \cot(6x)\sin(x)\,dx = \int \cot(6x)\sin(x)\,dx$$

input
```
int(cot(6*x)*sin(x),x)
```

output
```
int(cot(6*x)*sin(x),x)
```

3.32 $\int \sec(2x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \sec(2x)\sin(x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \sec(2x)\sin(x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}-\tan\left(\frac{x}{2}\right)\right)+\operatorname{arctanh}\left(\sqrt{2}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{2}}$$

input
```
Integrate[Sec[2*x]*Sin[x],x]
```

output
```
(ArcTanh[Sqrt[2] - Tan[x/2]] + ArcTanh[Sqrt[2] + Tan[x/2]])/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4857, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sec(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\cos(2x)}dx$$

$$\Big\downarrow \text{4857}$$

$$-\int \frac{1}{2\cos^2(x)-1}d\cos(x)$$

$$\Big\downarrow \text{220}$$

$$\frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

input
```
Int[Sec[2*x]*Sin[x],x]
```

output
```
ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\operatorname{arctanh}\left(\sqrt{2}\,\cos(x)\right)\sqrt{2}}{2}$	13
risch	$\frac{\sqrt{2}\,\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+1\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-\sqrt{2}\,e^{ix}+1\right)}{4}$	47

input
```
int(sec(2*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(2^(1/2)*cos(x))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \sec(2x)\sin(x)\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\,\cos(x)^2+2\sqrt{2}\cos(x)+1}{2\,\cos(x)^2-1}\right)$$

input
```
integrate(sec(2*x)*sin(x),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

Sympy [F]

$$\int \sec(2x)\sin(x)\,dx = \int \sin(x)\sec(2x)\,dx$$

input
```
integrate(sec(2*x)*sin(x),x)
```

output
```
Integral(sin(x)*sec(2*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(12) = 24$.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 8.60

$$\int \sec(2x)\sin(x)\,dx = \frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x) + 2\left(\sqrt{2}\cos(x)+1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 1\right) - \frac{1}{8}\sqrt{2}\log\left(-2\sqrt{2}\sin(2x)\sin(x) - 2\left(\sqrt{2}\cos(x)-1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 1\right)$$

input
```
integrate(sec(2*x)*sin(x),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x
) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
 1) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*
cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*c
os(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.27

$$\int \sec(2x)\sin(x)\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}\right)$$

input
```
integrate(sec(2*x)*sin(x),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sq
rt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \sec(2x)\sin(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\cos(x)\right)}{2}$$

input
```
int(sin(x)/cos(2*x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*cos(x)))/2
```

Reduce [F]

$$\int \sec(2x)\sin(x)\,dx = \int \frac{\sin(x)}{\cos(2x)}dx - 1$$

input
```
int(sec(2*x)*sin(x),x)
```

output
```
int(sin(x)/cos(2*x),x) - 1
```

3.33 $\int \sec(3x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \sec(3x)\sin(x)\,dx = \frac{1}{3}\log(\cos(x)) - \frac{1}{6}\log\left(3-4\cos^2(x)\right)$$

output
```
1/3*ln(cos(x))-1/6*ln(3-4*cos(x)^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sec(3x)\sin(x)\,dx = -\frac{1}{3}\text{arctanh}\left(\frac{1}{3}\left(-5+8\sin^2(x)\right)\right)$$

input
```
Integrate[Sec[3*x]*Sin[x],x]
```

output
```
-1/3*ArcTanh[(-5 + 8*Sin[x]^2)/3]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4857, 25, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sec(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\cos(3x)}dx$$

$$\Big\downarrow \text{4857}$$

$$-\int -\frac{\sec(x)}{3-4\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sec(x)}{3-4\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sec(x)}{3-4\cos^2(x)}d\cos^2(x)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{3-4\cos^2(x)}d\cos^2(x)+\frac{1}{3}\int \sec(x)d\cos^2(x)\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{3-4\cos^2(x)}d\cos^2(x)+\frac{1}{3}\log\left(\cos^2(x)\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\log\left(\cos^2(x)\right)-\frac{1}{3}\log\left(3-4\cos^2(x)\right)\right)$$

input
```
Int[Sec[3*x]*Sin[x],x]
```

output
```
(Log[Cos[x]^2]/3 - Log[3 - 4*Cos[x]^2]/3)/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\ln(\cos(x))}{3}-\frac{\ln\left(4\cos(x)^2-3\right)}{6}$	18
risch	$\frac{\ln(e^{2ix}+1)}{3}-\frac{\ln(e^{4ix}-e^{2ix}+1)}{6}$	29

input
```
int(sec(3*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(cos(x))-1/6*ln(4*cos(x)^2-3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sec(3x)\sin(x)\,dx = -\frac{1}{6}\log\left(4\cos(x)^2-3\right)+\frac{1}{3}\log(-\cos(x))$$

input
```
integrate(sec(3*x)*sin(x),x, algorithm="fricas")
```

output
```
-1/6*log(4*cos(x)^2 - 3) + 1/3*log(-cos(x))
```

Sympy [F]

$$\int \sec(3x)\sin(x)\,dx = \int \sin(x)\sec(3x)\,dx$$

input
```
integrate(sec(3*x)*sin(x),x)
```

output
```
Integral(sin(x)*sec(3*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(17) = 34$.

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.86

$$\int \sec(3x)\sin(x)\,dx = -\frac{1}{12}\log\left(-2\left(\cos\left(2x\right)-1\right)\cos\left(4x\right)+\cos\left(4x\right)^2+\cos\left(2x\right)^2+\sin\left(4x\right)^2-2\sin\left(4x\right)\sin\left(2x\right)+\sin\left(2x\right)^2-2\cos\left(2x\right)+1\right)+\frac{1}{6}\log\left(\cos\left(2x\right)^2+\sin\left(2x\right)^2+2\cos\left(2x\right)+1\right)$$

input
```
integrate(sec(3*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/12*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^
2 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/6*log(cos(2*x)^
2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \sec(3x)\sin(x)\,dx = \frac{1}{6}\log\left(-\sin\left(x\right)^2+1\right)-\frac{1}{6}\log\left(\left|4\sin\left(x\right)^2-1\right|\right)$$

input
```
integrate(sec(3*x)*sin(x),x, algorithm="giac")
```

output
```
1/6*log(-sin(x)^2 + 1) - 1/6*log(abs(4*sin(x)^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sec(3x)\sin(x)\,dx = \frac{\ln(\cos(x))}{3} - \frac{\ln\left(\cos(x)^2 - \frac{3}{4}\right)}{6}$$

input
```
int(sin(x)/cos(3*x),x)
```

output
```
log(cos(x))/3 - log(cos(x)^2 - 3/4)/6
```

Reduce [F]

$$\int \sec(3x)\sin(x)\,dx = \int \frac{\sin(x)}{\cos(3x)}dx - 1$$

input
```
int(sec(3*x)*sin(x),x)
```

output
```
int(sin(x)/cos(3*x),x) - 1
```

3.34 $\int \sec(4x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 71

$$\int \sec(4x)\sin(x)\,dx = -\frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

output
```
-1/2*arctanh(2*cos(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/2*arctanh(2
*cos(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 51.79 (sec) , antiderivative size = 4814, normalized size of antiderivative = 67.80

$$\int \sec(4x)\sin(x)\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[4*x]*Sin[x],x]
```

output
```
((-2*(-1)^(3/8)*(1 + Sqrt[2])*x - (2*(-1)^(1/4)*(-2 - (1 - I)*(-1)^(5/8) +
 (-1)^(5/8)*Sqrt[2])*ArcTan[(-Cos[x] + (1 + Sqrt[2])*Sin[x])/(2*(-1)^(3/8)
 + Cos[x] - Sqrt[2]*Cos[x] + Sin[x])])/((-1 + I) + 2*(-1)^(3/8) + Sqrt[2])
 - (2*(1 - I)^(3/2)*2^(1/4)*((-3 - I) + 2*(-1)^(5/8) + (2 + I)*Sqrt[2] - (
2 + 2*I)*(-1)^(3/8)*Sqrt[2] + 2*(-1)^(5/8)*Sqrt[2])*ArcTan[((1 + I) + I*Sq
rt[2] + ((-1 + I) + 2*(-1)^(3/8) + Sqrt[2])*Tan[x/2])/(Sqrt[1 - I]*2^(3/4)
)])/((-1 + I) + 2*(-1)^(3/8) + Sqrt[2]) + 2*(-1)^(3/8)*Log[Sec[x/2]^2] + (
(-1)^(3/4)*(-2 - (1 - I)*(-1)^(5/8) + (-1)^(5/8)*Sqrt[2])*Log[-(Sec[x/2]^4
*(-2 + (1 - I)*Sqrt[2] + 2*(-1)^(3/8)*(-1 + Sqrt[2])*Cos[x] + Sqrt[2]*Cos[
2*x] - 2*(-1)^(3/8)*Sin[x] + Sqrt[2]*Sin[2*x]))])/((-1 + I) + 2*(-1)^(3/8)
 + Sqrt[2]))*((-1/2 - I/2)/(((-1 + I) + Sqrt[1 - I]*Sqrt[1 + I])*(-((-1 -
I)^(3/2)*(1 - I)^(1/4)*(1 + I)^(1/4)) - (1 + I)*Cos[x] + I*Sqrt[1 - I]*Sqr
t[1 + I]*Cos[x] + (1 - I)*Sin[x] + Sqrt[1 - I]*Sqrt[1 + I]*Sin[x])) - Sin[
x]/(Sqrt[-1 - I]*(1 - I)^(1/4)*(1 + I)^(1/4)*((-1 + I) + Sqrt[1 - I]*Sqrt[
1 + I])*(-((-1 - I)^(3/2)*(1 - I)^(1/4)*(1 + I)^(1/4)) - (1 + I)*Cos[x] +
I*Sqrt[1 - I]*Sqrt[1 + I]*Cos[x] + (1 - I)*Sin[x] + Sqrt[1 - I]*Sqrt[1 + I
]*Sin[x])) - ((I/2)*Sqrt[-1 - I]*(1 - I)^(1/4)*(1 + I)^(1/4)*Sin[x])/(((-1
 + I) + Sqrt[1 - I]*Sqrt[1 + I])*(-((-1 - I)^(3/2)*(1 - I)^(1/4)*(1 + I)^(
1/4)) - (1 + I)*Cos[x] + I*Sqrt[1 - I]*Sqrt[1 + I]*Cos[x] + (1 - I)*Sin[x]
 + Sqrt[1 - I]*Sqrt[1 + I]*Sin[x]))))/(-2*(-1)^(3/8)*(1 + Sqrt[2]) - (2...
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4857, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sec(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\cos(4x)}dx$$

$$\Big\downarrow \text{4857}$$

$$-\int \frac{1}{8\cos^4(x)-8\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \text{1406}$$

$$\sqrt{2}\int \frac{1}{8\cos^2(x)-2\left(2-\sqrt{2}\right)}d\cos(x)-\sqrt{2}\int \frac{1}{8\cos^2(x)-2\left(2+\sqrt{2}\right)}d\cos(x)$$

$$\Big\downarrow \text{220}$$

$$\frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}$$

input
```
Int[Sec[4*x]*Sin[x],x]
```

output
```
-1/2*ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[2]]]/Sqrt[2*(2 - Sqrt[2])] + ArcTanh
[(2*Cos[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

method	result	size
risch	$-i\left(\sum_{\_R=\mathrm{RootOf}(2048\_Z^4+128\_Z^2+1)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-512i\_R^3-24i\_R\right)\mathrm{e}^{ix}+1\right)\right)$	47
default	$-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}$	54

input
```
int(sec(4*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-I*sum(_R*ln(exp(2*I*x)+(-512*I*_R^3-24*I*_R)*exp(I*x)+1),_R=RootOf(2048*_
Z^4+128*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(49) = 98.

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

$$\begin{aligned}\int \sec(4x)\sin(x)\,dx = &-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)+2\cos(x)\right)\\ &+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)-2\cos(x)\right)\\ &+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}+2\cos(x)\right)\\ &-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}-2\cos(x)\right)\end{aligned}$$

input
```
integrate(sec(4*x)*sin(x),x, algorithm="fricas")
```

output
```
-1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) + 2*cos(x)) + 1
/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) - 2*cos(x)) + 1/8
*sqrt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + 2*cos(x)) - 1/8
*sqrt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 2*cos(x))
```

Sympy [F]

$$\int \sec(4x)\sin(x)\,dx = \int \sin(x)\sec(4x)\,dx$$

input
```
integrate(sec(4*x)*sin(x),x)
```

output
```
Integral(sin(x)*sec(4*x), x)
```

Maxima [F]

$$\int \sec(4x)\sin(x)\,dx = \int \sec(4x)\sin(x)\,dx$$

input
```
integrate(sec(4*x)*sin(x),x, algorithm="maxima")
```

output
```
integrate(sec(4*x)*sin(x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(49) = 98$.

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.87

$$\int \sec(4x)\sin(x)\,dx = -\frac{2.16139547686000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.0395661298966000\right)}{\frac{140(\cos(x)-1)}{\cos(x)+1}+28.1312524456150} - \frac{4.18450863968000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.446462692172000\right)}{\frac{140(\cos(x)-1)}{\cos(x)+1}+44.3876588494000} - \frac{20.9929814212000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-2.23982880884000\right)}{\frac{140(\cos(x)-1)}{\cos(x)+1}+404.466590643000} - \frac{1380.66111446200\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-25.2741423691000\right)}{\frac{140(\cos(x)-1)}{\cos(x)+1}-10892.9855019000}$$

input
```
integrate(sec(4*x)*sin(x),x, algorithm="giac")
```

output
```
-2.16139547686000*log(-(cos(x) - 1)/(cos(x) + 1) - 0.0395661298966000)/(14
0*(cos(x) - 1)/(cos(x) + 1) + 28.1312524456150) - 4.18450863968000*log(-(c
os(x) - 1)/(cos(x) + 1) - 0.446462692172000)/(140*(cos(x) - 1)/(cos(x) + 1
) + 44.3876588494000) - 20.9929814212000*log(-(cos(x) - 1)/(cos(x) + 1) -
2.23982880884000)/(140*(cos(x) - 1)/(cos(x) + 1) + 404.466590643000) - 138
0.66111446200*log(-(cos(x) - 1)/(cos(x) + 1) - 25.2741423691000)/(140*(cos
(x) - 1)/(cos(x) + 1) - 10892.9855019000)
```

Mupad [B] (verification not implemented)

Time = 17.63 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

$$\int \sec(4x)\sin(x)\,dx = \frac{\operatorname{atanh}\left(\frac{\cos(x)\sqrt{2-\sqrt{2}}}{64\left(\frac{\sqrt{2}}{128}-\frac{1}{64}\right)}-\frac{\sqrt{2}\cos(x)\sqrt{2-\sqrt{2}}}{64\left(\frac{\sqrt{2}}{128}-\frac{1}{64}\right)}\right)\sqrt{2-\sqrt{2}}}{4} - \frac{\operatorname{atanh}\left(\frac{\cos(x)\sqrt{\sqrt{2}+2}}{64\left(\frac{\sqrt{2}}{128}+\frac{1}{64}\right)}+\frac{\sqrt{2}\cos(x)\sqrt{\sqrt{2}+2}}{64\left(\frac{\sqrt{2}}{128}+\frac{1}{64}\right)}\right)\sqrt{\sqrt{2}+2}}{4}$$

input
```
int(sin(x)/cos(4*x),x)
```

output
```
(atanh((cos(x)*(2 - 2^(1/2))^(1/2))/(64*(2^(1/2)/128 - 1/64)) - (2^(1/2)*c
os(x)*(2 - 2^(1/2))^(1/2))/(64*(2^(1/2)/128 - 1/64)))*(2 - 2^(1/2))^(1/2))
/4 - (atanh((cos(x)*(2^(1/2) + 2)^(1/2))/(64*(2^(1/2)/128 + 1/64)) + (2^(1
/2)*cos(x)*(2^(1/2) + 2)^(1/2))/(64*(2^(1/2)/128 + 1/64)))*(2^(1/2) + 2)^(
1/2))/4
```

Reduce [F]

$$\int \sec(4x)\sin(x)\,dx = \int \frac{\sin(x)}{\cos(4x)}dx - 1$$

input
```
int(sec(4*x)*sin(x),x)
```

output
```
int(sin(x)/cos(4*x),x) - 1
```

3.35 $\int \sec(5x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 71

$$\int \sec(5x)\sin(x)\,dx = -\frac{1}{5}\log(\cos(x)) + \frac{\log\left(5-\sqrt{5}-8\cos^2(x)\right)}{\sqrt{5}\left(5-\sqrt{5}\right)} - \frac{\log\left(5+\sqrt{5}-8\cos^2(x)\right)}{\sqrt{5}\left(5+\sqrt{5}\right)}$$

output
```
-1/5*ln(cos(x))+1/5*ln(5-5^(1/2)-8*cos(x)^2)*5^(1/2)/(5-5^(1/2))-1/5*ln(5+
5^(1/2)-8*cos(x)^2)*5^(1/2)/(5+5^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \sec(5x)\sin(x)\,dx = \frac{1}{20}\left(-4\log(\cos(x)) - \left(-1+\sqrt{5}\right)\log\left(-1-\sqrt{5}+4\cos(2x)\right) + \left(1+\sqrt{5}\right)\log\left(-1+\sqrt{5}+4\cos(2x)\right)\right)$$

input
```
Integrate[Sec[5*x]*Sin[x],x]
```

output
```
(-4*Log[Cos[x]] - (-1 + Sqrt[5])*Log[-1 - Sqrt[5] + 4*Cos[2*x]] + (1 + Sqr
t[5])*Log[-1 + Sqrt[5] + 4*Cos[2*x]])/20
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4857, 1434, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sec(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\cos(5x)}dx$$

$$\Big\downarrow \text{4857}$$

$$-\int \frac{\sec(x)}{16\cos^4(x)-20\cos^2(x)+5}d\cos(x)$$

$$\Big\downarrow \text{1434}$$

$$-\frac{1}{2}\int \frac{\sec(x)}{16\cos^4(x)-20\cos^2(x)+5}d\cos^2(x)$$

$$\Big\downarrow \text{1141}$$

$$-8\int \left(\frac{\sec(x)}{80}+\frac{1}{\sqrt{5}\left(5-\sqrt{5}\right)\left(-8\cos^2(x)-\sqrt{5}+5\right)}-\frac{1}{\sqrt{5}\left(5+\sqrt{5}\right)\left(-8\cos^2(x)+\sqrt{5}+5\right)}\right)d\cos^2(x)$$

$$\Big\downarrow \text{2009}$$

$$-8\left(\frac{1}{80}\log\left(\cos^2(x)\right)-\frac{\log\left(-8\cos^2(x)-\sqrt{5}+5\right)}{8\sqrt{5}\left(5-\sqrt{5}\right)}+\frac{\log\left(-8\cos^2(x)+\sqrt{5}+5\right)}{8\sqrt{5}\left(5+\sqrt{5}\right)}\right)$$

input
```
Int[Sec[5*x]*Sin[x],x]
```

output
```
-8*(Log[Cos[x]^2]/80 - Log[5 - Sqrt[5] - 8*Cos[x]^2]/(8*Sqrt[5]*(5 - Sqrt[
5])) + Log[5 + Sqrt[5] - 8*Cos[x]^2]/(8*Sqrt[5]*(5 + Sqrt[5])))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

method	result
default	$-\frac{\ln(\cos(x))}{5}+\frac{\ln\left(16\cos(x)^4-20\cos(x)^2+5\right)}{20}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(32\cos(x)^2-20\right)\sqrt{5}}{20}\right)}{10}$
risch	$-\frac{\ln(e^{2ix}+1)}{5}+\frac{\ln\left(e^{4ix}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)e^{2ix}+1\right)}{20}+\frac{\ln\left(e^{4ix}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)e^{2ix}+1\right)\sqrt{5}}{20}+\frac{\ln\left(e^{4ix}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2ix}+1\right)}{20}-\frac{\ln\left(e^{4ix}+\right.}{}$

input
```
int(sec(5*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/5*ln(cos(x))+1/20*ln(16*cos(x)^4-20*cos(x)^2+5)+1/10*5^(1/2)*arctanh(1/
20*(32*cos(x)^2-20)*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int \sec(5x)\sin(x)\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{32\cos(x)^4+8\left(\sqrt{5}-5\right)\cos(x)^2-5\sqrt{5}+15}{16\cos(x)^4-20\cos(x)^2+5}\right) + \frac{1}{20}\log\left(16\cos(x)^4-20\cos(x)^2+5\right)-\frac{1}{5}\log\left(-\cos(x)\right)$$

input
```
integrate(sec(5*x)*sin(x),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log((32*cos(x)^4 + 8*(sqrt(5) - 5)*cos(x)^2 - 5*sqrt(5) + 15)
/(16*cos(x)^4 - 20*cos(x)^2 + 5)) + 1/20*log(16*cos(x)^4 - 20*cos(x)^2 + 5
) - 1/5*log(-cos(x))
```

Sympy [F]

$$\int \sec(5x)\sin(x)\,dx = \int \sin{(x)}\sec{(5x)}\,dx$$

input
```
integrate(sec(5*x)*sin(x),x)
```

output
```
Integral(sin(x)*sec(5*x), x)
```

Maxima [F]

$$\int \sec(5x)\sin(x)\,dx = \int \sec(5\,x)\sin(x)\,dx$$

input
```
integrate(sec(5*x)*sin(x),x, algorithm="maxima")
```

output
```
1/5*integrate(-(cos(4*x)*sin(8*x) - cos(8*x)*sin(4*x) + cos(3/2*arctan2(si
n(4*x), cos(4*x)))*sin(4*x) + cos(1/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x
) - cos(4*x)*sin(3/2*arctan2(sin(4*x), cos(4*x))) - cos(4*x)*sin(1/2*arcta
n2(sin(4*x), cos(4*x))) - sin(4*x))/(2*(cos(4*x) + 1)*cos(8*x) + cos(8*x)^
2 + cos(4*x)^2 - 2*(cos(8*x) + cos(4*x) - cos(1/2*arctan2(sin(4*x), cos(4*
x))) + 1)*cos(3/2*arctan2(sin(4*x), cos(4*x))) + cos(3/2*arctan2(sin(4*x),
 cos(4*x)))^2 - 2*(cos(8*x) + cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(
4*x))) + cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(8*x)^2 + 2*sin(8*x)*
sin(4*x) + sin(4*x)^2 - 2*(sin(8*x) + sin(4*x) - sin(1/2*arctan2(sin(4*x),
 cos(4*x))))*sin(3/2*arctan2(sin(4*x), cos(4*x))) + sin(3/2*arctan2(sin(4*
x), cos(4*x)))^2 - 2*(sin(8*x) + sin(4*x))*sin(1/2*arctan2(sin(4*x), cos(4
*x))) + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2 + 2*cos(4*x) + 1), x) + 4/5
*integrate(-(cos(2*x)*sin(8*x) - cos(2*x)*sin(6*x) + cos(2*x)*sin(4*x) - c
os(8*x)*sin(2*x) + cos(6*x)*sin(2*x) - cos(4*x)*sin(2*x) - sin(2*x))/(2*(c
os(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x) - c
os(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^
2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^2
+ 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)*
sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 2/5*integrate(-(cos(4/3*arct
an2(sin(6*x), cos(6*x)))*sin(6*x) + cos(2/3*arctan2(sin(6*x), cos(6*x))...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \sec(5x)\sin(x)\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{\left|32\sin(x)^2 - 4\sqrt{5} - 12\right|}{\left|32\sin(x)^2 + 4\sqrt{5} - 12\right|}\right) - \frac{1}{10}\log\left(-\sin(x)^2 + 1\right) + \frac{1}{20}\log\left(\left|16\sin(x)^4 - 12\sin(x)^2 + 1\right|\right)$$

input
```
integrate(sec(5*x)*sin(x),x, algorithm="giac")
```

output
```
1/20*sqrt(5)*log(abs(32*sin(x)^2 - 4*sqrt(5) - 12)/abs(32*sin(x)^2 + 4*sqr
t(5) - 12)) - 1/10*log(-sin(x)^2 + 1) + 1/20*log(abs(16*sin(x)^4 - 12*sin(
x)^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

$$\int \sec(5x)\sin(x)\,dx = \ln\left(\cos(x)^2 + \frac{\sqrt{5}}{8} - \frac{5}{8}\right)\left(\frac{\sqrt{5}}{20} + \frac{1}{20}\right) - \ln\left(\cos(x)^2 - \frac{\sqrt{5}}{8} - \frac{5}{8}\right)\left(\frac{\sqrt{5}}{20} - \frac{1}{20}\right) - \frac{\ln(\cos(x))}{5}$$

input
```
int(sin(x)/cos(5*x),x)
```

output
```
log(cos(x)^2 + 5^(1/2)/8 - 5/8)*(5^(1/2)/20 + 1/20) - log(cos(x)^2 - 5^(1/
2)/8 - 5/8)*(5^(1/2)/20 - 1/20) - log(cos(x))/5
```

Reduce [F]

$$\int \sec(5x)\sin(x)\,dx = \int \frac{\sin(x)}{\cos(5x)}dx - 1$$

input
```
int(sec(5*x)*sin(x),x)
```

output
```
int(sin(x)/cos(5*x),x) - 1
```

3.36 $\int \sec(6x)\sin(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 85

$$\int \sec(6x)\sin(x)\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}} + \frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

output
```
-1/6*arctanh(cos(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*cos(x)/(1/2*6^(1/2)-1/2
*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*cos(x)/(1/2*6^(1/2)+1/2
*2^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.43 (sec) , antiderivative size = 627, normalized size of antiderivative = 7.38

$$\int \sec(6x)\sin(x)\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[6*x]*Sin[x],x]
```

output
```
((-4 - 4*I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - (4 - 4*I)*(-1)^(
1/4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + (2*(1 + Sqrt[2])*(x + 2*Sqrt[3]*Arc
Tanh[(2 + (2 + Sqrt[2])*Tan[x/2])/Sqrt[6]] - Log[Sec[x/2]^2] + Log[-(Sec[x
/2]^2*(Sqrt[2] - 2*Cos[x] + 2*Sin[x]))]))/(2 + Sqrt[2]) - Sqrt[2]*(x - 2*S
qrt[3]*ArcTanh[(Sqrt[2] + (-1 + Sqrt[2])*Tan[x/2])/Sqrt[3]] - Log[Sec[x/2]
^2] + Log[Sec[x/2]^2*(1 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x])]) + (2*(2*(Sqrt
[2] + Sqrt[3])*ArcTanh[(2 + (2 + Sqrt[6])*Tan[x/2])/Sqrt[2]] + (3 + Sqrt[6
])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[6] - 2*Cos[x] + 2*Sin[x])
)]))*(1 + Sqrt[6]*Sin[x])*(3 + Sqrt[6] - (2 + Sqrt[6])*Cos[x] + (2 + Sqrt[
6])*Sin[x]))/((12 + 5*Sqrt[6])*Cos[2*x] + 2*Cos[x]*(5 + 2*Sqrt[6] + 5*Sqrt
[6]*Sin[x]) - 2*(12 + 5*Sqrt[6] + 4*(5 + 2*Sqrt[6])*Sin[x] - 6*Sin[2*x]))
+ ((-2*(-2 + Sqrt[6])*ArcTanh[Sqrt[2] + (Sqrt[2] - Sqrt[3])*Tan[x/2]] + (3
*Sqrt[2] - 2*Sqrt[3])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[3] + S
qrt[2]*Cos[x] - Sqrt[2]*Sin[x]))]))*(Sqrt[2] - 2*Sqrt[3]*Sin[x])*(-3 + Sqr
t[6] - (-2 + Sqrt[6])*Cos[x] + (-2 + Sqrt[6])*Sin[x]))/((-12 + 5*Sqrt[6])*
Cos[2*x] + 2*Cos[x]*(-5 + 2*Sqrt[6] + 5*Sqrt[6]*Sin[x]) - 2*(-12 + 5*Sqrt[
6] + 4*(-5 + 2*Sqrt[6])*Sin[x] + 6*Sin[2*x])))/24
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4857, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sec(6x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)}{\cos(6x)}dx$$

$$\Big\downarrow \text{ 4857}$$

$$-\int \frac{1}{32\cos^6(x)-48\cos^4(x)+18\cos^2(x)-1}d\cos(x)$$

$$\Big\downarrow \text{ 2460}$$

$$-\int \left(\frac{4(2\cos^2(x)-1)}{3(16\cos^4(x)-16\cos^2(x)+1)}-\frac{1}{3(2\cos^2(x)-1)}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\operatorname{arctanh}(\sqrt{2}\cos(x))}{3\sqrt{2}}+\frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

input
```
Int[Sec[6*x]*Sin[x],x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2] + ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[3]
]]/(6*Sqrt[2 - Sqrt[3]]) + ArcTanh[(2*Cos[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2
 + Sqrt[3]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

method	result
default	$-\frac{\operatorname{arctanh}\left(\sqrt{2}\,\cos(x)\right)\sqrt{2}}{6}+\frac{2\,\operatorname{arctanh}\left(\frac{8\cos(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{3\left(2\sqrt{6}-2\sqrt{2}\right)}+\frac{2\,\operatorname{arctanh}\left(\frac{8\cos(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{3\left(2\sqrt{6}+2\sqrt{2}\right)}$
risch	$-i\left(\sum_{\_R=\operatorname{RootOf}(20736\_Z^4+576\_Z^2+1)}\_R\ln\left(e^{2ix}+\left(-1728i\_R^3-48i\_R\right)e^{ix}+1\right)\right)+\frac{\sqrt{2}\,\ln\left(e^{2ix}-\sqrt{\ }\right.}{12}$

input
```
int(sec(6*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*arctanh(2^(1/2)*cos(x))*2^(1/2)+2/3/(2*6^(1/2)-2*2^(1/2))*arctanh(8*c
os(x)/(2*6^(1/2)-2*2^(1/2)))+2/3/(2*6^(1/2)+2*2^(1/2))*arctanh(8*cos(x)/(2
*6^(1/2)+2*2^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(67) = 134.

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.80

$$\begin{aligned}\int \sec(6x)\sin(x)\,dx = &-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+2\cos\left(x\right)\right)\\&+\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)-2\cos\left(x\right)\right)\\&+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}+2\cos\left(x\right)\right)\\&-\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}-2\cos\left(x\right)\right)\\&+\frac{1}{12}\sqrt{2}\log\left(\frac{2\cos\left(x\right)^2-2\sqrt{2}\cos\left(x\right)+1}{2\cos\left(x\right)^2-1}\right)\end{aligned}$$

input
```
integrate(sec(6*x)*sin(x),x, algorithm="fricas")
```

output
```
-1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + 2*cos(x)) +
1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) - 2*cos(x)) + 1
/12*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + 2*cos(x)) -
1/12*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) - 2*cos(x)) +
 1/12*sqrt(2)*log((2*cos(x)^2 - 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

Sympy [F]

$$\int \sec(6x)\sin(x)\,dx = \int \sin{(x)}\sec{(6x)}\,dx$$

input
```
integrate(sec(6*x)*sin(x),x)
```

output
```
Integral(sin(x)*sec(6*x), x)
```

Maxima [F]

$$\int \sec(6x)\sin(x)\,dx = \int \sec\left(6\,x\right)\sin\left(x\right)\,dx$$

input
```
integrate(sec(6*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/24*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2
*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x)
 + 1) + 1/24*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) -
1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2
)*cos(x) + 1) - integrate(1/3*((sin(7*x) - sin(5*x) + sin(3*x) - sin(x))*c
os(8*x) - (sin(3*x) - sin(x))*cos(4*x) - (cos(7*x) - cos(5*x) + cos(3*x) -
 cos(x))*sin(8*x) - (cos(4*x) - 1)*sin(7*x) + (cos(4*x) - 1)*sin(5*x) + (c
os(3*x) - cos(x))*sin(4*x) + cos(7*x)*sin(4*x) - cos(5*x)*sin(4*x) + sin(3
*x) - sin(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8*x)^2 - cos(4*x)^2 - sin(8
*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos(4*x) - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(67) = 134$.

Time = 0.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.14

$$\int \sec(6x)\sin(x)\,dx = -\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}\right) - \frac{2.39014968180000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.0173323801210000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1}+60.0540532247402} + \frac{5.82951931426000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.588790706481000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1}+121.584934401100} + \frac{16.8155413244667\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-1.69839637242000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1}+559.622604171000} - \frac{7956.25491093333\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-57.6954805410000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1}-168981.261592000}$$

input
```
integrate(sec(6*x)*sin(x),x, algorithm="giac")
```

output
```
-1/12*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*
sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)) - 2.39014968180000*log(-(cos(x
) - 1)/(cos(x) + 1) - 0.0173323801210000)/(268*(cos(x) - 1)/(cos(x) + 1) +
 60.0540532247402) + 5.82951931426000*log(-(cos(x) - 1)/(cos(x) + 1) - 0.5
88790706481000)/(268*(cos(x) - 1)/(cos(x) + 1) + 121.584934401100) + 16.81
55413244667*log(-(cos(x) - 1)/(cos(x) + 1) - 1.69839637242000)/(268*(cos(x
) - 1)/(cos(x) + 1) + 559.622604171000) - 7956.25491093333*log(-(cos(x) -
1)/(cos(x) + 1) - 57.6954805410000)/(268*(cos(x) - 1)/(cos(x) + 1) - 16898
1.261592000)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

$$\int \sec(6x)\sin(x)\,dx = \operatorname{atanh}\left(\frac{5\sqrt{2}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)} + \frac{3\sqrt{6}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}+\frac{\sqrt{6}}{12}\right) - \operatorname{atanh}\left(\frac{5\sqrt{2}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}-\frac{1}{1048576}\right)} - \frac{3\sqrt{6}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}-\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}-\frac{\sqrt{6}}{12}\right) - \frac{\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\cos(x)\right)}{6}$$

input
```
int(sin(x)/cos(6*x),x)
```

output
```
atanh((5*2^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576))
 + (3*6^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576)))*(
2^(1/2)/12 + 6^(1/2)/12) - atanh((5*2^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(
1/2))/4194304 - 1/1048576)) - (3*6^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2
))/4194304 - 1/1048576)))*(2^(1/2)/12 - 6^(1/2)/12) - (2^(1/2)*atanh(2^(1/
2)*cos(x)))/6
```

Reduce [F]

$$\int \sec(6x)\sin(x)\,dx = \int \frac{\sin(x)}{\cos(6x)}dx - 1$$

input
```
int(sec(6*x)*sin(x),x)
```

output
```
int(sin(x)/cos(6*x),x) - 1
```

3.37 $\int \cos(mx)\cos(nx)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int \cos(mx)\cos(nx)\,dx = \frac{\sin((m-n)x)}{2(m-n)} + \frac{\sin((m+n)x)}{2(m+n)}$$

output
```
sin((m-n)*x)/(2*m-2*n)+sin((m+n)*x)/(2*m+2*n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \cos(mx)\cos(nx)\,dx = \frac{\sin((m-n)x)}{2(m-n)} + \frac{\sin((m+n)x)}{2(m+n)}$$

input
```
Integrate[Cos[m*x]*Cos[n*x],x]
```

output
```
Sin[(m - n)*x]/(2*(m - n)) + Sin[(m + n)*x]/(2*(m + n))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(mx)\cos(nx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{2}\cos(x(m-n))+\frac{1}{2}\cos(x(m+n))\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(x(m-n))}{2(m-n)}+\frac{\sin(x(m+n))}{2(m+n)}$$

input
```
Int[Cos[m*x]*Cos[n*x],x]
```

output
```
Sin[(m - n)*x]/(2*(m - n)) + Sin[(m + n)*x]/(2*(m + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result
default	$\frac{\sin((m-n)x)}{2m-2n}+\frac{\sin((m+n)x)}{2m+2n}$
parallelrisch	$\frac{(m+n)\sin((m-n)x)+\sin((m+n)x)(m-n)}{2m^2-2n^2}$
risch	$\frac{\sin((m-n)x)m}{2(m-n)(m+n)}+\frac{\sin((m-n)x)n}{2(m-n)(m+n)}+\frac{\sin((m+n)x)m}{2(m-n)(m+n)}-\frac{\sin((m+n)x)n}{2(m-n)(m+n)}$
norman	$\frac{\frac{2m\tan\left(\frac{mx}{2}\right)}{m^2-n^2}-\frac{2n\tan\left(\frac{nx}{2}\right)}{m^2-n^2}-\frac{2m\tan\left(\frac{mx}{2}\right)\tan\left(\frac{nx}{2}\right)^2}{m^2-n^2}+\frac{2n\tan\left(\frac{mx}{2}\right)^2\tan\left(\frac{nx}{2}\right)}{m^2-n^2}}{\left(1+\tan\left(\frac{mx}{2}\right)^2\right)\left(1+\tan\left(\frac{nx}{2}\right)^2\right)}$
orering	$-\frac{2(m^2+n^2)(-m\sin(mx)\cos(nx)-\cos(mx)n\sin(nx))}{m^4-2m^2n^2+n^4}-\frac{3m^2\cos(mx)\sin(nx)n+\cos(nx)m^3\sin(mx)+3m\sin(mx)n^2\cos}{m^4-2m^2n^2+n^4}$

input
```
int(cos(m*x)*cos(n*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/(m-n)*sin((m-n)*x)+1/2/(m+n)*sin((m+n)*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int\cos(mx)\cos(nx)\,dx=\frac{m\cos(nx)\sin(mx)-n\cos(mx)\sin(nx)}{m^2-n^2}$$

input
```
integrate(cos(m*x)*cos(n*x),x, algorithm="fricas")
```

output
```
(m*cos(n*x)*sin(m*x) - n*cos(m*x)*sin(n*x))/(m^2 - n^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(26) = 52$.

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.14

$$\int \cos(mx)\cos(nx)\,dx = \begin{cases} x & \text{for } m = 0 \wedge n = 0 \\ \frac{x\sin^2(nx)}{2} + \frac{x\cos^2(nx)}{2} + \frac{\sin(nx)\cos(nx)}{2n} & \text{for } m = -n \vee m = n \\ \frac{m\sin(mx)\cos(nx)}{m^2-n^2} - \frac{n\sin(nx)\cos(mx)}{m^2-n^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(m*x)*cos(n*x),x)
```

output
```
Piecewise((x, Eq(m, 0) & Eq(n, 0)), (x*sin(n*x)**2/2 + x*cos(n*x)**2/2 + s
in(n*x)*cos(n*x)/(2*n), Eq(m, n) | Eq(m, -n)), (m*sin(m*x)*cos(n*x)/(m**2
- n**2) - n*sin(n*x)*cos(m*x)/(m**2 - n**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \cos(mx)\cos(nx)\,dx = \frac{\sin((m+n)x)}{2(m+n)} + \frac{\sin((m-n)x)}{2(m-n)}$$

input
```
integrate(cos(m*x)*cos(n*x),x, algorithm="maxima")
```

output
```
1/2*sin((m + n)*x)/(m + n) + 1/2*sin((m - n)*x)/(m - n)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \cos(mx)\cos(nx)\,dx = \frac{\sin(mx+nx)}{2(m+n)} + \frac{\sin(mx-nx)}{2(m-n)}$$

input
```
integrate(cos(m*x)*cos(n*x),x, algorithm="giac")
```

output
```
1/2*sin(m*x + n*x)/(m + n) + 1/2*sin(m*x - n*x)/(m - n)
```

Mupad [B] (verification not implemented)

Time = 17.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.09

$$\int \cos(mx)\cos(nx)\,dx$$
$$= \begin{cases} x & \text{if } m=0 \wedge n=0 \\ \frac{x}{2}+\frac{\sin(2\,n\,x)}{4\,n} & \text{if } (m=n \vee m+n=0) \wedge n \neq 0 \\ \frac{\sin(x\,(m+n))}{2\,m+2\,n}+\frac{\sin(x\,(m-n))}{2\,m-2\,n} & \text{if } m \neq n \wedge ((m \neq n \wedge m+n \neq 0) \vee n=0) \wedge m+n \neq 0 \wedge (m \neq 0 \vee n \neq \end{cases}$$

input
```
int(cos(m*x)*cos(n*x),x)
```

output
```
piecewise(m == 0 & n == 0, x, (m == n | m + n == 0) & n ~= 0, x/2 + sin(2*
n*x)/(4*n), m ~= n & (m ~= n & m + n ~= 0 | n == 0) & m + n ~= 0 & (m ~= 0
 | n ~= 0), sin(x*(m + n))/(2*m + 2*n) + sin(x*(m - n))/(2*m - 2*n))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \cos(mx)\cos(nx)\,dx = \frac{-\cos(mx)\sin(nx)\,n + \cos(nx)\sin(mx)\,m}{m^2 - n^2}$$

input
```
int(cos(m*x)*cos(n*x),x)
```

output
```
( - cos(m*x)*sin(n*x)*n + cos(n*x)*sin(m*x)*m)/(m**2 - n**2)
```

3.38 $\int \cos(x)\cos(nx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \cos(x)\cos(nx)\,dx = \frac{\sin((1-n)x)}{2(1-n)} + \frac{\sin((1+n)x)}{2(1+n)}$$

output
```
sin((1-n)*x)/(2-2*n)+sin((1+n)*x)/(2+2*n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cos(x)\cos(nx)\,dx = \frac{-\cos(nx)\sin(x) + n\cos(x)\sin(nx)}{-1+n^2}$$

input
```
Integrate[Cos[x]*Cos[n*x],x]
```

output
```
(-(Cos[n*x]*Sin[x]) + n*Cos[x]*Sin[n*x])/(-1 + n^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(nx)\,dx$$
$$\Big\downarrow \text{5081}$$
$$\int \left(\frac{1}{2}\cos((1-n)x)+\frac{1}{2}\cos((n+1)x)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sin((1-n)x)}{2(1-n)}+\frac{\sin((n+1)x)}{2(n+1)}$$

input
```
Int[Cos[x]*Cos[n*x],x]
```

output
```
Sin[(1 - n)*x]/(2*(1 - n)) + Sin[(1 + n)*x]/(2*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result
default	$\frac{\sin(x(-1+n))}{-2+2n}+\frac{\sin((1+n)x)}{2+2n}$
risch	$\frac{\sin(x(-1+n))}{-2+2n}+\frac{\sin((1+n)x)}{2+2n}$
parallelrisch	$\frac{(1+n)\sin(x(-1+n))+\sin((1+n)x)(-1+n)}{2n^2-2}$
orering	$-\frac{2(n^2+1)(-\sin(x)\cos(nx)-\cos(x)n\sin(nx))}{n^4-2n^2+1}-\frac{\sin(x)\cos(nx)+3\cos(x)n\sin(nx)+3\sin(x)n^2\cos(nx)+\cos(x)n^3\sin(nx)}{n^4-2n^2+1}$
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{n^2-1}+\frac{2n\tan\left(\frac{nx}{2}\right)}{n^2-1}+\frac{2\tan\left(\frac{x}{2}\right)\tan\left(\frac{nx}{2}\right)^2}{n^2-1}-\frac{2n\tan\left(\frac{x}{2}\right)^2\tan\left(\frac{nx}{2}\right)}{n^2-1}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan\left(\frac{nx}{2}\right)^2\right)}$

input
```
int(cos(x)*cos(n*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x*(-1+n))/(-1+n)+1/2*sin((1+n)*x)/(1+n)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cos(x)\cos(nx)\,dx=\frac{n\cos(x)\sin(nx)-\cos(nx)\sin(x)}{n^2-1}$$

input
```
integrate(cos(x)*cos(n*x),x, algorithm="fricas")
```

output
```
(n*cos(x)*sin(n*x) - cos(n*x)*sin(x))/(n^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(22) = 44.

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \cos(x)\cos(nx)\,dx = \begin{cases} \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} & \text{for } n = -1 \vee n = 1 \\ \frac{n\sin(nx)\cos(x)}{n^2-1} - \frac{\sin(x)\cos(nx)}{n^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(x)*cos(n*x),x)
```

output
```
Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(n, -1) | Eq
(n, 1)), (n*sin(n*x)*cos(x)/(n**2 - 1) - sin(x)*cos(n*x)/(n**2 - 1), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \cos(x)\cos(nx)\,dx = \frac{\sin((n+1)x)}{2(n+1)} - \frac{\sin(-(n-1)x)}{2(n-1)}$$

input
```
integrate(cos(x)*cos(n*x),x, algorithm="maxima")
```

output
```
1/2*sin((n + 1)*x)/(n + 1) - 1/2*sin(-(n - 1)*x)/(n - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \cos(x)\cos(nx)\,dx = \frac{\sin(nx+x)}{2(n+1)} + \frac{\sin(nx-x)}{2(n-1)}$$

input
```
integrate(cos(x)*cos(n*x),x, algorithm="giac")
```

output
```
1/2*sin(n*x + x)/(n + 1) + 1/2*sin(n*x - x)/(n - 1)
```

Mupad [B] (verification not implemented)

Time = 16.76 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int \cos(x)\cos(nx)\,dx = \begin{cases} \frac{x}{2}+\frac{\sin(2\,x)}{4} & \text{if } n=-1\vee n=1\\ \frac{\sin(x\,(n-1))}{2\,n-2}+\frac{\sin(x\,(n+1))}{2\,n+2} & \text{if } n\neq -1\wedge n\neq 1 \end{cases}$$

input
```
int(cos(n*x)*cos(x),x)
```

output
```
piecewise(n == -1 | n == 1, x/2 + sin(2*x)/4, n ~= -1 & n ~= 1, sin(x*(n -
 1))/(2*n - 2) + sin(x*(n + 1))/(2*n + 2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cos(x)\cos(nx)\,dx = \frac{-\cos(nx)\sin(x)+\cos(x)\sin(nx)\,n}{n^2-1}$$

input
```
int(cos(x)*cos(n*x),x)
```

output
```
( - cos(n*x)*sin(x) + cos(x)*sin(n*x)*n)/(n**2 - 1)
```

3.39 $\int \cos(x)\cos(mx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \cos(x)\cos(mx)\,dx = \frac{\sin((1-m)x)}{2(1-m)} + \frac{\sin((1+m)x)}{2(1+m)}$$

output
```
sin((1-m)*x)/(2-2*m)+sin((1+m)*x)/(2+2*m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cos(x)\cos(mx)\,dx = \frac{-\cos(mx)\sin(x) + m\cos(x)\sin(mx)}{-1+m^2}$$

input
```
Integrate[Cos[x]*Cos[m*x],x]
```

output
```
(-(Cos[m*x]*Sin[x]) + m*Cos[x]*Sin[m*x])/(-1 + m^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(mx)\,dx$$

$$\Big\downarrow \text{ 5081}$$

$$\int \left(\frac{1}{2}\cos((1-m)x)+\frac{1}{2}\cos((m+1)x)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sin((1-m)x)}{2(1-m)}+\frac{\sin((m+1)x)}{2(m+1)}$$

input
```
Int[Cos[x]*Cos[m*x],x]
```

output
```
Sin[(1 - m)*x]/(2*(1 - m)) + Sin[(1 + m)*x]/(2*(1 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result
default	$\frac{\sin(x(-1+m))}{-2+2m}+\frac{\sin((1+m)x)}{2+2m}$
risch	$\frac{\sin(x(-1+m))}{-2+2m}+\frac{\sin((1+m)x)}{2+2m}$
parallelrisch	$\frac{(1+m)\sin(x(-1+m))+\sin((1+m)x)(-1+m)}{2m^2-2}$
orering	$-\frac{2(m^2+1)(-\cos(mx)\sin(x)-\cos(x)m\sin(mx))}{m^4-2m^2+1}-\frac{3\cos(x)m\sin(mx)+3\sin(x)m^2\cos(mx)+\cos(mx)\sin(x)+\cos(x)m^3\,\mathrm{s}}{m^4-2m^2+1}$
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{m^2-1}+\frac{2m\tan\left(\frac{mx}{2}\right)}{m^2-1}+\frac{2\tan\left(\frac{x}{2}\right)\tan\left(\frac{mx}{2}\right)^2}{m^2-1}-\frac{2m\tan\left(\frac{x}{2}\right)^2\tan\left(\frac{mx}{2}\right)}{m^2-1}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan\left(\frac{mx}{2}\right)^2\right)}$

input
```
int(cos(x)*cos(m*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-1+m)*sin(x*(-1+m))+1/2/(1+m)*sin((1+m)*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cos(x)\cos(mx)\,dx=\frac{m\cos(x)\sin(mx)-\cos(mx)\sin(x)}{m^2-1}$$

input
```
integrate(cos(x)*cos(m*x),x, algorithm="fricas")
```

output
```
(m*cos(x)*sin(m*x) - cos(m*x)*sin(x))/(m^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(22) = 44$.

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \cos(x)\cos(mx)\,dx = \begin{cases} \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} & \text{for } m = -1 \vee m = 1 \\ \frac{m\sin(mx)\cos(x)}{m^2-1} - \frac{\sin(x)\cos(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(x)*cos(m*x),x)
```

output
```
Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, -1) | Eq
(m, 1)), (m*sin(m*x)*cos(x)/(m**2 - 1) - sin(x)*cos(m*x)/(m**2 - 1), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \cos(x)\cos(mx)\,dx = \frac{\sin((m+1)x)}{2(m+1)} - \frac{\sin(-(m-1)x)}{2(m-1)}$$

input
```
integrate(cos(x)*cos(m*x),x, algorithm="maxima")
```

output
```
1/2*sin((m + 1)*x)/(m + 1) - 1/2*sin(-(m - 1)*x)/(m - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \cos(x)\cos(mx)\,dx = \frac{\sin(mx+x)}{2(m+1)} + \frac{\sin(mx-x)}{2(m-1)}$$

input
```
integrate(cos(x)*cos(m*x),x, algorithm="giac")
```

output
```
1/2*sin(m*x + x)/(m + 1) + 1/2*sin(m*x - x)/(m - 1)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int \cos(x)\cos(mx)\,dx = \begin{cases} \frac{x}{2} + \frac{\sin(2\,x)}{4} & \text{if}\ \ m = -1 \vee m = 1 \\ \frac{\sin(x\,(m-1))}{2\,m-2} + \frac{\sin(x\,(m+1))}{2\,m+2} & \text{if}\ \ m \neq -1 \wedge m \neq 1 \end{cases}$$

input
```
int(cos(m*x)*cos(x),x)
```

output
```
piecewise(m == -1 | m == 1, x/2 + sin(2*x)/4, m ~= -1 & m ~= 1, sin(x*(m -
 1))/(2*m - 2) + sin(x*(m + 1))/(2*m + 2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cos(x)\cos(mx)\,dx = \frac{-\cos(mx)\sin(x) + \cos(x)\sin(mx)\,m}{m^2 - 1}$$

input
```
int(cos(x)*cos(m*x),x)
```

output
```
( - cos(m*x)*sin(x) + cos(x)*sin(m*x)*m)/(m**2 - 1)
```

3.40 $\int \cos(x)\cot(nx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 76

$$\int \cos(x)\cot(nx)\,dx = e^{-ix}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2inx}\right) - e^{ix}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),e^{2inx}\right)+i\sin(x)$$

output
```
hypergeom([1, -1/2/n],[1-1/2/n],exp(2*I*n*x))/exp(I*x)-exp(I*x)*hypergeom(
[1, 1/2/n],[1+1/2/n],exp(2*I*n*x))+I*sin(x)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 179 vs. 2(76) = 152.

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.36

$$\int \cos(x)\cot(nx)\,dx = \frac{1}{2}e^{-2ix}\left(-\frac{e^{i(x+2nx)}\,\text{Hypergeometric2F1}\left(1,1-\frac{1}{2n},2-\frac{1}{2n},e^{2inx}\right)}{-1+2n} - \frac{e^{i(3+2n)x}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{2n},2+\frac{1}{2n},e^{2inx}\right)}{1+2n} + e^{ix}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2inx}\right) - e^{3ix}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},1+\frac{1}{2n},e^{2inx}\right)\right)$$

input
```
Integrate[Cos[x]*Cot[n*x],x]
```

output
```
(-((E^(I*(x + 2*n*x))*Hypergeometric2F1[1, 1 - 1/(2*n), 2 - 1/(2*n), E^((2
*I)*n*x)])/(-1 + 2*n)) - (E^(I*(3 + 2*n)*x)*Hypergeometric2F1[1, 1 + 1/(2*
n), 2 + 1/(2*n), E^((2*I)*n*x)])/(1 + 2*n) + E^(I*x)*Hypergeometric2F1[1,
-1/2*1/n, 1 - 1/(2*n), E^((2*I)*n*x)] - E^((3*I)*x)*Hypergeometric2F1[1, 1
/(2*n), 1 + 1/(2*n), E^((2*I)*n*x)])/(2*E^((2*I)*x))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5069, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(nx)\,dx$$

$$\Big\downarrow \; 5069$$

$$\int \left(-\frac{ie^{-ix}}{1-e^{2inx}}-\frac{ie^{ix}}{1-e^{2inx}}+\frac{1}{2}ie^{-ix}+\frac{1}{2}ie^{ix}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$e^{-ix}\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2inx}\right)- e^{ix}\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),e^{2inx}\right)-\frac{e^{-ix}}{2}+\frac{e^{ix}}{2}$$

input
```
Int[Cos[x]*Cot[n*x],x]
```

output
```
-1/2*1/E^(I*x) + E^(I*x)/2 + Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), E
^((2*I)*n*x)]/E^(I*x) - E^(I*x)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))
/2, E^((2*I)*n*x)]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5069
```
Int[Cos[(a_.) + (b_.)*(x_)]*Cot[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[I*(1/
(E^(I*(a + b*x))*2)) + I*(E^(I*(a + b*x))/2) - I*(1/(E^(I*(a + b*x))*(1 - E
^(2*I*(c + d*x))))) - I*(E^(I*(a + b*x))/(1 - E^(2*I*(c + d*x)))), x] /; Fr
eeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [F]

$$\int \cos (x) \cot (nx)\, dx$$

input
```
int(cos(x)*cot(n*x),x)
```

output
```
int(cos(x)*cot(n*x),x)
```

Fricas [F]

$$\int \cos(x)\cot(nx)\,dx = \int \cos\left(x\right)\cot\left(nx\right)\,dx$$

input
```
integrate(cos(x)*cot(n*x),x, algorithm="fricas")
```

output
```
integral(cos(x)*cot(n*x), x)
```

Sympy [F]

$$\int \cos(x)\cot(nx)\,dx = \int \cos\left(x\right)\cot\left(nx\right)dx$$

input
```
integrate(cos(x)*cot(n*x),x)
```

output
```
Integral(cos(x)*cot(n*x), x)
```

Maxima [F]

$$\int \cos(x)\cot(nx)\,dx = \int \cos\left(x\right)\cot\left(nx\right)\,dx$$

input
```
integrate(cos(x)*cot(n*x),x, algorithm="maxima")
```

output
```
integrate(cos(x)*cot(n*x), x)
```

Giac [F]

$$\int \cos(x)\cot(nx)\,dx = \int \cos(x)\cot(nx)\,dx$$

input
```
integrate(cos(x)*cot(n*x),x, algorithm="giac")
```

output
```
integrate(cos(x)*cot(n*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(x)\cot(nx)\,dx = \int \cot(n\,x)\,\cos(x)\,dx$$

input
```
int(cot(n*x)*cos(x),x)
```

output
```
int(cot(n*x)*cos(x), x)
```

Reduce [F]

$$\int \cos(x)\cot(nx)\,dx = \int \cos(x)\cot(nx)\,dx$$

input
```
int(cos(x)*cot(n*x),x)
```

output
```
int(cos(x)*cot(n*x),x)
```

3.41 $\int \cos(x)\sin(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cos(x)\sin(2x)\,dx = -\frac{\cos(x)}{2} - \frac{1}{6}\cos(3x)$$

output
```
-1/2*cos(x)-1/6*cos(3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(2x)\,dx = -\frac{\cos(x)}{2} - \frac{1}{6}\cos(3x)$$

input
```
Integrate[Cos[x]*Sin[2*x],x]
```

output
```
-1/2*Cos[x] - Cos[3*x]/6
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2x)\cos(x)dx$$

$$\Big\downarrow \text{4772}$$

$$-\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

input
```
Int[Cos[x]*Sin[2*x],x]
```

output
```
-1/2*Cos[x] - Cos[3*x]/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$	12
risch	$-\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$	12
parallelrisch	$-\frac{\cos(3x)}{6}-\frac{\cos(x)}{2}+\frac{2}{3}$	13
orering	$-\frac{\sin(x)\sin(2x)}{3}-\frac{2\cos(x)\cos(2x)}{3}$	18
norman	$\frac{\frac{4\tan(x)^2}{3}+\frac{4\tan\left(\frac{x}{2}\right)^2}{3}-\frac{4\tan\left(\frac{x}{2}\right)\tan(x)}{3}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(\tan(x)^2+1\right)}$	43

input
```
int(cos(x)*sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(x)-1/6*cos(3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \cos(x)\sin(2x)\,dx = -\frac{2}{3}\cos{(x)}^3$$

input
```
integrate(cos(x)*sin(2*x),x, algorithm="fricas")
```

output
```
-2/3*cos(x)^3
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cos(x)\sin(2x)\,dx = -\frac{\sin(x)\sin(2x)}{3} - \frac{2\cos(x)\cos(2x)}{3}$$

input
```
integrate(cos(x)*sin(2*x),x)
```

output
```
-sin(x)*sin(2*x)/3 - 2*cos(x)*cos(2*x)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(x)\sin(2x)\,dx = -\frac{1}{6}\cos(3x) - \frac{1}{2}\cos(x)$$

input
```
integrate(cos(x)*sin(2*x),x, algorithm="maxima")
```

output
```
-1/6*cos(3*x) - 1/2*cos(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \cos(x)\sin(2x)\,dx = -\frac{2}{3}\cos(x)^3$$

input
```
integrate(cos(x)*sin(2*x),x, algorithm="giac")
```

output
```
-2/3*cos(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \cos(x)\sin(2x)\,dx = -\frac{2\cos(x)^3}{3}$$

input
```
int(sin(2*x)*cos(x),x)
```

output
```
-(2*cos(x)^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(x)\sin(2x)\,dx = -\frac{2\cos(2x)\cos(x)}{3} - \frac{\sin(2x)\sin(x)}{3}$$

input
```
int(cos(x)*sin(2*x),x)
```

output
```
( - 2*cos(2*x)*cos(x) - sin(2*x)*sin(x))/3
```

3.42 $\int \cos(x)\sin(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(x)\sin(3x)\,dx = -\frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x)$$

output
```
-1/4*cos(2*x)-1/8*cos(4*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(3x)\,dx = -\frac{1}{2}\cos^2(x) - \frac{1}{8}\cos(4x)$$

input
```
Integrate[Cos[x]*Sin[3*x],x]
```

output
```
-1/2*Cos[x]^2 - Cos[4*x]/8
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(3x)\cos(x)dx$$

$$\Big\downarrow \text{4772}$$

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

input
```
Int[Cos[x]*Sin[3*x],x]
```

output
```
-1/4*Cos[2*x] - Cos[4*x]/8
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$	14
risch	$-\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$	14
parallelrisch	$-\frac{\cos(4x)}{8}+\frac{3}{8}-\frac{\cos(2x)}{4}$	15
orering	$-\frac{3\cos(x)\cos(3x)}{8}-\frac{\sin(x)\sin(3x)}{8}$	18
norman	$\frac{\frac{3\tan\left(\frac{x}{2}\right)^{2}}{4}+\frac{3\tan\left(\frac{3x}{2}\right)^{2}}{4}-\frac{\tan\left(\frac{x}{2}\right)\tan\left(\frac{3x}{2}\right)}{2}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan\left(\frac{3x}{2}\right)^{2}\right)}$	49

input
```
int(sin(3*x)*cos(x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(2*x)-1/8*cos(4*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(3x)\,dx=-\cos\left(x\right)^{4}+\frac{1}{2}\cos\left(x\right)^{2}$$

input
```
integrate(cos(x)*sin(3*x),x, algorithm="fricas")
```

output
```
-cos(x)^4 + 1/2*cos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \cos(x)\sin(3x)\,dx = -\frac{\sin(x)\sin(3x)}{8} - \frac{3\cos(x)\cos(3x)}{8}$$

input
```
integrate(cos(x)*sin(3*x),x)
```

output
```
-sin(x)*sin(3*x)/8 - 3*cos(x)*cos(3*x)/8
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(3x)\,dx = -\frac{1}{8}\cos(4x) - \frac{1}{4}\cos(2x)$$

input
```
integrate(cos(x)*sin(3*x),x, algorithm="maxima")
```

output
```
-1/8*cos(4*x) - 1/4*cos(2*x)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(3x)\,dx = -\cos(x)^4 + \frac{1}{2}\cos(x)^2$$

input
```
integrate(cos(x)*sin(3*x),x, algorithm="giac")
```

output
```
-cos(x)^4 + 1/2*cos(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(3x)\,dx = \frac{\cos(x)^2}{2} - \cos(x)^4$$

input
```
int(sin(3*x)*cos(x),x)
```

output
```
cos(x)^2/2 - cos(x)^4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(3x)\,dx = -\frac{3\cos(3x)\cos(x)}{8} - \frac{\sin(3x)\sin(x)}{8}$$

input
```
int(cos(x)*sin(3*x),x)
```

output
```
( - 3*cos(3*x)*cos(x) - sin(3*x)*sin(x))/8
```

3.43 $\int \cos(x)\sin(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(x)\sin(4x)\,dx = -\frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

output
```
-1/6*cos(3*x)-1/10*cos(5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(4x)\,dx = -\frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

input
```
Integrate[Cos[x]*Sin[4*x],x]
```

output
```
-1/6*Cos[3*x] - Cos[5*x]/10
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(4x)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(4x)\cos(x)dx$$

$$\Big\downarrow \text{4772}$$

$$-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

input
```
Int[Cos[x]*Sin[4*x],x]
```

output
```
-1/6*Cos[3*x] - Cos[5*x]/10
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}$	14
risch	$-\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}$	14
parallelrisch	$-\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}+\frac{4}{15}$	15
orering	$-\frac{\sin(x)\sin(4x)}{15}-\frac{4\cos(x)\cos(4x)}{15}$	18
norman	$\frac{\frac{8\tan(2x)^{2}}{15}+\frac{8\tan\left(\frac{x}{2}\right)^{2}}{15}-\frac{4\tan\left(\frac{x}{2}\right)\tan(2x)}{15}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan(2x)^{2}\right)}$	49

input
```
int(cos(x)*sin(4*x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*cos(3*x)-1/10*cos(5*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(4x)\,dx = -\frac{8}{5}\cos(x)^{5}+\frac{4}{3}\cos(x)^{3}$$

input
```
integrate(cos(x)*sin(4*x),x, algorithm="fricas")
```

output
```
-8/5*cos(x)^5 + 4/3*cos(x)^3
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \cos(x)\sin(4x)\,dx = -\frac{\sin(x)\sin(4x)}{15} - \frac{4\cos(x)\cos(4x)}{15}$$

input
```
integrate(cos(x)*sin(4*x),x)
```

output
```
-sin(x)*sin(4*x)/15 - 4*cos(x)*cos(4*x)/15
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(4x)\,dx = -\frac{1}{10}\cos(5x) - \frac{1}{6}\cos(3x)$$

input
```
integrate(cos(x)*sin(4*x),x, algorithm="maxima")
```

output
```
-1/10*cos(5*x) - 1/6*cos(3*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(4x)\,dx = -\frac{8}{5}\cos(x)^5 + \frac{4}{3}\cos(x)^3$$

input
```
integrate(cos(x)*sin(4*x),x, algorithm="giac")
```

output
```
-8/5*cos(x)^5 + 4/3*cos(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cos(x)\sin(4x)\,dx = -\frac{4\cos(x)^3\left(6\cos(x)^2-5\right)}{15}$$

input
```
int(sin(4*x)*cos(x),x)
```

output
```
-(4*cos(x)^3*(6*cos(x)^2 - 5))/15
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(4x)\,dx = -\frac{4\cos(4x)\cos(x)}{15} - \frac{\sin(4x)\sin(x)}{15}$$

input
```
int(cos(x)*sin(4*x),x)
```

output
```
( - 4*cos(4*x)*cos(x) - sin(4*x)*sin(x))/15
```

3.44 $\int \cos(x)\cos(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cos(x)\cos(2x)\,dx = \frac{\sin(x)}{2} + \frac{1}{6}\sin(3x)$$

output
```
1/2*sin(x)+1/6*sin(3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(2x)\,dx = \frac{\sin(x)}{2} + \frac{1}{6}\sin(3x)$$

input
```
Integrate[Cos[x]*Cos[2*x],x]
```

output
```
Sin[x]/2 + Sin[3*x]/6
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(2x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(x)\cos(2x)dx$$

$$\Big\downarrow \; 4771$$

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

input
```
Int[Cos[x]*Cos[2*x],x]
```

output
```
Sin[x]/2 + Sin[3*x]/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$	12
risch	$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$	12
parallelrisch	$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$	12
orering	$-\frac{\cos(2x)\sin(x)}{3}+\frac{2\cos(x)\sin(2x)}{3}$	18
norman	$\frac{-\frac{4\tan(x)\tan\left(\frac{x}{2}\right)^{2}}{3}+\frac{2\tan(x)^{2}\tan\left(\frac{x}{2}\right)}{3}+\frac{4\tan(x)}{3}-\frac{2\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(\tan(x)^{2}+1\right)}$	51

input
```
int(cos(x)*cos(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)+1/6*sin(3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \cos(x)\cos(2x)\,dx = \frac{1}{3}\left(2\,\cos\left(x\right)^{2}+1\right)\sin\left(x\right)$$

input
```
integrate(cos(x)*cos(2*x),x, algorithm="fricas")
```

output
```
1/3*(2*cos(x)^2 + 1)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(x)\cos(2x)\,dx = -\frac{\sin(x)\cos(2x)}{3} + \frac{2\sin(2x)\cos(x)}{3}$$

input
```
integrate(cos(x)*cos(2*x),x)
```

output
```
-sin(x)*cos(2*x)/3 + 2*sin(2*x)*cos(x)/3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(x)*cos(2*x),x, algorithm="maxima")
```

output
```
1/6*sin(3*x) + 1/2*sin(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(x)*cos(2*x),x, algorithm="giac")
```

output
```
1/6*sin(3*x) + 1/2*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \cos(x)\cos(2x)\,dx = \sin(x) - \frac{2\sin(x)^3}{3}$$

input
```
int(cos(2*x)*cos(x),x)
```

output
```
sin(x) - (2*sin(x)^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(x)\cos(2x)\,dx = -\frac{\cos(2x)\sin(x)}{3} + \frac{2\cos(x)\sin(2x)}{3}$$

input
```
int(cos(x)*cos(2*x),x)
```

output
```
( - cos(2*x)*sin(x) + 2*cos(x)*sin(2*x))/3
```

3.45 $\int \cos(x)\cos(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(x)\cos(3x)\,dx = \frac{1}{4}\sin(2x) + \frac{1}{8}\sin(4x)$$

output
```
1/4*sin(2*x)+1/8*sin(4*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(3x)\,dx = \frac{1}{4}\sin(2x) + \frac{1}{8}\sin(4x)$$

input
```
Integrate[Cos[x]*Cos[3*x],x]
```

output
```
Sin[2*x]/4 + Sin[4*x]/8
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(x)\cos(3x)dx$$

$$\Big\downarrow \text{ 4771}$$

$$\frac{1}{4}\sin(2x) + \frac{1}{8}\sin(4x)$$

input
```
Int[Cos[x]*Cos[3*x],x]
```

output
```
Sin[2*x]/4 + Sin[4*x]/8
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sin(4x)}{8}+\frac{\sin(2x)}{4}$	14
risch	$\frac{\sin(4x)}{8}+\frac{\sin(2x)}{4}$	14
parallelrisch	$\frac{\sin(4x)}{8}+\frac{\sin(2x)}{4}$	14
orering	$-\frac{\cos(3x)\sin(x)}{8}+\frac{3\sin(3x)\cos(x)}{8}$	18
norman	$\frac{\frac{\tan(\frac{x}{2})\tan(\frac{3x}{2})^2}{4}-\frac{3\tan(\frac{x}{2})^2\tan(\frac{3x}{2})}{4}-\frac{\tan(\frac{x}{2})}{4}+\frac{3\tan(\frac{3x}{2})}{4}}{\left(1+\tan(\frac{x}{2})^2\right)\left(1+\tan(\frac{3x}{2})^2\right)}$	59

input
```
int(cos(x)*cos(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*sin(4*x)+1/4*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.41

$$\int \cos(x)\cos(3x)\,dx = \cos(x)^3\sin(x)$$

input
```
integrate(cos(x)*cos(3*x),x, algorithm="fricas")
```

output
```
cos(x)^3*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cos(x)\cos(3x)\,dx = -\frac{\sin(x)\cos(3x)}{8} + \frac{3\sin(3x)\cos(x)}{8}$$

input
```
integrate(cos(x)*cos(3*x),x)
```

output
```
-sin(x)*cos(3*x)/8 + 3*sin(3*x)*cos(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(3x)\,dx = \frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(cos(x)*cos(3*x),x, algorithm="maxima")
```

output
```
1/8*sin(4*x) + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(3x)\,dx = \frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(cos(x)*cos(3*x),x, algorithm="giac")
```

output
```
1/8*sin(4*x) + 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.41

$$\int \cos(x)\cos(3x)\,dx = \cos(x)^3\sin(x)$$

input
```
int(cos(3*x)*cos(x),x)
```

output
```
cos(x)^3*sin(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(3x)\,dx = -\frac{\cos(3x)\sin(x)}{8} + \frac{3\cos(x)\sin(3x)}{8}$$

input
```
int(cos(x)*cos(3*x),x)
```

output
```
( - cos(3*x)*sin(x) + 3*cos(x)*sin(3*x))/8
```

3.46 $\int \cos(x)\cos(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

output
```
1/6*sin(3*x)+1/10*sin(5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

input
```
Integrate[Cos[x]*Cos[4*x],x]
```

output
```
Sin[3*x]/6 + Sin[5*x]/10
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(4x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(x)\cos(4x)dx$$

$$\Big\downarrow \text{ 4771}$$

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

input
```
Int[Cos[x]*Cos[4*x],x]
```

output
```
Sin[3*x]/6 + Sin[5*x]/10
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
risch	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
parallelrisch	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
orering	$-\frac{\sin(x)\cos(4x)}{15}+\frac{4\cos(x)\sin(4x)}{15}$	18
norman	$\frac{-\frac{8\tan(2x)\tan\left(\frac{x}{2}\right)^{2}}{15}+\frac{2\tan(2x)^{2}\tan\left(\frac{x}{2}\right)}{15}+\frac{8\tan(2x)}{15}-\frac{2\tan\left(\frac{x}{2}\right)}{15}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan(2x)^{2}\right)}$	59

input
```
int(cos(x)*cos(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*sin(3*x)+1/10*sin(5*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{15}\left(24\,\cos\left(x\right)^{4}-8\,\cos\left(x\right)^{2}-1\right)\sin\left(x\right)$$

input
```
integrate(cos(x)*cos(4*x),x, algorithm="fricas")
```

output
```
1/15*(24*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cos(x)\cos(4x)\,dx = -\frac{\sin(x)\cos(4x)}{15} + \frac{4\sin(4x)\cos(x)}{15}$$

input
```
integrate(cos(x)*cos(4*x),x)
```

output
```
-sin(x)*cos(4*x)/15 + 4*sin(4*x)*cos(x)/15
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

input
```
integrate(cos(x)*cos(4*x),x, algorithm="maxima")
```

output
```
1/10*sin(5*x) + 1/6*sin(3*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

input
```
integrate(cos(x)*cos(4*x),x, algorithm="giac")
```

output
```
1/10*sin(5*x) + 1/6*sin(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{\sin(3\,x)}{6} + \frac{\sin(5\,x)}{10}$$

input
```
int(cos(4*x)*cos(x),x)
```

output
```
sin(3*x)/6 + sin(5*x)/10
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(4x)\,dx = -\frac{\cos(4x)\sin(x)}{15} + \frac{4\cos(x)\sin(4x)}{15}$$

input
```
int(cos(x)*cos(4*x),x)
```

output
```
( - cos(4*x)*sin(x) + 4*cos(x)*sin(4*x))/15
```

3.47 $\int \cos(x)\tan(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 20

$$\int \cos(x)\tan(2x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}} - \cos(x)$$

output
```
1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)-cos(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. $2(20) = 40$.

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.25

$$\int \cos(x)\tan(2x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}-\tan\left(\frac{x}{2}\right)\right)}{\sqrt{2}} + \frac{\operatorname{arctanh}\left(\sqrt{2}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{2}} - \cos(x)$$

input
```
Integrate[Cos[x]*Tan[2*x],x]
```

output
```
ArcTanh[Sqrt[2] - Tan[x/2]]/Sqrt[2] + ArcTanh[Sqrt[2] + Tan[x/2]]/Sqrt[2]
- Cos[x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4879, 27, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\tan(2x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(2x)}{\sec(x)}dx$$

$$\Big\downarrow \text{ 4879}$$

$$-\int -\frac{2\cos^2(x)}{1-2\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{ 27}$$

$$2\int \frac{\cos^2(x)}{1-2\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{ 262}$$

$$2\left(\frac{1}{2}\int \frac{1}{1-2\cos^2(x)}d\cos(x)-\frac{\cos(x)}{2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$2\left(\frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}-\frac{\cos(x)}{2}\right)$$

input
```
Int[Cos[x]*Tan[2*x],x]
```

output
```
2*(ArcTanh[Sqrt[2]*Cos[x]]/(2*Sqrt[2]) - Cos[x]/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\operatorname{arctanh}\left(\sqrt{2}\,\cos \left(x \right)\right) \sqrt{2}}{2}-\cos \left(x \right)$	18
risch	$-\frac{{\mathrm e}^{i x}}{2}-\frac{{\mathrm e}^{-i x}}{2}+\frac{\sqrt{2}\,\ln \left({\mathrm e}^{2 i x}+\sqrt{2}\,{\mathrm e}^{i x}+1\right)}{4}-\frac{\sqrt{2}\,\ln \left({\mathrm e}^{2 i x}-\sqrt{2}\,{\mathrm e}^{i x}+1\right)}{4}$	61

input
```
int(cos(x)*tan(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(2^(1/2)*cos(x))*2^(1/2)-cos(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \cos(x)\tan(2x)\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2 + 2\sqrt{2}\cos(x) + 1}{2\cos(x)^2 - 1}\right) - \cos(x)$$

input
```
integrate(cos(x)*tan(2*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - c
os(x)
```

Sympy [F]

$$\int \cos(x)\tan(2x)\,dx = \int \cos(x)\tan(2x)\,dx$$

input
```
integrate(cos(x)*tan(2*x),x)
```

output
```
Integral(cos(x)*tan(2*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 6.65

$$\int \cos(x)\tan(2x)\,dx = \frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x) + 2\left(\sqrt{2}\cos(x) + 1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 1\right) - \frac{1}{8}\sqrt{2}\log\left(-2\sqrt{2}\sin(2x)\sin(x) - 2\left(\sqrt{2}\cos(x) - 1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 1\right) - \cos(x)$$

input
```
integrate(cos(x)*tan(2*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x
) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
 1) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*
cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*c
os(x) + 1) - cos(x)
```

Giac [F]

$$\int \cos(x)\tan(2x)\,dx = \int \cos(x)\tan(2x)\,dx$$

input
```
integrate(cos(x)*tan(2*x),x, algorithm="giac")
```

output
```
integrate(cos(x)*tan(2*x), x)
```

Mupad [B] (verification not implemented)

Time = 16.78 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \cos(x)\tan(2x)\,dx = -\frac{\sqrt{2}\,\text{atanh}\left(\frac{8\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{12\tan\left(\frac{x}{2}\right)^2-4}\right)}{2} - \frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(tan(2*x)*cos(x),x)
```

output
```
- (2^(1/2)*atanh((8*2^(1/2)*tan(x/2)^2)/(12*tan(x/2)^2 - 4)))/2 - 2/(tan(x
/2)^2 + 1)
```

Reduce [F]

$$\int \cos(x)\tan(2x)\,dx = \int \cos(x)\tan(2x)\,dx$$

input
```
int(cos(x)*tan(2*x),x)
```

output
```
int(cos(x)*tan(2*x),x)
```

3.48 $\int \cos(x)\tan(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \cos(x)\tan(3x)\,dx = \frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}} - \cos(x)$$

output
```
1/3*arctanh(2/3*3^(1/2)*cos(x))*3^(1/2)-cos(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(21) = 42.

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int \cos(x)\tan(3x)\,dx = -\frac{\operatorname{arctanh}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\operatorname{arctanh}\left(\frac{2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}} - \cos(x)$$

input
```
Integrate[Cos[x]*Tan[3*x],x]
```

output
```
-(ArcTanh[(-2 + Tan[x/2])/Sqrt[3]]/Sqrt[3]) + ArcTanh[(2 + Tan[x/2])/Sqrt[
3]]/Sqrt[3] - Cos[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4879, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\tan(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(3x)}{\sec(x)}dx$$

$$\Big\downarrow \text{ 4879}$$

$$-\int \frac{1-4\cos^2(x)}{3-4\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{ 299}$$

$$2\int \frac{1}{3-4\cos^2(x)}d\cos(x)-\cos(x)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\cos(x)$$

input
```
Int[Cos[x]*Tan[3*x],x]
```

output
```
ArcTanh[(2*Cos[x])/Sqrt[3]]/Sqrt[3] - Cos[x]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{2\sqrt{3}\,\cos(x)}{3}\right)\sqrt{3}}{3}-\cos\left(x\right)$	19
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}+\frac{\sqrt{3}\,\ln\left(e^{2ix}+\sqrt{3}\,e^{ix}+1\right)}{6}-\frac{\sqrt{3}\,\ln\left(e^{2ix}-\sqrt{3}\,e^{ix}+1\right)}{6}$	61

input
```
int(cos(x)*tan(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*arctanh(2/3*3^(1/2)*cos(x))*3^(1/2)-cos(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \cos(x)\tan(3x)\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{4\cos(x)^2 + 4\sqrt{3}\cos(x) + 3}{4\cos(x)^2 - 3}\right) - \cos(x)$$

input
```
integrate(cos(x)*tan(3*x),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) - c
os(x)
```

Sympy [F]

$$\int \cos(x)\tan(3x)\,dx = \int \cos(x)\tan(3x)\,dx$$

input
```
integrate(cos(x)*tan(3*x),x)
```

output
```
Integral(cos(x)*tan(3*x), x)
```

Maxima [F]

$$\int \cos(x)\tan(3x)\,dx = \int \cos(x)\tan(3x)\,dx$$

input
```
integrate(cos(x)*tan(3*x),x, algorithm="maxima")
```

output
```
-cos(x) - integrate(((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*si
n(4*x) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + c
os(2*x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x
)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x)
```

Giac [F]

$$\int \cos(x)\tan(3x)\,dx = \int \cos\left(x\right)\tan\left(3\,x\right)\,dx$$

input
```
integrate(cos(x)*tan(3*x),x, algorithm="giac")
```

output
```
integrate(cos(x)*tan(3*x), x)
```

Mupad [B] (verification not implemented)

Time = 16.96 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \cos(x)\tan(3x)\,dx = -\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{32\,\sqrt{3}\,\tan\left(\frac{x}{2}\right)^2}{3\,\left(\frac{56\,\tan\left(\frac{x}{2}\right)^2}{3}-\frac{8}{3}\right)}\right)}{3}-\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(tan(3*x)*cos(x),x)
```

output
```
- (3^(1/2)*atanh((32*3^(1/2)*tan(x/2)^2)/(3*((56*tan(x/2)^2)/3 - 8/3))))/3
 - 2/(tan(x/2)^2 + 1)
```

Reduce [F]

$$\int \cos(x)\tan(3x)\,dx = \int \cos{(x)}\tan{(3x)}\,dx$$

input
```
int(cos(x)*tan(3*x),x)
```

output
```
int(cos(x)*tan(3*x),x)
```

3.49 $\int \cos(x)\tan(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 77

$$\int \cos(x)\tan(4x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2\left(2-\sqrt{2}\right)}\cos(x)\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{2\left(2+\sqrt{2}\right)}\cos(x)\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} - \cos(x)$$

output
```
1/2*arctanh((4-2*2^(1/2))^(1/2)*cos(x))/(4-2*2^(1/2))^(1/2)+1/2*arctanh((4
+2*2^(1/2))^(1/2)*cos(x))/(4+2*2^(1/2))^(1/2)-cos(x)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 55.22 (sec) , antiderivative size = 6161, normalized size of antiderivative = 80.01

$$\int \cos(x)\tan(4x)\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[x]*Tan[4*x],x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4879, 27, 1602, 27, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\tan(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(4x)}{\sec(x)}dx$$

$$\Big\downarrow \text{4879}$$

$$-\int -\frac{4\cos^2(x)\left(1-2\cos^2(x)\right)}{8\cos^4(x)-8\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{\cos^2(x)\left(1-2\cos^2(x)\right)}{8\cos^4(x)-8\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \text{1602}$$

$$4\left(-\frac{1}{8}\int -\frac{2\left(1-4\cos^2(x)\right)}{8\cos^4(x)-8\cos^2(x)+1}d\cos(x)-\frac{\cos(x)}{4}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{4}\int \frac{1-4\cos^2(x)}{8\cos^4(x)-8\cos^2(x)+1}d\cos(x)-\frac{\cos(x)}{4}\right)$$

$$\Big\downarrow \text{1480}$$

$$4\left(\frac{1}{4}\left(-\left(\left(2-\sqrt{2}\right)\int \frac{1}{8\cos^2(x)-2\left(2-\sqrt{2}\right)}d\cos(x)\right)-\left(2+\sqrt{2}\right)\int \frac{1}{8\cos^2(x)-2\left(2+\sqrt{2}\right)}d\cos(x)\right)-\frac{\cos(}{4}\right.$$

$$\Big\downarrow \textbf{220}$$

$$4\left(\frac{1}{4}\left(\frac{1}{4}\sqrt{2-\sqrt{2}}\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)\right)-\frac{\cos(x)}{4}\right)$$

input
```
Int[Cos[x]*Tan[4*x],x]
```

output
```
4*(((Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2
+ Sqrt[2]]*ArcTanh[(2*Cos[x])/Sqrt[2 + Sqrt[2]]])/4)/4 - Cos[x]/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}-i\left(\sum_{\_R=\operatorname{RootOf}(2048\_Z^4+128\_Z^2+1)}\_R\ln\left(e^{2ix}-8i\_R\,e^{ix}+1\right)\right)$
default	$-\frac{\sqrt{2}\,\sqrt{2+\sqrt{2}}\,\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{4}-\frac{\left(\sqrt{2}-2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}-\cos(x)+\frac{\sqrt{2}\left(3+2\sqrt{2}\right)\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{(-$

input
```
int(cos(x)*tan(4*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(I*x)-1/2*exp(-I*x)-I*sum(_R*ln(exp(2*I*x)-8*I*_R*exp(I*x)+1),_R=R
ootOf(2048*_Z^4+128*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

$$\int \cos(x)\tan(4x)\,dx = \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}+2\cos\left(x\right)\right) - \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}-2\cos\left(x\right)\right) + \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}+2\cos\left(x\right)\right) - \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}-2\cos\left(x\right)\right) - \cos\left(x\right)$$

input
```
integrate(cos(x)*tan(4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + 2*cos(x)) - 1/8*sqrt(sqrt(2)
 + 2)*log(sqrt(sqrt(2) + 2) - 2*cos(x)) + 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(
-sqrt(2) + 2) + 2*cos(x)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)
- 2*cos(x)) - cos(x)
```

Sympy [F]

$$\int \cos(x)\tan(4x)\,dx = \int \cos\left(x\right)\tan\left(4x\right)\,dx$$

input
```
integrate(cos(x)*tan(4*x),x)
```

output
```
Integral(cos(x)*tan(4*x), x)
```

Maxima [F]

$$\int \cos(x)\tan(4x)\,dx = \int \cos(x)\tan(4\,x)\,dx$$

input
```
integrate(cos(x)*tan(4*x),x, algorithm="maxima")
```

output
```
-cos(x) - integrate(-((sin(7*x) - sin(x))*cos(8*x) - (cos(7*x) - cos(x))*s
in(8*x) + sin(7*x) - sin(x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1), x
)
```

Giac [F]

$$\int \cos(x)\tan(4x)\,dx = \int \cos(x)\tan(4\,x)\,dx$$

input
```
integrate(cos(x)*tan(4*x),x, algorithm="giac")
```

output
```
integrate(cos(x)*tan(4*x), x)
```

Mupad [B] (verification not implemented)

Time = 16.88 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.83

$$\int \cos(x)\tan(4x)\,dx =$$

$$-\frac{\operatorname{atanh}\left(\frac{219747975168\tan\left(\frac{x}{2}\right)^2\sqrt{2-\sqrt{2}}}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)^2+386664497152\tan\left(\frac{x}{2}\right)^2-20887633920}-\frac{15971909\cdots}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\cdots\right.}\right.}{\cdots}$$

$$-\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

$$-\frac{\operatorname{atanh}\left(\frac{15971909632\sqrt{\sqrt{2}+2}}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)^2-386664497152\tan\left(\frac{x}{2}\right)^2+20887633920}-\frac{219747975168\cdots}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\cdots\right.}\right.}{\cdots}$$

input
```
int(tan(4*x)*cos(x),x)
```

output
```
- (atanh((219747975168*tan(x/2)^2*(2 - 2^(1/2))^(1/2))/(6098518016*2^(1/2)
 - 254015438848*2^(1/2)*tan(x/2)^2 + 386664497152*tan(x/2)^2 - 20887633920
) - (15971909632*(2 - 2^(1/2))^(1/2))/(6098518016*2^(1/2) - 254015438848*2
^(1/2)*tan(x/2)^2 + 386664497152*tan(x/2)^2 - 20887633920) - (130056978432
*2^(1/2)*tan(x/2)^2*(2 - 2^(1/2))^(1/2))/(6098518016*2^(1/2) - 25401543884
8*2^(1/2)*tan(x/2)^2 + 386664497152*tan(x/2)^2 - 20887633920))*(2 - 2^(1/2
))^(1/2))/4 - 2/(tan(x/2)^2 + 1) - (atanh((15971909632*(2^(1/2) + 2)^(1/2)
)/(6098518016*2^(1/2) - 254015438848*2^(1/2)*tan(x/2)^2 - 386664497152*tan
(x/2)^2 + 20887633920) - (219747975168*tan(x/2)^2*(2^(1/2) + 2)^(1/2))/(60
98518016*2^(1/2) - 254015438848*2^(1/2)*tan(x/2)^2 - 386664497152*tan(x/2)
^2 + 20887633920) - (130056978432*2^(1/2)*tan(x/2)^2*(2^(1/2) + 2)^(1/2))/
(6098518016*2^(1/2) - 254015438848*2^(1/2)*tan(x/2)^2 - 386664497152*tan(x
/2)^2 + 20887633920))*(2^(1/2) + 2)^(1/2))/4
```

Reduce [F]

$$\int \cos(x)\tan(4x)\,dx=\int \cos(x)\tan(4x)\,dx$$

input
```
int(cos(x)*tan(4*x),x)
```

output
```
int(cos(x)*tan(4*x),x)
```

3.50 $\int \cos(x)\tan(5x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 82

$$\int \cos(x)\tan(5x)\,dx = \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{5}\left(5-\sqrt{5}\right)}\cos(x)\right) + \sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\operatorname{arctanh}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cos(x)\right) - \cos(x)$$

output
```
1/10*(10+2*5^(1/2))^(1/2)*arctanh(1/5*(50-10*5^(1/2))^(1/2)*cos(x))+2^(1/2
)/(25+5*5^(1/2))^(1/2)*arctanh(1/5*(50+10*5^(1/2))^(1/2)*cos(x))-cos(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 215 vs. 2(82) = 164.

Time = 0.48 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.62

$$\int \cos(x)\tan(5x)\,dx = \frac{(1+\sqrt{5})\,\text{arctanh}\left(\frac{4-(-1+\sqrt{5})\tan(\frac{x}{2})}{\sqrt{2(5+\sqrt{5})}}\right)}{\sqrt{10\,(5+\sqrt{5})}} + \frac{(1+\sqrt{5})\,\text{arctanh}\left(\frac{4+(-1+\sqrt{5})\tan(\frac{x}{2})}{\sqrt{2(5+\sqrt{5})}}\right)}{\sqrt{10\,(5+\sqrt{5})}} + \frac{(-1+\sqrt{5})\,\text{arctanh}\left(\frac{4-(1+\sqrt{5})\tan(\frac{x}{2})}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{50-10\sqrt{5}}} + \frac{(-1+\sqrt{5})\,\text{arctanh}\left(\frac{4+(1+\sqrt{5})\tan(\frac{x}{2})}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{50-10\sqrt{5}}} - \cos(x)$$

input
```
Integrate[Cos[x]*Tan[5*x],x]
```

output
```
((1 + Sqrt[5])*ArcTanh[(4 - (-1 + Sqrt[5])*Tan[x/2])/Sqrt[2*(5 + Sqrt[5])]
])/Sqrt[10*(5 + Sqrt[5])] + ((1 + Sqrt[5])*ArcTanh[(4 + (-1 + Sqrt[5])*Tan
[x/2])/Sqrt[2*(5 + Sqrt[5])]])/Sqrt[10*(5 + Sqrt[5])] + ((-1 + Sqrt[5])*Ar
cTanh[(4 - (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*Sqrt[5]]])/Sqrt[50 - 10*Sqr
t[5]] + ((-1 + Sqrt[5])*ArcTanh[(4 + (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*S
qrt[5]]])/Sqrt[50 - 10*Sqrt[5]] - Cos[x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4879, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\tan(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(5x)}{\sec(x)}dx$$

$$\Big\downarrow \text{4879}$$

$$-\int \frac{16\cos^4(x)-12\cos^2(x)+1}{16\cos^4(x)-20\cos^2(x)+5}d\cos(x)$$

$$\Big\downarrow \text{2205}$$

$$-\int \left(1-\frac{4(1-2\cos^2(x))}{16\cos^4(x)-20\cos^2(x)+5}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\text{arctanh}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cos(x)\right)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cos(x)\right)-\cos(x)$$

input
```
Int[Cos[x]*Tan[5*x],x]
```

output
```
(Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Cos[x]])/5 + (Sqrt[
(5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Cos[x]])/5 - Cos[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

method	result	size
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}-i\left(\sum_{\_R=\operatorname{RootOf}\left(2000\_Z^{4}+100\_Z^{2}+1\right)}\_R\ln\left(e^{2ix}-10i\_R\,e^{ix}+1\right)\right)$	54

input
```
int(cos(x)*tan(5*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(I*x)-1/2*exp(-I*x)-I*sum(_R*ln(exp(2*I*x)-10*I*_R*exp(I*x)+1),_R=
RootOf(2000*_Z^4+100*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.33

$$\int \cos(x)\tan(5x)\,dx = \frac{1}{10}\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}+2\cos(x)\right) - \frac{1}{10}\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}-2\cos(x)\right) + \frac{1}{10}\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}+2\cos(x)\right) - \frac{1}{10}\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}-2\cos(x)\right) - \cos(x)$$

input
```
integrate(cos(x)*tan(5*x),x, algorithm="fricas")
```

output
```
1/10*sqrt(1/2*sqrt(5) + 5/2)*log(sqrt(1/2*sqrt(5) + 5/2) + 2*cos(x)) - 1/1
0*sqrt(1/2*sqrt(5) + 5/2)*log(sqrt(1/2*sqrt(5) + 5/2) - 2*cos(x)) + 1/10*s
qrt(-1/2*sqrt(5) + 5/2)*log(sqrt(-1/2*sqrt(5) + 5/2) + 2*cos(x)) - 1/10*sq
rt(-1/2*sqrt(5) + 5/2)*log(sqrt(-1/2*sqrt(5) + 5/2) - 2*cos(x)) - cos(x)
```

Sympy [F]

$$\int \cos(x)\tan(5x)\,dx = \int \cos(x)\tan(5x)\,dx$$

input
```
integrate(cos(x)*tan(5*x),x)
```

output
```
Integral(cos(x)*tan(5*x), x)
```

Maxima [F]

$$\int \cos(x)\tan(5x)\,dx = \int \cos{(x)}\tan{(5\,x)}\,\,dx$$

input
```
integrate(cos(x)*tan(5*x),x, algorithm="maxima")
```

output
```
-cos(x) - integrate(((sin(7*x) - sin(5*x) + sin(3*x) - sin(x))*cos(8*x) +
(sin(6*x) - sin(4*x) + sin(2*x))*cos(7*x) + (sin(5*x) - sin(3*x) + sin(x))
*cos(6*x) + (sin(4*x) - sin(2*x))*cos(5*x) + (sin(3*x) - sin(x))*cos(4*x)
- (cos(7*x) - cos(5*x) + cos(3*x) - cos(x))*sin(8*x) - (cos(6*x) - cos(4*x
) + cos(2*x) - 1)*sin(7*x) - (cos(5*x) - cos(3*x) + cos(x))*sin(6*x) - (co
s(4*x) - cos(2*x) + 1)*sin(5*x) - (cos(3*x) - cos(x))*sin(4*x) - (cos(2*x)
 - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos(2*x)*sin(x) - s
in(x))/(2*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(
cos(4*x) - cos(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x)
 - cos(4*x)^2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) -
 sin(8*x)^2 + 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 +
 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x)
```

Giac [F]

$$\int \cos(x)\tan(5x)\,dx = \int \cos{(x)}\tan{(5\,x)}\,\,dx$$

input
```
integrate(cos(x)*tan(5*x),x, algorithm="giac")
```

output
```
integrate(cos(x)*tan(5*x), x)
```

Mupad [B] (verification not implemented)

Time = 17.01 (sec) , antiderivative size = 407, normalized size of antiderivative = 4.96

$$\int \cos(x)\tan(5x)\,dx = \text{Too large to display}$$

input
```
int(tan(5*x)*cos(x),x)
```

output
```
(2^(1/2)*atanh((18032420192256*2^(1/2)*tan(x/2)^2*(5^(1/2) + 5)^(1/2))/((8
851927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/25 - (3334
33343574016*tan(x/2)^2)/5 + 2398739234816) - (867583393792*2^(1/2)*5^(1/2)
*(5^(1/2) + 5)^(1/2))/(25*((8851927597056*5^(1/2))/25 - (676375744741376*5
^(1/2)*tan(x/2)^2)/25 - (333433343574016*tan(x/2)^2)/5 + 2398739234816)) -
 (3805341024256*2^(1/2)*(5^(1/2) + 5)^(1/2))/(5*((8851927597056*5^(1/2))/2
5 - (676375744741376*5^(1/2)*tan(x/2)^2)/25 - (333433343574016*tan(x/2)^2)
/5 + 2398739234816)) + (6886980059136*2^(1/2)*5^(1/2)*tan(x/2)^2*(5^(1/2)
+ 5)^(1/2))/((8851927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2
)^2)/25 - (333433343574016*tan(x/2)^2)/5 + 2398739234816))*(5^(1/2) + 5)^(
1/2))/10 - (2^(1/2)*atanh((867583393792*2^(1/2)*5^(1/2)*(5 - 5^(1/2))^(1/2
))/(25*((8851927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/
25 + (333433343574016*tan(x/2)^2)/5 - 2398739234816)) - (3805341024256*2^(
1/2)*(5 - 5^(1/2))^(1/2))/(5*((8851927597056*5^(1/2))/25 - (67637574474137
6*5^(1/2)*tan(x/2)^2)/25 + (333433343574016*tan(x/2)^2)/5 - 2398739234816)
) + (18032420192256*2^(1/2)*tan(x/2)^2*(5 - 5^(1/2))^(1/2))/((885192759705
6*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/25 + (333433343574016
*tan(x/2)^2)/5 - 2398739234816) - (6886980059136*2^(1/2)*5^(1/2)*tan(x/2)^
2*(5 - 5^(1/2))^(1/2))/((8851927597056*5^(1/2))/25 - (676375744741376*5^(1
/2)*tan(x/2)^2)/25 + (333433343574016*tan(x/2)^2)/5 - 2398739234816))*(...
```

Reduce [F]

$$\int \cos(x)\tan(5x)\,dx = \int \cos(x)\tan(5x)\,dx$$

input
```
int(cos(x)*tan(5*x),x)
```

output
```
int(cos(x)*tan(5*x),x)
```

3.51 $\int \cos(x)\tan(6x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 89

$$\int \cos(x)\tan(6x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}} + \frac{\operatorname{arctanh}\left(2\sqrt{2-\sqrt{3}}\cos(x)\right)}{6\sqrt{2-\sqrt{3}}} + \frac{\operatorname{arctanh}\left(2\sqrt{2+\sqrt{3}}\cos(x)\right)}{6\sqrt{2+\sqrt{3}}} - \cos(x)$$

output
```
1/6*arctanh(cos(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*(1/2*6^(1/2)-1/2*2^(1/2)
)*cos(x))/(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*(1/2*6^(1/2)+1/2*2^(1/2)
)*cos(x))/(1/2*6^(1/2)+1/2*2^(1/2))-cos(x)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.01 (sec) , antiderivative size = 628, normalized size of antiderivative = 7.06

$$\int \cos(x)\tan(6x)\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[x]*Tan[6*x],x]
```

output
```
((4 + 4*I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] + (4 - 4*I)*(-1)^(1
/4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] - 24*Cos[x] - (2*(1 + Sqrt[2])*(x - 2*
Sqrt[3]*ArcTanh[(2 + (2 + Sqrt[2])*Tan[x/2])/Sqrt[6]] - Log[Sec[x/2]^2] +
Log[-(Sec[x/2]^2*(Sqrt[2] - 2*Cos[x] + 2*Sin[x]))]))/(2 + Sqrt[2]) + Sqrt[
2]*(x + 2*Sqrt[3]*ArcTanh[(Sqrt[2] + (-1 + Sqrt[2])*Tan[x/2])/Sqrt[3]] - L
og[Sec[x/2]^2] + Log[Sec[x/2]^2*(1 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x])]) -
(2*(2*(-2 + Sqrt[6])*ArcTanh[Sqrt[2] + (Sqrt[2] - Sqrt[3])*Tan[x/2]] + (3*
Sqrt[2] - 2*Sqrt[3])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[3] + Sq
rt[2]*Cos[x] - Sqrt[2]*Sin[x]))]))*(Sqrt[2] - Sqrt[3]*Sin[x])*(-3 + Sqrt[6
] - (-2 + Sqrt[6])*Cos[x] + (-2 + Sqrt[6])*Sin[x]))/(-36 + 15*Sqrt[6] + (2
0 - 8*Sqrt[6])*Cos[x] + (12 - 5*Sqrt[6])*Cos[2*x] - 50*Sin[x] + 20*Sqrt[6]
*Sin[x] + 12*Sin[2*x] - 5*Sqrt[6]*Sin[2*x]) + (2*(-2*(Sqrt[2] + Sqrt[3])*A
rcTanh[(2 + (2 + Sqrt[6])*Tan[x/2])/Sqrt[2]] + (3 + Sqrt[6])*(x - Log[Sec[
x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[6] - 2*Cos[x] + 2*Sin[x]))]))*(2 + Sqrt[6
]*Sin[x])*(3 + Sqrt[6] - (2 + Sqrt[6])*Cos[x] + (2 + Sqrt[6])*Sin[x]))/(-3
6 - 15*Sqrt[6] + 4*(5 + 2*Sqrt[6])*Cos[x] + (12 + 5*Sqrt[6])*Cos[2*x] - 50
*Sin[x] - 20*Sqrt[6]*Sin[x] + 12*Sin[2*x] + 5*Sqrt[6]*Sin[2*x]))/24
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4879, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\tan(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(6x)}{\sec(x)}dx$$

$$\Big\downarrow \text{4879}$$

$$-\int -\frac{2\cos^2(x)\left(16\cos^4(x)-16\cos^2(x)+3\right)}{-32\cos^6(x)+48\cos^4(x)-18\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\cos^2(x)\left(16\cos^4(x)-16\cos^2(x)+3\right)}{-32\cos^6(x)+48\cos^4(x)-18\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \text{2460}$$

$$2\int \left(\frac{1-8\cos^2(x)}{3\left(16\cos^4(x)-16\cos^2(x)+1\right)}-\frac{1}{6\left(2\cos^2(x)-1\right)}-\frac{1}{2}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{6\sqrt{2}}+\frac{1}{12}\sqrt{2-\sqrt{3}}\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{12}\sqrt{2+\sqrt{3}}\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)-\frac{\cos(x)}{2}\right)$$

input
```
Int[Cos[x]*Tan[6*x],x]
```

output
```
2*(ArcTanh[Sqrt[2]*Cos[x]]/(6*Sqrt[2]) + (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Cos
[x])/Sqrt[2 - Sqrt[3]]])/12 + (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Cos[x])/Sqrt[2
 + Sqrt[3]]])/12 - Cos[x]/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}-i\left(\sum_{\_R=\mathrm{RootOf}(20736\_Z^4+576\_Z^2+1)}\_R\ln\left(e^{2ix}-12i\_R\,e^{ix}+1\right)\right)+\frac{\sqrt{2}\,\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+\right.}{12}$

input
```
int(cos(x)*tan(6*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(I*x)-1/2*exp(-I*x)-I*sum(_R*ln(exp(2*I*x)-12*I*_R*exp(I*x)+1),_R=
RootOf(20736*_Z^4+576*_Z^2+1))+1/12*2^(1/2)*ln(exp(2*I*x)+2^(1/2)*exp(I*x)
+1)-1/12*2^(1/2)*ln(exp(2*I*x)-2^(1/2)*exp(I*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(65) = 130$.

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51

$$\begin{aligned}\int\cos(x)\tan(6x)\,dx &= \frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}+2\cos(x)\right)\\&\quad-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}-2\cos(x)\right)\\&\quad+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}+2\cos(x)\right)\\&\quad-\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}-2\cos(x)\right)\\&\quad+\frac{1}{12}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right)-\cos(x)\end{aligned}$$

input
```
integrate(cos(x)*tan(6*x),x, algorithm="fricas")
```

output
```
1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) + 2*cos(x)) - 1/12*sqrt(sqrt(
3) + 2)*log(sqrt(sqrt(3) + 2) - 2*cos(x)) + 1/12*sqrt(-sqrt(3) + 2)*log(sq
rt(-sqrt(3) + 2) + 2*cos(x)) - 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) +
 2) - 2*cos(x)) + 1/12*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2
*cos(x)^2 - 1)) - cos(x)
```

Sympy [F]

$$\int \cos(x)\tan(6x)\,dx = \int \cos(x)\tan(6x)\,dx$$

input
```
integrate(cos(x)*tan(6*x),x)
```

output
```
Integral(cos(x)*tan(6*x), x)
```

Maxima [F]

$$\int \cos(x)\tan(6x)\,dx = \int \cos(x)\tan(6\,x)\,dx$$

input
```
integrate(cos(x)*tan(6*x),x, algorithm="maxima")
```

output
```
1/24*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*
x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x)
+ 1) - 1/24*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1
)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)
*cos(x) + 1) - cos(x) - integrate(1/3*((2*sin(7*x) + sin(5*x) - sin(3*x) -
 2*sin(x))*cos(8*x) + (sin(3*x) + 2*sin(x))*cos(4*x) - (2*cos(7*x) + cos(5
*x) - cos(3*x) - 2*cos(x))*sin(8*x) - 2*(cos(4*x) - 1)*sin(7*x) - (cos(4*x
) - 1)*sin(5*x) - (cos(3*x) + 2*cos(x))*sin(4*x) + 2*cos(7*x)*sin(4*x) + c
os(5*x)*sin(4*x) - sin(3*x) - 2*sin(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8
*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos
(4*x) - 1), x)
```

Giac [F]

$$\int \cos(x)\tan(6x)\,dx = \int \cos(x)\tan(6x)\,dx$$

input
```
integrate(cos(x)*tan(6*x),x, algorithm="giac")
```

output
```
integrate(cos(x)*tan(6*x), x)
```

Mupad [B] (verification not implemented)

Time = 19.79 (sec) , antiderivative size = 787, normalized size of antiderivative = 8.84

$$\int \cos(x)\tan(6x)\,dx = \text{Too large to display}$$

input
```
int(tan(6*x)*cos(x),x)
```

output
```
(6^(1/2)*(atan((2^(1/2)*321030945816576i)/(213254896304333030400*tan(x/2)^
4 - 1292758292627954380800*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*888
405273481134080i)/(213254896304333030400*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 + 2176593611144037376) - (2^(1/2)*tan(x/2)^2*1871105472480256
0i)/(213254896304333030400*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 +
 2176593611144037376) + (2^(1/2)*tan(x/2)^4*10905601889064960i)/(213254896
304333030400*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 + 2176593611144
037376) - (6^(1/2)*tan(x/2)^2*52765833462352287744i)/(21325489630433303040
0*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 + 2176593611144037376) + (
6^(1/2)*tan(x/2)^4*8705465049710601216 0i)/(213254896304333030400*tan(x/2)^
4 - 129275829262795438080*tan(x/2)^2 + 2176593611144037376)) + atan((2^(1/
2)*14433255045898017881904843325 44i)/(58923240426226065065455386 6240*2^(1/
2)*6^(1/2) + 1191297171699098884409493395 86560*tan(x/2)^2 - 3436727172698 7
95994646686203904 0*2^(1/2)*6^(1/2)*tan(x/2)^2 - 208709030945079899783455 72
92544) - (6^(1/2)*852047139771204346616741888000i)/(5892324042622606506545
53866240*2^(1/2)*6^(1/2) + 119129717169909888440949339586560*tan(x/2)^2 -
34367271726987959946466862039040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 208709030945
0798997834557292544) - (2^(1/2)*tan(x/2)^2*841822835713053045435685824102 4
0i)/(589232404262226065065455386624 0*2^(1/2)*6^(1/2) + 1191297171699098884 4
094933958656 0*tan(x/2)^2 - 34367271726987959946466862039040*2^(1/2)*6^(...
```

Reduce [F]

$$\int \cos(x)\tan(6x)\,dx = \int \cos(x)\tan(6x)\,dx$$

input
```
int(cos(x)*tan(6*x),x)
```

output
```
int(cos(x)*tan(6*x),x)
```

3.52 $\int \cos(x)\cot(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \cos(x)\cot(2x)\,dx = -\frac{1}{2}\operatorname{arctanh}(\cos(x)) + \cos(x)$$

output
```
-1/2*arctanh(cos(x))+cos(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.50

$$\int \cos(x)\cot(2x)\,dx = \cos(x) - \frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Cos[x]*Cot[2*x],x]
```

output
```
Cos[x] - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4879, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\cot(2x)dx$$

$$\Big\downarrow \text{4879}$$

$$-\int -\frac{1-2\cos^2(x)}{2\left(1-\cos^2(x)\right)}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1-2\cos^2(x)}{1-\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{299}$$

$$\frac{1}{2}\left(2\cos(x)-\int \frac{1}{1-\cos^2(x)}d\cos(x)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}(2\cos(x)-\text{arctanh}(\cos(x)))$$

input
```
Int[Cos[x]*Cot[2*x],x]
```

output
```
(-ArcTanh[Cos[x]] + 2*Cos[x])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

method	result	size
default	$\cos(x) + \ln(-\cot(x) + \csc(x)) + \frac{\ln(\cot(x)+\csc(x))}{2}$	20
risch	$\frac{e^{ix}}{2} + \frac{e^{-ix}}{2} - \frac{\ln(e^{ix}+1)}{2} + \frac{\ln(e^{ix}-1)}{2}$	36

input
```
int(cos(x)*cot(2*x),x,method=_RETURNVERBOSE)
```

output
```
cos(x)+ln(-cot(x)+csc(x))+1/2*ln(cot(x)+csc(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \cos(x)\cot(2x)\,dx = \cos(x) - \frac{1}{4}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{4}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(cos(x)*cot(2*x),x, algorithm="fricas")
```

output
```
cos(x) - 1/4*log(1/2*cos(x) + 1/2) + 1/4*log(-1/2*cos(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \cos(x)\cot(2x)\,dx = \frac{\log(\cos(x) - 1)}{4} - \frac{\log(\cos(x) + 1)}{4} + \cos(x)$$

input
```
integrate(cos(x)*cot(2*x),x)
```

output
```
log(cos(x) - 1)/4 - log(cos(x) + 1)/4 + cos(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.70

$$\int \cos(x)\cot(2x)\,dx = \cos(x) - \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)$$

input
```
integrate(cos(x)*cot(2*x),x, algorithm="maxima")
```

output
```
cos(x) - 1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \cos(x)\cot(2x)\,dx = \cos(x) - \frac{1}{4}\log\left(\cos(x) + 1\right) + \frac{1}{4}\log\left(-\cos(x) + 1\right)$$

input
```
integrate(cos(x)*cot(2*x),x, algorithm="giac")
```

output
```
cos(x) - 1/4*log(cos(x) + 1) + 1/4*log(-cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 17.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \cos(x)\cot(2x)\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2} + \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

input
```
int(cot(2*x)*cos(x),x)
```

output
```
log(tan(x/2))/2 + 2/(tan(x/2)^2 + 1)
```

Reduce [F]

$$\int \cos(x)\cot(2x)\,dx = \int \cos\left(x\right)\cot\left(2x\right)dx$$

input
```
int(cos(x)*cot(2*x),x)
```

output
```
int(cos(x)*cot(2*x),x)
```

3.53 $\int \cos(x)\cot(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 20

$$\int \cos(x)\cot(3x)\,dx = -\frac{1}{3}\text{arctanh}\left(\frac{3\cos(x)}{2+\cos(2x)}\right)+\cos(x)$$

output
```
-1/3*arctanh(3*cos(x)/(2+cos(2*x)))+cos(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(20) = 40.

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \cos(x)\cot(3x)\,dx = \cos(x)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{6}\log(1-2\cos(x)) -\frac{1}{6}\log(1+2\cos(x))+\frac{1}{3}\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Cos[x]*Cot[3*x],x]
```

output
```
Cos[x] - Log[Cos[x/2]]/3 + Log[1 - 2*Cos[x]]/6 - Log[1 + 2*Cos[x]]/6 + Log
[Sin[x/2]]/3
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 55 vs. 2(20) = 40.

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4879, 1602, 27, 1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\cot(3x)dx$$

$$\Big\downarrow \text{4879}$$

$$-\int \frac{\cos^2(x)\left(3-4\cos^2(x)\right)}{4\cos^4(x)-5\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{4}\int -\frac{4\left(1-2\cos^2(x)\right)}{4\cos^4(x)-5\cos^2(x)+1}d\cos(x)+\cos(x)$$

$$\Big\downarrow \text{27}$$

$$\cos(x)-\int \frac{1-2\cos^2(x)}{4\cos^4(x)-5\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{4}\int \frac{1}{\cos^2(x)-\frac{\cos(x)}{2}-\frac{1}{2}}d\cos(x)+\frac{1}{4}\int \frac{1}{\cos^2(x)+\frac{\cos(x)}{2}-\frac{1}{2}}d\cos(x)+\cos(x)$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{4}\int \left(-\frac{2}{3(\cos(x)+1)}-\frac{4}{3(1-2\cos(x))}\right)d\cos(x)+\frac{1}{4}\int \left(-\frac{4}{3(2\cos(x)+1)}-\frac{2}{3(1-\cos(x))}\right)d\cos(x)+\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\cos(x)+\frac{1}{4}\left(\frac{2}{3}\log(1-2\cos(x))-\frac{2}{3}\log(\cos(x)+1)\right)+\frac{1}{4}\left(\frac{2}{3}\log(1-\cos(x))-\frac{2}{3}\log(2\cos(x)+1)\right)$$

input
```
Int[Cos[x]*Cot[3*x],x]
```

output
```
Cos[x] + ((2*Log[1 - 2*Cos[x]])/3 - (2*Log[1 + Cos[x]])/3)/4 + ((2*Log[1 -
 Cos[x]])/3 - (2*Log[1 + 2*Cos[x]])/3)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

method	result	size
default	$-\frac{\ln(1+\cos(x))}{6}+\frac{\ln(\cos(x)-1)}{6}-\frac{\ln(2\cos(x)+1)}{6}+\frac{\ln(2\cos(x)-1)}{6}+\cos\left(x\right)$	36
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+\frac{\ln(e^{ix}-1)}{3}-\frac{\ln(e^{ix}+1)}{3}-\frac{\ln(e^{2ix}+e^{ix}+1)}{6}+\frac{\ln(e^{2ix}-e^{ix}+1)}{6}$	68

input
```
int(cos(x)*cot(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(1+cos(x))+1/6*ln(cos(x)-1)-1/6*ln(2*cos(x)+1)+1/6*ln(2*cos(x)-1)+c
os(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \cos(x)\cot(3x)\,dx = \cos\left(x\right)-\frac{1}{6}\,\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{6}\,\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{6}\,\log\left(-2\,\cos\left(x\right)+1\right)-\frac{1}{6}\,\log\left(-2\,\cos\left(x\right)-1\right)$$

input
```
integrate(cos(x)*cot(3*x),x, algorithm="fricas")
```

output
```
cos(x) - 1/6*log(1/2*cos(x) + 1/2) + 1/6*log(-1/2*cos(x) + 1/2) + 1/6*log(
-2*cos(x) + 1) - 1/6*log(-2*cos(x) - 1)
```

Sympy [F]

$$\int \cos(x)\cot(3x)\,dx = \int \cos\left(x\right)\cot\left(3x\right)\,dx$$

input
```
integrate(cos(x)*cot(3*x),x)
```

output
```
Integral(cos(x)*cot(3*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(18) = 36.

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 6.55

$$\begin{aligned}\int \cos(x)\cot(3x)\,dx = \cos\left(x\right) &- \frac{1}{12}\log\left(2\left(\cos\left(x\right)+1\right)\cos\left(2x\right)+\cos\left(2x\right)^{2}+\cos\left(x\right)^{2}\right. \\ &\left.+\sin\left(2x\right)^{2}+2\sin\left(2x\right)\sin\left(x\right)+\sin\left(x\right)^{2}+2\cos\left(x\right)+1\right) \\ &+ \frac{1}{12}\log\left(-2\left(\cos\left(x\right)-1\right)\cos\left(2x\right)+\cos\left(2x\right)^{2}+\cos\left(x\right)^{2}\right. \\ &\left.+\sin\left(2x\right)^{2}-2\sin\left(2x\right)\sin\left(x\right)+\sin\left(x\right)^{2}-2\cos\left(x\right)+1\right) \\ &- \frac{1}{6}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\cos\left(x\right)+1\right) \\ &+ \frac{1}{6}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\cos\left(x\right)+1\right)\end{aligned}$$

input
```
integrate(cos(x)*cot(3*x),x, algorithm="maxima")
```

output
```
cos(x) - 1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*
x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(-2*(cos(x)
- 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + s
in(x)^2 - 2*cos(x) + 1) - 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/
6*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(18) = 36$.

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \cos(x)\cot(3x)\,dx = \cos(x) - \frac{1}{6}\log(\cos(x)+1) + \frac{1}{6}\log(-\cos(x)+1) - \frac{1}{6}\log(|2\cos(x)+1|) + \frac{1}{6}\log(|2\cos(x)-1|)$$

input
```
integrate(cos(x)*cot(3*x),x, algorithm="giac")
```

output
```
cos(x) - 1/6*log(cos(x) + 1) + 1/6*log(-cos(x) + 1) - 1/6*log(abs(2*cos(x)
 + 1)) + 1/6*log(abs(2*cos(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 16.89 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \cos(x)\cot(3x)\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{3} + \frac{\operatorname{atanh}\left(\frac{8}{183\left(\frac{488\tan\left(\frac{x}{2}\right)^2}{243}-\frac{56}{81}\right)} + \frac{121}{122}\right)}{3} + \frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(cot(3*x)*cos(x),x)
```

output
```
log(tan(x/2))/3 + atanh(8/(183*((488*tan(x/2)^2)/243 - 56/81)) + 121/122)/
3 + 2/(tan(x/2)^2 + 1)
```

Reduce [F]

$$\int \cos(x)\cot(3x)\,dx = \int \cos(x)\cot(3x)\,dx$$

input
```
int(cos(x)*cot(3*x),x)
```

output
```
int(cos(x)*cot(3*x),x)
```

3.54 $\int \cos(x)\cot(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 28

$$\int \cos(x)\cot(4x)\,dx = -\frac{1}{4}\text{arctanh}(\cos(x)) - \frac{\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}} + \cos(x)$$

output
```
-1/4*arctanh(cos(x))-1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)+cos(x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.61

$$\int \cos(x)\cot(4x)\,dx = \frac{1}{4}\left((-1-i)(-1)^{3/4}\text{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - (1-i)\sqrt[4]{-1}\text{arctanh}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) + 4\cos(x) - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cos[x]*Cot[4*x],x]
```

output
```
((-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - (1 - I)*(-1)^(1/4)
*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + 4*Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]
])/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4879, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(4x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(x)\cot(4x)dx$$

$$\Big\downarrow \; 4879$$

$$-\int -\frac{8\cos^4(x)-8\cos^2(x)+1}{4\left(2\cos^4(x)-3\cos^2(x)+1\right)}d\cos(x)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\int \frac{8\cos^4(x)-8\cos^2(x)+1}{2\cos^4(x)-3\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \; 2205$$

$$\frac{1}{4}\int \left(4-\frac{3-4\cos^2(x)}{2\cos^4(x)-3\cos^2(x)+1}\right)d\cos(x)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(-\text{arctanh}(\cos(x))-\sqrt{2}\text{arctanh}\left(\sqrt{2}\cos(x)\right)+4\cos(x)\right)$$

input
```
Int[Cos[x]*Cot[4*x],x]
```

output
```
(-ArcTanh[Cos[x]] - Sqrt[2]*ArcTanh[Sqrt[2]*Cos[x]] + 4*Cos[x])/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.89

method	result	size
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}-\frac{\ln\left(e^{ix}+1\right)}{4}+\frac{\ln\left(e^{ix}-1\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+1\right)}{8}+\frac{\sqrt{2}\,\ln\left(e^{2ix}-\sqrt{2}\,e^{ix}+1\right)}{8}$	81

input
```
int(cos(x)*cot(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(I*x)+1/2*exp(-I*x)-1/4*ln(exp(I*x)+1)+1/4*ln(exp(I*x)-1)-1/8*2^(1/
2)*ln(exp(2*I*x)+2^(1/2)*exp(I*x)+1)+1/8*2^(1/2)*ln(exp(2*I*x)-2^(1/2)*exp
(I*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \cos(x)\cot(4x)\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{2\cos(x)^2 - 2\sqrt{2}\cos(x) + 1}{2\cos(x)^2 - 1}\right) + \cos(x) - \frac{1}{8}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{8}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(cos(x)*cot(4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log((2*cos(x)^2 - 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) + co
s(x) - 1/8*log(1/2*cos(x) + 1/2) + 1/8*log(-1/2*cos(x) + 1/2)
```

Sympy [F]

$$\int \cos(x)\cot(4x)\,dx = \int \cos(x)\cot(4x)\,dx$$

input
```
integrate(cos(x)*cot(4*x),x)
```

output
```
Integral(cos(x)*cot(4*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(20) = 40$.

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 5.89

$$\int \cos(x)\cot(4x)\,dx = -\frac{1}{16}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x) + 2\left(\sqrt{2}\cos(x) + 1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 1\right) + \frac{1}{16}\sqrt{2}\log\left(-2\sqrt{2}\sin(2x)\sin(x) - 2\left(\sqrt{2}\cos(x) - 1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 1\right) + \cos(x) - \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)$$

input
```
integrate(cos(x)*cot(4*x),x, algorithm="maxima")
```

output
```
-1/16*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2
*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x)
 + 1) + 1/16*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) -
1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2
)*cos(x) + 1) + cos(x) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/8
*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(20) = 40$.

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \cos(x)\cot(4x)\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\cos(x)\right|}{\left|2\sqrt{2} + 4\cos(x)\right|}\right) + \cos(x) - \frac{1}{8}\log(\cos(x) + 1) + \frac{1}{8}\log(-\cos(x) + 1)$$

input
```
integrate(cos(x)*cot(4*x),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*cos(x))/abs(2*sqrt(2) + 4*cos(x))) + co
s(x) - 1/8*log(cos(x) + 1) + 1/8*log(-cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 16.46 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \cos(x)\cot(4x)\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4} - \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{7\sqrt{2}}{8\left(\frac{29\tan\left(\frac{x}{2}\right)^2}{4}-\frac{5}{4}\right)} - \frac{41\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{8\left(\frac{29\tan\left(\frac{x}{2}\right)^2}{4}-\frac{5}{4}\right)}\right)}{4} + \frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(cot(4*x)*cos(x),x)
```

output
```
log(tan(x/2))/4 - (2^(1/2)*atanh((7*2^(1/2))/(8*((29*tan(x/2)^2)/4 - 5/4))
 - (41*2^(1/2)*tan(x/2)^2)/(8*((29*tan(x/2)^2)/4 - 5/4))))/4 + 2/(tan(x/2)
^2 + 1)
```

Reduce [F]

$$\int \cos(x)\cot(4x)\,dx = \int \cos(x)\cot(4x)\,dx$$

input
```
int(cos(x)*cot(4*x),x)
```

output
```
int(cos(x)*cot(4*x),x)
```

3.55 $\int \cos(x)\cot(5x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 84

$$\int \cos(x)\cot(5x)\,dx = -\frac{1}{5}\operatorname{arctanh}(\cos(x)) - \frac{1}{5}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{2\left(3-\sqrt{5}\right)}\cos(x)\right) - \frac{1}{5}\sqrt{\frac{2}{3+\sqrt{5}}}\operatorname{arctanh}\left(\sqrt{2\left(3+\sqrt{5}\right)}\cos(x)\right) + \cos(x)$$

output
```
-1/5*arctanh(cos(x))-1/5*(1/2+1/2*5^(1/2))*arctanh((5^(1/2)-1)*cos(x))-1/5
*2^(1/2)/(3+5^(1/2))^(1/2)*arctanh((5^(1/2)+1)*cos(x))+cos(x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.58

$$\int \cos(x)\cot(5x)\,dx = \frac{1}{100}\Big(100\cos(x) - 20\log\left(\cos\left(\frac{x}{2}\right)\right) + \sqrt{5}\left(-5+\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cos(x)\right) + \sqrt{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\cos(x)\right) - \sqrt{5}\left(-5+\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cos(x)\right) - \sqrt{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cos(x)\right) + 20\log\left(\sin\left(\frac{x}{2}\right)\right)\Big)$$

input
```
Integrate[Cos[x]*Cot[5*x],x]
```

output
```
(100*Cos[x] - 20*Log[Cos[x/2]] + Sqrt[5]*(-5 + Sqrt[5])*Log[1 - Sqrt[5] -
4*Cos[x]] + Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cos[x]] - Sqrt[5]*(-
5 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cos[x]] - Sqrt[5]*(5 + Sqrt[5])*Log[1 + S
qrt[5] + 4*Cos[x]] + 20*Log[Sin[x/2]])/100
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4879, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(5x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(x)\cot(5x)dx$$

$$\Big\downarrow \; 4879$$

$$-\int \frac{\cos^2(x)\left(16\cos^4(x)-20\cos^2(x)+5\right)}{-16\cos^6(x)+28\cos^4(x)-13\cos^2(x)+1}d\cos(x)$$

$\big\downarrow$ 2460

$$-\int \left(\frac{2(\cos(x)-1)}{5\left(4\cos^2(x)+2\cos(x)-1\right)}-\frac{1}{5\left(\cos^2(x)-1\right)}-\frac{2(\cos(x)+1)}{5\left(4\cos^2(x)-2\cos(x)-1\right)}-1\right)d\cos(x)$$

$\big\downarrow$ 2009

$$-\frac{1}{5}\text{arctanh}(\cos(x))+\cos(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\cos(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\cos(x)+\sqrt{5}+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\cos(x)-\sqrt{5}+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(4\cos(x)+\sqrt{5}+1\right)$$

input
```
Int[Cos[x]*Cot[5*x],x]
```

output
```
-1/5*ArcTanh[Cos[x]] + Cos[x] + ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Cos[x]]
)/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cos[x]])/20 - ((1 - Sqrt[5])*Log
[1 - Sqrt[5] + 4*Cos[x]])/20 - ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Cos[x]])
/20
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.67

method	result
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}-\frac{\ln(e^{ix}+1)}{5}+\frac{\ln(e^{ix}-1)}{5}-\frac{\ln\left(e^{2ix}+\frac{(\sqrt{5}+1)e^{ix}}{2}+1\right)}{20}-\frac{\ln\left(e^{2ix}+\frac{(\sqrt{5}+1)e^{ix}}{2}+1\right)\sqrt{5}}{20}-\frac{\ln\left(e^{2ix}-\frac{(\sqrt{5}-1)e}{2}\right.}{20}$

input
```
int(cos(x)*cot(5*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(I*x)+1/2*exp(-I*x)-1/5*ln(exp(I*x)+1)+1/5*ln(exp(I*x)-1)-1/20*ln(e
xp(2*I*x)+1/2*(5^(1/2)+1)*exp(I*x)+1)-1/20*ln(exp(2*I*x)+1/2*(5^(1/2)+1)*e
xp(I*x)+1)*5^(1/2)-1/20*ln(exp(2*I*x)-1/2*(5^(1/2)-1)*exp(I*x)+1)+1/20*ln(
exp(2*I*x)-1/2*(5^(1/2)-1)*exp(I*x)+1)*5^(1/2)+1/20*ln(exp(2*I*x)+1/2*(5^(
1/2)-1)*exp(I*x)+1)-1/20*ln(exp(2*I*x)+1/2*(5^(1/2)-1)*exp(I*x)+1)*5^(1/2)
+1/20*ln(exp(2*I*x)-1/2*(5^(1/2)+1)*exp(I*x)+1)+1/20*ln(exp(2*I*x)-1/2*(5^
(1/2)+1)*exp(I*x)+1)*5^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(45) = 90$.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.63

$$\int \cos(x)\cot(5x)\,dx = \frac{1}{20}\sqrt{5}\log\left(-\frac{4\left(\sqrt{5}-1\right)\cos(x) - 8\cos(x)^2 + \sqrt{5} - 3}{4\cos(x)^2 + 2\cos(x) - 1}\right) + \frac{1}{20}\sqrt{5}\log\left(-\frac{4\left(\sqrt{5}+1\right)\cos(x) - 8\cos(x)^2 - \sqrt{5} - 3}{4\cos(x)^2 - 2\cos(x) - 1}\right) + \cos(x) - \frac{1}{20}\log\left(4\cos(x)^2 + 2\cos(x) - 1\right) + \frac{1}{20}\log\left(4\cos(x)^2 - 2\cos(x) - 1\right) - \frac{1}{10}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{10}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(cos(x)*cot(5*x),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log(-(4*(sqrt(5) - 1)*cos(x) - 8*cos(x)^2 + sqrt(5) - 3)/(4*c
os(x)^2 + 2*cos(x) - 1)) + 1/20*sqrt(5)*log(-(4*(sqrt(5) + 1)*cos(x) - 8*c
os(x)^2 - sqrt(5) - 3)/(4*cos(x)^2 - 2*cos(x) - 1)) + cos(x) - 1/20*log(4*
cos(x)^2 + 2*cos(x) - 1) + 1/20*log(4*cos(x)^2 - 2*cos(x) - 1) - 1/10*log(
1/2*cos(x) + 1/2) + 1/10*log(-1/2*cos(x) + 1/2)
```

Sympy [F]

$$\int \cos(x)\cot(5x)\,dx = \int \cos(x)\cot(5x)\,dx$$

input
```
integrate(cos(x)*cot(5*x),x)
```

output
```
Integral(cos(x)*cot(5*x), x)
```

Maxima [F]

$$\int \cos(x)\cot(5x)\,dx = \int \cos{(x)}\cot{(5\,x)}\,\,dx$$

input
```
integrate(cos(x)*cot(5*x),x, algorithm="maxima")
```

output
```
cos(x) + 1/10*integrate(-(cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) + cos(3/2*
arctan2(sin(2*x), cos(2*x)))*sin(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x)
))*sin(2*x) - cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) - cos(2*x)*sin
(1/2*arctan2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) +
 cos(4*x)^2 + cos(2*x)^2 - 2*(cos(4*x) + cos(2*x) - cos(1/2*arctan2(sin(2*
x), cos(2*x))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2
(sin(2*x), cos(2*x)))^2 - 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(
2*x), cos(2*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2
*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*(sin(4*x) + sin(2*x) - sin(1/2*arctan2
(sin(2*x), cos(2*x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arct
an2(sin(2*x), cos(2*x)))^2 - 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2
*x), cos(2*x))) + sin(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1)
, x) + 1/10*integrate((cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) - cos(3/2*arc
tan2(sin(2*x), cos(2*x)))*sin(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x)))*
sin(2*x) + cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) + cos(2*x)*sin(1/
2*arctan2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + co
s(4*x)^2 + cos(2*x)^2 + 2*(cos(4*x) + cos(2*x) + cos(1/2*arctan2(sin(2*x),
 cos(2*x))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(si
n(2*x), cos(2*x)))^2 + 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x
), cos(2*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(45) = 90.

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\int \cos(x)\cot(5x)\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{|-2\sqrt{5}+8\cos(x)+2|}{|2\sqrt{5}+8\cos(x)+2|}\right) + \frac{1}{20}\sqrt{5}\log\left(\frac{|-2\sqrt{5}+8\cos(x)-2|}{|2\sqrt{5}+8\cos(x)-2|}\right) + \cos(x) - \frac{1}{10}\log(\cos(x)+1) + \frac{1}{10}\log(-\cos(x)+1) - \frac{1}{20}\log\left(\left|4\cos(x)^2+2\cos(x)-1\right|\right) + \frac{1}{20}\log\left(\left|4\cos(x)^2-2\cos(x)-1\right|\right)$$

input
```
integrate(cos(x)*cot(5*x),x, algorithm="giac")
```

output
```
1/20*sqrt(5)*log(abs(-2*sqrt(5) + 8*cos(x) + 2)/abs(2*sqrt(5) + 8*cos(x) +
 2)) + 1/20*sqrt(5)*log(abs(-2*sqrt(5) + 8*cos(x) - 2)/abs(2*sqrt(5) + 8*c
os(x) - 2)) + cos(x) - 1/10*log(cos(x) + 1) + 1/10*log(-cos(x) + 1) - 1/20
*log(abs(4*cos(x)^2 + 2*cos(x) - 1)) + 1/20*log(abs(4*cos(x)^2 - 2*cos(x)
- 1))
```

Mupad [B] (verification not implemented)

Time = 17.32 (sec) , antiderivative size = 611, normalized size of antiderivative = 7.27

$$\int \cos(x)\cot(5x)\,dx = \text{Too large to display}$$

input
```
int(cot(5*x)*cos(x),x)
```

output
```
(atan((tan(x/2)^2*4813499234516992i)/(1220703125*((213485644414976*5^(1/2)
)/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (4959229
085483008*tan(x/2)^2)/1220703125 + 110872433262592/244140625)) - 954873235
37408i/(244140625*((213485644414976*5^(1/2))/1220703125 - (215264619868979
2*5^(1/2)*tan(x/2)^2)/1220703125 - (4959229085483008*tan(x/2)^2)/122070312
5 + 110872433262592/244140625)) - (5^(1/2)*247887795585024i)/(1220703125*(
(213485644414976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^
2)/1220703125 - (4959229085483008*tan(x/2)^2)/1220703125 + 110872433262592
/244140625)) + (5^(1/2)*tan(x/2)^2*2217818569310208i)/(1220703125*((213485
644414976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220
703125 - (4959229085483008*tan(x/2)^2)/1220703125 + 110872433262592/244140
625)))*1i)/10 + (atan(95487323537408i/(244140625*((213485644414976*5^(1/2)
)/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 + (4959229
085483008*tan(x/2)^2)/1220703125 - 110872433262592/244140625)) - (5^(1/2)*
247887795585024i)/(1220703125*((213485644414976*5^(1/2))/1220703125 - (215
2646198689792*5^(1/2)*tan(x/2)^2)/1220703125 + (4959229085483008*tan(x/2)^
2)/1220703125 - 110872433262592/244140625)) - (tan(x/2)^2*4813499234516992
i)/(1220703125*((213485644414976*5^(1/2))/1220703125 - (2152646198689792*5
^(1/2)*tan(x/2)^2)/1220703125 + (4959229085483008*tan(x/2)^2)/1220703125 -
 110872433262592/244140625)) + (5^(1/2)*tan(x/2)^2*2217818569310208i)/(...
```

Reduce [F]

$$\int \cos(x)\cot(5x)\,dx = \int \cos(x)\cot(5x)\,dx$$

input
```
int(cos(x)*cot(5*x),x)
```

output
```
int(cos(x)*cot(5*x),x)
```

3.56 $\int \cos(x)\cot(6x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 38

$$\int \cos(x)\cot(6x)\,dx = -\frac{1}{6}\text{arctanh}(\cos(x)) - \frac{1}{6}\text{arctanh}(2\cos(x)) - \frac{\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \cos(x)$$

output
```
-1/6*arctanh(cos(x))-1/6*arctanh(2*cos(x))-1/6*arctanh(2/3*3^(1/2)*cos(x))
*3^(1/2)+cos(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(38) = 76.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.29

$$\int \cos(x)\cot(6x)\,dx = \frac{1}{12}\left(2\sqrt{3}\text{arctanh}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) - 2\sqrt{3}\text{arctanh}\left(\frac{2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) + 12\cos(x) - 2\log\left(\cos\left(\frac{x}{2}\right)\right) + \log(1-2\cos(x)) - \log(1+2\cos(x)) + 2\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cos[x]*Cot[6*x],x]
```

output
```
(2*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] - 2*Sqrt[3]*ArcTanh[(2 + Tan[x
/2])/Sqrt[3]] + 12*Cos[x] - 2*Log[Cos[x/2]] + Log[1 - 2*Cos[x]] - Log[1 +
2*Cos[x]] + 2*Log[Sin[x/2]])/12
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4879, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\cot(6x)dx$$

$$\Big\downarrow \text{4879}$$

$$-\int -\frac{-32\cos^6(x)+48\cos^4(x)-18\cos^2(x)+1}{2\left(-16\cos^6(x)+32\cos^4(x)-19\cos^2(x)+3\right)}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{-32\cos^6(x)+48\cos^4(x)-18\cos^2(x)+1}{-16\cos^6(x)+32\cos^4(x)-19\cos^2(x)+3}d\cos(x)$$

$$\Big\downarrow \text{2460}$$

$$\frac{1}{2}\int \left(\frac{2}{4\cos^2(x)-3}+\frac{2}{3\left(4\cos^2(x)-1\right)}+2+\frac{1}{3\left(\cos^2(x)-1\right)}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{3}\text{arctanh}(\cos(x))-\frac{1}{3}\text{arctanh}(2\cos(x))-\frac{\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}}+2\cos(x)\right)$$

input
```
Int[Cos[x]*Cot[6*x],x]
```

output
```
(-1/3*ArcTanh[Cos[x]] - ArcTanh[2*Cos[x]]/3 - ArcTanh[(2*Cos[x])/Sqrt[3]]/
Sqrt[3] + 2*Cos[x])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.97

method	result
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}-\frac{\ln(e^{ix}+1)}{6}+\frac{\ln(e^{ix}-1)}{6}-\frac{\sqrt{3}\,\ln\left(e^{2ix}+\sqrt{3}\,e^{ix}+1\right)}{12}+\frac{\sqrt{3}\,\ln\left(e^{2ix}-\sqrt{3}\,e^{ix}+1\right)}{12}+\frac{\ln(e^{2ix}-e^{ix}+1)}{12}-\frac{\ln(e$

input
```
int(cos(x)*cot(6*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(I*x)+1/2*exp(-I*x)-1/6*ln(exp(I*x)+1)+1/6*ln(exp(I*x)-1)-1/12*3^(1
/2)*ln(exp(2*I*x)+3^(1/2)*exp(I*x)+1)+1/12*3^(1/2)*ln(exp(2*I*x)-3^(1/2)*e
xp(I*x)+1)+1/12*ln(exp(2*I*x)-exp(I*x)+1)-1/12*ln(exp(2*I*x)+exp(I*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(28) = 56.

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\begin{aligned}\int \cos(x)\cot(6x)\,dx &= \frac{1}{12}\sqrt{3}\log\left(\frac{4\,\cos(x)^2-4\sqrt{3}\cos(x)+3}{4\,\cos(x)^2-3}\right)+\cos(x)\\&\quad-\frac{1}{12}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{12}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)\\&\quad+\frac{1}{12}\log(-2\cos(x)+1)-\frac{1}{12}\log(-2\cos(x)-1)\end{aligned}$$

input
```
integrate(cos(x)*cot(6*x),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log((4*cos(x)^2 - 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) + c
os(x) - 1/12*log(1/2*cos(x) + 1/2) + 1/12*log(-1/2*cos(x) + 1/2) + 1/12*lo
g(-2*cos(x) + 1) - 1/12*log(-2*cos(x) - 1)
```

Sympy [F]

$$\int \cos(x)\cot(6x)\,dx = \int \cos{(x)}\cot{(6x)}\,dx$$

input
```
integrate(cos(x)*cot(6*x),x)
```

output
```
Integral(cos(x)*cot(6*x), x)
```

Maxima [F]

$$\int \cos(x)\cot(6x)\,dx = \int \cos{(x)}\cot{(6\,x)}\,dx$$

input
```
integrate(cos(x)*cot(6*x),x, algorithm="maxima")
```

output
```
cos(x) + integrate(1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))
*sin(4*x) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x)
+ cos(2*x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(
2*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1),
x) - 1/24*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2
 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/24*log(-2*(cos(x) - 1)
*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x
)^2 - 2*cos(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/12*
log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(28) = 56$.

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \cos(x)\cot(6x)\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\cos(x)\right|}{\left|4\sqrt{3}+8\cos(x)\right|}\right) + \cos(x) - \frac{1}{12}\log(\cos(x)+1) + \frac{1}{12}\log(-\cos(x)+1) - \frac{1}{12}\log(|2\cos(x)+1|) + \frac{1}{12}\log(|2\cos(x)-1|)$$

input
```
integrate(cos(x)*cot(6*x),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*cos(x))/abs(4*sqrt(3) + 8*cos(x))) + c
os(x) - 1/12*log(cos(x) + 1) + 1/12*log(-cos(x) + 1) - 1/12*log(abs(2*cos(
x) + 1)) + 1/12*log(abs(2*cos(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.26

$$\int \cos(x)\cot(6x)\,dx = \frac{\operatorname{atanh}\left(\frac{1073741824}{10761687\left(\frac{427973089951744\tan\left(\frac{x}{2}\right)^2}{14348907}-\frac{47552804159488}{4782969}\right)}+\frac{797161}{797162}\right)}{6} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{6} - \frac{\sqrt{3}\operatorname{atanh}\left(\frac{303181204553728\sqrt{3}}{4782969\left(\frac{7314051205955584\tan\left(\frac{x}{2}\right)^2}{4782969}-\frac{525125250187264}{4782969}\right)}-\frac{4222769432625152\sqrt{3}\tan\left(\frac{x}{2}\right)^2}{4782969\left(\frac{7314051205955584\tan\left(\frac{x}{2}\right)^2}{4782969}-\frac{525125250187264}{4782969}\right)}\right)}{6} + \frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(cot(6*x)*cos(x),x)
```

output
```
atanh(1073741824/(10761687*((427973089951744*tan(x/2)^2)/14348907 - 475528
04159488/4782969)) + 797161/797162)/6 + log(tan(x/2))/6 - (3^(1/2)*atanh((
303181204553728*3^(1/2))/(4782969*((7314051205955584*tan(x/2)^2)/4782969 -
 525125250187264/4782969)) - (4222769432625152*3^(1/2)*tan(x/2)^2)/(478296
9*((7314051205955584*tan(x/2)^2)/4782969 - 525125250187264/4782969))))/6 +
 2/(tan(x/2)^2 + 1)
```

Reduce [F]

$$\int \cos(x)\cot(6x)\,dx = \int \cos(x)\cot(6x)\,dx$$

input
```
int(cos(x)*cot(6*x),x)
```

output
```
int(cos(x)*cot(6*x),x)
```

3.57 $\int \cos(x)\sec(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cos(x)\sec(2x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(x)\sec(2x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}$$

input
```
Integrate[Cos[x]*Sec[2*x],x]
```

output
```
ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4856, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sec(2x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)}{\cos(2x)}dx$$

$$\Big\downarrow \text{ 4856}$$

$$\int \frac{1}{1-2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}$$

input
```
Int[Cos[x]*Sec[2*x],x]
```

output
```
ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\operatorname{arctanh}\left(\sqrt{2}\,\sin(x)\right)\sqrt{2}}{2}$	13
risch	$\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{4}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{4}$	50

input
```
int(cos(x)*sec(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(2^(1/2)*sin(x))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \cos(x)\sec(2x)\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\,\cos{(x)}^2-2\,\sqrt{2}\sin{(x)}-3}{2\,\cos{(x)}^2-1}\right)$$

input
```
integrate(cos(x)*sec(2*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1))
```

Sympy [F]

$$\int \cos(x)\sec(2x)\,dx = \int \cos(x)\sec(2x)\,dx$$

input
```
integrate(cos(x)*sec(2*x),x)
```

output
```
Integral(cos(x)*sec(2*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(12) = 24.

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 9.13

$$\begin{aligned}\int \cos(x)\sec(2x)\,dx = {} & \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) \\ & - \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) \\ & + \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) \\ & - \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right)\end{aligned}$$

input
```
integrate(cos(x)*sec(2*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \cos(x)\sec(2x)\,dx = \frac{1}{4}\sqrt{2}\log\left(\left|\frac{1}{2}\sqrt{2}+\sin(x)\right|\right) - \frac{1}{4}\sqrt{2}\log\left(\left|-\frac{1}{2}\sqrt{2}+\sin(x)\right|\right)$$

input
```
integrate(cos(x)*sec(2*x),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(1/2*sqrt(2) + sin(x))) - 1/4*sqrt(2)*log(abs(-1/2*sqrt
(2) + sin(x)))
```

Mupad [B] (verification not implemented)

Time = 16.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \cos(x)\sec(2x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\sin(x)\right)}{2}$$

input
```
int(cos(x)/cos(2*x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*sin(x)))/2
```

Reduce [F]

$$\int \cos(x)\sec(2x)\,dx = -4\left(\int \frac{1}{\tan\left(\frac{x}{2}\right)^2\tan(x)^2-\tan\left(\frac{x}{2}\right)^2+\tan(x)^2-1}dx\right) + \frac{\log(\tan(x)-1)}{2} - \frac{\log(\tan(x)+1)}{2} - \sin(x) - x$$

input
```
int(cos(x)*sec(2*x),x)
```

output

```
( - 8*int(1/(tan(x/2)**2*tan(x)**2 - tan(x/2)**2 + tan(x)**2 - 1),x) + log
(tan(x) - 1) - log(tan(x) + 1) - 2*sin(x) - 2*x)/2
```

3.58 $\int \cos(x)\sec(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 44

$$\int \cos(x)\sec(3x)\,dx = -\frac{\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{2\sqrt{3}}+\frac{\log\left(\cos(x)+\sqrt{3}\sin(x)\right)}{2\sqrt{3}}$$

output
```
-1/6*ln(cos(x)-sin(x)*3^(1/2))*3^(1/2)+1/6*ln(cos(x)+sin(x)*3^(1/2))*3^(1/
2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.34

$$\int \cos(x)\sec(3x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{3}\tan(x)\right)}{\sqrt{3}}$$

input
```
Integrate[Cos[x]*Sec[3*x],x]
```

output
```
ArcTanh[Sqrt[3]*Tan[x]]/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.34, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4889, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sec(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)}{\cos(3x)}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{1}{1-3\tan^2(x)}d\tan(x)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\sqrt{3}\tan(x)\right)}{\sqrt{3}}$$

input
```
Int[Cos[x]*Sec[3*x],x]
```

output
```
ArcTanh[Sqrt[3]*Tan[x]]/Sqrt[3]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

method	result	size
default	$\frac{\sqrt{3}\, \operatorname{arctanh}\left(\tan \left(x \right) \sqrt{3}\right)}{3}$	13
risch	$\frac{\sqrt{3}\, \ln \left({\mathrm e}^{2 i x}-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)}{6}-\frac{\sqrt{3}\, \ln \left({\mathrm e}^{2 i x}-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right)}{6}$	40

input
```
int(cos(x)*sec(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arctanh(tan(x)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \cos(x) \sec(3x)\, dx = \frac{1}{12}\sqrt{3}\log\left(-\frac{8\cos\left(x\right)^{4} + 4\left(2\sqrt{3}\cos\left(x\right)^{3} - 3\sqrt{3}\cos\left(x\right)\right)\sin\left(x\right) - 9}{16\cos\left(x\right)^{4} - 24\cos\left(x\right)^{2} + 9}\right)$$

input
```
integrate(cos(x)*sec(3*x),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(-(8*cos(x)^4 + 4*(2*sqrt(3)*cos(x)^3 - 3*sqrt(3)*cos(x))*
sin(x) - 9)/(16*cos(x)^4 - 24*cos(x)^2 + 9))
```

Sympy [F]

$$\int \cos(x)\sec(3x)\,dx = \int \cos(x)\sec(3x)\,dx$$

input
```
integrate(cos(x)*sec(3*x),x)
```

output
```
Integral(cos(x)*sec(3*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(32) = 64$.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.73

$$\int \cos(x)\sec(3x)\,dx$$
$$= \frac{1}{12}\sqrt{3}\left(\log\left(\frac{4}{3}\cos(2x)^2 + \frac{4}{3}\sin(2x)^2 + \frac{4}{3}\sqrt{3}\sin(2x) - \frac{4}{3}\cos(2x) + \frac{4}{3}\right) - \log\left(\frac{4}{3}\cos(2x)^2 + \frac{4}{3}\sin\right.\right.$$

input
```
integrate(cos(x)*sec(3*x),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*(log(4/3*cos(2*x)^2 + 4/3*sin(2*x)^2 + 4/3*sqrt(3)*sin(2*x) -
 4/3*cos(2*x) + 4/3) - log(4/3*cos(2*x)^2 + 4/3*sin(2*x)^2 - 4/3*sqrt(3)*s
in(2*x) - 4/3*cos(2*x) + 4/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \cos(x)\sec(3x)\,dx = -\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+6\tan(x)\right|}{\left|2\sqrt{3}+6\tan(x)\right|}\right)$$

input
```
integrate(cos(x)*sec(3*x),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*log(abs(-2*sqrt(3) + 6*tan(x))/abs(2*sqrt(3) + 6*tan(x)))
```

Mupad [B] (verification not implemented)

Time = 16.83 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.36

$$\int \cos(x)\sec(3x)\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\sin(x)}{\cos(x)}\right)}{3}$$

input
```
int(cos(x)/cos(3*x),x)
```

output
```
(3^(1/2)*atanh((3^(1/2)*sin(x))/cos(x)))/3
```

Reduce [F]

$$\int \cos(x)\sec(3x)\,dx = \int \frac{\cos(x)}{\cos(3x)}dx$$

input
```
int(cos(x)*sec(3*x),x)
```

output
```
int(cos(x)/cos(3*x),x)
```

3.59 $\int \cos(x)\sec(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 71

$$\int \cos(x)\sec(4x)\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

output
```
1/2*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/2*arctanh(2*
sin(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \cos(x)\sec(4x)\,dx=\frac{1}{4}\sqrt{2+\sqrt{2}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

input
```
Integrate[Cos[x]*Sec[4*x],x]
```

output
```
(Sqrt[2 + Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]])/4 - ArcTanh[(2*S
in[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4856, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sec(4x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)}{\cos(4x)}dx$$

$$\Big\downarrow \text{ 4856}$$

$$\int \frac{1}{8\sin^4(x)-8\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{ 1406}$$

$$\sqrt{2}\int \frac{1}{8\sin^2(x)-2\left(2+\sqrt{2}\right)}d\sin(x)-\sqrt{2}\int \frac{1}{8\sin^2(x)-2\left(2-\sqrt{2}\right)}d\sin(x)$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

input
```
Int[Cos[x]*Sec[4*x],x]
```

output
```
ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTanh[
(2*Sin[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.65

method	result	size
risch	$2\left(\sum_{\_R=\operatorname{RootOf}(32768\_Z^4-512\_Z^2+1)}\_R\ln\left(e^{2ix}+\left(4096i\_R^3-48i\_R\right)e^{ix}-1\right)\right)$	46
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}$	54

input
```
int(cos(x)*sec(4*x),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(exp(2*I*x)+(4096*I*_R^3-48*I*_R)*exp(I*x)-1),_R=RootOf(32768*_
Z^4-512*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(49) = 98$.

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

$$\begin{aligned}\int \cos(x)\sec(4x)\,dx =& \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)+2\,\sin\left(x\right)\right)\\&-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)-2\,\sin\left(x\right)\right)\\&-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}+2\,\sin\left(x\right)\right)\\&+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}-2\,\sin\left(x\right)\right)\end{aligned}$$

input
```
integrate(cos(x)*sec(4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) + 2*sin(x)) - 1/
8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) - 2*sin(x)) - 1/8*
sqrt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + 2*sin(x)) + 1/8*
sqrt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 2*sin(x))
```

Sympy [F]

$$\int \cos(x)\sec(4x)\,dx = \int \cos\left(x\right)\sec\left(4x\right)\,dx$$

input
```
integrate(cos(x)*sec(4*x),x)
```

output
```
Integral(cos(x)*sec(4*x), x)
```

Maxima [F]

$$\int \cos(x)\sec(4x)\,dx = \int \cos\left(x\right)\sec\left(4\,x\right)\,dx$$

input
```
integrate(cos(x)*sec(4*x),x, algorithm="maxima")
```

output
```
integrate(cos(x)*sec(4*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \cos(x)\sec(4x)\,dx = -\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left|\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin\left(x\right)\right|\right) + \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left|-\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin\left(x\right)\right|\right) + \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\left|\sqrt{-\frac{1}{4}\sqrt{2}+\frac{1}{2}}+\sin\left(x\right)\right|\right) - \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\left|-\sqrt{-\frac{1}{4}\sqrt{2}+\frac{1}{2}}+\sin\left(x\right)\right|\right)$$

input
```
integrate(cos(x)*sec(4*x),x, algorithm="giac")
```

output
```
-1/8*sqrt(-sqrt(2) + 2)*log(abs(1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sqr
t(-sqrt(2) + 2)*log(abs(-1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sqrt(sqrt(
2) + 2)*log(abs(sqrt(-1/4*sqrt(2) + 1/2) + sin(x))) - 1/8*sqrt(sqrt(2) + 2
)*log(abs(-sqrt(-1/4*sqrt(2) + 1/2) + sin(x)))
```

Mupad [B] (verification not implemented)

Time = 16.54 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.34

$$\int \cos(x)\sec(4x)\,dx = \frac{\operatorname{atanh}\left(\frac{2\sin(x)\sqrt{\sqrt{2}+2}+2\sqrt{2}\sin(x)\sqrt{\sqrt{2}+2}}{\sqrt{2}+2}\right)\sqrt{\sqrt{2}+2}}{4} - \frac{\operatorname{atanh}\left(\frac{2\sin(x)\sqrt{2-\sqrt{2}}-2\sqrt{2}\sin(x)\sqrt{2-\sqrt{2}}}{\sqrt{2}-2}\right)\sqrt{2-\sqrt{2}}}{4}$$

input
```
int(cos(x)/cos(4*x),x)
```

output
```
(atanh((2*sin(x)*(2^(1/2) + 2)^(1/2) + 2*2^(1/2)*sin(x)*(2^(1/2) + 2)^(1/2
))/(2^(1/2) + 2))*(2^(1/2) + 2)^(1/2))/4 - (atanh((2*sin(x)*(2 - 2^(1/2))^
(1/2) - 2*2^(1/2)*sin(x)*(2 - 2^(1/2))^(1/2))/(2^(1/2) - 2))*(2 - 2^(1/2))
^(1/2))/4
```

Reduce [F]

$$\int \cos(x)\sec(4x)\,dx = -4\left(\int \frac{1}{\tan\left(\frac{x}{2}\right)^2\tan(2x)^2-\tan\left(\frac{x}{2}\right)^2+\tan(2x)^2-1}dx\right) + \frac{\log(\tan(2x)-1)}{4} - \frac{\log(\tan(2x)+1)}{4} - \sin(x) - x$$

input
```
int(cos(x)*sec(4*x),x)
```

output
```
( - 16*int(1/(tan(x/2)**2*tan(2*x)**2 - tan(x/2)**2 + tan(2*x)**2 - 1),x)
+ log(tan(2*x) - 1) - log(tan(2*x) + 1) - 4*sin(x) - 4*x)/4
```

3.60 $\int \cos(x)\sec(5x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 73

$$\int \cos(x)\sec(5x)\,dx = -\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\operatorname{arctanh}\left(\sqrt{5-2\sqrt{5}}\tan(x)\right) + \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{5+2\sqrt{5}}\tan(x)\right)$$

output
```
-2^(1/2)/(25+5*5^(1/2))^(1/2)*arctanh((5-2*5^(1/2))^(1/2)*tan(x))+1/10*(10
+2*5^(1/2))^(1/2)*arctanh((5+2*5^(1/2))^(1/2)*tan(x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \cos(x)\sec(5x)\,dx = \frac{\sqrt{5+\sqrt{5}}\operatorname{arctanh}\left(\frac{\left(5+\sqrt{5}\right)\tan(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\operatorname{arctanh}\left(\frac{\left(-5+\sqrt{5}\right)\tan(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

input
```
Integrate[Cos[x]*Sec[5*x],x]
```

output
```
(Sqrt[5 + Sqrt[5]]*ArcTanh[((5 + Sqrt[5])*Tan[x])/Sqrt[10 - 2*Sqrt[5]]] +
Sqrt[5 - Sqrt[5]]*ArcTanh[((-5 + Sqrt[5])*Tan[x])/Sqrt[2*(5 + Sqrt[5])]])/
(5*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4889, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sec(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\cos(5x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tan^2(x)+1}{5\tan^4(x)-10\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(1+\sqrt{5}\right)\int \frac{1}{5\tan^2(x)-2\sqrt{5}-5}d\tan(x)+\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{1}{5\tan^2(x)+2\sqrt{5}-5}d\tan(x)$$

$$\Big\downarrow \text{220}$$

$$-\frac{(1+\sqrt{5})\,\text{arctanh}\left(\sqrt{5-2\sqrt{5}}\tan(x)\right)}{2\sqrt{5\left(5+2\sqrt{5}\right)}}-\frac{(1-\sqrt{5})\,\text{arctanh}\left(\sqrt{5+2\sqrt{5}}\tan(x)\right)}{2\sqrt{5\left(5-2\sqrt{5}\right)}}$$

input
```
Int[Cos[x]*Sec[5*x],x]
```

output
```
-1/2*((1 + Sqrt[5])*ArcTanh[Sqrt[5 - 2*Sqrt[5]]*Tan[x]])/Sqrt[5*(5 + 2*Sqr
t[5])] - ((1 - Sqrt[5])*ArcTanh[Sqrt[5 + 2*Sqrt[5]]*Tan[x]])/(2*Sqrt[5*(5
- 2*Sqrt[5])])
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

method	result	size
risch	$2\left(\sum_{\_R=\mathrm{RootOf}(32000\_Z^4-400\_Z^2+1)}\_R\ln\left(\mathrm{e}^{2ix}+4000i\_R^3-200\_R^2-30i\_R+1\right)\right)$	44
default	$-\frac{\left(5+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{5\tan(x)}{\sqrt{25+10\sqrt{5}}}\right)}{10\sqrt{25+10\sqrt{5}}}-\frac{\left(-5+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{5\tan(x)}{\sqrt{25-10\sqrt{5}}}\right)}{10\sqrt{25-10\sqrt{5}}}$	68

input
```
int(cos(x)*sec(5*x),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(exp(2*I*x)+4000*I*_R^3-200*_R^2-30*I*_R+1),_R=RootOf(32000*_Z^
4-400*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(52) = 104.

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.75

$$\int \cos(x)\sec(5x)\,dx$$
$$= -\frac{1}{20}\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(2\left(\sqrt{5}-1\right)\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\cos\left(x\right)\sin\left(x\right)+2\left(\sqrt{5}+1\right)\cos\left(x\right)^2-\sqrt{5}-5\right)+\frac{1}{20}\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(-2\left(\sqrt{5}-1\right)\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}\cos\left(x\right)\sin\left(x\right)+2\left(\sqrt{5}+1\right)\cos\left(x\right)^2-\sqrt{5}-5\right)-\frac{1}{20}\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(2\left(\sqrt{5}+1\right)\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\cos\left(x\right)\sin\left(x\right)+2\left(\sqrt{5}-1\right)\cos\left(x\right)^2-\sqrt{5}+5\right)+\frac{1}{20}\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\log\left(-2\left(\sqrt{5}+1\right)\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}\cos\left(x\right)\sin\left(x\right)+2\left(\sqrt{5}-1\right)\cos\left(x\right)^2-\sqrt{5}+5\right)$$

input
```
integrate(cos(x)*sec(5*x),x, algorithm="fricas")
```

output
```
-1/20*sqrt(1/2*sqrt(5) + 5/2)*log(2*(sqrt(5) - 1)*sqrt(1/2*sqrt(5) + 5/2)*
cos(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) - 5) + 1/20*sqrt(1/2*sq
rt(5) + 5/2)*log(-2*(sqrt(5) - 1)*sqrt(1/2*sqrt(5) + 5/2)*cos(x)*sin(x) +
2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) - 5) - 1/20*sqrt(-1/2*sqrt(5) + 5/2)*lo
g(2*(sqrt(5) + 1)*sqrt(-1/2*sqrt(5) + 5/2)*cos(x)*sin(x) + 2*(sqrt(5) - 1)
*cos(x)^2 - sqrt(5) + 5) + 1/20*sqrt(-1/2*sqrt(5) + 5/2)*log(-2*(sqrt(5) +
 1)*sqrt(-1/2*sqrt(5) + 5/2)*cos(x)*sin(x) + 2*(sqrt(5) - 1)*cos(x)^2 - sq
rt(5) + 5)
```

Sympy [F]

$$\int \cos(x)\sec(5x)\,dx = \int \cos(x)\sec(5x)\,dx$$

input
```
integrate(cos(x)*sec(5*x),x)
```

output
```
Integral(cos(x)*sec(5*x), x)
```

Maxima [F]

$$\int \cos(x)\sec(5x)\,dx = \int \cos(x)\sec(5\,x)\,dx$$

input
```
integrate(cos(x)*sec(5*x),x, algorithm="maxima")
```

output
```
integrate(cos(x)*sec(5*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(52) = 104$.

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int \cos(x)\sec(5x)\,dx = -\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|\sqrt{\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right) + \frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|-\sqrt{\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right) + \frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|\sqrt{-\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right) - \frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|-\sqrt{-\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right)$$

input
```
integrate(cos(x)*sec(5*x),x, algorithm="giac")
```

output
```
-1/20*sqrt(-2*sqrt(5) + 10)*log(abs(sqrt(2/5*sqrt(5) + 1) + tan(x))) + 1/2
0*sqrt(-2*sqrt(5) + 10)*log(abs(-sqrt(2/5*sqrt(5) + 1) + tan(x))) + 1/20*s
qrt(2*sqrt(5) + 10)*log(abs(sqrt(-2/5*sqrt(5) + 1) + tan(x))) - 1/20*sqrt(
2*sqrt(5) + 10)*log(abs(-sqrt(-2/5*sqrt(5) + 1) + tan(x)))
```

Mupad [B] (verification not implemented)

Time = 16.33 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.97

$$\int \cos(x)\sec(5x)\,dx$$

$$= \frac{\sqrt{2}\,\mathrm{atanh}\left(-\frac{34359738368\sqrt{2}\tan\left(\frac{x}{2}\right)\sqrt{5-\sqrt{5}}}{5\left(\frac{124554051584\sqrt{5}}{25}-\frac{124554051584\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}-\frac{55834574848\tan\left(\frac{x}{2}\right)^2}{5}+\frac{55834574848}{5}\right)} - \frac{7730941132}{25\left(\frac{124554051584\sqrt{5}}{25}-\frac{124554051584}{25}\right.}\right.}{10}$$

$$- \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{77309411328\sqrt{2}\sqrt{5}\tan\left(\frac{x}{2}\right)\sqrt{\sqrt{5}+5}}{25\left(\frac{124554051584\sqrt{5}}{25}-\frac{124554051584\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}+\frac{55834574848\tan\left(\frac{x}{2}\right)^2}{5}-\frac{55834574848}{5}\right)} - \frac{34359738}{5\left(\frac{124554051584\sqrt{5}}{25}-\frac{124554051584}{25}\right.}\right.}{10}$$

input
```
int(cos(x)/cos(5*x),x)
```

output
```
(2^(1/2)*atanh(- (34359738368*2^(1/2)*tan(x/2)*(5 - 5^(1/2))^(1/2))/(5*((1
24554051584*5^(1/2))/25 - (124554051584*5^(1/2)*tan(x/2)^2)/25 - (55834574
848*tan(x/2)^2)/5 + 55834574848/5)) - (77309411328*2^(1/2)*5^(1/2)*tan(x/2
)*(5 - 5^(1/2))^(1/2))/(25*((124554051584*5^(1/2))/25 - (124554051584*5^(1
/2)*tan(x/2)^2)/25 - (55834574848*tan(x/2)^2)/5 + 55834574848/5)))*(5 - 5^
(1/2))^(1/2))/10 - (2^(1/2)*atanh((77309411328*2^(1/2)*5^(1/2)*tan(x/2)*(5
^(1/2) + 5)^(1/2))/(25*((124554051584*5^(1/2))/25 - (124554051584*5^(1/2)*
tan(x/2)^2)/25 + (55834574848*tan(x/2)^2)/5 - 55834574848/5)) - (343597383
68*2^(1/2)*tan(x/2)*(5^(1/2) + 5)^(1/2))/(5*((124554051584*5^(1/2))/25 - (
124554051584*5^(1/2)*tan(x/2)^2)/25 + (55834574848*tan(x/2)^2)/5 - 5583457
4848/5)))*(5^(1/2) + 5)^(1/2))/10
```

Reduce [F]

$$\int \cos(x)\sec(5x)\,dx = \int \frac{\cos(x)}{\cos(5x)}dx$$

input
```
int(cos(x)*sec(5*x),x)
```

output
```
int(cos(x)/cos(5*x),x)
```

3.61 $\int \cos(x)\sec(6x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 85

$$\int \cos(x)\sec(6x)\,dx=-\frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

output
```
-1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*sin(x)/(1/2*6^(1/2)-1/2
*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*sin(x)/(1/2*6^(1/2)+1/2
*2^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \cos(x)\sec(6x)\,dx=\frac{1}{6}\left(-\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)+\sqrt{2+\sqrt{3}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2-\sqrt{3}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)\right)$$

input
```
Integrate[Cos[x]*Sec[6*x],x]
```

output
```
(-(Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]]) + Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])
/Sqrt[2 - Sqrt[3]]] + Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3
]]])/6
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4856, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sec(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\cos(6x)}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \frac{1}{-32\sin^6(x)+48\sin^4(x)-18\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{1}{3\left(2\sin^2(x)-1\right)}-\frac{4\left(2\sin^2(x)-1\right)}{3\left(16\sin^4(x)-16\sin^2(x)+1\right)}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

input
```
Int[Cos[x]*Sec[6*x],x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] + ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[3]
]]/(6*Sqrt[2 - Sqrt[3]]) + ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2
 + Sqrt[3]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

method	result
default	$\frac{2 \operatorname{arctanh}\left(\frac{8 \sin(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{3\left(2\sqrt{6}+2\sqrt{2}\right)}+\frac{2 \operatorname{arctanh}\left(\frac{8 \sin(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{3\left(2\sqrt{6}-2\sqrt{2}\right)}-\frac{\operatorname{arctanh}\left(\sqrt{2}\, \sin(x)\right)\sqrt{2}}{6}$
risch	$2\left(\sum_{\_R=\operatorname{RootOf}\left(331776\_Z^{4}-2304\_Z^{2}+1\right)}\_R \ln\left(\mathrm{e}^{2ix}+\left(-13824i\_R^{3}+96i\_R\right)\mathrm{e}^{ix}-1\right)\right)+\frac{\sqrt{2}\, \ln\left(\mathrm{e}^{2ix}-\right.}{12}$

input
```
int(cos(x)*sec(6*x),x,method=_RETURNVERBOSE)
```

output
```
2/3/(2*6^(1/2)+2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)+2*2^(1/2)))+2/3/(2*6
^(1/2)-2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)-2*2^(1/2)))-1/6*arctanh(2^(1
/2)*sin(x))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(67) = 134$.

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.81

$$\begin{aligned}\int \cos(x)\sec(6x)\,dx = & -\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+2\sin\left(x\right)\right) \\ & +\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)-2\sin\left(x\right)\right) \\ & +\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}+2\sin\left(x\right)\right) \\ & -\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}-2\sin\left(x\right)\right) \\ & +\frac{1}{12}\sqrt{2}\log\left(-\frac{2\cos\left(x\right)^{2}+2\sqrt{2}\sin\left(x\right)-3}{2\cos\left(x\right)^{2}-1}\right)\end{aligned}$$

input
```
integrate(cos(x)*sec(6*x),x, algorithm="fricas")
```

output
```
-1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + 2*sin(x)) +
1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) - 2*sin(x)) + 1
/12*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + 2*sin(x)) -
1/12*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) - 2*sin(x)) +
 1/12*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1))
```

Sympy [F]

$$\int \cos(x)\sec(6x)\,dx = \int \cos{(x)}\sec{(6x)}\,dx$$

input
```
integrate(cos(x)*sec(6*x),x)
```

output
```
Integral(cos(x)*sec(6*x), x)
```

Maxima [F]

$$\int \cos(x)\sec(6x)\,dx = \int \cos{(x)}\sec{(6\,x)}\,dx$$

input
```
integrate(cos(x)*sec(6*x),x, algorithm="maxima")
```

output
```
-1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*s
in(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) -
 2*sqrt(2)*sin(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt
(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x
)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + integrate(-1/3*((cos(7*x)
 + cos(5*x) + cos(3*x) + cos(x))*cos(8*x) - (cos(4*x) - 1)*cos(7*x) - (cos
(4*x) - 1)*cos(5*x) - (cos(3*x) + cos(x))*cos(4*x) + (sin(7*x) + sin(5*x)
+ sin(3*x) + sin(x))*sin(8*x) - (sin(3*x) + sin(x))*sin(4*x) - sin(7*x)*si
n(4*x) - sin(5*x)*sin(4*x) + cos(3*x) + cos(x))/(2*(cos(4*x) - 1)*cos(8*x)
 - cos(8*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2
 + 2*cos(4*x) - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.55

$$\int \cos(x)\sec(6x)\,dx = \frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\left|\frac{1}{4}\sqrt{6}+\frac{1}{4}\sqrt{2}+\sin(x)\right|\right) + \frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\left|\frac{1}{4}\sqrt{6}-\frac{1}{4}\sqrt{2}+\sin(x)\right|\right) - \frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\left|-\frac{1}{4}\sqrt{6}+\frac{1}{4}\sqrt{2}+\sin(x)\right|\right) - \frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\left|-\frac{1}{4}\sqrt{6}-\frac{1}{4}\sqrt{2}+\sin(x)\right|\right) + \frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right)$$

input
```
integrate(cos(x)*sec(6*x),x, algorithm="giac")
```

output
```
1/24*(sqrt(6) - sqrt(2))*log(abs(1/4*sqrt(6) + 1/4*sqrt(2) + sin(x))) + 1/
24*(sqrt(6) + sqrt(2))*log(abs(1/4*sqrt(6) - 1/4*sqrt(2) + sin(x))) - 1/24
*(sqrt(6) + sqrt(2))*log(abs(-1/4*sqrt(6) + 1/4*sqrt(2) + sin(x))) - 1/24*
(sqrt(6) - sqrt(2))*log(abs(-1/4*sqrt(6) - 1/4*sqrt(2) + sin(x))) + 1/12*s
qrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x)))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

$$\int \cos(x)\sec(6x)\,dx = \operatorname{atanh}\left(\frac{5\sqrt{2}\sin(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)} + \frac{3\sqrt{6}\sin(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}+\frac{\sqrt{6}}{12}\right) - \operatorname{atanh}\left(\frac{5\sqrt{2}\sin(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}-\frac{1}{1048576}\right)} - \frac{3\sqrt{6}\sin(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}-\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}-\frac{\sqrt{6}}{12}\right) - \frac{\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\sin(x)\right)}{6}$$

input
```
int(cos(x)/cos(6*x),x)
```

output
```
atanh((5*2^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576))
 + (3*6^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576)))*(
2^(1/2)/12 + 6^(1/2)/12) - atanh((5*2^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(
1/2))/4194304 - 1/1048576)) - (3*6^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2
))/4194304 - 1/1048576)))*(2^(1/2)/12 - 6^(1/2)/12) - (2^(1/2)*atanh(2^(1/
2)*sin(x)))/6
```

Reduce [F]

$$\int \cos(x)\sec(6x)\,dx = -4\left(\int \frac{1}{\tan\left(\frac{x}{2}\right)^2\tan(3x)^2 - \tan\left(\frac{x}{2}\right)^2 + \tan(3x)^2 - 1}dx\right) + \frac{\log(\tan(3x)-1)}{6} - \frac{\log(\tan(3x)+1)}{6} - \sin(x) - x$$

input
```
int(cos(x)*sec(6*x),x)
```

output
```
( - 24*int(1/(tan(x/2)**2*tan(3*x)**2 - tan(x/2)**2 + tan(3*x)**2 - 1),x)
+ log(tan(3*x) - 1) - log(tan(3*x) + 1) - 6*sin(x) - 6*x)/6
```

3.62 $\int \cos(2x)\sec(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \cos(2x)\sec(x)\,dx = -\operatorname{arctanh}(\sin(x)) + 2\sin(x)$$

output
```
-arctanh(sin(x))+2*sin(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(2x)\sec(x)\,dx = -\operatorname{arctanh}(\sin(x)) + 2\sin(x)$$

input
```
Integrate[Cos[2*x]*Sec[x],x]
```

output
```
-ArcTanh[Sin[x]] + 2*Sin[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4864, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(2x)\sec(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(2x)}{\cos(x)}dx$$

$$\Big\downarrow \text{ 4864}$$

$$\int \frac{1-2\sin^2(x)}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 299}$$

$$2\sin(x)-\int \frac{1}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 219}$$

$$2\sin(x)-\text{arctanh}(\sin(x))$$

input
```
Int[Cos[2*x]*Sec[x],x]
```

output
```
-ArcTanh[Sin[x]] + 2*Sin[x]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4864
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
/; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

method	result	size
default	$-\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)+2\sin\left(x\right)$	14
risch	$-i{\mathrm e}^{i x}+i{\mathrm e}^{-i x}+\ln\left({\mathrm e}^{i x}-i\right)-\ln\left({\mathrm e}^{i x}+i\right)$	38

input
```
int(cos(2*x)*sec(x),x,method=_RETURNVERBOSE)
```

output
```
-ln(sec(x)+tan(x))+2*sin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \cos(2x)\sec(x)\,dx = -\frac{1}{2}\,\log\left(\sin\left(x\right)+1\right)+\frac{1}{2}\,\log\left(-\sin\left(x\right)+1\right)+2\,\sin\left(x\right)$$

input
```
integrate(cos(2*x)*sec(x),x, algorithm="fricas")
```

output
```
-1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1) + 2*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \cos(2x)\sec(x)\,dx = \frac{\log\left(\sin\left(x\right)-1\right)}{2}-\frac{\log\left(\sin\left(x\right)+1\right)}{2}+2\sin\left(x\right)$$

input
```
integrate(cos(2*x)*sec(x),x)
```

output
```
log(sin(x) - 1)/2 - log(sin(x) + 1)/2 + 2*sin(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \cos(2x)\sec(x)\,dx = -\frac{1}{2}\,\log\left(\sin\left(x\right)+1\right)+\frac{1}{2}\,\log\left(\sin\left(x\right)-1\right)+2\,\sin\left(x\right)$$

input
```
integrate(cos(2*x)*sec(x),x, algorithm="maxima")
```

output
```
-1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1) + 2*sin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \cos(2x)\sec(x)\,dx = -\frac{1}{2}\log(\sin(x)+1) + \frac{1}{2}\log(-\sin(x)+1) + 2\sin(x)$$

input
```
integrate(cos(2*x)*sec(x),x, algorithm="giac")
```

output
```
-1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1) + 2*sin(x)
```

Mupad [B] (verification not implemented)

Time = 15.95 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(2x)\sec(x)\,dx = 2\sin(x) - \operatorname{atanh}(\sin(x))$$

input
```
int(cos(2*x)/cos(x),x)
```

output
```
2*sin(x) - atanh(sin(x))
```

Reduce [F]

$$\int \cos(2x)\sec(x)\,dx = -4\left(\int \frac{1}{\tan\left(\frac{x}{2}\right)^2\tan(x)^2+\tan\left(\frac{x}{2}\right)^2-\tan(x)^2-1}dx\right) + \log\left(\tan\left(\frac{x}{2}\right)-1\right) - \log\left(\tan\left(\frac{x}{2}\right)+1\right) - \frac{\sin(2x)}{2} - x$$

input
```
int(cos(2*x)*sec(x),x)
```

output
```
( - 8*int(1/(tan(x/2)**2*tan(x)**2 + tan(x/2)**2 - tan(x)**2 - 1),x) + 2*l
og(tan(x/2) - 1) - 2*log(tan(x/2) + 1) - sin(2*x) - 2*x)/2
```

3.63 $\int \cos(4x)\sec(2x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \cos(4x)\sec(2x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(2x)) + \sin(2x)$$

output
```
-1/2*arctanh(sin(2*x))+sin(2*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sec(2x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(2x)) + \sin(2x)$$

input
```
Integrate[Cos[4*x]*Sec[2*x],x]
```

output
```
-1/2*ArcTanh[Sin[2*x]] + Sin[2*x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4864, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(4x)\sec(2x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(4x)}{\cos(2x)}dx$$

$$\Big\downarrow \text{ 4864}$$

$$\frac{1}{2}\int \frac{1-2\sin^2(2x)}{1-\sin^2(2x)}d\sin(2x)$$

$$\Big\downarrow \text{ 299}$$

$$\frac{1}{2}\left(2\sin(2x)-\int \frac{1}{1-\sin^2(2x)}d\sin(2x)\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}(2\sin(2x)-\text{arctanh}(\sin(2x)))$$

input
```
Int[Cos[4*x]*Sec[2*x],x]
```

output
```
(-ArcTanh[Sin[2*x]] + 2*Sin[2*x])/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4864
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
/; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{\ln(\sec(2x)+\tan(2x))}{2}+\sin(2x)$	18
risch	$-\frac{i e^{2ix}}{2}+\frac{i e^{-2ix}}{2}+\frac{\ln(e^{2ix}-i)}{2}-\frac{\ln(e^{2ix}+i)}{2}$	40

input
```
int(cos(4*x)*sec(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(sec(2*x)+tan(2*x))+sin(2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \cos(4x)\sec(2x)\,dx = -\frac{1}{4}\log\left(\sin\left(2x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(2x\right)+1\right)+\sin\left(2x\right)$$

input
```
integrate(cos(4*x)*sec(2*x),x, algorithm="fricas")
```

output
```
-1/4*log(sin(2*x) + 1) + 1/4*log(-sin(2*x) + 1) + sin(2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(12) = 24$.

Time = 1.90 (sec) , antiderivative size = 427, normalized size of antiderivative = 30.50

$$\int \cos(4x)\sec(2x)\,dx = \text{Too large to display}$$

input
```
integrate(cos(4*x)*sec(2*x),x)
```

output
```
-4*x + 32*x*tan(x/2)**4/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 64*x*tan(x/
2)**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 32*x/(8*tan(x/2)**4 + 16*tan(
x/2)**2 + 8) - 3*log(tan(x/2)**2 - 2*tan(x/2) - 1)/2 + 3*log(tan(x/2)**2 +
 2*tan(x/2) - 1)/2 + 8*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**4/(8*ta
n(x/2)**4 + 16*tan(x/2)**2 + 8) + 16*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan
(x/2)**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 8*log(tan(x/2)**2 - 2*tan(
x/2) - 1)/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 8*log(tan(x/2)**2 + 2*tan
(x/2) - 1)*tan(x/2)**4/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 16*log(tan(x
/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8)
- 8*log(tan(x/2)**2 + 2*tan(x/2) - 1)/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8)
 - 32*tan(x/2)**3/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 32*tan(x/2)/(8*ta
n(x/2)**4 + 16*tan(x/2)**2 + 8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 9.21

$$\int \cos(4x)\sec(2x)\,dx = \frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) + \frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) + \sin(2x)$$

input
```
integrate(cos(4*x)*sec(2*x),x, algorithm="maxima")
```

output
```
1/4*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2)
 - 1/4*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) +
 2) - 1/4*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x
) + 2) + 1/4*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*si
n(x) + 2) + sin(2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \cos(4x)\sec(2x)\,dx = -\frac{1}{4}\log(\sin(2x) + 1) + \frac{1}{4}\log(-\sin(2x) + 1) + \sin(2x)$$

input
```
integrate(cos(4*x)*sec(2*x),x, algorithm="giac")
```

output
```
-1/4*log(sin(2*x) + 1) + 1/4*log(-sin(2*x) + 1) + sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \cos(4x)\sec(2x)\,dx = \sin(2\,x) - \frac{\mathrm{atanh}(\sin(2\,x))}{2}$$

input
```
int(cos(4*x)/cos(2*x),x)
```

output
```
sin(2*x) - atanh(sin(2*x))/2
```

Reduce [F]

$$\int \cos(4x)\sec(2x)\,dx = -4\left(\int \frac{1}{\tan(2x)^2\tan(x)^2 - \tan(2x)^2 + \tan(x)^2 - 1}dx\right) + \frac{\log(\tan(x)-1)}{2} - \frac{\log(\tan(x)+1)}{2} - \frac{\sin(4x)}{4} - x$$

input
```
int(cos(4*x)*sec(2*x),x)
```

output
```
( - 16*int(1/(tan(2*x)**2*tan(x)**2 - tan(2*x)**2 + tan(x)**2 - 1),x) + 2*
log(tan(x) - 1) - 2*log(tan(x) + 1) - sin(4*x) - 4*x)/4
```

3.64 $\int \cos(x)\csc(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \cos(x)\csc(2x)\,dx = -\frac{1}{2}\text{arctanh}(\cos(x))$$

output
```
-1/2*arctanh(cos(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \cos(x)\csc(2x)\,dx = -\frac{1}{2}\text{arctanh}(\cos(x))$$

input
```
Integrate[Cos[x]*Csc[2*x],x]
```

output
```
-1/2*ArcTanh[Cos[x]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4775, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\csc(2x)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(x)}{\sin(2x)}dx$$
$$\Big\downarrow \text{ 4775}$$
$$\frac{\int \csc(x)dx}{2}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \csc(x)dx}{2}$$
$$\Big\downarrow \text{ 4257}$$
$$-\frac{1}{2}\text{arctanh}(\cos(x))$$

input
```
Int[Cos[x]*Csc[2*x],x]
```

output
```
-1/2*ArcTanh[Cos[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{\ln(\cot(x)+\csc(x))}{2}$	9
risch	$-\frac{\ln(e^{ix}+1)}{2}+\frac{\ln(e^{ix}-1)}{2}$	22

input
```
int(cos(x)*csc(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(cot(x)+csc(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.71

$$\int \cos(x)\csc(2x)\,dx = -\frac{1}{4}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{4}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

input
```
integrate(cos(x)*csc(2*x),x, algorithm="fricas")
```

output
```
-1/4*log(1/2*cos(x) + 1/2) + 1/4*log(-1/2*cos(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.45 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.14

$$\int \cos(x)\csc(2x)\,dx = \frac{\log\left(\cos\left(x\right)-1\right)}{4} - \frac{\log\left(\cos\left(x\right)+1\right)}{4}$$

input
```
integrate(cos(x)*csc(2*x),x)
```

output
```
log(cos(x) - 1)/4 - log(cos(x) + 1)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(5) = 10$.

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 5.00

$$\begin{aligned}\int \cos(x)\csc(2x)\,dx = &-\frac{1}{4}\,\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right)\\&+\frac{1}{4}\,\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\cos\left(x\right)+1\right)\end{aligned}$$

input
```
integrate(cos(x)*csc(2*x),x, algorithm="maxima")
```

output
```
-1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
 - 2*cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \cos(x)\csc(2x)\,dx = -\frac{1}{4}\log\left(\cos\left(x\right)+1\right)+\frac{1}{4}\log\left(-\cos\left(x\right)+1\right)$$

input
```
integrate(cos(x)*csc(2*x),x, algorithm="giac")
```

output
```
-1/4*log(cos(x) + 1) + 1/4*log(-cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \cos(x)\csc(2x)\,dx = -\frac{\operatorname{atanh}(\cos\left(x\right))}{2}$$

input
```
int(cos(x)/sin(2*x),x)
```

output
```
-atanh(cos(x))/2
```

Reduce [F]

$$\int \cos(x)\csc(2x)\,dx = \int \cos\left(x\right)\csc\left(2x\right)dx$$

input
```
int(cos(x)*csc(2*x),x)
```

output
```
int(cos(x)*csc(2*x),x)
```

3.65 $\int \cos(x)\csc(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3 - 4\sin^2(x)\right)$$

output
```
1/3*ln(sin(x))-1/6*ln(3-4*sin(x)^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3 - 4\sin^2(x)\right)$$

input
```
Integrate[Cos[x]*Csc[3*x],x]
```

output
```
Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 4856, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\csc(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sin(3x)}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \frac{\csc(x)}{3-4\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\csc(x)}{3-4\sin^2(x)}d\sin^2(x)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{3-4\sin^2(x)}d\sin^2(x)+\frac{1}{3}\int \csc(x)d\sin^2(x)\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{3-4\sin^2(x)}d\sin^2(x)+\frac{1}{3}\log\left(\sin^2(x)\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\log\left(\sin^2(x)\right)-\frac{1}{3}\log\left(3-4\sin^2(x)\right)\right)$$

input
```
Int[Cos[x]*Csc[3*x],x]
```

output
```
(Log[Sin[x]^2]/3 - Log[3 - 4*Sin[x]^2]/3)/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

method	result	size
risch	$\frac{\ln\left(\mathrm{e}^{2 i x}-1\right)}{3}-\frac{\ln\left(\mathrm{e}^{4 i x}+\mathrm{e}^{2 i x}+1\right)}{6}$	27
default	$-\frac{\ln\left(2\cos\left(x\right)+1\right)}{6}+\frac{\ln\left(1+\cos\left(x\right)\right)}{6}+\frac{\ln\left(\cos\left(x\right)-1\right)}{6}-\frac{\ln\left(2\cos\left(x\right)-1\right)}{6}$	34

input
```
int(cos(x)*csc(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(exp(2*I*x)-1)-1/6*ln(exp(4*I*x)+exp(2*I*x)+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \cos(x)\csc(3x)\,dx = -\frac{1}{6}\log\left(4\cos(x)^2-1\right)+\frac{1}{3}\log\left(\frac{1}{2}\sin(x)\right)$$

input
```
integrate(cos(x)*csc(3*x),x, algorithm="fricas")
```

output
```
-1/6*log(4*cos(x)^2 - 1) + 1/3*log(1/2*sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\csc(3x)\,dx = -\frac{\log\left(4\sin^2(x)-3\right)}{6}+\frac{\log\left(\sin(x)\right)}{3}$$

input
```
integrate(cos(x)*csc(3*x),x)
```

output
```
-log(4*sin(x)**2 - 3)/6 + log(sin(x))/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 6.14

$$\int \cos(x)\csc(3x)\,dx = -\frac{1}{12}\log\left(2\left(\cos\left(x\right)+1\right)\cos\left(2x\right)+\cos\left(2x\right)^2+\cos\left(x\right)^2+\sin\left(2x\right)^2+2\sin\left(2x\right)\sin\left(x\right)+\sin\left(x\right)^2+2\cos\left(x\right)+1\right) - \frac{1}{12}\log\left(-2\left(\cos\left(x\right)-1\right)\cos\left(2x\right)+\cos\left(2x\right)^2+\cos\left(x\right)^2+\sin\left(2x\right)^2-2\sin\left(2x\right)\sin\left(x\right)+\sin\left(x\right)^2-2\cos\left(x\right)+1\right) + \frac{1}{6}\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\cos\left(x\right)+1\right) + \frac{1}{6}\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\cos\left(x\right)+1\right)$$

input
```
integrate(cos(x)*csc(3*x),x, algorithm="maxima")
```

output
```
-1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2
*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) - 1/12*log(-2*(cos(x) - 1)*cos
(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2
- 2*cos(x) + 1) + 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/6*log(co
s(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{6}\log\left(-\cos\left(x\right)^2+1\right) - \frac{1}{6}\log\left(\left|4\cos\left(x\right)^2-1\right|\right)$$

input
```
integrate(cos(x)*csc(3*x),x, algorithm="giac")
```

output
```
1/6*log(-cos(x)^2 + 1) - 1/6*log(abs(4*cos(x)^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 15.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\csc(3x)\,dx = \frac{\ln(\sin(x))}{3} - \frac{\ln\left(\frac{1}{4} - \cos(x)^2\right)}{6}$$

input
```
int(cos(x)/sin(3*x),x)
```

output
```
log(sin(x))/3 - log(1/4 - cos(x)^2)/6
```

Reduce [F]

$$\int \cos(x)\csc(3x)\,dx = \int \cos(x)\csc(3x)\,dx$$

input
```
int(cos(x)*csc(3*x),x)
```

output
```
int(cos(x)*csc(3*x),x)
```

3.66 $\int \cos(x)\csc(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \cos(x)\csc(4x)\,dx = -\frac{1}{4}\operatorname{arctanh}(\cos(x)) + \frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}$$

output
```
-1/4*arctanh(cos(x))+1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \cos(x)\csc(4x)\,dx = \frac{1}{4}\left((1+i)(-1)^{3/4}\operatorname{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) + \sqrt{2}\operatorname{arctanh}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cos[x]*Csc[4*x],x]
```

output
```
((1 + I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] + Sqrt[2]*ArcTanh[(1
+ Tan[x/2])/Sqrt[2]] - Log[Cos[x/2]] + Log[Sin[x/2]])/4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4879, 1406, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\csc(4x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)}{\sin(4x)}dx$$
$$\Big\downarrow \text{4879}$$
$$-\int \frac{1}{-8\cos^4(x)+12\cos^2(x)-4}d\cos(x)$$
$$\Big\downarrow \text{1406}$$
$$2\int \frac{1}{4-8\cos^2(x)}d\cos(x)-2\int \frac{1}{8-8\cos^2(x)}d\cos(x)$$
$$\Big\downarrow \text{219}$$
$$\frac{\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}-\frac{1}{4}\text{arctanh}(\cos(x))$$

input
```
Int[Cos[x]*Csc[4*x],x]
```

output
```
-1/4*ArcTanh[Cos[x]] + ArcTanh[Sqrt[2]*Cos[x]]/(2*Sqrt[2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\operatorname{arctanh}\left(\sqrt{2}\,\cos(x)\right)\sqrt{2}}{4}-\frac{\ln(1+\cos(x))}{8}+\frac{\ln(\cos(x)-1)}{8}$	28
risch	$\frac{\ln(\mathrm{e}^{ix}-1)}{4}-\frac{\ln(\mathrm{e}^{ix}+1)}{4}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-\sqrt{2}\,\mathrm{e}^{ix}+1\right)}{8}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+\sqrt{2}\,\mathrm{e}^{ix}+1\right)}{8}$	67

input
```
int(cos(x)*csc(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*arctanh(2^(1/2)*cos(x))*2^(1/2)-1/8*ln(1+cos(x))+1/8*ln(cos(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \cos(x)\csc(4x)\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2 + 2\sqrt{2}\cos(x) + 1}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{8}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(cos(x)*csc(4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - 1
/8*log(1/2*cos(x) + 1/2) + 1/8*log(-1/2*cos(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(22) = 44$.

Time = 3.22 (sec) , antiderivative size = 248, normalized size of antiderivative = 9.54

$$\int \cos(x)\csc(4x)\,dx = -\frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right) - 1 + \sqrt{2}\right)}{110880\sqrt{2} + 156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right) - 1 + \sqrt{2}\right)}{110880\sqrt{2} + 156808} + \frac{27720\log\left(\tan\left(\frac{x}{2}\right) + 1 + \sqrt{2}\right)}{110880\sqrt{2} + 156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right) + 1 + \sqrt{2}\right)}{110880\sqrt{2} + 156808} + \frac{27720\log\left(\tan\left(\frac{x}{2}\right) - \sqrt{2} - 1\right)}{110880\sqrt{2} + 156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right) - \sqrt{2} - 1\right)}{110880\sqrt{2} + 156808} - \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right) - \sqrt{2} + 1\right)}{110880\sqrt{2} + 156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right) - \sqrt{2} + 1\right)}{110880\sqrt{2} + 156808} + \frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)\right)}{110880\sqrt{2} + 156808} + \frac{39202\log\left(\tan\left(\frac{x}{2}\right)\right)}{110880\sqrt{2} + 156808}$$

input
```
integrate(cos(x)*csc(4*x),x)
```

output
```
-19601*sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) - 277
20*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(x
/2) + 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2)
+ 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(x/2) - sqrt(2) -
1)/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2) - sqrt(2) - 1)/(
110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(1108
80*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2) +
 156808) + 27720*sqrt(2)*log(tan(x/2))/(110880*sqrt(2) + 156808) + 39202*l
og(tan(x/2))/(110880*sqrt(2) + 156808)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(18) = 36$.

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 6.27

$$\begin{aligned}\int \cos(x)\csc(4x)\,dx = {} & \frac{1}{16}\sqrt{2}\log\left(2\sqrt{2}\sin\left(2x\right)\sin\left(x\right) + 2\left(\sqrt{2}\cos\left(x\right)+1\right)\cos\left(2x\right) \right.\\ & \left. +\cos\left(2x\right)^2+2\cos\left(x\right)^2+\sin\left(2x\right)^2+2\sin\left(x\right)^2+2\sqrt{2}\cos\left(x\right)+1\right) \\ & - \frac{1}{16}\sqrt{2}\log\left(-2\sqrt{2}\sin\left(2x\right)\sin\left(x\right) \right.\\ & - 2\left(\sqrt{2}\cos\left(x\right)-1\right)\cos\left(2x\right)+\cos\left(2x\right)^2+2\cos\left(x\right)^2 \\ & \left. +\sin\left(2x\right)^2+2\sin\left(x\right)^2-2\sqrt{2}\cos\left(x\right)+1\right) \\ & - \frac{1}{8}\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\cos\left(x\right)+1\right) \\ & + \frac{1}{8}\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\cos\left(x\right)+1\right)\end{aligned}$$

input
```
integrate(cos(x)*csc(4*x),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*
x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x)
+ 1) - 1/16*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1
)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)
*cos(x) + 1) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/8*log(cos(x
)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \cos(x)\csc(4x)\,dx = -\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\cos(x)\right|}{\left|2\sqrt{2}+4\cos(x)\right|}\right) - \frac{1}{8}\log(\cos(x)+1) + \frac{1}{8}\log(-\cos(x)+1)$$

input
```
integrate(cos(x)*csc(4*x),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*cos(x))/abs(2*sqrt(2) + 4*cos(x))) - 1
/8*log(cos(x) + 1) + 1/8*log(-cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 16.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.12

$$\int \cos(x)\csc(4x)\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4} + \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{41\sqrt{2}}{8\left(\frac{169\tan\left(\frac{x}{2}\right)^2}{4}-\frac{29}{4}\right)} - \frac{239\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{8\left(\frac{169\tan\left(\frac{x}{2}\right)^2}{4}-\frac{29}{4}\right)}\right)}{4}$$

input
```
int(cos(x)/sin(4*x),x)
```

output
```
log(tan(x/2))/4 + (2^(1/2)*atanh((41*2^(1/2))/(8*((169*tan(x/2)^2)/4 - 29/
4)) - (239*2^(1/2)*tan(x/2)^2)/(8*((169*tan(x/2)^2)/4 - 29/4))))/4
```

Reduce [F]

$$\int \cos(x)\csc(4x)\,dx = \int \cos(x)\csc(4x)\,dx$$

input
```
int(cos(x)*csc(4*x),x)
```

output
```
int(cos(x)*csc(4*x),x)
```

3.67 $\int \cos(x)\csc(5x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 71

$$\int \cos(x)\csc(5x)\,dx = \frac{1}{5}\log(\sin(x)) - \frac{\log\left(5-\sqrt{5}-8\sin^2(x)\right)}{\sqrt{5}\left(5-\sqrt{5}\right)} + \frac{\log\left(5+\sqrt{5}-8\sin^2(x)\right)}{\sqrt{5}\left(5+\sqrt{5}\right)}$$

output
```
1/5*ln(sin(x))-1/5*ln(5-5^(1/2)-8*sin(x)^2)*5^(1/2)/(5-5^(1/2))+1/5*ln(5+5
^(1/2)-8*sin(x)^2)*5^(1/2)/(5+5^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \cos(x)\csc(5x)\,dx = \frac{1}{20}\left(-\left(\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cos(2x)\right)\right) + \left(-1+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cos(2x)\right) + 4\log(\sin(x))\right)$$

input
```
Integrate[Cos[x]*Csc[5*x],x]
```

output
```
(-((1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cos[2*x]]) + (-1 + Sqrt[5])*Log[1 + S
qrt[5] + 4*Cos[2*x]] + 4*Log[Sin[x]])/20
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4856, 1434, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\csc(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sin(5x)}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \frac{\csc(x)}{16\sin^4(x)-20\sin^2(x)+5}d\sin(x)$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{\csc(x)}{16\sin^4(x)-20\sin^2(x)+5}d\sin^2(x)$$

$$\Big\downarrow \text{1141}$$

$$8\int \left(\frac{\csc(x)}{80}+\frac{1}{\sqrt{5}\left(5-\sqrt{5}\right)\left(-8\sin^2(x)-\sqrt{5}+5\right)}-\frac{1}{\sqrt{5}\left(5+\sqrt{5}\right)\left(-8\sin^2(x)+\sqrt{5}+5\right)}\right)d\sin^2(x)$$

$$\Big\downarrow \text{2009}$$

$$8\left(\frac{1}{80}\log\left(\sin^2(x)\right)-\frac{\log\left(-8\sin^2(x)-\sqrt{5}+5\right)}{8\sqrt{5}\left(5-\sqrt{5}\right)}+\frac{\log\left(-8\sin^2(x)+\sqrt{5}+5\right)}{8\sqrt{5}\left(5+\sqrt{5}\right)}\right)$$

input
```
Int[Cos[x]*Csc[5*x],x]
```

output
```
8*(Log[Sin[x]^2]/80 - Log[5 - Sqrt[5] - 8*Sin[x]^2]/(8*Sqrt[5]*(5 - Sqrt[5
])) + Log[5 + Sqrt[5] - 8*Sin[x]^2]/(8*Sqrt[5]*(5 + Sqrt[5])))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\ln\left(4\cos(x)^2-2\cos(x)-1\right)}{20}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(8\cos(x)-2)\sqrt{5}}{10}\right)}{10}+\frac{\ln(1+\cos(x))}{10}-\frac{\ln\left(4\cos(x)^2+2\cos(x)-1\right)}{20}-\sqrt{5}\,\operatorname{arctanh}($
risch	$\frac{\ln(e^{2ix}-1)}{5}-\frac{\ln\left(e^{4ix}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2ix}+1\right)}{20}-\frac{\ln\left(e^{4ix}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2ix}+1\right)\sqrt{5}}{20}-\frac{\ln\left(e^{4ix}+\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)e^{2ix}+1\right)}{20}+\ln\left(e^{4ix}+\left(\frac{\sqrt{5}}{2}\right.\right.$

input
```
int(cos(x)*csc(5*x),x,method=_RETURNVERBOSE)
```

output
```
-1/20*ln(4*cos(x)^2-2*cos(x)-1)+1/10*5^(1/2)*arctanh(1/10*(8*cos(x)-2)*5^(
1/2))+1/10*ln(1+cos(x))-1/20*ln(4*cos(x)^2+2*cos(x)-1)-1/10*5^(1/2)*arctan
h(1/10*(8*cos(x)+2)*5^(1/2))+1/10*ln(cos(x)-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int \cos(x)\csc(5x)\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{32\,\cos(x)^4+8\left(\sqrt{5}-3\right)\cos(x)^2-3\sqrt{5}+7}{16\,\cos(x)^4-12\,\cos(x)^2+1}\right) - \frac{1}{20}\log\left(16\,\cos(x)^4-12\,\cos(x)^2+1\right)+\frac{1}{5}\log\left(\frac{1}{2}\sin(x)\right)$$

input
```
integrate(cos(x)*csc(5*x),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log((32*cos(x)^4 + 8*(sqrt(5) - 3)*cos(x)^2 - 3*sqrt(5) + 7)/
(16*cos(x)^4 - 12*cos(x)^2 + 1)) - 1/20*log(16*cos(x)^4 - 12*cos(x)^2 + 1)
 + 1/5*log(1/2*sin(x))
```

Sympy [F]

$$\int \cos(x)\csc(5x)\,dx = \int \cos(x)\csc(5x)\,dx$$

input
```
integrate(cos(x)*csc(5*x),x)
```

output
```
Integral(cos(x)*csc(5*x), x)
```

Maxima [F]

$$\int \cos(x)\csc(5x)\,dx = \int \cos\left(x\right)\csc\left(5\,x\right)\,dx$$

input
```
integrate(cos(x)*csc(5*x),x, algorithm="maxima")
```

output
```
-1/10*integrate(-(cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) + cos(3/2*arctan2(
sin(2*x), cos(2*x)))*sin(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x)))*sin(2
*x) - cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) - cos(2*x)*sin(1/2*arc
tan2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x
)^2 + cos(2*x)^2 - 2*(cos(4*x) + cos(2*x) - cos(1/2*arctan2(sin(2*x), cos(
2*x))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x
), cos(2*x)))^2 - 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), co
s(2*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x
)*sin(2*x) + sin(2*x)^2 - 2*(sin(4*x) + sin(2*x) - sin(1/2*arctan2(sin(2*x
), cos(2*x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(
2*x), cos(2*x)))^2 - 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos
(2*x))) + sin(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) + 1
/10*integrate((cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) - cos(3/2*arctan2(sin
(2*x), cos(2*x)))*sin(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x)))*sin(2*x)
 + cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) + cos(2*x)*sin(1/2*arctan
2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2
 + cos(2*x)^2 + 2*(cos(4*x) + cos(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x
))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x),
cos(2*x)))^2 + 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2
*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x...
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \cos(x)\csc(5x)\,dx = -\frac{1}{20}\sqrt{5}\log\left(\frac{\left|32\cos\left(x\right)^{2}-4\sqrt{5}-12\right|}{\left|32\cos\left(x\right)^{2}+4\sqrt{5}-12\right|}\right) + \frac{1}{10}\log\left(-\cos\left(x\right)^{2}+1\right) - \frac{1}{20}\log\left(\left|16\cos\left(x\right)^{4}-12\cos\left(x\right)^{2}+1\right|\right)$$

input
```
integrate(cos(x)*csc(5*x),x, algorithm="giac")
```

output
```
-1/20*sqrt(5)*log(abs(32*cos(x)^2 - 4*sqrt(5) - 12)/abs(32*cos(x)^2 + 4*sq
rt(5) - 12)) + 1/10*log(-cos(x)^2 + 1) - 1/20*log(abs(16*cos(x)^4 - 12*cos
(x)^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int \cos(x)\csc(5x)\,dx = \frac{\ln(\sin(x))}{5} + \ln\left(-\cos(x)^2 - \frac{\sqrt{5}}{8} + \frac{3}{8}\right)\left(\frac{\sqrt{5}}{20} - \frac{1}{20}\right) - \ln\left(-\cos(x)^2 + \frac{\sqrt{5}}{8} + \frac{3}{8}\right)\left(\frac{\sqrt{5}}{20} + \frac{1}{20}\right)$$

input
```
int(cos(x)/sin(5*x),x)
```

output
```
log(sin(x))/5 + log(3/8 - 5^(1/2)/8 - cos(x)^2)*(5^(1/2)/20 - 1/20) - log(
5^(1/2)/8 - cos(x)^2 + 3/8)*(5^(1/2)/20 + 1/20)
```

Reduce [F]

$$\int \cos(x)\csc(5x)\,dx = \int \cos(x)\csc(5x)\,dx$$

input
```
int(cos(x)*csc(5*x),x)
```

output
```
int(cos(x)*csc(5*x),x)
```

3.68 $\int \cos(x)\csc(6x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 36

$$\int \cos(x)\csc(6x)\,dx = -\frac{1}{6}\text{arctanh}(\cos(x)) - \frac{1}{6}\text{arctanh}(2\cos(x)) + \frac{\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctanh(cos(x))-1/6*arctanh(2*cos(x))+1/6*arctanh(2/3*3^(1/2)*cos(x))
*3^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(36) = 72.

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.31

$$\int \cos(x)\csc(6x)\,dx = \frac{1}{12}\left(-2\sqrt{3}\text{arctanh}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) + 2\sqrt{3}\text{arctanh}\left(\frac{2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) - 2\log\left(\cos\left(\frac{x}{2}\right)\right) + \log(1-2\cos(x)) - \log(1+2\cos(x)) + 2\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cos[x]*Csc[6*x],x]
```

output
```
(-2*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] + 2*Sqrt[3]*ArcTanh[(2 + Tan[
x/2])/Sqrt[3]] - 2*Log[Cos[x/2]] + Log[1 - 2*Cos[x]] - Log[1 + 2*Cos[x]] +
 2*Log[Sin[x/2]])/12
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4879, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\csc(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sin(6x)}dx$$

$$\Big\downarrow \text{4879}$$

$$-\int \frac{1}{2\left(-16\cos^6(x)+32\cos^4(x)-19\cos^2(x)+3\right)}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int \frac{1}{-16\cos^6(x)+32\cos^4(x)-19\cos^2(x)+3}d\cos(x)$$

$$\Big\downarrow \text{2460}$$

$$-\frac{1}{2}\int \left(\frac{2}{4\cos^2(x)-3}-\frac{2}{3\left(4\cos^2(x)-1\right)}-\frac{1}{3\left(\cos^2(x)-1\right)}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{3}\text{arctanh}(\cos(x))-\frac{1}{3}\text{arctanh}(2\cos(x))+\frac{\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

input
```
Int[Cos[x]*Csc[6*x],x]
```

output
```
(-1/3*ArcTanh[Cos[x]] - ArcTanh[2*Cos[x]]/3 + ArcTanh[(2*Cos[x])/Sqrt[3]]/
Sqrt[3])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

method	result	s
default	$-\frac{\ln(2\cos(x)+1)}{12}-\frac{\ln(1+\cos(x))}{12}+\frac{\ln(\cos(x)-1)}{12}+\frac{\operatorname{arctanh}\left(\frac{2\sqrt{3}\,\cos(x)}{3}\right)\sqrt{3}}{6}+\frac{\ln(2\cos(x)-1)}{12}$	4
risch	$\frac{\ln(e^{ix}-1)}{6}-\frac{\ln(e^{ix}+1)}{6}-\frac{\sqrt{3}\,\ln\left(e^{2ix}-\sqrt{3}\,e^{ix}+1\right)}{12}+\frac{\sqrt{3}\,\ln\left(e^{2ix}+\sqrt{3}\,e^{ix}+1\right)}{12}+\frac{\ln(e^{2ix}-e^{ix}+1)}{12}-\frac{\ln(e^{2ix}+e^{ix}+1)}{12}$	9

input
```
int(cos(x)*csc(6*x),x,method=_RETURNVERBOSE)
```

output
```
-1/12*ln(2*cos(x)+1)-1/12*ln(1+cos(x))+1/12*ln(cos(x)-1)+1/6*arctanh(2/3*3
^(1/2)*cos(x))*3^(1/2)+1/12*ln(2*cos(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(26) = 52.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.94

$$\int \cos(x)\csc(6x)\,dx = \frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\cos(x)+3}{4\cos(x)^2-3}\right) - \frac{1}{12}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{12}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right) + \frac{1}{12}\log(-2\cos(x)+1)-\frac{1}{12}\log(-2\cos(x)-1)$$

input
```
integrate(cos(x)*csc(6*x),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) -
1/12*log(1/2*cos(x) + 1/2) + 1/12*log(-1/2*cos(x) + 1/2) + 1/12*log(-2*cos
(x) + 1) - 1/12*log(-2*cos(x) - 1)
```

Sympy [F]

$$\int \cos(x)\csc(6x)\,dx = \int \cos{(x)}\csc{(6x)}\,dx$$

input
```
integrate(cos(x)*csc(6*x),x)
```

output
```
Integral(cos(x)*csc(6*x), x)
```

Maxima [F]

$$\int \cos(x)\csc(6x)\,dx = \int \cos\left(x\right)\csc\left(6\,x\right)\,dx$$

input
```
integrate(cos(x)*csc(6*x),x, algorithm="maxima")
```

output
```
-integrate(1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin(4*x
) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos(2*
x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 -
 sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 1/2
4*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*sin
(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/24*log(-2*(cos(x) - 1)*cos(2*x
) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2*
cos(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(cos(
x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.89

$$\int \cos(x)\csc(6x)\,dx = -\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\cos(x)\right|}{\left|4\sqrt{3}+8\cos(x)\right|}\right) - \frac{1}{12}\log(\cos(x)+1) + \frac{1}{12}\log(-\cos(x)+1) - \frac{1}{12}\log(|2\cos(x)+1|) + \frac{1}{12}\log(|2\cos(x)-1|)$$

input
```
integrate(cos(x)*csc(6*x),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*cos(x))/abs(4*sqrt(3) + 8*cos(x))) -
1/12*log(cos(x) + 1) + 1/12*log(-cos(x) + 1) - 1/12*log(abs(2*cos(x) + 1))
 + 1/12*log(abs(2*cos(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 16.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.06

$$\int \cos(x)\csc(6x)\,dx = \frac{\operatorname{atanh}\left(\frac{1073741824}{10761687\left(\frac{427973089951744\tan\left(\frac{x}{2}\right)^2}{14348907}-\frac{47552804159488}{4782969}\right)}+\frac{797161}{797162}\right)}{6} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{6} + \frac{\sqrt{3}\operatorname{atanh}\left(\frac{4222769432625152\sqrt{3}}{4782969\left(\frac{101871591633190912\tan\left(\frac{x}{2}\right)^2}{4782969}-\frac{7314051205955584}{4782969}\right)} - \frac{19605196950732800\sqrt{3}\tan\left(\frac{x}{2}\right)^2}{1594323\left(\frac{101871591633190912\tan\left(\frac{x}{2}\right)^2}{4782969}-\frac{7314051205955584}{4782969}\right)}\right)}{6}$$

input
```
int(cos(x)/sin(6*x),x)
```

output
```
atanh(1073741824/(10761687*((427973089951744*tan(x/2)^2)/14348907 - 475528
04159488/4782969)) + 797161/797162)/6 + log(tan(x/2))/6 + (3^(1/2)*atanh((
4222769432625152*3^(1/2))/(4782969*((101871591633190912*tan(x/2)^2)/478296
9 - 7314051205955584/4782969)) - (19605196950732800*3^(1/2)*tan(x/2)^2)/(1
594323*((101871591633190912*tan(x/2)^2)/4782969 - 7314051205955584/4782969
))))/6
```

Reduce [F]

$$\int \cos(x) \csc(6x)\, dx = \int \cos(x) \csc(6x)\, dx$$

input
```
int(cos(x)*csc(6*x),x)
```

output
```
int(cos(x)*csc(6*x),x)
```

3.69 $\int \cos^3(6x) \sin(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \cos^3(6x) \sin(x) \, dx = \frac{3}{40} \cos(5x) - \frac{3}{56} \cos(7x) + \frac{1}{136} \cos(17x) - \frac{1}{152} \cos(19x)$$

output
```
3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)-1/152*cos(19*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \cos^3(6x) \sin(x) \, dx = \frac{3}{40} \cos(5x) - \frac{3}{56} \cos(7x) + \frac{1}{136} \cos(17x) - \frac{1}{152} \cos(19x)$$

input
```
Integrate[Cos[6*x]^3*Sin[x],x]
```

output
```
(3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 - Cos[19*x]/152
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos^3(6x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)\cos(6x)^3dx$$

$$\Big\downarrow \text{ 4854}$$

$$\int \left(-\frac{3}{8}\sin(5x)+\frac{3}{8}\sin(7x)-\frac{1}{8}\sin(17x)+\frac{1}{8}\sin(19x)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)$$

input
```
Int[Cos[6*x]^3*Sin[x],x]
```

output
```
(3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 - Cos[19*x]/152
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}-\frac{\cos(19x)}{152}$	26
risch	$\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}-\frac{\cos(19x)}{152}$	26
parallelrisch	$-\frac{37}{11305}+\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}-\frac{\cos(19x)}{152}$	27
orering	$\frac{1926\cos(6x)^{2}\sin(x)\sin(6x)}{11305}+\frac{251\cos(6x)^{3}\cos(x)}{11305}+\frac{1296\sin(6x)^{3}\sin(x)}{11305}+\frac{216\cos(6x)\cos(x)\sin(6x)^{2}}{11305}$	50

input
```
int(cos(6*x)^3*sin(x),x,method=_RETURNVERBOSE)
```

output
```
3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)-1/152*cos(19*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int\cos^3(6x)\sin(x)\,dx=-\frac{32768}{19}\cos(x)^{19}+\frac{147456}{17}\cos(x)^{17}-18432\cos(x)^{15}+21504\cos(x)^{13}-14976\cos(x)^{11}+6336\cos(x)^{9}-\frac{11112}{7}\cos(x)^{7}+\frac{1116}{5}\cos(x)^{5}-18\cos(x)^{3}+\cos(x)$$

input
```
integrate(cos(6*x)^3*sin(x),x, algorithm="fricas")
```

output
```
-32768/19*cos(x)^19 + 147456/17*cos(x)^17 - 18432*cos(x)^15 + 21504*cos(x)
^13 - 14976*cos(x)^11 + 6336*cos(x)^9 - 11112/7*cos(x)^7 + 1116/5*cos(x)^5
 - 18*cos(x)^3 + cos(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

Time = 0.59 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \cos^3(6x)\sin(x)\,dx = \frac{1296\sin(x)\sin^3(6x)}{11305} + \frac{1926\sin(x)\sin(6x)\cos^2(6x)}{11305} + \frac{216\sin^2(6x)\cos(x)\cos(6x)}{11305} + \frac{251\cos(x)\cos^3(6x)}{11305}$$

input
```
integrate(cos(6*x)**3*sin(x),x)
```

output
```
1296*sin(x)*sin(6*x)**3/11305 + 1926*sin(x)*sin(6*x)*cos(6*x)**2/11305 + 2
16*sin(6*x)**2*cos(x)*cos(6*x)/11305 + 251*cos(x)*cos(6*x)**3/11305
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^3(6x)\sin(x)\,dx = -\frac{1}{152}\cos(19x) + \frac{1}{136}\cos(17x) - \frac{3}{56}\cos(7x) + \frac{3}{40}\cos(5x)$$

input
```
integrate(cos(6*x)^3*sin(x),x, algorithm="maxima")
```

output
```
-1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) + 3/40*cos(5*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^3(6x)\sin(x)\,dx = -\frac{1}{152}\cos(19x) + \frac{1}{136}\cos(17x) - \frac{3}{56}\cos(7x) + \frac{3}{40}\cos(5x)$$

input
```
integrate(cos(6*x)^3*sin(x),x, algorithm="giac")
```

output
```
-1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) + 3/40*cos(5*x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \cos^3(6x)\sin(x)\,dx = -\frac{32768\cos(x)^{19}}{19} + \frac{147456\cos(x)^{17}}{17} - 18432\cos(x)^{15} + 21504\cos(x)^{13} - 14976\cos(x)^{11} + 6336\cos(x)^9 - \frac{11112\cos(x)^7}{7} + \frac{1116\cos(x)^5}{5} - 18\cos(x)^3 + \cos(x)$$

input
```
int(cos(6*x)^3*sin(x),x)
```

output
```
cos(x) - 18*cos(x)^3 + (1116*cos(x)^5)/5 - (11112*cos(x)^7)/7 + 6336*cos(x
)^9 - 14976*cos(x)^11 + 21504*cos(x)^13 - 18432*cos(x)^15 + (147456*cos(x)
^17)/17 - (32768*cos(x)^19)/19
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \cos^3(6x)\sin(x)\,dx = -\frac{\cos(6x)\cos(x)\sin(6x)^2}{323} + \frac{251\cos(6x)\cos(x)}{11305} - \frac{18\sin(6x)^3\sin(x)}{323} + \frac{1926\sin(6x)\sin(x)}{11305} + \frac{251}{11305}$$

input
```
int(cos(6*x)^3*sin(x),x)
```

output
```
( - 35*cos(6*x)*cos(x)*sin(6*x)**2 + 251*cos(6*x)*cos(x) - 630*sin(6*x)**3
*sin(x) + 1926*sin(6*x)*sin(x) + 251)/11305
```

3.70 $\int \cos^3(6x)\sin(9x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

output
```
-1/8*cos(3*x)+1/72*cos(9*x)-1/40*cos(15*x)-1/216*cos(27*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

input
```
Integrate[Cos[6*x]^3*Sin[9*x],x]
```

output
```
-1/8*Cos[3*x] + Cos[9*x]/72 - Cos[15*x]/40 - Cos[27*x]/216
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(9x)\cos^3(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(9x)\cos(6x)^3dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(\frac{3}{8}\sin(3x)-\frac{1}{8}\sin(9x)+\frac{3}{8}\sin(15x)+\frac{1}{8}\sin(27x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

input
```
Int[Cos[6*x]^3*Sin[9*x],x]
```

output
```
-1/8*Cos[3*x] + Cos[9*x]/72 - Cos[15*x]/40 - Cos[27*x]/216
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\cos(3x)}{8}+\frac{\cos(9x)}{72}-\frac{\cos(15x)}{40}-\frac{\cos(27x)}{216}$	26
risch	$-\frac{\cos(3x)}{8}+\frac{\cos(9x)}{72}-\frac{\cos(15x)}{40}-\frac{\cos(27x)}{216}$	26
parallelrisch	$\frac{13}{135}+\frac{\cos(9x)}{72}-\frac{\cos(3x)}{8}-\frac{\cos(15x)}{40}-\frac{\cos(27x)}{216}$	27
orering	$-\frac{2\cos(6x)^2\sin(9x)\sin(6x)}{45}-\frac{19\cos(6x)^3\cos(9x)}{135}-\frac{16\sin(6x)^3\sin(9x)}{135}-\frac{8\cos(6x)\cos(9x)\sin(6x)^2}{45}$	58

input
```
int(cos(6*x)^3*sin(9*x),x,method=_RETURNVERBOSE)
```

output
```
-1/8*cos(3*x)+1/72*cos(9*x)-1/40*cos(15*x)-1/216*cos(27*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{32}{27}\cos(3x)^9+\frac{8}{3}\cos(3x)^7 -\frac{12}{5}\cos(3x)^5+\frac{10}{9}\cos(3x)^3-\frac{1}{3}\cos(3x)$$

input
```
integrate(cos(6*x)^3*sin(9*x),x, algorithm="fricas")
```

output
```
-32/27*cos(3*x)^9 + 8/3*cos(3*x)^7 - 12/5*cos(3*x)^5 + 10/9*cos(3*x)^3 - 1
/3*cos(3*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(26) = 52$.

Time = 0.63 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.15

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{16\sin^3(6x)\sin(9x)}{135} - \frac{8\sin^2(6x)\cos(6x)\cos(9x)}{45} - \frac{2\sin(6x)\sin(9x)\cos^2(6x)}{45} - \frac{19\cos^3(6x)\cos(9x)}{135}$$

input
```
integrate(cos(6*x)**3*sin(9*x),x)
```

output
```
-16*sin(6*x)**3*sin(9*x)/135 - 8*sin(6*x)**2*cos(6*x)*cos(9*x)/45 - 2*sin(
6*x)*sin(9*x)*cos(6*x)**2/45 - 19*cos(6*x)**3*cos(9*x)/135
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{1}{216}\cos(27x) - \frac{1}{40}\cos(15x) + \frac{1}{72}\cos(9x) - \frac{1}{8}\cos(3x)$$

input
```
integrate(cos(6*x)^3*sin(9*x),x, algorithm="maxima")
```

output
```
-1/216*cos(27*x) - 1/40*cos(15*x) + 1/72*cos(9*x) - 1/8*cos(3*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{1}{216}\cos(27x) - \frac{1}{40}\cos(15x) + \frac{1}{72}\cos(9x) - \frac{1}{8}\cos(3x)$$

input
```
integrate(cos(6*x)^3*sin(9*x),x, algorithm="giac")
```

output
```
-1/216*cos(27*x) - 1/40*cos(15*x) + 1/72*cos(9*x) - 1/8*cos(3*x)
```

Mupad [B] (verification not implemented)

Time = 16.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{2\left(135\tan\left(\frac{3x}{2}\right)^{16} - 900\tan\left(\frac{3x}{2}\right)^{14} + 5640\tan\left(\frac{3x}{2}\right)^{12} - 13140\tan\left(\frac{3x}{2}\right)^{10} + 15534\tan\left(\frac{3x}{2}\right)^{8} - 4044\tan\right.}{135\left(\tan\left(\frac{3x}{2}\right)^2 + 1\right)^9}$$

input
```
int(cos(6*x)^3*sin(9*x),x)
```

output
```
-(2*(36*tan((3*x)/2)^2 + 1584*tan((3*x)/2)^4 - 4044*tan((3*x)/2)^6 + 15534
*tan((3*x)/2)^8 - 13140*tan((3*x)/2)^10 + 5640*tan((3*x)/2)^12 - 900*tan((
3*x)/2)^14 + 135*tan((3*x)/2)^16 + 19))/(135*(tan((3*x)/2)^2 + 1)^9)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{\cos(9x)\cos(6x)\sin(6x)^2}{27} - \frac{19\cos(9x)\cos(6x)}{135} - \frac{2\sin(9x)\sin(6x)^3}{27} - \frac{2\sin(9x)\sin(6x)}{45} - \frac{19}{135}$$

input
```
int(cos(6*x)^3*sin(9*x),x)
```

output
```
( - 5*cos(9*x)*cos(6*x)*sin(6*x)**2 - 19*cos(9*x)*cos(6*x) - 10*sin(9*x)*s
in(6*x)**3 - 6*sin(9*x)*sin(6*x) - 19)/135
```

3.71 $\int \cos(2x)\sin^2(6x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \cos(2x)\sin^2(6x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{40}\sin(10x) - \frac{1}{56}\sin(14x)$$

output
```
1/4*sin(2*x)-1/40*sin(10*x)-1/56*sin(14*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cos(2x)\sin^2(6x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{40}\sin(10x) - \frac{1}{56}\sin(14x)$$

input
```
Integrate[Cos[2*x]*Sin[6*x]^2,x]
```

output
```
Sin[2*x]/4 - Sin[10*x]/40 - Sin[14*x]/56
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(6x)\cos(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(6x)^2\cos(2x)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(\frac{1}{2}\cos(2x)-\frac{1}{4}\cos(10x)-\frac{1}{4}\cos(14x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\sin(2x)-\frac{1}{40}\sin(10x)-\frac{1}{56}\sin(14x)$$

input
```
Int[Cos[2*x]*Sin[6*x]^2,x]
```

output
```
Sin[2*x]/4 - Sin[10*x]/40 - Sin[14*x]/56
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sin(2x)}{4}-\frac{\sin(10x)}{40}-\frac{\sin(14x)}{56}$	20
risch	$\frac{\sin(2x)}{4}-\frac{\sin(10x)}{40}-\frac{\sin(14x)}{56}$	20
parallelrisch	$\frac{\sin(2x)}{4}-\frac{\sin(10x)}{40}-\frac{\sin(14x)}{56}$	20
orering	$\frac{17\sin(2x)\sin(6x)^2}{70}-\frac{3\cos(2x)\sin(6x)\cos(6x)}{35}+\frac{9\sin(2x)\cos(6x)^2}{35}$	40
norman	$\frac{\frac{6\tan(3x)^3}{35}+\frac{32\tan(x)\tan(3x)^2}{35}+\frac{18\tan(x)\tan(3x)^4}{35}+\frac{6\tan(x)^2\tan(3x)}{35}-\frac{6\tan(x)^2\tan(3x)^3}{35}+\frac{18\tan(x)}{35}-\frac{6\tan(3x)}{35}}{\left(\tan(x)^2+1\right)\left(1+\tan(3x)^2\right)^2}$	81

input
```
int(cos(2*x)*sin(6*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(2*x)-1/40*sin(10*x)-1/56*sin(14*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \cos(2x)\sin^2(6x)\,dx = -\frac{1}{70}\left(80\cos(2x)^6-72\cos(2x)^4+9\cos(2x)^2-17\right)\sin(2x)$$

input
```
integrate(cos(2*x)*sin(6*x)^2,x, algorithm="fricas")
```

output
```
-1/70*(80*cos(2*x)^6 - 72*cos(2*x)^4 + 9*cos(2*x)^2 - 17)*sin(2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \cos(2x)\sin^2(6x)\,dx = \frac{17\sin(2x)\sin^2(6x)}{70} + \frac{9\sin(2x)\cos^2(6x)}{35} - \frac{3\sin(6x)\cos(2x)\cos(6x)}{35}$$

input
```
integrate(cos(2*x)*sin(6*x)**2,x)
```

output
```
17*sin(2*x)*sin(6*x)**2/70 + 9*sin(2*x)*cos(6*x)**2/35 - 3*sin(6*x)*cos(2*
x)*cos(6*x)/35
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos(2x)\sin^2(6x)\,dx = -\frac{1}{56}\sin(14x) - \frac{1}{40}\sin(10x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(cos(2*x)*sin(6*x)^2,x, algorithm="maxima")
```

output
```
-1/56*sin(14*x) - 1/40*sin(10*x) + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos(2x)\sin^2(6x)\,dx = -\frac{1}{56}\sin(14x) - \frac{1}{40}\sin(10x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(cos(2*x)*sin(6*x)^2,x, algorithm="giac")
```

output
```
-1/56*sin(14*x) - 1/40*sin(10*x) + 1/4*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cos(2x)\sin^2(6x)\,dx = \frac{8\sin(2x)^7}{7} - \frac{12\sin(2x)^5}{5} + \frac{3\sin(2x)^3}{2}$$

input
```
int(cos(2*x)*sin(6*x)^2,x)
```

output
```
(3*sin(2*x)^3)/2 - (12*sin(2*x)^5)/5 + (8*sin(2*x)^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \cos(2x)\sin^2(6x)\,dx = -\frac{3\cos(6x)\cos(2x)\sin(6x)}{35} - \frac{\sin(6x)^2\sin(2x)}{70} + \frac{9\sin(2x)}{35}$$

input
```
int(cos(2*x)*sin(6*x)^2,x)
```

output
```
( - 6*cos(6*x)*cos(2*x)*sin(6*x) - sin(6*x)**2*sin(2*x) + 18*sin(2*x))/70
```

3.72 $\int \cos(x)\sin^2(6x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int \cos(x)\sin^2(6x)\,dx = \frac{\sin(x)}{2} - \frac{1}{44}\sin(11x) - \frac{1}{52}\sin(13x)$$

output
```
1/2*sin(x)-1/44*sin(11*x)-1/52*sin(13*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin^2(6x)\,dx = \frac{\sin(x)}{2} - \frac{1}{44}\sin(11x) - \frac{1}{52}\sin(13x)$$

input
```
Integrate[Cos[x]*Sin[6*x]^2,x]
```

output
```
Sin[x]/2 - Sin[11*x]/44 - Sin[13*x]/52
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(6x)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(6x)^2\cos(x)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(\frac{\cos(x)}{2}-\frac{1}{4}\cos(11x)-\frac{1}{4}\cos(13x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(x)}{2}-\frac{1}{44}\sin(11x)-\frac{1}{52}\sin(13x)$$

input
```
Int[Cos[x]*Sin[6*x]^2,x]
```

output
```
Sin[x]/2 - Sin[11*x]/44 - Sin[13*x]/52
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	s
default	$\frac{\sin(x)}{2}-\frac{\sin(11x)}{44}-\frac{\sin(13x)}{52}$	1
risch	$\frac{\sin(x)}{2}-\frac{\sin(11x)}{44}-\frac{\sin(13x)}{52}$	1
orering	$\frac{71\sin(x)\sin(6x)^2}{143}-\frac{12\cos(x)\sin(6x)\cos(6x)}{143}+\frac{72\cos(6x)^2\sin(x)}{143}$	3
parallelrisch	$-\frac{\left(\sin\left(\frac{3x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{3x}{2}\right)+3\cos\left(\frac{x}{2}\right)\right)(213+295\cos(2x)+46\cos(8x)+11\cos(10x)+188\cos(4x)+105\cos(6x))}{143}$	5
norman	$\frac{\frac{24\tan(3x)^3}{143}+\frac{24\tan(3x)\tan\left(\frac{x}{2}\right)^2}{143}+\frac{280\tan(3x)^2\tan\left(\frac{x}{2}\right)}{143}-\frac{24\tan(3x)^3\tan\left(\frac{x}{2}\right)^2}{143}+\frac{144\tan(3x)^4\tan\left(\frac{x}{2}\right)}{143}-\frac{24\tan(3x)}{143}+\frac{144\tan\left(\frac{x}{2}\right)}{143}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan(3x)^2\right)^2}$	9

input
```
int(cos(x)*sin(6*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)-1/44*sin(11*x)-1/52*sin(13*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(17) = 34.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \cos(x)\sin^2(6x)\,dx =$$
$$-\frac{4}{143}\left(2816\cos(x)^{12}-6912\cos(x)^{10}+6048\cos(x)^8-2240\cos(x)^6+315\cos(x)^4-9\cos(x)^2-18\right.$$

input
```
integrate(cos(x)*sin(6*x)^2,x, algorithm="fricas")
```

output
```
-4/143*(2816*cos(x)^12 - 6912*cos(x)^10 + 6048*cos(x)^8 - 2240*cos(x)^6 +
315*cos(x)^4 - 9*cos(x)^2 - 18)*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \cos(x)\sin^2(6x)\,dx = \frac{71\sin(x)\sin^2(6x)}{143} + \frac{72\sin(x)\cos^2(6x)}{143} - \frac{12\sin(6x)\cos(x)\cos(6x)}{143}$$

input
```
integrate(cos(x)*sin(6*x)**2,x)
```

output
```
71*sin(x)*sin(6*x)**2/143 + 72*sin(x)*cos(6*x)**2/143 - 12*sin(6*x)*cos(x)
*cos(6*x)/143
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \cos(x)\sin^2(6x)\,dx = -\frac{1}{52}\sin(13x) - \frac{1}{44}\sin(11x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(x)*sin(6*x)^2,x, algorithm="maxima")
```

output
```
-1/52*sin(13*x) - 1/44*sin(11*x) + 1/2*sin(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \cos(x)\sin^2(6x)\,dx = -\frac{1}{52}\sin(13x) - \frac{1}{44}\sin(11x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(x)*sin(6*x)^2,x, algorithm="giac")
```

output
```
-1/52*sin(13*x) - 1/44*sin(11*x) + 1/2*sin(x)
```

Mupad [B] (verification not implemented)

Time = 16.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \cos(x)\sin^2(6x)\,dx = \frac{\sin(x)}{2} - \frac{\sin(13x)}{52} - \frac{\sin(11x)}{44}$$

input
```
int(sin(6*x)^2*cos(x),x)
```

output
```
sin(x)/2 - sin(13*x)/52 - sin(11*x)/44
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \cos(x)\sin^2(6x)\,dx = -\frac{12\cos(6x)\cos(x)\sin(6x)}{143} - \frac{\sin(6x)^2\sin(x)}{143} + \frac{72\sin(x)}{143}$$

input
```
int(cos(x)*sin(6*x)^2,x)
```

output
```
( - 12*cos(6*x)*cos(x)*sin(6*x) - sin(6*x)**2*sin(x) + 72*sin(x))/143
```

3.73 $\int \cos(x)\sin^3(6x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \cos(x)\sin^3(6x)\,dx = -\frac{3}{40}\cos(5x) - \frac{3}{56}\cos(7x) + \frac{1}{136}\cos(17x) + \frac{1}{152}\cos(19x)$$

output
```
-3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)+1/152*cos(19*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin^3(6x)\,dx = -\frac{3}{40}\cos(5x) - \frac{3}{56}\cos(7x) + \frac{1}{136}\cos(17x) + \frac{1}{152}\cos(19x)$$

input
```
Integrate[Cos[x]*Sin[6*x]^3,x]
```

output
```
(-3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 + Cos[19*x]/152
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(6x)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(6x)^3\cos(x)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(\frac{3}{8}\sin(5x)+\frac{3}{8}\sin(7x)-\frac{1}{8}\sin(17x)-\frac{1}{8}\sin(19x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)$$

input
```
Int[Cos[x]*Sin[6*x]^3,x]
```

output
```
(-3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 + Cos[19*x]/152
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}+\frac{\cos(19x)}{152}$	26
risch	$-\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}+\frac{\cos(19x)}{152}$	26
parallelrisch	$\frac{1272}{11305}-\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}+\frac{\cos(19x)}{152}$	27
orering	$-\frac{251\sin(6x)^3\sin(x)}{11305}-\frac{1926\cos(6x)\cos(x)\sin(6x)^2}{11305}-\frac{216\cos(6x)^2\sin(x)\sin(6x)}{11305}-\frac{1296\cos(6x)^3\cos(x)}{11305}$	50

input
```
int(cos(x)*sin(6*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)+1/152*cos(19*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \cos(x)\sin^3(6x)\,dx = \frac{32768}{19}\cos(x)^{19}-\frac{131072}{17}\cos(x)^{17}+14336\cos(x)^{15}-14336\cos(x)^{13}+8320\cos(x)^{11}-2816\cos(x)^{9}+\frac{3672}{7}\cos(x)^{7}-\frac{216}{5}\cos(x)^{5}$$

input
```
integrate(cos(x)*sin(6*x)^3,x, algorithm="fricas")
```

output
```
32768/19*cos(x)^19 - 131072/17*cos(x)^17 + 14336*cos(x)^15 - 14336*cos(x)^
13 + 8320*cos(x)^11 - 2816*cos(x)^9 + 3672/7*cos(x)^7 - 216/5*cos(x)^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time = 0.59 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.97

$$\int \cos(x)\sin^3(6x)\,dx = -\frac{251\sin(x)\sin^3(6x)}{11305} - \frac{216\sin(x)\sin(6x)\cos^2(6x)}{11305} - \frac{1926\sin^2(6x)\cos(x)\cos(6x)}{11305} - \frac{1296\cos(x)\cos^3(6x)}{11305}$$

input
```
integrate(cos(x)*sin(6*x)**3,x)
```

output
```
-251*sin(x)*sin(6*x)**3/11305 - 216*sin(x)*sin(6*x)*cos(6*x)**2/11305 - 19
26*sin(6*x)**2*cos(x)*cos(6*x)/11305 - 1296*cos(x)*cos(6*x)**3/11305
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin^3(6x)\,dx = \frac{1}{152}\cos(19x) + \frac{1}{136}\cos(17x) - \frac{3}{56}\cos(7x) - \frac{3}{40}\cos(5x)$$

input
```
integrate(cos(x)*sin(6*x)^3,x, algorithm="maxima")
```

output
```
1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) - 3/40*cos(5*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \cos(x)\sin^3(6x)\,dx = \frac{32768}{19}\cos(x)^{19} - \frac{131072}{17}\cos(x)^{17} + 14336\cos(x)^{15} - 14336\cos(x)^{13} + 8320\cos(x)^{11} - 2816\cos(x)^{9} + \frac{3672}{7}\cos(x)^{7} - \frac{216}{5}\cos(x)^{5}$$

input
```
integrate(cos(x)*sin(6*x)^3,x, algorithm="giac")
```

output
```
32768/19*cos(x)^19 - 131072/17*cos(x)^17 + 14336*cos(x)^15 - 14336*cos(x)^
13 + 8320*cos(x)^11 - 2816*cos(x)^9 + 3672/7*cos(x)^7 - 216/5*cos(x)^5
```

Mupad [B] (verification not implemented)

Time = 17.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 4.55

$$\int \cos(x)\sin^3(6x)\,dx = -\frac{32\left(305235\tan\left(\frac{x}{2}\right)^{34} - 9665775\tan\left(\frac{x}{2}\right)^{32} + 153838440\tan\left(\frac{x}{2}\right)^{30} - 1348695544\tan\left(\frac{x}{2}\right)^{28} + 708381248\right.}{}$$

input
```
int(sin(6*x)^3*cos(x),x)
```

output
```
-(32*(1539*tan(x/2)^2 - 291384*tan(x/2)^4 + 9744264*tan(x/2)^6 - 153524484
*tan(x/2)^8 + 1349637412*tan(x/2)^10 - 7081614792*tan(x/2)^12 + 2358290959
2*tan(x/2)^14 - 51607368282*tan(x/2)^16 + 75935973762*tan(x/2)^18 - 759284
91144*tan(x/2)^20 + 51613490424*tan(x/2)^22 - 23578828164*tan(x/2)^24 + 70
83812484*tan(x/2)^26 - 1348695544*tan(x/2)^28 + 153838440*tan(x/2)^30 - 96
65775*tan(x/2)^32 + 305235*tan(x/2)^34 + 81))/(11305*(tan(x/2)^2 + 1)^19)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \cos(x)\sin^3(6x)\,dx = -\frac{18\cos(6x)\cos(x)\sin(6x)^2}{323} - \frac{1296\cos(6x)\cos(x)}{11305} - \frac{\sin(6x)^3\sin(x)}{323} - \frac{216\sin(6x)\sin(x)}{11305} + \frac{1272}{11305}$$

input
```
int(cos(x)*sin(6*x)^3,x)
```

output
```
( - 630*cos(6*x)*cos(x)*sin(6*x)**2 - 1296*cos(6*x)*cos(x) - 35*sin(6*x)**
3*sin(x) - 216*sin(6*x)*sin(x) + 1272)/11305
```

3.74 $\int \cos(7x)\sin^3(6x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \cos(7x)\sin^3(6x)\,dx = \frac{3\cos(x)}{8} + \frac{1}{88}\cos(11x) - \frac{3}{104}\cos(13x) + \frac{1}{200}\cos(25x)$$

output
```
3/8*cos(x)+1/88*cos(11*x)-3/104*cos(13*x)+1/200*cos(25*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \cos(7x)\sin^3(6x)\,dx = \frac{3\cos(x)}{8} + \frac{1}{88}\cos(11x) - \frac{3}{104}\cos(13x) + \frac{1}{200}\cos(25x)$$

input
```
Integrate[Cos[7*x]*Sin[6*x]^3,x]
```

output
```
(3*Cos[x])/8 + Cos[11*x]/88 - (3*Cos[13*x])/104 + Cos[25*x]/200
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(6x)\cos(7x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(6x)^3\cos(7x)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(-\frac{3\sin(x)}{8}-\frac{1}{8}\sin(11x)+\frac{3}{8}\sin(13x)-\frac{1}{8}\sin(25x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\cos(x)}{8}+\frac{1}{88}\cos(11x)-\frac{3}{104}\cos(13x)+\frac{1}{200}\cos(25x)$$

input
```
Int[Cos[7*x]*Sin[6*x]^3,x]
```

output
```
(3*Cos[x])/8 + Cos[11*x]/88 - (3*Cos[13*x])/104 + Cos[25*x]/200
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result
default	$\frac{3\cos(x)}{8}+\frac{\cos(11x)}{88}-\frac{3\cos(13x)}{104}+\frac{\cos(25x)}{200}$
risch	$\frac{3\cos(x)}{8}+\frac{\cos(11x)}{88}-\frac{3\cos(13x)}{104}+\frac{\cos(25x)}{200}$
orering	$\frac{1421\sin(7x)\sin(6x)^3}{3575}+\frac{1062\cos(7x)\sin(6x)^2\cos(6x)}{3575}+\frac{1512\cos(6x)^2\sin(6x)\sin(7x)}{3575}+\frac{1296\cos(7x)\cos(6x)^3}{3575}$
parallelrisch	$\frac{\left(1176\tan(3x)^6-720\tan(3x)^2-1416\right)\tan\left(\frac{7x}{2}\right)^2+\left(6048\tan(3x)^5+10640\tan(3x)^3+6048\tan(3x)\right)\tan\left(\frac{7x}{2}\right)-1416\tan(3x)^6-}{3575\left(1+\tan(3x)^2\right)^3\left(1+\tan\left(\frac{7x}{2}\right)^2\right)}$

input
```
int(cos(7*x)*sin(6*x)^3,x,method=_RETURNVERBOSE)
```

output
```
3/8*cos(x)+1/88*cos(11*x)-3/104*cos(13*x)+1/200*cos(25*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(23) = 46$.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.16

$$\begin{aligned}\int \cos(7x)\sin^3(6x)\,dx = {} & \frac{2097152}{25}\cos(x)^{25} - 524288\cos(x)^{23} + 1441792\cos(x)^{21} \\ & - 2293760\cos(x)^{19} + 2334720\cos(x)^{17} - \frac{7938048}{5}\cos(x)^{15} \\ & + \frac{9503232}{13}\cos(x)^{13} - \frac{2484992}{11}\cos(x)^{11} \\ & + 45248\cos(x)^{9} - 5400\cos(x)^{7} + \frac{1512}{5}\cos(x)^{5}\end{aligned}$$

input
```
integrate(cos(7*x)*sin(6*x)^3,x, algorithm="fricas")
```

output
```
2097152/25*cos(x)^25 - 524288*cos(x)^23 + 1441792*cos(x)^21 - 2293760*cos(
x)^19 + 2334720*cos(x)^17 - 7938048/5*cos(x)^15 + 9503232/13*cos(x)^13 - 2
484992/11*cos(x)^11 + 45248*cos(x)^9 - 5400*cos(x)^7 + 1512/5*cos(x)^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(27) = 54$.

Time = 0.59 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \cos(7x)\sin^3(6x)\,dx = \frac{1421\sin^3(6x)\sin(7x)}{3575} + \frac{1062\sin^2(6x)\cos(6x)\cos(7x)}{3575} + \frac{1512\sin(6x)\sin(7x)\cos^2(6x)}{3575} + \frac{1296\cos^3(6x)\cos(7x)}{3575}$$

input
```
integrate(cos(7*x)*sin(6*x)**3,x)
```

output
```
1421*sin(6*x)**3*sin(7*x)/3575 + 1062*sin(6*x)**2*cos(6*x)*cos(7*x)/3575 +
 1512*sin(6*x)*sin(7*x)*cos(6*x)**2/3575 + 1296*cos(6*x)**3*cos(7*x)/3575
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \cos(7x)\sin^3(6x)\,dx = \frac{1}{200}\cos(25x) - \frac{3}{104}\cos(13x) + \frac{1}{88}\cos(11x) + \frac{3}{8}\cos(x)$$

input
```
integrate(cos(7*x)*sin(6*x)^3,x, algorithm="maxima")
```

output
```
1/200*cos(25*x) - 3/104*cos(13*x) + 1/88*cos(11*x) + 3/8*cos(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \cos(7x)\sin^3(6x)\,dx = \frac{1}{200}\cos(25x) - \frac{3}{104}\cos(13x) + \frac{1}{88}\cos(11x) + \frac{3}{8}\cos(x)$$

input
```
integrate(cos(7*x)*sin(6*x)^3,x, algorithm="giac")
```

output
```
1/200*cos(25*x) - 3/104*cos(13*x) + 1/88*cos(11*x) + 3/8*cos(x)
```

Mupad [B] (verification not implemented)

Time = 18.63 (sec) , antiderivative size = 198, normalized size of antiderivative = 6.39

$$\int \cos(7x)\sin^3(6x)\,dx$$

$$= \frac{32\left(-96525\tan\left(\frac{x}{2}\right)^{46} + 8655075\tan\left(\frac{x}{2}\right)^{44} - 300482325\tan\left(\frac{x}{2}\right)^{42} + 5743927475\tan\left(\frac{x}{2}\right)^{40} - 6779248547\right.}{}$$

input
```
int(cos(7*x)*sin(6*x)^3,x)
```

output
```
(32*(2025*tan(x/2)^2 + 120825*tan(x/2)^4 - 8468775*tan(x/2)^6 + 301506975*
tan(x/2)^8 - 5739623945*tan(x/2)^10 + 67806830575*tan(x/2)^12 - 5238294762
25*tan(x/2)^14 + 2750536240650*tan(x/2)^16 - 10084340561350*tan(x/2)^18 +
26326043727610*tan(x/2)^20 - 49575456537350*tan(x/2)^22 + 67896209197950*t
an(x/2)^24 - 67895787973650*tan(x/2)^26 + 49575817586750*tan(x/2)^28 - 263
25778958050*tan(x/2)^30 + 10084506042325*tan(x/2)^32 - 2750448633075*tan(x
/2)^34 + 523868412925*tan(x/2)^36 - 67792485475*tan(x/2)^38 + 5743927475*t
an(x/2)^40 - 300482325*tan(x/2)^42 + 8655075*tan(x/2)^44 - 96525*tan(x/2)^
46 + 81))/(3575*(tan(x/2)^2 + 1)^25)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \cos(7x)\sin^3(6x)\,dx = -\frac{18\cos(7x)\cos(6x)\sin(6x)^2}{275} + \frac{1296\cos(7x)\cos(6x)}{3575} - \frac{7\sin(7x)\sin(6x)^3}{275} + \frac{1512\sin(7x)\sin(6x)}{3575} - \frac{24}{715}$$

input
```
int(cos(7*x)*sin(6*x)^3,x)
```

output
```
( - 234*cos(7*x)*cos(6*x)*sin(6*x)**2 + 1296*cos(7*x)*cos(6*x) - 91*sin(7*
x)*sin(6*x)**3 + 1512*sin(7*x)*sin(6*x) - 120)/3575
```

3.75 $\int \cos^2(3x)\sin^3(2x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \cos^2(3x)\sin^3(2x)\,dx = -\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x) -\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

output
```
-3/16*cos(2*x)+3/64*cos(4*x)+1/48*cos(6*x)-3/128*cos(8*x)+1/192*cos(12*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \cos^2(3x)\sin^3(2x)\,dx = -\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x) -\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

input
```
Integrate[Cos[3*x]^2*Sin[2*x]^3,x]
```

output
```
(-3*Cos[2*x])/16 + (3*Cos[4*x])/64 + Cos[6*x]/48 - (3*Cos[8*x])/128 + Cos[
12*x]/192
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2x)\cos^2(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2x)^3\cos(3x)^2dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(\frac{3}{8}\sin(2x)-\frac{3}{16}\sin(4x)-\frac{1}{8}\sin(6x)+\frac{3}{16}\sin(8x)-\frac{1}{16}\sin(12x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

input
```
Int[Cos[3*x]^2*Sin[2*x]^3,x]
```

output
```
(-3*Cos[2*x])/16 + (3*Cos[4*x])/64 + Cos[6*x]/48 - (3*Cos[8*x])/128 + Cos[
12*x]/192
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

method	result
default	$-\frac{3\cos(2x)}{16}+\frac{3\cos(4x)}{64}+\frac{\cos(6x)}{48}-\frac{3\cos(8x)}{128}+\frac{\cos(12x)}{192}$
risch	$-\frac{3\cos(2x)}{16}+\frac{3\cos(4x)}{64}+\frac{\cos(6x)}{48}-\frac{3\cos(8x)}{128}+\frac{\cos(12x)}{192}$
parallelrisch	$\frac{377}{1920}+\frac{\cos(12x)}{192}-\frac{3\cos(2x)}{16}-\frac{3\cos(8x)}{128}+\frac{3\cos(4x)}{64}+\frac{\cos(6x)}{48}$
orering	$\frac{29\cos(3x)\sin(2x)^3\sin(3x)}{192}-\frac{31\cos(3x)^2\sin(2x)^2\cos(2x)}{128}-\frac{33\cos(2x)\sin(2x)^2\sin(3x)^2}{128}+\frac{7\cos(2x)^2\cos(3x)\sin(2x)\,\mathrm{s}}{64}$

input
```
int(cos(3*x)^2*sin(2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/16*cos(2*x)+3/64*cos(4*x)+1/48*cos(6*x)-3/128*cos(8*x)+1/192*cos(12*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \cos^2(3x)\sin^3(2x)\,dx=\frac{32}{3}\cos(x)^{12}-32\cos(x)^{10}+33\cos(x)^8-12\cos(x)^6$$

input
```
integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="fricas")
```

output
```
32/3*cos(x)^12 - 32*cos(x)^10 + 33*cos(x)^8 - 12*cos(x)^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(37) = 74$.

Time = 1.35 (sec) , antiderivative size = 228, normalized size of antiderivative = 5.56

$$\int \cos^2(3x)\sin^3(2x)\,dx = -\frac{x\sin^3(2x)\sin^2(3x)}{16} + \frac{x\sin^3(2x)\cos^2(3x)}{16} - \frac{3x\sin^2(2x)\sin(3x)\cos(2x)\cos(3x)}{8} + \frac{3x\sin(2x)\sin^2(3x)\cos^2(2x)}{16} - \frac{3x\sin(2x)\cos^2(2x)\cos^2(3x)}{16} + \frac{x\sin(3x)\cos^3(2x)\cos(3x)}{8} + \frac{5\sin^3(2x)\sin(3x)\cos(3x)}{16} - \frac{\sin^2(2x)\sin^2(3x)\cos(2x)}{2} - \frac{3\sin(2x)\sin(3x)\cos^2(2x)\cos(3x)}{8} - \frac{11\sin^2(3x)\cos^3(2x)}{96} - \frac{7\cos^3(2x)\cos^2(3x)}{32}$$

input
```
integrate(cos(3*x)**2*sin(2*x)**3,x)
```

output
```
-x*sin(2*x)**3*sin(3*x)**2/16 + x*sin(2*x)**3*cos(3*x)**2/16 - 3*x*sin(2*x
)**2*sin(3*x)*cos(2*x)*cos(3*x)/8 + 3*x*sin(2*x)*sin(3*x)**2*cos(2*x)**2/1
6 - 3*x*sin(2*x)*cos(2*x)**2*cos(3*x)**2/16 + x*sin(3*x)*cos(2*x)**3*cos(3
*x)/8 + 5*sin(2*x)**3*sin(3*x)*cos(3*x)/16 - sin(2*x)**2*sin(3*x)**2*cos(2
*x)/2 - 3*sin(2*x)*sin(3*x)*cos(2*x)**2*cos(3*x)/8 - 11*sin(3*x)**2*cos(2*
x)**3/96 - 7*cos(2*x)**3*cos(3*x)**2/32
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \cos^2(3x)\sin^3(2x)\,dx = \frac{1}{192}\cos(12x) - \frac{3}{128}\cos(8x) + \frac{1}{48}\cos(6x) + \frac{3}{64}\cos(4x) - \frac{3}{16}\cos(2x)$$

input
```
integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="maxima")
```

output
```
1/192*cos(12*x) - 3/128*cos(8*x) + 1/48*cos(6*x) + 3/64*cos(4*x) - 3/16*co
s(2*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \cos^2(3x)\sin^3(2x)\,dx = \frac{1}{192}\,\cos\left(12\,x\right) - \frac{3}{128}\,\cos\left(8\,x\right) + \frac{1}{48}\,\cos\left(6\,x\right) + \frac{3}{64}\,\cos\left(4\,x\right) - \frac{3}{16}\,\cos\left(2\,x\right)$$

input
```
integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="giac")
```

output
```
1/192*cos(12*x) - 3/128*cos(8*x) + 1/48*cos(6*x) + 3/64*cos(4*x) - 3/16*co
s(2*x)
```

Mupad [B] (verification not implemented)

Time = 18.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \cos^2(3x)\sin^3(2x)\,dx = \frac{32\cos\left(x\right)^{12}}{3} - 32\cos\left(x\right)^{10} + 33\cos\left(x\right)^{8} - 12\cos\left(x\right)^{6}$$

input
```
int(cos(3*x)^2*sin(2*x)^3,x)
```

output
```
33*cos(x)^8 - 12*cos(x)^6 - 32*cos(x)^10 + (32*cos(x)^12)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 4350, normalized size of antiderivative = 106.10

$$\int \cos^2(3x)\sin^3(2x)\,dx = \text{Too large to display}$$

input
```
int(cos(3*x)^2*sin(2*x)^3,x)
```

output
```
(3024*cos(3*x)*cos(2*x)*sin(3*x)*sin(2*x)*tan((3*x)/2)**4*tan(x)**6 + 9072
*cos(3*x)*cos(2*x)*sin(3*x)*sin(2*x)*tan((3*x)/2)**4*tan(x)**4 + 9072*cos(
3*x)*cos(2*x)*sin(3*x)*sin(2*x)*tan((3*x)/2)**4*tan(x)**2 + 3024*cos(3*x)*
cos(2*x)*sin(3*x)*sin(2*x)*tan((3*x)/2)**4 + 6048*cos(3*x)*cos(2*x)*sin(3*
x)*sin(2*x)*tan((3*x)/2)**2*tan(x)**6 + 18144*cos(3*x)*cos(2*x)*sin(3*x)*s
in(2*x)*tan((3*x)/2)**2*tan(x)**4 + 18144*cos(3*x)*cos(2*x)*sin(3*x)*sin(2
*x)*tan((3*x)/2)**2*tan(x)**2 + 6048*cos(3*x)*cos(2*x)*sin(3*x)*sin(2*x)*t
an((3*x)/2)**2 + 3024*cos(3*x)*cos(2*x)*sin(3*x)*sin(2*x)*tan(x)**6 + 9072
*cos(3*x)*cos(2*x)*sin(3*x)*sin(2*x)*tan(x)**4 + 9072*cos(3*x)*cos(2*x)*si
n(3*x)*sin(2*x)*tan(x)**2 + 3024*cos(3*x)*cos(2*x)*sin(3*x)*sin(2*x) + 560
*cos(3*x)*cos(2*x)*sin(2*x)**2*tan((3*x)/2)**4*tan(x)**6 + 1680*cos(3*x)*c
os(2*x)*sin(2*x)**2*tan((3*x)/2)**4*tan(x)**4 + 1680*cos(3*x)*cos(2*x)*sin
(2*x)**2*tan((3*x)/2)**4*tan(x)**2 + 560*cos(3*x)*cos(2*x)*sin(2*x)**2*tan
((3*x)/2)**4 + 1120*cos(3*x)*cos(2*x)*sin(2*x)**2*tan((3*x)/2)**2*tan(x)**
6 + 3360*cos(3*x)*cos(2*x)*sin(2*x)**2*tan((3*x)/2)**2*tan(x)**4 + 3360*co
s(3*x)*cos(2*x)*sin(2*x)**2*tan((3*x)/2)**2*tan(x)**2 + 1120*cos(3*x)*cos(
2*x)*sin(2*x)**2*tan((3*x)/2)**2 + 560*cos(3*x)*cos(2*x)*sin(2*x)**2*tan(x
)**6 + 1680*cos(3*x)*cos(2*x)*sin(2*x)**2*tan(x)**4 + 1680*cos(3*x)*cos(2*
x)*sin(2*x)**2*tan(x)**2 + 560*cos(3*x)*cos(2*x)*sin(2*x)**2 + 11200*cos(3
*x)*cos(2*x)*tan((3*x)/2)**4*tan(x)**6 + 33600*cos(3*x)*cos(2*x)*tan((3...
```

3.76 $\int \sin(a+bx)\sin(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \sin(a+bx)\sin(c+bx)\,dx = \frac{1}{2}x\cos(a-c) - \frac{\sin(a+c+2bx)}{4b}$$

output
```
1/2*x*cos(a-c)-1/4*sin(2*b*x+a+c)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sin(a+bx)\sin(c+bx)\,dx = -\frac{-2bx\cos(a-c)+\sin(a+c+2bx)}{4b}$$

input
```
Integrate[Sin[a + b*x]*Sin[c + b*x],x]
```

output
```
-1/4*(-2*b*x*Cos[a - c] + Sin[a + c + 2*b*x])/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin(bx+c)\,dx$$
$$\Big\downarrow \ 5080$$
$$\int \left(\frac{1}{2}\cos(a-c)-\frac{1}{2}\cos(a+2bx+c)\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{2}x\cos(a-c)-\frac{\sin(a+2bx+c)}{4b}$$

input
```
Int[Sin[a + b*x]*Sin[c + b*x],x]
```

output
```
(x*Cos[a - c])/2 - Sin[a + c + 2*b*x]/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cos(a-c)}{2}-\frac{\sin(2bx+a+c)}{4b}$
risch	$\frac{x\cos(a-c)}{2}-\frac{\sin(2bx+a+c)}{4b}$
parallelrisch	$\frac{2x\cos(a-c)b-\sin(2bx+a+c)+\sin(a-c)}{4b}$
orering	$x\sin(bx+a)\sin(bx+c)-\frac{b\cos(bx+a)\sin(bx+c)+\sin(bx+a)b\cos(bx+c)}{4b^2}+\frac{x(-2\sin(bx+c)\sin(bx+a)b^2+2\cos}{4b^2}$
norman	$\frac{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}+\frac{x}{2}-\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{2}-\frac{x\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2}{2}+2x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)+\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2}{2}+\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(1+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(sin(b*x+a)*sin(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*cos(a-c)-1/4*sin(2*b*x+a+c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(23) = 46$.

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \sin(a+bx)\sin(c+bx)\,dx$$
$$=\frac{bx\cos(-a+c)-\cos(bx+c)\cos(-a+c)\sin(bx+c)+\cos(bx+c)^2\sin(-a+c)}{2\,b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="fricas")
```

output
```
1/2*(b*x*cos(-a + c) - cos(b*x + c)*cos(-a + c)*sin(b*x + c) + cos(b*x + c
)^2*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \sin(a+bx)\sin(c+bx)\,dx = \begin{cases} \frac{x\sin(a+bx)\sin(bx+c)}{2} + \frac{x\cos(a+bx)\cos(bx+c)}{2} - \frac{\sin(bx+c)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin(a)\sin(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c),x)
```

output
```
Piecewise((x*sin(a + b*x)*sin(b*x + c)/2 + x*cos(a + b*x)*cos(b*x + c)/2 -
 sin(b*x + c)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*sin(a)*sin(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sin(a+bx)\sin(c+bx)\,dx = \frac{1}{2}\,x\cos(-a+c) - \frac{\sin(2\,bx+a+c)}{4\,b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="maxima")
```

output
```
1/2*x*cos(-a + c) - 1/4*sin(2*b*x + a + c)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sin(a+bx)\sin(c+bx)\,dx = \frac{1}{2}\,x\cos\left(a-c\right) - \frac{\sin\left(2\,bx+a+c\right)}{4\,b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="giac")
```

output
```
1/2*x*cos(a - c) - 1/4*sin(2*b*x + a + c)/b
```

Mupad [B] (verification not implemented)

Time = 17.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \sin(a+bx)\sin(c+bx)\,dx = \begin{cases} x\,\sin\left(a\right)\,\sin\left(c\right) & \text{if } b=0 \\ \frac{x\,\cos(a-c)}{2}-\frac{\sin(a+c+2\,b\,x)}{4\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int(sin(a + b*x)*sin(c + b*x),x)
```

output
```
piecewise(b == 0, x*sin(a)*sin(c), b ~= 0, (x*cos(a - c))/2 - sin(a + c +
2*b*x)/(4*b))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \sin(a+bx)\sin(c+bx)\,dx = \frac{\cos\left(bx+c\right)\cos\left(bx+a\right)bx-\cos\left(bx+c\right)\sin\left(bx+a\right)+\sin\left(bx+c\right)\sin\left(bx+a\right)bx}{2b}$$

input
```
int(sin(b*x+a)*sin(b*x+c),x)
```

output
```
(cos(b*x + c)*cos(a + b*x)*b*x - cos(b*x + c)*sin(a + b*x) + sin(b*x + c)*
sin(a + b*x)*b*x)/(2*b)
```

3.77 $\int \sin(c-bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \sin(c-bx)\sin(a+bx)\,dx = -\frac{1}{2}x\cos(a+c)+\frac{\sin(a-c+2bx)}{4b}$$

output
```
-1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sin(c-bx)\sin(a+bx)\,dx = \frac{-2bx\cos(a+c)+\sin(a-c+2bx)}{4b}$$

input
```
Integrate[Sin[c - b*x]*Sin[a + b*x],x]
```

output
```
(-2*b*x*Cos[a + c] + Sin[a - c + 2*b*x])/(4*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin(c-bx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(\frac{1}{2}\cos(a+2bx-c)-\frac{1}{2}\cos(a+c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a+2bx-c)}{4b}-\frac{1}{2}x\cos(a+c)$$

input
```
Int[Sin[c - b*x]*Sin[a + b*x],x]
```

output
```
-1/2*(x*Cos[a + c]) + Sin[a - c + 2*b*x]/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$-\frac{x\cos(a+c)}{2}+\frac{\sin(2bx+a-c)}{4b}$
risch	$-\frac{x\cos(a+c)}{2}+\frac{\sin(2bx+a-c)}{4b}$
parallelrisch	$-\frac{2x\cos(a+c)b+\sin(a+c)-\sin(2bx+a-c)}{4b}$
orering	$-x\sin\left(bx-c\right)\sin\left(bx+a\right)-\frac{-b\cos(bx-c)\sin(bx+a)-\sin(bx-c)b\cos(bx+a)}{4b^{2}}+\frac{x(2\sin(bx+a)\sin(bx-c)b^{2}-2}{4b^{2}}$
norman	$\frac{\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}-\frac{x}{2}+\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}}{2}+\frac{x\tan\left(\frac{bx}{2}-\frac{c}{2}\right)^{2}}{2}-2x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}-\frac{c}{2}\right)-\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}\tan\left(\frac{bx}{2}-\frac{c}{2}\right)^{2}}{2}-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\mathrm{ta}}{b}}{\left(1+\tan\left(\frac{bx}{2}-\frac{c}{2}\right)^{2}\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}\right)}$

input
```
int(-sin(b*x-c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \sin(c-bx)\sin(a+bx)\,dx$$
$$=-\frac{bx\cos\left(a+c\right)-\cos\left(bx+a\right)\cos\left(a+c\right)\sin\left(bx+a\right)+\cos\left(bx+a\right)^{2}\sin\left(a+c\right)}{2\,b}$$

input
```
integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(b*x*cos(a + c) - cos(b*x + a)*cos(a + c)*sin(b*x + a) + cos(b*x + a)
^2*sin(a + c))/b
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \sin(c-bx)\sin(a+bx)\,dx = -\begin{cases} \frac{x\sin(a+bx)\sin(bx-c)}{2} + \frac{x\cos(a+bx)\cos(bx-c)}{2} - \frac{\sin(bx-c)\cos(a+bx)}{2b} & \text{for } b\neq 0 \\ -x\sin(a)\sin(c) & \text{otherwise} \end{cases}$$

input
```
integrate(-sin(b*x-c)*sin(b*x+a),x)
```

output
```
-Piecewise((x*sin(a + b*x)*sin(b*x - c)/2 + x*cos(a + b*x)*cos(b*x - c)/2
- sin(b*x - c)*cos(a + b*x)/(2*b), Ne(b, 0)), (-x*sin(a)*sin(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sin(c-bx)\sin(a+bx)\,dx = -\frac{1}{2}\,x\cos(a+c) + \frac{\sin(2\,bx+a-c)}{4\,b}$$

input
```
integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sin(c-bx)\sin(a+bx)\,dx = -\frac{1}{2}\,x\cos(a+c) + \frac{\sin(2\,bx+a-c)}{4\,b}$$

input
```
integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

Mupad [B] (verification not implemented)

Time = 16.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \sin(c-bx)\sin(a+bx)\,dx = \begin{cases} x\,\sin(a)\,\sin(c) & \text{if}\ \ b=0 \\ \frac{\sin(a-c+2\,b\,x)}{4\,b}-\frac{x\,\cos(a+c)}{2} & \text{if}\ \ b\neq 0 \end{cases}$$

input
```
int(sin(a + b*x)*sin(c - b*x),x)
```

output
```
piecewise(b == 0, x*sin(a)*sin(c), b ~= 0, sin(a - c + 2*b*x)/(4*b) - (x*c
os(a + c))/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \sin(c-bx)\sin(a+bx)\,dx$$
$$= \frac{-\cos(bx-c)\cos(bx+a)\,bx+\cos(bx-c)\sin(bx+a)-\sin(bx-c)\sin(bx+a)\,bx}{2b}$$

input
```
int(-sin(b*x-c)*sin(b*x+a),x)
```

output
```
( - cos(b*x - c)*cos(a + b*x)*b*x + cos(b*x - c)*sin(a + b*x) - sin(b*x -
c)*sin(a + b*x)*b*x)/(2*b)
```

3.78 $\int \sin(a+bx)\sin^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \sin(a+bx)\sin^3(c+bx)\,dx = \frac{3}{8}x\cos(a-c)+\frac{\sin(a-3c-2bx)}{16b} - \frac{3\sin(a+c+2bx)}{16b}+\frac{\sin(a+3c+4bx)}{32b}$$

output
```
3/8*x*cos(a-c)+1/16*sin(-2*b*x+a-3*c)/b-3/16*sin(2*b*x+a+c)/b+1/32*sin(4*b
*x+a+3*c)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \sin(a+bx)\sin^3(c+bx)\,dx = \frac{12bx\cos(a-c)+2\sin(a-3c-2bx)-6\sin(a+c+2bx)+\sin(a+3c+4bx)}{32b}$$

input
```
Integrate[Sin[a + b*x]*Sin[c + b*x]^3,x]
```

output
```
(12*b*x*Cos[a - c] + 2*Sin[a - 3*c - 2*b*x] - 6*Sin[a + c + 2*b*x] + Sin[a
 + 3*c + 4*b*x])/(32*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^3(bx+c)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{1}{8}\cos(a-2bx-3c)-\frac{3}{8}\cos(a+2bx+c)+\frac{1}{8}\cos(a+4bx+3c)+\frac{3}{8}\cos(a-c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a-2bx-3c)}{16b}-\frac{3\sin(a+2bx+c)}{16b}+\frac{\sin(a+4bx+3c)}{32b}+\frac{3}{8}x\cos(a-c)$$

input
```
Int[Sin[a + b*x]*Sin[c + b*x]^3,x]
```

output
```
(3*x*Cos[a - c])/8 + Sin[a - 3*c - 2*b*x]/(16*b) - (3*Sin[a + c + 2*b*x])/
(16*b) + Sin[a + 3*c + 4*b*x]/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result
default	$\frac{3x\cos(a-c)}{8}+\frac{\sin(-2bx+a-3c)}{16b}-\frac{3\sin(2bx+a+c)}{16b}+\frac{\sin(4bx+a+3c)}{32b}$
risch	$\frac{3x\cos(a-c)}{8}+\frac{\sin(-2bx+a-3c)}{16b}-\frac{3\sin(2bx+a+c)}{16b}+\frac{\sin(4bx+a+3c)}{32b}$
parallelrisch	$\frac{12x\cos(a-c)b+2\sin(-2bx+a-3c)-6\sin(2bx+a+c)+\sin(4bx+a+3c)+3\sin(a-c)}{32b}$
orering	$x\sin(bx+a)\sin(bx+c)^3-\frac{5\left(b\cos(bx+a)\sin(bx+c)^3+3\sin(bx+a)\sin(bx+c)^2b\cos(bx+c)\right)}{16b^2}+\frac{5x\left(-4\sin(bx+\right.}{}$

input
```
int(sin(b*x+a)*sin(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
3/8*x*cos(a-c)+1/16*sin(-2*b*x+a-3*c)/b-3/16*sin(2*b*x+a+c)/b+1/32*sin(4*b
*x+a+3*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.33

$$\int \sin(a+bx)\sin^3(c+bx)\,dx$$
$$=\frac{3\,bx\cos(-a+c)+\left(2\,\cos(bx+c)^3\cos(-a+c)-5\,\cos(bx+c)\cos(-a+c)\right)\sin(bx+c)-2\left(\cos(bx\right.}{8\,b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/8*(3*b*x*cos(-a + c) + (2*cos(b*x + c)^3*cos(-a + c) - 5*cos(b*x + c)*co
s(-a + c))*sin(b*x + c) - 2*(cos(b*x + c)^4 - 2*cos(b*x + c)^2)*sin(-a + c
))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(54) = 108$.

Time = 0.97 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.79

$$\int \sin(a+bx)\sin^3(c+bx)\,dx$$
$$= \begin{cases} \frac{3x\sin{(a+bx)}\sin^3{(bx+c)}}{8} + \frac{3x\sin{(a+bx)}\sin{(bx+c)}\cos^2{(bx+c)}}{8} + \frac{3x\sin^2{(bx+c)}\cos{(a+bx)}\cos{(bx+c)}}{8} + \frac{3x\cos{(a+bx)}\cos^3{(bx+c)}}{8} \\ x\sin{(a)}\sin^3{(c)} \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c)**3,x)
```

output
```
Piecewise((3*x*sin(a + b*x)*sin(b*x + c)**3/8 + 3*x*sin(a + b*x)*sin(b*x +
 c)*cos(b*x + c)**2/8 + 3*x*sin(b*x + c)**2*cos(a + b*x)*cos(b*x + c)/8 +
3*x*cos(a + b*x)*cos(b*x + c)**3/8 - 3*sin(a + b*x)*sin(b*x + c)**2*cos(b*
x + c)/(4*b) - 3*sin(a + b*x)*cos(b*x + c)**3/(8*b) + sin(b*x + c)**3*cos(
a + b*x)/(8*b), Ne(b, 0)), (x*sin(a)*sin(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \sin(a+bx)\sin^3(c+bx)\,dx$$
$$= \frac{12\,bx\cos{(-a+c)} + \sin{(4\,bx+a+3\,c)} - 6\,\sin{(2\,bx+a+c)} - 2\,\sin{(2\,bx-a+3\,c)}}{32\,b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c)^3,x, algorithm="maxima")
```

output
```
1/32*(12*b*x*cos(-a + c) + sin(4*b*x + a + 3*c) - 6*sin(2*b*x + a + c) - 2
*sin(2*b*x - a + 3*c))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \sin(a+bx)\sin^3(c+bx)\,dx = \frac{3}{8}\,x\cos{(a-c)} + \frac{\sin{(4\,bx+a+3\,c)}}{32\,b} - \frac{3\,\sin{(2\,bx+a+c)}}{16\,b} + \frac{\sin{(-2\,bx+a-3\,c)}}{16\,b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c)^3,x, algorithm="giac")
```

output
```
3/8*x*cos(a - c) + 1/32*sin(4*b*x + a + 3*c)/b - 3/16*sin(2*b*x + a + c)/b
 + 1/16*sin(-2*b*x + a - 3*c)/b
```

Mupad [B] (verification not implemented)

Time = 19.81 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \sin(a+bx)\sin^3(c+bx)\,dx = \frac{3\,x\,\cos{(a-c)}}{8} - \frac{\frac{\sin(3\,c-a+2\,b\,x)}{16} - \frac{\sin(a+3\,c+4\,b\,x)}{32} + \frac{3\,\sin(a+c+2\,b\,x)}{16}}{b}$$

input
```
int(sin(a + b*x)*sin(c + b*x)^3,x)
```

output
```
(3*x*cos(a - c))/8 - (sin(3*c - a + 2*b*x)/16 - sin(a + 3*c + 4*b*x)/32 +
(3*sin(a + c + 2*b*x))/16)/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.69

$$\int \sin(a+bx)\sin^3(c+bx)\,dx$$

$$= \frac{3\cos(bx+c)\cos(bx+a)\,bx - 3\cos(bx+c)\sin(bx+c)^2\sin(bx+a) + 12\cos(bx+c)\sin(bx+a) + \mathrm{co}}{8b}$$

input
```
int(sin(b*x+a)*sin(b*x+c)^3,x)
```

output
```
(3*cos(b*x + c)*cos(a + b*x)*b*x - 3*cos(b*x + c)*sin(b*x + c)**2*sin(a +
b*x) + 12*cos(b*x + c)*sin(a + b*x) + cos(a + b*x)*sin(b*x + c)**3 - 15*co
s(a + b*x)*sin(b*x + c) + 3*sin(b*x + c)*sin(a + b*x)*b*x)/(8*b)
```

3.79 $\int \sin(a+bx)\sin^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \sin(a+bx)\sin^2(c+bx)\,dx = -\frac{\cos(a-2c-bx)}{4b}-\frac{\cos(a+bx)}{2b}+\frac{\cos(a+2c+3bx)}{12b}$$

output
```
-1/4*cos(-b*x+a-2*c)/b-1/2*cos(b*x+a)/b+1/12*cos(3*b*x+a+2*c)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \sin(a+bx)\sin^2(c+bx)\,dx = \frac{-3\cos(a-2c-bx)-6\cos(a+bx)+\cos(a+2c+3bx)}{12b}$$

input
```
Integrate[Sin[a + b*x]*Sin[c + b*x]^2,x]
```

output
```
(-3*Cos[a - 2*c - b*x] - 6*Cos[a + b*x] + Cos[a + 2*c + 3*b*x])/(12*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^2(bx+c)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{1}{4}\sin(a-bx-2c)-\frac{1}{4}\sin(a+3bx+2c)+\frac{1}{2}\sin(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(a-bx-2c)}{4b}+\frac{\cos(a+3bx+2c)}{12b}-\frac{\cos(a+bx)}{2b}$$

input
```
Int[Sin[a + b*x]*Sin[c + b*x]^2,x]
```

output
```
-1/4*Cos[a - 2*c - b*x]/b - Cos[a + b*x]/(2*b) + Cos[a + 2*c + 3*b*x]/(12*
b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\cos(-bx+a-2c)}{4b}-\frac{\cos(bx+a)}{2b}+\frac{\cos(3bx+a+2c)}{12b}$
risch	$-\frac{\cos(-bx+a-2c)}{4b}-\frac{\cos(bx+a)}{2b}+\frac{\cos(3bx+a+2c)}{12b}$
parallelrisch	$\frac{-8\cos(a-c)-6\cos(bx+a)-3\cos(-bx+a-2c)+\cos(3bx+a+2c)}{12b}$
norman	$\frac{-\frac{4}{3b}+\frac{4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{3b}-\frac{16\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)}{3b}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(1+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\right)^2}$
orering	$-\frac{10\left(b\cos(bx+a)\sin(bx+c)^2+2\sin(bx+a)\sin(bx+c)b\cos(bx+c)\right)}{9b^2}-\frac{-7b^3\cos(bx+a)\sin(bx+c)^2-14b^3\sin(bx+a)\sin(bx+}{9b^4}$

input
```
int(sin(b*x+a)*sin(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(-b*x+a-2*c)/b-1/2*cos(b*x+a)/b+1/12*cos(3*b*x+a+2*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \sin(a+bx)\sin^2(c+bx)\,dx$$
$$=\frac{\cos(bx+c)^3\cos(-a+c)+\left(\cos(bx+c)^2-1\right)\sin(bx+c)\sin(-a+c)-3\cos(bx+c)\cos(-a+c)}{3b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/3*(cos(b*x + c)^3*cos(-a + c) + (cos(b*x + c)^2 - 1)*sin(b*x + c)*sin(-a
 + c) - 3*cos(b*x + c)*cos(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(37) = 74$.

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56

$$\int \sin(a+bx)\sin^2(c+bx)\,dx = \begin{cases} -\frac{2\sin(a+bx)\sin(bx+c)\cos(bx+c)}{3b} - \frac{\sin^2(bx+c)\cos(a+bx)}{3b} - \frac{2\cos(a+bx)\cos^2(bx+c)}{3b} & \text{for } b \neq 0 \\ x\sin(a)\sin^2(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c)**2,x)
```

output
```
Piecewise((-2*sin(a + b*x)*sin(b*x + c)*cos(b*x + c)/(3*b) - sin(b*x + c)*
*2*cos(a + b*x)/(3*b) - 2*cos(a + b*x)*cos(b*x + c)**2/(3*b), Ne(b, 0)), (
x*sin(a)*sin(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \sin(a+bx)\sin^2(c+bx)\,dx = \frac{\cos(3\,bx+a+2\,c) - 6\,\cos(bx+a) - 3\,\cos(bx-a+2\,c)}{12\,b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/12*(cos(3*b*x + a + 2*c) - 6*cos(b*x + a) - 3*cos(b*x - a + 2*c))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \sin(a+bx)\sin^2(c+bx)\,dx = \frac{\cos(3\,bx+a+2\,c)}{12\,b} - \frac{\cos(bx+a)}{2\,b} - \frac{\cos(-bx+a-2\,c)}{4\,b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c)^2,x, algorithm="giac")
```

output
```
1/12*cos(3*b*x + a + 2*c)/b - 1/2*cos(b*x + a)/b - 1/4*cos(-b*x + a - 2*c)
/b
```

Mupad [B] (verification not implemented)

Time = 18.70 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \sin(a+bx)\sin^2(c+bx)\,dx = -\frac{6\,\cos(a+b\,x)-\cos(a+2\,c+3\,b\,x)+3\,\cos(2\,c-a+b\,x)}{12\,b}$$

input
```
int(sin(a + b*x)*sin(c + b*x)^2,x)
```

output
```
-(6*cos(a + b*x) - cos(a + 2*c + 3*b*x) + 3*cos(2*c - a + b*x))/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

$$\int \sin(a+bx)\sin^2(c+bx)\,dx = \frac{2\cos(bx+c)\cos(bx+a) - 2\cos(bx+c)\sin(bx+c)\sin(bx+a) + \cos(bx+a)\sin(bx+c)^2 - 2\cos(bx}{3b}$$

input
```
int(sin(b*x+a)*sin(b*x+c)^2,x)
```

output
```
(2*cos(b*x + c)*cos(a + b*x) - 2*cos(b*x + c)*sin(b*x + c)*sin(a + b*x) +
cos(a + b*x)*sin(b*x + c)**2 - 2*cos(a + b*x) + 2*sin(b*x + c)*sin(a + b*x
))/(3*b)
```

3.80 $\int \sin(a+bx)\sin(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \sin(a+bx)\sin(c+bx)\,dx = \frac{1}{2}x\cos(a-c) - \frac{\sin(a+c+2bx)}{4b}$$

output
```
1/2*x*cos(a-c)-1/4*sin(2*b*x+a+c)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sin(a+bx)\sin(c+bx)\,dx = -\frac{-2bx\cos(a-c)+\sin(a+c+2bx)}{4b}$$

input
```
Integrate[Sin[a + b*x]*Sin[c + b*x],x]
```

output
```
-1/4*(-2*b*x*Cos[a - c] + Sin[a + c + 2*b*x])/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin(bx+c)\,dx$$

$$\Big\downarrow \ 5080$$

$$\int \left(\frac{1}{2}\cos(a-c)-\frac{1}{2}\cos(a+2bx+c)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}x\cos(a-c)-\frac{\sin(a+2bx+c)}{4b}$$

input
```
Int[Sin[a + b*x]*Sin[c + b*x],x]
```

output
```
(x*Cos[a - c])/2 - Sin[a + c + 2*b*x]/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cos(a-c)}{2}-\frac{\sin(2bx+a+c)}{4b}$
risch	$\frac{x\cos(a-c)}{2}-\frac{\sin(2bx+a+c)}{4b}$
parallelrisch	$\frac{2x\cos(a-c)b-\sin(2bx+a+c)+\sin(a-c)}{4b}$
orering	$x\sin(bx+a)\sin(bx+c)-\frac{b\cos(bx+a)\sin(bx+c)+\sin(bx+a)b\cos(bx+c)}{4b^2}+\frac{x(-2\sin(bx+c)\sin(bx+a)b^2+2\cos}{4b^2}$
norman	$\frac{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}+\frac{x}{2}-\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{2}-\frac{x\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2}{2}+2x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)+\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2}{2}+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(1+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(sin(b*x+a)*sin(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*cos(a-c)-1/4*sin(2*b*x+a+c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(23) = 46.

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \sin(a+bx)\sin(c+bx)\,dx$$
$$=\frac{bx\cos(-a+c)-\cos(bx+c)\cos(-a+c)\sin(bx+c)+\cos(bx+c)^2\sin(-a+c)}{2b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="fricas")
```

output
```
1/2*(b*x*cos(-a + c) - cos(b*x + c)*cos(-a + c)*sin(b*x + c) + cos(b*x + c
)^2*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \sin(a+bx)\sin(c+bx)\,dx = \begin{cases} \frac{x\sin(a+bx)\sin(bx+c)}{2} + \frac{x\cos(a+bx)\cos(bx+c)}{2} - \frac{\sin(bx+c)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin(a)\sin(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c),x)
```

output
```
Piecewise((x*sin(a + b*x)*sin(b*x + c)/2 + x*cos(a + b*x)*cos(b*x + c)/2 -
 sin(b*x + c)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*sin(a)*sin(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sin(a+bx)\sin(c+bx)\,dx = \frac{1}{2}\,x\cos(-a+c) - \frac{\sin(2\,bx+a+c)}{4\,b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="maxima")
```

output
```
1/2*x*cos(-a + c) - 1/4*sin(2*b*x + a + c)/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sin(a+bx)\sin(c+bx)\,dx = \frac{1}{2}\,x\cos\left(a-c\right) - \frac{\sin\left(2\,bx+a+c\right)}{4\,b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="giac")
```

output
```
1/2*x*cos(a - c) - 1/4*sin(2*b*x + a + c)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \sin(a+bx)\sin(c+bx)\,dx = \begin{cases} x\,\sin\left(a\right)\,\sin\left(c\right) & \text{if } b=0 \\ \frac{x\,\cos(a-c)}{2}-\frac{\sin(a+c+2\,b\,x)}{4\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int(sin(a + b*x)*sin(c + b*x),x)
```

output
```
piecewise(b == 0, x*sin(a)*sin(c), b ~= 0, (x*cos(a - c))/2 - sin(a + c +
2*b*x)/(4*b))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \sin(a+bx)\sin(c+bx)\,dx = \frac{\cos\left(bx+c\right)\cos\left(bx+a\right)bx-\cos\left(bx+c\right)\sin\left(bx+a\right)+\sin\left(bx+c\right)\sin\left(bx+a\right)bx}{2b}$$

input
```
int(sin(b*x+a)*sin(b*x+c),x)
```

output
```
(cos(b*x + c)*cos(a + b*x)*b*x - cos(b*x + c)*sin(a + b*x) + sin(b*x + c)*
sin(a + b*x)*b*x)/(2*b)
```

3.81 $\int \csc(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \csc(c+bx)\sin(a+bx)\,dx = x\cos(a-c)+\frac{\log(\sin(c+bx))\sin(a-c)}{b}$$

output
```
x*cos(a-c)+ln(sin(b*x+c))*sin(a-c)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc(c+bx)\sin(a+bx)\,dx = x\cos(a-c)+\frac{\log(\sin(c+bx))\sin(a-c)}{b}$$

input
```
Integrate[Csc[c + b*x]*Sin[a + b*x],x]
```

output
```
x*Cos[a - c] + (Log[Sin[c + b*x]]*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5093, 24, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \ 5093$$

$$\sin(a-c)\int \cot(c+bx)dx+\cos(a-c)\int 1dx$$

$$\Big\downarrow \ 24$$

$$\sin(a-c)\int \cot(c+bx)dx+x\cos(a-c)$$

$$\Big\downarrow \ 3042$$

$$\sin(a-c)\int -\tan\left(c+bx+\frac{\pi}{2}\right)dx+x\cos(a-c)$$

$$\Big\downarrow \ 25$$

$$x\cos(a-c)-\sin(a-c)\int \tan\left(\frac{1}{2}(2c+\pi)+bx\right)dx$$

$$\Big\downarrow \ 3956$$

$$\frac{\sin(a-c)\log(-\sin(bx+c))}{b}+x\cos(a-c)$$

input
```
Int[Csc[c + b*x]*Sin[a + b*x],x]
```

output
```
x*Cos[a - c] + (Log[-Sin[c + b*x]]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

method	result
risch	$-2 i \sin \left(a -c \right) x -\frac{2 i \sin \left(a -c \right) a}{b}+x \,{\mathrm e}^{i \left(a -c \right)}+\frac{\ln \left({\mathrm e}^{2 i \left(b x +a \right)}-{\mathrm e}^{2 i \left(a -c \right)}\right) \sin \left(a -c \right)}{b}$
default	$\frac{\frac{\frac{\left(-\sin \left(a \right) \cos \left(c \right)+\cos \left(a \right) \sin \left(c \right)\right) \ln \left(\tan \left(b x +a \right)^{2}+1\right)}{2}+\left(\cos \left(a \right) \cos \left(c \right)+\sin \left(a \right) \sin \left(c \right)\right) \arctan \left(\tan \left(b x +a \right)\right)}{\left(\cos \left(c \right)^{2}+\sin \left(c \right)^{2}\right) \left(\cos \left(a \right)^{2}+\sin \left(a \right)^{2}\right)}-\frac{\left(-\sin \left(a \right) \cos \left(c \right)+\cos \left(a \right) \sin \left(c \right)\right) \ln \left(\tan \left(b x +a \right)\right.}{\cos \left(a \right)^{2} \cos \left(c \right)^{2}+\sin \left(\right.}}{b}$

input
```
int(csc(b*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*I*sin(a-c)*x-2*I/b*sin(a-c)*a+x*exp(I*(a-c))+ln(exp(2*I*(b*x+a))-exp(2*
I*(a-c)))/b*sin(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \csc(c+bx)\sin(a+bx)\,dx = \frac{bx\cos(-a+c) - \log\left(\frac{1}{2}\sin(bx+c)\right)\sin(-a+c)}{b}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
(b*x*cos(-a + c) - log(1/2*sin(b*x + c))*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(20) = 40.

Time = 4.37 (sec) , antiderivative size = 333, normalized size of antiderivative = 12.81

$$\int \csc(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x, Eq(c, 0)), (0, Eq(b, 0)), (-b*x*ta
n(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2*log(tan(c/2) +
 tan(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2))
*tan(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c
/2)**2 + b), True))*cos(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (log(
sin(b*x))/b, Eq(c, 0)), (x/sin(c), Eq(b, 0)), (2*b*x*tan(c/2)/(b*tan(c/2)*
*2 + b) - log(tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) + log
(tan(c/2) + tan(b*x/2))/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - 1/tan(c/2))
*tan(c/2)**2/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - 1/tan(c/2))/(b*tan(c/2
)**2 + b) + log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(t
an(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b), True))*sin(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(26) = 52$.

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.15

$$\int \csc(c+bx)\sin(a+bx)\,dx$$
$$= \frac{2\,bx\cos(-a+c) - \log\left(\cos(bx)^2 + 2\,\cos(bx)\cos(c) + \cos(c)^2 + \sin(bx)^2 - 2\,\sin(bx)\sin(c) + \sin(c)^2\right)}{}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*b*x*cos(-a + c) - log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + s
in(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2)*sin(-a + c) - log(cos(b*x)^2 - 2
*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*s
in(-a + c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 236, normalized size of antiderivative = 9.08

$$\int \csc(c+bx)\sin(a+bx)\,dx$$
$$= \frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}a\right)^2 + 4\,\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}c\right)^2 + 1\right)(bx+c)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1} - \frac{2\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right) - \tan\right.}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan}}{b}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan
(1/2*c)^2 + 1)*(b*x + c)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1
/2*c)^2 + 1) - 2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(
1/2*a) - tan(1/2*c))*log(tan(1/2*b*x + 1/2*c)^2 + 1)/(tan(1/2*a)^2*tan(1/2
*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^2*tan(1/2*c) - ta
n(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2
*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1))/b
```

Mupad [B] (verification not implemented)

Time = 17.87 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.27

$$\int \csc(c+bx)\sin(a+bx)\,dx = x\left(\frac{e^{-a\,1i+c\,1i}}{2}-\frac{e^{a\,1i-c\,1i}}{2}\right)+x\left(\frac{e^{-a\,1i+c\,1i}}{2}+\frac{e^{a\,1i-c\,1i}}{2}\right) + \frac{\ln\left(-e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}-\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)}{b}$$

input
```
int(sin(a + b*x)/sin(c + b*x),x)
```

output
```
x*(exp(c*1i - a*1i)/2 - exp(a*1i - c*1i)/2) + x*(exp(c*1i - a*1i)/2 + exp(
a*1i - c*1i)/2) + (log(exp(a*2i + b*x*2i) - exp(a*2i - c*2i))*((exp(c*1i -
 a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2))/b
```

Reduce [F]

$$\int \csc(c+bx)\sin(a+bx)\,dx = \int \csc(bx+c)\sin(bx+a)\,dx$$

input
```
int(csc(b*x+c)*sin(b*x+a),x)
```

output
```
int(csc(b*x + c)*sin(a + b*x),x)
```

3.82 $\int \csc^2(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \csc^2(c+bx)\sin(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(c+bx))\cos(a-c)}{b}-\frac{\csc(c+bx)\sin(a-c)}{b}$$

output
```
-arctanh(cos(b*x+c))*cos(a-c)/b-csc(b*x+c)*sin(a-c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = -\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b} - \frac{\csc(c+bx)\sin(a-c)}{b}$$

input
```
Integrate[Csc[c + b*x]^2*Sin[a + b*x],x]
```

output
```
((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x
)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Csc
[c + b*x]*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5093, 3042, 25, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \text{5093}$$

$$\cos(a-c)\int \csc(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\cos(a-c)\int \csc(c+bx)dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc(c+bx)dx-\frac{\sin(a-c)\int 1d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$\cos(a-c)\int \csc(c+bx)dx-\frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\cos(a-c)\text{arctanh}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}$$

input
```
Int[Csc[c + b*x]^2*Sin[a + b*x],x]
```

output
```
-((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) - (Csc[c + b*x]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.19

method	result
risch	$\frac{e^{i(bx+3a)}-e^{i(bx+a+2c)}}{b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos(a-c)}{b}+\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos(a-c)}{b}$
default	$\frac{4(-2\cos(a)\cos(c)-2\sin(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+8\sin(a)\cos(c)-8\cos(a)\sin}{\left(-4\cos(c)^2\sin(a)^2-4\cos(a)^2\cos(c)^2-4\sin(a)^2\sin(c)^2-4\sin(c)^2\cos(a)^2\right)\left(\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+2\tan\left(\frac{a}{2}+\frac{bx}{2}\right.\right.}$

input
```
int(csc(b*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))*(exp(I*(b*x+3*a))-exp(I*(b*x+a+2*c)))
-ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)+ln(exp(I*(b*x+a))-exp(I*(a-c))
)/b*cos(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.97

$$\int \csc^2(c+bx)\sin(a+bx)\,dx =$$

$$-\frac{\cos(-a+c)\log\left(\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)-\cos(-a+c)\log\left(-\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c}{2\,b\sin(bx+c)}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cos(-a + c)*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - cos(-a + c)*l
og(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - 2*sin(-a + c))/(b*sin(b*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(29) = 58$.

Time = 64.00 (sec) , antiderivative size = 3264, normalized size of antiderivative = 90.67

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)**2*sin(b*x+a),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), (log(tan(b*x/2))/b, Eq(c, 0)), (0, Eq(
b, 0)), (-log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4
*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan
(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(c
/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)*
*2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2))
 + log(tan(c/2) + tan(b*x/2))*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*ta
n(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan
(c/2) - b*tan(b*x/2)) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**2*tan(b*x/2
)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3
+ b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(c/2) + t
an(b*x/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**
3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) -
b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x
/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)*
*2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(b*x/2)/(b
*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*
tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/t
an(c/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*
tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) -...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(36) = 72.

Time = 0.06 (sec) , antiderivative size = 454, normalized size of antiderivative = 12.61

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*(cos(b*x + 2*a) - cos(b*x + 2*c))*cos(2*b*x + a + 2*c) - 2*cos(b*x
 + 2*a)*cos(a) + 2*cos(b*x + 2*c)*cos(a) + (cos(2*b*x + a + 2*c)^2*cos(-a
+ c) - 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(2*b*x +
 a + 2*c)^2 - 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(
a)^2)*cos(-a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x
)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*b*x + a + 2*c)^2*cos(-a + c)
- 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(2*b*x + a +
2*c)^2 - 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)
*cos(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 +
 2*sin(b*x)*sin(c) + sin(c)^2) + 2*(sin(b*x + 2*a) - sin(b*x + 2*c))*sin(2
*b*x + a + 2*c) - 2*sin(b*x + 2*a)*sin(a) + 2*sin(b*x + 2*c)*sin(a))/(b*co
s(2*b*x + a + 2*c)^2 - 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a +
 2*c)^2 - 2*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(36) = 72.

Time = 0.14 (sec) , antiderivative size = 349, normalized size of antiderivative = 9.69

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan
(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 +
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan
(1/2*c) - tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*b*x + 1/2
*c)*tan(1/2*a) - tan(1/2*b*x + 1/2*c)*tan(1/2*c))/(tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2
*tan(1/2*c)^2 - tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2 + 4*tan(1/2*b*x + 1/2*c)
*tan(1/2*a)*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c) - tan(1/2*b*x + 1/2*c)*ta
n(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*b*x + 1/2*c) + tan(1/2*a) -
 tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1
)*tan(1/2*b*x + 1/2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 22.27 (sec) , antiderivative size = 252, normalized size of antiderivative = 7.00

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = -\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}} + \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}} + \frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}-1\right)}{b\,\left(e^{a\,2i-c\,2i}-e^{a\,2i+b\,x\,2i}\right)}$$

input
```
int(sin(a + b*x)/sin(c + b*x)^2,x)
```

output
```
(log((exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-
c*2i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i))*(exp(
a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log(- exp(a*1i)*exp(b*x
*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i) - (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp
(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) + 1))/(2*
b*exp(a*2i - c*2i)^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) - 1))/(b
*(exp(a*2i - c*2i) - exp(a*2i + b*x*2i)))
```

Reduce [F]

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = \int \csc(bx+c)^2 \sin(bx+a)\,dx$$

input
```
int(csc(b*x+c)^2*sin(b*x+a),x)
```

output
```
int(csc(b*x + c)**2*sin(a + b*x),x)
```

3.83 $\int \csc^3(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \csc^3(c+bx)\sin(a+bx)\,dx=-\frac{\cos(a-c)\cot(c+bx)}{b}-\frac{\csc^2(c+bx)\sin(a-c)}{2b}$$

output
```
-cos(a-c)*cot(b*x+c)/b-1/2*csc(b*x+c)^2*sin(a-c)/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \csc^3(c+bx)\sin(a+bx)\,dx=\frac{(\cos(a)-\cos(a-c)\cos(c+2bx))\csc(c)\csc^2(c+bx)}{2b}$$

input
```
Integrate[Csc[c + b*x]^3*Sin[a + b*x],x]
```

output
```
((Cos[a] - Cos[a - c]*Cos[c + 2*b*x])*Csc[c]*Csc[c + b*x]^2)/(2*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5093, 3042, 25, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^3(bx+c)\,dx$$

$$\Big\downarrow \text{5093}$$

$$\cos(a-c)\int \csc^2(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc^2(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)^2dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^2\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\cos(a-c)\int \csc(c+bx)^2dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc(c+bx)^2dx-\frac{\sin(a-c)\int \csc(c+bx)d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(a-c)\int \csc(c+bx)^2dx-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\cos(a-c)\int 1d\cot(c+bx)}{b}-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\cos(a-c)\cot(bx+c)}{b}-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

input
```
Int[Csc[c + b*x]^3*Sin[a + b*x],x]
```

output
```
-((Cos[a - c]*Cot[c + b*x])/b) - (Csc[c + b*x]^2*Sin[a - c])/(2*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

method	result
risch	$\frac{i\left(-2\,\mathrm{e}^{i(2bx+5a+c)}+\mathrm{e}^{i(5a-c)}+\mathrm{e}^{i(3a+c)}\right)}{\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{2}b}$
parallelrisch	$-\frac{\csc\left(\frac{bx}{2}+\frac{c}{2}\right)\left(\sin(bx+a)\left(-\frac{\sec\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}}{2}+1\right)\csc\left(\frac{bx}{2}+\frac{c}{2}\right)+\sec\left(\frac{bx}{2}+\frac{c}{2}\right)\cos(bx+a)\right)}{4b}$
default	$\frac{-\frac{\sin(a)\cos(c)-\cos(a)\sin(c)}{2(\cos(a)\cos(c)+\sin(a)\sin(c))^{2}(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))^{2}}-\frac{\quad}{(\cos(a)\cos(c)+\sin(a)\sin}}{b}$

input
```
int(csc(b*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
I/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^2/b*(-2*exp(I*(2*b*x+5*a+c))+exp(I*(5*a
-c))+exp(I*(3*a+c)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \frac{2\,\cos\left(bx+c\right)\cos\left(-a+c\right)\sin\left(bx+c\right)-\sin\left(-a+c\right)}{2\left(b\cos\left(bx+c\right)^{2}-b\right)}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c) - sin(-a + c))/(b*cos(b*x + c
)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**3*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(37) = 74$.

Time = 0.05 (sec) , antiderivative size = 399, normalized size of antiderivative = 10.23

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \frac{(2\sin(2bx+2a+2c)-\sin(2a)-\sin(2c))\cos(4bx+a+5c)-2(2\sin(2bx+2a+2c)-\sin(2a)}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2-4b\cos(2}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
((2*sin(2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(4*b*x + a + 5*c) - 2
*(2*sin(2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(2*b*x + a + 3*c) - (
sin(2*a) + sin(2*c))*cos(a + c) - (2*cos(2*b*x + 2*a + 2*c) - cos(2*a) - c
os(2*c))*sin(4*b*x + a + 5*c) + 2*cos(a + c)*sin(2*b*x + 2*a + 2*c) + 2*(2
*cos(2*b*x + 2*a + 2*c) - cos(2*a) - cos(2*c))*sin(2*b*x + a + 3*c) + (cos
(2*a) + cos(2*c))*sin(a + c) - 2*cos(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos
(4*b*x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 - 4*b*cos(2*b*x + a + 3*c
)*cos(a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x +
 a + 3*c)^2 - 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(2*
b*cos(2*b*x + a + 3*c) - b*cos(a + c))*cos(4*b*x + a + 5*c) - 2*(2*b*sin(2
*b*x + a + 3*c) - b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(37) = 74$.

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.72

$$\int \csc^3(c+bx)\sin(a+bx)\,dx =$$

$$-\frac{\tan(bx+c)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2 - \tan(bx+c)\tan\left(\frac{1}{2}a\right)^2 + 4\tan(bx+c)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right) + \tan(}{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-(tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + c)*tan(1/2*a)^2 + 4*t
an(b*x + c)*tan(1/2*a)*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c) - tan(b*x + c)
*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^2 + tan(b*x + c) + tan(1/2*a) - tan(
1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b*t
an(b*x + c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/sin(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \frac{-\cos(bx+c)\sin(bx+a)-\cos(bx+a)\sin(bx+c)}{2\sin(bx+c)^2 b}$$

input
```
int(csc(b*x+c)^3*sin(b*x+a),x)
```

output
```
( - (cos(b*x + c)*sin(a + b*x) + cos(a + b*x)*sin(b*x + c)))/(2*sin(b*x +
c)**2*b)
```

3.84 $\int \csc^4(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} - \frac{\csc^3(c+bx)\sin(a-c)}{3b}$$

output
```
-1/2*arctanh(cos(b*x+c))*cos(a-c)/b-1/2*cos(a-c)*cot(b*x+c)*csc(b*x+c)/b-1
/3*csc(b*x+c)^3*sin(a-c)/b
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = -\frac{6\operatorname{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+3\cos(a-c)\cot(c+bx)\csc(c+bx)+2\csc^3(c+bx)\,\mathrm{s}}{6b}$$

input
```
Integrate[Csc[c + b*x]^4*Sin[a + b*x],x]
```

output
```
-1/6*(6*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + 3*Cos[a - c]*Co
t[c + b*x]*Csc[c + b*x] + 2*Csc[c + b*x]^3*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5093, 3042, 25, 3086, 15, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^4(bx+c)\,dx$$

$$\Big\downarrow \text{ 5093}$$

$$\cos(a-c)\int \csc^3(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc^3(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos(a-c)\int \csc(c+bx)^3dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^3\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 25}$$

$$\cos(a-c)\int \csc(c+bx)^3dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^3\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{ 3086}$$

$$\cos(a-c)\int \csc(c+bx)^3dx-\frac{\sin(a-c)\int \csc^2(c+bx)d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\cos(a-c)\int \csc(c+bx)^3dx-\frac{\sin(a-c)\csc^3(bx+c)}{3b}$$

$$\Big\downarrow \text{ 4255}$$

$$\cos(a-c)\left(\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\sin(a-c)\csc^3(bx+c)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$\cos(a-c)\left(\frac{1}{2}\int \csc(c+bx)dx - \frac{\cot(bx+c)\csc(bx+c)}{2b}\right) - \frac{\sin(a-c)\csc^3(bx+c)}{3b}$$

$$\Big\downarrow \textbf{4257}$$

$$\cos(a-c)\left(-\frac{\operatorname{arctanh}(\cos(bx+c))}{2b} - \frac{\cot(bx+c)\csc(bx+c)}{2b}\right) - \frac{\sin(a-c)\csc^3(bx+c)}{3b}$$

input
```
Int[Csc[c + b*x]^4*Sin[a + b*x],x]
```

output
```
Cos[a - c]*(-1/2*ArcTanh[Cos[c + b*x]]/b - (Cot[c + b*x]*Csc[c + b*x])/(2*
b)) - (Csc[c + b*x]^3*Sin[a - c])/(3*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.41 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.82

method	result
risch	$\frac{-3\,\mathrm{e}^{i(5bx+7a+4c)}-3\,\mathrm{e}^{i(5bx+5a+6c)}-8\,\mathrm{e}^{i(3bx+7a+2c)}+8\,\mathrm{e}^{i(3bx+5a+4c)}+3\,\mathrm{e}^{i(bx+7a)}+3\,\mathrm{e}^{i(bx+5a+2c)}}{6b\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{3}}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}+\mathrm{e}^{i(a-c)}\right)\cos(a}{2b}$
default	Expression too large to display

input
```
int(csc(b*x+c)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^3*(-3*exp(I*(5*b*x+7*a+4*c))-3*exp(
I*(5*b*x+5*a+6*c))-8*exp(I*(3*b*x+7*a+2*c))+8*exp(I*(3*b*x+5*a+4*c))+3*exp
(I*(b*x+7*a))+3*exp(I*(b*x+5*a+2*c)))-1/2*ln(exp(I*(b*x+a))+exp(I*(a-c)))/
b*cos(a-c)+1/2*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*cos(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(61) = 122.

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.10

$$\int \csc^4(c+bx)\sin(a+bx)\,dx$$
$$=\frac{6\cos(bx+c)\cos(-a+c)\sin(bx+c)-3\left(\cos(bx+c)^2\cos(-a+c)-\cos(-a+c)\right)\log\left(\frac{1}{2}\cos(bx+c\right.}{12\left(b\cos\right.}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(6*cos(b*x + c)*cos(-a + c)*sin(b*x + c) - 3*(cos(b*x + c)^2*cos(-a +
 c) - cos(-a + c))*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) + 3*(cos(b*x +
 c)^2*cos(-a + c) - cos(-a + c))*log(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c)
 - 4*sin(-a + c))/((b*cos(b*x + c)^2 - b)*sin(b*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**4*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1773 vs. $2(61) = 122$.

Time = 0.09 (sec) , antiderivative size = 1773, normalized size of antiderivative = 26.46

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/12*(2*(3*cos(5*b*x + 2*a + 4*c) + 3*cos(5*b*x + 6*c) + 8*cos(3*b*x + 2*a
 + 2*c) - 8*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(6*
b*x + a + 6*c) - 6*(3*cos(4*b*x + a + 4*c) - 3*cos(2*b*x + a + 2*c) + cos(
a))*cos(5*b*x + 2*a + 4*c) - 6*(3*cos(4*b*x + a + 4*c) - 3*cos(2*b*x + a +
 2*c) + cos(a))*cos(5*b*x + 6*c) - 6*(8*cos(3*b*x + 2*a + 2*c) - 8*cos(3*b
*x + 4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) + 16
*(3*cos(2*b*x + a + 2*c) - cos(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*cos(2*b*
x + a + 2*c) - cos(a))*cos(3*b*x + 4*c) - 18*(cos(b*x + 2*a) + cos(b*x + 2
*c))*cos(2*b*x + a + 2*c) + 6*cos(b*x + 2*a)*cos(a) + 6*cos(b*x + 2*c)*cos
(a) - 3*(cos(6*b*x + a + 6*c)^2*cos(-a + c) + 9*cos(4*b*x + a + 4*c)^2*cos
(-a + c) + 9*cos(2*b*x + a + 2*c)^2*cos(-a + c) - 6*cos(2*b*x + a + 2*c)*c
os(a)*cos(-a + c) + cos(-a + c)*sin(6*b*x + a + 6*c)^2 + 9*cos(-a + c)*sin
(4*b*x + a + 4*c)^2 + 9*cos(-a + c)*sin(2*b*x + a + 2*c)^2 - 6*cos(-a + c)
*sin(2*b*x + a + 2*c)*sin(a) - 2*(3*cos(4*b*x + a + 4*c)*cos(-a + c) - 3*c
os(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(-a + c))*cos(6*b*x + a + 6*c)
 - 6*(3*cos(2*b*x + a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(4*b*x +
 a + 4*c) + (cos(a)^2 + sin(a)^2)*cos(-a + c) - 2*(3*cos(-a + c)*sin(4*b*x
 + a + 4*c) - 3*cos(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)*sin(a))*sin
(6*b*x + a + 6*c) - 6*(3*cos(-a + c)*sin(2*b*x + a + 2*c) - cos(-a + c)*si
n(a))*sin(4*b*x + a + 4*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2221 vs. $2(61) = 122$.

Time = 0.16 (sec) , antiderivative size = 2221, normalized size of antiderivative = 33.15

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

output
```
1/24*(12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*
c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(1/
2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (2*tan(1/2*b*x + 1/2*c)^3*tan(
1/2*a)^6*tan(1/2*c)^5 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^6
 - 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^6 + 4*tan(1/2*b*x + 1/
2*c)^3*tan(1/2*a)^6*tan(1/2*c)^3 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5*t
an(1/2*c)^4 - 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^4 + 2*tan(1
/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^5 - 12*tan(1/2*b*x + 1/2*c)^2*ta
n(1/2*a)^5*tan(1/2*c)^5 + 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^6*tan(1/2*c)^5
 - 4*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^6 - 3*tan(1/2*b*x + 1/
2*c)^2*tan(1/2*a)^4*tan(1/2*c)^6 - 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^5*tan
(1/2*c)^6 + 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^6*tan(1/2*c) + 2*tan(1/2*b
*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^2 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2
*a)^6*tan(1/2*c)^2 + 4*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^3 -
24*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^5*tan(1/2*c)^3 + 12*tan(1/2*b*x + 1/2
*c)*tan(1/2*a)^6*tan(1/2*c)^3 - 4*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^3*tan(
1/2*c)^4 - 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*
b*x + 1/2*c)*tan(1/2*a)^5*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*
a)^2*tan(1/2*c)^5 - 24*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c)^5 +
6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^5 - 2*tan(1/2*b*x + 1/2*...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/sin(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^4(c+bx)\sin(a+bx)\,dx$$

$$= \frac{-2\cos(bx+c)\cos(bx+a)\sin(bx+c) - 4\cos(bx+c)\sin(bx+c)^2\sin(bx+a) - 2\cos(bx+c)\sin(bx}{}$$

input
```
int(csc(b*x+c)^4*sin(b*x+a),x)
```

output
```
( - 2*cos(b*x + c)*cos(a + b*x)*sin(b*x + c) - 4*cos(b*x + c)*sin(b*x + c)
**2*sin(a + b*x) - 2*cos(b*x + c)*sin(b*x + c) - 4*cos(b*x + c)*sin(a + b*
x) - 4*cos(a + b*x)*sin(b*x + c) - 6*int(tan((b*x + c)/2)/(tan((a + b*x)/2
)**2 + 1),x)*sin(b*x + c)**3*b - 2*int(1/(tan((b*x + c)/2)**3*tan((a + b*x
)/2)**2 + tan((b*x + c)/2)**3),x)*sin(b*x + c)**3*b + 4*log(tan((b*x + c)/
2)**2 + 1)*sin(b*x + c)**3 - 2*log(tan((b*x + c)/2))*sin(b*x + c)**3 + 3*s
in(b*x + c)**3 + 2*sin(b*x + c)**2*sin(a + b*x) - 2*sin(b*x + c))/(12*sin(
b*x + c)**3*b)
```

3.85 $\int \csc^5(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \csc^5(c+bx)\sin(a+bx)\,dx = -\frac{\cos(a-c)\cot(c+bx)}{b}-\frac{\cos(a-c)\cot^3(c+bx)}{3b}-\frac{\csc^4(c+bx)\sin(a-c)}{4b}$$

output
```
-cos(a-c)*cot(b*x+c)/b-1/3*cos(a-c)*cot(b*x+c)^3/b-1/4*csc(b*x+c)^4*sin(a-
c)/b
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \csc^5(c+bx)\sin(a+bx)\,dx = \frac{(3\cos(a)+\cos(a-c)(-4\cos(c+2bx)+\cos(3c+4bx)))\csc\left(\frac{c}{2}\right)\csc^4(c+bx)\sec\left(\frac{c}{2}\right)}{24b}$$

input
```
Integrate[Csc[c + b*x]^5*Sin[a + b*x],x]
```

output
```
((3*Cos[a] + Cos[a - c]*(-4*Cos[c + 2*b*x] + Cos[3*c + 4*b*x]))*Csc[c/2]*C
sc[c + b*x]^4*Sec[c/2])/(24*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5093, 3042, 25, 3086, 15, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^5(bx+c)\,dx$$

$$\Big\downarrow \text{5093}$$

$$\cos(a-c)\int \csc^4(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc^4(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)^4dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^4\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\cos(a-c)\int \csc(c+bx)^4dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^4\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc(c+bx)^4dx-\frac{\sin(a-c)\int \csc^3(c+bx)d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(a-c)\int \csc(c+bx)^4dx-\frac{\sin(a-c)\csc^4(bx+c)}{4b}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\cos(a-c)\int \left(\cot^2(c+bx)+1\right)d\cot(c+bx)}{b}-\frac{\sin(a-c)\csc^4(bx+c)}{4b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(a-c)\left(\frac{1}{3}\cot^3(bx+c)+\cot(bx+c)\right)}{b}-\frac{\sin(a-c)\csc^4(bx+c)}{4b}$$

input
```
Int[Csc[c + b*x]^5*Sin[a + b*x],x]
```

output
```
-((Cos[a - c]*(Cot[c + b*x] + Cot[c + b*x]^3/3))/b) - (Csc[c + b*x]^4*Sin[
a - c])/(4*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.58 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

method	result
risch	$\frac{2i\left(6\,\mathrm{e}^{i(4bx+9a+3c)}-4\,\mathrm{e}^{i(2bx+9a+c)}-4\,\mathrm{e}^{i(2bx+7a+3c)}+\mathrm{e}^{i(9a-c)}+\mathrm{e}^{i(7a+c)}\right)}{3\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{4}b}$
parallelrisch	$\left(\cot\left(\frac{bx}{2}+\frac{c}{2}\right)^{3}+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^{3}+19\cot\left(\frac{bx}{2}+\frac{c}{2}\right)+19\tan\left(\frac{bx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+\left(-3\cot\left(\frac{bx}{2}+\frac{c}{2}\right)^{4}+3\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^{4}-22\,\mathrm{c}\right.$ … / $96\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right.$ …
default	$-\frac{-3\cos(a)\sin(c)+3\sin(a)\cos(c)}{2(\cos(a)\cos(c)+\sin(a)\sin(c))^{4}(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))^{2}}+\frac{(-\sin(a)\cos(c)+\mathrm{c}}{4(\cos(a)\cos(c)+\sin(a)\,\mathrm{si}}$ …

input
```
int(csc(b*x+c)^5*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^4/b*(6*exp(I*(4*b*x+9*a+3*c))-4*exp
(I*(2*b*x+9*a+c))-4*exp(I*(2*b*x+7*a+3*c))+exp(I*(9*a-c))+exp(I*(7*a+c)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.25

$$\int \csc^5(c+bx)\sin(a+bx)\,dx = \frac{4\left(2\,\cos\left(bx+c\right)^{3}\cos\left(-a+c\right)-3\,\cos\left(bx+c\right)\cos\left(-a+c\right)\right)\sin\left(bx+c\right)+3\,\sin\left(-a+c\right)}{12\left(b\cos\left(bx+c\right)^{4}-2\,b\cos\left(bx+c\right)^{2}+b\right)}$$

input
```
integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(4*(2*cos(b*x + c)^3*cos(-a + c) - 3*cos(b*x + c)*cos(-a + c))*sin(b*
x + c) + 3*sin(-a + c))/(b*cos(b*x + c)^4 - 2*b*cos(b*x + c)^2 + b)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**5*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(56) = 112$.

Time = 0.05 (sec) , antiderivative size = 1076, normalized size of antiderivative = 17.93

$$\int \csc^5(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="maxima")
```

output
```
-2/3*((6*sin(4*b*x + 2*a + 4*c) - 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x +
 4*c) + sin(2*a) + sin(2*c))*cos(8*b*x + a + 9*c) - 4*(6*sin(4*b*x + 2*a +
 4*c) - 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) + sin(2*c
))*cos(6*b*x + a + 7*c) + 6*(4*sin(2*b*x + a + 3*c) - sin(a + c))*cos(4*b*
x + 2*a + 4*c) + 6*(6*sin(4*b*x + 2*a + 4*c) - 4*sin(2*b*x + 2*a + 2*c) -
4*sin(2*b*x + 4*c) + sin(2*a) + sin(2*c))*cos(4*b*x + a + 5*c) + 4*(4*sin(
2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(2*b*x + a + 3*c) - 4*(4*sin(
2*b*x + a + 3*c) - sin(a + c))*cos(2*b*x + 4*c) + (sin(2*a) + sin(2*c))*co
s(a + c) - (6*cos(4*b*x + 2*a + 4*c) - 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*
b*x + 4*c) + cos(2*a) + cos(2*c))*sin(8*b*x + a + 9*c) + 4*(6*cos(4*b*x +
2*a + 4*c) - 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4*c) + cos(2*a) + co
s(2*c))*sin(6*b*x + a + 7*c) - 6*(4*cos(2*b*x + a + 3*c) - cos(a + c))*sin
(4*b*x + 2*a + 4*c) - 6*(6*cos(4*b*x + 2*a + 4*c) - 4*cos(2*b*x + 2*a + 2*
c) - 4*cos(2*b*x + 4*c) + cos(2*a) + cos(2*c))*sin(4*b*x + a + 5*c) - 4*co
s(a + c)*sin(2*b*x + 2*a + 2*c) - 4*(4*cos(2*b*x + 2*a + 2*c) - cos(2*a) -
 cos(2*c))*sin(2*b*x + a + 3*c) + 4*(4*cos(2*b*x + a + 3*c) - cos(a + c))*
sin(2*b*x + 4*c) - (cos(2*a) + cos(2*c))*sin(a + c) + 4*cos(2*b*x + 2*a +
2*c)*sin(a + c))/(b*cos(8*b*x + a + 9*c)^2 + 16*b*cos(6*b*x + a + 7*c)^2 +
 36*b*cos(4*b*x + a + 5*c)^2 + 16*b*cos(2*b*x + a + 3*c)^2 - 8*b*cos(2*b*x
 + a + 3*c)*cos(a + c) + b*cos(a + c)^2 + b*sin(8*b*x + a + 9*c)^2 + 16...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(56) = 112$.

Time = 0.15 (sec) , antiderivative size = 301, normalized size of antiderivative = 5.02

$$\int \csc^5(c+bx)\sin(a+bx)\,dx =$$

$$-\frac{6\,\tan(bx+c)^3\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2 - 6\,\tan(bx+c)^3\tan\left(\frac{1}{2}\,a\right)^2 + 24\,\tan(bx+c)^3\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)}{}$$

input
```
integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/6*(6*tan(b*x + c)^3*tan(1/2*a)^2*tan(1/2*c)^2 - 6*tan(b*x + c)^3*tan(1/
2*a)^2 + 24*tan(b*x + c)^3*tan(1/2*a)*tan(1/2*c) + 6*tan(b*x + c)^2*tan(1/
2*a)^2*tan(1/2*c) - 6*tan(b*x + c)^3*tan(1/2*c)^2 - 6*tan(b*x + c)^2*tan(1
/2*a)*tan(1/2*c)^2 + 2*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(b*x
+ c)^3 + 6*tan(b*x + c)^2*tan(1/2*a) - 2*tan(b*x + c)*tan(1/2*a)^2 - 6*tan
(b*x + c)^2*tan(1/2*c) + 8*tan(b*x + c)*tan(1/2*a)*tan(1/2*c) + 3*tan(1/2*
a)^2*tan(1/2*c) - 2*tan(b*x + c)*tan(1/2*c)^2 - 3*tan(1/2*a)*tan(1/2*c)^2
+ 2*tan(b*x + c) + 3*tan(1/2*a) - 3*tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^
2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b*tan(b*x + c)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/sin(c + b*x)^5,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \csc^5(c+bx)\sin(a+bx)\,dx$$
$$= \frac{-4\cos(bx+c)\sin(bx+c)^2\sin(bx+a) - 3\cos(bx+c)\sin(bx+a) - 4\cos(bx+a)\sin(bx+c)^3 - \cos}{12\sin(bx+c)^4 b}$$

input
```
int(csc(b*x+c)^5*sin(b*x+a),x)
```

output
```
( - 4*cos(b*x + c)*sin(b*x + c)**2*sin(a + b*x) - 3*cos(b*x + c)*sin(a + b
*x) - 4*cos(a + b*x)*sin(b*x + c)**3 - cos(a + b*x)*sin(b*x + c))/(12*sin(
b*x + c)**4*b)
```

3.86 $\int \csc^6(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = -\frac{3\text{arctanh}(\cos(c+bx))\cos(a-c)}{8b} - \frac{3\cos(a-c)\cot(c+bx)\csc(c+bx)}{8b} - \frac{\cos(a-c)\cot(c+bx)\csc^3(c+bx)}{4b} - \frac{\csc^5(c+bx)\sin(a-c)}{5b}$$

output
```
-3/8*arctanh(cos(b*x+c))*cos(a-c)/b-3/8*cos(a-c)*cot(b*x+c)*csc(b*x+c)/b-1
/4*cos(a-c)*cot(b*x+c)*csc(b*x+c)^3/b-1/5*csc(b*x+c)^5*sin(a-c)/b
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = -\frac{480\text{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+2\csc^5(c+bx)(64\sin(a-c)+5\cos(a-c)(14\sin(2}{640b}$$

input
```
Integrate[Csc[c + b*x]^6*Sin[a + b*x],x]
```

output
```
-1/640*(480*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + 2*Csc[c + b
*x]^5*(64*Sin[a - c] + 5*Cos[a - c]*(14*Sin[2*(c + b*x)] - 3*Sin[4*(c + b*
x)])))/b
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5093, 3042, 25, 3086, 15, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^6(bx+c)\,dx$$

$$\Big\downarrow \ 5093$$

$$\cos(a-c)\int \csc^5(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc^5(c+bx)dx$$

$$\Big\downarrow \ 3042$$

$$\cos(a-c)\int \csc(c+bx)^5dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^5\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \ 25$$

$$\cos(a-c)\int \csc(c+bx)^5dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^5\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \ 3086$$

$$\cos(a-c)\int \csc(c+bx)^5dx-\frac{\sin(a-c)\int \csc^4(c+bx)d\csc(c+bx)}{b}$$

$$\Big\downarrow \ 15$$

$$\cos(a-c)\int \csc(c+bx)^5dx-\frac{\sin(a-c)\csc^5(bx+c)}{5b}$$

$$\Big\downarrow \ 4255$$

$$\cos(a-c)\left(\frac{3}{4}\int \csc^3(c+bx)dx-\frac{\cot(bx+c)\csc^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\csc^5(bx+c)}{5b}$$

$$\Big\downarrow \ 3042$$

$$\cos(a-c)\left(\frac{3}{4}\int \csc(c+bx)^3dx-\frac{\cot(bx+c)\csc^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\csc^5(bx+c)}{5b}$$

$$\Big\downarrow \ 4255$$

$$\cos(a-c)\left(\frac{3}{4}\left(\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cot(bx+c)\csc^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\csc^5(bx+c)}{5b}$$

$$\Big\downarrow \ 3042$$

$$\cos(a-c)\left(\frac{3}{4}\left(\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cot(bx+c)\csc^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\csc^5(bx+c)}{5b}$$

$$\Big\downarrow \ 4257$$

$$\cos(a-c)\left(\frac{3}{4}\left(-\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cot(bx+c)\csc^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\csc^5(bx+c)}{5b}$$

input
```
Int[Csc[c + b*x]^6*Sin[a + b*x],x]
```

output
```
Cos[a - c]*(-1/4*(Cot[c + b*x]*Csc[c + b*x]^3)/b + (3*(-1/2*ArcTanh[Cos[c
+ b*x]]/b - (Cot[c + b*x]*Csc[c + b*x])/(2*b)))/4) - (Csc[c + b*x]^5*Sin[a
 - c])/(5*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 19.34 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.73

method	result
risch	$\frac{-15\,\mathrm{e}^{i(9bx+11a+8c)}-15\,\mathrm{e}^{i(9bx+9a+10c)}+70\,\mathrm{e}^{i(7bx+11a+6c)}+70\,\mathrm{e}^{i(7bx+9a+8c)}+128\,\mathrm{e}^{i(5bx+11a+4c)}-128\,\mathrm{e}^{i(5bx+9a+6c)}-70\,\mathrm{e}^{i(3bx+1}}{40b\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{5}}$
default	Expression too large to display

input
```
int(csc(b*x+c)^6*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/40/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^5*(-15*exp(I*(9*b*x+11*a+8*c))-15*
exp(I*(9*b*x+9*a+10*c))+70*exp(I*(7*b*x+11*a+6*c))+70*exp(I*(7*b*x+9*a+8*c
))+128*exp(I*(5*b*x+11*a+4*c))-128*exp(I*(5*b*x+9*a+6*c))-70*exp(I*(3*b*x+
11*a+2*c))-70*exp(I*(3*b*x+9*a+4*c))+15*exp(I*(b*x+11*a))+15*exp(I*(b*x+9*
a+2*c)))+3/8*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*cos(a-c)-3/8*ln(exp(I*(b*x+
a))+exp(I*(a-c)))/b*cos(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(86) = 172.

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.10

$$\int \csc^6(c+bx)\sin(a+bx)\,dx =$$

$$-\frac{15\left(\cos\left(bx+c\right)^{4}\cos\left(-a+c\right)-2\,\cos\left(bx+c\right)^{2}\cos\left(-a+c\right)+\cos\left(-a+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(bx+c\right)+\frac{1}{2}\right)\,\mathrm{s}}{}$$

input
```
integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/80*(15*(cos(b*x + c)^4*cos(-a + c) - 2*cos(b*x + c)^2*cos(-a + c) + cos
(-a + c))*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - 15*(cos(b*x + c)^4*co
s(-a + c) - 2*cos(b*x + c)^2*cos(-a + c) + cos(-a + c))*log(-1/2*cos(b*x +
 c) + 1/2)*sin(b*x + c) - 10*(3*cos(b*x + c)^3*cos(-a + c) - 5*cos(b*x + c
)*cos(-a + c))*sin(b*x + c) - 16*sin(-a + c))/((b*cos(b*x + c)^4 - 2*b*cos
(b*x + c)^2 + b)*sin(b*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**6*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3879 vs. $2(86) = 172$.

Time $= 0.21$ (sec) , antiderivative size $= 3879$, normalized size of antiderivative $= 41.27$

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/80*(2*(15*cos(9*b*x + 2*a + 8*c) + 15*cos(9*b*x + 10*c) - 70*cos(7*b*x +
 2*a + 6*c) - 70*cos(7*b*x + 8*c) - 128*cos(5*b*x + 2*a + 4*c) + 128*cos(5
*b*x + 6*c) + 70*cos(3*b*x + 2*a + 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*x
 + 2*a) - 15*cos(b*x + 2*c))*cos(10*b*x + a + 10*c) - 30*(5*cos(8*b*x + a
+ 8*c) - 10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) - 5*cos(2*b*x +
 a + 2*c) + cos(a))*cos(9*b*x + 2*a + 8*c) - 30*(5*cos(8*b*x + a + 8*c) -
10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) - 5*cos(2*b*x + a + 2*c)
 + cos(a))*cos(9*b*x + 10*c) + 10*(70*cos(7*b*x + 2*a + 6*c) + 70*cos(7*b*
x + 8*c) + 128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*x + 6*c) - 70*cos(3*b*
x + 2*a + 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x + 2*a) + 15*cos(b*x + 2*
c))*cos(8*b*x + a + 8*c) - 140*(10*cos(6*b*x + a + 6*c) - 10*cos(4*b*x + a
 + 4*c) + 5*cos(2*b*x + a + 2*c) - cos(a))*cos(7*b*x + 2*a + 6*c) - 140*(1
0*cos(6*b*x + a + 6*c) - 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c)
- cos(a))*cos(7*b*x + 8*c) - 20*(128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*
x + 6*c) - 70*cos(3*b*x + 2*a + 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x +
2*a) + 15*cos(b*x + 2*c))*cos(6*b*x + a + 6*c) + 256*(10*cos(4*b*x + a + 4
*c) - 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 2*a + 4*c) - 256*(10*co
s(4*b*x + a + 4*c) - 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 6*c) - 1
00*(14*cos(3*b*x + 2*a + 2*c) + 14*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) - 3
*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) + 140*(5*cos(2*b*x + a + 2*c) - c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8035 vs. $2(86) = 172$.

Time = 0.19 (sec) , antiderivative size = 8035, normalized size of antiderivative = 85.48

$$\int \csc^6(c + bx) \sin(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="giac")
```

output
```
1/320*(120*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/
2*c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(
1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (4*tan(1/2*b*x + 1/2*c)^5*ta
n(1/2*a)^10*tan(1/2*c)^9 - 4*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^9*tan(1/2*c
)^10 - 5*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^10*tan(1/2*c)^10 + 16*tan(1/2*b
*x + 1/2*c)^5*tan(1/2*a)^10*tan(1/2*c)^7 - 12*tan(1/2*b*x + 1/2*c)^5*tan(1
/2*a)^9*tan(1/2*c)^8 - 15*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^10*tan(1/2*c)^
8 + 12*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^8*tan(1/2*c)^9 - 20*tan(1/2*b*x +
 1/2*c)^4*tan(1/2*a)^9*tan(1/2*c)^9 + 20*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)
^10*tan(1/2*c)^9 - 16*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^7*tan(1/2*c)^10 -
15*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^8*tan(1/2*c)^10 - 20*tan(1/2*b*x + 1/
2*c)^3*tan(1/2*a)^9*tan(1/2*c)^10 - 40*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^1
0*tan(1/2*c)^10 + 24*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^10*tan(1/2*c)^5 - 8
*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^9*tan(1/2*c)^6 - 10*tan(1/2*b*x + 1/2*c
)^4*tan(1/2*a)^10*tan(1/2*c)^6 + 48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^8*ta
n(1/2*c)^7 - 80*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^9*tan(1/2*c)^7 + 80*tan(
1/2*b*x + 1/2*c)^3*tan(1/2*a)^10*tan(1/2*c)^7 - 48*tan(1/2*b*x + 1/2*c)^5*
tan(1/2*a)^7*tan(1/2*c)^8 - 45*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^8*tan(1/2
*c)^8 - 60*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^9*tan(1/2*c)^8 - 120*tan(1/2*
b*x + 1/2*c)^2*tan(1/2*a)^10*tan(1/2*c)^8 + 8*tan(1/2*b*x + 1/2*c)^5*ta...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/sin(c + b*x)^6,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^6(c+bx)\sin(a+bx)\,dx$$

$$= \frac{-28\cos(bx+c)\cos(bx+a)\sin(bx+c)^3+24\cos(bx+c)\cos(bx+a)\sin(bx+c)-36\cos(bx+c)\sin$$

input
```
int(csc(b*x+c)^6*sin(b*x+a),x)
```

output
```
( - 28*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)**3 + 24*cos(b*x + c)*cos(a +
 b*x)*sin(b*x + c) - 36*cos(b*x + c)*sin(b*x + c)**4*sin(a + b*x) - 24*cos
(b*x + c)*sin(b*x + c)**3 - 48*cos(b*x + c)*sin(b*x + c)**2*sin(a + b*x) +
 24*cos(b*x + c)*sin(b*x + c) - 32*cos(b*x + c)*sin(a + b*x) - 60*cos(a +
b*x)*sin(b*x + c)**3 + 16*cos(a + b*x)*sin(b*x + c) - 60*int(tan((b*x + c)
/2)/(tan((a + b*x)/2)**2 + 1),x)*sin(b*x + c)**5*b + 12*int(1/(tan((b*x +
c)/2)**5*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**5),x)*sin(b*x + c)**5*b +
 36*log(tan((b*x + c)/2)**2 + 1)*sin(b*x + c)**5 - 12*log(tan((b*x + c)/2)
)*sin(b*x + c)**5 + 33*sin(b*x + c)**5 + 28*sin(b*x + c)**4*sin(a + b*x) -
 36*sin(b*x + c)**3 - 8*sin(b*x + c)**2*sin(a + b*x) + 24*sin(b*x + c))/(1
60*sin(b*x + c)**5*b)
```

3.87 $\int \sin^2(a+bx)\sin^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 103

$$\int \sin^2(a+bx)\sin^3(c+bx)\,dx = -\frac{\cos(2a-3c-bx)}{16b}-\frac{3\cos(2a-c+bx)}{16b}-\frac{3\cos(c+bx)}{8b}+\frac{\cos(2a+c+3bx)}{16b}+\frac{\cos(3c+3bx)}{24b}-\frac{\cos(2a+3c+5bx)}{80b}$$

output
```
-1/16*cos(-b*x+2*a-3*c)/b-3/16*cos(b*x+2*a-c)/b-3/8*cos(b*x+c)/b+1/16*cos(
3*b*x+2*a+c)/b+1/24*cos(3*b*x+3*c)/b-1/80*cos(5*b*x+2*a+3*c)/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

$$\int \sin^2(a+bx)\sin^3(c+bx)\,dx = -\frac{15\cos(2a-3c-bx)+45\cos(2a-c+bx)+90\cos(c+bx)-10\cos(3(c+bx))-15\cos(2a+c+3b}{240b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[c + b*x]^3,x]
```

output
```
-1/240*(15*Cos[2*a - 3*c - b*x] + 45*Cos[2*a - c + b*x] + 90*Cos[c + b*x]
- 10*Cos[3*(c + b*x)] - 15*Cos[2*a + c + 3*b*x] + 3*Cos[2*a + 3*c + 5*b*x]
)/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^3(bx+c)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{1}{16}\sin(2a-bx-3c)+\frac{3}{16}\sin(2a+bx-c)-\frac{3}{16}\sin(2a+3bx+c)+\frac{1}{16}\sin(2a+5bx+3c)+\frac{3}{8}\sin(bx+c\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(2a-bx-3c)}{16b}-\frac{3\cos(2a+bx-c)}{16b}+\frac{\cos(2a+3bx+c)}{16b}-\frac{\cos(2a+5bx+3c)}{80b}-\frac{3\cos(bx+c)}{8b}+\frac{\cos(3bx+3c)}{24b}$$

input
```
Int[Sin[a + b*x]^2*Sin[c + b*x]^3,x]
```

output
```
-1/16*Cos[2*a - 3*c - b*x]/b - (3*Cos[2*a - c + b*x])/(16*b) - (3*Cos[c +
b*x])/(8*b) + Cos[2*a + c + 3*b*x]/(16*b) + Cos[3*c + 3*b*x]/(24*b) - Cos[
2*a + 3*c + 5*b*x]/(80*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 3.82 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\cos(-bx+2a-3c)}{16b}-\frac{3\cos(bx+2a-c)}{16b}-\frac{3\cos(bx+c)}{8b}+\frac{\cos(3bx+2a+c)}{16b}+\frac{\cos(3bx+3c)}{24b}-\frac{\cos(5bx+2a+3c)}{80b}$
risch	$-\frac{\cos(-bx+2a-3c)}{16b}-\frac{3\cos(bx+2a-c)}{16b}-\frac{3\cos(bx+c)}{8b}+\frac{\cos(3bx+2a+c)}{16b}+\frac{\cos(3bx+3c)}{24b}-\frac{\cos(5bx+2a+3c)}{80b}$
parallelrisch	$\frac{-\frac{16}{15}+\frac{16\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^6}{15}-\frac{64\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^5\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{15}+\frac{16\left(6\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+1\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^4}{15}+\frac{64\left(-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\mathrm{ta}}{15}}{b\left(1+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\right)^3}$
orering	$-\frac{259\left(2\sin(bx+a)\sin(bx+c)^3b\cos(bx+a)+3\sin(bx+a)^2\sin(bx+c)^2b\cos(bx+c)\right)}{225b^2}-\frac{7\left(-26b^3\cos(bx+a)\sin(bx+c)^3\sin(ba\right.}{}$

input
```
int(sin(b*x+a)^2*sin(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*cos(-b*x+2*a-3*c)/b-3/16*cos(b*x+2*a-c)/b-3/8*cos(b*x+c)/b+1/16*cos(
3*b*x+2*a+c)/b+1/24*cos(3*b*x+3*c)/b-1/80*cos(5*b*x+2*a+3*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int \sin^2(a+bx)\sin^3(c+bx)\,dx =$$

$$-\frac{3\left(2\cos\left(-a+c\right)^2-1\right)\cos\left(bx+c\right)^5-5\left(3\cos\left(-a+c\right)^2-1\right)\cos\left(bx+c\right)^3+15\cos\left(bx+c\right)\cos\left(-a\right.}{}$$

input
```
integrate(sin(b*x+a)^2*sin(b*x+c)^3,x, algorithm="fricas")
```

output
```
-1/15*(3*(2*cos(-a + c)^2 - 1)*cos(b*x + c)^5 - 5*(3*cos(-a + c)^2 - 1)*co
s(b*x + c)^3 + 15*cos(b*x + c)*cos(-a + c)^2 + 6*(cos(b*x + c)^4*cos(-a +
c) - 2*cos(b*x + c)^2*cos(-a + c) + cos(-a + c))*sin(b*x + c)*sin(-a + c))
/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(87) = 174$.

Time = 2.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\int \sin^2(a+bx)\sin^3(c+bx)\,dx$$
$$= \begin{cases} -\frac{3\sin^2(a+bx)\sin^2(bx+c)\cos(bx+c)}{5b} - \frac{2\sin^2(a+bx)\cos^3(bx+c)}{15b} - \frac{2\sin(a+bx)\sin^3(bx+c)\cos(a+bx)}{5b} - \frac{4\sin(a+bx)\sin(bx+c)\cos}{5b} \\ x\sin^2(a)\sin^3(c) \end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(b*x+c)**3,x)
```

output
```
Piecewise((-3*sin(a + b*x)**2*sin(b*x + c)**2*cos(b*x + c)/(5*b) - 2*sin(a
 + b*x)**2*cos(b*x + c)**3/(15*b) - 2*sin(a + b*x)*sin(b*x + c)**3*cos(a +
 b*x)/(5*b) - 4*sin(a + b*x)*sin(b*x + c)*cos(a + b*x)*cos(b*x + c)**2/(5*
b) - 2*sin(b*x + c)**2*cos(a + b*x)**2*cos(b*x + c)/(5*b) - 8*cos(a + b*x)
**2*cos(b*x + c)**3/(15*b), Ne(b, 0)), (x*sin(a)**2*sin(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \sin^2(a+bx)\sin^3(c+bx)\,dx =$$
$$-\frac{3\cos(5bx+2a+3c) - 15\cos(3bx+2a+c) - 10\cos(3bx+3c) + 45\cos(bx+2a-c) + 15\cos}{240b}$$

input
```
integrate(sin(b*x+a)^2*sin(b*x+c)^3,x, algorithm="maxima")
```

output
```
-1/240*(3*cos(5*b*x + 2*a + 3*c) - 15*cos(3*b*x + 2*a + c) - 10*cos(3*b*x
+ 3*c) + 45*cos(b*x + 2*a - c) + 15*cos(b*x - 2*a + 3*c) + 90*cos(b*x + c)
)/b
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \sin^2(a+bx)\sin^3(c+bx)\,dx = -\frac{\cos(5\,bx+2\,a+3\,c)}{80\,b}+\frac{\cos(3\,bx+2\,a+c)}{16\,b}+\frac{\cos(3\,bx+3\,c)}{24\,b}-\frac{3\,\cos(bx+2\,a-c)}{16\,b}-\frac{3\,\cos(bx+c)}{8\,b}-\frac{\cos(-bx+2\,a-3\,c)}{16\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/80*cos(5*b*x + 2*a + 3*c)/b + 1/16*cos(3*b*x + 2*a + c)/b + 1/24*cos(3*
b*x + 3*c)/b - 3/16*cos(b*x + 2*a - c)/b - 3/8*cos(b*x + c)/b - 1/16*cos(-
b*x + 2*a - 3*c)/b
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \sin^2(a+bx)\sin^3(c+bx)\,dx = -\frac{90\,\cos(c+b\,x)-15\,\cos(2\,a+c+3\,b\,x)+45\,\cos(2\,a-c+b\,x)+15\,\cos(3\,c-2\,a+b\,x)+3\,\cos(2}{240\,b}$$

input
```
int(sin(a + b*x)^2*sin(c + b*x)^3,x)
```

output
```
-(90*cos(c + b*x) - 15*cos(2*a + c + 3*b*x) + 45*cos(2*a - c + b*x) + 15*c
os(3*c - 2*a + b*x) + 3*cos(2*a + 3*c + 5*b*x) - 10*cos(3*c + 3*b*x))/(240
*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.37

$$\int \sin^2(a+bx)\sin^3(c+bx)\,dx$$
$$= \frac{-12\cos(bx+c)\cos(bx+a) - 9\cos(bx+c)\sin(bx+c)^2\sin(bx+a)^2 + 2\cos(bx+c)\sin(bx+c)^2 + 6}{}$$

input
```
int(sin(b*x+a)^2*sin(b*x+c)^3,x)
```

output
```
( - 12*cos(b*x + c)*cos(a + b*x) - 9*cos(b*x + c)*sin(b*x + c)**2*sin(a +
b*x)**2 + 2*cos(b*x + c)*sin(b*x + c)**2 + 6*cos(b*x + c)*sin(a + b*x)**2
- 8*cos(b*x + c) + 6*cos(a + b*x)*sin(b*x + c)**3*sin(a + b*x) - 12*cos(a
+ b*x)*sin(b*x + c)*sin(a + b*x) - 12*sin(b*x + c)*sin(a + b*x) - 4)/(15*b
)
```

3.88 $\int \sin^2(a+bx)\sin^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \sin^2(a+bx)\sin^2(c+bx)\,dx = \frac{1}{8}x(2+\cos(2(a-c))) - \frac{\sin(2a+2bx)}{8b} - \frac{\sin(2c+2bx)}{8b} + \frac{\sin(2(a+c)+4bx)}{32b}$$

output
```
1/8*x*(2+cos(2*a-2*c))-1/8*sin(2*b*x+2*a)/b-1/8*sin(2*b*x+2*c)/b+1/32*sin(
4*b*x+2*a+2*c)/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \sin^2(a+bx)\sin^2(c+bx)\,dx = \frac{8bx+4bx\cos(2(a-c))-4\sin(2(a+bx))-4\sin(2(c+bx))+\sin(2(a+c+2bx))}{32b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[c + b*x]^2,x]
```

output
```
(8*b*x + 4*b*x*Cos[2*(a - c)] - 4*Sin[2*(a + b*x)] - 4*Sin[2*(c + b*x)] +
Sin[2*(a + c + 2*b*x)])/(32*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^2(bx+c)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(\frac{1}{8}\cos(2(a+c)+4bx)-\frac{1}{4}\cos(2a+2bx)+\frac{1}{8}\cos(2a-2c)-\frac{1}{4}\cos(2bx+2c)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(2(a+c)+4bx)}{32b}-\frac{\sin(2a+2bx)}{8b}+\frac{1}{8}x(\cos(2(a-c))+2)-\frac{\sin(2bx+2c)}{8b}$$

input
```
Int[Sin[a + b*x]^2*Sin[c + b*x]^2,x]
```

output
```
(x*(2 + Cos[2*(a - c)]))/8 - Sin[2*a + 2*b*x]/(8*b) - Sin[2*c + 2*b*x]/(8*
b) + Sin[2*(a + c) + 4*b*x]/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

method	result
default	$\frac{x}{4}+\frac{x\cos(2a-2c)}{8}-\frac{\sin(2bx+2a)}{8b}-\frac{\sin(2bx+2c)}{8b}+\frac{\sin(4bx+2a+2c)}{32b}$
risch	$\frac{x}{4}+\frac{x\cos(2a-2c)}{8}-\frac{\sin(2bx+2a)}{8b}-\frac{\sin(2bx+2c)}{8b}+\frac{\sin(4bx+2a+2c)}{32b}$
parallelrisch	$\frac{8bx+4bx\cos(2a-2c)+\sin(4bx+2a+2c)+3\sin(2a-2c)-4\sin(2bx+2a)-4\sin(2bx+2c)}{32b}$
orering	$x\sin\left(bx+a\right)^{2}\sin\left(bx+c\right)^{2}-\frac{5\left(2\sin(bx+c)^{2}\cos(bx+a)b\sin(bx+a)+2b\sin(bx+a)^{2}\cos(bx+c)\sin(bx+c)\right)}{16b^{2}}+$

input
```
int(sin(b*x+a)^2*sin(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x+1/8*x*cos(2*a-2*c)-1/8*sin(2*b*x+2*a)/b-1/8*sin(2*b*x+2*c)/b+1/32*si
n(4*b*x+2*a+2*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.68

$$\int \sin^2(a+bx)\sin^2(c+bx)\,dx$$
$$=\frac{2\,bx\cos\left(-a+c\right)^{2}+bx+\left(2\left(2\,\cos\left(-a+c\right)^{2}-1\right)\cos\left(bx+c\right)^{3}-\left(6\,\cos\left(-a+c\right)^{2}-1\right)\cos\left(bx+c\right)\right)\sin}{8\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/8*(2*b*x*cos(-a + c)^2 + b*x + (2*(2*cos(-a + c)^2 - 1)*cos(b*x + c)^3 -
 (6*cos(-a + c)^2 - 1)*cos(b*x + c))*sin(b*x + c) - 4*(cos(b*x + c)^4*cos(
-a + c) - 2*cos(b*x + c)^2*cos(-a + c))*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(56) = 112$.

Time = 0.92 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.09

$$\int \sin^2(a+bx)\sin^2(c+bx)\,dx$$
$$= \begin{cases} \frac{3x\sin^2(a+bx)\sin^2(bx+c)}{8} + \frac{x\sin^2(a+bx)\cos^2(bx+c)}{8} + \frac{x\sin(a+bx)\sin(bx+c)\cos(a+bx)\cos(bx+c)}{2} + \frac{x\sin^2(bx+c)\cos^2(a+bx)}{8} + \\ x\sin^2(a)\sin^2(c) \end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(b*x+c)**2,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**2*sin(b*x + c)**2/8 + x*sin(a + b*x)**2*cos(b
*x + c)**2/8 + x*sin(a + b*x)*sin(b*x + c)*cos(a + b*x)*cos(b*x + c)/2 + x
*sin(b*x + c)**2*cos(a + b*x)**2/8 + 3*x*cos(a + b*x)**2*cos(b*x + c)**2/8
 - sin(a + b*x)**2*sin(b*x + c)*cos(b*x + c)/(8*b) - sin(a + b*x)*sin(b*x
+ c)**2*cos(a + b*x)/(2*b) - 3*sin(b*x + c)*cos(a + b*x)**2*cos(b*x + c)/(
8*b), Ne(b, 0)), (x*sin(a)**2*sin(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \sin^2(a+bx)\sin^2(c+bx)\,dx$$
$$= \frac{4(b\cos(-2a+2c)+2b)x+\sin(4bx+2a+2c)-4\sin(2bx+2a)-4\sin(2bx+2c)}{32b}$$

input
```
integrate(sin(b*x+a)^2*sin(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/32*(4*(b*cos(-2*a + 2*c) + 2*b)*x + sin(4*b*x + 2*a + 2*c) - 4*sin(2*b*x
 + 2*a) - 4*sin(2*b*x + 2*c))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \sin^2(a+bx)\sin^2(c+bx)\,dx = \frac{1}{8}\,x\cos\left(2\,a-2\,c\right)+\frac{1}{4}\,x+\frac{\sin\left(4\,bx+2\,a+2\,c\right)}{32\,b} - \frac{\sin\left(2\,bx+2\,a\right)}{8\,b}-\frac{\sin\left(2\,bx+2\,c\right)}{8\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(b*x+c)^2,x, algorithm="giac")
```

output
```
1/8*x*cos(2*a - 2*c) + 1/4*x + 1/32*sin(4*b*x + 2*a + 2*c)/b - 1/8*sin(2*b
*x + 2*a)/b - 1/8*sin(2*b*x + 2*c)/b
```

Mupad [B] (verification not implemented)

Time = 18.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \sin^2(a+bx)\sin^2(c+bx)\,dx = \frac{\frac{\sin(2\,a+2\,c+4\,b\,x)}{4}-\sin\left(2\,a+2\,b\,x\right)-\sin\left(2\,c+2\,b\,x\right)+2\,b\,x+b\,x\,\cos\left(2\,a-2\,c\right)}{8\,b}$$

input
```
int(sin(a + b*x)^2*sin(c + b*x)^2,x)
```

output
```
(sin(2*a + 2*c + 4*b*x)/4 - sin(2*a + 2*b*x) - sin(2*c + 2*b*x) + 2*b*x +
b*x*cos(2*a - 2*c))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.76

$$\int \sin^2(a+bx)\sin^2(c+bx)\,dx$$
$$= \frac{12\cos(bx+c)\cos(bx+a)\sin(bx+c)\sin(bx+a)\,bx - 10\cos(bx+c)\sin(bx+c)\sin(bx+a)^2 - \cos(b$$

input
```
int(sin(b*x+a)^2*sin(b*x+c)^2,x)
```

output
```
(12*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)*sin(a + b*x)*b*x - 10*cos(b*x +
 c)*sin(b*x + c)*sin(a + b*x)**2 - cos(b*x + c)*sin(b*x + c) + 8*cos(b*x +
 c)*sin(a + b*x) + 4*cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x) - 8*cos(a +
 b*x)*sin(b*x + c) - 8*cos(a + b*x)*sin(a + b*x) + 12*sin(b*x + c)**2*sin(
a + b*x)**2*b*x - 6*sin(b*x + c)**2*b*x - 6*sin(a + b*x)**2*b*x + 9*b*x)/(
24*b)
```

3.89 $\int \sin^2(a+bx)\sin(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \sin^2(a+bx)\sin(c+bx)\,dx = -\frac{\cos(2a-c+bx)}{4b} - \frac{\cos(c+bx)}{2b} + \frac{\cos(2a+c+3bx)}{12b}$$

output
```
-1/4*cos(b*x+2*a-c)/b-1/2*cos(b*x+c)/b+1/12*cos(3*b*x+2*a+c)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \sin^2(a+bx)\sin(c+bx)\,dx = \frac{-3\cos(2a-c+bx) - 6\cos(c+bx) + \cos(2a+c+3bx)}{12b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[c + b*x],x]
```

output
```
(-3*Cos[2*a - c + b*x] - 6*Cos[c + b*x] + Cos[2*a + c + 3*b*x])/(12*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin(bx+c)\,dx$$

$$\Big\downarrow \; 5080$$

$$\int \left(\frac{1}{4}\sin(2a+bx-c)-\frac{1}{4}\sin(2a+3bx+c)+\frac{1}{2}\sin(bx+c)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\cos(2a+bx-c)}{4b}+\frac{\cos(2a+3bx+c)}{12b}-\frac{\cos(bx+c)}{2b}$$

input
```
Int[Sin[a + b*x]^2*Sin[c + b*x],x]
```

output
```
-1/4*Cos[2*a - c + b*x]/b - Cos[c + b*x]/(2*b) + Cos[2*a + c + 3*b*x]/(12*
b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\cos(bx+2a-c)}{4b}-\frac{\cos(bx+c)}{2b}+\frac{\cos(3bx+2a+c)}{12b}$
risch	$-\frac{\cos(bx+2a-c)}{4b}-\frac{\cos(bx+c)}{2b}+\frac{\cos(3bx+2a+c)}{12b}$
parallelrisch	$\frac{-3\cos(bx+2a-c)-8\cos(a-c)+\cos(3bx+2a+c)-6\cos(bx+c)}{12b}$
norman	$\frac{-\frac{4}{3b}+\frac{4\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2}{3b}-\frac{16\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)}{3b}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^2\left(1+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\right)}$
orering	$-\frac{10\left(2\sin(bx+a)\sin(bx+c)b\cos(bx+a)+\sin(bx+a)^2b\cos(bx+c)\right)}{9b^2}-\frac{-14b^3\cos(bx+a)\sin(bx+c)\sin(bx+a)+6b^3\cos(bx+c}{9b^4}$

input
```
int(sin(b*x+a)^2*sin(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(b*x+2*a-c)/b-1/2*cos(b*x+c)/b+1/12*cos(3*b*x+2*a+c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.65

$$\int \sin^2(a+bx)\sin(c+bx)\,dx$$
$$=\frac{\left(2\,\cos\left(-a+c\right)^2-1\right)\cos\left(bx+c\right)^3-3\,\cos\left(bx+c\right)\cos\left(-a+c\right)^2+2\left(\cos\left(bx+c\right)^2\cos\left(-a+c\right)-\cos\left(\right.\right.}{3\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(b*x+c),x, algorithm="fricas")
```

output
```
1/3*((2*cos(-a + c)^2 - 1)*cos(b*x + c)^3 - 3*cos(b*x + c)*cos(-a + c)^2 +
 2*(cos(b*x + c)^2*cos(-a + c) - cos(-a + c))*sin(b*x + c)*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(37) = 74$.

Time = 0.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \sin^2(a+bx)\sin(c+bx)\,dx = \begin{cases} -\frac{\sin^2(a+bx)\cos(bx+c)}{3b} - \frac{2\sin(a+bx)\sin(bx+c)\cos(a+bx)}{3b} - \frac{2\cos^2(a+bx)\cos(bx+c)}{3b} & \text{for } b \neq 0 \\ x\sin^2(a)\sin(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(b*x+c),x)
```

output
```
Piecewise((-sin(a + b*x)**2*cos(b*x + c)/(3*b) - 2*sin(a + b*x)*sin(b*x +
c)*cos(a + b*x)/(3*b) - 2*cos(a + b*x)**2*cos(b*x + c)/(3*b), Ne(b, 0)), (
x*sin(a)**2*sin(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \sin^2(a+bx)\sin(c+bx)\,dx = \frac{\cos(3bx+2a+c) - 3\cos(bx+2a-c) - 6\cos(bx+c)}{12b}$$

input
```
integrate(sin(b*x+a)^2*sin(b*x+c),x, algorithm="maxima")
```

output
```
1/12*(cos(3*b*x + 2*a + c) - 3*cos(b*x + 2*a - c) - 6*cos(b*x + c))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \sin^2(a+bx)\sin(c+bx)\,dx = \frac{\cos(3\,bx+2\,a+c)}{12\,b} - \frac{\cos(bx+2\,a-c)}{4\,b} - \frac{\cos(bx+c)}{2\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(b*x+c),x, algorithm="giac")
```

output
```
1/12*cos(3*b*x + 2*a + c)/b - 1/4*cos(b*x + 2*a - c)/b - 1/2*cos(b*x + c)/
b
```

Mupad [B] (verification not implemented)

Time = 17.91 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \sin^2(a+bx)\sin(c+bx)\,dx = -\frac{6\,\cos(c+b\,x)-\cos(2\,a+c+3\,b\,x)+3\,\cos(2\,a-c+b\,x)}{12\,b}$$

input
```
int(sin(a + b*x)^2*sin(c + b*x),x)
```

output
```
-(6*cos(c + b*x) - cos(2*a + c + 3*b*x) + 3*cos(2*a - c + b*x))/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \sin^2(a+bx)\sin(c+bx)\,dx = \frac{2\cos(bx+c)\cos(bx+a)+\cos(bx+c)\sin(bx+a)^2-2\cos(bx+c)-2\cos(bx+a)\sin(bx+c)\sin(b}{3b}$$

input
```
int(sin(b*x+a)^2*sin(b*x+c),x)
```

output
```
(2*cos(b*x + c)*cos(a + b*x) + cos(b*x + c)*sin(a + b*x)**2 - 2*cos(b*x +
c) - 2*cos(a + b*x)*sin(b*x + c)*sin(a + b*x) + 2*sin(b*x + c)*sin(a + b*x
) + 4)/(3*b)
```

3.90 $\int \csc(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \csc(c+bx)\sin^2(a+bx)\,dx = -\frac{\cos(2a-c+bx)}{b} - \frac{\operatorname{arctanh}(\cos(c+bx))\sin^2(a-c)}{b}$$

output
```
-cos(b*x+2*a-c)/b-arctanh(cos(b*x+c))*sin(a-c)^2/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int \csc(c+bx)\sin^2(a+bx)\,dx = \frac{-\cos(2a-c+bx)+\left(-\log\left(\cos\left(\frac{1}{2}(c+bx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+bx)\right)\right)\right)\sin^2(a-c)}{b}$$

input
```
Integrate[Csc[c + b*x]*Sin[a + b*x]^2,x]
```

output
```
(-Cos[2*a - c + b*x] + (-Log[Cos[(c + b*x)/2]] + Log[Sin[(c + b*x)/2]])*Si
n[a - c]^2)/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin^2(a+bx)\csc(bx+c)dx$$

input
```
Int[Csc[c + b*x]*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.73

method	result
risch	$-\frac{\ln\left(\mathrm{e}^{i(bx+a)}+\mathrm{e}^{i(a-c)}\right)}{2b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+\mathrm{e}^{i(a-c)}\right)\cos(2a-2c)}{2b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}-\mathrm{e}^{i(a-c)}\right)}{2b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-\mathrm{e}^{i(a-c)}\right)\cos(2a-2c)}{2b}-$
default	$\frac{\frac{8(\sin(a)\cos(c)-\cos(a)\sin(c))^2\arctan\left(\frac{2(\sin(a)\cos(c)-\cos(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2\cos(a)\cos(c)+2\sin(a)\sin(c)}{2\sqrt{-\cos(c)^2\sin(a)^2-\cos(a)^2\cos(c)^2-\sin(a)^2\sin(c)^2-\sin(c)^2\cos(a)^2}}\right)}{\left(4\cos(c)^2\sin(a)^2+4\cos(a)^2\cos(c)^2+4\sin(a)^2\sin(c)^2+4\sin(c)^2\cos(a)^2\right)\sqrt{-\cos(c)^2\sin(a)^2-\cos(a)^2\cos(c)^2-\sin(a)^2\sin(c)^2-\sin(c)^2\cos(a)^2}}-}{b}$

input
```
int(csc(b*x+c)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))+1/2/b*ln(exp(I*(b*x+a))+exp(I*(a-c)
))*cos(2*a-2*c)+1/2/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))-1/2/b*ln(exp(I*(b*x+
a))-exp(I*(a-c)))*cos(2*a-2*c)-cos(b*x+2*a-c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(37) = 74.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.46

$$\int \csc(c+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{4\cos(-a+c)\sin(bx+c)\sin(-a+c)+2\left(2\cos(-a+c)^2-1\right)\cos(bx+c)-\left(\cos(-a+c)^2-1\right)\mathrm{l}}{2b}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(4*cos(-a + c)*sin(b*x + c)*sin(-a + c) + 2*(2*cos(-a + c)^2 - 1)*cos
(b*x + c) - (cos(-a + c)^2 - 1)*log(1/2*cos(b*x + c) + 1/2) + (cos(-a + c)
^2 - 1)*log(-1/2*cos(b*x + c) + 1/2))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. 2(29) = 58.

Time = 10.43 (sec) , antiderivative size = 3215, normalized size of antiderivative = 86.89

$$\int \csc(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a)**2,x)
```

output
```
2*Piecewise((0, Eq(b, 0) & Eq(c, 0)), (sin(b*x)/b, Eq(c, 0)), (0, Eq(b, 0)
), (2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*
tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/
2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**3/(b
*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2
 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(c/2) + tan(b*x/2))*t
an(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*t
an(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(
tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)*
*4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b
) - 2*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**
4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(
c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**
3/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2
)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - 1/tan(c
/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 +
 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) +
2*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*t
an(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2
)**2 + b) - 2*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*t...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(37) = 74$.

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.19

$$\int \csc(c+bx)\sin^2(a+bx)\,dx$$
$$= \frac{(\cos(-2a+2c)-1)\log\left(\cos(bx)^2+2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2-2\sin(bx)\sin(c)+\sin(c\right.}{}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*((cos(-2*a + 2*c) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 +
 sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(-2*a + 2*c) - 1)*log(co
s(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) +
 sin(c)^2) - 4*cos(b*x + 2*a - c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(37) = 74$.

Time = 0.15 (sec) , antiderivative size = 688, normalized size of antiderivative = 18.59

$$\int \csc(c + bx) \sin^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
2*(2*(tan(1/2*a)^4*tan(1/2*c)^2 - 2*tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)
^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2*a)^2*tan(1/2*c)^2
+ 2*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2 - 2*tan(1/2*a)*tan(1/2*c) + tan
(1/2*c)^2)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*t
an(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*
tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2
 + 1) + (4*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^3 - 4*tan(1/2*b*x
+ 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c)^4 - 4*tan(1/2
*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c) + 24*tan(1/2*b*x + 1/2*c)*tan(1/2*a)
^3*tan(1/2*c)^2 + 6*tan(1/2*a)^4*tan(1/2*c)^2 - 24*tan(1/2*b*x + 1/2*c)*ta
n(1/2*a)^2*tan(1/2*c)^3 - 16*tan(1/2*a)^3*tan(1/2*c)^3 + 4*tan(1/2*b*x + 1
/2*c)*tan(1/2*a)*tan(1/2*c)^4 + 6*tan(1/2*a)^2*tan(1/2*c)^4 - 4*tan(1/2*b*
x + 1/2*c)*tan(1/2*a)^3 - tan(1/2*a)^4 + 24*tan(1/2*b*x + 1/2*c)*tan(1/2*a
)^2*tan(1/2*c) + 16*tan(1/2*a)^3*tan(1/2*c) - 24*tan(1/2*b*x + 1/2*c)*tan(
1/2*a)*tan(1/2*c)^2 - 36*tan(1/2*a)^2*tan(1/2*c)^2 + 4*tan(1/2*b*x + 1/2*c
)*tan(1/2*c)^3 + 16*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + 4*tan(1/2*b*x
 + 1/2*c)*tan(1/2*a) + 6*tan(1/2*a)^2 - 4*tan(1/2*b*x + 1/2*c)*tan(1/2*c)
- 16*tan(1/2*a)*tan(1/2*c) + 6*tan(1/2*c)^2 - 1)/((tan(1/2*a)^4*tan(1/2*c)
^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a
)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*t...
```

Mupad [B] (verification not implemented)

Time = 18.44 (sec) , antiderivative size = 223, normalized size of antiderivative = 6.03

$$\int \csc(c+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{e^{-a\,2i+c\,1i-b\,x\,1i}}{2\,b} - \frac{e^{a\,2i-c\,1i+b\,x\,1i}}{2\,b}$$
$$-\frac{e^{-a\,2i+c\,2i}\,\ln\left(-\frac{\left(e^{a\,2i}\,e^{-c\,2i}-1\right)^2\,1i}{2}+\frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(-e^{a\,2i}\,e^{-c\,2i}\,2i+e^{a\,4i}\,e^{-c\,4i}\,1i+1i\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}-1\right)^2}{4\,b}$$
$$+\frac{e^{-a\,2i+c\,2i}\,\ln\left(\frac{\left(e^{a\,2i}\,e^{-c\,2i}-1\right)^2\,1i}{2}+\frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(-e^{a\,2i}\,e^{-c\,2i}\,2i+e^{a\,4i}\,e^{-c\,4i}\,1i+1i\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}-1\right)^2}{4\,b}$$

input
```
int(sin(a + b*x)^2/sin(c + b*x),x)
```

output
```
(exp(c*2i - a*2i)*log(((exp(a*2i)*exp(-c*2i) - 1)^2*1i)/2 + (exp(c*1i)*exp
(b*x*1i)*(exp(a*4i)*exp(-c*4i)*1i - exp(a*2i)*exp(-c*2i)*2i + 1i))/2)*(exp
(a*2i - c*2i) - 1)^2)/(4*b) - exp(a*2i - c*1i + b*x*1i)/(2*b) - (exp(c*2i
- a*2i)*log((exp(c*1i)*exp(b*x*1i)*(exp(a*4i)*exp(-c*4i)*1i - exp(a*2i)*ex
p(-c*2i)*2i + 1i))/2 - ((exp(a*2i)*exp(-c*2i) - 1)^2*1i)/2)*(exp(a*2i - c*
2i) - 1)^2)/(4*b) - exp(c*1i - a*2i - b*x*1i)/(2*b)
```

Reduce [F]

$$\int \csc(c+bx)\sin^2(a+bx)\,dx = \int \csc(bx+c)\sin(bx+a)^2\,dx$$

input
```
int(csc(b*x+c)*sin(b*x+a)^2,x)
```

output
```
int(csc(b*x + c)*sin(a + b*x)**2,x)
```

3.91 $\int \csc^2(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx = x\cos(2(a-c)) - \frac{\cot(c+bx)\sin^2(a-c)}{b} + \frac{\log(\sin(c+bx))\sin(2(a-c))}{b}$$

output
```
x*cos(2*a-2*c)-cot(b*x+c)*sin(a-c)^2/b+ln(sin(b*x+c))*sin(2*a-2*c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(49) = 98.

Time = 0.44 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.69

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx = -\frac{\csc(c)\csc(c+bx)(bx\cos(2a-4c-bx) - bx\cos(2a-2c-bx) + bx\cos(2a+bx) - bx\cos(2a-2c+$$

input
```
Integrate[Csc[c + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
-1/4*(Csc[c]*Csc[c + b*x]*(b*x*Cos[2*a - 4*c - b*x] - b*x*Cos[2*a - 2*c -
b*x] + b*x*Cos[2*a + b*x] - b*x*Cos[2*a - 2*c + b*x] - 2*Sin[b*x] + Log[Si
n[c + b*x]]*Sin[2*a - 4*c - b*x] - Sin[2*a - 2*c - b*x] - Log[Sin[c + b*x]
]*Sin[2*a - 2*c - b*x] + Log[Sin[c + b*x]]*Sin[2*a + b*x] + Sin[2*a - 2*c
+ b*x] - Log[Sin[c + b*x]]*Sin[2*a - 2*c + b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \sin^2(a+bx)\csc^2(bx+c)dx$$

input
```
Int[Csc[c + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.53

method	result
risch	$x\,\mathrm{e}^{2i(a-c)}-2i\sin\left(2a-2c\right)x-\frac{2i\sin(2a-2c)a}{b}-\frac{i\mathrm{e}^{2i(2a-c)}}{2b\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{i\mathrm{e}^{2ia}}{b\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}-\frac{i\mathrm{e}}{2b\left(-\mathrm{e}^{2i(bx}\right.}$
default	$\frac{\frac{\left(2\cos(a)^{2}\cos(c)\sin(c)-2\cos(c)^{2}\cos(a)\sin(a)+2\cos(a)\sin(a)\sin(c)^{2}-2\sin(a)^{2}\cos(c)\sin(c)\right)\ln\left(\tan(bx+a)^{2}+1\right)}{2}+\left(\cos(a)^{2}\cos(c)^{2}-\sin(c)^{2}\cos(a\right.}{\left(\cos(c)^{2}+\sin(c)^{2}\right)^{2}\left(\cos(a)^{2}+\sin(a)^{2}\right)^{2}}$

input
```
int(csc(b*x+c)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*exp(2*I*(a-c))-2*I*sin(2*a-2*c)*x-2*I/b*sin(2*a-2*c)*a-1/2*I/b/(-exp(2*I
*(b*x+a+c))+exp(2*I*a))*exp(2*I*(2*a-c))+I/b/(-exp(2*I*(b*x+a+c))+exp(2*I*
a))*exp(2*I*a)-1/2*I/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))*exp(2*I*c)+ln(exp(
2*I*(b*x+a))-exp(2*I*(a-c)))/b*sin(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx = -\frac{2\cos\left(-a+c\right)\log\left(\frac{1}{2}\sin\left(bx+c\right)\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-\left(\cos\left(-a+c\right)^{2}-1\right)\cos\left(bx+c\right)-\left(2\,bx\right.}{b\sin\left(bx+c\right)}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*cos(-a + c)*log(1/2*sin(b*x + c))*sin(b*x + c)*sin(-a + c) - (cos(-a +
 c)^2 - 1)*cos(b*x + c) - (2*b*x*cos(-a + c)^2 - b*x)*sin(b*x + c))/(b*sin
(b*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**2*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(49) = 98$.

Time = 0.07 (sec) , antiderivative size = 711, normalized size of antiderivative = 14.51

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(b*cos(2*a + 2*c)*cos(4*c) + b*sin(2*a + 2*c)*sin(4*c))*x - (2*b*x*
cos(4*c) + sin(4*a) - 2*sin(2*a + 2*c) + sin(4*c))*cos(2*b*x + 2*a + 4*c)
+ 2*(b*x*cos(2*b*x + 2*a + 4*c) - b*x*cos(2*a + 2*c))*cos(2*b*x + 6*c) + (
sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(2*b*x + 2*a + 4*c)^2*sin(-2*a +
 2*c) - 2*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c)*sin(-2*a + 2*c) + cos(2*a
+ 2*c)^2*sin(-2*a + 2*c) + sin(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c) - 2*si
n(2*b*x + 2*a + 4*c)*sin(2*a + 2*c)*sin(-2*a + 2*c) + sin(2*a + 2*c)^2*sin
(-2*a + 2*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 -
 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c)
 - 2*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c)*sin(-2*a + 2*c) + cos(2*a + 2*c
)^2*sin(-2*a + 2*c) + sin(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c) - 2*sin(2*b
*x + 2*a + 4*c)*sin(2*a + 2*c)*sin(-2*a + 2*c) + sin(2*a + 2*c)^2*sin(-2*a
 + 2*c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*si
n(b*x)*sin(c) + sin(c)^2) - (2*b*x*sin(4*c) - cos(4*a) + 2*cos(2*a + 2*c)
- cos(4*c))*sin(2*b*x + 2*a + 4*c) + 2*(b*x*sin(2*b*x + 2*a + 4*c) - b*x*s
in(2*a + 2*c))*sin(2*b*x + 6*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))/(b
*cos(2*b*x + 2*a + 4*c)^2 - 2*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c) + b*
cos(2*a + 2*c)^2 + b*sin(2*b*x + 2*a + 4*c)^2 - 2*b*sin(2*b*x + 2*a + 4*c)
*sin(2*a + 2*c) + b*sin(2*a + 2*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(49) = 98$.

Time = 0.16 (sec) , antiderivative size = 1402, normalized size of antiderivative = 28.61

$$\int \csc^2(c + bx) \sin^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
((tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*a)^4*tan(1/2*c)^2 + 16*tan(1/2*a)^
3*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 16*tan(1/2*a
)^3*tan(1/2*c) + 36*tan(1/2*a)^2*tan(1/2*c)^2 - 16*tan(1/2*a)*tan(1/2*c)^3
 + tan(1/2*c)^4 - 6*tan(1/2*a)^2 + 16*tan(1/2*a)*tan(1/2*c) - 6*tan(1/2*c)
^2 + 1)*(b*x + c)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2
 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^
2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 4*(tan(1/2*a)^4*
tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*tan
(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*
c)^4 - tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)^
2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))*log(tan(1/2*b*x + 1/2*c)^2 + 1
)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^
2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4
 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) + 4*(tan(1/2*a)^4*tan(1/2*c)^3 - t
an(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*tan(1/2*a)^3*tan(1/
2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*a
)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3
 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^4*t
an(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 +
tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a...
```

Mupad [B] (verification not implemented)

Time = 17.98 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.02

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx$$
$$= x\left(\cos\left(2a-2c\right)-\sin\left(2a-2c\right)\,1\mathrm{i}\right)-\frac{\left(1+\mathrm{e}^{a\,4\mathrm{i}-c\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}\right)\,1\mathrm{i}}{2\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}$$
$$+\frac{\mathrm{e}^{-a\,4\mathrm{i}+c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\right)\,\left(2\,b\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-2\,b\,\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$

input
```
int(sin(a + b*x)^2/sin(c + b*x)^2,x)
```

output
```
x*(cos(2*a - 2*c) - sin(2*a - 2*c)*1i) - ((exp(a*4i - c*4i) - 2*exp(a*2i -
 c*2i) + 1)*1i)/(2*b*(exp(a*2i - c*2i) - exp(a*2i + b*x*2i))) + (exp(c*4i
- a*4i)*log(exp(a*2i)*exp(b*x*2i) - exp(a*2i)*exp(-c*2i))*(2*b*exp(a*2i -
c*2i) - 2*b*exp(a*6i - c*6i))*1i)/(4*b^2)
```

Reduce [F]

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx = \int \csc(bx+c)^2 \sin(bx+a)^2\,dx$$

input
```
int(csc(b*x+c)^2*sin(b*x+a)^2,x)
```

output
```
int(csc(b*x + c)**2*sin(a + b*x)**2,x)
```

3.92 $\int \csc^3(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+bx))\cos(2(a-c))}{b} - \frac{\operatorname{arctanh}(\cos(c+bx))\sin^2(a-c)}{2b} - \frac{\cot(c+bx)\csc(c+bx)\sin^2(a-c)}{2b} - \frac{\csc(c+bx)\sin(2(a-c))}{b}$$

output
```
-arctanh(cos(b*x+c))*cos(2*a-2*c)/b-1/2*arctanh(cos(b*x+c))*sin(a-c)^2/b-1
/2*cot(b*x+c)*csc(b*x+c)*sin(a-c)^2/b-csc(b*x+c)*sin(2*a-2*c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 247 vs. 2(89) = 178.

Time = 2.85 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.78

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx = \frac{\left(\cos\left(2a-2c-\frac{bx}{2}\right)-\cos\left(2a-2c+\frac{bx}{2}\right)\right)\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{bx}{2}\right)}{4b} + \frac{(-1+\cos(2a-2c))\csc^2\left(\frac{c}{2}+\frac{bx}{2}\right)}{16b} + \frac{(-1-3\cos(2a-2c))\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{4b} + \frac{(1+3\cos(2a-2c))\log\left(\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{4b} + \frac{\left(-\cos\left(2a-2c-\frac{bx}{2}\right)+\cos\left(2a-2c+\frac{bx}{2}\right)\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{bx}{2}\right)}{4b} + \frac{(1-\cos(2a-2c))\sec^2\left(\frac{c}{2}+\frac{bx}{2}\right)}{16b}$$

input
```
Integrate[Csc[c + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
((Cos[2*a - 2*c - (b*x)/2] - Cos[2*a - 2*c + (b*x)/2])*Csc[c/2]*Csc[c/2 +
(b*x)/2])/(4*b) + ((-1 + Cos[2*a - 2*c])*Csc[c/2 + (b*x)/2]^2)/(16*b) + ((
-1 - 3*Cos[2*a - 2*c])*Log[Cos[c/2 + (b*x)/2]])/(4*b) + ((1 + 3*Cos[2*a -
2*c])*Log[Sin[c/2 + (b*x)/2]])/(4*b) + ((-Cos[2*a - 2*c - (b*x)/2] + Cos[2
*a - 2*c + (b*x)/2])*Sec[c/2]*Sec[c/2 + (b*x)/2])/(4*b) + ((1 - Cos[2*a -
2*c])*Sec[c/2 + (b*x)/2]^2)/(16*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^3(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin^2(a+bx)\csc^3(bx+c)dx$$

input
```
Int[Csc[c + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.72 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.73

method	result
risch	$\frac{-5\,\mathrm{e}^{i(3bx+6a+c)}+2\,\mathrm{e}^{i(3bx+4a+3c)}+3\,\mathrm{e}^{i(3bx+2a+5c)}+3\,\mathrm{e}^{i(bx+6a-c)}+2\,\mathrm{e}^{i(bx+4a+c)}-5\,\mathrm{e}^{i(bx+2a+3c)}}{4\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{2}b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}+\mathrm{e}^{i(a-c)}\right)}{4b}-\frac{3}{}$
default	Expression too large to display

input
```
int(csc(b*x+c)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^2/b*(-5*exp(I*(3*b*x+6*a+c))+2*exp(I*
(3*b*x+4*a+3*c))+3*exp(I*(3*b*x+2*a+5*c))+3*exp(I*(b*x+6*a-c))+2*exp(I*(b*
x+4*a+c))-5*exp(I*(b*x+2*a+3*c)))-1/4/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))-3/
4/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))*cos(2*a-2*c)+1/4/b*ln(exp(I*(b*x+a))-e
xp(I*(a-c)))+3/4/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))*cos(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{8\,\cos\left(-a+c\right)\sin\left(bx+c\right)\sin\left(-a+c\right)+2\,\left(\cos\left(-a+c\right)^{2}-1\right)\cos\left(bx+c\right)+\left(\left(3\,\cos\left(-a+c\right)^{2}-1\right)\right.}{}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(8*cos(-a + c)*sin(b*x + c)*sin(-a + c) + 2*(cos(-a + c)^2 - 1)*cos(b
*x + c) + ((3*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - 3*cos(-a + c)^2 + 1)*log
(1/2*cos(b*x + c) + 1/2) - ((3*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - 3*cos(-
a + c)^2 + 1)*log(-1/2*cos(b*x + c) + 1/2))/(b*cos(b*x + c)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1595 vs. $2(85) = 170$.

Time = 0.10 (sec) , antiderivative size = 1595, normalized size of antiderivative = 17.92

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*(2*(5*cos(3*b*x + 4*a + 2*c) - 2*cos(3*b*x + 2*a + 4*c) - 3*cos(3*b*x
 + 6*c) - 3*cos(b*x + 4*a) - 2*cos(b*x + 2*a + 2*c) + 5*cos(b*x + 4*c))*co
s(4*b*x + 2*a + 5*c) - 10*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + c))*cos(3*
b*x + 4*a + 2*c) + 4*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + c))*cos(3*b*x +
 2*a + 4*c) + 6*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + c))*cos(3*b*x + 6*c)
 + 4*(3*cos(b*x + 4*a) + 2*cos(b*x + 2*a + 2*c) - 5*cos(b*x + 4*c))*cos(2*
b*x + 2*a + 3*c) - 6*cos(b*x + 4*a)*cos(2*a + c) - 4*cos(b*x + 2*a + 2*c)*
cos(2*a + c) + 10*cos(b*x + 4*c)*cos(2*a + c) + ((3*cos(-2*a + 2*c) + 1)*c
os(4*b*x + 2*a + 5*c)^2 + 4*(3*cos(-2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c)
^2 + (3*cos(-2*a + 2*c) + 1)*sin(4*b*x + 2*a + 5*c)^2 + 4*(3*cos(-2*a + 2*
c) + 1)*sin(2*b*x + 2*a + 3*c)^2 - 2*(2*(3*cos(-2*a + 2*c) + 1)*cos(2*b*x
+ 2*a + 3*c) - 3*cos(2*a + c)*cos(-2*a + 2*c) - cos(2*a + c))*cos(4*b*x +
2*a + 5*c) - 4*(3*cos(2*a + c)*cos(-2*a + 2*c) + cos(2*a + c))*cos(2*b*x +
 2*a + 3*c) + cos(2*a + c)^2 + 3*(cos(2*a + c)^2 + sin(2*a + c)^2)*cos(-2*
a + 2*c) - 2*(2*(3*cos(-2*a + 2*c) + 1)*sin(2*b*x + 2*a + 3*c) - 3*cos(-2*
a + 2*c)*sin(2*a + c) - sin(2*a + c))*sin(4*b*x + 2*a + 5*c) - 4*(3*cos(-2
*a + 2*c)*sin(2*a + c) + sin(2*a + c))*sin(2*b*x + 2*a + 3*c) + sin(2*a +
c)^2)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b
*x)*sin(c) + sin(c)^2) - ((3*cos(-2*a + 2*c) + 1)*cos(4*b*x + 2*a + 5*c)^2
 + 4*(3*cos(-2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c)^2 + (3*cos(-2*a + 2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2867 vs. $2(85) = 170$.

Time = 0.18 (sec) , antiderivative size = 2867, normalized size of antiderivative = 32.21

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(2*(tan(1/2*a)^4*tan(1/2*c)^4 - 4*tan(1/2*a)^4*tan(1/2*c)^2 + 12*tan(1
/2*a)^3*tan(1/2*c)^3 - 4*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 12*tan
(1/2*a)^3*tan(1/2*c) + 28*tan(1/2*a)^2*tan(1/2*c)^2 - 12*tan(1/2*a)*tan(1/
2*c)^3 + tan(1/2*c)^4 - 4*tan(1/2*a)^2 + 12*tan(1/2*a)*tan(1/2*c) - 4*tan(
1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^4*tan(1/2*c)^4 +
2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 +
 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c
)^2 + 1) + (tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^8*tan(1/2*c)^6 - 2*tan(1/2*b
*x + 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c)^7 - 4*tan(1/2*b*x + 1/2*c)*tan(1/2*a
)^8*tan(1/2*c)^7 + tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^8 + 4*ta
n(1/2*b*x + 1/2*c)*tan(1/2*a)^7*tan(1/2*c)^8 + 2*tan(1/2*b*x + 1/2*c)^2*ta
n(1/2*a)^8*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c)
^5 - 4*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^8*tan(1/2*c)^5 - 16*tan(1/2*b*x + 1
/2*c)*tan(1/2*a)^7*tan(1/2*c)^6 - 2*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^5*ta
n(1/2*c)^7 + 16*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^6*tan(1/2*c)^7 + 2*tan(1/2
*b*x + 1/2*c)^2*tan(1/2*a)^4*tan(1/2*c)^8 + 4*tan(1/2*b*x + 1/2*c)*tan(1/2
*a)^5*tan(1/2*c)^8 + tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^8*tan(1/2*c)^2 + 2*
tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c)^3 + 4*tan(1/2*b*x + 1/2*c)*
tan(1/2*a)^8*tan(1/2*c)^3 - 2*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*
c)^4 - 40*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^7*tan(1/2*c)^4 - 2*tan(1/2*b*...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^2/sin(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx = \int \csc(bx+c)^3\sin(bx+a)^2\,dx$$

input
```
int(csc(b*x+c)^3*sin(b*x+a)^2,x)
```

output
```
int(csc(b*x + c)**3*sin(a + b*x)**2,x)
```

3.93 $\int \csc^4(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx = -\frac{\cot(c+bx)}{2b} - \frac{\cot^3(c+bx)}{6b} + \frac{\cos(2a+c+3bx)\csc^3(c+bx)}{6b}$$

output
```
-1/2*cot(b*x+c)/b-1/6*cot(b*x+c)^3/b+1/6*cos(3*b*x+2*a+c)*csc(b*x+c)^3/b
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.48

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx = -\frac{\csc(c)\csc^3(c+bx)(-3\sin(bx)-\sin(2a-4c-3bx)+3\sin(2a-2c-bx)-3\sin(2a+bx)+\sin(2a -}{12b}$$

input
```
Integrate[Csc[c + b*x]^4*Sin[a + b*x]^2,x]
```

output
```
-1/12*(Csc[c]*Csc[c + b*x]^3*(-3*Sin[b*x] - Sin[2*a - 4*c - 3*b*x] + 3*Sin
[2*a - 2*c - b*x] - 3*Sin[2*a + b*x] + Sin[2*a + 3*b*x] + Sin[2*c + 3*b*x]
))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^4(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin^2(a+bx)\csc^4(bx+c)dx$$

input
```
Int[Csc[c + b*x]^4*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{\sec\left(\frac{bx}{2}+\frac{c}{2}\right)^{3}\csc\left(\frac{bx}{2}+\frac{c}{2}\right)^{3}\left(-3\cos(bx+c)+2\cos(3bx+2a+c)+\cos(3bx+3c)\right)}{96b}$
risch	$\frac{2i\left(3\,\mathrm{e}^{2i(2bx+4a+c)}-3\,\mathrm{e}^{2i(bx+4a)}-3\,\mathrm{e}^{2i(bx+3a+c)}+\mathrm{e}^{2i(4a-c)}+\mathrm{e}^{6ia}+\mathrm{e}^{2i(2a+c)}\right)}{3\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{3}b}$
default	$-\frac{-2\cos(a)\sin(c)+2\sin(a)\cos(c)}{2(\cos(a)\cos(c)+\sin(a)\sin(c))^{3}(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))^{2}}-\frac{\ldots}{3(\cos(a)\cos(c)+\sin(a)\sin\ldots}$

input
```
int(csc(b*x+c)^4*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/96*sec(1/2*b*x+1/2*c)^3*csc(1/2*b*x+1/2*c)^3*(-3*cos(b*x+c)+2*cos(3*b*x+
2*a+c)+cos(3*b*x+3*c))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.56

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx = -\frac{\left(4\cos\left(-a+c\right)^2-1\right)\cos\left(bx+c\right)^3-3\cos\left(bx+c\right)\cos\left(-a+c\right)^2+3\cos\left(-a+c\right)\sin\left(bx+c\right)\sin\left(-\right.}{3\left(b\cos\left(bx+c\right)^2-b\right)\sin\left(bx+c\right)}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3*((4*cos(-a + c)^2 - 1)*cos(b*x + c)^3 - 3*cos(b*x + c)*cos(-a + c)^2
+ 3*cos(-a + c)*sin(b*x + c)*sin(-a + c))/((b*cos(b*x + c)^2 - b)*sin(b*x
+ c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**4*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(48) = 96$.

Time = 0.05 (sec) , antiderivative size = 900, normalized size of antiderivative = 16.67

$$\int \csc^4(c + bx) \sin^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
2/3*((3*sin(4*b*x + 4*a + 4*c) - 3*sin(2*b*x + 4*a + 2*c) - 3*sin(2*b*x +
2*a + 4*c) + sin(4*a) + sin(2*a + 2*c) + sin(4*c))*cos(6*b*x + 2*a + 8*c)
- 3*(3*sin(2*b*x + 2*a + 4*c) - sin(2*a + 2*c))*cos(4*b*x + 4*a + 4*c) - 3
*(3*sin(4*b*x + 4*a + 4*c) - 3*sin(2*b*x + 4*a + 2*c) - 3*sin(2*b*x + 2*a
+ 4*c) + sin(4*a) + sin(2*a + 2*c) + sin(4*c))*cos(4*b*x + 2*a + 6*c) - 3*
(3*sin(2*b*x + 4*a + 2*c) - sin(4*a) - sin(4*c))*cos(2*b*x + 2*a + 4*c) -
(sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (3*cos(4*b*x + 4*a + 4*c) - 3*cos(2
*b*x + 4*a + 2*c) - 3*cos(2*b*x + 2*a + 4*c) + cos(4*a) + cos(2*a + 2*c) +
 cos(4*c))*sin(6*b*x + 2*a + 8*c) + 3*(3*cos(2*b*x + 2*a + 4*c) - cos(2*a
+ 2*c))*sin(4*b*x + 4*a + 4*c) + 3*(3*cos(4*b*x + 4*a + 4*c) - 3*cos(2*b*x
 + 4*a + 2*c) - 3*cos(2*b*x + 2*a + 4*c) + cos(4*a) + cos(2*a + 2*c) + cos
(4*c))*sin(4*b*x + 2*a + 6*c) + 3*cos(2*a + 2*c)*sin(2*b*x + 4*a + 2*c) +
3*(3*cos(2*b*x + 4*a + 2*c) - cos(4*a) - cos(4*c))*sin(2*b*x + 2*a + 4*c)
+ (cos(4*a) + cos(4*c))*sin(2*a + 2*c) - 3*cos(2*b*x + 4*a + 2*c)*sin(2*a
+ 2*c))/(b*cos(6*b*x + 2*a + 8*c)^2 + 9*b*cos(4*b*x + 2*a + 6*c)^2 + 9*b*c
os(2*b*x + 2*a + 4*c)^2 - 6*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c) + b*co
s(2*a + 2*c)^2 + b*sin(6*b*x + 2*a + 8*c)^2 + 9*b*sin(4*b*x + 2*a + 6*c)^2
 + 9*b*sin(2*b*x + 2*a + 4*c)^2 - 6*b*sin(2*b*x + 2*a + 4*c)*sin(2*a + 2*c
) + b*sin(2*a + 2*c)^2 - 2*(3*b*cos(4*b*x + 2*a + 6*c) - 3*b*cos(2*b*x + 2
*a + 4*c) + b*cos(2*a + 2*c))*cos(6*b*x + 2*a + 8*c) - 6*(3*b*cos(2*b*x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(48) = 96$.

Time = 0.16 (sec) , antiderivative size = 653, normalized size of antiderivative = 12.09

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/3*(3*tan(b*x + c)^2*tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(b*x + c)^2*tan(1/
2*a)^4*tan(1/2*c)^2 + 24*tan(b*x + c)^2*tan(1/2*a)^3*tan(1/2*c)^3 + 6*tan(
b*x + c)*tan(1/2*a)^4*tan(1/2*c)^3 - 6*tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2
*c)^4 - 6*tan(b*x + c)*tan(1/2*a)^3*tan(1/2*c)^4 + 3*tan(b*x + c)^2*tan(1/
2*a)^4 - 24*tan(b*x + c)^2*tan(1/2*a)^3*tan(1/2*c) - 6*tan(b*x + c)*tan(1/
2*a)^4*tan(1/2*c) + 60*tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 36*tan(b
*x + c)*tan(1/2*a)^3*tan(1/2*c)^2 + 4*tan(1/2*a)^4*tan(1/2*c)^2 - 24*tan(b
*x + c)^2*tan(1/2*a)*tan(1/2*c)^3 - 36*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c
)^3 - 8*tan(1/2*a)^3*tan(1/2*c)^3 + 3*tan(b*x + c)^2*tan(1/2*c)^4 + 6*tan(
b*x + c)*tan(1/2*a)*tan(1/2*c)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^4 - 6*tan(b*x
 + c)^2*tan(1/2*a)^2 - 6*tan(b*x + c)*tan(1/2*a)^3 + 24*tan(b*x + c)^2*tan
(1/2*a)*tan(1/2*c) + 36*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c) + 8*tan(1/2*a
)^3*tan(1/2*c) - 6*tan(b*x + c)^2*tan(1/2*c)^2 - 36*tan(b*x + c)*tan(1/2*a
)*tan(1/2*c)^2 - 16*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(b*x + c)*tan(1/2*c)^
3 + 8*tan(1/2*a)*tan(1/2*c)^3 + 3*tan(b*x + c)^2 + 6*tan(b*x + c)*tan(1/2*
a) + 4*tan(1/2*a)^2 - 6*tan(b*x + c)*tan(1/2*c) - 8*tan(1/2*a)*tan(1/2*c)
+ 4*tan(1/2*c)^2)/((tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^
2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1)*b*tan(b*x + c)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^2/sin(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx$$
$$= \frac{-\cos(bx+c)\sin(bx+c)^2-\cos(bx+c)\sin(bx+a)^2-\cos(bx+a)\sin(bx+c)\sin(bx+a)}{3\sin(bx+c)^3 b}$$

input
```
int(csc(b*x+c)^4*sin(b*x+a)^2,x)
```

output
```
( - (cos(b*x + c)*sin(b*x + c)**2 + cos(b*x + c)*sin(a + b*x)**2 + cos(a +
 b*x)*sin(b*x + c)*sin(a + b*x)))/(3*sin(b*x + c)**3*b)
```

3.94 $\int \sin^3(a+bx)\sin^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 126

$$\begin{aligned}\int \sin^3(a+bx)\sin^3(c+bx)\,dx = {} & \frac{1}{32}x(9\cos(a-c)+\cos(3(a-c))) \\ & + \frac{3\sin(a-3c-2bx)}{64b} - \frac{3\sin(3a-c+2bx)}{64b} \\ & - \frac{9\sin(a+c+2bx)}{64b} + \frac{3\sin(3a+c+4bx)}{128b} \\ & + \frac{3\sin(a+3c+4bx)}{128b} - \frac{\sin(3(a+c)+6bx)}{192b}\end{aligned}$$

output
```
1/32*x*(9*cos(a-c)+cos(3*a-3*c))+3/64*sin(-2*b*x+a-3*c)/b-3/64*sin(2*b*x+3
*a-c)/b-9/64*sin(2*b*x+a+c)/b+3/128*sin(4*b*x+3*a+c)/b+3/128*sin(4*b*x+a+3
*c)/b-1/192*sin(6*b*x+3*a+3*c)/b
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81

$$\int \sin^3(a+bx)\sin^3(c+bx)\,dx$$
$$= \frac{108bx\cos(a-c)+12bx\cos(3(a-c))+18\sin(a-3c-2bx)-18\sin(3a-c+2bx)-54\sin(a+c+2b}{384b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[c + b*x]^3,x]
```

output
```
(108*b*x*Cos[a - c] + 12*b*x*Cos[3*(a - c)] + 18*Sin[a - 3*c - 2*b*x] - 18
*Sin[3*a - c + 2*b*x] - 54*Sin[a + c + 2*b*x] - 2*Sin[3*(a + c + 2*b*x)] +
 9*Sin[3*a + c + 4*b*x] + 9*Sin[a + 3*c + 4*b*x])/(384*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^3(bx+c)\,dx$$
$$\Big\downarrow \text{5080}$$
$$\int \left(-\frac{3}{32}\cos(a-2bx-3c)-\frac{3}{32}\cos(3a+2bx-c)-\frac{9}{32}\cos(a+2bx+c)+\frac{3}{32}\cos(3a+4bx+c)+\frac{3}{32}\cos(a+\right.$$
$$\Big\downarrow \text{2009}$$
$$\frac{3\sin(a-2bx-3c)}{64b}-\frac{3\sin(3a+2bx-c)}{64b}-\frac{9\sin(a+2bx+c)}{64b}+\frac{3\sin(3a+4bx+c)}{128b}+\frac{3\sin(a+4bx+3c)}{128b}-\frac{\sin(3(a+c)+6bx)}{192b}+\frac{1}{32}x(9\cos(a-c)+\cos(3(a-c)))$$

input
```
Int[Sin[a + b*x]^3*Sin[c + b*x]^3,x]
```

output
```
(x*(9*Cos[a - c] + Cos[3*(a - c)]))/32 + (3*Sin[a - 3*c - 2*b*x])/(64*b) -
 (3*Sin[3*a - c + 2*b*x])/(64*b) - (9*Sin[a + c + 2*b*x])/(64*b) + (3*Sin[
3*a + c + 4*b*x])/(128*b) + (3*Sin[a + 3*c + 4*b*x])/(128*b) - Sin[3*(a +
c) + 6*b*x]/(192*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 8.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

method	result
default	$\frac{9x\cos(a-c)}{32}+\frac{x\cos(3a-3c)}{32}+\frac{3\sin(-2bx+a-3c)}{64b}-\frac{9\sin(2bx+a+c)}{64b}-\frac{3\sin(2bx+3a-c)}{64b}+\frac{3\sin(4bx+a+3c)}{128b}+$ 3 s
risch	$\frac{9x\cos(a-c)}{32}+\frac{x\cos(3a-3c)}{32}+\frac{3\sin(-2bx+a-3c)}{64b}-\frac{9\sin(2bx+a+c)}{64b}-\frac{3\sin(2bx+3a-c)}{64b}+\frac{3\sin(4bx+a+3c)}{128b}+$ 3 s
parallelrisch	$\frac{4\sin(3a-3c)+12bx\cos(3a-3c)+108x\cos(a-c)b-18\sin(2bx+3a-c)+9\sin(4bx+3a+c)-2\sin(6bx+3a+3c)-54\sin(2bx+a+}{384b}$
orering	Expression too large to display

input
```
int(sin(b*x+a)^3*sin(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
9/32*x*cos(a-c)+1/32*x*cos(3*a-3*c)+3/64*sin(-2*b*x+a-3*c)/b-9/64*sin(2*b*
x+a+c)/b-3/64*sin(2*b*x+3*a-c)/b+3/128*sin(4*b*x+a+3*c)/b+3/128*sin(4*b*x+
3*a+c)/b-1/192*sin(6*b*x+3*a+3*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.53

$$\int \sin^3(a+bx)\sin^3(c+bx)\,dx$$
$$= \frac{6\,bx\cos\left(-a+c\right)^3+9\,bx\cos\left(-a+c\right)-\left(8\left(4\cos\left(-a+c\right)^3-3\cos\left(-a+c\right)\right)\cos\left(bx+c\right)^5-2\left(34\cos\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^3*sin(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/48*(6*b*x*cos(-a + c)^3 + 9*b*x*cos(-a + c) - (8*(4*cos(-a + c)^3 - 3*co
s(-a + c))*cos(b*x + c)^5 - 2*(34*cos(-a + c)^3 - 21*cos(-a + c))*cos(b*x
+ c)^3 + 3*(14*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c))*sin(b*x + c) +
 4*(2*(4*cos(-a + c)^2 - 1)*cos(b*x + c)^6 - 3*(7*cos(-a + c)^2 - 1)*cos(b
*x + c)^4 + 18*cos(b*x + c)^2*cos(-a + c)^2)*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(117) = 234.

Time = 4.87 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.21

$$\int \sin^3(a+bx)\sin^3(c+bx)\,dx$$
$$= \begin{cases} \frac{5x\sin^3\left(a+bx\right)\sin^3\left(bx+c\right)}{16}+\frac{3x\sin^3\left(a+bx\right)\sin\left(bx+c\right)\cos^2\left(bx+c\right)}{16}+\frac{9x\sin^2\left(a+bx\right)\sin^2\left(bx+c\right)\cos\left(a+bx\right)\cos\left(bx+c\right)}{16}+\frac{3x\sin^2\left(a+bx\right.}{} \\ x\sin^3\left(a\right)\sin^3\left(c\right) \end{cases}$$

input
```
integrate(sin(b*x+a)**3*sin(b*x+c)**3,x)
```

output
```
Piecewise((5*x*sin(a + b*x)**3*sin(b*x + c)**3/16 + 3*x*sin(a + b*x)**3*si
n(b*x + c)*cos(b*x + c)**2/16 + 9*x*sin(a + b*x)**2*sin(b*x + c)**2*cos(a
+ b*x)*cos(b*x + c)/16 + 3*x*sin(a + b*x)**2*cos(a + b*x)*cos(b*x + c)**3/
16 + 3*x*sin(a + b*x)*sin(b*x + c)**3*cos(a + b*x)**2/16 + 9*x*sin(a + b*x
)*sin(b*x + c)*cos(a + b*x)**2*cos(b*x + c)**2/16 + 3*x*sin(b*x + c)**2*co
s(a + b*x)**3*cos(b*x + c)/16 + 5*x*cos(a + b*x)**3*cos(b*x + c)**3/16 + s
in(a + b*x)**3*cos(b*x + c)**3/(16*b) - 11*sin(a + b*x)**2*sin(b*x + c)**3
*cos(a + b*x)/(16*b) - 3*sin(a + b*x)*sin(b*x + c)**2*cos(a + b*x)**2*cos(
b*x + c)/(4*b) + 3*sin(a + b*x)*cos(a + b*x)**2*cos(b*x + c)**3/(16*b) - 7
*sin(b*x + c)**3*cos(a + b*x)**3/(48*b) - sin(b*x + c)*cos(a + b*x)**3*cos
(b*x + c)**2/(2*b), Ne(b, 0)), (x*sin(a)**3*sin(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \sin^3(a+bx)\sin^3(c+bx)\,dx = \frac{12\,(9\,b\cos(-a+c)+b\cos(-3\,a+3\,c))x-2\,\sin(6\,bx+3\,a+3\,c)+9\,\sin(4\,bx+3\,a+c)+9\,\sin(4\,bx}{384\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(b*x+c)^3,x, algorithm="maxima")
```

output
```
1/384*(12*(9*b*cos(-a + c) + b*cos(-3*a + 3*c))*x - 2*sin(6*b*x + 3*a + 3*
c) + 9*sin(4*b*x + 3*a + c) + 9*sin(4*b*x + a + 3*c) - 18*sin(2*b*x + 3*a
- c) - 54*sin(2*b*x + a + c) - 18*sin(2*b*x - a + 3*c))/b
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \sin^3(a+bx)\sin^3(c+bx)\,dx = {} & \frac{1}{32}\,x\cos(3\,a-3\,c)+\frac{9}{32}\,x\cos(a-c) \\ & -\frac{\sin(6\,bx+3\,a+3\,c)}{192\,b}+\frac{3\,\sin(4\,bx+3\,a+c)}{128\,b} \\ & +\frac{3\,\sin(4\,bx+a+3\,c)}{128\,b}-\frac{3\,\sin(2\,bx+3\,a-c)}{64\,b} \\ & -\frac{9\,\sin(2\,bx+a+c)}{64\,b}+\frac{3\,\sin(-2\,bx+a-3\,c)}{64\,b}\end{aligned}$$

input
```
integrate(sin(b*x+a)^3*sin(b*x+c)^3,x, algorithm="giac")
```

output
```
1/32*x*cos(3*a - 3*c) + 9/32*x*cos(a - c) - 1/192*sin(6*b*x + 3*a + 3*c)/b
 + 3/128*sin(4*b*x + 3*a + c)/b + 3/128*sin(4*b*x + a + 3*c)/b - 3/64*sin(
2*b*x + 3*a - c)/b - 9/64*sin(2*b*x + a + c)/b + 3/64*sin(-2*b*x + a - 3*c
)/b
```

Mupad [B] (verification not implemented)

Time = 21.93 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \sin^3(a+bx)\sin^3(c+bx)\,dx = \frac{\frac{9\sin(a+3c+4bx)}{8}+\frac{9\sin(3a+c+4bx)}{8}-\frac{9\sin(3c-a+2bx)}{4}-\frac{9\sin(3a-c+2bx)}{4}-\frac{\sin(3a+3c+6bx)}{4}-\frac{27\sin(a+c+2bx)}{4}+\frac{27b}{}}{48b}$$

input
```
int(sin(a + b*x)^3*sin(c + b*x)^3,x)
```

output
```
((9*sin(a + 3*c + 4*b*x))/8 + (9*sin(3*a + c + 4*b*x))/8 - (9*sin(3*c - a
+ 2*b*x))/4 - (9*sin(3*a - c + 2*b*x))/4 - sin(3*a + 3*c + 6*b*x)/4 - (27*
sin(a + c + 2*b*x))/4 + (27*b*x*cos(a - c))/2 + (3*b*x*cos(3*a - 3*c))/2)/
(48*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.30

$$\int \sin^3(a+bx)\sin^3(c+bx)\,dx = \frac{24\cos(bx+c)\cos(bx+a)\sin(bx+c)^2\sin(bx+a)^2bx-6\cos(bx+c)\cos(bx+a)\sin(bx+c)^2bx-6}{}$$

input
```
int(sin(b*x+a)^3*sin(b*x+c)^3,x)
```

output
```
(24*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x)**2*b*x - 6*cos(
b*x + c)*cos(a + b*x)*sin(b*x + c)**2*b*x - 6*cos(b*x + c)*cos(a + b*x)*si
n(a + b*x)**2*b*x + 15*cos(b*x + c)*cos(a + b*x)*b*x - 18*cos(b*x + c)*sin
(b*x + c)**2*sin(a + b*x)**3 + 9*cos(b*x + c)*sin(a + b*x)**3 + 10*cos(a +
 b*x)*sin(b*x + c)**3*sin(a + b*x)**2 + 2*cos(a + b*x)*sin(b*x + c)**3 - 2
1*cos(a + b*x)*sin(b*x + c)*sin(a + b*x)**2 - 15*cos(a + b*x)*sin(b*x + c)
 + 24*sin(b*x + c)**3*sin(a + b*x)**3*b*x - 18*sin(b*x + c)**3*sin(a + b*x
)*b*x - 18*sin(b*x + c)*sin(a + b*x)**3*b*x + 27*sin(b*x + c)*sin(a + b*x)
*b*x)/(48*b)
```

3.95 $\int \sin^3(a+bx)\sin^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \sin^3(a+bx)\sin^2(c+bx)\,dx = -\frac{3\cos(a-2c-bx)}{16b}-\frac{3\cos(a+bx)}{8b}-\frac{\cos(3a-2c+bx)}{16b}+\frac{\cos(3a+3bx)}{24b}+\frac{\cos(a+2c+3bx)}{16b}-\frac{\cos(3a+2c+5bx)}{80b}$$

output
```
-3/16*cos(-b*x+a-2*c)/b-3/8*cos(b*x+a)/b-1/16*cos(b*x+3*a-2*c)/b+1/24*cos(
3*b*x+3*a)/b+1/16*cos(3*b*x+a+2*c)/b-1/80*cos(5*b*x+3*a+2*c)/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \sin^3(a+bx)\sin^2(c+bx)\,dx = -\frac{45\cos(a-2c-bx)+90\cos(a+bx)-10\cos(3(a+bx))+15\cos(3a-2c+bx)-15\cos(a+2c+3b}{240b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[c + b*x]^2,x]
```

output
```
-1/240*(45*Cos[a - 2*c - b*x] + 90*Cos[a + b*x] - 10*Cos[3*(a + b*x)] + 15
*Cos[3*a - 2*c + b*x] - 15*Cos[a + 2*c + 3*b*x] + 3*Cos[3*a + 2*c + 5*b*x]
)/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^2(bx+c)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{3}{16}\sin(a-bx-2c)+\frac{1}{16}\sin(3a+bx-2c)-\frac{3}{16}\sin(a+3bx+2c)+\frac{1}{16}\sin(3a+5bx+2c)+\frac{3}{8}\sin(a+b$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\cos(a-bx-2c)}{16b}-\frac{\cos(3a+bx-2c)}{16b}+\frac{\cos(a+3bx+2c)}{16b}-\frac{\cos(3a+5bx+2c)}{80b}-\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}$$

input
```
Int[Sin[a + b*x]^3*Sin[c + b*x]^2,x]
```

output
```
(-3*Cos[a - 2*c - b*x])/(16*b) - (3*Cos[a + b*x])/(8*b) - Cos[3*a - 2*c +
b*x]/(16*b) + Cos[3*a + 3*b*x]/(24*b) + Cos[a + 2*c + 3*b*x]/(16*b) - Cos[
3*a + 2*c + 5*b*x]/(80*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 3.81 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

method	result
default	$-\frac{3\cos(-bx+a-2c)}{16b}-\frac{3\cos(bx+a)}{8b}-\frac{\cos(bx+3a-2c)}{16b}+\frac{\cos(3bx+3a)}{24b}+\frac{\cos(3bx+a+2c)}{16b}-\frac{\cos(5bx+3a+2c)}{80b}$
risch	$-\frac{3\cos(-bx+a-2c)}{16b}-\frac{3\cos(bx+a)}{8b}-\frac{\cos(bx+3a-2c)}{16b}+\frac{\cos(3bx+3a)}{24b}+\frac{\cos(3bx+a+2c)}{16b}-\frac{\cos(5bx+3a+2c)}{80b}$
parallelrisch	$\frac{-16\cos(2a-2c)-90\cos(bx+a)-45\cos(-bx+a-2c)-15\cos(bx+3a-2c)+10\cos(3bx+3a)+15\cos(3bx+a+2c)-3\cos(5bx+3}{240b}$
orering	$-\frac{259\left(3\sin(bx+a)^2\sin(bx+c)^2b\cos(bx+a)+2\sin(bx+a)^3\sin(bx+c)b\cos(bx+c)\right)}{225b^2}-\frac{7\left(6b^3\cos(bx+a)^3\sin(bx+c)^2+36\sin(\right.}{}$

input
```
int(sin(b*x+a)^3*sin(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/16*cos(-b*x+a-2*c)/b-3/8*cos(b*x+a)/b-1/16*cos(b*x+3*a-2*c)/b+1/24*cos(
3*b*x+3*a)/b+1/16*cos(3*b*x+a+2*c)/b-1/80*cos(5*b*x+3*a+2*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.48

$$\int \sin^3(a+bx)\sin^2(c+bx)\,dx =$$

$$-\frac{3\left(4\cos(-a+c)^3-3\cos(-a+c)\right)\cos(bx+c)^5-5\left(5\cos(-a+c)^3-3\cos(-a+c)\right)\cos(bx+c)}{}$$

input
```
integrate(sin(b*x+a)^3*sin(b*x+c)^2,x, algorithm="fricas")
```

output
```
-1/15*(3*(4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^5 - 5*(5*cos(-a +
c)^3 - 3*cos(-a + c))*cos(b*x + c)^3 + 15*cos(b*x + c)*cos(-a + c)^3 + (3*
(4*cos(-a + c)^2 - 1)*cos(b*x + c)^4 - (19*cos(-a + c)^2 - 1)*cos(b*x + c)
^2 + 7*cos(-a + c)^2 + 2)*sin(b*x + c)*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(87) = 174$.

Time = 2.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.73

$$\int \sin^3(a+bx)\sin^2(c+bx)\,dx = \begin{cases} -\frac{2\sin^3(a+bx)\sin(bx+c)\cos(bx+c)}{5b} - \frac{3\sin^2(a+bx)\sin^2(bx+c)\cos(a+bx)}{5b} - \frac{2\sin^2(a+bx)\cos(a+bx)\cos^2(bx+c)}{5b} - \frac{4\sin(a+bx)\sin}{} \\ x\sin^3(a)\sin^2(c) \end{cases}$$

input
```
integrate(sin(b*x+a)**3*sin(b*x+c)**2,x)
```

output
```
Piecewise((-2*sin(a + b*x)**3*sin(b*x + c)*cos(b*x + c)/(5*b) - 3*sin(a +
b*x)**2*sin(b*x + c)**2*cos(a + b*x)/(5*b) - 2*sin(a + b*x)**2*cos(a + b*x
)*cos(b*x + c)**2/(5*b) - 4*sin(a + b*x)*sin(b*x + c)*cos(a + b*x)**2*cos(
b*x + c)/(5*b) - 2*sin(b*x + c)**2*cos(a + b*x)**3/(15*b) - 8*cos(a + b*x)
**3*cos(b*x + c)**2/(15*b), Ne(b, 0)), (x*sin(a)**3*sin(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \sin^3(a+bx)\sin^2(c+bx)\,dx = -\frac{3\cos(5bx+3a+2c) - 10\cos(3bx+3a) - 15\cos(3bx+a+2c) + 15\cos(bx+3a-2c) + 90\text{ c}}{240b}$$

input
```
integrate(sin(b*x+a)^3*sin(b*x+c)^2,x, algorithm="maxima")
```

output
```
-1/240*(3*cos(5*b*x + 3*a + 2*c) - 10*cos(3*b*x + 3*a) - 15*cos(3*b*x + a
+ 2*c) + 15*cos(b*x + 3*a - 2*c) + 90*cos(b*x + a) + 45*cos(b*x - a + 2*c)
)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \sin^3(a+bx)\sin^2(c+bx)\,dx = -\frac{\cos(5\,bx+3\,a+2\,c)}{80\,b} + \frac{\cos(3\,bx+3\,a)}{24\,b} + \frac{\cos(3\,bx+a+2\,c)}{16\,b} - \frac{\cos(bx+3\,a-2\,c)}{16\,b} - \frac{3\,\cos(bx+a)}{8\,b} - \frac{3\,\cos(-bx+a-2\,c)}{16\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(b*x+c)^2,x, algorithm="giac")
```

output
```
-1/80*cos(5*b*x + 3*a + 2*c)/b + 1/24*cos(3*b*x + 3*a)/b + 1/16*cos(3*b*x
+ a + 2*c)/b - 1/16*cos(b*x + 3*a - 2*c)/b - 3/8*cos(b*x + a)/b - 3/16*cos
(-b*x + a - 2*c)/b
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \sin^3(a+bx)\sin^2(c+bx)\,dx = -\frac{90\,\cos(a+b\,x) - 15\,\cos(a+2\,c+3\,b\,x) + 45\,\cos(2\,c-a+b\,x) + 15\,\cos(3\,a-2\,c+b\,x) + 3\,\cos(3}{240\,b}$$

input
```
int(sin(a + b*x)^3*sin(c + b*x)^2,x)
```

output
```
-(90*cos(a + b*x) - 15*cos(a + 2*c + 3*b*x) + 45*cos(2*c - a + b*x) + 15*c
os(3*a - 2*c + b*x) + 3*cos(3*a + 2*c + 5*b*x) - 10*cos(3*a + 3*b*x))/(240
*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.75

$$\int \sin^3(a+bx)\sin^2(c+bx)\,dx$$
$$= \frac{-12\cos(bx+c)\cos(bx+a)\sin(bx+c)\sin(bx+a)+6\cos(bx+c)\sin(bx+c)\sin(bx+a)^3-12\cos(\ldots}{\ldots}$$

input
```
int(sin(b*x+a)^3*sin(b*x+c)^2,x)
```

output
```
( - 12*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)*sin(a + b*x) + 6*cos(b*x + c
)*sin(b*x + c)*sin(a + b*x)**3 - 12*cos(b*x + c)*sin(b*x + c)*sin(a + b*x)
 - 9*cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x)**2 + 6*cos(a + b*x)*sin(b*x
 + c)**2 + 2*cos(a + b*x)*sin(a + b*x)**2 - 8*cos(a + b*x) - 12*sin(b*x +
c)**2*sin(a + b*x)**2 + 6*sin(b*x + c)**2 + 6*sin(a + b*x)**2 + 32)/(15*b)
```

3.96 $\int \sin^3(a+bx)\sin(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \sin^3(a+bx)\sin(c+bx)\,dx = \frac{3}{8}x\cos(a-c) - \frac{\sin(3a-c+2bx)}{16b} - \frac{3\sin(a+c+2bx)}{16b} + \frac{\sin(3a+c+4bx)}{32b}$$

output
```
3/8*x*cos(a-c)-1/16*sin(2*b*x+3*a-c)/b-3/16*sin(2*b*x+a+c)/b+1/32*sin(4*b*
x+3*a+c)/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \sin^3(a+bx)\sin(c+bx)\,dx = \frac{12bx\cos(a-c) - 2\sin(3a-c+2bx) - 6\sin(a+c+2bx) + \sin(3a+c+4bx)}{32b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[c + b*x],x]
```

output
```
(12*b*x*Cos[a - c] - 2*Sin[3*a - c + 2*b*x] - 6*Sin[a + c + 2*b*x] + Sin[3
*a + c + 4*b*x])/(32*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin(bx+c)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{1}{8}\cos(3a+2bx-c)-\frac{3}{8}\cos(a+2bx+c)+\frac{1}{8}\cos(3a+4bx+c)+\frac{3}{8}\cos(a-c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(3a+2bx-c)}{16b}-\frac{3\sin(a+2bx+c)}{16b}+\frac{\sin(3a+4bx+c)}{32b}+\frac{3}{8}x\cos(a-c)$$

input
```
Int[Sin[a + b*x]^3*Sin[c + b*x],x]
```

output
```
(3*x*Cos[a - c])/8 - Sin[3*a - c + 2*b*x]/(16*b) - (3*Sin[a + c + 2*b*x])/
(16*b) + Sin[3*a + c + 4*b*x]/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result
default	$\frac{3x\cos(a-c)}{8}-\frac{\sin(2bx+3a-c)}{16b}-\frac{3\sin(2bx+a+c)}{16b}+\frac{\sin(4bx+3a+c)}{32b}$
risch	$\frac{3x\cos(a-c)}{8}-\frac{\sin(2bx+3a-c)}{16b}-\frac{3\sin(2bx+a+c)}{16b}+\frac{\sin(4bx+3a+c)}{32b}$
parallelrisch	$\frac{12x\cos(a-c)b-2\sin(2bx+3a-c)-6\sin(2bx+a+c)+\sin(4bx+3a+c)+9\sin(a-c)}{32b}$
orering	$x\sin(bx+a)^3\sin(bx+c)-\frac{5\left(3\sin(bx+a)^2\sin(bx+c)b\cos(bx+a)+\sin(bx+a)^3b\cos(bx+c)\right)}{16b^2}+\frac{5x\left(6\sin(bx+c)\right.}{}$

input
```
int(sin(b*x+a)^3*sin(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
3/8*x*cos(a-c)-1/16*sin(2*b*x+3*a-c)/b-3/16*sin(2*b*x+a+c)/b+1/32*sin(4*b*
x+3*a+c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(55) = 110.

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.03

$$\int \sin^3(a+bx)\sin(c+bx)\,dx$$
$$=\frac{3\,bx\cos\left(-a+c\right)+\left(2\left(4\cos\left(-a+c\right)^3-3\cos\left(-a+c\right)\right)\cos\left(bx+c\right)^3-\left(8\cos\left(-a+c\right)^3-3\cos\left(-a\right.\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^3*sin(b*x+c),x, algorithm="fricas")
```

output
```
1/8*(3*b*x*cos(-a + c) + (2*(4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)
^3 - (8*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c))*sin(b*x + c) - 2*((4*
cos(-a + c)^2 - 1)*cos(b*x + c)^4 - 6*cos(b*x + c)^2*cos(-a + c)^2)*sin(-a
 + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(54) = 108$.

Time = 0.88 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.73

$$\int \sin^3(a+bx)\sin(c+bx)\,dx$$
$$= \begin{cases} \frac{3x\sin^3(a+bx)\sin(bx+c)}{8} + \frac{3x\sin^2(a+bx)\cos(a+bx)\cos(bx+c)}{8} + \frac{3x\sin(a+bx)\sin(bx+c)\cos^2(a+bx)}{8} + \frac{3x\cos^3(a+bx)\cos(bx+c)}{8} \\ x\sin^3(a)\sin(c) \end{cases}$$

input
```
integrate(sin(b*x+a)**3*sin(b*x+c),x)
```

output
```
Piecewise((3*x*sin(a + b*x)**3*sin(b*x + c)/8 + 3*x*sin(a + b*x)**2*cos(a
+ b*x)*cos(b*x + c)/8 + 3*x*sin(a + b*x)*sin(b*x + c)*cos(a + b*x)**2/8 +
3*x*cos(a + b*x)**3*cos(b*x + c)/8 - 5*sin(a + b*x)**3*cos(b*x + c)/(8*b)
- 3*sin(a + b*x)*cos(a + b*x)**2*cos(b*x + c)/(4*b) + 3*sin(b*x + c)*cos(a
 + b*x)**3/(8*b), Ne(b, 0)), (x*sin(a)**3*sin(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \sin^3(a+bx)\sin(c+bx)\,dx$$
$$= \frac{12\,bx\cos(-a+c) + \sin(4\,bx+3\,a+c) - 2\,\sin(2\,bx+3\,a-c) - 6\,\sin(2\,bx+a+c)}{32\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(b*x+c),x, algorithm="maxima")
```

output
```
1/32*(12*b*x*cos(-a + c) + sin(4*b*x + 3*a + c) - 2*sin(2*b*x + 3*a - c) -
 6*sin(2*b*x + a + c))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \sin^3(a+bx)\sin(c+bx)\,dx = \frac{3}{8}\,x\cos\left(a-c\right) + \frac{\sin\left(4\,bx+3\,a+c\right)}{32\,b} - \frac{\sin\left(2\,bx+3\,a-c\right)}{16\,b} - \frac{3\,\sin\left(2\,bx+a+c\right)}{16\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(b*x+c),x, algorithm="giac")
```

output
```
3/8*x*cos(a - c) + 1/32*sin(4*b*x + 3*a + c)/b - 1/16*sin(2*b*x + 3*a - c)
/b - 3/16*sin(2*b*x + a + c)/b
```

Mupad [B] (verification not implemented)

Time = 18.75 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \sin^3(a+bx)\sin(c+bx)\,dx = \frac{3\,x\,\cos\left(a-c\right)}{8} - \frac{\frac{\sin(3\,a-c+2\,b\,x)}{16} - \frac{\sin(3\,a+c+4\,b\,x)}{32} + \frac{3\,\sin(a+c+2\,b\,x)}{16}}{b}$$

input
```
int(sin(a + b*x)^3*sin(c + b*x),x)
```

output
```
(3*x*cos(a - c))/8 - (sin(3*a - c + 2*b*x)/16 - sin(3*a + c + 4*b*x)/32 +
(3*sin(a + c + 2*b*x))/16)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.41

$$\int \sin^3(a+bx)\sin(c+bx)\,dx$$
$$= \frac{3\cos(bx+c)\cos(bx+a)\,bx+\cos(bx+c)\sin(bx+a)^3-3\cos(bx+a)\sin(bx+c)\sin(bx+a)^2-3\,\mathrm{co}}{8b}$$

input
```
int(sin(b*x+a)^3*sin(b*x+c),x)
```

output
```
(3*cos(b*x + c)*cos(a + b*x)*b*x + cos(b*x + c)*sin(a + b*x)**3 - 3*cos(a
+ b*x)*sin(b*x + c)*sin(a + b*x)**2 - 3*cos(a + b*x)*sin(b*x + c) + 3*sin(
b*x + c)*sin(a + b*x)*b*x)/(8*b)
```

3.97 $\int \csc(c+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \csc(c+bx)\sin^3(a+bx)\,dx = \frac{3}{4}x\cos(a-c) - \frac{1}{4}x\cos(3(a-c)) + \frac{\log(\sin(c+bx))\sin^3(a-c)}{b} - \frac{\sin(3a-c+2bx)}{4b}$$

output
```
3/4*x*cos(a-c)-1/4*x*cos(3*a-3*c)+ln(sin(b*x+c))*sin(a-c)^3/b-1/4*sin(2*b*
x+3*a-c)/b
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \csc(c+bx)\sin^3(a+bx)\,dx = -\frac{-3bx\cos(a-c) + bx\cos(3(a-c)) - 3\log(\sin(c+bx))\sin(a-c) + \log(\sin(c+bx))\sin(3(a-c)) + \text{s}}{4b}$$

input
```
Integrate[Csc[c + b*x]*Sin[a + b*x]^3,x]
```

output
```
-1/4*(-3*b*x*Cos[a - c] + b*x*Cos[3*(a - c)] - 3*Log[Sin[c + b*x]]*Sin[a -
 c] + Log[Sin[c + b*x]]*Sin[3*(a - c)] + Sin[3*a - c + 2*b*x])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin^3(a+bx)\csc(bx+c)dx$$

input
```
Int[Csc[c + b*x]*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.56

method	result
risch	$-\frac{x\,\mathrm{e}^{3i(a-c)}}{4}+\frac{3x\,\mathrm{e}^{i(a-c)}}{4}-\frac{3i\sin(a-c)x}{2}+\frac{i\sin(3a-3c)x}{2}-\frac{3i\sin(a-c)a}{2b}+\frac{i\sin(3a-3c)a}{2b}+\frac{3\ln\left(\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{2i(a-c)}\right)\mathrm{si}}{4b}$
default	Expression too large to display

input
```
int(csc(b*x+c)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*x*exp(3*I*(a-c))+3/4*x*exp(I*(a-c))-3/2*I*sin(a-c)*x+1/2*I*sin(3*a-3*
c)*x-3/2*I/b*sin(a-c)*a+1/2*I/b*sin(3*a-3*c)*a+3/4*ln(exp(2*I*(b*x+a))-exp
(2*I*(a-c)))/b*sin(a-c)-1/4/b*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*sin(3*a-
3*c)-1/4*sin(2*b*x+3*a-c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(57) = 114$.

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.83

$$\int \csc(c+bx)\sin^3(a+bx)\,dx = -\frac{2\,bx\cos(-a+c)^3 - \left(4\,\cos(-a+c)^2-1\right)\cos(bx+c)^2\sin(-a+c) - 3\,bx\cos(-a+c) + \left(4\,\cos(-a\right.}{}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b*x*cos(-a + c)^3 - (4*cos(-a + c)^2 - 1)*cos(b*x + c)^2*sin(-a +
c) - 3*b*x*cos(-a + c) + (4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)*si
n(b*x + c) - 2*(cos(-a + c)^2 - 1)*log(1/2*sin(b*x + c))*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6078 vs. $2(0) = 0$.

Time = 28.34 (sec) , antiderivative size = 33056, normalized size of antiderivative = 524.70

$$\int \csc(c+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a)**3,x)
```

output
```
3*Piecewise((0, Eq(b, 0) & Eq(c, 0)), (sin(b*x)**2/(2*b), Eq(c, 0)), (0, E
q(b, 0)), (-b*x*tan(c/2)**5*tan(b*x/2)**4/(b*tan(c/2)**6*tan(b*x/2)**4 + 2
*b*tan(c/2)**6*tan(b*x/2)**2 + b*tan(c/2)**6 + 3*b*tan(c/2)**4*tan(b*x/2)*
*4 + 6*b*tan(c/2)**4*tan(b*x/2)**2 + 3*b*tan(c/2)**4 + 3*b*tan(c/2)**2*tan
(b*x/2)**4 + 6*b*tan(c/2)**2*tan(b*x/2)**2 + 3*b*tan(c/2)**2 + b*tan(b*x/2
)**4 + 2*b*tan(b*x/2)**2 + b) - 2*b*x*tan(c/2)**5*tan(b*x/2)**2/(b*tan(c/2
)**6*tan(b*x/2)**4 + 2*b*tan(c/2)**6*tan(b*x/2)**2 + b*tan(c/2)**6 + 3*b*t
an(c/2)**4*tan(b*x/2)**4 + 6*b*tan(c/2)**4*tan(b*x/2)**2 + 3*b*tan(c/2)**4
 + 3*b*tan(c/2)**2*tan(b*x/2)**4 + 6*b*tan(c/2)**2*tan(b*x/2)**2 + 3*b*tan
(c/2)**2 + b*tan(b*x/2)**4 + 2*b*tan(b*x/2)**2 + b) - b*x*tan(c/2)**5/(b*t
an(c/2)**6*tan(b*x/2)**4 + 2*b*tan(c/2)**6*tan(b*x/2)**2 + b*tan(c/2)**6 +
 3*b*tan(c/2)**4*tan(b*x/2)**4 + 6*b*tan(c/2)**4*tan(b*x/2)**2 + 3*b*tan(c
/2)**4 + 3*b*tan(c/2)**2*tan(b*x/2)**4 + 6*b*tan(c/2)**2*tan(b*x/2)**2 + 3
*b*tan(c/2)**2 + b*tan(b*x/2)**4 + 2*b*tan(b*x/2)**2 + b) + 6*b*x*tan(c/2)
**3*tan(b*x/2)**4/(b*tan(c/2)**6*tan(b*x/2)**4 + 2*b*tan(c/2)**6*tan(b*x/2
)**2 + b*tan(c/2)**6 + 3*b*tan(c/2)**4*tan(b*x/2)**4 + 6*b*tan(c/2)**4*tan
(b*x/2)**2 + 3*b*tan(c/2)**4 + 3*b*tan(c/2)**2*tan(b*x/2)**4 + 6*b*tan(c/2
)**2*tan(b*x/2)**2 + 3*b*tan(c/2)**2 + b*tan(b*x/2)**4 + 2*b*tan(b*x/2)**2
 + b) + 12*b*x*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**4 + 2*
b*tan(c/2)**6*tan(b*x/2)**2 + b*tan(c/2)**6 + 3*b*tan(c/2)**4*tan(b*x/2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(57) = 114$.

Time = 0.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.57

$$\int \csc(c+bx)\sin^3(a+bx)\,dx$$
$$= \frac{2\,(3\,b\cos\left(-a+c\right)-b\cos\left(-3\,a+3\,c\right))x-(3\,\sin\left(-a+c\right)-\sin\left(-3\,a+3\,c\right))\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\right.}{}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*(2*(3*b*cos(-a + c) - b*cos(-3*a + 3*c))*x - (3*sin(-a + c) - sin(-3*a
 + 3*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*si
n(b*x)*sin(c) + sin(c)^2) - (3*sin(-a + c) - sin(-3*a + 3*c))*log(cos(b*x)
^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c
)^2) - 2*sin(2*b*x + 3*a - c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2488 vs. $2(57) = 114$.

Time = 0.16 (sec) , antiderivative size = 2488, normalized size of antiderivative = 39.49

$$\int \csc(c+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2*((tan(1/2*a)^6*tan(1/2*c)^6 + 9*tan(1/2*a)^6*tan(1/2*c)^4 - 12*tan(1/2
*a)^5*tan(1/2*c)^5 + 9*tan(1/2*a)^4*tan(1/2*c)^6 - 9*tan(1/2*a)^6*tan(1/2*
c)^2 + 72*tan(1/2*a)^5*tan(1/2*c)^3 - 111*tan(1/2*a)^4*tan(1/2*c)^4 + 72*t
an(1/2*a)^3*tan(1/2*c)^5 - 9*tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^6 - 12
*tan(1/2*a)^5*tan(1/2*c) + 111*tan(1/2*a)^4*tan(1/2*c)^2 - 176*tan(1/2*a)^
3*tan(1/2*c)^3 + 111*tan(1/2*a)^2*tan(1/2*c)^4 - 12*tan(1/2*a)*tan(1/2*c)^
5 - tan(1/2*c)^6 - 9*tan(1/2*a)^4 + 72*tan(1/2*a)^3*tan(1/2*c) - 111*tan(1
/2*a)^2*tan(1/2*c)^2 + 72*tan(1/2*a)*tan(1/2*c)^3 - 9*tan(1/2*c)^4 + 9*tan
(1/2*a)^2 - 12*tan(1/2*a)*tan(1/2*c) + 9*tan(1/2*c)^2 + 1)*(b*x + c)/(tan(
1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a)^4*tan(1
/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^4 + 3*ta
n(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^2 + 9*t
an(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2*a)^2*
tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1) - 8*(
tan(1/2*a)^6*tan(1/2*c)^3 - 3*tan(1/2*a)^5*tan(1/2*c)^4 + 3*tan(1/2*a)^4*t
an(1/2*c)^5 - tan(1/2*a)^3*tan(1/2*c)^6 + 3*tan(1/2*a)^5*tan(1/2*c)^2 - 9*
tan(1/2*a)^4*tan(1/2*c)^3 + 9*tan(1/2*a)^3*tan(1/2*c)^4 - 3*tan(1/2*a)^2*t
an(1/2*c)^5 + 3*tan(1/2*a)^4*tan(1/2*c) - 9*tan(1/2*a)^3*tan(1/2*c)^2 + 9*
tan(1/2*a)^2*tan(1/2*c)^3 - 3*tan(1/2*a)*tan(1/2*c)^4 + tan(1/2*a)^3 - 3*t
an(1/2*a)^2*tan(1/2*c) + 3*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3)*log(...
```

Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.68

$$\int \csc(c+bx)\sin^3(a+bx)\,dx$$
$$= -\frac{e^{-a\,3i+c\,1i-b\,x\,2i}\,1i}{8\,b}+\frac{e^{a\,3i-c\,1i+b\,x\,2i}\,1i}{8\,b}+\frac{x\,e^{-a\,3i+c\,1i}\,(3\,e^{a\,2i}-e^{c\,2i})}{4}$$
$$-\frac{e^{-a\,6i+c\,6i}\,\ln\left(e^{a\,2i}\,e^{b\,x\,2i}-e^{a\,2i}\,e^{-c\,2i}\right)\,(b\,e^{a\,3i-c\,3i}\,8i-b\,e^{a\,5i-c\,5i}\,24i+b\,e^{a\,7i-c\,7i}\,24i-b\,e^{a\,9i-c\,9i}\,8i)}{64\,b^2}$$

input
```
int(sin(a + b*x)^3/sin(c + b*x),x)
```

output
```
(exp(a*3i - c*1i + b*x*2i)*1i)/(8*b) - (exp(c*1i - a*3i - b*x*2i)*1i)/(8*b
) + (x*exp(c*1i - a*3i)*(3*exp(a*2i) - exp(c*2i)))/4 - (exp(c*6i - a*6i)*l
og(exp(a*2i)*exp(b*x*2i) - exp(a*2i)*exp(-c*2i))*(b*exp(a*3i - c*3i)*8i -
b*exp(a*5i - c*5i)*24i + b*exp(a*7i - c*7i)*24i - b*exp(a*9i - c*9i)*8i))/
(64*b^2)
```

Reduce [F]

$$\int \csc(c+bx)\sin^3(a+bx)\,dx = \int \csc\left(bx+c\right)\sin\left(bx+a\right)^3 dx$$

input
```
int(csc(b*x+c)*sin(b*x+a)^3,x)
```

output
```
int(csc(b*x + c)*sin(a + b*x)**3,x)
```

3.98 $\int \csc^2(c+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 353

$$\begin{aligned}\int \csc^2(c+bx)\sin^3(a+bx)\,dx = & -\frac{e^{-3ia+2ic-ibx}}{2b}-\frac{e^{i(3a-2c)+ibx}}{2b}+\frac{3e^{ia+ibx}}{4b\left(1-e^{2i(c+bx)}\right)} \\ & -\frac{3e^{-i(a-2c)+ibx}}{4b\left(1-e^{2i(c+bx)}\right)}-\frac{e^{i(3a-2c)+ibx}}{4b\left(1-e^{2i(c+bx)}\right)} \\ & +\frac{e^{-3ia+4ic+ibx}}{4b\left(1-e^{2i(c+bx)}\right)}-\frac{3e^{-i(a-c)}\operatorname{arctanh}\left(e^{ic+ibx}\right)}{4b} \\ & -\frac{3e^{i(a-c)}\operatorname{arctanh}\left(e^{ic+ibx}\right)}{4b}+\frac{3e^{-3i(a-c)}\operatorname{arctanh}\left(e^{ic+ibx}\right)}{4b} \\ & +\frac{3e^{3i(a-c)}\operatorname{arctanh}\left(e^{ic+ibx}\right)}{4b}\end{aligned}$$

output
```
-1/2*exp(-3*I*a+2*I*c-I*b*x)/b-1/2*exp(I*(3*a-2*c)+I*b*x)/b+3/4*exp(I*a+I*
b*x)/b/(1-exp(2*I*(b*x+c)))-3/4*exp(-I*(a-2*c)+I*b*x)/b/(1-exp(2*I*(b*x+c)
))-1/4*exp(I*(3*a-2*c)+I*b*x)/b/(1-exp(2*I*(b*x+c)))+1/4*exp(-3*I*a+4*I*c+
I*b*x)/b/(1-exp(2*I*(b*x+c)))-3/4*arctanh(exp(I*c+I*b*x))/b/exp(I*(a-c))-3
/4*exp(I*(a-c))*arctanh(exp(I*c+I*b*x))/b+3/4*arctanh(exp(I*c+I*b*x))/b/ex
p(3*I*(a-c))+3/4*exp(3*I*(a-c))*arctanh(exp(I*c+I*b*x))/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.38

$$\int \csc^2(c+bx)\sin^3(a+bx)\,dx$$
$$= -\frac{\cos(3a-2c)\cos(bx)}{b} - \frac{6i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)\sin^2(a-c)}{b} - \frac{\csc(c+bx)\sin^3(a-c)}{b} + \frac{\sin(3a-2c)\sin(bx)}{b}$$

input
```
Integrate[Csc[c + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
-((Cos[3*a - 2*c]*Cos[b*x])/b) - ((6*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c
]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/
2]*Sin[c])]*Cos[a - c]*Sin[a - c]^2)/b - (Csc[c + b*x]*Sin[a - c]^3)/b + (
Sin[3*a - 2*c]*Sin[b*x])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^2(bx+c)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \sin^3(a+bx)\csc^2(bx+c)dx$$

input
```
Int[Csc[c + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{e^{i(bx+5a-2c)}-3\,e^{i(bx+3a)}+3\,e^{i(bx+a+2c)}-e^{-i(-bx+a-4c)}}{4\left(-e^{2i(bx+a+c)}+e^{2ia}\right)b}+\frac{3\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos(a-c)}{4b}-\frac{3\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos(3}{4b}$
default	Expression too large to display

input
```
int(csc(b*x+c)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(-exp(2*I*(b*x+a+c))+exp(2*I*a))/b*(exp(I*(b*x+5*a-2*c))-3*exp(I*(b*x
+3*a))+3*exp(I*(b*x+a+2*c))-exp(-I*(-b*x+a-4*c)))+3/4*ln(exp(I*(b*x+a))-ex
p(I*(a-c)))/b*cos(a-c)-3/4*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*cos(3*a-3*c)-
3/4*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)+3/4*ln(exp(I*(b*x+a))+exp(I
*(a-c)))/b*cos(3*a-3*c)-cos(b*x+3*a-2*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.45

$$\int \csc^2(c+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{2\left(4\cos\left(-a+c\right)^3-3\cos\left(-a+c\right)\right)\cos\left(bx+c\right)\sin\left(bx+c\right)-3\left(\cos\left(-a+c\right)^3-\cos\left(-a+c\right)\right)\log\left(\right.}{}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)*sin(b*x + c) - 3*(c
os(-a + c)^3 - cos(-a + c))*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) + 3*(
cos(-a + c)^3 - cos(-a + c))*log(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - 2
*((4*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - 5*cos(-a + c)^2 + 2)*sin(-a + c))
/(b*sin(b*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(c+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**2*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1066 vs. $2(244) = 488$.

Time = 0.08 (sec) , antiderivative size = 1066, normalized size of antiderivative = 3.02

$$\int \csc^2(c+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(4*(cos(3*b*x + 3*a + 4*c) - cos(b*x + 3*a + 2*c))*cos(4*b*x + 6*a +
2*c) - 2*(3*cos(2*b*x + 6*a) - 3*cos(2*b*x + 4*a + 2*c) + 3*cos(2*b*x + 2*
a + 4*c) - 3*cos(2*b*x + 6*c) + 2*cos(4*c))*cos(3*b*x + 3*a + 4*c) + 6*cos
(2*b*x + 6*a)*cos(b*x + 3*a + 2*c) - 6*cos(2*b*x + 4*a + 2*c)*cos(b*x + 3*
a + 2*c) + 6*cos(2*b*x + 2*a + 4*c)*cos(b*x + 3*a + 2*c) - 6*cos(2*b*x + 6
*c)*cos(b*x + 3*a + 2*c) + 4*cos(b*x + 3*a + 2*c)*cos(4*c) + 3*((cos(-a +
c) - cos(-3*a + 3*c))*cos(3*b*x + 3*a + 4*c)^2 - 2*(cos(-a + c) - cos(-3*a
 + 3*c))*cos(3*b*x + 3*a + 4*c)*cos(b*x + 3*a + 2*c) + (cos(-a + c) - cos(
-3*a + 3*c))*cos(b*x + 3*a + 2*c)^2 + (cos(-a + c) - cos(-3*a + 3*c))*sin(
3*b*x + 3*a + 4*c)^2 - 2*(cos(-a + c) - cos(-3*a + 3*c))*sin(3*b*x + 3*a +
 4*c)*sin(b*x + 3*a + 2*c) + (cos(-a + c) - cos(-3*a + 3*c))*sin(b*x + 3*a
 + 2*c)^2)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*
sin(b*x)*sin(c) + sin(c)^2) - 3*((cos(-a + c) - cos(-3*a + 3*c))*cos(3*b*x
 + 3*a + 4*c)^2 - 2*(cos(-a + c) - cos(-3*a + 3*c))*cos(3*b*x + 3*a + 4*c)
*cos(b*x + 3*a + 2*c) + (cos(-a + c) - cos(-3*a + 3*c))*cos(b*x + 3*a + 2*
c)^2 + (cos(-a + c) - cos(-3*a + 3*c))*sin(3*b*x + 3*a + 4*c)^2 - 2*(cos(-
a + c) - cos(-3*a + 3*c))*sin(3*b*x + 3*a + 4*c)*sin(b*x + 3*a + 2*c) + (c
os(-a + c) - cos(-3*a + 3*c))*sin(b*x + 3*a + 2*c)^2)*log(cos(b*x)^2 - 2*c
os(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) + 4
*(sin(3*b*x + 3*a + 4*c) - sin(b*x + 3*a + 2*c))*sin(4*b*x + 6*a + 2*c)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2919 vs. $2(244) = 488$.

Time = 0.19 (sec) , antiderivative size = 2919, normalized size of antiderivative = 8.27

$$\int \csc^2(c+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
2*(6*(tan(1/2*a)^6*tan(1/2*c)^4 - 2*tan(1/2*a)^5*tan(1/2*c)^5 + tan(1/2*a)
^4*tan(1/2*c)^6 - tan(1/2*a)^6*tan(1/2*c)^2 + 8*tan(1/2*a)^5*tan(1/2*c)^3
- 14*tan(1/2*a)^4*tan(1/2*c)^4 + 8*tan(1/2*a)^3*tan(1/2*c)^5 - tan(1/2*a)^
2*tan(1/2*c)^6 - 2*tan(1/2*a)^5*tan(1/2*c) + 14*tan(1/2*a)^4*tan(1/2*c)^2
- 24*tan(1/2*a)^3*tan(1/2*c)^3 + 14*tan(1/2*a)^2*tan(1/2*c)^4 - 2*tan(1/2*
a)*tan(1/2*c)^5 - tan(1/2*a)^4 + 8*tan(1/2*a)^3*tan(1/2*c) - 14*tan(1/2*a)
^2*tan(1/2*c)^2 + 8*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + tan(1/2*a)^2
- 2*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2)*log(abs(tan(1/2*b*x + 1/2*c)))/(
tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a)^4*t
an(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^4 +
3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^2 +
 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2*a
)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1) -
 2*(tan(1/2*b*x + 1/2*c)*tan(1/2*a)^6*tan(1/2*c)^3 - 3*tan(1/2*b*x + 1/2*c
)*tan(1/2*a)^5*tan(1/2*c)^4 + 3*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*
c)^5 - tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^6 + 3*tan(1/2*b*x + 1/
2*c)*tan(1/2*a)^5*tan(1/2*c)^2 - 9*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1
/2*c)^3 + 9*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^4 - 3*tan(1/2*b*x
 + 1/2*c)*tan(1/2*a)^2*tan(1/2*c)^5 + 3*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^4*
tan(1/2*c) - 9*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^2 + 9*tan(1...
```

Mupad [B] (verification not implemented)

Time = 24.17 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.10

$$\int \csc^2(c+bx)\sin^3(a+bx)\,dx = -\frac{e^{-a\,3i+c\,2i-b\,x\,1i}}{2\,b} - \frac{e^{a\,3i-c\,2i+b\,x\,1i}}{2\,b} - \frac{e^{-a\,1i+c\,2i+b\,x\,1i}\,\left(3\,e^{a\,2i-c\,2i}-3\,e^{a\,4i-c\,4i}+e^{a\,6i-c\,6i}-1\right)\,1i}{4\,b\,\left(e^{a\,2i-c\,2i}\,1i-e^{a\,2i+b\,x\,2i}\,1i\right)} - \frac{3\,\sin\left(2\,a-2\,c\right)\,\ln\left(\frac{3\,e^{a\,1i}\,e^{b\,x\,1i}\,\left(\sin(2\,a-2\,c)-\sin(2\,a-2\,c)\,e^{a\,2i}\,e^{-c\,2i}\right)}{2}-\frac{\sin(2\,a-2\,c)\,e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,3i}{2\,\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}\right.}{4\,b\,\sqrt{-e^{a\,2i-c\,2i}}} + \frac{3\,\sin\left(2\,a-2\,c\right)\,\ln\left(\frac{3\,e^{a\,1i}\,e^{b\,x\,1i}\,\left(\sin(2\,a-2\,c)-\sin(2\,a-2\,c)\,e^{a\,2i}\,e^{-c\,2i}\right)}{2}+\frac{\sin(2\,a-2\,c)\,e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,3i}{2\,\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}\right.}{4\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

input
```
int(sin(a + b*x)^3/sin(c + b*x)^2,x)
```

output
```
(3*sin(2*a - 2*c)*log((3*exp(a*1i)*exp(b*x*1i)*(sin(2*a - 2*c) - sin(2*a -
 2*c)*exp(a*2i)*exp(-c*2i)))/2 + (sin(2*a - 2*c)*exp(a*2i)*exp(-c*2i)*(exp
(a*2i)*exp(-c*2i) - 1)*3i)/(2*(-exp(a*2i)*exp(-c*2i))^(1/2)))*(exp(a*2i -
c*2i) - 1))/(4*b*(-exp(a*2i - c*2i))^(1/2)) - exp(a*3i - c*2i + b*x*1i)/(2
*b) - (exp(c*2i - a*1i + b*x*1i)*(3*exp(a*2i - c*2i) - 3*exp(a*4i - c*4i)
+ exp(a*6i - c*6i) - 1)*1i)/(4*b*(exp(a*2i - c*2i)*1i - exp(a*2i + b*x*2i)
*1i)) - (3*sin(2*a - 2*c)*log((3*exp(a*1i)*exp(b*x*1i)*(sin(2*a - 2*c) - s
in(2*a - 2*c)*exp(a*2i)*exp(-c*2i)))/2 - (sin(2*a - 2*c)*exp(a*2i)*exp(-c*
2i)*(exp(a*2i)*exp(-c*2i) - 1)*3i)/(2*(-exp(a*2i)*exp(-c*2i))^(1/2)))*(exp
(a*2i - c*2i) - 1))/(4*b*(-exp(a*2i - c*2i))^(1/2)) - exp(c*2i - a*3i - b*
x*1i)/(2*b)
```

Reduce [F]

$$\int \csc^2(c+bx)\sin^3(a+bx)\,dx = \int \csc(bx+c)^2 \sin(bx+a)^3\,dx$$

input
```
int(csc(b*x+c)^2*sin(b*x+a)^3,x)
```

output
```
int(csc(b*x + c)**2*sin(a + b*x)**3,x)
```

3.99 $\int \csc^3(c+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 313

$$\int \csc^3(c+bx)\sin^3(a+bx)\,dx = \frac{3ie^{-i(a-c)}}{4b\left(1-e^{2i(c+bx)}\right)^2}-\frac{ie^{-3i(a-c)}}{4b\left(1-e^{2i(c+bx)}\right)^2}+\frac{ie^{3i(a-c)}}{4b\left(1-e^{2i(c+bx)}\right)^2}-\frac{3ie^{i(a+3c)+4ibx}}{4b\left(1-e^{2i(c+bx)}\right)^2}-\frac{ie^{-3i(a-c)}}{2b\left(1-e^{2i(c+bx)}\right)}-\frac{ie^{3i(a-c)}}{b\left(1-e^{2i(c+bx)}\right)}+e^{-3i(a-c)}x+\frac{ie^{-3i(a-c)}\log\left(1-e^{2i(c+bx)}\right)}{2b}-\frac{ie^{3i(a-c)}\log\left(1-e^{2i(c+bx)}\right)}{2b}$$

output
```
3/4*I/b/exp(I*(a-c))/(1-exp(2*I*(b*x+c)))^2-1/4*I/b/exp(3*I*(a-c))/(1-exp(
2*I*(b*x+c)))^2+1/4*I*exp(3*I*(a-c))/b/(1-exp(2*I*(b*x+c)))^2-3/4*I*exp(I*
(a+3*c)+4*I*b*x)/b/(1-exp(2*I*(b*x+c)))^2-1/2*I/b/exp(3*I*(a-c))/(1-exp(2*
I*(b*x+c)))-I*exp(3*I*(a-c))/b/(1-exp(2*I*(b*x+c)))+x/exp(3*I*(a-c))+1/2*I
*ln(1-exp(2*I*(b*x+c)))/b/exp(3*I*(a-c))-1/2*I*exp(3*I*(a-c))*ln(1-exp(2*I
*(b*x+c)))/b
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.94

$$\int \csc^3(c+bx)\sin^3(a+bx)\,dx =$$
$$-\frac{\csc\left(\frac{c}{2}\right)\csc^2(c+bx)\sec\left(\frac{c}{2}\right)(-6\cos(a)+4\cos(3a-2c)-3\cos(3a-4c-2bx)+3\cos(a+2bx)-3\,\mathrm{c}}{}$$

input
```
Integrate[Csc[c + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
-1/32*(Csc[c/2]*Csc[c + b*x]^2*Sec[c/2]*(-6*Cos[a] + 4*Cos[3*a - 2*c] - 3*
Cos[3*a - 4*c - 2*b*x] + 3*Cos[a + 2*b*x] - 3*Cos[3*a - 2*c + 2*b*x] + 3*C
os[a - 2*(c + b*x)] + Cos[3*a - 4*c]*(2 - 4*Log[Sin[c + b*x]]) + 4*Cos[3*a
 - 2*c]*Log[Sin[c + b*x]] + 2*Cos[3*a - 6*c - 2*b*x]*Log[Sin[c + b*x]] - 2
*Cos[3*a - 4*c - 2*b*x]*Log[Sin[c + b*x]] - 2*Cos[3*a + 2*b*x]*Log[Sin[c +
 b*x]] + 2*Cos[3*a - 2*c + 2*b*x]*Log[Sin[c + b*x]] + 4*b*x*Sin[3*a - 4*c]
 - 4*b*x*Sin[3*a - 2*c] - 2*b*x*Sin[3*a - 6*c - 2*b*x] + 2*b*x*Sin[3*a - 4
*c - 2*b*x] + 2*b*x*Sin[3*a + 2*b*x] - 2*b*x*Sin[3*a - 2*c + 2*b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^3(bx+c)\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \sin^3(a+bx)\csc^3(bx+c)dx$$

input
```
Int[Csc[c + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.63

method	result
risch	$x\,\mathrm{e}^{3i(a-c)} - 2i\sin(3a-3c)\,x - \frac{2i\sin(3a-3c)a}{b} - \frac{i\left(-4\,\mathrm{e}^{i(2bx+7a-c)}+6\,\mathrm{e}^{i(2bx+5a+c)}-2\,\mathrm{e}^{i(2bx+a+5c)}+3\,\mathrm{e}^{i(7a-3c)}-3\,\mathrm{e}\right.}{4\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{2}b}$
default	Expression too large to display

input
```
int(csc(b*x+c)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
x*exp(3*I*(a-c))-2*I*sin(3*a-3*c)*x-2*I/b*sin(3*a-3*c)*a-1/4*I/(-exp(2*I*(
b*x+a+c))+exp(2*I*a))^2/b*(-4*exp(I*(2*b*x+7*a-c))+6*exp(I*(2*b*x+5*a+c))-
2*exp(I*(2*b*x+a+5*c))+3*exp(I*(7*a-3*c))-3*exp(I*(5*a-c))-3*exp(I*(3*a+c)
)+3*exp(I*(a+3*c)))+1/b*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*sin(3*a-3*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.55

$$\int \csc^3(c+bx)\sin^3(a+bx)\,dx =$$
$$-\frac{8\,bx\cos(-a+c)^3 - 2\left(4\,bx\cos(-a+c)^3 - 3\,bx\cos(-a+c)\right)\cos(bx+c)^2 - 6\,bx\cos(-a+c) + 6\,(c}{}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(8*b*x*cos(-a + c)^3 - 2*(4*b*x*cos(-a + c)^3 - 3*b*x*cos(-a + c))*co
s(b*x + c)^2 - 6*b*x*cos(-a + c) + 6*(cos(-a + c)^3 - cos(-a + c))*cos(b*x
 + c)*sin(b*x + c) + 2*((4*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - 4*cos(-a +
c)^2 + 1)*log(1/2*sin(b*x + c))*sin(-a + c) - (cos(-a + c)^2 - 1)*sin(-a +
 c))/(b*cos(b*x + c)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sin^3(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(csc(b*x+c)**3*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1738 vs. $2(225) = 450$.

Time = 0.10 (sec) , antiderivative size = 1738, normalized size of antiderivative = 5.55

$$\int \csc^3(c+bx)\sin^3(a+bx)\,dx=\text{Too large to display}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(4*(b*cos(3*a + 3*c)*cos(6*c) + b*sin(3*a + 3*c)*sin(6*c))*x - (8*b*x*
cos(2*b*x + 8*c) - 4*b*x*cos(6*c) + 4*sin(2*b*x + 6*a + 2*c) - 6*sin(2*b*x
 + 4*a + 4*c) + 2*sin(2*b*x + 8*c) - 3*sin(6*a) + 3*sin(4*a + 2*c) + 3*sin
(2*a + 4*c) - 3*sin(6*c))*cos(4*b*x + 3*a + 7*c) + 4*(b*x*cos(4*b*x + 3*a
+ 7*c) - 2*b*x*cos(2*b*x + 3*a + 5*c) + b*x*cos(3*a + 3*c))*cos(4*b*x + 10
*c) - 2*(4*b*x*cos(6*c) - 4*sin(2*b*x + 6*a + 2*c) + 6*sin(2*b*x + 4*a + 4
*c) + 3*sin(6*a) - 3*sin(4*a + 2*c) - 3*sin(2*a + 4*c) + 3*sin(6*c))*cos(2
*b*x + 3*a + 5*c) + 2*(8*b*x*cos(2*b*x + 3*a + 5*c) - 4*b*x*cos(3*a + 3*c)
 - 2*sin(2*b*x + 3*a + 5*c) + sin(3*a + 3*c))*cos(2*b*x + 8*c) + 3*(sin(6*
a) - sin(4*a + 2*c) + sin(6*c))*cos(3*a + 3*c) - 2*(cos(4*b*x + 3*a + 7*c)
^2*sin(-3*a + 3*c) + 4*cos(2*b*x + 3*a + 5*c)^2*sin(-3*a + 3*c) - 4*cos(2*
b*x + 3*a + 5*c)*cos(3*a + 3*c)*sin(-3*a + 3*c) + cos(3*a + 3*c)^2*sin(-3*
a + 3*c) + sin(4*b*x + 3*a + 7*c)^2*sin(-3*a + 3*c) + 4*sin(2*b*x + 3*a +
5*c)^2*sin(-3*a + 3*c) - 4*sin(2*b*x + 3*a + 5*c)*sin(3*a + 3*c)*sin(-3*a
+ 3*c) + sin(3*a + 3*c)^2*sin(-3*a + 3*c) - 2*(2*cos(2*b*x + 3*a + 5*c)*si
n(-3*a + 3*c) - cos(3*a + 3*c)*sin(-3*a + 3*c))*cos(4*b*x + 3*a + 7*c) - 2
*(2*sin(2*b*x + 3*a + 5*c)*sin(-3*a + 3*c) - sin(3*a + 3*c)*sin(-3*a + 3*c
))*sin(4*b*x + 3*a + 7*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 +
 sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - 2*(cos(4*b*x + 3*a + 7*c)^2*
sin(-3*a + 3*c) + 4*cos(2*b*x + 3*a + 5*c)^2*sin(-3*a + 3*c) - 4*cos(2*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6427 vs. $2(225) = 450$.

Time = 0.25 (sec) , antiderivative size = 6427, normalized size of antiderivative = 20.53

$$\int \csc^3(c + bx) \sin^3(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
((tan(1/2*a)^6*tan(1/2*c)^6 - 15*tan(1/2*a)^6*tan(1/2*c)^4 + 36*tan(1/2*a)
^5*tan(1/2*c)^5 - 15*tan(1/2*a)^4*tan(1/2*c)^6 + 15*tan(1/2*a)^6*tan(1/2*c
)^2 - 120*tan(1/2*a)^5*tan(1/2*c)^3 + 225*tan(1/2*a)^4*tan(1/2*c)^4 - 120*
tan(1/2*a)^3*tan(1/2*c)^5 + 15*tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^6 +
36*tan(1/2*a)^5*tan(1/2*c) - 225*tan(1/2*a)^4*tan(1/2*c)^2 + 400*tan(1/2*a
)^3*tan(1/2*c)^3 - 225*tan(1/2*a)^2*tan(1/2*c)^4 + 36*tan(1/2*a)*tan(1/2*c
)^5 - tan(1/2*c)^6 + 15*tan(1/2*a)^4 - 120*tan(1/2*a)^3*tan(1/2*c) + 225*t
an(1/2*a)^2*tan(1/2*c)^2 - 120*tan(1/2*a)*tan(1/2*c)^3 + 15*tan(1/2*c)^4 -
 15*tan(1/2*a)^2 + 36*tan(1/2*a)*tan(1/2*c) - 15*tan(1/2*c)^2 + 1)*(b*x +
c)/(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a)
^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^
4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)
^2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1
/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 +
1) - 2*(3*tan(1/2*a)^6*tan(1/2*c)^5 - 3*tan(1/2*a)^5*tan(1/2*c)^6 - 10*tan
(1/2*a)^6*tan(1/2*c)^3 + 45*tan(1/2*a)^5*tan(1/2*c)^4 - 45*tan(1/2*a)^4*ta
n(1/2*c)^5 + 10*tan(1/2*a)^3*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c) - 45
*tan(1/2*a)^5*tan(1/2*c)^2 + 150*tan(1/2*a)^4*tan(1/2*c)^3 - 150*tan(1/2*a
)^3*tan(1/2*c)^4 + 45*tan(1/2*a)^2*tan(1/2*c)^5 - 3*tan(1/2*a)*tan(1/2*c)^
6 + 3*tan(1/2*a)^5 - 45*tan(1/2*a)^4*tan(1/2*c) + 150*tan(1/2*a)^3*tan(...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sin^3(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^3/sin(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^3(c+bx)\sin^3(a+bx)\,dx = \int \csc(bx+c)^3 \sin(bx+a)^3\,dx$$

input
```
int(csc(b*x+c)^3*sin(b*x+a)^3,x)
```

output
```
int(csc(b*x + c)**3*sin(a + b*x)**3,x)
```

3.100 $\int \csc^4(c+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 627

$$\begin{aligned}\int \csc^4(c+bx)\sin^3(a+bx)\,dx &= \frac{e^{-i(a-2c)+ibx}}{b\left(1-e^{2i(c+bx)}\right)^3}-\frac{e^{-3ia+4ic+ibx}}{3b\left(1-e^{2i(c+bx)}\right)^3}-\frac{e^{i(a+2c)+3ibx}}{b\left(1-e^{2i(c+bx)}\right)^3}\\
&\quad+\frac{e^{i(3a+2c)+5ibx}}{3b\left(1-e^{2i(c+bx)}\right)^3}+\frac{3e^{ia+ibx}}{4b\left(1-e^{2i(c+bx)}\right)^2}\\
&\quad-\frac{e^{-i(a-2c)+ibx}}{4b\left(1-e^{2i(c+bx)}\right)^2}-\frac{5e^{-3ia+4ic+ibx}}{12b\left(1-e^{2i(c+bx)}\right)^2}\\
&\quad-\frac{5e^{3ia+3ibx}}{12b\left(1-e^{2i(c+bx)}\right)^2}-\frac{3e^{ia+ibx}}{8b\left(1-e^{2i(c+bx)}\right)}\\
&\quad-\frac{3e^{-i(a-2c)+ibx}}{8b\left(1-e^{2i(c+bx)}\right)}+\frac{5e^{i(3a-2c)+ibx}}{8b\left(1-e^{2i(c+bx)}\right)}\\
&\quad-\frac{5e^{-3ia+4ic+ibx}}{8b\left(1-e^{2i(c+bx)}\right)}-\frac{3e^{-i(a-c)}\operatorname{arctanh}\left(e^{ic+ibx}\right)}{8b}\\
&\quad-\frac{3e^{i(a-c)}\operatorname{arctanh}\left(e^{ic+ibx}\right)}{8b}-\frac{5e^{-3i(a-c)}\operatorname{arctanh}\left(e^{ic+ibx}\right)}{8b}\\
&\quad-\frac{5e^{3i(a-c)}\operatorname{arctanh}\left(e^{ic+ibx}\right)}{8b}\end{aligned}$$

output
```
exp(-I*(a-2*c)+I*b*x)/b/(1-exp(2*I*(b*x+c)))^3-1/3*exp(-3*I*a+4*I*c+I*b*x)
/b/(1-exp(2*I*(b*x+c)))^3-exp(I*(a+2*c)+3*I*b*x)/b/(1-exp(2*I*(b*x+c)))^3+
1/3*exp(I*(3*a+2*c)+5*I*b*x)/b/(1-exp(2*I*(b*x+c)))^3+3/4*exp(I*a+I*b*x)/b
/(1-exp(2*I*(b*x+c)))^2-1/4*exp(-I*(a-2*c)+I*b*x)/b/(1-exp(2*I*(b*x+c)))^2
-5/12*exp(-3*I*a+4*I*c+I*b*x)/b/(1-exp(2*I*(b*x+c)))^2-5/12*exp(3*I*a+3*I*
b*x)/b/(1-exp(2*I*(b*x+c)))^2-3/8*exp(I*a+I*b*x)/b/(1-exp(2*I*(b*x+c)))-3/
8*exp(-I*(a-2*c)+I*b*x)/b/(1-exp(2*I*(b*x+c)))+5/8*exp(I*(3*a-2*c)+I*b*x)/
b/(1-exp(2*I*(b*x+c)))-5/8*exp(-3*I*a+4*I*c+I*b*x)/b/(1-exp(2*I*(b*x+c)))-
3/8*arctanh(exp(I*c+I*b*x))/b/exp(I*(a-c))-3/8*exp(I*(a-c))*arctanh(exp(I*
c+I*b*x))/b-5/8*arctanh(exp(I*c+I*b*x))/b/exp(3*I*(a-c))-5/8*exp(3*I*(a-c)
)*arctanh(exp(I*c+I*b*x))/b
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.17

$$\int \csc^4(c+bx)\sin^3(a+bx)\,dx$$
$$= \frac{-12\text{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)(3\cos(a-c)+5\cos(3(a-c)))+(-32-40\cos(2(a-c))+15\,\text{co}}{48b}$$

input
```
Integrate[Csc[c + b*x]^4*Sin[a + b*x]^3,x]
```

output
```
(-12*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*(3*Cos[a - c] + 5*Cos[3*(a - c)
]) + (-32 - 40*Cos[2*(a - c)] + 15*Cos[2*(a - 2*c - b*x)] + 33*Cos[2*(a +
b*x)] + 24*Cos[2*(c + b*x)])*Csc[c + b*x]^3*Sin[a - c])/(48*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^4(bx+c)\,dx$$
$$\Big\downarrow \ 7299$$

$$\int \sin^3(a+bx)\csc^4(bx+c)dx$$

input
```
Int[Csc[c + b*x]^4*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 6.58 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.56

method	result
risch	$\frac{33\,\mathrm{e}^{i(5bx+9a+2c)}-9\,\mathrm{e}^{i(5bx+7a+4c)}-9\,\mathrm{e}^{i(5bx+5a+6c)}-15\,\mathrm{e}^{i(5bx+3a+8c)}-40\,\mathrm{e}^{3i(bx+3a)}-24\,\mathrm{e}^{i(3bx+7a+2c)}+24\,\mathrm{e}^{i(3bx+5a+4c)}+40\,\mathrm{e}^{3i(b}}{24\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{3}b}$
default	Expression too large to display

input
```
int(csc(b*x+c)^4*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/24/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^3/b*(33*exp(I*(5*b*x+9*a+2*c))-9*exp
(I*(5*b*x+7*a+4*c))-9*exp(I*(5*b*x+5*a+6*c))-15*exp(I*(5*b*x+3*a+8*c))-40*
exp(3*I*(b*x+3*a))-24*exp(I*(3*b*x+7*a+2*c))+24*exp(I*(3*b*x+5*a+4*c))+40*
exp(3*I*(b*x+a+2*c))+15*exp(I*(b*x+9*a-2*c))+9*exp(I*(b*x+7*a))+9*exp(I*(b
*x+5*a+2*c))-33*exp(I*(b*x+3*a+4*c)))-3/8*ln(exp(I*(b*x+a))+exp(I*(a-c)))/
b*cos(a-c)-5/8*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(3*a-3*c)+3/8*ln(exp(I
*(b*x+a))-exp(I*(a-c)))/b*cos(a-c)+5/8*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*c
os(3*a-3*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.37

$$\int \csc^4(c+bx)\sin^3(a+bx)\,dx = -\frac{18\left(\cos(-a+c)^3-\cos(-a+c)\right)\cos(bx+c)\sin(bx+c)+3\left(\left(5\cos(-a+c)^3-3\cos(-a+c)\right)\cos\right.}{}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/12*(18*(cos(-a + c)^3 - cos(-a + c))*cos(b*x + c)*sin(b*x + c) + 3*((5*
cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^2 - 5*cos(-a + c)^3 + 3*cos(-a
 + c))*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - 3*((5*cos(-a + c)^3 - 3*
cos(-a + c))*cos(b*x + c)^2 - 5*cos(-a + c)^3 + 3*cos(-a + c))*log(-1/2*co
s(b*x + c) + 1/2)*sin(b*x + c) - 4*(3*(4*cos(-a + c)^2 - 1)*cos(b*x + c)^2
 - 11*cos(-a + c)^2 + 2)*sin(-a + c))/((b*cos(b*x + c)^2 - b)*sin(b*x + c)
)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(c+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**4*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3532 vs. 2(443) = 886.

Time = 0.19 (sec) , antiderivative size = 3532, normalized size of antiderivative = 5.63

$$\int \csc^4(c+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate(csc(b*x+c)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/48*(2*(33*cos(5*b*x + 6*a + 4*c) - 9*cos(5*b*x + 4*a + 6*c) - 9*cos(5*b
*x + 2*a + 8*c) - 15*cos(5*b*x + 10*c) - 40*cos(3*b*x + 6*a + 2*c) - 24*co
s(3*b*x + 4*a + 4*c) + 24*cos(3*b*x + 2*a + 6*c) + 40*cos(3*b*x + 8*c) + 1
5*cos(b*x + 6*a) + 9*cos(b*x + 4*a + 2*c) + 9*cos(b*x + 2*a + 4*c) - 33*co
s(b*x + 6*c))*cos(6*b*x + 3*a + 8*c) - 66*(3*cos(4*b*x + 3*a + 6*c) - 3*co
s(2*b*x + 3*a + 4*c) + cos(3*a + 2*c))*cos(5*b*x + 6*a + 4*c) + 18*(3*cos(
4*b*x + 3*a + 6*c) - 3*cos(2*b*x + 3*a + 4*c) + cos(3*a + 2*c))*cos(5*b*x
+ 4*a + 6*c) + 18*(3*cos(4*b*x + 3*a + 6*c) - 3*cos(2*b*x + 3*a + 4*c) + c
os(3*a + 2*c))*cos(5*b*x + 2*a + 8*c) + 30*(3*cos(4*b*x + 3*a + 6*c) - 3*c
os(2*b*x + 3*a + 4*c) + cos(3*a + 2*c))*cos(5*b*x + 10*c) + 6*(40*cos(3*b*
x + 6*a + 2*c) + 24*cos(3*b*x + 4*a + 4*c) - 24*cos(3*b*x + 2*a + 6*c) - 4
0*cos(3*b*x + 8*c) - 15*cos(b*x + 6*a) - 9*cos(b*x + 4*a + 2*c) - 9*cos(b*
x + 2*a + 4*c) + 33*cos(b*x + 6*c))*cos(4*b*x + 3*a + 6*c) - 80*(3*cos(2*b
*x + 3*a + 4*c) - cos(3*a + 2*c))*cos(3*b*x + 6*a + 2*c) - 48*(3*cos(2*b*x
 + 3*a + 4*c) - cos(3*a + 2*c))*cos(3*b*x + 4*a + 4*c) + 48*(3*cos(2*b*x +
 3*a + 4*c) - cos(3*a + 2*c))*cos(3*b*x + 2*a + 6*c) + 80*(3*cos(2*b*x + 3
*a + 4*c) - cos(3*a + 2*c))*cos(3*b*x + 8*c) + 18*(5*cos(b*x + 6*a) + 3*co
s(b*x + 4*a + 2*c) + 3*cos(b*x + 2*a + 4*c) - 11*cos(b*x + 6*c))*cos(2*b*x
 + 3*a + 4*c) - 30*cos(b*x + 6*a)*cos(3*a + 2*c) - 18*cos(b*x + 4*a + 2*c)
*cos(3*a + 2*c) - 18*cos(b*x + 2*a + 4*c)*cos(3*a + 2*c) + 66*cos(b*x +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16081 vs. $2(443) = 886$.

Time = 0.31 (sec) , antiderivative size = 16081, normalized size of antiderivative = 25.65

$$\int \csc^4(c+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/6*(6*(tan(1/2*a)^6*tan(1/2*c)^6 - 9*tan(1/2*a)^6*tan(1/2*c)^4 + 24*tan(1
/2*a)^5*tan(1/2*c)^5 - 9*tan(1/2*a)^4*tan(1/2*c)^6 + 9*tan(1/2*a)^6*tan(1/
2*c)^2 - 72*tan(1/2*a)^5*tan(1/2*c)^3 + 141*tan(1/2*a)^4*tan(1/2*c)^4 - 72
*tan(1/2*a)^3*tan(1/2*c)^5 + 9*tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^6 +
24*tan(1/2*a)^5*tan(1/2*c) - 141*tan(1/2*a)^4*tan(1/2*c)^2 + 256*tan(1/2*a
)^3*tan(1/2*c)^3 - 141*tan(1/2*a)^2*tan(1/2*c)^4 + 24*tan(1/2*a)*tan(1/2*c
)^5 - tan(1/2*c)^6 + 9*tan(1/2*a)^4 - 72*tan(1/2*a)^3*tan(1/2*c) + 141*tan
(1/2*a)^2*tan(1/2*c)^2 - 72*tan(1/2*a)*tan(1/2*c)^3 + 9*tan(1/2*c)^4 - 9*t
an(1/2*a)^2 + 24*tan(1/2*a)*tan(1/2*c) - 9*tan(1/2*c)^2 + 1)*log(abs(tan(1
/2*b*x + 1/2*c)))/(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4
 + 3*tan(1/2*a)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a
)^4*tan(1/2*c)^4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*
a)^4*tan(1/2*c)^2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2
*a)^4 + 9*tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*
tan(1/2*c)^2 + 1) - (2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^18*tan(1/2*c)^15
- 6*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^17*tan(1/2*c)^16 - 9*tan(1/2*b*x + 1
/2*c)^2*tan(1/2*a)^18*tan(1/2*c)^16 + 6*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^
16*tan(1/2*c)^17 + 18*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^17*tan(1/2*c)^17 +
 18*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^18*tan(1/2*c)^17 - 2*tan(1/2*b*x + 1/2
*c)^3*tan(1/2*a)^15*tan(1/2*c)^18 - 9*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+bx)\sin^3(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^3/sin(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^4(c+bx)\sin^3(a+bx)\,dx = \int \csc(bx+c)^4 \sin(bx+a)^3\,dx$$

input
```
int(csc(b*x+c)^4*sin(b*x+a)^3,x)
```

output
```
int(csc(b*x + c)**4*sin(a + b*x)**3,x)
```

3.101 $\int \csc(a+bx)\csc(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \csc(a+bx)\csc(c+bx)\,dx = -\frac{\csc(a-c)\log(\sin(a+bx))}{b} + \frac{\csc(a-c)\log(\sin(c+bx))}{b}$$

output
```
-csc(a-c)*ln(sin(b*x+a))/b+csc(a-c)*ln(sin(b*x+c))/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \csc(a+bx)\csc(c+bx)\,dx = -\frac{\csc(a-c)(\log(\sin(a+bx)) - \log(\sin(c+bx)))}{b}$$

input
```
Integrate[Csc[a + b*x]*Csc[c + b*x],x]
```

output
```
-((Csc[a - c]*(Log[Sin[a + b*x]] - Log[Sin[c + b*x]]))/b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5122, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \text{5122}$$

$$\csc(a-c)\int \cot(c+bx)dx-\csc(a-c)\int \cot(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a-c)\int -\tan\left(c+bx+\frac{\pi}{2}\right)dx-\csc(a-c)\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\csc(a-c)\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\csc(a-c)\int \tan\left(\frac{1}{2}(2c+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\csc(a-c)\log(-\sin(bx+c))}{b}-\frac{\csc(a-c)\log(-\sin(a+bx))}{b}$$

input
```
Int[Csc[a + b*x]*Csc[c + b*x],x]
```

output
```
-((Csc[a - c]*Log[-Sin[a + b*x]])/b) + (Csc[a - c]*Log[-Sin[c + b*x]])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5122
```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[Csc[(
b*c - a*d)/b]   Int[Cot[a + b*x], x], x] - Simp[Csc[(b*c - a*d)/d]   Int[Co
t[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*
c - a*d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.51 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

method	result	size
default	$\frac{-\frac{\ln(\tan(bx+a))}{\sin(a)\cos(c)-\cos(a)\sin(c)}+\frac{\ln(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))}{\sin(a)\cos(c)-\cos(a)\sin(c)}}{b}$	79
risch	$-\frac{2i\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)\mathrm{e}^{i(a+c)}}{\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)b}+\frac{2i\ln\left(\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{i(a+c)}}{\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)b}$	92

input
```
int(csc(b*x+a)*csc(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/(sin(a)*cos(c)-cos(a)*sin(c))*ln(tan(b*x+a))+1/(sin(a)*cos(c)-cos(
a)*sin(c))*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)-sin(a)*cos
(c)+cos(a)*sin(c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.06

$$\int \csc(a+bx)\csc(c+bx)\,dx =$$

$$-\frac{\log\left(-\frac{1}{4}\cos\left(bx+c\right)^{2}+\frac{1}{4}\right)-\log\left(-\frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(2\cos(-a+c)^{2}-1\right)\cos(bx+c)^{2}-\cos(-a+\cdots}{\cos(-a+c)^{2}+2\cos(-a+c)+1}\cdots\right.}{2\,b\sin\left(-a+c\right)}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(log(-1/4*cos(b*x + c)^2 + 1/4) - log(-(2*cos(b*x + c)*cos(-a + c)*si
n(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c
)^2)/(cos(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*sin(-a + c))
```

Sympy [F]

$$\int \csc(a+bx)\csc(c+bx)\,dx = \int \csc\left(a+bx\right)\csc\left(bx+c\right)\,dx$$

input
```
integrate(csc(b*x+a)*csc(b*x+c),x)
```

output
```
Integral(csc(a + b*x)*csc(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(36) = 72$.

Time = 0.06 (sec) , antiderivative size = 564, normalized size of antiderivative = 15.67

$$\int \csc(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="maxima")
```

output
```
-(2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*
arctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + 2*((cos(2*a) - cos(2*c))*c
os(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(b*x) - sin(a), c
os(b*x) + cos(a)) - 2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(
2*c))*sin(a + c))*arctan2(sin(b*x) + sin(c), cos(b*x) - cos(c)) - 2*((cos(
2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(si
n(b*x) - sin(c), cos(b*x) + cos(c)) - ((sin(2*a) - sin(2*c))*cos(a + c) -
(cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos
(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - ((sin(2*a) - sin(2*c)
)*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 - 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + ((sin(
2*a) - sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*
x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin
(c)^2) + ((sin(2*a) - sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a +
 c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*
x)*sin(c) + sin(c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a
)*sin(2*c) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(36) = 72$.

Time = 0.17 (sec) , antiderivative size = 396, normalized size of antiderivative = 11.00

$$\int \csc(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="giac")
```

output
```
1/2*((tan(1/2*a)^4*tan(1/2*c)^4 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)
^4 + 4*tan(1/2*a)^3*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4
+ 4*tan(1/2*a)*tan(1/2*c) + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c
)^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2
*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/
2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(1/
2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*tan(1/2*a
)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 -
 tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)^2 + ta
n(1/2*c)^3 + tan(1/2*a) - tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 + tan(1
/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)))/(tan(1/2*a)^2*tan(1/2*c
) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 22.11 (sec) , antiderivative size = 249, normalized size of antiderivative = 6.92

$$\int \csc(a+bx)\csc(c+bx)\,dx$$
$$= \frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}\left(\ln\left(\frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}\,\left(-4\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,4\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\right)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}-1\right)}-\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\,4\mathrm{i}\right)-\ln\left(\frac{2\sqrt{}\ldots}{}\right.\right.}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-1\right)}$$

input
```
int(1/(sin(a + b*x)*sin(c + b*x)),x)
```

output
```
(2*(-exp(a*2i - c*2i))^(1/2)*(log((2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(2*b*ex
p(a*2i)*exp(b*x*2i) - 4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*4i)*exp(-c*2i)*
exp(b*x*2i)))/(b*(exp(a*2i)*exp(-c*2i) - 1)) - exp(a*1i)*exp(a*2i)*exp(-c*
1i)*exp(b*x*2i)*4i) - log((2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(2*b*exp(a*2i)*
exp(b*x*2i) - 4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*
2i)))/(b - b*exp(a*2i)*exp(-c*2i)) - exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*
x*2i)*4i)))/(b*(exp(a*2i - c*2i) - 1))
```

Reduce [F]

$$\int \csc(a+bx)\csc(c+bx)\,dx = \int \csc(bx+c)\csc(bx+a)\,dx$$

input
```
int(csc(b*x+a)*csc(b*x+c),x)
```

output
```
int(csc(b*x + c)*csc(a + b*x),x)
```

3.102 $\int \csc(a+bx)\csc^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \csc(a+bx)\csc^2(c+bx)\,dx = \frac{\operatorname{arctanh}(\cos(c+bx))\cot(a-c)\csc(a-c)}{b} - \frac{\operatorname{arctanh}(\cos(a+bx))\csc^2(a-c)}{b} - \frac{\csc(a-c)\csc(c+bx)}{b}$$

output
```
arctanh(cos(b*x+c))*cot(a-c)*csc(a-c)/b-arctanh(cos(b*x+a))*csc(a-c)^2/b-c
sc(a-c)*csc(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \csc(a+bx)\csc^2(c+bx)\,dx = -\frac{\csc(a-c)\left(-2\operatorname{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cot(a-c)+\csc(c+bx)+\csc(a-c)\left(\log\left(\cos\left(\frac{1}{2}(a\right.\right.\right.}{b}$$

input
```
Integrate[Csc[a + b*x]*Csc[c + b*x]^2,x]
```

output
```
-((Csc[a - c]*(-2*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cot[a - c] + Csc[c
 + b*x] + Csc[a - c]*(Log[Cos[(a + b*x)/2]] - Log[Sin[(a + b*x)/2]])))/b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \csc(a+bx)\csc^2(bx+c)dx$$

input
```
Int[Csc[a + b*x]*Csc[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(61) = 122$.

Time = 1.02 (sec) , antiderivative size = 300, normalized size of antiderivative = 4.92

method	result
default	$\frac{\frac{\frac{4\left(\left(\frac{\cos(a)\cos(c)}{2}+\frac{\sin(a)\sin(c)}{2}\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-\frac{\sin(a)\cos(c)}{2}+\frac{\cos(a)\sin(c)}{2}\right)}{\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\cos(a)\cos(c)+2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c)}-\frac{2(\cdots}{\cdots}}{(\sin(a)\cos(c)-\cos(a)\sin(c))^2}}{b}$
risch	$-\frac{4\,e^{i(bx+3a+2c)}}{\left(-e^{2i(bx+a+c)}+e^{2ia}\right)\left(e^{2ia}-e^{2ic}\right)b}-\frac{4\ln\left(e^{i(bx+a)}-1\right)e^{2i(a+c)}}{b\left(e^{4ia}-2\,e^{2i(a+c)}+e^{4ic}\right)}-\frac{2\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)e^{i(3a+c)}}{b\left(e^{4ia}-2\,e^{2i(a+c)}+e^{4ic}\right)}-\frac{2\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cdots}{b\left(e^{4ia}-2\,e^{2i(a+c)}+\cdots\right.}$

input
```
int(csc(b*x+a)*csc(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(4/(sin(a)*cos(c)-cos(a)*sin(c))^2*(((1/2*cos(a)*cos(c)+1/2*sin(a)*sin
(c))*tan(1/2*a+1/2*b*x)-1/2*sin(a)*cos(c)+1/2*cos(a)*sin(c))/(cos(c)*sin(a
)*tan(1/2*a+1/2*b*x)^2-sin(c)*cos(a)*tan(1/2*a+1/2*b*x)^2+2*tan(1/2*a+1/2*
b*x)*cos(a)*cos(c)+2*tan(1/2*a+1/2*b*x)*sin(a)*sin(c)-sin(a)*cos(c)+cos(a)
*sin(c))-1/2*(cos(a)*cos(c)+sin(a)*sin(c))/(-cos(c)^2*sin(a)^2-cos(a)^2*co
s(c)^2-sin(a)^2*sin(c)^2-sin(c)^2*cos(a)^2)^(1/2)*arctan(1/2*(2*(sin(a)*co
s(c)-cos(a)*sin(c))*tan(1/2*a+1/2*b*x)+2*cos(a)*cos(c)+2*sin(a)*sin(c))/(-
cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-sin(c)^2*cos(a)^2)^(
1/2)))+1/(sin(a)*cos(c)-cos(a)*sin(c))^2*ln(tan(1/2*a+1/2*b*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(61) = 122.

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.95

$$\int \csc(a+bx)\csc^2(c+bx)\,dx =$$

$$-\frac{\cos(-a+c)\log\left(\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)-\cos(-a+c)\log\left(-\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c}{2(}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(cos(-a + c)*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - cos(-a + c)*l
og(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - log((cos(b*x + c)*cos(-a + c) +
 sin(b*x + c)*sin(-a + c) + 1)/(cos(-a + c) + 1))*sin(b*x + c) + log(-(cos
(b*x + c)*cos(-a + c) + sin(b*x + c)*sin(-a + c) - 1)/(cos(-a + c) + 1))*s
in(b*x + c) + 2*sin(-a + c))/((b*cos(-a + c)^2 - b)*sin(b*x + c))
```

Sympy [F]

$$\int \csc(a+bx)\csc^2(c+bx)\,dx = \int \csc(a+bx)\csc^2(bx+c)\,dx$$

input
```
integrate(csc(b*x+a)*csc(b*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)*csc(b*x + c)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19853 vs. 2(61) = 122.

Time = 0.37 (sec) , antiderivative size = 19853, normalized size of antiderivative = 325.46

$$\int \csc(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c)^2,x, algorithm="maxima")
```

output
```
(4*(cos(4*a)^2 - 4*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)
^2 + 2*cos(4*a)*cos(4*c) + cos(4*c)^2 + sin(4*a)^2 - 4*(sin(4*a) + sin(4*c
))*sin(2*a + 2*c) + 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)
*cos(2*b*x + 2*a + 2*c)*cos(b*x + a + 2*c) - 4*(cos(4*a)^2 - 4*(cos(4*a) +
 cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)^2 + 2*cos(4*a)*cos(4*c) + cos
(4*c)^2 + sin(4*a)^2 - 4*(sin(4*a) + sin(4*c))*sin(2*a + 2*c) + 4*sin(2*a
+ 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)*cos(2*b*x + 4*c)*cos(b*x + a
+ 2*c) + 4*(cos(4*a)^2 - 4*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*
a + 2*c)^2 + 2*cos(4*a)*cos(4*c) + cos(4*c)^2 + sin(4*a)^2 - 4*(sin(4*a) +
 sin(4*c))*sin(2*a + 2*c) + 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin
(4*c)^2)*sin(2*b*x + 2*a + 2*c)*sin(b*x + a + 2*c) - 4*(cos(4*a)^2 - 4*(co
s(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)^2 + 2*cos(4*a)*cos(4*
c) + cos(4*c)^2 + sin(4*a)^2 - 4*(sin(4*a) + sin(4*c))*sin(2*a + 2*c) + 4*
sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)*sin(2*b*x + 4*c)*sin(
b*x + a + 2*c) - 4*(((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + co
s(4*c))*sin(2*a + 2*c))*cos(2*b*x + 2*a + 2*c)^2 + ((sin(4*a) + sin(4*c))*
cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*cos(2*b*x + 4*c)^2
+ ((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a +
2*c))*sin(2*b*x + 2*a + 2*c)^2 + ((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (
cos(4*a) + cos(4*c))*sin(2*a + 2*c))*sin(2*b*x + 4*c)^2 - 2*((cos(2*a)*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1917 vs. $2(61) = 122$.

Time = 0.40 (sec) , antiderivative size = 1917, normalized size of antiderivative = 31.43

$$\int \csc(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c)^2,x, algorithm="giac")
```

output
```
1/4*((tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*
a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c
)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*a)))/
(tan(1/2*a)^4*tan(1/2*c)^2 - 2*tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^2*ta
n(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2*a)^2*tan(1/2*c)^2 + 2*t
an(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2 - 2*tan(1/2*a)*tan(1/2*c) + tan(1/2*
c)^2) - (tan(1/2*a)^6*tan(1/2*c)^6 + tan(1/2*a)^6*tan(1/2*c)^4 + 4*tan(1/2
*a)^5*tan(1/2*c)^5 + tan(1/2*a)^4*tan(1/2*c)^6 - tan(1/2*a)^6*tan(1/2*c)^2
 + 8*tan(1/2*a)^5*tan(1/2*c)^3 + tan(1/2*a)^4*tan(1/2*c)^4 + 8*tan(1/2*a)^
3*tan(1/2*c)^5 - tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^6 + 4*tan(1/2*a)^5
*tan(1/2*c) - tan(1/2*a)^4*tan(1/2*c)^2 + 16*tan(1/2*a)^3*tan(1/2*c)^3 - t
an(1/2*a)^2*tan(1/2*c)^4 + 4*tan(1/2*a)*tan(1/2*c)^5 - tan(1/2*c)^6 - tan(
1/2*a)^4 + 8*tan(1/2*a)^3*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c)^2 + 8*tan(1
/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c)
 + tan(1/2*c)^2 + 1)*log(abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c
) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a
)*tan(1/2*a) - 2*tan(1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c) + 4*tan(
1/2*a)*tan(1/2*c) - 2*tan(1/2*c)^2)/abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^
2*tan(1/2*c) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*
a)^2*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) - 2*tan(1/2*b*x +...
```

Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\csc^2(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(sin(a + b*x)*sin(c + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc(a+bx)\csc^2(c+bx)\,dx = \int \csc{(bx+c)}^2 \csc{(bx+a)}\,dx$$

input
```
int(csc(b*x+a)*csc(b*x+c)^2,x)
```

output
```
int(csc(b*x + c)**2*csc(a + b*x),x)
```

3.103 $\int \csc(a+bx)\csc^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \csc(a+bx)\csc^3(c+bx)\,dx = \frac{\cot(a-c)\cot(c+bx)\csc(a-c)}{b} - \frac{\csc(a-c)\csc^2(c+bx)}{2b} - \frac{\csc^3(a-c)\log(\sin(a+bx))}{b} + \frac{\csc^3(a-c)\log(\sin(c+bx))}{b}$$

output
```
cot(a-c)*cot(b*x+c)*csc(a-c)/b-1/2*csc(a-c)*csc(b*x+c)^2/b-csc(a-c)^3*ln(s
in(b*x+a))/b+csc(a-c)^3*ln(sin(b*x+c))/b
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.87

$$\int \csc(a+bx)\csc^3(c+bx)\,dx = -\csc^3(a-c)\csc(c)\csc^2(c+bx)(2\sin(2a-3c) + (2+4\log(\sin(a+bx)) - 4\log(\sin(c+bx)))\sin(c) - \mathrm{s}$$

input
```
Integrate[Csc[a + b*x]*Csc[c + b*x]^3,x]
```

output
```
-1/8*(Csc[a - c]^3*Csc[c]*Csc[c + b*x]^2*(2*Sin[2*a - 3*c] + (2 + 4*Log[Si
n[a + b*x]] - 4*Log[Sin[c + b*x]])*Sin[c] - Sin[2*a - 3*c - 2*b*x] - Sin[2
*a - c + 2*b*x] + 2*Log[Sin[a + b*x]]*Sin[c + 2*b*x] - 2*Log[Sin[c + b*x]]
*Sin[c + 2*b*x] - 2*Log[Sin[a + b*x]]*Sin[3*c + 2*b*x] + 2*Log[Sin[c + b*x
]]*Sin[3*c + 2*b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc^3(bx+c)\,dx$$
$$\Big\downarrow \ 7299$$
$$\int \csc(a+bx)\csc^3(bx+c)dx$$

input
```
Int[Csc[a + b*x]*Csc[c + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.59

method	result
risch	$\frac{4i\left(-2\,\mathrm{e}^{i(2bx+5a+5c)}+\mathrm{e}^{i(7a+c)}+\mathrm{e}^{i(5a+3c)}\right)}{\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^2\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)^2 b}+\frac{8i\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)\mathrm{e}^{3i(a+c)}}{\left(\mathrm{e}^{6ia}-3\,\mathrm{e}^{2i(2a+c)}+3\,\mathrm{e}^{2i(a+2c)}-\mathrm{e}^{6ic}\right)b}-\frac{8i\ln\left(\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{3i(a+c)}}{\left(\mathrm{e}^{6ia}-3\,\mathrm{e}^{2i(2a+c)}+3\,\mathrm{e}^{2i(a+2c)}-\mathrm{e}^{6ic}\right)b}$
default	$\frac{-\frac{\ln(\tan(bx+a))}{(\sin(a)\cos(c)-\cos(a)\sin(c))^3}+\frac{\ln(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))}{(\sin(a)\cos(c)-\cos(a)\sin(c))^3}-\frac{}{2(\cos(a)\cos(c)+\sin(a)\sin(c))}}{}$

input
```
int(csc(b*x+a)*csc(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
4*I/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^2/(exp(2*I*a)-exp(2*I*c))^2/b*(-2*exp
(I*(2*b*x+5*a+5*c))+exp(I*(7*a+c))+exp(I*(5*a+3*c)))+8*I*ln(exp(2*I*(b*x+a
))-1)/(exp(6*I*a)-3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))-exp(6*I*c))/b*exp(
3*I*(a+c))-8*I*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))/(exp(6*I*a)-3*exp(2*I*(
2*a+c))+3*exp(2*I*(a+2*c))-exp(6*I*c))/b*exp(3*I*(a+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(81) = 162$.

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.45

$$\int \csc(a+bx)\csc^3(c+bx)\,dx = \frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)-\cos(-a+c)^2+\left(\cos(bx+c)^2-1\right)\log\left(-\frac{1}{4}\cos(b\right.}{2\left(\left(b\cos(-a+c)^2-b\right)\cos(bx\right.}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) - cos(-a + c)^2 +
 (cos(b*x + c)^2 - 1)*log(-1/4*cos(b*x + c)^2 + 1/4) - (cos(b*x + c)^2 - 1
)*log(-(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) + (2*cos(-a +
c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2)/(cos(-a + c)^2 + 2*cos(-a + c) +
 1)) + 1)/(((b*cos(-a + c)^2 - b)*cos(b*x + c)^2 - b*cos(-a + c)^2 + b)*si
n(-a + c))
```

Sympy [F]

$$\int \csc(a+bx)\csc^3(c+bx)\,dx = \int \csc(a+bx)\csc^3(bx+c)\,dx$$

input
```
integrate(csc(b*x+a)*csc(b*x+c)**3,x)
```

output
```
Integral(csc(a + b*x)*csc(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105891 vs. $2(81) = 162$.

Time = 1.97 (sec) , antiderivative size = 105891, normalized size of antiderivative = 1275.80

$$\int \csc(a+bx)\csc^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c)^3,x, algorithm="maxima")
```

output
```
4*(9*((sin(4*a) + sin(4*c))*cos(3*a + c) - (cos(4*a) + cos(4*c))*sin(3*a +
 c) + 2*cos(2*a + 2*c)*sin(3*a + c) - 2*cos(3*a + c)*sin(2*a + 2*c))*cos(4
*a + 2*c)^2 + 9*((sin(4*a) + sin(4*c))*cos(3*a + c) + (sin(4*a) - 2*sin(2*
a + 2*c) + sin(4*c))*cos(a + 3*c) - (cos(4*a) + cos(4*c))*sin(3*a + c) + 2
*cos(2*a + 2*c)*sin(3*a + c) - 2*cos(3*a + c)*sin(2*a + 2*c) - (cos(4*a) -
 2*cos(2*a + 2*c) + cos(4*c))*sin(a + 3*c))*cos(2*a + 4*c)^2 + 9*((sin(4*a
) + sin(4*c))*cos(3*a + c) - (cos(4*a) + cos(4*c))*sin(3*a + c) + 2*cos(2*
a + 2*c)*sin(3*a + c) - 2*cos(3*a + c)*sin(2*a + 2*c))*sin(4*a + 2*c)^2 +
2*(cos(6*a)^2 - 2*cos(6*a)*cos(6*c) + cos(6*c)^2 + sin(6*a)^2 - 2*sin(6*a)
*sin(6*c) + sin(6*c)^2)*cos(2*a + 2*c)*sin(3*a + c) + 9*((sin(4*a) + sin(4
*c))*cos(3*a + c) + (sin(4*a) - 2*sin(2*a + 2*c) + sin(4*c))*cos(a + 3*c)
- (cos(4*a) + cos(4*c))*sin(3*a + c) + 2*cos(2*a + 2*c)*sin(3*a + c) - 2*c
os(3*a + c)*sin(2*a + 2*c) - (cos(4*a) - 2*cos(2*a + 2*c) + cos(4*c))*sin(
a + 3*c))*sin(2*a + 4*c)^2 - 2*(cos(6*a)^2 - 2*cos(6*a)*cos(6*c) + cos(6*c
)^2 + sin(6*a)^2 - 2*sin(6*a)*sin(6*c) + sin(6*c)^2)*cos(3*a + c)*sin(2*a
+ 2*c) - 2*(((cos(6*a) - 3*cos(4*a + 2*c) - cos(6*c))*cos(3*a + 3*c) + 3*c
os(3*a + 3*c)*cos(2*a + 4*c) + (sin(6*a) - 3*sin(4*a + 2*c) - sin(6*c))*si
n(3*a + 3*c) + 3*sin(3*a + 3*c)*sin(2*a + 4*c))*cos(4*b*x + 4*a + 4*c)^2 +
 4*((cos(6*a) - 3*cos(4*a + 2*c) - cos(6*c))*cos(3*a + 3*c) + 3*cos(3*a +
3*c)*cos(2*a + 4*c) + (sin(6*a) - 3*sin(4*a + 2*c) - sin(6*c))*sin(3*a ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7053 vs. $2(81) = 162$.

Time = 0.47 (sec) , antiderivative size = 7053, normalized size of antiderivative = 84.98

$$\int \csc(a + bx)\csc^3(c + bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c)^3,x, algorithm="giac")
```

output
```
1/16*(2*(tan(1/2*a)^8*tan(1/2*c)^8 + 2*tan(1/2*a)^8*tan(1/2*c)^6 + 4*tan(1
/2*a)^7*tan(1/2*c)^7 + 2*tan(1/2*a)^6*tan(1/2*c)^8 + 12*tan(1/2*a)^7*tan(1
/2*c)^5 + 4*tan(1/2*a)^6*tan(1/2*c)^6 + 12*tan(1/2*a)^5*tan(1/2*c)^7 - 2*t
an(1/2*a)^8*tan(1/2*c)^2 + 12*tan(1/2*a)^7*tan(1/2*c)^3 + 36*tan(1/2*a)^5*
tan(1/2*c)^5 + 12*tan(1/2*a)^3*tan(1/2*c)^7 - 2*tan(1/2*a)^2*tan(1/2*c)^8
- tan(1/2*a)^8 + 4*tan(1/2*a)^7*tan(1/2*c) - 4*tan(1/2*a)^6*tan(1/2*c)^2 +
 36*tan(1/2*a)^5*tan(1/2*c)^3 + 36*tan(1/2*a)^3*tan(1/2*c)^5 - 4*tan(1/2*a
)^2*tan(1/2*c)^6 + 4*tan(1/2*a)*tan(1/2*c)^7 - tan(1/2*c)^8 - 2*tan(1/2*a)
^6 + 12*tan(1/2*a)^5*tan(1/2*c) + 36*tan(1/2*a)^3*tan(1/2*c)^3 + 12*tan(1/
2*a)*tan(1/2*c)^5 - 2*tan(1/2*c)^6 + 12*tan(1/2*a)^3*tan(1/2*c) + 4*tan(1/
2*a)^2*tan(1/2*c)^2 + 12*tan(1/2*a)*tan(1/2*c)^3 + 2*tan(1/2*a)^2 + 4*tan(
1/2*a)*tan(1/2*c) + 2*tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*
tan(1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*tan(1
/2*c) - 2*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*
a)*tan(1/2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1/2*a)
^8*tan(1/2*c)^5 - 3*tan(1/2*a)^7*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^
7 - tan(1/2*a)^5*tan(1/2*c)^8 - tan(1/2*a)^8*tan(1/2*c)^3 + 10*tan(1/2*a)^
7*tan(1/2*c)^4 - 25*tan(1/2*a)^6*tan(1/2*c)^5 + 25*tan(1/2*a)^5*tan(1/2*c)
^6 - 10*tan(1/2*a)^4*tan(1/2*c)^7 + tan(1/2*a)^3*tan(1/2*c)^8 - 3*tan(1/2*
a)^7*tan(1/2*c)^2 + 25*tan(1/2*a)^6*tan(1/2*c)^3 - 60*tan(1/2*a)^5*tan(...
```

Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\csc^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(sin(a + b*x)*sin(c + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc(a+bx)\csc^3(c+bx)\,dx = \int \csc(bx+c)^3 \csc(bx+a)\,dx$$

input
```
int(csc(b*x+a)*csc(b*x+c)^3,x)
```

output
```
int(csc(b*x + c)**3*csc(a + b*x),x)
```

3.104 $\int \csc^2(a+bx)\csc^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \csc^2(a+bx)\csc^2(c+bx)\,dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 6.40 (sec) , antiderivative size = 404, normalized size of antiderivative = 404.00

$$\int \csc^2(a+bx)\csc^2(c+bx)\,dx = \frac{2i\arctan(\tan(c+bx))\cot(a-c)\csc^2(a-c)}{b} + \frac{2\cot(a-c)\csc^2(a-c)\log(\sin(a+bx))}{b} - \frac{\cot(a-c)\csc^2(a-c)\log\left(\sin^2(c+bx)\right)}{b} + x\left(-2\cot(a-c)\csc(a)\csc(a-c)\csc(c) - \frac{\cos(a)\cos(c)\cot(a)}{(\cos(c)\sin(a)-\cos(a)\sin(c))^3} - \frac{\cos(c)\csc(a)}{(\cos(c)\sin(a)-\cos(a)\sin(c))^3} + \frac{\cos(c)\sin(a)}{(\cos(c)\sin(a)-\cos(a)\sin(c))^3} - \frac{2\cos(a)\sin(c)}{(\cos(c)\sin(a)-\cos(a)\sin(c))^3} + \frac{2i\cos(a)\cos(c)}{(-\cos(c)\sin(a)+\cos(a)\sin(c))^3} - \frac{\cos(a)\cos(c)\cot(c)}{(-\cos(c)\sin(a)+\cos(a)\sin(c))^3} - \frac{\cos(a)\csc(c)}{(-\cos(c)\sin(a)+\cos(a)\sin(c))^3} - \frac{2\cos(c)\sin(a)}{(-\cos(c)\sin(a)+\cos(a)\sin(c))^3} - \frac{i\cos(c)\cot(c)\sin(a)}{(-\cos(c)\sin(a)+\cos(a)\sin(c))^3} + \frac{i\csc(c)\sin(a)}{(-\cos(c)\sin(a)+\cos(a)\sin(c))^3} + \frac{\cos(a)\sin(c)}{(-\cos(c)\sin(a)+\cos(a)\sin(c))^3} + \frac{i\sin(a)\sin(c)}{(-\cos(c)\sin(a)+\cos(a)\sin(c))^3}\right) + \frac{\csc(a)\csc^2(a-c)\csc(a+bx)\sin(bx)}{b} + \frac{\csc^2(a-c)\csc(c)\csc(c+bx)\sin(bx)}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[c + b*x]^2,x]
```

output
```
((2*I)*ArcTan[Tan[c + b*x]]*Cot[a - c]*Csc[a - c]^2)/b + (2*Cot[a - c]*Csc
[a - c]^2*Log[Sin[a + b*x]])/b - (Cot[a - c]*Csc[a - c]^2*Log[Sin[c + b*x]
^2])/b + x*(-2*Cot[a - c]*Csc[a]*Csc[a - c]*Csc[c] - (Cos[a]*Cos[c]*Cot[a]
)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^3 - (Cos[c]*Csc[a])/(Cos[c]*Sin[a] - Cos
[a]*Sin[c])^3 + (Cos[c]*Sin[a])/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^3 - (2*Cos
[a]*Sin[c])/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^3 + ((2*I)*Cos[a]*Cos[c])/(-(C
os[c]*Sin[a]) + Cos[a]*Sin[c])^3 - (Cos[a]*Cos[c]*Cot[c])/(-(Cos[c]*Sin[a]
) + Cos[a]*Sin[c])^3 - (Cos[a]*Csc[c])/(-(Cos[c]*Sin[a]) + Cos[a]*Sin[c])^
3 - (2*Cos[c]*Sin[a])/(-(Cos[c]*Sin[a]) + Cos[a]*Sin[c])^3 - (I*Cos[c]*Cot
[c]*Sin[a])/(-(Cos[c]*Sin[a]) + Cos[a]*Sin[c])^3 + (I*Csc[c]*Sin[a])/(-(Co
s[c]*Sin[a]) + Cos[a]*Sin[c])^3 + (Cos[a]*Sin[c])/(-(Cos[c]*Sin[a]) + Cos[
a]*Sin[c])^3 + (I*Sin[a]*Sin[c])/(-(Cos[c]*Sin[a]) + Cos[a]*Sin[c])^3) + (
Csc[a]*Csc[a - c]^2*Csc[a + b*x]*Sin[b*x])/b + (Csc[a - c]^2*Csc[c]*Csc[c
+ b*x]*Sin[b*x])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \csc^2(a+bx)\csc^2(bx+c)dx$$

input
```
Int[Csc[a + b*x]^2*Csc[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.07 (sec) , antiderivative size = 261, normalized size of antiderivative = 261.00

method	result
default	$-\frac{1}{(\sin(a)\cos(c)-\cos(a)\sin(c))^2\tan(bx+a)}+\frac{(2\cos(a)\cos(c)+2\sin(a)\sin(c))\ln(\tan(bx+a))}{(\sin(a)\cos(c)-\cos(a)\sin(c))^3}+\frac{(-2\cos(a)^2\cos(c)^2-4\cos(a)\cos(c)\sin(a)\sin(c)-2\,\mathrm{s}}{(\sin(a)}$
risch	$\frac{8i(\mathrm{e}^{2i(bx+3a+c)}+\mathrm{e}^{2i(bx+2a+2c)}-2\,\mathrm{e}^{2i(2a+c)})}{(\mathrm{e}^{2i(bx+a)}-1)(\mathrm{e}^{2i(bx+a+c)}-\mathrm{e}^{2ia})(-\mathrm{e}^{2ia}+\mathrm{e}^{2ic})^2b}-\frac{8i\ln(\mathrm{e}^{2i(bx+a)}-1)\mathrm{e}^{2i(2a+c)}}{(\mathrm{e}^{6ia}-3\,\mathrm{e}^{2i(2a+c)}+3\,\mathrm{e}^{2i(a+2c)}-\mathrm{e}^{6ic})b}-\frac{8i\ln(\mathrm{e}^{2i(bx+a)}-1)\mathrm{e}^{2i(a+2c)}}{(\mathrm{e}^{6ia}-3\,\mathrm{e}^{2i(2a+c)}+3\,\mathrm{e}^{2i(a+2c)}-}$

input
```
int(csc(b*x+a)^2*csc(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/(sin(a)*cos(c)-cos(a)*sin(c))^2/tan(b*x+a)+1/(sin(a)*cos(c)-cos(a)
*sin(c))^3*(2*cos(a)*cos(c)+2*sin(a)*sin(c))*ln(tan(b*x+a))+(-2*cos(a)^2*c
os(c)^2-4*cos(a)*cos(c)*sin(a)*sin(c)-2*sin(a)^2*sin(c)^2)/(sin(a)*cos(c)-
cos(a)*sin(c))^3/(cos(a)*cos(c)+sin(a)*sin(c))*ln(tan(b*x+a)*cos(a)*cos(c)
+tan(b*x+a)*sin(a)*sin(c)-sin(a)*cos(c)+cos(a)*sin(c))-(cos(a)^2*cos(c)^2+
sin(c)^2*cos(a)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(sin(a)*cos(c)-cos(
a)*sin(c))^2/(cos(a)*cos(c)+sin(a)*sin(c))/(tan(b*x+a)*cos(a)*cos(c)+tan(b
*x+a)*sin(a)*sin(c)-sin(a)*cos(c)+cos(a)*sin(c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.09 (sec) , antiderivative size = 358, normalized size of antiderivative = 358.00

$$\int \csc^2(a+bx)\csc^2(c+bx)\,dx$$

$$=\frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+2\left(\cos(-a+c)^2-1\right)\cos(bx+c)^2-\cos(-a+c)}{}$$

input
```
integrate(csc(b*x+a)^2*csc(b*x+c)^2,x, algorithm="fricas")
```

output
```
(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) + 2*(cos(-a + c)^2 -
1)*cos(b*x + c)^2 - cos(-a + c)^2 + (cos(b*x + c)^2*cos(-a + c)^2 + cos(b*
x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) - cos(-a + c)^2)*log(-1/4*cos(
b*x + c)^2 + 1/4) - (cos(b*x + c)^2*cos(-a + c)^2 + cos(b*x + c)*cos(-a +
c)*sin(b*x + c)*sin(-a + c) - cos(-a + c)^2)*log(-(2*cos(b*x + c)*cos(-a +
 c)*sin(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(
-a + c)^2)/(cos(-a + c)^2 + 2*cos(-a + c) + 1)) + 1)/((b*cos(-a + c)^4 - 2
*b*cos(-a + c)^2 + b)*cos(b*x + c)*sin(b*x + c) + (b*cos(-a + c)^3 - (b*co
s(-a + c)^3 - b*cos(-a + c))*cos(b*x + c)^2 - b*cos(-a + c))*sin(-a + c))
```

Sympy [F]

$$\int \csc^2(a+bx)\csc^2(c+bx)\,dx = \int \csc^2{(a+bx)}\csc^2{(bx+c)}\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(b*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**2*csc(b*x + c)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 3.88 (sec) , antiderivative size = 146269, normalized size of antiderivative = 146269.00

$$\int \csc^2(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(b*x+c)^2,x, algorithm="maxima")
```

output
```
-4*(36*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2
*a + 2*c))*cos(4*a + 2*c)^2 + 36*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (
cos(4*a) + cos(4*c))*sin(2*a + 2*c))*cos(2*a + 4*c)^2 + 36*((sin(4*a) + si
n(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*sin(4*a + 2
*c)^2 + 36*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*s
in(2*a + 2*c))*sin(2*a + 4*c)^2 - 2*(((cos(6*a) - cos(6*c))*cos(4*a + 2*c)
 - 3*cos(4*a + 2*c)^2 + (cos(6*a) - cos(6*c))*cos(2*a + 4*c) + 3*cos(2*a +
 4*c)^2 + (sin(6*a) - sin(6*c))*sin(4*a + 2*c) - 3*sin(4*a + 2*c)^2 + (sin
(6*a) - sin(6*c))*sin(2*a + 4*c) + 3*sin(2*a + 4*c)^2)*cos(4*b*x + 6*a + 2
*c)^2 + 4*((cos(6*a) - cos(6*c))*cos(4*a + 2*c) - 3*cos(4*a + 2*c)^2 + (co
s(6*a) - cos(6*c))*cos(2*a + 4*c) + 3*cos(2*a + 4*c)^2 + (sin(6*a) - sin(6
*c))*sin(4*a + 2*c) - 3*sin(4*a + 2*c)^2 + (sin(6*a) - sin(6*c))*sin(2*a +
 4*c) + 3*sin(2*a + 4*c)^2)*cos(4*b*x + 4*a + 4*c)^2 + ((cos(6*a) - cos(6*
c))*cos(4*a + 2*c) - 3*cos(4*a + 2*c)^2 + (cos(6*a) - cos(6*c))*cos(2*a +
4*c) + 3*cos(2*a + 4*c)^2 + (sin(6*a) - sin(6*c))*sin(4*a + 2*c) - 3*sin(4
*a + 2*c)^2 + (sin(6*a) - sin(6*c))*sin(2*a + 4*c) + 3*sin(2*a + 4*c)^2)*c
os(4*b*x + 2*a + 6*c)^2 + ((cos(6*a) - cos(6*c))*cos(4*a + 2*c) - 3*cos(4*
a + 2*c)^2 + (cos(6*a) - cos(6*c))*cos(2*a + 4*c) + 3*cos(2*a + 4*c)^2 + (
sin(6*a) - sin(6*c))*sin(4*a + 2*c) - 3*sin(4*a + 2*c)^2 + (sin(6*a) - sin
(6*c))*sin(2*a + 4*c) + 3*sin(2*a + 4*c)^2)*cos(2*b*x + 6*a)^2 + ((cos(...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.27 (sec) , antiderivative size = 3043, normalized size of antiderivative = 3043.00

$$\int \csc^2(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(b*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*((tan(1/2*a)^8*tan(1/2*c)^8 + 8*tan(1/2*a)^7*tan(1/2*c)^7 - 2*tan(1/2
*a)^8*tan(1/2*c)^4 + 8*tan(1/2*a)^7*tan(1/2*c)^5 + 16*tan(1/2*a)^6*tan(1/2
*c)^6 + 8*tan(1/2*a)^5*tan(1/2*c)^7 - 2*tan(1/2*a)^4*tan(1/2*c)^8 - 8*tan(
1/2*a)^7*tan(1/2*c)^3 + 32*tan(1/2*a)^6*tan(1/2*c)^4 + 8*tan(1/2*a)^5*tan(
1/2*c)^5 + 32*tan(1/2*a)^4*tan(1/2*c)^6 - 8*tan(1/2*a)^3*tan(1/2*c)^7 + ta
n(1/2*a)^8 - 8*tan(1/2*a)^7*tan(1/2*c) + 16*tan(1/2*a)^6*tan(1/2*c)^2 - 8*
tan(1/2*a)^5*tan(1/2*c)^3 + 68*tan(1/2*a)^4*tan(1/2*c)^4 - 8*tan(1/2*a)^3*
tan(1/2*c)^5 + 16*tan(1/2*a)^2*tan(1/2*c)^6 - 8*tan(1/2*a)*tan(1/2*c)^7 +
tan(1/2*c)^8 - 8*tan(1/2*a)^5*tan(1/2*c) + 32*tan(1/2*a)^4*tan(1/2*c)^2 +
8*tan(1/2*a)^3*tan(1/2*c)^3 + 32*tan(1/2*a)^2*tan(1/2*c)^4 - 8*tan(1/2*a)*
tan(1/2*c)^5 - 2*tan(1/2*a)^4 + 8*tan(1/2*a)^3*tan(1/2*c) + 16*tan(1/2*a)^
2*tan(1/2*c)^2 + 8*tan(1/2*a)*tan(1/2*c)^3 - 2*tan(1/2*c)^4 + 8*tan(1/2*a)
*tan(1/2*c) + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x
+ a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*a)^2*
tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^2 + tan(b
*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1/2*a)^8*tan(1/2*c)^5 - 3*tan
(1/2*a)^7*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^7 - tan(1/2*a)^5*tan(1/
2*c)^8 - tan(1/2*a)^8*tan(1/2*c)^3 + 10*tan(1/2*a)^7*tan(1/2*c)^4 - 25*tan
(1/2*a)^6*tan(1/2*c)^5 + 25*tan(1/2*a)^5*tan(1/2*c)^6 - 10*tan(1/2*a)^4*ta
n(1/2*c)^7 + tan(1/2*a)^3*tan(1/2*c)^8 - 3*tan(1/2*a)^7*tan(1/2*c)^2 + ...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\csc^2(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(sin(a + b*x)^2*sin(c + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^2(a+bx)\csc^2(c+bx)\,dx = \int \csc(bx+c)^2 \csc(bx+a)^2\,dx$$

input

```
int(csc(b*x+a)^2*csc(b*x+c)^2,x)
```

output

```
int(csc(b*x + c)**2*csc(a + b*x)**2,x)
```

3.105 $\int \csc^2(a+bx)\csc^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \csc^2(a+bx)\csc^3(c+bx)\,dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 8.75 (sec) , antiderivative size = 382, normalized size of antiderivative = 382.00

$$\int \csc^2(a+bx)\csc^3(c+bx)\,dx$$
$$= \frac{6i\arctan\left(\frac{(\cos(a)-i\sin(a))\left(\cos(a)\cos\left(\frac{bx}{2}\right)-\sin(a)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(a)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(a)}\right)\cos(a-c)}{\frac{3b}{8}+\frac{1}{8}b\cos(4a-4c)-\frac{1}{2}b\cos(2a-2c)} - \frac{\csc^2(a-c)\csc^2\left(\frac{c}{2}+\frac{bx}{2}\right)}{8b} + \frac{\csc^3(a-c)\csc(a+bx)}{b} - \frac{3(3+\cos(2a-2c))\csc^4(a-c)\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{4b} + \frac{3(3+\cos(2a-2c))\csc^4(a-c)\log\left(\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{4b} + \frac{\csc^2(a-c)\sec^2\left(\frac{c}{2}+\frac{bx}{2}\right)}{8b} - \frac{\csc^3(a-c)\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{bx}{2}\right)\left(\sin\left(a-c-\frac{bx}{2}\right)-\sin\left(a-c+\frac{bx}{2}\right)\right)}{2b} - \frac{\csc^3(a-c)\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{bx}{2}\right)\left(-\sin\left(a-c-\frac{bx}{2}\right)+\sin\left(a-c+\frac{bx}{2}\right)\right)}{2b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[c + b*x]^3,x]
```

output
```
((6*I)*ArcTan[((Cos[a] - I*Sin[a])*(Cos[a]*Cos[(b*x)/2] - Sin[a]*Sin[(b*x)
/2]))/(I*Cos[a]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[a])]*Cos[a - c])/((3*b)/8
+ (b*Cos[4*a - 4*c])/8 - (b*Cos[2*a - 2*c])/2) - (Csc[a - c]^2*Csc[c/2 + (
b*x)/2]^2)/(8*b) + (Csc[a - c]^3*Csc[a + b*x])/b - (3*(3 + Cos[2*a - 2*c])
*Csc[a - c]^4*Log[Cos[c/2 + (b*x)/2]])/(4*b) + (3*(3 + Cos[2*a - 2*c])*Csc
[a - c]^4*Log[Sin[c/2 + (b*x)/2]])/(4*b) + (Csc[a - c]^2*Sec[c/2 + (b*x)/2
]^2)/(8*b) - (Csc[a - c]^3*Sec[c/2]*Sec[c/2 + (b*x)/2]*(Sin[a - c - (b*x)/
2] - Sin[a - c + (b*x)/2]))/(2*b) - (Csc[a - c]^3*Csc[c/2]*Csc[c/2 + (b*x)
/2]*(-Sin[a - c - (b*x)/2] + Sin[a - c + (b*x)/2]))/(2*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^3(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc^2(a+bx)\csc^3(bx+c)dx$$

input
```
Int[Csc[a + b*x]^2*Csc[c + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 2.25 (sec) , antiderivative size = 770, normalized size of antiderivative = 770.00

method	result	size
default	Expression too large to display	770
risch	Expression too large to display	928

input
```
int(csc(b*x+a)^2*csc(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-4/(-sin(a)*cos(c)+cos(a)*sin(c))^4*(((-1/4*sin(c)^3*cos(a)^3-9/4*cos
(c)*sin(c)^2*sin(a)*cos(a)^2-3/2*sin(c)^3*sin(a)^2*cos(a)+3/2*cos(c)*sin(c
)^2*sin(a)^3+9/4*cos(c)^2*sin(c)*sin(a)^2*cos(a)-3/2*cos(c)^2*sin(c)*cos(a
)^3+3/2*cos(c)^3*sin(a)*cos(a)^2+1/4*cos(c)^3*sin(a)^3)*tan(1/2*a+1/2*b*x)
^3+(-5/4*sin(c)^3*sin(a)*cos(a)^2-5/4*cos(c)*sin(c)^2*cos(a)^3+5/2*sin(c)^
3*sin(a)^3+10*cos(c)*sin(c)^2*cos(a)*sin(a)^2+10*cos(c)^2*sin(c)*cos(a)^2*
sin(a)+5/2*cos(c)^3*cos(a)^3-5/4*cos(c)^2*sin(c)*sin(a)^3-5/4*cos(c)^3*sin
(a)^2*cos(a))*tan(1/2*a+1/2*b*x)^2+(-1/4*sin(c)^3*cos(a)^3+7/2*sin(c)^3*si
n(a)^2*cos(a)+31/4*cos(c)*sin(c)^2*sin(a)*cos(a)^2-7/2*cos(c)*sin(c)^2*sin
(a)^3+7/2*cos(c)^2*sin(c)*cos(a)^3-31/4*cos(c)^2*sin(c)*sin(a)^2*cos(a)-7/
2*cos(c)^3*sin(a)*cos(a)^2+1/4*cos(c)^3*sin(a)^3)*tan(1/2*a+1/2*b*x)+5/4*s
in(c)^3*sin(a)*cos(a)^2+5/4*cos(c)*sin(c)^2*cos(a)^3-5/2*cos(c)*sin(c)^2*c
os(a)*sin(a)^2-5/2*cos(c)^2*sin(c)*cos(a)^2*sin(a)+5/4*cos(c)^2*sin(c)*sin
(a)^3+5/4*cos(c)^3*sin(a)^2*cos(a))/(sin(c)*cos(a)*tan(1/2*a+1/2*b*x)^2-co
s(c)*sin(a)*tan(1/2*a+1/2*b*x)^2-2*tan(1/2*a+1/2*b*x)*sin(a)*sin(c)-2*tan(
1/2*a+1/2*b*x)*cos(a)*cos(c)-cos(a)*sin(c)+sin(a)*cos(c))^2+3/4*(sin(c)^2*
cos(a)^2+2*sin(a)^2*sin(c)^2+2*cos(a)*cos(c)*sin(a)*sin(c)+2*cos(a)^2*cos(
c)^2+cos(c)^2*sin(a)^2)/(-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin
(c)^2-sin(c)^2*cos(a)^2)^(1/2)*arctan(1/2*(2*(-sin(a)*cos(c)+cos(a)*sin(c)
)*tan(1/2*a+1/2*b*x)-2*cos(a)*cos(c)-2*sin(a)*sin(c))/(-cos(c)^2*sin(a)...
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.14 (sec) , antiderivative size = 678, normalized size of antiderivative = 678.00

$$\int \csc^2(a+bx)\csc^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(b*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(6*(cos(-a + c)^3 - cos(-a + c))*cos(b*x + c)*sin(b*x + c) + 6*((cos(
b*x + c)^2*cos(-a + c)^2 - cos(-a + c)^2)*sin(b*x + c) - (cos(b*x + c)^3*c
os(-a + c) - cos(b*x + c)*cos(-a + c))*sin(-a + c))*log((cos(b*x + c)*cos(
-a + c) + sin(b*x + c)*sin(-a + c) + 1)/(cos(-a + c) + 1)) - 6*((cos(b*x +
 c)^2*cos(-a + c)^2 - cos(-a + c)^2)*sin(b*x + c) - (cos(b*x + c)^3*cos(-a
 + c) - cos(b*x + c)*cos(-a + c))*sin(-a + c))*log(-(cos(b*x + c)*cos(-a +
 c) + sin(b*x + c)*sin(-a + c) - 1)/(cos(-a + c) + 1)) - 3*(((cos(-a + c)^
3 + cos(-a + c))*cos(b*x + c)^2 - cos(-a + c)^3 - cos(-a + c))*sin(b*x + c
) - ((cos(-a + c)^2 + 1)*cos(b*x + c)^3 - (cos(-a + c)^2 + 1)*cos(b*x + c)
)*sin(-a + c))*log(1/2*cos(b*x + c) + 1/2) + 3*(((cos(-a + c)^3 + cos(-a +
 c))*cos(b*x + c)^2 - cos(-a + c)^3 - cos(-a + c))*sin(b*x + c) - ((cos(-a
 + c)^2 + 1)*cos(b*x + c)^3 - (cos(-a + c)^2 + 1)*cos(b*x + c))*sin(-a + c
))*log(-1/2*cos(b*x + c) + 1/2) - 2*(3*(cos(-a + c)^2 + 1)*cos(b*x + c)^2
- 4*cos(-a + c)^2 - 2)*sin(-a + c))/((b*cos(-a + c)^5 - 2*b*cos(-a + c)^3
- (b*cos(-a + c)^5 - 2*b*cos(-a + c)^3 + b*cos(-a + c))*cos(b*x + c)^2 + b
*cos(-a + c))*sin(b*x + c) + ((b*cos(-a + c)^4 - 2*b*cos(-a + c)^2 + b)*co
s(b*x + c)^3 - (b*cos(-a + c)^4 - 2*b*cos(-a + c)^2 + b)*cos(b*x + c))*sin
(-a + c))
```

Sympy [F]

$$\int \csc^2(a+bx)\csc^3(c+bx)\,dx = \int \csc^2{(a+bx)}\csc^3{(bx+c)}\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(b*x+c)**3,x)
```

output
```
Integral(csc(a + b*x)**2*csc(b*x + c)**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 32.84 (sec) , antiderivative size = 810260, normalized size of antiderivative = 810260.00

$$\int \csc^2(a+bx)\csc^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(b*x+c)^3,x, algorithm="maxima")
```

output
```
-(24*(((sin(8*a) - 4*sin(6*a + 2*c) + sin(8*c))*cos(5*a + 3*c) + (sin(8*a)
 - 4*sin(6*a + 2*c) + 6*sin(4*a + 4*c) + sin(8*c))*cos(3*a + 5*c) + 4*(sin
(5*a + 3*c) + sin(3*a + 5*c))*cos(2*a + 6*c) - (cos(8*a) - 4*cos(6*a + 2*c
) + cos(8*c))*sin(5*a + 3*c) - 6*cos(4*a + 4*c)*sin(5*a + 3*c) + 6*cos(5*a
 + 3*c)*sin(4*a + 4*c) - (cos(8*a) - 4*cos(6*a + 2*c) + 6*cos(4*a + 4*c) +
 cos(8*c))*sin(3*a + 5*c) - 4*(cos(5*a + 3*c) + cos(3*a + 5*c))*sin(2*a +
6*c))*cos(6*b*x + 8*a + 4*c)^2 + 9*((sin(8*a) - 4*sin(6*a + 2*c) + sin(8*c
))*cos(5*a + 3*c) + (sin(8*a) - 4*sin(6*a + 2*c) + 6*sin(4*a + 4*c) + sin(
8*c))*cos(3*a + 5*c) + 4*(sin(5*a + 3*c) + sin(3*a + 5*c))*cos(2*a + 6*c)
- (cos(8*a) - 4*cos(6*a + 2*c) + cos(8*c))*sin(5*a + 3*c) - 6*cos(4*a + 4*
c)*sin(5*a + 3*c) + 6*cos(5*a + 3*c)*sin(4*a + 4*c) - (cos(8*a) - 4*cos(6*
a + 2*c) + 6*cos(4*a + 4*c) + cos(8*c))*sin(3*a + 5*c) - 4*(cos(5*a + 3*c)
 + cos(3*a + 5*c))*sin(2*a + 6*c))*cos(6*b*x + 6*a + 6*c)^2 + 9*((sin(8*a)
 - 4*sin(6*a + 2*c) + sin(8*c))*cos(5*a + 3*c) + (sin(8*a) - 4*sin(6*a + 2
*c) + 6*sin(4*a + 4*c) + sin(8*c))*cos(3*a + 5*c) + 4*(sin(5*a + 3*c) + si
n(3*a + 5*c))*cos(2*a + 6*c) - (cos(8*a) - 4*cos(6*a + 2*c) + cos(8*c))*si
n(5*a + 3*c) - 6*cos(4*a + 4*c)*sin(5*a + 3*c) + 6*cos(5*a + 3*c)*sin(4*a
+ 4*c) - (cos(8*a) - 4*cos(6*a + 2*c) + 6*cos(4*a + 4*c) + cos(8*c))*sin(3
*a + 5*c) - 4*(cos(5*a + 3*c) + cos(3*a + 5*c))*sin(2*a + 6*c))*cos(6*b*x
+ 4*a + 8*c)^2 + ((sin(8*a) - 4*sin(6*a + 2*c) + sin(8*c))*cos(5*a + 3*...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 2.32 (sec) , antiderivative size = 10037, normalized size of antiderivative = 10037.00

$$\int \csc^2(a+bx)\csc^3(c+bx)\,dx = \text{Too large to display}$$

input

```
integrate(csc(b*x+a)^2*csc(b*x+c)^3,x, algorithm="giac")
```

output

```
-1/32*(6*(tan(1/2*a)^8*tan(1/2*c)^8 + 2*tan(1/2*a)^8*tan(1/2*c)^6 + 4*tan(
1/2*a)^7*tan(1/2*c)^7 + 2*tan(1/2*a)^6*tan(1/2*c)^8 + 12*tan(1/2*a)^7*tan(
1/2*c)^5 + 4*tan(1/2*a)^6*tan(1/2*c)^6 + 12*tan(1/2*a)^5*tan(1/2*c)^7 - 2*
tan(1/2*a)^8*tan(1/2*c)^2 + 12*tan(1/2*a)^7*tan(1/2*c)^3 + 36*tan(1/2*a)^5
*tan(1/2*c)^5 + 12*tan(1/2*a)^3*tan(1/2*c)^7 - 2*tan(1/2*a)^2*tan(1/2*c)^8
 - tan(1/2*a)^8 + 4*tan(1/2*a)^7*tan(1/2*c) - 4*tan(1/2*a)^6*tan(1/2*c)^2
+ 36*tan(1/2*a)^5*tan(1/2*c)^3 + 36*tan(1/2*a)^3*tan(1/2*c)^5 - 4*tan(1/2*
a)^2*tan(1/2*c)^6 + 4*tan(1/2*a)*tan(1/2*c)^7 - tan(1/2*c)^8 - 2*tan(1/2*a
)^6 + 12*tan(1/2*a)^5*tan(1/2*c) + 36*tan(1/2*a)^3*tan(1/2*c)^3 + 12*tan(1
/2*a)*tan(1/2*c)^5 - 2*tan(1/2*c)^6 + 12*tan(1/2*a)^3*tan(1/2*c) + 4*tan(1
/2*a)^2*tan(1/2*c)^2 + 12*tan(1/2*a)*tan(1/2*c)^3 + 2*tan(1/2*a)^2 + 4*tan
(1/2*a)*tan(1/2*c) + 2*tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*a)))/(t
an(1/2*a)^8*tan(1/2*c)^4 - 4*tan(1/2*a)^7*tan(1/2*c)^5 + 6*tan(1/2*a)^6*ta
n(1/2*c)^6 - 4*tan(1/2*a)^5*tan(1/2*c)^7 + tan(1/2*a)^4*tan(1/2*c)^8 + 4*t
an(1/2*a)^7*tan(1/2*c)^3 - 16*tan(1/2*a)^6*tan(1/2*c)^4 + 24*tan(1/2*a)^5*
tan(1/2*c)^5 - 16*tan(1/2*a)^4*tan(1/2*c)^6 + 4*tan(1/2*a)^3*tan(1/2*c)^7
+ 6*tan(1/2*a)^6*tan(1/2*c)^2 - 24*tan(1/2*a)^5*tan(1/2*c)^3 + 36*tan(1/2*
a)^4*tan(1/2*c)^4 - 24*tan(1/2*a)^3*tan(1/2*c)^5 + 6*tan(1/2*a)^2*tan(1/2*
c)^6 + 4*tan(1/2*a)^5*tan(1/2*c) - 16*tan(1/2*a)^4*tan(1/2*c)^2 + 24*tan(1
/2*a)^3*tan(1/2*c)^3 - 16*tan(1/2*a)^2*tan(1/2*c)^4 + 4*tan(1/2*a)*tan(...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\csc^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(sin(a + b*x)^2*sin(c + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^2(a+bx)\csc^3(c+bx)\,dx = \int \csc(bx+c)^3 \csc(bx+a)^2\,dx$$

input
```
int(csc(b*x+a)^2*csc(b*x+c)^3,x)
```

output
```
int(csc(b*x + c)**3*csc(a + b*x)**2,x)
```

3.106 $\int \csc^2(a+bx)\csc^4(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \csc^2(a+bx)\csc^4(c+bx)\,dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 4.28 (sec) , antiderivative size = 328, normalized size of antiderivative = 328.00

$$\int \csc^2(a+bx)\csc^4(c+bx)\,dx = \frac{\csc^4(a-c)\,(-192i\arctan(\tan(a+bx))\cot(a-c)+192i\arctan(\tan(c+bx))\cot(a-c)+96\cot(a-c)}{}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[c + b*x]^4,x]
```

output
```
(Csc[a - c]^4*((-192*I)*ArcTan[Tan[a + b*x]]*Cot[a - c] + (192*I)*ArcTan[T
an[c + b*x]]*Cot[a - c] + 96*Cot[a - c]*Log[Sin[a + b*x]^2] - 96*Cot[a - c
]*Log[Sin[c + b*x]^2] + Csc[a]*Csc[c]*Csc[a + b*x]*Csc[c + b*x]^3*(15*Sin[
2*a] - 3*Sin[2*(a - 2*c)] + 9*Sin[2*(a - c)] + 3*Sin[4*(a - c)] + 18*Sin[2
*c] + 25*Sin[2*b*x] - Sin[4*a - 4*c - 2*b*x] - 7*Sin[2*a - 2*c - 2*b*x] -
2*Sin[2*(a - 2*c - b*x)] - 16*Sin[2*(a + b*x)] + Sin[4*(a + b*x)] + 3*Sin[
2*(a - c + b*x)] - 7*Sin[2*(c + b*x)] + 4*Sin[4*(c + b*x)] - 6*Sin[2*(a +
c + b*x)] - Sin[2*(a + 2*b*x)] - 3*Sin[4*a - 2*c + 2*b*x] - 10*Sin[2*(c +
2*b*x)] + 7*Sin[2*(a + c + 2*b*x)] - 6*Sin[4*c + 2*b*x] + Sin[2*(a - 2*(c
+ b*x))])))/(48*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^4(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc^2(a+bx)\csc^4(bx+c)dx$$

input
```
Int[Csc[a + b*x]^2*Csc[c + b*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 3.85 (sec) , antiderivative size = 627, normalized size of antiderivative = 627.00

method	result
risch	$\frac{32i\left(6\,\mathrm{e}^{2i(3bx+6a+4c)}+6\,\mathrm{e}^{2i(3bx+5a+5c)}-15\,\mathrm{e}^{2i(2bx+6a+3c)}-18\,\mathrm{e}^{2i(2bx+5a+4c)}-3\,\mathrm{e}^{2i(2bx+4a+5c)}+\mathrm{e}^{2i(bx+7a+c)}+7\,\mathrm{e}^{2i(bx+6a+2c)}+\right.}{3\left(\mathrm{e}^{2i(bx+a)}-1\right)\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{3}\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)^{4}b}$
default	Expression too large to display

input
```
int(csc(b*x+a)^2*csc(b*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
32/3*I/(exp(2*I*(b*x+a))-1)/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^3/(exp(2*I*a)
-exp(2*I*c))^4/b*(6*exp(2*I*(3*b*x+6*a+4*c))+6*exp(2*I*(3*b*x+5*a+5*c))-15
*exp(2*I*(2*b*x+6*a+3*c))-18*exp(2*I*(2*b*x+5*a+4*c))-3*exp(2*I*(2*b*x+4*a
+5*c))+exp(2*I*(b*x+7*a+c))+7*exp(2*I*(b*x+6*a+2*c))+25*exp(2*I*(b*x+5*a+3
*c))+3*exp(2*I*(b*x+4*a+4*c))-exp(2*I*(6*a+c))-10*exp(2*I*(5*a+2*c))-exp(2
*I*(4*a+3*c)))+64*I*ln(exp(2*I*(b*x+a))-1)/(exp(10*I*a)-5*exp(2*I*(4*a+c))
+10*exp(2*I*(3*a+2*c))-10*exp(2*I*(2*a+3*c))+5*exp(2*I*(a+4*c))-exp(10*I*c
))/b*exp(2*I*(3*a+2*c))+64*I*ln(exp(2*I*(b*x+a))-1)/(exp(10*I*a)-5*exp(2*I
*(4*a+c))+10*exp(2*I*(3*a+2*c))-10*exp(2*I*(2*a+3*c))+5*exp(2*I*(a+4*c))-e
xp(10*I*c))/b*exp(2*I*(2*a+3*c))-64*I*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))/
(exp(10*I*a)-5*exp(2*I*(4*a+c))+10*exp(2*I*(3*a+2*c))-10*exp(2*I*(2*a+3*c)
)+5*exp(2*I*(a+4*c))-exp(10*I*c))/b*exp(2*I*(3*a+2*c))-64*I*ln(exp(2*I*(b*
x+a))-exp(2*I*(a-c)))/(exp(10*I*a)-5*exp(2*I*(4*a+c))+10*exp(2*I*(3*a+2*c)
)-10*exp(2*I*(2*a+3*c))+5*exp(2*I*(a+4*c))-exp(10*I*c))/b*exp(2*I*(2*a+3*c
))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.11 (sec) , antiderivative size = 624, normalized size of antiderivative = 624.00

$$\int \csc^2(a+bx)\csc^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(b*x+c)^4,x, algorithm="fricas")
```

output
```
-1/3*(4*(cos(-a + c)^4 + cos(-a + c)^2 - 2)*cos(b*x + c)^4 + 3*cos(-a + c)
^4 - 6*(cos(-a + c)^4 + cos(-a + c)^2 - 2)*cos(b*x + c)^2 + 2*(2*(cos(-a +
 c)^3 + 2*cos(-a + c))*cos(b*x + c)^3 - 3*(cos(-a + c)^3 + cos(-a + c))*co
s(b*x + c))*sin(b*x + c)*sin(-a + c) + 6*(cos(b*x + c)^4*cos(-a + c)^2 - 2
*cos(b*x + c)^2*cos(-a + c)^2 + (cos(b*x + c)^3*cos(-a + c) - cos(b*x + c)
*cos(-a + c))*sin(b*x + c)*sin(-a + c) + cos(-a + c)^2)*log(-1/4*cos(b*x +
 c)^2 + 1/4) - 6*(cos(b*x + c)^4*cos(-a + c)^2 - 2*cos(b*x + c)^2*cos(-a +
 c)^2 + (cos(b*x + c)^3*cos(-a + c) - cos(b*x + c)*cos(-a + c))*sin(b*x +
c)*sin(-a + c) + cos(-a + c)^2)*log(-(2*cos(b*x + c)*cos(-a + c)*sin(b*x +
 c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2)/(c
os(-a + c)^2 + 2*cos(-a + c) + 1)) - 3)/(((b*cos(-a + c)^6 - 3*b*cos(-a +
c)^4 + 3*b*cos(-a + c)^2 - b)*cos(b*x + c)^3 - (b*cos(-a + c)^6 - 3*b*cos(
-a + c)^4 + 3*b*cos(-a + c)^2 - b)*cos(b*x + c))*sin(b*x + c) - (b*cos(-a
+ c)^5 + (b*cos(-a + c)^5 - 2*b*cos(-a + c)^3 + b*cos(-a + c))*cos(b*x + c
)^4 - 2*b*cos(-a + c)^3 - 2*(b*cos(-a + c)^5 - 2*b*cos(-a + c)^3 + b*cos(-
a + c))*cos(b*x + c)^2 + b*cos(-a + c))*sin(-a + c))
```

Sympy [F]

$$\int \csc^2(a+bx)\csc^4(c+bx)\,dx = \int \csc^2{(a+bx)}\csc^4{(bx+c)}\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(b*x+c)**4,x)
```

output
```
Integral(csc(a + b*x)**2*csc(b*x + c)**4, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 80.19 (sec) , antiderivative size = 2049662, normalized size of antiderivative = 2049662.00

$$\int \csc^2(a+bx)\csc^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(b*x+c)^4,x, algorithm="maxima")
```

output
```
32/3*(25*((sin(8*a) + sin(8*c))*cos(6*a + 2*c) - (cos(8*a) + cos(8*c))*sin
(6*a + 2*c))*cos(8*a + 2*c)^2 + 100*((sin(8*a) + sin(8*c))*cos(6*a + 2*c)
+ 2*(5*sin(8*a) - 23*sin(6*a + 2*c) + 5*sin(8*c))*cos(4*a + 4*c) - (cos(8*
a) + cos(8*c))*sin(6*a + 2*c) - 2*(5*cos(8*a) - 23*cos(6*a + 2*c) + 5*cos(
8*c))*sin(4*a + 4*c))*cos(6*a + 4*c)^2 + 100*((sin(8*a) + sin(8*c))*cos(6*
a + 2*c) + 2*(5*sin(8*a) - 23*sin(6*a + 2*c) + 5*sin(8*c))*cos(4*a + 4*c)
+ (sin(8*a) + 46*sin(4*a + 4*c) + sin(8*c))*cos(2*a + 6*c) - (cos(8*a) + c
os(8*c))*sin(6*a + 2*c) - 2*(5*cos(8*a) - 23*cos(6*a + 2*c) + 5*cos(8*c))*
sin(4*a + 4*c) - (cos(8*a) + 46*cos(4*a + 4*c) + cos(8*c))*sin(2*a + 6*c))
*cos(4*a + 6*c)^2 + 25*((sin(8*a) + sin(8*c))*cos(6*a + 2*c) + 2*(5*sin(8*
a) - 23*sin(6*a + 2*c) + 5*sin(8*c))*cos(4*a + 4*c) + (sin(8*a) + 46*sin(4
*a + 4*c) + sin(8*c))*cos(2*a + 6*c) - (cos(8*a) + cos(8*c))*sin(6*a + 2*c
) - 2*(5*cos(8*a) - 23*cos(6*a + 2*c) + 5*cos(8*c))*sin(4*a + 4*c) - (cos(
8*a) + 46*cos(4*a + 4*c) + cos(8*c))*sin(2*a + 6*c))*cos(2*a + 8*c)^2 + 25
*((sin(8*a) + sin(8*c))*cos(6*a + 2*c) - (cos(8*a) + cos(8*c))*sin(6*a + 2
*c))*sin(8*a + 2*c)^2 + 100*((sin(8*a) + sin(8*c))*cos(6*a + 2*c) + 2*(5*s
in(8*a) - 23*sin(6*a + 2*c) + 5*sin(8*c))*cos(4*a + 4*c) - (cos(8*a) + cos
(8*c))*sin(6*a + 2*c) - 2*(5*cos(8*a) - 23*cos(6*a + 2*c) + 5*cos(8*c))*si
n(4*a + 4*c))*sin(6*a + 4*c)^2 + 100*((sin(8*a) + sin(8*c))*cos(6*a + 2*c)
 + 2*(5*sin(8*a) - 23*sin(6*a + 2*c) + 5*sin(8*c))*cos(4*a + 4*c) + (si...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.49 (sec) , antiderivative size = 27104, normalized size of antiderivative = 27104.00

$$\int \csc^2(a+bx)\csc^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(b*x+c)^4,x, algorithm="giac")
```

output
```
-1/48*(6*(tan(1/2*a)^12*tan(1/2*c)^12 + 2*tan(1/2*a)^12*tan(1/2*c)^10 + 8*
tan(1/2*a)^11*tan(1/2*c)^11 + 2*tan(1/2*a)^10*tan(1/2*c)^12 - tan(1/2*a)^1
2*tan(1/2*c)^8 + 24*tan(1/2*a)^11*tan(1/2*c)^9 + 20*tan(1/2*a)^10*tan(1/2*
c)^10 + 24*tan(1/2*a)^9*tan(1/2*c)^11 - tan(1/2*a)^8*tan(1/2*c)^12 - 4*tan
(1/2*a)^12*tan(1/2*c)^6 + 16*tan(1/2*a)^11*tan(1/2*c)^7 + 62*tan(1/2*a)^10
*tan(1/2*c)^8 + 72*tan(1/2*a)^9*tan(1/2*c)^9 + 62*tan(1/2*a)^8*tan(1/2*c)^
10 + 16*tan(1/2*a)^7*tan(1/2*c)^11 - 4*tan(1/2*a)^6*tan(1/2*c)^12 - tan(1/
2*a)^12*tan(1/2*c)^4 - 16*tan(1/2*a)^11*tan(1/2*c)^5 + 88*tan(1/2*a)^10*ta
n(1/2*c)^6 + 48*tan(1/2*a)^9*tan(1/2*c)^7 + 257*tan(1/2*a)^8*tan(1/2*c)^8
+ 48*tan(1/2*a)^7*tan(1/2*c)^9 + 88*tan(1/2*a)^6*tan(1/2*c)^10 - 16*tan(1/
2*a)^5*tan(1/2*c)^11 - tan(1/2*a)^4*tan(1/2*c)^12 + 2*tan(1/2*a)^12*tan(1/
2*c)^2 - 24*tan(1/2*a)^11*tan(1/2*c)^3 + 62*tan(1/2*a)^10*tan(1/2*c)^4 - 4
8*tan(1/2*a)^9*tan(1/2*c)^5 + 388*tan(1/2*a)^8*tan(1/2*c)^6 + 32*tan(1/2*a
)^7*tan(1/2*c)^7 + 388*tan(1/2*a)^6*tan(1/2*c)^8 - 48*tan(1/2*a)^5*tan(1/2
*c)^9 + 62*tan(1/2*a)^4*tan(1/2*c)^10 - 24*tan(1/2*a)^3*tan(1/2*c)^11 + 2*
tan(1/2*a)^2*tan(1/2*c)^12 + tan(1/2*a)^12 - 8*tan(1/2*a)^11*tan(1/2*c) +
20*tan(1/2*a)^10*tan(1/2*c)^2 - 72*tan(1/2*a)^9*tan(1/2*c)^3 + 257*tan(1/2
*a)^8*tan(1/2*c)^4 - 32*tan(1/2*a)^7*tan(1/2*c)^5 + 592*tan(1/2*a)^6*tan(1
/2*c)^6 - 32*tan(1/2*a)^5*tan(1/2*c)^7 + 257*tan(1/2*a)^4*tan(1/2*c)^8 - 7
2*tan(1/2*a)^3*tan(1/2*c)^9 + 20*tan(1/2*a)^2*tan(1/2*c)^10 - 8*tan(1/2...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\csc^4(c+bx)\,dx=\text{Hanged}$$

input
```
int(1/(sin(a + b*x)^2*sin(c + b*x)^4),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^2(a+bx)\csc^4(c+bx)\,dx = \int \csc{(bx+c)}^4 \csc{(bx+a)}^2\,dx$$

input
```
int(csc(b*x+a)^2*csc(b*x+c)^4,x)
```

output
```
int(csc(b*x + c)**4*csc(a + b*x)**2,x)
```

3.107 $\int \csc^3(a+bx)\csc^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \csc^3(a+bx)\csc^3(c+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 4.09 (sec) , antiderivative size = 347, normalized size of antiderivative = 347.00

$$\int \csc^3(a+bx)\csc^3(c+bx)\,dx$$
$$=\frac{\csc^4(a-c)\left(64i\arctan(\tan(a+bx))(2+\cos(2(a-c)))\csc(a-c)-64i\arctan(\tan(c+bx))(2+\cos(2(\cdots\right.}{\cdots}$$

input
```
Integrate[Csc[a + b*x]^3*Csc[c + b*x]^3,x]
```

output
```
(Csc[a - c]^4*((64*I)*ArcTan[Tan[a + b*x]]*(2 + Cos[2*(a - c)])*Csc[a - c]
 - (64*I)*ArcTan[Tan[c + b*x]]*(2 + Cos[2*(a - c)])*Csc[a - c] - 32*(2 + C
os[2*(a - c)])*Csc[a - c]*Log[Sin[a + b*x]^2] + 32*(2 + Cos[2*(a - c)])*Cs
c[a - c]*Log[Sin[c + b*x]^2] + Csc[a]*Csc[c]*Csc[a + b*x]^2*Csc[c + b*x]^2
*(-15*Sin[2*a] + 3*Sin[2*(a - 2*c)] - 3*Sin[4*a - 2*c] - 15*Sin[2*c] - 16*
Sin[2*b*x] + 10*Sin[2*a - 2*c - 2*b*x] - 4*Sin[2*(a - 2*c - b*x)] + 8*Sin[
2*(a + b*x)] - 3*Sin[4*(a + b*x)] - 10*Sin[2*(a - c + b*x)] + 8*Sin[2*(c +
 b*x)] - 3*Sin[4*(c + b*x)] + 8*Sin[2*(a + c + b*x)] + 6*Sin[2*(a + 2*b*x)
] + 2*Sin[4*a + 2*b*x] + 4*Sin[4*a - 2*c + 2*b*x] + 6*Sin[2*(c + 2*b*x)] -
 6*Sin[2*(a + c + 2*b*x)] + 2*Sin[4*c + 2*b*x])))/(32*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\csc^3(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc^3(a+bx)\csc^3(bx+c)dx$$

input
```
Int[Csc[a + b*x]^3*Csc[c + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 2.73 (sec) , antiderivative size = 749, normalized size of antiderivative = 749.00

method	result
default	Expression too large to display
risch	$\frac{16i\left(2\,e^{i(6bx+13a+5c)}+8\,e^{i(6bx+11a+7c)}+2\,e^{3i(2bx+3a+3c)}-3\,e^{i(4bx+13a+3c)}-15\,e^{i(4bx+11a+5c)}-15\,e^{i(4bx+9a+7c)}-3\,e^{i(4bx+7a+9c)}\right.}{\left(e^{2i(bx+a+c)}-e^{2ia}\right)^{2}\left(e^{2i(bx+a)}-1\right)^{2}\left(-e^{2ia}+e^{2ic}\right.}$

input
```
int(csc(b*x+a)^3*csc(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-(-3*cos(a)*cos(c)-3*sin(a)*sin(c))/(sin(a)*cos(c)-cos(a)*sin(c))^4/t
an(b*x+a)+1/(sin(a)*cos(c)-cos(a)*sin(c))^5*(-2*cos(c)^2*sin(a)^2-6*cos(a)
^2*cos(c)^2-8*cos(a)*cos(c)*sin(a)*sin(c)-6*sin(a)^2*sin(c)^2-2*sin(c)^2*c
os(a)^2)*ln(tan(b*x+a))+1/2/(sin(a)*cos(c)-cos(a)*sin(c))^3/tan(b*x+a)^2-(
2*cos(c)^3*sin(a)^2*cos(a)+6*cos(c)^3*cos(a)^3+2*cos(c)^2*sin(c)*sin(a)^3+
14*cos(c)^2*sin(c)*cos(a)^2*sin(a)+14*cos(c)*sin(c)^2*cos(a)*sin(a)^2+2*co
s(c)*sin(c)^2*cos(a)^3+6*sin(c)^3*sin(a)^3+2*sin(c)^3*sin(a)*cos(a)^2)/(si
n(a)*cos(c)-cos(a)*sin(c))^5/(-cos(a)*cos(c)-sin(a)*sin(c))*ln(-tan(b*x+a)
*sin(a)*sin(c)-tan(b*x+a)*cos(a)*cos(c)+sin(a)*cos(c)-cos(a)*sin(c))+1/2*(
sin(a)^4*cos(c)^4+2*cos(a)^2*sin(a)^2*cos(c)^4+cos(a)^4*cos(c)^4+2*sin(a)^
4*cos(c)^2*sin(c)^2+4*cos(a)^2*sin(a)^2*cos(c)^2*sin(c)^2+2*cos(a)^4*cos(c
)^2*sin(c)^2+sin(a)^4*sin(c)^4+2*cos(a)^2*sin(a)^2*sin(c)^4+cos(a)^4*sin(c
)^4)/(sin(a)*cos(c)-cos(a)*sin(c))^3/(cos(a)*cos(c)+sin(a)*sin(c))/(-cos(a
)*cos(c)-sin(a)*sin(c))/(-tan(b*x+a)*sin(a)*sin(c)-tan(b*x+a)*cos(a)*cos(c
)+sin(a)*cos(c)-cos(a)*sin(c))^2-(sin(a)^4*cos(c)^4-2*cos(a)^2*sin(a)^2*co
s(c)^4-3*cos(a)^4*cos(c)^4-8*cos(a)*sin(a)^3*cos(c)^3*sin(c)-8*cos(a)^3*co
s(c)^3*sin(a)*sin(c)-2*sin(a)^4*cos(c)^2*sin(c)^2-4*cos(a)^2*sin(a)^2*cos(
c)^2*sin(c)^2-2*cos(a)^4*cos(c)^2*sin(c)^2-8*cos(a)*sin(a)^3*cos(c)*sin(c)
^3-8*cos(a)^3*sin(a)*cos(c)*sin(c)^3-3*sin(a)^4*sin(c)^4-2*cos(a)^2*sin(a)
^2*sin(c)^4+cos(a)^4*sin(c)^4)/(sin(a)*cos(c)-cos(a)*sin(c))^4/(cos(a)*...
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.13 (sec) , antiderivative size = 772, normalized size of antiderivative = 772.00

$$\int \csc^3(a+bx)\csc^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*csc(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(24*(cos(-a + c)^4 - cos(-a + c)^2)*cos(b*x + c)^4 + 7*cos(-a + c)^4 -
 2*(16*cos(-a + c)^4 - 17*cos(-a + c)^2 + 1)*cos(b*x + c)^2 + 4*(3*(2*cos(
-a + c)^3 - cos(-a + c))*cos(b*x + c)^3 - (5*cos(-a + c)^3 - 2*cos(-a + c)
)*cos(b*x + c))*sin(b*x + c)*sin(-a + c) - 8*cos(-a + c)^2 + 2*((4*cos(-a
+ c)^4 - 1)*cos(b*x + c)^4 + 2*cos(-a + c)^4 - (6*cos(-a + c)^4 + cos(-a +
 c)^2 - 1)*cos(b*x + c)^2 + 2*((2*cos(-a + c)^3 + cos(-a + c))*cos(b*x + c
)^3 - (2*cos(-a + c)^3 + cos(-a + c))*cos(b*x + c))*sin(b*x + c)*sin(-a +
c) + cos(-a + c)^2)*log(-1/4*cos(b*x + c)^2 + 1/4) - 2*((4*cos(-a + c)^4 -
 1)*cos(b*x + c)^4 + 2*cos(-a + c)^4 - (6*cos(-a + c)^4 + cos(-a + c)^2 -
1)*cos(b*x + c)^2 + 2*((2*cos(-a + c)^3 + cos(-a + c))*cos(b*x + c)^3 - (2
*cos(-a + c)^3 + cos(-a + c))*cos(b*x + c))*sin(b*x + c)*sin(-a + c) + cos
(-a + c)^2)*log(-(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) + (2
*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2)/(cos(-a + c)^2 + 2*cos
(-a + c) + 1)) + 1)/(2*((b*cos(-a + c)^7 - 3*b*cos(-a + c)^5 + 3*b*cos(-a
+ c)^3 - b*cos(-a + c))*cos(b*x + c)^3 - (b*cos(-a + c)^7 - 3*b*cos(-a + c
)^5 + 3*b*cos(-a + c)^3 - b*cos(-a + c))*cos(b*x + c))*sin(b*x + c) - (b*c
os(-a + c)^6 + (2*b*cos(-a + c)^6 - 5*b*cos(-a + c)^4 + 4*b*cos(-a + c)^2
- b)*cos(b*x + c)^4 - 2*b*cos(-a + c)^4 - (3*b*cos(-a + c)^6 - 7*b*cos(-a
+ c)^4 + 5*b*cos(-a + c)^2 - b)*cos(b*x + c)^2 + b*cos(-a + c)^2)*sin(-a +
 c))
```

Sympy [F]

$$\int \csc^3(a+bx)\csc^3(c+bx)\,dx = \int \csc^3(a+bx)\csc^3(bx+c)\,dx$$

input
```
integrate(csc(b*x+a)**3*csc(b*x+c)**3,x)
```

output
```
Integral(csc(a + b*x)**3*csc(b*x + c)**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\csc^3(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)^3*csc(b*x+c)^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.60 (sec) , antiderivative size = 13206, normalized size of antiderivative = 13206.00

$$\int \csc^3(a+bx)\csc^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*csc(b*x+c)^3,x, algorithm="giac")
```

output
```
1/16*((3*tan(1/2*a)^12*tan(1/2*c)^12 + 4*tan(1/2*a)^12*tan(1/2*c)^10 + 28*
tan(1/2*a)^11*tan(1/2*c)^11 + 4*tan(1/2*a)^10*tan(1/2*c)^12 - tan(1/2*a)^1
2*tan(1/2*c)^8 + 44*tan(1/2*a)^11*tan(1/2*c)^9 + 112*tan(1/2*a)^10*tan(1/2
*c)^10 + 44*tan(1/2*a)^9*tan(1/2*c)^11 - tan(1/2*a)^8*tan(1/2*c)^12 - 8*ta
n(1/2*a)^11*tan(1/2*c)^7 + 212*tan(1/2*a)^10*tan(1/2*c)^8 + 252*tan(1/2*a)
^9*tan(1/2*c)^9 + 212*tan(1/2*a)^8*tan(1/2*c)^10 - 8*tan(1/2*a)^7*tan(1/2*
c)^11 + tan(1/2*a)^12*tan(1/2*c)^4 - 8*tan(1/2*a)^11*tan(1/2*c)^5 + 536*ta
n(1/2*a)^9*tan(1/2*c)^7 + 427*tan(1/2*a)^8*tan(1/2*c)^8 + 536*tan(1/2*a)^7
*tan(1/2*c)^9 - 8*tan(1/2*a)^5*tan(1/2*c)^11 + tan(1/2*a)^4*tan(1/2*c)^12
- 4*tan(1/2*a)^12*tan(1/2*c)^2 + 44*tan(1/2*a)^11*tan(1/2*c)^3 - 212*tan(1
/2*a)^10*tan(1/2*c)^4 + 536*tan(1/2*a)^9*tan(1/2*c)^5 + 1648*tan(1/2*a)^7*
tan(1/2*c)^7 + 536*tan(1/2*a)^5*tan(1/2*c)^9 - 212*tan(1/2*a)^4*tan(1/2*c)
^10 + 44*tan(1/2*a)^3*tan(1/2*c)^11 - 4*tan(1/2*a)^2*tan(1/2*c)^12 - 3*tan
(1/2*a)^12 + 28*tan(1/2*a)^11*tan(1/2*c) - 112*tan(1/2*a)^10*tan(1/2*c)^2
+ 252*tan(1/2*a)^9*tan(1/2*c)^3 - 427*tan(1/2*a)^8*tan(1/2*c)^4 + 1648*tan
(1/2*a)^7*tan(1/2*c)^5 + 1648*tan(1/2*a)^5*tan(1/2*c)^7 - 427*tan(1/2*a)^4
*tan(1/2*c)^8 + 252*tan(1/2*a)^3*tan(1/2*c)^9 - 112*tan(1/2*a)^2*tan(1/2*c
)^10 + 28*tan(1/2*a)*tan(1/2*c)^11 - 3*tan(1/2*c)^12 - 4*tan(1/2*a)^10 + 4
4*tan(1/2*a)^9*tan(1/2*c) - 212*tan(1/2*a)^8*tan(1/2*c)^2 + 536*tan(1/2*a)
^7*tan(1/2*c)^3 + 1648*tan(1/2*a)^5*tan(1/2*c)^5 + 536*tan(1/2*a)^3*tan...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\csc^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(sin(a + b*x)^3*sin(c + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^3(a+bx)\csc^3(c+bx)\,dx = \int \csc(bx+c)^3\csc(bx+a)^3\,dx$$

input
```
int(csc(b*x+a)^3*csc(b*x+c)^3,x)
```

output
```
int(csc(b*x + c)**3*csc(a + b*x)**3,x)
```

3.108 $\int \csc^3(a+bx)\csc^4(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \csc^3(a+bx)\csc^4(c+bx)\,dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 15.32 (sec) , antiderivative size = 221, normalized size of antiderivative = 221.00

$$\int \csc^3(a+bx)\csc^4(c+bx)\,dx$$
$$= \frac{\csc^5(a-c)\left(240\text{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)(15\cos(a-c)+\cos(3(a-c)))\csc(a-c)-(349+33\ldots}{}$$

input
```
Integrate[Csc[a + b*x]^3*Csc[c + b*x]^4,x]
```

output
```
(Csc[a - c]^5*(240*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*(15*Cos[a - c] +
Cos[3*(a - c)])*Csc[a - c] - (349 + 338*Cos[2*(a - c)] + 33*Cos[4*(a - c)]
 - 120*Cos[2*(a - 2*c - b*x)] - 400*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)]
 - 400*Cos[2*(c + b*x)] + 75*Cos[4*(c + b*x)] - 40*Cos[4*a - 2*c + 2*b*x]
+ 150*Cos[2*(a + c + 2*b*x)])*Csc[a + b*x]^2*Csc[c + b*x]^3 - 240*(5 + 3*C
os[2*(a - c)])*Csc[a - c]*(Log[Cos[(a + b*x)/2]] - Log[Sin[(a + b*x)/2]]))
)/(192*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\csc^4(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc^3(a+bx)\csc^4(bx+c)dx$$

input
```
Int[Csc[a + b*x]^3*Csc[c + b*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 4.27 (sec) , antiderivative size = 1740, normalized size of antiderivative = 1740.00

method	result	size
risch	Expression too large to display	1740
default	Expression too large to display	1947

input
```
int(csc(b*x+a)^3*csc(b*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-8/3/(exp(2*I*(b*x+a))-1)^2/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^3/(exp(2*I*a)
-exp(2*I*c))^5/b*(15*exp(I*(9*b*x+17*a+8*c))+150*exp(I*(9*b*x+15*a+10*c))+
75*exp(I*(9*b*x+13*a+12*c))-40*exp(I*(7*b*x+17*a+6*c))-400*exp(I*(7*b*x+15
*a+8*c))-400*exp(I*(7*b*x+13*a+10*c))-120*exp(I*(7*b*x+11*a+12*c))+33*exp(
I*(5*b*x+17*a+4*c))+338*exp(I*(5*b*x+15*a+6*c))+698*exp(I*(5*b*x+13*a+8*c)
)+338*exp(I*(5*b*x+11*a+10*c))+33*exp(I*(5*b*x+9*a+12*c))-120*exp(I*(3*b*x
+15*a+4*c))-400*exp(I*(3*b*x+13*a+6*c))-400*exp(I*(3*b*x+11*a+8*c))-40*exp
(I*(3*b*x+9*a+10*c))+75*exp(I*(b*x+13*a+4*c))+150*exp(I*(b*x+11*a+6*c))+15
*exp(I*(b*x+9*a+8*c)))+120*ln(exp(I*(b*x+a))+1)/b/(exp(12*I*a)-6*exp(2*I*(
5*a+c))+15*exp(4*I*(2*a+c))-20*exp(6*I*(a+c))+15*exp(4*I*(a+2*c))-6*exp(2*
I*(a+5*c))+exp(12*I*c))*exp(4*I*(2*a+c))+400*ln(exp(I*(b*x+a))+1)/b/(exp(1
2*I*a)-6*exp(2*I*(5*a+c))+15*exp(4*I*(2*a+c))-20*exp(6*I*(a+c))+15*exp(4*I
*(a+2*c))-6*exp(2*I*(a+5*c))+exp(12*I*c))*exp(6*I*(a+c))+120*ln(exp(I*(b*x
+a))+1)/b/(exp(12*I*a)-6*exp(2*I*(5*a+c))+15*exp(4*I*(2*a+c))-20*exp(6*I*(
a+c))+15*exp(4*I*(a+2*c))-6*exp(2*I*(a+5*c))+exp(12*I*c))*exp(4*I*(a+2*c))
+20*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b/(exp(12*I*a)-6*exp(2*I*(5*a+c))+15*e
xp(4*I*(2*a+c))-20*exp(6*I*(a+c))+15*exp(4*I*(a+2*c))-6*exp(2*I*(a+5*c))+e
xp(12*I*c))*exp(3*I*(3*a+c))+300*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b/(exp(12
*I*a)-6*exp(2*I*(5*a+c))+15*exp(4*I*(2*a+c))-20*exp(6*I*(a+c))+15*exp(4*I*
(a+2*c))-6*exp(2*I*(a+5*c))+exp(12*I*c))*exp(I*(7*a+5*c))+300*ln(exp(I*...
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.22 (sec) , antiderivative size = 1339, normalized size of antiderivative = 1339.00

$$\int \csc^3(a+bx)\csc^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*csc(b*x+c)^4,x, algorithm="fricas")
```

output
```
1/12*(15*(((6*cos(-a + c)^4 - cos(-a + c)^2 - 1)*cos(b*x + c)^4 + 3*cos(-a
 + c)^4 - (9*cos(-a + c)^4 - 1)*cos(b*x + c)^2 + cos(-a + c)^2)*sin(b*x +
c) - 2*((3*cos(-a + c)^3 + cos(-a + c))*cos(b*x + c)^5 - 2*(3*cos(-a + c)^
3 + cos(-a + c))*cos(b*x + c)^3 + (3*cos(-a + c)^3 + cos(-a + c))*cos(b*x
+ c))*sin(-a + c))*log((cos(b*x + c)*cos(-a + c) + sin(b*x + c)*sin(-a + c
) + 1)/(cos(-a + c) + 1)) - 15*(((6*cos(-a + c)^4 - cos(-a + c)^2 - 1)*cos
(b*x + c)^4 + 3*cos(-a + c)^4 - (9*cos(-a + c)^4 - 1)*cos(b*x + c)^2 + cos
(-a + c)^2)*sin(b*x + c) - 2*((3*cos(-a + c)^3 + cos(-a + c))*cos(b*x + c)
^5 - 2*(3*cos(-a + c)^3 + cos(-a + c))*cos(b*x + c)^3 + (3*cos(-a + c)^3 +
 cos(-a + c))*cos(b*x + c))*sin(-a + c))*log(-(cos(b*x + c)*cos(-a + c) +
sin(b*x + c)*sin(-a + c) - 1)/(cos(-a + c) + 1)) - 15*(((2*cos(-a + c)^5 +
 5*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^4 + cos(-a + c)^5 - (3*cos(
-a + c)^5 + 8*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^2 + 3*cos(-a + c
)^3)*sin(b*x + c) - 2*((cos(-a + c)^4 + 3*cos(-a + c)^2)*cos(b*x + c)^5 -
2*(cos(-a + c)^4 + 3*cos(-a + c)^2)*cos(b*x + c)^3 + (cos(-a + c)^4 + 3*co
s(-a + c)^2)*cos(b*x + c))*sin(-a + c))*log(1/2*cos(b*x + c) + 1/2) + 15*(
((2*cos(-a + c)^5 + 5*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^4 + cos(
-a + c)^5 - (3*cos(-a + c)^5 + 8*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x +
c)^2 + 3*cos(-a + c)^3)*sin(b*x + c) - 2*((cos(-a + c)^4 + 3*cos(-a + c)^2
)*cos(b*x + c)^5 - 2*(cos(-a + c)^4 + 3*cos(-a + c)^2)*cos(b*x + c)^3 +...
```

Sympy [F]

$$\int \csc^3(a+bx)\csc^4(c+bx)\,dx = \int \csc^3(a+bx)\csc^4(bx+c)\,dx$$

input
```
integrate(csc(b*x+a)**3*csc(b*x+c)**4,x)
```

output
```
Integral(csc(a + b*x)**3*csc(b*x + c)**4, x)
```

Maxima [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\csc^4(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)^3*csc(b*x+c)^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 16.75 (sec) , antiderivative size = 41036, normalized size of antiderivative = 41036.00

$$\int \csc^3(a+bx)\csc^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*csc(b*x+c)^4,x, algorithm="giac")
```

output
```
1/384*(60*(tan(1/2*a)^12*tan(1/2*c)^12 + 3*tan(1/2*a)^12*tan(1/2*c)^10 + 6
*tan(1/2*a)^11*tan(1/2*c)^11 + 3*tan(1/2*a)^10*tan(1/2*c)^12 + 3*tan(1/2*a
)^12*tan(1/2*c)^8 + 18*tan(1/2*a)^11*tan(1/2*c)^9 + 24*tan(1/2*a)^10*tan(1
/2*c)^10 + 18*tan(1/2*a)^9*tan(1/2*c)^11 + 3*tan(1/2*a)^8*tan(1/2*c)^12 +
2*tan(1/2*a)^12*tan(1/2*c)^6 + 12*tan(1/2*a)^11*tan(1/2*c)^7 + 69*tan(1/2*
a)^10*tan(1/2*c)^8 + 54*tan(1/2*a)^9*tan(1/2*c)^9 + 69*tan(1/2*a)^8*tan(1/
2*c)^10 + 12*tan(1/2*a)^7*tan(1/2*c)^11 + 2*tan(1/2*a)^6*tan(1/2*c)^12 + 3
*tan(1/2*a)^12*tan(1/2*c)^4 - 12*tan(1/2*a)^11*tan(1/2*c)^5 + 96*tan(1/2*a
)^10*tan(1/2*c)^6 + 36*tan(1/2*a)^9*tan(1/2*c)^7 + 249*tan(1/2*a)^8*tan(1/
2*c)^8 + 36*tan(1/2*a)^7*tan(1/2*c)^9 + 96*tan(1/2*a)^6*tan(1/2*c)^10 - 12
*tan(1/2*a)^5*tan(1/2*c)^11 + 3*tan(1/2*a)^4*tan(1/2*c)^12 + 3*tan(1/2*a)^
12*tan(1/2*c)^2 - 18*tan(1/2*a)^11*tan(1/2*c)^3 + 69*tan(1/2*a)^10*tan(1/2
*c)^4 - 36*tan(1/2*a)^9*tan(1/2*c)^5 + 366*tan(1/2*a)^8*tan(1/2*c)^6 + 24*
tan(1/2*a)^7*tan(1/2*c)^7 + 366*tan(1/2*a)^6*tan(1/2*c)^8 - 36*tan(1/2*a)^
5*tan(1/2*c)^9 + 69*tan(1/2*a)^4*tan(1/2*c)^10 - 18*tan(1/2*a)^3*tan(1/2*c
)^11 + 3*tan(1/2*a)^2*tan(1/2*c)^12 + tan(1/2*a)^12 - 6*tan(1/2*a)^11*tan(
1/2*c) + 24*tan(1/2*a)^10*tan(1/2*c)^2 - 54*tan(1/2*a)^9*tan(1/2*c)^3 + 24
9*tan(1/2*a)^8*tan(1/2*c)^4 - 24*tan(1/2*a)^7*tan(1/2*c)^5 + 544*tan(1/2*a
)^6*tan(1/2*c)^6 - 24*tan(1/2*a)^5*tan(1/2*c)^7 + 249*tan(1/2*a)^4*tan(1/2
*c)^8 - 54*tan(1/2*a)^3*tan(1/2*c)^9 + 24*tan(1/2*a)^2*tan(1/2*c)^10 - ...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\csc^4(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(sin(a + b*x)^3*sin(c + b*x)^4),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^3(a+bx)\csc^4(c+bx)\,dx = \int \csc(bx+c)^4 \csc(bx+a)^3\,dx$$

input
```
int(csc(b*x+a)^3*csc(b*x+c)^4,x)
```

output
```
int(csc(b*x + c)**4*csc(a + b*x)**3,x)
```

3.109 $\int \csc^3(a+bx)\csc^5(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \csc^3(a+bx)\csc^5(c+bx)\,dx = 0$$

output
```
0
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 6.84 (sec) , antiderivative size = 1966, normalized size of antiderivative = 1966.00

$$\int \csc^3(a+bx)\csc^5(c+bx)\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[a + b*x]^3*Csc[c + b*x]^5,x]
```

output
```
((3*I)*ArcTan[Tan[a + b*x]]*(3 + 2*Cos[2*a - 2*c])*Csc[a - c]^7)/b - ((3*I
)*ArcTan[Tan[c + b*x]]*(3 + 2*Cos[2*a - 2*c])*Csc[a - c]^7)/b - (3*(3 + 2*
Cos[2*a - 2*c])*Csc[a - c]^7*Log[Sin[a + b*x]^2])/(2*b) + (3*(3 + 2*Cos[2*
a - 2*c])*Csc[a - c]^7*Log[Sin[c + b*x]^2])/(2*b) + x*((-9*I)/(Cos[c]*Sin[
a] - Cos[a]*Sin[c])^7 + ((3*I)*Cos[c]^2)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^7
 - ((9*I)*Cos[a]^2*Cos[c]^2)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^7 + (9*Cot[a]
)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^7 + (3*Cos[c]^2*Cot[a])/(Cos[c]*Sin[a] -
 Cos[a]*Sin[c])^7 + (3*Cos[a]^2*Cos[c]^2*Cot[a])/(Cos[c]*Sin[a] - Cos[a]*S
in[c])^7 - (9*Cos[a]*Cos[c]^2*Sin[a])/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^7 +
((3*I)*Cos[c]^2*Sin[a]^2)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^7 + (6*Cos[c]*Si
n[c])/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^7 + (18*Cos[a]^2*Cos[c]*Sin[c])/(Cos
[c]*Sin[a] - Cos[a]*Sin[c])^7 - ((6*I)*Cos[c]*Cot[a]*Sin[c])/(Cos[c]*Sin[a
] - Cos[a]*Sin[c])^7 + ((6*I)*Cos[a]^2*Cos[c]*Cot[a]*Sin[c])/(Cos[c]*Sin[a
] - Cos[a]*Sin[c])^7 - ((18*I)*Cos[a]*Cos[c]*Sin[a]*Sin[c])/(Cos[c]*Sin[a]
 - Cos[a]*Sin[c])^7 - (6*Cos[c]*Sin[a]^2*Sin[c])/(Cos[c]*Sin[a] - Cos[a]*S
in[c])^7 - ((3*I)*Sin[c]^2)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^7 + ((9*I)*Cos
[a]^2*Sin[c]^2)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^7 - (3*Cot[a]*Sin[c]^2)/(C
os[c]*Sin[a] - Cos[a]*Sin[c])^7 - (3*Cos[a]^2*Cot[a]*Sin[c]^2)/(Cos[c]*Sin
[a] - Cos[a]*Sin[c])^7 + (9*Cos[a]*Sin[a]*Sin[c]^2)/(Cos[c]*Sin[a] - Cos[a
]*Sin[c])^7 - ((3*I)*Sin[a]^2*Sin[c]^2)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\csc^5(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc^3(a+bx)\csc^5(bx+c)dx$$

input
```
Int[Csc[a + b*x]^3*Csc[c + b*x]^5,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 7.97 (sec) , antiderivative size = 1222, normalized size of antiderivative = 1222.00

method	result	size
risch	Expression too large to display	1222
default	Expression too large to display	1795

input
```
int(csc(b*x+a)^3*csc(b*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
32*I/(exp(2*I*(b*x+a))-1)^2/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^4/(exp(2*I*a)
-exp(2*I*c))^6/b*(-2*exp(I*(2*b*x+19*a+3*c))+18*exp(I*(8*b*x+13*a+15*c))-2
48*exp(I*(6*b*x+15*a+11*c))-12*exp(5*I*(2*b*x+3*a+3*c))-4*exp(I*(2*b*x+11*
a+11*c))-50*exp(I*(2*b*x+17*a+5*c))-36*exp(I*(10*b*x+17*a+13*c))+42*exp(I*
(8*b*x+19*a+9*c))+6*exp(I*(4*b*x+11*a+13*c))+exp(I*(4*b*x+21*a+3*c))+exp(I
*(17*a+3*c))+19*exp(I*(4*b*x+19*a+5*c))-220*exp(I*(6*b*x+17*a+9*c))+263*ex
p(I*(4*b*x+15*a+9*c))+exp(I*(11*a+9*c))-12*exp(I*(10*b*x+19*a+11*c))+29*ex
p(5*I*(3*a+c))+96*exp(I*(8*b*x+15*a+13*c))-76*exp(I*(6*b*x+13*a+13*c))+140
*exp(I*(4*b*x+13*a+11*c))+171*exp(I*(4*b*x+17*a+7*c))-106*exp(I*(2*b*x+13*
a+9*c))+144*exp(I*(8*b*x+17*a+11*c))-4*exp(I*(6*b*x+11*a+15*c))-138*exp(I*
(2*b*x+15*a+7*c))-52*exp(I*(6*b*x+19*a+7*c))+29*exp(I*(13*a+7*c)))+384*I*l
n(exp(2*I*(b*x+a))-1)/(exp(14*I*a)-7*exp(2*I*(6*a+c))+21*exp(2*I*(5*a+2*c)
)-35*exp(2*I*(4*a+3*c))+35*exp(2*I*(3*a+4*c))-21*exp(2*I*(2*a+5*c))+7*exp(
2*I*(a+6*c))-exp(14*I*c))/b*exp(I*(5*c+9*a))+1152*I*ln(exp(2*I*(b*x+a))-1)
/(exp(14*I*a)-7*exp(2*I*(6*a+c))+21*exp(2*I*(5*a+2*c))-35*exp(2*I*(4*a+3*c
))+35*exp(2*I*(3*a+4*c))-21*exp(2*I*(2*a+5*c))+7*exp(2*I*(a+6*c))-exp(14*I
*c))/b*exp(7*I*(a+c))+384*I*ln(exp(2*I*(b*x+a))-1)/(exp(14*I*a)-7*exp(2*I*
(6*a+c))+21*exp(2*I*(5*a+2*c))-35*exp(2*I*(4*a+3*c))+35*exp(2*I*(3*a+4*c))
-21*exp(2*I*(2*a+5*c))+7*exp(2*I*(a+6*c))-exp(14*I*c))/b*exp(I*(5*a+9*c))-
384*I*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))/(exp(14*I*a)-7*exp(2*I*(6*a+c...
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.15 (sec) , antiderivative size = 1195, normalized size of antiderivative = 1195.00

$$\int \csc^3(a+bx)\csc^5(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*csc(b*x+c)^5,x, algorithm="fricas")
```

output
```
-1/4*(8*(2*cos(-a + c)^6 + 11*cos(-a + c)^4 - 13*cos(-a + c)^2)*cos(b*x +
c)^6 - 7*cos(-a + c)^6 - 2*(20*cos(-a + c)^6 + 98*cos(-a + c)^4 - 121*cos(
-a + c)^2 + 3)*cos(b*x + c)^4 - 18*cos(-a + c)^4 + 3*(10*cos(-a + c)^6 + 4
3*cos(-a + c)^4 - 56*cos(-a + c)^2 + 3)*cos(b*x + c)^2 + 2*(2*(4*cos(-a +
c)^5 + 24*cos(-a + c)^3 - 13*cos(-a + c))*cos(b*x + c)^5 - 2*(8*cos(-a + c
)^5 + 43*cos(-a + c)^3 - 21*cos(-a + c))*cos(b*x + c)^3 + 3*(3*cos(-a + c)
^5 + 12*cos(-a + c)^3 - 5*cos(-a + c))*cos(b*x + c))*sin(b*x + c)*sin(-a +
 c) + 27*cos(-a + c)^2 + 6*((8*cos(-a + c)^4 - 2*cos(-a + c)^2 - 1)*cos(b*
x + c)^6 - (20*cos(-a + c)^4 - 3*cos(-a + c)^2 - 2)*cos(b*x + c)^4 - 4*cos
(-a + c)^4 + (16*cos(-a + c)^4 - 1)*cos(b*x + c)^2 + 2*((4*cos(-a + c)^3 +
 cos(-a + c))*cos(b*x + c)^5 - 2*(4*cos(-a + c)^3 + cos(-a + c))*cos(b*x +
 c)^3 + (4*cos(-a + c)^3 + cos(-a + c))*cos(b*x + c))*sin(b*x + c)*sin(-a
+ c) - cos(-a + c)^2)*log(-1/4*cos(b*x + c)^2 + 1/4) - 6*((8*cos(-a + c)^4
 - 2*cos(-a + c)^2 - 1)*cos(b*x + c)^6 - (20*cos(-a + c)^4 - 3*cos(-a + c)
^2 - 2)*cos(b*x + c)^4 - 4*cos(-a + c)^4 + (16*cos(-a + c)^4 - 1)*cos(b*x
+ c)^2 + 2*((4*cos(-a + c)^3 + cos(-a + c))*cos(b*x + c)^5 - 2*(4*cos(-a +
 c)^3 + cos(-a + c))*cos(b*x + c)^3 + (4*cos(-a + c)^3 + cos(-a + c))*cos(
b*x + c))*sin(b*x + c)*sin(-a + c) - cos(-a + c)^2)*log(-(2*cos(b*x + c)*c
os(-a + c)*sin(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2
 - cos(-a + c)^2)/(cos(-a + c)^2 + 2*cos(-a + c) + 1)) - 2)/(2*((b*cos(...
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\csc^5(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*csc(b*x+c)**5,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\csc^5(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)^3*csc(b*x+c)^5,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 7.00 (sec) , antiderivative size = 57794, normalized size of antiderivative = 57794.00

$$\int \csc^3(a+bx)\csc^5(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*csc(b*x+c)^5,x, algorithm="giac")
```

output
```
1/512*(12*(5*tan(1/2*a)^16*tan(1/2*c)^16 + 14*tan(1/2*a)^16*tan(1/2*c)^14
+ 52*tan(1/2*a)^15*tan(1/2*c)^15 + 14*tan(1/2*a)^14*tan(1/2*c)^16 + 6*tan(
1/2*a)^16*tan(1/2*c)^12 + 172*tan(1/2*a)^15*tan(1/2*c)^13 + 244*tan(1/2*a)
^14*tan(1/2*c)^14 + 172*tan(1/2*a)^13*tan(1/2*c)^15 + 6*tan(1/2*a)^12*tan(
1/2*c)^16 - 10*tan(1/2*a)^16*tan(1/2*c)^10 + 132*tan(1/2*a)^15*tan(1/2*c)^
11 + 836*tan(1/2*a)^14*tan(1/2*c)^12 + 884*tan(1/2*a)^13*tan(1/2*c)^13 + 8
36*tan(1/2*a)^12*tan(1/2*c)^14 + 132*tan(1/2*a)^11*tan(1/2*c)^15 - 10*tan(
1/2*a)^10*tan(1/2*c)^16 - 100*tan(1/2*a)^15*tan(1/2*c)^9 + 996*tan(1/2*a)^
14*tan(1/2*c)^10 + 2012*tan(1/2*a)^13*tan(1/2*c)^11 + 3284*tan(1/2*a)^12*t
an(1/2*c)^12 + 2012*tan(1/2*a)^11*tan(1/2*c)^13 + 996*tan(1/2*a)^10*tan(1/
2*c)^14 - 100*tan(1/2*a)^9*tan(1/2*c)^15 + 10*tan(1/2*a)^16*tan(1/2*c)^6 -
 100*tan(1/2*a)^15*tan(1/2*c)^7 + 2820*tan(1/2*a)^13*tan(1/2*c)^9 + 4084*t
an(1/2*a)^12*tan(1/2*c)^10 + 8212*tan(1/2*a)^11*tan(1/2*c)^11 + 4084*tan(1
/2*a)^10*tan(1/2*c)^12 + 2820*tan(1/2*a)^9*tan(1/2*c)^13 - 100*tan(1/2*a)^
7*tan(1/2*c)^15 + 10*tan(1/2*a)^6*tan(1/2*c)^16 - 6*tan(1/2*a)^16*tan(1/2*
c)^4 + 132*tan(1/2*a)^15*tan(1/2*c)^5 - 996*tan(1/2*a)^14*tan(1/2*c)^6 + 2
820*tan(1/2*a)^13*tan(1/2*c)^7 + 15500*tan(1/2*a)^11*tan(1/2*c)^9 + 5140*t
an(1/2*a)^10*tan(1/2*c)^10 + 15500*tan(1/2*a)^9*tan(1/2*c)^11 + 2820*tan(1
/2*a)^7*tan(1/2*c)^13 - 996*tan(1/2*a)^6*tan(1/2*c)^14 + 132*tan(1/2*a)^5*
tan(1/2*c)^15 - 6*tan(1/2*a)^4*tan(1/2*c)^16 - 14*tan(1/2*a)^16*tan(1/2...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\csc^5(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(sin(a + b*x)^3*sin(c + b*x)^5),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^3(a+bx)\csc^5(c+bx)\,dx = \int \csc(bx+c)^5 \csc(bx+a)^3\,dx$$

input
```
int(csc(b*x+a)^3*csc(b*x+c)^5,x)
```

output
```
int(csc(b*x + c)**5*csc(a + b*x)**3,x)
```

3.110 $\int \sin(a+bx)\sin^3(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \sin(a+bx)\sin^3(c+dx)\,dx = -\frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)} -\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(a+3c+(b+3d)x)}{8(b+3d)}$$

output
```
-1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3*sin(a-c+(b-d)*x)/(8*b-8*d)-3*sin(a+c+(
b+d)*x)/(8*b+8*d)+sin(a+3*c+(b+3*d)*x)/(8*b+24*d)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \sin(a+bx)\sin^3(c+dx)\,dx = \frac{1}{8}\left(-\frac{\sin(a-3c+bx-3dx)}{b-3d}+\frac{3\sin(a-c+bx-dx)}{b-d} +\frac{\sin(a+3c+bx+3dx)}{b+3d}-\frac{3\sin(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Sin[a + b*x]*Sin[c + d*x]^3,x]
```

output
```
(-(Sin[a - 3*c + b*x - 3*d*x]/(b - 3*d)) + (3*Sin[a - c + b*x - d*x])/(b -
 d) + Sin[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Sin[a + c + (b + d)*x])/(b
 + d))/8
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^3(c+dx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{1}{8}\cos(a+x(b-3d)-3c)+\frac{3}{8}\cos(a+x(b-d)-c)-\frac{3}{8}\cos(a+x(b+d)+c)+\frac{1}{8}\cos(a+x(b+3d)+3c\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

input
```
Int[Sin[a + b*x]*Sin[c + d*x]^3,x]
```

output
```
-1/8*Sin[a - 3*c + (b - 3*d)*x]/(b - 3*d) + (3*Sin[a - c + (b - d)*x])/(8*
(b - d)) - (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[a + 3*c + (b + 3*d
)*x]/(8*(b + 3*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(a+3c+(b+3d)x)}{8b+24d}$
risch	$-\frac{\sin(bx-3dx+a-3c)}{8(b-3d)}+\frac{3\sin(bx-dx+a-c)}{8(b-d)}-\frac{3\sin(bx+dx+a+c)}{8(b+d)}+\frac{\sin(bx+3dx+a+3c)}{8b+24d}$
parallelrisch	$\frac{12d^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+12b\,d^2\left(-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+12(-2b^2d+3d^3)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+}{(b-d)(b+}$
orering	Expression too large to display

input
```
int(sin(b*x+a)*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8/(b-d)*sin(a-c+(b-d)*x)-3/8/(b+d)*sin
(a+c+(b+d)*x)+1/8/(b+3*d)*sin(a+3*c+(b+3*d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.34

$$\int \sin(a+bx)\sin^3(c+dx)\,dx =$$
$$-\frac{3\left(\left(b^2d-d^3\right)\cos\left(dx+c\right)^3-\left(b^2d-3\,d^3\right)\cos\left(dx+c\right)\right)\sin\left(bx+a\right)-\left(\left(b^3-bd^2\right)\cos\left(bx+a\right)\cos\left(dx-\right.\right.}{b^4-10\,b^2d^2+9\,d^4}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
-(3*((b^2*d - d^3)*cos(d*x + c)^3 - (b^2*d - 3*d^3)*cos(d*x + c))*sin(b*x
+ a) - ((b^3 - b*d^2)*cos(b*x + a)*cos(d*x + c)^2 - (b^3 - 7*b*d^2)*cos(b*
x + a))*sin(d*x + c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(76) = 152$.

Time = 2.00 (sec) , antiderivative size = 921, normalized size of antiderivative = 10.12

$$\int \sin(a+bx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)**3,x)
```

output
```
Piecewise((x*sin(a)*sin(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x)*sin
(c + d*x)**3/8 - 3*x*sin(a - 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*s
in(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/8 + x*cos(a - 3*d*x)*cos(c + d*
x)**3/8 + sin(a - 3*d*x)*cos(c + d*x)**3/(8*d) + 7*sin(c + d*x)**3*cos(a -
 3*d*x)/(24*d) + sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b,
-3*d)), (3*x*sin(a - d*x)*sin(c + d*x)**3/8 + 3*x*sin(a - d*x)*sin(c + d*x
)*cos(c + d*x)**2/8 - 3*x*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/8 - 3*
x*cos(a - d*x)*cos(c + d*x)**3/8 + 3*sin(a - d*x)*cos(c + d*x)**3/(8*d) +
5*sin(c + d*x)**3*cos(a - d*x)/(8*d) + 3*sin(c + d*x)*cos(a - d*x)*cos(c +
 d*x)**2/(4*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**3/8 + 3*x*sin(
a + d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a + d*x)
*cos(c + d*x)/8 + 3*x*cos(a + d*x)*cos(c + d*x)**3/8 + 3*sin(a + d*x)*cos(
c + d*x)**3/(8*d) - 5*sin(c + d*x)**3*cos(a + d*x)/(8*d) - 3*sin(c + d*x)*
cos(a + d*x)*cos(c + d*x)**2/(4*d), Eq(b, d)), (x*sin(a + 3*d*x)*sin(c + d
*x)**3/8 - 3*x*sin(a + 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c +
 d*x)**2*cos(a + 3*d*x)*cos(c + d*x)/8 - x*cos(a + 3*d*x)*cos(c + d*x)**3/
8 + sin(a + 3*d*x)*cos(c + d*x)**3/(8*d) - 7*sin(c + d*x)**3*cos(a + 3*d*x
)/(24*d) - sin(c + d*x)*cos(a + 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b, 3*d)),
 (-b**3*sin(c + d*x)**3*cos(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 3*b*
*2*d*sin(a + b*x)*sin(c + d*x)**2*cos(c + d*x)/(b**4 - 10*b**2*d**2 + 9...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(83) = 166$.

Time = 0.11 (sec) , antiderivative size = 916, normalized size of antiderivative = 10.07

$$\int \sin(a + bx) \sin^3(c + dx) \, dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/16*((b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))
*cos((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin
(3*c) + 3*d^3*sin(3*c))*cos((b + 3*d)*x + a) - 3*(b^3*sin(3*c) - b^2*d*sin
(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x + a + 4*c) + 3*(b
^3*sin(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b +
 d)*x + a - 2*c) - 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9
*d^3*sin(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*sin(3*c) + b^2*d*sin(3*c
) - 9*b*d^2*sin(3*c) - 9*d^3*sin(3*c))*cos(-(b - d)*x - a - 2*c) + (b^3*si
n(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))*cos(-(b - 3*d
)*x - a + 6*c) - (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3
*sin(3*c))*cos(-(b - 3*d)*x - a) - (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^
2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*cos(3*c) -
3*b^2*d*cos(3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a) +
 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*sin
((b + d)*x + a + 4*c) + 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c
) + 9*d^3*cos(3*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos
(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 3*(
b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b
 - d)*x - a - 2*c) - (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3
*d^3*cos(3*c))*sin(-(b - 3*d)*x - a + 6*c) - (b^3*cos(3*c) + 3*b^2*d*co...
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\int \sin(a+bx)\sin^3(c+dx)\,dx = \frac{\sin(bx+3\,dx+a+3\,c)}{8\,(b+3\,d)} - \frac{3\,\sin(bx+dx+a+c)}{8\,(b+d)} + \frac{3\,\sin(bx-dx+a-c)}{8\,(b-d)} - \frac{\sin(bx-3\,dx+a-3\,c)}{8\,(b-3\,d)}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*sin(b*x + d*x + a + c)/(b +
 d) + 3/8*sin(b*x - d*x + a - c)/(b - d) - 1/8*sin(b*x - 3*d*x + a - 3*c)/
(b - 3*d)
```

Mupad [B] (verification not implemented)

Time = 18.53 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.42

$$\int \sin(a+bx)\sin^3(c+dx)\,dx = -\mathrm{e}^{a\,1\mathrm{i}-c\,3\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,3\mathrm{i}}\left(\frac{b+3\,d}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}} + \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(b-3\,d)}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}\right) + \mathrm{e}^{a\,1\mathrm{i}+c\,3\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,3\mathrm{i}}\left(\frac{b-3\,d}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}} + \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(b+3\,d)}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}\right) + \mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{3\,b+3\,d}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}} + \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(3\,b-3\,d)}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}\right) - \mathrm{e}^{a\,1\mathrm{i}+c\,1\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,1\mathrm{i}}\left(\frac{3\,b-3\,d}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}} + \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(3\,b+3\,d)}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}\right)$$

input
```
int(sin(a + b*x)*sin(c + d*x)^3,x)
```

output
```
exp(a*1i + c*3i + b*x*1i + d*x*3i)*((b - 3*d)/(b^2*16i - d^2*144i) + (exp(
- a*2i - b*x*2i)*(b + 3*d))/(b^2*16i - d^2*144i)) - exp(a*1i - c*3i + b*x*
1i - d*x*3i)*((b + 3*d)/(b^2*16i - d^2*144i) + (exp(- a*2i - b*x*2i)*(b -
3*d))/(b^2*16i - d^2*144i)) + exp(a*1i - c*1i + b*x*1i - d*x*1i)*((3*b + 3
*d)/(b^2*16i - d^2*16i) + (exp(- a*2i - b*x*2i)*(3*b - 3*d))/(b^2*16i - d^
2*16i)) - exp(a*1i + c*1i + b*x*1i + d*x*1i)*((3*b - 3*d)/(b^2*16i - d^2*1
6i) + (exp(- a*2i - b*x*2i)*(3*b + 3*d))/(b^2*16i - d^2*16i))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.59

$$\int \sin(a+bx)\sin^3(c+dx)\,dx$$

$$= \frac{-\cos(bx+a)\sin(dx+c)^3 b^3 + \cos(bx+a)\sin(dx+c)^3 b\,d^2 + 6\cos(bx+a)\sin(dx+c)\,b\,d^2 + 3\cos(a}{b^4 - 1}$$

input
```
int(sin(b*x+a)*sin(d*x+c)^3,x)
```

output
```
( - cos(a + b*x)*sin(c + d*x)**3*b**3 + cos(a + b*x)*sin(c + d*x)**3*b*d**
2 + 6*cos(a + b*x)*sin(c + d*x)*b*d**2 + 3*cos(c + d*x)*sin(a + b*x)*sin(c
 + d*x)**2*b**2*d - 3*cos(c + d*x)*sin(a + b*x)*sin(c + d*x)**2*d**3 - 6*c
os(c + d*x)*sin(a + b*x)*d**3)/(b**4 - 10*b**2*d**2 + 9*d**4)
```

3.111 $\int \sin(a+bx)\sin^2(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \sin(a+bx)\sin^2(c+dx)\,dx = -\frac{\cos(a+bx)}{2b}+\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}$$

output
```
-1/2*cos(b*x+a)/b+cos(a-2*c+(b-2*d)*x)/(4*b-8*d)+cos(a+2*c+(b+2*d)*x)/(4*b
+8*d)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \sin(a+bx)\sin^2(c+dx)\,dx = \frac{1}{4}\left(-\frac{2\cos(a)\cos(bx)}{b}+\frac{\cos(a-2c+bx-2dx)}{b-2d}+\frac{\cos(a+2c+bx+2dx)}{b+2d}+\frac{2\sin(a)\sin(bx)}{b}\right)$$

input
```
Integrate[Sin[a + b*x]*Sin[c + d*x]^2,x]
```

output
```
((-2*Cos[a]*Cos[b*x])/b + Cos[a - 2*c + b*x - 2*d*x]/(b - 2*d) + Cos[a + 2
*c + b*x + 2*d*x]/(b + 2*d) + (2*Sin[a]*Sin[b*x])/b)/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^2(c+dx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{1}{4}\sin(a+x(b-2d)-2c)-\frac{1}{4}\sin(a+x(b+2d)+2c)+\frac{1}{2}\sin(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

input
```
Int[Sin[a + b*x]*Sin[c + d*x]^2,x]
```

output
```
-1/2*Cos[a + b*x]/b + Cos[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cos[a + 2
*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\cos(bx+a)}{2b}+\frac{\cos(a-2c+(b-2d)x)}{4b-8d}+\frac{\cos(a+2c+(b+2d)x)}{4b+8d}$
risch	$-\frac{\cos(bx+a)}{2b}+\frac{\cos(bx-2dx+a-2c)}{4b-8d}+\frac{\cos(bx+2dx+a+2c)}{4b+8d}$
parallelrisch	$\frac{b(b+2d)\cos(a-2c+(b-2d)x)+b(b-2d)\cos(a+2c+(b+2d)x)+(-2b^2+8d^2)\cos(bx+a)-8d^2}{4b^3-16b\,d^2}$
norman	$\frac{\frac{4d^2}{b(b^2-4d^2)}+\frac{4d^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{b(b^2-4d^2)}+\frac{8d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-4d^2}-\frac{8d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{b^2-4d^2}+\frac{4b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{b^2-4d^2}+2(-2}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$
orering	$-\frac{(3b^4+16d^4)\left(b\cos(bx+a)\sin(dx+c)^2+2\sin(bx+a)\sin(dx+c)d\cos(dx+c)\right)}{b^2(b^4-8b^2d^2+16d^4)}-\frac{(3b^2+8d^2)\left(-b^3\cos(bx+a)\sin(dx+c)^2-6\right.}{}$

input
```
int(sin(b*x+a)*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(b*x+a)/b+1/4/(b-2*d)*cos(a-2*c+(b-2*d)*x)+1/4/(b+2*d)*cos(a+2*c+(
b+2*d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \sin(a+bx)\sin^2(c+dx)\,dx$$
$$=\frac{b^2\cos(bx+a)\cos(dx+c)^2+2\,bd\cos(dx+c)\sin(bx+a)\sin(dx+c)-(b^2-2\,d^2)\cos(bx+a)}{b^3-4\,bd^2}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
(b^2*cos(b*x + a)*cos(d*x + c)^2 + 2*b*d*cos(d*x + c)*sin(b*x + a)*sin(d*x
 + c) - (b^2 - 2*d^2)*cos(b*x + a))/(b^3 - 4*b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(49) = 98$.

Time = 0.72 (sec) , antiderivative size = 408, normalized size of antiderivative = 6.58

$$\int \sin(a+bx)\sin^2(c+dx)\,dx$$
$$= \begin{cases} x\sin(a)\sin^2(c) \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\sin(a) \\ \frac{x\sin(a-2dx)\sin^2(c+dx)}{4}-\frac{x\sin(a-2dx)\cos^2(c+dx)}{4}-\frac{x\sin(c+dx)\cos(a-2dx)\cos(c+dx)}{2}-\frac{3\sin(a-2dx)\sin(c+dx)\cos(c+dx)}{4d}+ \\ \frac{x\sin(a+2dx)\sin^2(c+dx)}{4}-\frac{x\sin(a+2dx)\cos^2(c+dx)}{4}+\frac{x\sin(c+dx)\cos(a+2dx)\cos(c+dx)}{2}-\frac{3\sin(a+2dx)\sin(c+dx)\cos(c+dx)}{4d}- \\ -\frac{b^2\sin^2(c+dx)\cos(a+bx)}{b^3-4bd^2}+\frac{2bd\sin(a+bx)\sin(c+dx)\cos(c+dx)}{b^3-4bd^2}+\frac{2d^2\sin^2(c+dx)\cos(a+bx)}{b^3-4bd^2}+\frac{2d^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2} \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)**2,x)
```

output
```
Piecewise((x*sin(a)*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
 + x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a), Eq(b, 0)
), (x*sin(a - 2*d*x)*sin(c + d*x)**2/4 - x*sin(a - 2*d*x)*cos(c + d*x)**2/
4 - x*sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/2 - 3*sin(a - 2*d*x)*sin(c
+ d*x)*cos(c + d*x)/(4*d) + cos(a - 2*d*x)*cos(c + d*x)**2/(2*d), Eq(b, -2
*d)), (x*sin(a + 2*d*x)*sin(c + d*x)**2/4 - x*sin(a + 2*d*x)*cos(c + d*x)*
*2/4 + x*sin(c + d*x)*cos(a + 2*d*x)*cos(c + d*x)/2 - 3*sin(a + 2*d*x)*sin
(c + d*x)*cos(c + d*x)/(4*d) - cos(a + 2*d*x)*cos(c + d*x)**2/(2*d), Eq(b,
 2*d)), (-b**2*sin(c + d*x)**2*cos(a + b*x)/(b**3 - 4*b*d**2) + 2*b*d*sin(
a + b*x)*sin(c + d*x)*cos(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*sin(c + d*x)
**2*cos(a + b*x)/(b**3 - 4*b*d**2) + 2*d**2*cos(a + b*x)*cos(c + d*x)**2/(
b**3 - 4*b*d**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(56) = 112$.

Time = 0.06 (sec) , antiderivative size = 414, normalized size of antiderivative = 6.68

$$\int \sin(a+bx)\sin^2(c+dx)\,dx = \frac{(b^2\cos(2c) - 2bd\cos(2c))\cos((b+2d)x+a+4c) + (b^2\cos(2c) - 2bd\cos(2c))\cos((b+2d)x+a)}{}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/8*((b^2*cos(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a + 4*c) + (b^2*cos
(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(2
*c))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*cos(-(b
 - 2*d)*x - a) - 2*(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a + 2*c) - 2*
(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a - 2*c) + (b^2*sin(2*c) - 2*b*d
*sin(2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*sin(2*c) - 2*b*d*sin(2*c))*si
n((b + 2*d)*x + a) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a
+ 4*c) - (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a) - 2*(b^2*si
n(2*c) - 4*d^2*sin(2*c))*sin(b*x + a + 2*c) + 2*(b^2*sin(2*c) - 4*d^2*sin(
2*c))*sin(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c)
^2 + b*sin(2*c)^2)*d^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \sin(a+bx)\sin^2(c+dx)\,dx = \frac{\cos(bx+2dx+a+2c)}{4(b+2d)} + \frac{\cos(bx-2dx+a-2c)}{4(b-2d)} - \frac{\cos(bx+a)}{2b}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*cos(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*cos(b*x - 2*d*x + a - 2*c)/
(b - 2*d) - 1/2*cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.58

$$\int \sin(a+bx)\sin^2(c+dx)\,dx =$$
$$-\frac{d\,(2\,b\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)-2\,b\,\cos\left(a+2\,c+b\,x+2\,d\,x\right))+b^2\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)+b^2\,\mathrm{c}}{16\,b\,d^2-4\,b^3}$$
$$-\frac{\cos\left(a+b\,x\right)}{2\,b}$$

input
```
int(sin(a + b*x)*sin(c + d*x)^2,x)
```

output
```
- (d*(2*b*cos(a - 2*c + b*x - 2*d*x) - 2*b*cos(a + 2*c + b*x + 2*d*x)) + b
^2*cos(a - 2*c + b*x - 2*d*x) + b^2*cos(a + 2*c + b*x + 2*d*x))/(16*b*d^2
- 4*b^3) - cos(a + b*x)/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \sin(a+bx)\sin^2(c+dx)\,dx$$
$$=\frac{-\cos\left(bx+a\right)\sin\left(dx+c\right)^2b^2+2\cos\left(bx+a\right)d^2+2\cos\left(dx+c\right)\sin\left(bx+a\right)\sin\left(dx+c\right)bd-2d^2}{b\left(b^2-4d^2\right)}$$

input
```
int(sin(b*x+a)*sin(d*x+c)^2,x)
```

output
```
( - cos(a + b*x)*sin(c + d*x)**2*b**2 + 2*cos(a + b*x)*d**2 + 2*cos(c + d*
x)*sin(a + b*x)*sin(c + d*x)*b*d - 2*d**2)/(b*(b**2 - 4*d**2))
```

3.112 $\int \sin(a+bx)\sin(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \sin(a+bx)\sin(c+dx)\,dx=\frac{\sin(a-c+(b-d)x)}{2(b-d)}-\frac{\sin(a+c+(b+d)x)}{2(b+d)}$$

output
```
sin(a-c+(b-d)*x)/(2*b-2*d)-sin(a+c+(b+d)*x)/(2*b+2*d)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\sin(c+dx)\,dx=\frac{\sin(a-c+(b-d)x)}{2(b-d)}-\frac{\sin(a+c+(b+d)x)}{2(b+d)}$$

input
```
Integrate[Sin[a + b*x]*Sin[c + d*x],x]
```

output
```
Sin[a - c + (b - d)*x]/(2*(b - d)) - Sin[a + c + (b + d)*x]/(2*(b + d))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin(c+dx)\,dx$$

$$\Big\downarrow \ 5080$$

$$\int \left(\frac{1}{2}\cos(a+x(b-d)-c)-\frac{1}{2}\cos(a+x(b+d)+c)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}-\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

input
```
Int[Sin[a + b*x]*Sin[c + d*x],x]
```

output
```
Sin[a - c + (b - d)*x]/(2*(b - d)) - Sin[a + c + (b + d)*x]/(2*(b + d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result
default	$\frac{\sin(a-c+(b-d)x)}{2b-2d}-\frac{\sin(a+c+(b+d)x)}{2(b+d)}$
risch	$\frac{\sin(bx-dx+a-c)}{2b-2d}-\frac{\sin(bx+dx+a+c)}{2(b+d)}$
parallelrisch	$\frac{(b+d)\sin(a-c+(b-d)x)-\sin(a+c+(b+d)x)(b-d)}{2b^2-2d^2}$
norman	$\frac{-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-d^2}+\frac{2d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b^2-d^2}+\frac{2b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-d^2}-\frac{2d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{b^2-d^2}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
orering	$-\frac{2(b^2+d^2)(\cos(bx+a)\sin(dx+c)b+\sin(bx+a)\cos(dx+c)d)}{b^4-2b^2d^2+d^4}-\frac{-b^3\cos(bx+a)\sin(dx+c)-3b^2\sin(bx+a)d\cos(dx+c)-3\cos}{b^4-2b^2d^2+d}$

input
```
int(sin(b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/(b-d)*sin(a-c+(b-d)*x)-1/2/(b+d)*sin(a+c+(b+d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \sin(a+bx)\sin(c+dx)\,dx=\frac{d\cos(dx+c)\sin(bx+a)-b\cos(bx+a)\sin(dx+c)}{b^2-d^2}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

output
```
(d*cos(d*x + c)*sin(b*x + a) - b*cos(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(32) = 64$.

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.56

$$\int \sin(a+bx)\sin(c+dx)\,dx = \begin{cases} x\sin(a)\sin(c) & \text{for } b=0 \wedge d=0 \\ \frac{x\sin(a-dx)\sin(c+dx)}{2} - \frac{x\cos(a-dx)\cos(c+dx)}{2} - \frac{\sin(a-dx)\cos(c+dx)}{2d} & \text{for } b=-d \\ \frac{x\sin(a+dx)\sin(c+dx)}{2} + \frac{x\cos(a+dx)\cos(c+dx)}{2} - \frac{\sin(a+dx)\cos(c+dx)}{2d} & \text{for } b=d \\ -\frac{b\sin(c+dx)\cos(a+bx)}{b^2-d^2} + \frac{d\sin(a+bx)\cos(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c),x)
```

output
```
Piecewise((x*sin(a)*sin(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x)*sin(c +
d*x)/2 - x*cos(a - d*x)*cos(c + d*x)/2 - sin(a - d*x)*cos(c + d*x)/(2*d),
Eq(b, -d)), (x*sin(a + d*x)*sin(c + d*x)/2 + x*cos(a + d*x)*cos(c + d*x)/2
 - sin(a + d*x)*cos(c + d*x)/(2*d), Eq(b, d)), (-b*sin(c + d*x)*cos(a + b*
x)/(b**2 - d**2) + d*sin(a + b*x)*cos(c + d*x)/(b**2 - d**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \sin(a+bx)\sin(c+dx)\,dx = -\frac{\sin(bx+dx+a+c)}{2(b+d)} - \frac{\sin(-bx+dx-a+c)}{2(b-d)}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*sin(b*x + d*x + a + c)/(b + d) - 1/2*sin(-b*x + d*x - a + c)/(b - d)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \sin(a+bx)\sin(c+dx)\,dx = -\frac{\sin(bx+dx+a+c)}{2(b+d)} + \frac{\sin(bx-dx+a-c)}{2(b-d)}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

output
```
-1/2*sin(b*x + d*x + a + c)/(b + d) + 1/2*sin(b*x - d*x + a - c)/(b - d)
```

Mupad [B] (verification not implemented)

Time = 17.47 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.95

$$\int \sin(a+bx)\sin(c+dx)\,dx = \frac{d\left(\frac{\sin(a+c+b\,x+d\,x)}{2}+\frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2-d^2} - \frac{b\left(\frac{\sin(a+c+b\,x+d\,x)}{2}-\frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2-d^2}$$

input
```
int(sin(a + b*x)*sin(c + d*x),x)
```

output
```
(d*(sin(a + c + b*x + d*x)/2 + sin(a - c + b*x - d*x)/2))/(b^2 - d^2) - (b
*(sin(a + c + b*x + d*x)/2 - sin(a - c + b*x - d*x)/2))/(b^2 - d^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \sin(a+bx)\sin(c+dx)\,dx = \frac{-\cos(bx+a)\sin(dx+c)\,b+\cos(dx+c)\sin(bx+a)\,d}{b^2-d^2}$$

input
```
int(sin(b*x+a)*sin(d*x+c),x)
```

output
```
( - cos(a + b*x)*sin(c + d*x)*b + cos(c + d*x)*sin(a + b*x)*d)/(b**2 - d**
2)
```

3.113 $\int \csc(c+dx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 138

$$\int \csc(c+dx)\sin(a+bx)\,dx = \frac{ie^{-ia-ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2d},\frac{1}{2}\left(3-\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b-d} + \frac{ie^{ia+ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b+d}$$

output
```
I*exp(-I*a-I*b*x+I*(d*x+c))*hypergeom([1, -1/2*(b-d)/d],[3/2-1/2*b/d],exp(
2*I*(d*x+c)))/(b-d)+I*exp(I*a+I*b*x+I*(d*x+c))*hypergeom([1, 1/2*(b+d)/d],
[3/2+1/2*b/d],exp(2*I*(d*x+c)))/(b+d)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int \csc(c+dx)\sin(a+bx)\,dx = \frac{ie^{-i(a-c+(b-d)x)}\,\text{Hypergeometric2F1}\left(1,\frac{-b+d}{2d},\frac{3}{2}-\frac{b}{2d},e^{2i(c+dx)}\right)}{b-d} + \frac{ie^{i(a+c+(b+d)x)}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b+d}$$

input
```
Integrate[Csc[c + d*x]*Sin[a + b*x],x]
```

output
```
(I*Hypergeometric2F1[1, (-b + d)/(2*d), 3/2 - b/(2*d), E^((2*I)*(c + d*x))
])/((b - d)*E^(I*(a - c + (b - d)*x))) + (I*E^(I*(a + c + (b + d)*x))*Hype
rgeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, E^((2*I)*(c + d*x))])/(b + d)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sin(a+bx)\csc(c+dx)dx$$

input
```
Int[Csc[c + d*x]*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \csc(dx+c)\sin(bx+a)\,dx$$

input
```
int(csc(d*x+c)*sin(b*x+a),x)
```

output
```
int(csc(d*x+c)*sin(b*x+a),x)
```

Fricas [F]

$$\int \csc(c+dx)\sin(a+bx)\,dx = \int \csc(dx+c)\sin(bx+a)\,\,dx$$

input
```
integrate(csc(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)*sin(b*x + a), x)
```

Sympy [F]

$$\int \csc(c+dx)\sin(a+bx)\,dx = \int \sin(a+bx)\csc(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)*sin(b*x+a),x)
```

output
```
Integral(sin(a + b*x)*csc(c + d*x), x)
```

Maxima [F]

$$\int \csc(c+dx)\sin(a+bx)\,dx = \int \csc(dx+c)\sin(bx+a)\,dx$$

input
```
integrate(csc(d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(csc(d*x + c)*sin(b*x + a), x)
```

Giac [F]

$$\int \csc(c+dx)\sin(a+bx)\,dx = \int \csc(dx+c)\sin(bx+a)\,dx$$

input
```
integrate(csc(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(c+dx)\sin(a+bx)\,dx = \int \frac{\sin(a+b\,x)}{\sin(c+d\,x)}\,dx$$

input
```
int(sin(a + b*x)/sin(c + d*x),x)
```

output
```
int(sin(a + b*x)/sin(c + d*x), x)
```

Reduce [F]

$$\int \csc(c+dx)\sin(a+bx)\,dx=\int \csc\left(dx+c\right)\sin\left(bx+a\right)dx$$

input
```
int(csc(d*x+c)*sin(b*x+a),x)
```

output
```
int(csc(c + d*x)*sin(a + b*x),x)
```

3.114 $\int \csc^2(c+dx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 131

$$\int \csc^2(c+dx)\sin(a+bx)\,dx = \frac{2e^{-ia-ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{b}{2d},2-\frac{b}{2d},e^{2i(c+dx)}\right)}{b-2d} + \frac{2e^{ia+ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{b}{2d},2+\frac{b}{2d},e^{2i(c+dx)}\right)}{b+2d}$$

output
```
2*exp(-I*a-I*b*x+2*I*(d*x+c))*hypergeom([2, 1-1/2*b/d],[2-1/2*b/d],exp(2*I
*(d*x+c)))/(b-2*d)+2*exp(I*a+I*b*x+2*I*(d*x+c))*hypergeom([2, 1+1/2*b/d],[
2+1/2*b/d],exp(2*I*(d*x+c)))/(b+2*d)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 273 vs. 2(131) = 262.

Time = 4.32 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.08

$$\int \csc^2(c+dx)\sin(a+bx)\,dx = \frac{\frac{e^{-i(a-2c+bx)}\left(-be^{2idx}\,\text{Hypergeometric2F1}\left(1,1-\frac{b}{2d},2-\frac{b}{2d},e^{2i(c+dx)}\right)+(b-2d)\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)\right)}{(b-2d)(-1+e^{2ic})} + \frac{e^{i(a+2c+bx}}{}}{}$$

input
```
Integrate[Csc[c + d*x]^2*Sin[a + b*x],x]
```

output
```
((-(b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 - b/(2*d), 2 - b/(2*d), E^((2*I
)*(c + d*x))]) + (b - 2*d)*Hypergeometric2F1[1, -1/2*b/d, 1 - b/(2*d), E^(
(2*I)*(c + d*x))])/((b - 2*d)*E^(I*(a - 2*c + b*x))*(-1 + E^((2*I)*c))) +
(E^(I*(a + 2*c + b*x))*(b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 + b/(2*d),
2 + b/(2*d), E^((2*I)*(c + d*x))] - (b + 2*d)*Hypergeometric2F1[1, b/(2*d)
, 1 + b/(2*d), E^((2*I)*(c + d*x))]))/((b + 2*d)*(-1 + E^((2*I)*c))) + Cos
[b*x]*Csc[c]*Csc[c + d*x]*Sin[a]*Sin[d*x] + Cos[a]*Csc[c]*Csc[c + d*x]*Sin
[b*x]*Sin[d*x])/d
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^2(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin(a+bx)\csc^2(c+dx)dx$$

input
```
Int[Csc[c + d*x]^2*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \csc (dx+c)^2 \sin (bx+a)\,dx$$

input
```
int(csc(d*x+c)^2*sin(b*x+a),x)
```

output
```
int(csc(d*x+c)^2*sin(b*x+a),x)
```

Fricas [F]

$$\int \csc^2(c+dx)\sin(a+bx)\,dx=\int \csc (dx+c)^2 \sin (bx+a)\,dx$$

input
```
integrate(csc(d*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^2*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(c+dx)\sin(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(csc(d*x+c)**2*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^2(c+dx)\sin(a+bx)\,dx = \int \csc\left(dx+c\right)^2\sin\left(bx+a\right)\,dx$$

input
```
integrate(csc(d*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-((cos(2*b*x + 2*a) - 1)*cos((b + 2*d)*x + a + 2*c) - cos(2*b*x + 2*a)*cos
(b*x + a) + (d*cos((b + 2*d)*x + a + 2*c)^2 - 2*d*cos((b + 2*d)*x + a + 2*
c)*cos(b*x + a) + d*cos(b*x + a)^2 + d*sin((b + 2*d)*x + a + 2*c)^2 - 2*d*
sin((b + 2*d)*x + a + 2*c)*sin(b*x + a) + d*sin(b*x + a)^2)*integrate(-1/2
*(b*cos((b + d)*x + a + c)*sin(2*b*x + 2*a) + b*cos(b*x + a)*sin(2*b*x + 2
*a) - b*cos(2*b*x + 2*a)*sin(b*x + a) - (b*cos(2*b*x + 2*a) + b)*sin((b +
d)*x + a + c) - b*sin(b*x + a))/(d*cos((b + d)*x + a + c)^2 + 2*d*cos((b +
 d)*x + a + c)*cos(b*x + a) + d*cos(b*x + a)^2 + d*sin((b + d)*x + a + c)^
2 + 2*d*sin((b + d)*x + a + c)*sin(b*x + a) + d*sin(b*x + a)^2), x) - (d*c
os((b + 2*d)*x + a + 2*c)^2 - 2*d*cos((b + 2*d)*x + a + 2*c)*cos(b*x + a)
+ d*cos(b*x + a)^2 + d*sin((b + 2*d)*x + a + 2*c)^2 - 2*d*sin((b + 2*d)*x
+ a + 2*c)*sin(b*x + a) + d*sin(b*x + a)^2)*integrate(-1/2*(b*cos((b + d)*
x + a + c)*sin(2*b*x + 2*a) - b*cos(b*x + a)*sin(2*b*x + 2*a) + b*cos(2*b*
x + 2*a)*sin(b*x + a) - (b*cos(2*b*x + 2*a) + b)*sin((b + d)*x + a + c) +
b*sin(b*x + a))/(d*cos((b + d)*x + a + c)^2 - 2*d*cos((b + d)*x + a + c)*c
os(b*x + a) + d*cos(b*x + a)^2 + d*sin((b + d)*x + a + c)^2 - 2*d*sin((b +
 d)*x + a + c)*sin(b*x + a) + d*sin(b*x + a)^2), x) + sin((b + 2*d)*x + a
+ 2*c)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(d
*cos((b + 2*d)*x + a + 2*c)^2 - 2*d*cos((b + 2*d)*x + a + 2*c)*cos(b*x + a
) + d*cos(b*x + a)^2 + d*sin((b + 2*d)*x + a + 2*c)^2 - 2*d*sin((b + 2*...
```

Giac [F]

$$\int \csc^2(c+dx)\sin(a+bx)\,dx = \int \csc\left(dx+c\right)^2\sin\left(bx+a\right)\,dx$$

input
```
integrate(csc(d*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^2*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(c+dx)\sin(a+bx)\,dx = \int \frac{\sin(a+b\,x)}{\sin(c+d\,x)^2}\,dx$$

input
```
int(sin(a + b*x)/sin(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)/sin(c + d*x)^2, x)
```

Reduce [F]

$$\int \csc^2(c+dx)\sin(a+bx)\,dx = \int \csc(dx+c)^2\sin(bx+a)\,dx$$

input
```
int(csc(d*x+c)^2*sin(b*x+a),x)
```

output
```
int(csc(c + d*x)**2*sin(a + b*x),x)
```

3.115 $\int \csc^3(c+dx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 141

$$\begin{aligned}\int \csc^3(c+dx)\sin(a+bx)\,dx &= -\frac{4ie^{-ia-ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{b}{d}\right),\frac{1}{2}\left(5-\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b-3d}\\ &\quad-\frac{4ie^{ia+ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{b}{d}\right),\frac{1}{2}\left(5+\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b+3d}\end{aligned}$$

output
```
-4*I*exp(-I*a-I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2-1/2*b/d],[5/2-1/2*b/d],
exp(2*I*(d*x+c)))/(b-3*d)-4*I*exp(I*a+I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2
+1/2*b/d],[5/2+1/2*b/d],exp(2*I*(d*x+c)))/(b+3*d)
```

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.21

$$\int \csc^3(c+dx)\sin(a+bx)\,dx = -\frac{8i(b+d)e^{-i(a-c+(b-d)x)}\,\text{Hypergeometric2F1}\left(1,\frac{-b+d}{2d},\frac{3}{2}-\frac{b}{2d},e^{2i(c+dx)}\right)+8i(b-d)e^{i(a+c+(b+d)x)}\,\text{Hyper}}{}$$

input
```
Integrate[Csc[c + d*x]^3*Sin[a + b*x],x]
```

output
```
-1/16*(((8*I)*(b + d)*Hypergeometric2F1[1, (-b + d)/(2*d), 3/2 - b/(2*d),
E^((2*I)*(c + d*x))])/E^(I*(a - c + (b - d)*x)) + (8*I)*(b - d)*E^(I*(a +
c + (b + d)*x))*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, E^((2*I)*
(c + d*x))] + 4*Csc[c + d*x]^2*((-b + d)*Sin[a - c + b*x - d*x] + (b + d)*
Sin[a + c + (b + d)*x]))/d^2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^3(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin(a+bx)\csc^3(c+dx)dx$$

input
```
Int[Csc[c + d*x]^3*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \csc(dx+c)^3 \sin(bx+a)\,dx$$

input
```
int(csc(d*x+c)^3*sin(b*x+a),x)
```

output
```
int(csc(d*x+c)^3*sin(b*x+a),x)
```

Fricas [F]

$$\int \csc^3(c+dx)\sin(a+bx)\,dx = \int \csc(dx+c)^3 \sin(bx+a)\,dx$$

input
```
integrate(csc(d*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^3*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(c+dx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**3*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^3(c+dx)\sin(a+bx)\,dx = \int \csc(dx+c)^3\sin(bx+a)\,dx$$

input
```
integrate(csc(d*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*((b - d)*cos(b*x + a)*sin((2*b + d)*x + 2*a + c) - (b - d)*cos((2*b +
 d)*x + 2*a + c)*sin(b*x + a) + (b - d)*cos(3*d*x + 3*c)*sin(b*x + a) - (b
 + d)*cos(d*x + c)*sin(b*x + a) - (b - d)*cos(b*x + a)*sin(3*d*x + 3*c) +
(b + d)*cos(b*x + a)*sin(d*x + c) - (2*(b + d)*sin((b + 2*d)*x + a + 2*c)
- (b + d)*sin(b*x + a))*cos((2*b + 3*d)*x + 2*a + 3*c) - ((b + d)*sin((2*b
 + 3*d)*x + 2*a + 3*c) - (b - d)*sin((2*b + d)*x + 2*a + c) + (b - d)*sin(
3*d*x + 3*c) - (b + d)*sin(d*x + c))*cos((b + 4*d)*x + a + 4*c) - 2*((b -
d)*sin((2*b + d)*x + 2*a + c) - (b - d)*sin(3*d*x + 3*c) + (b + d)*sin(d*x
 + c))*cos((b + 2*d)*x + a + 2*c) - 2*(d^2*cos((b + 4*d)*x + a + 4*c)^2 +
4*d^2*cos((b + 2*d)*x + a + 2*c)^2 - 4*d^2*cos((b + 2*d)*x + a + 2*c)*cos(
b*x + a) + d^2*cos(b*x + a)^2 + d^2*sin((b + 4*d)*x + a + 4*c)^2 + 4*d^2*s
in((b + 2*d)*x + a + 2*c)^2 - 4*d^2*sin((b + 2*d)*x + a + 2*c)*sin(b*x + a
) + d^2*sin(b*x + a)^2 - 2*(2*d^2*cos((b + 2*d)*x + a + 2*c) - d^2*cos(b*x
 + a))*cos((b + 4*d)*x + a + 4*c) - 2*(2*d^2*sin((b + 2*d)*x + a + 2*c) -
d^2*sin(b*x + a))*sin((b + 4*d)*x + a + 4*c))*integrate(-1/4*((b^2 - d^2)*
cos(2*b*x + 2*a)*cos(b*x + a) + (b^2 - d^2)*sin((b + d)*x + a + c)*sin(2*b
*x + 2*a) + (b^2 - d^2)*sin(2*b*x + 2*a)*sin(b*x + a) - (b^2 - d^2 - (b^2
- d^2)*cos(2*b*x + 2*a))*cos((b + d)*x + a + c) - (b^2 - d^2)*cos(b*x + a)
)/(d^2*cos((b + d)*x + a + c)^2 + 2*d^2*cos((b + d)*x + a + c)*cos(b*x + a
) + d^2*cos(b*x + a)^2 + d^2*sin((b + d)*x + a + c)^2 + 2*d^2*sin((b + ...
```

Giac [F]

$$\int \csc^3(c+dx)\sin(a+bx)\,dx = \int \csc(dx+c)^3\sin(bx+a)\,dx$$

input
```
integrate(csc(d*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^3*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+dx)\sin(a+bx)\,dx = \int \frac{\sin(a+b\,x)}{\sin(c+d\,x)^3}\,dx$$

input
```
int(sin(a + b*x)/sin(c + d*x)^3,x)
```

output
```
int(sin(a + b*x)/sin(c + d*x)^3, x)
```

Reduce [F]

$$\int \csc^3(c+dx)\sin(a+bx)\,dx = \int \csc(dx+c)^3 \sin(bx+a)\,dx$$

input
```
int(csc(d*x+c)^3*sin(b*x+a),x)
```

output
```
int(csc(c + d*x)**3*sin(a + b*x),x)
```

3.116 $\int \sin^2(a+bx)\sin^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx = \frac{\cos(2a-3c+(2b-3d)x)}{16(2b-3d)} - \frac{3\cos(2a-c+(2b-d)x)}{16(2b-d)} - \frac{3\cos(c+dx)}{8d} + \frac{\cos(3c+3dx)}{24d} + \frac{3\cos(2a+c+(2b+d)x)}{16(2b+d)} - \frac{\cos(2a+3c+(2b+3d)x)}{16(2b+3d)}$$

output
```
cos(2*a-3*c+(2*b-3*d)*x)/(32*b-48*d)-3*cos(2*a-c+(2*b-d)*x)/(32*b-16*d)-3/
8*cos(d*x+c)/d+1/24*cos(3*d*x+3*c)/d+3*cos(2*a+c+(2*b+d)*x)/(32*b+16*d)-co
s(2*a+3*c+(2*b+3*d)*x)/(32*b+48*d)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx = \frac{1}{48}\left(-\frac{18\cos(c)\cos(dx)}{d}+\frac{2\cos(3c)\cos(3dx)}{d}+\frac{3\cos(2a-3c+2bx-3dx)}{2b-3d}-\frac{9\cos(2a-c+2bx-dx)}{2b-d}+\frac{9\cos(2a+c+2bx+dx)}{2b+d}-\frac{3\cos(2a+3c+2bx+3dx)}{2b+3d}+\frac{18\sin(c)\sin(dx)}{d}-\frac{2\sin(3c)\sin(3dx)}{d}\right)$$

input
```
Integrate[Sin[a + b*x]^2*Sin[c + d*x]^3,x]
```

output
```
((-18*Cos[c]*Cos[d*x])/d + (2*Cos[3*c]*Cos[3*d*x])/d + (3*Cos[2*a - 3*c +
2*b*x - 3*d*x])/(2*b - 3*d) - (9*Cos[2*a - c + 2*b*x - d*x])/(2*b - d) + (
9*Cos[2*a + c + 2*b*x + d*x])/(2*b + d) - (3*Cos[2*a + 3*c + 2*b*x + 3*d*x
])/(2*b + 3*d) + (18*Sin[c]*Sin[d*x])/d - (2*Sin[3*c]*Sin[3*d*x])/d)/48
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{1}{16}\sin(2a+x(2b-3d)-3c)+\frac{3}{16}\sin(2a+x(2b-d)-c)-\frac{3}{16}\sin(2a+x(2b+d)+c)+\frac{1}{16}\sin(2a+x(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos(2a+x(2b-3d)-3c)}{16(2b-3d)}-\frac{3\cos(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\cos(2a+x(2b+d)+c)}{16(2b+d)}-\frac{\cos(2a+x(2b+3d)+3c)}{16(2b+3d)}-\frac{3\cos(c+dx)}{8d}+\frac{\cos(3c+3dx)}{24d}$$

input
```
Int[Sin[a + b*x]^2*Sin[c + d*x]^3,x]
```

output
```
Cos[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) - (3*Cos[2*a - c + (2*b -
d)*x])/(16*(2*b - d)) - (3*Cos[c + d*x])/(8*d) + Cos[3*c + 3*d*x]/(24*d) +
 (3*Cos[2*a + c + (2*b + d)*x])/(16*(2*b + d)) - Cos[2*a + 3*c + (2*b + 3*
d)*x]/(16*(2*b + 3*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 13.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

method	result
default	$-\frac{3\cos(dx+c)}{8d}+\frac{\cos(3dx+3c)}{24d}+\frac{\cos(2a-3c+(2b-3d)x)}{32b-48d}-\frac{3\cos(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\cos(2a+c+(2b+d)x)}{16(2b+d)}-\frac{\cos(2a}{}$
parallelrisch	$\frac{(24b^3d+36b^2d^2-6b\,d^3-9d^4)\cos(2a-3c+(2b-3d)x)+(-72b^3d-36b^2d^2+162b\,d^3+81d^4)\cos(2a-c+(2b-d)x)+(-24b^3d+36}{}$
risch	$-\frac{3\cos(dx+c)b^2}{2(2b+d)d(2b-d)}+\frac{3d\cos(dx+c)}{8(2b+d)(2b-d)}+\frac{\cos(2bx-3dx+2a-3c)b}{8(2b+3d)(2b-3d)}+\frac{3d\cos(2bx-3dx+2a-3c)}{16(2b+3d)(2b-3d)}-\frac{3\cos(2bx-dx+2a-c)b}{8(2b+d)(2b-d)}$
orering	Expression too large to display

input
```
int(sin(b*x+a)^2*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*cos(d*x+c)/d+1/24*cos(3*d*x+3*c)/d+1/16/(2*b-3*d)*cos(2*a-3*c+(2*b-3*
d)*x)-3/16*cos(2*a-c+(2*b-d)*x)/(2*b-d)+3/16/(2*b+d)*cos(2*a+c+(2*b+d)*x)-
1/16/(2*b+3*d)*cos(2*a+3*c+(2*b+3*d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.33

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx$$
$$= \frac{\left(8\,b^4-38\,b^2d^2+9\,d^4+9\left(4\,b^2d^2-d^4\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)^3+6\left(\left(4\,b^3d-bd^3\right)\cos\left(bx+a\right)\cos\left(dx\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/3*((8*b^4 - 38*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cos(b*x + a)^2)*cos
(d*x + c)^3 + 6*((4*b^3*d - b*d^3)*cos(b*x + a)*cos(d*x + c)^2 - (4*b^3*d
- 7*b*d^3)*cos(b*x + a))*sin(b*x + a)*sin(d*x + c) - 3*(8*b^4 - 26*b^2*d^2
 + 9*d^4 + 3*(4*b^2*d^2 - 3*d^4)*cos(b*x + a)^2)*cos(d*x + c))/(16*b^4*d -
 40*b^2*d^3 + 9*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2004 vs. $2(116) = 232$.

Time = 5.71 (sec) , antiderivative size = 2004, normalized size of antiderivative = 13.92

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)**2*sin(d*x+c)**3,x)
```

output
```
Piecewise((x*sin(a)**2*sin(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x/2
)**2*sin(c + d*x)**3/16 - 3*x*sin(a - 3*d*x/2)**2*sin(c + d*x)*cos(c + d*x
)**2/16 - 3*x*sin(a - 3*d*x/2)*sin(c + d*x)**2*cos(a - 3*d*x/2)*cos(c + d*
x)/8 + x*sin(a - 3*d*x/2)*cos(a - 3*d*x/2)*cos(c + d*x)**3/8 - x*sin(c + d
*x)**3*cos(a - 3*d*x/2)**2/16 + 3*x*sin(c + d*x)*cos(a - 3*d*x/2)**2*cos(c
 + d*x)**2/16 - sin(a - 3*d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/d - 5*sin
(a - 3*d*x/2)**2*cos(c + d*x)**3/(48*d) - sin(a - 3*d*x/2)*sin(c + d*x)**3
*cos(a - 3*d*x/2)/(24*d) + 5*sin(a - 3*d*x/2)*sin(c + d*x)*cos(a - 3*d*x/2
)*cos(c + d*x)**2/(4*d) - 9*cos(a - 3*d*x/2)**2*cos(c + d*x)**3/(16*d), Eq
(b, -3*d/2)), (3*x*sin(a - d*x/2)**2*sin(c + d*x)**3/16 + 3*x*sin(a - d*x/
2)**2*sin(c + d*x)*cos(c + d*x)**2/16 - 3*x*sin(a - d*x/2)*sin(c + d*x)**2
*cos(a - d*x/2)*cos(c + d*x)/8 - 3*x*sin(a - d*x/2)*cos(a - d*x/2)*cos(c +
 d*x)**3/8 - 3*x*sin(c + d*x)**3*cos(a - d*x/2)**2/16 - 3*x*sin(c + d*x)*c
os(a - d*x/2)**2*cos(c + d*x)**2/16 - sin(a - d*x/2)**2*sin(c + d*x)**2*co
s(c + d*x)/d - 31*sin(a - d*x/2)**2*cos(c + d*x)**3/(48*d) - 3*sin(a - d*x
/2)*sin(c + d*x)**3*cos(a - d*x/2)/(8*d) - sin(a - d*x/2)*sin(c + d*x)*cos
(a - d*x/2)*cos(c + d*x)**2/(4*d) - cos(a - d*x/2)**2*cos(c + d*x)**3/(48*
d), Eq(b, -d/2)), (3*x*sin(a + d*x/2)**2*sin(c + d*x)**3/16 + 3*x*sin(a +
d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/16 + 3*x*sin(a + d*x/2)*sin(c + d*x
)**2*cos(a + d*x/2)*cos(c + d*x)/8 + 3*x*sin(a + d*x/2)*cos(a + d*x/2)*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(132) = 264$.

Time = 0.10 (sec) , antiderivative size = 1362, normalized size of antiderivative = 9.46

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/96*(3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^
4*cos(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) + 3*(8*b^3*d*cos(3*c) - 12*b^2*
d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*cos((2*b + 3*d)*x + 2*a)
 - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*co
s(3*c))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos
(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*c))*cos((2*b + d)*x + 2*a - 2*c) +
 9*(8*b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) - 9*d^4*cos(
3*c))*cos(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*cos(3*c) + 4*b^2*d^2*cos(
3*c) - 18*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) -
 3*(8*b^3*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) - 3*d^4*cos(
3*c))*cos(-(2*b - 3*d)*x - 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) + 12*b^2*d^2*c
os(3*c) - 2*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*cos(-(2*b - 3*d)*x - 2*a) - 2
*(16*b^4*cos(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(3*d*x) - 2*(
16*b^4*cos(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(3*d*x + 6*c) +
 18*(16*b^4*cos(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(d*x + 4*c
) + 18*(16*b^4*cos(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(d*x -
2*c) + 3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^
4*sin(3*c))*sin((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*
d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*sin((2*b + 3*d)*x + 2*a)
 - 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4...
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx = -\frac{\cos(2\,bx+3\,dx+2\,a+3\,c)}{16\,(2\,b+3\,d)} + \frac{3\,\cos(2\,bx+dx+2\,a+c)}{16\,(2\,b+d)} - \frac{3\,\cos(2\,bx-dx+2\,a-c)}{16\,(2\,b-d)} + \frac{\cos(2\,bx-3\,dx+2\,a-3\,c)}{16\,(2\,b-3\,d)} + \frac{\cos(3\,dx+3\,c)}{24\,d} - \frac{3\,\cos(dx+c)}{8\,d}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/16*cos(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/16*cos(2*b*x + d*x +
2*a + c)/(2*b + d) - 3/16*cos(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/16*cos(
2*b*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*cos(3*d*x + 3*c)/d - 3/8*cos
(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 18.59 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.26

$$\begin{aligned}\int \sin^2(a+bx)\sin^3(c+dx)\,dx = {} & \mathrm{e}^{a\,2\mathrm{i}-c\,3\mathrm{i}+b\,x\,2\mathrm{i}-d\,x\,3\mathrm{i}}\left(\frac{3\,d\,(2\,b+3\,d)}{384\,b^2\,d-864\,d^3}\right.\\&\left.+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(8\,b^2-18\,d^2)}{384\,b^2\,d-864\,d^3}-\frac{3\,d\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(2\,b-3\,d)}{384\,b^2\,d-864\,d^3}\right)\\&+\mathrm{e}^{a\,2\mathrm{i}+c\,3\mathrm{i}+b\,x\,2\mathrm{i}+d\,x\,3\mathrm{i}}\left(-\frac{3\,d\,(2\,b-3\,d)}{384\,b^2\,d-864\,d^3}\right.\\&\left.+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(8\,b^2-18\,d^2)}{384\,b^2\,d-864\,d^3}+\frac{3\,d\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(2\,b+3\,d)}{384\,b^2\,d-864\,d^3}\right)\\&-\mathrm{e}^{a\,2\mathrm{i}-c\,1\mathrm{i}+b\,x\,2\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{3\,(2\,b+d)}{32\,(4\,b^2-d^2)}\right.\\&\left.-\frac{3\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(2\,b-d)}{32\,(4\,b^2-d^2)}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(24\,b^2-6\,d^2)}{32\,d\,(4\,b^2-d^2)}\right)\\&-\mathrm{e}^{a\,2\mathrm{i}+c\,1\mathrm{i}+b\,x\,2\mathrm{i}+d\,x\,1\mathrm{i}}\left(-\frac{3\,(2\,b-d)}{32\,(4\,b^2-d^2)}\right.\\&\left.+\frac{3\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(2\,b+d)}{32\,(4\,b^2-d^2)}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(24\,b^2-6\,d^2)}{32\,d\,(4\,b^2-d^2)}\right)\end{aligned}$$

input
```
int(sin(a + b*x)^2*sin(c + d*x)^3,x)
```

output
```
exp(a*2i - c*3i + b*x*2i - d*x*3i)*((3*d*(2*b + 3*d))/(384*b^2*d - 864*d^3
) + (exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(384*b^2*d - 864*d^3) - (3*d*e
xp(- a*4i - b*x*4i)*(2*b - 3*d))/(384*b^2*d - 864*d^3)) + exp(a*2i + c*3i
+ b*x*2i + d*x*3i)*((exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(384*b^2*d - 8
64*d^3) - (3*d*(2*b - 3*d))/(384*b^2*d - 864*d^3) + (3*d*exp(- a*4i - b*x*
4i)*(2*b + 3*d))/(384*b^2*d - 864*d^3)) - exp(a*2i - c*1i + b*x*2i - d*x*1
i)*((3*(2*b + d))/(32*(4*b^2 - d^2)) - (3*exp(- a*4i - b*x*4i)*(2*b - d))/
(32*(4*b^2 - d^2)) + (exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(32*d*(4*b^2
- d^2))) - exp(a*2i + c*1i + b*x*2i + d*x*1i)*((3*exp(- a*4i - b*x*4i)*(2*
b + d))/(32*(4*b^2 - d^2)) - (3*(2*b - d))/(32*(4*b^2 - d^2)) + (exp(- a*2
i - b*x*2i)*(24*b^2 - 6*d^2))/(32*d*(4*b^2 - d^2)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.83

$$\int \sin^2(a+bx)\sin^3(c+dx)\,dx$$
$$= \frac{-24\cos(bx+a)\sin(bx+a)\sin(dx+c)^3 b^3 d + 6\cos(bx+a)\sin(bx+a)\sin(dx+c)^3 b\,d^3 + 36\cos(bx$$

input
```
int(sin(b*x+a)^2*sin(d*x+c)^3,x)
```

output
```
( - 24*cos(a + b*x)*sin(a + b*x)*sin(c + d*x)**3*b**3*d + 6*cos(a + b*x)*s
in(a + b*x)*sin(c + d*x)**3*b*d**3 + 36*cos(a + b*x)*sin(a + b*x)*sin(c +
d*x)*b*d**3 + 36*cos(c + d*x)*sin(a + b*x)**2*sin(c + d*x)**2*b**2*d**2 -
9*cos(c + d*x)*sin(a + b*x)**2*sin(c + d*x)**2*d**4 - 18*cos(c + d*x)*sin(
a + b*x)**2*d**4 - 8*cos(c + d*x)*sin(c + d*x)**2*b**4 + 2*cos(c + d*x)*si
n(c + d*x)**2*b**2*d**2 - 16*cos(c + d*x)*b**4 + 40*cos(c + d*x)*b**2*d**2
 + 16*b**4 - 112*b**2*d**2 + 36*d**4)/(3*d*(16*b**4 - 40*b**2*d**2 + 9*d**
4))
```

3.117 $\int \sin^2(a+bx)\sin^2(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx = \frac{x}{4}-\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)} -\frac{\sin(2c+2dx)}{8d}+\frac{\sin(2(a+c)+2(b+d)x)}{16(b+d)}$$

output
```
1/4*x-1/8*sin(2*b*x+2*a)/b+sin(2*a-2*c+2*(b-d)*x)/(16*b-16*d)-1/8*sin(2*d*
x+2*c)/d+sin(2*a+2*c+2*(b+d)*x)/(16*b+16*d)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx = \frac{(-2b^2d+2d^3)\sin(2(a+bx))+bd(b+d)\sin(2(a-c+(b-d)x))+b(b-d)(-2(b+d)\sin(2(c+dx))-}{16b(b-d)d(b+d)}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[c + d*x]^2,x]
```

output
```
((-2*b^2*d + 2*d^3)*Sin[2*(a + b*x)] + b*d*(b + d)*Sin[2*(a - c + (b - d)*
x)] + b*(b - d)*(-2*(b + d)*Sin[2*(c + d*x)] + d*(4*(b + d)*x + Sin[2*(a +
 c + (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(\frac{1}{8}\cos(2(a-c)+2x(b-d))+\frac{1}{8}\cos(2(a+c)+2x(b+d))-\frac{1}{4}\cos(2a+2bx)-\frac{1}{4}\cos(2c+2dx)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}-\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

input
```
Int[Sin[a + b*x]^2*Sin[c + d*x]^2,x]
```

output
```
x/4 - Sin[2*a + 2*b*x]/(8*b) + Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) -
 Sin[2*c + 2*d*x]/(8*d) + Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 5.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
default	$\frac{x}{4}-\frac{\sin(2bx+2a)}{8b}-\frac{\sin(2dx+2c)}{8d}+\frac{\sin((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sin((2b+2d)x+2a+2c)}{16b+16d}$
parallelrisch	$\frac{bd(b+d)\sin((2b-2d)x+2a-2c)+4(b-d)\left(\frac{bd\sin((2b+2d)x+2a+2c)}{4}+\left(-\frac{d\sin(2bx+2a)}{2}+b\left(dx-\frac{\sin(2dx+2c)}{2}\right)\right)(b+d)\right)}{16b^3d-16b\,d^3}$
risch	$\frac{x}{4}-\frac{\sin(2bx+2a)}{8b}-\frac{\sin(2dx+2c)b^2}{8d(b-d)(b+d)}+\frac{d\sin(2dx+2c)}{8(b-d)(b+d)}+\frac{\sin(2bx-2dx+2a-2c)b}{16(b-d)(b+d)}+\frac{d\sin(2bx-2dx+2a-2c)}{16(b-d)(b+d)}+\frac{\sin(2b}{16}$
orering	Expression too large to display

input
```
int(sin(b*x+a)^2*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x-1/8*sin(2*b*x+2*a)/b-1/8*sin(2*d*x+2*c)/d+1/8/(2*b-2*d)*sin((2*b-2*d
)*x+2*a-2*c)+1/8/(2*b+2*d)*sin((2*b+2*d)*x+2*a+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.34

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx =$$
$$-\frac{\left(2\,bd^2\cos\left(bx+a\right)^2+b^3-2\,bd^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(b^3d-bd^3\right)x-\left(2\,b^2d\cos\left(bx+a\right)\cos\left(dx\right.\right.}{4\left(b^3d-bd^3\right)}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/4*((2*b*d^2*cos(b*x + a)^2 + b^3 - 2*b*d^2)*cos(d*x + c)*sin(d*x + c) -
 (b^3*d - b*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - (2*b^2*d - d^3
)*cos(b*x + a))*sin(b*x + a))/(b^3*d - b*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(76) = 152$.

Time = 1.61 (sec) , antiderivative size = 1027, normalized size of antiderivative = 11.67

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)**2*sin(d*x+c)**2,x)
```

output
```
Piecewise((x*sin(a)**2*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**
2/2 + x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**2, Eq
(b, 0)), (3*x*sin(a - d*x)**2*sin(c + d*x)**2/8 + x*sin(a - d*x)**2*cos(c
+ d*x)**2/8 - x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + x*
sin(c + d*x)**2*cos(a - d*x)**2/8 + 3*x*cos(a - d*x)**2*cos(c + d*x)**2/8
- sin(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(2*d) + sin(a - d*x)*sin(c + d
*x)**2*cos(a - d*x)/(8*d) + 3*sin(a - d*x)*cos(a - d*x)*cos(c + d*x)**2/(8
*d), Eq(b, -d)), (3*x*sin(a + d*x)**2*sin(c + d*x)**2/8 + x*sin(a + d*x)**
2*cos(c + d*x)**2/8 + x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x
)/2 + x*sin(c + d*x)**2*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)**2*cos(c + d*
x)**2/8 - 5*sin(a + d*x)*sin(c + d*x)**2*cos(a + d*x)/(8*d) + sin(a + d*x)
*cos(a + d*x)*cos(c + d*x)**2/(8*d) - sin(c + d*x)*cos(a + d*x)**2*cos(c +
 d*x)/(2*d), Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(
a + b*x)*cos(a + b*x)/(2*b))*sin(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)*
*2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c
+ d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2
/(4*b**3*d - 4*b*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*
d - 4*b*d**3) - b**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d -
 4*b*d**3) - b**3*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*
b*d**3) - 2*b**2*d*sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4*b**3*d ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(78) = 156$.

Time = 0.07 (sec) , antiderivative size = 620, normalized size of antiderivative = 7.05

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c
)^2)*d)*x + (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c)
 - (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) - (b^2*d*sin(2
*c) + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) + (b^2*d*sin(2*c) + b*
d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*
cos(2*b*x + 2*a + 2*c) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a
 - 2*c) + 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) - 2*(b^3*sin(2*c) -
 b*d^2*sin(2*c))*cos(2*d*x + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(
2*(b + d)*x + 2*a + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)
*x + 2*a) + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c
) + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) + 2*(b^2*d*c
os(2*c) - d^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) + 2*(b^2*d*cos(2*c) - d^3*c
os(2*c))*sin(2*b*x + 2*a - 2*c) + 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*
d*x) + 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2
+ b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx = \frac{1}{4}x + \frac{\sin(2bx+2dx+2a+2c)}{16(b+d)} + \frac{\sin(2bx-2dx+2a-2c)}{16(b-d)} - \frac{\sin(2bx+2a)}{8b} - \frac{\sin(2dx+2c)}{8d}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*x + 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/16*sin(2*b*x - 2*d
*x + 2*a - 2*c)/(b - d) - 1/8*sin(2*b*x + 2*a)/b - 1/8*sin(2*d*x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 17.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.01

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx$$
$$= \frac{2\,d^3\,\sin\left(2\,a+2\,b\,x\right)-2\,b^3\,\sin\left(2\,c+2\,d\,x\right)+b\,d^2\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)-b\,d^2\,\sin\left(2\,a+2\,c+2\,b\right.}{}$$

input
```
int(sin(a + b*x)^2*sin(c + d*x)^2,x)
```

output
```
(2*d^3*sin(2*a + 2*b*x) - 2*b^3*sin(2*c + 2*d*x) + b*d^2*sin(2*a - 2*c + 2
*b*x - 2*d*x) - b*d^2*sin(2*a + 2*c + 2*b*x + 2*d*x) + b^2*d*sin(2*a - 2*c
 + 2*b*x - 2*d*x) + b^2*d*sin(2*a + 2*c + 2*b*x + 2*d*x) - 2*b^2*d*sin(2*a
 + 2*b*x) + 2*b*d^2*sin(2*c + 2*d*x) - 4*b*d^3*x + 4*b^3*d*x)/(16*b*d*(b^2
 - d^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.34

$$\int \sin^2(a+bx)\sin^2(c+dx)\,dx$$
$$= \frac{-2\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(dx+c\right)^2 b^2 d+\cos\left(bx+a\right)\sin\left(bx+a\right)d^3+2\cos\left(dx+c\right)\sin\left(bx+a\right)^2 s}{4bd\left(b^2-d^2\right)}$$

input
```
int(sin(b*x+a)^2*sin(d*x+c)^2,x)
```

output
```
( - 2*cos(a + b*x)*sin(a + b*x)*sin(c + d*x)**2*b**2*d + cos(a + b*x)*sin(
a + b*x)*d**3 + 2*cos(c + d*x)*sin(a + b*x)**2*sin(c + d*x)*b*d**2 - cos(c
 + d*x)*sin(c + d*x)*b**3 + b**3*d*x - b*d**3*x)/(4*b*d*(b**2 - d**2))
```

3.118 $\int \sin^2(a+bx)\sin(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \sin^2(a+bx)\sin(c+dx)\,dx = -\frac{\cos(2a-c+(2b-d)x)}{4(2b-d)} - \frac{\cos(c+dx)}{2d} + \frac{\cos(2a+c+(2b+d)x)}{4(2b+d)}$$

output
```
-1/4*cos(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*cos(d*x+c)/d+cos(2*a+c+(2*b+d)*x)/(8
*b+4*d)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \sin^2(a+bx)\sin(c+dx)\,dx = -\frac{\cos(2a-c+2bx-dx)}{4(2b-d)} + \frac{\cos(2a+c+(2b+d)x)}{4(2b+d)} + \frac{1}{2}\left(-\frac{\cos(c)\cos(dx)}{d} + \frac{\sin(c)\sin(dx)}{d}\right)$$

input
```
Integrate[Sin[a + b*x]^2*Sin[c + d*x],x]
```

output
```
-1/4*Cos[2*a - c + 2*b*x - d*x]/(2*b - d) + Cos[2*a + c + (2*b + d)*x]/(4*
(2*b + d)) + (-((Cos[c]*Cos[d*x])/d) + (Sin[c]*Sin[d*x])/d)/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{5080, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin(c+dx)\,dx$$

$$\Big\downarrow \; 5080$$

$$\int \left(\frac{1}{4}\sin(2a+x(2b-d)-c)-\frac{1}{4}\sin(2a+x(2b+d)+c)+\frac{1}{2}\sin(c+dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\cos(2a+x(2b-d)-c)}{4(2b-d)}+\frac{\cos(2a+x(2b+d)+c)}{4(2b+d)}-\frac{\cos(c+dx)}{2d}$$

input
```
Int[Sin[a + b*x]^2*Sin[c + d*x],x]
```

output
```
-1/4*Cos[2*a - c + (2*b - d)*x]/(2*b - d) - Cos[c + d*x]/(2*d) + Cos[2*a +
 c + (2*b + d)*x]/(4*(2*b + d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\cos(2a-c+(2b-d)x)}{4(2b-d)}-\frac{\cos(dx+c)}{2d}+\frac{\cos(2a+c+(2b+d)x)}{8b+4d}$
parallelrisch	$\frac{(-2bd-d^2)\cos(2a-c+(2b-d)x)+(2bd-d^2)\cos(2a+c+(2b+d)x)+(-8b^2+2d^2)\cos(dx+c)+8b^2}{16b^2d-4d^3}$
risch	$-\frac{2\cos(dx+c)b^2}{(2b+d)d(2b-d)}+\frac{d\cos(dx+c)}{2(2b+d)(2b-d)}-\frac{\cos(2bx-dx+2a-c)b}{2(2b+d)(2b-d)}-\frac{d\cos(2bx-dx+2a-c)}{4(2b+d)(2b-d)}+\frac{\cos(2bx+dx+2a+c)b}{2(2b+d)(2b-d)}-\cdots$
norman	$\frac{-\frac{4b^2}{d(4b^2-d^2)}-\frac{4b^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{d(4b^2-d^2)}-\frac{8b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4b^2-d^2}+\frac{8b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4b^2-d^2}-\frac{4d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4b^2-d^2}+\cdots}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
orering	$-\frac{(16b^4+3d^4)\left(2\sin(bx+a)\sin(dx+c)b\cos(bx+a)+\sin(bx+a)^2d\cos(dx+c)\right)}{d^2(16b^4-8b^2d^2+d^4)}-\frac{(8b^2+3d^2)\left(-8b^3\cos(bx+a)\sin(dx+c)\sin\cdots\right.}{\cdots}$

input
```
int(sin(b*x+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(d*x+c)/d-1/4*cos(2*a-c+(2*b-d)*x)/(2*b-d)+1/4/(2*b+d)*cos(2*a+c+(
2*b+d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \sin^2(a+bx)\sin(c+dx)\,dx = -\frac{2\,bd\cos(bx+a)\sin(bx+a)\sin(dx+c)+\left(d^2\cos(bx+a)^2+2\,b^2-d^2\right)\cos(dx+c)}{4\,b^2d-d^3}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

output
```
-(2*b*d*cos(b*x + a)*sin(b*x + a)*sin(d*x + c) + (d^2*cos(b*x + a)^2 + 2*b
^2 - d^2)*cos(d*x + c))/(4*b^2*d - d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(49) = 98$.

Time = 0.73 (sec) , antiderivative size = 410, normalized size of antiderivative = 6.03

$$\int \sin^2(a+bx)\sin(c+dx)\,dx$$

$$= \begin{cases} x\sin^2(a)\sin(c) \\ \frac{x\sin^2\left(a-\frac{dx}{2}\right)\sin(c+dx)}{4} - \frac{x\sin\left(a-\frac{dx}{2}\right)\cos\left(a-\frac{dx}{2}\right)\cos(c+dx)}{2} - \frac{x\sin(c+dx)\cos^2\left(a-\frac{dx}{2}\right)}{4} - \frac{\sin^2\left(a-\frac{dx}{2}\right)\cos(c+dx)}{d} - \sin\left(a-\right. \\ \frac{x\sin^2\left(a+\frac{dx}{2}\right)\sin(c+dx)}{4} + \frac{x\sin\left(a+\frac{dx}{2}\right)\cos\left(a+\frac{dx}{2}\right)\cos(c+dx)}{2} - \frac{x\sin(c+dx)\cos^2\left(a+\frac{dx}{2}\right)}{4} - \frac{3\sin\left(a+\frac{dx}{2}\right)\sin(c+dx)\cos\left(a+\frac{dx}{2}\right.}{2d} \\ \left(\frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b}\right)\sin(c) \\ -\frac{2b^2\sin^2(a+bx)\cos(c+dx)}{4b^2d-d^3} - \frac{2b^2\cos^2(a+bx)\cos(c+dx)}{4b^2d-d^3} - \frac{2bd\sin(a+bx)\sin(c+dx)\cos(a+bx)}{4b^2d-d^3} + \frac{d^2\sin^2(a+bx)\cos(c+dx)}{4b^2d-d^3} \end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(d*x+c),x)
```

output
```
Piecewise((x*sin(a)**2*sin(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x/2)**2*
sin(c + d*x)/4 - x*sin(a - d*x/2)*cos(a - d*x/2)*cos(c + d*x)/2 - x*sin(c
+ d*x)*cos(a - d*x/2)**2/4 - sin(a - d*x/2)**2*cos(c + d*x)/d - sin(a - d*
x/2)*sin(c + d*x)*cos(a - d*x/2)/(2*d), Eq(b, -d/2)), (x*sin(a + d*x/2)**2
*sin(c + d*x)/4 + x*sin(a + d*x/2)*cos(a + d*x/2)*cos(c + d*x)/2 - x*sin(c
 + d*x)*cos(a + d*x/2)**2/4 - 3*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)
/(2*d) - cos(a + d*x/2)**2*cos(c + d*x)/d, Eq(b, d/2)), ((x*sin(a + b*x)**
2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a + b*x)/(2*b))*sin(c), Eq(d,
 0)), (-2*b**2*sin(a + b*x)**2*cos(c + d*x)/(4*b**2*d - d**3) - 2*b**2*cos
(a + b*x)**2*cos(c + d*x)/(4*b**2*d - d**3) - 2*b*d*sin(a + b*x)*sin(c + d
*x)*cos(a + b*x)/(4*b**2*d - d**3) + d**2*sin(a + b*x)**2*cos(c + d*x)/(4*
b**2*d - d**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(62) = 124$.

Time = 0.05 (sec) , antiderivative size = 371, normalized size of antiderivative = 5.46

$$\int \sin^2(a+bx)\sin(c+dx)\,dx = -\frac{(2\,bd\cos(c) - d^2\cos(c))\cos((2\,b+d)x + 2\,a + 2\,c) + (2\,bd\cos(c) - d^2\cos(c))\cos((2\,b+d)x + 2\,a)}{}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

output
```
-1/8*((2*b*d*cos(c) - d^2*cos(c))*cos((2*b + d)*x + 2*a + 2*c) + (2*b*d*co
s(c) - d^2*cos(c))*cos((2*b + d)*x + 2*a) - (2*b*d*cos(c) + d^2*cos(c))*co
s(-(2*b - d)*x - 2*a + 2*c) - (2*b*d*cos(c) + d^2*cos(c))*cos(-(2*b - d)*x
 - 2*a) - 2*(4*b^2*cos(c) - d^2*cos(c))*cos(d*x + 2*c) - 2*(4*b^2*cos(c) -
 d^2*cos(c))*cos(d*x) + (2*b*d*sin(c) - d^2*sin(c))*sin((2*b + d)*x + 2*a
+ 2*c) - (2*b*d*sin(c) - d^2*sin(c))*sin((2*b + d)*x + 2*a) - (2*b*d*sin(c
) + d^2*sin(c))*sin(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*sin(c) + d^2*sin(c)
)*sin(-(2*b - d)*x - 2*a) - 2*(4*b^2*sin(c) - d^2*sin(c))*sin(d*x + 2*c) +
 2*(4*b^2*sin(c) - d^2*sin(c))*sin(d*x))/((cos(c)^2 + sin(c)^2)*d^3 - 4*(b
^2*cos(c)^2 + b^2*sin(c)^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \sin^2(a+bx)\sin(c+dx)\,dx = \frac{\cos(2\,bx + dx + 2\,a + c)}{4\,(2\,b+d)} - \frac{\cos(2\,bx - dx + 2\,a - c)}{4\,(2\,b-d)} - \frac{\cos(dx+c)}{2\,d}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

output
```
1/4*cos(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*cos(2*b*x - d*x + 2*a - c)/
(2*b - d) - 1/2*cos(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 17.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.54

$$\int \sin^2(a+bx)\sin(c+dx)\,dx = -\frac{d^2\,\cos\left(2\,a+c+2\,b\,x+d\,x\right)-b\,\left(2\,d\,\cos\left(2\,a+c+2\,b\,x+d\,x\right)-2\,d\,\cos\left(2\,a-c+2\,b\,x-d\,x\right)\right)+d^2}{16\,b^2\,d-4\,d^3} - \frac{\cos\left(c+d\,x\right)}{2\,d}$$

input
```
int(sin(a + b*x)^2*sin(c + d*x),x)
```

output
```
- (d^2*cos(2*a + c + 2*b*x + d*x) - b*(2*d*cos(2*a + c + 2*b*x + d*x) - 2*
d*cos(2*a - c + 2*b*x - d*x)) + d^2*cos(2*a - c + 2*b*x - d*x))/(16*b^2*d
- 4*d^3) - cos(c + d*x)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.16

$$\int \sin^2(a+bx)\sin(c+dx)\,dx = \frac{-2\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(dx+c\right)bd+\cos\left(dx+c\right)\sin\left(bx+a\right)^2d^2-2\cos\left(dx+c\right)b^2+2b^2-2d^2}{d\left(4b^2-d^2\right)}$$

input
```
int(sin(b*x+a)^2*sin(d*x+c),x)
```

output
```
( - 2*cos(a + b*x)*sin(a + b*x)*sin(c + d*x)*b*d + cos(c + d*x)*sin(a + b*
x)**2*d**2 - 2*cos(c + d*x)*b**2 + 2*b**2 - 2*d**2)/(d*(4*b**2 - d**2))
```

3.119 $\int \csc(c+dx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 149

$$\begin{aligned}\int \csc(c+dx)\sin^2(a+bx)\,dx &= -\frac{\operatorname{arctanh}(\cos(c+dx))}{2d} \\ &\quad - \frac{e^{-2ia-2ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{b}{d},\frac{3}{2}-\frac{b}{d},e^{2i(c+dx)}\right)}{2(2b-d)} \\ &\quad + \frac{e^{2ia+2ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{b}{d},\frac{3}{2}+\frac{b}{d},e^{2i(c+dx)}\right)}{2(2b+d)}\end{aligned}$$

output
```
-1/2*arctanh(cos(d*x+c))/d-exp(-2*I*a-2*I*b*x+I*(d*x+c))*hypergeom([1, 1/2
-b/d],[3/2-b/d],exp(2*I*(d*x+c)))/(4*b-2*d)+exp(2*I*a+2*I*b*x+I*(d*x+c))*h
ypergeom([1, 1/2+b/d],[3/2+b/d],exp(2*I*(d*x+c)))/(4*b+2*d)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07

$$\int \csc(c+dx)\sin^2(a+bx)\,dx$$
$$= -\frac{e^{-i(2a-c+2bx-dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{b}{d},\frac{3}{2}-\frac{b}{d},e^{2i(c+dx)}\right)}{2(2b-d)}$$
$$+\frac{e^{i(2a+c+2bx+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{b}{d},\frac{3}{2}+\frac{b}{d},e^{2i(c+dx)}\right)}{2(2b+d)}$$
$$+\frac{-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

input
```
Integrate[Csc[c + d*x]*Sin[a + b*x]^2,x]
```

output
```
-1/2*Hypergeometric2F1[1, 1/2 - b/d, 3/2 - b/d, E^((2*I)*(c + d*x))]/((2*b
 - d)*E^(I*(2*a - c + 2*b*x - d*x))) + (E^(I*(2*a + c + 2*b*x + d*x))*Hype
rgeometric2F1[1, 1/2 + b/d, 3/2 + b/d, E^((2*I)*(c + d*x))])/(2*(2*b + d))
 + (-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])/(2*d)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc(c+dx)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \sin^2(a+bx)\csc(c+dx)dx$$

input
```
Int[Csc[c + d*x]*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \csc (dx+c)\sin (bx+a)^2\, dx$$

input
```
int(csc(d*x+c)*sin(b*x+a)^2,x)
```

output
```
int(csc(d*x+c)*sin(b*x+a)^2,x)
```

Fricas [F]

$$\int \csc(c+dx)\sin^2(a+bx)\, dx = \int \csc (dx+c)\sin (bx+a)^2\, dx$$

input
```
integrate(csc(d*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*csc(d*x + c), x)
```

Sympy [F]

$$\int \csc(c+dx)\sin^2(a+bx)\, dx = \int \sin^2 (a+bx)\csc (c+dx)\, dx$$

input
```
integrate(csc(d*x+c)*sin(b*x+a)**2,x)
```

output
```
Integral(sin(a + b*x)**2*csc(c + d*x), x)
```

Maxima [F]

$$\int \csc(c+dx)\sin^2(a+bx)\,dx = \int \csc(dx+c)\sin(bx+a)^2\,dx$$

input
```
integrate(csc(d*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(csc(d*x + c)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int \csc(c+dx)\sin^2(a+bx)\,dx = \int \csc(dx+c)\sin(bx+a)^2\,dx$$

input
```
integrate(csc(d*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(c+dx)\sin^2(a+bx)\,dx = \int \frac{\sin(a+b\,x)^2}{\sin(c+d\,x)}\,dx$$

input
```
int(sin(a + b*x)^2/sin(c + d*x),x)
```

output
```
int(sin(a + b*x)^2/sin(c + d*x), x)
```

Reduce [F]

$$\int \csc(c+dx)\sin^2(a+bx)\,dx = \int \csc(dx+c)\sin(bx+a)^2\,dx$$

input
```
int(csc(d*x+c)*sin(b*x+a)^2,x)
```

output
```
int(csc(c + d*x)*sin(a + b*x)**2,x)
```

3.120 $\int \csc^2(c+dx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 140

$$\int \csc^2(c+dx)\sin^2(a+bx)\,dx = -\frac{\cot(c+dx)}{2d} + \frac{ie^{-2ia-2ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{b}{d},2-\frac{b}{d},e^{2i(c+dx)}\right)}{2(b-d)} - \frac{ie^{2ia+2ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,\frac{b+d}{d},2+\frac{b}{d},e^{2i(c+dx)}\right)}{2(b+d)}$$

output
```
-1/2*cot(d*x+c)/d+1/2*I*exp(-2*I*a-2*I*b*x+2*I*(d*x+c))*hypergeom([2, 1-b/
d],[2-b/d],exp(2*I*(d*x+c)))/(b-d)-1/2*I*exp(2*I*a+2*I*b*x+2*I*(d*x+c))*hy
pergeom([2, (b+d)/d],[2+b/d],exp(2*I*(d*x+c)))/(b+d)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 290 vs. 2(140) = 280.

Time = 3.30 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.07

$$\int \csc^2(c+dx)\sin^2(a+bx)\,dx$$
$$= -\frac{ibe^{-2i(a-c)}\left(\frac{e^{-2i(b-d)x}\,\text{Hypergeometric2F1}\left(1,1-\frac{b}{d},2-\frac{b}{d},e^{2i(c+dx)}\right)}{b-d}-\frac{e^{-2ibx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{d},1-\frac{b}{d},e^{2i(c+dx)}\right)}{b}\right)}{-1+e^{2ic}}+\frac{ie^{2i(a+c)}\left((b+d)e^{2\cdots}\right.}{\cdots}$$

input
```
Integrate[Csc[c + d*x]^2*Sin[a + b*x]^2,x]
```

output
```
(((-I)*b*(Hypergeometric2F1[1, 1 - b/d, 2 - b/d, E^((2*I)*(c + d*x))]/((b
- d)*E^((2*I)*(b - d)*x)) - Hypergeometric2F1[1, -(b/d), 1 - b/d, E^((2*I)
*(c + d*x))]/(b*E^((2*I)*b*x))))/(E^((2*I)*(a - c))*(-1 + E^((2*I)*c)) +
(I*E^((2*I)*(a + c))*((b + d)*E^((2*I)*b*x)*Hypergeometric2F1[1, b/d, (b +
 d)/d, E^((2*I)*(c + d*x))] - b*E^((2*I)*(b + d)*x)*Hypergeometric2F1[1, (
b + d)/d, 2 + b/d, E^((2*I)*(c + d*x))]))/((b + d)*(-1 + E^((2*I)*c)) + C
sc[c]*Csc[c + d*x]*Sin[d*x] - Cos[2*a]*Cos[2*b*x]*Csc[c]*Csc[c + d*x]*Sin[
d*x] + Csc[c]*Csc[c + d*x]*Sin[2*a]*Sin[2*b*x]*Sin[d*x])/(2*d)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^2(c+dx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \sin^2(a+bx)\csc^2(c+dx)dx$$

input
```
Int[Csc[c + d*x]^2*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \csc(dx+c)^2 \sin(bx+a)^2\, dx$$

input
```
int(csc(d*x+c)^2*sin(b*x+a)^2,x)
```

output
```
int(csc(d*x+c)^2*sin(b*x+a)^2,x)
```

Fricas [F]

$$\int \csc^2(c+dx)\sin^2(a+bx)\, dx = \int \csc(dx+c)^2 \sin(bx+a)^2\, dx$$

input
```
integrate(csc(d*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*csc(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(c+dx)\sin^2(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**2*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^2(c+dx)\sin^2(a+bx)\,dx = \int \csc\left(dx+c\right)^2\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(csc(d*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*cos(2*(b + d)*x + 2*a + 2*c)
 + 2*(d*cos(2*(b + d)*x + 2*a + 2*c)^2 - 2*d*cos(2*(b + d)*x + 2*a + 2*c)*
cos(2*b*x + 2*a) + d*cos(2*b*x + 2*a)^2 + d*sin(2*(b + d)*x + 2*a + 2*c)^2
 - 2*d*sin(2*(b + d)*x + 2*a + 2*c)*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^
2)*integrate(1/2*(b*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + b*sin((2*b + d)*x
+ 2*a + c)*sin(4*b*x + 4*a) + b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b*cos
(4*b*x + 4*a) - b)*cos((2*b + d)*x + 2*a + c) - b*cos(2*b*x + 2*a))/(d*cos
((2*b + d)*x + 2*a + c)^2 + 2*d*cos((2*b + d)*x + 2*a + c)*cos(2*b*x + 2*a
) + d*cos(2*b*x + 2*a)^2 + d*sin((2*b + d)*x + 2*a + c)^2 + 2*d*sin((2*b +
 d)*x + 2*a + c)*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^2), x) - 2*(d*cos(2
*(b + d)*x + 2*a + 2*c)^2 - 2*d*cos(2*(b + d)*x + 2*a + 2*c)*cos(2*b*x + 2
*a) + d*cos(2*b*x + 2*a)^2 + d*sin(2*(b + d)*x + 2*a + 2*c)^2 - 2*d*sin(2*
(b + d)*x + 2*a + 2*c)*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^2)*integrate(
-1/2*(b*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) - b*sin((2*b + d)*x + 2*a + c)*s
in(4*b*x + 4*a) + b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - (b*cos(4*b*x + 4*a
) - b)*cos((2*b + d)*x + 2*a + c) - b*cos(2*b*x + 2*a))/(d*cos((2*b + d)*x
 + 2*a + c)^2 - 2*d*cos((2*b + d)*x + 2*a + c)*cos(2*b*x + 2*a) + d*cos(2*
b*x + 2*a)^2 + d*sin((2*b + d)*x + 2*a + c)^2 - 2*d*sin((2*b + d)*x + 2*a
+ c)*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^2), x) - (cos(4*b*x + 4*a) - 2*
cos(2*b*x + 2*a) + 1)*sin(2*(b + d)*x + 2*a + 2*c) - cos(2*b*x + 2*a)*s...
```

Giac [F]

$$\int \csc^2(c+dx)\sin^2(a+bx)\,dx = \int \csc\left(dx+c\right)^2\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(csc(d*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^2*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(c+dx)\sin^2(a+bx)\,dx=\int \frac{\sin\left(a+b\,x\right)^2}{\sin\left(c+d\,x\right)^2}\,dx$$

input
```
int(sin(a + b*x)^2/sin(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)^2/sin(c + d*x)^2, x)
```

Reduce [F]

$$\int \csc^2(c+dx)\sin^2(a+bx)\,dx=\int \csc\left(dx+c\right)^2\sin\left(bx+a\right)^2dx$$

input
```
int(csc(d*x+c)^2*sin(b*x+a)^2,x)
```

output
```
int(csc(c + d*x)**2*sin(a + b*x)**2,x)
```

3.121 $\int \csc^3(c+dx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 166

$$\begin{aligned}\int \csc^3(c+dx)\sin^2(a+bx)\,dx &= -\frac{\operatorname{arctanh}(\cos(c+dx))}{4d}-\frac{\cot(c+dx)\csc(c+dx)}{4d}\\&\quad+\frac{2e^{-2ia-2ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3}{2}-\frac{b}{d},\frac{5}{2}-\frac{b}{d},e^{2i(c+dx)}\right)}{2b-3d}\\&\quad-\frac{2e^{2ia+2ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3}{2}+\frac{b}{d},\frac{5}{2}+\frac{b}{d},e^{2i(c+dx)}\right)}{2b+3d}\end{aligned}$$

output
```
-1/4*arctanh(cos(d*x+c))/d-1/4*cot(d*x+c)*csc(d*x+c)/d+2*exp(-2*I*a-2*I*b*
x+3*I*(d*x+c))*hypergeom([3, 3/2-b/d],[5/2-b/d],exp(2*I*(d*x+c)))/(2*b-3*d
)-2*exp(2*I*a+2*I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2+b/d],[5/2+b/d],exp(2*
I*(d*x+c)))/(2*b+3*d)
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.24

$$\int \csc^3(c+dx)\sin^2(a+bx)\,dx$$
$$= \frac{16(2b+d)e^{-i(2a-c+2bx-dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{b}{d},\frac{3}{2}-\frac{b}{d},e^{2i(c+dx)}\right)-16(2b-d)e^{i(2a+c+(2b+d)x)}\,\text{Hy}}{}$$

input
```
Integrate[Csc[c + d*x]^3*Sin[a + b*x]^2,x]
```

output
```
((16*(2*b + d)*Hypergeometric2F1[1, 1/2 - b/d, 3/2 - b/d, E^((2*I)*(c + d*
x))])/E^(I*(2*a - c + 2*b*x - d*x)) - 16*(2*b - d)*E^(I*(2*a + c + (2*b +
d)*x))*Hypergeometric2F1[1, 1/2 + b/d, 3/2 + b/d, E^((2*I)*(c + d*x))] - 1
6*d*Log[Cos[(c + d*x)/2]] + 16*d*Log[Sin[(c + d*x)/2]] - 16*Csc[c + d*x]^2
*Sin[a + b*x]*((-2*b + d)*Sin[a - c + b*x - d*x] + (2*b + d)*Sin[a + c + (
b + d)*x]))/(64*d^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^3(c+dx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \sin^2(a+bx)\csc^3(c+dx)dx$$

input
```
Int[Csc[c + d*x]^3*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \csc (dx+c)^3 \sin (bx+a)^2 \, dx$$

input
```
int(csc(d*x+c)^3*sin(b*x+a)^2,x)
```

output
```
int(csc(d*x+c)^3*sin(b*x+a)^2,x)
```

Fricas [F]

$$\int \csc^3(c+dx)\sin^2(a+bx)\,dx = \int \csc (dx+c)^3 \sin (bx+a)^2 \, dx$$

input
```
integrate(csc(d*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*csc(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(c+dx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^3(c+dx)\sin^2(a+bx)\,dx=\int \csc\left(dx+c\right)^3\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(csc(d*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*((2*b - d)*cos((4*b + d)*x + 4*a + c)*cos(2*b*x + 2*a) + 2*d*cos((2*b
+ d)*x + 2*a + c)*cos(2*b*x + 2*a) + (2*b - d)*cos(2*b*x + 2*a)*cos(3*d*x
+ 3*c) - (2*b + d)*cos(2*b*x + 2*a)*cos(d*x + c) + (2*b - d)*sin((4*b + d)
*x + 4*a + c)*sin(2*b*x + 2*a) + 2*d*sin((2*b + d)*x + 2*a + c)*sin(2*b*x
+ 2*a) + (2*b - d)*sin(2*b*x + 2*a)*sin(3*d*x + 3*c) - (2*b + d)*sin(2*b*x
 + 2*a)*sin(d*x + c) + (2*(2*b + d)*cos(2*(b + d)*x + 2*a + 2*c) - (2*b +
d)*cos(2*b*x + 2*a))*cos((4*b + 3*d)*x + 4*a + 3*c) - 2*(2*d*cos(2*(b + d)
*x + 2*a + 2*c) - d*cos(2*b*x + 2*a))*cos((2*b + 3*d)*x + 2*a + 3*c) - ((2
*b + d)*cos((4*b + 3*d)*x + 4*a + 3*c) - (2*b - d)*cos((4*b + d)*x + 4*a +
 c) - 2*d*cos((2*b + 3*d)*x + 2*a + 3*c) - 2*d*cos((2*b + d)*x + 2*a + c)
- (2*b - d)*cos(3*d*x + 3*c) + (2*b + d)*cos(d*x + c))*cos(2*(b + 2*d)*x +
 2*a + 4*c) - 2*((2*b - d)*cos((4*b + d)*x + 4*a + c) + 2*d*cos((2*b + d)*
x + 2*a + c) + (2*b - d)*cos(3*d*x + 3*c) - (2*b + d)*cos(d*x + c))*cos(2*
(b + d)*x + 2*a + 2*c) - 4*(d^2*cos(2*(b + 2*d)*x + 2*a + 4*c)^2 + 4*d^2*c
os(2*(b + d)*x + 2*a + 2*c)^2 - 4*d^2*cos(2*(b + d)*x + 2*a + 2*c)*cos(2*b
*x + 2*a) + d^2*cos(2*b*x + 2*a)^2 + d^2*sin(2*(b + 2*d)*x + 2*a + 4*c)^2
+ 4*d^2*sin(2*(b + d)*x + 2*a + 2*c)^2 - 4*d^2*sin(2*(b + d)*x + 2*a + 2*c
)*sin(2*b*x + 2*a) + d^2*sin(2*b*x + 2*a)^2 - 2*(2*d^2*cos(2*(b + d)*x + 2
*a + 2*c) - d^2*cos(2*b*x + 2*a))*cos(2*(b + 2*d)*x + 2*a + 4*c) - 2*(2*d^
2*sin(2*(b + d)*x + 2*a + 2*c) - d^2*sin(2*b*x + 2*a))*sin(2*(b + 2*d)*...
```

Giac [F]

$$\int \csc^3(c+dx)\sin^2(a+bx)\,dx=\int \csc\left(dx+c\right)^3\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(csc(d*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^3*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+dx)\sin^2(a+bx)\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\sin\left(c+d\,x\right)^3}\,dx$$

input
```
int(sin(a + b*x)^2/sin(c + d*x)^3,x)
```

output
```
int(sin(a + b*x)^2/sin(c + d*x)^3, x)
```

Reduce [F]

$$\int \csc^3(c+dx)\sin^2(a+bx)\,dx = \int \csc\left(dx+c\right)^3\sin\left(bx+a\right)^2dx$$

input
```
int(csc(d*x+c)^3*sin(b*x+a)^2,x)
```

output
```
int(csc(c + d*x)**3*sin(a + b*x)**2,x)
```

3.122 $\int \csc^4(c+dx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 153

$$\begin{aligned}\int \csc^4(c+dx)\sin^2(a+bx)\,dx &= -\frac{\cot(c+dx)}{2d}-\frac{\cot^3(c+dx)}{6d}\\ &\quad-\frac{2ie^{-2ia-2ibx+4i(c+dx)}\,\text{Hypergeometric2F1}\left(4,2-\frac{b}{d},3-\frac{b}{d},e^{2i(c+dx)}\right)}{b-2d}\\ &\quad+\frac{2ie^{2ia+2ibx+4i(c+dx)}\,\text{Hypergeometric2F1}\left(4,2+\frac{b}{d},3+\frac{b}{d},e^{2i(c+dx)}\right)}{b+2d}\end{aligned}$$

output
```
-1/2*cot(d*x+c)/d-1/6*cot(d*x+c)^3/d-2*I*exp(-2*I*a-2*I*b*x+4*I*(d*x+c))*h
ypergeom([4, 2-b/d],[3-b/d],exp(2*I*(d*x+c)))/(b-2*d)+2*I*exp(2*I*a+2*I*b*
x+4*I*(d*x+c))*hypergeom([4, 2+b/d],[3+b/d],exp(2*I*(d*x+c)))/(b+2*d)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 563 vs. 2(153) = 306.

Time = 2.82 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.68

$$\int \csc^4(c+dx)\sin^2(a+bx)\,dx = -\frac{i\left(16(b+d)e^{-2i(a-c+2bx-dx)}\left(be^{2ibx}\,\text{Hypergeometric2F1}\left(1,1-\frac{b}{d},2-\frac{b}{d},e^{2i(c+dx)}\right)-(b-d)e^{2i(b-d)x}\,\text{Hyp}\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4*Sin[a + b*x]^2,x]
```

output
```
((-1/48*I)*((16*(b + d)*(b*E^((2*I)*b*x)*Hypergeometric2F1[1, 1 - b/d, 2 -
 b/d, E^((2*I)*(c + d*x))] - (b - d)*E^((2*I)*(b - d)*x)*Hypergeometric2F1
[1, -(b/d), 1 - b/d, E^((2*I)*(c + d*x))]))/E^((2*I)*(a - c + 2*b*x - d*x)
) - 16*(b - d)*E^((2*I)*(a + c))*((b + d)*E^((2*I)*b*x)*Hypergeometric2F1[
1, b/d, (b + d)/d, E^((2*I)*(c + d*x))] - b*E^((2*I)*(b + d)*x)*Hypergeome
tric2F1[1, (b + d)/d, 2 + b/d, E^((2*I)*(c + d*x))]) - (2*I)*(-1 + E^((2*I
)*c))*Csc[c]*Csc[c + d*x]^3*(Cos[2*(a + b*x)] - I*Sin[2*(a + b*x)])*(Cos[2
*(a + b*x)] + I*Sin[2*(a + b*x)])*(6*d^2*Sin[d*x] + (b^2 - d^2)*Sin[2*a -
2*c + 2*b*x - 3*d*x] - 2*b^2*Sin[2*a + 2*b*x - d*x] - b*d*Sin[2*a + 2*b*x
- d*x] + 3*d^2*Sin[2*a + 2*b*x - d*x] - b^2*Sin[2*a - 2*c + 2*b*x - d*x] +
 b*d*Sin[2*a - 2*c + 2*b*x - d*x] + 2*b^2*Sin[2*a + 2*b*x + d*x] - b*d*Sin
[2*a + 2*b*x + d*x] - 3*d^2*Sin[2*a + 2*b*x + d*x] + b^2*Sin[2*a + 2*c + 2
*b*x + d*x] + b*d*Sin[2*a + 2*c + 2*b*x + d*x] - 2*d^2*Sin[2*c + 3*d*x] -
b^2*Sin[2*a + 2*c + 2*b*x + 3*d*x] + d^2*Sin[2*a + 2*c + 2*b*x + 3*d*x]))
/(d^3*(1 - E^((2*I)*c)))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^4(c+dx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin^2(a+bx)\csc^4(c+dx)dx$$

input
```
Int[Csc[c + d*x]^4*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \csc(dx+c)^4 \sin(bx+a)^2\,dx$$

input
```
int(csc(d*x+c)^4*sin(b*x+a)^2,x)
```

output
```
int(csc(d*x+c)^4*sin(b*x+a)^2,x)
```

Fricas [F]

$$\int \csc^4(c+dx)\sin^2(a+bx)\,dx = \int \csc(dx+c)^4 \sin(bx+a)^2\,dx$$

input
```
integrate(csc(d*x+c)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*csc(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(c+dx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**4*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^4(c+dx)\sin^2(a+bx)\,dx = \int \csc\left(dx+c\right)^4\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(csc(d*x+c)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3*((b^2 - d^2)*cos(2*b*x + 2*a)*sin(4*b*x + 4*a) - (b^2 - d^2)*cos(4*b*
x + 4*a)*sin(2*b*x + 2*a) - (b^2 - b*d)*cos(4*d*x + 4*c)*sin(2*b*x + 2*a)
+ (2*b^2 - b*d - 3*d^2)*cos(2*d*x + 2*c)*sin(2*b*x + 2*a) + (b^2 - b*d)*co
s(2*b*x + 2*a)*sin(4*d*x + 4*c) - (2*b^2 - b*d - 3*d^2)*cos(2*b*x + 2*a)*s
in(2*d*x + 2*c) - (3*(2*b^2 + b*d - 3*d^2)*sin(2*(b + d)*x + 2*a + 2*c) -
(2*b^2 + b*d - 3*d^2)*sin(2*b*x + 2*a))*cos(2*(2*b + d)*x + 4*a + 2*c) + (
6*d^2*sin(2*(b + d)*x + 2*a + 2*c) - 2*d^2*sin(2*b*x + 2*a) + (2*b^2 + b*d
 - 3*d^2)*sin(2*(2*b + d)*x + 4*a + 2*c) - (b^2 + b*d)*sin(4*(b + d)*x + 4
*a + 4*c) - (b^2 - d^2)*sin(4*b*x + 4*a) - (b^2 - b*d)*sin(4*d*x + 4*c) +
(2*b^2 - b*d - 3*d^2)*sin(2*d*x + 2*c))*cos(2*(b + 3*d)*x + 2*a + 6*c) - 3
*(6*d^2*sin(2*(b + d)*x + 2*a + 2*c) - 2*d^2*sin(2*b*x + 2*a) + (2*b^2 + b
*d - 3*d^2)*sin(2*(2*b + d)*x + 4*a + 2*c) - (b^2 - d^2)*sin(4*b*x + 4*a)
- (b^2 - b*d)*sin(4*d*x + 4*c) + (2*b^2 - b*d - 3*d^2)*sin(2*d*x + 2*c))*c
os(2*(b + 2*d)*x + 2*a + 4*c) - (3*(b^2 + b*d)*sin(2*(b + 2*d)*x + 2*a + 4
*c) - 3*(b^2 + b*d)*sin(2*(b + d)*x + 2*a + 2*c) + (b^2 + b*d)*sin(2*b*x +
 2*a))*cos(4*(b + d)*x + 4*a + 4*c) - 3*((b^2 - d^2)*sin(4*b*x + 4*a) + (b
^2 - b*d)*sin(4*d*x + 4*c) - (2*b^2 - b*d - 3*d^2)*sin(2*d*x + 2*c))*cos(2
*(b + d)*x + 2*a + 2*c) + 3*(d^3*cos(2*(b + 3*d)*x + 2*a + 6*c)^2 + 9*d^3*
cos(2*(b + 2*d)*x + 2*a + 4*c)^2 + 9*d^3*cos(2*(b + d)*x + 2*a + 2*c)^2 -
6*d^3*cos(2*(b + d)*x + 2*a + 2*c)*cos(2*b*x + 2*a) + d^3*cos(2*b*x + 2...
```

Giac [F]

$$\int \csc^4(c+dx)\sin^2(a+bx)\,dx = \int \csc(dx+c)^4\sin(bx+a)^2\,dx$$

input
```
integrate(csc(d*x+c)^4*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^4*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+dx)\sin^2(a+bx)\,dx = \int \frac{\sin(a+b\,x)^2}{\sin(c+d\,x)^4}\,dx$$

input
```
int(sin(a + b*x)^2/sin(c + d*x)^4,x)
```

output
```
int(sin(a + b*x)^2/sin(c + d*x)^4, x)
```

Reduce [F]

$$\int \csc^4(c+dx)\sin^2(a+bx)\,dx = \int \csc(dx+c)^4\sin(bx+a)^2\,dx$$

input
```
int(csc(d*x+c)^4*sin(b*x+a)^2,x)
```

output
```
int(csc(c + d*x)**4*sin(a + b*x)**2,x)
```

3.123 $\int \sin^3(a+bx)\sin^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 195

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = -\frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}+\frac{\sin(3(a-c)+3(b-d)x)}{96(b-d)}-\frac{3\sin(3a-c+(3b-d)x)}{32(3b-d)}-\frac{9\sin(a+c+(b+d)x)}{32(b+d)}-\frac{\sin(3(a+c)+3(b+d)x)}{96(b+d)}+\frac{3\sin(3a+c+(3b+d)x)}{32(3b+d)}+\frac{3\sin(a+3c+(b+3d)x)}{32(b+3d)}$$

output

```
-3*sin(a-3*c+(b-3*d)*x)/(32*b-96*d)+9*sin(a-c+(b-d)*x)/(32*b-32*d)+sin(3*a
-3*c+3*(b-d)*x)/(96*b-96*d)-3*sin(3*a-c+(3*b-d)*x)/(96*b-32*d)-9*sin(a+c+(
b+d)*x)/(32*b+32*d)-sin(3*a+3*c+3*(b+d)*x)/(96*b+96*d)+3*sin(3*a+c+(3*b+d)
*x)/(96*b+32*d)+3*sin(a+3*c+(b+3*d)*x)/(32*b+96*d)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = \frac{1}{96}\left(-\frac{9\sin(a-3c+bx-3dx)}{b-3d} + \frac{27\sin(a-c+bx-dx)}{b-d} + \frac{\sin(3(a-c+bx-dx))}{b-d} - \frac{9\sin(3a-c+3bx-dx)}{3b-d} + \frac{9\sin(3a+c+3bx+dx)}{3b+d} + \frac{9\sin(a+3c+bx+3dx)}{b+3d} - \frac{27\sin(a+c+(b+d)x)}{b+d} - \frac{\sin(3(a+c+(b+d)x))}{b+d}\right)$$

input
```
Integrate[Sin[a + b*x]^3*Sin[c + d*x]^3,x]
```

output
```
((-9*Sin[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sin[a - c + b*x - d*x])/(
b - d) + Sin[3*(a - c + b*x - d*x)]/(b - d) - (9*Sin[3*a - c + 3*b*x - d*x
])/(3*b - d) + (9*Sin[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sin[a + 3*c +
 b*x + 3*d*x])/(b + 3*d) - (27*Sin[a + c + (b + d)*x])/(b + d) - Sin[3*(a
+ c + (b + d)*x)]/(b + d))/96
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx$$

$$\big\downarrow \text{ 5080}$$

$$\int \left(-\frac{3}{32}\cos(a+x(b-3d)-3c)+\frac{9}{32}\cos(a+x(b-d)-c)+\frac{1}{32}\cos(3(a-c)+3x(b-d))-\frac{3}{32}\cos(3a+x(3b$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}-\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\sin(a+x(b+d)+c)}{32(b+d)}-\frac{\sin(3(a+c)+3x(b+d))}{96(b+d)}+\frac{3\sin(3a+x(3b+d)+c)}{32(3b+d)}+\frac{3\sin(a+x(b+3d)+3c)}{32(b+3d)}$$

input
```
Int[Sin[a + b*x]^3*Sin[c + d*x]^3,x]
```

output
```
(-3*Sin[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sin[a - c + (b - d)*x]
)/(32*(b - d)) + Sin[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) - (3*Sin[3*a -
c + (3*b - d)*x])/(32*(3*b - d)) - (9*Sin[a + c + (b + d)*x])/(32*(b + d))
 - Sin[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sin[3*a + c + (3*b + d)*
x])/(32*(3*b + d)) + (3*Sin[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 43.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.97

method	result
default	$-\frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}-\frac{9\sin(a+c+(b+d)x)}{32(b+d)}+\frac{3\sin(a+3c+(b+3d)x)}{32(b+3d)}+\frac{\sin((3b-3d)x+3a-3}{96b-96d}$
parallelrisch	$\frac{-\frac{9\left(b+\frac{d}{3}\right)(b-3d)(b+3d)(b-d)(b+d)\sin(3a-c+(3b-d)x)}{32}+9\left(\frac{\left(b+\frac{d}{3}\right)(b-3d)(b+3d)(b+d)\sin((3b-3d)x+3a-3c)}{3}-\frac{\left(b+\frac{d}{3}\right)(b-3d)(b+3d)(b-}{}\right.}{}$
risch	Expression too large to display
orering	Expression too large to display

input
```
int(sin(b*x+a)^3*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32/(b-d)*sin(a-c+(b-d)*x)-9/32/(b+d)*
sin(a+c+(b+d)*x)+3/32/(b+3*d)*sin(a+3*c+(b+3*d)*x)+1/32/(3*b-3*d)*sin((3*b
-3*d)*x+3*a-3*c)-3/32/(3*b-d)*sin(3*a-c+(3*b-d)*x)+3/32/(3*b+d)*sin(3*a+c+
(3*b+d)*x)-1/32/(3*b+3*d)*sin((3*b+3*d)*x+3*a+3*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.49

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = -\frac{\left(\left(63\,b^4d-88\,b^2d^3+9\,d^5-\left(9\,b^4d-82\,b^2d^3+9\,d^5\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)^3-3\left(21\,b^4d-70\,b^2d^3-\right.}{}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/3*(((63*b^4*d - 88*b^2*d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cos
(b*x + a)^2)*cos(d*x + c)^3 - 3*(21*b^4*d - 70*b^2*d^3 + 9*d^5 - (3*b^4*d
- 28*b^2*d^3 + 9*d^5)*cos(b*x + a)^2)*cos(d*x + c))*sin(b*x + a) - ((9*b^5
 - 88*b^3*d^2 + 63*b*d^4)*cos(b*x + a)^3 - ((9*b^5 - 82*b^3*d^2 + 9*b*d^4)
*cos(b*x + a)^3 - 3*(9*b^5 - 28*b^3*d^2 + 3*b*d^4)*cos(b*x + a))*cos(d*x +
 c)^2 - 3*(9*b^5 - 70*b^3*d^2 + 21*b*d^4)*cos(b*x + a))*sin(d*x + c))/(9*b
^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3580 vs. $2(172) = 344$.

Time = 17.81 (sec) , antiderivative size = 3580, normalized size of antiderivative = 18.36

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)**3*sin(d*x+c)**3,x)
```

output
```
Piecewise((x*sin(a)**3*sin(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - 3*d*x
)**3*sin(c + d*x)**3/32 - 9*x*sin(a - 3*d*x)**3*sin(c + d*x)*cos(c + d*x)*
*2/32 - 9*x*sin(a - 3*d*x)**2*sin(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/
32 + 3*x*sin(a - 3*d*x)**2*cos(a - 3*d*x)*cos(c + d*x)**3/32 + 3*x*sin(a -
 3*d*x)*sin(c + d*x)**3*cos(a - 3*d*x)**2/32 - 9*x*sin(a - 3*d*x)*sin(c +
d*x)*cos(a - 3*d*x)**2*cos(c + d*x)**2/32 - 9*x*sin(c + d*x)**2*cos(a - 3*
d*x)**3*cos(c + d*x)/32 + 3*x*cos(a - 3*d*x)**3*cos(c + d*x)**3/32 + 13*si
n(a - 3*d*x)**3*sin(c + d*x)**2*cos(c + d*x)/(320*d) + sin(a - 3*d*x)**3*c
os(c + d*x)**3/(12*d) + 101*sin(a - 3*d*x)**2*sin(c + d*x)**3*cos(a - 3*d*
x)/(320*d) + 3*sin(a - 3*d*x)**2*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)*
*2/(20*d) + 27*sin(a - 3*d*x)*cos(a - 3*d*x)**2*cos(c + d*x)**3/(320*d) +
sin(c + d*x)**3*cos(a - 3*d*x)**3/(5*d) + 51*sin(c + d*x)*cos(a - 3*d*x)**
3*cos(c + d*x)**2/(320*d), Eq(b, -3*d)), (5*x*sin(a - d*x)**3*sin(c + d*x)
**3/16 + 3*x*sin(a - d*x)**3*sin(c + d*x)*cos(c + d*x)**2/16 - 9*x*sin(a -
 d*x)**2*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/16 - 3*x*sin(a - d*x)**
2*cos(a - d*x)*cos(c + d*x)**3/16 + 3*x*sin(a - d*x)*sin(c + d*x)**3*cos(a
 - d*x)**2/16 + 9*x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)
**2/16 - 3*x*sin(c + d*x)**2*cos(a - d*x)**3*cos(c + d*x)/16 - 5*x*cos(a -
 d*x)**3*cos(c + d*x)**3/16 - 11*sin(a - d*x)**3*sin(c + d*x)**2*cos(c + d
*x)/(16*d) - 7*sin(a - d*x)**3*cos(c + d*x)**3/(48*d) + 3*sin(a - d*x)*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2612 vs. $2(179) = 358$.

Time = 0.21 (sec) , antiderivative size = 2612, normalized size of antiderivative = 13.39

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/192*(9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a +
 4*c) - 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a -
 2*c) + 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a
+ 4*c) - 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2
*d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a
 - 2*c) + 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30
*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a +
 6*c) - 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b
^2*d^3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a) -
(9*b^5*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(
3*c) + 9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a + 6*c) + (
9*b^5*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3
*c) + 9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a) - 27*(9*b^
5*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c)
+ 9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((b + d)*x + a + 4*c) + 27*(9*b^5*
sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) +
9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = -\frac{\sin(3\,bx+3\,dx+3\,a+3\,c)}{96\,(b+d)} + \frac{3\,\sin(3\,bx+dx+3\,a+c)}{32\,(3\,b+d)} - \frac{3\,\sin(3\,bx-dx+3\,a-c)}{32\,(3\,b-d)} + \frac{\sin(3\,bx-3\,dx+3\,a-3\,c)}{96\,(b-d)} + \frac{3\,\sin(bx+3\,dx+a+3\,c)}{32\,(b+3\,d)} - \frac{9\,\sin(bx+dx+a+c)}{32\,(b+d)} + \frac{9\,\sin(bx-dx+a-c)}{32\,(b-d)} - \frac{3\,\sin(bx-3\,dx+a-3\,c)}{32\,(b-3\,d)}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/96*sin(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*sin(3*b*x + d*x + 3*a
+ c)/(3*b + d) - 3/32*sin(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*sin(3*b*
x - 3*d*x + 3*a - 3*c)/(b - d) + 3/32*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d)
 - 9/32*sin(b*x + d*x + a + c)/(b + d) + 9/32*sin(b*x - d*x + a - c)/(b -
d) - 3/32*sin(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

Mupad [B] (verification not implemented)

Time = 21.09 (sec) , antiderivative size = 997, normalized size of antiderivative = 5.11

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
int(sin(a + b*x)^3*sin(c + d*x)^3,x)
```

output
```
exp(a*3i - c*1i + b*x*3i - d*x*1i)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(b
^4*576i + d^4*64i - b^2*d^2*640i) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + 3*b^2
*d - 9*b^3 - 3*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*2i - b
*x*2i)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^
2*640i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(b^
4*576i + d^4*64i - b^2*d^2*640i)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((9
*b*d^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(b^4*576i + d^4*64i - b^2*d^2*640i) + (e
xp(- a*6i - b*x*6i)*(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3))/(b^4*576i + d^4*6
4i - b^2*d^2*640i) - (exp(- a*2i - b*x*2i)*(9*b*d^2 + 81*b^2*d - 81*b^3 -
9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*4i - b*x*4i)*(9*b*d
^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i)) - ex
p(a*3i - c*3i + b*x*3i - d*x*3i)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(b^4*192
i + d^4*1728i - b^2*d^2*1920i) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + b^2*d -
b^3 - 9*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i) - (exp(- a*2i - b*x*2
i)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*
1920i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(b^4
*192i + d^4*1728i - b^2*d^2*1920i)) + exp(a*3i + c*3i + b*x*3i + d*x*3i)*(
(9*b*d^2 + b^2*d - b^3 - 9*d^3)/(b^4*192i + d^4*1728i - b^2*d^2*1920i) + (
exp(- a*6i - b*x*6i)*(9*b*d^2 - b^2*d - b^3 + 9*d^3))/(b^4*192i + d^4*1728
i - b^2*d^2*1920i) - (exp(- a*2i - b*x*2i)*(9*b*d^2 + 27*b^2*d - 9*b^3 ...
```

Reduce [F]

$$\int \sin^3(a+bx)\sin^3(c+dx)\,dx = \int \sin(bx+a)^3 \sin(dx+c)^3\,dx$$

input
```
int(sin(b*x+a)^3*sin(d*x+c)^3,x)
```

output
```
int(sin(b*x+a)^3*sin(d*x+c)^3,x)
```

3.124 $\int \sin^3(a+bx)\sin^2(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 138

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx = -\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}+\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}-\frac{\cos(3a-2c+(3b-2d)x)}{16(3b-2d)}+\frac{3\cos(a+2c+(b+2d)x)}{16(b+2d)}-\frac{\cos(3a+2c+(3b+2d)x)}{16(3b+2d)}$$

output
```
-3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b+3*cos(a-2*c+(b-2*d)*x)/(16*b-32*d)
-cos(3*a-2*c+(3*b-2*d)*x)/(48*b-32*d)+3*cos(a+2*c+(b+2*d)*x)/(16*b+32*d)-c
os(3*a+2*c+(3*b+2*d)*x)/(48*b+32*d)
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx = \frac{1}{48}\left(-\frac{18\cos(a)\cos(bx)}{b}+\frac{2\cos(3a)\cos(3bx)}{b}+\frac{9\cos(a-2c+bx-2dx)}{b-2d}-\frac{3\cos(3a-2c+3bx-2dx)}{3b-2d}+\frac{9\cos(a+2c+bx+2dx)}{b+2d}-\frac{3\cos(3a+2c+3bx+2dx)}{3b+2d}+\frac{18\sin(a)\sin(bx)}{b}-\frac{2\sin(3a)\sin(3bx)}{b}\right)$$

input
```
Integrate[Sin[a + b*x]^3*Sin[c + d*x]^2,x]
```

output
```
((-18*Cos[a]*Cos[b*x])/b + (2*Cos[3*a]*Cos[3*b*x])/b + (9*Cos[a - 2*c + b*
x - 2*d*x])/(b - 2*d) - (3*Cos[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d) + (
9*Cos[a + 2*c + b*x + 2*d*x])/(b + 2*d) - (3*Cos[3*a + 2*c + 3*b*x + 2*d*x
])/(3*b + 2*d) + (18*Sin[a]*Sin[b*x])/b - (2*Sin[3*a]*Sin[3*b*x])/b)/48
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(-\frac{3}{16}\sin(a+x(b-2d)-2c)+\frac{1}{16}\sin(3a+x(3b-2d)-2c)-\frac{3}{16}\sin(a+x(b+2d)+2c)+\frac{1}{16}\sin(3a+x(\right.$$

$$\downarrow \text{2009}$$

$$\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}-\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}+\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}-\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}-\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}$$

input
```
Int[Sin[a + b*x]^3*Sin[c + d*x]^2,x]
```

output
```
(-3*Cos[a + b*x])/(8*b) + Cos[3*a + 3*b*x]/(24*b) + (3*Cos[a - 2*c + (b -
2*d)*x])/(16*(b - 2*d)) - Cos[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d))
+ (3*Cos[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) - Cos[3*a + 2*c + (3*b + 2
*d)*x]/(16*(3*b + 2*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 13.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

method	result
default	$-\frac{3\cos(bx+a)}{8b}+\frac{\cos(3bx+3a)}{24b}+\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}+\frac{3\cos(a+2c+(b+2d)x)}{16(b+2d)}-\frac{\cos(3a-2c+(3b-2d)x)}{16(3b-2d)}-\frac{\cos(3a}{}$
parallelrisch	$\frac{-9\left(b+\frac{2d}{3}\right)(b+2d)b(b-2d)\cos(3a-2c+(3b-2d)x)-9(b+2d)\left(b-\frac{2d}{3}\right)b(b-2d)\cos(3a+2c+(3b+2d)x)+81\left(b+\frac{2d}{3}\right)(b+2d)\left(b-\right.}{}$
risch	$-\frac{3\cos(bx+a)}{8b}+\frac{27\cos(bx-2dx+a-2c)b^3}{16(b+2d)(3b+2d)(3b-2d)(b-2d)}+\frac{27\cos(bx-2dx+a-2c)b^2d}{8(b+2d)(3b+2d)(3b-2d)(b-2d)}-\frac{3\cos(bx-2dx+a-2c)b\,d^2}{4(b+2d)(3b+2d)(3b-2d)(b-2}$
orering	Expression too large to display

input
```
int(sin(b*x+a)^3*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b+3/16/(b-2*d)*cos(a-2*c+(b-2*d)*x)+
3/16/(b+2*d)*cos(a+2*c+(b+2*d)*x)-1/16/(3*b-2*d)*cos(3*a-2*c+(3*b-2*d)*x)-
1/16/(3*b+2*d)*cos(3*a+2*c+(3*b+2*d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.37

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx$$
$$= \frac{(9\,b^4-38\,b^2d^2+8\,d^4)\cos{(bx+a)}^3+6\left(7\,b^3d-4\,bd^3-(b^3d-4\,bd^3)\cos{(bx+a)}^2\right)\cos{(dx+c)}\sin{(bx}}{}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/3*((9*b^4 - 38*b^2*d^2 + 8*d^4)*cos(b*x + a)^3 + 6*(7*b^3*d - 4*b*d^3 -
(b^3*d - 4*b*d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a)*sin(d*x + c) -
 9*((b^4 - 4*b^2*d^2)*cos(b*x + a)^3 - (3*b^4 - 4*b^2*d^2)*cos(b*x + a))*c
os(d*x + c)^2 - 3*(9*b^4 - 26*b^2*d^2 + 8*d^4)*cos(b*x + a))/(9*b^5 - 40*b
^3*d^2 + 16*b*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. 2(116) = 232.

Time = 5.81 (sec) , antiderivative size = 2030, normalized size of antiderivative = 14.71

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)**3*sin(d*x+c)**2,x)
```

output
```
Piecewise((x*sin(a)**3*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**
2/2 + x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**3, Eq
(b, 0)), (3*x*sin(a - 2*d*x)**3*sin(c + d*x)**2/16 - 3*x*sin(a - 2*d*x)**3
*cos(c + d*x)**2/16 - 3*x*sin(a - 2*d*x)**2*sin(c + d*x)*cos(a - 2*d*x)*co
s(c + d*x)/8 + 3*x*sin(a - 2*d*x)*sin(c + d*x)**2*cos(a - 2*d*x)**2/16 - 3
*x*sin(a - 2*d*x)*cos(a - 2*d*x)**2*cos(c + d*x)**2/16 - 3*x*sin(c + d*x)*
cos(a - 2*d*x)**3*cos(c + d*x)/8 - 13*sin(a - 2*d*x)**3*sin(c + d*x)*cos(c
 + d*x)/(16*d) + sin(a - 2*d*x)**2*cos(a - 2*d*x)*cos(c + d*x)**2/(2*d) -
7*sin(a - 2*d*x)*sin(c + d*x)*cos(a - 2*d*x)**2*cos(c + d*x)/(8*d) - 17*si
n(c + d*x)**2*cos(a - 2*d*x)**3/(96*d) + 49*cos(a - 2*d*x)**3*cos(c + d*x)
**2/(96*d), Eq(b, -2*d)), (x*sin(a - 2*d*x/3)**3*sin(c + d*x)**2/16 - x*si
n(a - 2*d*x/3)**3*cos(c + d*x)**2/16 - 3*x*sin(a - 2*d*x/3)**2*sin(c + d*x
)*cos(a - 2*d*x/3)*cos(c + d*x)/8 - 3*x*sin(a - 2*d*x/3)*sin(c + d*x)**2*c
os(a - 2*d*x/3)**2/16 + 3*x*sin(a - 2*d*x/3)*cos(a - 2*d*x/3)**2*cos(c + d
*x)**2/16 + x*sin(c + d*x)*cos(a - 2*d*x/3)**3*cos(c + d*x)/8 - 15*sin(a -
 2*d*x/3)**3*sin(c + d*x)*cos(c + d*x)/(16*d) + 3*sin(a - 2*d*x/3)**2*cos(
a - 2*d*x/3)*cos(c + d*x)**2/(2*d) + 9*sin(a - 2*d*x/3)*sin(c + d*x)*cos(a
 - 2*d*x/3)**2*cos(c + d*x)/(8*d) + 21*sin(c + d*x)**2*cos(a - 2*d*x/3)**3
/(32*d) + 11*cos(a - 2*d*x/3)**3*cos(c + d*x)**2/(32*d), Eq(b, -2*d/3)), (
x*sin(a + 2*d*x/3)**3*sin(c + d*x)**2/16 - x*sin(a + 2*d*x/3)**3*cos(c ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(126) = 252$.

Time = 0.10 (sec) , antiderivative size = 1360, normalized size of antiderivative = 9.86

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/96*(3*(3*b^4*cos(2*c) - 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^
3*cos(2*c))*cos((3*b + 2*d)*x + 3*a + 4*c) + 3*(3*b^4*cos(2*c) - 2*b^3*d*c
os(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((3*b + 2*d)*x + 3*a)
 + 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*co
s(2*c))*cos(-(3*b - 2*d)*x - 3*a + 4*c) + 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(
2*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(3*b - 2*d)*x - 3*a) -
 9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(
2*c))*cos((b + 2*d)*x + a + 4*c) - 9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) -
 4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((b + 2*d)*x + a) - 9*(9*b^4*co
s(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(
b - 2*d)*x - a + 4*c) - 9*(9*b^4*cos(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*
cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(b - 2*d)*x - a) - 2*(9*b^4*cos(2*c) - 4
0*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a + 2*c) - 2*(9*b^4*co
s(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a - 2*c) + 1
8*(9*b^4*cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a + 2
*c) + 18*(9*b^4*cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x
+ a - 2*c) + 3*(3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) +
8*b*d^3*sin(2*c))*sin((3*b + 2*d)*x + 3*a + 4*c) - 3*(3*b^4*sin(2*c) - 2*b
^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((3*b + 2*d)*x
+ 3*a) + 3*(3*b^4*sin(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) - 8...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx = -\frac{\cos(3\,bx+2\,dx+3\,a+2\,c)}{16\,(3\,b+2\,d)} - \frac{\cos(3\,bx-2\,dx+3\,a-2\,c)}{16\,(3\,b-2\,d)} + \frac{\cos(3\,bx+3\,a)}{24\,b} + \frac{3\,\cos(bx+2\,dx+a+2\,c)}{16\,(b+2\,d)} + \frac{3\,\cos(bx-2\,dx+a-2\,c)}{16\,(b-2\,d)} - \frac{3\,\cos(bx+a)}{8\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/16*cos(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) - 1/16*cos(3*b*x - 2*d*x
+ 3*a - 2*c)/(3*b - 2*d) + 1/24*cos(3*b*x + 3*a)/b + 3/16*cos(b*x + 2*d*x
+ a + 2*c)/(b + 2*d) + 3/16*cos(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/8*cos
(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 19.16 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.17

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx$$
$$= \frac{81\,b^4\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)+81\,b^4\,\cos\left(a+2\,c+b\,x+2\,d\,x\right)-162\,b^4\,\cos\left(a+b\,x\right)-288\,d^4\,\cos\left(a+\right.}{}$$

input
```
int(sin(a + b*x)^3*sin(c + d*x)^2,x)
```

output
```
(81*b^4*cos(a - 2*c + b*x - 2*d*x) + 81*b^4*cos(a + 2*c + b*x + 2*d*x) - 1
62*b^4*cos(a + b*x) - 288*d^4*cos(a + b*x) - 9*b^4*cos(3*a - 2*c + 3*b*x -
 2*d*x) - 9*b^4*cos(3*a + 2*c + 3*b*x + 2*d*x) + 18*b^4*cos(3*a + 3*b*x) +
 32*d^4*cos(3*a + 3*b*x) + 24*b*d^3*cos(3*a - 2*c + 3*b*x - 2*d*x) - 24*b*
d^3*cos(3*a + 2*c + 3*b*x + 2*d*x) - 6*b^3*d*cos(3*a - 2*c + 3*b*x - 2*d*x
) + 6*b^3*d*cos(3*a + 2*c + 3*b*x + 2*d*x) - 36*b^2*d^2*cos(a - 2*c + b*x
- 2*d*x) - 36*b^2*d^2*cos(a + 2*c + b*x + 2*d*x) + 720*b^2*d^2*cos(a + b*x
) + 36*b^2*d^2*cos(3*a - 2*c + 3*b*x - 2*d*x) + 36*b^2*d^2*cos(3*a + 2*c +
 3*b*x + 2*d*x) - 80*b^2*d^2*cos(3*a + 3*b*x) - 72*b*d^3*cos(a - 2*c + b*x
 - 2*d*x) + 72*b*d^3*cos(a + 2*c + b*x + 2*d*x) + 162*b^3*d*cos(a - 2*c +
b*x - 2*d*x) - 162*b^3*d*cos(a + 2*c + b*x + 2*d*x))/(48*(16*b*d^4 + 9*b^5
 - 40*b^3*d^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.87

$$\int \sin^3(a+bx)\sin^2(c+dx)\,dx$$
$$= \frac{-9\cos(bx+a)\sin(bx+a)^2\sin(dx+c)^2b^4+36\cos(bx+a)\sin(bx+a)^2\sin(dx+c)^2b^2d^2+2\cos(bx}{}$$

input
```
int(sin(b*x+a)^3*sin(d*x+c)^2,x)
```

output
```
( - 9*cos(a + b*x)*sin(a + b*x)**2*sin(c + d*x)**2*b**4 + 36*cos(a + b*x)*
sin(a + b*x)**2*sin(c + d*x)**2*b**2*d**2 + 2*cos(a + b*x)*sin(a + b*x)**2
*b**2*d**2 - 8*cos(a + b*x)*sin(a + b*x)**2*d**4 - 18*cos(a + b*x)*sin(c +
 d*x)**2*b**4 + 40*cos(a + b*x)*b**2*d**2 - 16*cos(a + b*x)*d**4 + 6*cos(c
 + d*x)*sin(a + b*x)**3*sin(c + d*x)*b**3*d - 24*cos(c + d*x)*sin(a + b*x)
**3*sin(c + d*x)*b*d**3 + 36*cos(c + d*x)*sin(a + b*x)*sin(c + d*x)*b**3*d
 - 16*b**2*d**2 + 16*d**4)/(3*b*(9*b**4 - 40*b**2*d**2 + 16*d**4))
```

3.125 $\int \sin^3(a+bx)\sin(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \sin^3(a+bx)\sin(c+dx)\,dx = \frac{3\sin(a-c+(b-d)x)}{8(b-d)} - \frac{\sin(3a-c+(3b-d)x)}{8(3b-d)} - \frac{3\sin(a+c+(b+d)x)}{8(b+d)} + \frac{\sin(3a+c+(3b+d)x)}{8(3b+d)}$$

output
```
3*sin(a-c+(b-d)*x)/(8*b-8*d)-sin(3*a-c+(3*b-d)*x)/(24*b-8*d)-3*sin(a+c+(b+
d)*x)/(8*b+8*d)+sin(3*a+c+(3*b+d)*x)/(24*b+8*d)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \sin^3(a+bx)\sin(c+dx)\,dx = \frac{1}{8}\left(\frac{3\sin(a-c+bx-dx)}{b-d} - \frac{\sin(3a-c+3bx-dx)}{3b-d} + \frac{\sin(3a+c+3bx+dx)}{3b+d} - \frac{3\sin(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Sin[a + b*x]^3*Sin[c + d*x],x]
```

output
```
((3*Sin[a - c + b*x - d*x])/(b - d) - Sin[3*a - c + 3*b*x - d*x]/(3*b - d)
 + Sin[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Sin[a + c + (b + d)*x])/(b +
d))/8
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin(c+dx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(\frac{3}{8}\cos(a+x(b-d)-c)-\frac{1}{8}\cos(3a+x(3b-d)-c)-\frac{3}{8}\cos(a+x(b+d)+c)+\frac{1}{8}\cos(3a+x(3b+d)+c)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{\sin(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(3a+x(3b+d)+c)}{8(3b+d)}$$

input
```
Int[Sin[a + b*x]^3*Sin[c + d*x],x]
```

output
```
(3*Sin[a - c + (b - d)*x])/(8*(b - d)) - Sin[3*a - c + (3*b - d)*x]/(8*(3*
b - d)) - (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[3*a + c + (3*b + d)
*x]/(8*(3*b + d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 3.94 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
default	$\frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{3\sin(a+c+(b+d)x)}{8(b+d)}-\frac{\sin(3a-c+(3b-d)x)}{8(3b-d)}+\frac{\sin(3a+c+(3b+d)x)}{24b+8d}$
parallelrisch	$\frac{-3\left(b+\frac{d}{3}\right)(b-d)(b+d)\sin(3a-c+(3b-d)x)+27\left(\left(b+\frac{d}{3}\right)(b+d)\sin(a-c+(b-d)x)-\left(\left(-\frac{b}{9}-\frac{d}{9}\right)\sin(3a+c+(3b+d)x)+\left(b+\frac{d}{3}\right)s\right.\right.}{72b^4-80b^2d^2+8d^4}$
risch	$\frac{27\sin(bx-dx+a-c)b^3}{8(-b+d)(-3b+d)(3b+d)(b+d)}+\frac{27\sin(bx-dx+a-c)b^2d}{8(-b+d)(-3b+d)(3b+d)(b+d)}-\frac{3\sin(bx-dx+a-c)b\,d^2}{8(-b+d)(-3b+d)(3b+d)(b+d)}-\frac{3\sin(bx-dx\cdot}{8(-b+d)(-3b+d}$
orering	Expression too large to display

input
```
int(sin(b*x+a)^3*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
3/8/(b-d)*sin(a-c+(b-d)*x)-3/8/(b+d)*sin(a+c+(b+d)*x)-1/8/(3*b-d)*sin(3*a-
c+(3*b-d)*x)+1/8/(3*b+d)*sin(3*a+c+(3*b+d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.19

$$\int \sin^3(a+bx)\sin(c+dx)\,dx$$
$$=\frac{(7b^2d-d^3-(b^2d-d^3)\cos(bx+a)^2)\cos(dx+c)\sin(bx+a)+3\left((b^3-bd^2)\cos(bx+a)^3-(3b^3-bd\right.}{9b^4-10b^2d^2+d^4}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="fricas")
```

output
```
((7*b^2*d - d^3 - (b^2*d - d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a)
+ 3*((b^3 - b*d^2)*cos(b*x + a)^3 - (3*b^3 - b*d^2)*cos(b*x + a))*sin(d*x
+ c))/(9*b^4 - 10*b^2*d^2 + d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(76) = 152$.

Time = 2.05 (sec) , antiderivative size = 933, normalized size of antiderivative = 9.62

$$\int \sin^3(a+bx)\sin(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)**3*sin(d*x+c),x)
```

output
```
Piecewise((x*sin(a)**3*sin(c), Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - d*x)**3*
sin(c + d*x)/8 - 3*x*sin(a - d*x)**2*cos(a - d*x)*cos(c + d*x)/8 + 3*x*sin
(a - d*x)*sin(c + d*x)*cos(a - d*x)**2/8 - 3*x*cos(a - d*x)**3*cos(c + d*x
)/8 + sin(a - d*x)**3*cos(c + d*x)/(8*d) + 3*sin(a - d*x)**2*sin(c + d*x)*
cos(a - d*x)/(4*d) + 3*sin(c + d*x)*cos(a - d*x)**3/(8*d), Eq(b, -d)), (x*
sin(a - d*x/3)**3*sin(c + d*x)/8 - 3*x*sin(a - d*x/3)**2*cos(a - d*x/3)*co
s(c + d*x)/8 - 3*x*sin(a - d*x/3)*sin(c + d*x)*cos(a - d*x/3)**2/8 + x*cos
(a - d*x/3)**3*cos(c + d*x)/8 - 9*sin(a - d*x/3)**3*cos(c + d*x)/(8*d) - 3
*sin(a - d*x/3)**2*sin(c + d*x)*cos(a - d*x/3)/(4*d) - sin(c + d*x)*cos(a
- d*x/3)**3/(8*d), Eq(b, -d/3)), (x*sin(a + d*x/3)**3*sin(c + d*x)/8 + 3*x
*sin(a + d*x/3)**2*cos(a + d*x/3)*cos(c + d*x)/8 - 3*x*sin(a + d*x/3)*sin(
c + d*x)*cos(a + d*x/3)**2/8 - x*cos(a + d*x/3)**3*cos(c + d*x)/8 - 9*sin(
a + d*x/3)**3*cos(c + d*x)/(8*d) + 3*sin(a + d*x/3)**2*sin(c + d*x)*cos(a
+ d*x/3)/(4*d) + sin(c + d*x)*cos(a + d*x/3)**3/(8*d), Eq(b, d/3)), (3*x*s
in(a + d*x)**3*sin(c + d*x)/8 + 3*x*sin(a + d*x)**2*cos(a + d*x)*cos(c + d
*x)/8 + 3*x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)
**3*cos(c + d*x)/8 - 5*sin(a + d*x)**3*cos(c + d*x)/(8*d) - 3*sin(a + d*x)
*cos(a + d*x)**2*cos(c + d*x)/(4*d) + 3*sin(c + d*x)*cos(a + d*x)**3/(8*d)
, Eq(b, d)), (-9*b**3*sin(a + b*x)**2*sin(c + d*x)*cos(a + b*x)/(9*b**4 -
10*b**2*d**2 + d**4) - 6*b**3*sin(c + d*x)*cos(a + b*x)**3/(9*b**4 - 10...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(89) = 178$.

Time $= 0.08$ (sec) , antiderivative size $= 789$, normalized size of antiderivative $= 8.13$

$$\int \sin^3(a+bx)\sin(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="maxima")
```

output
```
-1/16*((3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c) + d^3*sin(c))*cos((3*
b + d)*x + 3*a + 2*c) - (3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c) + d^
3*sin(c))*cos((3*b + d)*x + 3*a) + (3*b^3*sin(c) + b^2*d*sin(c) - 3*b*d^2*
sin(c) - d^3*sin(c))*cos(-(3*b - d)*x - 3*a + 2*c) - (3*b^3*sin(c) + b^2*d
*sin(c) - 3*b*d^2*sin(c) - d^3*sin(c))*cos(-(3*b - d)*x - 3*a) - 3*(9*b^3*
sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*cos((b + d)*x + a + 2
*c) + 3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*cos((b
 + d)*x + a) - 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*sin(c) - d^3*sin(c
))*cos(-(b - d)*x - a + 2*c) + 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*si
n(c) - d^3*sin(c))*cos(-(b - d)*x - a) - (3*b^3*cos(c) - b^2*d*cos(c) - 3*
b*d^2*cos(c) + d^3*cos(c))*sin((3*b + d)*x + 3*a + 2*c) - (3*b^3*cos(c) -
b^2*d*cos(c) - 3*b*d^2*cos(c) + d^3*cos(c))*sin((3*b + d)*x + 3*a) - (3*b^
3*cos(c) + b^2*d*cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*sin(-(3*b - d)*x -
3*a + 2*c) - (3*b^3*cos(c) + b^2*d*cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*s
in(-(3*b - d)*x - 3*a) + 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) +
 d^3*cos(c))*sin((b + d)*x + a + 2*c) + 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) -
 b*d^2*cos(c) + d^3*cos(c))*sin((b + d)*x + a) + 3*(9*b^3*cos(c) + 9*b^2*d
*cos(c) - b*d^2*cos(c) - d^3*cos(c))*sin(-(b - d)*x - a + 2*c) + 3*(9*b^3*
cos(c) + 9*b^2*d*cos(c) - b*d^2*cos(c) - d^3*cos(c))*sin(-(b - d)*x - a))/
(9*b^4*cos(c)^2 + 9*b^4*sin(c)^2 + (cos(c)^2 + sin(c)^2)*d^4 - 10*(b^2*...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \sin^3(a+bx)\sin(c+dx)\,dx = \frac{\sin(3\,bx+dx+3\,a+c)}{8\,(3\,b+d)} - \frac{\sin(3\,bx-dx+3\,a-c)}{8\,(3\,b-d)} - \frac{3\,\sin(bx+dx+a+c)}{8\,(b+d)} + \frac{3\,\sin(bx-dx+a-c)}{8\,(b-d)}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="giac")
```

output
```
1/8*sin(3*b*x + d*x + 3*a + c)/(3*b + d) - 1/8*sin(3*b*x - d*x + 3*a - c)/
(3*b - d) - 3/8*sin(b*x + d*x + a + c)/(b + d) + 3/8*sin(b*x - d*x + a - c
)/(b - d)
```

Mupad [B] (verification not implemented)

Time = 17.85 (sec) , antiderivative size = 494, normalized size of antiderivative = 5.09

$$\int \sin^3(a+bx)\sin(c+dx)\,dx = \mathrm{e}^{a\,3\mathrm{i}-c\,1\mathrm{i}+b\,x\,3\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{-3\,b^3-b^2\,d+3\,b\,d^2+d^3}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}} + \frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-3\,b^3+b^2\,d+3\,b\,d^2-d^3)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}} - \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}} - \frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}}\right) - \mathrm{e}^{a\,3\mathrm{i}+c\,1\mathrm{i}+b\,x\,3\mathrm{i}+d\,x\,1\mathrm{i}}\left(\frac{-3\,b^3+b^2\,d+3\,b\,d^2-d^3}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}} + \frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-3\,b^3-b^2\,d+3\,b\,d^2+d^3)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}} - \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}} - \frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}}\right)$$

input
```
int(sin(a + b*x)^3*sin(c + d*x),x)
```

output
```
exp(a*3i - c*1i + b*x*3i - d*x*1i)*((3*b*d^2 - b^2*d - 3*b^3 + d^3)/(b^4*1
44i + d^4*16i - b^2*d^2*160i) + (exp(- a*6i - b*x*6i)*(3*b*d^2 + b^2*d - 3
*b^3 - d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i) - (exp(- a*2i - b*x*2i)*(
3*b*d^2 - 27*b^2*d - 27*b^3 + 3*d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i)
- (exp(- a*4i - b*x*4i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(b^4*144i +
 d^4*16i - b^2*d^2*160i)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((3*b*d^2 +
 b^2*d - 3*b^3 - d^3)/(b^4*144i + d^4*16i - b^2*d^2*160i) + (exp(- a*6i -
b*x*6i)*(3*b*d^2 - b^2*d - 3*b^3 + d^3))/(b^4*144i + d^4*16i - b^2*d^2*160
i) - (exp(- a*2i - b*x*2i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(b^4*144
i + d^4*16i - b^2*d^2*160i) - (exp(- a*4i - b*x*4i)*(3*b*d^2 - 27*b^2*d -
27*b^3 + 3*d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.49

$$\int \sin^3(a+bx)\sin(c+dx)\,dx$$

$$= \frac{-3\cos(bx+a)\sin(bx+a)^2\sin(dx+c)\,b^3 + 3\cos(bx+a)\sin(bx+a)^2\sin(dx+c)\,b\,d^2 - 6\cos(bx+a}{9b^4 -}$$

input
```
int(sin(b*x+a)^3*sin(d*x+c),x)
```

output
```
( - 3*cos(a + b*x)*sin(a + b*x)**2*sin(c + d*x)*b**3 + 3*cos(a + b*x)*sin(
a + b*x)**2*sin(c + d*x)*b*d**2 - 6*cos(a + b*x)*sin(c + d*x)*b**3 + cos(c
 + d*x)*sin(a + b*x)**3*b**2*d - cos(c + d*x)*sin(a + b*x)**3*d**3 + 6*cos
(c + d*x)*sin(a + b*x)*b**2*d)/(9*b**4 - 10*b**2*d**2 + d**4)
```

3.126 $\int \csc(c+dx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 290

$$\int \csc(c+dx)\sin^3(a+bx)\,dx = \frac{3ie^{-ia-ibx+i(c+dx)}\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2d},\frac{1}{2}\left(3-\frac{b}{d}\right),e^{2i(c+dx)}\right)}{4(b-d)} - \frac{ie^{-3ia-3ibx+i(c+dx)}\text{Hypergeometric2F1}\left(1,-\frac{3b-d}{2d},\frac{3}{2}\left(1-\frac{b}{d}\right),e^{2i(c+dx)}\right)}{4(3b-d)} + \frac{3ie^{ia+ibx+i(c+dx)}\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),e^{2i(c+dx)}\right)}{4(b+d)} - \frac{ie^{3ia+3ibx+i(c+dx)}\text{Hypergeometric2F1}\left(1,\frac{3b+d}{2d},\frac{3(b+d)}{2d},e^{2i(c+dx)}\right)}{4(3b+d)}$$

output
```
3/4*I*exp(-I*a-I*b*x+I*(d*x+c))*hypergeom([1, -1/2*(b-d)/d],[3/2-1/2*b/d],
exp(2*I*(d*x+c)))/(b-d)-1/4*I*exp(-3*I*a-3*I*b*x+I*(d*x+c))*hypergeom([1,
-1/2*(3*b-d)/d],[3/2-3/2*b/d],exp(2*I*(d*x+c)))/(3*b-d)+3/4*I*exp(I*a+I*b*
x+I*(d*x+c))*hypergeom([1, 1/2*(b+d)/d],[3/2+1/2*b/d],exp(2*I*(d*x+c)))/(b
+d)-1/4*I*exp(3*I*a+3*I*b*x+I*(d*x+c))*hypergeom([1, 1/2*(3*b+d)/d],[3/2*(
b+d)/d],exp(2*I*(d*x+c)))/(3*b+d)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.90

$$\int \csc(c+dx)\sin^3(a+bx)\,dx$$
$$= -\frac{1}{4}ie^{-3ia+ic}\left(\frac{e^{-3ibx+idx}\,\text{Hypergeometric2F1}\left(1,\frac{-3b+d}{2d},\frac{3}{2}-\frac{3b}{2d},e^{2i(c+dx)}\right)}{3b-d}\right.$$
$$-\frac{3e^{i(2a-bx+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{-b+d}{2d},\frac{3}{2}-\frac{b}{2d},e^{2i(c+dx)}\right)}{b-d}$$
$$-\frac{3e^{i(4a+(b+d)x)}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b+d}$$
$$\left.+\frac{e^{i(6a+3bx+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{3b+d}{2d},\frac{3(b+d)}{2d},e^{2i(c+dx)}\right)}{3b+d}\right)$$

input
```
Integrate[Csc[c + d*x]*Sin[a + b*x]^3,x]
```

output
```
(-1/4*I)*E^((-3*I)*a + I*c)*((E^((-3*I)*b*x + I*d*x)*Hypergeometric2F1[1,
(-3*b + d)/(2*d), 3/2 - (3*b)/(2*d), E^((2*I)*(c + d*x))])/(3*b - d) - (3*
E^(I*(2*a - b*x + d*x))*Hypergeometric2F1[1, (-b + d)/(2*d), 3/2 - b/(2*d)
, E^((2*I)*(c + d*x))])/(b - d) - (3*E^(I*(4*a + (b + d)*x))*Hypergeometri
c2F1[1, (b + d)/(2*d), (3 + b/d)/2, E^((2*I)*(c + d*x))])/(b + d) + (E^(I*
(6*a + 3*b*x + d*x))*Hypergeometric2F1[1, (3*b + d)/(2*d), (3*(b + d))/(2*
d), E^((2*I)*(c + d*x))])/(3*b + d))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc(c+dx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \sin^3(a+bx)\csc(c+dx)dx$$

input
```
Int[Csc[c + d*x]*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \csc (dx + c) \sin (bx + a)^3 \, dx$$

input
```
int(csc(d*x+c)*sin(b*x+a)^3,x)
```

output
```
int(csc(d*x+c)*sin(b*x+a)^3,x)
```

Fricas [F]

$$\int \csc(c + dx) \sin^3(a + bx) \, dx = \int \csc (dx + c) \sin (bx + a)^3 \, dx$$

input
```
integrate(csc(d*x+c)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*csc(d*x + c)*sin(b*x + a), x)
```

Sympy [F]

$$\int \csc(c+dx)\sin^3(a+bx)\,dx = \int \sin^3{(a+bx)}\csc{(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)*sin(b*x+a)**3,x)
```

output
```
Integral(sin(a + b*x)**3*csc(c + d*x), x)
```

Maxima [F]

$$\int \csc(c+dx)\sin^3(a+bx)\,dx = \int \csc(dx+c)\sin(bx+a)^3\,dx$$

input
```
integrate(csc(d*x+c)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(csc(d*x + c)*sin(b*x + a)^3, x)
```

Giac [F]

$$\int \csc(c+dx)\sin^3(a+bx)\,dx = \int \csc(dx+c)\sin(bx+a)^3\,dx$$

input
```
integrate(csc(d*x+c)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(c+dx)\sin^3(a+bx)\,dx = \int \frac{\sin(a+b\,x)^3}{\sin(c+d\,x)}\,dx$$

input
```
int(sin(a + b*x)^3/sin(c + d*x),x)
```

output
```
int(sin(a + b*x)^3/sin(c + d*x), x)
```

Reduce [F]

$$\int \csc(c+dx)\sin^3(a+bx)\,dx = \int \csc(dx+c)\sin(bx+a)^3\,dx$$

input
```
int(csc(d*x+c)*sin(b*x+a)^3,x)
```

output
```
int(csc(c + d*x)*sin(a + b*x)**3,x)
```

3.127 $\int \csc^2(c+dx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 273

$$\begin{aligned}\int \csc^2(c+dx)\sin^3(a+bx)\,dx &= -\frac{e^{-3ia-3ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{3b}{2d},2-\frac{3b}{2d},e^{2i(c+dx)}\right)}{2(3b-2d)} \\ &\quad+\frac{3e^{-ia-ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{b}{2d},2-\frac{b}{2d},e^{2i(c+dx)}\right)}{2(b-2d)} \\ &\quad+\frac{3e^{ia+ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{b}{2d},2+\frac{b}{2d},e^{2i(c+dx)}\right)}{2(b+2d)} \\ &\quad-\frac{e^{3ia+3ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{3b}{2d},2+\frac{3b}{2d},e^{2i(c+dx)}\right)}{2(3b+2d)}\end{aligned}$$

output

```
-1/2*exp(-3*I*a-3*I*b*x+2*I*(d*x+c))*hypergeom([2, 1-3/2*b/d],[2-3/2*b/d],
exp(2*I*(d*x+c)))/(3*b-2*d)+3*exp(-I*a-I*b*x+2*I*(d*x+c))*hypergeom([2, 1-
1/2*b/d],[2-1/2*b/d],exp(2*I*(d*x+c)))/(2*b-4*d)+3*exp(I*a+I*b*x+2*I*(d*x+
c))*hypergeom([2, 1+1/2*b/d],[2+1/2*b/d],exp(2*I*(d*x+c)))/(2*b+4*d)-exp(3
*I*a+3*I*b*x+2*I*(d*x+c))*hypergeom([2, 1+3/2*b/d],[2+3/2*b/d],exp(2*I*(d*
x+c)))/(6*b+4*d)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 570 vs. $2(273) = 546$.

Time = 4.03 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.09

$$\int \csc^2(c+dx)\sin^3(a+bx)\,dx$$
$$= \frac{\frac{e^{-i(3a-2c+3bx)}\left(3be^{2idx}\,\text{Hypergeometric2F1}\left(1,1-\frac{3b}{2d},2-\frac{3b}{2d},e^{2i(c+dx)}\right)+(-3b+2d)\,\text{Hypergeometric2F1}\left(1,-\frac{3b}{2d},1-\frac{3b}{2d},e^{2i(c+dx)}\right)\right)}{(3b-2d)(-1+e^{2ic})}+\frac{e^{-i(a-}}{}}{}$$

input
```
Integrate[Csc[c + d*x]^2*Sin[a + b*x]^3,x]
```

output
```
((3*b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 - (3*b)/(2*d), 2 - (3*b)/(2*d),
 E^((2*I)*(c + d*x))] + (-3*b + 2*d)*Hypergeometric2F1[1, (-3*b)/(2*d), 1
- (3*b)/(2*d), E^((2*I)*(c + d*x))])/((3*b - 2*d)*E^(I*(3*a - 2*c + 3*b*x)
)*(-1 + E^((2*I)*c))) + (-3*b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 - b/(2*
d), 2 - b/(2*d), E^((2*I)*(c + d*x))] + 3*(b - 2*d)*Hypergeometric2F1[1, -
1/2*b/d, 1 - b/(2*d), E^((2*I)*(c + d*x))])/((b - 2*d)*E^(I*(a - 2*c + b*x
))*(-1 + E^((2*I)*c))) + (3*E^(I*(a + 2*c + b*x))*(b*E^((2*I)*d*x)*Hyperge
ometric2F1[1, 1 + b/(2*d), 2 + b/(2*d), E^((2*I)*(c + d*x))] - (b + 2*d)*H
ypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x))]))/((b + 2*d
)*(-1 + E^((2*I)*c))) + (E^(I*(3*a + 2*c + 3*b*x))*(-3*b*E^((2*I)*d*x)*Hyp
ergeometric2F1[1, 1 + (3*b)/(2*d), 2 + (3*b)/(2*d), E^((2*I)*(c + d*x))] +
 (3*b + 2*d)*Hypergeometric2F1[1, (3*b)/(2*d), 1 + (3*b)/(2*d), E^((2*I)*(
c + d*x))]))/((3*b + 2*d)*(-1 + E^((2*I)*c))) + 3*Cos[b*x]*Csc[c]*Csc[c +
d*x]*Sin[a]*Sin[d*x] - Cos[3*b*x]*Csc[c]*Csc[c + d*x]*Sin[3*a]*Sin[d*x] +
3*Cos[a]*Csc[c]*Csc[c + d*x]*Sin[b*x]*Sin[d*x] - Cos[3*a]*Csc[c]*Csc[c + d
*x]*Sin[3*b*x]*Sin[d*x])/(4*d)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^2(c+dx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin^3(a+bx)\csc^2(c+dx)dx$$

input
```
Int[Csc[c + d*x]^2*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \csc(dx+c)^2 \sin(bx+a)^3\,dx$$

input
```
int(csc(d*x+c)^2*sin(b*x+a)^3,x)
```

output
```
int(csc(d*x+c)^2*sin(b*x+a)^3,x)
```

Fricas [F]

$$\int \csc^2(c+dx)\sin^3(a+bx)\,dx = \int \csc(dx+c)^2 \sin(bx+a)^3\,dx$$

input
```
integrate(csc(d*x+c)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*csc(d*x + c)^2*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(c+dx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**2*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^2(c+dx)\sin^3(a+bx)\,dx = \int \csc(dx+c)^2 \sin(bx+a)^3\,dx$$

input
```
integrate(csc(d*x+c)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*((cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos(
(3*b + 2*d)*x + 3*a + 2*c) - (3*cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a) - c
os(6*b*x + 6*a)*cos(3*b*x + 3*a) + 3*cos(4*b*x + 4*a)*cos(3*b*x + 3*a) + 4
*(d*cos((3*b + 2*d)*x + 3*a + 2*c)^2 - 2*d*cos((3*b + 2*d)*x + 3*a + 2*c)*
cos(3*b*x + 3*a) + d*cos(3*b*x + 3*a)^2 + d*sin((3*b + 2*d)*x + 3*a + 2*c)
^2 - 2*d*sin((3*b + 2*d)*x + 3*a + 2*c)*sin(3*b*x + 3*a) + d*sin(3*b*x + 3
*a)^2)*integrate(-3/8*(b*cos(3*b*x + 3*a)*sin(6*b*x + 6*a) - b*cos(3*b*x +
 3*a)*sin(4*b*x + 4*a) - b*cos(6*b*x + 6*a)*sin(3*b*x + 3*a) + b*cos(4*b*x
 + 4*a)*sin(3*b*x + 3*a) - b*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + (b*sin(6*
b*x + 6*a) - b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*cos((3*b + d)*x + 3*
a + c) - (b*cos(6*b*x + 6*a) - b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) + b
)*sin((3*b + d)*x + 3*a + c) + (b*cos(2*b*x + 2*a) - b)*sin(3*b*x + 3*a))/
(d*cos((3*b + d)*x + 3*a + c)^2 + 2*d*cos((3*b + d)*x + 3*a + c)*cos(3*b*x
 + 3*a) + d*cos(3*b*x + 3*a)^2 + d*sin((3*b + d)*x + 3*a + c)^2 + 2*d*sin(
(3*b + d)*x + 3*a + c)*sin(3*b*x + 3*a) + d*sin(3*b*x + 3*a)^2), x) - 4*(d
*cos((3*b + 2*d)*x + 3*a + 2*c)^2 - 2*d*cos((3*b + 2*d)*x + 3*a + 2*c)*cos
(3*b*x + 3*a) + d*cos(3*b*x + 3*a)^2 + d*sin((3*b + 2*d)*x + 3*a + 2*c)^2
- 2*d*sin((3*b + 2*d)*x + 3*a + 2*c)*sin(3*b*x + 3*a) + d*sin(3*b*x + 3*a)
^2)*integrate(3/8*(b*cos(3*b*x + 3*a)*sin(6*b*x + 6*a) - b*cos(3*b*x + 3*a
)*sin(4*b*x + 4*a) - b*cos(6*b*x + 6*a)*sin(3*b*x + 3*a) + b*cos(4*b*x ...
```

Giac [F]

$$\int \csc^2(c+dx)\sin^3(a+bx)\,dx=\int \csc\left(dx+c\right)^2\sin\left(bx+a\right)^3\,dx$$

input
```
integrate(csc(d*x+c)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^2*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(c+dx)\sin^3(a+bx)\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\sin\left(c+d\,x\right)^2}\,dx$$

input
```
int(sin(a + b*x)^3/sin(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)^3/sin(c + d*x)^2, x)
```

Reduce [F]

$$\int \csc^2(c+dx)\sin^3(a+bx)\,dx = \int \csc\left(dx+c\right)^2\sin\left(bx+a\right)^3dx$$

input
```
int(csc(d*x+c)^2*sin(b*x+a)^3,x)
```

output
```
int(csc(c + d*x)**2*sin(a + b*x)**3,x)
```

3.128 $\int \csc^3(c+dx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 283

$$\begin{aligned}
&\int \csc^3(c+dx)\sin^3(a+bx)\,dx\\
&=\frac{ie^{-3ia-3ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3}{2}\left(1-\frac{b}{d}\right),\frac{1}{2}\left(5-\frac{3b}{d}\right),e^{2i(c+dx)}\right)}{3(b-d)}\\
&\quad-\frac{3ie^{-ia-ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{b}{d}\right),\frac{1}{2}\left(5-\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b-3d}\\
&\quad-\frac{3ie^{ia+ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{b}{d}\right),\frac{1}{2}\left(5+\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b+3d}\\
&\quad+\frac{ie^{3ia+3ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3(b+d)}{2d},\frac{1}{2}\left(5+\frac{3b}{d}\right),e^{2i(c+dx)}\right)}{3(b+d)}
\end{aligned}$$

output
```
1/3*I*exp(-3*I*a-3*I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2-3/2*b/d],[5/2-3/2*
b/d],exp(2*I*(d*x+c)))/(b-d)-3*I*exp(-I*a-I*b*x+3*I*(d*x+c))*hypergeom([3,
 3/2-1/2*b/d],[5/2-1/2*b/d],exp(2*I*(d*x+c)))/(b-3*d)-3*I*exp(I*a+I*b*x+3*
I*(d*x+c))*hypergeom([3, 3/2+1/2*b/d],[5/2+1/2*b/d],exp(2*I*(d*x+c)))/(b+3
*d)+1/3*I*exp(3*I*a+3*I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2*(b+d)/d],[5/2+3
/2*b/d],exp(2*I*(d*x+c)))/(b+d)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.10

$$\int \csc^3(c+dx)\sin^3(a+bx)\,dx$$
$$= \frac{i\Big(4(3b+d)e^{-i(3a-c+3bx-dx)}\,\text{Hypergeometric2F1}\left(1,\frac{-3b+d}{2d},\frac{3}{2}-\frac{3b}{2d},e^{2i(c+dx)}\right)-12(b+d)e^{-i(a-c+(b-d)x)}\,\text{Hy}}{}$$

input
```
Integrate[Csc[c + d*x]^3*Sin[a + b*x]^3,x]
```

output
```
((I/32)*((4*(3*b + d)*Hypergeometric2F1[1, (-3*b + d)/(2*d), 3/2 - (3*b)/(
2*d), E^((2*I)*(c + d*x))])/E^(I*(3*a - c + 3*b*x - d*x)) - (12*(b + d)*Hy
pergeometric2F1[1, (-b + d)/(2*d), 3/2 - b/(2*d), E^((2*I)*(c + d*x))])/E^
(I*(a - c + (b - d)*x)) - 12*(b - d)*E^(I*(a + c + (b + d)*x))*Hypergeomet
ric2F1[1, (b + d)/(2*d), (3 + b/d)/2, E^((2*I)*(c + d*x))] + 4*(3*b - d)*E
^(I*(3*a + c + (3*b + d)*x))*Hypergeometric2F1[1, (3*b + d)/(2*d), (3*(b +
 d))/(2*d), E^((2*I)*(c + d*x))] - (8*I)*Csc[c + d*x]^2*Sin[a + b*x]^2*((3
*b - d)*Sin[a - c + b*x - d*x] - (3*b + d)*Sin[a + c + (b + d)*x])))/d^2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^3(c+dx)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \sin^3(a+bx)\csc^3(c+dx)dx$$

input
```
Int[Csc[c + d*x]^3*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \csc (dx+c)^3 \sin (bx+a)^3 \, dx$$

input
```
int(csc(d*x+c)^3*sin(b*x+a)^3,x)
```

output
```
int(csc(d*x+c)^3*sin(b*x+a)^3,x)
```

Fricas [F]

$$\int \csc^3(c+dx)\sin^3(a+bx)\,dx=\int \csc (dx+c)^3 \sin (bx+a)^3 \,dx$$

input
```
integrate(csc(d*x+c)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*csc(d*x + c)^3*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(c+dx)\sin^3(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(csc(d*x+c)**3*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^3(c+dx)\sin^3(a+bx)\,dx = \int \csc(dx+c)^3 \sin(bx+a)^3 \,dx$$

input
```
integrate(csc(d*x+c)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*((3*b - d)*cos(3*b*x + 3*a)*sin((6*b + d)*x + 6*a + c) - 3*(b - d)*cos
(3*b*x + 3*a)*sin((4*b + d)*x + 4*a + c) - 3*(b + d)*cos(3*b*x + 3*a)*sin(
(2*b + d)*x + 2*a + c) - (3*b - d)*cos((6*b + d)*x + 6*a + c)*sin(3*b*x +
3*a) + 3*(b - d)*cos((4*b + d)*x + 4*a + c)*sin(3*b*x + 3*a) + 3*(b + d)*c
os((2*b + d)*x + 2*a + c)*sin(3*b*x + 3*a) + (3*b - d)*cos(3*d*x + 3*c)*si
n(3*b*x + 3*a) - (3*b + d)*cos(d*x + c)*sin(3*b*x + 3*a) - (3*b - d)*cos(3
*b*x + 3*a)*sin(3*d*x + 3*c) + (3*b + d)*cos(3*b*x + 3*a)*sin(d*x + c) + 3
*(2*(b + d)*sin((3*b + 2*d)*x + 3*a + 2*c) - (b + d)*sin(3*b*x + 3*a))*cos
((4*b + 3*d)*x + 4*a + 3*c) + ((3*b - d)*sin((6*b + d)*x + 6*a + c) + 3*(b
 + d)*sin((4*b + 3*d)*x + 4*a + 3*c) - 3*(b - d)*sin((4*b + d)*x + 4*a + c
) + 3*(b - d)*sin((2*b + 3*d)*x + 2*a + 3*c) - (3*b + d)*sin(3*(2*b + d)*x
 + 6*a + 3*c) - 3*(b + d)*sin((2*b + d)*x + 2*a + c) - (3*b - d)*sin(3*d*x
 + 3*c) + (3*b + d)*sin(d*x + c))*cos((3*b + 4*d)*x + 3*a + 4*c) - 2*((3*b
 - d)*sin((6*b + d)*x + 6*a + c) - 3*(b - d)*sin((4*b + d)*x + 4*a + c) -
3*(b + d)*sin((2*b + d)*x + 2*a + c) - (3*b - d)*sin(3*d*x + 3*c) + (3*b +
 d)*sin(d*x + c))*cos((3*b + 2*d)*x + 3*a + 2*c) + 3*(2*(b - d)*sin((3*b +
 2*d)*x + 3*a + 2*c) - (b - d)*sin(3*b*x + 3*a))*cos((2*b + 3*d)*x + 2*a +
 3*c) - (2*(3*b + d)*sin((3*b + 2*d)*x + 3*a + 2*c) - (3*b + d)*sin(3*b*x
+ 3*a))*cos(3*(2*b + d)*x + 6*a + 3*c) - 8*(d^2*cos((3*b + 4*d)*x + 3*a +
4*c)^2 + 4*d^2*cos((3*b + 2*d)*x + 3*a + 2*c)^2 - 4*d^2*cos((3*b + 2*d)...
```

Giac [F]

$$\int \csc^3(c+dx)\sin^3(a+bx)\,dx = \int \csc(dx+c)^3 \sin(bx+a)^3 \,dx$$

input
```
integrate(csc(d*x+c)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^3*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+dx)\sin^3(a+bx)\,dx = \int \frac{\sin(a+b\,x)^3}{\sin(c+d\,x)^3}\,dx$$

input
```
int(sin(a + b*x)^3/sin(c + d*x)^3,x)
```

output
```
int(sin(a + b*x)^3/sin(c + d*x)^3, x)
```

Reduce [F]

$$\int \csc^3(c+dx)\sin^3(a+bx)\,dx = \int \csc(dx+c)^3\sin(bx+a)^3\,dx$$

input
```
int(csc(d*x+c)^3*sin(b*x+a)^3,x)
```

output
```
int(csc(d*x+c)^3*sin(b*x+a)^3,x)
```

3.129 $\int \csc^4(c+dx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 265

$$\int \csc^4(c+dx)\sin^3(a+bx)\,dx = \frac{2e^{-3ia-3ibx+4i(c+dx)}\text{Hypergeometric2F1}\left(4,2-\frac{3b}{2d},3-\frac{3b}{2d},e^{2i(c+dx)}\right)}{3b-4d} - \frac{6e^{-ia-ibx+4i(c+dx)}\text{Hypergeometric2F1}\left(4,2-\frac{b}{2d},3-\frac{b}{2d},e^{2i(c+dx)}\right)}{b-4d} - \frac{6e^{ia+ibx+4i(c+dx)}\text{Hypergeometric2F1}\left(4,2+\frac{b}{2d},3+\frac{b}{2d},e^{2i(c+dx)}\right)}{b+4d} + \frac{2e^{3ia+3ibx+4i(c+dx)}\text{Hypergeometric2F1}\left(4,2+\frac{3b}{2d},3+\frac{3b}{2d},e^{2i(c+dx)}\right)}{3b+4d}$$

output
```
2*exp(-3*I*a-3*I*b*x+4*I*(d*x+c))*hypergeom([4, 2-3/2*b/d],[3-3/2*b/d],exp
(2*I*(d*x+c)))/(3*b-4*d)-6*exp(-I*a-I*b*x+4*I*(d*x+c))*hypergeom([4, 2-1/2
*b/d],[3-1/2*b/d],exp(2*I*(d*x+c)))/(b-4*d)-6*exp(I*a+I*b*x+4*I*(d*x+c))*h
ypergeom([4, 2+1/2*b/d],[3+1/2*b/d],exp(2*I*(d*x+c)))/(b+4*d)+2*exp(3*I*a+
3*I*b*x+4*I*(d*x+c))*hypergeom([4, 2+3/2*b/d],[3+3/2*b/d],exp(2*I*(d*x+c))
)/(3*b+4*d)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 913 vs. $2(265) = 530$.

Time = 5.99 (sec) , antiderivative size = 913, normalized size of antiderivative = 3.45

$$\int \csc^4(c + dx) \sin^3(a + bx) \, dx = \text{Too large to display}$$

input
```
Integrate[Csc[c + d*x]^4*Sin[a + b*x]^3,x]
```

output
```
((4*(3*b + 2*d)*(3*b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 - (3*b)/(2*d), 2
 - (3*b)/(2*d), E^((2*I)*(c + d*x))] + (-3*b + 2*d)*Hypergeometric2F1[1, (
-3*b)/(2*d), 1 - (3*b)/(2*d), E^((2*I)*(c + d*x))]))/E^(I*(3*a - 2*c + 3*b
*x)) - (12*(b + 2*d)*(b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 - b/(2*d), 2
- b/(2*d), E^((2*I)*(c + d*x))] - (b - 2*d)*Hypergeometric2F1[1, -1/2*b/d,
 1 - b/(2*d), E^((2*I)*(c + d*x))]))/E^(I*(a - 2*c + b*x)) + 12*(b - 2*d)*
E^(I*(a + 2*c + b*x))*(b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 + b/(2*d), 2
 + b/(2*d), E^((2*I)*(c + d*x))] - (b + 2*d)*Hypergeometric2F1[1, b/(2*d),
 1 + b/(2*d), E^((2*I)*(c + d*x))]) - 4*(3*b - 2*d)*E^(I*(3*a + 2*c + 3*b*
x))*(3*b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 + (3*b)/(2*d), 2 + (3*b)/(2*
d), E^((2*I)*(c + d*x))] - (3*b + 2*d)*Hypergeometric2F1[1, (3*b)/(2*d), 1
 + (3*b)/(2*d), E^((2*I)*(c + d*x))]) + (6*I)*(-1 + E^((2*I)*c))*Csc[c + d
*x]*(Cos[a] - I*Sin[a])*(Cos[b*x] - I*Sin[b*x])*(d*((-I)*b + 2*d*Cot[c])*C
sc[c + d*x] + (b^2 - 4*d^2)*Csc[c]*Sin[d*x] - 2*d^2*Csc[c]*Csc[c + d*x]^2*
Sin[d*x]) + 2*(-1 + E^((2*I)*c))*Csc[c + d*x]*(I*Cos[3*a] + Sin[3*a])*(Cos
[3*b*x] - I*Sin[3*b*x])*(d*((3*I)*b - 2*d*Cot[c])*Csc[c + d*x] + (-9*b^2 +
 4*d^2)*Csc[c]*Sin[d*x] + 2*d^2*Csc[c]*Csc[c + d*x]^2*Sin[d*x]) + (I/2)*(-
1 + E^((2*I)*c))*Csc[c]*Csc[c + d*x]^3*(Cos[3*a] + I*Sin[3*a])*(Cos[3*b*x]
 + I*Sin[3*b*x])*((6*I)*b*d*Cos[d*x] - (6*I)*b*d*Cos[2*c + d*x] + 18*b^2*S
in[d*x] - 12*d^2*Sin[d*x] + 9*b^2*Sin[2*c + d*x] - 9*b^2*Sin[2*c + 3*d*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^4(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sin^3(a+bx)\csc^4(c+dx)dx$$

input
```
Int[Csc[c + d*x]^4*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \csc(dx+c)^4\sin(bx+a)^3\,dx$$

input
```
int(csc(d*x+c)^4*sin(b*x+a)^3,x)
```

output
```
int(csc(d*x+c)^4*sin(b*x+a)^3,x)
```

Fricas [F]

$$\int \csc^4(c+dx)\sin^3(a+bx)\,dx = \int \csc(dx+c)^4 \sin(bx+a)^3 \,dx$$

input
```
integrate(csc(d*x+c)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*csc(d*x + c)^4*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(c+dx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**4*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^4(c+dx)\sin^3(a+bx)\,dx = \int \csc(dx+c)^4 \sin(bx+a)^3 \,dx$$

input
```
integrate(csc(d*x+c)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/24*(6*(b^2 - b*d - 6*d^2)*cos(2*(b + d)*x + 2*a + 2*c)*cos(3*b*x + 3*a)
 - (9*b^2 - 4*d^2)*cos(6*b*x + 6*a)*cos(3*b*x + 3*a) + 3*(b^2 - 4*d^2)*cos
(4*b*x + 4*a)*cos(3*b*x + 3*a) + 3*(3*b^2 - 2*b*d)*cos(3*b*x + 3*a)*cos(4*
d*x + 4*c) - 6*(3*b^2 - b*d - 2*d^2)*cos(3*b*x + 3*a)*cos(2*d*x + 2*c) + 6
*(b^2 - b*d - 6*d^2)*sin(2*(b + d)*x + 2*a + 2*c)*sin(3*b*x + 3*a) - (9*b^
2 - 4*d^2)*sin(6*b*x + 6*a)*sin(3*b*x + 3*a) + 3*(b^2 - 4*d^2)*sin(4*b*x +
 4*a)*sin(3*b*x + 3*a) - 3*(b^2 - 4*d^2)*sin(3*b*x + 3*a)*sin(2*b*x + 2*a)
 + 3*(3*b^2 - 2*b*d)*sin(3*b*x + 3*a)*sin(4*d*x + 4*c) - 6*(3*b^2 - b*d -
2*d^2)*sin(3*b*x + 3*a)*sin(2*d*x + 2*c) + 3*(9*b^2 - 4*d^2 + 6*(3*b^2 + b
*d - 2*d^2)*cos(2*(3*b + d)*x + 6*a + 2*c) - 6*(b^2 + b*d - 6*d^2)*cos(2*(
2*b + d)*x + 4*a + 2*c) - 3*(b^2 - 2*b*d)*cos(2*(b + 2*d)*x + 2*a + 4*c) +
 6*(b^2 - b*d - 6*d^2)*cos(2*(b + d)*x + 2*a + 2*c) - (9*b^2 - 4*d^2)*cos(
6*b*x + 6*a) + 3*(b^2 - 4*d^2)*cos(4*b*x + 4*a) - 3*(b^2 - 4*d^2)*cos(2*b*
x + 2*a) + 3*(3*b^2 - 2*b*d)*cos(4*d*x + 4*c) - 6*(3*b^2 - b*d - 2*d^2)*co
s(2*d*x + 2*c))*cos((3*b + 4*d)*x + 3*a + 4*c) - 3*(3*(3*b^2 + 2*b*d)*cos(
(3*b + 4*d)*x + 3*a + 4*c) - 3*(3*b^2 + 2*b*d)*cos((3*b + 2*d)*x + 3*a + 2
*c) + (3*b^2 + 2*b*d)*cos(3*b*x + 3*a))*cos(2*(3*b + 2*d)*x + 6*a + 4*c) -
 3*(9*b^2 - 4*d^2 + 6*(b^2 - b*d - 6*d^2)*cos(2*(b + d)*x + 2*a + 2*c) - (
9*b^2 - 4*d^2)*cos(6*b*x + 6*a) + 3*(b^2 - 4*d^2)*cos(4*b*x + 4*a) - 3*(b^
2 - 4*d^2)*cos(2*b*x + 2*a) + 3*(3*b^2 - 2*b*d)*cos(4*d*x + 4*c) - 6*(3...
```

Giac [F]

$$\int \csc^4(c+dx)\sin^3(a+bx)\,dx = \int \csc(dx+c)^4\sin(bx+a)^3\,dx$$

input
```
integrate(csc(d*x+c)^4*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^4*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+dx)\sin^3(a+bx)\,dx = \int \frac{\sin(a+b\,x)^3}{\sin(c+d\,x)^4}\,dx$$

input
```
int(sin(a + b*x)^3/sin(c + d*x)^4,x)
```

output
```
int(sin(a + b*x)^3/sin(c + d*x)^4, x)
```

Reduce [F]

$$\int \csc^4(c+dx)\sin^3(a+bx)\,dx = \int \csc(dx+c)^4\sin(bx+a)^3\,dx$$

input
```
int(csc(d*x+c)^4*sin(b*x+a)^3,x)
```

output
```
int(csc(d*x+c)^4*sin(b*x+a)^3,x)
```

3.130 $\int \csc(a+bx)\csc(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \csc(a+bx)\csc(c+dx)\,dx = \text{Int}(\csc(a+bx)\csc(c+dx),x)$$

output
```
Defer(Int)(csc(b*x+a)*csc(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 14.84 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \csc(a+bx)\csc(c+dx)\,dx = \int \csc(a+bx)\csc(c+dx)\,dx$$

input
```
Integrate[Csc[a + b*x]*Csc[c + d*x],x]
```

output
```
Integrate[Csc[a + b*x]*Csc[c + d*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc(a+bx)\csc(c+dx)dx$$

input
```
Int[Csc[a + b*x]*Csc[c + d*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc(bx+a)\csc(dx+c)\,dx$$

input
```
int(csc(b*x+a)*csc(d*x+c),x)
```

output
```
int(csc(b*x+a)*csc(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \csc(a+bx)\csc(c+dx)\,dx = \int \csc\left(bx+a\right)\csc\left(dx+c\right)\,dx$$

input
```
integrate(csc(b*x+a)*csc(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*csc(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \csc(a+bx)\csc(c+dx)\,dx = \int \csc\left(a+bx\right)\csc\left(c+dx\right)dx$$

input
```
integrate(csc(b*x+a)*csc(d*x+c),x)
```

output
```
Integral(csc(a + b*x)*csc(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \csc(a+bx)\csc(c+dx)\,dx=\int \csc(bx+a)\csc(dx+c)\,dx$$

input
```
integrate(csc(b*x+a)*csc(d*x+c),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*csc(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \csc(a+bx)\csc(c+dx)\,dx=\int \csc(bx+a)\csc(dx+c)\,dx$$

input
```
integrate(csc(b*x+a)*csc(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*csc(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 16.98 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \csc(a+bx)\csc(c+dx)\,dx=\int \frac{1}{\sin(a+b\,x)\,\sin(c+d\,x)}\,dx$$

input
```
int(1/(sin(a + b*x)*sin(c + d*x)),x)
```

output
```
int(1/(sin(a + b*x)*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \csc(a+bx)\csc(c+dx)\,dx = \int \csc\left(bx+a\right)\csc\left(dx+c\right)dx$$

input
```
int(csc(b*x+a)*csc(d*x+c),x)
```

output
```
int(csc(a + b*x)*csc(c + d*x),x)
```

3.131 $\int \csc(a+bx)\csc^2(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \csc(a+bx)\csc^2(c+dx)\,dx = \text{Int}\big(\csc(a+bx)\csc^2(c+dx),x\big)$$

output
```
Defer(Int)(csc(b*x+a)*csc(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 14.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\csc^2(c+dx)\,dx = \int \csc(a+bx)\csc^2(c+dx)\,dx$$

input
```
Integrate[Csc[a + b*x]*Csc[c + d*x]^2,x]
```

output
```
Integrate[Csc[a + b*x]*Csc[c + d*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc^2(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \csc(a+bx)\csc^2(c+dx)dx$$

input
```
Int[Csc[a + b*x]*Csc[c + d*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc\left(bx+a\right)\csc\left(dx+c\right)^{2}dx$$

input
```
int(csc(b*x+a)*csc(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)*csc(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\csc^2(c+dx)\,dx=\int \csc\left(bx+a\right)\csc\left(dx+c\right)^2\,dx$$

input
```
integrate(csc(b*x+a)*csc(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*csc(d*x + c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\csc^2(c+dx)\,dx=\int \csc\left(a+bx\right)\csc^2\left(c+dx\right)dx$$

input
```
integrate(csc(b*x+a)*csc(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)*csc(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 2521, normalized size of antiderivative = 168.07

$$\int \csc(a+bx)\csc^2(c+dx)\,dx = \int \csc(bx+a)\csc(dx+c)^2\,dx$$

input
```
integrate(csc(b*x+a)*csc(d*x+c)^2,x, algorithm="maxima")
```

output
```
(4*cos(2*(b + d)*x + 2*a + 2*c)*cos(b*x + a) - 4*cos(2*b*x + 2*a)*cos(b*x
+ a) - 4*cos(b*x + a)*cos(2*d*x + 2*c) - (d*cos(2*(b + d)*x + 2*a + 2*c)^2
 + d*cos(2*b*x + 2*a)^2 + d*cos(2*d*x + 2*c)^2 + d*sin(2*(b + d)*x + 2*a +
 2*c)^2 + d*sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a)*sin(2*d*x + 2*c) + d
*sin(2*d*x + 2*c)^2 - 2*(d*cos(2*b*x + 2*a) + d*cos(2*d*x + 2*c) - d)*cos(
2*(b + d)*x + 2*a + 2*c) - 2*d*cos(2*b*x + 2*a) + 2*(d*cos(2*b*x + 2*a) -
d)*cos(2*d*x + 2*c) - 2*(d*sin(2*b*x + 2*a) + d*sin(2*d*x + 2*c))*sin(2*(b
 + d)*x + 2*a + 2*c) + d)*integrate(-2*(2*b*cos(3*b*x + 3*a)*sin(2*b*x + 2
*a) + 2*b*cos(b*x + a)*sin(2*b*x + 2*a) - 2*b*cos(2*b*x + 2*a)*sin(b*x + a
) + (b*sin(3*b*x + 3*a) + b*sin(b*x + a))*cos((4*b + d)*x + 4*a + c) - 2*(
b*sin(3*b*x + 3*a) + b*sin(b*x + a))*cos((2*b + d)*x + 2*a + c) + (b*sin(3
*b*x + 3*a) + b*sin(b*x + a))*cos(4*b*x + 4*a) + (b*sin(3*b*x + 3*a) + b*s
in(b*x + a))*cos(d*x + c) - (b*cos(3*b*x + 3*a) + b*cos(b*x + a))*sin((4*b
 + d)*x + 4*a + c) + 2*(b*cos(3*b*x + 3*a) + b*cos(b*x + a))*sin((2*b + d)
*x + 2*a + c) - (b*cos(3*b*x + 3*a) + b*cos(b*x + a))*sin(4*b*x + 4*a) - (
2*b*cos(2*b*x + 2*a) - b)*sin(3*b*x + 3*a) + b*sin(b*x + a) - (b*cos(3*b*x
 + 3*a) + b*cos(b*x + a))*sin(d*x + c))/(d*cos((4*b + d)*x + 4*a + c)^2 +
4*d*cos((2*b + d)*x + 2*a + c)^2 + d*cos(4*b*x + 4*a)^2 + 4*d*cos(2*b*x +
2*a)^2 + d*cos(d*x + c)^2 + d*sin((4*b + d)*x + 4*a + c)^2 + 4*d*sin((2*b
+ d)*x + 2*a + c)^2 + d*sin(4*b*x + 4*a)^2 - 4*d*sin(4*b*x + 4*a)*sin(2...
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\csc^2(c+dx)\,dx = \int \csc(bx+a)\csc(dx+c)^2\,dx$$

input
```
integrate(csc(b*x+a)*csc(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*csc(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 16.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \csc(a+bx)\csc^2(c+dx)\,dx = \int \frac{1}{\sin(a+b\,x)\,\sin(c+d\,x)^2}\,dx$$

input
```
int(1/(sin(a + b*x)*sin(c + d*x)^2),x)
```

output
```
int(1/(sin(a + b*x)*sin(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\csc^2(c+dx)\,dx = \int \csc(bx+a)\csc(dx+c)^2\,dx$$

input
```
int(csc(b*x+a)*csc(d*x+c)^2,x)
```

output
```
int(csc(a + b*x)*csc(c + d*x)**2,x)
```

3.132 $\int \csc(a+bx)\csc^3(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \csc(a+bx)\csc^3(c+dx)\,dx = \text{Int}\left(\csc(a+bx)\csc^3(c+dx),x\right)$$

output
```
Defer(Int)(csc(b*x+a)*csc(d*x+c)^3,x)
```

Mathematica [F(-1)]

Timed out.

$$\int \csc(a+bx)\csc^3(c+dx)\,dx = \$\text{Aborted}$$

input
```
Integrate[Csc[a + b*x]*Csc[c + d*x]^3,x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc^3(c+dx)\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \csc(a+bx)\csc^3(c+dx)dx$$

input
```
Int[Csc[a + b*x]*Csc[c + d*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc\left(bx+a\right)\csc\left(dx+c\right)^{3}dx$$

input
```
int(csc(b*x+a)*csc(d*x+c)^3,x)
```

output
```
int(csc(b*x+a)*csc(d*x+c)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\csc^3(c+dx)\,dx = \int \csc\left(bx+a\right)\csc\left(dx+c\right)^3\,dx$$

input
```
integrate(csc(b*x+a)*csc(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*csc(d*x + c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\csc^3(c+dx)\,dx = \int \csc\left(a+bx\right)\csc^3\left(c+dx\right)dx$$

input
```
integrate(csc(b*x+a)*csc(d*x+c)**3,x)
```

output
```
Integral(csc(a + b*x)*csc(c + d*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 10.97 (sec) , antiderivative size = 9659, normalized size of antiderivative = 643.93

$$\int \csc(a+bx)\csc^3(c+dx)\,dx = \int \csc(bx+a)\csc(dx+c)^3\,dx$$

input
```
integrate(csc(b*x+a)*csc(d*x+c)^3,x, algorithm="maxima")
```

output
```
(2*((b + d)*sin(4*b*x + 4*a) - 2*(b + d)*sin(2*b*x + 2*a) + (b + d)*sin(4*
d*x + 4*c) - 2*(b + d)*sin(2*d*x + 2*c))*cos((3*b + d)*x + 3*a + c) + 4*((
b + d)*sin((3*b + d)*x + 3*a + c) + (b - d)*sin((b + d)*x + a + c))*cos(2*
(2*b + d)*x + 4*a + 2*c) + 2*(2*(b + d)*sin(2*(2*b + d)*x + 4*a + 2*c) - 4
*(b + d)*sin(2*(b + d)*x + 2*a + 2*c) - (b + d)*sin(4*b*x + 4*a) + 2*(b +
d)*sin(2*b*x + 2*a) - (b + d)*sin(4*d*x + 4*c) + 2*(b + d)*sin(2*d*x + 2*c
))*cos((b + 3*d)*x + a + 3*c) + 4*((b + d)*sin((3*b + d)*x + 3*a + c) - (b
 + d)*sin((b + 3*d)*x + a + 3*c) - (b - d)*sin(3*(b + d)*x + 3*a + 3*c) +
(b - d)*sin((b + d)*x + a + c))*cos(2*(b + 2*d)*x + 2*a + 4*c) - 2*((b + d
)*sin((3*b + d)*x + 3*a + c) - (b + d)*sin((b + 3*d)*x + a + 3*c) - (b - d
)*sin(3*(b + d)*x + 3*a + 3*c) + (b - d)*sin((b + d)*x + a + c))*cos(4*(b
+ d)*x + 4*a + 4*c) + 2*(2*(b - d)*sin(2*(2*b + d)*x + 4*a + 2*c) - 4*(b -
 d)*sin(2*(b + d)*x + 2*a + 2*c) - (b - d)*sin(4*b*x + 4*a) + 2*(b - d)*si
n(2*b*x + 2*a) - (b - d)*sin(4*d*x + 4*c) + 2*(b - d)*sin(2*d*x + 2*c))*co
s(3*(b + d)*x + 3*a + 3*c) - 8*((b + d)*sin((3*b + d)*x + 3*a + c) + (b -
d)*sin((b + d)*x + a + c))*cos(2*(b + d)*x + 2*a + 2*c) + 2*((b - d)*sin(4
*b*x + 4*a) - 2*(b - d)*sin(2*b*x + 2*a) + (b - d)*sin(4*d*x + 4*c) - 2*(b
 - d)*sin(2*d*x + 2*c))*cos((b + d)*x + a + c) - (4*d^2*cos(2*(2*b + d)*x
+ 4*a + 2*c)^2 + 4*d^2*cos(2*(b + 2*d)*x + 2*a + 4*c)^2 + d^2*cos(4*(b + d
)*x + 4*a + 4*c)^2 + 16*d^2*cos(2*(b + d)*x + 2*a + 2*c)^2 + d^2*cos(4*...
```

Giac [N/A]

Not integrable

Time = 10.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\csc^3(c+dx)\,dx = \int \csc(bx+a)\csc(dx+c)^3\,dx$$

input
```
integrate(csc(b*x+a)*csc(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*csc(d*x + c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 16.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \csc(a+bx)\csc^3(c+dx)\,dx = \int \frac{1}{\sin(a+b\,x)\,\sin(c+d\,x)^3}\,dx$$

input
```
int(1/(sin(a + b*x)*sin(c + d*x)^3),x)
```

output
```
int(1/(sin(a + b*x)*sin(c + d*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\csc^3(c+dx)\,dx = \int \csc(bx+a)\csc(dx+c)^3\,dx$$

input
```
int(csc(b*x+a)*csc(d*x+c)^3,x)
```

output
```
int(csc(a + b*x)*csc(c + d*x)**3,x)
```

3.133 $\int \csc^2(a+bx)\csc^2(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \csc^2(a+bx)\csc^2(c+dx)\,dx = \text{Int}\big(\csc^2(a+bx)\csc^2(c+dx),x\big)$$

output
```
Defer(Int)(csc(b*x+a)^2*csc(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 14.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^2(c+dx)\,dx = \int \csc^2(a+bx)\csc^2(c+dx)\,dx$$

input
```
Integrate[Csc[a + b*x]^2*Csc[c + d*x]^2,x]
```

output
```
Integrate[Csc[a + b*x]^2*Csc[c + d*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^2(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc^2(a+bx)\csc^2(c+dx)dx$$

input
```
Int[Csc[a + b*x]^2*Csc[c + d*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc(bx+a)^2 \csc(dx+c)^2\,dx$$

input
```
int(csc(b*x+a)^2*csc(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^2*csc(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^2(c+dx)\,dx=\int \csc\left(bx+a\right)^2\csc\left(dx+c\right)^2\,dx$$

input
```
integrate(csc(b*x+a)^2*csc(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*csc(d*x + c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 3.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\csc^2(c+dx)\,dx=\int \csc^2\left(a+bx\right)\csc^2\left(c+dx\right)dx$$

input
```
integrate(csc(b*x+a)**2*csc(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**2*csc(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 3.39 (sec) , antiderivative size = 4492, normalized size of antiderivative = 264.24

$$\int \csc^2(a+bx)\csc^2(c+dx)\,dx = \int \csc{(bx+a)}^2 \csc{(dx+c)}^2 \, dx$$

input
```
integrate(csc(b*x+a)^2*csc(d*x+c)^2,x, algorithm="maxima")
```

output
```
-((d*cos(2*(2*b + d)*x + 4*a + 2*c)^2 + 4*d*cos(2*(b + d)*x + 2*a + 2*c)^2
 + d*cos(4*b*x + 4*a)^2 + 4*d*cos(2*b*x + 2*a)^2 + d*cos(2*d*x + 2*c)^2 +
d*sin(2*(2*b + d)*x + 4*a + 2*c)^2 + 4*d*sin(2*(b + d)*x + 2*a + 2*c)^2 +
d*sin(4*b*x + 4*a)^2 - 4*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*d*sin(2*b
*x + 2*a)^2 + d*sin(2*d*x + 2*c)^2 - 2*(2*d*cos(2*(b + d)*x + 2*a + 2*c) +
 d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) - d*cos(2*d*x + 2*c) + d)*cos(2
*(2*b + d)*x + 4*a + 2*c) + 4*(d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) -
 d*cos(2*d*x + 2*c) + d)*cos(2*(b + d)*x + 2*a + 2*c) - 2*(2*d*cos(2*b*x +
 2*a) - d)*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) - 2*(d*cos(4*b*x + 4*a)
 - 2*d*cos(2*b*x + 2*a) + d)*cos(2*d*x + 2*c) - 2*(2*d*sin(2*(b + d)*x + 2
*a + 2*c) + d*sin(4*b*x + 4*a) - 2*d*sin(2*b*x + 2*a) - d*sin(2*d*x + 2*c)
)*sin(2*(2*b + d)*x + 4*a + 2*c) + 4*(d*sin(4*b*x + 4*a) - 2*d*sin(2*b*x +
 2*a) - d*sin(2*d*x + 2*c))*sin(2*(b + d)*x + 2*a + 2*c) - 2*(d*sin(4*b*x
+ 4*a) - 2*d*sin(2*b*x + 2*a))*sin(2*d*x + 2*c) + d)*integrate(8*(3*b*cos(
4*b*x + 4*a)^2 - 3*b*cos(2*b*x + 2*a)^2 + 3*b*sin(4*b*x + 4*a)^2 - 3*b*sin
(2*b*x + 2*a)^2 + (b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a))*cos((6*b + d)*
x + 6*a + c) - 3*(b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a))*cos((4*b + d)*x
 + 4*a + c) + 3*(b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a))*cos((2*b + d)*x
+ 2*a + c) - (b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a))*cos(6*b*x + 6*a) +
b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - (b*cos(4*b*x + 4*a) + b*cos(2...
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^2(c+dx)\,dx=\int \csc(bx+a)^2\csc(dx+c)^2\,dx$$

input
```
integrate(csc(b*x+a)^2*csc(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*csc(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 16.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^2(c+dx)\,dx=\int \frac{1}{\sin(a+b\,x)^2\sin(c+d\,x)^2}\,dx$$

input
```
int(1/(sin(a + b*x)^2*sin(c + d*x)^2),x)
```

output
```
int(1/(sin(a + b*x)^2*sin(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^2(c+dx)\,dx=\int \csc(bx+a)^2\csc(dx+c)^2\,dx$$

input
```
int(csc(b*x+a)^2*csc(d*x+c)^2,x)
```

output
```
int(csc(a + b*x)**2*csc(c + d*x)**2,x)
```

3.134 $\int \csc^2(a+bx)\csc^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \csc^2(a+bx)\csc^3(c+dx)\,dx=\text{Int}\left(\csc^2(a+bx)\csc^3(c+dx),x\right)$$

output
```
Defer(Int)(csc(b*x+a)^2*csc(d*x+c)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 78.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^3(c+dx)\,dx=\int \csc^2(a+bx)\csc^3(c+dx)\,dx$$

input
```
Integrate[Csc[a + b*x]^2*Csc[c + d*x]^3,x]
```

output
```
Integrate[Csc[a + b*x]^2*Csc[c + d*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^3(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc^2(a+bx)\csc^3(c+dx)dx$$

input
```
Int[Csc[a + b*x]^2*Csc[c + d*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc(bx+a)^2 \csc(dx+c)^3\,dx$$

input
```
int(csc(b*x+a)^2*csc(d*x+c)^3,x)
```

output
```
int(csc(b*x+a)^2*csc(d*x+c)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^3(c+dx)\,dx=\int \csc\left(bx+a\right)^2\csc\left(dx+c\right)^3\,dx$$

input
```
integrate(csc(b*x+a)^2*csc(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*csc(d*x + c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 10.87 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\csc^3(c+dx)\,dx=\int \csc^2\left(a+bx\right)\csc^3\left(c+dx\right)dx$$

input
```
integrate(csc(b*x+a)**2*csc(d*x+c)**3,x)
```

output
```
Integral(csc(a + b*x)**2*csc(c + d*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 18.56 (sec) , antiderivative size = 15005, normalized size of antiderivative = 882.65

$$\int \csc^2(a+bx)\csc^3(c+dx)\,dx = \int \csc(bx+a)^2\csc(dx+c)^3\,dx$$

input
```
integrate(csc(b*x+a)^2*csc(d*x+c)^3,x, algorithm="maxima")
```

output
```
-(4*(2*(2*b - d)*cos(2*(3*b + d)*x + 6*a + 2*c) - 6*(2*b - d)*cos(2*(2*b +
 d)*x + 4*a + 2*c) + 6*(2*b - d)*cos(2*(b + d)*x + 2*a + 2*c) - (2*b - d)*
cos(6*b*x + 6*a) + 3*(2*b - d)*cos(4*b*x + 4*a) - 3*(2*b - d)*cos(2*b*x +
2*a) + (2*b - d)*cos(4*d*x + 4*c) - 2*(2*b - d)*cos(2*d*x + 2*c) + 2*b - d
)*cos((4*b + 3*d)*x + 4*a + 3*c) + 4*((2*b + d)*cos(6*b*x + 6*a) - 3*(2*b
+ d)*cos(4*b*x + 4*a) + 3*(2*b + d)*cos(2*b*x + 2*a) - (2*b + d)*cos(4*d*x
 + 4*c) + 2*(2*b + d)*cos(2*d*x + 2*c) - 2*b - d)*cos((4*b + d)*x + 4*a +
c) - 4*((2*b - d)*cos((4*b + 3*d)*x + 4*a + 3*c) - (2*b + d)*cos((4*b + d)
*x + 4*a + c) + (2*b + d)*cos((2*b + 3*d)*x + 2*a + 3*c) - (2*b - d)*cos((
2*b + d)*x + 2*a + c))*cos(2*(3*b + 2*d)*x + 6*a + 4*c) - 8*((2*b + d)*cos
((4*b + d)*x + 4*a + c) + (2*b - d)*cos((2*b + d)*x + 2*a + c))*cos(2*(3*b
 + d)*x + 6*a + 2*c) + 4*(2*(2*b + d)*cos(2*(3*b + d)*x + 6*a + 2*c) - 6*(
2*b + d)*cos(2*(2*b + d)*x + 4*a + 2*c) + 6*(2*b + d)*cos(2*(b + d)*x + 2*
a + 2*c) - (2*b + d)*cos(6*b*x + 6*a) + 3*(2*b + d)*cos(4*b*x + 4*a) - 3*(
2*b + d)*cos(2*b*x + 2*a) + (2*b + d)*cos(4*d*x + 4*c) - 2*(2*b + d)*cos(2
*d*x + 2*c) + 2*b + d)*cos((2*b + 3*d)*x + 2*a + 3*c) + 24*((2*b + d)*cos(
(4*b + d)*x + 4*a + c) + (2*b - d)*cos((2*b + d)*x + 2*a + c))*cos(2*(2*b
+ d)*x + 4*a + 2*c) + 4*((2*b - d)*cos(6*b*x + 6*a) - 3*(2*b - d)*cos(4*b*
x + 4*a) + 3*(2*b - d)*cos(2*b*x + 2*a) - (2*b - d)*cos(4*d*x + 4*c) + 2*(
2*b - d)*cos(2*d*x + 2*c) - 2*b + d)*cos((2*b + d)*x + 2*a + c) - 12*((...
```

Giac [N/A]

Not integrable

Time = 56.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^3(c+dx)\,dx=\int \csc(bx+a)^2\csc(dx+c)^3\,dx$$

input
```
integrate(csc(b*x+a)^2*csc(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*csc(d*x + c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 16.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^3(c+dx)\,dx=\int \frac{1}{\sin(a+b\,x)^2\sin(c+d\,x)^3}\,dx$$

input
```
int(1/(sin(a + b*x)^2*sin(c + d*x)^3),x)
```

output
```
int(1/(sin(a + b*x)^2*sin(c + d*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 152.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^3(c+dx)\,dx=\int \csc(bx+a)^2\csc(dx+c)^3\,dx$$

input
```
int(csc(b*x+a)^2*csc(d*x+c)^3,x)
```

output
```
int(csc(a + b*x)**2*csc(c + d*x)**3,x)
```

3.135 $\int \csc^2(a+bx)\csc^4(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \csc^2(a+bx)\csc^4(c+dx)\,dx = \text{Int}\left(\csc^2(a+bx)\csc^4(c+dx),x\right)$$

output
```
Defer(Int)(csc(b*x+a)^2*csc(d*x+c)^4,x)
```

Mathematica [N/A]

Not integrable

Time = 25.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^4(c+dx)\,dx = \int \csc^2(a+bx)\csc^4(c+dx)\,dx$$

input
```
Integrate[Csc[a + b*x]^2*Csc[c + d*x]^4,x]
```

output
```
Integrate[Csc[a + b*x]^2*Csc[c + d*x]^4, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^4(c+dx)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \csc^2(a+bx)\csc^4(c+dx)dx$$

input
```
Int[Csc[a + b*x]^2*Csc[c + d*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc(bx+a)^2\csc(dx+c)^4\,dx$$

input
```
int(csc(b*x+a)^2*csc(d*x+c)^4,x)
```

output
```
int(csc(b*x+a)^2*csc(d*x+c)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^4(c+dx)\,dx = \int \csc(bx+a)^2\csc(dx+c)^4\,dx$$

input
```
integrate(csc(b*x+a)^2*csc(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*csc(d*x + c)^4, x)
```

Sympy [N/A]

Not integrable

Time = 26.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\csc^4(c+dx)\,dx = \int \csc^2(a+bx)\csc^4(c+dx)\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(d*x+c)**4,x)
```

output
```
Integral(csc(a + b*x)**2*csc(c + d*x)**4, x)
```

Maxima [N/A]

Not integrable

Time = 55.41 (sec) , antiderivative size = 36042, normalized size of antiderivative = 2120.12

$$\int \csc^2(a+bx)\csc^4(c+dx)\,dx=\int \csc{(bx+a)}^2\csc{(dx+c)}^4\,dx$$

input
```
integrate(csc(b*x+a)^2*csc(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/3*(32*(11*b^2 + d^2)*cos(6*b*x + 6*a)*sin(4*b*x + 4*a) - 32*(11*b^2 + d^
2)*cos(4*b*x + 4*a)*sin(2*b*x + 2*a) + 16*(4*b^2*sin(4*(b + d)*x + 4*a + 4
*c) + (b^2 - b*d)*sin(2*(3*b + 2*d)*x + 6*a + 4*c) - (2*b^2 - b*d - 3*d^2)
*sin(2*(3*b + d)*x + 6*a + 2*c) - 2*(4*b^2 + 3*d^2)*sin(2*(2*b + d)*x + 4*
a + 2*c) + (b^2 + b*d)*sin(2*(b + 2*d)*x + 2*a + 4*c) - (2*b^2 + b*d - 3*d
^2)*sin(2*(b + d)*x + 2*a + 2*c) + (b^2 - d^2)*sin(6*b*x + 6*a) + 2*(2*b^2
 + d^2)*sin(4*b*x + 4*a) + (b^2 - d^2)*sin(2*b*x + 2*a))*cos(2*(4*b + 3*d)
*x + 8*a + 6*c) - 48*((2*b^2 - b*d - 3*d^2)*sin(2*(3*b + d)*x + 6*a + 2*c)
 + 2*(4*b^2 + 3*d^2)*sin(2*(2*b + d)*x + 4*a + 2*c) + (2*b^2 + b*d - 3*d^2
)*sin(2*(b + d)*x + 2*a + 2*c) - (b^2 - d^2)*sin(6*b*x + 6*a) - 2*(2*b^2 +
 d^2)*sin(4*b*x + 4*a) - (b^2 - d^2)*sin(2*b*x + 2*a))*cos(2*(4*b + d)*x +
 8*a + 2*c) + 16*(24*b*d*sin(2*(b + 2*d)*x + 2*a + 4*c) - 3*(b^2 - b*d)*si
n(2*(4*b + d)*x + 8*a + 2*c) - 12*(b^2 - 3*d^2)*sin(2*(3*b + d)*x + 6*a +
2*c) - 6*(19*b^2 - 3*b*d + 12*d^2)*sin(2*(2*b + d)*x + 4*a + 2*c) + 6*(11*
b^2 - 3*b*d)*sin(4*(b + d)*x + 4*a + 4*c) - 12*(b^2 + 2*b*d - 3*d^2)*sin(2
*(b + d)*x + 2*a + 2*c) + (b^2 - b*d)*sin(8*b*x + 8*a) + 4*(2*b^2 + b*d -
3*d^2)*sin(6*b*x + 6*a) + 6*(9*b^2 - b*d + 4*d^2)*sin(4*b*x + 4*a) + 4*(2*
b^2 + b*d - 3*d^2)*sin(2*b*x + 2*a) - (b^2 - b*d)*sin(6*d*x + 6*c) + 3*(b^
2 - b*d)*sin(4*d*x + 4*c) - 3*(b^2 - b*d)*sin(2*d*x + 2*c))*cos(2*(3*b + 2
*d)*x + 6*a + 4*c) + 16*(24*b*d*sin(2*(b + d)*x + 2*a + 2*c) + 6*(22*b^...
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^4(c+dx)\,dx = \int \csc(bx+a)^2\csc(dx+c)^4\,dx$$

input
```
integrate(csc(b*x+a)^2*csc(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*csc(d*x + c)^4, x)
```

Mupad [N/A]

Not integrable

Time = 16.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^4(c+dx)\,dx = \int \frac{1}{\sin(a+b\,x)^2\sin(c+d\,x)^4}\,dx$$

input
```
int(1/(sin(a + b*x)^2*sin(c + d*x)^4),x)
```

output
```
int(1/(sin(a + b*x)^2*sin(c + d*x)^4), x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\csc^4(c+dx)\,dx = \int \csc(bx+a)^2\csc(dx+c)^4\,dx$$

input
```
int(csc(b*x+a)^2*csc(d*x+c)^4,x)
```

output
```
int(csc(b*x+a)^2*csc(d*x+c)^4,x)
```

3.136 $\int \sin^3(a+bx)\sqrt{\sin(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 477

$$\begin{aligned}
\int \sin^3(a+bx)\sqrt{\sin(c+dx)}\,dx =& -\frac{3e^{\frac{1}{2}i(2a-c)+\frac{1}{2}i(2b-d)x+\frac{1}{2}i(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{2b}{d}\right),\frac{1}{4}\left(3+\frac{2b}{d}\right),e^{2i(c+dx)}\right)\sqrt{\sin(c+dx)}}{4(2b-d)\sqrt{1-e^{2i(c+dx)}}} \\
&+\frac{e^{\frac{1}{2}i(6a-c)+\frac{1}{2}i(6b-d)x+\frac{1}{2}i(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{6b}{d}\right),\frac{3(2b+d)}{4d},e^{2i(c+dx)}\right)\sqrt{\sin(c+dx)}}{4(6b-d)\sqrt{1-e^{2i(c+dx)}}} \\
&-\frac{3e^{-\frac{1}{2}i(2a+c)-\frac{1}{2}i(2b+d)x+\frac{1}{2}i(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2b+d}{4d},\frac{1}{4}\left(3-\frac{2b}{d}\right),e^{2i(c+dx)}\right)\sqrt{\sin(c+dx)}}{4(2b+d)\sqrt{1-e^{2i(c+dx)}}} \\
&+\frac{e^{-\frac{1}{2}i(6a+c)-\frac{1}{2}i(6b+d)x+\frac{1}{2}i(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{6b+d}{4d},\frac{3}{4}\left(1-\frac{2b}{d}\right),e^{2i(c+dx)}\right)\sqrt{\sin(c+dx)}}{4(6b+d)\sqrt{1-e^{2i(c+dx)}}}
\end{aligned}$$

output
```
-3/4*exp(1/2*I*(2*a-c)+1/2*I*(2*b-d)*x+1/2*I*(d*x+c))*hypergeom([-1/2, -1/
4+1/2*b/d],[3/4+1/2*b/d],exp(2*I*(d*x+c)))*sin(d*x+c)^(1/2)/(2*b-d)/(1-exp
(2*I*(d*x+c)))^(1/2)+1/4*exp(1/2*I*(6*a-c)+1/2*I*(6*b-d)*x+1/2*I*(d*x+c))*
hypergeom([-1/2, -1/4+3/2*b/d],[3/4*(2*b+d)/d],exp(2*I*(d*x+c)))*sin(d*x+c
)^(1/2)/(6*b-d)/(1-exp(2*I*(d*x+c)))^(1/2)-3/4*exp(-1/2*I*(2*a+c)-1/2*I*(2
*b+d)*x+1/2*I*(d*x+c))*hypergeom([-1/2, -1/4*(2*b+d)/d],[3/4-1/2*b/d],exp(
2*I*(d*x+c)))*sin(d*x+c)^(1/2)/(2*b+d)/(1-exp(2*I*(d*x+c)))^(1/2)+1/4*exp(
-1/2*I*(6*a+c)-1/2*I*(6*b+d)*x+1/2*I*(d*x+c))*hypergeom([-1/2, -1/4*(6*b+d
)/d],[3/4-3/2*b/d],exp(2*I*(d*x+c)))*sin(d*x+c)^(1/2)/(6*b+d)/(1-exp(2*I*(
d*x+c)))^(1/2)
```

Mathematica [F]

$$\int \sin^3(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sin^3(a+bx)\sqrt{\sin(c+dx)}\,dx$$

input
```
Integrate[Sin[a + b*x]^3*Sqrt[Sin[c + d*x]],x]
```

output
```
Integrate[Sin[a + b*x]^3*Sqrt[Sin[c + d*x]], x]
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5064, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sqrt{\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{ 5064}$$

$$\frac{\int \left(3ie^{-ia-ibx}\sqrt{ie^{-i(c+dx)}-ie^{i(c+dx)}}-3ie^{ia+ibx}\sqrt{ie^{-i(c+dx)}-ie^{i(c+dx)}}-ie^{-3ia-3ibx}\sqrt{ie^{-i(c+dx)}-ie^{i(c+dx)}}+i\right.}{8\sqrt{2}}$$

$\big\downarrow$ 2009

$$-\frac{6\sqrt{ie^{-i(c+dx)}-ie^{i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(\frac{2b}{d}-1\right),\frac{1}{4}\left(\frac{2b}{d}+3\right),e^{2i(c+dx)}\right)\exp\left(\frac{1}{2}i(2a-c)+\frac{1}{2}ix(2b-d)+\frac{1}{2}i(c+dx)\right)}{(2b-d)\sqrt{1-e^{2ic+2idx}}}+\frac{2\sqrt{ie^{-i(c+dx)}}}{}$$

input
```
Int[Sin[a + b*x]^3*Sqrt[Sin[c + d*x]],x]
```

output
```
((-6*E^((I/2)*(2*a - c) + (I/2)*(2*b - d)*x + (I/2)*(c + d*x))*Sqrt[I/E^(I
*(c + d*x)) - I*E^(I*(c + d*x))]*Hypergeometric2F1[-1/2, (-1 + (2*b)/d)/4,
 (3 + (2*b)/d)/4, E^((2*I)*(c + d*x))])/((2*b - d)*Sqrt[1 - E^((2*I)*c + (
2*I)*d*x)]) + (2*E^((I/2)*(6*a - c) + (I/2)*(6*b - d)*x + (I/2)*(c + d*x))
*Sqrt[I/E^(I*(c + d*x)) - I*E^(I*(c + d*x))]*Hypergeometric2F1[-1/2, (-1 +
 (6*b)/d)/4, (3*(2*b + d))/(4*d), E^((2*I)*(c + d*x))])/((6*b - d)*Sqrt[1
- E^((2*I)*c + (2*I)*d*x)]) - (6*E^((-1/2*I)*(2*a + c) - (I/2)*(2*b + d)*x
 + (I/2)*(c + d*x))*Sqrt[I/E^(I*(c + d*x)) - I*E^(I*(c + d*x))]*Hypergeome
tric2F1[-1/2, -1/4*(2*b + d)/d, (3 - (2*b)/d)/4, E^((2*I)*(c + d*x))])/((2
*b + d)*Sqrt[1 - E^((2*I)*c + (2*I)*d*x)]) + (2*E^((-1/2*I)*(6*a + c) - (I
/2)*(6*b + d)*x + (I/2)*(c + d*x))*Sqrt[I/E^(I*(c + d*x)) - I*E^(I*(c + d*
x))]*Hypergeometric2F1[-1/2, -1/4*(6*b + d)/d, (3*(1 - (2*b)/d))/4, E^((2*
I)*(c + d*x))])/((6*b + d)*Sqrt[1 - E^((2*I)*c + (2*I)*d*x)]))/(8*Sqrt[2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5064
```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c +
d*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a
, b, c, d, q}, x] && IGtQ[p, 0] && !IntegerQ[q]
```

Maple [F]

$$\int \sin{(bx+a)}^3 \sqrt{\sin{(dx+c)}}dx$$

input
```
int(sin(b*x+a)^3*sin(d*x+c)^(1/2),x)
```

output
```
int(sin(b*x+a)^3*sin(d*x+c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sin^3(a+bx)\sqrt{\sin(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sqrt{\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*sin(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^3(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sin(bx+a)^3\sqrt{\sin(dx+c)}\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3*sqrt(sin(d*x + c)), x)
```

Giac [F]

$$\int \sin^3(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sin(bx+a)^3\sqrt{\sin(dx+c)}\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3*sqrt(sin(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sin(a+b\,x)^3\sqrt{\sin(c+d\,x)}\,dx$$

input
```
int(sin(a + b*x)^3*sin(c + d*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^3*sin(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sin^3(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sqrt{\sin(dx+c)}\,\sin(bx+a)^3\,dx$$

input
```
int(sin(b*x+a)^3*sin(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sin(c + d*x))*sin(a + b*x)**3,x)
```

3.137 $\int \sin^2(a+bx)\sqrt{\sin(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 263

$$\int \sin^2(a+bx)\sqrt{\sin(c+dx)}\,dx = \frac{E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d} - \frac{ie^{-\frac{1}{2}i(4a+c)-\frac{1}{2}i(4b+d)x+\frac{1}{2}i(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1-\frac{4b}{d}\right),\frac{1}{4}\left(3-\frac{4b}{d}\right),e^{2i(c+dx)}\right)\sqrt{\sin(c+dx)}}{2(4b+d)\sqrt{1-e^{2i(c+dx)}}} + \frac{ie^{\frac{1}{2}i(4a-c)+\frac{1}{2}i(4b-d)x+\frac{1}{2}i(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{4b}{d}\right),\frac{1}{4}\left(3+\frac{4b}{d}\right),e^{2i(c+dx)}\right)\sqrt{\sin(c+dx)}}{2(4b-d)\sqrt{1-e^{2i(c+dx)}}}$$

output

```
-EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d-1/2*I*exp(-1/2*I*(4*a+c)-1
/2*I*(4*b+d)*x+1/2*I*(d*x+c))*hypergeom([-1/2, -1/4-b/d],[3/4-b/d],exp(2*I
*(d*x+c)))*sin(d*x+c)^(1/2)/(4*b+d)/(1-exp(2*I*(d*x+c)))^(1/2)+1/2*I*exp(1
/2*I*(4*a-c)+1/2*I*(4*b-d)*x+1/2*I*(d*x+c))*hypergeom([-1/2, -1/4+b/d],[3/
4+b/d],exp(2*I*(d*x+c)))*sin(d*x+c)^(1/2)/(4*b-d)/(1-exp(2*I*(d*x+c)))^(1/
2)
```

Mathematica [F]

$$\int \sin^2(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sin^2(a+bx)\sqrt{\sin(c+dx)}\,dx$$

input
```
Integrate[Sin[a + b*x]^2*Sqrt[Sin[c + d*x]],x]
```

output
```
Integrate[Sin[a + b*x]^2*Sqrt[Sin[c + d*x]], x]
```

Rubi [A] (warning: unable to verify)

Time = 1.17 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.71, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5064, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sqrt{\sin(c+dx)}\,dx$$

$$\Big\downarrow \; 5064$$

$$\frac{\int \left(-e^{-2ia-2ibx}\sqrt{ie^{-i(c+dx)}-ie^{i(c+dx)}}-e^{2ia+2ibx}\sqrt{ie^{-i(c+dx)}-ie^{i(c+dx)}}+2\sqrt{ie^{-i(c+dx)}-ie^{i(c+dx)}}\right)dx}{4\sqrt{2}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2i\sqrt{ie^{-i(c+dx)}-ie^{i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-\frac{4b}{d}-1\right),\frac{1}{4}\left(3-\frac{4b}{d}\right),e^{2i(c+dx)}\right)\exp\left(-\frac{1}{2}i(4a+c)-\frac{1}{2}ix(4b+d)+\frac{1}{2}i(c+dx)\right)}{(4b+d)\sqrt{1-e^{2ic+2idx}}}+\frac{2i\sqrt{ie^{-i(}}}{}$$

input
```
Int[Sin[a + b*x]^2*Sqrt[Sin[c + d*x]],x]
```

output
```
(((4*I)*Sqrt[I/E^(I*(c + d*x)) - I*E^(I*(c + d*x))])/d - ((8*I)*Sqrt[E^(I*
(c + d*x))]*Sqrt[I/E^(I*(c + d*x)) - I*E^(I*(c + d*x))]*EllipticE[ArcSin[S
qrt[1 - E^(I*(c + d*x))]/Sqrt[2]], 2])/(d*Sqrt[1 - E^((2*I)*(c + d*x))]) -
 ((2*I)*E^((-1/2*I)*(4*a + c) - (I/2)*(4*b + d)*x + (I/2)*(c + d*x))*Sqrt[
I/E^(I*(c + d*x)) - I*E^(I*(c + d*x))]*Hypergeometric2F1[-1/2, (-1 - (4*b)
/d)/4, (3 - (4*b)/d)/4, E^((2*I)*(c + d*x))])/((4*b + d)*Sqrt[1 - E^((2*I)
*c + (2*I)*d*x)]) + ((2*I)*E^((I/2)*(4*a - c) + (I/2)*(4*b - d)*x + (I/2)*
(c + d*x))*Sqrt[I/E^(I*(c + d*x)) - I*E^(I*(c + d*x))]*Hypergeometric2F1[-
1/2, (-1 + (4*b)/d)/4, (3 + (4*b)/d)/4, E^((2*I)*(c + d*x))])/((4*b - d)*S
qrt[1 - E^((2*I)*c + (2*I)*d*x)]))/(4*Sqrt[2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5064
```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c +
d*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a
, b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \sin(bx+a)^2 \sqrt{\sin(dx+c)}dx$$

input
```
int(sin(b*x+a)^2*sin(d*x+c)^(1/2),x)
```

output
```
int(sin(b*x+a)^2*sin(d*x+c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sin^2(a+bx)\sqrt{\sin(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sin^2(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sin^2{(a+bx)}\sqrt{\sin{(c+dx)}}\,dx$$

input
```
integrate(sin(b*x+a)**2*sin(d*x+c)**(1/2),x)
```

output
```
Integral(sin(a + b*x)**2*sqrt(sin(c + d*x)), x)
```

Maxima [F]

$$\int \sin^2(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sin{(bx+a)}^2\sqrt{\sin{(dx+c)}}\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2*sqrt(sin(d*x + c)), x)
```

Giac [F]

$$\int \sin^2(a+bx)\sqrt{\sin(c+dx)}\,dx=\int \sin(bx+a)^2\sqrt{\sin(dx+c)}\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2*sqrt(sin(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sqrt{\sin(c+dx)}\,dx=\int \sin(a+b\,x)^2\sqrt{\sin(c+d\,x)}\,dx$$

input
```
int(sin(a + b*x)^2*sin(c + d*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^2*sin(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sin^2(a+bx)\sqrt{\sin(c+dx)}\,dx=\int \sqrt{\sin(dx+c)}\,\sin(bx+a)^2\,dx$$

input
```
int(sin(b*x+a)^2*sin(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sin(c + d*x))*sin(a + b*x)**2,x)
```

3.138 $\int \sin(a+bx)\sqrt{\sin(c+dx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 235

$$\int \sin(a+bx)\sqrt{\sin(c+dx)}\,dx = -\frac{e^{\frac{1}{2}i(2a-c)+\frac{1}{2}i(2b-d)x+\frac{1}{2}i(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{2b}{d}\right),\frac{1}{4}\left(3+\frac{2b}{d}\right),e^{2i(c+dx)}\right)\sqrt{\sin(c+dx)}}{(2b-d)\sqrt{1-e^{2i(c+dx)}}} - \frac{e^{-\frac{1}{2}i(2a+c)-\frac{1}{2}i(2b+d)x+\frac{1}{2}i(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2b+d}{4d},\frac{1}{4}\left(3-\frac{2b}{d}\right),e^{2i(c+dx)}\right)\sqrt{\sin(c+dx)}}{(2b+d)\sqrt{1-e^{2i(c+dx)}}}$$

output
```
-exp(1/2*I*(2*a-c)+1/2*I*(2*b-d)*x+1/2*I*(d*x+c))*hypergeom([-1/2, -1/4+1/
2*b/d],[3/4+1/2*b/d],exp(2*I*(d*x+c)))*sin(d*x+c)^(1/2)/(2*b-d)/(1-exp(2*I
*(d*x+c)))^(1/2)-exp(-1/2*I*(2*a+c)-1/2*I*(2*b+d)*x+1/2*I*(d*x+c))*hyperge
om([-1/2, -1/4*(2*b+d)/d],[3/4-1/2*b/d],exp(2*I*(d*x+c)))*sin(d*x+c)^(1/2)
/(2*b+d)/(1-exp(2*I*(d*x+c)))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 10.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.83

$$\int \sin(a+bx)\sqrt{\sin(c+dx)}\,dx = \frac{ie^{-\frac{1}{2}i(2a-2c+2bx-dx)}\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^{3/2}\left((2b-d)e^{\frac{idx}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{5}{4}-\frac{b}{2d},\frac{3}{4}-\frac{b}{2d},\right.\right.}{\sqrt{2}\left(4b^2-d^2\right)}$$

input
```
Integrate[Sin[a + b*x]*Sqrt[Sin[c + d*x]],x]
```

output
```
(I*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^(3/2)*((2*b - d)*E^
((I/2)*d*x)*Hypergeometric2F1[1, 5/4 - b/(2*d), 3/4 - b/(2*d), E^((2*I)*(c
 + d*x))] + (2*b + d)*E^((I/2)*(4*a + (4*b + d)*x))*Hypergeometric2F1[1, 5
/4 + b/(2*d), 3/4 + b/(2*d), E^((2*I)*(c + d*x))]))/(Sqrt[2]*(4*b^2 - d^2)
*E^((I/2)*(2*a - 2*c + 2*b*x - d*x)))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5064, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sqrt{\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{ 5064}$$

$$\frac{\int \left(ie^{-ia-ibx}\sqrt{ie^{-i(c+dx)}-ie^{i(c+dx)}}-ie^{ia+ibx}\sqrt{ie^{-i(c+dx)}-ie^{i(c+dx)}}\right)dx}{2\sqrt{2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{2\sqrt{ie^{-i(c+dx)}-ie^{i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(\frac{2b}{d}-1\right),\frac{1}{4}\left(\frac{2b}{d}+3\right),e^{2i(c+dx)}\right)\exp\left(\frac{1}{2}i(2a-c)+\frac{1}{2}ix(2b-d)+\frac{1}{2}i(c+dx)\right)}{(2b-d)\sqrt{1-e^{2ic+2idx}}}-\frac{2\sqrt{ie^{-i(c+dx)}}}{}}{2\sqrt{2}}$$

input
```
Int[Sin[a + b*x]*Sqrt[Sin[c + d*x]],x]
```

output
```
((-2*E^((I/2)*(2*a - c) + (I/2)*(2*b - d)*x + (I/2)*(c + d*x))*Sqrt[I/E^(I
*(c + d*x)) - I*E^(I*(c + d*x))]*Hypergeometric2F1[-1/2, (-1 + (2*b)/d)/4,
 (3 + (2*b)/d)/4, E^((2*I)*(c + d*x))])/((2*b - d)*Sqrt[1 - E^((2*I)*c + (
2*I)*d*x)]) - (2*E^((-1/2*I)*(2*a + c) - (I/2)*(2*b + d)*x + (I/2)*(c + d*
x))*Sqrt[I/E^(I*(c + d*x)) - I*E^(I*(c + d*x))]*Hypergeometric2F1[-1/2, -1
/4*(2*b + d)/d, (3 - (2*b)/d)/4, E^((2*I)*(c + d*x))])/((2*b + d)*Sqrt[1 -
 E^((2*I)*c + (2*I)*d*x)]))/(2*Sqrt[2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5064
```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c +
d*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a
, b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \sin(bx+a)\sqrt{\sin(dx+c)}dx$$

input
```
int(sin(b*x+a)*sin(d*x+c)^(1/2),x)
```

output
```
int(sin(b*x+a)*sin(d*x+c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sin(a+bx)\sqrt{\sin(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sin(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sin(a+bx)\sqrt{\sin(c+dx)}\,dx$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)**(1/2),x)
```

output
```
Integral(sin(a + b*x)*sqrt(sin(c + d*x)), x)
```

Maxima [F]

$$\int \sin(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sin(bx+a)\sqrt{\sin(dx+c)}\,dx$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)*sqrt(sin(d*x + c)), x)
```

Giac [F]

$$\int \sin(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sin(bx+a)\sqrt{\sin(dx+c)}\,dx$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*sqrt(sin(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sin(a+b\,x)\,\sqrt{\sin(c+d\,x)}\,dx$$

input
```
int(sin(a + b*x)*sin(c + d*x)^(1/2),x)
```

output
```
int(sin(a + b*x)*sin(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sin(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sqrt{\sin(dx+c)}\,\sin(bx+a)\,dx$$

input
```
int(sin(b*x+a)*sin(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sin(c + d*x))*sin(a + b*x),x)
```

3.139 $\int \csc(a+bx)\sqrt{\sin(c+dx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \csc(a+bx)\sqrt{\sin(c+dx)}\,dx=\text{Int}\Big(\csc(a+bx)\sqrt{\sin(c+dx)},x\Big)$$

output
```
Defer(Int)(csc(b*x+a)*sin(d*x+c)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 12.95 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc(a+bx)\sqrt{\sin(c+dx)}\,dx=\int \csc(a+bx)\sqrt{\sin(c+dx)}\,dx$$

input
```
Integrate[Csc[a + b*x]*Sqrt[Sin[c + d*x]],x]
```

output
```
Integrate[Csc[a + b*x]*Sqrt[Sin[c + d*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sqrt{\sin(c+dx)}\,dx$$

$$\Big\downarrow \textcolor{blue}{7299}$$

$$\int \csc(a+bx)\sqrt{\sin(c+dx)}dx$$

input
```
Int[Csc[a + b*x]*Sqrt[Sin[c + d*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \csc\left(bx+a\right)\sqrt{\sin\left(dx+c\right)}dx$$

input
```
int(csc(b*x+a)*sin(d*x+c)^(1/2),x)
```

output
```
int(csc(b*x+a)*sin(d*x+c)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sqrt{\sin(c+dx)}\,dx=\int \csc\left(bx+a\right)\sqrt{\sin\left(dx+c\right)}\,dx$$

input
```
integrate(csc(b*x+a)*sin(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*sqrt(sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sqrt{\sin(c+dx)}\,dx=\int \sqrt{\sin\left(c+dx\right)}\csc\left(a+bx\right)dx$$

input
```
integrate(csc(b*x+a)*sin(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(sin(c + d*x))*csc(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \csc(bx+a)\sqrt{\sin(dx+c)}\,dx$$

input
```
integrate(csc(b*x+a)*sin(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*sqrt(sin(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \csc(bx+a)\sqrt{\sin(dx+c)}\,dx$$

input
```
integrate(csc(b*x+a)*sin(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sqrt(sin(d*x + c)), x)
```

Mupad [N/A]

Not integrable

Time = 17.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \frac{\sqrt{\sin(c+d\,x)}}{\sin(a+b\,x)}\,dx$$

input
```
int(sin(c + d*x)^(1/2)/sin(a + b*x),x)
```

output
```
int(sin(c + d*x)^(1/2)/sin(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \csc(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sqrt{\sin(dx+c)}\,\csc(bx+a)\,dx$$

input
```
int(csc(b*x+a)*sin(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sin(c + d*x))*csc(a + b*x),x)
```

3.140 $\int \csc^2(a+bx)\sqrt{\sin(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \csc^2(a+bx)\sqrt{\sin(c+dx)}\,dx=\text{Int}\left(\csc^2(a+bx)\sqrt{\sin(c+dx)},x\right)$$

output
```
Defer(Int)(csc(b*x+a)^2*sin(d*x+c)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 20.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \csc^2(a+bx)\sqrt{\sin(c+dx)}\,dx=\int \csc^2(a+bx)\sqrt{\sin(c+dx)}\,dx$$

input
```
Integrate[Csc[a + b*x]^2*Sqrt[Sin[c + d*x]],x]
```

output
```
Integrate[Csc[a + b*x]^2*Sqrt[Sin[c + d*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sqrt{\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc^2(a+bx)\sqrt{\sin(c+dx)}dx$$

input
```
Int[Csc[a + b*x]^2*Sqrt[Sin[c + d*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \csc\left(bx+a\right)^{2}\sqrt{\sin\left(dx+c\right)}dx$$

input
```
int(csc(b*x+a)^2*sin(d*x+c)^(1/2),x)
```

output
```
int(csc(b*x+a)^2*sin(d*x+c)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \csc(bx+a)^2\sqrt{\sin(dx+c)}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sqrt(sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sqrt{\sin(c+dx)}\csc^2(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*sin(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(sin(c + d*x))*csc(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \csc(bx+a)^2\sqrt{\sin(dx+c)}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2*sqrt(sin(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \csc(bx+a)^2\sqrt{\sin(dx+c)}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sqrt(sin(d*x + c)), x)
```

Mupad [N/A]

Not integrable

Time = 16.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \frac{\sqrt{\sin(c+d\,x)}}{\sin(a+b\,x)^2}\,dx$$

input
```
int(sin(c + d*x)^(1/2)/sin(a + b*x)^2,x)
```

output
```
int(sin(c + d*x)^(1/2)/sin(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \csc^2(a+bx)\sqrt{\sin(c+dx)}\,dx = \int \sqrt{\sin\left(dx+c\right)}\,\csc\left(bx+a\right)^2dx$$

input
```
int(csc(b*x+a)^2*sin(d*x+c)^(1/2),x)
```

output
```
int(sqrt(sin(c + d*x))*csc(a + b*x)**2,x)
```

3.141 $\int \sin^3(a+bx)\sin^q(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 473

$$\int \sin^3(a+bx)\sin^q(c+dx)\,dx$$
$$= \frac{e^{i(3a-cq)+i(3b-dq)x+iq(c+dx)}\left(1-e^{2i(c+dx)}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{3b}{d}-q\right),-q,\frac{1}{2}\left(2+\frac{3b}{d}-q\right),e^{2i(c+dx)}\right)\text{s}}{8(3b-dq)}$$
$$-\frac{3e^{i(a-cq)+i(b-dq)x+iq(c+dx)}\left(1-e^{2i(c+dx)}\right)^{-q}\text{Hypergeometric2F1}\left(-q,\frac{b-dq}{2d},\frac{1}{2}\left(2+\frac{b}{d}-q\right),e^{2i(c+dx)}\right)\sin^q(c}{8(b-dq)}$$
$$-\frac{3e^{-i(a+cq)-i(b+dq)x+iq(c+dx)}\left(1-e^{2i(c+dx)}\right)^{-q}\text{Hypergeometric2F1}\left(-q,-\frac{b+dq}{2d},-\frac{b-d(2-q)}{2d},e^{2i(c+dx)}\right)\sin^q(c}{8(b+dq)}$$
$$+\frac{e^{-i(3a+cq)-i(3b+dq)x+iq(c+dx)}\left(1-e^{2i(c+dx)}\right)^{-q}\text{Hypergeometric2F1}\left(-q,-\frac{3b+dq}{2d},\frac{1}{2}\left(2-\frac{3b}{d}-q\right),e^{2i(c+dx)}\right)}{8(3b+dq)}$$

output

```
1/8*exp(I*(-c*q+3*a)+I*(-d*q+3*b)*x+I*q*(d*x+c))*hypergeom([-q, 3/2*b/d-1/
2*q],[1+3/2*b/d-1/2*q],exp(2*I*(d*x+c)))*sin(d*x+c)^q/((1-exp(2*I*(d*x+c))
)^q)/(-d*q+3*b)-3/8*exp(I*(-c*q+a)+I*(-d*q+b)*x+I*q*(d*x+c))*hypergeom([-q
, 1/2*(-d*q+b)/d],[1+1/2*b/d-1/2*q],exp(2*I*(d*x+c)))*sin(d*x+c)^q/((1-exp
(2*I*(d*x+c)))^q)/(-d*q+b)-3/8*exp(-I*(c*q+a)-I*(d*q+b)*x+I*q*(d*x+c))*hyp
ergeom([-q, -1/2*(d*q+b)/d],[-1/2*(b-d*(2-q))/d],exp(2*I*(d*x+c)))*sin(d*x
+c)^q/((1-exp(2*I*(d*x+c)))^q)/(d*q+b)+1/8*exp(-I*(c*q+3*a)-I*(d*q+3*b)*x+
I*q*(d*x+c))*hypergeom([-q, -1/2*(d*q+3*b)/d],[1-3/2*b/d-1/2*q],exp(2*I*(d
*x+c)))*sin(d*x+c)^q/((1-exp(2*I*(d*x+c)))^q)/(d*q+3*b)
```

Mathematica [F]

$$\int \sin^3(a+bx)\sin^q(c+dx)\,dx = \int \sin^3(a+bx)\sin^q(c+dx)\,dx$$

input
```
Integrate[Sin[a + b*x]^3*Sin[c + d*x]^q,x]
```

output
```
Integrate[Sin[a + b*x]^3*Sin[c + d*x]^q, x]
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5064, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^q(c+dx)\,dx$$

$$\Big\downarrow \ 5064$$

$$2^{-q-3}\int \left(3ie^{-ia-ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^q-3ie^{ia+ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^q-ie^{-3ia-3ibx}\left(ie^{-i(c+dx)}-ie^{i(\right.\right.$$

$$\Big\downarrow \ 2009$$

$$2^{-q-3}\left(\frac{\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^q\left(1-e^{2ic+2idx}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{3b}{d}-q\right),-q,\frac{1}{2}\left(\frac{3b}{d}-q+2\right),e^{2i(c+dx)}\right)\right.}{3b-dq}\right.$$

input
```
Int[Sin[a + b*x]^3*Sin[c + d*x]^q,x]
```

output
```
2^(-3 - q)*((E^(I*(3*a - c*q) + I*(3*b - d*q)*x + I*q*(c + d*x))*(I/E^(I*(
c + d*x)) - I*E^(I*(c + d*x)))^q*Hypergeometric2F1[((3*b)/d - q)/2, -q, (2
 + (3*b)/d - q)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^q*
(3*b - d*q)) - (3*E^(I*(a - c*q) + I*(b - d*q)*x + I*q*(c + d*x))*(I/E^(I*
(c + d*x)) - I*E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, (b - d*q)/(2*d), (
2 + b/d - q)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^q*(b
- d*q)) - (3*E^((-I)*(a + c*q) - I*(b + d*q)*x + I*q*(c + d*x))*(I/E^(I*(c
 + d*x)) - I*E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, -1/2*(b + d*q)/d, 1
- (b + d*q)/(2*d), E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^q*
(b + d*q)) + (E^((-I)*(3*a + c*q) - I*(3*b + d*q)*x + I*q*(c + d*x))*(I/E^
(I*(c + d*x)) - I*E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, -1/2*(3*b + d*q
)/d, (2 - (3*b)/d - q)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d
*x))^q*(3*b + d*q)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5064
```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c +
d*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a
, b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \sin (bx+a)^3 \sin (dx+c)^q \, dx$$

input
```
int(sin(b*x+a)^3*sin(d*x+c)^q,x)
```

output
```
int(sin(b*x+a)^3*sin(d*x+c)^q,x)
```

Fricas [F]

$$\int \sin^3(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q \sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^q,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sin(d*x + c)^q*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sin^q(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*sin(d*x+c)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^3(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q \sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^q,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)^q*sin(b*x + a)^3, x)
```

Giac [F]

$$\int \sin^3(a+bx)\sin^q(c+dx)\,dx=\int \sin{(dx+c)}^q\sin{(bx+a)}^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(d*x+c)^q,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^q*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sin^q(c+dx)\,dx=\int \sin{(a+b\,x)}^3\sin{(c+d\,x)}^q\,dx$$

input
```
int(sin(a + b*x)^3*sin(c + d*x)^q,x)
```

output
```
int(sin(a + b*x)^3*sin(c + d*x)^q, x)
```

Reduce [F]

$$\int \sin^3(a+bx)\sin^q(c+dx)\,dx=\int \sin{(dx+c)}^q\sin{(bx+a)}^3\,dx$$

input
```
int(sin(b*x+a)^3*sin(d*x+c)^q,x)
```

output
```
int(sin(c + d*x)**q*sin(a + b*x)**3,x)
```

3.142 $\int \sin^2(a+bx)\sin^q(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 314

$$\int \sin^2(a+bx)\sin^q(c+dx)\,dx = -\frac{ie^{-i(2a+cq)-i(2b+dq)x+iq(c+dx)}\left(1-e^{2i(c+dx)}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2b}{d}-q\right),-q,\frac{1}{2}\left(2-\frac{2b}{d}-q\right),e^{2i(c}\right.}{4(2b+dq)} + \frac{ie^{i(2a-cq)+i(2b-dq)x+iq(c+dx)}\left(1-e^{2i(c+dx)}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{2b}{d}-q\right),-q,\frac{1}{2}\left(2+\frac{2b}{d}-q\right),e^{2i(c+dx}\right.}{4(2b-dq)} + \frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+q}{2},\frac{3+q}{2},\sin^2(c+dx)\right)\sin^{1+q}(c+dx)}{2d(1+q)\sqrt{\cos^2(c+dx)}}$$

output
```
-1/4*I*exp(-I*(c*q+2*a)-I*(d*q+2*b)*x+I*q*(d*x+c))*hypergeom([-q, -b/d-1/2
*q],[1-b/d-1/2*q],exp(2*I*(d*x+c)))*sin(d*x+c)^q/((1-exp(2*I*(d*x+c)))^q)/
(d*q+2*b)+1/4*I*exp(I*(-c*q+2*a)+I*(-d*q+2*b)*x+I*q*(d*x+c))*hypergeom([-q
, b/d-1/2*q],[1+b/d-1/2*q],exp(2*I*(d*x+c)))*sin(d*x+c)^q/((1-exp(2*I*(d*x
+c)))^q)/(-d*q+2*b)+1/2*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*q],[3/2+1/2*q],
sin(d*x+c)^2)*sin(d*x+c)^(1+q)/d/(1+q)/(cos(d*x+c)^2)^(1/2)
```

Mathematica [F]

$$\int \sin^2(a+bx)\sin^q(c+dx)\,dx = \int \sin^2(a+bx)\sin^q(c+dx)\,dx$$

input
```
Integrate[Sin[a + b*x]^2*Sin[c + d*x]^q,x]
```

output
```
Integrate[Sin[a + b*x]^2*Sin[c + d*x]^q, x]
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5064, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^q(c+dx)\,dx$$

$$\Big\downarrow \text{5064}$$

$$2^{-q-2}\int \left(-e^{-2ia-2ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^q - e^{2ia+2ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^q + 2\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^q\right)$$

$$\Big\downarrow \text{2009}$$

$$2^{-q-2}\left(-\frac{i\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^q\left(1-e^{2ic+2idx}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2b}{d}-q\right),-q,\frac{1}{2}\left(-\frac{2b}{d}-q+2\right),e^{2i(c}\right.}{2b+dq}\right.$$

input
```
Int[Sin[a + b*x]^2*Sin[c + d*x]^q,x]
```

output
```
2^(-2 - q)*(((-I)*E^((-I)*(2*a + c*q) - I*(2*b + d*q)*x + I*q*(c + d*x))*(
I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^q*Hypergeometric2F1[((-2*b)/d - q)/
2, -q, (2 - (2*b)/d - q)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)
*d*x))^q*(2*b + d*q)) + (I*E^(I*(2*a - c*q) + I*(2*b - d*q)*x + I*q*(c + d
*x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^q*Hypergeometric2F1[((2*b)/d
- q)/2, -q, (2 + (2*b)/d - q)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c +
(2*I)*d*x))^q*(2*b - d*q)) + ((2*I)*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x))
)^q*Hypergeometric2F1[-q, -1/2*q, 1 - q/2, E^((2*I)*(c + d*x))])/(d*(1 - E
^((2*I)*(c + d*x)))^q*q))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5064
```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c +
d*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a
, b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \sin(bx+a)^2 \sin(dx+c)^q \, dx$$

input
```
int(sin(b*x+a)^2*sin(d*x+c)^q,x)
```

output
```
int(sin(b*x+a)^2*sin(d*x+c)^q,x)
```

Fricas [F]

$$\int \sin^2(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q \sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^q,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sin(d*x + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^q(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*sin(d*x+c)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^2(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q \sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^q,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)^q*sin(b*x + a)^2, x)
```

Giac [F]

$$\int \sin^2(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q \sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(d*x+c)^q,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^q*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^q(c+dx)\,dx = \int \sin(a+b\,x)^2 \sin(c+d\,x)^q\,dx$$

input
```
int(sin(a + b*x)^2*sin(c + d*x)^q,x)
```

output
```
int(sin(a + b*x)^2*sin(c + d*x)^q, x)
```

Reduce [F]

$$\int \sin^2(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q \sin(bx+a)^2\,dx$$

input
```
int(sin(b*x+a)^2*sin(d*x+c)^q,x)
```

output
```
int(sin(c + d*x)**q*sin(a + b*x)**2,x)
```

3.143 $\int \sin(a+bx)\sin^q(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 230

$$\int \sin(a+bx)\sin^q(c+dx)\,dx = -\frac{e^{i(a-cq)+i(b-dq)x+iq(c+dx)}\left(1-e^{2i(c+dx)}\right)^{-q}\text{Hypergeometric2F1}\left(-q,\frac{b-dq}{2d},\frac{1}{2}\left(2+\frac{b}{d}-q\right),e^{2i(c+dx)}\right)\sin^q(c}{2(b-dq)}$$
$$-\frac{e^{-i(a+cq)-i(b+dq)x+iq(c+dx)}\left(1-e^{2i(c+dx)}\right)^{-q}\text{Hypergeometric2F1}\left(-q,-\frac{b+dq}{2d},-\frac{b-d(2-q)}{2d},e^{2i(c+dx)}\right)\sin^q(c}{2(b+dq)}$$

output
```
-1/2*exp(I*(-c*q+a)+I*(-d*q+b)*x+I*q*(d*x+c))*hypergeom([-q, 1/2*(-d*q+b)/
d],[1+1/2*b/d-1/2*q],exp(2*I*(d*x+c)))*sin(d*x+c)^q/((1-exp(2*I*(d*x+c)))^
q)/(-d*q+b)-1/2*exp(-I*(c*q+a)-I*(d*q+b)*x+I*q*(d*x+c))*hypergeom([-q, -1/
2*(d*q+b)/d],[-1/2*(b-d*(2-q))/d],exp(2*I*(d*x+c)))*sin(d*x+c)^q/((1-exp(2
*I*(d*x+c)))^q)/(d*q+b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.50 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.79

$$\int \sin(a+bx)\sin^q(c+dx)\,dx$$
$$= \frac{i2^{-1-q}e^{-i(a-c+(b-d)x)}\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^{1+q}\left((b-dq)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{b}{d}+q\right),-\right.\right.}{(b-dq)(b-}$$

input
```
Integrate[Sin[a + b*x]*Sin[c + d*x]^q,x]
```

output
```
(I*2^(-1 - q)*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^(1 + q)*
((b - d*q)*Hypergeometric2F1[1, (2 - b/d + q)/2, -1/2*(b + d*(-2 + q))/d,
E^((2*I)*(c + d*x))] + E^((2*I)*(a + b*x))*(b + d*q)*Hypergeometric2F1[1,
(b + d*(2 + q))/(2*d), (2 + b/d - q)/2, E^((2*I)*(c + d*x))]))/(E^(I*(a -
c + (b - d)*x))*(b - d*q)*(b + d*q))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5064, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^q(c+dx)\,dx$$
$$\Big\downarrow \text{5064}$$
$$2^{-q-1}\int \left(ie^{-ia-ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^q-ie^{ia+ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^q\right)dx$$
$$\Big\downarrow \text{2009}$$
$$2^{-q-1}\left(-\frac{\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^q\left(1-e^{2ic+2idx}\right)^{-q}\text{Hypergeometric2F1}\left(-q,\frac{b-dq}{2d},\frac{1}{2}\left(\frac{b}{d}-q+2\right),e^{2i(c+dx)}\right)\exp(}{b-dq}\right.$$

input
```
Int[Sin[a + b*x]*Sin[c + d*x]^q,x]
```

output
```
2^(-1 - q)*(-((E^(I*(a - c*q) + I*(b - d*q)*x + I*q*(c + d*x))*(I/E^(I*(c
+ d*x)) - I*E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, (b - d*q)/(2*d), (2 +
 b/d - q)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^q*(b - d
*q))) - (E^((-I)*(a + c*q) - I*(b + d*q)*x + I*q*(c + d*x))*(I/E^(I*(c + d
*x)) - I*E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, -1/2*(b + d*q)/d, 1 - (b
 + d*q)/(2*d), E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^q*(b +
 d*q)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5064
```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c +
d*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a
, b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \sin (bx+a)\sin (dx+c)^q\,dx$$

input
```
int(sin(b*x+a)*sin(d*x+c)^q,x)
```

output
```
int(sin(b*x+a)*sin(d*x+c)^q,x)
```

Fricas [F]

$$\int \sin(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q \sin(bx+a)\,dx$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^q,x, algorithm="fricas")
```

output
```
integral(sin(d*x + c)^q*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^q(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q \sin(bx+a)\,dx$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^q,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)^q*sin(b*x + a), x)
```

Giac [F]

$$\int \sin(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q \sin(bx+a)\,dx$$

input
```
integrate(sin(b*x+a)*sin(d*x+c)^q,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^q*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^q(c+dx)\,dx = \int \sin(a+b\,x)\,\sin(c+d\,x)^q\,dx$$

input
```
int(sin(a + b*x)*sin(c + d*x)^q,x)
```

output
```
int(sin(a + b*x)*sin(c + d*x)^q, x)
```

Reduce [F]

$$\int \sin(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q \sin(bx+a)\,dx$$

input
```
int(sin(b*x+a)*sin(d*x+c)^q,x)
```

output
```
int(sin(c + d*x)**q*sin(a + b*x),x)
```

3.144 $\int \csc(a+bx)\sin^q(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \csc(a+bx)\sin^q(c+dx)\,dx = \text{Int}(\csc(a+bx)\sin^q(c+dx),x)$$

output
```
Defer(Int)(csc(b*x+a)*sin(d*x+c)^q,x)
```

Mathematica [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\sin^q(c+dx)\,dx = \int \csc(a+bx)\sin^q(c+dx)\,dx$$

input
```
Integrate[Csc[a + b*x]*Sin[c + d*x]^q,x]
```

output
```
Integrate[Csc[a + b*x]*Sin[c + d*x]^q, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sin^q(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc(a+bx)\sin^q(c+dx)dx$$

input
```
Int[Csc[a + b*x]*Sin[c + d*x]^q,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc\left(bx+a\right)\sin\left(dx+c\right)^{q}\,dx$$

input
```
int(csc(b*x+a)*sin(d*x+c)^q,x)
```

output
```
int(csc(b*x+a)*sin(d*x+c)^q,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\sin^q(c+dx)\,dx = \int \sin\left(dx+c\right)^q \csc\left(bx+a\right)\,dx$$

input
```
integrate(csc(b*x+a)*sin(d*x+c)^q,x, algorithm="fricas")
```

output
```
integral(sin(d*x + c)^q*csc(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin^q(c+dx)\,dx = \int \sin^q\left(c+dx\right)\csc\left(a+bx\right)dx$$

input
```
integrate(csc(b*x+a)*sin(d*x+c)**q,x)
```

output
```
Integral(sin(c + d*x)**q*csc(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*sin(d*x+c)^q,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)^q*csc(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*sin(d*x+c)^q,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^q*csc(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 16.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \csc(a+bx)\sin^q(c+dx)\,dx = \int \frac{\sin(c+d\,x)^q}{\sin(a+b\,x)}\,dx$$

input
```
int(sin(c + d*x)^q/sin(a + b*x),x)
```

output
```
int(sin(c + d*x)^q/sin(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\sin^q(c+dx)\,dx=\int \sin(dx+c)^q\csc(bx+a)\,dx$$

input
```
int(csc(b*x+a)*sin(d*x+c)^q,x)
```

output
```
int(sin(c + d*x)**q*csc(a + b*x),x)
```

3.145 $\int \csc^2(a+bx)\sin^q(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \csc^2(a+bx)\sin^q(c+dx)\,dx = \text{Int}\left(\csc^2(a+bx)\sin^q(c+dx),x\right)$$

output
```
Defer(Int)(csc(b*x+a)^2*sin(d*x+c)^q,x)
```

Mathematica [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\sin^q(c+dx)\,dx = \int \csc^2(a+bx)\sin^q(c+dx)\,dx$$

input
```
Integrate[Csc[a + b*x]^2*Sin[c + d*x]^q,x]
```

output
```
Integrate[Csc[a + b*x]^2*Sin[c + d*x]^q, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sin^q(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \csc^2(a+bx)\sin^q(c+dx)dx$$

input
```
Int[Csc[a + b*x]^2*Sin[c + d*x]^q,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc(bx+a)^2 \sin(dx+c)^q\,dx$$

input
```
int(csc(b*x+a)^2*sin(d*x+c)^q,x)
```

output
```
int(csc(b*x+a)^2*sin(d*x+c)^q,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q \csc(bx+a)^2\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(d*x+c)^q,x, algorithm="fricas")
```

output
```
integral(sin(d*x + c)^q*csc(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 10.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sin^q(c+dx)\,dx = \int \sin^q(c+dx)\csc^2(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*sin(d*x+c)**q,x)
```

output
```
Integral(sin(c + d*x)**q*csc(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\sin^q(c+dx)\,dx = \int \sin{(dx+c)}^q \csc{(bx+a)}^2 \,dx$$

input
```
integrate(csc(b*x+a)^2*sin(d*x+c)^q,x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)^q*csc(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\sin^q(c+dx)\,dx = \int \sin{(dx+c)}^q \csc{(bx+a)}^2 \,dx$$

input
```
integrate(csc(b*x+a)^2*sin(d*x+c)^q,x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^q*csc(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 17.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\sin^q(c+dx)\,dx = \int \frac{\sin{(c+d\,x)}^q}{\sin{(a+b\,x)}^2}\,dx$$

input
```
int(sin(c + d*x)^q/sin(a + b*x)^2,x)
```

output
```
int(sin(c + d*x)^q/sin(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(a+bx)\sin^q(c+dx)\,dx = \int \sin(dx+c)^q \csc(bx+a)^2\,dx$$

input
```
int(csc(b*x+a)^2*sin(d*x+c)^q,x)
```

output
```
int(sin(c + d*x)**q*csc(a + b*x)**2,x)
```

3.146 $\int \cos^3(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \cos^3(c+bx)\sin(a+bx)\,dx = \frac{\cos(a-3c-2bx)}{16b} - \frac{3\cos(a+c+2bx)}{16b} - \frac{\cos(a+3c+4bx)}{32b} + \frac{3}{8}x\sin(a-c)$$

output
```
1/16*cos(-2*b*x+a-3*c)/b-3/16*cos(2*b*x+a+c)/b-1/32*cos(4*b*x+a+3*c)/b+3/8
*x*sin(a-c)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \cos^3(c+bx)\sin(a+bx)\,dx = -\frac{-2\cos(a-3c-2bx)+6\cos(a+c+2bx)+\cos(a+3c+4bx)-12bx\sin(a-c)}{32b}$$

input
```
Integrate[Cos[c + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/32*(-2*Cos[a - 3*c - 2*b*x] + 6*Cos[a + c + 2*b*x] + Cos[a + 3*c + 4*b*
x] - 12*b*x*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^3(bx+c)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{1}{8}\sin(a-2bx-3c)+\frac{3}{8}\sin(a+2bx+c)+\frac{1}{8}\sin(a+4bx+3c)+\frac{3}{8}\sin(a-c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos(a-2bx-3c)}{16b}-\frac{3\cos(a+2bx+c)}{16b}-\frac{\cos(a+4bx+3c)}{32b}+\frac{3}{8}x\sin(a-c)$$

input
```
Int[Cos[c + b*x]^3*Sin[a + b*x],x]
```

output
```
Cos[a - 3*c - 2*b*x]/(16*b) - (3*Cos[a + c + 2*b*x])/(16*b) - Cos[a + 3*c
+ 4*b*x]/(32*b) + (3*x*Sin[a - c])/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result
default	$\frac{\cos(-2bx+a-3c)}{16b}-\frac{3\cos(2bx+a+c)}{16b}-\frac{\cos(4bx+a+3c)}{32b}+\frac{3x\sin(a-c)}{8}$
risch	$\frac{\cos(-2bx+a-3c)}{16b}-\frac{3\cos(2bx+a+c)}{16b}-\frac{\cos(4bx+a+3c)}{32b}+\frac{3x\sin(a-c)}{8}$
parallelrisch	$\frac{12x\sin(a-c)b+2\cos(-2bx+a-3c)-6\cos(2bx+a+c)-\cos(4bx+a+3c)+5\cos(a-c)}{32b}$
orering	$x\cos(bx+c)^3\sin(bx+a)-\frac{5\left(-3\cos(bx+c)^2\sin(bx+a)b\sin(bx+c)+\cos(bx+c)^3b\cos(bx+a)\right)}{16b^2}+\frac{5x\left(6\cos(bx+\right.}{}$

input
```
int(cos(b*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/16*cos(-2*b*x+a-3*c)/b-3/16*cos(2*b*x+a+c)/b-1/32*cos(4*b*x+a+3*c)/b+3/8
*x*sin(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \cos^3(c+bx)\sin(a+bx)\,dx =$$
$$-\frac{2\cos(bx+c)^4\cos(-a+c)+3\,bx\sin(-a+c)+\left(2\cos(bx+c)^3+3\cos(bx+c)\right)\sin(bx+c)\sin(-}{8\,b}$$

input
```
integrate(cos(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(2*cos(b*x + c)^4*cos(-a + c) + 3*b*x*sin(-a + c) + (2*cos(b*x + c)^3
 + 3*cos(b*x + c))*sin(b*x + c)*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(54) = 108$.

Time = 0.85 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.82

$$\int \cos^3(c+bx)\sin(a+bx)\,dx$$
$$=\begin{cases} \frac{3x\sin{(a+bx)}\sin^2{(bx+c)}\cos{(bx+c)}}{8} + \frac{3x\sin{(a+bx)}\cos^3{(bx+c)}}{8} - \frac{3x\sin^3{(bx+c)}\cos{(a+bx)}}{8} - \frac{3x\sin{(bx+c)}\cos{(a+bx)}\cos^2{(bx+c)}}{8} \\ x\sin{(a)}\cos^3{(c)} \end{cases}$$

input
```
integrate(cos(b*x+c)**3*sin(b*x+a),x)
```

output
```
Piecewise((3*x*sin(a + b*x)*sin(b*x + c)**2*cos(b*x + c)/8 + 3*x*sin(a + b
*x)*cos(b*x + c)**3/8 - 3*x*sin(b*x + c)**3*cos(a + b*x)/8 - 3*x*sin(b*x +
 c)*cos(a + b*x)*cos(b*x + c)**2/8 - 3*sin(a + b*x)*sin(b*x + c)**3/(8*b)
- 3*sin(b*x + c)**2*cos(a + b*x)*cos(b*x + c)/(4*b) - 5*cos(a + b*x)*cos(b
*x + c)**3/(8*b), Ne(b, 0)), (x*sin(a)*cos(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \cos^3(c+bx)\sin(a+bx)\,dx = -\frac{12\,bx\sin{(-a+c)} + \cos{(4\,bx+a+3\,c)} + 6\,\cos{(2\,bx+a+c)} - 2\,\cos{(2\,bx-a+3\,c)}}{32\,b}$$

input
```
integrate(cos(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/32*(12*b*x*sin(-a + c) + cos(4*b*x + a + 3*c) + 6*cos(2*b*x + a + c) -
2*cos(2*b*x - a + 3*c))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \cos^3(c+bx)\sin(a+bx)\,dx = \frac{3}{8}\,x\sin\left(a-c\right) - \frac{\cos\left(4\,bx+a+3\,c\right)}{32\,b} - \frac{3\,\cos\left(2\,bx+a+c\right)}{16\,b} + \frac{\cos\left(-2\,bx+a-3\,c\right)}{16\,b}$$

input
```
integrate(cos(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
3/8*x*sin(a - c) - 1/32*cos(4*b*x + a + 3*c)/b - 3/16*cos(2*b*x + a + c)/b
 + 1/16*cos(-2*b*x + a - 3*c)/b
```

Mupad [B] (verification not implemented)

Time = 19.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \cos^3(c+bx)\sin(a+bx)\,dx = \frac{3\,x\,\sin\left(a-c\right)}{8} - \frac{\frac{\cos\left(a+3\,c+4\,b\,x\right)}{32} - \frac{\cos\left(3\,c-a+2\,b\,x\right)}{16} + \frac{3\,\cos\left(a+c+2\,b\,x\right)}{16}}{b}$$

input
```
int(cos(c + b*x)^3*sin(a + b*x),x)
```

output
```
(3*x*sin(a - c))/8 - (cos(a + 3*c + 4*b*x)/32 - cos(3*c - a + 2*b*x)/16 +
(3*cos(a + c + 2*b*x))/16)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

$$\int \cos^3(c+bx)\sin(a+bx)\,dx$$
$$= \frac{-\cos(bx+c)\cos(bx+a)\sin(bx+c)^2 - 5\cos(bx+c)\cos(bx+a) + 3\cos(bx+c)\sin(bx+a)\,bx - 3\,\mathrm{c}}{8b}$$

input
```
int(cos(b*x+c)^3*sin(b*x+a),x)
```

output
```
( - cos(b*x + c)*cos(a + b*x)*sin(b*x + c)**2 - 5*cos(b*x + c)*cos(a + b*x
) + 3*cos(b*x + c)*sin(a + b*x)*b*x - 3*cos(a + b*x)*sin(b*x + c)*b*x - 3*
sin(b*x + c)**3*sin(a + b*x) + 5)/(8*b)
```

3.147 $\int \cos^2(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \cos^2(c+bx)\sin(a+bx)\,dx=\frac{\cos(a-2c-bx)}{4b}-\frac{\cos(a+bx)}{2b}-\frac{\cos(a+2c+3bx)}{12b}$$

output
```
1/4*cos(-b*x+a-2*c)/b-1/2*cos(b*x+a)/b-1/12*cos(3*b*x+a+2*c)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \cos^2(c+bx)\sin(a+bx)\,dx =-\frac{-3\cos(a-2c-bx)+6\cos(a+bx)+\cos(a+2c+3bx)}{12b}$$

input
```
Integrate[Cos[c + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/12*(-3*Cos[a - 2*c - b*x] + 6*Cos[a + b*x] + Cos[a + 2*c + 3*b*x])/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^2(bx+c)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{1}{4}\sin(a-bx-2c)+\frac{1}{4}\sin(a+3bx+2c)+\frac{1}{2}\sin(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos(a-bx-2c)}{4b}-\frac{\cos(a+3bx+2c)}{12b}-\frac{\cos(a+bx)}{2b}$$

input
```
Int[Cos[c + b*x]^2*Sin[a + b*x],x]
```

output
```
Cos[a - 2*c - b*x]/(4*b) - Cos[a + b*x]/(2*b) - Cos[a + 2*c + 3*b*x]/(12*b
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

method	result
default	$\frac{\cos(-bx+a-2c)}{4b}-\frac{\cos(bx+a)}{2b}-\frac{\cos(3bx+a+2c)}{12b}$
risch	$\frac{\cos(-bx+a-2c)}{4b}-\frac{\cos(bx+a)}{2b}-\frac{\cos(3bx+a+2c)}{12b}$
parallelrisch	$\frac{-12+8\cos(a-c)+3\cos(-bx+a-2c)-\cos(3bx+a+2c)-6\cos(bx+a)}{12b}$
norman	$\frac{-\frac{4\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}}{b}-\frac{2\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^{4}}{b}-\frac{2}{3b}-\frac{4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}}{3b}+\frac{16\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)}{3b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}\right)^{2}\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}\right)}$
orering	$-\frac{10\left(-2\sin(bx+a)\sin(bx+c)b\cos(bx+c)+\cos(bx+c)^{2}b\cos(bx+a)\right)}{9b^{2}}-\frac{14b^{3}\sin(bx+a)\sin(bx+c)\cos(bx+c)-7b^{3}\cos(bx+a}{9b^{4}}$

input
```
int(cos(b*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*cos(-b*x+a-2*c)/b-1/2*cos(b*x+a)/b-1/12*cos(3*b*x+a+2*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \cos^2(c+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+c)^3\cos(-a+c)+\left(\cos(bx+c)^2+2\right)\sin(bx+c)\sin(-a+c)}{3\,b}$$

input
```
integrate(cos(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/3*(cos(b*x + c)^3*cos(-a + c) + (cos(b*x + c)^2 + 2)*sin(b*x + c)*sin(-
a + c))/b
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \cos^2(c+bx)\sin(a+bx)\,dx = \begin{cases} \frac{2\sin(a+bx)\sin(bx+c)\cos(bx+c)}{3b} - \frac{2\sin^2(bx+c)\cos(a+bx)}{3b} - \frac{\cos(a+bx)\cos^2(bx+c)}{3b} & \text{for } b \neq 0 \\ x\sin(a)\cos^2(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+c)**2*sin(b*x+a),x)
```

output
```
Piecewise((2*sin(a + b*x)*sin(b*x + c)*cos(b*x + c)/(3*b) - 2*sin(b*x + c)
**2*cos(a + b*x)/(3*b) - cos(a + b*x)*cos(b*x + c)**2/(3*b), Ne(b, 0)), (x
*sin(a)*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \cos^2(c+bx)\sin(a+bx)\,dx = -\frac{\cos(3\,bx+a+2\,c)+6\,\cos(bx+a)-3\,\cos(bx-a+2\,c)}{12\,b}$$

input
```
integrate(cos(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/12*(cos(3*b*x + a + 2*c) + 6*cos(b*x + a) - 3*cos(b*x - a + 2*c))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \cos^2(c+bx)\sin(a+bx)\,dx = -\frac{\cos(3\,bx+a+2\,c)}{12\,b} - \frac{\cos(bx+a)}{2\,b} + \frac{\cos(-bx+a-2\,c)}{4\,b}$$

input
```
integrate(cos(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/12*cos(3*b*x + a + 2*c)/b - 1/2*cos(b*x + a)/b + 1/4*cos(-b*x + a - 2*c
)/b
```

Mupad [B] (verification not implemented)

Time = 18.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \cos^2(c+bx)\sin(a+bx)\,dx = -\frac{\cos(a+2\,c+3\,b\,x)+6\,\cos(a+b\,x)-3\,\cos(2\,c-a+b\,x)}{12\,b}$$

input
```
int(cos(c + b*x)^2*sin(a + b*x),x)
```

output
```
-(cos(a + 2*c + 3*b*x) + 6*cos(a + b*x) - 3*cos(2*c - a + b*x))/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

$$\int \cos^2(c+bx)\sin(a+bx)\,dx = \frac{-2\cos(bx+c)\cos(bx+a)+2\cos(bx+c)\sin(bx+c)\sin(bx+a)-\cos(bx+a)\sin(bx+c)^2-\cos(b}{3b}$$

input
```
int(cos(b*x+c)^2*sin(b*x+a),x)
```

output
```
( - 2*cos(b*x + c)*cos(a + b*x) + 2*cos(b*x + c)*sin(b*x + c)*sin(a + b*x)
 - cos(a + b*x)*sin(b*x + c)**2 - cos(a + b*x) - 2*sin(b*x + c)*sin(a + b*
x) + 1)/(3*b)
```

3.148 $\int \cos(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \cos(c+bx)\sin(a+bx)\,dx = -\frac{\cos(a+c+2bx)}{4b}+\frac{1}{2}x\sin(a-c)$$

output
```
-1/4*cos(2*b*x+a+c)/b+1/2*x*sin(a-c)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \cos(c+bx)\sin(a+bx)\,dx = -\frac{\cos(a+c+2bx)-2bx\sin(a-c)}{4b}$$

input
```
Integrate[Cos[c + b*x]*Sin[a + b*x],x]
```

output
```
-1/4*(Cos[a + c + 2*b*x] - 2*b*x*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos(bx+c)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{1}{2}\sin(a+2bx+c)+\frac{1}{2}\sin(a-c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x\sin(a-c)-\frac{\cos(a+2bx+c)}{4b}$$

input
```
Int[Cos[c + b*x]*Sin[a + b*x],x]
```

output
```
-1/4*Cos[a + c + 2*b*x]/b + (x*Sin[a - c])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\cos(2bx+a+c)}{4b}+\frac{x\sin(a-c)}{2}$
risch	$-\frac{\cos(2bx+a+c)}{4b}+\frac{x\sin(a-c)}{2}$
parallelrisch	$\frac{2x\sin(a-c)b-\cos(2bx+a+c)+\cos(a-c)}{4b}$
orering	$x\cos\left(bx+c\right)\sin\left(bx+a\right)-\frac{-b\sin(bx+c)\sin(bx+a)+\cos(bx+c)b\cos(bx+a)}{4b^2}+\frac{x\left(-2\sin(bx+a)\cos(bx+c)b^2-2\cdots\right.}{4b^2}$
norman	$\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan\left(\frac{bx}{2}+\frac{c}{2}\right)-x\tan\left(\frac{bx}{2}+\frac{c}{2}\right)-x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2+\frac{2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)}{b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)}$

input
```
int(cos(b*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(2*b*x+a+c)/b+1/2*x*sin(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(23) = 46.

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int\cos(c+bx)\sin(a+bx)\,dx$$
$$=-\frac{\cos\left(bx+c\right)^2\cos\left(-a+c\right)+bx\sin\left(-a+c\right)+\cos\left(bx+c\right)\sin\left(bx+c\right)\sin\left(-a+c\right)}{2\,b}$$

input
```
integrate(cos(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cos(b*x + c)^2*cos(-a + c) + b*x*sin(-a + c) + cos(b*x + c)*sin(b*x
+ c)*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cos(c+bx)\sin(a+bx)\,dx = \begin{cases} \frac{x\sin(a+bx)\cos(bx+c)}{2} - \frac{x\sin(bx+c)\cos(a+bx)}{2} - \frac{\cos(a+bx)\cos(bx+c)}{2b} & \text{for } b\neq 0 \\ x\sin(a)\cos(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+c)*sin(b*x+a),x)
```

output
```
Piecewise((x*sin(a + b*x)*cos(b*x + c)/2 - x*sin(b*x + c)*cos(a + b*x)/2 -
 cos(a + b*x)*cos(b*x + c)/(2*b), Ne(b, 0)), (x*sin(a)*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cos(c+bx)\sin(a+bx)\,dx = -\frac{1}{2}\,x\sin(-a+c) - \frac{\cos(2\,bx+a+c)}{4\,b}$$

input
```
integrate(cos(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*x*sin(-a + c) - 1/4*cos(2*b*x + a + c)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cos(c+bx)\sin(a+bx)\,dx = \frac{1}{2}\,x\sin\left(a-c\right) - \frac{\cos\left(2\,bx+a+c\right)}{4\,b}$$

input
```
integrate(cos(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/2*x*sin(a - c) - 1/4*cos(2*b*x + a + c)/b
```

Mupad [B] (verification not implemented)

Time = 16.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \cos(c+bx)\sin(a+bx)\,dx = \begin{cases} x\,\cos\left(c\right)\,\sin\left(a\right) & \text{if}\ \ b=0 \\ \frac{x\,\sin(a-c)}{2}-\frac{\cos(a+c+2\,b\,x)}{4\,b} & \text{if}\ \ b\neq 0 \end{cases}$$

input
```
int(cos(c + b*x)*sin(a + b*x),x)
```

output
```
piecewise(b == 0, x*cos(c)*sin(a), b ~= 0, (x*sin(a - c))/2 - cos(a + c +
2*b*x)/(4*b))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int \cos(c+bx)\sin(a+bx)\,dx$$
$$= \frac{-\cos\left(bx+c\right)\cos\left(bx+a\right)+\cos\left(bx+c\right)\sin\left(bx+a\right)bx-\cos\left(bx+a\right)\sin\left(bx+c\right)bx}{2b}$$

input
```
int(cos(b*x+c)*sin(b*x+a),x)
```

output
```
( - cos(b*x + c)*cos(a + b*x) + cos(b*x + c)*sin(a + b*x)*b*x - cos(a + b*
x)*sin(b*x + c)*b*x)/(2*b)
```

3.149 $\int \sec(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \sec(c+bx)\sin(a+bx)\,dx = -\frac{\cos(a-c)\log(\cos(c+bx))}{b} + x\sin(a-c)$$

output
```
-cos(a-c)*ln(cos(b*x+c))/b+x*sin(a-c)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sec(c+bx)\sin(a+bx)\,dx = -\frac{\cos(a-c)\log(\cos(c+bx))}{b} + x\sin(a-c)$$

input
```
Integrate[Sec[c + b*x]*Sin[a + b*x],x]
```

output
```
-((Cos[a - c]*Log[Cos[c + b*x]])/b) + x*Sin[a - c]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5091, 24, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \text{5091}$$

$$\cos(a-c)\int \tan(c+bx)dx+\sin(a-c)\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\cos(a-c)\int \tan(c+bx)dx+x\sin(a-c)$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \tan(c+bx)dx+x\sin(a-c)$$

$$\Big\downarrow \text{3956}$$

$$x\sin(a-c)-\frac{\cos(a-c)\log(\cos(bx+c))}{b}$$

input
```
Int[Sec[c + b*x]*Sin[a + b*x],x]
```

output
```
-((Cos[a - c]*Log[Cos[c + b*x]])/b) + x*Sin[a - c]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.56

method	result
risch	$2i\cos\left(a-c\right)x-ix\,\mathrm{e}^{i(a-c)}+\frac{2i\cos(a-c)a}{b}-\frac{\ln\left(\mathrm{e}^{2i(bx+a)}+\mathrm{e}^{2i(a-c)}\right)\cos(a-c)}{b}$
default	$\frac{-\frac{(\cos(a)\cos(c)+\sin(a)\sin(c))\ln(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}{\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2}+\frac{\frac{(\cos(a)\cos(c)+\sin(a)\sin(c))\ln\left(\mathrm{t}\right.}{2}}{\left(\mathrm{co}\right.}}{b}$

input
```
int(sec(b*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*I*cos(a-c)*x-I*x*exp(I*(a-c))+2*I/b*cos(a-c)*a-ln(exp(2*I*(b*x+a))+exp(2
*I*(a-c)))/b*cos(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \sec(c+bx)\sin(a+bx)\,dx = -\frac{bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(-\cos\left(bx+c\right)\right)}{b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-(b*x*sin(-a + c) + cos(-a + c)*log(-cos(b*x + c)))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(20) = 40$.

Time = 4.83 (sec) , antiderivative size = 435, normalized size of antiderivative = 16.11

$$\int \sec(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a),x)
```

output
```
Piecewise((-x, Eq(c, pi/2)), (x, Eq(c, -pi/2)), (0, Eq(b, 0)), (-2*b*x*tan
(c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)
**2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - t
an(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b)
 - log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/2
)**2 + b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*t
an(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1)
- 1/(tan(c/2) + 1))/(b*tan(c/2)**2 + b), True))*cos(a) + Piecewise((-log(s
in(b*x))/b, Eq(c, pi/2)), (log(sin(b*x))/b, Eq(c, -pi/2)), (x/cos(c), Eq(b
, 0)), (-b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) + 2
*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) -
tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b) -
 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/(
b*tan(c/2)**2 + b), True))*sin(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(27) = 54.

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.70

$$\int \sec(c+bx)\sin(a+bx)\,dx =$$
$$-\frac{2\,bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\cos\left(2\,bx\right)^{2}+2\,\cos\left(2\,bx\right)\cos\left(2\,c\right)+\cos\left(2\,c\right)^{2}+\sin\left(2\,bx\right)^{2}-2\,\mathrm{si}}{2\,b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*b*x*sin(-a + c) + cos(-a + c)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(
2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(27) = 54.

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 5.85

$$\int \sec(c+bx)\sin(a+bx)\,dx$$
$$=\frac{\frac{4\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)-\tan\left(\frac{1}{2}\,c\right)\right)(bx+c)}{\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)^{2}+\tan\left(\frac{1}{2}\,c\right)^{2}+1}+\frac{\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,a\right)^{2}+4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)^{2}+\tan\left(\frac{1}{2}\,c\right)}}{2\,b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - t
an(1/2*c))*(b*x + c)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c
)^2 + 1) + (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/
2*c) - tan(1/2*c)^2 + 1)*log(tan(b*x + c)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^
2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1))/b
```

Mupad [B] (verification not implemented)

Time = 16.61 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.15

$$\int \sec(c+bx)\sin(a+bx)\,dx = x\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)+x\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}\,1\mathrm{i}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)-\frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}}{2}\right)}{b}$$

input
```
int(sin(a + b*x)/cos(c + b*x),x)
```

output
```
x*((exp(c*1i - a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2) + x*((exp(c*1i - a*1
i)*1i)/2 + (exp(a*1i - c*1i)*1i)/2) - (log(exp(a*2i - c*2i) + exp(a*2i + b
*x*2i))*(exp(c*1i - a*1i)/2 + exp(a*1i - c*1i)/2))/b
```

Reduce [F]

$$\int \sec(c+bx)\sin(a+bx)\,dx = \frac{\left(\int \frac{\sin(bx+a)}{\cos(bx+c)}dx\right)b-1}{b}$$

input
```
int(sec(b*x+c)*sin(b*x+a),x)
```

output
```
(int(sin(a + b*x)/cos(b*x + c),x)*b - 1)/b
```

3.150 $\int \sec^2(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \sec^2(c+bx)\sin(a+bx)\,dx=\frac{\cos(a-c)\sec(c+bx)}{b}+\frac{\operatorname{arctanh}(\sin(c+bx))\sin(a-c)}{b}$$

output
```
cos(a-c)*sec(b*x+c)/b+arctanh(sin(b*x+c))*sin(a-c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.59

$$\int \sec^2(c+bx)\sin(a+bx)\,dx \\ =\frac{\cos(a-c)\sec(c+bx)}{b} \\ -\frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b}$$

input
```
Integrate[Sec[c + b*x]^2*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x])/b - ((2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x
)/2]*Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*
Sin[c])]*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5091, 3042, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^2(bx+c)\,dx$$

$$\Big\downarrow \text{5091}$$

$$\sin(a-c)\int \sec(c+bx)dx+\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\cos(a-c)\int 1d\sec(c+bx)}{b}+\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{24}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\frac{\cos(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sin(a-c)\text{arctanh}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}$$

input
```
Int[Sec[c + b*x]^2*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.38

method	result
risch	$\frac{\mathrm{e}^{i(bx+3a)}+\mathrm{e}^{i(bx+a+2c)}}{b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a-c)}\right)\sin(a-c)}{b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a-c)}\right)\sin(a-c)}{b}$
default	$\frac{4(-2\cos(a)\sin(c)+2\sin(a)\cos(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+8\cos(a)\cos(c)+8\sin(a)\sin}{\left(-4\cos(c)^2\sin(a)^2-4\cos(a)^2\cos(c)^2-4\sin(a)^2\sin(c)^2-4\sin(c)^2\cos(a)^2\right)\left(\cos(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+\sin(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-2\cos(c)\sin(\right.}$

input
```
int(sec(b*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))+
ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*sin(a-c)-ln(exp(I*(b*x+a))-I*exp(I*(a-
c)))/b*sin(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int \sec^2(c+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+c)\log(\sin(bx+c)+1)\sin(-a+c) - \cos(bx+c)\log(-\sin(bx+c)+1)\sin(-a+c) - 2}{2\,b\cos(bx+c)}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cos(b*x + c)*log(sin(b*x + c) + 1)*sin(-a + c) - cos(b*x + c)*log(-s
in(b*x + c) + 1)*sin(-a + c) - 2*cos(-a + c))/(b*cos(b*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1448 vs. $2(27) = 54$.

Time = 95.38 (sec) , antiderivative size = 5545, normalized size of antiderivative = 163.09

$$\int \sec^2(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)**2*sin(b*x+a),x)
```

output
```
Piecewise((log(tan(b*x/2))/b, Eq(c, -pi/2) | Eq(c, pi/2)), (0, Eq(b, 0)),
(-2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)*
*3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/
2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(
tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**3/(b*ta
n(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b
*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 8*log(tan(b*x/2) - tan(c/2)/
(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*t
an(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*t
an(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) -
 1) - 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**
2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) -
 b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(ta
n(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*t
an(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2
*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3*
tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)*
*3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*log(tan
(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(c
/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(34) = 68$.

Time = 0.18 (sec) , antiderivative size = 387, normalized size of antiderivative = 11.38

$$\int \sec^2(c + bx)\sin(a + bx)\,dx$$
$$= \frac{2\left(\cos\left(bx + 2a\right) + \cos\left(bx + 2c\right)\right)\cos\left(2bx + a + 2c\right) + 2\cos\left(bx + 2a\right)\cos\left(a\right) + 2\cos\left(bx + 2c\right)\cos\left(a\right)}{}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*(cos(b*x + 2*a) + cos(b*x + 2*c))*cos(2*b*x + a + 2*c) + 2*cos(b*x
+ 2*a)*cos(a) + 2*cos(b*x + 2*c)*cos(a) + (cos(2*b*x + a + 2*c)^2*sin(-a +
 c) + 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(2*b*x + a + 2*c)^2*s
in(-a + c) + 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) + (cos(a)^2 + sin(a
)^2)*sin(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*
c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c
)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x +
2*c)*sin(c) + sin(c)^2)) + 2*(sin(b*x + 2*a) + sin(b*x + 2*c))*sin(2*b*x +
 a + 2*c) + 2*sin(b*x + 2*a)*sin(a) + 2*sin(b*x + 2*c)*sin(a))/(b*cos(2*b*
x + a + 2*c)^2 + 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^
2 + 2*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(34) = 68$.

Time = 0.15 (sec) , antiderivative size = 248, normalized size of antiderivative = 7.29

$$\int \sec^2(c+bx)\sin(a+bx)\,dx = \frac{2\left(\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)+1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1} - \frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+t\right.}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+t}\right.}{b}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
2*((tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1
/2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan
(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(
1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) - 1))/(ta
n(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*a)^2
*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)
/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b
*x + 1/2*c)^2 - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 21.60 (sec) , antiderivative size = 254, normalized size of antiderivative = 7.47

$$\int \sec^2(c+bx)\sin(a+bx)\,dx$$
$$= \frac{e^{a\,1i+b\,x\,1i}\left(e^{a\,2i-c\,2i}+1\right)}{b\left(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)} + \frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\left(e^{a\,2i}\,e^{-c\,2i}-1\right)1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}} - \frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\left(e^{a\,2i}\,e^{-c\,2i}-1\right)1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

input
```
int(sin(a + b*x)/cos(c + b*x)^2,x)
```

output
```
(exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) + 1))/(b*(exp(a*2i - c*2i) + exp(a*2
i + b*x*2i))) + (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i)
- (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*
2i))^(1/2))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log
(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) + (exp(a*2i)*exp(-c*
2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2
i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2))
```

Reduce [F]

$$\int \sec^2(c+bx)\sin(a+bx)\,dx = \int \sec(bx+c)^2\sin(bx+a)\,dx$$

input
```
int(sec(b*x+c)^2*sin(b*x+a),x)
```

output
```
int(sec(b*x + c)**2*sin(a + b*x),x)
```

3.151 $\int \sec^3(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sec^3(c+bx)\sin(a+bx)\,dx=\frac{\cos(a-c)\sec^2(c+bx)}{2b}+\frac{\sin(a-c)\tan(c+bx)}{b}$$

output
```
1/2*cos(a-c)*sec(b*x+c)^2/b+sin(a-c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \sec^3(c+bx)\sin(a+bx)\,dx=\frac{\sec(c)\sec^2(c+bx)(\cos(a)+\sin(a-c)\sin(c+2bx))}{2b}$$

input
```
Integrate[Sec[c + b*x]^3*Sin[a + b*x],x]
```

output
```
(Sec[c]*Sec[c + b*x]^2*(Cos[a] + Sin[a - c]*Sin[c + 2*b*x]))/(2*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5091, 3042, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^3(bx+c)\,dx$$

$$\Big\downarrow \text{5091}$$

$$\sin(a-c)\int \sec^2(c+bx)dx+\cos(a-c)\int \sec^2(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx+\cos(a-c)\int \sec(c+bx)^2\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\cos(a-c)\int \sec(c+bx)d\sec(c+bx)}{b}+\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{15}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx+\frac{\cos(a-c)\sec^2(bx+c)}{2b}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\cos(a-c)\sec^2(bx+c)}{2b}-\frac{\sin(a-c)\int 1d(-\tan(c+bx))}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(a-c)\tan(bx+c)}{b}+\frac{\cos(a-c)\sec^2(bx+c)}{2b}$$

input
```
Int[Sec[c + b*x]^3*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x]^2)/(2*b) + (Sin[a - c]*Tan[c + b*x])/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

method	result
parallelrisch	$\frac{1+\cos(2bx+2c)-2\cos(2bx+a+c)}{2b(1+\cos(2bx+2c))}$
risch	$\frac{2\,\mathrm{e}^{i(2bx+5a+c)}+\mathrm{e}^{i(5a-c)}-\mathrm{e}^{i(3a+c)}}{\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{2}b}$
default	$\frac{\frac{\cos(a)\cos(c)+\sin(a)\sin(c)}{2(\sin(a)\cos(c)-\cos(a)\sin(c))^{2}(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))^{2}}-\frac{}{(\sin(a)\cos(c)-\cos(a)\sin(c}}{b}$

input
```
int(sec(b*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(1+cos(2*b*x+2*c)-2*cos(2*b*x+a+c))/b/(1+cos(2*b*x+2*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \sec^3(c+bx)\sin(a+bx)\,dx = -\frac{2\,\cos\left(bx+c\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-\cos\left(-a+c\right)}{2\,b\cos\left(bx+c\right)^2}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*cos(b*x + c)*sin(b*x + c)*sin(-a + c) - cos(-a + c))/(b*cos(b*x +
c)^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \sec^3(c+bx)\sin(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+c)**3*sin(b*x+a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 391, normalized size of antiderivative = 10.29

$$\int \sec^3(c + bx) \sin(a + bx)\, dx = \frac{(2 \cos(2bx + 2a + 2c) + \cos(2a) - \cos(2c)) \cos(4bx + a + 5c) + 2(2 \cos(2bx + 2a + 2c) + \cos(2a}{b \cos(4bx + a + 5c)^2 + 4b \cos(2bx + a + 3c)^2 + 4b \cos(2}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
((2*cos(2*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(4*b*x + a + 5*c) + 2
*(2*cos(2*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(2*b*x + a + 3*c) + (
cos(2*a) - cos(2*c))*cos(a + c) + 2*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (2
*sin(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(4*b*x + a + 5*c) + 2*(2
*sin(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(2*b*x + a + 3*c) + (sin
(2*a) - sin(2*c))*sin(a + c) + 2*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos
(4*b*x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 + 4*b*cos(2*b*x + a + 3*c
)*cos(a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x +
 a + 3*c)^2 + 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(2*
b*cos(2*b*x + a + 3*c) + b*cos(a + c))*cos(4*b*x + a + 5*c) + 2*(2*b*sin(2
*b*x + a + 3*c) + b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 4.58

$$\int \sec^3(c + bx) \sin(a + bx)\, dx = \frac{\tan(bx + c)^2 \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 - \tan(bx + c)^2 \tan\left(\frac{1}{2}a\right)^2 + 4 \tan(bx + c)^2 \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right) + 4 \tan}{}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
1/2*(tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + c)^2*tan(1/2*a)^
2 + 4*tan(b*x + c)^2*tan(1/2*a)*tan(1/2*c) + 4*tan(b*x + c)*tan(1/2*a)^2*t
an(1/2*c) - tan(b*x + c)^2*tan(1/2*c)^2 - 4*tan(b*x + c)*tan(1/2*a)*tan(1/
2*c)^2 + tan(b*x + c)^2 + 4*tan(b*x + c)*tan(1/2*a) - 4*tan(b*x + c)*tan(1
/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/cos(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \sec^3(c+bx)\sin(a+bx)\,dx = \frac{\cos(bx+c)\cos(bx+a)-\sin(bx+c)\sin(bx+a)}{2b\left(\sin(bx+c)^2-1\right)}$$

input
```
int(sec(b*x+c)^3*sin(b*x+a),x)
```

output
```
(cos(b*x + c)*cos(a + b*x) - sin(b*x + c)*sin(a + b*x))/(2*b*(sin(b*x + c)
**2 - 1))
```

3.152 $\int \sec^4(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \sec^4(c+bx)\sin(a+bx)\,dx=\frac{\cos(a-c)\sec^3(c+bx)}{3b}+\frac{\operatorname{arctanh}(\sin(c+bx))\sin(a-c)}{2b}+\frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b}$$

output
```
1/3*cos(a-c)*sec(b*x+c)^3/b+1/2*arctanh(sin(b*x+c))*sin(a-c)/b+1/2*sec(b*x
+c)*sin(a-c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \sec^4(c+bx)\sin(a+bx)\,dx=\frac{12\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a-c)+\sec^3(c+bx)(4\cos(a-c)+3\sin(a-c)\sin(2(c+bx)))}{12b}$$

input
```
Integrate[Sec[c + b*x]^4*Sin[a + b*x],x]
```

output
```
(12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c] + Sec[c + b*x]^3*(4*C
os[a - c] + 3*Sin[a - c]*Sin[2*(c + b*x)]))/(12*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5091, 3042, 3086, 15, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^4(bx+c)\,dx$$

$$\Big\downarrow \text{5091}$$

$$\sin(a-c)\int \sec^3(c+bx)dx+\cos(a-c)\int \sec^3(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx+\cos(a-c)\int \sec(c+bx)^3\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\cos(a-c)\int \sec^2(c+bx)d\sec(c+bx)}{b}+\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{15}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx+\frac{\cos(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4255}$$

$$\sin(a-c)\left(\frac{1}{2}\int \sec(c+bx)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\cos(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\left(\frac{1}{2}\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\cos(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4257}$$

$$\sin(a-c)\left(\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\cos(a-c)\sec^3(bx+c)}{3b}$$

input
```
Int[Sec[c + b*x]^4*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x]^3)/(3*b) + Sin[a - c]*(ArcTanh[Sin[c + b*x]]/(2*b
) + (Sec[c + b*x]*Tan[c + b*x])/(2*b))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.85

method	result
risch	$\frac{-3\,\mathrm{e}^{i(5bx+7a+4c)}+3\,\mathrm{e}^{i(5bx+5a+6c)}+8\,\mathrm{e}^{i(3bx+7a+2c)}+8\,\mathrm{e}^{i(3bx+5a+4c)}+3\,\mathrm{e}^{i(bx+7a)}-3\,\mathrm{e}^{i(bx+5a+2c)}}{6b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^3}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a-c)}\right)\sin(a}{2b}$
default	Expression too large to display

input
```
int(sec(b*x+c)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))^3*(-3*exp(I*(5*b*x+7*a+4*c))+3*exp(I
*(5*b*x+5*a+6*c))+8*exp(I*(3*b*x+7*a+2*c))+8*exp(I*(3*b*x+5*a+4*c))+3*exp(
I*(b*x+7*a))-3*exp(I*(b*x+5*a+2*c)))+1/2*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))
/b*sin(a-c)-1/2*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \sec^4(c+bx)\sin(a+bx)\,dx =$$
$$-\frac{3\,\cos\left(bx+c\right)^3\log\left(\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right)-3\,\cos\left(bx+c\right)^3\log\left(-\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right.}{12\,b\cos\left(bx+c\right)^3}$$

input
```
integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/12*(3*cos(b*x + c)^3*log(sin(b*x + c) + 1)*sin(-a + c) - 3*cos(b*x + c)
^3*log(-sin(b*x + c) + 1)*sin(-a + c) + 6*cos(b*x + c)*sin(b*x + c)*sin(-a
 + c) - 4*cos(-a + c))/(b*cos(b*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+c)**4*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1424 vs. $2(61) = 122$.

Time = 0.21 (sec) , antiderivative size = 1424, normalized size of antiderivative = 21.25

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/12*(2*(3*cos(5*b*x + 2*a + 4*c) - 3*cos(5*b*x + 6*c) - 8*cos(3*b*x + 2*
a + 2*c) - 8*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) + 3*cos(b*x + 2*c))*cos(6
*b*x + a + 6*c) + 6*(3*cos(4*b*x + a + 4*c) + 3*cos(2*b*x + a + 2*c) + cos
(a))*cos(5*b*x + 2*a + 4*c) - 6*(3*cos(4*b*x + a + 4*c) + 3*cos(2*b*x + a
+ 2*c) + cos(a))*cos(5*b*x + 6*c) - 6*(8*cos(3*b*x + 2*a + 2*c) + 8*cos(3*
b*x + 4*c) + 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) - 1
6*(3*cos(2*b*x + a + 2*c) + cos(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*cos(2*b
*x + a + 2*c) + cos(a))*cos(3*b*x + 4*c) - 18*(cos(b*x + 2*a) - cos(b*x +
2*c))*cos(2*b*x + a + 2*c) - 6*cos(b*x + 2*a)*cos(a) + 6*cos(b*x + 2*c)*co
s(a) - 3*(cos(6*b*x + a + 6*c)^2*sin(-a + c) + 9*cos(4*b*x + a + 4*c)^2*si
n(-a + c) + 9*cos(2*b*x + a + 2*c)^2*sin(-a + c) + 6*cos(2*b*x + a + 2*c)*
cos(a)*sin(-a + c) + sin(6*b*x + a + 6*c)^2*sin(-a + c) + 9*sin(4*b*x + a
+ 4*c)^2*sin(-a + c) + 9*sin(2*b*x + a + 2*c)^2*sin(-a + c) + 6*sin(2*b*x
+ a + 2*c)*sin(a)*sin(-a + c) + 2*(3*cos(4*b*x + a + 4*c)*sin(-a + c) + 3*
cos(2*b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(6*b*x + a + 6*c
) + 6*(3*cos(2*b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(4*b*x
+ a + 4*c) + 2*(3*sin(4*b*x + a + 4*c)*sin(-a + c) + 3*sin(2*b*x + a + 2*c
)*sin(-a + c) + sin(a)*sin(-a + c))*sin(6*b*x + a + 6*c) + 6*(3*sin(2*b*x
+ a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(4*b*x + a + 4*c) + (cos(a
)^2 + sin(a)^2)*sin(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(61) = 122$.

Time = 0.14 (sec) , antiderivative size = 495, normalized size of antiderivative = 7.39

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

output
```
1/3*(3*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - t
an(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 +
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 3*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a
)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) - 1
))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(3*ta
n(1/2*b*x + 1/2*c)^5*tan(1/2*a)^2*tan(1/2*c) - 3*tan(1/2*b*x + 1/2*c)^5*ta
n(1/2*a)*tan(1/2*c)^2 - 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2*tan(1/2*c)^2
 + 3*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a) + 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*
a)^2 - 3*tan(1/2*b*x + 1/2*c)^5*tan(1/2*c) - 12*tan(1/2*b*x + 1/2*c)^4*tan
(1/2*a)*tan(1/2*c) + 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*c)^2 - 3*tan(1/2*b*x
 + 1/2*c)^4 - 3*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c) + 3*tan(1/2*b
*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(1/
2*b*x + 1/2*c)*tan(1/2*a) + tan(1/2*a)^2 + 3*tan(1/2*b*x + 1/2*c)*tan(1/2*
c) - 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1)/((tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b*x + 1/2*c)^2 - 1)^3))/b
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/cos(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \int \sec(bx+c)^4\sin(bx+a)\,dx$$

input
```
int(sec(b*x+c)^4*sin(b*x+a),x)
```

output
```
int(sec(b*x + c)**4*sin(a + b*x),x)
```

3.153 $\int \sec^5(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \sec^5(c+bx)\sin(a+bx)\,dx=\frac{\cos(a-c)\sec^4(c+bx)}{4b}+\frac{\sin(a-c)\tan(c+bx)}{b}+\frac{\sin(a-c)\tan^3(c+bx)}{3b}$$

output
```
1/4*cos(a-c)*sec(b*x+c)^4/b+sin(a-c)*tan(b*x+c)/b+1/3*sin(a-c)*tan(b*x+c)^
3/b
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \sec^5(c+bx)\sin(a+bx)\,dx=\frac{\sec(c)\sec^4(c+bx)(3\cos(a)+\sin(a-c)(4\sin(c+2bx)+\sin(3c+4bx)))}{12b}$$

input
```
Integrate[Sec[c + b*x]^5*Sin[a + b*x],x]
```

output
```
(Sec[c]*Sec[c + b*x]^4*(3*Cos[a] + Sin[a - c]*(4*Sin[c + 2*b*x] + Sin[3*c
+ 4*b*x])))/(12*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5091, 3042, 3086, 15, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^5(bx+c)\,dx$$

$$\Big\downarrow \text{ 5091}$$

$$\sin(a-c)\int \sec^4(c+bx)dx+\cos(a-c)\int \sec^4(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^4dx+\cos(a-c)\int \sec(c+bx)^4\tan(c+bx)dx$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{\cos(a-c)\int \sec^3(c+bx)d\sec(c+bx)}{b}+\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{ 15}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^4dx+\frac{\cos(a-c)\sec^4(bx+c)}{4b}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{\cos(a-c)\sec^4(bx+c)}{4b}-\frac{\sin(a-c)\int \left(\tan^2(c+bx)+1\right)d(-\tan(c+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\cos(a-c)\sec^4(bx+c)}{4b}-\frac{\sin(a-c)\left(-\frac{1}{3}\tan^3(bx+c)-\tan(bx+c)\right)}{b}$$

input
```
Int[Sec[c + b*x]^5*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x]^4)/(4*b) - (Sin[a - c]*(-Tan[c + b*x] - Tan[c + b
*x]^3/3))/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [A] (verified)

Time = 7.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

method	result
parallelrisch	$\frac{-5\cos(4bx+4c)-20\cos(2bx+2c)-15-32\cos(2bx+a+c)-8\cos(4bx+a+3c)}{12b(\cos(4bx+4c)+3+4\cos(2bx+2c))}$
risch	$\frac{4\,\mathrm{e}^{i(4bx+9a+3c)}+\frac{8\,\mathrm{e}^{i(2bx+9a+c)}}{3}-\frac{8\,\mathrm{e}^{i(2bx+7a+3c)}}{3}+\frac{2\,\mathrm{e}^{i(9a-c)}}{3}-\frac{2\,\mathrm{e}^{i(7a+c)}}{3}}{\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^4 b}$
default	$-\frac{1}{(\sin(a)\cos(c)-\cos(a)\sin(c))^4(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}+\frac{}{2(\sin(a)\cos(c)-\cos(a)\sin(c}$

input
```
int(sec(b*x+c)^5*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-5*cos(4*b*x+4*c)-20*cos(2*b*x+2*c)-15-32*cos(2*b*x+a+c)-8*cos(4*b*x
+a+3*c))/b/(cos(4*b*x+4*c)+3+4*cos(2*b*x+2*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \sec^5(c+bx)\sin(a+bx)\,dx = -\frac{4\left(2\cos(bx+c)^3+\cos(bx+c)\right)\sin(bx+c)\sin(-a+c)-3\cos(-a+c)}{12\,b\cos(bx+c)^4}$$

input
```
integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/12*(4*(2*cos(b*x + c)^3 + cos(b*x + c))*sin(b*x + c)*sin(-a + c) - 3*co
s(-a + c))/(b*cos(b*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^5(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+c)**5*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(55) = 110$.

Time = 0.05 (sec) , antiderivative size = 1074, normalized size of antiderivative = 18.20

$$\int \sec^5(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="maxima")
```

output
```
2/3*((6*cos(4*b*x + 2*a + 4*c) + 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x +
4*c) + cos(2*a) - cos(2*c))*cos(8*b*x + a + 9*c) + 4*(6*cos(4*b*x + 2*a +
4*c) + 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4*c) + cos(2*a) - cos(2*c)
)*cos(6*b*x + a + 7*c) + 6*(4*cos(2*b*x + a + 3*c) + cos(a + c))*cos(4*b*x
 + 2*a + 4*c) + 6*(6*cos(4*b*x + 2*a + 4*c) + 4*cos(2*b*x + 2*a + 2*c) - 4
*cos(2*b*x + 4*c) + cos(2*a) - cos(2*c))*cos(4*b*x + a + 5*c) + 4*(4*cos(2
*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(2*b*x + a + 3*c) - 4*(4*cos(2
*b*x + a + 3*c) + cos(a + c))*cos(2*b*x + 4*c) + (cos(2*a) - cos(2*c))*cos
(a + c) + 4*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (6*sin(4*b*x + 2*a + 4*c)
+ 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) - sin(2*c))*sin
(8*b*x + a + 9*c) + 4*(6*sin(4*b*x + 2*a + 4*c) + 4*sin(2*b*x + 2*a + 2*c)
 - 4*sin(2*b*x + 4*c) + sin(2*a) - sin(2*c))*sin(6*b*x + a + 7*c) + 6*(4*s
in(2*b*x + a + 3*c) + sin(a + c))*sin(4*b*x + 2*a + 4*c) + 6*(6*sin(4*b*x
+ 2*a + 4*c) + 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) -
sin(2*c))*sin(4*b*x + a + 5*c) + 4*(4*sin(2*b*x + 2*a + 2*c) + sin(2*a) -
sin(2*c))*sin(2*b*x + a + 3*c) - 4*(4*sin(2*b*x + a + 3*c) + sin(a + c))*s
in(2*b*x + 4*c) + (sin(2*a) - sin(2*c))*sin(a + c) + 4*sin(2*b*x + 2*a + 2
*c)*sin(a + c))/(b*cos(8*b*x + a + 9*c)^2 + 16*b*cos(6*b*x + a + 7*c)^2 +
36*b*cos(4*b*x + a + 5*c)^2 + 16*b*cos(2*b*x + a + 3*c)^2 + 8*b*cos(2*b*x
+ a + 3*c)*cos(a + c) + b*cos(a + c)^2 + b*sin(8*b*x + a + 9*c)^2 + 16*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(55) = 110$.

Time = 0.15 (sec) , antiderivative size = 327, normalized size of antiderivative = 5.54

$$\int \sec^5(c+bx)\sin(a+bx)\,dx$$
$$= \frac{3\tan(bx+c)^4\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2 - 3\tan(bx+c)^4\tan\left(\frac{1}{2}a\right)^2 + 12\tan(bx+c)^4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right) +}{}$$

input
```
integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="giac")
```

output
```
1/12*(3*tan(b*x + c)^4*tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(b*x + c)^4*tan(1/
2*a)^2 + 12*tan(b*x + c)^4*tan(1/2*a)*tan(1/2*c) + 8*tan(b*x + c)^3*tan(1/
2*a)^2*tan(1/2*c) - 3*tan(b*x + c)^4*tan(1/2*c)^2 - 8*tan(b*x + c)^3*tan(1
/2*a)*tan(1/2*c)^2 + 6*tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(b*
x + c)^4 + 8*tan(b*x + c)^3*tan(1/2*a) - 6*tan(b*x + c)^2*tan(1/2*a)^2 - 8
*tan(b*x + c)^3*tan(1/2*c) + 24*tan(b*x + c)^2*tan(1/2*a)*tan(1/2*c) + 24*
tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c) - 6*tan(b*x + c)^2*tan(1/2*c)^2 - 24*
tan(b*x + c)*tan(1/2*a)*tan(1/2*c)^2 + 6*tan(b*x + c)^2 + 24*tan(b*x + c)*
tan(1/2*a) - 24*tan(b*x + c)*tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan
(1/2*a)^2 + tan(1/2*c)^2 + 1)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^5(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/cos(c + b*x)^5,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.95

$$\int \sec^5(c+bx)\sin(a+bx)\,dx$$
$$= \frac{8\cos(bx+c)\cos(bx+a)\sin(bx+c)^2 - 10\cos(bx+c)\cos(bx+a) + 3\sin(bx+c)^4 - 8\sin(bx+c)^3 \text{si}}{24b\left(\sin(bx+c)^4 - 2\sin(bx+c)^2 + 1\right)}$$

input
```
int(sec(b*x+c)^5*sin(b*x+a),x)
```

output
```
(8*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)**2 - 10*cos(b*x + c)*cos(a + b*x
) + 3*sin(b*x + c)**4 - 8*sin(b*x + c)**3*sin(a + b*x) - 6*sin(b*x + c)**2
 + 14*sin(b*x + c)*sin(a + b*x) + 3)/(24*b*(sin(b*x + c)**4 - 2*sin(b*x +
c)**2 + 1))
```

3.154 $\int \sec^6(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \frac{\cos(a-c)\sec^5(c+bx)}{5b} + \frac{3\text{arctanh}(\sin(c+bx))\sin(a-c)}{8b} + \frac{3\sec(c+bx)\sin(a-c)\tan(c+bx)}{8b} + \frac{\sec^3(c+bx)\sin(a-c)\tan(c+bx)}{4b}$$

output
```
1/5*cos(a-c)*sec(b*x+c)^5/b+3/8*arctanh(sin(b*x+c))*sin(a-c)/b+3/8*sec(b*x
+c)*sin(a-c)*tan(b*x+c)/b+1/4*sec(b*x+c)^3*sin(a-c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \frac{480\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a-c)+2\sec^5(c+bx)(64\cos(a-c)+5\sin(a-c)(14\sin(2(c\cdots}{640b}$$

input
```
Integrate[Sec[c + b*x]^6*Sin[a + b*x],x]
```

output
```
(480*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c] + 2*Sec[c + b*x]^5*(
64*Cos[a - c] + 5*Sin[a - c]*(14*Sin[2*(c + b*x)] + 3*Sin[4*(c + b*x)])))/
(640*b)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5091, 3042, 3086, 15, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^6(bx+c)\,dx$$

$$\Big\downarrow \text{5091}$$

$$\sin(a-c)\int \sec^5(c+bx)dx+\cos(a-c)\int \sec^5(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^5dx+\cos(a-c)\int \sec(c+bx)^5\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\cos(a-c)\int \sec^4(c+bx)d\sec(c+bx)}{b}+\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{15}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^5dx+\frac{\cos(a-c)\sec^5(bx+c)}{5b}$$

$$\Big\downarrow \text{4255}$$

$$\sin(a-c)\left(\frac{3}{4}\int \sec^3(c+bx)dx+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)+\frac{\cos(a-c)\sec^5(bx+c)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\left(\frac{3}{4}\int\csc\left(c+bx+\frac{\pi}{2}\right)^3dx+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)+\frac{\cos(a-c)\sec^5(bx+c)}{5b}$$

$\Big\downarrow$ 4255

$$\sin(a-c)\left(\frac{3}{4}\left(\frac{1}{2}\int\sec(c+bx)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)+\frac{\cos(a-c)\sec^5(bx+c)}{5b}$$

$\Big\downarrow$ 3042

$$\sin(a-c)\left(\frac{3}{4}\left(\frac{1}{2}\int\csc\left(c+bx+\frac{\pi}{2}\right)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)+\frac{\cos(a-c)\sec^5(bx+c)}{5b}$$

$\Big\downarrow$ 4257

$$\sin(a-c)\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(bx+c))}{2b}+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)+\frac{\cos(a-c)\sec^5(bx+c)}{5b}$$

input
```
Int[Sec[c + b*x]^6*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x]^5)/(5*b) + Sin[a - c]*((Sec[c + b*x]^3*Tan[c + b*
x])/(4*b) + (3*(ArcTanh[Sin[c + b*x]]/(2*b) + (Sec[c + b*x]*Tan[c + b*x])/
(2*b)))/4)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 59.02 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.76

method	result
risch	$\frac{-15\,e^{i(9bx+11a+8c)}+15\,e^{i(9bx+9a+10c)}-70\,e^{i(7bx+11a+6c)}+70\,e^{i(7bx+9a+8c)}+128\,e^{i(5bx+11a+4c)}+128\,e^{i(5bx+9a+6c)}+70\,e^{i(3bx+1}}{40b\left(e^{2i(bx+a+c)}+e^{2ia}\right)^5}$
default	Expression too large to display

input
```
int(sec(b*x+c)^6*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/40/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))^5*(-15*exp(I*(9*b*x+11*a+8*c))+15*e
xp(I*(9*b*x+9*a+10*c))-70*exp(I*(7*b*x+11*a+6*c))+70*exp(I*(7*b*x+9*a+8*c)
)+128*exp(I*(5*b*x+11*a+4*c))+128*exp(I*(5*b*x+9*a+6*c))+70*exp(I*(3*b*x+1
1*a+2*c))-70*exp(I*(3*b*x+9*a+4*c))+15*exp(I*(b*x+11*a))-15*exp(I*(b*x+9*a
+2*c)))+3/8*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*sin(a-c)-3/8*ln(exp(I*(b*x
+a))-I*exp(I*(a-c)))/b*sin(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = -\frac{15\,\cos{(bx+c)}^5\log{(\sin{(bx+c)}+1)}\sin{(-a+c)} - 15\,\cos{(bx+c)}^5\log{(-\sin{(bx+c)}+1)}\sin{(-a+}}{80\,b\cos{(bx+c)}^5}$$

input
```
integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/80*(15*cos(b*x + c)^5*log(sin(b*x + c) + 1)*sin(-a + c) - 15*cos(b*x +
c)^5*log(-sin(b*x + c) + 1)*sin(-a + c) + 10*(3*cos(b*x + c)^3 + 2*cos(b*x
 + c))*sin(b*x + c)*sin(-a + c) - 16*cos(-a + c))/(b*cos(b*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+c)**6*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3096 vs. 2(86) = 172.

Time = 0.31 (sec) , antiderivative size = 3096, normalized size of antiderivative = 32.94

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/80*(2*(15*cos(9*b*x + 2*a + 8*c) - 15*cos(9*b*x + 10*c) + 70*cos(7*b*x
+ 2*a + 6*c) - 70*cos(7*b*x + 8*c) - 128*cos(5*b*x + 2*a + 4*c) - 128*cos(
5*b*x + 6*c) - 70*cos(3*b*x + 2*a + 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*
x + 2*a) + 15*cos(b*x + 2*c))*cos(10*b*x + a + 10*c) + 30*(5*cos(8*b*x + a
 + 8*c) + 10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x
+ a + 2*c) + cos(a))*cos(9*b*x + 2*a + 8*c) - 30*(5*cos(8*b*x + a + 8*c) +
 10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c
) + cos(a))*cos(9*b*x + 10*c) + 10*(70*cos(7*b*x + 2*a + 6*c) - 70*cos(7*b
*x + 8*c) - 128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*x + 6*c) - 70*cos(3*b
*x + 2*a + 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*x + 2*a) + 15*cos(b*x + 2
*c))*cos(8*b*x + a + 8*c) + 140*(10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x +
a + 4*c) + 5*cos(2*b*x + a + 2*c) + cos(a))*cos(7*b*x + 2*a + 6*c) - 140*(
10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c)
 + cos(a))*cos(7*b*x + 8*c) - 20*(128*cos(5*b*x + 2*a + 4*c) + 128*cos(5*b
*x + 6*c) + 70*cos(3*b*x + 2*a + 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x +
 2*a) - 15*cos(b*x + 2*c))*cos(6*b*x + a + 6*c) - 256*(10*cos(4*b*x + a +
4*c) + 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 2*a + 4*c) - 256*(10*c
os(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 6*c) -
100*(14*cos(3*b*x + 2*a + 2*c) - 14*cos(3*b*x + 4*c) + 3*cos(b*x + 2*a) -
3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) - 140*(5*cos(2*b*x + a + 2*c) + ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 756 vs. $2(86) = 172$.

Time = 0.15 (sec) , antiderivative size = 756, normalized size of antiderivative = 8.04

$$\int \sec^6(c + bx)\sin(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="giac")
```

output
```
1/20*(15*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) -
 tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2
 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 15*(tan(1/2*a)^2*tan(1/2*c) - tan(1/
2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c)
- 1))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(2
5*tan(1/2*b*x + 1/2*c)^9*tan(1/2*a)^2*tan(1/2*c) - 25*tan(1/2*b*x + 1/2*c)
^9*tan(1/2*a)*tan(1/2*c)^2 - 20*tan(1/2*b*x + 1/2*c)^8*tan(1/2*a)^2*tan(1/
2*c)^2 + 25*tan(1/2*b*x + 1/2*c)^9*tan(1/2*a) + 20*tan(1/2*b*x + 1/2*c)^8*
tan(1/2*a)^2 - 25*tan(1/2*b*x + 1/2*c)^9*tan(1/2*c) - 80*tan(1/2*b*x + 1/2
*c)^8*tan(1/2*a)*tan(1/2*c) - 10*tan(1/2*b*x + 1/2*c)^7*tan(1/2*a)^2*tan(1
/2*c) + 20*tan(1/2*b*x + 1/2*c)^8*tan(1/2*c)^2 + 10*tan(1/2*b*x + 1/2*c)^7
*tan(1/2*a)*tan(1/2*c)^2 - 20*tan(1/2*b*x + 1/2*c)^8 - 10*tan(1/2*b*x + 1/
2*c)^7*tan(1/2*a) + 10*tan(1/2*b*x + 1/2*c)^7*tan(1/2*c) - 40*tan(1/2*b*x
+ 1/2*c)^4*tan(1/2*a)^2*tan(1/2*c)^2 + 40*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a
)^2 - 160*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)*tan(1/2*c) + 10*tan(1/2*b*x +
1/2*c)^3*tan(1/2*a)^2*tan(1/2*c) + 40*tan(1/2*b*x + 1/2*c)^4*tan(1/2*c)^2
- 10*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)*tan(1/2*c)^2 - 40*tan(1/2*b*x + 1/2
*c)^4 + 10*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a) - 10*tan(1/2*b*x + 1/2*c)^3*t
an(1/2*c) - 25*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c) + 25*tan(1/2*b
*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 - 4*tan(1/2*a)^2*tan(1/2*c)^2 - 25*...
```

Mupad [F(-1)]

Timed out.

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/cos(c + b*x)^6,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \int \sec\left(bx+c\right)^6\sin\left(bx+a\right)dx$$

input
```
int(sec(b*x+c)^6*sin(b*x+a),x)
```

output
```
int(sec(b*x + c)**6*sin(a + b*x),x)
```

3.155 $\int \cos^3(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 103

$$\int \cos^3(c+bx)\sin^2(a+bx)\,dx = \frac{\sin(2a-3c-bx)}{16b} - \frac{3\sin(2a-c+bx)}{16b} + \frac{3\sin(c+bx)}{8b} - \frac{\sin(2a+c+3bx)}{16b} + \frac{\sin(3c+3bx)}{24b} - \frac{\sin(2a+3c+5bx)}{80b}$$

output
```
1/16*sin(-b*x+2*a-3*c)/b-3/16*sin(b*x+2*a-c)/b+3/8*sin(b*x+c)/b-1/16*sin(3
*b*x+2*a+c)/b+1/24*sin(3*b*x+3*c)/b-1/80*sin(5*b*x+2*a+3*c)/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

$$\int \cos^3(c+bx)\sin^2(a+bx)\,dx = \frac{15\sin(2a-3c-bx) - 45\sin(2a-c+bx) + 90\sin(c+bx) + 10\sin(3(c+bx)) - 15\sin(2a+c+3bx) -}{240b}$$

input
```
Integrate[Cos[c + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(15*Sin[2*a - 3*c - b*x] - 45*Sin[2*a - c + b*x] + 90*Sin[c + b*x] + 10*Si
n[3*(c + b*x)] - 15*Sin[2*a + c + 3*b*x] - 3*Sin[2*a + 3*c + 5*b*x])/(240*
b)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^3(bx+c)\,dx$$

$$\Big\downarrow \; 5085$$

$$\int \left(-\frac{1}{16}\cos(2a-bx-3c)-\frac{3}{16}\cos(2a+bx-c)-\frac{3}{16}\cos(2a+3bx+c)-\frac{1}{16}\cos(2a+5bx+3c)+\frac{3}{8}\cos(bx+$$

$$\Big\downarrow \; 2009$$

$$\frac{\sin(2a-bx-3c)}{16b}-\frac{3\sin(2a+bx-c)}{16b}-\frac{\sin(2a+3bx+c)}{16b}-\frac{\sin(2a+5bx+3c)}{80b}+\frac{3\sin(bx+c)}{8b}+\frac{\sin(3bx+3c)}{24b}$$

input
```
Int[Cos[c + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
Sin[2*a - 3*c - b*x]/(16*b) - (3*Sin[2*a - c + b*x])/(16*b) + (3*Sin[c + b
*x])/(8*b) - Sin[2*a + c + 3*b*x]/(16*b) + Sin[3*c + 3*b*x]/(24*b) - Sin[2
*a + 3*c + 5*b*x]/(80*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

method	result
default	$\frac{\sin(-bx+2a-3c)}{16b}-\frac{3\sin(bx+2a-c)}{16b}+\frac{3\sin(bx+c)}{8b}-\frac{\sin(3bx+2a+c)}{16b}+\frac{\sin(3bx+3c)}{24b}-\frac{\sin(5bx+2a+3c)}{80b}$
risch	$\frac{\sin(-bx+2a-3c)}{16b}-\frac{3\sin(bx+2a-c)}{16b}+\frac{3\sin(bx+c)}{8b}-\frac{\sin(3bx+2a+c)}{16b}+\frac{\sin(3bx+3c)}{24b}-\frac{\sin(5bx+2a+3c)}{80b}$
parallelrisch	$\frac{\left(80\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^3+64\tan\left(\frac{bx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4+\left(-240\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^4-80\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2+64\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3+\left(240\tan\right.}{}$
orering	$-\frac{259\left(-3\cos(bx+c)^2\sin(bx+a)^2b\sin(bx+c)+2\cos(bx+c)^3\sin(bx+a)b\cos(bx+a)\right)}{225b^2}-\frac{7\left(-6b^3\sin(bx+c)^3\sin(bx+a)^2+36\right.}{}$

input
```
int(cos(b*x+c)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*sin(-b*x+2*a-3*c)/b-3/16*sin(b*x+2*a-c)/b+3/8*sin(b*x+c)/b-1/16*sin(3
*b*x+2*a+c)/b+1/24*sin(3*b*x+3*c)/b-1/80*sin(5*b*x+2*a+3*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \cos^3(c+bx)\sin^2(a+bx)\,dx$$
$$=\frac{6\cos(bx+c)^5\cos(-a+c)\sin(-a+c)-\left(3\left(2\cos(-a+c)^2-1\right)\cos(bx+c)^4+\left(3\cos(-a+c)^2-4\right.\right.}{15\,b}$$

input
```
integrate(cos(b*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/15*(6*cos(b*x + c)^5*cos(-a + c)*sin(-a + c) - (3*(2*cos(-a + c)^2 - 1)*
cos(b*x + c)^4 + (3*cos(-a + c)^2 - 4)*cos(b*x + c)^2 + 6*cos(-a + c)^2 -
8)*sin(b*x + c))/b
```

Sympy [A] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.68

$$\int \cos^3(c+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{2\sin^2(a+bx)\sin^3(bx+c)}{15b} + \frac{3\sin^2(a+bx)\sin(bx+c)\cos^2(bx+c)}{5b} - \frac{4\sin(a+bx)\sin^2(bx+c)\cos(a+bx)\cos(bx+c)}{5b} - \frac{2\sin(a+bx)\cos(a\cdots}{5b} \\ x\sin^2(a)\cos^3(c) \end{cases}$$

input
```
integrate(cos(b*x+c)**3*sin(b*x+a)**2,x)
```

output
```
Piecewise((2*sin(a + b*x)**2*sin(b*x + c)**3/(15*b) + 3*sin(a + b*x)**2*si
n(b*x + c)*cos(b*x + c)**2/(5*b) - 4*sin(a + b*x)*sin(b*x + c)**2*cos(a +
b*x)*cos(b*x + c)/(5*b) - 2*sin(a + b*x)*cos(a + b*x)*cos(b*x + c)**3/(5*b
) + 8*sin(b*x + c)**3*cos(a + b*x)**2/(15*b) + 2*sin(b*x + c)*cos(a + b*x)
**2*cos(b*x + c)**2/(5*b), Ne(b, 0)), (x*sin(a)**2*cos(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \cos^3(c+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{3\sin(5bx+2a+3c)+15\sin(3bx+2a+c)-10\sin(3bx+3c)+45\sin(bx+2a-c)+15\sin(\cdots}{240b}$$

input
```
integrate(cos(b*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/240*(3*sin(5*b*x + 2*a + 3*c) + 15*sin(3*b*x + 2*a + c) - 10*sin(3*b*x
+ 3*c) + 45*sin(b*x + 2*a - c) + 15*sin(b*x - 2*a + 3*c) - 90*sin(b*x + c)
)/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \cos^3(c+bx)\sin^2(a+bx)\,dx = -\frac{\sin(5\,bx+2\,a+3\,c)}{80\,b} - \frac{\sin(3\,bx+2\,a+c)}{16\,b} + \frac{\sin(3\,bx+3\,c)}{24\,b} - \frac{3\,\sin(bx+2\,a-c)}{16\,b} + \frac{3\,\sin(bx+c)}{8\,b} + \frac{\sin(-bx+2\,a-3\,c)}{16\,b}$$

input
```
integrate(cos(b*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/80*sin(5*b*x + 2*a + 3*c)/b - 1/16*sin(3*b*x + 2*a + c)/b + 1/24*sin(3*
b*x + 3*c)/b - 3/16*sin(b*x + 2*a - c)/b + 3/8*sin(b*x + c)/b + 1/16*sin(-
b*x + 2*a - 3*c)/b
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \cos^3(c+bx)\sin^2(a+bx)\,dx = -\frac{15\,\sin(2\,a+c+3\,b\,x) - 90\,\sin(c+b\,x) + 45\,\sin(2\,a-c+b\,x) + 15\,\sin(3\,c-2\,a+b\,x) + 3\,\sin(2\,a}{240\,b}$$

input
```
int(cos(c + b*x)^3*sin(a + b*x)^2,x)
```

output
```
-(15*sin(2*a + c + 3*b*x) - 90*sin(c + b*x) + 45*sin(2*a - c + b*x) + 15*s
in(3*c - 2*a + b*x) + 3*sin(2*a + 3*c + 5*b*x) - 10*sin(3*c + 3*b*x))/(240
*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.30

$$\int \cos^3(c+bx)\sin^2(a+bx)\,dx$$
$$= \frac{-6\cos(bx+c)\cos(bx+a)\sin(bx+c)^2\sin(bx+a) - 6\cos(bx+c)\cos(bx+a)\sin(bx+a) - 30\cos(b$$

input
```
int(cos(b*x+c)^3*sin(b*x+a)^2,x)
```

output
```
( - 6*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x) - 6*cos(b*x +
 c)*cos(a + b*x)*sin(a + b*x) - 30*cos(b*x + c)*sin(a + b*x) + 30*cos(a +
b*x)*sin(b*x + c) - 9*sin(b*x + c)**3*sin(a + b*x)**2 + 2*sin(b*x + c)**3
+ 3*sin(b*x + c)*sin(a + b*x)**2 + 6*sin(b*x + c))/(15*b)
```

3.156 $\int \cos^2(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \cos^2(c+bx)\sin^2(a+bx)\,dx = \frac{1}{8}x(2-\cos(2(a-c)))-\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2bx)}{8b}-\frac{\sin(2(a+c)+4bx)}{32b}$$

output
```
1/8*x*(2-cos(2*a-2*c))-1/8*sin(2*b*x+2*a)/b+1/8*sin(2*b*x+2*c)/b-1/32*sin(
4*b*x+2*a+2*c)/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \cos^2(c+bx)\sin^2(a+bx)\,dx = -\frac{-8bx+4bx\cos(2(a-c))+4\sin(2(a+bx))-4\sin(2(c+bx))+\sin(2(a+c+2bx))}{32b}$$

input
```
Integrate[Cos[c + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
-1/32*(-8*b*x + 4*b*x*Cos[2*(a - c)] + 4*Sin[2*(a + b*x)] - 4*Sin[2*(c + b
*x)] + Sin[2*(a + c + 2*b*x)])/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^2(bx+c)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(-\frac{1}{8}\cos(2(a+c)+4bx)-\frac{1}{4}\cos(2a+2bx)-\frac{1}{8}\cos(2a-2c)+\frac{1}{4}\cos(2bx+2c)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(2(a+c)+4bx)}{32b}-\frac{\sin(2a+2bx)}{8b}+\frac{1}{8}x(2-\cos(2(a-c)))+\frac{\sin(2bx+2c)}{8b}$$

input
```
Int[Cos[c + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(x*(2 - Cos[2*(a - c)]))/8 - Sin[2*a + 2*b*x]/(8*b) + Sin[2*c + 2*b*x]/(8*
b) - Sin[2*(a + c) + 4*b*x]/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

method	result
default	$\frac{x}{4}-\frac{x\cos(2a-2c)}{8}-\frac{\sin(2bx+2a)}{8b}+\frac{\sin(2bx+2c)}{8b}-\frac{\sin(4bx+2a+2c)}{32b}$
risch	$\frac{x}{4}-\frac{x\cos(2a-2c)}{8}-\frac{\sin(2bx+2a)}{8b}+\frac{\sin(2bx+2c)}{8b}-\frac{\sin(4bx+2a+2c)}{32b}$
parallelrisch	$\frac{8bx-4bx\cos(2a-2c)-\sin(4bx+2a+2c)+5\sin(2a-2c)-4\sin(2bx+2a)+4\sin(2bx+2c)}{32b}$
orering	$x\cos(bx+c)^{2}\sin(bx+a)^{2}-\frac{5\left(-2b\sin(bx+a)^{2}\cos(bx+c)\sin(bx+c)+2\cos(bx+a)\cos(bx+c)^{2}b\sin(bx+a)\right)}{16b^{2}}+$

input
```
int(cos(b*x+c)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x-1/8*x*cos(2*a-2*c)-1/8*sin(2*b*x+2*a)/b+1/8*sin(2*b*x+2*c)/b-1/32*si
n(4*b*x+2*a+2*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\int \cos^2(c+bx)\sin^2(a+bx)\,dx$$
$$=\frac{4\cos(bx+c)^{4}\cos(-a+c)\sin(-a+c)-2\,bx\cos(-a+c)^{2}+3\,bx-\left(2\left(2\cos(-a+c)^{2}-1\right)\cos(bx+\right.}{8\,b}$$

input
```
integrate(cos(b*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*cos(b*x + c)^4*cos(-a + c)*sin(-a + c) - 2*b*x*cos(-a + c)^2 + 3*b*
x - (2*(2*cos(-a + c)^2 - 1)*cos(b*x + c)^3 + (2*cos(-a + c)^2 - 3)*cos(b*
x + c))*sin(b*x + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(56) = 112$.

Time = 0.87 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.00

$$\int \cos^2(c+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{x\sin^2(a+bx)\sin^2(bx+c)}{8} + \frac{3x\sin^2(a+bx)\cos^2(bx+c)}{8} - \frac{x\sin(a+bx)\sin(bx+c)\cos(a+bx)\cos(bx+c)}{2} + \frac{3x\sin^2(bx+c)\cos^2(a+bx)}{8} \\ x\sin^2(a)\cos^2(c) \end{cases}$$

input
```
integrate(cos(b*x+c)**2*sin(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a + b*x)**2*sin(b*x + c)**2/8 + 3*x*sin(a + b*x)**2*cos(b
*x + c)**2/8 - x*sin(a + b*x)*sin(b*x + c)*cos(a + b*x)*cos(b*x + c)/2 + 3
*x*sin(b*x + c)**2*cos(a + b*x)**2/8 + x*cos(a + b*x)**2*cos(b*x + c)**2/8
 + 5*sin(a + b*x)**2*sin(b*x + c)*cos(b*x + c)/(8*b) - sin(a + b*x)*sin(b*
x + c)**2*cos(a + b*x)/(2*b) - sin(b*x + c)*cos(a + b*x)**2*cos(b*x + c)/(
8*b), Ne(b, 0)), (x*sin(a)**2*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int \cos^2(c+bx)\sin^2(a+bx)\,dx = -\frac{4\,(b\cos(-2\,a+2\,c)-2\,b)x+\sin(4\,bx+2\,a+2\,c)+4\,\sin(2\,bx+2\,a)-4\,\sin(2\,bx+2\,c)}{32\,b}$$

input
```
integrate(cos(b*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/32*(4*(b*cos(-2*a + 2*c) - 2*b)*x + sin(4*b*x + 2*a + 2*c) + 4*sin(2*b*
x + 2*a) - 4*sin(2*b*x + 2*c))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \cos^2(c+bx)\sin^2(a+bx)\,dx = -\frac{1}{8}\,x\cos(2\,a-2\,c)+\frac{1}{4}\,x-\frac{\sin(4\,bx+2\,a+2\,c)}{32\,b}-\frac{\sin(2\,bx+2\,a)}{8\,b}+\frac{\sin(2\,bx+2\,c)}{8\,b}$$

input
```
integrate(cos(b*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/8*x*cos(2*a - 2*c) + 1/4*x - 1/32*sin(4*b*x + 2*a + 2*c)/b - 1/8*sin(2*
b*x + 2*a)/b + 1/8*sin(2*b*x + 2*c)/b
```

Mupad [B] (verification not implemented)

Time = 19.47 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \cos^2(c+bx)\sin^2(a+bx)\,dx = -\frac{\frac{\sin(2\,a+2\,c+4\,b\,x)}{4}+\sin(2\,a+2\,b\,x)-\sin(2\,c+2\,b\,x)-2\,b\,x+b\,x\,\cos(2\,a-2\,c)}{8\,b}$$

input
```
int(cos(c + b*x)^2*sin(a + b*x)^2,x)
```

output
```
-(sin(2*a + 2*c + 4*b*x)/4 + sin(2*a + 2*b*x) - sin(2*c + 2*b*x) - 2*b*x +
 b*x*cos(2*a - 2*c))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.68

$$\int \cos^2(c+bx)\sin^2(a+bx)\,dx$$
$$= \frac{-12\cos(bx+c)\cos(bx+a)\sin(bx+c)\sin(bx+a)\,bx+2\cos(bx+c)\sin(bx+c)\sin(bx+a)^2+5\cos\cdots}{}$$

input
```
int(cos(b*x+c)^2*sin(b*x+a)^2,x)
```

output
```
( - 12*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)*sin(a + b*x)*b*x + 2*cos(b*x
 + c)*sin(b*x + c)*sin(a + b*x)**2 + 5*cos(b*x + c)*sin(b*x + c) + 8*cos(b
*x + c)*sin(a + b*x) + 4*cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x) - 8*cos
(a + b*x)*sin(b*x + c) - 8*cos(a + b*x)*sin(a + b*x) - 12*sin(b*x + c)**2*
sin(a + b*x)**2*b*x + 6*sin(b*x + c)**2*b*x + 6*sin(a + b*x)**2*b*x + 3*b*
x)/(24*b)
```

3.157 $\int \cos(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \cos(c+bx)\sin^2(a+bx)\,dx=-\frac{\sin(2a-c+bx)}{4b}+\frac{\sin(c+bx)}{2b}-\frac{\sin(2a+c+3bx)}{12b}$$

output
```
-1/4*sin(b*x+2*a-c)/b+1/2*sin(b*x+c)/b-1/12*sin(3*b*x+2*a+c)/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \cos(c+bx)\sin^2(a+bx)\,dx=-\frac{3\sin(2a-c+bx)-6\sin(c+bx)+\sin(2a+c+3bx)}{12b}$$

input
```
Integrate[Cos[c + b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/12*(3*Sin[2*a - c + b*x] - 6*Sin[c + b*x] + Sin[2*a + c + 3*b*x])/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos(bx+c)\,dx$$

$$\Big\downarrow \ 5085$$

$$\int \left(-\frac{1}{4}\cos(2a+bx-c)-\frac{1}{4}\cos(2a+3bx+c)+\frac{1}{2}\cos(bx+c)\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sin(2a+bx-c)}{4b}-\frac{\sin(2a+3bx+c)}{12b}+\frac{\sin(bx+c)}{2b}$$

input
```
Int[Cos[c + b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/4*Sin[2*a - c + b*x]/b + Sin[c + b*x]/(2*b) - Sin[2*a + c + 3*b*x]/(12*
b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\sin(bx+2a-c)}{4b}+\frac{\sin(bx+c)}{2b}-\frac{\sin(3bx+2a+c)}{12b}$
risch	$-\frac{\sin(bx+2a-c)}{4b}+\frac{\sin(bx+c)}{2b}-\frac{\sin(3bx+2a+c)}{12b}$
parallelrisch	$\frac{6\sin(bx+c)-8\sin(a-c)-\sin(3bx+2a+c)-3\sin(bx+2a-c)}{12b}$
norman	$\frac{-\frac{8\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{3b}+\frac{8\tan\left(\frac{bx}{2}+\frac{c}{2}\right)}{3b}+\frac{8\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2}{3b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^2}$
orering	$-\frac{10\left(-b\sin(bx+c)\sin(bx+a)^2+2\cos(bx+c)\sin(bx+a)b\cos(bx+a)\right)}{9b^2}-\frac{7b^3\sin(bx+c)\sin(bx+a)^2-14b^3\cos(bx+c)\sin(bx+}{9b^4}$

input
```
int(cos(b*x+c)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*sin(b*x+2*a-c)/b+1/2*sin(b*x+c)/b-1/12*sin(3*b*x+2*a+c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \cos(c+bx)\sin^2(a+bx)\,dx$$
$$=\frac{2\cos(bx+c)^3\cos(-a+c)\sin(-a+c)-\left(\left(2\cos(-a+c)^2-1\right)\cos(bx+c)^2+\cos(-a+c)^2-2\right)\sin}{3b}$$

input
```
integrate(cos(b*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*(2*cos(b*x + c)^3*cos(-a + c)*sin(-a + c) - ((2*cos(-a + c)^2 - 1)*cos
(b*x + c)^2 + cos(-a + c)^2 - 2)*sin(b*x + c))/b
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int \cos(c+bx)\sin^2(a+bx)\,dx = \begin{cases} \frac{\sin^2(a+bx)\sin(bx+c)}{3b} - \frac{2\sin(a+bx)\cos(a+bx)\cos(bx+c)}{3b} + \frac{2\sin(bx+c)\cos^2(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin^2(a)\cos(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+c)*sin(b*x+a)**2,x)
```

output
```
Piecewise((sin(a + b*x)**2*sin(b*x + c)/(3*b) - 2*sin(a + b*x)*cos(a + b*x
)*cos(b*x + c)/(3*b) + 2*sin(b*x + c)*cos(a + b*x)**2/(3*b), Ne(b, 0)), (x
*sin(a)**2*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \cos(c+bx)\sin^2(a+bx)\,dx = -\frac{\sin(3\,bx+2\,a+c)+3\,\sin(bx+2\,a-c)-6\,\sin(bx+c)}{12\,b}$$

input
```
integrate(cos(b*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/12*(sin(3*b*x + 2*a + c) + 3*sin(b*x + 2*a - c) - 6*sin(b*x + c))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \cos(c+bx)\sin^2(a+bx)\,dx = -\frac{\sin(3\,bx+2\,a+c)}{12\,b} - \frac{\sin(bx+2\,a-c)}{4\,b} + \frac{\sin(bx+c)}{2\,b}$$

input
```
integrate(cos(b*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/12*sin(3*b*x + 2*a + c)/b - 1/4*sin(b*x + 2*a - c)/b + 1/2*sin(b*x + c)
/b
```

Mupad [B] (verification not implemented)

Time = 17.70 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \cos(c+bx)\sin^2(a+bx)\,dx = -\frac{\sin(2\,a+c+3\,b\,x)-6\,\sin(c+b\,x)+3\,\sin(2\,a-c+b\,x)}{12\,b}$$

input
```
int(cos(c + b*x)*sin(a + b*x)^2,x)
```

output
```
-(sin(2*a + c + 3*b*x) - 6*sin(c + b*x) + 3*sin(2*a - c + b*x))/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \cos(c+bx)\sin^2(a+bx)\,dx = \frac{-2\cos(bx+c)\cos(bx+a)\sin(bx+a)-2\cos(bx+c)\sin(bx+a)+2\cos(bx+a)\sin(bx+c)-\sin(b}{3b}$$

input
```
int(cos(b*x+c)*sin(b*x+a)^2,x)
```

output
```
( - 2*cos(b*x + c)*cos(a + b*x)*sin(a + b*x) - 2*cos(b*x + c)*sin(a + b*x)
 + 2*cos(a + b*x)*sin(b*x + c) - sin(b*x + c)*sin(a + b*x)**2 + 2*sin(b*x
+ c))/(3*b)
```

3.158 $\int \sec(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \sec(c+bx)\sin^2(a+bx)\,dx = \frac{\text{arctanh}(\sin(c+bx))\cos^2(a-c)}{b} - \frac{\sin(2a-c+bx)}{b}$$

output
```
arctanh(sin(b*x+c))*cos(a-c)^2/b-sin(b*x+2*a-c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 129 vs. 2(36) = 72.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.58

$$\begin{aligned}\int \sec(c+bx)\sin^2(a+bx)\,dx &= \frac{(-1-\cos(2a-2c))\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)-\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{2b} \\ &+ \frac{(1+\cos(2a-2c))\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)+\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{2b} \\ &- \frac{\cos(bx)\sin(2a-c)}{b} - \frac{\cos(2a-c)\sin(bx)}{b}\end{aligned}$$

input
```
Integrate[Sec[c + b*x]*Sin[a + b*x]^2,x]
```

output
```
((-1 - Cos[2*a - 2*c])*Log[Cos[c/2 + (b*x)/2] - Sin[c/2 + (b*x)/2]])/(2*b)
 + ((1 + Cos[2*a - 2*c])*Log[Cos[c/2 + (b*x)/2] + Sin[c/2 + (b*x)/2]])/(2*
b) - (Cos[b*x]*Sin[2*a - c])/b - (Cos[2*a - c]*Sin[b*x])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sin^2(a+bx)\sec(bx+c)dx$$

input
```
Int[Sec[c + b*x]*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.06

method	result
risch	$\frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)}{2b}+\frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\cos(2a-2c)}{2b}-\frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)}{2b}-\frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\cos(2a-2c)}{2b}-$
default	$\frac{\frac{2(-\cos(a)\cos(c)-\sin(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-2\sin(a)\cos(c)+2\cos(a)\sin(c)}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)}-\frac{8(\cos(a)\cos(c)+\sin(a)\sin(c))^2\arctan\left(\frac{2(c}{2}\right.}{\left(4\cos(c)^2\sin(a)^2+4\cos(a)^2\cos(c)^2+4\sin(a)^2\sin(c)^2+\right.}}{b}$

input
```
int(sec(b*x+c)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))+1/2/b*ln(exp(I*(b*x+a))+I*exp(I*(a
-c)))*cos(2*a-2*c)-1/2/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))-1/2/b*ln(exp(I*
(b*x+a))-I*exp(I*(a-c)))*cos(2*a-2*c)-sin(b*x+2*a-c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int \sec(c+bx)\sin^2(a+bx)\,dx = \frac{\cos(-a+c)^2\log(\sin(bx+c)+1)-\cos(-a+c)^2\log(-\sin(bx+c)+1)+4\cos(bx+c)\cos(-a+c)}{2b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(cos(-a + c)^2*log(sin(b*x + c) + 1) - cos(-a + c)^2*log(-sin(b*x + c)
 + 1) + 4*cos(b*x + c)*cos(-a + c)*sin(-a + c) - 2*(2*cos(-a + c)^2 - 1)*s
in(b*x + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1581 vs. $2(27) = 54$.

Time = 18.89 (sec) , antiderivative size = 3645, normalized size of antiderivative = 101.25

$$\int \sec(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a)**2,x)
```

output
```
2*Piecewise((-sin(b*x)/b, Eq(c, pi/2)), (sin(b*x)/b, Eq(c, -pi/2)), (0, Eq
(b, 0)), (-2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*
tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2
*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*
log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**3/(
b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**
2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(
tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*ta
n(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)
**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) -
1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 +
2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2
*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3*
tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)*
*2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/
2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(c/2)**
4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(
c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1
) - 1/(tan(c/2) + 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2
+ b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*t...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(36) = 72$.

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.89

$$\int \sec(c + bx) \sin^2(a + bx) \, dx =$$

$$-\frac{(\cos(-2a + 2c) + 1) \log\left(\frac{\cos(bx+2c)^2+\cos(c)^2-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2+2\cos(bx+2c)\sin(c)+\sin(c)^2}{\cos(bx+2c)^2+\cos(c)^2+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2-2\cos(bx+2c)\sin(c)+\sin(c)^2}\right) + 4\sin}{4b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*((cos(-2*a + 2*c) + 1)*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*si
n(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos
(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*
cos(b*x + 2*c)*sin(c) + sin(c)^2)) + 4*sin(b*x + 2*a - c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 961 vs. $2(36) = 72$.

Time = 0.14 (sec) , antiderivative size = 961, normalized size of antiderivative = 26.69

$$\int \sec(c + bx) \sin^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
((tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*a)^3
*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 8*tan(1/2*a)^
3*tan(1/2*c) + 20*tan(1/2*a)^2*tan(1/2*c)^2 - 8*tan(1/2*a)*tan(1/2*c)^3 +
tan(1/2*c)^4 - 2*tan(1/2*a)^2 + 8*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*c)^2 +
 1)*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(
1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan
(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 +
1) - (tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*
a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 8*tan(1/2
*a)^3*tan(1/2*c) + 20*tan(1/2*a)^2*tan(1/2*c)^2 - 8*tan(1/2*a)*tan(1/2*c)^
3 + tan(1/2*c)^4 - 2*tan(1/2*a)^2 + 8*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*c)
^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c) - 1))/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*
tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4
*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^
2 + 1) - 2*(tan(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*b*x
 + 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^2 + 16*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3
*tan(1/2*c)^3 + 4*tan(1/2*a)^4*tan(1/2*c)^3 - 6*tan(1/2*b*x + 1/2*c)*tan(1
/2*a)^2*tan(1/2*c)^4 - 4*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*b*x + 1/2*c)*
tan(1/2*a)^4 - 16*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2
*a)^4*tan(1/2*c) + 36*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c)^2 + ...
```

Mupad [B] (verification not implemented)

Time = 18.79 (sec) , antiderivative size = 217, normalized size of antiderivative = 6.03

$$
\begin{aligned}
&\int \sec(c+bx)\sin^2(a+bx)\,dx \\
&= -\frac{e^{-a\,2i+c\,1i-b\,x\,1i}\,1i}{2\,b}+\frac{e^{a\,2i-c\,1i+b\,x\,1i}\,1i}{2\,b} \\
&\quad+\frac{e^{-a\,2i+c\,2i}\,\ln\left(-\frac{\left(e^{a\,2i}\,e^{-c\,2i}+1\right)^2\,1i}{2}-\frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(2\,e^{a\,2i}\,e^{-c\,2i}+e^{a\,4i}\,e^{-c\,4i}+1\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}+1\right)^2}{4\,b} \\
&\quad-\frac{e^{-a\,2i+c\,2i}\,\ln\left(\frac{\left(e^{a\,2i}\,e^{-c\,2i}+1\right)^2\,1i}{2}-\frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(2\,e^{a\,2i}\,e^{-c\,2i}+e^{a\,4i}\,e^{-c\,4i}+1\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}+1\right)^2}{4\,b}
\end{aligned}
$$

input
```
int(sin(a + b*x)^2/cos(c + b*x),x)
```

output
```
(exp(a*2i - c*1i + b*x*1i)*1i)/(2*b) - (exp(c*1i - a*2i - b*x*1i)*1i)/(2*b
) + (exp(c*2i - a*2i)*log(- ((exp(a*2i)*exp(-c*2i) + 1)^2*1i)/2 - (exp(c*1
i)*exp(b*x*1i)*(2*exp(a*2i)*exp(-c*2i) + exp(a*4i)*exp(-c*4i) + 1))/2)*(ex
p(a*2i - c*2i) + 1)^2)/(4*b) - (exp(c*2i - a*2i)*log(((exp(a*2i)*exp(-c*2i
) + 1)^2*1i)/2 - (exp(c*1i)*exp(b*x*1i)*(2*exp(a*2i)*exp(-c*2i) + exp(a*4i
)*exp(-c*4i) + 1))/2)*(exp(a*2i - c*2i) + 1)^2)/(4*b)
```

Reduce [F]

$$\int \sec(c+bx)\sin^2(a+bx)\,dx = \int \sec\left(bx+c\right)\sin\left(bx+a\right)^2 dx$$

input
```
int(sec(b*x+c)*sin(b*x+a)^2,x)
```

output
```
int(sec(b*x + c)*sin(a + b*x)**2,x)
```

3.159 $\int \sec^2(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx = -x\cos(2(a-c)) - \frac{\log(\cos(c+bx))\sin(2(a-c))}{b} + \frac{\cos^2(a-c)\tan(c+bx)}{b}$$

output
```
-x*cos(2*a-2*c)-ln(cos(b*x+c))*sin(2*a-2*c)/b+cos(a-c)^2*tan(b*x+c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(50) = 100$.

Time = 0.42 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.54

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx = -\frac{\sec(c)\sec(c+bx)(bx\cos(2a-4c-bx)+bx\cos(2a-2c-bx)+bx\cos(2a+bx)+bx\cos(2a-2c+\ldots}{}$$

input
```
Integrate[Sec[c + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
-1/4*(Sec[c]*Sec[c + b*x]*(b*x*Cos[2*a - 4*c - b*x] + b*x*Cos[2*a - 2*c -
b*x] + b*x*Cos[2*a + b*x] + b*x*Cos[2*a - 2*c + b*x] - 2*Sin[b*x] + Log[Co
s[c + b*x]]*Sin[2*a - 4*c - b*x] + Sin[2*a - 2*c - b*x] + Log[Cos[c + b*x]
]*Sin[2*a - 2*c - b*x] + Log[Cos[c + b*x]]*Sin[2*a + b*x] - Sin[2*a - 2*c
+ b*x] + Log[Cos[c + b*x]]*Sin[2*a - 2*c + b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec^2(bx+c)\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \sin^2(a+bx)\sec^2(bx+c)dx$$

input
```
Int[Sec[c + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.34

method	result
risch	$-x\,\mathrm{e}^{2i(a-c)}+2i\sin\left(2a-2c\right)x+\frac{2i\sin(2a-2c)a}{b}+\frac{i\mathrm{e}^{2i(2a-c)}}{2b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{i\mathrm{e}^{2ia}}{b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{i\mathrm{e}^{2i}}{2b\left(\mathrm{e}^{2i(bx+a+}\right.}$
default	$-\frac{\left(2\cos(c)^{3}\sin(a)^{2}\cos(a)+2\cos(c)^{2}\sin(c)\sin(a)^{3}-4\cos(c)^{2}\sin(c)\cos(a)^{2}\sin(a)-4\cos(c)\sin(c)^{2}\cos(a)\sin(a)^{2}+2\cos(c)\sin(c)^{2}\cos(a)^{3}+2\sin(c)^{}}{\left(\cos(c)^{2}+\sin(c)^{2}\right)^{2}\left(\cos(a)^{2}+\sin(a)^{2}\right)^{2}\left(\sin(a)\cos(c)-\cos\right.}$

input
```
int(sec(b*x+c)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-x*exp(2*I*(a-c))+2*I*sin(2*a-2*c)*x+2*I/b*sin(2*a-2*c)*a+1/2*I/b/(exp(2*I
*(b*x+a+c))+exp(2*I*a))*exp(2*I*(2*a-c))+I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a
))*exp(2*I*a)+1/2*I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))*exp(2*I*c)-ln(exp(2*
I*(b*x+a))+exp(2*I*(a-c)))/b*sin(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.64

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx$$
$$=\frac{2\cos\left(bx+c\right)\cos\left(-a+c\right)\log\left(-\cos\left(bx+c\right)\right)\sin\left(-a+c\right)+\cos\left(-a+c\right)^{2}\sin\left(bx+c\right)-\left(2\,bx\cos\left(-a\right.\right.}{b\cos\left(bx+c\right)}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
(2*cos(b*x + c)*cos(-a + c)*log(-cos(b*x + c))*sin(-a + c) + cos(-a + c)^2
*sin(b*x + c) - (2*b*x*cos(-a + c)^2 - b*x)*cos(b*x + c))/(b*cos(b*x + c))
```

Sympy [F(-2)]

Exception generated.

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+c)**2*sin(b*x+a)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(50) = 100$.

Time = 0.06 (sec) , antiderivative size = 534, normalized size of antiderivative = 10.68

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(b*cos(2*a + 2*c)*cos(4*c) + b*sin(2*a + 2*c)*sin(4*c))*x + (2*b*x
*cos(4*c) + sin(4*a) + 2*sin(2*a + 2*c) + sin(4*c))*cos(2*b*x + 2*a + 4*c)
 + 2*(b*x*cos(2*b*x + 2*a + 4*c) + b*x*cos(2*a + 2*c))*cos(2*b*x + 6*c) +
(sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(2*b*x + 2*a + 4*c)^2*sin(-2*a
+ 2*c) + 2*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c)*sin(-2*a + 2*c) + cos(2*a
 + 2*c)^2*sin(-2*a + 2*c) + sin(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c) + 2*s
in(2*b*x + 2*a + 4*c)*sin(2*a + 2*c)*sin(-2*a + 2*c) + sin(2*a + 2*c)^2*si
n(-2*a + 2*c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin
(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2) + (2*b*x*sin(4*c) - cos(4*
a) - 2*cos(2*a + 2*c) - cos(4*c))*sin(2*b*x + 2*a + 4*c) + 2*(b*x*sin(2*b*
x + 2*a + 4*c) + b*x*sin(2*a + 2*c))*sin(2*b*x + 6*c) - (cos(4*a) + cos(4*
c))*sin(2*a + 2*c))/(b*cos(2*b*x + 2*a + 4*c)^2 + 2*b*cos(2*b*x + 2*a + 4*
c)*cos(2*a + 2*c) + b*cos(2*a + 2*c)^2 + b*sin(2*b*x + 2*a + 4*c)^2 + 2*b*
sin(2*b*x + 2*a + 4*c)*sin(2*a + 2*c) + b*sin(2*a + 2*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(50) = 100$.

Time = 0.13 (sec) , antiderivative size = 760, normalized size of antiderivative = 15.20

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-((tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*a)^4*tan(1/2*c)^2 + 16*tan(1/2*a)
^3*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 16*tan(1/2*
a)^3*tan(1/2*c) + 36*tan(1/2*a)^2*tan(1/2*c)^2 - 16*tan(1/2*a)*tan(1/2*c)^
3 + tan(1/2*c)^4 - 6*tan(1/2*a)^2 + 16*tan(1/2*a)*tan(1/2*c) - 6*tan(1/2*c
)^2 + 1)*(b*x + c)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^
2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4
*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*ta
n(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2
*c)^4 - tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)
^2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))*log(tan(b*x + c)^2 + 1)/(tan(
1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1
/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*ta
n(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - (tan(b*x + c)*tan(1/2*a)^4*tan(1/2*c)^4
 - 2*tan(b*x + c)*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(b*x + c)*tan(1/2*a)^3*
tan(1/2*c)^3 - 2*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^4 + tan(b*x + c)*tan
(1/2*a)^4 - 8*tan(b*x + c)*tan(1/2*a)^3*tan(1/2*c) + 20*tan(b*x + c)*tan(1
/2*a)^2*tan(1/2*c)^2 - 8*tan(b*x + c)*tan(1/2*a)*tan(1/2*c)^3 + tan(b*x +
c)*tan(1/2*c)^4 - 2*tan(b*x + c)*tan(1/2*a)^2 + 8*tan(b*x + c)*tan(1/2*a)*
tan(1/2*c) - 2*tan(b*x + c)*tan(1/2*c)^2 + tan(b*x + c))/(tan(1/2*a)^4*...
```

Mupad [B] (verification not implemented)

Time = 18.61 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.92

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx$$
$$= -x\left(\cos\left(2\,a-2\,c\right)-\sin\left(2\,a-2\,c\right)\,1\mathrm{i}\right)+\frac{\left(2\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}-c\,4\mathrm{i}}+1\right)\,1\mathrm{i}}{2\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}$$
$$-\frac{\mathrm{e}^{-a\,4\mathrm{i}+c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\right)\,\left(2\,b\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-2\,b\,\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$

input
```
int(sin(a + b*x)^2/cos(c + b*x)^2,x)
```

output
```
((2*exp(a*2i - c*2i) + exp(a*4i - c*4i) + 1)*1i)/(2*b*(exp(a*2i - c*2i) +
exp(a*2i + b*x*2i))) - x*(cos(2*a - 2*c) - sin(2*a - 2*c)*1i) - (exp(c*4i
- a*4i)*log(exp(a*2i)*exp(b*x*2i) + exp(a*2i)*exp(-c*2i))*(2*b*exp(a*2i -
c*2i) - 2*b*exp(a*6i - c*6i))*1i)/(4*b^2)
```

Reduce [F]

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
int(sec(b*x+c)^2*sin(b*x+a)^2,x)
```

output
```
(7*cos(b*x + c)*cos(a + b*x)*sin(a + b*x) + 96*cos(b*x + c)*int(tan((b*x +
 c)/2)**2/(tan((b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)**4
*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan((a
+ b*x)/2)**4 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x + c)
/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b + 96*cos(b*
x + c)*int(tan((a + b*x)/2)**2/(tan((b*x + c)/2)**4*tan((a + b*x)/2)**4 +
2*tan((b*x + c)/2)**4*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b
*x + c)/2)**2*tan((a + b*x)/2)**4 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)
**2 - 2*tan((b*x + c)/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2
+ 1),x)*b - 128*cos(b*x + c)*int((tan((b*x + c)/2)*tan((a + b*x)/2))/(tan(
(b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)**4*tan((a + b*x)/
2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan((a + b*x)/2)**4 -
4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x + c)/2)**2 + tan((a
 + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b - 32*cos(b*x + c)*int(1/(t
an((b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)**4*tan((a + b*
x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan((a + b*x)/2)**4
 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x + c)/2)**2 + tan
((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b + 16*cos(b*x + c)*sin(a
 + b*x) + 9*cos(b*x + c)*a + 9*cos(b*x + c)*b*x - 8*cos(a + b*x)*sin(b*x +
 c) + 8*cos(a + b*x)*sin(a + b*x) + 4*sin(b*x + c)*sin(a + b*x)**2 - 8*...
```

3.160 $\int \sec^3(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(c+bx))\cos^2(a-c)}{2b} - \frac{\operatorname{arctanh}(\sin(c+bx))\cos(2(a-c))}{b} + \frac{\sec(c+bx)\sin(2(a-c))}{b} + \frac{\cos^2(a-c)\sec(c+bx)\tan(c+bx)}{2b}$$

output
```
1/2*arctanh(sin(b*x+c))*cos(a-c)^2/b-arctanh(sin(b*x+c))*cos(2*a-2*c)/b+se
c(b*x+c)*sin(2*a-2*c)/b+1/2*cos(a-c)^2*sec(b*x+c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.88

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx = \frac{-\log\left(\cos\left(\frac{1}{2}(c+bx)\right)-\sin\left(\frac{1}{2}(c+bx)\right)\right)+3\cos(2(a-c))\left(\log\left(\cos\left(\frac{1}{2}(c+bx)\right)-\sin\left(\frac{1}{2}(c+bx)\right)\right)-\text{lo}\right.}{}$$

input
```
Integrate[Sec[c + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(-Log[Cos[(c + b*x)/2] - Sin[(c + b*x)/2]] + 3*Cos[2*(a - c)]*(Log[Cos[(c
+ b*x)/2] - Sin[(c + b*x)/2]] - Log[Cos[(c + b*x)/2] + Sin[(c + b*x)/2]])
+ Log[Cos[(c + b*x)/2] + Sin[(c + b*x)/2]] - 4*Sec[c]*Sin[2*(a - c)] + 4*S
ec[c + b*x]*Sin[2*(a - c)] + 2*Cos[a - c]^2*Sec[c + b*x]*Tan[c + b*x])/(4*
b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec^3(bx+c)\,dx$$
$$\Big\downarrow \ 7299$$
$$\int \sin^2(a+bx)\sec^3(bx+c)dx$$

input
```
Int[Sec[c + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.97 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.84

method	result
risch	$-\frac{i\left(5\,\mathrm{e}^{i(3bx+6a+c)}+2\,\mathrm{e}^{i(3bx+4a+3c)}-3\,\mathrm{e}^{i(3bx+2a+5c)}+3\,\mathrm{e}^{i(bx+6a-c)}-2\,\mathrm{e}^{i(bx+4a+c)}-5\,\mathrm{e}^{i(bx+2a+3c)}\right)}{4\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{2}b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a-c)}\right)}{4b}$
default	Expression too large to display

input
```
int(sec(b*x+c)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/(exp(2*I*(b*x+a+c))+exp(2*I*a))^2/b*(5*exp(I*(3*b*x+6*a+c))+2*exp(I
*(3*b*x+4*a+3*c))-3*exp(I*(3*b*x+2*a+5*c))+3*exp(I*(b*x+6*a-c))-2*exp(I*(b
*x+4*a+c))-5*exp(I*(b*x+2*a+3*c)))-1/4/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))
+3/4/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))*cos(2*a-2*c)+1/4/b*ln(exp(I*(b*x+
a))+I*exp(I*(a-c)))-3/4/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))*cos(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\left(3\,\cos\left(-a+c\right)^{2}-2\right)\cos\left(bx+c\right)^{2}\log\left(\sin\left(bx+c\right)+1\right)-\left(3\,\cos\left(-a+c\right)^{2}-2\right)\cos\left(bx+c\right)^{2}\log\left(-\right.}{4\,b\cos\left(bx+c\right)}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*((3*cos(-a + c)^2 - 2)*cos(b*x + c)^2*log(sin(b*x + c) + 1) - (3*cos(
-a + c)^2 - 2)*cos(b*x + c)^2*log(-sin(b*x + c) + 1) - 2*cos(-a + c)^2*sin
(b*x + c) + 8*cos(b*x + c)*cos(-a + c)*sin(-a + c))/(b*cos(b*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+c)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. $2(84) = 168$.

Time $= 0.22$ (sec) , antiderivative size $= 1263$, normalized size of antiderivative $= 14.35$

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/8*(2*(5*sin(3*b*x + 4*a + 2*c) + 2*sin(3*b*x + 2*a + 4*c) - 3*sin(3*b*x
+ 6*c) + 3*sin(b*x + 4*a) - 2*sin(b*x + 2*a + 2*c) - 5*sin(b*x + 4*c))*cos
(4*b*x + 2*a + 5*c) - 10*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + c))*cos(3*b
*x + 4*a + 2*c) - 4*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + c))*cos(3*b*x +
2*a + 4*c) + 6*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + c))*cos(3*b*x + 6*c)
+ 4*(3*sin(b*x + 4*a) - 2*sin(b*x + 2*a + 2*c) - 5*sin(b*x + 4*c))*cos(2*b
*x + 2*a + 3*c) + ((3*cos(-2*a + 2*c) - 1)*cos(4*b*x + 2*a + 5*c)^2 + 4*(3
*cos(-2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c)^2 + (3*cos(-2*a + 2*c) - 1)*s
in(4*b*x + 2*a + 5*c)^2 + 4*(3*cos(-2*a + 2*c) - 1)*sin(2*b*x + 2*a + 3*c)
^2 + 2*(2*(3*cos(-2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c) + 3*cos(2*a + c)*
cos(-2*a + 2*c) - cos(2*a + c))*cos(4*b*x + 2*a + 5*c) + 4*(3*cos(2*a + c)
*cos(-2*a + 2*c) - cos(2*a + c))*cos(2*b*x + 2*a + 3*c) - cos(2*a + c)^2 +
 3*(cos(2*a + c)^2 + sin(2*a + c)^2)*cos(-2*a + 2*c) + 2*(2*(3*cos(-2*a +
2*c) - 1)*sin(2*b*x + 2*a + 3*c) + 3*cos(-2*a + 2*c)*sin(2*a + c) - sin(2*
a + c))*sin(4*b*x + 2*a + 5*c) + 4*(3*cos(-2*a + 2*c)*sin(2*a + c) - sin(2
*a + c))*sin(2*b*x + 2*a + 3*c) - sin(2*a + c)^2)*log((cos(b*x + 2*c)^2 +
cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*s
in(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) +
 sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - 2*(5*cos(3*b*x
+ 4*a + 2*c) + 2*cos(3*b*x + 2*a + 4*c) - 3*cos(3*b*x + 6*c) + 3*cos(b*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1502 vs. $2(84) = 168$.

Time = 0.17 (sec) , antiderivative size = 1502, normalized size of antiderivative = 17.07

$$\int \sec^3(c + bx) \sin^2(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*((tan(1/2*a)^4*tan(1/2*c)^4 - 10*tan(1/2*a)^4*tan(1/2*c)^2 + 24*tan(1
/2*a)^3*tan(1/2*c)^3 - 10*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 24*ta
n(1/2*a)^3*tan(1/2*c) + 52*tan(1/2*a)^2*tan(1/2*c)^2 - 24*tan(1/2*a)*tan(1
/2*c)^3 + tan(1/2*c)^4 - 10*tan(1/2*a)^2 + 24*tan(1/2*a)*tan(1/2*c) - 10*t
an(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^4*tan(1/2*
c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2
*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*ta
n(1/2*c)^2 + 1) - (tan(1/2*a)^4*tan(1/2*c)^4 - 10*tan(1/2*a)^4*tan(1/2*c)^
2 + 24*tan(1/2*a)^3*tan(1/2*c)^3 - 10*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*
a)^4 - 24*tan(1/2*a)^3*tan(1/2*c) + 52*tan(1/2*a)^2*tan(1/2*c)^2 - 24*tan(
1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 10*tan(1/2*a)^2 + 24*tan(1/2*a)*tan(1
/2*c) - 10*tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c) - 1))/(tan(1/2*a
)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)
^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2
*a)^2 + 2*tan(1/2*c)^2 + 1) - 2*(tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1
/2*c)^4 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*b
*x + 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^3 - 8*tan(1/2*b*x + 1/2*c)^2*tan(1/2
*a)^4*tan(1/2*c)^3 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^2*tan(1/2*c)^4 +
8*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*b*x + 1/2*c)*
tan(1/2*a)^4*tan(1/2*c)^4 + tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4 - 8*tan...
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^2/cos(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx = \int \sec(bx+c)^3 \sin(bx+a)^2\,dx$$

input
```
int(sec(b*x+c)^3*sin(b*x+a)^2,x)
```

output
```
int(sec(b*x + c)**3*sin(a + b*x)**2,x)
```

3.161 $\int \cos^3(c+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 128

$$\int \cos^3(c+bx)\sin^3(a+bx)\,dx = \frac{3\cos(a-3c-2bx)}{64b}+\frac{3\cos(3a-c+2bx)}{64b} - \frac{9\cos(a+c+2bx)}{64b}+\frac{3\cos(3a+c+4bx)}{128b} - \frac{3\cos(a+3c+4bx)}{128b}+\frac{\cos(3(a+c)+6bx)}{192b} + \frac{1}{32}x(9\sin(a-c)-\sin(3(a-c)))$$

output
```
3/64*cos(-2*b*x+a-3*c)/b+3/64*cos(2*b*x+3*a-c)/b-9/64*cos(2*b*x+a+c)/b+3/1
28*cos(4*b*x+3*a+c)/b-3/128*cos(4*b*x+a+3*c)/b+1/192*cos(6*b*x+3*a+3*c)/b+
1/32*x*(9*sin(a-c)-sin(3*a-3*c))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \cos^3(c+bx)\sin^3(a+bx)\,dx = \frac{18\cos(a-3c-2bx)+18\cos(3a-c+2bx)-54\cos(a+c+2bx)+2\cos(3(a+c+2bx))+9\cos(3a+}{384b}$$

input
```
Integrate[Cos[c + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(18*Cos[a - 3*c - 2*b*x] + 18*Cos[3*a - c + 2*b*x] - 54*Cos[a + c + 2*b*x]
 + 2*Cos[3*(a + c + 2*b*x)] + 9*Cos[3*a + c + 4*b*x] - 9*Cos[a + 3*c + 4*b
*x] + 108*b*x*Sin[a - c] - 12*b*x*Sin[3*(a - c)])/(384*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^3(bx+c)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{3}{32}\sin(a-2bx-3c)-\frac{3}{32}\sin(3a+2bx-c)+\frac{9}{32}\sin(a+2bx+c)-\frac{3}{32}\sin(3a+4bx+c)+\frac{3}{32}\sin(a+4bx\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\cos(a-2bx-3c)}{64b}+\frac{3\cos(3a+2bx-c)}{64b}-\frac{9\cos(a+2bx+c)}{64b}+\frac{3\cos(3a+4bx+c)}{128b}-\frac{3\cos(a+4bx+3c)}{128b}+\frac{\cos(3(a+c)+6bx)}{192b}+\frac{1}{32}x(9\sin(a-c)-\sin(3(a-c)))$$

input
```
Int[Cos[c + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(3*Cos[a - 3*c - 2*b*x])/(64*b) + (3*Cos[3*a - c + 2*b*x])/(64*b) - (9*Cos
[a + c + 2*b*x])/(64*b) + (3*Cos[3*a + c + 4*b*x])/(128*b) - (3*Cos[a + 3*
c + 4*b*x])/(128*b) + Cos[3*(a + c) + 6*b*x]/(192*b) + (x*(9*Sin[a - c] -
Sin[3*(a - c)]))/32
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 7.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.89

method	result
default	$\frac{9x\sin(a-c)}{32}-\frac{x\sin(3a-3c)}{32}+\frac{3\cos(-2bx+a-3c)}{64b}-\frac{9\cos(2bx+a+c)}{64b}+\frac{3\cos(2bx+3a-c)}{64b}-\frac{3\cos(4bx+a+3c)}{128b}+\frac{3}{}$
risch	$\frac{9x\sin(a-c)}{32}-\frac{x\sin(3a-3c)}{32}+\frac{3\cos(-2bx+a-3c)}{64b}-\frac{9\cos(2bx+a+c)}{64b}+\frac{3\cos(2bx+3a-c)}{64b}-\frac{3\cos(4bx+a+3c)}{128b}+\frac{3}{}$
parallelrisch	$\frac{-12xb\sin(3a-3c)+108x\sin(a-c)b+18\cos(2bx+3a-c)+9\cos(4bx+3a+c)+2\cos(6bx+3a+3c)-29\cos(3a-3c)+45\cos(a-c)}{384b}$
orering	Expression too large to display

input
```
int(cos(b*x+c)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
9/32*x*sin(a-c)-1/32*x*sin(3*a-3*c)+3/64*cos(-2*b*x+a-3*c)/b-9/64*cos(2*b*
x+a+c)/b+3/64*cos(2*b*x+3*a-c)/b-3/128*cos(4*b*x+a+3*c)/b+3/128*cos(4*b*x+
3*a+c)/b+1/192*cos(6*b*x+3*a+3*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.22

$$\int \cos^3(c+bx)\sin^3(a+bx)\,dx$$
$$= \frac{8\left(4\,\cos\left(-a+c\right)^3-3\,\cos\left(-a+c\right)\right)\cos\left(bx+c\right)^6-12\,\cos\left(bx+c\right)^4\cos\left(-a+c\right)^3+\left(8\left(4\,\cos\left(-a+c\right)^2\right.\right.}{}$$

input
```
integrate(cos(b*x+c)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/48*(8*(4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^6 - 12*cos(b*x + c)
^4*cos(-a + c)^3 + (8*(4*cos(-a + c)^2 - 1)*cos(b*x + c)^5 + 2*(2*cos(-a +
 c)^2 - 5)*cos(b*x + c)^3 + 3*(2*cos(-a + c)^2 - 5)*cos(b*x + c))*sin(b*x
+ c)*sin(-a + c) + 3*(2*b*x*cos(-a + c)^2 - 5*b*x)*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(117) = 234.

Time = 5.12 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.16

$$\int \cos^3(c+bx)\sin^3(a+bx)\,dx$$
$$= \begin{cases} \frac{3x\sin^3(a+bx)\sin^2(bx+c)\cos(bx+c)}{16}+\frac{5x\sin^3(a+bx)\cos^3(bx+c)}{16}-\frac{3x\sin^2(a+bx)\sin^3(bx+c)\cos(a+bx)}{16}-\frac{9x\sin^2(a+bx)\sin(bx+c}{1} \\ x\sin^3(a)\cos^3(c) \end{cases}$$

input
```
integrate(cos(b*x+c)**3*sin(b*x+a)**3,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**3*sin(b*x + c)**2*cos(b*x + c)/16 + 5*x*sin(a
 + b*x)**3*cos(b*x + c)**3/16 - 3*x*sin(a + b*x)**2*sin(b*x + c)**3*cos(a
+ b*x)/16 - 9*x*sin(a + b*x)**2*sin(b*x + c)*cos(a + b*x)*cos(b*x + c)**2/
16 + 9*x*sin(a + b*x)*sin(b*x + c)**2*cos(a + b*x)**2*cos(b*x + c)/16 + 3*
x*sin(a + b*x)*cos(a + b*x)**2*cos(b*x + c)**3/16 - 5*x*sin(b*x + c)**3*co
s(a + b*x)**3/16 - 3*x*sin(b*x + c)*cos(a + b*x)**3*cos(b*x + c)**2/16 - s
in(a + b*x)**3*sin(b*x + c)**3/(16*b) - 11*sin(a + b*x)**2*cos(a + b*x)*co
s(b*x + c)**3/(16*b) - 3*sin(a + b*x)*sin(b*x + c)**3*cos(a + b*x)**2/(16*
b) + 3*sin(a + b*x)*sin(b*x + c)*cos(a + b*x)**2*cos(b*x + c)**2/(4*b) - s
in(b*x + c)**2*cos(a + b*x)**3*cos(b*x + c)/(2*b) - 7*cos(a + b*x)**3*cos(
b*x + c)**3/(48*b), Ne(b, 0)), (x*sin(a)**3*cos(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int \cos^3(c+bx)\sin^3(a+bx)\,dx = -\frac{12\,(9\,b\sin\left(-a+c\right)-b\sin\left(-3\,a+3\,c\right))x-2\,\cos\left(6\,bx+3\,a+3\,c\right)-9\,\cos\left(4\,bx+3\,a+c\right)+9\,\cos\left(4\right.}{384\,b}$$

input
```
integrate(cos(b*x+c)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/384*(12*(9*b*sin(-a + c) - b*sin(-3*a + 3*c))*x - 2*cos(6*b*x + 3*a + 3
*c) - 9*cos(4*b*x + 3*a + c) + 9*cos(4*b*x + a + 3*c) - 18*cos(2*b*x + 3*a
 - c) + 54*cos(2*b*x + a + c) - 18*cos(2*b*x - a + 3*c))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \cos^3(c+bx)\sin^3(a+bx)\,dx = &-\frac{1}{32}\,x\sin\left(3\,a-3\,c\right)+\frac{9}{32}\,x\sin\left(a-c\right)\\&+\frac{\cos\left(6\,bx+3\,a+3\,c\right)}{192\,b}+\frac{3\,\cos\left(4\,bx+3\,a+c\right)}{128\,b}\\&-\frac{3\,\cos\left(4\,bx+a+3\,c\right)}{128\,b}+\frac{3\,\cos\left(2\,bx+3\,a-c\right)}{64\,b}\\&-\frac{9\,\cos\left(2\,bx+a+c\right)}{64\,b}+\frac{3\,\cos\left(-2\,bx+a-3\,c\right)}{64\,b}\end{aligned}$$

input
```
integrate(cos(b*x+c)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/32*x*sin(3*a - 3*c) + 9/32*x*sin(a - c) + 1/192*cos(6*b*x + 3*a + 3*c)/
b + 3/128*cos(4*b*x + 3*a + c)/b - 3/128*cos(4*b*x + a + 3*c)/b + 3/64*cos
(2*b*x + 3*a - c)/b - 9/64*cos(2*b*x + a + c)/b + 3/64*cos(-2*b*x + a - 3*
c)/b
```

Mupad [B] (verification not implemented)

Time = 22.66 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.81

$$\int \cos^3(c+bx)\sin^3(a+bx)\,dx = \frac{\frac{9\,\cos(3\,a+c+4\,b\,x)}{8}-\frac{9\,\cos(a+3\,c+4\,b\,x)}{8}+\frac{9\,\cos(3\,c-a+2\,b\,x)}{4}+\frac{9\,\cos(3\,a-c+2\,b\,x)}{4}+\frac{\cos(3\,a+3\,c+6\,b\,x)}{4}-\frac{27\,\cos(a+c+2\,b\,x)}{4}+\frac{27}{}}{48\,b}$$

input
```
int(cos(c + b*x)^3*sin(a + b*x)^3,x)
```

output
```
((9*cos(3*a + c + 4*b*x))/8 - (9*cos(a + 3*c + 4*b*x))/8 + (9*cos(3*c - a
+ 2*b*x))/4 + (9*cos(3*a - c + 2*b*x))/4 + cos(3*a + 3*c + 6*b*x)/4 - (27*
cos(a + c + 2*b*x))/4 + (27*b*x*sin(a - c))/2 - (3*b*x*sin(3*a - 3*c))/2)/
(48*b)
```

Reduce [F]

$$\int \cos^3(c+bx)\sin^3(a+bx)\,dx = \int \cos(bx+c)^3\sin(bx+a)^3\,dx$$

input
```
int(cos(b*x+c)^3*sin(b*x+a)^3,x)
```

output
```
int(cos(b*x + c)**3*sin(a + b*x)**3,x)
```

3.162 $\int \cos^2(c+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \cos^2(c+bx)\sin^3(a+bx)\,dx = \frac{3\cos(a-2c-bx)}{16b} - \frac{3\cos(a+bx)}{8b} + \frac{\cos(3a-2c+bx)}{16b} + \frac{\cos(3a+3bx)}{24b} - \frac{\cos(a+2c+3bx)}{16b} + \frac{\cos(3a+2c+5bx)}{80b}$$

output
```
3/16*cos(-b*x+a-2*c)/b-3/8*cos(b*x+a)/b+1/16*cos(b*x+3*a-2*c)/b+1/24*cos(3
*b*x+3*a)/b-1/16*cos(3*b*x+a+2*c)/b+1/80*cos(5*b*x+3*a+2*c)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \cos^2(c+bx)\sin^3(a+bx)\,dx = \frac{45\cos(a-2c-bx) - 90\cos(a+bx) + 10\cos(3(a+bx)) + 15\cos(3a-2c+bx) - 15\cos(a+2c+3bx)}{240b}$$

input
```
Integrate[Cos[c + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(45*Cos[a - 2*c - b*x] - 90*Cos[a + b*x] + 10*Cos[3*(a + b*x)] + 15*Cos[3*
a - 2*c + b*x] - 15*Cos[a + 2*c + 3*b*x] + 3*Cos[3*a + 2*c + 5*b*x])/(240*
b)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{5085, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^2(bx+c)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{3}{16}\sin(a-bx-2c)-\frac{1}{16}\sin(3a+bx-2c)+\frac{3}{16}\sin(a+3bx+2c)-\frac{1}{16}\sin(3a+5bx+2c)+\frac{3}{8}\sin(a+bx)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\cos(a-bx-2c)}{16b}+\frac{\cos(3a+bx-2c)}{16b}-\frac{\cos(a+3bx+2c)}{16b}+\frac{\cos(3a+5bx+2c)}{80b}-\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}$$

input
```
Int[Cos[c + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(3*Cos[a - 2*c - b*x])/(16*b) - (3*Cos[a + b*x])/(8*b) + Cos[3*a - 2*c + b
*x]/(16*b) + Cos[3*a + 3*b*x]/(24*b) - Cos[a + 2*c + 3*b*x]/(16*b) + Cos[3
*a + 2*c + 5*b*x]/(80*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 3.69 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

method	result
default	$\frac{3\cos(-bx+a-2c)}{16b}-\frac{3\cos(bx+a)}{8b}+\frac{\cos(bx+3a-2c)}{16b}+\frac{\cos(3bx+3a)}{24b}-\frac{\cos(3bx+a+2c)}{16b}+\frac{\cos(5bx+3a+2c)}{80b}$
risch	$\frac{3\cos(-bx+a-2c)}{16b}-\frac{3\cos(bx+a)}{8b}+\frac{\cos(bx+3a-2c)}{16b}+\frac{\cos(3bx+3a)}{24b}-\frac{\cos(3bx+a+2c)}{16b}+\frac{\cos(5bx+3a+2c)}{80b}$
parallelrisch	$\frac{56\cos(2a-2c)-90\cos(bx+a)+45\cos(-bx+a-2c)+15\cos(bx+3a-2c)+10\cos(3bx+3a)-15\cos(3bx+a+2c)+3\cos(5bx+3a\cdots}{240b}$
orering	$-\frac{259\left(-2\cos(bx+c)\sin(bx+a)^3b\sin(bx+c)+3\cos(bx+c)^2\sin(bx+a)^2b\cos(bx+a)\right)}{225b^2}-\frac{7\left(26b^3\sin(bx+c)\sin(bx+a)^3\cos(b\cdots\right.}{\cdots}$

input
```
int(cos(b*x+c)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/16*cos(-b*x+a-2*c)/b-3/8*cos(b*x+a)/b+1/16*cos(b*x+3*a-2*c)/b+1/24*cos(3
*b*x+3*a)/b-1/16*cos(3*b*x+a+2*c)/b+1/80*cos(5*b*x+3*a+2*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.18

$$\int \cos^2(c+bx)\sin^3(a+bx)\,dx$$
$$=\frac{3\left(4\cos(-a+c)^3-3\cos(-a+c)\right)\cos(bx+c)^5-5\cos(bx+c)^3\cos(-a+c)^3+\left(3\left(4\cos(-a+c)^2\cdots\right.\right.}{15\cdots}$$

input
```
integrate(cos(b*x+c)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/15*(3*(4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^5 - 5*cos(b*x + c)^
3*cos(-a + c)^3 + (3*(4*cos(-a + c)^2 - 1)*cos(b*x + c)^4 + (cos(-a + c)^2
 - 4)*cos(b*x + c)^2 + 2*cos(-a + c)^2 - 8)*sin(b*x + c)*sin(-a + c))/b
```

Sympy [A] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.71

$$\int \cos^2(c+bx)\sin^3(a+bx)\,dx$$
$$= \begin{cases} \frac{2\sin^3(a+bx)\sin(bx+c)\cos(bx+c)}{5b} - \frac{2\sin^2(a+bx)\sin^2(bx+c)\cos(a+bx)}{5b} - \frac{3\sin^2(a+bx)\cos(a+bx)\cos^2(bx+c)}{5b} + \frac{4\sin(a+bx)\sin(b\cdots}{} \\ x\sin^3(a)\cos^2(c) \end{cases}$$

input
```
integrate(cos(b*x+c)**2*sin(b*x+a)**3,x)
```

output
```
Piecewise((2*sin(a + b*x)**3*sin(b*x + c)*cos(b*x + c)/(5*b) - 2*sin(a + b
*x)**2*sin(b*x + c)**2*cos(a + b*x)/(5*b) - 3*sin(a + b*x)**2*cos(a + b*x)
*cos(b*x + c)**2/(5*b) + 4*sin(a + b*x)*sin(b*x + c)*cos(a + b*x)**2*cos(b
*x + c)/(5*b) - 8*sin(b*x + c)**2*cos(a + b*x)**3/(15*b) - 2*cos(a + b*x)*
*3*cos(b*x + c)**2/(15*b), Ne(b, 0)), (x*sin(a)**3*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \cos^2(c+bx)\sin^3(a+bx)\,dx$$
$$= \frac{3\cos(5bx+3a+2c)+10\cos(3bx+3a)-15\cos(3bx+a+2c)+15\cos(bx+3a-2c)-90\cos\cdots}{240\,b}$$

input
```
integrate(cos(b*x+c)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/240*(3*cos(5*b*x + 3*a + 2*c) + 10*cos(3*b*x + 3*a) - 15*cos(3*b*x + a +
 2*c) + 15*cos(b*x + 3*a - 2*c) - 90*cos(b*x + a) + 45*cos(b*x - a + 2*c))
/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \cos^2(c+bx)\sin^3(a+bx)\,dx = \frac{\cos(5\,bx+3\,a+2\,c)}{80\,b} + \frac{\cos(3\,bx+3\,a)}{24\,b} - \frac{\cos(3\,bx+a+2\,c)}{16\,b} + \frac{\cos(bx+3\,a-2\,c)}{16\,b} - \frac{3\,\cos(bx+a)}{8\,b} + \frac{3\,\cos(-bx+a-2\,c)}{16\,b}$$

input
```
integrate(cos(b*x+c)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/80*cos(5*b*x + 3*a + 2*c)/b + 1/24*cos(3*b*x + 3*a)/b - 1/16*cos(3*b*x +
 a + 2*c)/b + 1/16*cos(b*x + 3*a - 2*c)/b - 3/8*cos(b*x + a)/b + 3/16*cos(
-b*x + a - 2*c)/b
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \cos^2(c+bx)\sin^3(a+bx)\,dx = \frac{45\,\cos(2\,c-a+b\,x)-90\,\cos(a+b\,x)-15\,\cos(a+2\,c+3\,b\,x)+15\,\cos(3\,a-2\,c+b\,x)+3\,\cos(3\,a}{240\,b}$$

input
```
int(cos(c + b*x)^2*sin(a + b*x)^3,x)
```

output
```
(45*cos(2*c - a + b*x) - 90*cos(a + b*x) - 15*cos(a + 2*c + 3*b*x) + 15*co
s(3*a - 2*c + b*x) + 3*cos(3*a + 2*c + 5*b*x) + 10*cos(3*a + 3*b*x))/(240*
b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.03

$$\int \cos^2(c+bx)\sin^3(a+bx)\,dx$$

$$= \frac{30\cos(bx+c)\cos(bx+a)\sin(bx+c)\sin(bx+a)+6\cos(bx+c)\cos(bx+a)-12\cos(bx+c)\sin(bx}{}$$

input
```
int(cos(b*x+c)^2*sin(b*x+a)^3,x)
```

output
```
(30*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)*sin(a + b*x) + 6*cos(b*x + c)*c
os(a + b*x) - 12*cos(b*x + c)*sin(b*x + c)*sin(a + b*x)**3 + 24*cos(b*x +
c)*sin(b*x + c)*sin(a + b*x) + 18*cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x
)**2 - 12*cos(a + b*x)*sin(b*x + c)**2 - 14*cos(a + b*x)*sin(a + b*x)**2 -
 4*cos(a + b*x) + 30*sin(b*x + c)**2*sin(a + b*x)**2 - 15*sin(b*x + c)**2
+ 6*sin(b*x + c)*sin(a + b*x) - 15*sin(a + b*x)**2 - 2)/(30*b)
```

3.163 $\int \cos(c+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \cos(c+bx)\sin^3(a+bx)\,dx=\frac{\cos(3a-c+2bx)}{16b}-\frac{3\cos(a+c+2bx)}{16b}+\frac{\cos(3a+c+4bx)}{32b}+\frac{3}{8}x\sin(a-c)$$

output
```
1/16*cos(2*b*x+3*a-c)/b-3/16*cos(2*b*x+a+c)/b+1/32*cos(4*b*x+3*a+c)/b+3/8*
x*sin(a-c)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \cos(c+bx)\sin^3(a+bx)\,dx=\frac{2\cos(3a-c+2bx)-6\cos(a+c+2bx)+\cos(3a+c+4bx)+12bx\sin(a-c)}{32b}$$

input
```
Integrate[Cos[c + b*x]*Sin[a + b*x]^3,x]
```

output
```
(2*Cos[3*a - c + 2*b*x] - 6*Cos[a + c + 2*b*x] + Cos[3*a + c + 4*b*x] + 12
*b*x*Sin[a - c])/(32*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos(bx+c)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(-\frac{1}{8}\sin(3a+2bx-c)+\frac{3}{8}\sin(a+2bx+c)-\frac{1}{8}\sin(3a+4bx+c)+\frac{3}{8}\sin(a-c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos(3a+2bx-c)}{16b}-\frac{3\cos(a+2bx+c)}{16b}+\frac{\cos(3a+4bx+c)}{32b}+\frac{3}{8}x\sin(a-c)$$

input
```
Int[Cos[c + b*x]*Sin[a + b*x]^3,x]
```

output
```
Cos[3*a - c + 2*b*x]/(16*b) - (3*Cos[a + c + 2*b*x])/(16*b) + Cos[3*a + c
+ 4*b*x]/(32*b) + (3*x*Sin[a - c])/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result
default	$\frac{\cos(2bx+3a-c)}{16b}-\frac{3\cos(2bx+a+c)}{16b}+\frac{\cos(4bx+3a+c)}{32b}+\frac{3x\sin(a-c)}{8}$
risch	$\frac{\cos(2bx+3a-c)}{16b}-\frac{3\cos(2bx+a+c)}{16b}+\frac{\cos(4bx+3a+c)}{32b}+\frac{3x\sin(a-c)}{8}$
parallelrisch	$\frac{12x\sin(a-c)b+3\cos(a-c)+2\cos(2bx+3a-c)-6\cos(2bx+a+c)+\cos(4bx+3a+c)}{32b}$
orering	$x\cos(bx+c)\sin(bx+a)^3-\frac{5\left(-b\sin(bx+c)\sin(bx+a)^3+3\cos(bx+c)\sin(bx+a)^2b\cos(bx+a)\right)}{16b^2}+\frac{5x\left(-4\sin(b\right.}{}$

input
```
int(cos(b*x+c)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*cos(2*b*x+3*a-c)/b-3/16*cos(2*b*x+a+c)/b+1/32*cos(4*b*x+3*a+c)/b+3/8*
x*sin(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.70

$$\int \cos(c+bx)\sin^3(a+bx)\,dx$$
$$=\frac{2\left(4\cos(-a+c)^3-3\cos(-a+c)\right)\cos(bx+c)^4-4\cos(bx+c)^2\cos(-a+c)^3-3\,bx\sin(-a+c)+}{8\,b}$$

input
```
integrate(cos(b*x+c)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*(2*(4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^4 - 4*cos(b*x + c)^2
*cos(-a + c)^3 - 3*b*x*sin(-a + c) + (2*(4*cos(-a + c)^2 - 1)*cos(b*x + c)
^3 - 3*cos(b*x + c))*sin(b*x + c)*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(54) = 108$.

Time = 0.88 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.73

$$\int \cos(c+bx)\sin^3(a+bx)\,dx$$
$$= \begin{cases} \frac{3x\sin^3(a+bx)\cos(bx+c)}{8} - \frac{3x\sin^2(a+bx)\sin(bx+c)\cos(a+bx)}{8} + \frac{3x\sin(a+bx)\cos^2(a+bx)\cos(bx+c)}{8} - \frac{3x\sin(bx+c)\cos^3(a+bx)}{8} \\ x\sin^3(a)\cos(c) \end{cases}$$

input
```
integrate(cos(b*x+c)*sin(b*x+a)**3,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**3*cos(b*x + c)/8 - 3*x*sin(a + b*x)**2*sin(b*
x + c)*cos(a + b*x)/8 + 3*x*sin(a + b*x)*cos(a + b*x)**2*cos(b*x + c)/8 -
3*x*sin(b*x + c)*cos(a + b*x)**3/8 + 5*sin(a + b*x)**3*sin(b*x + c)/(8*b)
+ 3*sin(a + b*x)*sin(b*x + c)*cos(a + b*x)**2/(4*b) + 3*cos(a + b*x)**3*co
s(b*x + c)/(8*b), Ne(b, 0)), (x*sin(a)**3*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \cos(c+bx)\sin^3(a+bx)\,dx =$$
$$-\frac{12\,bx\sin(-a+c) - \cos(4\,bx+3\,a+c) - 2\,\cos(2\,bx+3\,a-c) + 6\,\cos(2\,bx+a+c)}{32\,b}$$

input
```
integrate(cos(b*x+c)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/32*(12*b*x*sin(-a + c) - cos(4*b*x + 3*a + c) - 2*cos(2*b*x + 3*a - c)
+ 6*cos(2*b*x + a + c))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \cos(c+bx)\sin^3(a+bx)\,dx = \frac{3}{8}\,x\sin\left(a-c\right) + \frac{\cos\left(4\,bx+3\,a+c\right)}{32\,b} + \frac{\cos\left(2\,bx+3\,a-c\right)}{16\,b} - \frac{3\,\cos\left(2\,bx+a+c\right)}{16\,b}$$

input
```
integrate(cos(b*x+c)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
3/8*x*sin(a - c) + 1/32*cos(4*b*x + 3*a + c)/b + 1/16*cos(2*b*x + 3*a - c)
/b - 3/16*cos(2*b*x + a + c)/b
```

Mupad [B] (verification not implemented)

Time = 19.77 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \cos(c+bx)\sin^3(a+bx)\,dx = \frac{3\,x\,\sin\left(a-c\right)}{8} + \frac{\frac{\cos(3\,a+c+4\,b\,x)}{32}+\frac{\cos(3\,a-c+2\,b\,x)}{16}-\frac{3\,\cos(a+c+2\,b\,x)}{16}}{b}$$

input
```
int(cos(c + b*x)*sin(a + b*x)^3,x)
```

output
```
(3*x*sin(a - c))/8 + (cos(3*a + c + 4*b*x)/32 + cos(3*a - c + 2*b*x)/16 -
(3*cos(a + c + 2*b*x))/16)/b
```

Reduce [F]

$$\int \cos(c+bx)\sin^3(a+bx)\,dx=\int \cos(bx+c)\sin(bx+a)^3\,dx$$

input
```
int(cos(b*x+c)*sin(b*x+a)^3,x)
```

output
```
int(cos(b*x + c)*sin(a + b*x)**3,x)
```

3.164 $\int \sec(c+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \sec(c+bx)\sin^3(a+bx)\,dx = \frac{\cos(3a-c+2bx)}{4b} - \frac{\cos^3(a-c)\log(\cos(c+bx))}{b} + \frac{3}{4}x\sin(a-c) + \frac{1}{4}x\sin(3(a-c))$$

output
```
1/4*cos(2*b*x+3*a-c)/b-cos(a-c)^3*ln(cos(b*x+c))/b+3/4*x*sin(a-c)+1/4*x*si
n(3*a-3*c)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \sec(c+bx)\sin^3(a+bx)\,dx = \frac{\cos(3a-c+2bx) - 3\cos(a-c)\log(\cos(c+bx)) - \cos(3(a-c))\log(\cos(c+bx)) + 3bx\sin(a-c) + bx}{4b}$$

input
```
Integrate[Sec[c + b*x]*Sin[a + b*x]^3,x]
```

output
```
(Cos[3*a - c + 2*b*x] - 3*Cos[a - c]*Log[Cos[c + b*x]] - Cos[3*(a - c)]*Lo
g[Cos[c + b*x]] + 3*b*x*Sin[a - c] + b*x*Sin[3*(a - c)])/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin^3(a+bx)\sec(bx+c)dx$$

input
```
Int[Sec[c + b*x]*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.48

method	result
risch	$-\frac{ix\,\mathrm{e}^{3i(a-c)}}{4}-\frac{3ix\,\mathrm{e}^{i(a-c)}}{4}+\frac{3i\cos(a-c)x}{2}+\frac{i\cos(3a-3c)x}{2}+\frac{3i\cos(a-c)a}{2b}+\frac{i\cos(3a-3c)a}{2b}-\frac{3\ln\left(\mathrm{e}^{2i(bx+a)}+\mathrm{e}^{2i(a-c)}\right.}{4b}$
default	Expression too large to display

input
```
int(sec(b*x+c)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*x*exp(3*I*(a-c))-3/4*I*x*exp(I*(a-c))+3/2*I*cos(a-c)*x+1/2*I*cos(3*
a-3*c)*x+3/2*I/b*cos(a-c)*a+1/2*I/b*cos(3*a-3*c)*a-3/4*ln(exp(2*I*(b*x+a))
+exp(2*I*(a-c)))/b*cos(a-c)-1/4/b*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*cos(
3*a-3*c)+1/4*cos(2*b*x+3*a-c)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.70

$$\int \sec(c+bx)\sin^3(a+bx)\,dx = -\frac{2\cos(-a+c)^3\log(-\cos(bx+c)) - \left(4\cos(-a+c)^2-1\right)\cos(bx+c)\sin(bx+c)\sin(-a+c) - (4}{2b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*cos(-a + c)^3*log(-cos(b*x + c)) - (4*cos(-a + c)^2 - 1)*cos(b*x +
 c)*sin(b*x + c)*sin(-a + c) - (4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x +
 c)^2 + (2*b*x*cos(-a + c)^2 + b*x)*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10623 vs. $2(0) = 0$.

Time = 67.71 (sec) , antiderivative size = 34286, normalized size of antiderivative = 535.72

$$\int \sec(c+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a)**3,x)
```

output
```
3*Piecewise((-sin(b*x)**2/(2*b), Eq(c, pi/2)), (sin(b*x)**2/(2*b), Eq(c, -
pi/2)), (0, Eq(b, 0)), (-b*x*tan(c/2)**6*tan(b*x/2)**4/(2*b*tan(c/2)**6*ta
n(b*x/2)**4 + 4*b*tan(c/2)**6*tan(b*x/2)**2 + 2*b*tan(c/2)**6 + 6*b*tan(c/
2)**4*tan(b*x/2)**4 + 12*b*tan(c/2)**4*tan(b*x/2)**2 + 6*b*tan(c/2)**4 + 6
*b*tan(c/2)**2*tan(b*x/2)**4 + 12*b*tan(c/2)**2*tan(b*x/2)**2 + 6*b*tan(c/
2)**2 + 2*b*tan(b*x/2)**4 + 4*b*tan(b*x/2)**2 + 2*b) - 2*b*x*tan(c/2)**6*t
an(b*x/2)**2/(2*b*tan(c/2)**6*tan(b*x/2)**4 + 4*b*tan(c/2)**6*tan(b*x/2)**
2 + 2*b*tan(c/2)**6 + 6*b*tan(c/2)**4*tan(b*x/2)**4 + 12*b*tan(c/2)**4*tan
(b*x/2)**2 + 6*b*tan(c/2)**4 + 6*b*tan(c/2)**2*tan(b*x/2)**4 + 12*b*tan(c/
2)**2*tan(b*x/2)**2 + 6*b*tan(c/2)**2 + 2*b*tan(b*x/2)**4 + 4*b*tan(b*x/2)
**2 + 2*b) - b*x*tan(c/2)**6/(2*b*tan(c/2)**6*tan(b*x/2)**4 + 4*b*tan(c/2)
**6*tan(b*x/2)**2 + 2*b*tan(c/2)**6 + 6*b*tan(c/2)**4*tan(b*x/2)**4 + 12*b
*tan(c/2)**4*tan(b*x/2)**2 + 6*b*tan(c/2)**4 + 6*b*tan(c/2)**2*tan(b*x/2)*
*4 + 12*b*tan(c/2)**2*tan(b*x/2)**2 + 6*b*tan(c/2)**2 + 2*b*tan(b*x/2)**4
+ 4*b*tan(b*x/2)**2 + 2*b) + 7*b*x*tan(c/2)**4*tan(b*x/2)**4/(2*b*tan(c/2)
**6*tan(b*x/2)**4 + 4*b*tan(c/2)**6*tan(b*x/2)**2 + 2*b*tan(c/2)**6 + 6*b*
tan(c/2)**4*tan(b*x/2)**4 + 12*b*tan(c/2)**4*tan(b*x/2)**2 + 6*b*tan(c/2)*
*4 + 6*b*tan(c/2)**2*tan(b*x/2)**4 + 12*b*tan(c/2)**2*tan(b*x/2)**2 + 6*b*
tan(c/2)**2 + 2*b*tan(b*x/2)**4 + 4*b*tan(b*x/2)**2 + 2*b) + 14*b*x*tan(c/
2)**4*tan(b*x/2)**2/(2*b*tan(c/2)**6*tan(b*x/2)**4 + 4*b*tan(c/2)**6*ta...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.73

$$\int \sec(c+bx)\sin^3(a+bx)\,dx = -\frac{2\,(3\,b\sin\left(-a+c\right)+b\sin\left(-3\,a+3\,c\right))x+(3\,\cos\left(-a+c\right)+\cos\left(-3\,a+3\,c\right))\log\left(\cos\left(2\,bx\right)^{2}+2\,\cos\right.}{8\,b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(2*(3*b*sin(-a + c) + b*sin(-3*a + 3*c))*x + (3*cos(-a + c) + cos(-3*
a + 3*c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*
x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2) - 2*cos(2*b*x + 3*a - c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2306 vs. $2(58) = 116$.

Time = 0.16 (sec) , antiderivative size = 2306, normalized size of antiderivative = 36.03

$$\int \sec(c+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2*(2*(3*tan(1/2*a)^6*tan(1/2*c)^5 - 3*tan(1/2*a)^5*tan(1/2*c)^6 - 2*tan(
1/2*a)^6*tan(1/2*c)^3 + 21*tan(1/2*a)^5*tan(1/2*c)^4 - 21*tan(1/2*a)^4*tan
(1/2*c)^5 + 2*tan(1/2*a)^3*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c) - 21*t
an(1/2*a)^5*tan(1/2*c)^2 + 78*tan(1/2*a)^4*tan(1/2*c)^3 - 78*tan(1/2*a)^3*
tan(1/2*c)^4 + 21*tan(1/2*a)^2*tan(1/2*c)^5 - 3*tan(1/2*a)*tan(1/2*c)^6 +
3*tan(1/2*a)^5 - 21*tan(1/2*a)^4*tan(1/2*c) + 78*tan(1/2*a)^3*tan(1/2*c)^2
 - 78*tan(1/2*a)^2*tan(1/2*c)^3 + 21*tan(1/2*a)*tan(1/2*c)^4 - 3*tan(1/2*c
)^5 - 2*tan(1/2*a)^3 + 21*tan(1/2*a)^2*tan(1/2*c) - 21*tan(1/2*a)*tan(1/2*
c)^2 + 2*tan(1/2*c)^3 + 3*tan(1/2*a) - 3*tan(1/2*c))*(b*x + c)/(tan(1/2*a)
^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a)^4*tan(1/2*c)^
6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^4 + 3*tan(1/2*
a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^2 + 9*tan(1/2
*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2*a)^2*tan(1/
2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1) + (tan(1/2*
a)^6*tan(1/2*c)^6 - 3*tan(1/2*a)^6*tan(1/2*c)^4 + 12*tan(1/2*a)^5*tan(1/2*
c)^5 - 3*tan(1/2*a)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 - 24*tan(
1/2*a)^5*tan(1/2*c)^3 + 57*tan(1/2*a)^4*tan(1/2*c)^4 - 24*tan(1/2*a)^3*tan
(1/2*c)^5 + 3*tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^6 + 12*tan(1/2*a)^5*t
an(1/2*c) - 57*tan(1/2*a)^4*tan(1/2*c)^2 + 112*tan(1/2*a)^3*tan(1/2*c)^3 -
 57*tan(1/2*a)^2*tan(1/2*c)^4 + 12*tan(1/2*a)*tan(1/2*c)^5 - tan(1/2*c)...
```

Mupad [B] (verification not implemented)

Time = 17.94 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.56

$$\int \sec(c+bx)\sin^3(a+bx)\,dx = \frac{e^{-a\,3i+c\,1i-b\,x\,2i}}{8\,b}+\frac{e^{a\,3i-c\,1i+b\,x\,2i}}{8\,b}+\frac{x\,e^{-a\,3i+c\,1i}\,\left(e^{a\,2i}\,3i+e^{c\,2i}\,1i\right)}{4} - \frac{e^{-a\,6i+c\,6i}\,\ln\left(e^{a\,2i}\,e^{b\,x\,2i}+e^{a\,2i}\,e^{-c\,2i}\right)\,\left(8\,b\,e^{a\,3i-c\,3i}+24\,b\,e^{a\,5i-c\,5i}+24\,b\,e^{a\,7i-c\,7i}+8\,b\,e^{a\,9i-c\,9i}\right)}{64\,b^2}$$

input
```
int(sin(a + b*x)^3/cos(c + b*x),x)
```

output
```
exp(c*1i - a*3i - b*x*2i)/(8*b) + exp(a*3i - c*1i + b*x*2i)/(8*b) + (x*exp
(c*1i - a*3i)*(exp(a*2i)*3i + exp(c*2i)*1i))/4 - (exp(c*6i - a*6i)*log(exp
(a*2i)*exp(b*x*2i) + exp(a*2i)*exp(-c*2i))*(8*b*exp(a*3i - c*3i) + 24*b*ex
p(a*5i - c*5i) + 24*b*exp(a*7i - c*7i) + 8*b*exp(a*9i - c*9i)))/(64*b^2)
```

Reduce [F]

$$\int \sec(c+bx)\sin^3(a+bx)\,dx = \int \sec\left(bx+c\right)\sin\left(bx+a\right)^3 dx$$

input
```
int(sec(b*x+c)*sin(b*x+a)^3,x)
```

output
```
int(sec(b*x + c)*sin(a + b*x)**3,x)
```

3.165 $\int \sec^2(c+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \sec^2(c+bx)\sin^3(a+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 131.00

$$\begin{aligned}&\int \sec^2(c+bx)\sin^3(a+bx)\,dx\\&=\frac{\cos(3a-2c)\cos(bx)}{b}+\frac{\cos^3(a-c)\sec(c+bx)}{b}\\&\quad-\frac{6i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos^2(a-c)\sin(a-c)}{b}\\&\quad-\frac{\sin(3a-2c)\sin(bx)}{b}\end{aligned}$$

input
```
Integrate[Sec[c + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(Cos[3*a - 2*c]*Cos[b*x])/b + (Cos[a - c]^3*Sec[c + b*x])/b - ((6*I)*ArcTa
n[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c
]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c]^2*Sin[a - c])/b - (Sin
[3*a - 2*c]*Sin[b*x])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sec^2(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin^3(a+bx)\sec^2(bx+c)dx$$

input
```
Int[Sec[c + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 6.66 (sec) , antiderivative size = 235, normalized size of antiderivative = 235.00

method	result
risch	$\frac{e^{i(bx+5a-2c)}+3\,e^{i(bx+3a)}+3\,e^{i(bx+a+2c)}+e^{-i(-bx+a-4c)}}{4\left(e^{2i(bx+a+c)}+e^{2ia}\right)b}+\frac{3\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(3a-3c)}{4b}+\frac{3\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(}{4b}$
default	Expression too large to display

input
```
int(sec(b*x+c)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/(exp(2*I*(b*x+a+c))+exp(2*I*a))/b*(exp(I*(b*x+5*a-2*c))+3*exp(I*(b*x+3
*a))+3*exp(I*(b*x+a+2*c))+exp(-I*(-b*x+a-4*c)))+3/4*ln(exp(I*(b*x+a))+I*ex
p(I*(a-c)))/b*sin(3*a-3*c)+3/4*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*sin(a-c
)-3/4*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(3*a-3*c)-3/4*ln(exp(I*(b*x+a
))-I*exp(I*(a-c)))/b*sin(a-c)+cos(b*x+3*a-2*c)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 149.00

$$\int \sec^2(c+bx)\sin^3(a+bx)\,dx =$$
$$-\frac{3\,\cos\left(bx+c\right)\cos\left(-a+c\right)^2\log\left(\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right)-3\,\cos\left(bx+c\right)\cos\left(-a+c\right)^2\log\left(-\sin\right.}{}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(3*cos(b*x + c)*cos(-a + c)^2*log(sin(b*x + c) + 1)*sin(-a + c) - 3*c
os(b*x + c)*cos(-a + c)^2*log(-sin(b*x + c) + 1)*sin(-a + c) - 2*(4*cos(-a
 + c)^2 - 1)*cos(b*x + c)*sin(b*x + c)*sin(-a + c) - 2*(4*cos(-a + c)^3 -
3*cos(-a + c))*cos(b*x + c)^2 - 2*cos(-a + c)^3)/(b*cos(b*x + c))
```

Sympy [F(-2)]

Exception generated.

$$\int \sec^2(c+bx)\sin^3(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+c)**2*sin(b*x+a)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.22 (sec) , antiderivative size = 871, normalized size of antiderivative = 871.00

$$\int \sec^2(c+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*(4*(cos(3*b*x + 3*a + 4*c) + cos(b*x + 3*a + 2*c))*cos(4*b*x + 6*a + 2
*c) + 2*(3*cos(2*b*x + 6*a) + 3*cos(2*b*x + 4*a + 2*c) + 3*cos(2*b*x + 2*a
 + 4*c) + 3*cos(2*b*x + 6*c) + 2*cos(4*c))*cos(3*b*x + 3*a + 4*c) + 6*cos(
2*b*x + 6*a)*cos(b*x + 3*a + 2*c) + 6*cos(2*b*x + 4*a + 2*c)*cos(b*x + 3*a
 + 2*c) + 6*cos(2*b*x + 2*a + 4*c)*cos(b*x + 3*a + 2*c) + 6*cos(2*b*x + 6*
c)*cos(b*x + 3*a + 2*c) + 4*cos(b*x + 3*a + 2*c)*cos(4*c) + 3*((sin(-a + c
) + sin(-3*a + 3*c))*cos(3*b*x + 3*a + 4*c)^2 + 2*(sin(-a + c) + sin(-3*a
+ 3*c))*cos(3*b*x + 3*a + 4*c)*cos(b*x + 3*a + 2*c) + (sin(-a + c) + sin(-
3*a + 3*c))*cos(b*x + 3*a + 2*c)^2 + (sin(-a + c) + sin(-3*a + 3*c))*sin(3
*b*x + 3*a + 4*c)^2 + 2*(sin(-a + c) + sin(-3*a + 3*c))*sin(3*b*x + 3*a +
4*c)*sin(b*x + 3*a + 2*c) + (sin(-a + c) + sin(-3*a + 3*c))*sin(b*x + 3*a
+ 2*c)^2)*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin
(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + co
s(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin
(c) + sin(c)^2)) + 4*(sin(3*b*x + 3*a + 4*c) + sin(b*x + 3*a + 2*c))*sin(4
*b*x + 6*a + 2*c) + 2*(3*sin(2*b*x + 6*a) + 3*sin(2*b*x + 4*a + 2*c) + 3*s
in(2*b*x + 2*a + 4*c) + 3*sin(2*b*x + 6*c) + 2*sin(4*c))*sin(3*b*x + 3*a +
 4*c) + 6*sin(2*b*x + 6*a)*sin(b*x + 3*a + 2*c) + 6*sin(2*b*x + 4*a + 2*c)
*sin(b*x + 3*a + 2*c) + 6*sin(2*b*x + 2*a + 4*c)*sin(b*x + 3*a + 2*c) + 6*
sin(2*b*x + 6*c)*sin(b*x + 3*a + 2*c) + 4*sin(b*x + 3*a + 2*c)*sin(4*c)...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.18 (sec) , antiderivative size = 2956, normalized size of antiderivative = 2956.00

$$\int \sec^2(c+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
2*(3*(tan(1/2*a)^6*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^6 - 2*tan(1/2*a)
^6*tan(1/2*c)^3 + 11*tan(1/2*a)^5*tan(1/2*c)^4 - 11*tan(1/2*a)^4*tan(1/2*c
)^5 + 2*tan(1/2*a)^3*tan(1/2*c)^6 + tan(1/2*a)^6*tan(1/2*c) - 11*tan(1/2*a
)^5*tan(1/2*c)^2 + 38*tan(1/2*a)^4*tan(1/2*c)^3 - 38*tan(1/2*a)^3*tan(1/2*
c)^4 + 11*tan(1/2*a)^2*tan(1/2*c)^5 - tan(1/2*a)*tan(1/2*c)^6 + tan(1/2*a)
^5 - 11*tan(1/2*a)^4*tan(1/2*c) + 38*tan(1/2*a)^3*tan(1/2*c)^2 - 38*tan(1/
2*a)^2*tan(1/2*c)^3 + 11*tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*c)^5 - 2*tan(1/
2*a)^3 + 11*tan(1/2*a)^2*tan(1/2*c) - 11*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1
/2*c)^3 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan
(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a)^4*tan(
1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^4 + 3*t
an(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^2 + 9*
tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2*a)^2
*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1) - 3*
(tan(1/2*a)^6*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^6 - 2*tan(1/2*a)^6*ta
n(1/2*c)^3 + 11*tan(1/2*a)^5*tan(1/2*c)^4 - 11*tan(1/2*a)^4*tan(1/2*c)^5 +
 2*tan(1/2*a)^3*tan(1/2*c)^6 + tan(1/2*a)^6*tan(1/2*c) - 11*tan(1/2*a)^5*t
an(1/2*c)^2 + 38*tan(1/2*a)^4*tan(1/2*c)^3 - 38*tan(1/2*a)^3*tan(1/2*c)^4
+ 11*tan(1/2*a)^2*tan(1/2*c)^5 - tan(1/2*a)*tan(1/2*c)^6 + tan(1/2*a)^5 -
11*tan(1/2*a)^4*tan(1/2*c) + 38*tan(1/2*a)^3*tan(1/2*c)^2 - 38*tan(1/2*...
```

Mupad [B] (verification not implemented)

Time = 24.93 (sec) , antiderivative size = 382, normalized size of antiderivative = 382.00

$$\int \sec^2(c+bx)\sin^3(a+bx)\,dx = \frac{\mathrm{e}^{-a\,3\mathrm{i}+c\,2\mathrm{i}-b\,x\,1\mathrm{i}}}{2\,b}+\frac{\mathrm{e}^{a\,3\mathrm{i}-c\,2\mathrm{i}+b\,x\,1\mathrm{i}}}{2\,b}+\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,2\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(3\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+3\,\mathrm{e}^{a\,4\mathrm{i}-c\,4\mathrm{i}}+\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}+1\right)\,1\mathrm{i}}{4\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}\,1\mathrm{i}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}\right)} + \frac{3\,\sin\left(2\,a-2\,c\right)\,\ln\left(-\frac{3\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,\left(\sin(2\,a-2\,c)+\sin(2\,a-2\,c)\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\right)}{2}-\frac{\sin(2\,a-2\,c)\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}+1\right)\,3\mathrm{i}}{2\,\sqrt{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}}\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}-c}\right.}{4\,b\,\sqrt{\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}} - \frac{3\,\sin\left(2\,a-2\,c\right)\,\ln\left(-\frac{3\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,\left(\sin(2\,a-2\,c)+\sin(2\,a-2\,c)\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\right)}{2}+\frac{\sin(2\,a-2\,c)\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}+1\right)\,3\mathrm{i}}{2\,\sqrt{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}}\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}-c}\right.}{4\,b\,\sqrt{\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}}$$

input
```
int(sin(a + b*x)^3/cos(c + b*x)^2,x)
```

output
```
exp(c*2i - a*3i - b*x*1i)/(2*b) + exp(a*3i - c*2i + b*x*1i)/(2*b) + (exp(c
*2i - a*1i + b*x*1i)*(3*exp(a*2i - c*2i) + 3*exp(a*4i - c*4i) + exp(a*6i -
 c*6i) + 1)*1i)/(4*b*(exp(a*2i - c*2i)*1i + exp(a*2i + b*x*2i)*1i)) + (3*s
in(2*a - 2*c)*log(- (3*exp(a*1i)*exp(b*x*1i)*(sin(2*a - 2*c) + sin(2*a - 2
*c)*exp(a*2i)*exp(-c*2i)))/2 - (sin(2*a - 2*c)*exp(a*2i)*exp(-c*2i)*(exp(a
*2i)*exp(-c*2i) + 1)*3i)/(2*(exp(a*2i)*exp(-c*2i))^(1/2)))*(exp(a*2i - c*2
i) + 1))/(4*b*exp(a*2i - c*2i)^(1/2)) - (3*sin(2*a - 2*c)*log((sin(2*a - 2
*c)*exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*3i)/(2*(exp(a*2i)*exp(
-c*2i))^(1/2)) - (3*exp(a*1i)*exp(b*x*1i)*(sin(2*a - 2*c) + sin(2*a - 2*c)
*exp(a*2i)*exp(-c*2i)))/2)*(exp(a*2i - c*2i) + 1))/(4*b*exp(a*2i - c*2i)^(
1/2))
```

Reduce [F]

$$\int \sec^2(c+bx)\sin^3(a+bx)\,dx = \int \sec(bx+c)^2\sin(bx+a)^3\,dx$$

input
```
int(sec(b*x+c)^2*sin(b*x+a)^3,x)
```

output
```
int(sec(b*x + c)**2*sin(a + b*x)**3,x)
```

3.166 $\int \sec^3(c+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \sec^3(c+bx)\sin^3(a+bx)\,dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.75 (sec) , antiderivative size = 284, normalized size of antiderivative = 284.00

$$\int \sec^3(c+bx)\sin^3(a+bx)\,dx$$
$$= \frac{\sec(c)\sec^2(c+bx)(6\cos(a)+4\cos(3a-2c)+3\cos(3a-4c-2bx)-3\cos(a+2bx)-3\cos(3a-2c+$$

input
```
Integrate[Sec[c + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(Sec[c]*Sec[c + b*x]^2*(6*Cos[a] + 4*Cos[3*a - 2*c] + 3*Cos[3*a - 4*c - 2*
b*x] - 3*Cos[a + 2*b*x] - 3*Cos[3*a - 2*c + 2*b*x] + 3*Cos[a - 2*(c + b*x)
] + 4*Cos[3*a - 2*c]*Log[Cos[c + b*x]] + 2*Cos[3*a - 6*c - 2*b*x]*Log[Cos[
c + b*x]] + 2*Cos[3*a - 4*c - 2*b*x]*Log[Cos[c + b*x]] + 2*Cos[3*a + 2*b*x
]*Log[Cos[c + b*x]] + 2*Cos[3*a - 2*c + 2*b*x]*Log[Cos[c + b*x]] + Cos[3*a
 - 4*c]*(-2 + 4*Log[Cos[c + b*x]]) - 4*b*x*Sin[3*a - 4*c] - 4*b*x*Sin[3*a
- 2*c] - 2*b*x*Sin[3*a - 6*c - 2*b*x] - 2*b*x*Sin[3*a - 4*c - 2*b*x] - 2*b
*x*Sin[3*a + 2*b*x] - 2*b*x*Sin[3*a - 2*c + 2*b*x]))/(16*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sec^3(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin^3(a+bx)\sec^3(bx+c)dx$$

input
```
Int[Sec[c + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 17.54 (sec) , antiderivative size = 193, normalized size of antiderivative = 193.00

method	result
risch	$ix\,\mathrm{e}^{3i(a-c)}-2i\cos\left(3a-3c\right)x-\frac{2i\cos(3a-3c)a}{b}+\frac{4\,\mathrm{e}^{i(2bx+7a-c)}+6\,\mathrm{e}^{i(2bx+5a+c)}-2\,\mathrm{e}^{i(2bx+a+5c)}+3\,\mathrm{e}^{i(7a-3c)}+3\,\mathrm{e}^{i(}}{4\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{2}b}$
default	Expression too large to display

input
```
int(sec(b*x+c)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
I*x*exp(3*I*(a-c))-2*I*cos(3*a-3*c)*x-2*I/b*cos(3*a-3*c)*a+1/4/(exp(2*I*(b
*x+a+c))+exp(2*I*a))^2/b*(4*exp(I*(2*b*x+7*a-c))+6*exp(I*(2*b*x+5*a+c))-2*
exp(I*(2*b*x+a+5*c))+3*exp(I*(7*a-3*c))+3*exp(I*(5*a-c))-3*exp(I*(3*a+c))-
3*exp(I*(a+3*c)))+1/b*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*cos(3*a-3*c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 123.00

$$\int \sec^3(c+bx)\sin^3(a+bx)\,dx =$$
$$-\frac{6\,\cos\left(bx+c\right)\cos\left(-a+c\right)^{2}\sin\left(bx+c\right)\sin\left(-a+c\right)-2\left(4\,\cos\left(-a+c\right)^{3}-3\,\cos\left(-a+c\right)\right)\cos\left(bx+\right.}{2\,b\cos\left(bx\,-\right.}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(6*cos(b*x + c)*cos(-a + c)^2*sin(b*x + c)*sin(-a + c) - 2*(4*cos(-a
+ c)^3 - 3*cos(-a + c))*cos(b*x + c)^2*log(-cos(b*x + c)) - 2*(4*b*x*cos(-
a + c)^2 - b*x)*cos(b*x + c)^2*sin(-a + c) - cos(-a + c)^3)/(b*cos(b*x + c
)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+c)**3*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.09 (sec) , antiderivative size = 1403, normalized size of antiderivative = 1403.00

$$\int \sec^3(c+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(4*(b*cos(6*c)*sin(3*a + 3*c) - b*cos(3*a + 3*c)*sin(6*c))*x - (8*b*x
*sin(2*b*x + 8*c) + 4*b*x*sin(6*c) + 4*cos(2*b*x + 6*a + 2*c) + 6*cos(2*b*
x + 4*a + 4*c) - 2*cos(2*b*x + 8*c) + 3*cos(6*a) + 3*cos(4*a + 2*c) - 3*co
s(2*a + 4*c) - 3*cos(6*c))*cos(4*b*x + 3*a + 7*c) + 4*(b*x*sin(4*b*x + 3*a
 + 7*c) + 2*b*x*sin(2*b*x + 3*a + 5*c) + b*x*sin(3*a + 3*c))*cos(4*b*x + 1
0*c) - 2*(4*b*x*sin(6*c) + 4*cos(2*b*x + 6*a + 2*c) + 6*cos(2*b*x + 4*a +
4*c) + 3*cos(6*a) + 3*cos(4*a + 2*c) - 3*cos(2*a + 4*c) - 3*cos(6*c))*cos(
2*b*x + 3*a + 5*c) + 2*(8*b*x*sin(2*b*x + 3*a + 5*c) + 4*b*x*sin(3*a + 3*c
) + 2*cos(2*b*x + 3*a + 5*c) + cos(3*a + 3*c))*cos(2*b*x + 8*c) - 3*(cos(6
*a) + cos(4*a + 2*c) - cos(6*c))*cos(3*a + 3*c) - 4*cos(2*b*x + 6*a + 2*c)
*cos(3*a + 3*c) - 6*cos(2*b*x + 4*a + 4*c)*cos(3*a + 3*c) + 3*cos(3*a + 3*
c)*cos(2*a + 4*c) - 2*(cos(4*b*x + 3*a + 7*c)^2*cos(-3*a + 3*c) + 4*cos(2*
b*x + 3*a + 5*c)^2*cos(-3*a + 3*c) + 4*cos(2*b*x + 3*a + 5*c)*cos(3*a + 3*
c)*cos(-3*a + 3*c) + cos(3*a + 3*c)^2*cos(-3*a + 3*c) + cos(-3*a + 3*c)*si
n(4*b*x + 3*a + 7*c)^2 + 4*cos(-3*a + 3*c)*sin(2*b*x + 3*a + 5*c)^2 + 4*co
s(-3*a + 3*c)*sin(2*b*x + 3*a + 5*c)*sin(3*a + 3*c) + cos(-3*a + 3*c)*sin(
3*a + 3*c)^2 + 2*(2*cos(2*b*x + 3*a + 5*c)*cos(-3*a + 3*c) + cos(3*a + 3*c
)*cos(-3*a + 3*c))*cos(4*b*x + 3*a + 7*c) + 2*(2*cos(-3*a + 3*c)*sin(2*b*x
 + 3*a + 5*c) + cos(-3*a + 3*c)*sin(3*a + 3*c))*sin(4*b*x + 3*a + 7*c))*lo
g(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.19 (sec) , antiderivative size = 4224, normalized size of antiderivative = 4224.00

$$\int \sec^3(c+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*(4*(3*tan(1/2*a)^6*tan(1/2*c)^5 - 3*tan(1/2*a)^5*tan(1/2*c)^6 - 10*ta
n(1/2*a)^6*tan(1/2*c)^3 + 45*tan(1/2*a)^5*tan(1/2*c)^4 - 45*tan(1/2*a)^4*t
an(1/2*c)^5 + 10*tan(1/2*a)^3*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c) - 4
5*tan(1/2*a)^5*tan(1/2*c)^2 + 150*tan(1/2*a)^4*tan(1/2*c)^3 - 150*tan(1/2*
a)^3*tan(1/2*c)^4 + 45*tan(1/2*a)^2*tan(1/2*c)^5 - 3*tan(1/2*a)*tan(1/2*c)
^6 + 3*tan(1/2*a)^5 - 45*tan(1/2*a)^4*tan(1/2*c) + 150*tan(1/2*a)^3*tan(1/
2*c)^2 - 150*tan(1/2*a)^2*tan(1/2*c)^3 + 45*tan(1/2*a)*tan(1/2*c)^4 - 3*ta
n(1/2*c)^5 - 10*tan(1/2*a)^3 + 45*tan(1/2*a)^2*tan(1/2*c) - 45*tan(1/2*a)*
tan(1/2*c)^2 + 10*tan(1/2*c)^3 + 3*tan(1/2*a) - 3*tan(1/2*c))*(b*x + c)/(t
an(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a)^4*ta
n(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^4 + 3
*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^2 +
9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2*a)
^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1) +
(tan(1/2*a)^6*tan(1/2*c)^6 - 15*tan(1/2*a)^6*tan(1/2*c)^4 + 36*tan(1/2*a)^
5*tan(1/2*c)^5 - 15*tan(1/2*a)^4*tan(1/2*c)^6 + 15*tan(1/2*a)^6*tan(1/2*c)
^2 - 120*tan(1/2*a)^5*tan(1/2*c)^3 + 225*tan(1/2*a)^4*tan(1/2*c)^4 - 120*t
an(1/2*a)^3*tan(1/2*c)^5 + 15*tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^6 + 3
6*tan(1/2*a)^5*tan(1/2*c) - 225*tan(1/2*a)^4*tan(1/2*c)^2 + 400*tan(1/2*a)
^3*tan(1/2*c)^3 - 225*tan(1/2*a)^2*tan(1/2*c)^4 + 36*tan(1/2*a)*tan(1/2...
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+bx)\sin^3(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^3/cos(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^3(c+bx)\sin^3(a+bx)\,dx = \int \sec(bx+c)^3 \sin(bx+a)^3\,dx$$

input
```
int(sec(b*x+c)^3*sin(b*x+a)^3,x)
```

output
```
int(sec(b*x + c)**3*sin(a + b*x)**3,x)
```

3.167 $\int \cos^3(a+bx)\csc(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \cos^3(a+bx)\csc(c+bx)\,dx=\frac{\cos(3a-c+2bx)}{4b}+\frac{\cos^3(a-c)\log(\sin(c+bx))}{b}-\frac{3}{4}x\sin(a-c)-\frac{1}{4}x\sin(3(a-c))$$

output
```
1/4*cos(2*b*x+3*a-c)/b+cos(a-c)^3*ln(sin(b*x+c))/b-3/4*x*sin(a-c)-1/4*x*si
n(3*a-3*c)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

$$\int \cos^3(a+bx)\csc(c+bx)\,dx$$
$$=\frac{6ibx\cos(a-c)-8i\arctan(\tan(c+bx))\cos^3(a-c)+2\cos(3a-c+2bx)+3\cos(a-c)\log\left(\sin^2(c+b\right.}{8b}$$

input
```
Integrate[Cos[a + b*x]^3*Csc[c + b*x],x]
```

output
```
((6*I)*b*x*Cos[a - c] - (8*I)*ArcTan[Tan[c + b*x]]*Cos[a - c]^3 + 2*Cos[3*
a - c + 2*b*x] + 3*Cos[a - c]*Log[Sin[c + b*x]^2] + Cos[3*(a - c)]*((2*I)*
b*x + Log[Sin[c + b*x]^2]) - 6*b*x*Sin[a - c] - 2*b*x*Sin[3*(a - c)])/(8*b
)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \cos^3(a+bx)\csc(bx+c)dx$$

input
```
Int[Cos[a + b*x]^3*Csc[c + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.59

method	result
risch	$\frac{ix\,e^{3i(a-c)}}{4}+\frac{3ix\,e^{i(a-c)}}{4}-\frac{3i\cos(a-c)x}{2}-\frac{i\cos(3a-3c)x}{2}-\frac{3i\cos(a-c)a}{2b}-\frac{i\cos(3a-3c)a}{2b}+\frac{3\ln\left(e^{2i(bx+a)}-e^{2i(a-c)}\right)\cdots}{4b}$
default	Expression too large to display

input
```
int(cos(b*x+a)^3*csc(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*x*exp(3*I*(a-c))+3/4*I*x*exp(I*(a-c))-3/2*I*cos(a-c)*x-1/2*I*cos(3*a
-3*c)*x-3/2*I/b*cos(a-c)*a-1/2*I/b*cos(3*a-3*c)*a+3/4/b*ln(exp(2*I*(b*x+a)
)-exp(2*I*(a-c)))*cos(a-c)+1/4/b*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*cos(3
*a-3*c)+1/4*cos(2*b*x+3*a-c)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.70

$$\int \cos^3(a+bx)\csc(c+bx)\,dx = \frac{2\cos(-a+c)^3\log\left(\frac{1}{2}\sin(bx+c)\right)+\left(4\cos(-a+c)^2-1\right)\cos(bx+c)\sin(bx+c)\sin(-a+c)+(4\cos\ldots}{2b}$$

input
```
integrate(cos(b*x+a)^3*csc(b*x+c),x, algorithm="fricas")
```

output
```
1/2*(2*cos(-a + c)^3*log(1/2*sin(b*x + c)) + (4*cos(-a + c)^2 - 1)*cos(b*x
 + c)*sin(b*x + c)*sin(-a + c) + (4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x
 + c)^2 + (2*b*x*cos(-a + c)^2 + b*x)*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10487 vs. 2(0) = 0.

Time = 27.79 (sec) , antiderivative size = 33056, normalized size of antiderivative = 524.70

$$\int \cos^3(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**3*csc(b*x+c),x)
```

output
```
3*Piecewise((0, Eq(b, 0) & Eq(c, 0)), (sin(b*x)**2/(2*b), Eq(c, 0)), (0, E
q(b, 0)), (-b*x*tan(c/2)**5*tan(b*x/2)**4/(b*tan(c/2)**6*tan(b*x/2)**4 + 2
*b*tan(c/2)**6*tan(b*x/2)**2 + b*tan(c/2)**6 + 3*b*tan(c/2)**4*tan(b*x/2)*
*4 + 6*b*tan(c/2)**4*tan(b*x/2)**2 + 3*b*tan(c/2)**4 + 3*b*tan(c/2)**2*tan
(b*x/2)**4 + 6*b*tan(c/2)**2*tan(b*x/2)**2 + 3*b*tan(c/2)**2 + b*tan(b*x/2
)**4 + 2*b*tan(b*x/2)**2 + b) - 2*b*x*tan(c/2)**5*tan(b*x/2)**2/(b*tan(c/2
)**6*tan(b*x/2)**4 + 2*b*tan(c/2)**6*tan(b*x/2)**2 + b*tan(c/2)**6 + 3*b*t
an(c/2)**4*tan(b*x/2)**4 + 6*b*tan(c/2)**4*tan(b*x/2)**2 + 3*b*tan(c/2)**4
 + 3*b*tan(c/2)**2*tan(b*x/2)**4 + 6*b*tan(c/2)**2*tan(b*x/2)**2 + 3*b*tan
(c/2)**2 + b*tan(b*x/2)**4 + 2*b*tan(b*x/2)**2 + b) - b*x*tan(c/2)**5/(b*t
an(c/2)**6*tan(b*x/2)**4 + 2*b*tan(c/2)**6*tan(b*x/2)**2 + b*tan(c/2)**6 +
 3*b*tan(c/2)**4*tan(b*x/2)**4 + 6*b*tan(c/2)**4*tan(b*x/2)**2 + 3*b*tan(c
/2)**4 + 3*b*tan(c/2)**2*tan(b*x/2)**4 + 6*b*tan(c/2)**2*tan(b*x/2)**2 + 3
*b*tan(c/2)**2 + b*tan(b*x/2)**4 + 2*b*tan(b*x/2)**2 + b) + 6*b*x*tan(c/2)
**3*tan(b*x/2)**4/(b*tan(c/2)**6*tan(b*x/2)**4 + 2*b*tan(c/2)**6*tan(b*x/2
)**2 + b*tan(c/2)**6 + 3*b*tan(c/2)**4*tan(b*x/2)**4 + 6*b*tan(c/2)**4*tan
(b*x/2)**2 + 3*b*tan(c/2)**4 + 3*b*tan(c/2)**2*tan(b*x/2)**4 + 6*b*tan(c/2
)**2*tan(b*x/2)**2 + 3*b*tan(c/2)**2 + b*tan(b*x/2)**4 + 2*b*tan(b*x/2)**2
 + b) + 12*b*x*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**4 + 2*
b*tan(c/2)**6*tan(b*x/2)**2 + b*tan(c/2)**6 + 3*b*tan(c/2)**4*tan(b*x/2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(57) = 114$.

Time = 0.06 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.46

$$\int \cos^3(a+bx)\csc(c+bx)\,dx$$
$$= \frac{2\left(3\,b\sin\left(-a+c\right)+b\sin\left(-3\,a+3\,c\right)\right)x+\left(3\,\cos\left(-a+c\right)+\cos\left(-3\,a+3\,c\right)\right)\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\right.}{}$$

input
```
integrate(cos(b*x+a)^3*csc(b*x+c),x, algorithm="maxima")
```

output
```
1/8*(2*(3*b*sin(-a + c) + b*sin(-3*a + 3*c))*x + (3*cos(-a + c) + cos(-3*a
 + 3*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*si
n(b*x)*sin(c) + sin(c)^2) + (3*cos(-a + c) + cos(-3*a + 3*c))*log(cos(b*x)
^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c
)^2) + 2*cos(2*b*x + 3*a - c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3325 vs. $2(57) = 114$.

Time = 0.21 (sec) , antiderivative size = 3325, normalized size of antiderivative = 52.78

$$\int \cos^3(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*csc(b*x+c),x, algorithm="giac")
```

output
```
-1/2*(2*(3*tan(1/2*a)^6*tan(1/2*c)^5 - 3*tan(1/2*a)^5*tan(1/2*c)^6 - 2*tan
(1/2*a)^6*tan(1/2*c)^3 + 21*tan(1/2*a)^5*tan(1/2*c)^4 - 21*tan(1/2*a)^4*ta
n(1/2*c)^5 + 2*tan(1/2*a)^3*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c) - 21*
tan(1/2*a)^5*tan(1/2*c)^2 + 78*tan(1/2*a)^4*tan(1/2*c)^3 - 78*tan(1/2*a)^3
*tan(1/2*c)^4 + 21*tan(1/2*a)^2*tan(1/2*c)^5 - 3*tan(1/2*a)*tan(1/2*c)^6 +
 3*tan(1/2*a)^5 - 21*tan(1/2*a)^4*tan(1/2*c) + 78*tan(1/2*a)^3*tan(1/2*c)^
2 - 78*tan(1/2*a)^2*tan(1/2*c)^3 + 21*tan(1/2*a)*tan(1/2*c)^4 - 3*tan(1/2*
c)^5 - 2*tan(1/2*a)^3 + 21*tan(1/2*a)^2*tan(1/2*c) - 21*tan(1/2*a)*tan(1/2
*c)^2 + 2*tan(1/2*c)^3 + 3*tan(1/2*a) - 3*tan(1/2*c))*(b*x + a)/(tan(1/2*a
)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a)^4*tan(1/2*c)
^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^4 + 3*tan(1/2
*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^2 + 9*tan(1/
2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2*a)^2*tan(1
/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1) + (tan(1/2
*a)^6*tan(1/2*c)^6 - 3*tan(1/2*a)^6*tan(1/2*c)^4 + 12*tan(1/2*a)^5*tan(1/2
*c)^5 - 3*tan(1/2*a)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 - 24*tan
(1/2*a)^5*tan(1/2*c)^3 + 57*tan(1/2*a)^4*tan(1/2*c)^4 - 24*tan(1/2*a)^3*ta
n(1/2*c)^5 + 3*tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^6 + 12*tan(1/2*a)^5*
tan(1/2*c) - 57*tan(1/2*a)^4*tan(1/2*c)^2 + 112*tan(1/2*a)^3*tan(1/2*c)^3
- 57*tan(1/2*a)^2*tan(1/2*c)^4 + 12*tan(1/2*a)*tan(1/2*c)^5 - tan(1/2*c...
```

Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.62

$$\int \cos^3(a+bx)\csc(c+bx)\,dx = \frac{e^{-a\,3i+c\,1i-b\,x\,2i}}{8\,b}+\frac{e^{a\,3i-c\,1i+b\,x\,2i}}{8\,b}-\frac{x\,e^{-a\,3i+c\,1i}\,\left(e^{a\,2i}\,3i+e^{c\,2i}\,1i\right)}{4}+\frac{e^{-a\,6i+c\,6i}\,\ln\left(e^{a\,2i}\,e^{b\,x\,2i}-e^{a\,2i}\,e^{-c\,2i}\right)\,\left(8\,b\,e^{a\,3i-c\,3i}+24\,b\,e^{a\,5i-c\,5i}+24\,b\,e^{a\,7i-c\,7i}+8\,b\,e^{a\,9i-c\,9i}\right)}{64\,b^2}$$

input
```
int(cos(a + b*x)^3/sin(c + b*x),x)
```

output
```
exp(c*1i - a*3i - b*x*2i)/(8*b) + exp(a*3i - c*1i + b*x*2i)/(8*b) - (x*exp
(c*1i - a*3i)*(exp(a*2i)*3i + exp(c*2i)*1i))/4 + (exp(c*6i - a*6i)*log(exp
(a*2i)*exp(b*x*2i) - exp(a*2i)*exp(-c*2i))*(8*b*exp(a*3i - c*3i) + 24*b*ex
p(a*5i - c*5i) + 24*b*exp(a*7i - c*7i) + 8*b*exp(a*9i - c*9i)))/(64*b^2)
```

Reduce [F]

$$\int \cos^3(a+bx)\csc(c+bx)\,dx = \int \cos(bx+a)^3\csc(bx+c)\,dx$$

input
```
int(cos(b*x+a)^3*csc(b*x+c),x)
```

output
```
int(cos(a + b*x)**3*csc(b*x + c),x)
```

3.168 $\int \cos^2(a+bx)\csc(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \cos^2(a+bx)\csc(c+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(c+bx))\cos^2(a-c)}{b}+\frac{\cos(2a-c+bx)}{b}$$

output
```
-arctanh(cos(b*x+c))*cos(a-c)^2/b+cos(b*x+2*a-c)/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int \cos^2(a+bx)\csc(c+bx)\,dx =\frac{\cos(2a-c+bx)+\cos^2(a-c)\left(-\log\left(\cos\left(\frac{1}{2}(c+bx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+bx)\right)\right)\right)}{b}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c + b*x],x]
```

output
```
(Cos[2*a - c + b*x] + Cos[a - c]^2*(-Log[Cos[(c + b*x)/2]] + Log[Sin[(c +
b*x)/2]]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc(bx+c)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \cos^2(a+bx)\csc(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Csc[c + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.81

method	result
risch	$\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)}{2b}+\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos(2a-2c)}{2b}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)}{2b}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos(2a-2c)}{2b}+\text{co}$
default	$\frac{-\frac{2\left(-\cos(a)^2\cos(c)^2-2\cos(a)\cos(c)\sin(a)\sin(c)-\sin(a)^2\sin(c)^2\right)\arctan\left(\frac{2(\sin(a)\cos(c)-\cos(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2\cos(a)\cos(c)+2\sin(a)\,\text{si}}{2\sqrt{-\cos(c)^2\sin(a)^2-\cos(a)^2\cos(c)^2-\sin(a)^2\sin(c)^2-\sin(c)^2\cos(a}}\right.}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)\sqrt{-\cos(c)^2\sin(a)^2-\cos(a)^2\cos(c)^2-\sin(a)^2\sin(c)^2-\sin(c)^2\cos(a)^2}}}{b}$

input
```
int(cos(b*x+a)^2*csc(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))+1/2/b*ln(exp(I*(b*x+a))-exp(I*(a-c))
)*cos(2*a-2*c)-1/2/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))-1/2/b*ln(exp(I*(b*x+a
))+exp(I*(a-c)))*cos(2*a-2*c)+cos(b*x+2*a-c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.42

$$\int \cos^2(a+bx)\csc(c+bx)\,dx = -\frac{\cos(-a+c)^2\log\left(\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right) - \cos(-a+c)^2\log\left(-\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right) - 4\cos(-a+c)\sin}{2b}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(cos(-a + c)^2*log(1/2*cos(b*x + c) + 1/2) - cos(-a + c)^2*log(-1/2*c
os(b*x + c) + 1/2) - 4*cos(-a + c)*sin(b*x + c)*sin(-a + c) - 2*(2*cos(-a
+ c)^2 - 1)*cos(b*x + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1459 vs. $2(27) = 54$.

Time = 10.25 (sec) , antiderivative size = 3215, normalized size of antiderivative = 89.31

$$\int \cos^2(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**2*csc(b*x+c),x)
```

output
```
-2*Piecewise((0, Eq(b, 0) & Eq(c, 0)), (sin(b*x)/b, Eq(c, 0)), (0, Eq(b, 0
)), (2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4
*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c
/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**3/(
b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**
2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(c/2) + tan(b*x/2))*
tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*
tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log
(tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)
**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 +
b) - 2*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)*
*4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan
(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)*
*3/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/
2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - 1/tan(
c/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4
+ 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) +
 2*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*
tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/
2)**2 + b) - 2*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(36) = 72$.

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.28

$$\int \cos^2(a+bx)\csc(c+bx)\,dx = -\frac{(\cos(-2a+2c)+1)\log\left(\cos(bx)^2+2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2-2\sin(bx)\sin(c)+\sin\right.}{}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c),x, algorithm="maxima")
```

output
```
-1/4*((cos(-2*a + 2*c) + 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2
+ sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(-2*a + 2*c) + 1)*log(c
os(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c)
+ sin(c)^2) - 4*cos(b*x + 2*a - c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(36) = 72$.

Time = 0.28 (sec) , antiderivative size = 527, normalized size of antiderivative = 14.64

$$\int \cos^2(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c),x, algorithm="giac")
```

output
```
((tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*a)^3
*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 8*tan(1/2*a)^
3*tan(1/2*c) + 20*tan(1/2*a)^2*tan(1/2*c)^2 - 8*tan(1/2*a)*tan(1/2*c)^3 +
tan(1/2*c)^4 - 2*tan(1/2*a)^2 + 8*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*c)^2 +
 1)*log(abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(1/2*b*x
 + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) - 2*
tan(1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)
 - 2*tan(1/2*c)^2)/abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) - 2*
tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^2
 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c) +
 4*tan(1/2*a)*tan(1/2*c) + 2))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 +
 tan(1/2*c)^2 + 1)^2 - 2*(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) -
 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^
2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*a)^2 - 2*tan(1/2*b*x + 1/2
*a)*tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1)/((tan(1/2*a)^
2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b*x + 1/2*a)^2
+ 1)))/b
```

Mupad [B] (verification not implemented)

Time = 19.36 (sec) , antiderivative size = 223, normalized size of antiderivative = 6.19

$$\int \cos^2(a+bx)\csc(c+bx)\,dx$$
$$= \frac{e^{-a\,2i+c\,1i-b\,x\,1i}}{2\,b} + \frac{e^{a\,2i-c\,1i+b\,x\,1i}}{2\,b} - \frac{e^{-a\,2i+c\,2i}\,\ln\left(-\frac{\left(e^{a\,2i}\,e^{-c\,2i}+1\right)^2\,1i}{2} - \frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,2i+e^{a\,4i}\,e^{-c\,4i}\,1i+1i\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}+1\right)^2}{4\,b} + \frac{e^{-a\,2i+c\,2i}\,\ln\left(\frac{\left(e^{a\,2i}\,e^{-c\,2i}+1\right)^2\,1i}{2} - \frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,2i+e^{a\,4i}\,e^{-c\,4i}\,1i+1i\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}+1\right)^2}{4\,b}$$

input
```
int(cos(a + b*x)^2/sin(c + b*x),x)
```

output
```
exp(c*1i - a*2i - b*x*1i)/(2*b) + exp(a*2i - c*1i + b*x*1i)/(2*b) - (exp(c
*2i - a*2i)*log(- ((exp(a*2i)*exp(-c*2i) + 1)^2*1i)/2 - (exp(c*1i)*exp(b*x
*1i)*(exp(a*2i)*exp(-c*2i)*2i + exp(a*4i)*exp(-c*4i)*1i + 1i))/2)*(exp(a*2
i - c*2i) + 1)^2)/(4*b) + (exp(c*2i - a*2i)*log(((exp(a*2i)*exp(-c*2i) + 1
)^2*1i)/2 - (exp(c*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*2i + exp(a*4i)*ex
p(-c*4i)*1i + 1i))/2)*(exp(a*2i - c*2i) + 1)^2)/(4*b)
```

Reduce [F]

$$\int \cos^2(a+bx)\csc(c+bx)\,dx = \int \cos(bx+a)^2\csc(bx+c)\,dx$$

input
```
int(cos(b*x+a)^2*csc(b*x+c),x)
```

output
```
int(cos(a + b*x)**2*csc(b*x + c),x)
```

3.169 $\int \cos(a+bx)\csc(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \cos(a+bx)\csc(c+bx)\,dx = \frac{\cos(a-c)\log(\sin(c+bx))}{b} - x\sin(a-c)$$

output
```
cos(a-c)*ln(sin(b*x+c))/b-x*sin(a-c)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cos(a+bx)\csc(c+bx)\,dx$$
$$= \frac{-2i\arctan(\tan(c+bx))\cos(a-c)+\cos(a-c)\left(2ibx+\log\left(\sin^2(c+bx)\right)\right)-2bx\sin(a-c)}{2b}$$

input
```
Integrate[Cos[a + b*x]*Csc[c + b*x],x]
```

output
```
((-2*I)*ArcTan[Tan[c + b*x]]*Cos[a - c] + Cos[a - c]*((2*I)*b*x + Log[Sin[
c + b*x]^2]) - 2*b*x*Sin[a - c])/(2*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5092, 24, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \text{5092}$$

$$\cos(a-c)\int \cot(c+bx)dx-\sin(a-c)\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\cos(a-c)\int \cot(c+bx)dx-x\sin(a-c)$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int -\tan\left(c+bx+\frac{\pi}{2}\right)dx-x\sin(a-c)$$

$$\Big\downarrow \text{25}$$

$$-\cos(a-c)\int \tan\left(\frac{1}{2}(2c+\pi)+bx\right)dx-x\sin(a-c)$$

$$\Big\downarrow \text{3956}$$

$$\frac{\cos(a-c)\log(-\sin(bx+c))}{b}-x\sin(a-c)$$

input
```
Int[Cos[a + b*x]*Csc[c + b*x],x]
```

output
```
(Cos[a - c]*Log[-Sin[c + b*x]])/b - x*Sin[a - c]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5092
```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Simp[Cos[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] - Simp[Sin[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

method	result
risch	$-2i\cos\left(a-c\right)x+ix\,\mathrm{e}^{i(a-c)}-\frac{2i\cos(a-c)a}{b}+\frac{\ln\left(\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{2i(a-c)}\right)\cos(a-c)}{b}$
default	$\frac{\frac{\frac{(-\cos(a)\cos(c)-\sin(a)\sin(c))\ln\left(\tan(bx+a)^2+1\right)}{2}+(-\sin(a)\cos(c)+\cos(a)\sin(c))\arctan(\tan(bx+a))}{\left(\cos(c)^2+\sin(c)^2\right)\left(\cos(a)^2+\sin(a)^2\right)}+\frac{(\cos(a)\cos(c)+\sin(a)\sin(c))\ln(\tan(bx+a)}{\cos(a)^2\cos(c)^2+\sin}}{b}$

input
```
int(cos(b*x+a)*csc(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2*I*cos(a-c)*x+I*x*exp(I*(a-c))-2*I/b*cos(a-c)*a+1/b*ln(exp(2*I*(b*x+a))-
exp(2*I*(a-c)))*cos(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \cos(a+bx)\csc(c+bx)\,dx = \frac{bx\sin(-a+c)+\cos(-a+c)\log\left(\frac{1}{2}\sin(bx+c)\right)}{b}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c),x, algorithm="fricas")
```

output
```
(b*x*sin(-a + c) + cos(-a + c)*log(1/2*sin(b*x + c)))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(20) = 40$.

Time = 4.26 (sec) , antiderivative size = 333, normalized size of antiderivative = 12.33

$$\int \cos(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c),x)
```

output
```
-Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x, Eq(c, 0)), (0, Eq(b, 0)), (-b*x*t
an(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2*log(tan(c/2)
+ tan(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2)
)*tan(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(
c/2)**2 + b), True))*sin(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (log
(sin(b*x))/b, Eq(c, 0)), (x/sin(c), Eq(b, 0)), (2*b*x*tan(c/2)/(b*tan(c/2)
**2 + b) - log(tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) + lo
g(tan(c/2) + tan(b*x/2))/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - 1/tan(c/2)
)*tan(c/2)**2/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - 1/tan(c/2))/(b*tan(c/
2)**2 + b) + log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(
tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b), True))*cos(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(27) = 54.

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.93

$$\int \cos(a+bx)\csc(c+bx)\,dx = \frac{2\,bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\cos\left(bx\right)^{2}+2\,\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^{2}+\sin\left(bx\right)^{2}-2\,\sin\left(bx\right)\sin\left(\right.}{}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c),x, algorithm="maxima")
```

output
```
1/2*(2*b*x*sin(-a + c) + cos(-a + c)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) +
cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + cos(-a + c)*log(co
s(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) +
 sin(c)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(27) = 54.

Time = 0.15 (sec) , antiderivative size = 482, normalized size of antiderivative = 17.85

$$\int \cos(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c),x, algorithm="giac")
```

output
```
-1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) -
tan(1/2*c))*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*
c)^2 + 1) + (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1
/2*c) - tan(1/2*c)^2 + 1)*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4*tan(1/2*c)^4 - 2*t
an(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^2*ta
n(1/2*c)^4 + tan(1/2*a)^4 - 8*tan(1/2*a)^3*tan(1/2*c) + 20*tan(1/2*a)^2*ta
n(1/2*c)^2 - 8*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 2*tan(1/2*a)^2 + 8
*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)*tan(1/2*
a)^2*tan(1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*
tan(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan
(1/2*a)*tan(1/2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1
/2*a)^4*tan(1/2*c)^4 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^4 + 4*tan(
1/2*a)^3*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + 4*tan(1/2
*a)*tan(1/2*c) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 19.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 4.26

$$\int \cos(a+bx)\csc(c+bx)\,dx = -x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}-\frac{e^{a\,1i-c\,1i}\,1i}{2}\right) - x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}+\frac{e^{a\,1i-c\,1i}\,1i}{2}\right) + \frac{\ln\left(-e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i+c\,1i}}{2}+\frac{e^{a\,1i-c\,1i}}{2}\right)}{b}$$

input
```
int(cos(a + b*x)/sin(c + b*x),x)
```

output
```
(log(exp(a*2i + b*x*2i) - exp(a*2i - c*2i))*(exp(c*1i - a*1i)/2 + exp(a*1i
 - c*1i)/2))/b - x*((exp(c*1i - a*1i)*1i)/2 + (exp(a*1i - c*1i)*1i)/2) - x
*((exp(c*1i - a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2)
```

Reduce [F]

$$\int \cos(a+bx)\csc(c+bx)\,dx = \int \cos\left(bx+a\right)\csc\left(bx+c\right)dx$$

input
```
int(cos(b*x+a)*csc(b*x+c),x)
```

output
```
int(cos(a + b*x)*csc(b*x + c),x)
```

3.170 $\int \csc(c+bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \csc(c+bx)\sec(a+bx)\,dx = -\frac{\log(\cos(a+bx))\sec(a-c)}{b}+\frac{\log(\sin(c+bx))\sec(a-c)}{b}$$

output
```
-ln(cos(b*x+a))*sec(a-c)/b+ln(sin(b*x+c))*sec(a-c)/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \csc(c+bx)\sec(a+bx)\,dx = -\frac{(\log(\cos(a+bx))-\log(\sin(c+bx)))\sec(a-c)}{b}$$

input
```
Integrate[Csc[c + b*x]*Sec[a + b*x],x]
```

output
```
-(((Log[Cos[a + b*x]] - Log[Sin[c + b*x]])*Sec[a - c])/b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\csc(bx+c)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \sec(a+bx)\csc(bx+c)dx$$

input
```
Int[Csc[c + b*x]*Sec[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

method	result	size
default	$\frac{\ln(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))}{b(\cos(a)\cos(c)+\sin(a)\sin(c))}$	53
risch	$\frac{2\ln\left(e^{2i(bx+a)}-e^{2i(a-c)}\right)e^{i(a+c)}}{(e^{2ia}+e^{2ic})b}-\frac{2\ln\left(e^{2i(bx+a)}+1\right)e^{i(a+c)}}{(e^{2ia}+e^{2ic})b}$	86

input
```
int(csc(b*x+c)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b/(cos(a)*cos(c)+sin(a)*sin(c))*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*s
in(a)*sin(c)-sin(a)*cos(c)+cos(a)*sin(c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.08

$$\int \csc(c+bx)\sec(a+bx)\,dx$$
$$= \frac{\log\left(-\frac{1}{4}\cos(bx+c)^2+\frac{1}{4}\right)-\log\left(\frac{4\left(2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(2\cos(-a+c)^2-1\right)\cos(bx+c)^2-\cos(-a+c)^2}{\cos(-a+c)^2+2\cos(-a+c)+1}\right.}{2b\cos(-a+c)}$$

input
```
integrate(csc(b*x+c)*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(log(-1/4*cos(b*x + c)^2 + 1/4) - log(4*(2*cos(b*x + c)*cos(-a + c)*si
n(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c
)^2 + 1)/(cos(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*cos(-a + c))
```

Sympy [F]

$$\int \csc(c+bx)\sec(a+bx)\,dx = \int \csc(bx+c)\sec(a+bx)\,dx$$

input
```
integrate(csc(b*x+c)*sec(b*x+a),x)
```

output
```
Integral(csc(b*x + c)*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(36) = 72$.

Time = 0.06 (sec) , antiderivative size = 430, normalized size of antiderivative = 11.94

$$\int \csc(c + bx) \sec(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sec(b*x+a),x, algorithm="maxima")
```

output
```
-(2*((sin(2*a) + sin(2*c))*cos(a + c) - (cos(2*a) + cos(2*c))*sin(a + c))*
arctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - 2*((sin(2*a) + sin
(2*c))*cos(a + c) - (cos(2*a) + cos(2*c))*sin(a + c))*arctan2(sin(b*x) + s
in(c), cos(b*x) - cos(c)) - 2*((sin(2*a) + sin(2*c))*cos(a + c) - (cos(2*a
) + cos(2*c))*sin(a + c))*arctan2(sin(b*x) - sin(c), cos(b*x) + cos(c)) +
((cos(2*a) + cos(2*c))*cos(a + c) + (sin(2*a) + sin(2*c))*sin(a + c))*log(
cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2
*b*x)*sin(2*a) + sin(2*a)^2) - ((cos(2*a) + cos(2*c))*cos(a + c) + (sin(2*
a) + sin(2*c))*sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 +
 sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - ((cos(2*a) + cos(2*c))*cos(a
 + c) + (sin(2*a) + sin(2*c))*sin(a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(
c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2))/(2*b*cos(2*a)*
cos(2*c) + b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) + b*sin(2*c)^2 + (cos(2*a)
^2 + sin(2*a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(36) = 72$.

Time = 0.15 (sec) , antiderivative size = 391, normalized size of antiderivative = 10.86

$$\int \csc(c + bx) \sec(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sec(b*x+a),x, algorithm="giac")
```

output
```
-((tan(1/2*a)^4*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^4*ta
n(1/2*c) - tan(1/2*a)*tan(1/2*c)^4 + tan(1/2*a)^3 - tan(1/2*c)^3 + tan(1/2
*a) - tan(1/2*c))*log(abs(2*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b
*x + c)*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x +
c)*tan(1/2*a) + tan(1/2*a)^2 - 2*tan(b*x + c)*tan(1/2*c) - 4*tan(1/2*a)*ta
n(1/2*c) + tan(1/2*c)^2 - 1))/(tan(1/2*a)^4*tan(1/2*c)^3 - tan(1/2*a)^3*ta
n(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*tan(1/2*a)^3*tan(1/2*c)^2 - 6*tan
(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*a)^3 + 6*tan(1/
2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a)
- tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 +
 1)*log(abs(tan(b*x + c)))/(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*t
an(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1))/b
```

Mupad [B] (verification not implemented)

Time = 23.77 (sec) , antiderivative size = 311, normalized size of antiderivative = 8.64

$$\int \csc(c+bx)\sec(a+bx)\,dx = -\frac{4\sqrt{e^{a\,2i-c\,2i}}\,\mathrm{atan}\left(\frac{e^{a\,2i}\,e^{b\,x\,2i}\left(\frac{2\,e^{a\,1i}\,e^{-c\,1i}}{b\,\left(e^{a\,2i}\,e^{-c\,2i}\right)^{3/2}}-\frac{2\,e^{-a\,3i}\,e^{c\,3i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\left(b\,\sqrt{e^{a\,2i}\,e^{-c\,2i}}-b\,\left(e^{a\,2i}\,e^{-c\,2i}\right)^{3/2}\right)}{\sqrt{-b^2\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)^2}\,\sqrt{-b^2-2\,b^2\,e^{a\,2i}\,e^{-c\,2i}-b^2\,e^{a\,4i}\,e^{-c\,4i}}}\right)\sqrt{-b^2-2\,b^2\,e^{a\,2i}\,e}}{4}\right.}{\sqrt{-2\,b^2\,e^{a\,2i-c\,2i}-b^2\,e^{a\,4i-c\,4i}-b^2}}$$

input
```
int(1/(cos(a + b*x)*sin(c + b*x)),x)
```

output
```
-(4*exp(a*2i - c*2i)^(1/2)*atan((exp(a*2i)*exp(b*x*2i)*((2*exp(a*1i)*exp(-
c*1i))/(b*(exp(a*2i)*exp(-c*2i))^(3/2)) - (2*exp(-a*3i)*exp(c*3i)*(exp(a*2
i)*exp(-c*2i) - 1)*(b*(exp(a*2i)*exp(-c*2i))^(1/2) - b*(exp(a*2i)*exp(-c*2
i))^(3/2)))/((-b^2*(exp(a*2i)*exp(-c*2i) + 1)^2)^(1/2)*(- b^2 - 2*b^2*exp(
a*2i)*exp(-c*2i) - b^2*exp(a*4i)*exp(-c*4i))^(1/2)))*(- b^2 - 2*b^2*exp(a*
2i)*exp(-c*2i) - b^2*exp(a*4i)*exp(-c*4i))^(1/2))/4 + (b*exp(-a*3i)*exp(c*
3i)*(exp(a*2i)*exp(-c*2i) - 1)*(exp(a*2i)*exp(-c*2i))^(3/2))/(-b^2*(exp(a*
2i)*exp(-c*2i) + 1)^2)^(1/2)))/(- 2*b^2*exp(a*2i - c*2i) - b^2*exp(a*4i -
c*4i) - b^2)^(1/2)
```

Reduce [F]

$$\int \csc(c+bx)\sec(a+bx)\,dx$$
$$= \frac{\left(\int \frac{1}{\sin(bx+c)}dx\right)b+\left(\int \frac{1}{\cos(bx+a)\sin(bx+c)}dx\right)b-\log\left(\tan\left(\frac{bx}{2}+\frac{c}{2}\right)\right)}{b}$$

input
```
int(csc(b*x+c)*sec(b*x+a),x)
```

output
```
(int(1/sin(b*x + c),x)*b + int(1/(cos(a + b*x)*sin(b*x + c)),x)*b - log(ta
n((b*x + c)/2)))/b
```

3.171 $\int \csc(c+bx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \csc(c+bx)\sec^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+bx))\sec^2(a-c)}{b} + \frac{\sec(a-c)\sec(a+bx)}{b} + \frac{\operatorname{arctanh}(\sin(a+bx))\sec(a-c)\tan(a-c)}{b}$$

output
```
-arctanh(cos(b*x+c))*sec(a-c)^2/b+sec(a-c)*sec(b*x+a)/b+arctanh(sin(b*x+a)
)*sec(a-c)*tan(a-c)/b
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int \csc(c+bx)\sec^2(a+bx)\,dx = \frac{\sec^2(a-c)\left(-\log\left(\cos\left(\frac{1}{2}(c+bx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+bx)\right)\right)+\cos(a-c)\sec(a+bx)+2\operatorname{arctanh}(\sin(a)\right.}{b}$$

input
```
Integrate[Csc[c + b*x]*Sec[a + b*x]^2,x]
```

output
```
(Sec[a - c]^2*(-Log[Cos[(c + b*x)/2]] + Log[Sin[(c + b*x)/2]] + Cos[a - c]
*Sec[a + b*x] + 2*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]]*Sin[a - c]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \sec^2(a+bx)\csc(bx+c)dx$$

input
```
Int[Csc[c + b*x]*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(60) = 120$.

Time = 0.98 (sec) , antiderivative size = 282, normalized size of antiderivative = 4.70

method	result
default	$-\frac{\frac{2\sqrt{-\cos(c)^2\sin(a)^2-\cos(a)^2\cos(c)^2-\sin(a)^2\sin(c)^2-\sin(c)^2\cos(a)^2}\,\arctan\left(\frac{2(\sin(a)\cos(c)-\cos(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2\cos(a)\cos(c)+2\sin}{2\sqrt{-\cos(c)^2\sin(a)^2-\cos(a)^2\cos(c)^2-\sin(a)^2\sin(c)^2-\sin(c)^2}}\right.}{(\cos(a)\cos(c)+\sin(a)\sin(c))^2}}{}$
risch	$\frac{4\,e^{i(bx+2a+c)}}{\left(e^{2i(bx+a)}+1\right)\left(e^{2ia}+e^{2ic}\right)b}-\frac{2i\ln\left(e^{i(bx+a)}+i\right)e^{i(3a+c)}}{\left(e^{4ia}+2\,e^{2i(a+c)}+e^{4ic}\right)b}+\frac{2i\ln\left(e^{i(bx+a)}+i\right)e^{i(a+3c)}}{\left(e^{4ia}+2\,e^{2i(a+c)}+e^{4ic}\right)b}+\frac{4\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)e^{2i(a+c)}}{\left(e^{4ia}+2\,e^{2i(a+c)}+e^{4ic}\right)b}+\frac{2i}{(}$

input
```
int(csc(b*x+c)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-2*(-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-sin(c)^2*c
os(a)^2)^(1/2)/(cos(a)*cos(c)+sin(a)*sin(c))^2*arctan(1/2*(2*(sin(a)*cos(c
)-cos(a)*sin(c))*tan(1/2*a+1/2*b*x)+2*cos(a)*cos(c)+2*sin(a)*sin(c))/(-cos
(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-sin(c)^2*cos(a)^2)^(1/2
))-2/(2*cos(a)*cos(c)+2*sin(a)*sin(c))/(tan(1/2*a+1/2*b*x)-1)+1/(cos(a)*co
s(c)+sin(a)*sin(c))^2*(-sin(a)*cos(c)+cos(a)*sin(c))*ln(tan(1/2*a+1/2*b*x)
-1)+2/(2*cos(a)*cos(c)+2*sin(a)*sin(c))/(tan(1/2*a+1/2*b*x)+1)+(sin(a)*cos
(c)-cos(a)*sin(c))/(cos(a)*cos(c)+sin(a)*sin(c))^2*ln(tan(1/2*a+1/2*b*x)+1
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(60) = 120$.

Time = 0.11 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.90

$$\int \csc(c+bx)\sec^2(a+bx)\,dx = -\frac{\left(\cos\left(bx+c\right)\cos\left(-a+c\right)\sin\left(-a+c\right)-\left(\cos\left(-a+c\right)^2-1\right)\sin\left(bx+c\right)\right)\log\left(\frac{2\left(\cos(-a+c)\sin(bx+c)-\cos(b\right.}{\cos(-a+c)-}\right.}{}$$

input
```
integrate(csc(b*x+c)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*((cos(b*x + c)*cos(-a + c)*sin(-a + c) - (cos(-a + c)^2 - 1)*sin(b*x
+ c))*log(2*(cos(-a + c)*sin(b*x + c) - cos(b*x + c)*sin(-a + c) + 1)/(cos
(-a + c) + 1)) - (cos(b*x + c)*cos(-a + c)*sin(-a + c) - (cos(-a + c)^2 -
1)*sin(b*x + c))*log(-2*(cos(-a + c)*sin(b*x + c) - cos(b*x + c)*sin(-a +
c) - 1)/(cos(-a + c) + 1)) + (cos(b*x + c)*cos(-a + c) + sin(b*x + c)*sin(
-a + c))*log(1/2*cos(b*x + c) + 1/2) - (cos(b*x + c)*cos(-a + c) + sin(b*x
 + c)*sin(-a + c))*log(-1/2*cos(b*x + c) + 1/2) - 2*cos(-a + c))/(b*cos(b*
x + c)*cos(-a + c)^3 + b*cos(-a + c)^2*sin(b*x + c)*sin(-a + c))
```

Sympy [F]

$$\int \csc(c+bx)\sec^2(a+bx)\,dx = \int \csc(bx+c)\sec^2(a+bx)\,dx$$

input
```
integrate(csc(b*x+c)*sec(b*x+a)**2,x)
```

output
```
Integral(csc(b*x + c)*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14127 vs. 2(60) = 120.

Time = 0.35 (sec) , antiderivative size = 14127, normalized size of antiderivative = 235.45

$$\int \csc(c+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
(4*(cos(4*a)^2 + 4*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)
^2 + 2*cos(4*a)*cos(4*c) + cos(4*c)^2 + sin(4*a)^2 + 4*(sin(4*a) + sin(4*c
))*sin(2*a + 2*c) + 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)
*cos(2*b*x + 4*a)*cos(b*x + 2*a + c) + 4*(cos(4*a)^2 + 4*(cos(4*a) + cos(4
*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)^2 + 2*cos(4*a)*cos(4*c) + cos(4*c)^
2 + sin(4*a)^2 + 4*(sin(4*a) + sin(4*c))*sin(2*a + 2*c) + 4*sin(2*a + 2*c)
^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)*cos(2*b*x + 2*a + 2*c)*cos(b*x + 2*
a + c) + 4*(cos(4*a)^2 + 4*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*
a + 2*c)^2 + 2*cos(4*a)*cos(4*c) + cos(4*c)^2 + sin(4*a)^2 + 4*(sin(4*a) +
 sin(4*c))*sin(2*a + 2*c) + 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin
(4*c)^2)*sin(2*b*x + 4*a)*sin(b*x + 2*a + c) + 4*(cos(4*a)^2 + 4*(cos(4*a)
 + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)^2 + 2*cos(4*a)*cos(4*c) + c
os(4*c)^2 + sin(4*a)^2 + 4*(sin(4*a) + sin(4*c))*sin(2*a + 2*c) + 4*sin(2*
a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)*sin(2*b*x + 2*a + 2*c)*sin(
b*x + 2*a + c) + 2*(((cos(4*a) + cos(4*c))*cos(3*a + c) + 2*cos(3*a + c)*c
os(2*a + 2*c) - (cos(4*a) + 2*cos(2*a + 2*c) + cos(4*c))*cos(a + 3*c) + (s
in(4*a) + sin(4*c))*sin(3*a + c) + 2*sin(3*a + c)*sin(2*a + 2*c) - (sin(4*
a) + 2*sin(2*a + 2*c) + sin(4*c))*sin(a + 3*c))*cos(2*b*x + 4*a)^2 + ((cos
(4*a) + cos(4*c))*cos(3*a + c) + 2*cos(3*a + c)*cos(2*a + 2*c) - (cos(4*a)
 + 2*cos(2*a + 2*c) + cos(4*c))*cos(a + 3*c) + (sin(4*a) + sin(4*c))*si...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2933 vs. $2(60) = 120$.

Time = 0.37 (sec) , antiderivative size = 2933, normalized size of antiderivative = 48.88

$$\int \csc(c + bx) \sec^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
(2*(tan(1/2*a)^5*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c)^5 - tan(1/2*a)^5*t
an(1/2*c)^3 + 2*tan(1/2*a)^4*tan(1/2*c)^4 - tan(1/2*a)^3*tan(1/2*c)^5 + ta
n(1/2*a)^5*tan(1/2*c)^2 + tan(1/2*a)^4*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2
*c)^4 - tan(1/2*a)^2*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c) + tan(1/2*a)^4
*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^4 - tan(1/2*a)*tan(1/2*c)^5 + 2*ta
n(1/2*a)^4*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*a)^4 + tan(1/2
*a)^3*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + tan(1/2*a)^3 +
 tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/
2*a)^2 + 2*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))
*log(abs(-tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*c
)*tan(1/2*a) - tan(1/2*b*x + 1/2*c)*tan(1/2*c) - tan(1/2*a)*tan(1/2*c) - t
an(1/2*b*x + 1/2*c) - tan(1/2*a) + tan(1/2*c) - 1))/(tan(1/2*a)^5*tan(1/2*
c)^5 - tan(1/2*a)^5*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c)^5 - 2*tan(1/2*a
)^5*tan(1/2*c)^3 + 9*tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^3*tan(1/2*c)
^5 + 2*tan(1/2*a)^5*tan(1/2*c)^2 - 10*tan(1/2*a)^4*tan(1/2*c)^3 + 10*tan(1
/2*a)^3*tan(1/2*c)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^5 + tan(1/2*a)^5*tan(1/2*
c) - 10*tan(1/2*a)^4*tan(1/2*c)^2 + 28*tan(1/2*a)^3*tan(1/2*c)^3 - 10*tan(
1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)*tan(1/2*c)^5 - tan(1/2*a)^5 + 9*tan(1/2
*a)^4*tan(1/2*c) - 28*tan(1/2*a)^3*tan(1/2*c)^2 + 28*tan(1/2*a)^2*tan(1/2*
c)^3 - 9*tan(1/2*a)*tan(1/2*c)^4 + tan(1/2*c)^5 + tan(1/2*a)^4 - 10*tan...
```

Mupad [F(-1)]

Timed out.

$$\int \csc(c+bx)\sec^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)^2*sin(c + b*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc(c+bx)\sec^2(a+bx)\,dx = \int \csc(bx+c)\sec(bx+a)^2\,dx$$

input
```
int(csc(b*x+c)*sec(b*x+a)^2,x)
```

output
```
int(csc(b*x + c)*sec(a + b*x)**2,x)
```

3.172 $\int \csc(c+bx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 1

$$\int \csc(c+bx)\sec^3(a+bx)\,dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 151.00

$$\int \csc(c+bx)\sec^3(a+bx)\,dx =$$
$$-\frac{(-2\cos(3a-2c)+\cos(3a-2c+2bx)-\cos(a-2(c+bx))+2\cos(a+2bx)\log(\cos(a+bx))+2\cos}{}$$

input
```
Integrate[Csc[c + b*x]*Sec[a + b*x]^3,x]
```

output
```
-1/8*((-2*Cos[3*a - 2*c] + Cos[3*a - 2*c + 2*b*x] - Cos[a - 2*(c + b*x)] +
 2*Cos[a + 2*b*x]*Log[Cos[a + b*x]] + 2*Cos[3*a + 2*b*x]*Log[Cos[a + b*x]]
 + Cos[a]*(-2 + 4*Log[Cos[a + b*x]] - 4*Log[Sin[c + b*x]]) - 2*Cos[a + 2*b
*x]*Log[Sin[c + b*x]] - 2*Cos[3*a + 2*b*x]*Log[Sin[c + b*x]])*Sec[a]*Sec[a
 - c]^3*Sec[a + b*x]^2)/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sec^3(a+bx)\csc(bx+c)dx$$

input
```
Int[Csc[c + b*x]*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.45 (sec) , antiderivative size = 158, normalized size of antiderivative = 158.00

method	result
default	$\frac{\frac{\tan(bx+a)^2\cos(a)\cos(c)}{2}+\frac{\tan(bx+a)^2\sin(a)\sin(c)}{2}-\tan(bx+a)\cos(a)\sin(c)+\tan(bx+a)\sin(a)\cos(c)}{(\cos(a)\cos(c)+\sin(a)\sin(c))^2}+\frac{(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2}{}$ / b
risch	$-\frac{4\left(-2\,e^{i(2bx+5a+c)}-e^{i(3a+c)}+e^{i(a+3c)}\right)}{\left(e^{2i(bx+a)}+1\right)^2\left(e^{2ia}+e^{2ic}\right)^2 b}+\frac{8\ln\left(e^{2i(bx+a)}-e^{2i(a-c)}\right)e^{3i(a+c)}}{\left(e^{6ia}+3\,e^{2i(2a+c)}+3\,e^{2i(a+2c)}+e^{6ic}\right)b}-\frac{8\ln\left(e^{2i(bx+a)}+1\right)e^{3i(a+c)}}{\left(e^{6ia}+3\,e^{2i(2a+c)}+3\,e^{2i(a+2c)}+e^{6ic}\right)b}$

input
```
int(csc(b*x+c)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/(cos(a)*cos(c)+sin(a)*sin(c))^2*(1/2*tan(b*x+a)^2*cos(a)*cos(c)+1/2
*tan(b*x+a)^2*sin(a)*sin(c)-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos
(c))+(cos(a)^2*cos(c)^2+sin(c)^2*cos(a)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c
)^2)/(cos(a)*cos(c)+sin(a)*sin(c))^3*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a
)*sin(a)*sin(c)-sin(a)*cos(c)+cos(a)*sin(c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.10 (sec) , antiderivative size = 392, normalized size of antiderivative = 392.00

$$\int \csc(c+bx)\sec^3(a+bx)\,dx$$

$$= \frac{2\cos(-a+c)^4-2\left(2\cos(-a+c)^3-\cos(-a+c)\right)\cos(bx+c)\sin(bx+c)\sin(-a+c)-4\left(\cos(-a\right.}{}$$

input
```
integrate(csc(b*x+c)*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(2*cos(-a + c)^4 - 2*(2*cos(-a + c)^3 - cos(-a + c))*cos(b*x + c)*sin(
b*x + c)*sin(-a + c) - 4*(cos(-a + c)^4 - cos(-a + c)^2)*cos(b*x + c)^2 -
cos(-a + c)^2 + (2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) + (2*
cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)*log(-1/4*cos(b*x +
c)^2 + 1/4) - (2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) + (2*co
s(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)*log(4*(2*cos(b*x + c)
*cos(-a + c)*sin(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)
^2 - cos(-a + c)^2 + 1)/(cos(-a + c)^2 + 2*cos(-a + c) + 1)))/(2*b*cos(b*x
 + c)*cos(-a + c)^4*sin(b*x + c)*sin(-a + c) - b*cos(-a + c)^5 + b*cos(-a
+ c)^3 + (2*b*cos(-a + c)^5 - b*cos(-a + c)^3)*cos(b*x + c)^2)
```

Sympy [F]

$$\int \csc(c+bx)\sec^3(a+bx)\,dx = \int \csc(bx+c)\sec^3(a+bx)\,dx$$

input
```
integrate(csc(b*x+c)*sec(b*x+a)**3,x)
```

output
```
Integral(csc(b*x + c)*sec(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.26 (sec) , antiderivative size = 85407, normalized size of antiderivative = 85407.00

$$\int \csc(c+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
4*(9*((cos(4*a) + cos(4*c))*cos(3*a + c) + 2*cos(3*a + c)*cos(2*a + 2*c) +
 (sin(4*a) + sin(4*c))*sin(3*a + c) + 2*sin(3*a + c)*sin(2*a + 2*c))*cos(4
*a + 2*c)^2 + 9*((cos(4*a) + cos(4*c))*cos(3*a + c) + 2*cos(3*a + c)*cos(2
*a + 2*c) - (cos(4*a) + 2*cos(2*a + 2*c) + cos(4*c))*cos(a + 3*c) + (sin(4
*a) + sin(4*c))*sin(3*a + c) + 2*sin(3*a + c)*sin(2*a + 2*c) - (sin(4*a) +
 2*sin(2*a + 2*c) + sin(4*c))*sin(a + 3*c))*cos(2*a + 4*c)^2 + 2*(cos(6*a)
^2 + 2*cos(6*a)*cos(6*c) + cos(6*c)^2 + sin(6*a)^2 + 2*sin(6*a)*sin(6*c) +
 sin(6*c)^2)*cos(3*a + c)*cos(2*a + 2*c) + 9*((cos(4*a) + cos(4*c))*cos(3*
a + c) + 2*cos(3*a + c)*cos(2*a + 2*c) + (sin(4*a) + sin(4*c))*sin(3*a + c
) + 2*sin(3*a + c)*sin(2*a + 2*c))*sin(4*a + 2*c)^2 + 9*((cos(4*a) + cos(4
*c))*cos(3*a + c) + 2*cos(3*a + c)*cos(2*a + 2*c) - (cos(4*a) + 2*cos(2*a
+ 2*c) + cos(4*c))*cos(a + 3*c) + (sin(4*a) + sin(4*c))*sin(3*a + c) + 2*s
in(3*a + c)*sin(2*a + 2*c) - (sin(4*a) + 2*sin(2*a + 2*c) + sin(4*c))*sin(
a + 3*c))*sin(2*a + 4*c)^2 + 2*(cos(6*a)^2 + 2*cos(6*a)*cos(6*c) + cos(6*c
)^2 + sin(6*a)^2 + 2*sin(6*a)*sin(6*c) + sin(6*c)^2)*sin(3*a + c)*sin(2*a
+ 2*c) - 2*(((sin(6*a) + 3*sin(4*a + 2*c) + sin(6*c))*cos(3*a + 3*c) - (co
s(6*a) + 3*cos(4*a + 2*c) + cos(6*c))*sin(3*a + 3*c) - 3*cos(2*a + 4*c)*si
n(3*a + 3*c) + 3*cos(3*a + 3*c)*sin(2*a + 4*c))*cos(4*b*x + 8*a)^2 + 4*((s
in(6*a) + 3*sin(4*a + 2*c) + sin(6*c))*cos(3*a + 3*c) - (cos(6*a) + 3*cos(
4*a + 2*c) + cos(6*c))*sin(3*a + 3*c) - 3*cos(2*a + 4*c)*sin(3*a + 3*c)...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.37 (sec) , antiderivative size = 6981, normalized size of antiderivative = 6981.00

$$\int \csc(c + bx) \sec^3(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/8*(8*(tan(1/2*a)^8*tan(1/2*c)^7 - tan(1/2*a)^7*tan(1/2*c)^8 + 3*tan(1/2
*a)^8*tan(1/2*c)^5 - 2*tan(1/2*a)^7*tan(1/2*c)^6 + 2*tan(1/2*a)^6*tan(1/2*
c)^7 - 3*tan(1/2*a)^5*tan(1/2*c)^8 + 3*tan(1/2*a)^8*tan(1/2*c)^3 + 6*tan(1
/2*a)^6*tan(1/2*c)^5 - 6*tan(1/2*a)^5*tan(1/2*c)^6 - 3*tan(1/2*a)^3*tan(1/
2*c)^8 + tan(1/2*a)^8*tan(1/2*c) + 2*tan(1/2*a)^7*tan(1/2*c)^2 + 6*tan(1/2
*a)^6*tan(1/2*c)^3 - 6*tan(1/2*a)^3*tan(1/2*c)^6 - 2*tan(1/2*a)^2*tan(1/2*
c)^7 - tan(1/2*a)*tan(1/2*c)^8 + tan(1/2*a)^7 + 2*tan(1/2*a)^6*tan(1/2*c)
+ 6*tan(1/2*a)^5*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^5 - 2*tan(1/2*a)
*tan(1/2*c)^6 - tan(1/2*c)^7 + 3*tan(1/2*a)^5 + 6*tan(1/2*a)^3*tan(1/2*c)^
2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 - 3*tan(1/2*c)^5 + 3*tan(1/2*a)^3 - 2*tan(
1/2*a)^2*tan(1/2*c) + 2*tan(1/2*a)*tan(1/2*c)^2 - 3*tan(1/2*c)^3 + tan(1/2
*a) - tan(1/2*c))*log(abs(2*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b
*x + c)*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x +
c)*tan(1/2*a) + tan(1/2*a)^2 - 2*tan(b*x + c)*tan(1/2*c) - 4*tan(1/2*a)*ta
n(1/2*c) + tan(1/2*c)^2 - 1))/(tan(1/2*a)^8*tan(1/2*c)^7 - tan(1/2*a)^7*ta
n(1/2*c)^8 - 3*tan(1/2*a)^8*tan(1/2*c)^5 + 16*tan(1/2*a)^7*tan(1/2*c)^6 -
16*tan(1/2*a)^6*tan(1/2*c)^7 + 3*tan(1/2*a)^5*tan(1/2*c)^8 + 3*tan(1/2*a)^
8*tan(1/2*c)^3 - 30*tan(1/2*a)^7*tan(1/2*c)^4 + 96*tan(1/2*a)^6*tan(1/2*c)
^5 - 96*tan(1/2*a)^5*tan(1/2*c)^6 + 30*tan(1/2*a)^4*tan(1/2*c)^7 - 3*tan(1
/2*a)^3*tan(1/2*c)^8 - tan(1/2*a)^8*tan(1/2*c) + 16*tan(1/2*a)^7*tan(1/...
```

Mupad [F(-1)]

Timed out.

$$\int \csc(c+bx)\sec^3(a+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)^3*sin(c + b*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc(c+bx)\sec^3(a+bx)\,dx = \int \csc(bx+c)\sec(bx+a)^3\,dx$$

input
```
int(csc(b*x+c)*sec(b*x+a)^3,x)
```

output
```
int(csc(b*x + c)*sec(a + b*x)**3,x)
```

3.173 $\int \cos^3(a+bx)\csc^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \cos^3(a+bx)\csc^2(c+bx)\,dx=0$$

output

```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 134.00

$$\begin{aligned}&\int \cos^3(a+bx)\csc^2(c+bx)\,dx\\&=-\frac{\cos^3(a-c)\csc(c+bx)}{b}-\frac{\cos(bx)\sin(3a-2c)}{b}\\&\quad+\frac{6i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos^2(a-c)\sin(a-c)}{b}\\&\quad-\frac{\cos(3a-2c)\sin(bx)}{b}\end{aligned}$$

input
```
Integrate[Cos[a + b*x]^3*Csc[c + b*x]^2,x]
```

output
```
-((Cos[a - c]^3*Csc[c + b*x])/b) - (Cos[b*x]*Sin[3*a - 2*c])/b + ((6*I)*Ar
cTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]))/(I*
Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c]^2*Sin[a - c])/b - (
Cos[3*a - 2*c]*Sin[b*x])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos^3(a+bx)\csc^2(bx+c)dx$$

input
```
Int[Cos[a + b*x]^3*Csc[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 3.70 (sec) , antiderivative size = 231, normalized size of antiderivative = 231.00

method	result
risch	$\frac{i\left(\mathrm{e}^{i(bx+5a-2c)}+3\,\mathrm{e}^{i(bx+3a)}+3\,\mathrm{e}^{i(bx+a+2c)}+\mathrm{e}^{-i(-bx+a-4c)}\right)}{4\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)b}+\frac{3\ln\left(\mathrm{e}^{i(bx+a)}+\mathrm{e}^{i(a-c)}\right)\sin(a-c)}{4b}+\frac{3\ln\left(\mathrm{e}^{i(bx+a)}+\mathrm{e}^{i(a-c)}\right)\sin(3}{4b}$
default	Expression too large to display

input
```
int(cos(b*x+a)^3*csc(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/(-exp(2*I*(b*x+a+c))+exp(2*I*a))/b*(exp(I*(b*x+5*a-2*c))+3*exp(I*(b*
x+3*a))+3*exp(I*(b*x+a+2*c))+exp(-I*(-b*x+a-4*c)))+3/4*ln(exp(I*(b*x+a))+e
xp(I*(a-c)))/b*sin(a-c)+3/4*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin(3*a-3*c)
-3/4*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*sin(a-c)-3/4*ln(exp(I*(b*x+a))-exp(
I*(a-c)))/b*sin(3*a-3*c)-sin(b*x+3*a-2*c)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 159.00

$$\int \cos^3(a+bx)\csc^2(c+bx)\,dx = -\frac{3\cos(-a+c)^2\log\left(\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)\sin(-a+c)-3\cos(-a+c)^2\log\left(-\frac{1}{2}\cos(bx+\right.}{}$$

input
```
integrate(cos(b*x+a)^3*csc(b*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(3*cos(-a + c)^2*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c)*sin(-a + c)
 - 3*cos(-a + c)^2*log(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) -
 2*(4*cos(-a + c)^2 - 1)*cos(b*x + c)*sin(b*x + c)*sin(-a + c) - 2*(4*cos(
-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^2 + 10*cos(-a + c)^3 - 6*cos(-a +
c))/(b*sin(b*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^2(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*csc(b*x+c)**2,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.08 (sec) , antiderivative size = 1042, normalized size of antiderivative = 1042.00

$$\int \cos^3(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*csc(b*x+c)^2,x, algorithm="maxima")
```

output

```
1/8*(4*(sin(3*b*x + 3*a + 4*c) - sin(b*x + 3*a + 2*c))*cos(4*b*x + 6*a + 2
*c) + 2*(3*sin(2*b*x + 6*a) + 3*sin(2*b*x + 4*a + 2*c) + 3*sin(2*b*x + 2*a
 + 4*c) + 3*sin(2*b*x + 6*c) - 2*sin(4*c))*cos(3*b*x + 3*a + 4*c) - 3*((si
n(-a + c) + sin(-3*a + 3*c))*cos(3*b*x + 3*a + 4*c)^2 - 2*(sin(-a + c) + s
in(-3*a + 3*c))*cos(3*b*x + 3*a + 4*c)*cos(b*x + 3*a + 2*c) + (sin(-a + c)
 + sin(-3*a + 3*c))*cos(b*x + 3*a + 2*c)^2 + (sin(-a + c) + sin(-3*a + 3*c
))*sin(3*b*x + 3*a + 4*c)^2 - 2*(sin(-a + c) + sin(-3*a + 3*c))*sin(3*b*x
+ 3*a + 4*c)*sin(b*x + 3*a + 2*c) + (sin(-a + c) + sin(-3*a + 3*c))*sin(b*
x + 3*a + 2*c)^2)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)
^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + 3*((sin(-a + c) + sin(-3*a + 3*c))*co
s(3*b*x + 3*a + 4*c)^2 - 2*(sin(-a + c) + sin(-3*a + 3*c))*cos(3*b*x + 3*a
 + 4*c)*cos(b*x + 3*a + 2*c) + (sin(-a + c) + sin(-3*a + 3*c))*cos(b*x + 3
*a + 2*c)^2 + (sin(-a + c) + sin(-3*a + 3*c))*sin(3*b*x + 3*a + 4*c)^2 - 2
*(sin(-a + c) + sin(-3*a + 3*c))*sin(3*b*x + 3*a + 4*c)*sin(b*x + 3*a + 2*
c) + (sin(-a + c) + sin(-3*a + 3*c))*sin(b*x + 3*a + 2*c)^2)*log(cos(b*x)^
2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)
^2) - 4*(cos(3*b*x + 3*a + 4*c) - cos(b*x + 3*a + 2*c))*sin(4*b*x + 6*a +
2*c) - 2*(3*cos(2*b*x + 6*a) + 3*cos(2*b*x + 4*a + 2*c) + 3*cos(2*b*x + 2*
a + 4*c) + 3*cos(2*b*x + 6*c) - 2*cos(4*c))*sin(3*b*x + 3*a + 4*c) - 6*cos
(b*x + 3*a + 2*c)*sin(2*b*x + 6*a) - 6*cos(b*x + 3*a + 2*c)*sin(2*b*x +...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.59 (sec) , antiderivative size = 5372, normalized size of antiderivative = 5372.00

$$\int \cos^3(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)^3*csc(b*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(12*(tan(1/2*a)^7*tan(1/2*c)^6 - tan(1/2*a)^6*tan(1/2*c)^7 - 2*tan(1/
2*a)^7*tan(1/2*c)^4 + 12*tan(1/2*a)^6*tan(1/2*c)^5 - 12*tan(1/2*a)^5*tan(1
/2*c)^6 + 2*tan(1/2*a)^4*tan(1/2*c)^7 + tan(1/2*a)^7*tan(1/2*c)^2 - 13*tan
(1/2*a)^6*tan(1/2*c)^3 + 49*tan(1/2*a)^5*tan(1/2*c)^4 - 49*tan(1/2*a)^4*ta
n(1/2*c)^5 + 13*tan(1/2*a)^3*tan(1/2*c)^6 - tan(1/2*a)^2*tan(1/2*c)^7 + 2*
tan(1/2*a)^6*tan(1/2*c) - 22*tan(1/2*a)^5*tan(1/2*c)^2 + 76*tan(1/2*a)^4*t
an(1/2*c)^3 - 76*tan(1/2*a)^3*tan(1/2*c)^4 + 22*tan(1/2*a)^2*tan(1/2*c)^5
- 2*tan(1/2*a)*tan(1/2*c)^6 + tan(1/2*a)^5 - 13*tan(1/2*a)^4*tan(1/2*c) +
49*tan(1/2*a)^3*tan(1/2*c)^2 - 49*tan(1/2*a)^2*tan(1/2*c)^3 + 13*tan(1/2*a
)*tan(1/2*c)^4 - tan(1/2*c)^5 - 2*tan(1/2*a)^3 + 12*tan(1/2*a)^2*tan(1/2*c
) - 12*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))
*log(abs(tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)
 - tan(1/2*a) + tan(1/2*c)))/(tan(1/2*a)^7*tan(1/2*c)^7 + 3*tan(1/2*a)^7*t
an(1/2*c)^5 + tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^5*tan(1/2*c)^7 + 3*
tan(1/2*a)^7*tan(1/2*c)^3 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 9*tan(1/2*a)^5*t
an(1/2*c)^5 + 3*tan(1/2*a)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^3*tan(1/2*c)^7 +
tan(1/2*a)^7*tan(1/2*c) + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^5*tan
(1/2*c)^3 + 9*tan(1/2*a)^4*tan(1/2*c)^4 + 9*tan(1/2*a)^3*tan(1/2*c)^5 + 3*
tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)*tan(1/2*c)^7 + tan(1/2*a)^6 + 3*tan
(1/2*a)^5*tan(1/2*c) + 9*tan(1/2*a)^4*tan(1/2*c)^2 + 9*tan(1/2*a)^3*tan...
```

Mupad [B] (verification not implemented)

Time = 25.92 (sec) , antiderivative size = 393, normalized size of antiderivative = 393.00

$$\int \cos^3(a+bx)\csc^2(c+bx)\,dx = -\frac{e^{-a\,3i+c\,2i-b\,x\,1i}\,1i}{2\,b}+\frac{e^{a\,3i-c\,2i+b\,x\,1i}\,1i}{2\,b} - \frac{e^{-a\,1i+c\,2i+b\,x\,1i}\,\left(3\,e^{a\,2i-c\,2i}+3\,e^{a\,4i-c\,4i}+e^{a\,6i-c\,6i}+1\right)}{4\,b\,\left(e^{a\,2i-c\,2i}\,1i-e^{a\,2i+b\,x\,2i}\,1i\right)} - \frac{3\,\ln\left(\frac{3\,e^{a\,1i}\,e^{b\,x\,1i}\,\left(\sin(2\,a-2\,c)\,1i+\sin(2\,a-2\,c)\,e^{a\,2i}\,e^{-c\,2i}\,1i\right)}{2}-\frac{\sin(2\,a-2\,c)\,e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,3i}{2\,\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\sin\left(2\,a-2\,c\right)\,\left(e^{a\,2i-}\right.}{4\,b\,\sqrt{e^{a\,2i-c\,2i}}} + \frac{3\,\ln\left(\frac{3\,e^{a\,1i}\,e^{b\,x\,1i}\,\left(\sin(2\,a-2\,c)\,1i+\sin(2\,a-2\,c)\,e^{a\,2i}\,e^{-c\,2i}\,1i\right)}{2}+\frac{\sin(2\,a-2\,c)\,e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,3i}{2\,\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\sin\left(2\,a-2\,c\right)\,\left(e^{a\,2i-}\right.}{4\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

input
```
int(cos(a + b*x)^3/sin(c + b*x)^2,x)
```

output
```
(exp(a*3i - c*2i + b*x*1i)*1i)/(2*b) - (exp(c*2i - a*3i - b*x*1i)*1i)/(2*b
) - (exp(c*2i - a*1i + b*x*1i)*(3*exp(a*2i - c*2i) + 3*exp(a*4i - c*4i) +
exp(a*6i - c*6i) + 1))/(4*b*(exp(a*2i - c*2i)*1i - exp(a*2i + b*x*2i)*1i))
 - (3*log((3*exp(a*1i)*exp(b*x*1i)*(sin(2*a - 2*c)*1i + sin(2*a - 2*c)*exp
(a*2i)*exp(-c*2i)*1i))/2 - (sin(2*a - 2*c)*exp(a*2i)*exp(-c*2i)*(exp(a*2i)
*exp(-c*2i) + 1)*3i)/(2*(exp(a*2i)*exp(-c*2i))^(1/2)))*sin(2*a - 2*c)*(exp
(a*2i - c*2i) + 1))/(4*b*exp(a*2i - c*2i)^(1/2)) + (3*log((3*exp(a*1i)*exp
(b*x*1i)*(sin(2*a - 2*c)*1i + sin(2*a - 2*c)*exp(a*2i)*exp(-c*2i)*1i))/2 +
 (sin(2*a - 2*c)*exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*3i)/(2*(e
xp(a*2i)*exp(-c*2i))^(1/2)))*sin(2*a - 2*c)*(exp(a*2i - c*2i) + 1))/(4*b*e
xp(a*2i - c*2i)^(1/2))
```

Reduce [F]

$$\int \cos^3(a+bx)\csc^2(c+bx)\,dx = \int \cos(bx+a)^3 \csc(bx+c)^2\,dx$$

input
```
int(cos(b*x+a)^3*csc(b*x+c)^2,x)
```

output
```
int(cos(a + b*x)**3*csc(b*x + c)**2,x)
```

3.174 $\int \cos^2(a+bx)\csc^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = -x\cos(2(a-c)) - \frac{\cos^2(a-c)\cot(c+bx)}{b} - \frac{\log(\sin(c+bx))\sin(2(a-c))}{b}$$

output
```
-x*cos(2*a-2*c)-cos(a-c)^2*cot(b*x+c)/b-ln(sin(b*x+c))*sin(2*a-2*c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(51) = 102.

Time = 0.40 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.55

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = \frac{\csc(c)\csc(c+bx)(bx\cos(2a-4c-bx) - bx\cos(2a-2c-bx) + bx\cos(2a+bx) - bx\cos(2a-2c+bx$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c + b*x]^2,x]
```

output
```
(Csc[c]*Csc[c + b*x]*(b*x*Cos[2*a - 4*c - b*x] - b*x*Cos[2*a - 2*c - b*x]
+ b*x*Cos[2*a + b*x] - b*x*Cos[2*a - 2*c + b*x] + 2*Sin[b*x] + Log[Sin[c +
 b*x]]*Sin[2*a - 4*c - b*x] - Sin[2*a - 2*c - b*x] - Log[Sin[c + b*x]]*Sin
[2*a - 2*c - b*x] + Log[Sin[c + b*x]]*Sin[2*a + b*x] + Sin[2*a - 2*c + b*x
] - Log[Sin[c + b*x]]*Sin[2*a - 2*c + b*x]))/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^2(bx+c)\,dx$$
$$\Big\downarrow \ 7299$$
$$\int \cos^2(a+bx)\csc^2(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Csc[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.43

method	result
risch	$-x\,\mathrm{e}^{2i(a-c)}+2i\sin\left(2a-2c\right)x+\frac{2i\sin(2a-2c)a}{b}+\frac{i\mathrm{e}^{2i(2a-c)}}{2b\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{i\mathrm{e}^{2ia}}{b\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{}{2b(-\mathrm{e}^{2i(}}$
default	$\frac{\frac{\left(-2\cos(a)^2\cos(c)\sin(c)+2\cos(c)^2\cos(a)\sin(a)-2\cos(a)\sin(a)\sin(c)^2+2\sin(a)^2\cos(c)\sin(c)\right)\ln\left(\tan(bx+a)^2+1\right)}{2}+\left(-\cos(a)^2\cos(c)^2+\cos(c)^2\,\mathrm{s}\right.}{\left(\cos(c)^2+\sin(c)^2\right)^2\left(\cos(a)^2+\sin(a)^2\right)^2}$

input
```
int(cos(b*x+a)^2*csc(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-x*exp(2*I*(a-c))+2*I*sin(2*a-2*c)*x+2*I/b*sin(2*a-2*c)*a+1/2*I/b/(-exp(2*
I*(b*x+a+c))+exp(2*I*a))*exp(2*I*(2*a-c))+I/b/(-exp(2*I*(b*x+a+c))+exp(2*I
*a))*exp(2*I*a)+1/2*I/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))*exp(2*I*c)-ln(exp
(2*I*(b*x+a))-exp(2*I*(a-c)))/b*sin(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx$$
$$=\frac{2\cos(-a+c)\log\left(\frac{1}{2}\sin(bx+c)\right)\sin(bx+c)\sin(-a+c)-\cos(bx+c)\cos(-a+c)^2-\left(2\,bx\cos(-a\right.}{b\sin(bx+c)}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^2,x, algorithm="fricas")
```

output
```
(2*cos(-a + c)*log(1/2*sin(b*x + c))*sin(b*x + c)*sin(-a + c) - cos(b*x +
c)*cos(-a + c)^2 - (2*b*x*cos(-a + c)^2 - b*x)*sin(b*x + c))/(b*sin(b*x +
c))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*csc(b*x+c)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(51) = 102$.

Time = 0.06 (sec) , antiderivative size = 711, normalized size of antiderivative = 13.94

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(b*cos(2*a + 2*c)*cos(4*c) + b*sin(2*a + 2*c)*sin(4*c))*x - (2*b*x
*cos(4*c) + sin(4*a) + 2*sin(2*a + 2*c) + sin(4*c))*cos(2*b*x + 2*a + 4*c)
 + 2*(b*x*cos(2*b*x + 2*a + 4*c) - b*x*cos(2*a + 2*c))*cos(2*b*x + 6*c) +
(sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(2*b*x + 2*a + 4*c)^2*sin(-2*a
+ 2*c) - 2*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c)*sin(-2*a + 2*c) + cos(2*a
 + 2*c)^2*sin(-2*a + 2*c) + sin(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c) - 2*s
in(2*b*x + 2*a + 4*c)*sin(2*a + 2*c)*sin(-2*a + 2*c) + sin(2*a + 2*c)^2*si
n(-2*a + 2*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2
- 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c
) - 2*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c)*sin(-2*a + 2*c) + cos(2*a + 2*
c)^2*sin(-2*a + 2*c) + sin(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c) - 2*sin(2*
b*x + 2*a + 4*c)*sin(2*a + 2*c)*sin(-2*a + 2*c) + sin(2*a + 2*c)^2*sin(-2*
a + 2*c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*s
in(b*x)*sin(c) + sin(c)^2) - (2*b*x*sin(4*c) - cos(4*a) - 2*cos(2*a + 2*c)
 - cos(4*c))*sin(2*b*x + 2*a + 4*c) + 2*(b*x*sin(2*b*x + 2*a + 4*c) - b*x*
sin(2*a + 2*c))*sin(2*b*x + 6*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))/(
b*cos(2*b*x + 2*a + 4*c)^2 - 2*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c) + b
*cos(2*a + 2*c)^2 + b*sin(2*b*x + 2*a + 4*c)^2 - 2*b*sin(2*b*x + 2*a + 4*c
)*sin(2*a + 2*c) + b*sin(2*a + 2*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2042 vs. $2(51) = 102$.

Time = 0.20 (sec) , antiderivative size = 2042, normalized size of antiderivative = 40.04

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^2,x, algorithm="giac")
```

output
```
-((tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*a)^4*tan(1/2*c)^2 + 16*tan(1/2*a)
^3*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 16*tan(1/2*
a)^3*tan(1/2*c) + 36*tan(1/2*a)^2*tan(1/2*c)^2 - 16*tan(1/2*a)*tan(1/2*c)^
3 + tan(1/2*c)^4 - 6*tan(1/2*a)^2 + 16*tan(1/2*a)*tan(1/2*c) - 6*tan(1/2*c
)^2 + 1)*(b*x + a)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^
2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4
*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*ta
n(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2
*c)^4 - tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)
^2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))*log(tan(b*x + a)^2 + 1)/(tan(
1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1
/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*ta
n(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) + 4*(tan(1/2*a)^6*tan(1/2*c)^5 - tan(1/2*
a)^5*tan(1/2*c)^6 - 2*tan(1/2*a)^6*tan(1/2*c)^3 + 11*tan(1/2*a)^5*tan(1/2*
c)^4 - 11*tan(1/2*a)^4*tan(1/2*c)^5 + 2*tan(1/2*a)^3*tan(1/2*c)^6 + tan(1/
2*a)^6*tan(1/2*c) - 11*tan(1/2*a)^5*tan(1/2*c)^2 + 38*tan(1/2*a)^4*tan(1/2
*c)^3 - 38*tan(1/2*a)^3*tan(1/2*c)^4 + 11*tan(1/2*a)^2*tan(1/2*c)^5 - tan(
1/2*a)*tan(1/2*c)^6 + tan(1/2*a)^5 - 11*tan(1/2*a)^4*tan(1/2*c) + 38*tan(1
/2*a)^3*tan(1/2*c)^2 - 38*tan(1/2*a)^2*tan(1/2*c)^3 + 11*tan(1/2*a)*tan...
```

Mupad [B] (verification not implemented)

Time = 18.82 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.92

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = -x\left(\cos\left(2a-2c\right)-\sin\left(2a-2c\right)\,1\mathrm{i}\right)+\frac{\left(2\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}-c\,4\mathrm{i}}+1\right)\,1\mathrm{i}}{2\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)} - \frac{\mathrm{e}^{-a\,4\mathrm{i}+c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\right)\,\left(2\,b\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-2\,b\,\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$

input
```
int(cos(a + b*x)^2/sin(c + b*x)^2,x)
```

output
```
((2*exp(a*2i - c*2i) + exp(a*4i - c*4i) + 1)*1i)/(2*b*(exp(a*2i - c*2i) -
exp(a*2i + b*x*2i))) - x*(cos(2*a - 2*c) - sin(2*a - 2*c)*1i) - (exp(c*4i
- a*4i)*log(exp(a*2i)*exp(b*x*2i) - exp(a*2i)*exp(-c*2i))*(2*b*exp(a*2i -
c*2i) - 2*b*exp(a*6i - c*6i))*1i)/(4*b^2)
```

Reduce [F]

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = \int \cos(bx+a)^2\csc(bx+c)^2\,dx$$

input
```
int(cos(b*x+a)^2*csc(b*x+c)^2,x)
```

output
```
int(cos(a + b*x)**2*csc(b*x + c)**2,x)
```

3.175 $\int \cos(a+bx)\csc^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \cos(a+bx)\csc^2(c+bx)\,dx=-\frac{\cos(a-c)\csc(c+bx)}{b}+\frac{\operatorname{arctanh}(\cos(c+bx))\sin(a-c)}{b}$$

output
```
-cos(a-c)*csc(b*x+c)/b+arctanh(cos(b*x+c))*sin(a-c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.57

$$\begin{aligned}&\int \cos(a+bx)\csc^2(c+bx)\,dx\\&=-\frac{\cos(a-c)\csc(c+bx)}{b}\\&\quad+\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b}\end{aligned}$$

input
```
Integrate[Cos[a + b*x]*Csc[c + b*x]^2,x]
```

output
```
-((Cos[a - c]*Csc[c + b*x])/b) + ((2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c
]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/
2]*Sin[c])]*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5092, 3042, 25, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \; 5092$$

$$\cos(a-c)\int \cot(c+bx)\csc(c+bx)dx - \sin(a-c)\int \csc(c+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\cos(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx - \sin(a-c)\int \csc(c+bx)dx$$

$$\Big\downarrow \; 25$$

$$-\sin(a-c)\int \csc(c+bx)dx - \cos(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \; 3086$$

$$-\frac{\cos(a-c)\int 1d\csc(c+bx)}{b} - \sin(a-c)\int \csc(c+bx)dx$$

$$\Big\downarrow \; 24$$

$$-\sin(a-c)\int \csc(c+bx)dx - \frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \; 4257$$

$$\frac{\sin(a-c)\text{arctanh}(\cos(bx+c))}{b} - \frac{\cos(a-c)\csc(bx+c)}{b}$$

input
```
Int[Cos[a + b*x]*Csc[c + b*x]^2,x]
```

output
```
-((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5092
```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Simp[Cos[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] - Simp[Sin[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.29

method	result
risch	$\frac{i\left(\mathrm{e}^{i(bx+3a)}+\mathrm{e}^{i(bx+a+2c)}\right)}{b\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+\mathrm{e}^{i(a-c)}\right)\sin(a-c)}{b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-\mathrm{e}^{i(a-c)}\right)\sin(a-c)}{b}$
default	$-\frac{2\left(-\frac{\left(\cos(a)^2\cos(c)^2+2\cos(a)\cos(c)\sin(a)\sin(c)+\sin(a)^2\sin(c)^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)(\sin(a)\cos(c)-\cos(a)\sin(c))}+\frac{\cos(a)\cos(c)+\sin(a)\mathrm{si}}{\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2}\right.}{\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\cos(a)\cos(c)+2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\sin(a)\sin(c)-\sin(a)\cos(c)+\mathrm{c}}$

input
```
int(cos(b*x+a)*csc(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
I/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))
+ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin(a-c)-ln(exp(I*(b*x+a))-exp(I*(a-c))
)/b*sin(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(35) = 70.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.03

$$\int \cos(a+bx)\csc^2(c+bx)\,dx = -\frac{\log\left(\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)\sin(-a+c)-\log\left(-\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)\sin(-a+c)}{2\,b\sin(bx+c)}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) - log(-1/2*cos(
b*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) + 2*cos(-a + c))/(b*sin(b*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1572 vs. $2(27) = 54$.

Time = 61.22 (sec) , antiderivative size = 3264, normalized size of antiderivative = 93.26

$$\int \cos(a + bx) \csc^2(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)**2,x)
```

output
```
-Piecewise((0, Eq(b, 0) & Eq(c, 0)), (log(tan(b*x/2))/b, Eq(c, 0)), (0, Eq
(b, 0)), (-log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**
4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*ta
n(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(
c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)
**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)
) + log(tan(c/2) + tan(b*x/2))*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*t
an(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*ta
n(c/2) - b*tan(b*x/2)) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**2*tan(b*x/
2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3
 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(c/2) +
tan(b*x/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)*
*3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) -
 b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*
x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)
**2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(b*x/2)/(
b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b
*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/
tan(c/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3
*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(35) = 70$.

Time = 0.06 (sec) , antiderivative size = 450, normalized size of antiderivative = 12.86

$$\int \cos(a + bx) \csc^2(c + bx)\, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(sin(b*x + 2*a) + sin(b*x + 2*c))*cos(2*b*x + a + 2*c) - (cos(2*b*x
 + a + 2*c)^2*sin(-a + c) - 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + si
n(2*b*x + a + 2*c)^2*sin(-a + c) - 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a +
c) + (cos(a)^2 + sin(a)^2)*sin(-a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c)
 + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + (cos(2*b*x + a
+ 2*c)^2*sin(-a + c) - 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(2*b
*x + a + 2*c)^2*sin(-a + c) - 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) +
(cos(a)^2 + sin(a)^2)*sin(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + co
s(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*(cos(b*x + 2*a) +
cos(b*x + 2*c))*sin(2*b*x + a + 2*c) - 2*cos(a)*sin(b*x + 2*a) - 2*cos(a)*
sin(b*x + 2*c) + 2*cos(b*x + 2*a)*sin(a) + 2*cos(b*x + 2*c)*sin(a))/(b*cos
(2*b*x + a + 2*c)^2 - 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a +
2*c)^2 - 2*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(35) = 70$.

Time = 0.25 (sec) , antiderivative size = 893, normalized size of antiderivative = 25.51

$$\int \cos(a + bx) \csc^2(c + bx)\, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) -
tan(1/2*c))*log(abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan
(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2
*a) - 2*tan(1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c) + 4*tan(1/2*a)*ta
n(1/2*c) - 2*tan(1/2*c)^2)/abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2
*c) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(
1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) - 2*tan(1/2*b*x + 1/2*a)*tan(
1/2*c) + 4*tan(1/2*a)*tan(1/2*c) + 2))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/
2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)
^4 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*b*x + 1/
2*a)*tan(1/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(1/2*b
*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(
1/2*b*x + 1/2*a)*tan(1/2*a)^4 - 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/
2*c) + 2*tan(1/2*a)^4*tan(1/2*c) + 20*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*ta
n(1/2*c)^2 - 12*tan(1/2*a)^3*tan(1/2*c)^2 - 8*tan(1/2*b*x + 1/2*a)*tan(1/2
*a)*tan(1/2*c)^3 + 12*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*b*x + 1/2*a)*tan
(1/2*c)^4 - 2*tan(1/2*a)*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^
2 + 2*tan(1/2*a)^3 + 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - 12*tan
(1/2*a)^2*tan(1/2*c) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c)^2 + 12*tan(1/2*a)
*tan(1/2*c)^2 - 2*tan(1/2*c)^3 + tan(1/2*b*x + 1/2*a) - 2*tan(1/2*a) + ...
```

Mupad [B] (verification not implemented)

Time = 23.20 (sec) , antiderivative size = 252, normalized size of antiderivative = 7.20

$$\int \cos(a+bx)\csc^2(c+bx)\,dx = -\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{-c\,2i}-1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\left(e^{a\,2i}\,e^{-c\,2i}-1\right)1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}} + \frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{-c\,2i}-1\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\left(e^{a\,2i}\,e^{-c\,2i}-1\right)1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}} + \frac{e^{a\,1i+b\,x\,1i}\left(e^{a\,2i-c\,2i}+1\right)1i}{b\left(e^{a\,2i-c\,2i}-e^{a\,2i+b\,x\,2i}\right)}$$

input
```
int(cos(a + b*x)/sin(c + b*x)^2,x)
```

output
```
(log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) + (exp(a*2i)*exp(-c*
2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2
i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log(exp(a*1i)*exp(b*x*1
i)*(exp(a*2i)*exp(-c*2i) - 1) - (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i
) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) - 1))/(2*b*(-e
xp(a*2i - c*2i))^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) + 1)*1i)/(
b*(exp(a*2i - c*2i) - exp(a*2i + b*x*2i)))
```

Reduce [F]

$$\int \cos(a+bx)\csc^2(c+bx)\,dx=\int \cos\left(bx+a\right)\csc\left(bx+c\right)^2dx$$

input
```
int(cos(b*x+a)*csc(b*x+c)^2,x)
```

output
```
int(cos(a + b*x)*csc(b*x + c)**2,x)
```

3.176 $\int \csc^2(c+bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 1

$$\int \csc^2(c+bx)\sec(a+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 97.00

$$\int \csc^2(c+bx)\sec(a+bx)\,dx = -\frac{\sec(a-c)\left(\csc(c+bx)+\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a-\right.\right.\right.}{b}$$

input
```
Integrate[Csc[c + b*x]^2*Sec[a + b*x],x]
```

output
```
-((Sec[a - c]*(Csc[c + b*x] + (Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] -
Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]])*Sec[a - c] + 2*ArcTanh[Cos[c] -
Sin[c]*Tan[(b*x)/2]]*Tan[a - c]))/b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sec(a+bx)\csc^2(bx+c)dx$$

input
```
Int[Csc[c + b*x]^2*Sec[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.17 (sec) , antiderivative size = 357, normalized size of antiderivative = 357.00

method	result
default	$\frac{\frac{2\left(\frac{\left(\cos(a)^2\cos(c)^2+2\cos(a)\cos(c)\sin(a)\sin(c)+\sin(a)^2\sin(c)^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\sin(a)\cos(c)-\cos(a)\sin(c)}-\cos(a)\cos(c)-\sin(a)\sin(c)\right)}{\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\cos(a)\cos(c)+2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c)}+ \frac{2(s}{}}{(\cos(a)\cos(c)+\sin(a)\sin(c))^2}$
risch	$\frac{4ie^{i(bx+3a+2c)}}{\left(-e^{2i(bx+a+c)}+e^{2ia}\right)\left(e^{2ia}+e^{2ic}\right)b}-\frac{4\ln\left(e^{i(bx+a)}-i\right)e^{2i(a+c)}}{\left(e^{4ia}+2e^{2i(a+c)}+e^{4ic}\right)b}+\frac{2i\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)e^{i(3a+c)}}{\left(e^{4ia}+2e^{2i(a+c)}+e^{4ic}\right)b}-\frac{2i\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)e}{\left(e^{4ia}+2e^{2i(a+c)}+e^{4i\right.}}$

input
```
int(csc(b*x+c)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(2/(cos(a)*cos(c)+sin(a)*sin(c))^2*(((cos(a)^2*cos(c)^2+2*cos(a)*cos(c
)*sin(a)*sin(c)+sin(a)^2*sin(c)^2)/(sin(a)*cos(c)-cos(a)*sin(c))*tan(1/2*a
+1/2*b*x)-cos(a)*cos(c)-sin(a)*sin(c))/(cos(c)*sin(a)*tan(1/2*a+1/2*b*x)^2
-sin(c)*cos(a)*tan(1/2*a+1/2*b*x)^2+2*tan(1/2*a+1/2*b*x)*cos(a)*cos(c)+2*t
an(1/2*a+1/2*b*x)*sin(a)*sin(c)-sin(a)*cos(c)+cos(a)*sin(c))+(sin(a)*cos(c
)-cos(a)*sin(c))/(-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-s
in(c)^2*cos(a)^2)^(1/2)*arctan(1/2*(2*(sin(a)*cos(c)-cos(a)*sin(c))*tan(1/
2*a+1/2*b*x)+2*cos(a)*cos(c)+2*sin(a)*sin(c))/(-cos(c)^2*sin(a)^2-cos(a)^2
*cos(c)^2-sin(a)^2*sin(c)^2-sin(c)^2*cos(a)^2)^(1/2))-1/(cos(a)*cos(c)+si
n(a)*sin(c))^2*ln(tan(1/2*a+1/2*b*x)-1)+1/(cos(a)*cos(c)+sin(a)*sin(c))^2*
ln(tan(1/2*a+1/2*b*x)+1))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 178.00

$$\int \csc^2(c+bx)\sec(a+bx)\,dx$$

$$= \frac{\log\left(\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)\sin(-a+c)-\log\left(-\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)\sin(-a+c)+}{}$$

input
```
integrate(csc(b*x+c)^2*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) - log(-1/2*cos(b
*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) + log(2*(cos(-a + c)*sin(b*x + c)
- cos(b*x + c)*sin(-a + c) + 1)/(cos(-a + c) + 1))*sin(b*x + c) - log(-2*(
cos(-a + c)*sin(b*x + c) - cos(b*x + c)*sin(-a + c) - 1)/(cos(-a + c) + 1)
)*sin(b*x + c) - 2*cos(-a + c))/(b*cos(-a + c)^2*sin(b*x + c))
```

Sympy [F]

$$\int \csc^2(c+bx)\sec(a+bx)\,dx = \int \csc^2(bx+c)\sec(a+bx)\,dx$$

input
```
integrate(csc(b*x+c)**2*sec(b*x+a),x)
```

output
```
Integral(csc(b*x + c)**2*sec(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.41 (sec) , antiderivative size = 17016, normalized size of antiderivative = 17016.00

$$\int \csc^2(c+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sec(b*x+a),x, algorithm="maxima")
```

output
```
-(4*(cos(4*a)^2 + 4*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c
)^2 + 2*cos(4*a)*cos(4*c) + cos(4*c)^2 + sin(4*a)^2 + 4*(sin(4*a) + sin(4*
c))*sin(2*a + 2*c) + 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2
)*cos(b*x + a + 2*c)*sin(2*b*x + 2*a + 2*c) + 4*(cos(4*a)^2 + 4*(cos(4*a)
+ cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)^2 + 2*cos(4*a)*cos(4*c) + co
s(4*c)^2 + sin(4*a)^2 + 4*(sin(4*a) + sin(4*c))*sin(2*a + 2*c) + 4*sin(2*a
 + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)*cos(b*x + a + 2*c)*sin(2*b*x
 + 4*c) - 4*(cos(4*a)^2 + 4*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2
*a + 2*c)^2 + 2*cos(4*a)*cos(4*c) + cos(4*c)^2 + sin(4*a)^2 + 4*(sin(4*a)
+ sin(4*c))*sin(2*a + 2*c) + 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + si
n(4*c)^2)*cos(2*b*x + 2*a + 2*c)*sin(b*x + a + 2*c) - 4*(cos(4*a)^2 + 4*(c
os(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)^2 + 2*cos(4*a)*cos(4
*c) + cos(4*c)^2 + sin(4*a)^2 + 4*(sin(4*a) + sin(4*c))*sin(2*a + 2*c) + 4
*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)*cos(2*b*x + 4*c)*sin
(b*x + a + 2*c) - 4*(((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + c
os(4*c))*sin(2*a + 2*c))*cos(2*b*x + 2*a + 2*c)^2 + ((sin(4*a) + sin(4*c))
*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*cos(2*b*x + 4*c)^2
 + ((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a +
 2*c))*sin(2*b*x + 2*a + 2*c)^2 + ((sin(4*a) + sin(4*c))*cos(2*a + 2*c) -
(cos(4*a) + cos(4*c))*sin(2*a + 2*c))*sin(2*b*x + 4*c)^2 - 2*((cos(2*a)...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.23 (sec) , antiderivative size = 2468, normalized size of antiderivative = 2468.00

$$\int \csc^2(c+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sec(b*x+a),x, algorithm="giac")
```

output
```
1/2*(2*(tan(1/2*a)^5*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^4 + tan(1/2*a)
^4*tan(1/2*c)^5 + 2*tan(1/2*a)^5*tan(1/2*c)^3 + tan(1/2*a)^4*tan(1/2*c)^4
+ 2*tan(1/2*a)^3*tan(1/2*c)^5 - 2*tan(1/2*a)^5*tan(1/2*c)^2 + 2*tan(1/2*a)
^4*tan(1/2*c)^3 - 2*tan(1/2*a)^3*tan(1/2*c)^4 + 2*tan(1/2*a)^2*tan(1/2*c)^
5 + tan(1/2*a)^5*tan(1/2*c) + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 4*tan(1/2*a)^3
*tan(1/2*c)^3 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)*tan(1/2*c)^5 - ta
n(1/2*a)^5 + tan(1/2*a)^4*tan(1/2*c) - 4*tan(1/2*a)^3*tan(1/2*c)^2 + 4*tan
(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 + tan(1/2*c)^5 + tan(1/2*
a)^4 + 2*tan(1/2*a)^3*tan(1/2*c) + 4*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2
*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 2*tan(1/2*a)^3 + 2*tan(1/2*a)^2*tan(1/2*
c) - 2*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*c)^3 + 2*tan(1/2*a)^2 + tan(1/2
*a)*tan(1/2*c) + 2*tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1)*log(abs(-ta
n(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*c)*tan(1/2*a)
 - tan(1/2*b*x + 1/2*c)*tan(1/2*c) - tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x +
 1/2*c) - tan(1/2*a) + tan(1/2*c) - 1))/(tan(1/2*a)^5*tan(1/2*c)^5 - tan(1
/2*a)^5*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c)^5 - 2*tan(1/2*a)^5*tan(1/2*
c)^3 + 9*tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^3*tan(1/2*c)^5 + 2*tan(1
/2*a)^5*tan(1/2*c)^2 - 10*tan(1/2*a)^4*tan(1/2*c)^3 + 10*tan(1/2*a)^3*tan(
1/2*c)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^5 + tan(1/2*a)^5*tan(1/2*c) - 10*tan(
1/2*a)^4*tan(1/2*c)^2 + 28*tan(1/2*a)^3*tan(1/2*c)^3 - 10*tan(1/2*a)^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(c+bx)\sec(a+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)*sin(c + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^2(c+bx)\sec(a+bx)\,dx$$

$$= \frac{2\cos(bx+c) - \left(\int \frac{\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}dx\right)\sin(bx+c)\,b - \left(\int \frac{1}{\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2}dx\right)\sin(bx+}{2\sin(bx+c}$$

input
```
int(csc(b*x+c)^2*sec(b*x+a),x)
```

output
```
(2*cos(b*x + c) - int(tan((b*x + c)/2)**2/(tan((a + b*x)/2)**2 - 1),x)*sin
(b*x + c)*b - int(1/(tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - tan((b*x +
c)/2)**2),x)*sin(b*x + c)*b - log(tan((a + b*x)/2) - 1)*sin(b*x + c) + log
(tan((a + b*x)/2) + 1)*sin(b*x + c) + sin(b*x + c)*b*x)/(2*sin(b*x + c)*b)
```

3.177 $\int \csc^2(c+bx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \csc^2(c+bx)\sec^2(a+bx)\,dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.86 (sec) , antiderivative size = 64, normalized size of antiderivative = 64.00

$$\int \csc^2(c+bx)\sec^2(a+bx)\,dx = \frac{\sec^2(a-c)(\csc(c)\csc(c+bx)\sin(bx)+\sec(a)\sec(a+bx)\sin(bx)+2(-\log(\cos(a+bx))+\log(\sin(c+}{b}$$

input
```
Integrate[Csc[c + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
(Sec[a - c]^2*(Csc[c]*Csc[c + b*x]*Sin[b*x] + Sec[a]*Sec[a + b*x]*Sin[b*x]
 + 2*(-Log[Cos[a + b*x]] + Log[Sin[c + b*x]])*Tan[a - c]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sec^2(a+bx)\csc^2(bx+c)dx$$

input
```
Int[Csc[c + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.79 (sec) , antiderivative size = 176, normalized size of antiderivative = 176.00

method	result
default	$\frac{\frac{\tan(bx+a)}{(\cos(a)\cos(c)+\sin(a)\sin(c))^2}-\frac{\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2}{(\cos(a)\cos(c)+\sin(a)\sin(c))^3(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))}+\frac{(-2 \cdots}{}}{b}$
risch	$\frac{8i\left(-e^{2i(bx+3a+c)}+e^{2i(bx+2a+2c)}-2\,e^{2i(2a+c)}\right)}{\left(e^{2i(bx+a)}+1\right)\left(e^{2i(bx+a+c)}-e^{2ia}\right)\left(e^{2ia}+e^{2ic}\right)^2 b}+\frac{8i\ln\left(e^{2i(bx+a)}+1\right)e^{2i(2a+c)}}{\left(e^{6ia}+3\,e^{2i(2a+c)}+3\,e^{2i(a+2c)}+e^{6ic}\right)b}-\frac{8i\ln\left(e^{2i(bx+a)}+1\right)e^{2i(a+2c)}}{\left(e^{6ia}+3\,e^{2i(2a+c)}+3\,e^{2i(a+2c)}+e^{6 \cdots}\right)}$

input
```
int(csc(b*x+c)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(tan(b*x+a)/(cos(a)*cos(c)+sin(a)*sin(c))^2-1/(cos(a)*cos(c)+sin(a)*si
n(c))^3*(cos(a)^2*cos(c)^2+sin(c)^2*cos(a)^2+cos(c)^2*sin(a)^2+sin(a)^2*si
n(c)^2)/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)-sin(a)*cos(c)+c
os(a)*sin(c))+(-2*cos(a)*sin(c)+2*sin(a)*cos(c))/(cos(a)*cos(c)+sin(a)*sin
(c))^3*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)-sin(a)*cos(c)+
cos(a)*sin(c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.10 (sec) , antiderivative size = 330, normalized size of antiderivative = 330.00

$$\int \csc^2(c+bx)\sec^2(a+bx)\,dx =$$

$$-\frac{2\cos(bx+c)^2\cos(-a+c)^2+2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)-\cos(-a+c)^2+}{}$$

input
```
integrate(csc(b*x+c)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*cos(b*x + c)^2*cos(-a + c)^2 + 2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)
*sin(-a + c) - cos(-a + c)^2 + (cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(
-a + c) + (cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)*log(-1/4
*cos(b*x + c)^2 + 1/4) - (cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c
) + (cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)*log(4*(2*cos(b
*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b
*x + c)^2 - cos(-a + c)^2 + 1)/(cos(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*co
s(b*x + c)*cos(-a + c)^4*sin(b*x + c) - (b*cos(b*x + c)^2*cos(-a + c)^3 -
b*cos(-a + c)^3)*sin(-a + c))
```

Sympy [F]

$$\int \csc^2(c+bx)\sec^2(a+bx)\,dx = \int \csc^2(bx+c)\sec^2(a+bx)\,dx$$

input
```
integrate(csc(b*x+c)**2*sec(b*x+a)**2,x)
```

output
```
Integral(csc(b*x + c)**2*sec(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 2.55 (sec) , antiderivative size = 114453, normalized size of antiderivative = 114453.00

$$\int \csc^2(c+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
4*(36*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*
a + 2*c))*cos(4*a + 2*c)^2 + 36*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (c
os(4*a) + cos(4*c))*sin(2*a + 2*c))*cos(2*a + 4*c)^2 + 36*((sin(4*a) + sin
(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*sin(4*a + 2*
c)^2 + 36*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*si
n(2*a + 2*c))*sin(2*a + 4*c)^2 - 2*(((cos(6*a) + cos(6*c))*cos(4*a + 2*c)
+ 3*cos(4*a + 2*c)^2 - (cos(6*a) + cos(6*c))*cos(2*a + 4*c) - 3*cos(2*a +
4*c)^2 + (sin(6*a) + sin(6*c))*sin(4*a + 2*c) + 3*sin(4*a + 2*c)^2 - (sin(
6*a) + sin(6*c))*sin(2*a + 4*c) - 3*sin(2*a + 4*c)^2)*cos(4*b*x + 6*a + 2*
c)^2 + 4*((cos(6*a) + cos(6*c))*cos(4*a + 2*c) + 3*cos(4*a + 2*c)^2 - (cos
(6*a) + cos(6*c))*cos(2*a + 4*c) - 3*cos(2*a + 4*c)^2 + (sin(6*a) + sin(6*
c))*sin(4*a + 2*c) + 3*sin(4*a + 2*c)^2 - (sin(6*a) + sin(6*c))*sin(2*a +
4*c) - 3*sin(2*a + 4*c)^2)*cos(4*b*x + 4*a + 4*c)^2 + ((cos(6*a) + cos(6*c
))*cos(4*a + 2*c) + 3*cos(4*a + 2*c)^2 - (cos(6*a) + cos(6*c))*cos(2*a + 4
*c) - 3*cos(2*a + 4*c)^2 + (sin(6*a) + sin(6*c))*sin(4*a + 2*c) + 3*sin(4*
a + 2*c)^2 - (sin(6*a) + sin(6*c))*sin(2*a + 4*c) - 3*sin(2*a + 4*c)^2)*co
s(4*b*x + 2*a + 6*c)^2 + ((cos(6*a) + cos(6*c))*cos(4*a + 2*c) + 3*cos(4*a
 + 2*c)^2 - (cos(6*a) + cos(6*c))*cos(2*a + 4*c) - 3*cos(2*a + 4*c)^2 + (s
in(6*a) + sin(6*c))*sin(4*a + 2*c) + 3*sin(4*a + 2*c)^2 - (sin(6*a) + sin(
6*c))*sin(2*a + 4*c) - 3*sin(2*a + 4*c)^2)*cos(2*b*x + 6*a)^2 + ((cos(6...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.24 (sec) , antiderivative size = 3227, normalized size of antiderivative = 3227.00

$$\int \csc^2(c+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(8*(tan(1/2*a)^8*tan(1/2*c)^6 - 2*tan(1/2*a)^7*tan(1/2*c)^7 + tan(1/2
*a)^6*tan(1/2*c)^8 + 2*tan(1/2*a)^8*tan(1/2*c)^4 - 2*tan(1/2*a)^7*tan(1/2*
c)^5 - 2*tan(1/2*a)^5*tan(1/2*c)^7 + 2*tan(1/2*a)^4*tan(1/2*c)^8 + tan(1/2
*a)^8*tan(1/2*c)^2 + 2*tan(1/2*a)^7*tan(1/2*c)^3 - 2*tan(1/2*a)^6*tan(1/2*
c)^4 - 2*tan(1/2*a)^5*tan(1/2*c)^5 - 2*tan(1/2*a)^4*tan(1/2*c)^6 + 2*tan(1
/2*a)^3*tan(1/2*c)^7 + tan(1/2*a)^2*tan(1/2*c)^8 + 2*tan(1/2*a)^7*tan(1/2*
c) + 2*tan(1/2*a)^5*tan(1/2*c)^3 - 8*tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2
*a)^3*tan(1/2*c)^5 + 2*tan(1/2*a)*tan(1/2*c)^7 + tan(1/2*a)^6 + 2*tan(1/2*
a)^5*tan(1/2*c) - 2*tan(1/2*a)^4*tan(1/2*c)^2 - 2*tan(1/2*a)^3*tan(1/2*c)^
3 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)*tan(1/2*c)^5 + tan(1/2*c)^6
 + 2*tan(1/2*a)^4 - 2*tan(1/2*a)^3*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c)^3
+ 2*tan(1/2*c)^4 + tan(1/2*a)^2 - 2*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2)*
log(abs(2*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + c)*tan(1/2*a)
*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + c)*tan(1/2*a) + ta
n(1/2*a)^2 - 2*tan(b*x + c)*tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c) + tan(1/2
*c)^2 - 1))/(tan(1/2*a)^8*tan(1/2*c)^7 - tan(1/2*a)^7*tan(1/2*c)^8 - 3*tan
(1/2*a)^8*tan(1/2*c)^5 + 16*tan(1/2*a)^7*tan(1/2*c)^6 - 16*tan(1/2*a)^6*ta
n(1/2*c)^7 + 3*tan(1/2*a)^5*tan(1/2*c)^8 + 3*tan(1/2*a)^8*tan(1/2*c)^3 - 3
0*tan(1/2*a)^7*tan(1/2*c)^4 + 96*tan(1/2*a)^6*tan(1/2*c)^5 - 96*tan(1/2*a)
^5*tan(1/2*c)^6 + 30*tan(1/2*a)^4*tan(1/2*c)^7 - 3*tan(1/2*a)^3*tan(1/2...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(c+bx)\sec^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)^2*sin(c + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^2(c+bx)\sec^2(a+bx)\,dx = \int \csc(bx+c)^2\sec(bx+a)^2\,dx$$

input
```
int(csc(b*x+c)^2*sec(b*x+a)^2,x)
```

output
```
int(csc(b*x + c)**2*sec(a + b*x)**2,x)
```

3.178 $\int \csc^2(c+bx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \csc^2(c+bx)\sec^3(a+bx)\,dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 6.60 (sec) , antiderivative size = 491, normalized size of antiderivative = 491.00

$$
\begin{aligned}
&\int \csc^2(c+bx)\sec^3(a+bx)\,dx\\
&=-\frac{\csc(c+bx)\sec^3(a-c)}{b}\\
&\quad+\frac{3(-3+\cos(2a-2c))\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sec^4(a-c)}{4b}\\
&\quad-\frac{3(-3+\cos(2a-2c))\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sec^4(a-c)}{4b}\\
&\quad-\frac{6i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{\frac{3b}{8}+\frac{1}{8}b\cos(4a-4c)+\frac{1}{2}b\cos(2a-2c)}\\
&\quad+\frac{\sec^2(a-c)}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2}\\
&\quad+\frac{\left(\cos\left(a-c-\frac{bx}{2}\right)-\cos\left(a-c+\frac{bx}{2}\right)\right)\sec^3(a-c)}{b\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}\\
&\quad-\frac{\sec^2(a-c)}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2}\\
&\quad+\frac{\left(-\cos\left(a-c-\frac{bx}{2}\right)+\cos\left(a-c+\frac{bx}{2}\right)\right)\sec^3(a-c)}{b\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}
\end{aligned}
$$

input
```
Integrate[Csc[c + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
-((Csc[c + b*x]*Sec[a - c]^3)/b) + (3*(-3 + Cos[2*a - 2*c])*Log[Cos[a/2 +
(b*x)/2] - Sin[a/2 + (b*x)/2]]*Sec[a - c]^4)/(4*b) - (3*(-3 + Cos[2*a - 2*
c])*Log[Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2]]*Sec[a - c]^4)/(4*b) - ((6
*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]
))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Sin[a - c])/((3*b)/8 + (
b*Cos[4*a - 4*c])/8 + (b*Cos[2*a - 2*c])/2) + Sec[a - c]^2/(4*b*(Cos[a/2 +
 (b*x)/2] - Sin[a/2 + (b*x)/2])^2) + ((Cos[a - c - (b*x)/2] - Cos[a - c +
(b*x)/2])*Sec[a - c]^3)/(b*(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin
[a/2 + (b*x)/2])) - Sec[a - c]^2/(4*b*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x
)/2])^2) + ((-Cos[a - c - (b*x)/2] + Cos[a - c + (b*x)/2])*Sec[a - c]^3)/(
b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec^3(a+bx)\csc^2(bx+c)dx$$

input
```
Int[Csc[c + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 4.13 (sec) , antiderivative size = 858, normalized size of antiderivative = 858.00

method	result	size
default	Expression too large to display	858
risch	Expression too large to display	922

input
```
int(csc(b*x+c)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(2/(cos(a)*cos(c)+sin(a)*sin(c))^4*(((sin(a)^4*cos(c)^2*sin(c)^2+sin(a
)^4*sin(c)^4+2*cos(a)*sin(a)^3*cos(c)^3*sin(c)+2*cos(a)*sin(a)^3*cos(c)*si
n(c)^3+cos(a)^2*sin(a)^2*cos(c)^4+2*cos(a)^2*sin(a)^2*cos(c)^2*sin(c)^2+co
s(a)^2*sin(a)^2*sin(c)^4+2*cos(a)^3*cos(c)^3*sin(a)*sin(c)+2*cos(a)^3*sin(
a)*cos(c)*sin(c)^3+cos(a)^4*cos(c)^4+cos(a)^4*cos(c)^2*sin(c)^2)/(sin(a)*c
os(c)-cos(a)*sin(c))*tan(1/2*a+1/2*b*x)-(cos(a)^2*cos(c)^2+sin(c)^2*cos(a)
^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)*(cos(a)*cos(c)+sin(a)*sin(c)))/(co
s(c)*sin(a)*tan(1/2*a+1/2*b*x)^2-sin(c)*cos(a)*tan(1/2*a+1/2*b*x)^2+2*tan(
1/2*a+1/2*b*x)*cos(a)*cos(c)+2*tan(1/2*a+1/2*b*x)*sin(a)*sin(c)-sin(a)*cos
(c)+cos(a)*sin(c))+3*(cos(c)^3*sin(a)^3+cos(c)*sin(c)^2*sin(a)^3-cos(c)^2*
sin(c)*sin(a)^2*cos(a)-sin(c)^3*sin(a)^2*cos(a)+cos(c)^3*sin(a)*cos(a)^2+c
os(c)*sin(c)^2*sin(a)*cos(a)^2-cos(c)^2*sin(c)*cos(a)^3-sin(c)^3*cos(a)^3)
/(-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-sin(c)^2*cos(a)^2
)^(1/2)*arctan(1/2*(2*(sin(a)*cos(c)-cos(a)*sin(c))*tan(1/2*a+1/2*b*x)+2*c
os(a)*cos(c)+2*sin(a)*sin(c))/(-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)
^2*sin(c)^2-sin(c)^2*cos(a)^2)^(1/2)))+1/2/(cos(a)*cos(c)+sin(a)*sin(c))^2
/(tan(1/2*a+1/2*b*x)-1)^2-1/2*(4*sin(a)*cos(c)-sin(a)*sin(c)-cos(a)*cos(c)
-4*cos(a)*sin(c))/(cos(a)*cos(c)+sin(a)*sin(c))^3/(tan(1/2*a+1/2*b*x)-1)+1
/2/(cos(a)*cos(c)+sin(a)*sin(c))^4*(-6*cos(c)^2*sin(a)^2-3*sin(a)^2*sin(c)
^2+6*cos(a)*cos(c)*sin(a)*sin(c)-3*cos(a)^2*cos(c)^2-6*sin(c)^2*cos(a)^...
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.15 (sec) , antiderivative size = 739, normalized size of antiderivative = 739.00

$$\int \csc^2(c+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(18*cos(b*x + c)*cos(-a + c)^2*sin(b*x + c)*sin(-a + c) + 6*(3*cos(-a
+ c)^3 - 2*cos(-a + c))*cos(b*x + c)^2 - 14*cos(-a + c)^3 - 3*((cos(-a + c
)^4 - (2*cos(-a + c)^4 - 5*cos(-a + c)^2 + 2)*cos(b*x + c)^2 - 3*cos(-a +
c)^2 + 2)*sin(b*x + c) + 2*((cos(-a + c)^3 - 2*cos(-a + c))*cos(b*x + c)^3
 - (cos(-a + c)^3 - 2*cos(-a + c))*cos(b*x + c))*sin(-a + c))*log(2*(cos(-
a + c)*sin(b*x + c) - cos(b*x + c)*sin(-a + c) + 1)/(cos(-a + c) + 1)) + 3
*((cos(-a + c)^4 - (2*cos(-a + c)^4 - 5*cos(-a + c)^2 + 2)*cos(b*x + c)^2
- 3*cos(-a + c)^2 + 2)*sin(b*x + c) + 2*((cos(-a + c)^3 - 2*cos(-a + c))*c
os(b*x + c)^3 - (cos(-a + c)^3 - 2*cos(-a + c))*cos(b*x + c))*sin(-a + c))
*log(-2*(cos(-a + c)*sin(b*x + c) - cos(b*x + c)*sin(-a + c) - 1)/(cos(-a
+ c) + 1)) - 6*(2*(cos(-a + c)^3 - cos(-a + c))*cos(b*x + c)^3 + ((2*cos(-
a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)*sin(b*x + c)*sin(-a + c)
 - 2*(cos(-a + c)^3 - cos(-a + c))*cos(b*x + c))*log(1/2*cos(b*x + c) + 1/
2) + 6*(2*(cos(-a + c)^3 - cos(-a + c))*cos(b*x + c)^3 + ((2*cos(-a + c)^2
 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)*sin(b*x + c)*sin(-a + c) - 2*(co
s(-a + c)^3 - cos(-a + c))*cos(b*x + c))*log(-1/2*cos(b*x + c) + 1/2) + 12
*cos(-a + c))/((b*cos(-a + c)^6 - b*cos(-a + c)^4 - (2*b*cos(-a + c)^6 - b
*cos(-a + c)^4)*cos(b*x + c)^2)*sin(b*x + c) + 2*(b*cos(b*x + c)^3*cos(-a
+ c)^5 - b*cos(b*x + c)*cos(-a + c)^5)*sin(-a + c))
```

Sympy [F]

$$\int \csc^2(c+bx)\sec^3(a+bx)\,dx = \int \csc^2{(bx+c)}\sec^3{(a+bx)}\,dx$$

input
```
integrate(csc(b*x+c)**2*sec(b*x+a)**3,x)
```

output
```
Integral(csc(b*x + c)**2*sec(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 20.64 (sec) , antiderivative size = 660194, normalized size of antiderivative = 660194.00

$$\int \csc^2(c+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-(6*(((sin(8*a) + sin(8*c))*cos(6*a + 2*c) - 6*(sin(8*a) + 5*sin(6*a + 2*c
) + sin(8*c))*cos(4*a + 4*c) + (sin(8*a) + 30*sin(4*a + 4*c) + sin(8*c))*c
os(2*a + 6*c) - (cos(8*a) + cos(8*c))*sin(6*a + 2*c) + 6*(cos(8*a) + 5*cos
(6*a + 2*c) + cos(8*c))*sin(4*a + 4*c) - (cos(8*a) + 30*cos(4*a + 4*c) + c
os(8*c))*sin(2*a + 6*c))*cos(6*b*x + 10*a + 2*c)^2 + 9*((sin(8*a) + sin(8*
c))*cos(6*a + 2*c) - 6*(sin(8*a) + 5*sin(6*a + 2*c) + sin(8*c))*cos(4*a +
4*c) + (sin(8*a) + 30*sin(4*a + 4*c) + sin(8*c))*cos(2*a + 6*c) - (cos(8*a
) + cos(8*c))*sin(6*a + 2*c) + 6*(cos(8*a) + 5*cos(6*a + 2*c) + cos(8*c))*
sin(4*a + 4*c) - (cos(8*a) + 30*cos(4*a + 4*c) + cos(8*c))*sin(2*a + 6*c))
*cos(6*b*x + 8*a + 4*c)^2 + 9*((sin(8*a) + sin(8*c))*cos(6*a + 2*c) - 6*(s
in(8*a) + 5*sin(6*a + 2*c) + sin(8*c))*cos(4*a + 4*c) + (sin(8*a) + 30*sin
(4*a + 4*c) + sin(8*c))*cos(2*a + 6*c) - (cos(8*a) + cos(8*c))*sin(6*a + 2
*c) + 6*(cos(8*a) + 5*cos(6*a + 2*c) + cos(8*c))*sin(4*a + 4*c) - (cos(8*a
) + 30*cos(4*a + 4*c) + cos(8*c))*sin(2*a + 6*c))*cos(6*b*x + 6*a + 6*c)^2
 + ((sin(8*a) + sin(8*c))*cos(6*a + 2*c) - 6*(sin(8*a) + 5*sin(6*a + 2*c)
+ sin(8*c))*cos(4*a + 4*c) + (sin(8*a) + 30*sin(4*a + 4*c) + sin(8*c))*cos
(2*a + 6*c) - (cos(8*a) + cos(8*c))*sin(6*a + 2*c) + 6*(cos(8*a) + 5*cos(6
*a + 2*c) + cos(8*c))*sin(4*a + 4*c) - (cos(8*a) + 30*cos(4*a + 4*c) + cos
(8*c))*sin(2*a + 6*c))*cos(6*b*x + 4*a + 8*c)^2 + ((sin(8*a) + sin(8*c))*c
os(6*a + 2*c) - 6*(sin(8*a) + 5*sin(6*a + 2*c) + sin(8*c))*cos(4*a + 4*...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 3.03 (sec) , antiderivative size = 14520, normalized size of antiderivative = 14520.00

$$\int \csc^2(c+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2*(3*(tan(1/2*a)^9*tan(1/2*c)^9 - tan(1/2*a)^9*tan(1/2*c)^8 + tan(1/2*a)
^8*tan(1/2*c)^9 + 8*tan(1/2*a)^9*tan(1/2*c)^7 - 7*tan(1/2*a)^8*tan(1/2*c)^
8 + 8*tan(1/2*a)^7*tan(1/2*c)^9 - 8*tan(1/2*a)^9*tan(1/2*c)^6 + 16*tan(1/2
*a)^8*tan(1/2*c)^7 - 16*tan(1/2*a)^7*tan(1/2*c)^8 + 8*tan(1/2*a)^6*tan(1/2
*c)^9 + 14*tan(1/2*a)^9*tan(1/2*c)^5 + 8*tan(1/2*a)^7*tan(1/2*c)^7 + 14*ta
n(1/2*a)^5*tan(1/2*c)^9 - 14*tan(1/2*a)^9*tan(1/2*c)^4 + 22*tan(1/2*a)^8*t
an(1/2*c)^5 - 24*tan(1/2*a)^7*tan(1/2*c)^6 + 24*tan(1/2*a)^6*tan(1/2*c)^7
- 22*tan(1/2*a)^5*tan(1/2*c)^8 + 14*tan(1/2*a)^4*tan(1/2*c)^9 + 8*tan(1/2*
a)^9*tan(1/2*c)^3 + 22*tan(1/2*a)^8*tan(1/2*c)^4 + 8*tan(1/2*a)^7*tan(1/2*
c)^5 + 8*tan(1/2*a)^6*tan(1/2*c)^6 + 8*tan(1/2*a)^5*tan(1/2*c)^7 + 22*tan(
1/2*a)^4*tan(1/2*c)^8 + 8*tan(1/2*a)^3*tan(1/2*c)^9 - 8*tan(1/2*a)^9*tan(1
/2*c)^2 - 8*tan(1/2*a)^7*tan(1/2*c)^4 + 24*tan(1/2*a)^6*tan(1/2*c)^5 - 24*
tan(1/2*a)^5*tan(1/2*c)^6 + 8*tan(1/2*a)^4*tan(1/2*c)^7 + 8*tan(1/2*a)^2*t
an(1/2*c)^9 + tan(1/2*a)^9*tan(1/2*c) + 16*tan(1/2*a)^8*tan(1/2*c)^2 + 24*
tan(1/2*a)^7*tan(1/2*c)^3 + 24*tan(1/2*a)^6*tan(1/2*c)^4 - 4*tan(1/2*a)^5*
tan(1/2*c)^5 + 24*tan(1/2*a)^4*tan(1/2*c)^6 + 24*tan(1/2*a)^3*tan(1/2*c)^7
 + 16*tan(1/2*a)^2*tan(1/2*c)^8 + tan(1/2*a)*tan(1/2*c)^9 - tan(1/2*a)^9 -
 7*tan(1/2*a)^8*tan(1/2*c) - 8*tan(1/2*a)^7*tan(1/2*c)^2 + 8*tan(1/2*a)^6*
tan(1/2*c)^3 + 4*tan(1/2*a)^5*tan(1/2*c)^4 - 4*tan(1/2*a)^4*tan(1/2*c)^5 -
 8*tan(1/2*a)^3*tan(1/2*c)^6 + 8*tan(1/2*a)^2*tan(1/2*c)^7 + 7*tan(1/2*...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(c+bx)\sec^3(a+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)^3*sin(c + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^2(c+bx)\sec^3(a+bx)\,dx = \int \csc(bx+c)^2 \sec(bx+a)^3\,dx$$

input
```
int(csc(b*x+c)^2*sec(b*x+a)^3,x)
```

output
```
int(csc(b*x + c)**2*sec(a + b*x)**3,x)
```

3.179 $\int \cos^3(a+bx)\csc^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \cos^3(a+bx)\csc^3(c+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 1.00 (sec) , antiderivative size = 437, normalized size of antiderivative = 437.00

$$\int \cos^3(a+bx)\csc^3(c+bx)\,dx$$
$$=\frac{\csc\left(\frac{c}{2}\right)\csc^2(c+bx)\sec\left(\frac{c}{2}\right)(4bx\cos(3a-4c)-4bx\cos(3a-2c)-2bx\cos(3a-6c-2bx)+2bx\cos(3a}{}$$

input
```
Integrate[Cos[a + b*x]^3*Csc[c + b*x]^3,x]
```

output
```
(Csc[c/2]*Csc[c + b*x]^2*Sec[c/2]*(4*b*x*Cos[3*a - 4*c] - 4*b*x*Cos[3*a -
2*c] - 2*b*x*Cos[3*a - 6*c - 2*b*x] + 2*b*x*Cos[3*a - 4*c - 2*b*x] + 2*b*x
*Cos[3*a + 2*b*x] - 2*b*x*Cos[3*a - 2*c + 2*b*x] - 6*Sin[a] - 2*Sin[3*a -
4*c] + (4*I)*b*x*Sin[3*a - 4*c] + 2*Log[Sin[c + b*x]^2]*Sin[3*a - 4*c] - 4
*Sin[3*a - 2*c] - (4*I)*b*x*Sin[3*a - 2*c] - 2*Log[Sin[c + b*x]^2]*Sin[3*a
 - 2*c] - (2*I)*b*x*Sin[3*a - 6*c - 2*b*x] - Log[Sin[c + b*x]^2]*Sin[3*a -
 6*c - 2*b*x] + 3*Sin[3*a - 4*c - 2*b*x] + (2*I)*b*x*Sin[3*a - 4*c - 2*b*x
] + Log[Sin[c + b*x]^2]*Sin[3*a - 4*c - 2*b*x] + (16*I)*ArcTan[Tan[c + b*x
]]*Cos[3*(a - c)]*Sin[c]*Sin[c + b*x]^2 + 3*Sin[a + 2*b*x] + (2*I)*b*x*Sin
[3*a + 2*b*x] + Log[Sin[c + b*x]^2]*Sin[3*a + 2*b*x] + 3*Sin[3*a - 2*c + 2
*b*x] - (2*I)*b*x*Sin[3*a - 2*c + 2*b*x] - Log[Sin[c + b*x]^2]*Sin[3*a - 2
*c + 2*b*x] + 3*Sin[a - 2*(c + b*x)]))/(32*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc^3(bx+c)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \cos^3(a+bx)\csc^3(bx+c)dx$$

input
```
Int[Cos[a + b*x]^3*Csc[c + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 7.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 198.00

method	result
risch	$-ix\,e^{3i(a-c)}+2i\cos\left(3a-3c\right)x+\frac{2i\cos(3a-3c)a}{b}-\frac{-4\,e^{i(2bx+7a-c)}-6\,e^{i(2bx+5a+c)}+2\,e^{i(2bx+a+5c)}+3\,e^{i(7a-3c)}+}{4\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^{2}b}$
default	Expression too large to display

input
```
int(cos(b*x+a)^3*csc(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-I*x*exp(3*I*(a-c))+2*I*cos(3*a-3*c)*x+2*I/b*cos(3*a-3*c)*a-1/4/(-exp(2*I*
(b*x+a+c))+exp(2*I*a))^2/b*(-4*exp(I*(2*b*x+7*a-c))-6*exp(I*(2*b*x+5*a+c))
+2*exp(I*(2*b*x+a+5*c))+3*exp(I*(7*a-3*c))+3*exp(I*(5*a-c))-3*exp(I*(3*a+c
))-3*exp(I*(a+3*c)))-ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))/b*cos(3*a-3*c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 165.00

$$\int \cos^3(a+bx)\csc^3(c+bx)\,dx = \frac{6\cos\left(bx+c\right)\cos\left(-a+c\right)^2\sin\left(bx+c\right)\sin\left(-a+c\right)+\cos\left(-a+c\right)^3-2\left(\left(4\cos\left(-a+c\right)^3-3\cos\left(-a\right.\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3*csc(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(6*cos(b*x + c)*cos(-a + c)^2*sin(b*x + c)*sin(-a + c) + cos(-a + c)^3
 - 2*((4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^2 - 4*cos(-a + c)^3 +
 3*cos(-a + c))*log(1/2*sin(b*x + c)) + 2*(4*b*x*cos(-a + c)^2 - (4*b*x*co
s(-a + c)^2 - b*x)*cos(b*x + c)^2 - b*x)*sin(-a + c))/(b*cos(b*x + c)^2 -
b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^3(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*csc(b*x+c)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.10 (sec) , antiderivative size = 1738, normalized size of antiderivative = 1738.00

$$\int \cos^3(a+bx)\csc^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*csc(b*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*(4*(b*cos(6*c)*sin(3*a + 3*c) - b*cos(3*a + 3*c)*sin(6*c))*x + (8*b*x*
sin(2*b*x + 8*c) - 4*b*x*sin(6*c) + 4*cos(2*b*x + 6*a + 2*c) + 6*cos(2*b*x
 + 4*a + 4*c) - 2*cos(2*b*x + 8*c) - 3*cos(6*a) - 3*cos(4*a + 2*c) + 3*cos
(2*a + 4*c) + 3*cos(6*c))*cos(4*b*x + 3*a + 7*c) + 4*(b*x*sin(4*b*x + 3*a
+ 7*c) - 2*b*x*sin(2*b*x + 3*a + 5*c) + b*x*sin(3*a + 3*c))*cos(4*b*x + 10
*c) + 2*(4*b*x*sin(6*c) - 4*cos(2*b*x + 6*a + 2*c) - 6*cos(2*b*x + 4*a + 4
*c) + 3*cos(6*a) + 3*cos(4*a + 2*c) - 3*cos(2*a + 4*c) - 3*cos(6*c))*cos(2
*b*x + 3*a + 5*c) + 2*(8*b*x*sin(2*b*x + 3*a + 5*c) - 4*b*x*sin(3*a + 3*c)
 + 2*cos(2*b*x + 3*a + 5*c) - cos(3*a + 3*c))*cos(2*b*x + 8*c) - 3*(cos(6*
a) + cos(4*a + 2*c) - cos(6*c))*cos(3*a + 3*c) + 4*cos(2*b*x + 6*a + 2*c)*
cos(3*a + 3*c) + 6*cos(2*b*x + 4*a + 4*c)*cos(3*a + 3*c) + 3*cos(3*a + 3*c
)*cos(2*a + 4*c) - 2*(cos(4*b*x + 3*a + 7*c)^2*cos(-3*a + 3*c) + 4*cos(2*b
*x + 3*a + 5*c)^2*cos(-3*a + 3*c) - 4*cos(2*b*x + 3*a + 5*c)*cos(3*a + 3*c
)*cos(-3*a + 3*c) + cos(3*a + 3*c)^2*cos(-3*a + 3*c) + cos(-3*a + 3*c)*sin
(4*b*x + 3*a + 7*c)^2 + 4*cos(-3*a + 3*c)*sin(2*b*x + 3*a + 5*c)^2 - 4*cos
(-3*a + 3*c)*sin(2*b*x + 3*a + 5*c)*sin(3*a + 3*c) + cos(-3*a + 3*c)*sin(3
*a + 3*c)^2 - 2*(2*cos(2*b*x + 3*a + 5*c)*cos(-3*a + 3*c) - cos(3*a + 3*c)
*cos(-3*a + 3*c))*cos(4*b*x + 3*a + 7*c) - 2*(2*cos(-3*a + 3*c)*sin(2*b*x
+ 3*a + 5*c) - cos(-3*a + 3*c)*sin(3*a + 3*c))*sin(4*b*x + 3*a + 7*c))*log
(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*si...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.28 (sec) , antiderivative size = 5339, normalized size of antiderivative = 5339.00

$$\int \cos^3(a+bx)\csc^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*csc(b*x+c)^3,x, algorithm="giac")
```

output
```
1/2*(4*(3*tan(1/2*a)^6*tan(1/2*c)^5 - 3*tan(1/2*a)^5*tan(1/2*c)^6 - 10*tan
(1/2*a)^6*tan(1/2*c)^3 + 45*tan(1/2*a)^5*tan(1/2*c)^4 - 45*tan(1/2*a)^4*ta
n(1/2*c)^5 + 10*tan(1/2*a)^3*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c) - 45
*tan(1/2*a)^5*tan(1/2*c)^2 + 150*tan(1/2*a)^4*tan(1/2*c)^3 - 150*tan(1/2*a
)^3*tan(1/2*c)^4 + 45*tan(1/2*a)^2*tan(1/2*c)^5 - 3*tan(1/2*a)*tan(1/2*c)^
6 + 3*tan(1/2*a)^5 - 45*tan(1/2*a)^4*tan(1/2*c) + 150*tan(1/2*a)^3*tan(1/2
*c)^2 - 150*tan(1/2*a)^2*tan(1/2*c)^3 + 45*tan(1/2*a)*tan(1/2*c)^4 - 3*tan
(1/2*c)^5 - 10*tan(1/2*a)^3 + 45*tan(1/2*a)^2*tan(1/2*c) - 45*tan(1/2*a)*t
an(1/2*c)^2 + 10*tan(1/2*c)^3 + 3*tan(1/2*a) - 3*tan(1/2*c))*(b*x + a)/(ta
n(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a)^4*tan
(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^4 + 3*
tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^2 + 9
*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2*a)^
2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1) + (
tan(1/2*a)^6*tan(1/2*c)^6 - 15*tan(1/2*a)^6*tan(1/2*c)^4 + 36*tan(1/2*a)^5
*tan(1/2*c)^5 - 15*tan(1/2*a)^4*tan(1/2*c)^6 + 15*tan(1/2*a)^6*tan(1/2*c)^
2 - 120*tan(1/2*a)^5*tan(1/2*c)^3 + 225*tan(1/2*a)^4*tan(1/2*c)^4 - 120*ta
n(1/2*a)^3*tan(1/2*c)^5 + 15*tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^6 + 36
*tan(1/2*a)^5*tan(1/2*c) - 225*tan(1/2*a)^4*tan(1/2*c)^2 + 400*tan(1/2*a)^
3*tan(1/2*c)^3 - 225*tan(1/2*a)^2*tan(1/2*c)^4 + 36*tan(1/2*a)*tan(1/2*...
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)^3/sin(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos^3(a+bx)\csc^3(c+bx)\,dx = \int \cos(bx+a)^3 \csc(bx+c)^3\,dx$$

input
```
int(cos(b*x+a)^3*csc(b*x+c)^3,x)
```

output
```
int(cos(a + b*x)**3*csc(b*x + c)**3,x)
```

3.180 $\int \cos^2(a+bx)\csc^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+bx))\cos^2(a-c)}{2b} + \frac{\operatorname{arctanh}(\cos(c+bx))\cos(2(a-c))}{b} - \frac{\cos^2(a-c)\cot(c+bx)\csc(c+bx)}{2b} + \frac{\csc(c+bx)\sin(2(a-c))}{b}$$

output
```
-1/2*arctanh(cos(b*x+c))*cos(a-c)^2/b+arctanh(cos(b*x+c))*cos(2*a-2*c)/b-1
/2*cos(a-c)^2*cot(b*x+c)*csc(b*x+c)/b+csc(b*x+c)*sin(2*a-2*c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 247 vs. 2(87) = 174.

Time = 2.81 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.84

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx$$
$$= \frac{\left(-\cos\left(2a-2c-\frac{bx}{2}\right)+\cos\left(2a-2c+\frac{bx}{2}\right)\right)\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{bx}{2}\right)}{4b}$$
$$+\frac{(-1-\cos(2a-2c))\csc^2\left(\frac{c}{2}+\frac{bx}{2}\right)}{16b}+\frac{(-1+3\cos(2a-2c))\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{4b}$$
$$+\frac{(1-3\cos(2a-2c))\log\left(\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{4b}$$
$$+\frac{\left(\cos\left(2a-2c-\frac{bx}{2}\right)-\cos\left(2a-2c+\frac{bx}{2}\right)\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{bx}{2}\right)}{4b}$$
$$+\frac{(1+\cos(2a-2c))\sec^2\left(\frac{c}{2}+\frac{bx}{2}\right)}{16b}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c + b*x]^3,x]
```

output
```
((-Cos[2*a - 2*c - (b*x)/2] + Cos[2*a - 2*c + (b*x)/2])*Csc[c/2]*Csc[c/2 +
 (b*x)/2])/(4*b) + ((-1 - Cos[2*a - 2*c])*Csc[c/2 + (b*x)/2]^2)/(16*b) + (
(-1 + 3*Cos[2*a - 2*c])*Log[Cos[c/2 + (b*x)/2]])/(4*b) + ((1 - 3*Cos[2*a -
 2*c])*Log[Sin[c/2 + (b*x)/2]])/(4*b) + ((Cos[2*a - 2*c - (b*x)/2] - Cos[2
*a - 2*c + (b*x)/2])*Sec[c/2]*Sec[c/2 + (b*x)/2])/(4*b) + ((1 + Cos[2*a -
2*c])*Sec[c/2 + (b*x)/2]^2)/(16*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^3(bx+c)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \cos^2(a+bx)\csc^3(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Csc[c + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.79

method	result
risch	$-\frac{-5\,\mathrm{e}^{i(3bx+6a+c)}-2\,\mathrm{e}^{i(3bx+4a+3c)}+3\,\mathrm{e}^{i(3bx+2a+5c)}+3\,\mathrm{e}^{i(bx+6a-c)}-2\,\mathrm{e}^{i(bx+4a+c)}-5\,\mathrm{e}^{i(bx+2a+3c)}}{4\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{2}b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}-\mathrm{e}^{i(a-c)}\right)}{4b}-$
default	Expression too large to display

input
```
int(cos(b*x+a)^2*csc(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^2/b*(-5*exp(I*(3*b*x+6*a+c))-2*exp(I
*(3*b*x+4*a+3*c))+3*exp(I*(3*b*x+2*a+5*c))+3*exp(I*(b*x+6*a-c))-2*exp(I*(b
*x+4*a+c))-5*exp(I*(b*x+2*a+3*c)))+1/4/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))-3
/4/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))*cos(2*a-2*c)-1/4/b*ln(exp(I*(b*x+a))+
exp(I*(a-c)))+3/4/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))*cos(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.68

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx$$
$$=\frac{2\,\cos\left(bx+c\right)\cos\left(-a+c\right)^{2}+8\,\cos\left(-a+c\right)\sin\left(bx+c\right)\sin\left(-a+c\right)+\left(\left(3\,\cos\left(-a+c\right)^{2}-2\right)\cos\left(bx+\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(2*cos(b*x + c)*cos(-a + c)^2 + 8*cos(-a + c)*sin(b*x + c)*sin(-a + c)
 + ((3*cos(-a + c)^2 - 2)*cos(b*x + c)^2 - 3*cos(-a + c)^2 + 2)*log(1/2*co
s(b*x + c) + 1/2) - ((3*cos(-a + c)^2 - 2)*cos(b*x + c)^2 - 3*cos(-a + c)^
2 + 2)*log(-1/2*cos(b*x + c) + 1/2))/(b*cos(b*x + c)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*csc(b*x+c)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(83) = 166$.

Time = 0.10 (sec) , antiderivative size = 1603, normalized size of antiderivative = 18.43

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^3,x, algorithm="maxima")
```

output
```
1/8*(2*(5*cos(3*b*x + 4*a + 2*c) + 2*cos(3*b*x + 2*a + 4*c) - 3*cos(3*b*x
+ 6*c) - 3*cos(b*x + 4*a) + 2*cos(b*x + 2*a + 2*c) + 5*cos(b*x + 4*c))*cos
(4*b*x + 2*a + 5*c) - 10*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + c))*cos(3*b
*x + 4*a + 2*c) - 4*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + c))*cos(3*b*x +
2*a + 4*c) + 6*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + c))*cos(3*b*x + 6*c)
+ 4*(3*cos(b*x + 4*a) - 2*cos(b*x + 2*a + 2*c) - 5*cos(b*x + 4*c))*cos(2*b
*x + 2*a + 3*c) - 6*cos(b*x + 4*a)*cos(2*a + c) + 4*cos(b*x + 2*a + 2*c)*c
os(2*a + c) + 10*cos(b*x + 4*c)*cos(2*a + c) + ((3*cos(-2*a + 2*c) - 1)*co
s(4*b*x + 2*a + 5*c)^2 + 4*(3*cos(-2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c)^
2 + (3*cos(-2*a + 2*c) - 1)*sin(4*b*x + 2*a + 5*c)^2 + 4*(3*cos(-2*a + 2*c
) - 1)*sin(2*b*x + 2*a + 3*c)^2 - 2*(2*(3*cos(-2*a + 2*c) - 1)*cos(2*b*x +
 2*a + 3*c) - 3*cos(2*a + c)*cos(-2*a + 2*c) + cos(2*a + c))*cos(4*b*x + 2
*a + 5*c) - 4*(3*cos(2*a + c)*cos(-2*a + 2*c) - cos(2*a + c))*cos(2*b*x +
2*a + 3*c) - cos(2*a + c)^2 + 3*(cos(2*a + c)^2 + sin(2*a + c)^2)*cos(-2*a
 + 2*c) - 2*(2*(3*cos(-2*a + 2*c) - 1)*sin(2*b*x + 2*a + 3*c) - 3*cos(-2*a
 + 2*c)*sin(2*a + c) + sin(2*a + c))*sin(4*b*x + 2*a + 5*c) - 4*(3*cos(-2*
a + 2*c)*sin(2*a + c) - sin(2*a + c))*sin(2*b*x + 2*a + 3*c) - sin(2*a + c
)^2)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*
x)*sin(c) + sin(c)^2) - ((3*cos(-2*a + 2*c) - 1)*cos(4*b*x + 2*a + 5*c)^2
+ 4*(3*cos(-2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c)^2 + (3*cos(-2*a + 2*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5628 vs. $2(83) = 166$.

Time = 0.44 (sec) , antiderivative size = 5628, normalized size of antiderivative = 64.69

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/8*(4*(tan(1/2*a)^4*tan(1/2*c)^4 - 10*tan(1/2*a)^4*tan(1/2*c)^2 + 24*tan
(1/2*a)^3*tan(1/2*c)^3 - 10*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 24*
tan(1/2*a)^3*tan(1/2*c) + 52*tan(1/2*a)^2*tan(1/2*c)^2 - 24*tan(1/2*a)*tan
(1/2*c)^3 + tan(1/2*c)^4 - 10*tan(1/2*a)^2 + 24*tan(1/2*a)*tan(1/2*c) - 10
*tan(1/2*c)^2 + 1)*log(abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)
- 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*
tan(1/2*a) - 2*tan(1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c) + 4*tan(1/
2*a)*tan(1/2*c) - 2*tan(1/2*c)^2)/abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*
tan(1/2*c) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*a)
^2*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) - 2*tan(1/2*b*x + 1/2*
a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) + 2))/(tan(1/2*a)^2*tan(1/2*c)^2 +
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1)^2 + (2*tan(1/2*b*x + 1/2*a)^3*tan(1/2*a)
^10*tan(1/2*c)^9 - 2*tan(1/2*b*x + 1/2*a)^3*tan(1/2*a)^9*tan(1/2*c)^10 + t
an(1/2*b*x + 1/2*a)^2*tan(1/2*a)^10*tan(1/2*c)^10 + 12*tan(1/2*b*x + 1/2*a
)^3*tan(1/2*a)^10*tan(1/2*c)^7 - 18*tan(1/2*b*x + 1/2*a)^3*tan(1/2*a)^9*ta
n(1/2*c)^8 + 9*tan(1/2*b*x + 1/2*a)^2*tan(1/2*a)^10*tan(1/2*c)^8 + 18*tan(
1/2*b*x + 1/2*a)^3*tan(1/2*a)^8*tan(1/2*c)^9 - 8*tan(1/2*b*x + 1/2*a)^2*ta
n(1/2*a)^9*tan(1/2*c)^9 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^10*tan(1/2*c)^
9 - 12*tan(1/2*b*x + 1/2*a)^3*tan(1/2*a)^7*tan(1/2*c)^10 + 9*tan(1/2*b*x +
 1/2*a)^2*tan(1/2*a)^8*tan(1/2*c)^10 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a...
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)^2/sin(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx = \int \cos(bx+a)^2\csc(bx+c)^3\,dx$$

input
```
int(cos(b*x+a)^2*csc(b*x+c)^3,x)
```

output
```
int(cos(a + b*x)**2*csc(b*x + c)**3,x)
```

3.181 $\int \cos(a+bx)\csc^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = -\frac{\cos(a-c)\csc^2(c+bx)}{2b} + \frac{\cot(c+bx)\sin(a-c)}{b}$$

output
```
-1/2*cos(a-c)*csc(b*x+c)^2/b+cot(b*x+c)*sin(a-c)/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = -\frac{\csc(c)\csc^2(c+bx)(\sin(a)-\cos(c+2bx)\sin(a-c))}{2b}$$

input
```
Integrate[Cos[a + b*x]*Csc[c + b*x]^3,x]
```

output
```
-1/2*(Csc[c]*Csc[c + b*x]^2*(Sin[a] - Cos[c + 2*b*x]*Sin[a - c]))/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5092, 3042, 25, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^3(bx+c)\,dx$$

$$\Big\downarrow \text{5092}$$

$$\cos(a-c)\int \cot(c+bx)\csc^2(c+bx)dx-\sin(a-c)\int \csc^2(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^2\tan\left(c+bx-\frac{\pi}{2}\right)dx-\sin(a-c)\int \csc(c+bx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\sin(a-c)\int \csc(c+bx)^2dx-\cos(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(a-c)\int \csc(c+bx)d\csc(c+bx)}{b}-\sin(a-c)\int \csc(c+bx)^2dx$$

$$\Big\downarrow \text{15}$$

$$-\sin(a-c)\int \csc(c+bx)^2dx-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sin(a-c)\int 1d\cot(c+bx)}{b}-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(a-c)\cot(bx+c)}{b}-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

input
```
Int[Cos[a + b*x]*Csc[c + b*x]^3,x]
```

output
```
-1/2*(Cos[a - c]*Csc[c + b*x]^2)/b + (Cot[c + b*x]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5092
```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Simp[Cos[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] - Simp[Sin[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$-\frac{\sec\left(\frac{bx}{2}+\frac{c}{2}\right)^{2} \csc\left(\frac{bx}{2}+\frac{c}{2}\right)^{2} \cos(2bx+a+c)}{8b}$	36
default	$-\frac{1}{2b(\cos(a)\cos(c)+\sin(a)\sin(c))(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))^{2}}$	55
risch	$-\frac{-2\,e^{i(2bx+5a+c)}+e^{i(5a-c)}-e^{i(3a+c)}}{\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^{2} b}$	64

input
```
int(cos(b*x+a)*csc(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/b*sec(1/2*b*x+1/2*c)^2*csc(1/2*b*x+1/2*c)^2*cos(2*b*x+a+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \frac{2\cos(bx+c)\sin(bx+c)\sin(-a+c)+\cos(-a+c)}{2\left(b\cos(bx+c)^2-b\right)}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + c)*sin(b*x + c)*sin(-a + c) + cos(-a + c))/(b*cos(b*x + c
)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(36) = 72.

Time = 0.04 (sec) , antiderivative size = 395, normalized size of antiderivative = 10.39

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \frac{(2\cos(2bx+2a+2c)-\cos(2a)+\cos(2c))\cos(4bx+a+5c)-2(2\cos(2bx+2a+2c)-\cos(2a}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2-4b\cos(2}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^3,x, algorithm="maxima")
```

output
```
((2*cos(2*b*x + 2*a + 2*c) - cos(2*a) + cos(2*c))*cos(4*b*x + a + 5*c) - 2
*(2*cos(2*b*x + 2*a + 2*c) - cos(2*a) + cos(2*c))*cos(2*b*x + a + 3*c) - (
cos(2*a) - cos(2*c))*cos(a + c) + 2*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (2
*sin(2*b*x + 2*a + 2*c) - sin(2*a) + sin(2*c))*sin(4*b*x + a + 5*c) - 2*(2
*sin(2*b*x + 2*a + 2*c) - sin(2*a) + sin(2*c))*sin(2*b*x + a + 3*c) - (sin
(2*a) - sin(2*c))*sin(a + c) + 2*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos
(4*b*x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 - 4*b*cos(2*b*x + a + 3*c
)*cos(a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x +
 a + 3*c)^2 - 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(2*
b*cos(2*b*x + a + 3*c) - b*cos(a + c))*cos(4*b*x + a + 5*c) - 2*(2*b*sin(2
*b*x + a + 3*c) - b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(36) = 72.

Time = 0.15 (sec) , antiderivative size = 327, normalized size of antiderivative = 8.61

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = -\frac{\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^2+9\,\mathrm{t}}{2\left(\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2+4\tan(bx+a)\tan\left(\frac{1}{2}a\right)\tan\left(\right.\right.}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/2*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*
a)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c
)^4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*
c)^2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan
(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2
+ 1)/((tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2
+ 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c) - tan(b
*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^2 + tan(b*x + a) - 2*tan(1/
2*a) + 2*tan(1/2*c))^2*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1
/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/sin(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \frac{-\cos(bx+c)\cos(bx+a)+\sin(bx+c)\sin(bx+a)}{2\sin(bx+c)^2 b}$$

input
```
int(cos(b*x+a)*csc(b*x+c)^3,x)
```

output
```
( - cos(b*x + c)*cos(a + b*x) + sin(b*x + c)*sin(a + b*x))/(2*sin(b*x + c)
**2*b)
```

3.182 $\int \csc^3(c+bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 1

$$\int \csc^3(c+bx)\sec(a+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.64 (sec) , antiderivative size = 165, normalized size of antiderivative = 165.00

$$\int \csc^3(c+bx)\sec(a+bx)\,dx = \frac{\csc\left(\frac{c}{2}\right)\csc^2(c+bx)\sec^3(a-c)\sec\left(\frac{c}{2}\right)(2\sin(2a-3c)+(-2-4\log(\cos(a+bx))+4\log(\sin(c+bx)))\,\mathrm{s}}{}$$

input
```
Integrate[Csc[c + b*x]^3*Sec[a + b*x],x]
```

output
```
(Csc[c/2]*Csc[c + b*x]^2*Sec[a - c]^3*Sec[c/2]*(2*Sin[2*a - 3*c] + (-2 - 4
*Log[Cos[a + b*x]] + 4*Log[Sin[c + b*x]])*Sin[c] - Sin[2*a - 3*c - 2*b*x]
- Sin[2*a - c + 2*b*x] - 2*Log[Cos[a + b*x]]*Sin[c + 2*b*x] + 2*Log[Sin[c
+ b*x]]*Sin[c + 2*b*x] + 2*Log[Cos[a + b*x]]*Sin[3*c + 2*b*x] - 2*Log[Sin[
c + b*x]]*Sin[3*c + 2*b*x]))/(16*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\csc^3(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sec(a+bx)\csc^3(bx+c)dx$$

input
```
Int[Csc[c + b*x]^3*Sec[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 2.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 207.00

method	result
default	$-\frac{-2\cos(a)\sin(c)+2\sin(a)\cos(c)}{(\cos(a)\cos(c)+\sin(a)\sin(c))^3(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))}-\frac{\cos(a)^2\cos(}{2(\cos(a)\cos(c)+\sin(a)\sin(c))^3(\tan}$
risch	$\frac{8\,e^{i(2bx+5a+5c)}+4\,e^{i(7a+c)}-4\,e^{i(5a+3c)}}{(-e^{2i(bx+a+c)}+e^{2ia})^2(e^{2ia}+e^{2ic})^2b}+\frac{8\ln(e^{2i(bx+a)}-e^{2i(a-c)})e^{3i(a+c)}}{(e^{6ia}+3\,e^{2i(2a+c)}+3\,e^{2i(a+2c)}+e^{6ic})b}-\frac{8\ln(e^{2i(bx+a)}+1)e^{3i(a+c)}}{(e^{6ia}+3\,e^{2i(2a+c)}+3\,e^{2i(a+2c)}+e^{6ic})b}$

input
```
int(csc(b*x+c)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-(-2*cos(a)*sin(c)+2*sin(a)*cos(c))/(cos(a)*cos(c)+sin(a)*sin(c))^3/(
tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)-sin(a)*cos(c)+cos(a)*sin
(c))-1/2*(cos(a)^2*cos(c)^2+sin(c)^2*cos(a)^2+cos(c)^2*sin(a)^2+sin(a)^2*s
in(c)^2)/(cos(a)*cos(c)+sin(a)*sin(c))^3/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x
+a)*sin(a)*sin(c)-sin(a)*cos(c)+cos(a)*sin(c))^2+1/(cos(a)*cos(c)+sin(a)*s
in(c))^3*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)-sin(a)*cos(c
)+cos(a)*sin(c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 189.00

$$\int \csc^3(c+bx)\sec(a+bx)\,dx =$$

$$-\frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)-\cos(-a+c)^2-\left(\cos(bx+c)^2-1\right)\log\left(-\frac{1}{4}\cos\right.}{2\left(b\cos(bx+c)^2\right.}$$

input
```
integrate(csc(b*x+c)^3*sec(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) - cos(-a + c)^2
- (cos(b*x + c)^2 - 1)*log(-1/4*cos(b*x + c)^2 + 1/4) + (cos(b*x + c)^2 -
1)*log(4*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) + (2*cos(-a
+ c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)/(cos(-a + c)^2 + 2*cos(-a
+ c) + 1)))/(b*cos(b*x + c)^2*cos(-a + c)^3 - b*cos(-a + c)^3)
```

Sympy [F]

$$\int \csc^3(c+bx)\sec(a+bx)\,dx = \int \csc^3(bx+c)\sec(a+bx)\,dx$$

input
```
integrate(csc(b*x+c)**3*sec(b*x+a),x)
```

output
```
Integral(csc(b*x + c)**3*sec(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.38 (sec) , antiderivative size = 85585, normalized size of antiderivative = 85585.00

$$\int \csc^3(c+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^3*sec(b*x+a),x, algorithm="maxima")
```

output
```
4*(9*((cos(4*a) + cos(4*c))*cos(3*a + c) + 2*cos(3*a + c)*cos(2*a + 2*c) +
 (sin(4*a) + sin(4*c))*sin(3*a + c) + 2*sin(3*a + c)*sin(2*a + 2*c))*cos(4
*a + 2*c)^2 + 9*((cos(4*a) + cos(4*c))*cos(3*a + c) + 2*cos(3*a + c)*cos(2
*a + 2*c) - (cos(4*a) + 2*cos(2*a + 2*c) + cos(4*c))*cos(a + 3*c) + (sin(4
*a) + sin(4*c))*sin(3*a + c) + 2*sin(3*a + c)*sin(2*a + 2*c) - (sin(4*a) +
 2*sin(2*a + 2*c) + sin(4*c))*sin(a + 3*c))*cos(2*a + 4*c)^2 + 2*(cos(6*a)
^2 + 2*cos(6*a)*cos(6*c) + cos(6*c)^2 + sin(6*a)^2 + 2*sin(6*a)*sin(6*c) +
 sin(6*c)^2)*cos(3*a + c)*cos(2*a + 2*c) + 9*((cos(4*a) + cos(4*c))*cos(3*
a + c) + 2*cos(3*a + c)*cos(2*a + 2*c) + (sin(4*a) + sin(4*c))*sin(3*a + c
) + 2*sin(3*a + c)*sin(2*a + 2*c))*sin(4*a + 2*c)^2 + 9*((cos(4*a) + cos(4
*c))*cos(3*a + c) + 2*cos(3*a + c)*cos(2*a + 2*c) - (cos(4*a) + 2*cos(2*a
+ 2*c) + cos(4*c))*cos(a + 3*c) + (sin(4*a) + sin(4*c))*sin(3*a + c) + 2*s
in(3*a + c)*sin(2*a + 2*c) - (sin(4*a) + 2*sin(2*a + 2*c) + sin(4*c))*sin(
a + 3*c))*sin(2*a + 4*c)^2 + 2*(cos(6*a)^2 + 2*cos(6*a)*cos(6*c) + cos(6*c
)^2 + sin(6*a)^2 + 2*sin(6*a)*sin(6*c) + sin(6*c)^2)*sin(3*a + c)*sin(2*a
+ 2*c) - 2*(((sin(6*a) + 3*sin(4*a + 2*c) + sin(6*c))*cos(3*a + 3*c) - (co
s(6*a) + 3*cos(4*a + 2*c) + cos(6*c))*sin(3*a + 3*c) - 3*cos(2*a + 4*c)*si
n(3*a + 3*c) + 3*cos(3*a + 3*c)*sin(2*a + 4*c))*cos(4*b*x + 4*a + 4*c)^2 +
 4*((sin(6*a) + 3*sin(4*a + 2*c) + sin(6*c))*cos(3*a + 3*c) - (cos(6*a) +
3*cos(4*a + 2*c) + cos(6*c))*sin(3*a + 3*c) - 3*cos(2*a + 4*c)*sin(3*a ...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.21 (sec) , antiderivative size = 2551, normalized size of antiderivative = 2551.00

$$\int \csc^3(c + bx)\sec(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^3*sec(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(2*(tan(1/2*a)^8*tan(1/2*c)^7 - tan(1/2*a)^7*tan(1/2*c)^8 + 3*tan(1/2
*a)^8*tan(1/2*c)^5 - 2*tan(1/2*a)^7*tan(1/2*c)^6 + 2*tan(1/2*a)^6*tan(1/2*
c)^7 - 3*tan(1/2*a)^5*tan(1/2*c)^8 + 3*tan(1/2*a)^8*tan(1/2*c)^3 + 6*tan(1
/2*a)^6*tan(1/2*c)^5 - 6*tan(1/2*a)^5*tan(1/2*c)^6 - 3*tan(1/2*a)^3*tan(1/
2*c)^8 + tan(1/2*a)^8*tan(1/2*c) + 2*tan(1/2*a)^7*tan(1/2*c)^2 + 6*tan(1/2
*a)^6*tan(1/2*c)^3 - 6*tan(1/2*a)^3*tan(1/2*c)^6 - 2*tan(1/2*a)^2*tan(1/2*
c)^7 - tan(1/2*a)*tan(1/2*c)^8 + tan(1/2*a)^7 + 2*tan(1/2*a)^6*tan(1/2*c)
+ 6*tan(1/2*a)^5*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^5 - 2*tan(1/2*a)
*tan(1/2*c)^6 - tan(1/2*c)^7 + 3*tan(1/2*a)^5 + 6*tan(1/2*a)^3*tan(1/2*c)^
2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 - 3*tan(1/2*c)^5 + 3*tan(1/2*a)^3 - 2*tan(
1/2*a)^2*tan(1/2*c) + 2*tan(1/2*a)*tan(1/2*c)^2 - 3*tan(1/2*c)^3 + tan(1/2
*a) - tan(1/2*c))*log(abs(2*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b
*x + c)*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x +
c)*tan(1/2*a) + tan(1/2*a)^2 - 2*tan(b*x + c)*tan(1/2*c) - 4*tan(1/2*a)*ta
n(1/2*c) + tan(1/2*c)^2 - 1))/(tan(1/2*a)^8*tan(1/2*c)^7 - tan(1/2*a)^7*ta
n(1/2*c)^8 - 3*tan(1/2*a)^8*tan(1/2*c)^5 + 16*tan(1/2*a)^7*tan(1/2*c)^6 -
16*tan(1/2*a)^6*tan(1/2*c)^7 + 3*tan(1/2*a)^5*tan(1/2*c)^8 + 3*tan(1/2*a)^
8*tan(1/2*c)^3 - 30*tan(1/2*a)^7*tan(1/2*c)^4 + 96*tan(1/2*a)^6*tan(1/2*c)
^5 - 96*tan(1/2*a)^5*tan(1/2*c)^6 + 30*tan(1/2*a)^4*tan(1/2*c)^7 - 3*tan(1
/2*a)^3*tan(1/2*c)^8 - tan(1/2*a)^8*tan(1/2*c) + 16*tan(1/2*a)^7*tan(1/...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sec(a+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)*sin(c + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^3(c+bx)\sec(a+bx)\,dx$$
$$= \frac{\cos(bx+c) + 2\left(\int \frac{1}{\sin(bx+c)^3}dx\right)\sin(bx+c)^2 b + 2\left(\int \frac{1}{\cos(bx+a)\sin(bx+c)^3}dx\right)\sin(bx+c)^2 b - \log\left(\tan\left(\frac{bx}{2}\right.\right.}{2\sin(bx+c)^2 b}$$

input
```
int(csc(b*x+c)^3*sec(b*x+a),x)
```

output
```
(cos(b*x + c) + 2*int(1/sin(b*x + c)**3,x)*sin(b*x + c)**2*b + 2*int(1/(co
s(a + b*x)*sin(b*x + c)**3),x)*sin(b*x + c)**2*b - log(tan((b*x + c)/2))*s
in(b*x + c)**2)/(2*sin(b*x + c)**2*b)
```

3.183 $\int \csc^3(c+bx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \csc^3(c+bx)\sec^2(a+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 6.61 (sec) , antiderivative size = 381, normalized size of antiderivative = 381.00

$$
\begin{aligned}
&\int \csc^3(c+bx)\sec^2(a+bx)\,dx \\
&= -\frac{\csc^2\left(\frac{c}{2}+\frac{bx}{2}\right)\sec^2(a-c)}{8b} \\
&\quad+\frac{\left(\cos\left(a-c-\frac{bx}{2}\right)-\cos\left(a-c+\frac{bx}{2}\right)\right)\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{bx}{2}\right)\sec^3(a-c)}{2b} \\
&\quad+\frac{3(-3+\cos(2a-2c))\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)\right)\sec^4(a-c)}{4b} \\
&\quad-\frac{3(-3+\cos(2a-2c))\log\left(\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)\sec^4(a-c)}{4b} \\
&\quad+\frac{\left(-\cos\left(a-c-\frac{bx}{2}\right)+\cos\left(a-c+\frac{bx}{2}\right)\right)\sec^3(a-c)\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{bx}{2}\right)}{2b} \\
&\quad+\frac{\sec^2(a-c)\sec^2\left(\frac{c}{2}+\frac{bx}{2}\right)}{8b}+\frac{\sec^3(a-c)\sec(a+bx)}{b} \\
&\quad-\frac{6i\arctan\left(\frac{(i\cos(a)+\sin(a))\left(\cos\left(\frac{bx}{2}\right)\sin(a)+\cos(a)\sin\left(\frac{bx}{2}\right)\right)}{\cos(a)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(a)}\right)\sin(a-c)}{\frac{3b}{8}+\frac{1}{8}b\cos(4a-4c)+\frac{1}{2}b\cos(2a-2c)}
\end{aligned}
$$

input
```
Integrate[Csc[c + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
-1/8*(Csc[c/2 + (b*x)/2]^2*Sec[a - c]^2)/b + ((Cos[a - c - (b*x)/2] - Cos[
a - c + (b*x)/2])*Csc[c/2]*Csc[c/2 + (b*x)/2]*Sec[a - c]^3)/(2*b) + (3*(-3
 + Cos[2*a - 2*c])*Log[Cos[c/2 + (b*x)/2]]*Sec[a - c]^4)/(4*b) - (3*(-3 +
Cos[2*a - 2*c])*Log[Sin[c/2 + (b*x)/2]]*Sec[a - c]^4)/(4*b) + ((-Cos[a - c
 - (b*x)/2] + Cos[a - c + (b*x)/2])*Sec[a - c]^3*Sec[c/2]*Sec[c/2 + (b*x)/
2])/(2*b) + (Sec[a - c]^2*Sec[c/2 + (b*x)/2]^2)/(8*b) + (Sec[a - c]^3*Sec[
a + b*x])/b - ((6*I)*ArcTan[((I*Cos[a] + Sin[a])*(Cos[(b*x)/2]*Sin[a] + Co
s[a]*Sin[(b*x)/2]))/(Cos[a]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[a])]*Sin[a -
 c])/((3*b)/8 + (b*Cos[4*a - 4*c])/8 + (b*Cos[2*a - 2*c])/2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\csc^3(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec^2(a+bx)\csc^3(bx+c)dx$$

input
```
Int[Csc[c + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 5.50 (sec) , antiderivative size = 934, normalized size of antiderivative = 934.00

method	result	size
risch	Expression too large to display	934
default	Expression too large to display	1072

input
```
int(csc(b*x+c)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/(exp(2*I*(b*x+a))+1)/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^2/(exp(2*I*a)+exp(
2*I*c))^3/b*(-3*exp(5*I*(b*x+2*a+c))+9*exp(I*(5*b*x+8*a+7*c))+5*exp(I*(3*b
*x+10*a+3*c))-14*exp(I*(3*b*x+8*a+5*c))+5*exp(I*(3*b*x+6*a+7*c))+9*exp(I*(
b*x+8*a+3*c))-3*exp(I*(b*x+6*a+5*c)))-24*I*ln(exp(I*(b*x+a))+I)/(exp(8*I*a
)+4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))+4*exp(2*I*(a+3*c))+exp(8*I*c))/b*exp
(I*(5*a+3*c))+24*I*ln(exp(I*(b*x+a))+I)/(exp(8*I*a)+4*exp(2*I*(3*a+c))+6*e
xp(4*I*(a+c))+4*exp(2*I*(a+3*c))+exp(8*I*c))/b*exp(I*(3*a+5*c))+24*I*ln(ex
p(I*(b*x+a))-I)/(exp(8*I*a)+4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))+4*exp(2*I*
(a+3*c))+exp(8*I*c))/b*exp(I*(5*a+3*c))-24*I*ln(exp(I*(b*x+a))-I)/(exp(8*I
*a)+4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))+4*exp(2*I*(a+3*c))+exp(8*I*c))/b*e
xp(I*(3*a+5*c))-6*ln(exp(I*(b*x+a))-exp(I*(a-c)))/(exp(8*I*a)+4*exp(2*I*(3
*a+c))+6*exp(4*I*(a+c))+4*exp(2*I*(a+3*c))+exp(8*I*c))/b*exp(2*I*(3*a+c))+
36*ln(exp(I*(b*x+a))-exp(I*(a-c)))/(exp(8*I*a)+4*exp(2*I*(3*a+c))+6*exp(4*
I*(a+c))+4*exp(2*I*(a+3*c))+exp(8*I*c))/b*exp(4*I*(a+c))-6*ln(exp(I*(b*x+a
))-exp(I*(a-c)))/(exp(8*I*a)+4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))+4*exp(2*I
*(a+3*c))+exp(8*I*c))/b*exp(2*I*(a+3*c))+6*ln(exp(I*(b*x+a))+exp(I*(a-c)))
/(exp(8*I*a)+4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))+4*exp(2*I*(a+3*c))+exp(8*
I*c))/b*exp(2*I*(3*a+c))-36*ln(exp(I*(b*x+a))+exp(I*(a-c)))/(exp(8*I*a)+4*
exp(2*I*(3*a+c))+6*exp(4*I*(a+c))+4*exp(2*I*(a+3*c))+exp(8*I*c))/b*exp(4*I
*(a+c))+6*ln(exp(I*(b*x+a))+exp(I*(a-c)))/(exp(8*I*a)+4*exp(2*I*(3*a+c)...
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.14 (sec) , antiderivative size = 617, normalized size of antiderivative = 617.00

$$\int \csc^3(c+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(6*cos(b*x + c)*cos(-a + c)^2*sin(b*x + c)*sin(-a + c) + 6*(cos(-a +
c)^3 - 2*cos(-a + c))*cos(b*x + c)^2 - 8*cos(-a + c)^3 - 6*(((cos(-a + c)^
2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)*sin(b*x + c) - (cos(b*x + c)^3*
cos(-a + c) - cos(b*x + c)*cos(-a + c))*sin(-a + c))*log(2*(cos(-a + c)*si
n(b*x + c) - cos(b*x + c)*sin(-a + c) + 1)/(cos(-a + c) + 1)) + 6*(((cos(-
a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)*sin(b*x + c) - (cos(b*x
+ c)^3*cos(-a + c) - cos(b*x + c)*cos(-a + c))*sin(-a + c))*log(-2*(cos(-a
 + c)*sin(b*x + c) - cos(b*x + c)*sin(-a + c) - 1)/(cos(-a + c) + 1)) - 3*
((cos(-a + c)^3 - 2*cos(-a + c))*cos(b*x + c)^3 + ((cos(-a + c)^2 - 2)*cos
(b*x + c)^2 - cos(-a + c)^2 + 2)*sin(b*x + c)*sin(-a + c) - (cos(-a + c)^3
 - 2*cos(-a + c))*cos(b*x + c))*log(1/2*cos(b*x + c) + 1/2) + 3*((cos(-a +
 c)^3 - 2*cos(-a + c))*cos(b*x + c)^3 + ((cos(-a + c)^2 - 2)*cos(b*x + c)^
2 - cos(-a + c)^2 + 2)*sin(b*x + c)*sin(-a + c) - (cos(-a + c)^3 - 2*cos(-
a + c))*cos(b*x + c))*log(-1/2*cos(b*x + c) + 1/2) + 12*cos(-a + c))/(b*co
s(b*x + c)^3*cos(-a + c)^5 - b*cos(b*x + c)*cos(-a + c)^5 + (b*cos(b*x + c
)^2*cos(-a + c)^4 - b*cos(-a + c)^4)*sin(b*x + c)*sin(-a + c))
```

Sympy [F]

$$\int \csc^3(c+bx)\sec^2(a+bx)\,dx = \int \csc^3(bx+c)\sec^2(a+bx)\,dx$$

input
```
integrate(csc(b*x+c)**3*sec(b*x+a)**2,x)
```

output
```
Integral(csc(b*x + c)**3*sec(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 22.86 (sec) , antiderivative size = 643576, normalized size of antiderivative = 643576.00

$$\int \csc^3(c+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
(24*(((cos(8*a) + 4*cos(6*a + 2*c) + cos(8*c))*cos(5*a + 3*c) + 6*cos(5*a
+ 3*c)*cos(4*a + 4*c) - (cos(8*a) + 4*cos(6*a + 2*c) + 6*cos(4*a + 4*c) +
cos(8*c))*cos(3*a + 5*c) + 4*(cos(5*a + 3*c) - cos(3*a + 5*c))*cos(2*a + 6
*c) + (sin(8*a) + 4*sin(6*a + 2*c) + sin(8*c))*sin(5*a + 3*c) + 6*sin(5*a
+ 3*c)*sin(4*a + 4*c) - (sin(8*a) + 4*sin(6*a + 2*c) + 6*sin(4*a + 4*c) +
sin(8*c))*sin(3*a + 5*c) + 4*(sin(5*a + 3*c) - sin(3*a + 5*c))*sin(2*a + 6
*c))*cos(6*b*x + 8*a + 4*c)^2 + 9*((cos(8*a) + 4*cos(6*a + 2*c) + cos(8*c)
)*cos(5*a + 3*c) + 6*cos(5*a + 3*c)*cos(4*a + 4*c) - (cos(8*a) + 4*cos(6*a
 + 2*c) + 6*cos(4*a + 4*c) + cos(8*c))*cos(3*a + 5*c) + 4*(cos(5*a + 3*c)
- cos(3*a + 5*c))*cos(2*a + 6*c) + (sin(8*a) + 4*sin(6*a + 2*c) + sin(8*c)
)*sin(5*a + 3*c) + 6*sin(5*a + 3*c)*sin(4*a + 4*c) - (sin(8*a) + 4*sin(6*a
 + 2*c) + 6*sin(4*a + 4*c) + sin(8*c))*sin(3*a + 5*c) + 4*(sin(5*a + 3*c)
- sin(3*a + 5*c))*sin(2*a + 6*c))*cos(6*b*x + 6*a + 6*c)^2 + 9*((cos(8*a)
+ 4*cos(6*a + 2*c) + cos(8*c))*cos(5*a + 3*c) + 6*cos(5*a + 3*c)*cos(4*a +
 4*c) - (cos(8*a) + 4*cos(6*a + 2*c) + 6*cos(4*a + 4*c) + cos(8*c))*cos(3*
a + 5*c) + 4*(cos(5*a + 3*c) - cos(3*a + 5*c))*cos(2*a + 6*c) + (sin(8*a)
+ 4*sin(6*a + 2*c) + sin(8*c))*sin(5*a + 3*c) + 6*sin(5*a + 3*c)*sin(4*a +
 4*c) - (sin(8*a) + 4*sin(6*a + 2*c) + 6*sin(4*a + 4*c) + sin(8*c))*sin(3*
a + 5*c) + 4*(sin(5*a + 3*c) - sin(3*a + 5*c))*sin(2*a + 6*c))*cos(6*b*x +
 4*a + 8*c)^2 + ((cos(8*a) + 4*cos(6*a + 2*c) + cos(8*c))*cos(5*a + 3*c...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.77 (sec) , antiderivative size = 13498, normalized size of antiderivative = 13498.00

$$\int \csc^3(c+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
1/8*(48*(tan(1/2*a)^9*tan(1/2*c)^8 - tan(1/2*a)^8*tan(1/2*c)^9 - tan(1/2*a
)^9*tan(1/2*c)^7 + 2*tan(1/2*a)^8*tan(1/2*c)^8 - tan(1/2*a)^7*tan(1/2*c)^9
 + 3*tan(1/2*a)^9*tan(1/2*c)^6 - tan(1/2*a)^8*tan(1/2*c)^7 + tan(1/2*a)^7*
tan(1/2*c)^8 - 3*tan(1/2*a)^6*tan(1/2*c)^9 - 3*tan(1/2*a)^9*tan(1/2*c)^5 +
 5*tan(1/2*a)^8*tan(1/2*c)^6 - 4*tan(1/2*a)^7*tan(1/2*c)^7 + 5*tan(1/2*a)^
6*tan(1/2*c)^8 - 3*tan(1/2*a)^5*tan(1/2*c)^9 + 3*tan(1/2*a)^9*tan(1/2*c)^4
 + 3*tan(1/2*a)^8*tan(1/2*c)^5 + 4*tan(1/2*a)^7*tan(1/2*c)^6 - 4*tan(1/2*a
)^6*tan(1/2*c)^7 - 3*tan(1/2*a)^5*tan(1/2*c)^8 - 3*tan(1/2*a)^4*tan(1/2*c)
^9 - 3*tan(1/2*a)^9*tan(1/2*c)^3 + 3*tan(1/2*a)^8*tan(1/2*c)^4 - 6*tan(1/2
*a)^7*tan(1/2*c)^5 + 12*tan(1/2*a)^6*tan(1/2*c)^6 - 6*tan(1/2*a)^5*tan(1/2
*c)^7 + 3*tan(1/2*a)^4*tan(1/2*c)^8 - 3*tan(1/2*a)^3*tan(1/2*c)^9 + tan(1/
2*a)^9*tan(1/2*c)^2 + 5*tan(1/2*a)^8*tan(1/2*c)^3 + 6*tan(1/2*a)^7*tan(1/2
*c)^4 + 6*tan(1/2*a)^6*tan(1/2*c)^5 - 6*tan(1/2*a)^5*tan(1/2*c)^6 - 6*tan(
1/2*a)^4*tan(1/2*c)^7 - 5*tan(1/2*a)^3*tan(1/2*c)^8 - tan(1/2*a)^2*tan(1/2
*c)^9 - tan(1/2*a)^9*tan(1/2*c) - tan(1/2*a)^8*tan(1/2*c)^2 - 4*tan(1/2*a)
^7*tan(1/2*c)^3 + 6*tan(1/2*a)^6*tan(1/2*c)^4 + 6*tan(1/2*a)^4*tan(1/2*c)^
6 - 4*tan(1/2*a)^3*tan(1/2*c)^7 - tan(1/2*a)^2*tan(1/2*c)^8 - tan(1/2*a)*t
an(1/2*c)^9 + 2*tan(1/2*a)^8*tan(1/2*c) + 4*tan(1/2*a)^7*tan(1/2*c)^2 + 12
*tan(1/2*a)^6*tan(1/2*c)^3 - 12*tan(1/2*a)^3*tan(1/2*c)^6 - 4*tan(1/2*a)^2
*tan(1/2*c)^7 - 2*tan(1/2*a)*tan(1/2*c)^8 - tan(1/2*a)^8 - tan(1/2*a)^7...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sec^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)^2*sin(c + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^3(c+bx)\sec^2(a+bx)\,dx=\int \csc\left(bx+c\right)^3\sec\left(bx+a\right)^2dx$$

input
```
int(csc(b*x+c)^3*sec(b*x+a)^2,x)
```

output
```
int(csc(b*x + c)**3*sec(a + b*x)**2,x)
```

3.184 $\int \csc^3(c+bx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \csc^3(c+bx)\sec^3(a+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 6.67 (sec) , antiderivative size = 1451, normalized size of antiderivative = 1451.00

$$\int \csc^3(c+bx)\sec^3(a+bx)\,dx=\text{Too large to display}$$

input
```
Integrate[Csc[c + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
((-2*I)*ArcTan[Tan[a + b*x]]*(-2 + Cos[2*a - 2*c])*Sec[a - c]^5)/b + ((2*I
)*ArcTan[Tan[c + b*x]]*(-2 + Cos[2*a - 2*c])*Sec[a - c]^5)/b + ((-2 + Cos[
2*a - 2*c])*Log[Cos[a + b*x]^2]*Sec[a - c]^5)/b - ((-2 + Cos[2*a - 2*c])*L
og[Sin[c + b*x]^2]*Sec[a - c]^5)/b + (3*(Cos[a - c - b*x] - Cos[a - c + b*
x])*Csc[c/2]*Csc[c + b*x]*Sec[c/2])/(4*b*(Cos[a/2 - c/2] - Sin[a/2 - c/2])
^4*(Cos[a/2 - c/2] + Sin[a/2 - c/2])^4) - (3*(-Cos[a - c - b*x] + Cos[a -
c + b*x])*Sec[a + b*x])/(2*b*(Cos[a/2] - Sin[a/2])*(Cos[a/2] + Sin[a/2])*(
Cos[a/2 - c/2] - Sin[a/2 - c/2])^4*(Cos[a/2 - c/2] + Sin[a/2 - c/2])^4) -
Csc[c + b*x]^2/(2*b*(Cos[a/2 - c/2] - Sin[a/2 - c/2])^3*(Cos[a/2 - c/2] +
Sin[a/2 - c/2])^3) + Sec[a + b*x]^2/(2*b*(Cos[a/2 - c/2] - Sin[a/2 - c/2])
^3*(Cos[a/2 - c/2] + Sin[a/2 - c/2])^3) + x*((I*Cos[a]^2)/(Cos[a]*Cos[c] +
 Sin[a]*Sin[c])^5 + (I*Cos[c]^2)/(Cos[a]*Cos[c] + Sin[a]*Sin[c])^5 - ((2*I
)*Cos[a]^2*Cos[c]^2)/(Cos[a]*Cos[c] + Sin[a]*Sin[c])^5 - (4*Cot[c])/(Cos[a
]*Cos[c] + Sin[a]*Sin[c])^5 + (Cos[a]^2*Cot[c])/(Cos[a]*Cos[c] + Sin[a]*Si
n[c])^5 + (Cos[a]^2*Cos[c]^2*Cot[c])/(Cos[a]*Cos[c] + Sin[a]*Sin[c])^5 + (
2*Cos[a]*Sin[a])/(Cos[a]*Cos[c] + Sin[a]*Sin[c])^5 + (9*Cos[a]*Cos[c]^2*Si
n[a])/(Cos[a]*Cos[c] + Sin[a]*Sin[c])^5 - ((2*I)*Cos[a]*Cot[c]*Sin[a])/(Co
s[a]*Cos[c] + Sin[a]*Sin[c])^5 + ((2*I)*Cos[a]*Cos[c]^2*Cot[c]*Sin[a])/(Co
s[a]*Cos[c] + Sin[a]*Sin[c])^5 - (I*Sin[a]^2)/(Cos[a]*Cos[c] + Sin[a]*Sin[
c])^5 - (Cot[c]*Sin[a]^2)/(Cos[a]*Cos[c] + Sin[a]*Sin[c])^5 - (Cos[c]^2...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)\csc^3(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sec^3(a+bx)\csc^3(bx+c)dx$$

input
```
Int[Csc[c + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 8.55 (sec) , antiderivative size = 483, normalized size of antiderivative = 483.00

method	result
default	$\frac{\frac{\frac{\tan(bx+a)^2\cos(a)\cos(c)}{2}+\frac{\tan(bx+a)^2\sin(a)\sin(c)}{2}+3\tan(bx+a)\sin(a)\cos(c)-3\tan(bx+a)\cos(a)\sin(c)}{(\cos(a)\cos(c)+\sin(a)\sin(c))^4}+\frac{(2\cos(a)^2\cos(c)^2+6\cos(c)^2\sin(a)^2-8}{}}{}$
risch	$-\frac{16(2\,e^{i(6bx+13a+5c)}-8\,e^{i(6bx+11a+7c)}+2\,e^{3i(2bx+3a+3c)}-3\,e^{i(4bx+13a+3c)}+15\,e^{i(4bx+11a+5c)}-15\,e^{i(4bx+9a+7c)}+3\,e^{i(4bx+7a+9}}{(e^{2i(bx+a)}+1)^2(e^{2i(bx+a+c)}-e^{2ia})^2(e^{2ia}+e^{2i}}$

input
```
int(csc(b*x+c)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/(cos(a)*cos(c)+sin(a)*sin(c))^4*(1/2*tan(b*x+a)^2*cos(a)*cos(c)+1/2
*tan(b*x+a)^2*sin(a)*sin(c)+3*tan(b*x+a)*sin(a)*cos(c)-3*tan(b*x+a)*cos(a)
*sin(c))+(2*cos(a)^2*cos(c)^2+6*cos(c)^2*sin(a)^2-8*cos(a)*cos(c)*sin(a)*s
in(c)+6*sin(c)^2*cos(a)^2+2*sin(a)^2*sin(c)^2)/(cos(a)*cos(c)+sin(a)*sin(c
))^5*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)-sin(a)*cos(c)+co
s(a)*sin(c))-(4*cos(c)^3*sin(a)*cos(a)^2+4*cos(c)^3*sin(a)^3-4*cos(c)^2*si
n(c)*cos(a)^3-4*cos(c)^2*sin(c)*sin(a)^2*cos(a)+4*cos(c)*sin(c)^2*sin(a)*c
os(a)^2+4*cos(c)*sin(c)^2*sin(a)^3-4*sin(c)^3*cos(a)^3-4*sin(c)^3*sin(a)^2
*cos(a))/(cos(a)*cos(c)+sin(a)*sin(c))^5/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x
+a)*sin(a)*sin(c)-sin(a)*cos(c)+cos(a)*sin(c))-1/2/(cos(a)*cos(c)+sin(a)*s
in(c))^5*(sin(a)^4*cos(c)^4+2*cos(a)^2*sin(a)^2*cos(c)^4+cos(a)^4*cos(c)^4
+2*sin(a)^4*cos(c)^2*sin(c)^2+4*cos(a)^2*sin(a)^2*cos(c)^2*sin(c)^2+2*cos(
a)^4*cos(c)^2*sin(c)^2+sin(a)^4*sin(c)^4+2*cos(a)^2*sin(a)^2*sin(c)^4+cos(
a)^4*sin(c)^4)/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)-sin(a)*c
os(c)+cos(a)*sin(c))^2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.12 (sec) , antiderivative size = 702, normalized size of antiderivative = 702.00

$$\int \csc^3(c+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(24*(cos(-a + c)^4 - cos(-a + c)^2)*cos(b*x + c)^4 + 7*cos(-a + c)^4
- 2*(16*cos(-a + c)^4 - 15*cos(-a + c)^2)*cos(b*x + c)^2 + 4*(3*(2*cos(-a
+ c)^3 - cos(-a + c))*cos(b*x + c)^3 - (5*cos(-a + c)^3 - 3*cos(-a + c))*c
os(b*x + c))*sin(b*x + c)*sin(-a + c) - 6*cos(-a + c)^2 + 2*((4*cos(-a + c
)^4 - 8*cos(-a + c)^2 + 3)*cos(b*x + c)^4 + 2*cos(-a + c)^4 - (6*cos(-a +
c)^4 - 13*cos(-a + c)^2 + 6)*cos(b*x + c)^2 + 2*((2*cos(-a + c)^3 - 3*cos(
-a + c))*cos(b*x + c)^3 - (2*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c))*
sin(b*x + c)*sin(-a + c) - 5*cos(-a + c)^2 + 3)*log(-1/4*cos(b*x + c)^2 +
1/4) - 2*((4*cos(-a + c)^4 - 8*cos(-a + c)^2 + 3)*cos(b*x + c)^4 + 2*cos(-
a + c)^4 - (6*cos(-a + c)^4 - 13*cos(-a + c)^2 + 6)*cos(b*x + c)^2 + 2*((2
*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^3 - (2*cos(-a + c)^3 - 3*cos(
-a + c))*cos(b*x + c))*sin(b*x + c)*sin(-a + c) - 5*cos(-a + c)^2 + 3)*log
(4*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2
 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)/(cos(-a + c)^2 + 2*cos(-a + c) +
 1)))/(b*cos(-a + c)^7 - b*cos(-a + c)^5 + (2*b*cos(-a + c)^7 - b*cos(-a +
 c)^5)*cos(b*x + c)^4 - (3*b*cos(-a + c)^7 - 2*b*cos(-a + c)^5)*cos(b*x +
c)^2 + 2*(b*cos(b*x + c)^3*cos(-a + c)^6 - b*cos(b*x + c)*cos(-a + c)^6)*s
in(b*x + c)*sin(-a + c))
```

Sympy [F]

$$\int \csc^3(c+bx)\sec^3(a+bx)\,dx = \int \csc^3(bx+c)\sec^3(a+bx)\,dx$$

input
```
integrate(csc(b*x+c)**3*sec(b*x+a)**3,x)
```

output
```
Integral(csc(b*x + c)**3*sec(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 103.29 (sec) , antiderivative size = 2494919, normalized size of antiderivative = 2494919.00

$$\int \csc^3(c+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
16*(600*((cos(8*a) + 4*cos(6*a + 2*c) + cos(8*c))*cos(5*a + 3*c) + 6*cos(5
*a + 3*c)*cos(4*a + 4*c) + (sin(8*a) + 4*sin(6*a + 2*c) + sin(8*c))*sin(5*
a + 3*c) + 6*sin(5*a + 3*c)*sin(4*a + 4*c))*cos(6*a + 4*c)^2 + 600*((cos(8
*a) + 4*cos(6*a + 2*c) + cos(8*c))*cos(5*a + 3*c) + 6*cos(5*a + 3*c)*cos(4
*a + 4*c) - (cos(8*a) + 4*cos(6*a + 2*c) + 6*cos(4*a + 4*c) + cos(8*c))*co
s(3*a + 5*c) + 4*(cos(5*a + 3*c) - cos(3*a + 5*c))*cos(2*a + 6*c) + (sin(8
*a) + 4*sin(6*a + 2*c) + sin(8*c))*sin(5*a + 3*c) + 6*sin(5*a + 3*c)*sin(4
*a + 4*c) - (sin(8*a) + 4*sin(6*a + 2*c) + 6*sin(4*a + 4*c) + sin(8*c))*si
n(3*a + 5*c) + 4*(sin(5*a + 3*c) - sin(3*a + 5*c))*sin(2*a + 6*c))*cos(4*a
 + 6*c)^2 + 36*(cos(10*a)^2 + 10*(cos(10*a) + cos(10*c))*cos(8*a + 2*c) +
25*cos(8*a + 2*c)^2 + 2*cos(10*a)*cos(10*c) + cos(10*c)^2 + sin(10*a)^2 +
10*(sin(10*a) + sin(10*c))*sin(8*a + 2*c) + 25*sin(8*a + 2*c)^2 + 2*sin(10
*a)*sin(10*c) + sin(10*c)^2)*cos(5*a + 3*c)*cos(4*a + 4*c) + 150*((cos(8*a
) + 4*cos(6*a + 2*c) + cos(8*c))*cos(5*a + 3*c) + 6*cos(5*a + 3*c)*cos(4*a
 + 4*c) - (cos(8*a) + 4*cos(6*a + 2*c) + 6*cos(4*a + 4*c) + cos(8*c))*cos(
3*a + 5*c) + 4*(cos(5*a + 3*c) - cos(3*a + 5*c))*cos(2*a + 6*c) + (sin(8*a
) + 4*sin(6*a + 2*c) + sin(8*c))*sin(5*a + 3*c) + 6*sin(5*a + 3*c)*sin(4*a
 + 4*c) - (sin(8*a) + 4*sin(6*a + 2*c) + 6*sin(4*a + 4*c) + sin(8*c))*sin(
3*a + 5*c) + 4*(sin(5*a + 3*c) - sin(3*a + 5*c))*sin(2*a + 6*c))*cos(2*a +
 8*c)^2 + 600*((cos(8*a) + 4*cos(6*a + 2*c) + cos(8*c))*cos(5*a + 3*c) ...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.51 (sec) , antiderivative size = 13903, normalized size of antiderivative = 13903.00

$$\int \csc^3(c+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate(csc(b*x+c)^3*sec(b*x+a)^3,x, algorithm="giac")
```

output

```
-1/8*(16*(tan(1/2*a)^12*tan(1/2*c)^11 - tan(1/2*a)^11*tan(1/2*c)^12 + 13*t
an(1/2*a)^12*tan(1/2*c)^9 - 28*tan(1/2*a)^11*tan(1/2*c)^10 + 28*tan(1/2*a)
^10*tan(1/2*c)^11 - 13*tan(1/2*a)^9*tan(1/2*c)^12 + 34*tan(1/2*a)^12*tan(1
/2*c)^7 - 53*tan(1/2*a)^11*tan(1/2*c)^8 + 44*tan(1/2*a)^10*tan(1/2*c)^9 -
44*tan(1/2*a)^9*tan(1/2*c)^10 + 53*tan(1/2*a)^8*tan(1/2*c)^11 - 34*tan(1/2
*a)^7*tan(1/2*c)^12 + 34*tan(1/2*a)^12*tan(1/2*c)^5 - 8*tan(1/2*a)^10*tan(
1/2*c)^7 - 49*tan(1/2*a)^9*tan(1/2*c)^8 + 49*tan(1/2*a)^8*tan(1/2*c)^9 + 8
*tan(1/2*a)^7*tan(1/2*c)^10 - 34*tan(1/2*a)^5*tan(1/2*c)^12 + 13*tan(1/2*a
)^12*tan(1/2*c)^3 + 53*tan(1/2*a)^11*tan(1/2*c)^4 - 8*tan(1/2*a)^10*tan(1/
2*c)^5 - 118*tan(1/2*a)^8*tan(1/2*c)^7 + 118*tan(1/2*a)^7*tan(1/2*c)^8 + 8
*tan(1/2*a)^5*tan(1/2*c)^10 - 53*tan(1/2*a)^4*tan(1/2*c)^11 - 13*tan(1/2*a
)^3*tan(1/2*c)^12 + tan(1/2*a)^12*tan(1/2*c) + 28*tan(1/2*a)^11*tan(1/2*c)
^2 + 44*tan(1/2*a)^10*tan(1/2*c)^3 + 49*tan(1/2*a)^9*tan(1/2*c)^4 - 118*ta
n(1/2*a)^8*tan(1/2*c)^5 + 118*tan(1/2*a)^5*tan(1/2*c)^8 - 49*tan(1/2*a)^4*
tan(1/2*c)^9 - 44*tan(1/2*a)^3*tan(1/2*c)^10 - 28*tan(1/2*a)^2*tan(1/2*c)^
11 - tan(1/2*a)*tan(1/2*c)^12 + tan(1/2*a)^11 + 28*tan(1/2*a)^10*tan(1/2*c
) + 44*tan(1/2*a)^9*tan(1/2*c)^2 + 49*tan(1/2*a)^8*tan(1/2*c)^3 - 118*tan(
1/2*a)^7*tan(1/2*c)^4 + 118*tan(1/2*a)^4*tan(1/2*c)^7 - 49*tan(1/2*a)^3*ta
n(1/2*c)^8 - 44*tan(1/2*a)^2*tan(1/2*c)^9 - 28*tan(1/2*a)*tan(1/2*c)^10 -
tan(1/2*c)^11 + 13*tan(1/2*a)^9 + 53*tan(1/2*a)^8*tan(1/2*c) - 8*tan(1/...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sec^3(a+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)^3*sin(c + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^3(c+bx)\sec^3(a+bx)\,dx = \int \csc{(bx+c)}^3 \sec{(bx+a)}^3\,dx$$

input
```
int(csc(b*x+c)^3*sec(b*x+a)^3,x)
```

output
```
int(csc(b*x + c)**3*sec(a + b*x)**3,x)
```

3.185 $\int \cos^3(c+dx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \cos^3(c+dx)\sin(a+bx)\,dx = -\frac{\cos(a-3c+(b-3d)x)}{8(b-3d)} - \frac{3\cos(a-c+(b-d)x)}{8(b-d)} - \frac{3\cos(a+c+(b+d)x)}{8(b+d)} - \frac{\cos(a+3c+(b+3d)x)}{8(b+3d)}$$

output
```
-1/8*cos(a-3*c+(b-3*d)*x)/(b-3*d)-3*cos(a-c+(b-d)*x)/(8*b-8*d)-3*cos(a+c+(
b+d)*x)/(8*b+8*d)-cos(a+3*c+(b+3*d)*x)/(8*b+24*d)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \cos^3(c+dx)\sin(a+bx)\,dx = \frac{1}{8}\left(-\frac{\cos(a-3c+bx-3dx)}{b-3d} - \frac{3\cos(a-c+bx-dx)}{b-d} - \frac{\cos(a+3c+bx+3dx)}{b+3d} - \frac{3\cos(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Cos[c + d*x]^3*Sin[a + b*x],x]
```

output
```
(-(Cos[a - 3*c + b*x - 3*d*x]/(b - 3*d)) - (3*Cos[a - c + b*x - d*x])/(b -
 d) - Cos[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Cos[a + c + (b + d)*x])/(b
 + d))/8
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{1}{8}\sin(a+x(b-3d)-3c)+\frac{3}{8}\sin(a+x(b-d)-c)+\frac{3}{8}\sin(a+x(b+d)+c)+\frac{1}{8}\sin(a+x(b+3d)+3c)\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(a+x(b-3d)-3c)}{8(b-3d)}-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}-\frac{\cos(a+x(b+3d)+3c)}{8(b+3d)}$$

input
```
Int[Cos[c + d*x]^3*Sin[a + b*x],x]
```

output
```
-1/8*Cos[a - 3*c + (b - 3*d)*x]/(b - 3*d) - (3*Cos[a - c + (b - d)*x])/(8*
(b - d)) - (3*Cos[a + c + (b + d)*x])/(8*(b + d)) - Cos[a + 3*c + (b + 3*d
)*x]/(8*(b + 3*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\cos(a-3c+(b-3d)x)}{8(b-3d)}-\frac{3\cos(a-c+(b-d)x)}{8(b-d)}-\frac{3\cos(a+c+(b+d)x)}{8(b+d)}-\frac{\cos(a+3c+(b+3d)x)}{8(b+3d)}$
risch	$-\frac{\cos(bx-3dx+a-3c)}{8(b-3d)}-\frac{3\cos(bx-dx+a-c)}{8(b-d)}-\frac{3\cos(bx+dx+a+c)}{8(b+d)}-\frac{\cos(bx+3dx+a+3c)}{8(b+3d)}$
parallelrisch	$\frac{-(b-d)(b+3d)(b+d)\cos(a-3c+(b-3d)x)-3(b+3d)(b-3d)(b+d)\cos(a-c+(b-d)x)-(b-d)(b-3d)(b+d)\cos(a+3c+(b+3d)x}{8b^4-80b^2d^2+72d^4}$
orering	Expression too large to display

input
```
int(cos(d*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8*cos(a-3*c+(b-3*d)*x)/(b-3*d)-3/8*cos(a-c+(b-d)*x)/(b-d)-3/8/(b+d)*cos
(a+c+(b+d)*x)-1/8/(b+3*d)*cos(a+3*c+(b+3*d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16

$$\int \cos^3(c+dx)\sin(a+bx)\,dx$$
$$=\frac{6\,bd^2\cos(bx+a)\cos(dx+c)-(b^3-bd^2)\cos(bx+a)\cos(dx+c)^3+3\left(2\,d^3-(b^2d-d^3)\cos(dx+c)^2\right.}{b^4-10\,b^2d^2+9\,d^4}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
(6*b*d^2*cos(b*x + a)*cos(d*x + c) - (b^3 - b*d^2)*cos(b*x + a)*cos(d*x +
c)^3 + 3*(2*d^3 - (b^2*d - d^3)*cos(d*x + c)^2)*sin(b*x + a)*sin(d*x + c))
/(b^4 - 10*b^2*d^2 + 9*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(78) = 156$.

Time = 2.04 (sec) , antiderivative size = 918, normalized size of antiderivative = 10.09

$$\int \cos^3(c + dx)\sin(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**3*sin(b*x+a),x)
```

output
```
Piecewise((x*sin(a)*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*sin(a - 3*d*x)*
sin(c + d*x)**2*cos(c + d*x)/8 + x*sin(a - 3*d*x)*cos(c + d*x)**3/8 - x*si
n(c + d*x)**3*cos(a - 3*d*x)/8 + 3*x*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d
*x)**2/8 - sin(a - 3*d*x)*sin(c + d*x)**3/(24*d) - sin(a - 3*d*x)*sin(c +
d*x)*cos(c + d*x)**2/(4*d) + 3*cos(a - 3*d*x)*cos(c + d*x)**3/(8*d), Eq(b,
 -3*d)), (3*x*sin(a - d*x)*sin(c + d*x)**2*cos(c + d*x)/8 + 3*x*sin(a - d*
x)*cos(c + d*x)**3/8 + 3*x*sin(c + d*x)**3*cos(a - d*x)/8 + 3*x*sin(c + d*
x)*cos(a - d*x)*cos(c + d*x)**2/8 + 3*sin(a - d*x)*sin(c + d*x)**3/(8*d) +
 3*sin(a - d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) - cos(a - d*x)*cos(c +
d*x)**3/(8*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**2*cos(c + d*x)/
8 + 3*x*sin(a + d*x)*cos(c + d*x)**3/8 - 3*x*sin(c + d*x)**3*cos(a + d*x)/
8 - 3*x*sin(c + d*x)*cos(a + d*x)*cos(c + d*x)**2/8 + 3*sin(a + d*x)*sin(c
 + d*x)**3/(8*d) + 3*sin(a + d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) + cos
(a + d*x)*cos(c + d*x)**3/(8*d), Eq(b, d)), (-3*x*sin(a + 3*d*x)*sin(c + d
*x)**2*cos(c + d*x)/8 + x*sin(a + 3*d*x)*cos(c + d*x)**3/8 + x*sin(c + d*x
)**3*cos(a + 3*d*x)/8 - 3*x*sin(c + d*x)*cos(a + 3*d*x)*cos(c + d*x)**2/8
- sin(a + 3*d*x)*sin(c + d*x)**3/(24*d) - sin(a + 3*d*x)*sin(c + d*x)*cos(
c + d*x)**2/(4*d) - 3*cos(a + 3*d*x)*cos(c + d*x)**3/(8*d), Eq(b, 3*d)), (
-b**3*cos(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2
*d*sin(a + b*x)*sin(c + d*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(83) = 166$.

Time = 0.09 (sec) , antiderivative size = 912, normalized size of antiderivative = 10.02

$$\int \cos^3(c+dx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/16*((b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))
*cos((b + 3*d)*x + a + 6*c) + (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos
(3*c) + 3*d^3*cos(3*c))*cos((b + 3*d)*x + a) + 3*(b^3*cos(3*c) - b^2*d*cos
(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*cos((b + d)*x + a + 4*c) + 3*(b
^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*cos((b +
 d)*x + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9
*d^3*cos(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c
) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*cos(-(b - d)*x - a - 2*c) + (b^3*co
s(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))*cos(-(b - 3*d
)*x - a + 6*c) + (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3
*cos(3*c))*cos(-(b - 3*d)*x - a) + (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^
2*sin(3*c) + 3*d^3*sin(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) -
3*b^2*d*sin(3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))*sin((b + 3*d)*x + a) +
 3*(b^3*sin(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*sin
((b + d)*x + a + 4*c) - 3*(b^3*sin(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c
) + 9*d^3*sin(3*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*sin(3*c) + b^2*d*sin
(3*c) - 9*b*d^2*sin(3*c) - 9*d^3*sin(3*c))*sin(-(b - d)*x - a + 4*c) - 3*(
b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9*d^3*sin(3*c))*sin(-(b
 - d)*x - a - 2*c) + (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3
*d^3*sin(3*c))*sin(-(b - 3*d)*x - a + 6*c) - (b^3*sin(3*c) + 3*b^2*d*si...
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\int \cos^3(c+dx)\sin(a+bx)\,dx = -\frac{\cos(bx+3\,dx+a+3\,c)}{8\,(b+3\,d)} - \frac{3\,\cos(bx+dx+a+c)}{8\,(b+d)} - \frac{3\,\cos(bx-dx+a-c)}{8\,(b-d)} - \frac{\cos(bx-3\,dx+a-3\,c)}{8\,(b-3\,d)}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/8*cos(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*cos(b*x + d*x + a + c)/(b
+ d) - 3/8*cos(b*x - d*x + a - c)/(b - d) - 1/8*cos(b*x - 3*d*x + a - 3*c)
/(b - 3*d)
```

Mupad [B] (verification not implemented)

Time = 18.09 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.26

$$\int \cos^3(c+dx)\sin(a+bx)\,dx = -\mathrm{e}^{a\,1\mathrm{i}-c\,3\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,3\mathrm{i}}\left(\frac{b+3\,d}{16\,b^2-144\,d^2}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(b-3\,d)}{16\,b^2-144\,d^2}\right) - \mathrm{e}^{a\,1\mathrm{i}+c\,3\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,3\mathrm{i}}\left(\frac{b-3\,d}{16\,b^2-144\,d^2}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(b+3\,d)}{16\,b^2-144\,d^2}\right) - \mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{3\,b+3\,d}{16\,b^2-16\,d^2}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(3\,b-3\,d)}{16\,b^2-16\,d^2}\right) - \mathrm{e}^{a\,1\mathrm{i}+c\,1\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,1\mathrm{i}}\left(\frac{3\,b-3\,d}{16\,b^2-16\,d^2}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(3\,b+3\,d)}{16\,b^2-16\,d^2}\right)$$

input
```
int(cos(c + d*x)^3*sin(a + b*x),x)
```

output
```
- exp(a*1i - c*3i + b*x*1i - d*x*3i)*((b + 3*d)/(16*b^2 - 144*d^2) + (exp(
- a*2i - b*x*2i)*(b - 3*d))/(16*b^2 - 144*d^2)) - exp(a*1i + c*3i + b*x*1i
 + d*x*3i)*((b - 3*d)/(16*b^2 - 144*d^2) + (exp(- a*2i - b*x*2i)*(b + 3*d)
)/(16*b^2 - 144*d^2)) - exp(a*1i - c*1i + b*x*1i - d*x*1i)*((3*b + 3*d)/(1
6*b^2 - 16*d^2) + (exp(- a*2i - b*x*2i)*(3*b - 3*d))/(16*b^2 - 16*d^2)) -
exp(a*1i + c*1i + b*x*1i + d*x*1i)*((3*b - 3*d)/(16*b^2 - 16*d^2) + (exp(-
 a*2i - b*x*2i)*(3*b + 3*d))/(16*b^2 - 16*d^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.10

$$\int \cos^3(c+dx)\sin(a+bx)\,dx$$
$$= \frac{\cos(bx+a)\cos(dx+c)\sin(dx+c)^2 b^3 - \cos(bx+a)\cos(dx+c)\sin(dx+c)^2 b\,d^2 - \cos(bx+a)\cos(\ldots}{}$$

input
```
int(cos(d*x+c)^3*sin(b*x+a),x)
```

output
```
(cos(a + b*x)*cos(c + d*x)*sin(c + d*x)**2*b**3 - cos(a + b*x)*cos(c + d*x
)*sin(c + d*x)**2*b*d**2 - cos(a + b*x)*cos(c + d*x)*b**3 + 7*cos(a + b*x)
*cos(c + d*x)*b*d**2 + 3*sin(a + b*x)*sin(c + d*x)**3*b**2*d - 3*sin(a + b
*x)*sin(c + d*x)**3*d**3 - 3*sin(a + b*x)*sin(c + d*x)*b**2*d + 9*sin(a +
b*x)*sin(c + d*x)*d**3 - b**3 + 7*b*d**2)/(b**4 - 10*b**2*d**2 + 9*d**4)
```

3.186 $\int \cos^2(c+dx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \cos^2(c+dx)\sin(a+bx)\,dx = -\frac{\cos(a+bx)}{2b}-\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)} -\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}$$

output
```
-1/2*cos(b*x+a)/b-cos(a-2*c+(b-2*d)*x)/(4*b-8*d)-cos(a+2*c+(b+2*d)*x)/(4*b
+8*d)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \cos^2(c+dx)\sin(a+bx)\,dx = \frac{1}{4}\left(-\frac{2\cos(a)\cos(bx)}{b}-\frac{\cos(a-2c+bx-2dx)}{b-2d} -\frac{\cos(a+2c+bx+2dx)}{b+2d}+\frac{2\sin(a)\sin(bx)}{b}\right)$$

input
```
Integrate[Cos[c + d*x]^2*Sin[a + b*x],x]
```

output
```
((-2*Cos[a]*Cos[b*x])/b - Cos[a - 2*c + b*x - 2*d*x]/(b - 2*d) - Cos[a + 2
*c + b*x + 2*d*x]/(b + 2*d) + (2*Sin[a]*Sin[b*x])/b)/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^2(c+dx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{1}{4}\sin(a+x(b-2d)-2c)+\frac{1}{4}\sin(a+x(b+2d)+2c)+\frac{1}{2}\sin(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}-\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

input
```
Int[Cos[c + d*x]^2*Sin[a + b*x],x]
```

output
```
-1/2*Cos[a + b*x]/b - Cos[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) - Cos[a + 2
*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\cos(bx+a)}{2b}-\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)}-\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}$
risch	$-\frac{\cos(bx+a)}{2b}-\frac{\cos(bx-2dx+a-2c)}{4(b-2d)}-\frac{\cos(bx+2dx+a+2c)}{4(b+2d)}$
parallelrisch	$\frac{-b(b+2d)\cos(a-2c+(b-2d)x)-b(b-2d)\cos(a+2c+(b+2d)x)+(-2b^2+8d^2)\cos(bx+a)-4b^2+8d^2}{4b^3-16b\,d^2}$
norman	$\frac{\frac{-2b^2+4d^2}{b(b^2-4d^2)}+\frac{(-2b^2+4d^2)\tan(\frac{dx}{2}+\frac{c}{2})^4}{b(b^2-4d^2)}+\frac{8d^2\tan(\frac{dx}{2}+\frac{c}{2})^2}{b(b^2-4d^2)}-\frac{8d\tan(\frac{a}{2}+\frac{bx}{2})\tan(\frac{dx}{2}+\frac{c}{2})}{b^2-4d^2}+\frac{8d\tan(\frac{a}{2}+\frac{bx}{2})\tan(\frac{dx}{2}+\frac{c}{2})^3}{b^2-4d^2}-\frac{4b\tan(\frac{a}{2}}{}}{(1+\tan(\frac{dx}{2}+\frac{c}{2})^2)^2(1+\tan(\frac{a}{2}+\frac{bx}{2})^2)}$
orering	$-\frac{(3b^4+16d^4)(-2\sin(bx+a)\sin(dx+c)d\cos(dx+c)+\cos(dx+c)^2b\cos(bx+a))}{b^2(b^4-8b^2d^2+16d^4)}-\frac{(3b^2+8d^2)(6b^2\sin(bx+a)\cos(dx+c)\sin}{}$

input
```
int(cos(d*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(b*x+a)/b-1/4/(b-2*d)*cos(a-2*c+(b-2*d)*x)-1/4/(b+2*d)*cos(a+2*c+(
b+2*d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \cos^2(c+dx)\sin(a+bx)\,dx = -\frac{b^2\cos(bx+a)\cos(dx+c)^2+2\,bd\cos(dx+c)\sin(bx+a)\sin(dx+c)-2\,d^2\cos(bx+a)}{b^3-4\,bd^2}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-(b^2*cos(b*x + a)*cos(d*x + c)^2 + 2*b*d*cos(d*x + c)*sin(b*x + a)*sin(d*
x + c) - 2*d^2*cos(b*x + a))/(b^3 - 4*b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(51) = 102$.

Time = 0.74 (sec) , antiderivative size = 405, normalized size of antiderivative = 6.53

$$\int \cos^2(c+dx)\sin(a+bx)\,dx$$
$$= \begin{cases} x\sin(a)\cos^2(c) \\ \left(\frac{x\sin^2(c+dx)}{2} + \frac{x\cos^2(c+dx)}{2} + \frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\sin(a) \\ -\frac{x\sin(a-2dx)\sin^2(c+dx)}{4} + \frac{x\sin(a-2dx)\cos^2(c+dx)}{4} + \frac{x\sin(c+dx)\cos(a-2dx)\cos(c+dx)}{2} - \frac{\sin(a-2dx)\sin(c+dx)\cos(c+dx)}{4d} \\ -\frac{x\sin(a+2dx)\sin^2(c+dx)}{4} + \frac{x\sin(a+2dx)\cos^2(c+dx)}{4} - \frac{x\sin(c+dx)\cos(a+2dx)\cos(c+dx)}{2} - \frac{\sin(a+2dx)\sin(c+dx)\cos(c+dx)}{4d} \\ -\frac{b^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2} - \frac{2bd\sin(a+bx)\sin(c+dx)\cos(c+dx)}{b^3-4bd^2} + \frac{2d^2\sin^2(c+dx)\cos(a+bx)}{b^3-4bd^2} + \frac{2d^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*sin(b*x+a),x)
```

output
```
Piecewise((x*sin(a)*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a), Eq(b, 0)
), (-x*sin(a - 2*d*x)*sin(c + d*x)**2/4 + x*sin(a - 2*d*x)*cos(c + d*x)**2
/4 + x*sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/2 - sin(a - 2*d*x)*sin(c +
 d*x)*cos(c + d*x)/(4*d) + cos(a - 2*d*x)*cos(c + d*x)**2/(2*d), Eq(b, -2*
d)), (-x*sin(a + 2*d*x)*sin(c + d*x)**2/4 + x*sin(a + 2*d*x)*cos(c + d*x)*
*2/4 - x*sin(c + d*x)*cos(a + 2*d*x)*cos(c + d*x)/2 - sin(a + 2*d*x)*sin(c
 + d*x)*cos(c + d*x)/(4*d) - cos(a + 2*d*x)*cos(c + d*x)**2/(2*d), Eq(b, 2
*d)), (-b**2*cos(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*sin(a
+ b*x)*sin(c + d*x)*cos(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*sin(c + d*x)**
2*cos(a + b*x)/(b**3 - 4*b*d**2) + 2*d**2*cos(a + b*x)*cos(c + d*x)**2/(b*
*3 - 4*b*d**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(56) = 112$.

Time = 0.06 (sec) , antiderivative size = 414, normalized size of antiderivative = 6.68

$$\int \cos^2(c+dx)\sin(a+bx)\,dx = -\frac{(b^2\cos(2c) - 2bd\cos(2c))\cos((b+2d)x+a+4c) + (b^2\cos(2c) - 2bd\cos(2c))\cos((b+2d)x+a}{}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/8*((b^2*cos(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a + 4*c) + (b^2*co
s(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(
2*c))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*cos(-(
b - 2*d)*x - a) + 2*(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a + 2*c) + 2
*(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a - 2*c) + (b^2*sin(2*c) - 2*b*
d*sin(2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*sin(2*c) - 2*b*d*sin(2*c))*s
in((b + 2*d)*x + a) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a
 + 4*c) - (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a) + 2*(b^2*s
in(2*c) - 4*d^2*sin(2*c))*sin(b*x + a + 2*c) - 2*(b^2*sin(2*c) - 4*d^2*sin
(2*c))*sin(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c
)^2 + b*sin(2*c)^2)*d^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \cos^2(c+dx)\sin(a+bx)\,dx = -\frac{\cos(bx+2dx+a+2c)}{4(b+2d)} - \frac{\cos(bx-2dx+a-2c)}{4(b-2d)} - \frac{\cos(bx+a)}{2b}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/4*cos(b*x + 2*d*x + a + 2*c)/(b + 2*d) - 1/4*cos(b*x - 2*d*x + a - 2*c)
/(b - 2*d) - 1/2*cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 17.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

$$\int \cos^2(c+dx)\sin(a+bx)\,dx$$
$$= \frac{d\,(2\,b\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)-2\,b\,\cos\left(a+2\,c+b\,x+2\,d\,x\right))+b^2\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)+b^2\,\cos}{16\,b\,d^2-4\,b^3}$$
$$-\frac{\cos\left(a+b\,x\right)}{2\,b}$$

input
```
int(cos(c + d*x)^2*sin(a + b*x),x)
```

output
```
(d*(2*b*cos(a - 2*c + b*x - 2*d*x) - 2*b*cos(a + 2*c + b*x + 2*d*x)) + b^2
*cos(a - 2*c + b*x - 2*d*x) + b^2*cos(a + 2*c + b*x + 2*d*x))/(16*b*d^2 -
4*b^3) - cos(a + b*x)/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.39

$$\int \cos^2(c+dx)\sin(a+bx)\,dx$$
$$= \frac{\cos\left(bx+a\right)\sin\left(dx+c\right)^2 b^2-\cos\left(bx+a\right)b^2+2\cos\left(bx+a\right)d^2-2\cos\left(dx+c\right)\sin\left(bx+a\right)\sin\left(dx+c\right)}{b\left(b^2-4d^2\right)}$$

input
```
int(cos(d*x+c)^2*sin(b*x+a),x)
```

output
```
(cos(a + b*x)*sin(c + d*x)**2*b**2 - cos(a + b*x)*b**2 + 2*cos(a + b*x)*d*
*2 - 2*cos(c + d*x)*sin(a + b*x)*sin(c + d*x)*b*d + b**2 - 2*d**2)/(b*(b**
2 - 4*d**2))
```

3.187 $\int \cos(c+dx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \cos(c+dx)\sin(a+bx)\,dx = -\frac{\cos(a-c+(b-d)x)}{2(b-d)} - \frac{\cos(a+c+(b+d)x)}{2(b+d)}$$

output
```
-1/2*cos(a-c+(b-d)*x)/(b-d)-cos(a+c+(b+d)*x)/(2*b+2*d)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)\sin(a+bx)\,dx = -\frac{\cos(a-c+(b-d)x)}{2(b-d)} - \frac{\cos(a+c+(b+d)x)}{2(b+d)}$$

input
```
Integrate[Cos[c + d*x]*Sin[a + b*x],x]
```

output
```
-1/2*Cos[a - c + (b - d)*x]/(b - d) - Cos[a + c + (b + d)*x]/(2*(b + d))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos(c+dx)\,dx$$

$$\Big\downarrow \text{ 5085}$$

$$\int \left(\frac{1}{2}\sin(a+x(b-d)-c)+\frac{1}{2}\sin(a+x(b+d)+c)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\cos(a+x(b-d)-c)}{2(b-d)}-\frac{\cos(a+x(b+d)+c)}{2(b+d)}$$

input
```
Int[Cos[c + d*x]*Sin[a + b*x],x]
```

output
```
-1/2*Cos[a - c + (b - d)*x]/(b - d) - Cos[a + c + (b + d)*x]/(2*(b + d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\cos(a-c+(b-d)x)}{2(b-d)}-\frac{\cos(a+c+(b+d)x)}{2(b+d)}$
risch	$-\frac{\cos(bx-dx+a-c)}{2(b-d)}-\frac{\cos(bx+dx+a+c)}{2(b+d)}$
parallelrisch	$\frac{2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}b-4d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{(b-d)(b+d)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}\right)}$
norman	$\frac{\frac{2b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}}{b^{2}-d^{2}}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{b^{2}-d^{2}}-\frac{4d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^{2}-d^{2}}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}\right)}$
orering	$-\frac{2(b^{2}+d^{2})(-\sin(bx+a)\sin(dx+c)d+\cos(dx+c)\cos(bx+a)b)}{b^{4}-2b^{2}d^{2}+d^{4}}-\frac{-3d^{2}\cos(dx+c)\cos(bx+a)b+3d\sin(dx+c)b^{2}\sin(bx+a)+}{b^{4}-2b^{2}d^{2}+}$

input
```
int(cos(d*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(a-c+(b-d)*x)/(b-d)-1/2/(b+d)*cos(a+c+(b+d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \cos(c+dx)\sin(a+bx)\,dx=-\frac{b\cos\left(bx+a\right)\cos\left(dx+c\right)+d\sin\left(bx+a\right)\sin\left(dx+c\right)}{b^{2}-d^{2}}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-(b*cos(b*x + a)*cos(d*x + c) + d*sin(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(34) = 68$.

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.60

$$\int \cos(c+dx)\sin(a+bx)\,dx = \begin{cases} x\sin(a)\cos(c) & \text{for } b=0 \wedge d=0 \\ \frac{x\sin(a-dx)\cos(c+dx)}{2} + \frac{x\sin(c+dx)\cos(a-dx)}{2} + \frac{\cos(a-dx)\cos(c+dx)}{2d} & \text{for } b=-d \\ \frac{x\sin(a+dx)\cos(c+dx)}{2} - \frac{x\sin(c+dx)\cos(a+dx)}{2} - \frac{\cos(a+dx)\cos(c+dx)}{2d} & \text{for } b=d \\ -\frac{b\cos(a+bx)\cos(c+dx)}{b^2-d^2} - \frac{d\sin(a+bx)\sin(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a),x)
```

output
```
Piecewise((x*sin(a)*cos(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x)*cos(c +
d*x)/2 + x*sin(c + d*x)*cos(a - d*x)/2 + cos(a - d*x)*cos(c + d*x)/(2*d),
Eq(b, -d)), (x*sin(a + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)*cos(a + d*x)/2
 - cos(a + d*x)*cos(c + d*x)/(2*d), Eq(b, d)), (-b*cos(a + b*x)*cos(c + d*
x)/(b**2 - d**2) - d*sin(a + b*x)*sin(c + d*x)/(b**2 - d**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \cos(c+dx)\sin(a+bx)\,dx = -\frac{\cos(bx+dx+a+c)}{2(b+d)} - \frac{\cos(-bx+dx-a+c)}{2(b-d)}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*cos(b*x + d*x + a + c)/(b + d) - 1/2*cos(-b*x + d*x - a + c)/(b - d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \cos(c+dx)\sin(a+bx)\,dx = -\frac{\cos(bx+dx+a+c)}{2(b+d)} - \frac{\cos(bx-dx+a-c)}{2(b-d)}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/2*cos(b*x + d*x + a + c)/(b + d) - 1/2*cos(b*x - d*x + a - c)/(b - d)
```

Mupad [B] (verification not implemented)

Time = 17.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \cos(c+dx)\sin(a+bx)\,dx = -\frac{b\left(\frac{\cos(a-c+b\,x-d\,x)}{2}+\frac{\cos(a+c+b\,x+d\,x)}{2}\right)}{b^2-d^2} - \frac{d\left(\frac{\cos(a-c+b\,x-d\,x)}{2}-\frac{\cos(a+c+b\,x+d\,x)}{2}\right)}{b^2-d^2}$$

input
```
int(cos(c + d*x)*sin(a + b*x),x)
```

output
```
- (b*(cos(a - c + b*x - d*x)/2 + cos(a + c + b*x + d*x)/2))/(b^2 - d^2) -
(d*(cos(a - c + b*x - d*x)/2 - cos(a + c + b*x + d*x)/2))/(b^2 - d^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)\sin(a+bx)\,dx = \frac{-\cos(bx+a)\cos(dx+c)\,b-\sin(bx+a)\sin(dx+c)\,d}{b^2-d^2}$$

input
```
int(cos(d*x+c)*sin(b*x+a),x)
```

output
```
( - (cos(a + b*x)*cos(c + d*x)*b + sin(a + b*x)*sin(c + d*x)*d))/(b**2 - d
**2)
```

3.188 $\int \sec(c+dx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 138

$$\int \sec(c+dx)\sin(a+bx)\,dx = -\frac{e^{-ia-ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2d},\frac{1}{2}\left(3-\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{b-d} - \frac{e^{ia+ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{b+d}$$

output
```
-exp(-I*a-I*b*x+I*(d*x+c))*hypergeom([1, -1/2*(b-d)/d],[3/2-1/2*b/d],-exp(
2*I*(d*x+c)))/(b-d)-exp(I*a+I*b*x+I*(d*x+c))*hypergeom([1, 1/2*(b+d)/d],[3
/2+1/2*b/d],-exp(2*I*(d*x+c)))/(b+d)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int \sec(c+dx)\sin(a+bx)\,dx = -\frac{e^{-i(a-c+(b-d)x)}\,\text{Hypergeometric2F1}\left(1,\frac{-b+d}{2d},\frac{3}{2}-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b-d} - \frac{e^{i(a+c+(b+d)x)}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{b+d}$$

input
```
Integrate[Sec[c + d*x]*Sin[a + b*x],x]
```

output
```
-(Hypergeometric2F1[1, (-b + d)/(2*d), 3/2 - b/(2*d), -E^((2*I)*(c + d*x))
]/((b - d)*E^(I*(a - c + (b - d)*x)))) - (E^(I*(a + c + (b + d)*x))*Hyperg
eometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, -E^((2*I)*(c + d*x))])/(b + d)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sin(a+bx)\sec(c+dx)dx$$

input
```
Int[Sec[c + d*x]*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \sec (dx + c) \sin (bx + a) \, dx$$

input
```
int(sec(d*x+c)*sin(b*x+a),x)
```

output
```
int(sec(d*x+c)*sin(b*x+a),x)
```

Fricas [F]

$$\int \sec(c + dx) \sin(a + bx) \, dx = \int \sec (dx + c) \sin (bx + a) \, dx$$

input
```
integrate(sec(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)*sin(b*x + a), x)
```

Sympy [F]

$$\int \sec(c + dx) \sin(a + bx) \, dx = \int \sin (a + bx) \sec (c + dx) \, dx$$

input
```
integrate(sec(d*x+c)*sin(b*x+a),x)
```

output
```
Integral(sin(a + b*x)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)\sin(a+bx)\,dx = \int \sec\left(dx+c\right)\sin\left(bx+a\right)\,dx$$

input
```
integrate(sec(d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)*sin(b*x + a), x)
```

Giac [F]

$$\int \sec(c+dx)\sin(a+bx)\,dx = \int \sec\left(dx+c\right)\sin\left(bx+a\right)\,dx$$

input
```
integrate(sec(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)\sin(a+bx)\,dx = \int \frac{\sin\left(a+b\,x\right)}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(sin(a + b*x)/cos(c + d*x),x)
```

output
```
int(sin(a + b*x)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)\sin(a+bx)\,dx = \frac{\left(\int \frac{\sin(bx+a)}{\cos(dx+c)}dx\right)b-1}{b}$$

input
```
int(sec(d*x+c)*sin(b*x+a),x)
```

output
```
(int(sin(a + b*x)/cos(c + d*x),x)*b - 1)/b
```

3.189 $\int \sec^2(c+dx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 135

$$\int \sec^2(c+dx)\sin(a+bx)\,dx = -\frac{2e^{-ia-ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{b}{2d},2-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b-2d} - \frac{2e^{ia+ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{b}{2d},2+\frac{b}{2d},-e^{2i(c+dx)}\right)}{b+2d}$$

output
```
-2*exp(-I*a-I*b*x+2*I*(d*x+c))*hypergeom([2, 1-1/2*b/d],[2-1/2*b/d],-exp(2
*I*(d*x+c)))/(b-2*d)-2*exp(I*a+I*b*x+2*I*(d*x+c))*hypergeom([2, 1+1/2*b/d]
,[2+1/2*b/d],-exp(2*I*(d*x+c)))/(b+2*d)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 289 vs. $2(135) = 270$.

Time = 3.98 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.14

$$\int \sec^2(c+dx)\sin(a+bx)\,dx$$
$$= \frac{-\frac{be^{-i(a-2c)}\left(\frac{e^{-i(b-2d)x}\,\text{Hypergeometric2F1}\left(1,1-\frac{b}{2d},2-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b-2d}-\frac{e^{-ibx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b}\right)}{1+e^{2ic}}+\frac{e^{i(a+2c+bx)}\left(\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[Sec[c + d*x]^2*Sin[a + b*x],x]
```

output
```
(-((b*(Hypergeometric2F1[1, 1 - b/(2*d), 2 - b/(2*d), -E^((2*I)*(c + d*x))
]/((b - 2*d)*E^(I*(b - 2*d)*x)) - Hypergeometric2F1[1, -1/2*b/d, 1 - b/(2*
d), -E^((2*I)*(c + d*x))]/(b*E^(I*b*x))))/(E^(I*(a - 2*c))*(1 + E^((2*I)*c
)))) + (E^(I*(a + 2*c + b*x))*(b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 + b/
(2*d), 2 + b/(2*d), -E^((2*I)*(c + d*x))] - (b + 2*d)*Hypergeometric2F1[1,
 b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))]))/((b + 2*d)*(1 + E^((2*I)*c)
)) + Cos[b*x]*Sec[c]*Sec[c + d*x]*Sin[a]*Sin[d*x] + Cos[a]*Sec[c]*Sec[c +
d*x]*Sin[b*x]*Sin[d*x])/d
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^2(c+dx)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \sin(a+bx)\sec^2(c+dx)dx$$

input
```
Int[Sec[c + d*x]^2*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \sec\left(dx+c\right)^{2} \sin\left(bx+a\right) dx$$

input
```
int(sec(d*x+c)^2*sin(b*x+a),x)
```

output
```
int(sec(d*x+c)^2*sin(b*x+a),x)
```

Fricas [F]

$$\int \sec^2(c+dx)\sin(a+bx)\,dx = \int \sec\left(dx+c\right)^{2} \sin\left(bx+a\right)\, dx$$

input
```
integrate(sec(d*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^2*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^2(c+dx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**2*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^2(c+dx)\sin(a+bx)\,dx=\int \sec\left(dx+c\right)^2\sin\left(bx+a\right)\,dx$$

input
```
integrate(sec(d*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
((cos(2*b*x + 2*a) - 1)*cos((b + 2*d)*x + a + 2*c) + cos(2*b*x + 2*a)*cos(
b*x + a) - (d*cos((b + 2*d)*x + a + 2*c)^2 + 2*d*cos((b + 2*d)*x + a + 2*c
)*cos(b*x + a) + d*cos(b*x + a)^2 + d*sin((b + 2*d)*x + a + 2*c)^2 + 2*d*s
in((b + 2*d)*x + a + 2*c)*sin(b*x + a) + d*sin(b*x + a)^2)*integrate(-(b*c
os((b + 2*d)*x + a + 2*c)*sin(2*b*x + 2*a) + b*cos(b*x + a)*sin(2*b*x + 2*
a) - b*cos(2*b*x + 2*a)*sin(b*x + a) - (b*cos(2*b*x + 2*a) + b)*sin((b + 2
*d)*x + a + 2*c) - b*sin(b*x + a))/(d*cos((b + 2*d)*x + a + 2*c)^2 + 2*d*c
os((b + 2*d)*x + a + 2*c)*cos(b*x + a) + d*cos(b*x + a)^2 + d*sin((b + 2*d
)*x + a + 2*c)^2 + 2*d*sin((b + 2*d)*x + a + 2*c)*sin(b*x + a) + d*sin(b*x
 + a)^2), x) + sin((b + 2*d)*x + a + 2*c)*sin(2*b*x + 2*a) + sin(2*b*x + 2
*a)*sin(b*x + a) - cos(b*x + a))/(d*cos((b + 2*d)*x + a + 2*c)^2 + 2*d*cos
((b + 2*d)*x + a + 2*c)*cos(b*x + a) + d*cos(b*x + a)^2 + d*sin((b + 2*d)*
x + a + 2*c)^2 + 2*d*sin((b + 2*d)*x + a + 2*c)*sin(b*x + a) + d*sin(b*x +
 a)^2)
```

Giac [F]

$$\int \sec^2(c+dx)\sin(a+bx)\,dx=\int \sec\left(dx+c\right)^2\sin\left(bx+a\right)\,dx$$

input
```
integrate(sec(d*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)\sin(a+bx)\,dx = \int \frac{\sin(a+b\,x)}{\cos(c+d\,x)^2}\,dx$$

input
```
int(sin(a + b*x)/cos(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^2*sin(b*x+a),x)
```

output
```
(cos(a + b*x)*cos(c + d*x)*b + 2*cos(a + b*x)*b + 8*cos(c + d*x)*int(tan((
a + b*x)/2)/(tan((a + b*x)/2)**2*tan((c + d*x)/2)**4*b**2 + 2*tan((a + b*x
)/2)**2*tan((c + d*x)/2)**4*d**2 - 2*tan((a + b*x)/2)**2*tan((c + d*x)/2)*
*2*b**2 - 4*tan((a + b*x)/2)**2*tan((c + d*x)/2)**2*d**2 + tan((a + b*x)/2
)**2*b**2 + 2*tan((a + b*x)/2)**2*d**2 + tan((c + d*x)/2)**4*b**2 + 2*tan(
(c + d*x)/2)**4*d**2 - 2*tan((c + d*x)/2)**2*b**2 - 4*tan((c + d*x)/2)**2*
d**2 + b**2 + 2*d**2),x)*b**4 + 16*cos(c + d*x)*int(tan((a + b*x)/2)/(tan(
(a + b*x)/2)**2*tan((c + d*x)/2)**4*b**2 + 2*tan((a + b*x)/2)**2*tan((c +
d*x)/2)**4*d**2 - 2*tan((a + b*x)/2)**2*tan((c + d*x)/2)**2*b**2 - 4*tan((
a + b*x)/2)**2*tan((c + d*x)/2)**2*d**2 + tan((a + b*x)/2)**2*b**2 + 2*tan
((a + b*x)/2)**2*d**2 + tan((c + d*x)/2)**4*b**2 + 2*tan((c + d*x)/2)**4*d
**2 - 2*tan((c + d*x)/2)**2*b**2 - 4*tan((c + d*x)/2)**2*d**2 + b**2 + 2*d
**2),x)*b**2*d**2 - 16*cos(c + d*x)*int(tan((c + d*x)/2)/(tan((a + b*x)/2)
**2*tan((c + d*x)/2)**4*b**2 + 2*tan((a + b*x)/2)**2*tan((c + d*x)/2)**4*d
**2 - 2*tan((a + b*x)/2)**2*tan((c + d*x)/2)**2*b**2 - 4*tan((a + b*x)/2)*
*2*tan((c + d*x)/2)**2*d**2 + tan((a + b*x)/2)**2*b**2 + 2*tan((a + b*x)/2
)**2*d**2 + tan((c + d*x)/2)**4*b**2 + 2*tan((c + d*x)/2)**4*d**2 - 2*tan(
(c + d*x)/2)**2*b**2 - 4*tan((c + d*x)/2)**2*d**2 + b**2 + 2*d**2),x)*b**3
*d - 32*cos(c + d*x)*int(tan((c + d*x)/2)/(tan((a + b*x)/2)**2*tan((c + d*
x)/2)**4*b**2 + 2*tan((a + b*x)/2)**2*tan((c + d*x)/2)**4*d**2 - 2*tan(...
```

3.190 $\int \sec^3(c+dx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 141

$$\int \sec^3(c+dx)\sin(a+bx)\,dx = -\frac{4e^{-ia-ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{b}{d}\right),\frac{1}{2}\left(5-\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{b-3d} - \frac{4e^{ia+ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{b}{d}\right),\frac{1}{2}\left(5+\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{b+3d}$$

output
```
-4*exp(-I*a-I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2-1/2*b/d],[5/2-1/2*b/d],-e
xp(2*I*(d*x+c)))/(b-3*d)-4*exp(I*a+I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2+1/
2*b/d],[5/2+1/2*b/d],-exp(2*I*(d*x+c)))/(b+3*d)
```

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.21

$$\int \sec^3(c+dx)\sin(a+bx)\,dx = \frac{2(b+d)e^{-i(a-c+(b-d)x)}\,\text{Hypergeometric2F1}\left(1,\frac{-b+d}{2d},\frac{3}{2}-\frac{b}{2d},-e^{2i(c+dx)}\right)+2(b-d)e^{i(a+c+(b+d)x)}\,\text{Hyperge}$$

input
```
Integrate[Sec[c + d*x]^3*Sin[a + b*x],x]
```

output
```
((2*(b + d)*Hypergeometric2F1[1, (-b + d)/(2*d), 3/2 - b/(2*d), -E^((2*I)*
(c + d*x))])/E^(I*(a - c + (b - d)*x)) + 2*(b - d)*E^(I*(a + c + (b + d)*x
))*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, -E^((2*I)*(c + d*x))]
- ((b - d)*Cos[a - c + b*x - d*x] + (b + d)*Cos[a + c + (b + d)*x])*Sec[c
+ d*x]^2)/(4*d^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^3(c+dx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \sin(a+bx)\sec^3(c+dx)dx$$

input
```
Int[Sec[c + d*x]^3*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \sec(dx+c)^3 \sin(bx+a)\,dx$$

input
```
int(sec(d*x+c)^3*sin(b*x+a),x)
```

output
```
int(sec(d*x+c)^3*sin(b*x+a),x)
```

Fricas [F]

$$\int \sec^3(c+dx)\sin(a+bx)\,dx = \int \sec(dx+c)^3 \sin(bx+a)\,dx$$

input
```
integrate(sec(d*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^3*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^3(c+dx)\sin(a+bx)\,dx = \int \sec(dx+c)^3 \sin(bx+a)\,dx$$

input
```
integrate(sec(d*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*((b - d)*cos((2*b + d)*x + 2*a + c)*cos(b*x + a) + (b - d)*cos(b*x +
a)*cos(3*d*x + 3*c) + (b + d)*cos(b*x + a)*cos(d*x + c) + (b - d)*sin((2*b
 + d)*x + 2*a + c)*sin(b*x + a) + (b - d)*sin(b*x + a)*sin(3*d*x + 3*c) +
(b + d)*sin(b*x + a)*sin(d*x + c) + (2*(b + d)*cos((b + 2*d)*x + a + 2*c)
+ (b + d)*cos(b*x + a))*cos((2*b + 3*d)*x + 2*a + 3*c) + ((b + d)*cos((2*b
 + 3*d)*x + 2*a + 3*c) + (b - d)*cos((2*b + d)*x + 2*a + c) + (b - d)*cos(
3*d*x + 3*c) + (b + d)*cos(d*x + c))*cos((b + 4*d)*x + a + 4*c) + 2*((b -
d)*cos((2*b + d)*x + 2*a + c) + (b - d)*cos(3*d*x + 3*c) + (b + d)*cos(d*x
 + c))*cos((b + 2*d)*x + a + 2*c) + 2*(d^2*cos((b + 4*d)*x + a + 4*c)^2 +
4*d^2*cos((b + 2*d)*x + a + 2*c)^2 + 4*d^2*cos((b + 2*d)*x + a + 2*c)*cos(
b*x + a) + d^2*cos(b*x + a)^2 + d^2*sin((b + 4*d)*x + a + 4*c)^2 + 4*d^2*s
in((b + 2*d)*x + a + 2*c)^2 + 4*d^2*sin((b + 2*d)*x + a + 2*c)*sin(b*x + a
) + d^2*sin(b*x + a)^2 + 2*(2*d^2*cos((b + 2*d)*x + a + 2*c) + d^2*cos(b*x
 + a))*cos((b + 4*d)*x + a + 4*c) + 2*(2*d^2*sin((b + 2*d)*x + a + 2*c) +
d^2*sin(b*x + a))*sin((b + 4*d)*x + a + 4*c))*integrate(1/2*((b^2 - d^2)*c
os(b*x + a)*sin((2*b + d)*x + 2*a + c) - (b^2 - d^2)*cos((2*b + d)*x + 2*a
 + c)*sin(b*x + a) + (b^2 - d^2)*cos(d*x + c)*sin(b*x + a) - (b^2 - d^2)*c
os(b*x + a)*sin(d*x + c) + ((b^2 - d^2)*sin((2*b + d)*x + 2*a + c) - (b^2
- d^2)*sin(d*x + c))*cos((b + 2*d)*x + a + 2*c) - ((b^2 - d^2)*cos((2*b +
d)*x + 2*a + c) - (b^2 - d^2)*cos(d*x + c))*sin((b + 2*d)*x + a + 2*c))...
```

Giac [F]

$$\int \sec^3(c+dx)\sin(a+bx)\,dx = \int \sec(dx+c)^3 \sin(bx+a)\,dx$$

input
```
integrate(sec(d*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)\sin(a+bx)\,dx = \int \frac{\sin(a+b\,x)}{\cos(c+d\,x)^3}\,dx$$

input
```
int(sin(a + b*x)/cos(c + d*x)^3,x)
```

output
```
int(sin(a + b*x)/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sec^3(c+dx)\sin(a+bx)\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^3*sin(b*x+a),x)
```

output
```
( - 6*cos(a + b*x)*cos(c + d*x)*tan((a + b*x)/2)**2*tan((c + d*x)/2)**4*b*
*6*d**2 + 24*cos(a + b*x)*cos(c + d*x)*tan((a + b*x)/2)**2*tan((c + d*x)/2
)**4*b**4*d**4 + 72*cos(a + b*x)*cos(c + d*x)*tan((a + b*x)/2)**2*tan((c +
 d*x)/2)**4*b**2*d**6 + 12*cos(a + b*x)*cos(c + d*x)*tan((a + b*x)/2)**2*t
an((c + d*x)/2)**2*b**6*d**2 - 48*cos(a + b*x)*cos(c + d*x)*tan((a + b*x)/
2)**2*tan((c + d*x)/2)**2*b**4*d**4 - 144*cos(a + b*x)*cos(c + d*x)*tan((a
 + b*x)/2)**2*tan((c + d*x)/2)**2*b**2*d**6 - 6*cos(a + b*x)*cos(c + d*x)*
tan((a + b*x)/2)**2*b**6*d**2 + 24*cos(a + b*x)*cos(c + d*x)*tan((a + b*x)
/2)**2*b**4*d**4 + 72*cos(a + b*x)*cos(c + d*x)*tan((a + b*x)/2)**2*b**2*d
**6 - 6*cos(a + b*x)*cos(c + d*x)*tan((c + d*x)/2)**4*b**6*d**2 + 24*cos(a
 + b*x)*cos(c + d*x)*tan((c + d*x)/2)**4*b**4*d**4 + 72*cos(a + b*x)*cos(c
 + d*x)*tan((c + d*x)/2)**4*b**2*d**6 + 12*cos(a + b*x)*cos(c + d*x)*tan((
c + d*x)/2)**2*b**6*d**2 - 48*cos(a + b*x)*cos(c + d*x)*tan((c + d*x)/2)**
2*b**4*d**4 - 144*cos(a + b*x)*cos(c + d*x)*tan((c + d*x)/2)**2*b**2*d**6
- 6*cos(a + b*x)*cos(c + d*x)*b**6*d**2 + 24*cos(a + b*x)*cos(c + d*x)*b**
4*d**4 + 72*cos(a + b*x)*cos(c + d*x)*b**2*d**6 + 2*cos(a + b*x)*sin(c + d
*x)**2*tan((a + b*x)/2)**2*tan((c + d*x)/2)**4*b**6*d**2 - 2*cos(a + b*x)*
sin(c + d*x)**2*tan((a + b*x)/2)**2*tan((c + d*x)/2)**4*b**4*d**4 - 8*cos(
a + b*x)*sin(c + d*x)**2*tan((a + b*x)/2)**2*tan((c + d*x)/2)**4*b**2*d**6
 + 8*cos(a + b*x)*sin(c + d*x)**2*tan((a + b*x)/2)**2*tan((c + d*x)/2)*...
```

3.191 $\int \cos^3(c+dx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx = -\frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)} - \frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)} + \frac{3\sin(c+dx)}{8d} + \frac{\sin(3c+3dx)}{24d} - \frac{3\sin(2a+c+(2b+d)x)}{16(2b+d)} - \frac{\sin(2a+3c+(2b+3d)x)}{16(2b+3d)}$$

output
```
-1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3*sin(2*a-c+(2*b-d)*x)/(32*b-16*d
)+3/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d-3*sin(2*a+c+(2*b+d)*x)/(32*b+16*d
)-sin(2*a+3*c+(2*b+3*d)*x)/(32*b+48*d)
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx = \frac{1}{48}\left(\frac{18\cos(dx)\sin(c)}{d}+\frac{2\cos(3dx)\sin(3c)}{d}+\frac{18\cos(c)\sin(dx)}{d}+\frac{2\cos(3c)\sin(3dx)}{d}-\frac{3\sin(2a-3c+2bx-3dx)}{2b-3d}-\frac{9\sin(2a-c+2bx-dx)}{2b-d}-\frac{9\sin(2a+c+2bx+dx)}{2b+d}-\frac{3\sin(2a+3c+2bx+3dx)}{2b+3d}\right)$$

input

```
Integrate[Cos[c + d*x]^3*Sin[a + b*x]^2,x]
```

output

```
((18*Cos[d*x]*Sin[c])/d + (2*Cos[3*d*x]*Sin[3*c])/d + (18*Cos[c]*Sin[d*x])
/d + (2*Cos[3*c]*Sin[3*d*x])/d - (3*Sin[2*a - 3*c + 2*b*x - 3*d*x])/(2*b -
 3*d) - (9*Sin[2*a - c + 2*b*x - d*x])/(2*b - d) - (9*Sin[2*a + c + 2*b*x
+ d*x])/(2*b + d) - (3*Sin[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d))/48
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \text{ 5085}$$

$$\int \left(-\frac{1}{16}\cos(2a + x(2b-3d) - 3c) - \frac{3}{16}\cos(2a + x(2b-d) - c) - \frac{3}{16}\cos(2a + x(2b+d) + c) - \frac{1}{16}\cos(2a + x\right.$$

$$\downarrow \text{2009}$$

$$-\frac{\sin(2a + x(2b-3d) - 3c)}{16(2b-3d)} - \frac{3\sin(2a + x(2b-d) - c)}{16(2b-d)} - \frac{3\sin(2a + x(2b+d) + c)}{16(2b+d)} - \frac{\sin(2a + x(2b+3d) + 3c)}{16(2b+3d)} + \frac{3\sin(c+dx)}{8d} + \frac{\sin(3c+3dx)}{24d}$$

input
```
Int[Cos[c + d*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/16*Sin[2*a - 3*c + (2*b - 3*d)*x]/(2*b - 3*d) - (3*Sin[2*a - c + (2*b -
 d)*x])/(16*(2*b - d)) + (3*Sin[c + d*x])/(8*d) + Sin[3*c + 3*d*x]/(24*d)
- (3*Sin[2*a + c + (2*b + d)*x])/(16*(2*b + d)) - Sin[2*a + 3*c + (2*b + 3
*d)*x]/(16*(2*b + 3*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 13.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

method	result
default	$\frac{3\sin(dx+c)}{8d}+\frac{\sin(3dx+3c)}{24d}-\frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)}-\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}-\frac{3\sin(2a+c+(2b+d)x)}{16(2b+d)}-\frac{\sin(2a+3c}{16(2}$
parallelrisch	$\frac{(-24d\,b^3-36b^2d^2+6b\,d^3+9d^4)\sin(2a-3c+(2b-3d)x)-72\left(\left(b+\frac{3d}{2}\right)d\left(b+\frac{d}{2}\right)\sin(2a-c+(2b-d)x)+\left(b-\frac{d}{2}\right)\left(\frac{d\left(b+\frac{d}{2}\right)\sin(2a+}{3}\right.\right.}{768b^4d-1920d^3b^2+432d^5}$
risch	$\frac{3\sin(dx+c)b^2}{2d(2b-d)(2b+d)}-\frac{3d\sin(dx+c)}{8(2b-d)(2b+d)}-\frac{\sin(2bx-3dx+2a-3c)b}{8(2b-3d)(2b+3d)}-\frac{3d\sin(2bx-3dx+2a-3c)}{16(2b-3d)(2b+3d)}-\frac{3\sin(2bx-dx+2a-c)b}{8(2b-d)(2b+d)}-$
orering	Expression too large to display

input
```
int(cos(d*x+c)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d-1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-
3*d)-3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)-3/16/(2*b+d)*sin(2*a+c+(2*b+d)*x)-1
/16*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.21

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx$$
$$=\frac{6\left(6\,bd^3\cos(bx+a)\cos(dx+c)-(4\,b^3d-bd^3)\cos(bx+a)\cos(dx+c)^3\right)\sin(bx+a)-\left(18\,d^4\cos(bx\right.}{3\,(16\,b^4d}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*(6*(6*b*d^3*cos(b*x + a)*cos(d*x + c) - (4*b^3*d - b*d^3)*cos(b*x + a)
*cos(d*x + c)^3)*sin(b*x + a) - (18*d^4*cos(b*x + a)^2 - 16*b^4 + 40*b^2*d
^2 - 18*d^4 - (8*b^4 - 38*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cos(b*x +
a)^2)*cos(d*x + c)^2)*sin(d*x + c))/(16*b^4*d - 40*b^2*d^3 + 9*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2004 vs. 2(116) = 232.

Time = 5.68 (sec) , antiderivative size = 2004, normalized size of antiderivative = 13.92

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**3*sin(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a)**2*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*sin(a - 3*d*
x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 + x*sin(a - 3*d*x/2)**2*cos(c + d*
x)**3/16 - x*sin(a - 3*d*x/2)*sin(c + d*x)**3*cos(a - 3*d*x/2)/8 + 3*x*sin
(a - 3*d*x/2)*sin(c + d*x)*cos(a - 3*d*x/2)*cos(c + d*x)**2/8 + 3*x*sin(c
+ d*x)**2*cos(a - 3*d*x/2)**2*cos(c + d*x)/16 - x*cos(a - 3*d*x/2)**2*cos(
c + d*x)**3/16 + 5*sin(a - 3*d*x/2)**2*sin(c + d*x)**3/(48*d) + sin(a - 3*
d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d + 5*sin(a - 3*d*x/2)*sin(c + d*x)
**2*cos(a - 3*d*x/2)*cos(c + d*x)/(4*d) - sin(a - 3*d*x/2)*cos(a - 3*d*x/2
)*cos(c + d*x)**3/(24*d) + 9*sin(c + d*x)**3*cos(a - 3*d*x/2)**2/(16*d), E
q(b, -3*d/2)), (3*x*sin(a - d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 + 3*
x*sin(a - d*x/2)**2*cos(c + d*x)**3/16 + 3*x*sin(a - d*x/2)*sin(c + d*x)**
3*cos(a - d*x/2)/8 + 3*x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)*cos(c
+ d*x)**2/8 - 3*x*sin(c + d*x)**2*cos(a - d*x/2)**2*cos(c + d*x)/16 - 3*x*
cos(a - d*x/2)**2*cos(c + d*x)**3/16 + 31*sin(a - d*x/2)**2*sin(c + d*x)**
3/(48*d) + sin(a - d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d - sin(a - d*x/
2)*sin(c + d*x)**2*cos(a - d*x/2)*cos(c + d*x)/(4*d) - 3*sin(a - d*x/2)*co
s(a - d*x/2)*cos(c + d*x)**3/(8*d) + sin(c + d*x)**3*cos(a - d*x/2)**2/(48
*d), Eq(b, -d/2)), (3*x*sin(a + d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16
+ 3*x*sin(a + d*x/2)**2*cos(c + d*x)**3/16 - 3*x*sin(a + d*x/2)*sin(c + d*
x)**3*cos(a + d*x/2)/8 - 3*x*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(132) = 264$.

Time = 0.19 (sec) , antiderivative size = 1362, normalized size of antiderivative = 9.46

$$\int \cos^3(c + dx) \sin^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/96*(3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4
*sin(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*d
^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*cos((2*b + 3*d)*x + 2*a)
+ 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin
(3*c))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(
3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c))*cos((2*b + d)*x + 2*a - 2*c) -
9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3
*c))*cos(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3
*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) -
3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3
*c))*cos(-(2*b - 3*d)*x - 2*a + 6*c) + 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*si
n(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(-(2*b - 3*d)*x - 2*a) + 2*
(16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x) - 2*(1
6*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x + 6*c) -
18*(16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x + 4*c)
 + 18*(16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x - 2
*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4
*cos(3*c))*sin((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*d
^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((2*b + 3*d)*x + 2*a)
- 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx = -\frac{\sin(2\,bx+3\,dx+2\,a+3\,c)}{16\,(2\,b+3\,d)} - \frac{3\,\sin(2\,bx+dx+2\,a+c)}{16\,(2\,b+d)} - \frac{3\,\sin(2\,bx-dx+2\,a-c)}{16\,(2\,b-d)} - \frac{\sin(2\,bx-3\,dx+2\,a-3\,c)}{16\,(2\,b-3\,d)} + \frac{\sin(3\,dx+3\,c)}{24\,d} + \frac{3\,\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/16*sin(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) - 3/16*sin(2*b*x + d*x +
2*a + c)/(2*b + d) - 3/16*sin(2*b*x - d*x + 2*a - c)/(2*b - d) - 1/16*sin(
2*b*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*sin(3*d*x + 3*c)/d + 3/8*sin
(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 18.99 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.44

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx = -e^{a\,2i-c\,1i+b\,x\,2i-d\,x\,1i}\left(\frac{e^{-a\,2i-b\,x\,2i}\,(24\,b^2-6\,d^2)}{b^2\,d\,128i-d^3\,32i}+\frac{3\,d\,(2\,b+d)}{b^2\,d\,128i-d^3\,32i}-\frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b-d)}{b^2\,d\,128i-d^3\,32i}\right)+e^{a\,2i+c\,1i+b\,x\,2i+d\,x\,1i}\left(-\frac{3\,d\,(2\,b-d)}{b^2\,d\,128i-d^3\,32i}+\frac{e^{-a\,2i-b\,x\,2i}\,(24\,b^2-6\,d^2)}{b^2\,d\,128i-d^3\,32i}+\frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b+d)}{b^2\,d\,128i-d^3\,32i}\right)-e^{a\,2i-c\,3i+b\,x\,2i-d\,x\,3i}\left(\frac{3\,d\,(2\,b+3\,d)}{b^2\,d\,384i-d^3\,864i}+\frac{e^{-a\,2i-b\,x\,2i}\,(8\,b^2-18\,d^2)}{b^2\,d\,384i-d^3\,864i}-\frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b-3\,d)}{b^2\,d\,384i-d^3\,864i}\right)+e^{a\,2i+c\,3i+b\,x\,2i+d\,x\,3i}\left(-\frac{3\,d\,(2\,b-3\,d)}{b^2\,d\,384i-d^3\,864i}+\frac{e^{-a\,2i-b\,x\,2i}\,(8\,b^2-18\,d^2)}{b^2\,d\,384i-d^3\,864i}+\frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b+3\,d)}{b^2\,d\,384i-d^3\,864i}\right)$$

input
```
int(cos(c + d*x)^3*sin(a + b*x)^2,x)
```

output
```
exp(a*2i + c*1i + b*x*2i + d*x*1i)*((exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2)
)/(b^2*d*128i - d^3*32i) - (3*d*(2*b - d))/(b^2*d*128i - d^3*32i) + (3*d*e
xp(- a*4i - b*x*4i)*(2*b + d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*1i +
 b*x*2i - d*x*1i)*((exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(b^2*d*128i - d
^3*32i) + (3*d*(2*b + d))/(b^2*d*128i - d^3*32i) - (3*d*exp(- a*4i - b*x*4
i)*(2*b - d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*3i + b*x*2i - d*x*3i)
*((3*d*(2*b + 3*d))/(b^2*d*384i - d^3*864i) + (exp(- a*2i - b*x*2i)*(8*b^2
 - 18*d^2))/(b^2*d*384i - d^3*864i) - (3*d*exp(- a*4i - b*x*4i)*(2*b - 3*d
))/(b^2*d*384i - d^3*864i)) + exp(a*2i + c*3i + b*x*2i + d*x*3i)*((exp(- a
*2i - b*x*2i)*(8*b^2 - 18*d^2))/(b^2*d*384i - d^3*864i) - (3*d*(2*b - 3*d)
)/(b^2*d*384i - d^3*864i) + (3*d*exp(- a*4i - b*x*4i)*(2*b + 3*d))/(b^2*d*
384i - d^3*864i))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.94

$$\int \cos^3(c+dx)\sin^2(a+bx)\,dx$$
$$= \frac{24\cos(bx+a)\cos(dx+c)\sin(bx+a)\sin(dx+c)^2 b^3 d - 6\cos(bx+a)\cos(dx+c)\sin(bx+a)\sin(dx}{}$$

input
```
int(cos(d*x+c)^3*sin(b*x+a)^2,x)
```

output
```
(24*cos(a + b*x)*cos(c + d*x)*sin(a + b*x)*sin(c + d*x)**2*b**3*d - 6*cos(
a + b*x)*cos(c + d*x)*sin(a + b*x)*sin(c + d*x)**2*b*d**3 - 24*cos(a + b*x
)*cos(c + d*x)*sin(a + b*x)*b**3*d + 42*cos(a + b*x)*cos(c + d*x)*sin(a +
b*x)*b*d**3 + 36*sin(a + b*x)**2*sin(c + d*x)**3*b**2*d**2 - 9*sin(a + b*x
)**2*sin(c + d*x)**3*d**4 - 36*sin(a + b*x)**2*sin(c + d*x)*b**2*d**2 + 27
*sin(a + b*x)**2*sin(c + d*x)*d**4 - 8*sin(c + d*x)**3*b**4 + 2*sin(c + d*
x)**3*b**2*d**2 + 24*sin(c + d*x)*b**4 - 42*sin(c + d*x)*b**2*d**2)/(3*d*(
16*b**4 - 40*b**2*d**2 + 9*d**4))
```

3.192 $\int \cos^2(c+dx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx = \frac{x}{4} - \frac{\sin(2a+2bx)}{8b} - \frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)} + \frac{\sin(2c+2dx)}{8d} - \frac{\sin(2(a+c)+2(b+d)x)}{16(b+d)}$$

output
```
1/4*x-1/8*sin(2*b*x+2*a)/b-sin(2*a-2*c+2*(b-d)*x)/(16*b-16*d)+1/8*sin(2*d*
x+2*c)/d-sin(2*a+2*c+2*(b+d)*x)/(16*b+16*d)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx = \frac{(-2b^2d+2d^3)\sin(2(a+bx)) - bd(b+d)\sin(2(a-c+(b-d)x)) + b(b-d)(4d(b+d)x+2(b+d)\sin(}{16b(b-d)d(b+d)}$$

input
```
Integrate[Cos[c + d*x]^2*Sin[a + b*x]^2,x]
```

output
```
((-2*b^2*d + 2*d^3)*Sin[2*(a + b*x)] - b*d*(b + d)*Sin[2*(a - c + (b - d)*
x)] + b*(b - d)*(4*d*(b + d)*x + 2*(b + d)*Sin[2*(c + d*x)] - d*Sin[2*(a +
 c + (b + d)*x)]))/(16*b*(b - d)*d*(b + d))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{5085, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^2(c+dx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(-\frac{1}{8}\cos(2(a-c)+2x(b-d))-\frac{1}{8}\cos(2(a+c)+2x(b+d))-\frac{1}{4}\cos(2a+2bx)+\frac{1}{4}\cos(2c+2dx)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}-\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

input
```
Int[Cos[c + d*x]^2*Sin[a + b*x]^2,x]
```

output
```
x/4 - Sin[2*a + 2*b*x]/(8*b) - Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) +
 Sin[2*c + 2*d*x]/(8*d) - Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 5.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
default	$\frac{x}{4}-\frac{\sin(2bx+2a)}{8b}+\frac{\sin(2dx+2c)}{8d}-\frac{\sin((2b-2d)x+2a-2c)}{8(2b-2d)}-\frac{\sin((2b+2d)x+2a+2c)}{8(2b+2d)}$
parallelrisch	$\frac{-bd(b+d)\sin((2b-2d)x+2a-2c)+4(b-d)\left(-\frac{bd\sin((2b+2d)x+2a+2c)}{4}+\left(-\frac{d\sin(2bx+2a)}{2}+b\left(dx+\frac{\sin(2dx+2c)}{2}\right)\right)(b+d)\right)}{16d\,b^3-16b\,d^3}$
risch	$\frac{x}{4}-\frac{\sin(2bx+2a)}{8b}+\frac{\sin(2dx+2c)b^2}{8d(b-d)(b+d)}-\frac{d\sin(2dx+2c)}{8(b-d)(b+d)}-\frac{\sin(2bx-2dx+2a-2c)b}{16(b-d)(b+d)}-\frac{d\sin(2bx-2dx+2a-2c)}{16(b-d)(b+d)}-\frac{\sin(2b}{1(}$
orering	Expression too large to display

input
```
int(cos(d*x+c)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x-1/8*sin(2*b*x+2*a)/b+1/8*sin(2*d*x+2*c)/d-1/8/(2*b-2*d)*sin((2*b-2*d
)*x+2*a-2*c)-1/8/(2*b+2*d)*sin((2*b+2*d)*x+2*a+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx$$
$$=\frac{\left(2\,bd^2\cos\left(bx+a\right)^2+b^3-2\,bd^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(b^3d-bd^3\right)x-\left(2\,b^2d\cos\left(bx+a\right)\cos\left(dx+\right.\right.}{4\left(b^3d-bd^3\right)}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*((2*b*d^2*cos(b*x + a)^2 + b^3 - 2*b*d^2)*cos(d*x + c)*sin(d*x + c) +
(b^3*d - b*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - d^3*cos(b*x + a
))*sin(b*x + a))/(b^3*d - b*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(76) = 152$.

Time = 1.61 (sec) , antiderivative size = 1027, normalized size of antiderivative = 11.67

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2*sin(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a)**2*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**
2/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**2, Eq
(b, 0)), (x*sin(a - d*x)**2*sin(c + d*x)**2/8 + 3*x*sin(a - d*x)**2*cos(c
+ d*x)**2/8 + x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + 3*
x*sin(c + d*x)**2*cos(a - d*x)**2/8 + x*cos(a - d*x)**2*cos(c + d*x)**2/8
+ sin(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*sin(a - d*x)*sin(c +
 d*x)**2*cos(a - d*x)/(8*d) + sin(a - d*x)*cos(a - d*x)*cos(c + d*x)**2/(8
*d), Eq(b, -d)), (x*sin(a + d*x)**2*sin(c + d*x)**2/8 + 3*x*sin(a + d*x)**
2*cos(c + d*x)**2/8 - x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x
)/2 + 3*x*sin(c + d*x)**2*cos(a + d*x)**2/8 + x*cos(a + d*x)**2*cos(c + d*
x)**2/8 + sin(a + d*x)*sin(c + d*x)**2*cos(a + d*x)/(8*d) - 5*sin(a + d*x)
*cos(a + d*x)*cos(c + d*x)**2/(8*d) + sin(c + d*x)*cos(a + d*x)**2*cos(c +
 d*x)/(2*d), Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(
a + b*x)*cos(a + b*x)/(2*b))*cos(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)*
*2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c
+ d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2
/(4*b**3*d - 4*b*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*
d - 4*b*d**3) + b**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d -
 4*b*d**3) + b**3*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*
b*d**3) - 2*b**2*d*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**3*d ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(78) = 156$.

Time = 0.08 (sec) , antiderivative size = 620, normalized size of antiderivative = 7.05

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c
)^2)*d)*x - (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c)
 + (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) + (b^2*d*sin(2
*c) + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) - (b^2*d*sin(2*c) + b*
d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*
cos(2*b*x + 2*a + 2*c) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a
 - 2*c) - 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) + 2*(b^3*sin(2*c) -
 b*d^2*sin(2*c))*cos(2*d*x + 4*c) + (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(
2*(b + d)*x + 2*a + 4*c) + (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)
*x + 2*a) - (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c
) - (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) + 2*(b^2*d*c
os(2*c) - d^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) + 2*(b^2*d*cos(2*c) - d^3*c
os(2*c))*sin(2*b*x + 2*a - 2*c) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*
d*x) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2
+ b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx = \frac{1}{4}x - \frac{\sin(2\,bx + 2\,dx + 2\,a + 2\,c)}{16\,(b+d)} - \frac{\sin(2\,bx - 2\,dx + 2\,a - 2\,c)}{16\,(b-d)} - \frac{\sin(2\,bx + 2\,a)}{8\,b} + \frac{\sin(2\,dx + 2\,c)}{8\,d}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*x - 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) - 1/16*sin(2*b*x - 2*d
*x + 2*a - 2*c)/(b - d) - 1/8*sin(2*b*x + 2*a)/b + 1/8*sin(2*d*x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 17.87 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.01

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx = -\frac{b\,d^2\,\sin(2\,a-2\,c+2\,b\,x-2\,d\,x)-2\,b^3\,\sin(2\,c+2\,d\,x)-2\,d^3\,\sin(2\,a+2\,b\,x)-b\,d^2\,\sin(2\,a+2\,c+}{}$$

input
```
int(cos(c + d*x)^2*sin(a + b*x)^2,x)
```

output
```
-(b*d^2*sin(2*a - 2*c + 2*b*x - 2*d*x) - 2*b^3*sin(2*c + 2*d*x) - 2*d^3*si
n(2*a + 2*b*x) - b*d^2*sin(2*a + 2*c + 2*b*x + 2*d*x) + b^2*d*sin(2*a - 2*
c + 2*b*x - 2*d*x) + b^2*d*sin(2*a + 2*c + 2*b*x + 2*d*x) + 2*b^2*d*sin(2*
a + 2*b*x) + 2*b*d^2*sin(2*c + 2*d*x) + 4*b*d^3*x - 4*b^3*d*x)/(16*b*d*(b^
2 - d^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.53

$$\int \cos^2(c+dx)\sin^2(a+bx)\,dx = \frac{2\cos(bx+a)\sin(bx+a)\sin(dx+c)^2 b^2d-2\cos(bx+a)\sin(bx+a)b^2d+\cos(bx+a)\sin(bx+a)d^3}{4bd(b^2-d^2)}$$

input
```
int(cos(d*x+c)^2*sin(b*x+a)^2,x)
```

output
```
(2*cos(a + b*x)*sin(a + b*x)*sin(c + d*x)**2*b**2*d - 2*cos(a + b*x)*sin(a
 + b*x)*b**2*d + cos(a + b*x)*sin(a + b*x)*d**3 - 2*cos(c + d*x)*sin(a + b
*x)**2*sin(c + d*x)*b*d**2 + cos(c + d*x)*sin(c + d*x)*b**3 + b**3*d*x - b
*d**3*x)/(4*b*d*(b**2 - d**2))
```

3.193 $\int \cos(c+dx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \cos(c+dx)\sin^2(a+bx)\,dx = -\frac{\sin(2a-c+(2b-d)x)}{4(2b-d)} + \frac{\sin(c+dx)}{2d} - \frac{\sin(2a+c+(2b+d)x)}{4(2b+d)}$$

output
```
-1/4*sin(2*a-c+(2*b-d)*x)/(2*b-d)+1/2*sin(d*x+c)/d-sin(2*a+c+(2*b+d)*x)/(8
*b+4*d)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \cos(c+dx)\sin^2(a+bx)\,dx = \frac{1}{4}\left(\frac{2\cos(dx)\sin(c)}{d} + \frac{2\cos(c)\sin(dx)}{d} - \frac{\sin(2a-c+2bx-dx)}{2b-d} - \frac{\sin(2a+c+2bx+dx)}{2b+d}\right)$$

input
```
Integrate[Cos[c + d*x]*Sin[a + b*x]^2,x]
```

output
```
((2*Cos[d*x]*Sin[c])/d + (2*Cos[c]*Sin[d*x])/d - Sin[2*a - c + 2*b*x - d*x
]/(2*b - d) - Sin[2*a + c + 2*b*x + d*x]/(2*b + d))/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos(c+dx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(-\frac{1}{4}\cos(2a+x(2b-d)-c)-\frac{1}{4}\cos(2a+x(2b+d)+c)+\frac{1}{2}\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(2a+x(2b-d)-c)}{4(2b-d)}-\frac{\sin(2a+x(2b+d)+c)}{4(2b+d)}+\frac{\sin(c+dx)}{2d}$$

input
```
Int[Cos[c + d*x]*Sin[a + b*x]^2,x]
```

output
```
-1/4*Sin[2*a - c + (2*b - d)*x]/(2*b - d) + Sin[c + d*x]/(2*d) - Sin[2*a +
 c + (2*b + d)*x]/(4*(2*b + d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

method	result
default	$\frac{\sin(dx+c)}{2d}-\frac{\sin(2a-c+(2b-d)x)}{4(2b-d)}-\frac{\sin(2a+c+(2b+d)x)}{4(2b+d)}$
parallelrisch	$\frac{(-2bd-d^2)\sin(2a-c+(2b-d)x)+(-2bd+d^2)\sin(2a+c+(2b+d)x)+(8b^2-2d^2)\sin(dx+c)}{16b^2d-4d^3}$
risch	$\frac{2\sin(dx+c)b^2}{d(2b-d)(2b+d)}-\frac{d\sin(dx+c)}{2(2b-d)(2b+d)}-\frac{\sin(2bx-dx+2a-c)b}{2(2b-d)(2b+d)}-\frac{d\sin(2bx-dx+2a-c)}{4(2b-d)(2b+d)}-\frac{\sin(2bx+dx+2a+c)b}{2(2b-d)(2b+d)}+$
norman	$\frac{-\frac{4b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{4b^2-d^2}+\frac{4b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{4b^2-d^2}+\frac{4b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4b^2-d^2}-\frac{4b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4b^2-d^2}+\frac{4b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(4b^2-d^2\right)}+}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^2}$
orering	$-\frac{(16b^4+3d^4)\left(-d\sin(dx+c)\sin(bx+a)^2+2\cos(dx+c)\sin(bx+a)b\cos(bx+a)\right)}{d^2(16b^4-8b^2d^2+d^4)}-$

input
```
int(cos(d*x+c)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(d*x+c)/d-1/4*sin(2*a-c+(2*b-d)*x)/(2*b-d)-1/4/(2*b+d)*sin(2*a+c+(2
*b+d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \cos(c+dx)\sin^2(a+bx)\,dx = -\frac{2\,bd\cos\left(bx+a\right)\cos\left(dx+c\right)\sin\left(bx+a\right)-\left(d^2\cos\left(bx+a\right)^2+2\,b^2-d^2\right)\sin\left(dx+c\right)}{4\,b^2d-d^3}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b*d*cos(b*x + a)*cos(d*x + c)*sin(b*x + a) - (d^2*cos(b*x + a)^2 + 2*b
^2 - d^2)*sin(d*x + c))/(4*b^2*d - d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(49) = 98$.

Time = 0.77 (sec) , antiderivative size = 410, normalized size of antiderivative = 6.03

$$\int \cos(c+dx)\sin^2(a+bx)\,dx$$

$$= \begin{cases} x\sin^2(a)\cos(c) \\ \frac{x\sin^2\left(a-\frac{dx}{2}\right)\cos(c+dx)}{4} + \frac{x\sin\left(a-\frac{dx}{2}\right)\sin(c+dx)\cos\left(a-\frac{dx}{2}\right)}{2} - \frac{x\cos^2\left(a-\frac{dx}{2}\right)\cos(c+dx)}{4} + \frac{\sin^2\left(a-\frac{dx}{2}\right)\sin(c+dx)}{d} - \frac{\sin\left(a-\right.}{} \\ \frac{x\sin^2\left(a+\frac{dx}{2}\right)\cos(c+dx)}{4} - \frac{x\sin\left(a+\frac{dx}{2}\right)\sin(c+dx)\cos\left(a+\frac{dx}{2}\right)}{2} - \frac{x\cos^2\left(a+\frac{dx}{2}\right)\cos(c+dx)}{4} - \frac{3\sin\left(a+\frac{dx}{2}\right)\cos\left(a+\frac{dx}{2}\right)\cos(c+d}{2d} \\ \left(\frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b}\right)\cos(c) \\ \frac{2b^2\sin^2(a+bx)\sin(c+dx)}{4b^2d-d^3} + \frac{2b^2\sin(c+dx)\cos^2(a+bx)}{4b^2d-d^3} - \frac{2bd\sin(a+bx)\cos(a+bx)\cos(c+dx)}{4b^2d-d^3} - \frac{d^2\sin^2(a+bx)\sin(c+dx)}{4b^2d-d^3} \end{cases}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a)**2*cos(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x/2)**2*
cos(c + d*x)/4 + x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)/2 - x*cos(a
- d*x/2)**2*cos(c + d*x)/4 + sin(a - d*x/2)**2*sin(c + d*x)/d - sin(a - d*
x/2)*cos(a - d*x/2)*cos(c + d*x)/(2*d), Eq(b, -d/2)), (x*sin(a + d*x/2)**2
*cos(c + d*x)/4 - x*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)/2 - x*cos(a
 + d*x/2)**2*cos(c + d*x)/4 - 3*sin(a + d*x/2)*cos(a + d*x/2)*cos(c + d*x)
/(2*d) + sin(c + d*x)*cos(a + d*x/2)**2/d, Eq(b, d/2)), ((x*sin(a + b*x)**
2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a + b*x)/(2*b))*cos(c), Eq(d,
 0)), (2*b**2*sin(a + b*x)**2*sin(c + d*x)/(4*b**2*d - d**3) + 2*b**2*sin(
c + d*x)*cos(a + b*x)**2/(4*b**2*d - d**3) - 2*b*d*sin(a + b*x)*cos(a + b*
x)*cos(c + d*x)/(4*b**2*d - d**3) - d**2*sin(a + b*x)**2*sin(c + d*x)/(4*b
**2*d - d**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(62) = 124$.

Time = 0.08 (sec) , antiderivative size = 371, normalized size of antiderivative = 5.46

$$\int \cos(c+dx)\sin^2(a+bx)\,dx = -\frac{(2\,bd\sin(c) - d^2\sin(c))\cos((2\,b+d)x + 2\,a + 2\,c) - (2\,bd\sin(c) - d^2\sin(c))\cos((2\,b+d)x + 2\,a) -$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*((2*b*d*sin(c) - d^2*sin(c))*cos((2*b + d)*x + 2*a + 2*c) - (2*b*d*si
n(c) - d^2*sin(c))*cos((2*b + d)*x + 2*a) - (2*b*d*sin(c) + d^2*sin(c))*co
s(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*sin(c) + d^2*sin(c))*cos(-(2*b - d)*x
 - 2*a) - 2*(4*b^2*sin(c) - d^2*sin(c))*cos(d*x + 2*c) + 2*(4*b^2*sin(c) -
 d^2*sin(c))*cos(d*x) - (2*b*d*cos(c) - d^2*cos(c))*sin((2*b + d)*x + 2*a
+ 2*c) - (2*b*d*cos(c) - d^2*cos(c))*sin((2*b + d)*x + 2*a) + (2*b*d*cos(c
) + d^2*cos(c))*sin(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*cos(c) + d^2*cos(c)
)*sin(-(2*b - d)*x - 2*a) + 2*(4*b^2*cos(c) - d^2*cos(c))*sin(d*x + 2*c) +
 2*(4*b^2*cos(c) - d^2*cos(c))*sin(d*x))/((cos(c)^2 + sin(c)^2)*d^3 - 4*(b
^2*cos(c)^2 + b^2*sin(c)^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \cos(c+dx)\sin^2(a+bx)\,dx = -\frac{\sin(2\,bx + dx + 2\,a + c)}{4\,(2\,b+d)} - \frac{\sin(2\,bx - dx + 2\,a - c)}{4\,(2\,b-d)} + \frac{\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/4*sin(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*sin(2*b*x - d*x + 2*a - c)
/(2*b - d) + 1/2*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 17.78 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.54

$$\int \cos(c+dx)\sin^2(a+bx)\,dx = \frac{\sin(c+d\,x)}{2\,d} - \frac{b\,(2\,d\,\sin(2\,a+c+2\,b\,x+d\,x)+2\,d\,\sin(2\,a-c+2\,b\,x-d\,x))-d^2\,\sin(2\,a+c+2\,b\,x+d\,x)+d^2\,\mathrm{s}}{16\,b^2\,d-4\,d^3}$$

input
```
int(cos(c + d*x)*sin(a + b*x)^2,x)
```

output
```
sin(c + d*x)/(2*d) - (b*(2*d*sin(2*a + c + 2*b*x + d*x) + 2*d*sin(2*a - c
+ 2*b*x - d*x)) - d^2*sin(2*a + c + 2*b*x + d*x) + d^2*sin(2*a - c + 2*b*x
 - d*x))/(16*b^2*d - 4*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \cos(c+dx)\sin^2(a+bx)\,dx = \frac{-2\cos(bx+a)\cos(dx+c)\sin(bx+a)\,bd-\sin(bx+a)^2\sin(dx+c)\,d^2+2\sin(dx+c)\,b^2}{d\,(4b^2-d^2)}$$

input
```
int(cos(d*x+c)*sin(b*x+a)^2,x)
```

output
```
( - 2*cos(a + b*x)*cos(c + d*x)*sin(a + b*x)*b*d - sin(a + b*x)**2*sin(c +
 d*x)*d**2 + 2*sin(c + d*x)*b**2)/(d*(4*b**2 - d**2))
```

3.194 $\int \sec(c+dx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 157

$$\int \sec(c+dx)\sin^2(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{ie^{-2ia-2ibx+i(c+dx)}\operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{b}{d},\frac{3}{2}-\frac{b}{d},-e^{2i(c+dx)}\right)}{2(2b-d)} + \frac{ie^{2ia+2ibx+i(c+dx)}\operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{b}{d},\frac{3}{2}+\frac{b}{d},-e^{2i(c+dx)}\right)}{2(2b+d)}$$

output
```
1/2*arctanh(sin(d*x+c))/d-1/2*I*exp(-2*I*a-2*I*b*x+I*(d*x+c))*hypergeom([1
, 1/2-b/d],[3/2-b/d],-exp(2*I*(d*x+c)))/(2*b-d)+1/2*I*exp(2*I*a+2*I*b*x+I*
(d*x+c))*hypergeom([1, 1/2+b/d],[3/2+b/d],-exp(2*I*(d*x+c)))/(2*b+d)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.22

$$\int \sec(c+dx)\sin^2(a+bx)\,dx$$
$$= -\frac{ie^{-i(2a-c+2bx-dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{b}{d},\frac{3}{2}-\frac{b}{d},-e^{2i(c+dx)}\right)}{2(2b-d)} + \frac{ie^{i(2a+c+2bx+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{b}{d},\frac{3}{2}+\frac{b}{d},-e^{2i(c+dx)}\right)}{2(2b+d)} + \frac{-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

input
```
Integrate[Sec[c + d*x]*Sin[a + b*x]^2,x]
```

output
```
((-1/2*I)*Hypergeometric2F1[1, 1/2 - b/d, 3/2 - b/d, -E^((2*I)*(c + d*x))]
)/((2*b - d)*E^(I*(2*a - c + 2*b*x - d*x))) + ((I/2)*E^(I*(2*a + c + 2*b*x
 + d*x))*Hypergeometric2F1[1, 1/2 + b/d, 3/2 + b/d, -E^((2*I)*(c + d*x))])
/(2*b + d) + (-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]])/(2*d)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec(c+dx)\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \sin^2(a+bx)\sec(c+dx)dx$$

input
```
Int[Sec[c + d*x]*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \sec(dx+c)\sin(bx+a)^2\, dx$$

input
```
int(sec(d*x+c)*sin(b*x+a)^2,x)
```

output
```
int(sec(d*x+c)*sin(b*x+a)^2,x)
```

Fricas [F]

$$\int \sec(c+dx)\sin^2(a+bx)\, dx = \int \sec(dx+c)\sin(bx+a)^2\, dx$$

input
```
integrate(sec(d*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sec(d*x + c), x)
```

Sympy [F]

$$\int \sec(c+dx)\sin^2(a+bx)\, dx = \int \sin^2(a+bx)\sec(c+dx)\, dx$$

input
```
integrate(sec(d*x+c)*sin(b*x+a)**2,x)
```

output
```
Integral(sin(a + b*x)**2*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)\sin^2(a+bx)\,dx = \int \sec(dx+c)\sin(bx+a)^2\,dx$$

input
```
integrate(sec(d*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int \sec(c+dx)\sin^2(a+bx)\,dx = \int \sec(dx+c)\sin(bx+a)^2\,dx$$

input
```
integrate(sec(d*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)\sin^2(a+bx)\,dx = \int \frac{\sin(a+b\,x)^2}{\cos(c+d\,x)}\,dx$$

input
```
int(sin(a + b*x)^2/cos(c + d*x),x)
```

output
```
int(sin(a + b*x)^2/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)\sin^2(a+bx)\,dx = \int \frac{\sin(bx+a)^2}{\cos(dx+c)}dx$$

input
```
int(sec(d*x+c)*sin(b*x+a)^2,x)
```

output
```
int(sin(a + b*x)**2/cos(c + d*x),x)
```

3.195 $\int \sec^2(c+dx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \sec^2(c+dx)\sin^2(a+bx)\,dx = -\frac{ie^{-2ia-2ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{b}{d},2-\frac{b}{d},-e^{2i(c+dx)}\right)}{2(b-d)} + \frac{ie^{2ia+2ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,\frac{b+d}{d},2+\frac{b}{d},-e^{2i(c+dx)}\right)}{2(b+d)} + \frac{\tan(c+dx)}{2d}$$

output
```
-1/2*I*exp(-2*I*a-2*I*b*x+2*I*(d*x+c))*hypergeom([2, 1-b/d],[2-b/d],-exp(2
*I*(d*x+c)))/(b-d)+1/2*I*exp(2*I*a+2*I*b*x+2*I*(d*x+c))*hypergeom([2, (b+d
)/d],[2+b/d],-exp(2*I*(d*x+c)))/(b+d)+1/2*tan(d*x+c)/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 295 vs. 2(144) = 288.

Time = 2.74 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.05

$$\int \sec^2(c+dx)\sin^2(a+bx)\,dx = \frac{-\frac{ie^{-2i(a-c)}\left(\frac{be^{-2i(b-d)x}\,\text{Hypergeometric2F1}\left(1,1-\frac{b}{d},2-\frac{b}{d},-e^{2i(c+dx)}\right)}{b-d}-e^{-2ibx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{d},1-\frac{b}{d},-e^{2i(c+dx)}\right)\right)}{1+e^{2ic}}+\frac{ie^{2i(a+c)}\left((\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[Sec[c + d*x]^2*Sin[a + b*x]^2,x]
```

output
```
(((-I)*((b*Hypergeometric2F1[1, 1 - b/d, 2 - b/d, -E^((2*I)*(c + d*x))])/(
(b - d)*E^((2*I)*(b - d)*x)) - Hypergeometric2F1[1, -(b/d), 1 - b/d, -E^((
2*I)*(c + d*x))]/E^((2*I)*b*x)))/(E^((2*I)*(a - c))*(1 + E^((2*I)*c))) + (
I*E^((2*I)*(a + c))*((b + d)*E^((2*I)*b*x)*Hypergeometric2F1[1, b/d, (b +
d)/d, -E^((2*I)*(c + d*x))] - b*E^((2*I)*(b + d)*x)*Hypergeometric2F1[1, (
b + d)/d, 2 + b/d, -E^((2*I)*(c + d*x))]))/((b + d)*(1 + E^((2*I)*c))) + S
ec[c]*Sec[c + d*x]*Sin[d*x] - Cos[2*a]*Cos[2*b*x]*Sec[c]*Sec[c + d*x]*Sin[
d*x] + Sec[c]*Sec[c + d*x]*Sin[2*a]*Sin[2*b*x]*Sin[d*x])/(2*d)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec^2(c+dx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin^2(a+bx)\sec^2(c+dx)dx$$

input
```
Int[Sec[c + d*x]^2*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \sec (dx+c)^2 \sin (bx+a)^2 \, dx$$

input
```
int(sec(d*x+c)^2*sin(b*x+a)^2,x)
```

output
```
int(sec(d*x+c)^2*sin(b*x+a)^2,x)
```

Fricas [F]

$$\int \sec^2(c+dx) \sin^2(a+bx) \, dx = \int \sec (dx+c)^2 \sin (bx+a)^2 \, dx$$

input
```
integrate(sec(d*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sec(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^2(c+dx) \sin^2(a+bx) \, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**2*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^2(c+dx)\sin^2(a+bx)\,dx = \int \sec\left(dx+c\right)^2 \sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*((sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*cos(2*(b + d)*x + 2*a + 2*c)
- 2*(d*cos(2*(b + d)*x + 2*a + 2*c)^2 + 2*d*cos(2*(b + d)*x + 2*a + 2*c)*c
os(2*b*x + 2*a) + d*cos(2*b*x + 2*a)^2 + d*sin(2*(b + d)*x + 2*a + 2*c)^2
+ 2*d*sin(2*(b + d)*x + 2*a + 2*c)*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^2
)*integrate((b*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + b*sin(2*(b + d)*x + 2*a
 + 2*c)*sin(4*b*x + 4*a) + b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b*cos(4*
b*x + 4*a) - b)*cos(2*(b + d)*x + 2*a + 2*c) - b*cos(2*b*x + 2*a))/(d*cos(
2*(b + d)*x + 2*a + 2*c)^2 + 2*d*cos(2*(b + d)*x + 2*a + 2*c)*cos(2*b*x +
2*a) + d*cos(2*b*x + 2*a)^2 + d*sin(2*(b + d)*x + 2*a + 2*c)^2 + 2*d*sin(2
*(b + d)*x + 2*a + 2*c)*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^2), x) - (co
s(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*sin(2*(b + d)*x + 2*a + 2*c) + co
s(2*b*x + 2*a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin(
2*b*x + 2*a))/(d*cos(2*(b + d)*x + 2*a + 2*c)^2 + 2*d*cos(2*(b + d)*x + 2*
a + 2*c)*cos(2*b*x + 2*a) + d*cos(2*b*x + 2*a)^2 + d*sin(2*(b + d)*x + 2*a
 + 2*c)^2 + 2*d*sin(2*(b + d)*x + 2*a + 2*c)*sin(2*b*x + 2*a) + d*sin(2*b*
x + 2*a)^2)
```

Giac [F]

$$\int \sec^2(c+dx)\sin^2(a+bx)\,dx = \int \sec\left(dx+c\right)^2 \sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)\sin^2(a+bx)\,dx=\int \frac{\sin(a+b\,x)^2}{\cos(c+d\,x)^2}\,dx$$

input
```
int(sin(a + b*x)^2/cos(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)^2/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)\sin^2(a+bx)\,dx=\int \sec(dx+c)^2\sin(bx+a)^2\,dx$$

input
```
int(sec(d*x+c)^2*sin(b*x+a)^2,x)
```

output
```
int(sec(c + d*x)**2*sin(a + b*x)**2,x)
```

3.196 $\int \sec^3(c+dx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 174

$$\int \sec^3(c+dx)\sin^2(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{4d} - \frac{2ie^{-2ia-2ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3}{2}-\frac{b}{d},\frac{5}{2}-\frac{b}{d},-e^{2i(c+dx)}\right)}{2b-3d} + \frac{2ie^{2ia+2ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3}{2}+\frac{b}{d},\frac{5}{2}+\frac{b}{d},-e^{2i(c+dx)}\right)}{2b+3d} + \frac{\sec(c+dx)\tan(c+dx)}{4d}$$

output
```
1/4*arctanh(sin(d*x+c))/d-2*I*exp(-2*I*a-2*I*b*x+3*I*(d*x+c))*hypergeom([3
, 3/2-b/d],[5/2-b/d],-exp(2*I*(d*x+c)))/(2*b-3*d)+2*I*exp(2*I*a+2*I*b*x+3*
I*(d*x+c))*hypergeom([3, 3/2+b/d],[5/2+b/d],-exp(2*I*(d*x+c)))/(2*b+3*d)+1
/4*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 4.64 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.36

$$\int \sec^3(c+dx)\sin^2(a+bx)\,dx$$
$$= \frac{i(2b+d)e^{-i(2a-c+2bx-dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{b}{d},\frac{3}{2}-\frac{b}{d},-e^{2i(c+dx)}\right)+i(-2b+d)e^{i(2a+c+(2b+d)x)}\,\text{Hy}}{}$$

input
```
Integrate[Sec[c + d*x]^3*Sin[a + b*x]^2,x]
```

output
```
((I*(2*b + d)*Hypergeometric2F1[1, 1/2 - b/d, 3/2 - b/d, -E^((2*I)*(c + d*
x))])/E^(I*(2*a - c + 2*b*x - d*x)) + I*(-2*b + d)*E^(I*(2*a + c + (2*b +
d)*x))*Hypergeometric2F1[1, 1/2 + b/d, 3/2 + b/d, -E^((2*I)*(c + d*x))] -
d*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + d*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] - ((2*b - d)*Cos[a - c + b*x - d*x] + (2*b + d)*Cos[a + c +
(b + d)*x])*Sec[c + d*x]^2*Sin[a + b*x])/(4*d^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec^3(c+dx)\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \sin^2(a+bx)\sec^3(c+dx)dx$$

input
```
Int[Sec[c + d*x]^3*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \sec(dx+c)^3 \sin(bx+a)^2 \, dx$$

input
```
int(sec(d*x+c)^3*sin(b*x+a)^2,x)
```

output
```
int(sec(d*x+c)^3*sin(b*x+a)^2,x)
```

Fricas [F]

$$\int \sec^3(c+dx)\sin^2(a+bx)\,dx = \int \sec(dx+c)^3 \sin(bx+a)^2 \, dx$$

input
```
integrate(sec(d*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sec(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^3(c+dx)\sin^2(a+bx)\,dx = \int \sec\left(dx+c\right)^3\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*((2*b - d)*cos(2*b*x + 2*a)*sin((4*b + d)*x + 4*a + c) + 2*d*cos(2*b*
x + 2*a)*sin((2*b + d)*x + 2*a + c) - (2*b - d)*cos((4*b + d)*x + 4*a + c)
*sin(2*b*x + 2*a) - 2*d*cos((2*b + d)*x + 2*a + c)*sin(2*b*x + 2*a) + (2*b
 - d)*cos(3*d*x + 3*c)*sin(2*b*x + 2*a) + (2*b + d)*cos(d*x + c)*sin(2*b*x
 + 2*a) - (2*b - d)*cos(2*b*x + 2*a)*sin(3*d*x + 3*c) - (2*b + d)*cos(2*b*
x + 2*a)*sin(d*x + c) - (2*(2*b + d)*sin(2*(b + d)*x + 2*a + 2*c) + (2*b +
 d)*sin(2*b*x + 2*a))*cos((4*b + 3*d)*x + 4*a + 3*c) + 2*(2*d*sin(2*(b + d
)*x + 2*a + 2*c) + d*sin(2*b*x + 2*a))*cos((2*b + 3*d)*x + 2*a + 3*c) + ((
2*b + d)*sin((4*b + 3*d)*x + 4*a + 3*c) + (2*b - d)*sin((4*b + d)*x + 4*a
+ c) - 2*d*sin((2*b + 3*d)*x + 2*a + 3*c) + 2*d*sin((2*b + d)*x + 2*a + c)
 - (2*b - d)*sin(3*d*x + 3*c) - (2*b + d)*sin(d*x + c))*cos(2*(b + 2*d)*x
+ 2*a + 4*c) + 2*((2*b - d)*sin((4*b + d)*x + 4*a + c) + 2*d*sin((2*b + d)
*x + 2*a + c) - (2*b - d)*sin(3*d*x + 3*c) - (2*b + d)*sin(d*x + c))*cos(2
*(b + d)*x + 2*a + 2*c) + 4*(d^2*cos(2*(b + 2*d)*x + 2*a + 4*c)^2 + 4*d^2*
cos(2*(b + d)*x + 2*a + 2*c)^2 + 4*d^2*cos(2*(b + d)*x + 2*a + 2*c)*cos(2*
b*x + 2*a) + d^2*cos(2*b*x + 2*a)^2 + d^2*sin(2*(b + 2*d)*x + 2*a + 4*c)^2
 + 4*d^2*sin(2*(b + d)*x + 2*a + 2*c)^2 + 4*d^2*sin(2*(b + d)*x + 2*a + 2*
c)*sin(2*b*x + 2*a) + d^2*sin(2*b*x + 2*a)^2 + 2*(2*d^2*cos(2*(b + d)*x +
2*a + 2*c) + d^2*cos(2*b*x + 2*a))*cos(2*(b + 2*d)*x + 2*a + 4*c) + 2*(2*d
^2*sin(2*(b + d)*x + 2*a + 2*c) + d^2*sin(2*b*x + 2*a))*sin(2*(b + 2*d)...
```

Giac [F]

$$\int \sec^3(c+dx)\sin^2(a+bx)\,dx = \int \sec\left(dx+c\right)^3\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)\sin^2(a+bx)\,dx = \int \frac{\sin(a+b\,x)^2}{\cos(c+d\,x)^3}\,dx$$

input
```
int(sin(a + b*x)^2/cos(c + d*x)^3,x)
```

output
```
int(sin(a + b*x)^2/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sec^3(c+dx)\sin^2(a+bx)\,dx = \int \sec(dx+c)^3\sin(bx+a)^2\,dx$$

input
```
int(sec(d*x+c)^3*sin(b*x+a)^2,x)
```

output
```
int(sec(c + d*x)**3*sin(a + b*x)**2,x)
```

3.197 $\int \cos^3(c+dx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 195

$$\begin{aligned}\int \cos^3(c+dx)\sin^3(a+bx)\,dx =& -\frac{3\cos(a-3c+(b-3d)x)}{32(b-3d)}-\frac{9\cos(a-c+(b-d)x)}{32(b-d)} \\ &+\frac{\cos(3(a-c)+3(b-d)x)}{96(b-d)} \\ &+\frac{3\cos(3a-c+(3b-d)x)}{32(3b-d)}-\frac{9\cos(a+c+(b+d)x)}{32(b+d)} \\ &+\frac{\cos(3(a+c)+3(b+d)x)}{96(b+d)} \\ &+\frac{3\cos(3a+c+(3b+d)x)}{32(3b+d)} \\ &-\frac{3\cos(a+3c+(b+3d)x)}{32(b+3d)}\end{aligned}$$

output

```
-3*cos(a-3*c+(b-3*d)*x)/(32*b-96*d)-9*cos(a-c+(b-d)*x)/(32*b-32*d)+cos(3*a
-3*c+3*(b-d)*x)/(96*b-96*d)+3*cos(3*a-c+(3*b-d)*x)/(96*b-32*d)-9*cos(a+c+(
b+d)*x)/(32*b+32*d)+cos(3*a+3*c+3*(b+d)*x)/(96*b+96*d)+3*cos(3*a+c+(3*b+d)
*x)/(96*b+32*d)-3*cos(a+3*c+(b+3*d)*x)/(32*b+96*d)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

$$\int \cos^3(c+dx)\sin^3(a+bx)\,dx = \frac{1}{96}\left(-\frac{9\cos(a-3c+bx-3dx)}{b-3d} - \frac{27\cos(a-c+bx-dx)}{b-d} + \frac{\cos(3(a-c+bx-dx))}{b-d} + \frac{9\cos(3a-c+3bx-dx)}{3b-d} + \frac{9\cos(3a+c+3bx+dx)}{3b+d} - \frac{9\cos(a+3c+bx+3dx)}{b+3d} - \frac{27\cos(a+c+(b+d)x)}{b+d} + \frac{\cos(3(a+c+(b+d)x))}{b+d}\right)$$

input
```
Integrate[Cos[c + d*x]^3*Sin[a + b*x]^3,x]
```

output
```
((-9*Cos[a - 3*c + b*x - 3*d*x])/(b - 3*d) - (27*Cos[a - c + b*x - d*x])/(
b - d) + Cos[3*(a - c + b*x - d*x)]/(b - d) + (9*Cos[3*a - c + 3*b*x - d*x
])/(3*b - d) + (9*Cos[3*a + c + 3*b*x + d*x])/(3*b + d) - (9*Cos[a + 3*c +
 b*x + 3*d*x])/(b + 3*d) - (27*Cos[a + c + (b + d)*x])/(b + d) + Cos[3*(a
+ c + (b + d)*x)]/(b + d))/96
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \; 5085$$

$$\int \left(\frac{3}{32}\sin(a+x(b-3d)-3c)+\frac{9}{32}\sin(a+x(b-d)-c)-\frac{1}{32}\sin(3(a-c)+3x(b-d))-\frac{3}{32}\sin(3a+x(3b-\right.$$

$$\Big\downarrow 2009$$

$$-\frac{3\cos(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\cos(a+x(b-d)-c)}{32(b-d)}+\frac{\cos(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\cos(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\cos(a+x(b+d)+c)}{32(b+d)}+\frac{\cos(3(a+c)+3x(b+d))}{96(b+d)}+\frac{3\cos(3a+x(3b+d)+c)}{32(3b+d)}-\frac{3\cos(a+x(b+3d)+3c)}{32(b+3d)}$$

input
```
Int[Cos[c + d*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-3*Cos[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Cos[a - c + (b - d)*x]
)/(32*(b - d)) + Cos[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Cos[3*a -
c + (3*b - d)*x])/(32*(3*b - d)) - (9*Cos[a + c + (b + d)*x])/(32*(b + d))
 + Cos[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Cos[3*a + c + (3*b + d)*
x])/(32*(3*b + d)) - (3*Cos[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 40.98 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.97

method	result
default	$-\frac{3\cos(a-3c+(b-3d)x)}{32(b-3d)}-\frac{9\cos(a-c+(b-d)x)}{32(b-d)}-\frac{9\cos(a+c+(b+d)x)}{32(b+d)}-\frac{3\cos(a+3c+(b+3d)x)}{32(b+3d)}+\frac{\cos((3b-3d)x+3a-}{96b-96d}$
parallelrisch	$\frac{-36\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\left(b^2-\frac{61d^2}{9}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\frac{40\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2d^2}{3}+3b^2-7d^2\right)b^3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6-216d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(b^2-\frac{7d}{3}\right.\right.}{}$
risch	Expression too large to display
orering	Expression too large to display

input
```
int(cos(d*x+c)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/32*cos(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cos(a-c+(b-d)*x)/(b-d)-9/32/(b+d)*
cos(a+c+(b+d)*x)-3/32/(b+3*d)*cos(a+3*c+(b+3*d)*x)+1/32/(3*b-3*d)*cos((3*b
-3*d)*x+3*a-3*c)+3/32/(3*b-d)*cos(3*a-c+(3*b-d)*x)+3/32/(3*b+d)*cos(3*a+c+
(3*b+d)*x)+1/32*cos((3*b+3*d)*x+3*a+3*c)/(3*b+3*d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.35

$$\int\cos^3(c+dx)\sin^3(a+bx)\,dx = \frac{\left((9\,b^5-82\,b^3d^2+9\,bd^4)\cos\left(bx+a\right)^3-3\left(9\,b^5-28\,b^3d^2+3\,bd^4\right)\cos\left(bx+a\right)\right)\cos\left(dx+c\right)^3+\left(122\,b^2d^3\right.}{}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/3*(((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cos(b*x + a)^3 - 3*(9*b^5 - 28*b^3*d^
2 + 3*b*d^4)*cos(b*x + a))*cos(d*x + c)^3 + (122*b^2*d^3 - 18*d^5 - 2*(b^2
*d^3 - 9*d^5)*cos(b*x + a)^2 - (63*b^4*d - 88*b^2*d^3 + 9*d^5 - (9*b^4*d -
 82*b^2*d^3 + 9*d^5)*cos(b*x + a)^2)*cos(d*x + c)^2)*sin(b*x + a)*sin(d*x
+ c) - 6*((b^3*d^2 - 9*b*d^4)*cos(b*x + a)^3 - 3*(7*b^3*d^2 - 3*b*d^4)*cos
(b*x + a))*cos(d*x + c))/(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3580 vs. $2(172) = 344$.

Time = 17.76 (sec) , antiderivative size = 3580, normalized size of antiderivative = 18.36

$$\int \cos^3(c + dx)\sin^3(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**3*sin(b*x+a)**3,x)
```

output
```
Piecewise((x*sin(a)**3*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-9*x*sin(a - 3*d*
x)**3*sin(c + d*x)**2*cos(c + d*x)/32 + 3*x*sin(a - 3*d*x)**3*cos(c + d*x)
**3/32 - 3*x*sin(a - 3*d*x)**2*sin(c + d*x)**3*cos(a - 3*d*x)/32 + 9*x*sin
(a - 3*d*x)**2*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/32 - 9*x*sin(a
- 3*d*x)*sin(c + d*x)**2*cos(a - 3*d*x)**2*cos(c + d*x)/32 + 3*x*sin(a - 3
*d*x)*cos(a - 3*d*x)**2*cos(c + d*x)**3/32 - 3*x*sin(c + d*x)**3*cos(a - 3
*d*x)**3/32 + 9*x*sin(c + d*x)*cos(a - 3*d*x)**3*cos(c + d*x)**2/32 - sin(
a - 3*d*x)**3*sin(c + d*x)**3/(12*d) - 13*sin(a - 3*d*x)**3*sin(c + d*x)*c
os(c + d*x)**2/(320*d) + 3*sin(a - 3*d*x)**2*sin(c + d*x)**2*cos(a - 3*d*x
)*cos(c + d*x)/(20*d) + 101*sin(a - 3*d*x)**2*cos(a - 3*d*x)*cos(c + d*x)*
*3/(320*d) - 27*sin(a - 3*d*x)*sin(c + d*x)**3*cos(a - 3*d*x)**2/(320*d) +
 51*sin(c + d*x)**2*cos(a - 3*d*x)**3*cos(c + d*x)/(320*d) + cos(a - 3*d*x
)**3*cos(c + d*x)**3/(5*d), Eq(b, -3*d)), (3*x*sin(a - d*x)**3*sin(c + d*x
)**2*cos(c + d*x)/16 + 5*x*sin(a - d*x)**3*cos(c + d*x)**3/16 + 3*x*sin(a
- d*x)**2*sin(c + d*x)**3*cos(a - d*x)/16 + 9*x*sin(a - d*x)**2*sin(c + d*
x)*cos(a - d*x)*cos(c + d*x)**2/16 + 9*x*sin(a - d*x)*sin(c + d*x)**2*cos(
a - d*x)**2*cos(c + d*x)/16 + 3*x*sin(a - d*x)*cos(a - d*x)**2*cos(c + d*x
)**3/16 + 5*x*sin(c + d*x)**3*cos(a - d*x)**3/16 + 3*x*sin(c + d*x)*cos(a
- d*x)**3*cos(c + d*x)**2/16 + sin(a - d*x)**3*sin(c + d*x)**3/(16*d) + 3*
sin(a - d*x)**3*sin(c + d*x)*cos(c + d*x)**2/(16*d) + sin(a - d*x)**2*c...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2612 vs. $2(179) = 358$.

Time = 0.45 (sec) , antiderivative size = 2612, normalized size of antiderivative = 13.39

$$\int \cos^3(c + dx) \sin^3(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/192*(9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*b^2*d
^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((3*b + d)*x + 3*a +
4*c) + 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*b^2*d
^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((3*b + d)*x + 3*a -
2*c) + 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2*d
^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*cos(-(3*b - d)*x - 3*a +
 4*c) + 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2*
d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*cos(-(3*b - d)*x - 3*a
- 2*c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*
b^2*d^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*cos((b + 3*d)*x + a +
6*c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^
2*d^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*cos((b + 3*d)*x + a) + (
9*b^5*cos(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3
*c) + 9*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos(3*(b + d)*x + 3*a + 6*c) + (9
*b^5*cos(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*
c) + 9*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos(3*(b + d)*x + 3*a) - 27*(9*b^5
*cos(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) +
 9*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((b + d)*x + a + 4*c) - 27*(9*b^5*c
os(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9
*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5*...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93

$$\int \cos^3(c+dx)\sin^3(a+bx)\,dx = \frac{\cos(3\,bx+3\,dx+3\,a+3\,c)}{96\,(b+d)} + \frac{3\,\cos(3\,bx+dx+3\,a+c)}{32\,(3\,b+d)} + \frac{3\,\cos(3\,bx-dx+3\,a-c)}{32\,(3\,b-d)} + \frac{\cos(3\,bx-3\,dx+3\,a-3\,c)}{96\,(b-d)} - \frac{3\,\cos(bx+3\,dx+a+3\,c)}{32\,(b+3\,d)} - \frac{9\,\cos(bx+dx+a+c)}{32\,(b+d)} - \frac{9\,\cos(bx-dx+a-c)}{32\,(b-d)} - \frac{3\,\cos(bx-3\,dx+a-3\,c)}{32\,(b-3\,d)}$$

input
```
integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/96*cos(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*cos(3*b*x + d*x + 3*a +
 c)/(3*b + d) + 3/32*cos(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*cos(3*b*x
 - 3*d*x + 3*a - 3*c)/(b - d) - 3/32*cos(b*x + 3*d*x + a + 3*c)/(b + 3*d)
- 9/32*cos(b*x + d*x + a + c)/(b + d) - 9/32*cos(b*x - d*x + a - c)/(b - d
) - 3/32*cos(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

Mupad [B] (verification not implemented)

Time = 20.62 (sec) , antiderivative size = 951, normalized size of antiderivative = 4.88

$$\int \cos^3(c+dx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^3*sin(a + b*x)^3,x)
```

output
```
- exp(a*3i - c*1i + b*x*3i - d*x*1i)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/
(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + 3*b^2*
d - 9*b^3 - 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- a*2i - b*x*2
i)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2)
 - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 +
 64*d^4 - 640*b^2*d^2)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((9*b*d^2 + 3
*b^2*d - 9*b^3 - 3*d^3)/(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- a*6i - b
*x*6i)*(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^
2) - (exp(- a*2i - b*x*2i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4
 + 64*d^4 - 640*b^2*d^2) - (exp(- a*4i - b*x*4i)*(9*b*d^2 - 81*b^2*d - 81*
b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2)) - exp(a*3i - c*3i + b*x*3i
 - d*x*3i)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2
*d^2) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + b^2*d - b^3 - 9*d^3))/(192*b^4 +
1728*d^4 - 1920*b^2*d^2) - (exp(- a*2i - b*x*2i)*(9*b*d^2 - 27*b^2*d - 9*b
^3 + 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- a*4i - b*x*4i)*
(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2)
) - exp(a*3i + c*3i + b*x*3i + d*x*3i)*((9*b*d^2 + b^2*d - b^3 - 9*d^3)/(1
92*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- a*6i - b*x*6i)*(9*b*d^2 - b^2*d
 - b^3 + 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- a*2i - b*x*2
i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^...
```

Reduce [F]

$$\int \cos^3(c+dx)\sin^3(a+bx)\,dx = \int \cos(dx+c)^3\sin(bx+a)^3\,dx$$

input
```
int(cos(d*x+c)^3*sin(b*x+a)^3,x)
```

output
```
int(cos(d*x+c)^3*sin(b*x+a)^3,x)
```

3.198 $\int \cos^2(c+dx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 138

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx = -\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b} - \frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)} + \frac{\cos(3a-2c+(3b-2d)x)}{16(3b-2d)} - \frac{3\cos(a+2c+(b+2d)x)}{16(b+2d)} + \frac{\cos(3a+2c+(3b+2d)x)}{16(3b+2d)}$$

output
```
-3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b-3*cos(a-2*c+(b-2*d)*x)/(16*b-32*d)
+cos(3*a-2*c+(3*b-2*d)*x)/(48*b-32*d)-3*cos(a+2*c+(b+2*d)*x)/(16*b+32*d)+c
os(3*a+2*c+(3*b+2*d)*x)/(48*b+32*d)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx = \frac{1}{48}\left(-\frac{18\cos(a)\cos(bx)}{b}+\frac{2\cos(3a)\cos(3bx)}{b}-\frac{9\cos(a-2c+bx-2dx)}{b-2d}+\frac{3\cos(3a-2c+3bx-2dx)}{3b-2d}-\frac{9\cos(a+2c+bx+2dx)}{b+2d}+\frac{3\cos(3a+2c+3bx+2dx)}{3b+2d}+\frac{18\sin(a)\sin(bx)}{b}-\frac{2\sin(3a)\sin(3bx)}{b}\right)$$

input
```
Integrate[Cos[c + d*x]^2*Sin[a + b*x]^3,x]
```

output
```
((-18*Cos[a]*Cos[b*x])/b + (2*Cos[3*a]*Cos[3*b*x])/b - (9*Cos[a - 2*c + b*
x - 2*d*x])/(b - 2*d) + (3*Cos[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d) - (
9*Cos[a + 2*c + b*x + 2*d*x])/(b + 2*d) + (3*Cos[3*a + 2*c + 3*b*x + 2*d*x
])/(3*b + 2*d) + (18*Sin[a]*Sin[b*x])/b - (2*Sin[3*a]*Sin[3*b*x])/b)/48
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^2(c+dx)\,dx$$

$$\Big\downarrow \; 5085$$

$$\int \left(\frac{3}{16}\sin(a+x(b-2d)-2c)-\frac{1}{16}\sin(3a+x(3b-2d)-2c)+\frac{3}{16}\sin(a+x(b+2d)+2c)-\frac{1}{16}\sin(3a+x(3b$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}+\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}-\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}+\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}-\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}$$

input
```
Int[Cos[c + d*x]^2*Sin[a + b*x]^3,x]
```

output
```
(-3*Cos[a + b*x])/(8*b) + Cos[3*a + 3*b*x]/(24*b) - (3*Cos[a - 2*c + (b -
2*d)*x])/(16*(b - 2*d)) + Cos[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d))
- (3*Cos[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) + Cos[3*a + 2*c + (3*b + 2
*d)*x]/(16*(3*b + 2*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 12.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

method	result
default	$-\frac{3\cos(bx+a)}{8b}+\frac{\cos(3bx+3a)}{24b}-\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}-\frac{3\cos(a+2c+(b+2d)x)}{16(b+2d)}+\frac{\cos(3a-2c+(3b-2d)x)}{48b-32d}+\frac{\cos(3a$
parallelrisch	$\frac{9\left(b+\frac{2d}{3}\right)(b+2d)b(b-2d)\cos(3a-2c+(3b-2d)x)+9(b+2d)\left(b-\frac{2d}{3}\right)b(b-2d)\cos(3a+2c+(3b+2d)x)-81\left(b+\frac{2d}{3}\right)(b+2d)\left(b-\frac{2d}{3}\right.}{}$
risch	$-\frac{3\cos(bx+a)}{8b}-\frac{27\cos(bx-2dx+a-2c)b^3}{16(b+2d)(3b+2d)(3b-2d)(b-2d)}-\frac{27\cos(bx-2dx+a-2c)b^2d}{8(b+2d)(3b+2d)(3b-2d)(b-2d)}+\frac{3\cos(bx-2dx+a-2c)bd^2}{4(b+2d)(3b+2d)(3b-2d)(b-2d}$
orering	Expression too large to display

input
```
int(cos(d*x+c)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b-3/16/(b-2*d)*cos(a-2*c+(b-2*d)*x)-
3/16/(b+2*d)*cos(a+2*c+(b+2*d)*x)+1/16/(3*b-2*d)*cos(3*a-2*c+(3*b-2*d)*x)+
1/16/(3*b+2*d)*cos(3*a+2*c+(3*b+2*d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.30

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx = -\frac{2\left(b^2d^2-4\,d^4\right)\cos\left(bx+a\right)^3+6\left(7\,b^3d-4\,bd^3-\left(b^3d-4\,bd^3\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)\sin\left(bx+a\right)\mathrm{s}}{3\,(9\,b}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/3*(2*(b^2*d^2 - 4*d^4)*cos(b*x + a)^3 + 6*(7*b^3*d - 4*b*d^3 - (b^3*d -
 4*b*d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a)*sin(d*x + c) - 9*((b^4
 - 4*b^2*d^2)*cos(b*x + a)^3 - (3*b^4 - 4*b^2*d^2)*cos(b*x + a))*cos(d*x +
 c)^2 - 6*(7*b^2*d^2 - 4*d^4)*cos(b*x + a))/(9*b^5 - 40*b^3*d^2 + 16*b*d^4
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2020 vs. 2(116) = 232.

Time = 5.99 (sec) , antiderivative size = 2020, normalized size of antiderivative = 14.64

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2*sin(b*x+a)**3,x)
```

output
```
Piecewise((x*sin(a)**3*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**
2/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**3, Eq
(b, 0)), (-3*x*sin(a - 2*d*x)**3*sin(c + d*x)**2/16 + 3*x*sin(a - 2*d*x)**
3*cos(c + d*x)**2/16 + 3*x*sin(a - 2*d*x)**2*sin(c + d*x)*cos(a - 2*d*x)*c
os(c + d*x)/8 - 3*x*sin(a - 2*d*x)*sin(c + d*x)**2*cos(a - 2*d*x)**2/16 +
3*x*sin(a - 2*d*x)*cos(a - 2*d*x)**2*cos(c + d*x)**2/16 + 3*x*sin(c + d*x)
*cos(a - 2*d*x)**3*cos(c + d*x)/8 - 3*sin(a - 2*d*x)**3*sin(c + d*x)*cos(c
 + d*x)/(16*d) + sin(a - 2*d*x)**2*cos(a - 2*d*x)*cos(c + d*x)**2/(2*d) -
sin(a - 2*d*x)*sin(c + d*x)*cos(a - 2*d*x)**2*cos(c + d*x)/(8*d) + sin(c +
 d*x)**2*cos(a - 2*d*x)**3/(96*d) + 31*cos(a - 2*d*x)**3*cos(c + d*x)**2/(
96*d), Eq(b, -2*d)), (-x*sin(a - 2*d*x/3)**3*sin(c + d*x)**2/16 + x*sin(a
- 2*d*x/3)**3*cos(c + d*x)**2/16 + 3*x*sin(a - 2*d*x/3)**2*sin(c + d*x)*co
s(a - 2*d*x/3)*cos(c + d*x)/8 + 3*x*sin(a - 2*d*x/3)*sin(c + d*x)**2*cos(a
 - 2*d*x/3)**2/16 - 3*x*sin(a - 2*d*x/3)*cos(a - 2*d*x/3)**2*cos(c + d*x)*
*2/16 - x*sin(c + d*x)*cos(a - 2*d*x/3)**3*cos(c + d*x)/8 - sin(a - 2*d*x/
3)**3*sin(c + d*x)*cos(c + d*x)/(16*d) + 3*sin(a - 2*d*x/3)**2*cos(a - 2*d
*x/3)*cos(c + d*x)**2/(2*d) + 15*sin(a - 2*d*x/3)*sin(c + d*x)*cos(a - 2*d
*x/3)**2*cos(c + d*x)/(8*d) + 27*sin(c + d*x)**2*cos(a - 2*d*x/3)**3/(32*d
) + 5*cos(a - 2*d*x/3)**3*cos(c + d*x)**2/(32*d), Eq(b, -2*d/3)), (-x*sin(
a + 2*d*x/3)**3*sin(c + d*x)**2/16 + x*sin(a + 2*d*x/3)**3*cos(c + d*x)...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(126) = 252$.

Time = 0.15 (sec) , antiderivative size = 1360, normalized size of antiderivative = 9.86

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/96*(3*(3*b^4*cos(2*c) - 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3
*cos(2*c))*cos((3*b + 2*d)*x + 3*a + 4*c) + 3*(3*b^4*cos(2*c) - 2*b^3*d*co
s(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((3*b + 2*d)*x + 3*a)
+ 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos
(2*c))*cos(-(3*b - 2*d)*x - 3*a + 4*c) + 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2
*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(3*b - 2*d)*x - 3*a) -
9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2
*c))*cos((b + 2*d)*x + a + 4*c) - 9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) -
4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((b + 2*d)*x + a) - 9*(9*b^4*cos
(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(b
 - 2*d)*x - a + 4*c) - 9*(9*b^4*cos(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*c
os(2*c) - 8*b*d^3*cos(2*c))*cos(-(b - 2*d)*x - a) + 2*(9*b^4*cos(2*c) - 40
*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a + 2*c) + 2*(9*b^4*cos
(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a - 2*c) - 18
*(9*b^4*cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a + 2*
c) - 18*(9*b^4*cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x +
 a - 2*c) + 3*(3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) + 8
*b*d^3*sin(2*c))*sin((3*b + 2*d)*x + 3*a + 4*c) - 3*(3*b^4*sin(2*c) - 2*b^
3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((3*b + 2*d)*x +
 3*a) + 3*(3*b^4*sin(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) - 8*...
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx = \frac{\cos(3\,bx+2\,dx+3\,a+2\,c)}{16\,(3\,b+2\,d)} + \frac{\cos(3\,bx-2\,dx+3\,a-2\,c)}{16\,(3\,b-2\,d)} + \frac{\cos(3\,bx+3\,a)}{24\,b} - \frac{3\,\cos(bx+2\,dx+a+2\,c)}{16\,(b+2\,d)} - \frac{3\,\cos(bx-2\,dx+a-2\,c)}{16\,(b-2\,d)} - \frac{3\,\cos(bx+a)}{8\,b}$$

input
```
integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/16*cos(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) + 1/16*cos(3*b*x - 2*d*x +
 3*a - 2*c)/(3*b - 2*d) + 1/24*cos(3*b*x + 3*a)/b - 3/16*cos(b*x + 2*d*x +
 a + 2*c)/(b + 2*d) - 3/16*cos(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/8*cos(
b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 18.38 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.17

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx = -\frac{81\,b^4\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)+81\,b^4\,\cos\left(a+2\,c+b\,x+2\,d\,x\right)+162\,b^4\,\cos\left(a+b\,x\right)+288\,d^4\,\cos\left(\right.}{}$$

input
```
int(cos(c + d*x)^2*sin(a + b*x)^3,x)
```

output
```
-(81*b^4*cos(a - 2*c + b*x - 2*d*x) + 81*b^4*cos(a + 2*c + b*x + 2*d*x) +
162*b^4*cos(a + b*x) + 288*d^4*cos(a + b*x) - 9*b^4*cos(3*a - 2*c + 3*b*x
- 2*d*x) - 9*b^4*cos(3*a + 2*c + 3*b*x + 2*d*x) - 18*b^4*cos(3*a + 3*b*x)
- 32*d^4*cos(3*a + 3*b*x) + 24*b*d^3*cos(3*a - 2*c + 3*b*x - 2*d*x) - 24*b
*d^3*cos(3*a + 2*c + 3*b*x + 2*d*x) - 6*b^3*d*cos(3*a - 2*c + 3*b*x - 2*d*
x) + 6*b^3*d*cos(3*a + 2*c + 3*b*x + 2*d*x) - 36*b^2*d^2*cos(a - 2*c + b*x
 - 2*d*x) - 36*b^2*d^2*cos(a + 2*c + b*x + 2*d*x) - 720*b^2*d^2*cos(a + b*
x) + 36*b^2*d^2*cos(3*a - 2*c + 3*b*x - 2*d*x) + 36*b^2*d^2*cos(3*a + 2*c
+ 3*b*x + 2*d*x) + 80*b^2*d^2*cos(3*a + 3*b*x) - 72*b*d^3*cos(a - 2*c + b*
x - 2*d*x) + 72*b*d^3*cos(a + 2*c + b*x + 2*d*x) + 162*b^3*d*cos(a - 2*c +
 b*x - 2*d*x) - 162*b^3*d*cos(a + 2*c + b*x + 2*d*x))/(48*(16*b*d^4 + 9*b^
5 - 40*b^3*d^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.12

$$\int \cos^2(c+dx)\sin^3(a+bx)\,dx$$
$$= \frac{9\cos(bx+a)\sin(bx+a)^2\sin(dx+c)^2b^4 - 36\cos(bx+a)\sin(bx+a)^2\sin(dx+c)^2b^2d^2 - 9\cos(bx+}{}$$

input
```
int(cos(d*x+c)^2*sin(b*x+a)^3,x)
```

output
```
(9*cos(a + b*x)*sin(a + b*x)**2*sin(c + d*x)**2*b**4 - 36*cos(a + b*x)*sin
(a + b*x)**2*sin(c + d*x)**2*b**2*d**2 - 9*cos(a + b*x)*sin(a + b*x)**2*b*
*4 + 38*cos(a + b*x)*sin(a + b*x)**2*b**2*d**2 - 8*cos(a + b*x)*sin(a + b*
x)**2*d**4 + 18*cos(a + b*x)*sin(c + d*x)**2*b**4 - 18*cos(a + b*x)*b**4 +
 40*cos(a + b*x)*b**2*d**2 - 16*cos(a + b*x)*d**4 - 6*cos(c + d*x)*sin(a +
 b*x)**3*sin(c + d*x)*b**3*d + 24*cos(c + d*x)*sin(a + b*x)**3*sin(c + d*x
)*b*d**3 - 36*cos(c + d*x)*sin(a + b*x)*sin(c + d*x)*b**3*d + 18*b**4 - 64
*b**2*d**2 + 16*d**4)/(3*b*(9*b**4 - 40*b**2*d**2 + 16*d**4))
```

3.199 $\int \cos(c+dx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \cos(c+dx)\sin^3(a+bx)\,dx = -\frac{3\cos(a-c+(b-d)x)}{8(b-d)} + \frac{\cos(3a-c+(3b-d)x)}{8(3b-d)} - \frac{3\cos(a+c+(b+d)x)}{8(b+d)} + \frac{\cos(3a+c+(3b+d)x)}{8(3b+d)}$$

output
```
-3*cos(a-c+(b-d)*x)/(8*b-8*d)+cos(3*a-c+(3*b-d)*x)/(24*b-8*d)-3*cos(a+c+(b
+d)*x)/(8*b+8*d)+cos(3*a+c+(3*b+d)*x)/(24*b+8*d)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \cos(c+dx)\sin^3(a+bx)\,dx = \frac{1}{8}\left(-\frac{3\cos(a-c+bx-dx)}{b-d} + \frac{\cos(3a-c+3bx-dx)}{3b-d} + \frac{\cos(3a+c+3bx+dx)}{3b+d} - \frac{3\cos(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Cos[c + d*x]*Sin[a + b*x]^3,x]
```

output
```
((-3*Cos[a - c + b*x - d*x])/(b - d) + Cos[3*a - c + 3*b*x - d*x]/(3*b - d
) + Cos[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Cos[a + c + (b + d)*x])/(b +
 d))/8
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos(c+dx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{3}{8}\sin(a+x(b-d)-c)-\frac{1}{8}\sin(3a+x(3b-d)-c)+\frac{3}{8}\sin(a+x(b+d)+c)-\frac{1}{8}\sin(3a+x(3b+d)+c)\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}+\frac{\cos(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}+\frac{\cos(3a+x(3b+d)+c)}{8(3b+d)}$$

input
```
Int[Cos[c + d*x]*Sin[a + b*x]^3,x]
```

output
```
(-3*Cos[a - c + (b - d)*x])/(8*(b - d)) + Cos[3*a - c + (3*b - d)*x]/(8*(3
*b - d)) - (3*Cos[a + c + (b + d)*x])/(8*(b + d)) + Cos[3*a + c + (3*b + d
)*x]/(8*(3*b + d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
default	$-\frac{3\cos(a-c+(b-d)x)}{8(b-d)}-\frac{3\cos(a+c+(b+d)x)}{8(b+d)}+\frac{\cos(3a-c+(3b-d)x)}{24b-8d}+\frac{\cos(3a+c+(3b+d)x)}{24b+8d}$
parallelrisch	$\frac{-12\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{6}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b^{3}-24b^{2}d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{5}+12\left(\left(-3b^{3}+b\,d^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-b\,d^{2}\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}}{(b-d)(b+d)(3b-d)(3b+d)\left(\right.}$
risch	$-\frac{27\cos(bx-dx+a-c)b^{3}}{8(b+d)(3b+d)(-b+d)(-3b+d)}-\frac{27\cos(bx-dx+a-c)b^{2}d}{8(b+d)(3b+d)(-b+d)(-3b+d)}+\frac{3\cos(bx-dx+a-c)b\,d^{2}}{8(b+d)(3b+d)(-b+d)(-3b+d)}+\frac{3\cos(bx-}{8(b+d)(3b+d)(}$
orering	Expression too large to display

input
```
int(cos(d*x+c)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*cos(a-c+(b-d)*x)/(b-d)-3/8/(b+d)*cos(a+c+(b+d)*x)+1/8/(3*b-d)*cos(3*a
-c+(3*b-d)*x)+1/8/(3*b+d)*cos(3*a+c+(3*b+d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.20

$$\int\cos(c+dx)\sin^3(a+bx)\,dx =$$
$$-\frac{\left(7\,b^2d-d^3-\left(b^2d-d^3\right)\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)\sin\left(dx+c\right)-3\left(\left(b^3-bd^2\right)\cos\left(bx+a\right)^3-\left(3\,b^3-\right.\right.}{9\,b^4-10\,b^2d^2+d^4}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-((7*b^2*d - d^3 - (b^2*d - d^3)*cos(b*x + a)^2)*sin(b*x + a)*sin(d*x + c)
 - 3*((b^3 - b*d^2)*cos(b*x + a)^3 - (3*b^3 - b*d^2)*cos(b*x + a))*cos(d*x
 + c))/(9*b^4 - 10*b^2*d^2 + d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(76) = 152$.

Time = 2.09 (sec) , antiderivative size = 933, normalized size of antiderivative = 9.62

$$\int \cos(c + dx)\sin^3(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)**3,x)
```

output
```
Piecewise((x*sin(a)**3*cos(c), Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - d*x)**3*
cos(c + d*x)/8 + 3*x*sin(a - d*x)**2*sin(c + d*x)*cos(a - d*x)/8 + 3*x*sin
(a - d*x)*cos(a - d*x)**2*cos(c + d*x)/8 + 3*x*sin(c + d*x)*cos(a - d*x)**
3/8 - sin(a - d*x)**3*sin(c + d*x)/(8*d) + 3*sin(a - d*x)**2*cos(a - d*x)*
cos(c + d*x)/(4*d) + 3*cos(a - d*x)**3*cos(c + d*x)/(8*d), Eq(b, -d)), (x*
sin(a - d*x/3)**3*cos(c + d*x)/8 + 3*x*sin(a - d*x/3)**2*sin(c + d*x)*cos(
a - d*x/3)/8 - 3*x*sin(a - d*x/3)*cos(a - d*x/3)**2*cos(c + d*x)/8 - x*sin
(c + d*x)*cos(a - d*x/3)**3/8 + 9*sin(a - d*x/3)**3*sin(c + d*x)/(8*d) - 3
*sin(a - d*x/3)**2*cos(a - d*x/3)*cos(c + d*x)/(4*d) - cos(a - d*x/3)**3*c
os(c + d*x)/(8*d), Eq(b, -d/3)), (x*sin(a + d*x/3)**3*cos(c + d*x)/8 - 3*x
*sin(a + d*x/3)**2*sin(c + d*x)*cos(a + d*x/3)/8 - 3*x*sin(a + d*x/3)*cos(
a + d*x/3)**2*cos(c + d*x)/8 + x*sin(c + d*x)*cos(a + d*x/3)**3/8 + 9*sin(
a + d*x/3)**3*sin(c + d*x)/(8*d) + 3*sin(a + d*x/3)**2*cos(a + d*x/3)*cos(
c + d*x)/(4*d) + cos(a + d*x/3)**3*cos(c + d*x)/(8*d), Eq(b, d/3)), (3*x*s
in(a + d*x)**3*cos(c + d*x)/8 - 3*x*sin(a + d*x)**2*sin(c + d*x)*cos(a + d
*x)/8 + 3*x*sin(a + d*x)*cos(a + d*x)**2*cos(c + d*x)/8 - 3*x*sin(c + d*x)
*cos(a + d*x)**3/8 + 5*sin(a + d*x)**3*sin(c + d*x)/(8*d) + 3*sin(a + d*x)
*sin(c + d*x)*cos(a + d*x)**2/(4*d) + 3*cos(a + d*x)**3*cos(c + d*x)/(8*d)
, Eq(b, d)), (-9*b**3*sin(a + b*x)**2*cos(a + b*x)*cos(c + d*x)/(9*b**4 -
10*b**2*d**2 + d**4) - 6*b**3*cos(a + b*x)**3*cos(c + d*x)/(9*b**4 - 10...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(89) = 178$.

Time = 0.12 (sec) , antiderivative size = 785, normalized size of antiderivative = 8.09

$$\int \cos(c + dx)\sin^3(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/16*((3*b^3*cos(c) - b^2*d*cos(c) - 3*b*d^2*cos(c) + d^3*cos(c))*cos((3*b
 + d)*x + 3*a + 2*c) + (3*b^3*cos(c) - b^2*d*cos(c) - 3*b*d^2*cos(c) + d^3
*cos(c))*cos((3*b + d)*x + 3*a) + (3*b^3*cos(c) + b^2*d*cos(c) - 3*b*d^2*c
os(c) - d^3*cos(c))*cos(-(3*b - d)*x - 3*a + 2*c) + (3*b^3*cos(c) + b^2*d*
cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*cos(-(3*b - d)*x - 3*a) - 3*(9*b^3*c
os(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) + d^3*cos(c))*cos((b + d)*x + a + 2*
c) - 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) + d^3*cos(c))*cos((b
+ d)*x + a) - 3*(9*b^3*cos(c) + 9*b^2*d*cos(c) - b*d^2*cos(c) - d^3*cos(c)
)*cos(-(b - d)*x - a + 2*c) - 3*(9*b^3*cos(c) + 9*b^2*d*cos(c) - b*d^2*cos
(c) - d^3*cos(c))*cos(-(b - d)*x - a) + (3*b^3*sin(c) - b^2*d*sin(c) - 3*b
*d^2*sin(c) + d^3*sin(c))*sin((3*b + d)*x + 3*a + 2*c) - (3*b^3*sin(c) - b
^2*d*sin(c) - 3*b*d^2*sin(c) + d^3*sin(c))*sin((3*b + d)*x + 3*a) + (3*b^3
*sin(c) + b^2*d*sin(c) - 3*b*d^2*sin(c) - d^3*sin(c))*sin(-(3*b - d)*x - 3
*a + 2*c) - (3*b^3*sin(c) + b^2*d*sin(c) - 3*b*d^2*sin(c) - d^3*sin(c))*si
n(-(3*b - d)*x - 3*a) - 3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) +
d^3*sin(c))*sin((b + d)*x + a + 2*c) + 3*(9*b^3*sin(c) - 9*b^2*d*sin(c) -
b*d^2*sin(c) + d^3*sin(c))*sin((b + d)*x + a) - 3*(9*b^3*sin(c) + 9*b^2*d*
sin(c) - b*d^2*sin(c) - d^3*sin(c))*sin(-(b - d)*x - a + 2*c) + 3*(9*b^3*s
in(c) + 9*b^2*d*sin(c) - b*d^2*sin(c) - d^3*sin(c))*sin(-(b - d)*x - a))/(
9*b^4*cos(c)^2 + 9*b^4*sin(c)^2 + (cos(c)^2 + sin(c)^2)*d^4 - 10*(b^2*c...
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \cos(c+dx)\sin^3(a+bx)\,dx = \frac{\cos(3\,bx+dx+3\,a+c)}{8\,(3\,b+d)} + \frac{\cos(3\,bx-dx+3\,a-c)}{8\,(3\,b-d)} - \frac{3\,\cos(bx+dx+a+c)}{8\,(b+d)} - \frac{3\,\cos(bx-dx+a-c)}{8\,(b-d)}$$

input
```
integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*cos(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/8*cos(3*b*x - d*x + 3*a - c)/
(3*b - d) - 3/8*cos(b*x + d*x + a + c)/(b + d) - 3/8*cos(b*x - d*x + a - c
)/(b - d)
```

Mupad [B] (verification not implemented)

Time = 18.12 (sec) , antiderivative size = 471, normalized size of antiderivative = 4.86

$$\begin{aligned}\int \cos(c+dx)\sin^3(a+bx)\,dx = &-\mathrm{e}^{a\,3\mathrm{i}-c\,1\mathrm{i}+b\,x\,3\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{-3\,b^3-b^2\,d+3\,b\,d^2+d^3}{144\,b^4-160\,b^2\,d^2+16\,d^4}\right.\\&+\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,\left(-3\,b^3+b^2\,d+3\,b\,d^2-d^3\right)}{144\,b^4-160\,b^2\,d^2+16\,d^4}\\&-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3\right)}{144\,b^4-160\,b^2\,d^2+16\,d^4}\\&\left.-\frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3\right)}{144\,b^4-160\,b^2\,d^2+16\,d^4}\right)\\&-\mathrm{e}^{a\,3\mathrm{i}+c\,1\mathrm{i}+b\,x\,3\mathrm{i}+d\,x\,1\mathrm{i}}\left(\frac{-3\,b^3+b^2\,d+3\,b\,d^2-d^3}{144\,b^4-160\,b^2\,d^2+16\,d^4}\right.\\&+\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,\left(-3\,b^3-b^2\,d+3\,b\,d^2+d^3\right)}{144\,b^4-160\,b^2\,d^2+16\,d^4}\\&-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3\right)}{144\,b^4-160\,b^2\,d^2+16\,d^4}\\&\left.-\frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3\right)}{144\,b^4-160\,b^2\,d^2+16\,d^4}\right)\end{aligned}$$

input
```
int(cos(c + d*x)*sin(a + b*x)^3,x)
```

output
```
- exp(a*3i - c*1i + b*x*3i - d*x*1i)*((3*b*d^2 - b^2*d - 3*b^3 + d^3)/(144
*b^4 + 16*d^4 - 160*b^2*d^2) + (exp(- a*6i - b*x*6i)*(3*b*d^2 + b^2*d - 3*
b^3 - d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2) - (exp(- a*2i - b*x*2i)*(3*b*
d^2 - 27*b^2*d - 27*b^3 + 3*d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2) - (exp(
- a*4i - b*x*4i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(144*b^4 + 16*d^4
- 160*b^2*d^2)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((3*b*d^2 + b^2*d - 3
*b^3 - d^3)/(144*b^4 + 16*d^4 - 160*b^2*d^2) + (exp(- a*6i - b*x*6i)*(3*b*
d^2 - b^2*d - 3*b^3 + d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2) - (exp(- a*2i
 - b*x*2i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(144*b^4 + 16*d^4 - 160*
b^2*d^2) - (exp(- a*4i - b*x*4i)*(3*b*d^2 - 27*b^2*d - 27*b^3 + 3*d^3))/(1
44*b^4 + 16*d^4 - 160*b^2*d^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.61

$$\int \cos(c+dx)\sin^3(a+bx)\,dx$$
$$= \frac{-3\cos(bx+a)\cos(dx+c)\sin(bx+a)^2 b^3 + 3\cos(bx+a)\cos(dx+c)\sin(bx+a)^2 b\,d^2 - 6\cos(bx+a}{}$$

input
```
int(cos(d*x+c)*sin(b*x+a)^3,x)
```

output
```
( - 3*cos(a + b*x)*cos(c + d*x)*sin(a + b*x)**2*b**3 + 3*cos(a + b*x)*cos(
c + d*x)*sin(a + b*x)**2*b*d**2 - 6*cos(a + b*x)*cos(c + d*x)*b**3 - sin(a
 + b*x)**3*sin(c + d*x)*b**2*d + sin(a + b*x)**3*sin(c + d*x)*d**3 - 6*sin
(a + b*x)*sin(c + d*x)*b**2*d - 6*b**3 + 4*b*d**2)/(9*b**4 - 10*b**2*d**2
+ d**4)
```

3.200 $\int \sec(c+dx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 290

$$\begin{aligned}\int \sec(c+dx)\sin^3(a+bx)\,dx &= -\frac{3e^{-ia-ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2d},\frac{1}{2}\left(3-\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{4(b-d)}\\ &\quad+\frac{e^{-3ia-3ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,-\frac{3b-d}{2d},\frac{3}{2}\left(1-\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{4(3b-d)}\\ &\quad-\frac{3e^{ia+ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{4(b+d)}\\ &\quad+\frac{e^{3ia+3ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{3b+d}{2d},\frac{3(b+d)}{2d},-e^{2i(c+dx)}\right)}{4(3b+d)}\end{aligned}$$

output
```
-3*exp(-I*a-I*b*x+I*(d*x+c))*hypergeom([1, -1/2*(b-d)/d],[3/2-1/2*b/d],-ex
p(2*I*(d*x+c)))/(4*b-4*d)+exp(-3*I*a-3*I*b*x+I*(d*x+c))*hypergeom([1, -1/2
*(3*b-d)/d],[3/2-3/2*b/d],-exp(2*I*(d*x+c)))/(12*b-4*d)-3*exp(I*a+I*b*x+I*
(d*x+c))*hypergeom([1, 1/2*(b+d)/d],[3/2+1/2*b/d],-exp(2*I*(d*x+c)))/(4*b+
4*d)+exp(3*I*a+3*I*b*x+I*(d*x+c))*hypergeom([1, 1/2*(3*b+d)/d],[3/2*(b+d)/
d],-exp(2*I*(d*x+c)))/(12*b+4*d)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.92

$$\int \sec(c+dx)\sin^3(a+bx)\,dx$$
$$= \frac{e^{-i(3a-c+3bx-dx)}\,\text{Hypergeometric2F1}\left(1,\frac{-3b+d}{2d},\frac{3}{2}-\frac{3b}{2d},-e^{2i(c+dx)}\right)}{4(3b-d)} - \frac{3e^{-i(a-c+(b-d)x)}\,\text{Hypergeometric2F1}\left(1,\frac{-b+d}{2d},\frac{3}{2}-\frac{b}{2d},-e^{2i(c+dx)}\right)}{4(b-d)} - \frac{3e^{i(a+c+(b+d)x)}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{4(b+d)} + \frac{e^{i(3a+c+(3b+d)x)}\,\text{Hypergeometric2F1}\left(1,\frac{3b+d}{2d},\frac{3(b+d)}{2d},-e^{2i(c+dx)}\right)}{4(3b+d)}$$

input
```
Integrate[Sec[c + d*x]*Sin[a + b*x]^3,x]
```

output
```
Hypergeometric2F1[1, (-3*b + d)/(2*d), 3/2 - (3*b)/(2*d), -E^((2*I)*(c + d
*x))]/(4*(3*b - d)*E^(I*(3*a - c + 3*b*x - d*x))) - (3*Hypergeometric2F1[1
, (-b + d)/(2*d), 3/2 - b/(2*d), -E^((2*I)*(c + d*x))])/(4*(b - d)*E^(I*(a
 - c + (b - d)*x))) - (3*E^(I*(a + c + (b + d)*x))*Hypergeometric2F1[1, (b
 + d)/(2*d), (3 + b/d)/2, -E^((2*I)*(c + d*x))])/(4*(b + d)) + (E^(I*(3*a
+ c + (3*b + d)*x))*Hypergeometric2F1[1, (3*b + d)/(2*d), (3*(b + d))/(2*d
), -E^((2*I)*(c + d*x))])/(4*(3*b + d))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sec(c+dx)\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \sin^3(a+bx)\sec(c+dx)dx$$

input
```
Int[Sec[c + d*x]*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \sec (dx + c) \sin (bx + a)^3 \, dx$$

input
```
int(sec(d*x+c)*sin(b*x+a)^3,x)
```

output
```
int(sec(d*x+c)*sin(b*x+a)^3,x)
```

Fricas [F]

$$\int \sec(c + dx) \sin^3(a + bx) \, dx = \int \sec (dx + c) \sin (bx + a)^3 \, dx$$

input
```
integrate(sec(d*x+c)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sec(d*x + c)*sin(b*x + a), x)
```

Sympy [F]

$$\int \sec(c+dx)\sin^3(a+bx)\,dx = \int \sin^3{(a+bx)}\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*sin(b*x+a)**3,x)
```

output
```
Integral(sin(a + b*x)**3*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)\sin^3(a+bx)\,dx = \int \sec{(dx+c)}\sin{(bx+a)}^3\,dx$$

input
```
integrate(sec(d*x+c)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)*sin(b*x + a)^3, x)
```

Giac [F]

$$\int \sec(c+dx)\sin^3(a+bx)\,dx = \int \sec{(dx+c)}\sin{(bx+a)}^3\,dx$$

input
```
integrate(sec(d*x+c)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)\sin^3(a+bx)\,dx=\int \frac{\sin\left(a+b\,x\right)^3}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(sin(a + b*x)^3/cos(c + d*x),x)
```

output
```
int(sin(a + b*x)^3/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)\sin^3(a+bx)\,dx=\int \frac{\sin\left(bx+a\right)^3}{\cos\left(dx+c\right)}dx$$

input
```
int(sec(d*x+c)*sin(b*x+a)^3,x)
```

output
```
int(sin(a + b*x)**3/cos(c + d*x),x)
```

3.201 $\int \sec^2(c+dx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 281

$$\int \sec^2(c+dx)\sin^3(a+bx)\,dx = \frac{e^{-3ia-3ibx+2i(c+dx)}\operatorname{Hypergeometric2F1}\left(2,1-\frac{3b}{2d},2-\frac{3b}{2d},-e^{2i(c+dx)}\right)}{2(3b-2d)} - \frac{3e^{-ia-ibx+2i(c+dx)}\operatorname{Hypergeometric2F1}\left(2,1-\frac{b}{2d},2-\frac{b}{2d},-e^{2i(c+dx)}\right)}{2(b-2d)} - \frac{3e^{ia+ibx+2i(c+dx)}\operatorname{Hypergeometric2F1}\left(2,1+\frac{b}{2d},2+\frac{b}{2d},-e^{2i(c+dx)}\right)}{2(b+2d)} + \frac{e^{3ia+3ibx+2i(c+dx)}\operatorname{Hypergeometric2F1}\left(2,1+\frac{3b}{2d},2+\frac{3b}{2d},-e^{2i(c+dx)}\right)}{2(3b+2d)}$$

output

```
exp(-3*I*a-3*I*b*x+2*I*(d*x+c))*hypergeom([2, 1-3/2*b/d],[2-3/2*b/d],-exp(
2*I*(d*x+c)))/(6*b-4*d)-3*exp(-I*a-I*b*x+2*I*(d*x+c))*hypergeom([2, 1-1/2*
b/d],[2-1/2*b/d],-exp(2*I*(d*x+c)))/(2*b-4*d)-3*exp(I*a+I*b*x+2*I*(d*x+c))
*hypergeom([2, 1+1/2*b/d],[2+1/2*b/d],-exp(2*I*(d*x+c)))/(2*b+4*d)+exp(3*I
*a+3*I*b*x+2*I*(d*x+c))*hypergeom([2, 1+3/2*b/d],[2+3/2*b/d],-exp(2*I*(d*x
+c)))/(6*b+4*d)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 593 vs. $2(281) = 562$.

Time = 3.56 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.11

$$\int \sec^2(c+dx)\sin^3(a+bx)\,dx$$

$$= \frac{\frac{e^{-i(3a-2c+3bx)}\left(3be^{2idx}\,\text{Hypergeometric2F1}\left(1,1-\frac{3b}{2d},2-\frac{3b}{2d},-e^{2i(c+dx)}\right)+(-3b+2d)\,\text{Hypergeometric2F1}\left(1,-\frac{3b}{2d},1-\frac{3b}{2d},-e^{2i(c+dx)}\right)\right)}{(3b-2d)(1+e^{2ic})}-\frac{3be}{}}{}$$

input
```
Integrate[Sec[c + d*x]^2*Sin[a + b*x]^3,x]
```

output
```
((3*b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 - (3*b)/(2*d), 2 - (3*b)/(2*d),
 -E^((2*I)*(c + d*x))] + (-3*b + 2*d)*Hypergeometric2F1[1, (-3*b)/(2*d), 1
 - (3*b)/(2*d), -E^((2*I)*(c + d*x))])/((3*b - 2*d)*E^(I*(3*a - 2*c + 3*b*
x))*(1 + E^((2*I)*c))) - (3*b*(Hypergeometric2F1[1, 1 - b/(2*d), 2 - b/(2*
d), -E^((2*I)*(c + d*x))]/((b - 2*d)*E^(I*(b - 2*d)*x)) - Hypergeometric2F
1[1, -1/2*b/d, 1 - b/(2*d), -E^((2*I)*(c + d*x))]/(b*E^(I*b*x))))/(E^(I*(a
 - 2*c))*(1 + E^((2*I)*c))) + (3*E^(I*(a + 2*c + b*x))*(b*E^((2*I)*d*x)*Hy
pergeometric2F1[1, 1 + b/(2*d), 2 + b/(2*d), -E^((2*I)*(c + d*x))] - (b +
2*d)*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))]))/((
b + 2*d)*(1 + E^((2*I)*c))) + (E^(I*(3*a + 2*c + 3*b*x))*(-3*b*E^((2*I)*d*
x)*Hypergeometric2F1[1, 1 + (3*b)/(2*d), 2 + (3*b)/(2*d), -E^((2*I)*(c + d
*x))] + (3*b + 2*d)*Hypergeometric2F1[1, (3*b)/(2*d), 1 + (3*b)/(2*d), -E^
((2*I)*(c + d*x))]))/((3*b + 2*d)*(1 + E^((2*I)*c))) + 3*Cos[b*x]*Sec[c]*S
ec[c + d*x]*Sin[a]*Sin[d*x] - Cos[3*b*x]*Sec[c]*Sec[c + d*x]*Sin[3*a]*Sin[
d*x] + 3*Cos[a]*Sec[c]*Sec[c + d*x]*Sin[b*x]*Sin[d*x] - Cos[3*a]*Sec[c]*Se
c[c + d*x]*Sin[3*b*x]*Sin[d*x])/(4*d)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sec^2(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sin^3(a+bx)\sec^2(c+dx)dx$$

input
```
Int[Sec[c + d*x]^2*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \sec(dx+c)^2 \sin(bx+a)^3\,dx$$

input
```
int(sec(d*x+c)^2*sin(b*x+a)^3,x)
```

output
```
int(sec(d*x+c)^2*sin(b*x+a)^3,x)
```

Fricas [F]

$$\int \sec^2(c+dx)\sin^3(a+bx)\,dx = \int \sec(dx+c)^2 \sin(bx+a)^3 \, dx$$

input
```
integrate(sec(d*x+c)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sec(d*x + c)^2*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^2(c+dx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**2*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^2(c+dx)\sin^3(a+bx)\,dx = \int \sec(dx+c)^2 \sin(bx+a)^3 \, dx$$

input
```
integrate(sec(d*x+c)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*((cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos
((3*b + 2*d)*x + 3*a + 2*c) + (3*cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a) +
cos(6*b*x + 6*a)*cos(3*b*x + 3*a) - 3*cos(4*b*x + 4*a)*cos(3*b*x + 3*a) -
4*(d*cos((3*b + 2*d)*x + 3*a + 2*c)^2 + 2*d*cos((3*b + 2*d)*x + 3*a + 2*c)
*cos(3*b*x + 3*a) + d*cos(3*b*x + 3*a)^2 + d*sin((3*b + 2*d)*x + 3*a + 2*c
)^2 + 2*d*sin((3*b + 2*d)*x + 3*a + 2*c)*sin(3*b*x + 3*a) + d*sin(3*b*x +
3*a)^2)*integrate(-3/4*(b*cos(3*b*x + 3*a)*sin(6*b*x + 6*a) - b*cos(3*b*x
+ 3*a)*sin(4*b*x + 4*a) - b*cos(6*b*x + 6*a)*sin(3*b*x + 3*a) + b*cos(4*b*
x + 4*a)*sin(3*b*x + 3*a) - b*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + (b*sin(6
*b*x + 6*a) - b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*cos((3*b + 2*d)*x +
 3*a + 2*c) - (b*cos(6*b*x + 6*a) - b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a
) + b)*sin((3*b + 2*d)*x + 3*a + 2*c) + (b*cos(2*b*x + 2*a) - b)*sin(3*b*x
 + 3*a))/(d*cos((3*b + 2*d)*x + 3*a + 2*c)^2 + 2*d*cos((3*b + 2*d)*x + 3*a
 + 2*c)*cos(3*b*x + 3*a) + d*cos(3*b*x + 3*a)^2 + d*sin((3*b + 2*d)*x + 3*
a + 2*c)^2 + 2*d*sin((3*b + 2*d)*x + 3*a + 2*c)*sin(3*b*x + 3*a) + d*sin(3
*b*x + 3*a)^2), x) + (sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x
+ 2*a))*sin((3*b + 2*d)*x + 3*a + 2*c) + sin(6*b*x + 6*a)*sin(3*b*x + 3*a)
 - 3*sin(4*b*x + 4*a)*sin(3*b*x + 3*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*
a))/(d*cos((3*b + 2*d)*x + 3*a + 2*c)^2 + 2*d*cos((3*b + 2*d)*x + 3*a + 2*
c)*cos(3*b*x + 3*a) + d*cos(3*b*x + 3*a)^2 + d*sin((3*b + 2*d)*x + 3*a ...
```

Giac [F]

$$\int \sec^2(c+dx)\sin^3(a+bx)\,dx = \int \sec(dx+c)^2\sin(bx+a)^3\,dx$$

input
```
integrate(sec(d*x+c)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)\sin^3(a+bx)\,dx = \int \frac{\sin(a+b\,x)^3}{\cos(c+d\,x)^2}\,dx$$

input
```
int(sin(a + b*x)^3/cos(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)^3/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)\sin^3(a+bx)\,dx = \int \sec(dx+c)^2\sin(bx+a)^3\,dx$$

input
```
int(sec(d*x+c)^2*sin(b*x+a)^3,x)
```

output
```
int(sec(c + d*x)**2*sin(a + b*x)**3,x)
```

3.202 $\int \sec^3(c+dx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 283

$$\begin{aligned}\int \sec^3(c+dx)\sin^3(a+bx)\,dx &= \frac{e^{-3ia-3ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3}{2}\left(1-\frac{b}{d}\right),\frac{1}{2}\left(5-\frac{3b}{d}\right),-e^{2i(c+dx)}\right)}{3(b-d)}\\ &\quad-\frac{3e^{-ia-ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{b}{d}\right),\frac{1}{2}\left(5-\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{b-3d}\\ &\quad-\frac{3e^{ia+ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{b}{d}\right),\frac{1}{2}\left(5+\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{b+3d}\\ &\quad+\frac{e^{3ia+3ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3(b+d)}{2d},\frac{1}{2}\left(5+\frac{3b}{d}\right),-e^{2i(c+dx)}\right)}{3(b+d)}\end{aligned}$$

output
```
exp(-3*I*a-3*I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2-3/2*b/d],[5/2-3/2*b/d],-
exp(2*I*(d*x+c)))/(3*b-3*d)-3*exp(-I*a-I*b*x+3*I*(d*x+c))*hypergeom([3, 3/
2-1/2*b/d],[5/2-1/2*b/d],-exp(2*I*(d*x+c)))/(b-3*d)-3*exp(I*a+I*b*x+3*I*(d
*x+c))*hypergeom([3, 3/2+1/2*b/d],[5/2+1/2*b/d],-exp(2*I*(d*x+c)))/(b+3*d)
+exp(3*I*a+3*I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2*(b+d)/d],[5/2+3/2*b/d],-
exp(2*I*(d*x+c)))/(3*b+3*d)
```

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.11

$$\int \sec^3(c+dx)\sin^3(a+bx)\,dx$$
$$= \frac{-4(3b+d)e^{-i(3a-c+3bx-dx)}\operatorname{Hypergeometric2F1}\left(1,\frac{-3b+d}{2d},\frac{3}{2}-\frac{3b}{2d},-e^{2i(c+dx)}\right)+12(b+d)e^{-i(a-c+(b-d)x)}\operatorname{H}}{}$$

input
```
Integrate[Sec[c + d*x]^3*Sin[a + b*x]^3,x]
```

output
```
((-4*(3*b + d)*Hypergeometric2F1[1, (-3*b + d)/(2*d), 3/2 - (3*b)/(2*d), -
E^((2*I)*(c + d*x))])/E^(I*(3*a - c + 3*b*x - d*x)) + (12*(b + d)*Hypergeo
metric2F1[1, (-b + d)/(2*d), 3/2 - b/(2*d), -E^((2*I)*(c + d*x))])/E^(I*(a
 - c + (b - d)*x)) + 12*(b - d)*E^(I*(a + c + (b + d)*x))*Hypergeometric2F
1[1, (b + d)/(2*d), (3 + b/d)/2, -E^((2*I)*(c + d*x))] - 4*(3*b - d)*E^(I*
(3*a + c + (3*b + d)*x))*Hypergeometric2F1[1, (3*b + d)/(2*d), (3*(b + d))
/(2*d), -E^((2*I)*(c + d*x))] - 8*((3*b - d)*Cos[a - c + b*x - d*x] + (3*b
 + d)*Cos[a + c + (b + d)*x])*Sec[c + d*x]^2*Sin[a + b*x]^2)/(32*d^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sec^3(c+dx)\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \sin^3(a+bx)\sec^3(c+dx)dx$$

input
```
Int[Sec[c + d*x]^3*Sin[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \sec (dx+c)^{3} \sin (bx+a)^{3} dx$$

input
```
int(sec(d*x+c)^3*sin(b*x+a)^3,x)
```

output
```
int(sec(d*x+c)^3*sin(b*x+a)^3,x)
```

Fricas [F]

$$\int \sec^3(c+dx) \sin^3(a+bx) \, dx = \int \sec (dx+c)^{3} \sin (bx+a)^{3} \, dx$$

input
```
integrate(sec(d*x+c)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sec(d*x + c)^3*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx) \sin^3(a+bx) \, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^3(c+dx)\sin^3(a+bx)\,dx = \int \sec(dx+c)^3\sin(bx+a)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*((3*b - d)*cos((6*b + d)*x + 6*a + c)*cos(3*b*x + 3*a) - 3*(b - d)*cos
((4*b + d)*x + 4*a + c)*cos(3*b*x + 3*a) - 3*(b + d)*cos((2*b + d)*x + 2*a
 + c)*cos(3*b*x + 3*a) + (3*b - d)*cos(3*b*x + 3*a)*cos(3*d*x + 3*c) + (3*
b + d)*cos(3*b*x + 3*a)*cos(d*x + c) + (3*b - d)*sin((6*b + d)*x + 6*a + c
)*sin(3*b*x + 3*a) - 3*(b - d)*sin((4*b + d)*x + 4*a + c)*sin(3*b*x + 3*a)
 - 3*(b + d)*sin((2*b + d)*x + 2*a + c)*sin(3*b*x + 3*a) + (3*b - d)*sin(3
*b*x + 3*a)*sin(3*d*x + 3*c) + (3*b + d)*sin(3*b*x + 3*a)*sin(d*x + c) - 3
*(2*(b + d)*cos((3*b + 2*d)*x + 3*a + 2*c) + (b + d)*cos(3*b*x + 3*a))*cos
((4*b + 3*d)*x + 4*a + 3*c) + ((3*b - d)*cos((6*b + d)*x + 6*a + c) - 3*(b
 + d)*cos((4*b + 3*d)*x + 4*a + 3*c) - 3*(b - d)*cos((4*b + d)*x + 4*a + c
) - 3*(b - d)*cos((2*b + 3*d)*x + 2*a + 3*c) + (3*b + d)*cos(3*(2*b + d)*x
 + 6*a + 3*c) - 3*(b + d)*cos((2*b + d)*x + 2*a + c) + (3*b - d)*cos(3*d*x
 + 3*c) + (3*b + d)*cos(d*x + c))*cos((3*b + 4*d)*x + 3*a + 4*c) + 2*((3*b
 - d)*cos((6*b + d)*x + 6*a + c) - 3*(b - d)*cos((4*b + d)*x + 4*a + c) -
3*(b + d)*cos((2*b + d)*x + 2*a + c) + (3*b - d)*cos(3*d*x + 3*c) + (3*b +
 d)*cos(d*x + c))*cos((3*b + 2*d)*x + 3*a + 2*c) - 3*(2*(b - d)*cos((3*b +
 2*d)*x + 3*a + 2*c) + (b - d)*cos(3*b*x + 3*a))*cos((2*b + 3*d)*x + 2*a +
 3*c) + (2*(3*b + d)*cos((3*b + 2*d)*x + 3*a + 2*c) + (3*b + d)*cos(3*b*x
+ 3*a))*cos(3*(2*b + d)*x + 6*a + 3*c) + 8*(d^2*cos((3*b + 4*d)*x + 3*a +
4*c)^2 + 4*d^2*cos((3*b + 2*d)*x + 3*a + 2*c)^2 + 4*d^2*cos((3*b + 2*d)...
```

Giac [F]

$$\int \sec^3(c+dx)\sin^3(a+bx)\,dx = \int \sec(dx+c)^3\sin(bx+a)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)\sin^3(a+bx)\,dx = \int \frac{\sin(a+b\,x)^3}{\cos(c+d\,x)^3}\,dx$$

input
```
int(sin(a + b*x)^3/cos(c + d*x)^3,x)
```

output
```
int(sin(a + b*x)^3/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sec^3(c+dx)\sin^3(a+bx)\,dx = \int \sec(dx+c)^3\sin(bx+a)^3\,dx$$

input
```
int(sec(d*x+c)^3*sin(b*x+a)^3,x)
```

output
```
int(sec(c + d*x)**3*sin(a + b*x)**3,x)
```

3.203 $\int \cos^3(a+bx)\csc(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 282

$$\int \cos^3(a+bx)\csc(c+dx)\,dx = \frac{3e^{-ia-ibx+i(c+dx)}\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2d},\frac{1}{2}\left(3-\frac{b}{d}\right),e^{2i(c+dx)}\right)}{4(b-d)} + \frac{e^{-3ia-3ibx+i(c+dx)}\text{Hypergeometric2F1}\left(1,-\frac{3b-d}{2d},\frac{3}{2}\left(1-\frac{b}{d}\right),e^{2i(c+dx)}\right)}{4(3b-d)} - \frac{3e^{ia+ibx+i(c+dx)}\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),e^{2i(c+dx)}\right)}{4(b+d)} - \frac{e^{3ia+3ibx+i(c+dx)}\text{Hypergeometric2F1}\left(1,\frac{3b+d}{2d},\frac{3(b+d)}{2d},e^{2i(c+dx)}\right)}{4(3b+d)}$$

output
```
3*exp(-I*a-I*b*x+I*(d*x+c))*hypergeom([1, -1/2*(b-d)/d],[3/2-1/2*b/d],exp(
2*I*(d*x+c)))/(4*b-4*d)+exp(-3*I*a-3*I*b*x+I*(d*x+c))*hypergeom([1, -1/2*(
3*b-d)/d],[3/2-3/2*b/d],exp(2*I*(d*x+c)))/(12*b-4*d)-3*exp(I*a+I*b*x+I*(d*
x+c))*hypergeom([1, 1/2*(b+d)/d],[3/2+1/2*b/d],exp(2*I*(d*x+c)))/(4*b+4*d)
-exp(3*I*a+3*I*b*x+I*(d*x+c))*hypergeom([1, 1/2*(3*b+d)/d],[3/2*(b+d)/d],e
xp(2*I*(d*x+c)))/(12*b+4*d)
```

Mathematica [A] (verified)

Time = 20.66 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.03

$$\int \cos^3(a+bx)\csc(c+dx)\,dx$$
$$= \frac{e^{-3i(a+bx)}\left(-\text{Hypergeometric2F1}\left(1,-\frac{3b}{d},1-\frac{3b}{d},-e^{i(c+dx)}\right)+\text{Hypergeometric2F1}\left(1,-\frac{3b}{d},1-\frac{3b}{d},e^{i(c+dx}\right.\right.}{}$$

input
```
Integrate[Cos[a + b*x]^3*Csc[c + d*x],x]
```

output
```
(-Hypergeometric2F1[1, (-3*b)/d, 1 - (3*b)/d, -E^(I*(c + d*x))] + Hypergeo
metric2F1[1, (-3*b)/d, 1 - (3*b)/d, E^(I*(c + d*x))] + E^((2*I)*(a + b*x))
*(-9*Hypergeometric2F1[1, -(b/d), 1 - b/d, -E^(I*(c + d*x))] + 9*Hypergeom
etric2F1[1, -(b/d), 1 - b/d, E^(I*(c + d*x))] + E^((2*I)*(a + b*x))*(9*Hyp
ergeometric2F1[1, b/d, (b + d)/d, -E^(I*(c + d*x))] - 9*Hypergeometric2F1[
1, b/d, (b + d)/d, E^(I*(c + d*x))] + E^((2*I)*(a + b*x))*(Hypergeometric2
F1[1, (3*b)/d, 1 + (3*b)/d, -E^(I*(c + d*x))] - Hypergeometric2F1[1, (3*b)
/d, 1 + (3*b)/d, E^(I*(c + d*x))]))))/(24*b*E^((3*I)*(a + b*x)))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc(c+dx)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \cos^3(a+bx)\csc(c+dx)dx$$

input
```
Int[Cos[a + b*x]^3*Csc[c + d*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos(bx+a)^3 \csc(dx+c)\, dx$$

input
```
int(cos(b*x+a)^3*csc(d*x+c),x)
```

output
```
int(cos(b*x+a)^3*csc(d*x+c),x)
```

Fricas [F]

$$\int \cos^3(a+bx)\csc(c+dx)\, dx = \int \cos(bx+a)^3 \csc(dx+c)\, dx$$

input
```
integrate(cos(b*x+a)^3*csc(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^3*csc(d*x + c), x)
```

Sympy [F]

$$\int \cos^3(a+bx)\csc(c+dx)\, dx = \int \cos^3(a+bx)\csc(c+dx)\, dx$$

input
```
integrate(cos(b*x+a)**3*csc(d*x+c),x)
```

output
```
Integral(cos(a + b*x)**3*csc(c + d*x), x)
```

Maxima [F]

$$\int \cos^3(a+bx)\csc(c+dx)\,dx = \int \cos\left(bx+a\right)^3\csc\left(dx+c\right)\,dx$$

input
```
integrate(cos(b*x+a)^3*csc(d*x+c),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3*csc(d*x + c), x)
```

Giac [F]

$$\int \cos^3(a+bx)\csc(c+dx)\,dx = \int \cos\left(bx+a\right)^3\csc\left(dx+c\right)\,dx$$

input
```
integrate(cos(b*x+a)^3*csc(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3*csc(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc(c+dx)\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\sin\left(c+d\,x\right)}\,dx$$

input
```
int(cos(a + b*x)^3/sin(c + d*x),x)
```

output
```
int(cos(a + b*x)^3/sin(c + d*x), x)
```

Reduce [F]

$$\int \cos^3(a+bx)\csc(c+dx)\,dx=\int \cos(bx+a)^3\csc(dx+c)\,dx$$

input
```
int(cos(b*x+a)^3*csc(d*x+c),x)
```

output
```
int(cos(a + b*x)**3*csc(c + d*x),x)
```

3.204 $\int \cos^2(a+bx)\csc(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 149

$$\int \cos^2(a+bx)\csc(c+dx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{2d} + \frac{e^{-2ia-2ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{b}{d},\frac{3}{2}-\frac{b}{d},e^{2i(c+dx)}\right)}{2(2b-d)} - \frac{e^{2ia+2ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{b}{d},\frac{3}{2}+\frac{b}{d},e^{2i(c+dx)}\right)}{2(2b+d)}$$

output
```
-1/2*arctanh(cos(d*x+c))/d+exp(-2*I*a-2*I*b*x+I*(d*x+c))*hypergeom([1, 1/2
-b/d],[3/2-b/d],exp(2*I*(d*x+c)))/(4*b-2*d)-exp(2*I*a+2*I*b*x+I*(d*x+c))*h
ypergeom([1, 1/2+b/d],[3/2+b/d],exp(2*I*(d*x+c)))/(4*b+2*d)
```

Mathematica [A] (verified)

Time = 13.36 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.43

$$\int \cos^2(a+bx)\csc(c+dx)\,dx$$
$$= \frac{e^{-2i(a+bx)}\left(-d\,\text{Hypergeometric2F1}\left(1,-\frac{2b}{d},1-\frac{2b}{d},-e^{i(c+dx)}\right)+d\,\text{Hypergeometric2F1}\left(1,-\frac{2b}{d},1-\frac{2b}{d},e^{i(c}\right.\right.}{}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c + d*x],x]
```

output
```
(-(d*Hypergeometric2F1[1, (-2*b)/d, 1 - (2*b)/d, -E^(I*(c + d*x))]) + d*Hy
pergeometric2F1[1, (-2*b)/d, 1 - (2*b)/d, E^(I*(c + d*x))] + d*E^((4*I)*(a
 + b*x))*Hypergeometric2F1[1, (2*b)/d, 1 + (2*b)/d, -E^(I*(c + d*x))] - d*
E^((4*I)*(a + b*x))*Hypergeometric2F1[1, (2*b)/d, 1 + (2*b)/d, E^(I*(c + d
*x))] + 4*b*E^((2*I)*(a + b*x))*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(
c + d*x))]))/(8*b*d*E^((2*I)*(a + b*x)))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc(c+dx)\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \cos^2(a+bx)\csc(c+dx)dx$$

input
```
Int[Cos[a + b*x]^2*Csc[c + d*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos \left(b x+a\right)^{2} \csc \left(d x+c\right)d x$$

input
```
int(cos(b*x+a)^2*csc(d*x+c),x)
```

output
```
int(cos(b*x+a)^2*csc(d*x+c),x)
```

Fricas [F]

$$\int \cos^2(a+bx)\csc(c+dx)\,dx=\int \cos \left(bx+a\right)^{2} \csc \left(dx+c\right)\,dx$$

input
```
integrate(cos(b*x+a)^2*csc(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*csc(d*x + c), x)
```

Sympy [F]

$$\int \cos^2(a+bx)\csc(c+dx)\,dx=\int \cos^{2}\left(a+bx\right)\csc\left(c+dx\right)dx$$

input
```
integrate(cos(b*x+a)**2*csc(d*x+c),x)
```

output
```
Integral(cos(a + b*x)**2*csc(c + d*x), x)
```

Maxima [F]

$$\int \cos^2(a+bx)\csc(c+dx)\,dx=\int \cos{(bx+a)}^2\csc{(dx+c)}\,\,dx$$

input
```
integrate(cos(b*x+a)^2*csc(d*x+c),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2*csc(d*x + c), x)
```

Giac [F]

$$\int \cos^2(a+bx)\csc(c+dx)\,dx=\int \cos{(bx+a)}^2\csc{(dx+c)}\,\,dx$$

input
```
integrate(cos(b*x+a)^2*csc(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*csc(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc(c+dx)\,dx=\int \frac{{\cos\left(a+b\,x\right)}^2}{\sin\left(c+d\,x\right)}\,dx$$

input
```
int(cos(a + b*x)^2/sin(c + d*x),x)
```

output
```
int(cos(a + b*x)^2/sin(c + d*x), x)
```

Reduce [F]

$$\int \cos^2(a+bx)\csc(c+dx)\,dx = \int \cos\left(bx+a\right)^2\csc\left(dx+c\right)dx$$

input
```
int(cos(b*x+a)^2*csc(d*x+c),x)
```

output
```
int(cos(a + b*x)**2*csc(c + d*x),x)
```

3.205 $\int \cos(a+bx)\csc(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 133

$$\int \cos(a+bx)\csc(c+dx)\,dx = \frac{e^{-ia-ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2d},\frac{1}{2}\left(3-\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b-d} - \frac{e^{ia+ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b+d}$$

output
```
exp(-I*a-I*b*x+I*(d*x+c))*hypergeom([1, -1/2*(b-d)/d],[3/2-1/2*b/d],exp(2*
I*(d*x+c)))/(b-d)-exp(I*a+I*b*x+I*(d*x+c))*hypergeom([1, 1/2*(b+d)/d],[3/2
+1/2*b/d],exp(2*I*(d*x+c)))/(b+d)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

$$\int \cos(a+bx)\csc(c+dx)\,dx = \frac{e^{-i(a-c+(b-d)x)}\left((b+d)\,\text{Hypergeometric2F1}\left(1,\frac{-b+d}{2d},\frac{3}{2}-\frac{b}{2d},e^{2i(c+dx)}\right)-(b-d)e^{2i(a+bx)}\,\text{Hypergeometric}\right.}{(b-d)(b+d)}$$

input
```
Integrate[Cos[a + b*x]*Csc[c + d*x],x]
```

output
```
((b + d)*Hypergeometric2F1[1, (-b + d)/(2*d), 3/2 - b/(2*d), E^((2*I)*(c +
 d*x))] - (b - d)*E^((2*I)*(a + b*x))*Hypergeometric2F1[1, (b + d)/(2*d),
(3 + b/d)/2, E^((2*I)*(c + d*x))])/((b - d)*(b + d)*E^(I*(a - c + (b - d)*
x)))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \cos(a+bx)\csc(c+dx)dx$$

input
```
Int[Cos[a + b*x]*Csc[c + d*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos (bx+a) \csc (dx+c) \, dx$$

input
```
int(cos(b*x+a)*csc(d*x+c),x)
```

output
```
int(cos(b*x+a)*csc(d*x+c),x)
```

Fricas [F]

$$\int \cos(a+bx) \csc(c+dx) \, dx = \int \cos (bx+a) \csc (dx+c) \, dx$$

input
```
integrate(cos(b*x+a)*csc(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*csc(d*x + c), x)
```

Sympy [F]

$$\int \cos(a+bx) \csc(c+dx) \, dx = \int \cos (a+bx) \csc (c+dx) \, dx$$

input
```
integrate(cos(b*x+a)*csc(d*x+c),x)
```

output
```
Integral(cos(a + b*x)*csc(c + d*x), x)
```

Maxima [F]

$$\int \cos(a+bx)\csc(c+dx)\,dx = \int \cos(bx+a)\csc(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)*csc(d*x+c),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*csc(d*x + c), x)
```

Giac [F]

$$\int \cos(a+bx)\csc(c+dx)\,dx = \int \cos(bx+a)\csc(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)*csc(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*csc(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc(c+dx)\,dx = \int \frac{\cos(a+b\,x)}{\sin(c+d\,x)}\,dx$$

input
```
int(cos(a + b*x)/sin(c + d*x),x)
```

output
```
int(cos(a + b*x)/sin(c + d*x), x)
```

Reduce [F]

$$\int \cos(a+bx)\csc(c+dx)\,dx = \int \cos(bx+a)\csc(dx+c)\,dx$$

input
```
int(cos(b*x+a)*csc(d*x+c),x)
```

output
```
int(cos(a + b*x)*csc(c + d*x),x)
```

3.206 $\int \csc(c+dx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \csc(c+dx)\sec(a+bx)\,dx = \text{Int}(\csc(c+dx)\sec(a+bx),x)$$

output
```
Defer(Int)(csc(d*x+c)*sec(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 16.96 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \csc(c+dx)\sec(a+bx)\,dx = \int \csc(c+dx)\sec(a+bx)\,dx$$

input
```
Integrate[Csc[c + d*x]*Sec[a + b*x],x]
```

output
```
Integrate[Csc[c + d*x]*Sec[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\csc(c+dx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sec(a+bx)\csc(c+dx)dx$$

input
```
Int[Csc[c + d*x]*Sec[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc(dx+c)\sec(bx+a)\,dx$$

input
```
int(csc(d*x+c)*sec(b*x+a),x)
```

output
```
int(csc(d*x+c)*sec(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \csc(c+dx)\sec(a+bx)\,dx = \int \csc\left(dx+c\right)\sec\left(bx+a\right)\,dx$$

input
```
integrate(csc(d*x+c)*sec(b*x+a),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)*sec(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \csc(c+dx)\sec(a+bx)\,dx = \int \csc\left(c+dx\right)\sec\left(a+bx\right)dx$$

input
```
integrate(csc(d*x+c)*sec(b*x+a),x)
```

output
```
Integral(csc(c + d*x)*sec(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \csc(c+dx)\sec(a+bx)\,dx = \int \csc(dx+c)\sec(bx+a)\,dx$$

input
```
integrate(csc(d*x+c)*sec(b*x+a),x, algorithm="maxima")
```

output
```
integrate(csc(d*x + c)*sec(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \csc(c+dx)\sec(a+bx)\,dx = \int \csc(dx+c)\sec(bx+a)\,dx$$

input
```
integrate(csc(d*x+c)*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)*sec(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 17.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \csc(c+dx)\sec(a+bx)\,dx = \int \frac{1}{\cos(a+b\,x)\,\sin(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \csc(c+dx)\sec(a+bx)\,dx=\int \csc\left(dx+c\right)\sec\left(bx+a\right)dx$$

input
```
int(csc(d*x+c)*sec(b*x+a),x)
```

output
```
int(csc(c + d*x)*sec(a + b*x),x)
```

3.207 $\int \csc(c+dx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \csc(c+dx)\sec^2(a+bx)\,dx = \text{Int}\big(\csc(c+dx)\sec^2(a+bx),x\big)$$

output
```
Defer(Int)(csc(d*x+c)*sec(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 13.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(c+dx)\sec^2(a+bx)\,dx = \int \csc(c+dx)\sec^2(a+bx)\,dx$$

input
```
Integrate[Csc[c + d*x]*Sec[a + b*x]^2,x]
```

output
```
Integrate[Csc[c + d*x]*Sec[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\csc(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec^2(a+bx)\csc(c+dx)dx$$

input
```
Int[Csc[c + d*x]*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc(dx+c)\sec(bx+a)^2\,dx$$

input
```
int(csc(d*x+c)*sec(b*x+a)^2,x)
```

output
```
int(csc(d*x+c)*sec(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(c+dx)\sec^2(a+bx)\,dx = \int \csc\left(dx+c\right)\sec\left(bx+a\right)^2\,dx$$

input
```
integrate(csc(d*x+c)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)*sec(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc(c+dx)\sec^2(a+bx)\,dx = \int \csc\left(c+dx\right)\sec^2\left(a+bx\right)dx$$

input
```
integrate(csc(d*x+c)*sec(b*x+a)**2,x)
```

output
```
Integral(csc(c + d*x)*sec(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(c+dx)\sec^2(a+bx)\,dx = \int \csc(dx+c)\sec(bx+a)^2\,dx$$

input
```
integrate(csc(d*x+c)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(csc(d*x + c)*sec(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(c+dx)\sec^2(a+bx)\,dx = \int \csc(dx+c)\sec(bx+a)^2\,dx$$

input
```
integrate(csc(d*x+c)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)*sec(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 17.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \csc(c+dx)\sec^2(a+bx)\,dx = \int \frac{1}{\cos(a+b\,x)^2\,\sin(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(c+dx)\sec^2(a+bx)\,dx = \int \csc(dx+c)\sec(bx+a)^2\,dx$$

input
```
int(csc(d*x+c)*sec(b*x+a)^2,x)
```

output
```
int(csc(c + d*x)*sec(a + b*x)**2,x)
```

3.208 $\int \csc(c+dx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \csc(c+dx)\sec^3(a+bx)\,dx=\text{Int}\left(\csc(c+dx)\sec^3(a+bx),x\right)$$

output
```
Defer(Int)(csc(d*x+c)*sec(b*x+a)^3,x)
```

Mathematica [F(-1)]

Timed out.

$$\int \csc(c+dx)\sec^3(a+bx)\,dx=\$\text{Aborted}$$

input
```
Integrate[Csc[c + d*x]*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)\csc(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec^3(a+bx)\csc(c+dx)dx$$

input
```
Int[Csc[c + d*x]*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc(dx+c)\sec(bx+a)^3\,dx$$

input
```
int(csc(d*x+c)*sec(b*x+a)^3,x)
```

output
```
int(csc(d*x+c)*sec(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(c+dx)\sec^3(a+bx)\,dx = \int \csc(dx+c)\sec(bx+a)^3\,dx$$

input
```
integrate(csc(d*x+c)*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)*sec(b*x + a)^3, x)
```

Sympy [N/A]

Not integrable

Time = 4.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc(c+dx)\sec^3(a+bx)\,dx = \int \csc(c+dx)\sec^3(a+bx)\,dx$$

input
```
integrate(csc(d*x+c)*sec(b*x+a)**3,x)
```

output
```
Integral(csc(c + d*x)*sec(a + b*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(c+dx)\sec^3(a+bx)\,dx = \int \csc(dx+c)\sec(bx+a)^3\,dx$$

input
```
integrate(csc(d*x+c)*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(csc(d*x + c)*sec(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(c+dx)\sec^3(a+bx)\,dx = \int \csc(dx+c)\sec(bx+a)^3\,dx$$

input
```
integrate(csc(d*x+c)*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)*sec(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 16.76 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \csc(c+dx)\sec^3(a+bx)\,dx = \int \frac{1}{\cos(a+b\,x)^3\,\sin(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc(c+dx)\sec^3(a+bx)\,dx=\int \csc\left(dx+c\right)\sec\left(bx+a\right)^3dx$$

input
```
int(csc(d*x+c)*sec(b*x+a)^3,x)
```

output
```
int(csc(c + d*x)*sec(a + b*x)**3,x)
```

3.209 $\int \cos^3(a+bx)\csc^2(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 281

$$\begin{aligned}\int \cos^3(a+bx)\csc^2(c+dx)\,dx &= -\frac{ie^{-3ia-3ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{3b}{2d},2-\frac{3b}{2d},e^{2i(c+dx)}\right)}{2(3b-2d)} \\ &\quad - \frac{3ie^{-ia-ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{b}{2d},2-\frac{b}{2d},e^{2i(c+dx)}\right)}{2(b-2d)} \\ &\quad + \frac{3ie^{ia+ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{b}{2d},2+\frac{b}{2d},e^{2i(c+dx)}\right)}{2(b+2d)} \\ &\quad + \frac{ie^{3ia+3ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{3b}{2d},2+\frac{3b}{2d},e^{2i(c+dx)}\right)}{2(3b+2d)}\end{aligned}$$

output
```
-1/2*I*exp(-3*I*a-3*I*b*x+2*I*(d*x+c))*hypergeom([2, 1-3/2*b/d],[2-3/2*b/d
],exp(2*I*(d*x+c)))/(3*b-2*d)-3/2*I*exp(-I*a-I*b*x+2*I*(d*x+c))*hypergeom(
[2, 1-1/2*b/d],[2-1/2*b/d],exp(2*I*(d*x+c)))/(b-2*d)+3/2*I*exp(I*a+I*b*x+2
*I*(d*x+c))*hypergeom([2, 1+1/2*b/d],[2+1/2*b/d],exp(2*I*(d*x+c)))/(b+2*d)
+1/2*I*exp(3*I*a+3*I*b*x+2*I*(d*x+c))*hypergeom([2, 1+3/2*b/d],[2+3/2*b/d]
,exp(2*I*(d*x+c)))/(3*b+2*d)
```

Mathematica [A] (verified)

Time = 6.91 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.32

$$\int \cos^3(a+bx)\csc^2(c+dx)\,dx =$$
$$-\frac{ie^{-3i(a+bx)}\left(1+3e^{2i(a+bx)}+3e^{4i(a+bx)}+e^{6i(a+bx)}+(-1+e^{2ic})\,\text{Hypergeometric2F1}\left(1,-\frac{3b}{2d},1-\frac{3b}{2d},e^{2i(}\right.\right.}{}$$
$$+\frac{\cos^3(a+bx)\csc(c)\csc(c+dx)\sin(dx)}{d}$$

input
```
Integrate[Cos[a + b*x]^3*Csc[c + d*x]^2,x]
```

output
```
((-1/4*I)*(1 + 3*E^((2*I)*(a + b*x)) + 3*E^((4*I)*(a + b*x)) + E^((6*I)*(a
 + b*x)) + (-1 + E^((2*I)*c))*Hypergeometric2F1[1, (-3*b)/(2*d), 1 - (3*b)
/(2*d), E^((2*I)*(c + d*x))] + 3*E^((2*I)*(a + b*x))*(-1 + E^((2*I)*c))*Hy
pergeometric2F1[1, -1/2*b/d, 1 - b/(2*d), E^((2*I)*(c + d*x))] - 3*E^((4*I
)*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x)
)] + 3*E^((2*I)*(2*a + c + 2*b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*
d), E^((2*I)*(c + d*x))] - E^((6*I)*(a + b*x))*Hypergeometric2F1[1, (3*b)/
(2*d), 1 + (3*b)/(2*d), E^((2*I)*(c + d*x))] + E^((2*I)*(3*a + c + 3*b*x))
*Hypergeometric2F1[1, (3*b)/(2*d), 1 + (3*b)/(2*d), E^((2*I)*(c + d*x))]))
/(d*E^((3*I)*(a + b*x))*(-1 + E^((2*I)*c))) + (Cos[a + b*x]^3*Csc[c]*Csc[c
 + d*x]*Sin[d*x])/d
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc^2(c+dx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \cos^3(a+bx)\csc^2(c+dx)dx$$

input
```
Int[Cos[a + b*x]^3*Csc[c + d*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos (bx+a)^3 \csc (dx+c)^2 \, dx$$

input
```
int(cos(b*x+a)^3*csc(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)^3*csc(d*x+c)^2,x)
```

Fricas [F]

$$\int \cos^3(a+bx)\csc^2(c+dx)\,dx = \int \cos (bx+a)^3 \csc (dx+c)^2 \, dx$$

input
```
integrate(cos(b*x+a)^3*csc(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^3*csc(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*csc(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(a+bx)\csc^2(c+dx)\,dx = \int \cos(bx+a)^3\csc(dx+c)^2\,dx$$

input
```
integrate(cos(b*x+a)^3*csc(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*((sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*cos((3*b
 + 2*d)*x + 3*a + 2*c) + 4*(d*cos((3*b + 2*d)*x + 3*a + 2*c)^2 - 2*d*cos((
3*b + 2*d)*x + 3*a + 2*c)*cos(3*b*x + 3*a) + d*cos(3*b*x + 3*a)^2 + d*sin(
(3*b + 2*d)*x + 3*a + 2*c)^2 - 2*d*sin((3*b + 2*d)*x + 3*a + 2*c)*sin(3*b*
x + 3*a) + d*sin(3*b*x + 3*a)^2)*integrate(3/8*(b*cos(6*b*x + 6*a)*cos(3*b
*x + 3*a) + b*cos(4*b*x + 4*a)*cos(3*b*x + 3*a) + b*sin(6*b*x + 6*a)*sin(3
*b*x + 3*a) + b*sin(4*b*x + 4*a)*sin(3*b*x + 3*a) - b*sin(3*b*x + 3*a)*sin
(2*b*x + 2*a) + (b*cos(6*b*x + 6*a) + b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2
*a) - b)*cos((3*b + d)*x + 3*a + c) - (b*cos(2*b*x + 2*a) + b)*cos(3*b*x +
 3*a) + (b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin
((3*b + d)*x + 3*a + c))/(d*cos((3*b + d)*x + 3*a + c)^2 + 2*d*cos((3*b +
d)*x + 3*a + c)*cos(3*b*x + 3*a) + d*cos(3*b*x + 3*a)^2 + d*sin((3*b + d)*
x + 3*a + c)^2 + 2*d*sin((3*b + d)*x + 3*a + c)*sin(3*b*x + 3*a) + d*sin(3
*b*x + 3*a)^2), x) - 4*(d*cos((3*b + 2*d)*x + 3*a + 2*c)^2 - 2*d*cos((3*b
+ 2*d)*x + 3*a + 2*c)*cos(3*b*x + 3*a) + d*cos(3*b*x + 3*a)^2 + d*sin((3*b
 + 2*d)*x + 3*a + 2*c)^2 - 2*d*sin((3*b + 2*d)*x + 3*a + 2*c)*sin(3*b*x +
3*a) + d*sin(3*b*x + 3*a)^2)*integrate(-3/8*(b*cos(6*b*x + 6*a)*cos(3*b*x
+ 3*a) + b*cos(4*b*x + 4*a)*cos(3*b*x + 3*a) + b*sin(6*b*x + 6*a)*sin(3*b*
x + 3*a) + b*sin(4*b*x + 4*a)*sin(3*b*x + 3*a) - b*sin(3*b*x + 3*a)*sin(2*
b*x + 2*a) - (b*cos(6*b*x + 6*a) + b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2...
```

Giac [F]

$$\int \cos^3(a+bx)\csc^2(c+dx)\,dx = \int \cos(bx+a)^3\csc(dx+c)^2\,dx$$

input
```
integrate(cos(b*x+a)^3*csc(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3*csc(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^2(c+dx)\,dx = \int \frac{\cos(a+b\,x)^3}{\sin(c+d\,x)^2}\,dx$$

input
```
int(cos(a + b*x)^3/sin(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)^3/sin(c + d*x)^2, x)
```

Reduce [F]

$$\int \cos^3(a+bx)\csc^2(c+dx)\,dx = \int \cos(bx+a)^3\csc(dx+c)^2\,dx$$

input
```
int(cos(b*x+a)^3*csc(d*x+c)^2,x)
```

output
```
int(cos(a + b*x)**3*csc(c + d*x)**2,x)
```

3.210 $\int \cos^2(a+bx)\csc^2(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 140

$$\int \cos^2(a+bx)\csc^2(c+dx)\,dx = -\frac{\cot(c+dx)}{2d} - \frac{ie^{-2ia-2ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{b}{d},2-\frac{b}{d},e^{2i(c+dx)}\right)}{2(b-d)} + \frac{ie^{2ia+2ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,\frac{b+d}{d},2+\frac{b}{d},e^{2i(c+dx)}\right)}{2(b+d)}$$

output
```
-1/2*cot(d*x+c)/d-1/2*I*exp(-2*I*a-2*I*b*x+2*I*(d*x+c))*hypergeom([2, 1-b/
d],[2-b/d],exp(2*I*(d*x+c)))/(b-d)+1/2*I*exp(2*I*a+2*I*b*x+2*I*(d*x+c))*hy
pergeom([2, (b+d)/d],[2+b/d],exp(2*I*(d*x+c)))/(b+d)
```

Mathematica [A] (verified)

Time = 3.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09

$$\int \cos^2(a+bx)\csc^2(c+dx)\,dx = -\frac{ie^{-2i(a+bx)}\left(1+e^{4i(a+bx)}+\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(1,-\frac{b}{d},1-\frac{b}{d},e^{2i(c+dx)}\right)+e^{4i(a+bx)}\left(-1+e^{2\cdots}\right.\right.}{2d\left(-1+e^{2ic}\right)} + \frac{\cos^2(a+bx)\csc(c)\csc(c+dx)\sin(dx)}{d}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c + d*x]^2,x]
```

output
```
((-1/2*I)*(1 + E^((4*I)*(a + b*x)) + (-1 + E^((2*I)*c))*Hypergeometric2F1[
1, -(b/d), 1 - b/d, E^((2*I)*(c + d*x))] + E^((4*I)*(a + b*x))*(-1 + E^((2
*I)*c))*Hypergeometric2F1[1, b/d, (b + d)/d, E^((2*I)*(c + d*x))]))/(d*E^(
(2*I)*(a + b*x))*(-1 + E^((2*I)*c))) + (Cos[a + b*x]^2*Csc[c]*Csc[c + d*x]
*Sin[d*x])/d
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^2(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos^2(a+bx)\csc^2(c+dx)dx$$

input
```
Int[Cos[a + b*x]^2*Csc[c + d*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos(bx+a)^2 \csc(dx+c)^2\, dx$$

input
```
int(cos(b*x+a)^2*csc(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)^2*csc(d*x+c)^2,x)
```

Fricas [F]

$$\int \cos^2(a+bx)\csc^2(c+dx)\, dx = \int \cos(bx+a)^2 \csc(dx+c)^2\, dx$$

input
```
integrate(cos(b*x+a)^2*csc(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*csc(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^2(c+dx)\, dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*csc(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(a+bx)\csc^2(c+dx)\,dx=\int \cos\left(bx+a\right)^2\csc\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*csc(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*((sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(2*(b + d)*x + 2*a + 2*c)
+ 2*(d*cos(2*(b + d)*x + 2*a + 2*c)^2 - 2*d*cos(2*(b + d)*x + 2*a + 2*c)*c
os(2*b*x + 2*a) + d*cos(2*b*x + 2*a)^2 + d*sin(2*(b + d)*x + 2*a + 2*c)^2
- 2*d*sin(2*(b + d)*x + 2*a + 2*c)*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^2
)*integrate(1/2*(b*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + b*sin((2*b + d)*x +
 2*a + c)*sin(4*b*x + 4*a) + b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b*cos(
4*b*x + 4*a) - b)*cos((2*b + d)*x + 2*a + c) - b*cos(2*b*x + 2*a))/(d*cos(
(2*b + d)*x + 2*a + c)^2 + 2*d*cos((2*b + d)*x + 2*a + c)*cos(2*b*x + 2*a)
 + d*cos(2*b*x + 2*a)^2 + d*sin((2*b + d)*x + 2*a + c)^2 + 2*d*sin((2*b +
d)*x + 2*a + c)*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^2), x) - 2*(d*cos(2*
(b + d)*x + 2*a + 2*c)^2 - 2*d*cos(2*(b + d)*x + 2*a + 2*c)*cos(2*b*x + 2*
a) + d*cos(2*b*x + 2*a)^2 + d*sin(2*(b + d)*x + 2*a + 2*c)^2 - 2*d*sin(2*(
b + d)*x + 2*a + 2*c)*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^2)*integrate(-
1/2*(b*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) - b*sin((2*b + d)*x + 2*a + c)*si
n(4*b*x + 4*a) + b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - (b*cos(4*b*x + 4*a)
 - b)*cos((2*b + d)*x + 2*a + c) - b*cos(2*b*x + 2*a))/(d*cos((2*b + d)*x
+ 2*a + c)^2 - 2*d*cos((2*b + d)*x + 2*a + c)*cos(2*b*x + 2*a) + d*cos(2*b
*x + 2*a)^2 + d*sin((2*b + d)*x + 2*a + c)^2 - 2*d*sin((2*b + d)*x + 2*a +
 c)*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^2), x) - (cos(4*b*x + 4*a) + 2*c
os(2*b*x + 2*a) + 1)*sin(2*(b + d)*x + 2*a + 2*c) - cos(2*b*x + 2*a)*si...
```

Giac [F]

$$\int \cos^2(a+bx)\csc^2(c+dx)\,dx=\int \cos\left(bx+a\right)^2\csc\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*csc(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*csc(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^2(c+dx)\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\sin\left(c+d\,x\right)^2}\,dx$$

input
```
int(cos(a + b*x)^2/sin(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)^2/sin(c + d*x)^2, x)
```

Reduce [F]

$$\int \cos^2(a+bx)\csc^2(c+dx)\,dx = \int \cos\left(bx+a\right)^2\csc\left(dx+c\right)^2dx$$

input
```
int(cos(b*x+a)^2*csc(d*x+c)^2,x)
```

output
```
int(cos(a + b*x)**2*csc(c + d*x)**2,x)
```

3.211 $\int \cos(a+bx)\csc^2(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 135

$$\int \cos(a+bx)\csc^2(c+dx)\,dx = -\frac{2ie^{-ia-ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{b}{2d},2-\frac{b}{2d},e^{2i(c+dx)}\right)}{b-2d} + \frac{2ie^{ia+ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{b}{2d},2+\frac{b}{2d},e^{2i(c+dx)}\right)}{b+2d}$$

output
```
-2*I*exp(-I*a-I*b*x+2*I*(d*x+c))*hypergeom([2, 1-1/2*b/d],[2-1/2*b/d],exp(
2*I*(d*x+c)))/(b-2*d)+2*I*exp(I*a+I*b*x+2*I*(d*x+c))*hypergeom([2, 1+1/2*b
/d],[2+1/2*b/d],exp(2*I*(d*x+c)))/(b+2*d)
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.17

$$\int \cos(a+bx)\csc^2(c+dx)\,dx = -\frac{ie^{-i(a+bx)}\left(1+e^{2i(a+bx)}+\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)+e^{2i(a+bx)}(-1+e\right.}{d\left(-1+e^{2ic}\right)} + \frac{\cos(a+bx)\csc(c)\csc(c+dx)\sin(dx)}{d}$$

input
```
Integrate[Cos[a + b*x]*Csc[c + d*x]^2,x]
```

output
```
((-I)*(1 + E^((2*I)*(a + b*x)) + (-1 + E^((2*I)*c))*Hypergeometric2F1[1, -
1/2*b/d, 1 - b/(2*d), E^((2*I)*(c + d*x))] + E^((2*I)*(a + b*x))*(-1 + E^(
(2*I)*c))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x))])
)/(d*E^(I*(a + b*x))*(-1 + E^((2*I)*c))) + (Cos[a + b*x]*Csc[c]*Csc[c + d*
x]*Sin[d*x])/d
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^2(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos(a+bx)\csc^2(c+dx)dx$$

input
```
Int[Cos[a + b*x]*Csc[c + d*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos (bx + a) \csc (dx + c)^{2} dx$$

input
```
int(cos(b*x+a)*csc(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)*csc(d*x+c)^2,x)
```

Fricas [F]

$$\int \cos(a + bx) \csc^2(c + dx) \, dx = \int \cos (bx + a) \csc (dx + c)^{2} \, dx$$

input
```
integrate(cos(b*x+a)*csc(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*csc(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a + bx) \csc^2(c + dx) \, dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*csc(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(a+bx)\csc^2(c+dx)\,dx = \int \cos(bx+a)\csc(dx+c)^2\,dx$$

input
```
integrate(cos(b*x+a)*csc(d*x+c)^2,x, algorithm="maxima")
```

output
```
((d*cos((b + 2*d)*x + a + 2*c)^2 - 2*d*cos((b + 2*d)*x + a + 2*c)*cos(b*x
+ a) + d*cos(b*x + a)^2 + d*sin((b + 2*d)*x + a + 2*c)^2 - 2*d*sin((b + 2*
d)*x + a + 2*c)*sin(b*x + a) + d*sin(b*x + a)^2)*integrate(1/2*(b*cos(2*b*
x + 2*a)*cos(b*x + a) + b*sin((b + d)*x + a + c)*sin(2*b*x + 2*a) + b*sin(
2*b*x + 2*a)*sin(b*x + a) + (b*cos(2*b*x + 2*a) - b)*cos((b + d)*x + a + c
) - b*cos(b*x + a))/(d*cos((b + d)*x + a + c)^2 + 2*d*cos((b + d)*x + a +
c)*cos(b*x + a) + d*cos(b*x + a)^2 + d*sin((b + d)*x + a + c)^2 + 2*d*sin(
(b + d)*x + a + c)*sin(b*x + a) + d*sin(b*x + a)^2), x) - (d*cos((b + 2*d)
*x + a + 2*c)^2 - 2*d*cos((b + 2*d)*x + a + 2*c)*cos(b*x + a) + d*cos(b*x
+ a)^2 + d*sin((b + 2*d)*x + a + 2*c)^2 - 2*d*sin((b + 2*d)*x + a + 2*c)*s
in(b*x + a) + d*sin(b*x + a)^2)*integrate(-1/2*(b*cos(2*b*x + 2*a)*cos(b*x
 + a) - b*sin((b + d)*x + a + c)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a)*sin
(b*x + a) - (b*cos(2*b*x + 2*a) - b)*cos((b + d)*x + a + c) - b*cos(b*x +
a))/(d*cos((b + d)*x + a + c)^2 - 2*d*cos((b + d)*x + a + c)*cos(b*x + a)
+ d*cos(b*x + a)^2 + d*sin((b + d)*x + a + c)^2 - 2*d*sin((b + d)*x + a +
c)*sin(b*x + a) + d*sin(b*x + a)^2), x) - (cos(2*b*x + 2*a) + 1)*sin((b +
2*d)*x + a + 2*c) + cos((b + 2*d)*x + a + 2*c)*sin(2*b*x + 2*a) - cos(b*x
+ a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))/(d*c
os((b + 2*d)*x + a + 2*c)^2 - 2*d*cos((b + 2*d)*x + a + 2*c)*cos(b*x + a)
+ d*cos(b*x + a)^2 + d*sin((b + 2*d)*x + a + 2*c)^2 - 2*d*sin((b + 2*d)...
```

Giac [F]

$$\int \cos(a+bx)\csc^2(c+dx)\,dx = \int \cos(bx+a)\csc(dx+c)^2\,dx$$

input
```
integrate(cos(b*x+a)*csc(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*csc(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^2(c+dx)\,dx = \int \frac{\cos(a+b\,x)}{\sin(c+d\,x)^2}\,dx$$

input
```
int(cos(a + b*x)/sin(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)/sin(c + d*x)^2, x)
```

Reduce [F]

$$\int \cos(a+bx)\csc^2(c+dx)\,dx = \int \cos(bx+a)\csc(dx+c)^2\,dx$$

input
```
int(cos(b*x+a)*csc(d*x+c)^2,x)
```

output
```
int(cos(a + b*x)*csc(c + d*x)**2,x)
```

3.212 $\int \csc^2(c+dx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \csc^2(c+dx)\sec(a+bx)\,dx = \operatorname{Int}\left(\csc^2(c+dx)\sec(a+bx),x\right)$$

output
```
Defer(Int)(csc(d*x+c)^2*sec(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 20.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc^2(c+dx)\sec(a+bx)\,dx = \int \csc^2(c+dx)\sec(a+bx)\,dx$$

input
```
Integrate[Csc[c + d*x]^2*Sec[a + b*x],x]
```

output
```
Integrate[Csc[c + d*x]^2*Sec[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\csc^2(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec(a+bx)\csc^2(c+dx)dx$$

input
```
Int[Csc[c + d*x]^2*Sec[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc(dx+c)^2 \sec(bx+a)\,dx$$

input
```
int(csc(d*x+c)^2*sec(b*x+a),x)
```

output
```
int(csc(d*x+c)^2*sec(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc^2(c+dx)\sec(a+bx)\,dx=\int \csc\left(dx+c\right)^2\sec\left(bx+a\right)\,dx$$

input
```
integrate(csc(d*x+c)^2*sec(b*x+a),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^2*sec(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc^2(c+dx)\sec(a+bx)\,dx=\int \csc^2\left(c+dx\right)\sec\left(a+bx\right)dx$$

input
```
integrate(csc(d*x+c)**2*sec(b*x+a),x)
```

output
```
Integral(csc(c + d*x)**2*sec(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 4.00 (sec) , antiderivative size = 2521, normalized size of antiderivative = 168.07

$$\int \csc^2(c+dx)\sec(a+bx)\,dx = \int \csc(dx+c)^2\sec(bx+a)\,dx$$

input
```
integrate(csc(d*x+c)^2*sec(b*x+a),x, algorithm="maxima")
```

output
```
((d*cos(2*(b + d)*x + 2*a + 2*c)^2 + d*cos(2*b*x + 2*a)^2 + d*cos(2*d*x +
2*c)^2 + d*sin(2*(b + d)*x + 2*a + 2*c)^2 + d*sin(2*b*x + 2*a)^2 - 2*d*sin
(2*b*x + 2*a)*sin(2*d*x + 2*c) + d*sin(2*d*x + 2*c)^2 - 2*(d*cos(2*b*x + 2
*a) - d*cos(2*d*x + 2*c) + d)*cos(2*(b + d)*x + 2*a + 2*c) + 2*d*cos(2*b*x
 + 2*a) - 2*(d*cos(2*b*x + 2*a) + d)*cos(2*d*x + 2*c) - 2*(d*sin(2*b*x + 2
*a) - d*sin(2*d*x + 2*c))*sin(2*(b + d)*x + 2*a + 2*c) + d)*integrate(2*(2
*b*cos(2*b*x + 2*a)*cos(b*x + a) - 2*b*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) +
 2*b*sin(2*b*x + 2*a)*sin(b*x + a) + (b*cos(3*b*x + 3*a) - b*cos(b*x + a))
*cos((4*b + d)*x + 4*a + c) + 2*(b*cos(3*b*x + 3*a) - b*cos(b*x + a))*cos(
(2*b + d)*x + 2*a + c) - (b*cos(3*b*x + 3*a) - b*cos(b*x + a))*cos(4*b*x +
 4*a) - (2*b*cos(2*b*x + 2*a) + b)*cos(3*b*x + 3*a) + b*cos(b*x + a) + (b*
cos(3*b*x + 3*a) - b*cos(b*x + a))*cos(d*x + c) + (b*sin(3*b*x + 3*a) - b*
sin(b*x + a))*sin((4*b + d)*x + 4*a + c) + 2*(b*sin(3*b*x + 3*a) - b*sin(b
*x + a))*sin((2*b + d)*x + 2*a + c) - (b*sin(3*b*x + 3*a) - b*sin(b*x + a)
)*sin(4*b*x + 4*a) + (b*sin(3*b*x + 3*a) - b*sin(b*x + a))*sin(d*x + c))/(
d*cos((4*b + d)*x + 4*a + c)^2 + 4*d*cos((2*b + d)*x + 2*a + c)^2 + d*cos(
4*b*x + 4*a)^2 + 4*d*cos(2*b*x + 2*a)^2 + d*cos(d*x + c)^2 + d*sin((4*b +
d)*x + 4*a + c)^2 + 4*d*sin((2*b + d)*x + 2*a + c)^2 + d*sin(4*b*x + 4*a)^
2 + 4*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*d*sin(2*b*x + 2*a)^2 + d*sin
(d*x + c)^2 + 2*(2*d*cos((2*b + d)*x + 2*a + c) - d*cos(4*b*x + 4*a) - ...
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc^2(c+dx)\sec(a+bx)\,dx = \int \csc\left(dx+c\right)^2\sec\left(bx+a\right)\,dx$$

input
```
integrate(csc(d*x+c)^2*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^2*sec(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 16.71 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \csc^2(c+dx)\sec(a+bx)\,dx = \int \frac{1}{\cos\left(a+b\,x\right)\,\sin\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)*sin(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc^2(c+dx)\sec(a+bx)\,dx = \int \csc\left(dx+c\right)^2\sec\left(bx+a\right)dx$$

input
```
int(csc(d*x+c)^2*sec(b*x+a),x)
```

output
```
int(csc(c + d*x)**2*sec(a + b*x),x)
```

3.213 $\int \csc^2(c+dx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \csc^2(c+dx)\sec^2(a+bx)\,dx = \text{Int}\left(\csc^2(c+dx)\sec^2(a+bx),x\right)$$

output
```
Defer(Int)(csc(d*x+c)^2*sec(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 16.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(c+dx)\sec^2(a+bx)\,dx = \int \csc^2(c+dx)\sec^2(a+bx)\,dx$$

input
```
Integrate[Csc[c + d*x]^2*Sec[a + b*x]^2,x]
```

output
```
Integrate[Csc[c + d*x]^2*Sec[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\csc^2(c+dx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sec^2(a+bx)\csc^2(c+dx)dx$$

input
```
Int[Csc[c + d*x]^2*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc(dx+c)^2 \sec(bx+a)^2\,dx$$

input
```
int(csc(d*x+c)^2*sec(b*x+a)^2,x)
```

output
```
int(csc(d*x+c)^2*sec(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(c+dx)\sec^2(a+bx)\,dx=\int \csc\left(dx+c\right)^2\sec\left(bx+a\right)^2\,dx$$

input
```
integrate(csc(d*x+c)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^2*sec(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 4.74 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc^2(c+dx)\sec^2(a+bx)\,dx=\int \csc^2\left(c+dx\right)\sec^2\left(a+bx\right)dx$$

input
```
integrate(csc(d*x+c)**2*sec(b*x+a)**2,x)
```

output
```
Integral(csc(c + d*x)**2*sec(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 7.73 (sec) , antiderivative size = 4492, normalized size of antiderivative = 264.24

$$\int \csc^2(c+dx)\sec^2(a+bx)\,dx = \int \csc(dx+c)^2\sec(bx+a)^2\,dx$$

input
```
integrate(csc(d*x+c)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
-((d*cos(2*(2*b + d)*x + 4*a + 2*c)^2 + 4*d*cos(2*(b + d)*x + 2*a + 2*c)^2
 + d*cos(4*b*x + 4*a)^2 + 4*d*cos(2*b*x + 2*a)^2 + d*cos(2*d*x + 2*c)^2 +
d*sin(2*(2*b + d)*x + 4*a + 2*c)^2 + 4*d*sin(2*(b + d)*x + 2*a + 2*c)^2 +
d*sin(4*b*x + 4*a)^2 + 4*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*d*sin(2*b
*x + 2*a)^2 + d*sin(2*d*x + 2*c)^2 + 2*(2*d*cos(2*(b + d)*x + 2*a + 2*c) -
 d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + d*cos(2*d*x + 2*c) - d)*cos(2
*(2*b + d)*x + 4*a + 2*c) - 4*(d*cos(4*b*x + 4*a) + 2*d*cos(2*b*x + 2*a) -
 d*cos(2*d*x + 2*c) + d)*cos(2*(b + d)*x + 2*a + 2*c) + 2*(2*d*cos(2*b*x +
 2*a) + d)*cos(4*b*x + 4*a) + 4*d*cos(2*b*x + 2*a) - 2*(d*cos(4*b*x + 4*a)
 + 2*d*cos(2*b*x + 2*a) + d)*cos(2*d*x + 2*c) + 2*(2*d*sin(2*(b + d)*x + 2
*a + 2*c) - d*sin(4*b*x + 4*a) - 2*d*sin(2*b*x + 2*a) + d*sin(2*d*x + 2*c)
)*sin(2*(2*b + d)*x + 4*a + 2*c) - 4*(d*sin(4*b*x + 4*a) + 2*d*sin(2*b*x +
 2*a) - d*sin(2*d*x + 2*c))*sin(2*(b + d)*x + 2*a + 2*c) - 2*(d*sin(4*b*x
+ 4*a) + 2*d*sin(2*b*x + 2*a))*sin(2*d*x + 2*c) + d)*integrate(8*(3*b*cos(
4*b*x + 4*a)^2 - 3*b*cos(2*b*x + 2*a)^2 + 3*b*sin(4*b*x + 4*a)^2 - 3*b*sin
(2*b*x + 2*a)^2 + (b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a))*cos((6*b + d)*
x + 6*a + c) + 3*(b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a))*cos((4*b + d)*x
 + 4*a + c) + 3*(b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a))*cos((2*b + d)*x
+ 2*a + c) + (b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a))*cos(6*b*x + 6*a) +
b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) + (b*cos(4*b*x + 4*a) - b*cos(2...
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(c+dx)\sec^2(a+bx)\,dx = \int \csc(dx+c)^2\sec(bx+a)^2\,dx$$

input
```
integrate(csc(d*x+c)^2*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^2*sec(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 16.76 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(c+dx)\sec^2(a+bx)\,dx = \int \frac{1}{\cos(a+b\,x)^2\sin(c+d\,x)^2}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(c+dx)\sec^2(a+bx)\,dx = \int \csc(dx+c)^2\sec(bx+a)^2\,dx$$

input
```
int(csc(d*x+c)^2*sec(b*x+a)^2,x)
```

output
```
int(csc(c + d*x)**2*sec(a + b*x)**2,x)
```

3.214 $\int \csc^2(c+dx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \csc^2(c+dx)\sec^3(a+bx)\,dx = \text{Int}\left(\csc^2(c+dx)\sec^3(a+bx),x\right)$$

output
```
Defer(Int)(csc(d*x+c)^2*sec(b*x+a)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 118.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(c+dx)\sec^3(a+bx)\,dx = \int \csc^2(c+dx)\sec^3(a+bx)\,dx$$

input
```
Integrate[Csc[c + d*x]^2*Sec[a + b*x]^3,x]
```

output
```
Integrate[Csc[c + d*x]^2*Sec[a + b*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)\csc^2(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec^3(a+bx)\csc^2(c+dx)dx$$

input
```
Int[Csc[c + d*x]^2*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc(dx+c)^2\sec(bx+a)^3\,dx$$

input
```
int(csc(d*x+c)^2*sec(b*x+a)^3,x)
```

output
```
int(csc(d*x+c)^2*sec(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(c+dx)\sec^3(a+bx)\,dx = \int \csc\left(dx+c\right)^2 \sec\left(bx+a\right)^3\,dx$$

input
```
integrate(csc(d*x+c)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^2*sec(b*x + a)^3, x)
```

Sympy [N/A]

Not integrable

Time = 14.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc^2(c+dx)\sec^3(a+bx)\,dx = \int \csc^2\left(c+dx\right)\sec^3\left(a+bx\right)dx$$

input
```
integrate(csc(d*x+c)**2*sec(b*x+a)**3,x)
```

output
```
Integral(csc(c + d*x)**2*sec(a + b*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 16.83 (sec) , antiderivative size = 7340, normalized size of antiderivative = 431.76

$$\int \csc^2(c+dx)\sec^3(a+bx)\,dx=\int \csc{(dx+c)}^2\sec{(bx+a)}^3\,dx$$

input
```
integrate(csc(d*x+c)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
((d*cos(2*(3*b + d)*x + 6*a + 2*c)^2 + 9*d*cos(2*(2*b + d)*x + 4*a + 2*c)^
2 + 9*d*cos(2*(b + d)*x + 2*a + 2*c)^2 + d*cos(6*b*x + 6*a)^2 + 9*d*cos(4*
b*x + 4*a)^2 + 9*d*cos(2*b*x + 2*a)^2 + d*cos(2*d*x + 2*c)^2 + d*sin(2*(3*
b + d)*x + 6*a + 2*c)^2 + 9*d*sin(2*(2*b + d)*x + 4*a + 2*c)^2 + 9*d*sin(2
*(b + d)*x + 2*a + 2*c)^2 + d*sin(6*b*x + 6*a)^2 + 9*d*sin(4*b*x + 4*a)^2
+ 18*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*d*sin(2*b*x + 2*a)^2 + d*sin(
2*d*x + 2*c)^2 + 2*(3*d*cos(2*(2*b + d)*x + 4*a + 2*c) + 3*d*cos(2*(b + d)
*x + 2*a + 2*c) - d*cos(6*b*x + 6*a) - 3*d*cos(4*b*x + 4*a) - 3*d*cos(2*b*
x + 2*a) + d*cos(2*d*x + 2*c) - d)*cos(2*(3*b + d)*x + 6*a + 2*c) + 6*(3*d
*cos(2*(b + d)*x + 2*a + 2*c) - d*cos(6*b*x + 6*a) - 3*d*cos(4*b*x + 4*a)
- 3*d*cos(2*b*x + 2*a) + d*cos(2*d*x + 2*c) - d)*cos(2*(2*b + d)*x + 4*a +
 2*c) - 6*(d*cos(6*b*x + 6*a) + 3*d*cos(4*b*x + 4*a) + 3*d*cos(2*b*x + 2*a
) - d*cos(2*d*x + 2*c) + d)*cos(2*(b + d)*x + 2*a + 2*c) + 2*(3*d*cos(4*b*
x + 4*a) + 3*d*cos(2*b*x + 2*a) + d)*cos(6*b*x + 6*a) + 6*(3*d*cos(2*b*x +
 2*a) + d)*cos(4*b*x + 4*a) + 6*d*cos(2*b*x + 2*a) - 2*(d*cos(6*b*x + 6*a)
 + 3*d*cos(4*b*x + 4*a) + 3*d*cos(2*b*x + 2*a) + d)*cos(2*d*x + 2*c) + 2*(
3*d*sin(2*(2*b + d)*x + 4*a + 2*c) + 3*d*sin(2*(b + d)*x + 2*a + 2*c) - d*
sin(6*b*x + 6*a) - 3*d*sin(4*b*x + 4*a) - 3*d*sin(2*b*x + 2*a) + d*sin(2*d
*x + 2*c))*sin(2*(3*b + d)*x + 6*a + 2*c) + 6*(3*d*sin(2*(b + d)*x + 2*a +
 2*c) - d*sin(6*b*x + 6*a) - 3*d*sin(4*b*x + 4*a) - 3*d*sin(2*b*x + 2*a...
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(c+dx)\sec^3(a+bx)\,dx = \int \csc(dx+c)^2\sec(bx+a)^3\,dx$$

input
```
integrate(csc(d*x+c)^2*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^2*sec(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 16.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(c+dx)\sec^3(a+bx)\,dx = \int \frac{1}{\cos(a+b\,x)^3\sin(c+d\,x)^2}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^2(c+dx)\sec^3(a+bx)\,dx = \int \csc(dx+c)^2\sec(bx+a)^3\,dx$$

input
```
int(csc(d*x+c)^2*sec(b*x+a)^3,x)
```

output
```
int(csc(c + d*x)**2*sec(a + b*x)**3,x)
```

3.215 $\int \cos^3(a+bx)\csc^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 275

$$\begin{aligned}\int \cos^3(a+bx)\csc^3(c+dx)\,dx &= -\frac{e^{-3ia-3ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3}{2}\left(1-\frac{b}{d}\right),\frac{1}{2}\left(5-\frac{3b}{d}\right),e^{2i(c+dx)}\right)}{3(b-d)}\\
&\quad-\frac{3e^{-ia-ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{b}{d}\right),\frac{1}{2}\left(5-\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b-3d}\\
&\quad+\frac{3e^{ia+ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{b}{d}\right),\frac{1}{2}\left(5+\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b+3d}\\
&\quad+\frac{e^{3ia+3ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3(b+d)}{2d},\frac{1}{2}\left(5+\frac{3b}{d}\right),e^{2i(c+dx)}\right)}{3(b+d)}\end{aligned}$$

output
```
-1/3*exp(-3*I*a-3*I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2-3/2*b/d],[5/2-3/2*b
/d],exp(2*I*(d*x+c)))/(b-d)-3*exp(-I*a-I*b*x+3*I*(d*x+c))*hypergeom([3, 3/
2-1/2*b/d],[5/2-1/2*b/d],exp(2*I*(d*x+c)))/(b-3*d)+3*exp(I*a+I*b*x+3*I*(d*
x+c))*hypergeom([3, 3/2+1/2*b/d],[5/2+1/2*b/d],exp(2*I*(d*x+c)))/(b+3*d)+e
xp(3*I*a+3*I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2*(b+d)/d],[5/2+3/2*b/d],exp
(2*I*(d*x+c)))/(3*b+3*d)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1795 vs. 2(275) = 550.

Time = 12.27 (sec) , antiderivative size = 1795, normalized size of antiderivative = 6.53

$$\int \cos^3(a+bx)\csc^3(c+dx)\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[a + b*x]^3*Csc[c + d*x]^3,x]
```

output
```
-1/4*(Cos[a + b*x]^2*((-3*b + d)*Cos[a - c + b*x - d*x] + (3*b + d)*Cos[a
+ c + (b + d)*x])*Csc[c + d*x]^2)/d^2 + ((-4*E^(((3*I)/2)*c)*(-1 + E^((2*I
)*(a + b*x)))*(9*b^2*(1 + E^((2*I)*(a + b*x)))^2 - d^2*(1 + 10*E^((2*I)*(a
 + b*x)) + E^((4*I)*(a + b*x)))))/(-1 + E^((2*I)*c)) + ((9*I)*b^2*E^(I*c)*
Csc[c/2]*(3*b*E^(I*(6*a + (6*b + d)*x))*Hypergeometric2F1[1, 1 + (3*b)/d,
2 + (3*b)/d, E^(I*(c + d*x))] + (3*b + d)*Hypergeometric2F1[1, (-3*b)/d, 1
 - (3*b)/d, E^(I*(c + d*x))]))/(3*b + d) - (I*d^2*E^(I*c)*Csc[c/2]*(3*b*E^
(I*(6*a + (6*b + d)*x))*Hypergeometric2F1[1, 1 + (3*b)/d, 2 + (3*b)/d, E^(
I*(c + d*x))] + (3*b + d)*Hypergeometric2F1[1, (-3*b)/d, 1 - (3*b)/d, E^(I
*(c + d*x))]))/(3*b + d) - (9*I)*b^2*E^((2*I)*(a + b*x))*Csc[c/2]*(-1 + Hy
pergeometric2F1[1, -(b/d), 1 - b/d, E^(I*(c + d*x))] + E^(I*(2*a + c + 2*b
*x))*Hypergeometric2F1[1, b/d, (b + d)/d, E^(I*(c + d*x))]) + (9*I)*d^2*E^
((2*I)*(a + b*x))*Csc[c/2]*(-1 + Hypergeometric2F1[1, -(b/d), 1 - b/d, E^(
I*(c + d*x))] + E^(I*(2*a + c + 2*b*x))*Hypergeometric2F1[1, b/d, (b + d)/
d, E^(I*(c + d*x))]) - (9*I)*b^2*Csc[c/2]*(-1 + Hypergeometric2F1[1, (-3*b
)/d, 1 - (3*b)/d, E^(I*(c + d*x))] + E^(I*(6*a + c + 6*b*x))*Hypergeometri
c2F1[1, (3*b)/d, 1 + (3*b)/d, E^(I*(c + d*x))]) + I*d^2*Csc[c/2]*(-1 + Hyp
ergeometric2F1[1, (-3*b)/d, 1 - (3*b)/d, E^(I*(c + d*x))] + E^(I*(6*a + c
+ 6*b*x))*Hypergeometric2F1[1, (3*b)/d, 1 + (3*b)/d, E^(I*(c + d*x))]) + (
(9*I)*b^2*E^(I*(2*a + c + 2*b*x))*Csc[c/2]*((b + d)*Hypergeometric2F1[1...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc^3(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos^3(a+bx)\csc^3(c+dx)dx$$

input
```
Int[Cos[a + b*x]^3*Csc[c + d*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos(bx+a)^3 \csc(dx+c)^3\,dx$$

input
```
int(cos(b*x+a)^3*csc(d*x+c)^3,x)
```

output
```
int(cos(b*x+a)^3*csc(d*x+c)^3,x)
```

Fricas [F]

$$\int \cos^3(a+bx)\csc^3(c+dx)\,dx = \int \cos(bx+a)^3\csc(dx+c)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*csc(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^3*csc(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*csc(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(a+bx)\csc^3(c+dx)\,dx = \int \cos(bx+a)^3\csc(dx+c)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*csc(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/8*((3*b - d)*cos((6*b + d)*x + 6*a + c)*cos(3*b*x + 3*a) + 3*(b - d)*co
s((4*b + d)*x + 4*a + c)*cos(3*b*x + 3*a) - 3*(b + d)*cos((2*b + d)*x + 2*
a + c)*cos(3*b*x + 3*a) + (3*b - d)*cos(3*b*x + 3*a)*cos(3*d*x + 3*c) - (3
*b + d)*cos(3*b*x + 3*a)*cos(d*x + c) + (3*b - d)*sin((6*b + d)*x + 6*a +
c)*sin(3*b*x + 3*a) + 3*(b - d)*sin((4*b + d)*x + 4*a + c)*sin(3*b*x + 3*a
) - 3*(b + d)*sin((2*b + d)*x + 2*a + c)*sin(3*b*x + 3*a) + (3*b - d)*sin(
3*b*x + 3*a)*sin(3*d*x + 3*c) - (3*b + d)*sin(3*b*x + 3*a)*sin(d*x + c) +
3*(2*(b + d)*cos((3*b + 2*d)*x + 3*a + 2*c) - (b + d)*cos(3*b*x + 3*a))*co
s((4*b + 3*d)*x + 4*a + 3*c) + ((3*b - d)*cos((6*b + d)*x + 6*a + c) - 3*(
b + d)*cos((4*b + 3*d)*x + 4*a + 3*c) + 3*(b - d)*cos((4*b + d)*x + 4*a +
c) + 3*(b - d)*cos((2*b + 3*d)*x + 2*a + 3*c) - (3*b + d)*cos(3*(2*b + d)*
x + 6*a + 3*c) - 3*(b + d)*cos((2*b + d)*x + 2*a + c) + (3*b - d)*cos(3*d*
x + 3*c) - (3*b + d)*cos(d*x + c))*cos((3*b + 4*d)*x + 3*a + 4*c) - 2*((3*
b - d)*cos((6*b + d)*x + 6*a + c) + 3*(b - d)*cos((4*b + d)*x + 4*a + c) -
 3*(b + d)*cos((2*b + d)*x + 2*a + c) + (3*b - d)*cos(3*d*x + 3*c) - (3*b
+ d)*cos(d*x + c))*cos((3*b + 2*d)*x + 3*a + 2*c) - 3*(2*(b - d)*cos((3*b
+ 2*d)*x + 3*a + 2*c) - (b - d)*cos(3*b*x + 3*a))*cos((2*b + 3*d)*x + 2*a
+ 3*c) + (2*(3*b + d)*cos((3*b + 2*d)*x + 3*a + 2*c) - (3*b + d)*cos(3*b*x
 + 3*a))*cos(3*(2*b + d)*x + 6*a + 3*c) - 8*(d^2*cos((3*b + 4*d)*x + 3*a +
 4*c)^2 + 4*d^2*cos((3*b + 2*d)*x + 3*a + 2*c)^2 - 4*d^2*cos((3*b + 2*d...
```

Giac [F]

$$\int \cos^3(a+bx)\csc^3(c+dx)\,dx=\int \cos\left(bx+a\right)^3\csc\left(dx+c\right)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*csc(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3*csc(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^3(c+dx)\,dx=\int \frac{\cos\left(a+b\,x\right)^3}{\sin\left(c+d\,x\right)^3}\,dx$$

input
```
int(cos(a + b*x)^3/sin(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)^3/sin(c + d*x)^3, x)
```

Reduce [F]

$$\int \cos^3(a+bx)\csc^3(c+dx)\,dx=\int \cos\left(bx+a\right)^3\csc\left(dx+c\right)^3dx$$

input
```
int(cos(b*x+a)^3*csc(d*x+c)^3,x)
```

output
```
int(cos(b*x+a)^3*csc(d*x+c)^3,x)
```

3.216 $\int \cos^2(a+bx)\csc^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 166

$$\begin{aligned}\int \cos^2(a+bx)\csc^3(c+dx)\,dx &= -\frac{\operatorname{arctanh}(\cos(c+dx))}{4d}-\frac{\cot(c+dx)\csc(c+dx)}{4d}\\ &\quad-\frac{2e^{-2ia-2ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3}{2}-\frac{b}{d},\frac{5}{2}-\frac{b}{d},e^{2i(c+dx)}\right)}{2b-3d}\\ &\quad+\frac{2e^{2ia+2ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3}{2}+\frac{b}{d},\frac{5}{2}+\frac{b}{d},e^{2i(c+dx)}\right)}{2b+3d}\end{aligned}$$

output
```
-1/4*arctanh(cos(d*x+c))/d-1/4*cot(d*x+c)*csc(d*x+c)/d-2*exp(-2*I*a-2*I*b*
x+3*I*(d*x+c))*hypergeom([3, 3/2-b/d],[5/2-b/d],exp(2*I*(d*x+c)))/(2*b-3*d
)+2*exp(2*I*a+2*I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2+b/d],[5/2+b/d],exp(2*
I*(d*x+c)))/(2*b+3*d)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 365 vs. $2(166) = 332$.

Time = 12.12 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.20

$$\int \cos^2(a+bx)\csc^3(c+dx)\,dx$$
$$= \frac{\left(-b\cos\left(2a+2bx-\frac{dx}{2}\right)+b\cos\left(2a+2bx+\frac{dx}{2}\right)\right)\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d^2}$$
$$+\frac{(-1-\cos(2a+2bx))\csc^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{16d}$$
$$-\frac{e^{-2i(a+bx)}\left(2de^{2i(a+bx)}\operatorname{arctanh}\left(e^{i(c+dx)}\right)+(2b+d)e^{i(c+dx)}\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{b}{d},\frac{3}{2}-\frac{b}{d},e^{2i(c+dx)}\right.\right.}{4d^2}$$
$$+\frac{\left(b\cos\left(2a+2bx-\frac{dx}{2}\right)-b\cos\left(2a+2bx+\frac{dx}{2}\right)\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d^2}$$
$$+\frac{(1+\cos(2a+2bx))\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{16d}+\frac{b\csc(c)\sin(2a+2bx)}{2d^2}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c + d*x]^3,x]
```

output
```
((-(b*Cos[2*a + 2*b*x - (d*x)/2]) + b*Cos[2*a + 2*b*x + (d*x)/2])*Csc[c/2]
*Csc[c/2 + (d*x)/2])/(8*d^2) + ((-1 - Cos[2*a + 2*b*x])*Csc[c/2 + (d*x)/2]
^2)/(16*d) - (2*d*E^((2*I)*(a + b*x))*ArcTanh[E^(I*(c + d*x))] + (2*b + d)
*E^(I*(c + d*x))*Hypergeometric2F1[1, 1/2 - b/d, 3/2 - b/d, E^((2*I)*(c +
d*x))] - (2*b - d)*E^(I*(4*a + c + 4*b*x + d*x))*Hypergeometric2F1[1, 1/2
+ b/d, 3/2 + b/d, E^((2*I)*(c + d*x))])/(4*d^2*E^((2*I)*(a + b*x))) + ((b*
Cos[2*a + 2*b*x - (d*x)/2] - b*Cos[2*a + 2*b*x + (d*x)/2])*Sec[c/2]*Sec[c/
2 + (d*x)/2])/(8*d^2) + ((1 + Cos[2*a + 2*b*x])*Sec[c/2 + (d*x)/2]^2)/(16*
d) + (b*Csc[c]*Sin[2*a + 2*b*x])/(2*d^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^3(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \cos^2(a+bx)\csc^3(c+dx)dx$$

input
```
Int[Cos[a + b*x]^2*Csc[c + d*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos(bx+a)^2\csc(dx+c)^3\,dx$$

input
```
int(cos(b*x+a)^2*csc(d*x+c)^3,x)
```

output
```
int(cos(b*x+a)^2*csc(d*x+c)^3,x)
```

Fricas [F]

$$\int \cos^2(a+bx)\csc^3(c+dx)\,dx = \int \cos(bx+a)^2 \csc(dx+c)^3 \, dx$$

input
```
integrate(cos(b*x+a)^2*csc(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*csc(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*csc(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(a+bx)\csc^3(c+dx)\,dx = \int \cos(bx+a)^2 \csc(dx+c)^3 \, dx$$

input
```
integrate(cos(b*x+a)^2*csc(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*((2*b - d)*cos((4*b + d)*x + 4*a + c)*cos(2*b*x + 2*a) - 2*d*cos((2*b
 + d)*x + 2*a + c)*cos(2*b*x + 2*a) + (2*b - d)*cos(2*b*x + 2*a)*cos(3*d*x
 + 3*c) - (2*b + d)*cos(2*b*x + 2*a)*cos(d*x + c) + (2*b - d)*sin((4*b + d
)*x + 4*a + c)*sin(2*b*x + 2*a) - 2*d*sin((2*b + d)*x + 2*a + c)*sin(2*b*x
 + 2*a) + (2*b - d)*sin(2*b*x + 2*a)*sin(3*d*x + 3*c) - (2*b + d)*sin(2*b*
x + 2*a)*sin(d*x + c) + (2*(2*b + d)*cos(2*(b + d)*x + 2*a + 2*c) - (2*b +
 d)*cos(2*b*x + 2*a))*cos((4*b + 3*d)*x + 4*a + 3*c) + 2*(2*d*cos(2*(b + d
)*x + 2*a + 2*c) - d*cos(2*b*x + 2*a))*cos((2*b + 3*d)*x + 2*a + 3*c) - ((
2*b + d)*cos((4*b + 3*d)*x + 4*a + 3*c) - (2*b - d)*cos((4*b + d)*x + 4*a
+ c) + 2*d*cos((2*b + 3*d)*x + 2*a + 3*c) + 2*d*cos((2*b + d)*x + 2*a + c)
 - (2*b - d)*cos(3*d*x + 3*c) + (2*b + d)*cos(d*x + c))*cos(2*(b + 2*d)*x
+ 2*a + 4*c) - 2*((2*b - d)*cos((4*b + d)*x + 4*a + c) - 2*d*cos((2*b + d)
*x + 2*a + c) + (2*b - d)*cos(3*d*x + 3*c) - (2*b + d)*cos(d*x + c))*cos(2
*(b + d)*x + 2*a + 2*c) - 4*(d^2*cos(2*(b + 2*d)*x + 2*a + 4*c)^2 + 4*d^2*
cos(2*(b + d)*x + 2*a + 2*c)^2 - 4*d^2*cos(2*(b + d)*x + 2*a + 2*c)*cos(2*
b*x + 2*a) + d^2*cos(2*b*x + 2*a)^2 + d^2*sin(2*(b + 2*d)*x + 2*a + 4*c)^2
 + 4*d^2*sin(2*(b + d)*x + 2*a + 2*c)^2 - 4*d^2*sin(2*(b + d)*x + 2*a + 2*
c)*sin(2*b*x + 2*a) + d^2*sin(2*b*x + 2*a)^2 - 2*(2*d^2*cos(2*(b + d)*x +
2*a + 2*c) - d^2*cos(2*b*x + 2*a))*cos(2*(b + 2*d)*x + 2*a + 4*c) - 2*(2*d
^2*sin(2*(b + d)*x + 2*a + 2*c) - d^2*sin(2*b*x + 2*a))*sin(2*(b + 2*d)...
```

Giac [F]

$$\int \cos^2(a+bx)\csc^3(c+dx)\,dx = \int \cos\left(bx+a\right)^2\csc\left(dx+c\right)^3\,dx$$

input
```
integrate(cos(b*x+a)^2*csc(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*csc(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^3(c+dx)\,dx = \int \frac{\cos(a+b\,x)^2}{\sin(c+d\,x)^3}\,dx$$

input
```
int(cos(a + b*x)^2/sin(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)^2/sin(c + d*x)^3, x)
```

Reduce [F]

$$\int \cos^2(a+bx)\csc^3(c+dx)\,dx = \int \cos(bx+a)^2\csc(dx+c)^3\,dx$$

input
```
int(cos(b*x+a)^2*csc(d*x+c)^3,x)
```

output
```
int(cos(a + b*x)**2*csc(c + d*x)**3,x)
```

3.217 $\int \cos(a+bx)\csc^3(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 137

$$\int \cos(a+bx)\csc^3(c+dx)\,dx = -\frac{4e^{-ia-ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{b}{d}\right),\frac{1}{2}\left(5-\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b-3d} + \frac{4e^{ia+ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{b}{d}\right),\frac{1}{2}\left(5+\frac{b}{d}\right),e^{2i(c+dx)}\right)}{b+3d}$$

output
```
-4*exp(-I*a-I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2-1/2*b/d],[5/2-1/2*b/d],ex
p(2*I*(d*x+c)))/(b-3*d)+4*exp(I*a+I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2+1/2
*b/d],[5/2+1/2*b/d],exp(2*I*(d*x+c)))/(b+3*d)
```

Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.39

$$\int \cos(a+bx)\csc^3(c+dx)\,dx = \frac{e^{-i(a+(b-d)x)}\left(-4(b+d)e^{ic}\,\text{Hypergeometric2F1}\left(1,\frac{-b+d}{2d},\frac{3}{2}-\frac{b}{2d},e^{2i(c+dx)}\right)+e^{ia}\left(-2e^{i(b-d)x}((-b+d)\cos(\right.\right.}{}$$

input
```
Integrate[Cos[a + b*x]*Csc[c + d*x]^3,x]
```

output
```
(-4*(b + d)*E^(I*c)*Hypergeometric2F1[1, (-b + d)/(2*d), 3/2 - b/(2*d), E^
((2*I)*(c + d*x))] + E^(I*a)*(-2*E^(I*(b - d)*x)*((-b + d)*Cos[a - c + b*x
 - d*x] + (b + d)*Cos[a + c + (b + d)*x])*Csc[c + d*x]^2 + 4*(b - d)*E^(I*
(a + c + 2*b*x))*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, E^((2*I)
*(c + d*x))]))/(8*d^2*E^(I*(a + (b - d)*x)))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^3(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos(a+bx)\csc^3(c+dx)dx$$

input
```
Int[Cos[a + b*x]*Csc[c + d*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos(bx+a)\csc(dx+c)^3\,dx$$

input
```
int(cos(b*x+a)*csc(d*x+c)^3,x)
```

output
```
int(cos(b*x+a)*csc(d*x+c)^3,x)
```

Fricas [F]

$$\int \cos(a+bx)\csc^3(c+dx)\,dx = \int \cos(bx+a)\csc(dx+c)^3\,dx$$

input
```
integrate(cos(b*x+a)*csc(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*csc(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*csc(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(a+bx)\csc^3(c+dx)\,dx = \int \cos\left(bx+a\right)\csc\left(dx+c\right)^3\,dx$$

input
```
integrate(cos(b*x+a)*csc(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*((b - d)*cos((2*b + d)*x + 2*a + c)*cos(b*x + a) + (b - d)*cos(b*x +
a)*cos(3*d*x + 3*c) - (b + d)*cos(b*x + a)*cos(d*x + c) + (b - d)*sin((2*b
 + d)*x + 2*a + c)*sin(b*x + a) + (b - d)*sin(b*x + a)*sin(3*d*x + 3*c) -
(b + d)*sin(b*x + a)*sin(d*x + c) + (2*(b + d)*cos((b + 2*d)*x + a + 2*c)
- (b + d)*cos(b*x + a))*cos((2*b + 3*d)*x + 2*a + 3*c) - ((b + d)*cos((2*b
 + 3*d)*x + 2*a + 3*c) - (b - d)*cos((2*b + d)*x + 2*a + c) - (b - d)*cos(
3*d*x + 3*c) + (b + d)*cos(d*x + c))*cos((b + 4*d)*x + a + 4*c) - 2*((b -
d)*cos((2*b + d)*x + 2*a + c) + (b - d)*cos(3*d*x + 3*c) - (b + d)*cos(d*x
 + c))*cos((b + 2*d)*x + a + 2*c) - 2*(d^2*cos((b + 4*d)*x + a + 4*c)^2 +
4*d^2*cos((b + 2*d)*x + a + 2*c)^2 - 4*d^2*cos((b + 2*d)*x + a + 2*c)*cos(
b*x + a) + d^2*cos(b*x + a)^2 + d^2*sin((b + 4*d)*x + a + 4*c)^2 + 4*d^2*s
in((b + 2*d)*x + a + 2*c)^2 - 4*d^2*sin((b + 2*d)*x + a + 2*c)*sin(b*x + a
) + d^2*sin(b*x + a)^2 - 2*(2*d^2*cos((b + 2*d)*x + a + 2*c) - d^2*cos(b*x
 + a))*cos((b + 4*d)*x + a + 4*c) - 2*(2*d^2*sin((b + 2*d)*x + a + 2*c) -
d^2*sin(b*x + a))*sin((b + 4*d)*x + a + 4*c))*integrate(1/4*((b^2 - d^2)*c
os((b + d)*x + a + c)*sin(2*b*x + 2*a) + (b^2 - d^2)*cos(b*x + a)*sin(2*b*
x + 2*a) - (b^2 - d^2)*cos(2*b*x + 2*a)*sin(b*x + a) - (b^2 - d^2 + (b^2 -
 d^2)*cos(2*b*x + 2*a))*sin((b + d)*x + a + c) - (b^2 - d^2)*sin(b*x + a))
/(d^2*cos((b + d)*x + a + c)^2 + 2*d^2*cos((b + d)*x + a + c)*cos(b*x + a)
 + d^2*cos(b*x + a)^2 + d^2*sin((b + d)*x + a + c)^2 + 2*d^2*sin((b + d...
```

Giac [F]

$$\int \cos(a+bx)\csc^3(c+dx)\,dx = \int \cos\left(bx+a\right)\csc\left(dx+c\right)^3\,dx$$

input
```
integrate(cos(b*x+a)*csc(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*csc(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^3(c+dx)\,dx=\int \frac{\cos\left(a+b\,x\right)}{{\sin\left(c+d\,x\right)}^3}\,dx$$

input
```
int(cos(a + b*x)/sin(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)/sin(c + d*x)^3, x)
```

Reduce [F]

$$\int \cos(a+bx)\csc^3(c+dx)\,dx=\int \cos\left(bx+a\right)\csc\left(dx+c\right)^3dx$$

input
```
int(cos(b*x+a)*csc(d*x+c)^3,x)
```

output
```
int(cos(a + b*x)*csc(c + d*x)**3,x)
```

3.218 $\int \csc^3(c+dx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \csc^3(c+dx)\sec(a+bx)\,dx=\text{Int}\left(\csc^3(c+dx)\sec(a+bx),x\right)$$

output
```
Defer(Int)(csc(d*x+c)^3*sec(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 42.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc^3(c+dx)\sec(a+bx)\,dx=\int \csc^3(c+dx)\sec(a+bx)\,dx$$

input
```
Integrate[Csc[c + d*x]^3*Sec[a + b*x],x]
```

output
```
Integrate[Csc[c + d*x]^3*Sec[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\csc^3(c+dx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \sec(a+bx)\csc^3(c+dx)dx$$

input
```
Int[Csc[c + d*x]^3*Sec[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc\left(dx+c\right)^{3}\sec\left(bx+a\right)dx$$

input
```
int(csc(d*x+c)^3*sec(b*x+a),x)
```

output
```
int(csc(d*x+c)^3*sec(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc^3(c+dx)\sec(a+bx)\,dx = \int \csc\left(dx+c\right)^3\sec\left(bx+a\right)\,dx$$

input
```
integrate(csc(d*x+c)^3*sec(b*x+a),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^3*sec(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc^3(c+dx)\sec(a+bx)\,dx = \int \csc^3\left(c+dx\right)\sec\left(a+bx\right)dx$$

input
```
integrate(csc(d*x+c)**3*sec(b*x+a),x)
```

output
```
Integral(csc(c + d*x)**3*sec(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 21.53 (sec) , antiderivative size = 9655, normalized size of antiderivative = 643.67

$$\int \csc^3(c+dx)\sec(a+bx)\,dx = \int \csc(dx+c)^3 \sec(bx+a)\,dx$$

input
```
integrate(csc(d*x+c)^3*sec(b*x+a),x, algorithm="maxima")
```

output
```
(2*((b + d)*cos(4*b*x + 4*a) + 2*(b + d)*cos(2*b*x + 2*a) + (b + d)*cos(4*
d*x + 4*c) - 2*(b + d)*cos(2*d*x + 2*c) + b + d)*cos((3*b + d)*x + 3*a + c
) - 4*((b + d)*cos((3*b + d)*x + 3*a + c) - (b - d)*cos((b + d)*x + a + c)
)*cos(2*(2*b + d)*x + 4*a + 2*c) - 2*(2*(b + d)*cos(2*(2*b + d)*x + 4*a +
2*c) + 4*(b + d)*cos(2*(b + d)*x + 2*a + 2*c) - (b + d)*cos(4*b*x + 4*a) -
 2*(b + d)*cos(2*b*x + 2*a) - (b + d)*cos(4*d*x + 4*c) + 2*(b + d)*cos(2*d
*x + 2*c) - b - d)*cos((b + 3*d)*x + a + 3*c) + 4*((b + d)*cos((3*b + d)*x
 + 3*a + c) + (b + d)*cos((b + 3*d)*x + a + 3*c) - (b - d)*cos(3*(b + d)*x
 + 3*a + 3*c) - (b - d)*cos((b + d)*x + a + c))*cos(2*(b + 2*d)*x + 2*a +
4*c) + 2*((b + d)*cos((3*b + d)*x + 3*a + c) + (b + d)*cos((b + 3*d)*x + a
 + 3*c) - (b - d)*cos(3*(b + d)*x + 3*a + 3*c) - (b - d)*cos((b + d)*x + a
 + c))*cos(4*(b + d)*x + 4*a + 4*c) + 2*(2*(b - d)*cos(2*(2*b + d)*x + 4*a
 + 2*c) + 4*(b - d)*cos(2*(b + d)*x + 2*a + 2*c) - (b - d)*cos(4*b*x + 4*a
) - 2*(b - d)*cos(2*b*x + 2*a) - (b - d)*cos(4*d*x + 4*c) + 2*(b - d)*cos(
2*d*x + 2*c) - b + d)*cos(3*(b + d)*x + 3*a + 3*c) - 8*((b + d)*cos((3*b +
 d)*x + 3*a + c) - (b - d)*cos((b + d)*x + a + c))*cos(2*(b + d)*x + 2*a +
 2*c) - 2*((b - d)*cos(4*b*x + 4*a) + 2*(b - d)*cos(2*b*x + 2*a) + (b - d)
*cos(4*d*x + 4*c) - 2*(b - d)*cos(2*d*x + 2*c) + b - d)*cos((b + d)*x + a
+ c) + (4*d^2*cos(2*(2*b + d)*x + 4*a + 2*c)^2 + 4*d^2*cos(2*(b + 2*d)*x +
 2*a + 4*c)^2 + d^2*cos(4*(b + d)*x + 4*a + 4*c)^2 + 16*d^2*cos(2*(b + ...
```

Giac [N/A]

Not integrable

Time = 14.96 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc^3(c+dx)\sec(a+bx)\,dx = \int \csc(dx+c)^3\sec(bx+a)\,dx$$

input
```
integrate(csc(d*x+c)^3*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^3*sec(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 16.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \csc^3(c+dx)\sec(a+bx)\,dx = \int \frac{1}{\cos(a+b\,x)\,\sin(c+d\,x)^3}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(c + d*x)^3),x)
```

output
```
int(1/(cos(a + b*x)*sin(c + d*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc^3(c+dx)\sec(a+bx)\,dx = \int \csc(dx+c)^3\sec(bx+a)\,dx$$

input
```
int(csc(d*x+c)^3*sec(b*x+a),x)
```

output
```
int(csc(c + d*x)**3*sec(a + b*x),x)
```

3.219 $\int \csc^3(c+dx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \csc^3(c+dx)\sec^2(a+bx)\,dx = \text{Int}\left(\csc^3(c+dx)\sec^2(a+bx),x\right)$$

output
```
Defer(Int)(csc(d*x+c)^3*sec(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 17.65 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^3(c+dx)\sec^2(a+bx)\,dx = \int \csc^3(c+dx)\sec^2(a+bx)\,dx$$

input
```
Integrate[Csc[c + d*x]^3*Sec[a + b*x]^2,x]
```

output
```
Integrate[Csc[c + d*x]^3*Sec[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\csc^3(c+dx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sec^2(a+bx)\csc^3(c+dx)dx$$

input
```
Int[Csc[c + d*x]^3*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc\left(dx+c\right)^{3}\sec\left(bx+a\right)^{2}dx$$

input
```
int(csc(d*x+c)^3*sec(b*x+a)^2,x)
```

output
```
int(csc(d*x+c)^3*sec(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^3(c+dx)\sec^2(a+bx)\,dx = \int \csc(dx+c)^3 \sec(bx+a)^2\,dx$$

input
```
integrate(csc(d*x+c)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^3*sec(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 14.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc^3(c+dx)\sec^2(a+bx)\,dx = \int \csc^3(c+dx)\sec^2(a+bx)\,dx$$

input
```
integrate(csc(d*x+c)**3*sec(b*x+a)**2,x)
```

output
```
Integral(csc(c + d*x)**3*sec(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 42.40 (sec) , antiderivative size = 15013, normalized size of antiderivative = 883.12

$$\int \csc^3(c+dx)\sec^2(a+bx)\,dx = \int \csc(dx+c)^3\sec(bx+a)^2\,dx$$

input
```
integrate(csc(d*x+c)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
(4*(2*(2*b - d)*cos(2*(3*b + d)*x + 6*a + 2*c) + 6*(2*b - d)*cos(2*(2*b +
d)*x + 4*a + 2*c) + 6*(2*b - d)*cos(2*(b + d)*x + 2*a + 2*c) - (2*b - d)*c
os(6*b*x + 6*a) - 3*(2*b - d)*cos(4*b*x + 4*a) - 3*(2*b - d)*cos(2*b*x + 2
*a) - (2*b - d)*cos(4*d*x + 4*c) + 2*(2*b - d)*cos(2*d*x + 2*c) - 2*b + d)
*cos((4*b + 3*d)*x + 4*a + 3*c) + 4*((2*b + d)*cos(6*b*x + 6*a) + 3*(2*b +
 d)*cos(4*b*x + 4*a) + 3*(2*b + d)*cos(2*b*x + 2*a) + (2*b + d)*cos(4*d*x
+ 4*c) - 2*(2*b + d)*cos(2*d*x + 2*c) + 2*b + d)*cos((4*b + d)*x + 4*a + c
) - 4*((2*b - d)*cos((4*b + 3*d)*x + 4*a + 3*c) - (2*b + d)*cos((4*b + d)*
x + 4*a + c) - (2*b + d)*cos((2*b + 3*d)*x + 2*a + 3*c) + (2*b - d)*cos((2
*b + d)*x + 2*a + c))*cos(2*(3*b + 2*d)*x + 6*a + 4*c) - 8*((2*b + d)*cos(
(4*b + d)*x + 4*a + c) - (2*b - d)*cos((2*b + d)*x + 2*a + c))*cos(2*(3*b
+ d)*x + 6*a + 2*c) - 4*(2*(2*b + d)*cos(2*(3*b + d)*x + 6*a + 2*c) + 6*(2
*b + d)*cos(2*(2*b + d)*x + 4*a + 2*c) + 6*(2*b + d)*cos(2*(b + d)*x + 2*a
 + 2*c) - (2*b + d)*cos(6*b*x + 6*a) - 3*(2*b + d)*cos(4*b*x + 4*a) - 3*(2
*b + d)*cos(2*b*x + 2*a) - (2*b + d)*cos(4*d*x + 4*c) + 2*(2*b + d)*cos(2*
d*x + 2*c) - 2*b - d)*cos((2*b + 3*d)*x + 2*a + 3*c) - 24*((2*b + d)*cos((
4*b + d)*x + 4*a + c) - (2*b - d)*cos((2*b + d)*x + 2*a + c))*cos(2*(2*b +
 d)*x + 4*a + 2*c) - 4*((2*b - d)*cos(6*b*x + 6*a) + 3*(2*b - d)*cos(4*b*x
 + 4*a) + 3*(2*b - d)*cos(2*b*x + 2*a) + (2*b - d)*cos(4*d*x + 4*c) - 2*(2
*b - d)*cos(2*d*x + 2*c) + 2*b - d)*cos((2*b + d)*x + 2*a + c) - 12*((2...
```

Giac [N/A]

Not integrable

Time = 48.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^3(c+dx)\sec^2(a+bx)\,dx = \int \csc(dx+c)^3\sec(bx+a)^2\,dx$$

input
```
integrate(csc(d*x+c)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^3*sec(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 16.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^3(c+dx)\sec^2(a+bx)\,dx = \int \frac{1}{\cos(a+b\,x)^2\sin(c+d\,x)^3}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(c + d*x)^3),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(c + d*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 11.89 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^3(c+dx)\sec^2(a+bx)\,dx = \int \csc(dx+c)^3\sec(bx+a)^2\,dx$$

input
```
int(csc(d*x+c)^3*sec(b*x+a)^2,x)
```

output
```
int(csc(c + d*x)**3*sec(a + b*x)**2,x)
```

3.220 $\int \csc^3(c+dx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \csc^3(c+dx)\sec^3(a+bx)\,dx = \text{Int}\left(\csc^3(c+dx)\sec^3(a+bx),x\right)$$

output
```
Defer(Int)(csc(d*x+c)^3*sec(b*x+a)^3,x)
```

Mathematica [F(-1)]

Timed out.

$$\int \csc^3(c+dx)\sec^3(a+bx)\,dx = \$Aborted$$

input
```
Integrate[Csc[c + d*x]^3*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)\csc^3(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec^3(a+bx)\csc^3(c+dx)dx$$

input
```
Int[Csc[c + d*x]^3*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc\left(dx+c\right)^{3} \sec\left(bx+a\right)^{3} dx$$

input
```
int(csc(d*x+c)^3*sec(b*x+a)^3,x)
```

output
```
int(csc(d*x+c)^3*sec(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^3(c+dx)\sec^3(a+bx)\,dx = \int \csc\left(dx+c\right)^3\sec\left(bx+a\right)^3\,dx$$

input
```
integrate(csc(d*x+c)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^3*sec(b*x + a)^3, x)
```

Sympy [N/A]

Not integrable

Time = 43.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc^3(c+dx)\sec^3(a+bx)\,dx = \int \csc^3\left(c+dx\right)\sec^3\left(a+bx\right)dx$$

input
```
integrate(csc(d*x+c)**3*sec(b*x+a)**3,x)
```

output
```
Integral(csc(c + d*x)**3*sec(a + b*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 76.46 (sec) , antiderivative size = 22516, normalized size of antiderivative = 1324.47

$$\int \csc^3(c+dx)\sec^3(a+bx)\,dx = \int \csc(dx+c)^3\sec(bx+a)^3\,dx$$

input
```
integrate(csc(d*x+c)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
(8*(2*(3*b - d)*cos(2*(4*b + d)*x + 8*a + 2*c) + 8*(3*b - d)*cos(2*(3*b +
d)*x + 6*a + 2*c) + 12*(3*b - d)*cos(2*(2*b + d)*x + 4*a + 2*c) + 8*(3*b -
 d)*cos(2*(b + d)*x + 2*a + 2*c) - (3*b - d)*cos(8*b*x + 8*a) - 4*(3*b - d
)*cos(6*b*x + 6*a) - 6*(3*b - d)*cos(4*b*x + 4*a) - 4*(3*b - d)*cos(2*b*x
+ 2*a) - (3*b - d)*cos(4*d*x + 4*c) + 2*(3*b - d)*cos(2*d*x + 2*c) - 3*b +
 d)*cos((5*b + 3*d)*x + 5*a + 3*c) + 8*((3*b + d)*cos(8*b*x + 8*a) + 4*(3*
b + d)*cos(6*b*x + 6*a) + 6*(3*b + d)*cos(4*b*x + 4*a) + 4*(3*b + d)*cos(2
*b*x + 2*a) + (3*b + d)*cos(4*d*x + 4*c) - 2*(3*b + d)*cos(2*d*x + 2*c) +
3*b + d)*cos((5*b + d)*x + 5*a + c) - 16*((3*b + d)*cos((5*b + d)*x + 5*a
+ c) - (3*b - d)*cos((3*b + d)*x + 3*a + c))*cos(2*(4*b + d)*x + 8*a + 2*c
) - 32*((3*b - d)*cos((5*b + 3*d)*x + 5*a + 3*c) - (3*b + d)*cos((5*b + d)
*x + 5*a + c) + (3*b - d)*cos((3*b + d)*x + 3*a + c) - (3*b + d)*cos(3*(b
+ d)*x + 3*a + 3*c))*cos(2*(3*b + 2*d)*x + 6*a + 4*c) - 64*((3*b + d)*cos(
(5*b + d)*x + 5*a + c) - (3*b - d)*cos((3*b + d)*x + 3*a + c))*cos(2*(3*b
+ d)*x + 6*a + 2*c) - 8*((3*b - d)*cos(8*b*x + 8*a) + 4*(3*b - d)*cos(6*b*
x + 6*a) + 6*(3*b - d)*cos(4*b*x + 4*a) + 4*(3*b - d)*cos(2*b*x + 2*a) + (
3*b - d)*cos(4*d*x + 4*c) - 2*(3*b - d)*cos(2*d*x + 2*c) + 3*b - d)*cos((3
*b + d)*x + 3*a + c) - 8*((3*b - d)*cos((5*b + 3*d)*x + 5*a + 3*c) - (3*b
+ d)*cos((5*b + d)*x + 5*a + c) + (3*b - d)*cos((3*b + d)*x + 3*a + c) - (
3*b + d)*cos(3*(b + d)*x + 3*a + 3*c))*cos(4*(2*b + d)*x + 8*a + 4*c) -...
```

Giac [N/A]

Not integrable

Time = 115.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^3(c+dx)\sec^3(a+bx)\,dx = \int \csc(dx+c)^3\sec(bx+a)^3\,dx$$

input
```
integrate(csc(d*x+c)^3*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^3*sec(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 17.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^3(c+dx)\sec^3(a+bx)\,dx = \int \frac{1}{\cos(a+b\,x)^3\sin(c+d\,x)^3}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(c + d*x)^3),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(c + d*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \csc^3(c+dx)\sec^3(a+bx)\,dx = \int \csc(dx+c)^3\sec(bx+a)^3\,dx$$

input
```
int(csc(d*x+c)^3*sec(b*x+a)^3,x)
```

output
```
int(csc(d*x+c)^3*sec(b*x+a)^3,x)
```

3.221 $\int \sqrt{\cos(c+dx)}\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 477

$$\int \sqrt{\cos(c+dx)}\sin^3(a+bx)\,dx = -\frac{3e^{\frac{1}{2}i(2a-c)+\frac{1}{2}i(2b-d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{2b}{d}\right),\frac{1}{4}\left(3+\frac{2b}{d}\right),-e^{2i(c+dx)}\right)}{4(2b-d)\sqrt{1+e^{2i(c+dx)}}}$$
$$+\frac{e^{\frac{1}{2}i(6a-c)+\frac{1}{2}i(6b-d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{6b}{d}\right),\frac{3(2b+d)}{4d},-e^{2i(c+dx)}\right)}{4(6b-d)\sqrt{1+e^{2i(c+dx)}}}$$
$$-\frac{3e^{-\frac{1}{2}i(2a+c)-\frac{1}{2}i(2b+d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2b+d}{4d},\frac{1}{4}\left(3-\frac{2b}{d}\right),-e^{2i(c+dx)}\right)}{4(2b+d)\sqrt{1+e^{2i(c+dx)}}}$$
$$+\frac{e^{-\frac{1}{2}i(6a+c)-\frac{1}{2}i(6b+d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{6b+d}{4d},\frac{3}{4}\left(1-\frac{2b}{d}\right),-e^{2i(c+dx)}\right)}{4(6b+d)\sqrt{1+e^{2i(c+dx)}}}$$

output
```
-3/4*exp(1/2*I*(2*a-c)+1/2*I*(2*b-d)*x+1/2*I*(d*x+c))*cos(d*x+c)^(1/2)*hyp
ergeom([-1/2, -1/4+1/2*b/d],[3/4+1/2*b/d],-exp(2*I*(d*x+c)))/(2*b-d)/(1+ex
p(2*I*(d*x+c)))^(1/2)+1/4*exp(1/2*I*(6*a-c)+1/2*I*(6*b-d)*x+1/2*I*(d*x+c))
*cos(d*x+c)^(1/2)*hypergeom([-1/2, -1/4+3/2*b/d],[3/4*(2*b+d)/d],-exp(2*I*
(d*x+c)))/(6*b-d)/(1+exp(2*I*(d*x+c)))^(1/2)-3/4*exp(-1/2*I*(2*a+c)-1/2*I*
(2*b+d)*x+1/2*I*(d*x+c))*cos(d*x+c)^(1/2)*hypergeom([-1/2, -1/4*(2*b+d)/d]
,[3/4-1/2*b/d],-exp(2*I*(d*x+c)))/(2*b+d)/(1+exp(2*I*(d*x+c)))^(1/2)+1/4*e
xp(-1/2*I*(6*a+c)-1/2*I*(6*b+d)*x+1/2*I*(d*x+c))*cos(d*x+c)^(1/2)*hypergeo
m([-1/2, -1/4*(6*b+d)/d],[3/4-3/2*b/d],-exp(2*I*(d*x+c)))/(6*b+d)/(1+exp(2
*I*(d*x+c)))^(1/2)
```

Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.77

$$\int \sqrt{\cos(c+dx)}\sin^3(a+bx)\,dx$$

$$= \frac{e^{-3i(a+bx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(-\frac{3e^{2i(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2b+d}{4d},\frac{3}{4}-\frac{b}{2d},-e^{2i(c+dx)}\right)}{2b+d}+\frac{\text{Hypergeometric2F1}\left(\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sin[a + b*x]^3,x]
```

output
```
(Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*((-3*E^((2*I)*(a + b*x))*
Hypergeometric2F1[-1/2, -1/4*(2*b + d)/d, 3/4 - b/(2*d), -E^((2*I)*(c + d*
x))])/(2*b + d) + Hypergeometric2F1[-1/2, -1/4*(6*b + d)/d, 3/4 - (3*b)/(2
*d), -E^((2*I)*(c + d*x))]/(6*b + d) - (3*E^((4*I)*(a + b*x))*((2*b - d)*S
qrt[1 + E^((2*I)*(c + d*x))] + 2*d*Hypergeometric2F1[1/2, -1/4 + b/(2*d),
3/4 + b/(2*d), -E^((2*I)*(c + d*x))]))/(4*b^2 - d^2) + (E^((6*I)*(a + b*x)
)*((6*b - d)*Sqrt[1 + E^((2*I)*(c + d*x))] + 2*d*Hypergeometric2F1[1/2, -1
/4 + (3*b)/(2*d), (3*(2*b + d))/(4*d), -E^((2*I)*(c + d*x))]))/(36*b^2 - d
^2)))/(4*Sqrt[2]*E^((3*I)*(a + b*x))*Sqrt[1 + E^((2*I)*(c + d*x))])
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5066, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sqrt{\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{5066}$$

$$\frac{\int \left(3ie^{-ia-ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}-3ie^{ia+ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}-ie^{-3ia-3ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}+ie^{3ia+\cdots}\right.}{8\sqrt{2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(\frac{2b}{d}-1\right),\frac{1}{4}\left(\frac{2b}{d}+3\right),-e^{2i(c+dx)}\right)\exp\left(\frac{1}{2}i(2a-c)+\frac{1}{2}ix(2b-d)+\frac{1}{2}i(c+dx)\right)}{(2b-d)\sqrt{1+e^{2ic+2idx}}}+\frac{2\sqrt{e^{-i(c+dx)}\cdots}}{\cdots}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sin[a + b*x]^3,x]
```

output
```
((-6*E^((I/2)*(2*a - c) + (I/2)*(2*b - d)*x + (I/2)*(c + d*x))*Sqrt[E^((-I
)*(c + d*x)) + E^(I*(c + d*x))]*Hypergeometric2F1[-1/2, (-1 + (2*b)/d)/4,
(3 + (2*b)/d)/4, -E^((2*I)*(c + d*x))])/((2*b - d)*Sqrt[1 + E^((2*I)*c + (
2*I)*d*x)]) + (2*E^((I/2)*(6*a - c) + (I/2)*(6*b - d)*x + (I/2)*(c + d*x))
*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))]*Hypergeometric2F1[-1/2, (-1 +
(6*b)/d)/4, (3*(2*b + d))/(4*d), -E^((2*I)*(c + d*x))])/((6*b - d)*Sqrt[1
+ E^((2*I)*c + (2*I)*d*x)]) - (6*E^((-1/2*I)*(2*a + c) - (I/2)*(2*b + d)*x
 + (I/2)*(c + d*x))*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))]*Hypergeomet
ric2F1[-1/2, -1/4*(2*b + d)/d, (3 - (2*b)/d)/4, -E^((2*I)*(c + d*x))])/((2
*b + d)*Sqrt[1 + E^((2*I)*c + (2*I)*d*x)]) + (2*E^((-1/2*I)*(6*a + c) - (I
/2)*(6*b + d)*x + (I/2)*(c + d*x))*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x
))]*Hypergeometric2F1[-1/2, -1/4*(6*b + d)/d, (3*(1 - (2*b)/d))/4, -E^((2*
I)*(c + d*x))])/((6*b + d)*Sqrt[1 + E^((2*I)*c + (2*I)*d*x)]))/(8*Sqrt[2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5066
```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \sqrt{\cos(dx+c)}\, \sin(bx+a)^3 \, dx$$

input
```
int(cos(d*x+c)^(1/2)*sin(b*x+a)^3,x)
```

output
```
int(cos(d*x+c)^(1/2)*sin(b*x+a)^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\cos(c+dx)} \sin^3(a+bx) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(1/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}\sin^3(a+bx)\,dx = \int \sqrt{\cos(dx+c)}\sin(bx+a)^3\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))*sin(b*x + a)^3, x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}\sin^3(a+bx)\,dx = \int \sqrt{\cos(dx+c)}\sin(bx+a)^3\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sin^3(a+bx)\,dx = \int \sqrt{\cos\left(c+d\,x\right)}\,\sin\left(a+b\,x\right)^3\,dx$$

input
```
int(cos(c + d*x)^(1/2)*sin(a + b*x)^3,x)
```

output
```
int(cos(c + d*x)^(1/2)*sin(a + b*x)^3, x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}\sin^3(a+bx)\,dx = \int \sqrt{\cos\left(dx+c\right)}\,\sin\left(bx+a\right)^3dx$$

input
```
int(cos(d*x+c)^(1/2)*sin(b*x+a)^3,x)
```

output
```
int(sqrt(cos(c + d*x))*sin(a + b*x)**3,x)
```

3.222 $\int \sqrt{\cos(c+dx)}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 258

$$\int \sqrt{\cos(c+dx)}\sin^2(a+bx)\,dx = \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} - \frac{ie^{-\frac{1}{2}i(4a+c)-\frac{1}{2}i(4b+d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1-\frac{4b}{d}\right),\frac{1}{4}\left(3-\frac{4b}{d}\right),-e^{2i(c+dx}\right.}{2(4b+d)\sqrt{1+e^{2i(c+dx)}}} + \frac{ie^{\frac{1}{2}i(4a-c)+\frac{1}{2}i(4b-d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{4b}{d}\right),\frac{1}{4}\left(3+\frac{4b}{d}\right),-e^{2i(c+dx)}\right)}{2(4b-d)\sqrt{1+e^{2i(c+dx)}}}$$

output
```
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d-1/2*I*exp(-1/2*I*(4*a+c)-1/2*I*(4*
b+d)*x+1/2*I*(d*x+c))*cos(d*x+c)^(1/2)*hypergeom([-1/2, -1/4-b/d],[3/4-b/d
],-exp(2*I*(d*x+c)))/(4*b+d)/(1+exp(2*I*(d*x+c)))^(1/2)+1/2*I*exp(1/2*I*(4
*a-c)+1/2*I*(4*b-d)*x+1/2*I*(d*x+c))*cos(d*x+c)^(1/2)*hypergeom([-1/2, -1/
4+b/d],[3/4+b/d],-exp(2*I*(d*x+c)))/(4*b-d)/(1+exp(2*I*(d*x+c)))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 5.07 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.39

$$\int \sqrt{\cos(c+dx)}\sin^2(a+bx)\,dx$$
$$= \frac{ie^{-i(2a+c+(2b+d)x)}\left(-\left((4b-d)e^{2i(a+bx)}\sqrt{1+e^{2i(c+dx)}}\left(4b\left(-1+e^{2i(c+dx)}\right)-d\left(1+e^{2i(a+bx)}-e^{2i(c+dx)}+e^{2\cdots}\right.\right.\right.\right.}{\cdots}$$
$$-\frac{\cos(c)\csc(dx+\arctan(\tan(c)))\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sqrt{\sec^2(c)}\sqrt{\sin^2(dx+\text{arct}\cdots}}{2d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sin[a + b*x]^2,x]
```

output
```
((I/4)*(-((4*b - d)*E^((2*I)*(a + b*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*(4*b
*(-1 + E^((2*I)*(c + d*x))) - d*(1 + E^((2*I)*(a + b*x)) - E^((2*I)*(c + d
*x)) + E^((2*I)*(a + c + (b + d)*x))))) - (4*b - d)*d*(1 + E^((2*I)*(c + d
*x)))*Hypergeometric2F1[-1/2, -1/4 - b/d, 3/4 - b/d, -E^((2*I)*(c + d*x))]
 + 2*d^2*E^((4*I)*(a + b*x))*(1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1
/2, -1/4 + b/d, 3/4 + b/d, -E^((2*I)*(c + d*x))]))/((16*b^2*d - d^3)*E^(I*
(2*a + c + (2*b + d)*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[Cos[c + d*x]])
 - (Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]
]]^2])/(2*d*Sqrt[Cos[c + d*x]])
```

Rubi [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 646 vs. 2(258) = 516.

Time = 1.14 (sec) , antiderivative size = 646, normalized size of antiderivative = 2.50, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5066, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sqrt{\cos(c+dx)}\,dx$$

$$\Big\downarrow \; 5066$$

$$\frac{\int \left(-e^{-2ia-2ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}-e^{2ia+2ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}+2\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}\right)dx}{4\sqrt{2}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2i\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-\frac{4b}{d}-1\right),\frac{1}{4}\left(3-\frac{4b}{d}\right),-e^{2i(c+dx)}\right)\exp\left(-\frac{1}{2}i(4a+c)-\frac{1}{2}ix(4b+d)+\frac{1}{2}i(c+dx)\right)}{(4b+d)\sqrt{1+e^{2ic+2idx}}}+\frac{2i\sqrt{e^{-i(c}}}{}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sin[a + b*x]^2,x]
```

output
```
(((4*I)*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))])/d - ((8*I)*E^(I*(c + d
*x))*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))])/(d*(1 + E^(I*(c + d*x))))
 + ((8*I)*Sqrt[E^(I*(c + d*x))]*(1 + E^(I*(c + d*x)))*Sqrt[E^((-I)*(c + d*
x)) + E^(I*(c + d*x))]*Sqrt[(1 + E^((2*I)*(c + d*x)))/(1 + E^(I*(c + d*x))
)^2]*EllipticE[2*ArcTan[Sqrt[E^(I*(c + d*x))]], 1/2])/(d*(1 + E^((2*I)*(c
+ d*x)))) - ((4*I)*Sqrt[E^(I*(c + d*x))]*(1 + E^(I*(c + d*x)))*Sqrt[E^((-I
)*(c + d*x)) + E^(I*(c + d*x))]*Sqrt[(1 + E^((2*I)*(c + d*x)))/(1 + E^(I*(
c + d*x)))^2]*EllipticF[2*ArcTan[Sqrt[E^(I*(c + d*x))]], 1/2])/(d*(1 + E^(
(2*I)*(c + d*x)))) - ((2*I)*E^((-1/2*I)*(4*a + c) - (I/2)*(4*b + d)*x + (I
/2)*(c + d*x))*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))]*Hypergeometric2F
1[-1/2, (-1 - (4*b)/d)/4, (3 - (4*b)/d)/4, -E^((2*I)*(c + d*x))])/((4*b +
d)*Sqrt[1 + E^((2*I)*c + (2*I)*d*x)]) + ((2*I)*E^((I/2)*(4*a - c) + (I/2)*
(4*b - d)*x + (I/2)*(c + d*x))*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))]*
Hypergeometric2F1[-1/2, (-1 + (4*b)/d)/4, (3 + (4*b)/d)/4, -E^((2*I)*(c +
d*x))])/((4*b - d)*Sqrt[1 + E^((2*I)*c + (2*I)*d*x)]))/(4*Sqrt[2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5066
```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \sqrt{\cos\left(dx+c\right)}\,\sin\left(bx+a\right)^{2}dx$$

input
```
int(cos(d*x+c)^(1/2)*sin(b*x+a)^2,x)
```

output
```
int(cos(d*x+c)^(1/2)*sin(b*x+a)^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\cos(c+dx)}\sin^2(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(1/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}\sin^2(a+bx)\,dx = \int \sin^2(a+bx)\sqrt{\cos(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*sin(b*x+a)**2,x)
```

output
```
Integral(sin(a + b*x)**2*sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}\sin^2(a+bx)\,dx = \int \sqrt{\cos(dx+c)}\sin(bx+a)^2\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))*sin(b*x + a)^2, x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}\sin^2(a+bx)\,dx = \int \sqrt{\cos(dx+c)}\sin(bx+a)^2\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sin^2(a+bx)\,dx = \int \sqrt{\cos\left(c+d\,x\right)}\,\sin\left(a+b\,x\right)^2\,dx$$

input
```
int(cos(c + d*x)^(1/2)*sin(a + b*x)^2,x)
```

output
```
int(cos(c + d*x)^(1/2)*sin(a + b*x)^2, x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}\sin^2(a+bx)\,dx = \int \sqrt{\cos\left(dx+c\right)}\,\sin\left(bx+a\right)^2dx$$

input
```
int(cos(d*x+c)^(1/2)*sin(b*x+a)^2,x)
```

output
```
int(sqrt(cos(c + d*x))*sin(a + b*x)**2,x)
```

3.223 $\int \sqrt{\cos(c+dx)}\sin(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 235

$$\int \sqrt{\cos(c+dx)}\sin(a+bx)\,dx = -\frac{e^{\frac{1}{2}i(2a-c)+\frac{1}{2}i(2b-d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{2b}{d}\right),\frac{1}{4}\left(3+\frac{2b}{d}\right),-e^{2i(c+dx)}\right)}{(2b-d)\sqrt{1+e^{2i(c+dx)}}} - \frac{e^{-\frac{1}{2}i(2a+c)-\frac{1}{2}i(2b+d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2b+d}{4d},\frac{1}{4}\left(3-\frac{2b}{d}\right),-e^{2i(c+dx)}\right)}{(2b+d)\sqrt{1+e^{2i(c+dx)}}}$$

output
```
-exp(1/2*I*(2*a-c)+1/2*I*(2*b-d)*x+1/2*I*(d*x+c))*cos(d*x+c)^(1/2)*hyperge
om([-1/2, -1/4+1/2*b/d],[3/4+1/2*b/d],-exp(2*I*(d*x+c)))/(2*b-d)/(1+exp(2*
I*(d*x+c)))^(1/2)-exp(-1/2*I*(2*a+c)-1/2*I*(2*b+d)*x+1/2*I*(d*x+c))*cos(d*
x+c)^(1/2)*hypergeom([-1/2, -1/4*(2*b+d)/d],[3/4-1/2*b/d],-exp(2*I*(d*x+c)
))/(2*b+d)/(1+exp(2*I*(d*x+c)))^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.88

$$\int \sqrt{\cos(c+dx)}\sin(a+bx)\,dx$$
$$= \frac{e^{-i(a+bx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left((-2b+d)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2b+d}{4d},\frac{3}{4}-\frac{b}{2d},-e^{2i(c+dx)}\right)-e^{2i(a}\right.}{\sqrt{2}\left(4b^2-d^2\right)\sqrt{1+e^{2}}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sin[a + b*x],x]
```

output
```
(Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*((-2*b + d)*Hypergeometri
c2F1[-1/2, -1/4*(2*b + d)/d, 3/4 - b/(2*d), -E^((2*I)*(c + d*x))] - E^((2*
I)*(a + b*x))*((2*b - d)*Sqrt[1 + E^((2*I)*(c + d*x))] + 2*d*Hypergeometri
c2F1[1/2, -1/4 + b/(2*d), 3/4 + b/(2*d), -E^((2*I)*(c + d*x))])))/(Sqrt[2]
*(4*b^2 - d^2)*E^(I*(a + b*x))*Sqrt[1 + E^((2*I)*(c + d*x))])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5066, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sqrt{\cos(c+dx)}\,dx$$
$$\Big\downarrow \text{5066}$$
$$\frac{\int \left(ie^{-ia-ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}-ie^{ia+ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}\right)dx}{2\sqrt{2}}$$
$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(\frac{2b}{d}-1\right),\frac{1}{4}\left(\frac{2b}{d}+3\right),-e^{2i(c+dx)}\right)\exp\left(\frac{1}{2}i(2a-c)+\frac{1}{2}ix(2b-d)+\frac{1}{2}i(c+dx)\right)}{(2b-d)\sqrt{1+e^{2ic+2idx}}}-\frac{2\sqrt{e^{-i(c+dx)}}}{}}{2\sqrt{2}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sin[a + b*x],x]
```

output
```
((-2*E^((I/2)*(2*a - c) + (I/2)*(2*b - d)*x + (I/2)*(c + d*x))*Sqrt[E^((-I
)*(c + d*x)) + E^(I*(c + d*x))]*Hypergeometric2F1[-1/2, (-1 + (2*b)/d)/4,
(3 + (2*b)/d)/4, -E^((2*I)*(c + d*x))])/((2*b - d)*Sqrt[1 + E^((2*I)*c + (
2*I)*d*x)]) - (2*E^((-1/2*I)*(2*a + c) - (I/2)*(2*b + d)*x + (I/2)*(c + d*
x))*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))]*Hypergeometric2F1[-1/2, -1/
4*(2*b + d)/d, (3 - (2*b)/d)/4, -E^((2*I)*(c + d*x))])/((2*b + d)*Sqrt[1 +
 E^((2*I)*c + (2*I)*d*x)]))/(2*Sqrt[2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5066
```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \sqrt{\cos (dx+c)}\,\sin (bx+a)\,dx$$

input
```
int(cos(d*x+c)^(1/2)*sin(b*x+a),x)
```

output
```
int(cos(d*x+c)^(1/2)*sin(b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\cos(c+dx)}\sin(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(1/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}\sin(a+bx)\,dx = \int \sin(a+bx)\sqrt{\cos(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*sin(b*x+a),x)
```

output
```
Integral(sin(a + b*x)*sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}\sin(a+bx)\,dx = \int \sqrt{\cos(dx+c)}\sin(bx+a)\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))*sin(b*x + a), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}\sin(a+bx)\,dx = \int \sqrt{\cos\left(dx+c\right)}\sin\left(bx+a\right)\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sin(a+bx)\,dx = \int \sqrt{\cos\left(c+d\,x\right)}\,\sin\left(a+b\,x\right)\,dx$$

input
```
int(cos(c + d*x)^(1/2)*sin(a + b*x),x)
```

output
```
int(cos(c + d*x)^(1/2)*sin(a + b*x), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}\sin(a+bx)\,dx = \int \sqrt{\cos\left(dx+c\right)}\,\sin\left(bx+a\right)dx$$

input
```
int(cos(d*x+c)^(1/2)*sin(b*x+a),x)
```

output
```
int(sqrt(cos(c + d*x))*sin(a + b*x),x)
```

3.224 $\int \sqrt{\cos(c+dx)}\csc(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \sqrt{\cos(c+dx)}\csc(a+bx)\,dx = \text{Int}\left(\sqrt{\cos(c+dx)}\csc(a+bx),x\right)$$

output
```
Defer(Int)(cos(d*x+c)^(1/2)*csc(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 16.76 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sqrt{\cos(c+dx)}\csc(a+bx)\,dx = \int \sqrt{\cos(c+dx)}\csc(a+bx)\,dx$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Csc[a + b*x],x]
```

output
```
Integrate[Sqrt[Cos[c + d*x]]*Csc[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sqrt{\cos(c+dx)}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \csc(a+bx)\sqrt{\cos(c+dx)}dx$$

input
```
Int[Sqrt[Cos[c + d*x]]*Csc[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{\cos\left(dx+c\right)}\,\csc\left(bx+a\right)dx$$

input
```
int(cos(d*x+c)^(1/2)*csc(b*x+a),x)
```

output
```
int(cos(d*x+c)^(1/2)*csc(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\csc(a+bx)\,dx = \int \sqrt{\cos\left(dx+c\right)}\csc\left(bx+a\right)\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*csc(b*x+a),x, algorithm="fricas")
```

output
```
integral(sqrt(cos(d*x + c))*csc(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\csc(a+bx)\,dx = \int \sqrt{\cos\left(c+dx\right)}\csc\left(a+bx\right)dx$$

input
```
integrate(cos(d*x+c)**(1/2)*csc(b*x+a),x)
```

output
```
Integral(sqrt(cos(c + d*x))*csc(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\csc(a+bx)\,dx = \int \sqrt{\cos\left(dx+c\right)}\csc\left(bx+a\right)\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*csc(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))*csc(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\csc(a+bx)\,dx = \int \sqrt{\cos\left(dx+c\right)}\csc\left(bx+a\right)\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))*csc(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 16.96 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sqrt{\cos(c+dx)}\csc(a+bx)\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sin\left(a+b\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/sin(a + b*x),x)
```

output
```
int(cos(c + d*x)^(1/2)/sin(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \sqrt{\cos(c+dx)}\csc(a+bx)\,dx=\int \sqrt{\cos\left(dx+c\right)}\,\csc\left(bx+a\right)dx$$

input
```
int(cos(d*x+c)^(1/2)*csc(b*x+a),x)
```

output
```
int(sqrt(cos(c + d*x))*csc(a + b*x),x)
```

3.225 $\int \sqrt{\cos(c+dx)}\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \sqrt{\cos(c+dx)}\csc^2(a+bx)\,dx=\text{Int}\left(\sqrt{\cos(c+dx)}\csc^2(a+bx),x\right)$$

output
```
Defer(Int)(cos(d*x+c)^(1/2)*csc(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 26.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sqrt{\cos(c+dx)}\csc^2(a+bx)\,dx=\int \sqrt{\cos(c+dx)}\csc^2(a+bx)\,dx$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Csc[a + b*x]^2,x]
```

output
```
Integrate[Sqrt[Cos[c + d*x]]*Csc[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sqrt{\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \csc^2(a+bx)\sqrt{\cos(c+dx)}dx$$

input
```
Int[Sqrt[Cos[c + d*x]]*Csc[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \sqrt{\cos\left(dx+c\right)}\,\csc\left(bx+a\right)^{2}dx$$

input
```
int(cos(d*x+c)^(1/2)*csc(b*x+a)^2,x)
```

output
```
int(cos(d*x+c)^(1/2)*csc(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\csc^2(a+bx)\,dx = \int \sqrt{\cos(dx+c)}\csc(bx+a)^2\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(cos(d*x + c))*csc(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\csc^2(a+bx)\,dx = \int \sqrt{\cos(c+dx)}\csc^2(a+bx)\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*csc(b*x+a)**2,x)
```

output
```
Integral(sqrt(cos(c + d*x))*csc(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\csc^2(a+bx)\,dx = \int \sqrt{\cos(dx+c)}\csc(bx+a)^2\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))*csc(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\csc^2(a+bx)\,dx = \int \sqrt{\cos(dx+c)}\csc(bx+a)^2\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))*csc(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 16.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\csc^2(a+bx)\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{\sin(a+b\,x)^2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/sin(a + b*x)^2,x)
```

output
```
int(cos(c + d*x)^(1/2)/sin(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \sqrt{\cos(c+dx)}\csc^2(a+bx)\,dx = \int \sqrt{\cos(dx+c)}\,\csc(bx+a)^2\,dx$$

input
```
int(cos(d*x+c)^(1/2)*csc(b*x+a)^2,x)
```

output
```
int(sqrt(cos(c + d*x))*csc(a + b*x)**2,x)
```

3.226 $\int \cos^q(c+dx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 473

$$\int \cos^q(c+dx)\sin^3(a+bx)\,dx = \frac{e^{i(3a-cq)+i(3b-dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{3b}{d}-q\right),-q,\frac{1}{2}\left(2+\frac{3b}{d}-q\right.\right.}{8(3b-dq)} - \frac{3e^{i(a-cq)+i(b-dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(-q,\frac{b-dq}{2d},\frac{1}{2}\left(2+\frac{b}{d}-q\right),-\right.}{8(b-dq)} - \frac{3e^{-i(a+cq)-i(b+dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(-q,-\frac{b+dq}{2d},-\frac{b-d(2-q)}{2d},-\right.}{8(b+dq)} + \frac{e^{-i(3a+cq)-i(3b+dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(-q,-\frac{3b+dq}{2d},\frac{1}{2}\left(2-\frac{3b}{d}-\right.\right.}{8(3b+dq)}$$

output
```
1/8*exp(I*(-c*q+3*a)+I*(-d*q+3*b)*x+I*q*(d*x+c))*cos(d*x+c)^q*hypergeom([-
q, 3/2*b/d-1/2*q],[1+3/2*b/d-1/2*q],-exp(2*I*(d*x+c)))/((1+exp(2*I*(d*x+c)
))^q)/(-d*q+3*b)-3/8*exp(I*(-c*q+a)+I*(-d*q+b)*x+I*q*(d*x+c))*cos(d*x+c)^q
*hypergeom([-q, 1/2*(-d*q+b)/d],[1+1/2*b/d-1/2*q],-exp(2*I*(d*x+c)))/((1+e
xp(2*I*(d*x+c)))^q)/(-d*q+b)-3/8*exp(-I*(c*q+a)-I*(d*q+b)*x+I*q*(d*x+c))*c
os(d*x+c)^q*hypergeom([-q, -1/2*(d*q+b)/d],[-1/2*(b-d*(2-q))/d],-exp(2*I*(
d*x+c)))/((1+exp(2*I*(d*x+c)))^q)/(d*q+b)+1/8*exp(-I*(c*q+3*a)-I*(d*q+3*b)
*x+I*q*(d*x+c))*cos(d*x+c)^q*hypergeom([-q, -1/2*(d*q+3*b)/d],[1-3/2*b/d-1
/2*q],-exp(2*I*(d*x+c)))/((1+exp(2*I*(d*x+c)))^q)/(d*q+3*b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.57 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.70

$$\int \cos^q(c+dx)\sin^3(a+bx)\,dx = 2^{-3-q}e^{i(-3a+c+d(1+q)x)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{1+q}\left(\frac{e^{-i(3b+dq)x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{3b}{d}+q\right),1-\frac{3b}{2d}-\frac{q}{2},-e^{2i(c+dx)}\right)}{3b+dq}-\frac{3e^{2ia-i(b+dq)x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{b}{d}+q\right),-\frac{b+d(-2+q)}{2d},-e^{2i(c+dx)}\right)}{b+dq}+e^{i(4a+bx-dqx)}\left(\frac{e^{2i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2+\frac{3b}{d}+q\right),1+\frac{3b}{2d}-\frac{q}{2},-e^{2i(c+dx)}\right)}{3b-dq}-\frac{3\,\text{Hypergeometric2F1}\left(1,\frac{b+d(2+q)}{2d},\frac{1}{2}\left(2+\frac{b}{d}-q\right),-e^{2i(c+dx)}\right)}{b-dq}\right)\right)$$

input

```
Integrate[Cos[c + d*x]^q*Sin[a + b*x]^3,x]
```

output

```
2^(-3 - q)*E^(I*(-3*a + c + d*(1 + q)*x))*((1 + E^((2*I)*(c + d*x)))/E^(I*
(c + d*x)))^(1 + q)*(Hypergeometric2F1[1, (2 - (3*b)/d + q)/2, 1 - (3*b)/(
2*d) - q/2, -E^((2*I)*(c + d*x))]/(E^(I*(3*b + d*q)*x)*(3*b + d*q)) - (3*E
^((2*I)*a - I*(b + d*q)*x)*Hypergeometric2F1[1, (2 - b/d + q)/2, -1/2*(b +
 d*(-2 + q))/d, -E^((2*I)*(c + d*x))])/(b + d*q) + E^(I*(4*a + b*x - d*q*x
))*((E^((2*I)*(a + b*x))*Hypergeometric2F1[1, (2 + (3*b)/d + q)/2, 1 + (3*
b)/(2*d) - q/2, -E^((2*I)*(c + d*x))])/(3*b - d*q) - (3*Hypergeometric2F1[
1, (b + d*(2 + q))/(2*d), (2 + b/d - q)/2, -E^((2*I)*(c + d*x))])/(b - d*q
)))
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5066, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^q(c+dx)\,dx$$

$$\Big\downarrow \text{5066}$$

$$2^{-q-3}\int \left(3ie^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q-3ie^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q-ie^{-3ia-3ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$2^{-q-3}\left(\frac{\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q\left(1+e^{2ic+2idx}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{3b}{d}-q\right),-q,\frac{1}{2}\left(\frac{3b}{d}-q+2\right),-e^{2i(c+dx)}\right)}{3b-dq}\right.$$

input
```
Int[Cos[c + d*x]^q*Sin[a + b*x]^3,x]
```

output
```
2^(-3 - q)*((E^(I*(3*a - c*q) + I*(3*b - d*q)*x + I*q*(c + d*x))*(E^((-I)*
(c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[((3*b)/d - q)/2, -q, (2
+ (3*b)/d - q)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^q*
(3*b - d*q)) - (3*E^(I*(a - c*q) + I*(b - d*q)*x + I*q*(c + d*x))*(E^((-I)
*(c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, (b - d*q)/(2*d), (2
 + b/d - q)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^q*(b
- d*q)) - (3*E^((-I)*(a + c*q) - I*(b + d*q)*x + I*q*(c + d*x))*(E^((-I)*(
c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, -1/2*(b + d*q)/d, 1 -
 (b + d*q)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^q*
(b + d*q)) + (E^((-I)*(3*a + c*q) - I*(3*b + d*q)*x + I*q*(c + d*x))*(E^((
-I)*(c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, -1/2*(3*b + d*q)
/d, (2 - (3*b)/d - q)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d
*x))^q*(3*b + d*q)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5066
```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \cos(dx+c)^q \sin(bx+a)^3 \, dx$$

input
```
int(cos(d*x+c)^q*sin(b*x+a)^3,x)
```

output
```
int(cos(d*x+c)^q*sin(b*x+a)^3,x)
```

Fricas [F]

$$\int \cos^q(c+dx)\sin^3(a+bx)\,dx = \int \cos(dx+c)^q \sin(bx+a)^3 \, dx$$

input
```
integrate(cos(d*x+c)^q*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*cos(d*x + c)^q*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^q(c+dx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**q*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^q(c+dx)\sin^3(a+bx)\,dx = \int \cos(dx+c)^q \sin(bx+a)^3\,dx$$

input
```
integrate(cos(d*x+c)^q*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^q*sin(b*x + a)^3, x)
```

Giac [F]

$$\int \cos^q(c+dx)\sin^3(a+bx)\,dx = \int \cos(dx+c)^q \sin(bx+a)^3\,dx$$

input
```
integrate(cos(d*x+c)^q*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^q*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^q(c+dx)\sin^3(a+bx)\,dx=\int \cos\left(c+d\,x\right)^q\,\sin\left(a+b\,x\right)^3\,dx$$

input
```
int(cos(c + d*x)^q*sin(a + b*x)^3,x)
```

output
```
int(cos(c + d*x)^q*sin(a + b*x)^3, x)
```

Reduce [F]

$$\int \cos^q(c+dx)\sin^3(a+bx)\,dx=\int \cos\left(dx+c\right)^q\sin\left(bx+a\right)^3dx$$

input
```
int(cos(d*x+c)^q*sin(b*x+a)^3,x)
```

output
```
int(cos(c + d*x)**q*sin(a + b*x)**3,x)
```

3.227 $\int \cos^q(c+dx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 314

$$\int \cos^q(c+dx)\sin^2(a+bx)\,dx = -\frac{ie^{-i(2a+cq)-i(2b+dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2b}{d}-q\right),-q,\frac{1}{2}(2-}{4(2b+dq)}$$
$$+\frac{ie^{i(2a-cq)+i(2b-dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{2b}{d}-q\right),-q,\frac{1}{2}(2+\frac{2b}{d}}{4(2b-dq)}$$
$$-\frac{\cos^{1+q}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+q}{2},\frac{3+q}{2},\cos^2(c+dx)\right)\sin(c+dx)}{2d(1+q)\sqrt{\sin^2(c+dx)}}$$

output
```
-1/4*I*exp(-I*(c*q+2*a)-I*(d*q+2*b)*x+I*q*(d*x+c))*cos(d*x+c)^q*hypergeom(
[-q, -b/d-1/2*q],[1-b/d-1/2*q],-exp(2*I*(d*x+c)))/((1+exp(2*I*(d*x+c)))^q)
/(d*q+2*b)+1/4*I*exp(I*(-c*q+2*a)+I*(-d*q+2*b)*x+I*q*(d*x+c))*cos(d*x+c)^q
*hypergeom([-q, b/d-1/2*q],[1+b/d-1/2*q],-exp(2*I*(d*x+c)))/((1+exp(2*I*(d
*x+c)))^q)/(-d*q+2*b)-1/2*cos(d*x+c)^(1+q)*hypergeom([1/2, 1/2+1/2*q],[3/2
+1/2*q],cos(d*x+c)^2)*sin(d*x+c)/d/(1+q)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.77

$$\int \cos^q(c+dx)\sin^2(a+bx)\,dx = -\frac{i2^{-2-q}e^{-2i(a+bx)+i(c+dx)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{1+q}\left(dq(-2b+dq)\,\text{Hypergeometric2F1}\left(1,1-\frac{b}{d}+\frac{q}{2},\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^q*Sin[a + b*x]^2,x]
```

output
```
((-I)*2^(-2 - q)*E^((-2*I)*(a + b*x) + I*(c + d*x))*((1 + E^((2*I)*(c + d*
x)))/E^(I*(c + d*x)))^(1 + q)*(d*q*(-2*b + d*q)*Hypergeometric2F1[1, 1 - b
/d + q/2, 1 - b/d - q/2, -E^((2*I)*(c + d*x))] + E^((2*I)*(a + b*x))*(2*b
+ d*q)*(d*E^((2*I)*(a + b*x))*q*Hypergeometric2F1[1, 1 + b/d + q/2, 1 + b/
d - q/2, -E^((2*I)*(c + d*x))] + 2*(2*b - d*q)*Hypergeometric2F1[1, (2 + q
)/2, 1 - q/2, -E^((2*I)*(c + d*x))])))/(-4*b^2*d*q + d^3*q^3)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5066, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^q(c+dx)\,dx$$

$$\Big\downarrow \text{5066}$$

$$2^{-q-2}\int \left(-e^{-2ia-2ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q-e^{2ia+2ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q+2\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2^{-q-2}\left(-\frac{i\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q\left(1+e^{2ic+2idx}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2b}{d}-q\right),-q,\frac{1}{2}\left(-\frac{2b}{d}-q+2\right),-e^{2i(c}\right.}{2b+dq}\right.$$

input
```
Int[Cos[c + d*x]^q*Sin[a + b*x]^2,x]
```

output
```
2^(-2 - q)*(((-I)*E^((-I)*(2*a + c*q) - I*(2*b + d*q)*x + I*q*(c + d*x))*(
E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[((-2*b)/d - q)/2
, -q, (2 - (2*b)/d - q)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)
*d*x))^q*(2*b + d*q)) + (I*E^(I*(2*a - c*q) + I*(2*b - d*q)*x + I*q*(c + d
*x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[((2*b)/d -
 q)/2, -q, (2 + (2*b)/d - q)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c +
(2*I)*d*x))^q*(2*b - d*q)) + ((2*I)*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))
^q*Hypergeometric2F1[-q, -1/2*q, 1 - q/2, -E^((2*I)*(c + d*x))])/(d*(1 + E
^((2*I)*(c + d*x)))^q*q))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5066
```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \cos{(dx+c)}^q \sin{(bx+a)}^2\,dx$$

input
```
int(cos(d*x+c)^q*sin(b*x+a)^2,x)
```

output
```
int(cos(d*x+c)^q*sin(b*x+a)^2,x)
```

Fricas [F]

$$\int \cos^q(c+dx)\sin^2(a+bx)\,dx = \int \cos{(dx+c)}^q \sin{(bx+a)}^2\,dx$$

input
```
integrate(cos(d*x+c)^q*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*cos(d*x + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^q(c+dx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**q*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^q(c+dx)\sin^2(a+bx)\,dx = \int \cos{(dx+c)}^q \sin{(bx+a)}^2\,dx$$

input
```
integrate(cos(d*x+c)^q*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^q*sin(b*x + a)^2, x)
```

Giac [F]

$$\int \cos^q(c+dx)\sin^2(a+bx)\,dx = \int \cos(dx+c)^q \sin(bx+a)^2 \,dx$$

input
```
integrate(cos(d*x+c)^q*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^q*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^q(c+dx)\sin^2(a+bx)\,dx = \int \cos(c+d\,x)^q \sin(a+b\,x)^2\,dx$$

input
```
int(cos(c + d*x)^q*sin(a + b*x)^2,x)
```

output
```
int(cos(c + d*x)^q*sin(a + b*x)^2, x)
```

Reduce [F]

$$\int \cos^q(c+dx)\sin^2(a+bx)\,dx = \int \cos(dx+c)^q \sin(bx+a)^2\,dx$$

input
```
int(cos(d*x+c)^q*sin(b*x+a)^2,x)
```

output
```
int(cos(c + d*x)**q*sin(a + b*x)**2,x)
```

3.228 $\int \cos^q(c+dx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 230

$$\int \cos^q(c+dx)\sin(a+bx)\,dx = -\frac{e^{i(a-cq)+i(b-dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(-q,\frac{b-dq}{2d},\frac{1}{2}\left(2+\frac{b}{d}-q\right),-e\right.}{2(b-dq)} - \frac{e^{-i(a+cq)-i(b+dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(-q,-\frac{b+dq}{2d},-\frac{b-d(2-q)}{2d},-e^{2}\right.}{2(b+dq)}$$

output
```
-1/2*exp(I*(-c*q+a)+I*(-d*q+b)*x+I*q*(d*x+c))*cos(d*x+c)^q*hypergeom([-q,
1/2*(-d*q+b)/d],[1+1/2*b/d-1/2*q],-exp(2*I*(d*x+c)))/((1+exp(2*I*(d*x+c)))
^q)/(-d*q+b)-1/2*exp(-I*(c*q+a)-I*(d*q+b)*x+I*q*(d*x+c))*cos(d*x+c)^q*hype
rgeom([-q, -1/2*(d*q+b)/d],[-1/2*(b-d*(2-q))/d],-exp(2*I*(d*x+c)))/((1+exp
(2*I*(d*x+c)))^q)/(d*q+b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.45 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.79

$$\int \cos^q(c+dx)\sin(a+bx)\,dx = -\frac{2^{-1-q}e^{-i(a-c+(b-d)x)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{1+q}\left((b-dq)\,\text{Hypergeometric2F1}\left(1,\tfrac{1}{2}\left(2-\tfrac{b}{d}+q\right),-\tfrac{b+}{}\right.\right.}{(b-dq)(}$$

input
```
Integrate[Cos[c + d*x]^q*Sin[a + b*x],x]
```

output
```
-((2^(-1 - q)*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^(1 + q)*((b - d*
q)*Hypergeometric2F1[1, (2 - b/d + q)/2, -1/2*(b + d*(-2 + q))/d, -E^((2*I
)*(c + d*x))] + E^((2*I)*(a + b*x))*(b + d*q)*Hypergeometric2F1[1, (b + d*
(2 + q))/(2*d), (2 + b/d - q)/2, -E^((2*I)*(c + d*x))]))/(E^(I*(a - c + (b
 - d)*x))*(b - d*q)*(b + d*q)))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5066, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^q(c+dx)\,dx$$

$$\Big\downarrow \text{5066}$$

$$2^{-q-1}\int \left(ie^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q - ie^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2^{-q-1}\left(-\frac{\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q\left(1+e^{2ic+2idx}\right)^{-q}\text{Hypergeometric2F1}\left(-q,\frac{b-dq}{2d},\frac{1}{2}\left(\frac{b}{d}-q+2\right),-e^{2i(c+dx)}\right)\exp}{b-dq}\right.$$

input
```
Int[Cos[c + d*x]^q*Sin[a + b*x],x]
```

output
```
2^(-1 - q)*(-((E^(I*(a - c*q) + I*(b - d*q)*x + I*q*(c + d*x))*(E^((-I)*(c
 + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, (b - d*q)/(2*d), (2 +
b/d - q)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^q*(b - d
*q))) - (E^((-I)*(a + c*q) - I*(b + d*q)*x + I*q*(c + d*x))*(E^((-I)*(c +
d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, -1/2*(b + d*q)/d, 1 - (b
+ d*q)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^q*(b +
 d*q)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5066
```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \cos (dx+c)^q \sin (bx+a)\, dx$$

input
```
int(cos(d*x+c)^q*sin(b*x+a),x)
```

output
```
int(cos(d*x+c)^q*sin(b*x+a),x)
```

Fricas [F]

$$\int \cos^q(c+dx)\sin(a+bx)\,dx = \int \cos(dx+c)^q \sin(bx+a)\,dx$$

input
```
integrate(cos(d*x+c)^q*sin(b*x+a),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^q*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^q(c+dx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**q*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^q(c+dx)\sin(a+bx)\,dx = \int \cos(dx+c)^q \sin(bx+a)\,dx$$

input
```
integrate(cos(d*x+c)^q*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^q*sin(b*x + a), x)
```

Giac [F]

$$\int \cos^q(c+dx)\sin(a+bx)\,dx = \int \cos(dx+c)^q \sin(bx+a)\,dx$$

input
```
integrate(cos(d*x+c)^q*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^q*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^q(c+dx)\sin(a+bx)\,dx = \int \cos(c+d\,x)^q \sin(a+b\,x)\,dx$$

input
```
int(cos(c + d*x)^q*sin(a + b*x),x)
```

output
```
int(cos(c + d*x)^q*sin(a + b*x), x)
```

Reduce [F]

$$\int \cos^q(c+dx)\sin(a+bx)\,dx = \int \cos(dx+c)^q \sin(bx+a)\,dx$$

input
```
int(cos(d*x+c)^q*sin(b*x+a),x)
```

output
```
int(cos(c + d*x)**q*sin(a + b*x),x)
```

3.229 $\int \cos^q(c+dx)\csc(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos^q(c+dx)\csc(a+bx)\,dx = \text{Int}(\cos^q(c+dx)\csc(a+bx),x)$$

output
```
Defer(Int)(cos(d*x+c)^q*csc(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos^q(c+dx)\csc(a+bx)\,dx = \int \cos^q(c+dx)\csc(a+bx)\,dx$$

input
```
Integrate[Cos[c + d*x]^q*Csc[a + b*x],x]
```

output
```
Integrate[Cos[c + d*x]^q*Csc[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\cos^q(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc(a+bx)\cos^q(c+dx)dx$$

input
```
Int[Cos[c + d*x]^q*Csc[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos\left(dx+c\right)^{q} \csc\left(bx+a\right) dx$$

input
```
int(cos(d*x+c)^q*csc(b*x+a),x)
```

output
```
int(cos(d*x+c)^q*csc(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos^q(c+dx)\csc(a+bx)\,dx=\int \cos\left(dx+c\right)^q \csc\left(bx+a\right)\,dx$$

input
```
integrate(cos(d*x+c)^q*csc(b*x+a),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^q*csc(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos^q(c+dx)\csc(a+bx)\,dx=\int \cos^q\left(c+dx\right)\csc\left(a+bx\right)dx$$

input
```
integrate(cos(d*x+c)**q*csc(b*x+a),x)
```

output
```
Integral(cos(c + d*x)**q*csc(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos^q(c+dx)\csc(a+bx)\,dx = \int \cos{(dx+c)}^q \csc{(bx+a)}\,\,dx$$

input
```
integrate(cos(d*x+c)^q*csc(b*x+a),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^q*csc(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos^q(c+dx)\csc(a+bx)\,dx = \int \cos{(dx+c)}^q \csc{(bx+a)}\,\,dx$$

input
```
integrate(cos(d*x+c)^q*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^q*csc(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 16.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cos^q(c+dx)\csc(a+bx)\,dx = \int \frac{{\cos\left(c+d\,x\right)}^q}{\sin\left(a+b\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^q/sin(a + b*x),x)
```

output
```
int(cos(c + d*x)^q/sin(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos^q(c+dx)\csc(a+bx)\,dx=\int \cos\left(dx+c\right)^q\csc\left(bx+a\right)dx$$

input
```
int(cos(d*x+c)^q*csc(b*x+a),x)
```

output
```
int(cos(c + d*x)**q*csc(a + b*x),x)
```

3.230 $\int \cos^q(c+dx)\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \cos^q(c+dx)\csc^2(a+bx)\,dx = \mathrm{Int}\left(\cos^q(c+dx)\csc^2(a+bx),x\right)$$

output
```
Defer(Int)(cos(d*x+c)^q*csc(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cos^q(c+dx)\csc^2(a+bx)\,dx = \int \cos^q(c+dx)\csc^2(a+bx)\,dx$$

input
```
Integrate[Cos[c + d*x]^q*Csc[a + b*x]^2,x]
```

output
```
Integrate[Cos[c + d*x]^q*Csc[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\cos^q(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \csc^2(a+bx)\cos^q(c+dx)dx$$

input
```
Int[Cos[c + d*x]^q*Csc[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(dx+c)^q \csc(bx+a)^2\,dx$$

input
```
int(cos(d*x+c)^q*csc(b*x+a)^2,x)
```

output
```
int(cos(d*x+c)^q*csc(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cos^q(c+dx)\csc^2(a+bx)\,dx=\int \cos\left(dx+c\right)^q\csc\left(bx+a\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^q*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^q*csc(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 10.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos^q(c+dx)\csc^2(a+bx)\,dx=\int \cos^q\left(c+dx\right)\csc^2\left(a+bx\right)dx$$

input
```
integrate(cos(d*x+c)**q*csc(b*x+a)**2,x)
```

output
```
Integral(cos(c + d*x)**q*csc(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cos^q(c+dx)\csc^2(a+bx)\,dx = \int \cos(dx+c)^q\csc(bx+a)^2\,dx$$

input
```
integrate(cos(d*x+c)^q*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^q*csc(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cos^q(c+dx)\csc^2(a+bx)\,dx = \int \cos(dx+c)^q\csc(bx+a)^2\,dx$$

input
```
integrate(cos(d*x+c)^q*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^q*csc(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 16.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cos^q(c+dx)\csc^2(a+bx)\,dx = \int \frac{\cos(c+d\,x)^q}{\sin(a+b\,x)^2}\,dx$$

input
```
int(cos(c + d*x)^q/sin(a + b*x)^2,x)
```

output
```
int(cos(c + d*x)^q/sin(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cos^q(c+dx)\csc^2(a+bx)\,dx = \int \cos(dx+c)^q \csc(bx+a)^2\,dx$$

input
```
int(cos(d*x+c)^q*csc(b*x+a)^2,x)
```

output
```
int(cos(c + d*x)**q*csc(a + b*x)**2,x)
```

3.231 $\int \sin(a+bx)\tan^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \sin(a+bx)\tan^3(c+bx)\,dx = -\frac{3\text{arctanh}(\sin(c+bx))\cos(a-c)}{2b} + \frac{\sec(c+bx)\sin(a-c)}{b} + \frac{\sin(a+bx)}{b} + \frac{\cos(a-c)\sec(c+bx)\tan(c+bx)}{2b}$$

output
```
-3/2*arctanh(sin(b*x+c))*cos(a-c)/b+sec(b*x+c)*sin(a-c)/b+sin(b*x+a)/b+1/2
*cos(a-c)*sec(b*x+c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \sin(a+bx)\tan^3(c+bx)\,dx = \frac{-12\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+\sec^2(c+bx)(2\sin(a-2c-bx)+5\sin(a+bx)+\sin}{4b}$$

input
```
Integrate[Sin[a + b*x]*Tan[c + b*x]^3,x]
```

output
```
(-12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Cos[a - c] + Sec[c + b*x]^2*(2*
Sin[a - 2*c - b*x] + 5*Sin[a + b*x] + Sin[a + 2*c + 3*b*x]))/(4*b)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.867, Rules used = {5087, 3042, 3091, 3042, 4257, 5090, 3042, 3086, 24, 5087, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan^3(bx+c)\,dx$$

$$\Big\downarrow \text{5087}$$

$$\cos(a-c)\int \sec(c+bx)\tan^2(c+bx)dx-\int \cos(a+bx)\tan^2(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \sec(c+bx)\tan(c+bx)^2dx-\int \cos(a+bx)\tan^2(c+bx)dx$$

$$\Big\downarrow \text{3091}$$

$$\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{1}{2}\int \sec(c+bx)dx\right)-\int \cos(a+bx)\tan^2(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{1}{2}\int \csc\left(c+bx+\frac{\pi}{2}\right)dx\right)-\int \cos(a+bx)\tan^2(c+bx)dx$$

$$\Big\downarrow \text{4257}$$

$$\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)-\int \cos(a+bx)\tan^2(c+bx)dx$$

$$\Big\downarrow \text{5090}$$

$$-\int \sin(a+bx)\tan(c+bx)dx+\sin(a-c)\int \sec(c+bx)\tan(c+bx)dx+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \; 3042$$

$$-\int \sin(a+bx)\tan(c+bx)dx+\sin(a-c)\int \sec(c+bx)\tan(c+bx)dx+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \; 3086$$

$$-\int \sin(a+bx)\tan(c+bx)dx+\frac{\sin(a-c)\int 1d\sec(c+bx)}{b}+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \; 24$$

$$-\int \sin(a+bx)\tan(c+bx)dx+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \; 5087$$

$$-\cos(a-c)\int \sec(c+bx)dx+\int \cos(a+bx)dx+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\int \sin\left(a+bx+\frac{\pi}{2}\right)dx+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \; 3117$$

$$-\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\sin(a-c)\sec(bx+c)}{b}+\frac{\sin(a+bx)}{b}$$

$$\Big\downarrow \; 4257$$

$$-\frac{\cos(a-c)\text{arctanh}(\sin(bx+c))}{b}+\cos(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\sin(a-c)\sec(bx+c)}{b}+\frac{\sin(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]*Tan[c + b*x]^3,x]
```

output
```
-((ArcTanh[Sin[c + b*x]]*Cos[a - c])/b) + (Sec[c + b*x]*Sin[a - c])/b + Si
n[a + b*x]/b + Cos[a - c]*(-1/2*ArcTanh[Sin[c + b*x]]/b + (Sec[c + b*x]*Ta
n[c + b*x])/(2*b))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5087
```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5090
```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Sim
p[Sin[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.58

method	result
risch	$-\frac{i\,\mathrm{e}^{i(bx+a)}}{2b}+\frac{i\,\mathrm{e}^{-i(bx+a)}}{2b}-\frac{i\left(3\,\mathrm{e}^{i(3bx+5a+2c)}-\mathrm{e}^{i(3bx+3a+4c)}+\mathrm{e}^{i(bx+5a)}-3\,\mathrm{e}^{i(bx+3a+2c)}\right)}{2b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{2}}+\frac{3\ln\left(\mathrm{e}^{i(bx+a)}-i\,\mathrm{e}^{i(a-c)}\right)\cos(a-c}{2b}$

input
```
int(sin(b*x+a)*tan(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b*exp(I*(b*x+a))+1/2*I/b*exp(-I*(b*x+a))-1/2*I/b/(exp(2*I*(b*x+a+c)
)+exp(2*I*a))^2*(3*exp(I*(3*b*x+5*a+2*c))-exp(I*(3*b*x+3*a+4*c))+exp(I*(b*
x+5*a))-3*exp(I*(b*x+3*a+2*c)))+3/2*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*co
s(a-c)-3/2*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*cos(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(68) = 136.

Time = 0.09 (sec) , antiderivative size = 376, normalized size of antiderivative = 5.22

$$\int \sin(a+bx)\tan^3(c+bx)\,dx =$$

$$-\frac{3\sqrt{2}\left(2\left(\cos(-2a+2c)+1\right)\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-2\left(\cos(-2a+2c)^2+\cos(-2a+2c)\right)\cos(bx+a)^2+\cos(-2a+2c)^2-1\right)\log\Bigg(}{\sqrt{\cos(-2a}}$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="fricas")
```

output
```
-1/8*(3*sqrt(2)*(2*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a)*sin(-2*
a + 2*c) - 2*(cos(-2*a + 2*c)^2 + cos(-2*a + 2*c))*cos(b*x + a)^2 + cos(-2
*a + 2*c)^2 - 1)*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*s
in(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a
) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2
*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*s
in(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a + 2*c) + 1) - 4*(4*co
s(b*x + a)^2*cos(-2*a + 2*c) - 3*cos(-2*a + 2*c) + 5)*sin(b*x + a) - 4*(4*
cos(b*x + a)^3 - 5*cos(b*x + a))*sin(-2*a + 2*c))/(2*b*cos(b*x + a)^2*cos(
-2*a + 2*c) - 2*b*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a +
 2*c) + b)
```

Sympy [F]

$$\int \sin(a+bx)\tan^3(c+bx)\,dx=\int \sin(a+bx)\tan^3(bx+c)\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)*tan(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(68) = 136$.

Time = 0.19 (sec) , antiderivative size = 1027, normalized size of antiderivative = 14.26

$$\int \sin(a+bx)\tan^3(c+bx)\,dx=\text{Too large to display}$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*(sin(5*b*x + a + 4*c) + 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*cos
(6*b*x + 2*a + 4*c) - 2*(5*sin(4*b*x + 2*a + 2*c) - 2*sin(4*b*x + 4*c) + 2
*sin(2*b*x + 2*a) - 5*sin(2*b*x + 2*c))*cos(5*b*x + a + 4*c) + 10*(2*sin(3
*b*x + a + 2*c) + sin(b*x + a))*cos(4*b*x + 2*a + 2*c) - 4*(2*sin(3*b*x +
a + 2*c) + sin(b*x + a))*cos(4*b*x + 4*c) - 4*(2*sin(2*b*x + 2*a) - 5*sin(
2*b*x + 2*c))*cos(3*b*x + a + 2*c) - 3*(cos(5*b*x + a + 4*c)^2*cos(-a + c)
 + 4*cos(3*b*x + a + 2*c)^2*cos(-a + c) + 4*cos(3*b*x + a + 2*c)*cos(b*x +
 a)*cos(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x + a +
 4*c)^2 + 4*cos(-a + c)*sin(3*b*x + a + 2*c)^2 + 4*cos(-a + c)*sin(3*b*x +
 a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2 + 2*(2*cos(3*b*x + a +
 2*c)*cos(-a + c) + cos(b*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) + 2*(2*
cos(-a + c)*sin(3*b*x + a + 2*c) + cos(-a + c)*sin(b*x + a))*sin(5*b*x + a
 + 4*c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(
b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos
(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(
c) + sin(c)^2)) - 2*(cos(5*b*x + a + 4*c) + 2*cos(3*b*x + a + 2*c) + cos(b
*x + a))*sin(6*b*x + 2*a + 4*c) + 2*(5*cos(4*b*x + 2*a + 2*c) - 2*cos(4*b*
x + 4*c) + 2*cos(2*b*x + 2*a) - 5*cos(2*b*x + 2*c) - 1)*sin(5*b*x + a + 4*
c) - 10*(2*cos(3*b*x + a + 2*c) + cos(b*x + a))*sin(4*b*x + 2*a + 2*c) + 4
*(2*cos(3*b*x + a + 2*c) + cos(b*x + a))*sin(4*b*x + 4*c) + 4*(2*cos(2*...
```

Giac [F]

$$\int \sin(a+bx)\tan^3(c+bx)\,dx = \int \sin\left(bx+a\right)\tan\left(bx+c\right)^3\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*tan(b*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\tan^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)*tan(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sin(a+bx)\tan^3(c+bx)\,dx$$
$$= \frac{-\cos(bx+a)\tan(bx+c)-3\left(\int \sin(bx+a)\tan(bx+c)\,dx\right)b+\sin(bx+a)\tan(bx+c)^2+\sin(bx+}{2b}$$

input
```
int(sin(b*x+a)*tan(b*x+c)^3,x)
```

output
```
( - cos(a + b*x)*tan(b*x + c) - 3*int(sin(a + b*x)*tan(b*x + c),x)*b + sin
(a + b*x)*tan(b*x + c)**2 + sin(a + b*x))/(2*b)
```

3.232 $\int \sin(a+bx)\tan^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \sin(a+bx)\tan^2(c+bx)\,dx=\frac{\cos(a+bx)}{b}+\frac{\cos(a-c)\sec(c+bx)}{b}+\frac{\operatorname{arctanh}(\sin(c+bx))\sin(a-c)}{b}$$

output
```
cos(b*x+a)/b+cos(a-c)*sec(b*x+c)/b+arctanh(sin(b*x+c))*sin(a-c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.48

$$\int \sin(a+bx)\tan^2(c+bx)\,dx=\frac{\cos(a)\cos(bx)}{b}+\frac{\cos(a-c)\sec(c+bx)}{b}-\frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b}-\frac{\sin(a)\sin(bx)}{b}$$

input
```
Integrate[Sin[a + b*x]*Tan[c + b*x]^2,x]
```

output
```
(Cos[a]*Cos[b*x])/b + (Cos[a - c]*Sec[c + b*x])/b - ((2*I)*ArcTan[((I*Cos[
c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x
)/2] - I*Cos[(b*x)/2]*Sin[c])]*Sin[a - c])/b - (Sin[a]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5087, 3042, 3086, 24, 5090, 3042, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan^2(bx+c)\,dx$$

$$\Big\downarrow \text{5087}$$

$$\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx-\int \cos(a+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx-\int \cos(a+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\cos(a-c)\int 1d\sec(c+bx)}{b}-\int \cos(a+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{24}$$

$$\frac{\cos(a-c)\sec(bx+c)}{b}-\int \cos(a+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{5090}$$

$$\sin(a-c)\int \sec(c+bx)dx-\int \sin(a+bx)dx+\frac{\cos(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\int \sin(a+bx)dx+\frac{\cos(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\frac{\cos(a-c)\sec(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sin(a-c)\text{arctanh}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]*Tan[c + b*x]^2,x]
```

output
```
Cos[a + b*x]/b + (Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[
a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5087
```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5090
```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Sim
p[Sin[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.25

method	result	siz
risch	$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}+\frac{e^{i(bx+3a)}+e^{i(bx+a+2c)}}{b\left(e^{2i(bx+a+c)}+e^{2ia}\right)}+\frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(a-c)}{b}-\frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\sin(a-c)}{b}$	14

input
```
int(sin(b*x+a)*tan(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/b*exp(I*(b*x+a))+1/2/b*exp(-I*(b*x+a))+1/b/(exp(2*I*(b*x+a+c))+exp(2*I
*a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))+ln(exp(I*(b*x+a))+I*exp(I*(a-c)
))/b*sin(a-c)-ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(44) = 88$.

Time = 0.10 (sec) , antiderivative size = 315, normalized size of antiderivative = 7.16

$$\int \sin(a+bx)\tan^2(c+bx)\,dx =$$

$$-\frac{4\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)^{2}-4\,\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right)+\frac{\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\right.}{}}{}$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)^2 - 4*cos(b*x + a)*sin(b*x + a)
*sin(-2*a + 2*c) + sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(-2*a +
2*c) + (cos(-2*a + 2*c)^2 - 1)*sin(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2
*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(
-2*a + 2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*
a + 2*c) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2
*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(co
s(-2*a + 2*c) + 1) + 4*cos(-2*a + 2*c) + 4)/(b*sin(b*x + a)*sin(-2*a + 2*c
) - (b*cos(-2*a + 2*c) + b)*cos(b*x + a))
```

Sympy [F]

$$\int \sin(a+bx)\tan^2(c+bx)\,dx = \int \sin{\left(a+bx\right)}\tan^2{\left(bx+c\right)}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)*tan(b*x + c)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(44) = 88$.

Time = 0.19 (sec) , antiderivative size = 520, normalized size of antiderivative = 11.82

$$\int \sin(a+bx)\tan^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*((cos(3*b*x + a + 2*c) + cos(b*x + a))*cos(4*b*x + 2*a + 2*c) + (3*cos
(2*b*x + 2*a) + 3*cos(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 3*cos(2*b*x
 + 2*a)*cos(b*x + a) + 3*cos(2*b*x + 2*c)*cos(b*x + a) + (cos(3*b*x + a +
2*c)^2*sin(-a + c) + 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a + c) + cos
(b*x + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c)^2*sin(-a + c) + 2*sin(3*b*x
 + a + 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c))*log((co
s(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2
*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)
*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2))
+ (sin(3*b*x + a + 2*c) + sin(b*x + a))*sin(4*b*x + 2*a + 2*c) + 3*(sin(2*
b*x + 2*a) + sin(2*b*x + 2*c))*sin(3*b*x + a + 2*c) + 3*sin(2*b*x + 2*a)*s
in(b*x + a) + 3*sin(2*b*x + 2*c)*sin(b*x + a) + cos(b*x + a))/(b*cos(3*b*x
 + a + 2*c)^2 + 2*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 +
 b*sin(3*b*x + a + 2*c)^2 + 2*b*sin(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(
b*x + a)^2)
```

Giac [F]

$$\int \sin(a+bx)\tan^2(c+bx)\,dx = \int \sin\left(bx+a\right)\tan\left(bx+c\right)^2\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*tan(b*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 21.28 (sec) , antiderivative size = 294, normalized size of antiderivative = 6.68

$$\int \sin(a+bx)\tan^2(c+bx)\,dx$$
$$= \frac{e^{-a\,1i-b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}+1\right)\,1i}{b\,\left(e^{a\,2i-c\,2i}\,1i+e^{a\,2i+b\,x\,2i}\,1i\right)}$$
$$+\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$
$$-\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

input
```
int(sin(a + b*x)*tan(c + b*x)^2,x)
```

output
```
exp(- a*1i - b*x*1i)/(2*b) + exp(a*1i + b*x*1i)/(2*b) + (exp(a*1i + b*x*1i
)*(exp(a*2i - c*2i) + 1)*1i)/(b*(exp(a*2i - c*2i)*1i + exp(a*2i + b*x*2i)*
1i)) + (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) - (exp(a*
2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2
))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log(exp(a*1i
)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) + (exp(a*2i)*exp(-c*2i)*(exp(
a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i)
 - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2))
```

Reduce [F]

$$\int \sin(a+bx)\tan^2(c+bx)\,dx=\int \sin(bx+a)\tan(bx+c)^2\,dx$$

input
```
int(sin(b*x+a)*tan(b*x+c)^2,x)
```

output
```
int(sin(a + b*x)*tan(b*x + c)**2,x)
```

3.233 $\int \sin(a+bx)\tan(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \sin(a+bx)\tan(c+bx)\,dx=\frac{\operatorname{arctanh}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sin(a+bx)}{b}$$

output
```
arctanh(sin(b*x+c))*cos(a-c)/b-sin(b*x+a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.24

$$\begin{aligned}\int \sin(a+bx)\tan(c+bx)\,dx\\ =-\frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b}\\ -\frac{\cos(bx)\sin(a)}{b}-\frac{\cos(a)\sin(bx)}{b}\end{aligned}$$

input
```
Integrate[Sin[a + b*x]*Tan[c + b*x],x]
```

output
```
((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x
)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Cos
[b*x]*Sin[a])/b - (Cos[a]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5087, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan(bx+c)\,dx$$

$$\Big\downarrow \text{5087}$$

$$\cos(a-c)\int \sec(c+bx)dx-\int \cos(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\int \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\frac{\sin(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(a-c)\text{arctanh}(\sin(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]*Tan[c + b*x],x]
```

output
```
(ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - Sin[a + b*x]/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5087
```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.41

method	result	size
risch	$\frac{i e^{i(bx+a)}}{2b}-\frac{i e^{-i(bx+a)}}{2b}-\frac{\ln\left(e^{i(bx+a)}-i e^{i(a-c)}\right)\cos(a-c)}{b}+\frac{\ln\left(e^{i(bx+a)}+i e^{i(a-c)}\right)\cos(a-c)}{b}$	99

input
```
int(sin(b*x+a)*tan(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b*exp(I*(b*x+a))-1/2*I/b*exp(-I*(b*x+a))-ln(exp(I*(b*x+a))-I*exp(I*(
a-c)))/b*cos(a-c)+ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*cos(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 6.48

$$\int \sin(a+bx)\tan(c+bx)\,dx$$
$$= \frac{\sqrt{2}\sqrt{\cos(-2a+2c)+1}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\frac{2\sqrt{2}((\cos(-2a+2c)+1)\sin(bx+a)}{\sqrt{\cos(-2a+2}}}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos}\right.}{4b}$$

input
```
integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*sqrt(cos(-2*a + 2*c) + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*
c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a +
2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c
) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*
x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1)) - 4*sin(b*x +
a))/b
```

Sympy [F]

$$\int \sin(a+bx)\tan(c+bx)\,dx = \int \sin(a+bx)\tan(bx+c)\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+c),x)
```

output
```
Integral(sin(a + b*x)*tan(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(29) = 58$.

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.52

$$\int \sin(a+bx)\tan(c+bx)\,dx = -\frac{\cos(-a+c)\log\left(\frac{\cos(bx+2c)^2+\cos(c)^2-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2+2\cos(bx+2c)\sin(c)+\sin(c)^2}{\cos(bx+2c)^2+\cos(c)^2+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2-2\cos(bx+2c)\sin(c)+\sin(c)^2}\right)+2\sin(bx+a)}{2b}$$

input
```
integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

output
```
-1/2*(cos(-a + c)*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*
c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c
)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x +
2*c)*sin(c) + sin(c)^2)) + 2*sin(b*x + a))/b
```

Giac [F]

$$\int \sin(a+bx)\tan(c+bx)\,dx = \int \sin(bx+a)\tan(bx+c)\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*tan(b*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 21.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 7.83

$$\int \sin(a+bx)\tan(c+bx)\,dx$$
$$= -\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b} + \frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b} + \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right) - \frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}} - \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right) + \frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

input
```
int(sin(a + b*x)*tan(c + b*x),x)
```

output
```
(exp(a*1i + b*x*1i)*1i)/(2*b) - (exp(- a*1i - b*x*1i)*1i)/(2*b) + (log(- e
xp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1) - (exp(a*2i)*exp(-c*2i)*(e
xp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2
i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log((exp(a*2i)*exp(-c*2i)*(exp(a*
2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1
i)*(exp(a*2i)*exp(-c*2i) + 1))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2
i)^(1/2))
```

Reduce [F]

$$\int \sin(a+bx)\tan(c+bx)\,dx = \int \sin(bx+a)\tan(bx+c)\,dx$$

input
```
int(sin(b*x+a)*tan(b*x+c),x)
```

output
```
int(sin(a + b*x)*tan(b*x + c),x)
```

3.234 $\int \cot(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \cot(c+bx)\sin(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(c+bx))\sin(a-c)}{b}+\frac{\sin(a+bx)}{b}$$

output
```
-arctanh(cos(b*x+c))*sin(a-c)/b+sin(b*x+a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

$$\int \cot(c+bx)\sin(a+bx)\,dx$$
$$=\frac{\cos(bx)\sin(a)}{b}-\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b}+\frac{\cos(a)\sin(bx)}{b}$$

input
```
Integrate[Cot[c + b*x]*Sin[a + b*x],x]
```

output
```
(Cos[b*x]*Sin[a])/b - ((2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)
/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]
*Sin[a - c])/b + (Cos[a]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5089, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cot(bx+c)\,dx$$

$$\Big\downarrow \text{5089}$$

$$\sin(a-c)\int \csc(c+bx)dx+\int \cos(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc(c+bx)dx+\int \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\sin(a-c)\int \csc(c+bx)dx+\frac{\sin(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sin(a+bx)}{b}-\frac{\sin(a-c)\text{arctanh}(\cos(bx+c))}{b}$$

input
```
Int[Cot[c + b*x]*Sin[a + b*x],x]
```

output
```
-((ArcTanh[Cos[c + b*x]]*Sin[a - c])/b) + Sin[a + b*x]/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5089
```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Sim
p[Sin[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.28

method	result	size
risch	$-\frac{i \mathrm{e}^{i(bx+a)}}{2b}+\frac{i \mathrm{e}^{-i(bx+a)}}{2b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}-\mathrm{e}^{i(a-c)}\right)\sin(a-c)}{b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}+\mathrm{e}^{i(a-c)}\right)\sin(a-c)}{b}$	95

input
```
int(cot(b*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b*exp(I*(b*x+a))+1/2*I/b*exp(-I*(b*x+a))+ln(exp(I*(b*x+a))-exp(I*(a
-c)))/b*sin(a-c)-ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(29) = 58.

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 6.79

$$\int \cot(c+bx)\sin(a+bx)\,dx$$
$$= \frac{\frac{\sqrt{2}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)+\frac{2\sqrt{2}((\cos(-2a+2c)+1)\cos(bx+a)-\sin(bx+a)\sin(-2a+2c))}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)+3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)-1}\right)}{\sqrt{\cos(-2a+2c)+1}}}{4b}$$

input
```
integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*
x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a) - s
in(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) +
 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2
*a + 2*c) - cos(-2*a + 2*c) - 1))*sin(-2*a + 2*c)/sqrt(cos(-2*a + 2*c) + 1
) + 4*sin(b*x + a))/b
```

Sympy [F]

$$\int \cot(c+bx)\sin(a+bx)\,dx = \int \sin(a+bx)\cot(bx+c)\,dx$$

input
```
integrate(cot(b*x+c)*sin(b*x+a),x)
```

output
```
Integral(sin(a + b*x)*cot(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(29) = 58$.

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.62

$$\int \cot(c+bx)\sin(a+bx)\,dx$$
$$= \frac{\log\left(\cos(bx)^2 + 2\cos(bx)\cos(c) + \cos(c)^2 + \sin(bx)^2 - 2\sin(bx)\sin(c) + \sin(c)^2\right)\sin(-a+c) - \log}{}$$

input
```
integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*
x)*sin(c) + sin(c)^2)*sin(-a + c) - log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + c
os(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*sin(-a + c) + 2*sin(b
*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(29) = 58$.

Time = 0.15 (sec) , antiderivative size = 226, normalized size of antiderivative = 7.79

$$\int \cot(c+bx)\sin(a+bx)\,dx =$$
$$-\frac{2\left(\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^3 + \tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}c\right)^2\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right) - 1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3 + \tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right) + \tan\left(\frac{1}{2}c\right)^3 + \tan\left(\frac{1}{2}c\right)} - \frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right) - \tan}{\tan}\right.}{b}$$

input
```
integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
-2*((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*tan(
1/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(tan(1/2*a)^2
*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1/2*c)) - (ta
n(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))
*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*
a)^2 + tan(1/2*c)^2 + 1) + (tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*b*x) - 2*t
an(1/2*a))/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 20.98 (sec) , antiderivative size = 233, normalized size of antiderivative = 8.03

$$\int \cot(c+bx)\sin(a+bx)\,dx$$
$$= \frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b} - \frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b}$$
$$- \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right) - \frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$
$$+ \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right) + \frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

input
```
int(cot(c + b*x)*sin(a + b*x),x)
```

output
```
(exp(- a*1i - b*x*1i)*1i)/(2*b) - (exp(a*1i + b*x*1i)*1i)/(2*b) - (log(- e
xp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) - (exp(a*2i)*exp(-c*2i)*(e
xp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*
2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) + (log((exp(a*2i)*exp(-c*2i)*(ex
p(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(
b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2
i - c*2i))^(1/2))
```

Reduce [F]

$$\int \cot(c+bx)\sin(a+bx)\,dx = \int \cot(bx+c)\sin(bx+a)\,dx$$

input
```
int(cot(b*x+c)*sin(b*x+a),x)
```

output
```
int(cot(b*x + c)*sin(a + b*x),x)
```

3.235 $\int \cot^2(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \cot^2(c+bx)\sin(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+bx))\cos(a-c)}{b} + \frac{\cos(a+bx)}{b} - \frac{\csc(c+bx)\sin(a-c)}{b}$$

output
```
-arctanh(cos(b*x+c))*cos(a-c)/b+cos(b*x+a)/b-csc(b*x+c)*sin(a-c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.41

$$\int \cot^2(c+bx)\sin(a+bx)\,dx = -\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b} + \frac{\cos(a)\cos(bx)}{b} - \frac{\csc(c+bx)\sin(a-c)}{b} - \frac{\sin(a)\sin(bx)}{b}$$

input
```
Integrate[Cot[c + b*x]^2*Sin[a + b*x],x]
```

output
```
((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x
)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b + (Cos
[a]*Cos[b*x])/b - (Csc[c + b*x]*Sin[a - c])/b - (Sin[a]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5089, 3042, 25, 3086, 24, 5088, 3042, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cot^2(bx+c)\,dx$$

$$\Big\downarrow \text{5089}$$

$$\int \cos(a+bx)\cot(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)\cot(c+bx)dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\int \cos(a+bx)\cot(c+bx)dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\int \cos(a+bx)\cot(c+bx)dx-\frac{\sin(a-c)\int 1d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$\int \cos(a+bx)\cot(c+bx)dx-\frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{5088}$$

$$\cos(a-c)\int \csc(c+bx)dx - \int \sin(a+bx)dx - \frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)dx - \int \sin(a+bx)dx - \frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\cos(a-c)\int \csc(c+bx)dx - \frac{\sin(a-c)\csc(bx+c)}{b} + \frac{\cos(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\cos(a-c)\text{arctanh}(\cos(bx+c))}{b} - \frac{\sin(a-c)\csc(bx+c)}{b} + \frac{\cos(a+bx)}{b}$$

input
```
Int[Cot[c + b*x]^2*Sin[a + b*x],x]
```

output
```
-((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) + Cos[a + b*x]/b - (Csc[c + b*x]*S
in[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5088
```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5089
```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Sim
p[Sin[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.11

method	result	size
risch	$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}+\frac{e^{i(bx+3a)}-e^{i(bx+a+2c)}}{b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)}+\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos(a-c)}{b}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos(a-c)}{b}$	143

input
```
int(cot(b*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*exp(I*(b*x+a))+1/2/b*exp(-I*(b*x+a))+1/b/(-exp(2*I*(b*x+a+c))+exp(2*
I*a))*(exp(I*(b*x+3*a))-exp(I*(b*x+a+2*c)))+ln(exp(I*(b*x+a))-exp(I*(a-c))
)/b*cos(a-c)-ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(46) = 92$.

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 6.87

$$\int \cot^2(c+bx)\sin(a+bx)\,dx$$

$$= \frac{4\left(\cos\left(-2a+2c\right)+1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+\frac{\sqrt{2}\left(\left(\cos(-2a+2c)+1\right)\cos(bx+a)\sin(-2a+2c)+\left(\cos(-2a+2c)^2+2\,\text{co}\right.\right.}{}}{4\left(b\,\text{co}\right.}$$

input
```
integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*((cos(-2*
a + 2*c) + 1)*cos(b*x + a)*sin(-2*a + 2*c) + (cos(-2*a + 2*c)^2 + 2*cos(-2
*a + 2*c) + 1)*sin(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*co
s(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)
*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) -
cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*si
n(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a + 2*c) +
1) + 4*(cos(b*x + a)^2 + 1)*sin(-2*a + 2*c))/(b*cos(b*x + a)*sin(-2*a + 2*
c) + (b*cos(-2*a + 2*c) + b)*sin(b*x + a))
```

Sympy [F]

$$\int \cot^2(c+bx)\sin(a+bx)\,dx = \int \sin\left(a+bx\right)\cot^2\left(bx+c\right)dx$$

input
```
integrate(cot(b*x+c)**2*sin(b*x+a),x)
```

output
```
Integral(sin(a + b*x)*cot(b*x + c)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(46) = 92$.

Time = 0.06 (sec) , antiderivative size = 612, normalized size of antiderivative = 13.30

$$\int \cot^2(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/2*((cos(3*b*x + a + 2*c) - cos(b*x + a))*cos(4*b*x + 2*a + 2*c) - (3*cos
(2*b*x + 2*a) - 3*cos(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 3*cos(2*b*x
 + 2*a)*cos(b*x + a) - 3*cos(2*b*x + 2*c)*cos(b*x + a) - (cos(3*b*x + a +
2*c)^2*cos(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos(-a + c) + cos
(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b*x + a + 2*c)^2 - 2*cos(-a +
c)*sin(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2)*log(cos
(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) +
sin(c)^2) + (cos(3*b*x + a + 2*c)^2*cos(-a + c) - 2*cos(3*b*x + a + 2*c)*c
os(b*x + a)*cos(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b
*x + a + 2*c)^2 - 2*cos(-a + c)*sin(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a
 + c)*sin(b*x + a)^2)*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(
b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) + (sin(3*b*x + a + 2*c) - sin(b*x +
 a))*sin(4*b*x + 2*a + 2*c) - 3*(sin(2*b*x + 2*a) - sin(2*b*x + 2*c))*sin(
3*b*x + a + 2*c) + 3*sin(2*b*x + 2*a)*sin(b*x + a) - 3*sin(2*b*x + 2*c)*si
n(b*x + a) + cos(b*x + a))/(b*cos(3*b*x + a + 2*c)^2 - 2*b*cos(3*b*x + a +
 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + a + 2*c)^2 - 2*b*sin
(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(46) = 92$.

Time = 0.17 (sec) , antiderivative size = 577, normalized size of antiderivative = 12.54

$$\int \cot^2(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-((tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a)*tan(
1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1)
)/(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + ta
n(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1
/2*c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^
2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + (tan(1/2*b*x)^3*tan(1/
2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*b*x
)^3*tan(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^2 - t
an(1/2*b*x)^3*tan(1/2*c)^3 + 6*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^3 + 3*
tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c
)^4 - tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^3*tan(1/2*c) - 6*tan(1/2*b*
x)^2*tan(1/2*a)*tan(1/2*c) - 3*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c) - 2*ta
n(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^2 - 4*tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(1
/2*b*x)*tan(1/2*c)^3 + 2*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*b*x)*tan(1/2*a)
 + 3*tan(1/2*b*x)*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c) + 4*tan(1/2*c)^2)/(
(tan(1/2*b*x)^4*tan(1/2*c) + tan(1/2*b*x)^3*tan(1/2*c)^2 - tan(1/2*b*x)^3
+ tan(1/2*b*x)*tan(1/2*c)^2 - tan(1/2*b*x) - tan(1/2*c))*(tan(1/2*a)^2*tan
(1/2*c) + tan(1/2*c))))/b
```

Mupad [B] (verification not implemented)

Time = 23.09 (sec) , antiderivative size = 290, normalized size of antiderivative = 6.30

$$\int \cot^2(c+bx)\sin(a+bx)\,dx$$
$$= \frac{e^{-a\,1i-b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}-1\right)\,1i}{b\,\left(e^{a\,2i-c\,2i}\,1i-e^{a\,2i+b\,x\,2i}\,1i\right)}$$
$$-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$
$$+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

input
```
int(cot(c + b*x)^2*sin(a + b*x),x)
```

output
```
exp(- a*1i - b*x*1i)/(2*b) + exp(a*1i + b*x*1i)/(2*b) + (exp(a*1i + b*x*1i
)*(exp(a*2i - c*2i) - 1)*1i)/(b*(exp(a*2i - c*2i)*1i - exp(a*2i + b*x*2i)*
1i)) - (log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i) - (exp(
a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/
2))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) + (log((exp(a*2i)
*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) -
exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i))*(exp(a*2i - c*2i) +
1))/(2*b*exp(a*2i - c*2i)^(1/2))
```

Reduce [F]

$$\int \cot^2(c+bx)\sin(a+bx)\,dx=\int \cot\left(bx+c\right)^2\sin\left(bx+a\right)dx$$

input
```
int(cot(b*x+c)^2*sin(b*x+a),x)
```

output
```
int(cot(b*x + c)**2*sin(a + b*x),x)
```

3.236 $\int \cot^3(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \cot^3(c+bx)\sin(a+bx)\,dx = -\frac{\cos(a-c)\csc(c+bx)}{b} + \frac{3\text{arctanh}(\cos(c+bx))\sin(a-c)}{2b} - \frac{\cot(c+bx)\csc(c+bx)\sin(a-c)}{2b} - \frac{\sin(a+bx)}{b}$$

output
```
-cos(a-c)*csc(b*x+c)/b+3/2*arctanh(cos(b*x+c))*sin(a-c)/b-1/2*cot(b*x+c)*c
sc(b*x+c)*sin(a-c)/b-sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \cot^3(c+bx)\sin(a+bx)\,dx = \frac{12\text{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a-c)+\csc^2(c+bx)(2\sin(a-2c-bx)-5\sin(a+bx)+\sin(a}{4b}$$

input
```
Integrate[Cot[c + b*x]^3*Sin[a + b*x],x]
```

output
```
(12*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Sin[a - c] + Csc[c + b*x]^2*(2*S
in[a - 2*c - b*x] - 5*Sin[a + b*x] + Sin[a + 2*c + 3*b*x]))/(4*b)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.933, Rules used = {5089, 3042, 3091, 3042, 4257, 5088, 3042, 25, 3086, 24, 5089, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cot^3(bx+c)\,dx$$

$$\Big\downarrow \text{5089}$$

$$\int \cos(a+bx)\cot^2(c+bx)dx+\sin(a-c)\int \cot^2(c+bx)\csc(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)\cot^2(c+bx)dx+\sin(a-c)\int \sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\int \cos(a+bx)\cot^2(c+bx)dx+\sin(a-c)\left(-\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)\cot^2(c+bx)dx+\sin(a-c)\left(-\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{4257}$$

$$\int \cos(a+bx)\cot^2(c+bx)dx+\sin(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{5088}$$

$$-\int \cot(c+bx)\sin(a+bx)dx+\cos(a-c)\int \cot(c+bx)\csc(c+bx)dx+\sin(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \cot(c+bx)\sin(a+bx)dx+\cos(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx+\sin(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$-\int \cot(c+bx)\sin(a+bx)dx-\cos(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx+\sin(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(a-c)\int 1d\csc(c+bx)}{b}-\int \cot(c+bx)\sin(a+bx)dx+\sin(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{24}$$

$$-\int \cot(c+bx)\sin(a+bx)dx+\sin(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{5089}$$

$$-\sin(a-c)\int \csc(c+bx)dx-\int \cos(a+bx)dx+\sin(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\sin(a-c)\int \csc(c+bx)dx-\int \sin\left(a+bx+\frac{\pi}{2}\right)dx+\sin(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{3117}$$

$$-\sin(a-c)\int \csc(c+bx)dx+\sin(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cos(a-c)\csc(bx+c)}{b}-\frac{\sin(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sin(a-c)\text{arctanh}(\cos(bx+c))}{b}+\sin(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cos(a-c)\csc(bx+c)}{b}-\frac{\sin(a+bx)}{b}$$

input
```
Int[Cot[c + b*x]^3*Sin[a + b*x],x]
```

output
```
-((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b + (A
rcTanh[Cos[c + b*x]]/(2*b) - (Cot[c + b*x]*Csc[c + b*x])/(2*b))*Sin[a - c]
 - Sin[a + b*x]/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5088
```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5089
```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Sim
p[Sin[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.49

method	result
risch	$\frac{i e^{i(bx+a)}}{2b} - \frac{i e^{-i(bx+a)}}{2b} + \frac{i\left(-3\,e^{i(3bx+5a+2c)} - e^{i(3bx+3a+4c)} + e^{i(bx+5a)} + 3\,e^{i(bx+3a+2c)}\right)}{2b\left(-e^{2i(bx+a+c)} + e^{2ia}\right)^2} + \frac{3\ln\left(e^{i(bx+a)} + e^{i(a-c)}\right)\sin(a-c)}{2b}$

input
```
int(cot(b*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b*exp(I*(b*x+a))-1/2*I/b*exp(-I*(b*x+a))+1/2*I/b/(-exp(2*I*(b*x+a+c)
)+exp(2*I*a))^2*(-3*exp(I*(3*b*x+5*a+2*c))-exp(I*(3*b*x+3*a+4*c))+exp(I*(b
*x+5*a))+3*exp(I*(b*x+3*a+2*c)))+3/2*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin
(a-c)-3/2*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*sin(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(70) = 140$.

Time = 0.10 (sec) , antiderivative size = 372, normalized size of antiderivative = 5.03

$$\int \cot^3(c+bx)\sin(a+bx)\,dx$$

$$= \frac{3\sqrt{2}\left(2\left(\cos(-2a+2c)^2-1\right)\cos(bx+a)\sin(bx+a)+\left(2\cos(bx+a)^2\cos(-2a+2c)-\cos(-2a+2c)-1\right)\sin(-2a+2c)\right)\log\left(-\frac{2\cos(bx+a)^2\cos(-\cdots}{\cdots}\right)}{\sqrt{\cos(-2a+2c)+1}}\cdots$$

input
```
integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(3*sqrt(2)*(2*(cos(-2*a + 2*c)^2 - 1)*cos(b*x + a)*sin(b*x + a) + (2*c
os(b*x + a)^2*cos(-2*a + 2*c) - cos(-2*a + 2*c) - 1)*sin(-2*a + 2*c))*log(
-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a
+ 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(
-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x +
 a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(
-2*a + 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1) - 4*(4*cos(b*x + a)^2*cos(-2*a
 + 2*c) - 3*cos(-2*a + 2*c) - 5)*sin(b*x + a) - 4*(4*cos(b*x + a)^3 - 5*co
s(b*x + a))*sin(-2*a + 2*c))/(2*b*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos
(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) - b)
```

Sympy [F]

$$\int \cot^3(c+bx)\sin(a+bx)\,dx = \int \sin(a+bx)\cot^3(bx+c)\,dx$$

input
```
integrate(cot(b*x+c)**3*sin(b*x+a),x)
```

output
```
Integral(sin(a + b*x)*cot(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. $2(70) = 140$.

Time = 0.08 (sec) , antiderivative size = 1254, normalized size of antiderivative = 16.95

$$\int \cot^3(c + bx) \sin(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(2*(sin(5*b*x + a + 4*c) - 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*cos(
6*b*x + 2*a + 4*c) + 2*(5*sin(4*b*x + 2*a + 2*c) + 2*sin(4*b*x + 4*c) - 2*
sin(2*b*x + 2*a) - 5*sin(2*b*x + 2*c))*cos(5*b*x + a + 4*c) + 10*(2*sin(3*
b*x + a + 2*c) - sin(b*x + a))*cos(4*b*x + 2*a + 2*c) + 4*(2*sin(3*b*x + a
 + 2*c) - sin(b*x + a))*cos(4*b*x + 4*c) + 4*(2*sin(2*b*x + 2*a) + 5*sin(2
*b*x + 2*c))*cos(3*b*x + a + 2*c) - 3*(cos(5*b*x + a + 4*c)^2*sin(-a + c)
+ 4*cos(3*b*x + a + 2*c)^2*sin(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x +
a)*sin(-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*sin(-
a + c) + 4*sin(3*b*x + a + 2*c)^2*sin(-a + c) - 4*sin(3*b*x + a + 2*c)*sin
(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c) - 2*(2*cos(3*b*x + a +
2*c)*sin(-a + c) - cos(b*x + a)*sin(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*s
in(3*b*x + a + 2*c)*sin(-a + c) - sin(b*x + a)*sin(-a + c))*sin(5*b*x + a
+ 4*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin
(b*x)*sin(c) + sin(c)^2) + 3*(cos(5*b*x + a + 4*c)^2*sin(-a + c) + 4*cos(3
*b*x + a + 2*c)^2*sin(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a
 + c) + cos(b*x + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*sin(-a + c) +
4*sin(3*b*x + a + 2*c)^2*sin(-a + c) - 4*sin(3*b*x + a + 2*c)*sin(b*x + a)
*sin(-a + c) + sin(b*x + a)^2*sin(-a + c) - 2*(2*cos(3*b*x + a + 2*c)*sin(
-a + c) - cos(b*x + a)*sin(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*sin(3*b*x
+ a + 2*c)*sin(-a + c) - sin(b*x + a)*sin(-a + c))*sin(5*b*x + a + 4*c)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 870 vs. $2(70) = 140$.

Time = 0.19 (sec) , antiderivative size = 870, normalized size of antiderivative = 11.76

$$\int \cot^3(c + bx) \sin(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
1/4*(12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*
tan(1/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(tan(1/2*
a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1/2*c)) -
 12*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(
1/2*c))*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + t
an(1/2*a)^2 + tan(1/2*c)^2 + 1) + 8*(tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*b
*x) - 2*tan(1/2*a))/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)) - (2*tan(1/2
*b*x)^3*tan(1/2*a)*tan(1/2*c)^7 + tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^7
 + tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^8 - 4*tan(1/2*b*x)^3*tan(1/2*a)^2*
tan(1/2*c)^4 + 6*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^5 - 5*tan(1/2*b*x)^2
*tan(1/2*a)^2*tan(1/2*c)^5 + 2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^6 - 4*
tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*b*x)^2*tan(1/2*c)^7 - 2*t
an(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^7 - 6*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*
c)^3 + 5*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^3 + 4*tan(1/2*b*x)^3*tan(1
/2*c)^4 - 22*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^4 + 4*tan(1/2*b*x)*tan(1
/2*a)^2*tan(1/2*c)^4 + 5*tan(1/2*b*x)^2*tan(1/2*c)^5 - 14*tan(1/2*b*x)*tan
(1/2*a)*tan(1/2*c)^5 + 2*tan(1/2*a)^2*tan(1/2*c)^5 + 4*tan(1/2*b*x)*tan(1/
2*c)^6 + 2*tan(1/2*a)*tan(1/2*c)^6 - 2*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c
) - tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c) + 2*tan(1/2*b*x)^2*tan(1/2*a)*t
an(1/2*c)^2 - 4*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^2 - 5*tan(1/2*b*x)...
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(cot(c + b*x)^3*sin(a + b*x),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cot^3(c+bx)\sin(a+bx)\,dx$$
$$= \frac{-\cos(bx+a)\cot(bx+c)-\cot(bx+c)^2\sin(bx+a)-3\left(\int\cot(bx+c)\sin(bx+a)\,dx\right)b-\sin(bx+a}{2b}$$

input
```
int(cot(b*x+c)^3*sin(b*x+a),x)
```

output
```
( - cos(a + b*x)*cot(b*x + c) - cot(b*x + c)**2*sin(a + b*x) - 3*int(cot(b
*x + c)*sin(a + b*x),x)*b - sin(a + b*x))/(2*b)
```

3.237 $\int \sin^2(a+bx)\tan^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \sin^2(a+bx)\tan^3(c+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 1.62 (sec) , antiderivative size = 239, normalized size of antiderivative = 239.00

$$\int \sin^2(a+bx)\tan^3(c+bx)\,dx$$
$$=\frac{-\cos(2a)\cos(2bx)+2\log(\cos(c+bx))+2\cos(2a-2c-bx)\sec(c)\sec(c+bx)-2\cos(2a-2c+bx)\mathrm{se}}{}$$

input
```
Integrate[Sin[a + b*x]^2*Tan[c + b*x]^3,x]
```

output
```
(-(Cos[2*a]*Cos[2*b*x]) + 2*Log[Cos[c + b*x]] + 2*Cos[2*a - 2*c - b*x]*Sec
[c]*Sec[c + b*x] - 2*Cos[2*a - 2*c + b*x]*Sec[c]*Sec[c + b*x] + 2*Cos[a -
c]^2*Sec[c + b*x]^2 + 6*b*x*Cos[a]*Sin[a] - 6*b*x*Cos[a]*Cos[c]^2*Sin[a] -
 6*b*x*Sin[2*(a - c)] + 9*b*x*Cos[a]^2*Cos[c]*Sin[c] - 9*b*x*Cos[c]*Sin[a]
^2*Sin[c] + 9*b*x*Sin[2*a]*Sin[c]^2 + Sin[2*a]*Sin[2*b*x] - 3*b*x*Cos[a]^2
*Tan[c] + 3*b*x*Sin[a]^2*Tan[c] - 3*b*x*Cos[a]^2*Sin[c]^2*Tan[c] + 3*b*x*S
in[a]^2*Sin[c]^2*Tan[c] + 6*Cos[2*(a - c)]*(Log[Cos[c + b*x]] - b*x*Tan[c]
))/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\tan^3(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin^2(a+bx)\tan^3(bx+c)dx$$

input
```
Int[Sin[a + b*x]^2*Tan[c + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.40 (sec) , antiderivative size = 226, normalized size of antiderivative = 226.00

method	result
risch	$\frac{3ix\,\mathrm{e}^{2i(a-c)}}{2}-\frac{ix}{2}-\frac{\mathrm{e}^{2i(bx+a)}}{8b}-\frac{\mathrm{e}^{-2i(bx+a)}}{8b}-3i\cos\left(2a-2c\right)x-\frac{3i\cos(2a-2c)a}{b}-\frac{ia}{b}+\frac{3\,\mathrm{e}^{2i(bx+3a)}+2\,\mathrm{e}^{2i(bx+}}{2}$

input
```
int(sin(b*x+a)^2*tan(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
3/2*I*x*exp(2*I*(a-c))-1/2*I*x-1/8/b*exp(2*I*(b*x+a))-1/8/b*exp(-2*I*(b*x+
a))-3*I*cos(2*a-2*c)*x-3*I/b*cos(2*a-2*c)*a-I/b*a+1/2/b/(exp(2*I*(b*x+a+c)
)+exp(2*I*a))^2*(3*exp(2*I*(b*x+3*a))+2*exp(2*I*(b*x+2*a+c))-exp(2*I*(b*x+
a+2*c))+2*exp(2*I*(3*a-c))-2*exp(2*I*(a+c)))+1/2*ln(exp(2*I*(b*x+a))+exp(2
*I*(a-c)))/b+3/2*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))/b*cos(2*a-2*c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.10 (sec) , antiderivative size = 412, normalized size of antiderivative = 412.00

$$\int \sin^2(a+bx)\tan^3(c+bx)\,dx =$$

$$-\frac{4\cos\left(bx+a\right)^4\cos\left(-2\,a+2\,c\right)-2\left(4\cos\left(-2\,a+2\,c\right)^2+2\cos\left(-2\,a+2\,c\right)-5\right)\cos\left(bx+a\right)^2+4\,\mathrm{co}}{}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(4*cos(b*x + a)^4*cos(-2*a + 2*c) - 2*(4*cos(-2*a + 2*c)^2 + 2*cos(-2
*a + 2*c) - 5)*cos(b*x + a)^2 + 4*cos(-2*a + 2*c)^2 + (2*(3*cos(-2*a + 2*c
) + 1)*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*(3*cos(-2*a + 2*c)^2
+ cos(-2*a + 2*c))*cos(b*x + a)^2 + 3*cos(-2*a + 2*c)^2 - 2*cos(-2*a + 2*c
) - 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)
*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1)/(cos(-2*a + 2*c) + 1)) - 2*(6*(b*x
*cos(-2*a + 2*c)^2 - b*x)*cos(b*x + a) + (2*cos(b*x + a)^3 - (4*cos(-2*a +
 2*c) + 1)*cos(b*x + a))*sin(-2*a + 2*c))*sin(b*x + a) - 6*(2*b*x*cos(b*x
+ a)^2*cos(-2*a + 2*c) - b*x*cos(-2*a + 2*c) + b*x)*sin(-2*a + 2*c) - cos(
-2*a + 2*c) - 7)/(2*b*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos(b*x + a)*si
n(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) + b)
```

Sympy [F]

$$\int \sin^2(a+bx)\tan^3(c+bx)\,dx=\int \sin^2{(a+bx)}\tan^3{(bx+c)}\,dx$$

input
```
integrate(sin(b*x+a)**2*tan(b*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**2*tan(b*x + c)**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.12 (sec) , antiderivative size = 1755, normalized size of antiderivative = 1755.00

$$\int \sin^2(a+bx)\tan^3(c+bx)\,dx=\text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+c)^3,x, algorithm="maxima")
```

output
```
-1/8*((cos(6*b*x + 2*a + 6*c) + 2*cos(4*b*x + 2*a + 4*c) + cos(2*b*x + 2*a
 + 2*c))*cos(8*b*x + 4*a + 6*c) + 2*(2*cos(4*b*x + 2*a + 4*c) + cos(2*b*x
+ 2*a + 2*c))*cos(6*b*x + 4*a + 4*c) - (24*b*x*sin(4*b*x + 6*c) + 12*b*x*s
in(2*b*x + 4*c) - 2*cos(6*b*x + 4*a + 4*c) + 11*cos(4*b*x + 4*a + 2*c) + 8
*cos(4*b*x + 2*a + 4*c) - 5*cos(4*b*x + 6*c) + 8*cos(2*b*x + 4*a) - 10*cos
(2*b*x + 4*c) - cos(2*c))*cos(6*b*x + 2*a + 6*c) + 12*(b*x*sin(6*b*x + 2*a
 + 6*c) + 2*b*x*sin(4*b*x + 2*a + 4*c) + b*x*sin(2*b*x + 2*a + 2*c))*cos(6
*b*x + 8*c) - 2*(12*b*x*sin(2*b*x + 4*c) + 11*cos(4*b*x + 4*a + 2*c) + 8*c
os(2*b*x + 4*a) + 4*cos(2*b*x + 2*a + 2*c) - 10*cos(2*b*x + 4*c) - cos(2*c
))*cos(4*b*x + 2*a + 4*c) - 16*cos(4*b*x + 2*a + 4*c)^2 + (48*b*x*sin(4*b*
x + 2*a + 4*c) + 24*b*x*sin(2*b*x + 2*a + 2*c) + 10*cos(4*b*x + 2*a + 4*c)
 + 5*cos(2*b*x + 2*a + 2*c))*cos(4*b*x + 6*c) - (8*cos(2*b*x + 4*a) - cos(
2*c))*cos(2*b*x + 2*a + 2*c) - 11*cos(4*b*x + 4*a + 2*c)*cos(2*b*x + 2*a +
 2*c) + 2*(6*b*x*sin(2*b*x + 2*a + 2*c) + 5*cos(2*b*x + 2*a + 2*c))*cos(2*
b*x + 4*c) - 2*((3*cos(-2*a + 2*c) + 1)*cos(6*b*x + 2*a + 6*c)^2 + 4*(3*co
s(-2*a + 2*c) + 1)*cos(4*b*x + 2*a + 4*c)^2 + 4*(3*cos(-2*a + 2*c) + 1)*co
s(4*b*x + 2*a + 4*c)*cos(2*b*x + 2*a + 2*c) + (3*cos(-2*a + 2*c) + 1)*cos(
2*b*x + 2*a + 2*c)^2 + (3*cos(-2*a + 2*c) + 1)*sin(6*b*x + 2*a + 6*c)^2 +
4*(3*cos(-2*a + 2*c) + 1)*sin(4*b*x + 2*a + 4*c)^2 + 4*(3*cos(-2*a + 2*c)
+ 1)*sin(4*b*x + 2*a + 4*c)*sin(2*b*x + 2*a + 2*c) + (3*cos(-2*a + 2*c)...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 88.12 (sec) , antiderivative size = 1957357, normalized size of antiderivative = 1957357.00

$$\int \sin^2(a+bx)\tan^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/32*(48*b*x*tan(b*x)^4*tan(a + 3*c)^2*tan(a + 2*c)^2*tan(a + c)^2*tan(a
- c)^2*tan(a - 2*c)^2*tan(a - 3*c)^2*tan(a)^2*tan(c)^9 - 48*b*x*tan(b*x)^4
*tan(a + 3*c)^2*tan(a + 2*c)^2*tan(a + c)^2*tan(a - c)^2*tan(a - 2*c)^2*ta
n(a - 3*c)^2*tan(a)*tan(c)^10 + 48*b*x*tan(b*x)^4*tan(a + 3*c)^2*tan(a + 2
*c)^2*tan(a + c)^2*tan(a - c)*tan(a - 2*c)^2*tan(a - 3*c)^2*tan(a)^2*tan(c
)^10 - 8*log(4*(tan(b*x)^2*tan(c)^2 - 2*tan(b*x)*tan(c) + 1)/(tan(b*x)^2*t
an(c)^2 + tan(b*x)^2 + tan(c)^2 + 1))*tan(b*x)^4*tan(a + 3*c)^2*tan(a + 2*
c)^2*tan(a + c)^2*tan(a - c)^2*tan(a - 2*c)^2*tan(a - 3*c)^2*tan(a)^2*tan(
c)^10 + 48*arctan((tan(b*x) + tan(c))/(tan(b*x)*tan(c) - 1))*tan(b*x)^4*ta
n(a + 3*c)^2*tan(a + 2*c)^2*tan(a + c)^2*tan(a - c)^2*tan(a - 2*c)^2*tan(a
 - 3*c)^2*tan(a)^2*tan(c)^9 - 24*arctan((tan(b*x) + tan(c))/(tan(b*x)*tan(
c) - 1))*tan(b*x)^4*tan(a + 3*c)^2*tan(a + 2*c)^2*tan(a + c)^2*tan(a - c)^
2*tan(a - 2*c)^2*tan(a - 3*c)^2*tan(a)*tan(c)^10 + 24*arctan((tan(b*x) + t
an(c))/(tan(b*x)*tan(c) - 1))*tan(b*x)^4*tan(a + 3*c)^2*tan(a + 2*c)^2*tan
(a + c)^2*tan(a - c)*tan(a - 2*c)^2*tan(a - 3*c)^2*tan(a)^2*tan(c)^10 + 24
*arctan((tan(b*x) + tan(c))/(tan(b*x)*tan(c) - 1))*tan(b*x)^4*tan(a + 3*c)
^2*tan(a + 2*c)*tan(a + c)^2*tan(a - c)^2*tan(a - 2*c)^2*tan(a - 3*c)^2*ta
n(a)^2*tan(c)^10 - 24*arctan((tan(b*x) + tan(c))/(tan(b*x)*tan(c) - 1))*ta
n(b*x)^4*tan(a + 3*c)*tan(a + 2*c)^2*tan(a + c)^2*tan(a - c)^2*tan(a - 2*c
)^2*tan(a - 3*c)^2*tan(a)^2*tan(c)^10 - 8*tan(b*x)^4*tan(a + 3*c)^2*tan...
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\tan^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^2*tan(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sin^2(a+bx)\tan^3(c+bx)\,dx = \int \sin{(bx+a)}^2\tan{(bx+c)}^3\,dx$$

input
```
int(sin(b*x+a)^2*tan(b*x+c)^3,x)
```

output
```
int(sin(a + b*x)**2*tan(b*x + c)**3,x)
```

3.238 $\int \sin^2(a+bx)\tan^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \sin^2(a+bx)\tan^2(c+bx)\,dx = -\frac{x}{2} - x\cos(2(a-c)) - \frac{\log(\cos(c+bx))\sin(2(a-c))}{b} + \frac{\sec(c+bx)\sin(2a+c+3bx)}{8b} + \frac{(4+5\cos(2(a-c)))\tan(c+bx)}{8b}$$

output
```
-1/2*x-x*cos(2*a-2*c)-ln(cos(b*x+c))*sin(2*a-2*c)/b+1/8*sec(b*x+c)*sin(3*b
*x+2*a+c)/b+1/8*(4+5*cos(2*a-2*c))*tan(b*x+c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(85) = 170.

Time = 1.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.22

$$\int \sin^2(a+bx)\tan^2(c+bx)\,dx = -x\cos(2(a-c)) + \frac{\sec(c)\sec(c+bx)(-4bx\cos(bx) - 4bx\cos(2c+bx) + 8\sin(bx) - 4\log(\cos(c+bx))\sin(2a-4c-bx)}{}$$

input
```
Integrate[Sin[a + b*x]^2*Tan[c + b*x]^2,x]
```

output
```
-(x*Cos[2*(a - c)]) + (Sec[c]*Sec[c + b*x]*(-4*b*x*Cos[b*x] - 4*b*x*Cos[2*
c + b*x] + 8*Sin[b*x] - 4*Log[Cos[c + b*x]]*Sin[2*a - 4*c - b*x] - 4*Sin[2
*a - 2*c - b*x] - 4*Log[Cos[c + b*x]]*Sin[2*a - 2*c - b*x] + Sin[2*a + b*x
] - 4*Log[Cos[c + b*x]]*Sin[2*a + b*x] + 5*Sin[2*a - 2*c + b*x] - 4*Log[Co
s[c + b*x]]*Sin[2*a - 2*c + b*x] + Sin[2*a + 3*b*x] + Sin[2*a + 2*c + 3*b*
x]))/(16*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\tan^2(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sin^2(a+bx)\tan^2(bx+c)dx$$

input
```
Int[Sin[a + b*x]^2*Tan[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.35

method	result
risch	$-x\,e^{2i(a-c)}-\frac{x}{2}-\frac{i e^{2i(bx+a)}}{8b}+\frac{i e^{-2i(bx+a)}}{8b}+2i\sin\left(2a-2c\right)x+\frac{2i\sin(2a-2c)a}{b}+\frac{i e^{2i(2a-c)}}{2b\left(e^{2i(bx+a+c)}+e^{2ia}\right)}+$

input
```
int(sin(b*x+a)^2*tan(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-x*exp(2*I*(a-c))-1/2*x-1/8*I/b*exp(2*I*(b*x+a))+1/8*I/b*exp(-2*I*(b*x+a))
+2*I*sin(2*a-2*c)*x+2*I/b*sin(2*a-2*c)*a+1/2*I/b/(exp(2*I*(b*x+a+c))+exp(2
*I*a))*exp(2*I*(2*a-c))+I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))*exp(2*I*a)+1/2
*I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))*exp(2*I*c)-ln(exp(2*I*(b*x+a))+exp(2*
I*(a-c)))/b*sin(2*a-2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(79) = 158.

Time = 0.09 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.53

$$\int \sin^2(a+bx)\tan^2(c+bx)\,dx$$
$$=\frac{\left(2\,bx\cos\left(-2\,a+2\,c\right)^2+3\,bx\cos\left(-2\,a+2\,c\right)+bx\right)\cos\left(bx+a\right)-\left(\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)\sin\right.}{}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*((2*b*x*cos(-2*a + 2*c)^2 + 3*b*x*cos(-2*a + 2*c) + b*x)*cos(b*x + a)
- ((cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(-2*a + 2*c) + (cos(-2*a + 2*c)^2
 - 1)*sin(b*x + a))*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)
*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1)/(cos(-2*a + 2*c) + 1)
) - ((cos(-2*a + 2*c) + 1)*cos(b*x + a)^2 + cos(-2*a + 2*c)^2 + (2*b*x*cos
(-2*a + 2*c) + b*x)*sin(-2*a + 2*c) + 2*cos(-2*a + 2*c) + 1)*sin(b*x + a)
- (cos(b*x + a)^3 + cos(b*x + a)*cos(-2*a + 2*c))*sin(-2*a + 2*c))/(b*sin(
b*x + a)*sin(-2*a + 2*c) - (b*cos(-2*a + 2*c) + b)*cos(b*x + a))
```

Sympy [F]

$$\int \sin^2(a+bx)\tan^2(c+bx)\,dx=\int \sin^2{(a+bx)}\tan^2{(bx+c)}\,dx$$

input
```
integrate(sin(b*x+a)**2*tan(b*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**2*tan(b*x + c)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sin^2(a+bx)\tan^2(c+bx)\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300537 vs. $2(79) = 158$.

Time = 12.19 (sec) , antiderivative size = 300537, normalized size of antiderivative = 3535.73

$$\int \sin^2(a+bx)\tan^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+c)^2,x, algorithm="giac")
```

output
```
1/8*(pi - 4*b*x*tan(b*x)^3*tan(a + 2*c)^2*tan(a + c)^2*tan(a - c)^2*tan(a
- 2*c)^2*tan(a)^2*tan(c)^7 - 3*pi*sgn(2*tan(b*x)^2*tan(c) + 2*tan(b*x)*tan
(c)^2 - 2*tan(b*x) - 2*tan(c))*tan(b*x)^3*tan(a + 2*c)^2*tan(a + c)^2*tan(
a - c)^2*tan(a - 2*c)^2*tan(a)^2*tan(c)^7 - pi*tan(b*x)^3*tan(a + 2*c)^2*t
an(a + c)^2*tan(a - c)^2*tan(a - 2*c)^2*tan(a)^2*tan(c)^7 + 2*arctan((tan(
b*x)*tan(c) - 1)/(tan(b*x) + tan(c)))*tan(b*x)^3*tan(a + 2*c)^2*tan(a + c)
^2*tan(a - c)^2*tan(a - 2*c)^2*tan(a)^2*tan(c)^7 + 2*arctan((tan(b*x) + ta
n(c))/(tan(b*x)*tan(c) - 1))*tan(b*x)^3*tan(a + 2*c)^2*tan(a + c)^2*tan(a
- c)^2*tan(a - 2*c)^2*tan(a)^2*tan(c)^7 - 4*b*x*tan(b*x)^3*tan(a + 2*c)^2*
tan(a + c)^2*tan(a - c)^2*tan(a - 2*c)^2*tan(a)^2*tan(c)^5 - 9*pi*sgn(2*ta
n(b*x)^2*tan(c) + 2*tan(b*x)*tan(c)^2 - 2*tan(b*x) - 2*tan(c))*tan(b*x)^3*
tan(a + 2*c)^2*tan(a + c)^2*tan(a - c)^2*tan(a - 2*c)^2*tan(a)^2*tan(c)^5
- 16*b*x*tan(b*x)^3*tan(a + 2*c)^2*tan(a + c)^2*tan(a - c)^2*tan(a - 2*c)^
2*tan(a)*tan(c)^6 + 4*b*x*tan(b*x)^2*tan(a + 2*c)^2*tan(a + c)^2*tan(a - c
)^2*tan(a - 2*c)^2*tan(a)^2*tan(c)^6 + 3*pi*sgn(2*tan(b*x)^2*tan(c) + 2*ta
n(b*x)*tan(c)^2 - 2*tan(b*x) - 2*tan(c))*tan(b*x)^2*tan(a + 2*c)^2*tan(a +
 c)^2*tan(a - c)^2*tan(a - 2*c)^2*tan(a)^2*tan(c)^6 - 4*log(4*(tan(b*x)^2*
tan(c)^2 - 2*tan(b*x)*tan(c) + 1)/(tan(b*x)^2*tan(c)^2 + tan(b*x)^2 + tan(
c)^2 + 1))*tan(b*x)^3*tan(a + 2*c)^2*tan(a + c)^2*tan(a - c)^2*tan(a - 2*c
)^2*tan(a)^2*tan(c)^6 + 4*b*x*tan(b*x)^3*tan(a + 2*c)^2*tan(a + c)^2*ta...
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.41

$$\int \sin^2(a+bx)\tan^2(c+bx)\,dx$$
$$= -x\left(\cos(2a-2c)+\frac{1}{2}-\sin(2a-2c)\,1\mathrm{i}\right)+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{8\,b}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{8\,b}$$
$$-\frac{\mathrm{e}^{-a\,4\mathrm{i}+c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\right)\left(2\,b\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-2\,b\,\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$
$$-\frac{\mathrm{e}^{-a\,2\mathrm{i}+c\,2\mathrm{i}}\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+2\,\mathrm{e}^{a\,4\mathrm{i}-c\,4\mathrm{i}}+\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)}{2\,b\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}\,1\mathrm{i}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}\right)}$$

input
```
int(sin(a + b*x)^2*tan(c + b*x)^2,x)
```

output
```
(exp(- a*2i - b*x*2i)*1i)/(8*b) - x*(cos(2*a - 2*c) - sin(2*a - 2*c)*1i +
1/2) - (exp(a*2i + b*x*2i)*1i)/(8*b) - (exp(c*4i - a*4i)*log(exp(a*2i)*exp
(b*x*2i) + exp(a*2i)*exp(-c*2i))*(2*b*exp(a*2i - c*2i) - 2*b*exp(a*6i - c*
6i))*1i)/(4*b^2) - (exp(c*2i - a*2i)*(exp(a*2i - c*2i) + 2*exp(a*4i - c*4i
) + exp(a*6i - c*6i)))/(2*b*(exp(a*2i - c*2i)*1i + exp(a*2i + b*x*2i)*1i))
```

Reduce [F]

$$\int \sin^2(a+bx)\tan^2(c+bx)\,dx = \text{Too large to display}$$

input
```
int(sin(b*x+a)^2*tan(b*x+c)^2,x)
```

output
```
(5*cos(b*x + c)*cos(a + b*x)*sin(a + b*x) + 48*cos(b*x + c)*int(tan((b*x +
 c)/2)**2/(tan((b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)**4
*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan((a
+ b*x)/2)**4 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x + c)
/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b + 48*cos(b*
x + c)*int(tan((a + b*x)/2)**2/(tan((b*x + c)/2)**4*tan((a + b*x)/2)**4 +
2*tan((b*x + c)/2)**4*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b
*x + c)/2)**2*tan((a + b*x)/2)**4 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)
**2 - 2*tan((b*x + c)/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2
+ 1),x)*b - 64*cos(b*x + c)*int((tan((b*x + c)/2)*tan((a + b*x)/2))/(tan((
b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)**4*tan((a + b*x)/2
)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan((a + b*x)/2)**4 - 4
*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x + c)/2)**2 + tan((a
+ b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b - 16*cos(b*x + c)*int(1/(ta
n((b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)**4*tan((a + b*x
)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan((a + b*x)/2)**4
- 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x + c)/2)**2 + tan(
(a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b + 8*cos(b*x + c)*sin(a +
 b*x) + 3*cos(b*x + c)*a + 3*cos(b*x + c)*b*x - 4*cos(a + b*x)*sin(b*x + c
) + 4*cos(a + b*x)*sin(a + b*x) + 2*sin(b*x + c)*sin(a + b*x)**2 - 4*si...
```

3.239 $\int \sin^2(a+bx)\tan(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \sin^2(a+bx)\tan(c+bx)\,dx = \frac{\cos^2(a+bx)}{2b} - \frac{\cos^2(a-c)\log(\cos(c+bx))}{b} + \frac{1}{2}x\sin(2(a-c))$$

output
```
1/2*cos(b*x+a)^2/b-cos(a-c)^2*ln(cos(b*x+c))/b+1/2*x*sin(2*a-2*c)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \sin^2(a+bx)\tan(c+bx)\,dx = \frac{\cos(2(a+bx)) - 4\cos^2(a-c)\log(\cos(c+bx)) + 2bx\sin(2(a-c))}{4b}$$

input
```
Integrate[Sin[a + b*x]^2*Tan[c + b*x],x]
```

output
```
(Cos[2*(a + b*x)] - 4*Cos[a - c]^2*Log[Cos[c + b*x]] + 2*b*x*Sin[2*(a - c)
])/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\tan(bx+c)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \sin^2(a+bx)\tan(bx+c)dx$$

input
```
Int[Sin[a + b*x]^2*Tan[c + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

method	result
risch	$i\cos(2a-2c)\,x-\frac{ix\,e^{2i(a-c)}}{2}+\frac{i\cos(2a-2c)a}{b}+\frac{ix}{2}-\frac{\ln\left(e^{2i(bx+a)}+e^{2i(a-c)}\right)\cos(2a-2c)}{2b}+\frac{ia}{b}+\frac{e^{2i(bx+a)}}{8b}+e^{-}$

input
```
int(sin(b*x+a)^2*tan(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
I*cos(2*a-2*c)*x-1/2*I*x*exp(2*I*(a-c))+I/b*cos(2*a-2*c)*a+1/2*I*x-1/2*ln(
exp(2*I*(b*x+a))+exp(2*I*(a-c)))/b*cos(2*a-2*c)+I/b*a+1/8/b*exp(2*I*(b*x+a
))+1/8/b*exp(-2*I*(b*x+a))-1/2*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(45) = 90$.

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.14

$$\int \sin^2(a+bx)\tan(c+bx)\,dx = -\frac{2\,bx\sin\left(-2\,a+2\,c\right)-2\,\cos\left(bx+a\right)^2+\left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(\frac{2\,\cos(bx+a)^2\cos(-2\,a+2\,c)-2\,\cos(bx+a)\sin(}{\cos(-2\,a+2}\right.}{4\,b}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+c),x, algorithm="fricas")
```

output
```
-1/4*(2*b*x*sin(-2*a + 2*c) - 2*cos(b*x + a)^2 + (cos(-2*a + 2*c) + 1)*log
((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a
+ 2*c) - cos(-2*a + 2*c) + 1)/(cos(-2*a + 2*c) + 1)))/b
```

Sympy [F]

$$\int \sin^2(a+bx)\tan(c+bx)\,dx = \int \sin^2\left(a+bx\right)\tan\left(bx+c\right)dx$$

input
```
integrate(sin(b*x+a)**2*tan(b*x+c),x)
```

output
```
Integral(sin(a + b*x)**2*tan(b*x + c), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.84

$$\int \sin^2(a+bx)\tan(c+bx)\,dx = -\frac{2\,bx\sin\left(-2\,a+2\,c\right)+\left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(\cos\left(2\,bx\right)^2+2\,\cos\left(2\,bx\right)\cos\left(2\,c\right)+\cos\left(2\,c\right)^2+\sin\left(\right.\right.}{4\,b}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+c),x, algorithm="maxima")
```

output
```
-1/4*(2*b*x*sin(-2*a + 2*c) + (cos(-2*a + 2*c) + 1)*log(cos(2*b*x)^2 + 2*c
os(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + s
in(2*c)^2) - cos(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11424 vs. $2(45) = 90$.

Time = 0.64 (sec) , antiderivative size = 11424, normalized size of antiderivative = 233.14

$$\int \sin^2(a+bx)\tan(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+c),x, algorithm="giac")
```

output
```
1/8*(4*b*x*tan(b*x)^2*tan(a + c)^2*tan(a - c)^2*tan(a)^2*tan(c)^3 - 4*b*x*
tan(b*x)^2*tan(a + c)^2*tan(a - c)^2*tan(a)*tan(c)^4 + 4*b*x*tan(b*x)^2*ta
n(a + c)^2*tan(a - c)*tan(a)^2*tan(c)^4 - 2*log(4*(tan(b*x)^2*tan(c)^2 - 2
*tan(b*x)*tan(c) + 1)/(tan(b*x)^2*tan(c)^2 + tan(b*x)^2 + tan(c)^2 + 1))*t
an(b*x)^2*tan(a + c)^2*tan(a - c)^2*tan(a)^2*tan(c)^4 + 4*arctan((tan(b*x)
 + tan(c))/(tan(b*x)*tan(c) - 1))*tan(b*x)^2*tan(a + c)^2*tan(a - c)^2*tan
(a)^2*tan(c)^3 + 2*arctan((tan(b*x) + tan(c))/(tan(b*x)*tan(c) - 1))*tan(b
*x)^2*tan(a + c)^2*tan(a - c)*tan(a)^2*tan(c)^4 - 2*arctan((tan(b*x) + tan
(c))/(tan(b*x)*tan(c) - 1))*tan(b*x)^2*tan(a + c)*tan(a - c)^2*tan(a)^2*ta
n(c)^4 + 4*b*x*tan(b*x)^2*tan(a + c)^2*tan(a - c)^2*tan(a)^2*tan(c) + 8*b*
x*tan(b*x)^2*tan(a + c)^2*tan(a - c)*tan(a)^2*tan(c)^2 - 6*log(4*(tan(b*x)
^2*tan(c)^2 - 2*tan(b*x)*tan(c) + 1)/(tan(b*x)^2*tan(c)^2 + tan(b*x)^2 + t
an(c)^2 + 1))*tan(b*x)^2*tan(a + c)^2*tan(a - c)^2*tan(a)^2*tan(c)^2 - 4*b
*x*tan(b*x)^2*tan(a + c)^2*tan(a - c)^2*tan(c)^3 - 4*log(4*(tan(b*x)^2*tan
(c)^2 - 2*tan(b*x)*tan(c) + 1)/(tan(b*x)^2*tan(c)^2 + tan(b*x)^2 + tan(c)^
2 + 1))*tan(b*x)^2*tan(a + c)^2*tan(a - c)^2*tan(a)*tan(c)^3 + 4*b*x*tan(b
*x)^2*tan(a + c)^2*tan(a)^2*tan(c)^3 + 4*b*x*tan(b*x)^2*tan(a - c)^2*tan(a
)^2*tan(c)^3 + 4*log(4*(tan(b*x)^2*tan(c)^2 - 2*tan(b*x)*tan(c) + 1)/(tan(
b*x)^2*tan(c)^2 + tan(b*x)^2 + tan(c)^2 + 1))*tan(b*x)^2*tan(a + c)*tan(a
- c)^2*tan(a)^2*tan(c)^3 + 4*b*x*tan(a + c)^2*tan(a - c)^2*tan(a)^2*tan...
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.67

$$\int \sin^2(a+bx)\tan(c+bx)\,dx$$
$$= x\left(\frac{\mathrm{e}^{-a\,2\mathrm{i}+c\,2\mathrm{i}}\,1\mathrm{i}}{2}+\frac{1}{2}\mathrm{i}\right)+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}}{8\,b}+\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}{8\,b}$$
$$-\frac{\mathrm{e}^{-a\,4\mathrm{i}+c\,4\mathrm{i}}\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\right)\left(4\,b\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+8\,b\,\mathrm{e}^{a\,4\mathrm{i}-c\,4\mathrm{i}}+4\,b\,\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)}{16\,b^2}$$

input
```
int(sin(a + b*x)^2*tan(c + b*x),x)
```

output
```
x*((exp(c*2i - a*2i)*1i)/2 + 1i/2) + exp(- a*2i - b*x*2i)/(8*b) + exp(a*2i
 + b*x*2i)/(8*b) - (exp(c*4i - a*4i)*log(exp(a*2i)*exp(b*x*2i) + exp(a*2i)
*exp(-c*2i))*(4*b*exp(a*2i - c*2i) + 8*b*exp(a*4i - c*4i) + 4*b*exp(a*6i -
 c*6i)))/(16*b^2)
```

Reduce [F]

$$\int \sin^2(a+bx)\tan(c+bx)\,dx = \int \sin(bx+a)^2\tan(bx+c)\,dx$$

input
```
int(sin(b*x+a)^2*tan(b*x+c),x)
```

output
```
int(sin(a + b*x)**2*tan(b*x + c),x)
```

3.240 $\int \cot(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \cot(c+bx)\sin^2(a+bx)\,dx = -\frac{\cos(2(a+bx))}{4b}+\frac{\log(\sin(c+bx))\sin^2(a-c)}{b}+\frac{1}{2}x\sin(2(a-c))$$

output
```
-1/4*cos(2*b*x+2*a)/b+ln(sin(b*x+c))*sin(a-c)^2/b+1/2*x*sin(2*a-2*c)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int \cot(c+bx)\sin^2(a+bx)\,dx = \frac{2ibx-\cos(2(a+bx))+\cos(2(a-c))\left(-2ibx-\log\left(\sin^2(c+bx)\right)\right)+\log\left(\sin^2(c+bx)\right)-4i\arctan(\tan(}{4b}$$

input
```
Integrate[Cot[c + b*x]*Sin[a + b*x]^2,x]
```

output
```
((2*I)*b*x - Cos[2*(a + b*x)] + Cos[2*(a - c)]*((-2*I)*b*x - Log[Sin[c + b
*x]^2]) + Log[Sin[c + b*x]^2] - (4*I)*ArcTan[Tan[c + b*x]]*Sin[a - c]^2 +
2*b*x*Sin[2*(a - c)])/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cot(bx+c)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \sin^2(a+bx)\cot(bx+c)dx$$

input
```
Int[Cot[c + b*x]*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.98

method	result
risch	$i\cos(2a-2c)\,x-\frac{ix\,e^{2i(a-c)}}{2}+\frac{i\cos(2a-2c)a}{b}-\frac{ix}{2}-\frac{\ln\left(e^{2i(bx+a)}-e^{2i(a-c)}\right)\cos(2a-2c)}{2b}-\frac{ia}{b}-\frac{e^{2i(bx+a)}}{8b}-e^{-}$

input
```
int(cot(b*x+c)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
I*cos(2*a-2*c)*x-1/2*I*x*exp(2*I*(a-c))+I/b*cos(2*a-2*c)*a-1/2*I*x-1/2/b*l
n(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*cos(2*a-2*c)-I/b*a-1/8/b*exp(2*I*(b*x+a
))-1/8/b*exp(-2*I*(b*x+a))+1/2/b*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(45) = 90$.

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.21

$$\int \cot(c+bx)\sin^2(a+bx)\,dx = -\frac{2\,bx\sin\left(-2\,a+2\,c\right)+2\,\cos\left(bx+a\right)^2+\left(\cos\left(-2\,a+2\,c\right)-1\right)\log\left(-\frac{2\,\cos(bx+a)^2\cos(-2\,a+2\,c)-2\,\cos(bx+a)\,\mathrm{si}}{\cos(-2\,a-}\right.}{4\,b}$$

input
```
integrate(cot(b*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*b*x*sin(-2*a + 2*c) + 2*cos(b*x + a)^2 + (cos(-2*a + 2*c) - 1)*log
(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a
 + 2*c) - cos(-2*a + 2*c) - 1)/(cos(-2*a + 2*c) + 1)))/b
```

Sympy [F]

$$\int \cot(c+bx)\sin^2(a+bx)\,dx = \int \sin^2\left(a+bx\right)\cot\left(bx+c\right)\,dx$$

input
```
integrate(cot(b*x+c)*sin(b*x+a)**2,x)
```

output
```
Integral(sin(a + b*x)**2*cot(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(45) = 90.

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.60

$$\int \cot(c+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{2\,bx\sin\left(-2\,a+2\,c\right)+\left(\cos\left(-2\,a+2\,c\right)-1\right)\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^2+\sin\left(bx\right)^2-\right.}{}$$

input
```
integrate(cot(b*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x*sin(-2*a + 2*c) + (cos(-2*a + 2*c) - 1)*log(cos(b*x)^2 + 2*cos
(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + (co
s(-2*a + 2*c) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x
)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) + cos(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1224 vs. 2(45) = 90.

Time = 0.17 (sec) , antiderivative size = 1224, normalized size of antiderivative = 25.50

$$\int \cot(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(4*(tan(1/2*a)^4*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)
^4*tan(1/2*c) + 6*tan(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3
+ tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*t
an(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))*b*x/(tan(
1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1
/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*ta
n(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 4*(tan(1/2*a)^4*tan(1/2*c)^2 - 2*tan(1/
2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c
) - 4*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2
 - 2*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2)*log(tan(b*x)^2 + 1)/(tan(1/2*a)
^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^
4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*
a)^2 + 2*tan(1/2*c)^2 + 1) + 8*(tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^3
*tan(1/2*c)^5 + tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^4*tan(1/2*c)^2 + 4*
tan(1/2*a)^3*tan(1/2*c)^3 - 5*tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)*tan
(1/2*c)^5 - 2*tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - 4*ta
n(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - tan(1/2*a)^2 + 2*tan(1/2*a)*tan(1/2
*c) - tan(1/2*c)^2)*log(abs(tan(b*x)*tan(1/2*c)^2 - tan(b*x) - 2*tan(1/2*c
)))/(tan(1/2*a)^4*tan(1/2*c)^6 + tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^
2*tan(1/2*c)^6 - tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^...
```

Mupad [B] (verification not implemented)

Time = 17.72 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.75

$$\int \cot(c+bx)\sin^2(a+bx)\,dx$$
$$= x\left(\frac{\mathrm{e}^{-a\,2\mathrm{i}+c\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{1}{2}\mathrm{i}\right)-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}}{8\,b}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}{8\,b}$$
$$-\frac{\mathrm{e}^{-a\,4\mathrm{i}+c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\right)\,\left(4\,b\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-8\,b\,\mathrm{e}^{a\,4\mathrm{i}-c\,4\mathrm{i}}+4\,b\,\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)}{16\,b^2}$$

input
```
int(cot(c + b*x)*sin(a + b*x)^2,x)
```

output
```
x*((exp(c*2i - a*2i)*1i)/2 - 1i/2) - exp(- a*2i - b*x*2i)/(8*b) - exp(a*2i
 + b*x*2i)/(8*b) - (exp(c*4i - a*4i)*log(exp(a*2i)*exp(b*x*2i) - exp(a*2i)
*exp(-c*2i))*(4*b*exp(a*2i - c*2i) - 8*b*exp(a*4i - c*4i) + 4*b*exp(a*6i -
 c*6i)))/(16*b^2)
```

Reduce [F]

$$\int \cot(c+bx)\sin^2(a+bx)\,dx = \int \cot\left(bx+c\right)\sin\left(bx+a\right)^2 dx$$

input
```
int(cot(b*x+c)*sin(b*x+a)^2,x)
```

output
```
int(cot(b*x + c)*sin(a + b*x)**2,x)
```

3.241 $\int \cot^2(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 130

$$\int \cot^2(c+bx)\sin^2(a+bx)\,dx = -\frac{x}{2}+x\cos(2(a-c))-\frac{(4-5\cos(2(a-c)))\cot(c+bx)}{8b} + \frac{\log(\cos(c+bx))\sin(2(a-c))}{b} + \frac{\log(\tan(c+bx))\sin(2(a-c))}{b} + \frac{\csc(c)\csc(c+bx)\sin(2a+3bx)}{16b} - \frac{\csc(c)\csc(c+bx)\sin(2a+2c+3bx)}{16b}$$

output
```
-1/2*x+x*cos(2*a-2*c)-1/8*(4-5*cos(2*a-2*c))*cot(b*x+c)/b+ln(cos(b*x+c))*s
in(2*a-2*c)/b+ln(tan(b*x+c))*sin(2*a-2*c)/b+1/16*csc(c)*csc(b*x+c)*sin(3*b
*x+2*a)/b-1/16*csc(c)*csc(b*x+c)*sin(3*b*x+2*a+2*c)/b
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.46

$$\int \cot^2(c+bx)\sin^2(a+bx)\,dx = x\cos(2(a-c)) + \frac{\csc(c)\csc(c+bx)(-4bx\cos(bx)+4bx\cos(2c+bx)+8\sin(bx)-4\log(\sin(c+bx))\sin(2a-4c-bx)}{}$$

input
```
Integrate[Cot[c + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
x*Cos[2*(a - c)] + (Csc[c]*Csc[c + b*x]*(-4*b*x*Cos[b*x] + 4*b*x*Cos[2*c +
 b*x] + 8*Sin[b*x] - 4*Log[Sin[c + b*x]]*Sin[2*a - 4*c - b*x] + 4*Sin[2*a
- 2*c - b*x] + 4*Log[Sin[c + b*x]]*Sin[2*a - 2*c - b*x] + Sin[2*a + b*x] -
 4*Log[Sin[c + b*x]]*Sin[2*a + b*x] - 5*Sin[2*a - 2*c + b*x] + 4*Log[Sin[c
 + b*x]]*Sin[2*a - 2*c + b*x] + Sin[2*a + 3*b*x] - Sin[2*a + 2*c + 3*b*x])
)/(16*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cot^2(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin^2(a+bx)\cot^2(bx+c)dx$$

input
```
Int[Cot[c + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.58

method	result
risch	$x\,\mathrm{e}^{2i(a-c)}-\frac{x}{2}-\frac{i\mathrm{e}^{2i(bx+a)}}{8b}+\frac{i\mathrm{e}^{-2i(bx+a)}}{8b}-2i\sin\left(2a-2c\right)x-\frac{2i\sin(2a-2c)a}{b}-\frac{i\mathrm{e}^{2i(2a-c)}}{2b\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{}{b}$

input
```
int(cot(b*x+c)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*exp(2*I*(a-c))-1/2*x-1/8*I/b*exp(2*I*(b*x+a))+1/8*I/b*exp(-2*I*(b*x+a))-
2*I*sin(2*a-2*c)*x-2*I/b*sin(2*a-2*c)*a-1/2*I/b/(-exp(2*I*(b*x+a+c))+exp(2
*I*a))*exp(2*I*(2*a-c))+I/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))*exp(2*I*a)-1/
2*I/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))*exp(2*I*c)+ln(exp(2*I*(b*x+a))-exp(
2*I*(a-c)))/b*sin(2*a-2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(122) = 244.

Time = 0.09 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.32

$$\int \cot^2(c+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{(\cos\left(-2\,a+2\,c\right)+1)\cos\left(bx+a\right)^3-(2\,bx\cos\left(-2\,a+2\,c\right)-bx)\cos\left(bx+a\right)\sin\left(-2\,a+2\,c\right)-(\cos(}{}$$

input
```
integrate(cot(b*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*((cos(-2*a + 2*c) + 1)*cos(b*x + a)^3 - (2*b*x*cos(-2*a + 2*c) - b*x)
*cos(b*x + a)*sin(-2*a + 2*c) - (cos(-2*a + 2*c)^2 + cos(-2*a + 2*c))*cos(
b*x + a) + ((cos(-2*a + 2*c) + 1)*sin(b*x + a)*sin(-2*a + 2*c) - (cos(-2*a
 + 2*c)^2 - 1)*cos(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*co
s(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1)/(cos(-2*a +
 2*c) + 1)) - (2*b*x*cos(-2*a + 2*c)^2 + b*x*cos(-2*a + 2*c) - b*x + (cos(
b*x + a)^2 - cos(-2*a + 2*c) + 1)*sin(-2*a + 2*c))*sin(b*x + a))/(b*cos(b*
x + a)*sin(-2*a + 2*c) + (b*cos(-2*a + 2*c) + b)*sin(b*x + a))
```

Sympy [F]

$$\int \cot^2(c+bx)\sin^2(a+bx)\,dx = \int \sin^2(a+bx)\cot^2(bx+c)\,dx$$

input
```
integrate(cot(b*x+c)**2*sin(b*x+a)**2,x)
```

output
```
Integral(sin(a + b*x)**2*cot(b*x + c)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \cot^2(c+bx)\sin^2(a+bx)\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(b*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1401 vs. $2(122) = 244$.

Time = 0.20 (sec) , antiderivative size = 1401, normalized size of antiderivative = 10.78

$$\int \cot^2(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*((tan(1/2*a)^4*tan(1/2*c)^4 - 14*tan(1/2*a)^4*tan(1/2*c)^2 + 32*tan(1/
2*a)^3*tan(1/2*c)^3 - 14*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 32*tan
(1/2*a)^3*tan(1/2*c) + 68*tan(1/2*a)^2*tan(1/2*c)^2 - 32*tan(1/2*a)*tan(1/
2*c)^3 + tan(1/2*c)^4 - 14*tan(1/2*a)^2 + 32*tan(1/2*a)*tan(1/2*c) - 14*ta
n(1/2*c)^2 + 1)*b*x/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)
^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c
)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 4*(tan(1/2*a)^
4*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*t
an(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/
2*c)^4 - tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c
)^2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))*log(tan(b*x)^2 + 1)/(tan(1/2
*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*
c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1
/2*a)^2 + 2*tan(1/2*c)^2 + 1) + 8*(tan(1/2*a)^4*tan(1/2*c)^5 - tan(1/2*a)^
3*tan(1/2*c)^6 - 2*tan(1/2*a)^4*tan(1/2*c)^3 + 7*tan(1/2*a)^3*tan(1/2*c)^4
 - 6*tan(1/2*a)^2*tan(1/2*c)^5 + tan(1/2*a)*tan(1/2*c)^6 + tan(1/2*a)^4*ta
n(1/2*c) - 7*tan(1/2*a)^3*tan(1/2*c)^2 + 12*tan(1/2*a)^2*tan(1/2*c)^3 - 7*
tan(1/2*a)*tan(1/2*c)^4 + tan(1/2*c)^5 + tan(1/2*a)^3 - 6*tan(1/2*a)^2*tan
(1/2*c) + 7*tan(1/2*a)*tan(1/2*c)^2 - 2*tan(1/2*c)^3 - tan(1/2*a) + tan(1/
2*c))*log(abs(tan(b*x)*tan(1/2*c)^2 - tan(b*x) - 2*tan(1/2*c)))/(tan(1/...
```

Mupad [B] (verification not implemented)

Time = 18.51 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.60

$$\int \cot^2(c+bx)\sin^2(a+bx)\,dx$$
$$= -x\left(\frac{1}{2}-\cos(2a-2c)+\sin(2a-2c)\,1\mathrm{i}\right)+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{8\,b}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{8\,b}$$
$$+\frac{\mathrm{e}^{-a\,4\mathrm{i}+c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\right)\left(2\,b\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-2\,b\,\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$
$$+\frac{\mathrm{e}^{-a\,2\mathrm{i}+c\,2\mathrm{i}}\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-2\,\mathrm{e}^{a\,4\mathrm{i}-c\,4\mathrm{i}}+\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)}{2\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}\right)}$$

input
```
int(cot(c + b*x)^2*sin(a + b*x)^2,x)
```

output
```
(exp(- a*2i - b*x*2i)*1i)/(8*b) - x*(sin(2*a - 2*c)*1i - cos(2*a - 2*c) +
1/2) - (exp(a*2i + b*x*2i)*1i)/(8*b) + (exp(c*4i - a*4i)*log(exp(a*2i)*exp
(b*x*2i) - exp(a*2i)*exp(-c*2i))*(2*b*exp(a*2i - c*2i) - 2*b*exp(a*6i - c*
6i))*1i)/(4*b^2) + (exp(c*2i - a*2i)*(exp(a*2i - c*2i) - 2*exp(a*4i - c*4i
) + exp(a*6i - c*6i)))/(2*b*(exp(a*2i - c*2i)*1i - exp(a*2i + b*x*2i)*1i))
```

Reduce [F]

$$\int \cot^2(c+bx)\sin^2(a+bx)\,dx=\int \cot(bx+c)^2\sin(bx+a)^2\,dx$$

input
```
int(cot(b*x+c)^2*sin(b*x+a)^2,x)
```

output
```
int(cot(b*x + c)**2*sin(a + b*x)**2,x)
```

3.242 $\int \cot^3(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \cot^3(c+bx)\sin^2(a+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 6.28 (sec) , antiderivative size = 422, normalized size of antiderivative = 422.00

$$\int \cot^3(c+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{i\arctan(\tan(c+bx))(-1+3\cos(2a-2c))}{2b}+\frac{\cos(2a)\cos(2bx)}{4b}$$
$$+\frac{(\cos(2a-2c-bx)-\cos(2a-2c+bx))\csc(c)\csc(c+bx)}{2b}$$
$$+\frac{(-1+3\cos(2a-2c))\log\left(\sin^2(c+bx)\right)}{4b}$$
$$-3x\cos(a-c)\sin(a-c)-\frac{\csc^2(c+bx)\sin^2(a-c)}{2b}$$
$$+x\left(-\frac{i}{2}-\frac{3}{4}i\cos^2(a)+\frac{9}{4}i\cos^2(a)\cos^2(c)+\frac{\cot(c)}{2}-\frac{3}{4}\cos^2(a)\cot(c)\right.$$
$$+\frac{1}{2}(-1+3\cos(2a-2c))\cot(c)-\frac{3}{4}\cos^2(a)\cos^2(c)\cot(c)-\frac{3}{2}\cos(a)\sin(a)$$
$$-\frac{9}{2}\cos(a)\cos^2(c)\sin(a)+\frac{3}{2}i\cos(a)\cot(c)\sin(a)-\frac{3}{2}i\cos(a)\cos^2(c)\cot(c)\sin(a)$$
$$+\frac{3}{4}i\sin^2(a)-\frac{9}{4}i\cos^2(c)\sin^2(a)+\frac{3}{4}\cot(c)\sin^2(a)+\frac{3}{4}\cos^2(c)\cot(c)\sin^2(a)$$
$$+\frac{9}{4}\cos^2(a)\cos(c)\sin(c)+\frac{9}{2}i\cos(a)\cos(c)\sin(a)\sin(c)-\frac{9}{4}\cos(c)\sin^2(a)\sin(c)$$
$$\left.-\frac{3}{4}i\cos^2(a)\sin^2(c)+\frac{3}{2}\cos(a)\sin(a)\sin^2(c)+\frac{3}{4}i\sin^2(a)\sin^2(c)\right)-\frac{\sin(2a)\sin(2bx)}{4b}$$

input
```
Integrate[Cot[c + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
((-1/2*I)*ArcTan[Tan[c + b*x]]*(-1 + 3*Cos[2*a - 2*c]))/b + (Cos[2*a]*Cos[
2*b*x])/(4*b) + ((Cos[2*a - 2*c - b*x] - Cos[2*a - 2*c + b*x])*Csc[c]*Csc[
c + b*x])/(2*b) + ((-1 + 3*Cos[2*a - 2*c])*Log[Sin[c + b*x]^2])/(4*b) - 3*
x*Cos[a - c]*Sin[a - c] - (Csc[c + b*x]^2*Sin[a - c]^2)/(2*b) + x*(-1/2*I
- ((3*I)/4)*Cos[a]^2 + ((9*I)/4)*Cos[a]^2*Cos[c]^2 + Cot[c]/2 - (3*Cos[a]^
2*Cot[c])/4 + ((-1 + 3*Cos[2*a - 2*c])*Cot[c])/2 - (3*Cos[a]^2*Cos[c]^2*Co
t[c])/4 - (3*Cos[a]*Sin[a])/2 - (9*Cos[a]*Cos[c]^2*Sin[a])/2 + ((3*I)/2)*C
os[a]*Cot[c]*Sin[a] - ((3*I)/2)*Cos[a]*Cos[c]^2*Cot[c]*Sin[a] + ((3*I)/4)*
Sin[a]^2 - ((9*I)/4)*Cos[c]^2*Sin[a]^2 + (3*Cot[c]*Sin[a]^2)/4 + (3*Cos[c]
^2*Cot[c]*Sin[a]^2)/4 + (9*Cos[a]^2*Cos[c]*Sin[c])/4 + ((9*I)/2)*Cos[a]*Co
s[c]*Sin[a]*Sin[c] - (9*Cos[c]*Sin[a]^2*Sin[c])/4 - ((3*I)/4)*Cos[a]^2*Sin
[c]^2 + (3*Cos[a]*Sin[a]*Sin[c]^2)/2 + ((3*I)/4)*Sin[a]^2*Sin[c]^2) - (Sin
[2*a]*Sin[2*b*x])/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cot^3(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin^2(a+bx)\cot^3(bx+c)dx$$

input
```
Int[Cot[c + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.77 (sec) , antiderivative size = 230, normalized size of antiderivative = 230.00

method	result
risch	$\frac{3ix\,e^{2i(a-c)}}{2}+\frac{ix}{2}+\frac{e^{2i(bx+a)}}{8b}+\frac{e^{-2i(bx+a)}}{8b}-3i\cos(2a-2c)\,x-\frac{3i\cos(2a-2c)a}{b}+\frac{ia}{b}+\frac{-3\,e^{2i(bx+3a)}+2\,e^{2i(bx}}{2}$

input
```
int(cot(b*x+c)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/2*I*x*exp(2*I*(a-c))+1/2*I*x+1/8/b*exp(2*I*(b*x+a))+1/8/b*exp(-2*I*(b*x+
a))-3*I*cos(2*a-2*c)*x-3*I/b*cos(2*a-2*c)*a+I/b*a+1/2/b/(-exp(2*I*(b*x+a+c
))+exp(2*I*a))^2*(-3*exp(2*I*(b*x+3*a))+2*exp(2*I*(b*x+2*a+c))+exp(2*I*(b*
x+a+2*c))+2*exp(2*I*(3*a-c))-2*exp(2*I*(a+c)))-1/2/b*ln(exp(2*I*(b*x+a))-e
xp(2*I*(a-c)))+3/2/b*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*cos(2*a-2*c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.10 (sec) , antiderivative size = 419, normalized size of antiderivative = 419.00

$$\int \cot^3(c+bx)\sin^2(a+bx)\,dx$$

$$= \frac{4\cos(bx+a)^4\cos(-2a+2c)+2\left(4\cos(-2a+2c)^2-2\cos(-2a+2c)-5\right)\cos(bx+a)^2-4\cos(}{}$$

input
```
integrate(cot(b*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(4*cos(b*x + a)^4*cos(-2*a + 2*c) + 2*(4*cos(-2*a + 2*c)^2 - 2*cos(-2*
a + 2*c) - 5)*cos(b*x + a)^2 - 4*cos(-2*a + 2*c)^2 - (2*(3*cos(-2*a + 2*c)
 - 1)*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*(3*cos(-2*a + 2*c)^2 -
 cos(-2*a + 2*c))*cos(b*x + a)^2 + 3*cos(-2*a + 2*c)^2 + 2*cos(-2*a + 2*c)
 - 1)*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)
*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1)/(cos(-2*a + 2*c) + 1)) + 2*(6*(b*x
*cos(-2*a + 2*c)^2 - b*x)*cos(b*x + a) - (2*cos(b*x + a)^3 + (4*cos(-2*a +
 2*c) - 1)*cos(b*x + a))*sin(-2*a + 2*c))*sin(b*x + a) + 6*(2*b*x*cos(b*x
+ a)^2*cos(-2*a + 2*c) - b*x*cos(-2*a + 2*c) - b*x)*sin(-2*a + 2*c) - cos(
-2*a + 2*c) + 7)/(2*b*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos(b*x + a)*si
n(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) - b)
```

Sympy [F]

$$\int \cot^3(c+bx)\sin^2(a+bx)\,dx = \int \sin^2{(a+bx)}\cot^3{(bx+c)}\,dx$$

input
```
integrate(cot(b*x+c)**3*sin(b*x+a)**2,x)
```

output
```
Integral(sin(a + b*x)**2*cot(b*x + c)**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.13 (sec) , antiderivative size = 2160, normalized size of antiderivative = 2160.00

$$\int \cot^3(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/8*((cos(6*b*x + 2*a + 6*c) - 2*cos(4*b*x + 2*a + 4*c) + cos(2*b*x + 2*a
+ 2*c))*cos(8*b*x + 4*a + 6*c) + 2*(2*cos(4*b*x + 2*a + 4*c) - cos(2*b*x +
 2*a + 2*c))*cos(6*b*x + 4*a + 4*c) - (24*b*x*sin(4*b*x + 6*c) - 12*b*x*si
n(2*b*x + 4*c) + 2*cos(6*b*x + 4*a + 4*c) + 11*cos(4*b*x + 4*a + 2*c) - 8*
cos(4*b*x + 2*a + 4*c) - 5*cos(4*b*x + 6*c) - 8*cos(2*b*x + 4*a) + 10*cos(
2*b*x + 4*c) - cos(2*c))*cos(6*b*x + 2*a + 6*c) - 12*(b*x*sin(6*b*x + 2*a
+ 6*c) - 2*b*x*sin(4*b*x + 2*a + 4*c) + b*x*sin(2*b*x + 2*a + 2*c))*cos(6*
b*x + 8*c) - 2*(12*b*x*sin(2*b*x + 4*c) - 11*cos(4*b*x + 4*a + 2*c) + 8*co
s(2*b*x + 4*a) - 4*cos(2*b*x + 2*a + 2*c) - 10*cos(2*b*x + 4*c) + cos(2*c)
)*cos(4*b*x + 2*a + 4*c) - 16*cos(4*b*x + 2*a + 4*c)^2 - (48*b*x*sin(4*b*x
 + 2*a + 4*c) - 24*b*x*sin(2*b*x + 2*a + 2*c) + 10*cos(4*b*x + 2*a + 4*c)
- 5*cos(2*b*x + 2*a + 2*c))*cos(4*b*x + 6*c) + (8*cos(2*b*x + 4*a) + cos(2
*c))*cos(2*b*x + 2*a + 2*c) - 11*cos(4*b*x + 4*a + 2*c)*cos(2*b*x + 2*a +
2*c) - 2*(6*b*x*sin(2*b*x + 2*a + 2*c) + 5*cos(2*b*x + 2*a + 2*c))*cos(2*b
*x + 4*c) + 2*((3*cos(-2*a + 2*c) - 1)*cos(6*b*x + 2*a + 6*c)^2 + 4*(3*cos
(-2*a + 2*c) - 1)*cos(4*b*x + 2*a + 4*c)^2 - 4*(3*cos(-2*a + 2*c) - 1)*cos
(4*b*x + 2*a + 4*c)*cos(2*b*x + 2*a + 2*c) + (3*cos(-2*a + 2*c) - 1)*cos(2
*b*x + 2*a + 2*c)^2 + (3*cos(-2*a + 2*c) - 1)*sin(6*b*x + 2*a + 6*c)^2 + 4
*(3*cos(-2*a + 2*c) - 1)*sin(4*b*x + 2*a + 4*c)^2 - 4*(3*cos(-2*a + 2*c) -
 1)*sin(4*b*x + 2*a + 4*c)*sin(2*b*x + 2*a + 2*c) + (3*cos(-2*a + 2*c) ...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.20 (sec) , antiderivative size = 2856, normalized size of antiderivative = 2856.00

$$\int \cot^3(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(12*(tan(1/2*a)^4*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*
a)^4*tan(1/2*c) + 6*tan(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^
3 + tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6
*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))*b*x/(ta
n(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan
(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*
tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) + (tan(1/2*a)^4*tan(1/2*c)^4 - 10*tan(1
/2*a)^4*tan(1/2*c)^2 + 24*tan(1/2*a)^3*tan(1/2*c)^3 - 10*tan(1/2*a)^2*tan(
1/2*c)^4 + tan(1/2*a)^4 - 24*tan(1/2*a)^3*tan(1/2*c) + 52*tan(1/2*a)^2*tan
(1/2*c)^2 - 24*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 10*tan(1/2*a)^2 +
24*tan(1/2*a)*tan(1/2*c) - 10*tan(1/2*c)^2 + 1)*log(tan(b*x)^2 + 1)/(tan(1
/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/
2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan
(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4*tan(1/2*c)^6 - 11*tan(1/
2*a)^4*tan(1/2*c)^4 + 24*tan(1/2*a)^3*tan(1/2*c)^5 - 10*tan(1/2*a)^2*tan(1
/2*c)^6 + 11*tan(1/2*a)^4*tan(1/2*c)^2 - 48*tan(1/2*a)^3*tan(1/2*c)^3 + 62
*tan(1/2*a)^2*tan(1/2*c)^4 - 24*tan(1/2*a)*tan(1/2*c)^5 + tan(1/2*c)^6 - t
an(1/2*a)^4 + 24*tan(1/2*a)^3*tan(1/2*c) - 62*tan(1/2*a)^2*tan(1/2*c)^2 +
48*tan(1/2*a)*tan(1/2*c)^3 - 11*tan(1/2*c)^4 + 10*tan(1/2*a)^2 - 24*tan(1/
2*a)*tan(1/2*c) + 11*tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*c)^2 - ...
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+bx)\sin^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(cot(c + b*x)^3*sin(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cot^3(c+bx)\sin^2(a+bx)\,dx = \int \cot(bx+c)^3 \sin(bx+a)^2\,dx$$

input
```
int(cot(b*x+c)^3*sin(b*x+a)^2,x)
```

output
```
int(cot(b*x + c)**3*sin(a + b*x)**2,x)
```

3.243 $\int \sin^4(a+bx)\tan(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 106

$$\int \sin^4(a+bx)\tan(c+bx)\,dx = \frac{\cos(2(a+bx))}{4b}-\frac{\cos(4(a+bx))}{32b}+\frac{\cos(4a-2c+2bx)}{8b}-\frac{\cos^4(a-c)\log(\cos(c+bx))}{b}+\frac{3}{4}x\cos(a-c)\sin(a-c)+\frac{1}{4}x\cos(a-c)\sin(3(a-c))$$

output
```
1/4*cos(2*b*x+2*a)/b-1/32*cos(4*b*x+4*a)/b+1/8*cos(2*b*x+4*a-2*c)/b-cos(a-
c)^4*ln(cos(b*x+c))/b+3/4*x*cos(a-c)*sin(a-c)+1/4*x*cos(a-c)*sin(3*a-3*c)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int \sin^4(a+bx)\tan(c+bx)\,dx = \frac{8\cos(2(a+bx))-\cos(4(a+bx))+4\cos(4a-2c+2bx)+8\cos(a-c)(-3\cos(a-c)\log(\cos(c+bx))}{32b}$$

input
```
Integrate[Sin[a + b*x]^4*Tan[c + b*x],x]
```

output
```
(8*Cos[2*(a + b*x)] - Cos[4*(a + b*x)] + 4*Cos[4*a - 2*c + 2*b*x] + 8*Cos[
a - c]*(-3*Cos[a - c]*Log[Cos[c + b*x]] - Cos[3*(a - c)]*Log[Cos[c + b*x]]
 + b*x*(3*Sin[a - c] + Sin[3*(a - c)])))/(32*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\tan(bx+c)\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \sin^4(a+bx)\tan(bx+c)dx$$

input
```
Int[Sin[a + b*x]^4*Tan[c + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.29

method	result
risch	$\frac{e^{-2i(bx+a)}}{8b}+\frac{e^{2i(bx+a)}}{8b}+\frac{i\cos(2a-2c)a}{b}-\frac{ix\,e^{2i(a-c)}}{2}+i\cos(2a-2c)\,x-\frac{ix\,e^{4i(a-c)}}{8}+\frac{3ix}{8}+\frac{i\cos(4a-4c)a}{4b}+$

input
```
int(sin(b*x+a)^4*tan(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*exp(-2*I*(b*x+a))+1/8/b*exp(2*I*(b*x+a))+I/b*cos(2*a-2*c)*a-1/2*I*x*
exp(2*I*(a-c))+I*cos(2*a-2*c)*x-1/8*I*x*exp(4*I*(a-c))+3/8*I*x+1/4*I/b*cos
(4*a-4*c)*a+3/4*I/b*a+1/4*I*cos(4*a-4*c)*x-3/8*ln(exp(2*I*(b*x+a))+exp(2*I
*(a-c)))/b-1/8/b*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*cos(4*a-4*c)-1/2*ln(e
xp(2*I*(b*x+a))+exp(2*I*(a-c)))/b*cos(2*a-2*c)-1/32*cos(4*b*x+4*a)/b+1/8*c
os(2*b*x+4*a-2*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.63

$$\int \sin^4(a+bx)\tan(c+bx)\,dx =$$

$$-\frac{2\cos(bx+a)^4 - 2(\cos(-2a+2c)+3)\cos(bx+a)^2 - 2\cos(bx+a)\sin(bx+a)\sin(-2a+2c) + \cdots}{\cdots}$$

input
```
integrate(sin(b*x+a)^4*tan(b*x+c),x, algorithm="fricas")
```

output
```
-1/8*(2*cos(b*x + a)^4 - 2*(cos(-2*a + 2*c) + 3)*cos(b*x + a)^2 - 2*cos(b*
x + a)*sin(b*x + a)*sin(-2*a + 2*c) + (cos(-2*a + 2*c)^2 + 2*cos(-2*a + 2*
c) + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a
)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1)/(cos(-2*a + 2*c) + 1)) + 2*(b*x*c
os(-2*a + 2*c) + 2*b*x)*sin(-2*a + 2*c))/b
```

Sympy [F]

$$\int \sin^4(a+bx)\tan(c+bx)\,dx = \int \sin^4(a+bx)\tan(bx+c)\,dx$$

input
```
integrate(sin(b*x+a)**4*tan(b*x+c),x)
```

output
```
Integral(sin(a + b*x)**4*tan(b*x + c), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.29

$$\int \sin^4(a+bx)\tan(c+bx)\,dx =$$
$$-\frac{4\,(4\,b\sin\left(-2\,a+2\,c\right)+b\sin\left(-4\,a+4\,c\right))x+2\,(4\,\cos\left(-2\,a+2\,c\right)+\cos\left(-4\,a+4\,c\right)+3)\log\left(\cos\left(2\,b\right.\right.}{}$$

input
```
integrate(sin(b*x+a)^4*tan(b*x+c),x, algorithm="maxima")
```

output
```
-1/32*(4*(4*b*sin(-2*a + 2*c) + b*sin(-4*a + 4*c))*x + 2*(4*cos(-2*a + 2*c
) + cos(-4*a + 4*c) + 3)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*
c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2) + cos(4*b*x + 4*
a) - 4*cos(2*b*x + 4*a - 2*c) - 8*cos(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454260 vs. $2(98) = 196$.

Time = 19.00 (sec) , antiderivative size = 454260, normalized size of antiderivative = 4285.47

$$\int \sin^4(a+bx)\tan(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^4*tan(b*x+c),x, algorithm="giac")
```

output
```
1/32*(12*b*x*tan(b*x)^4*tan(2*a)^2*tan(2*a + c)^2*tan(2*a - c)^2*tan(a + c
)^2*tan(a - c)^2*tan(a)^2*tan(c)^5 - 16*b*x*tan(b*x)^4*tan(2*a)^2*tan(2*a
+ c)^2*tan(2*a - c)^2*tan(a + c)^2*tan(a - c)^2*tan(a)*tan(c)^6 + 16*b*x*t
an(b*x)^4*tan(2*a)^2*tan(2*a + c)^2*tan(2*a - c)^2*tan(a + c)^2*tan(a - c)
*tan(a)^2*tan(c)^6 - 4*b*x*tan(b*x)^4*tan(2*a)^2*tan(2*a + c)^2*tan(2*a -
c)*tan(a + c)^2*tan(a - c)^2*tan(a)^2*tan(c)^6 + 4*b*x*tan(b*x)^4*tan(2*a)
*tan(2*a + c)^2*tan(2*a - c)^2*tan(a + c)^2*tan(a - c)^2*tan(a)^2*tan(c)^6
 - 6*log(4*(tan(b*x)^2*tan(c)^2 - 2*tan(b*x)*tan(c) + 1)/(tan(b*x)^2*tan(c
)^2 + tan(b*x)^2 + tan(c)^2 + 1))*tan(b*x)^4*tan(2*a)^2*tan(2*a + c)^2*tan
(2*a - c)^2*tan(a + c)^2*tan(a - c)^2*tan(a)^2*tan(c)^6 + 12*arctan((tan(b
*x) + tan(c))/(tan(b*x)*tan(c) - 1))*tan(b*x)^4*tan(2*a)^2*tan(2*a + c)^2*
tan(2*a - c)^2*tan(a + c)^2*tan(a - c)^2*tan(a)^2*tan(c)^5 + 8*arctan((tan
(b*x) + tan(c))/(tan(b*x)*tan(c) - 1))*tan(b*x)^4*tan(2*a)^2*tan(2*a + c)^
2*tan(2*a - c)^2*tan(a + c)^2*tan(a - c)*tan(a)^2*tan(c)^6 - 8*arctan((tan
(b*x) + tan(c))/(tan(b*x)*tan(c) - 1))*tan(b*x)^4*tan(2*a)^2*tan(2*a + c)^
2*tan(2*a - c)^2*tan(a + c)*tan(a - c)^2*tan(a)^2*tan(c)^6 - 2*arctan((tan
(b*x) + tan(c))/(tan(b*x)*tan(c) - 1))*tan(b*x)^4*tan(2*a)^2*tan(2*a + c)^
2*tan(2*a - c)*tan(a + c)^2*tan(a - c)^2*tan(a)^2*tan(c)^6 + 2*arctan((tan
(b*x) + tan(c))/(tan(b*x)*tan(c) - 1))*tan(b*x)^4*tan(2*a)^2*tan(2*a + c)*
tan(2*a - c)^2*tan(a + c)^2*tan(a - c)^2*tan(a)^2*tan(c)^6 + 40*b*x*tan...
```

Mupad [B] (verification not implemented)

Time = 40.60 (sec) , antiderivative size = 12617, normalized size of antiderivative = 119.03

$$\int \sin^4(a+bx)\tan(c+bx)\,dx = \text{Too large to display}$$

input
```
int(sin(a + b*x)^4*tan(c + b*x),x)
```

output
```
(log(tan((b*x)/2)^2 + 1)*(4*cos(2*a) + cos(4*a) + 6*tan(c)^2 + 3*tan(c)^4
+ 8*sin(2*a)*tan(c) + 4*sin(4*a)*tan(c) - 4*cos(2*a)*tan(c)^4 - 6*cos(4*a)
*tan(c)^2 + cos(4*a)*tan(c)^4 + 8*sin(2*a)*tan(c)^3 - 4*sin(4*a)*tan(c)^3
+ 3))/(2*b*(8*tan(c)^2 + 4*tan(c)^4 + 4)) - ((tan((b*x)/2)*(sin(2*a) + sin
(4*a)/4 - cos(4*a)*tan(c) + sin(2*a)*tan(c)^2 - (3*sin(4*a)*tan(c)^2)/4))/
(tan(c)^2 + 1) + (4*tan((b*x)/2)^4*(cos(2*a) + cos(4*a) + sin(4*a)*tan(c)
+ cos(2*a)*tan(c)^2))/(tan(c)^2 + 1) - (tan((b*x)/2)^7*(sin(2*a) + sin(4*a
)/4 - cos(4*a)*tan(c) + sin(2*a)*tan(c)^2 - (3*sin(4*a)*tan(c)^2)/4))/(tan
(c)^2 + 1) + (tan((b*x)/2)^3*(sin(2*a) + (9*sin(4*a))/4 - cos(4*a)*tan(c)
+ sin(2*a)*tan(c)^2 + (5*sin(4*a)*tan(c)^2)/4))/(tan(c)^2 + 1) - (tan((b*x
)/2)^5*(sin(2*a) + (9*sin(4*a))/4 - cos(4*a)*tan(c) + sin(2*a)*tan(c)^2 +
(5*sin(4*a)*tan(c)^2)/4))/(tan(c)^2 + 1) + (2*tan((b*x)/2)^2*(cos(2*a) + s
in(4*a)*tan(c) + cos(2*a)*tan(c)^2 - cos(4*a)*tan(c)^2))/(tan(c)^2 + 1) +
(2*tan((b*x)/2)^6*(cos(2*a) + sin(4*a)*tan(c) + cos(2*a)*tan(c)^2 - cos(4*
a)*tan(c)^2))/(tan(c)^2 + 1))/(b*(4*tan((b*x)/2)^2 + 6*tan((b*x)/2)^4 + 4*
tan((b*x)/2)^6 + tan((b*x)/2)^8 + 1)) - (log(tan((b*x)/2)^2*cos(c) - cos(c
) + 2*tan((b*x)/2)*sin(c))*(cos(2*a)/2 + cos(4*a)/8 + (3*tan(c)^2)/4 + (3*
tan(c)^4)/8 + sin(2*a)*tan(c) + (sin(4*a)*tan(c))/2 - (cos(2*a)*tan(c)^4)/
2 - (3*cos(4*a)*tan(c)^2)/4 + (cos(4*a)*tan(c)^4)/8 + sin(2*a)*tan(c)^3 -
(sin(4*a)*tan(c)^3)/2 + 3/8))/(b*(2*tan(c)^2 + tan(c)^4 + 1)) + (atan((...
```

Reduce [F]

$$\int \sin^4(a+bx)\tan(c+bx)\,dx = \int \sin(bx+a)^4 \tan(bx+c)\,dx$$

input
```
int(sin(b*x+a)^4*tan(b*x+c),x)
```

output
```
int(sin(a + b*x)**4*tan(b*x + c),x)
```

3.244 $\int \csc(a+bx)\tan^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 1

$$\int \csc(a+bx)\tan^2(c+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 80.00

$$\int \csc(a+bx)\tan^2(c+bx)\,dx$$
$$=\frac{\sec(a-c)\sec(c+bx)-2\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\sec(a-c)\tan(a-c)+\left(-\log\left(\cos\left(\frac{1}{2}(a+b\right.\right.\right.}{b}$$

input
```
Integrate[Csc[a + b*x]*Tan[c + b*x]^2,x]
```

output
```
(Sec[a - c]*Sec[c + b*x] - 2*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sec[a -
 c]*Tan[a - c] + (-Log[Cos[(a + b*x)/2]] + Log[Sin[(a + b*x)/2]])*Tan[a -
c]^2)/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\tan^2(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \csc(a+bx)\tan^2(bx+c)dx$$

input
```
Int[Csc[a + b*x]*Tan[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.21 (sec) , antiderivative size = 558, normalized size of antiderivative = 558.00

method	result
risch	$\frac{4\,\mathrm{e}^{i(bx+3a+2c)}}{\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)\left(\mathrm{e}^{2ia}+\mathrm{e}^{2ic}\right)b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-1\right)\mathrm{e}^{4ia}}{\left(\mathrm{e}^{4ia}+2\,\mathrm{e}^{2i(a+c)}+\mathrm{e}^{4ic}\right)b}+\frac{2\ln\left(\mathrm{e}^{i(bx+a)}-1\right)\mathrm{e}^{2i(a+c)}}{\left(\mathrm{e}^{4ia}+2\,\mathrm{e}^{2i(a+c)}+\mathrm{e}^{4ic}\right)b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-1\right)\mathrm{e}^{4ic}}{\left(\mathrm{e}^{4ia}+2\,\mathrm{e}^{2i(a+c)}+\mathrm{e}^{4ic}\right)b}+\frac{2i\ln\ldots}{\ldots}$

input
```
int(csc(b*x+a)*tan(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
4/(exp(2*I*(b*x+a+c))+exp(2*I*a))/(exp(2*I*a)+exp(2*I*c))/b*exp(I*(b*x+3*a
+2*c))-1/(exp(4*I*a)+2*exp(2*I*(a+c))+exp(4*I*c))/b*ln(exp(I*(b*x+a))-1)*e
xp(4*I*a)+2/(exp(4*I*a)+2*exp(2*I*(a+c))+exp(4*I*c))/b*ln(exp(I*(b*x+a))-1
)*exp(2*I*(a+c))-1/(exp(4*I*a)+2*exp(2*I*(a+c))+exp(4*I*c))/b*ln(exp(I*(b*
x+a))-1)*exp(4*I*c)+2*I*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b/(exp(4*I*a)+2*
exp(2*I*(a+c))+exp(4*I*c))*exp(I*(3*a+c))-2*I*ln(exp(I*(b*x+a))+I*exp(I*(a
-c)))/b/(exp(4*I*a)+2*exp(2*I*(a+c))+exp(4*I*c))*exp(I*(a+3*c))-2*I*ln(exp
(I*(b*x+a))-I*exp(I*(a-c)))/b/(exp(4*I*a)+2*exp(2*I*(a+c))+exp(4*I*c))*exp
(I*(3*a+c))+2*I*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b/(exp(4*I*a)+2*exp(2*I*
(a+c))+exp(4*I*c))*exp(I*(a+3*c))+1/(exp(4*I*a)+2*exp(2*I*(a+c))+exp(4*I*c
))/b*ln(exp(I*(b*x+a))+1)*exp(4*I*a)-2/(exp(4*I*a)+2*exp(2*I*(a+c))+exp(4*
I*c))/b*ln(exp(I*(b*x+a))+1)*exp(2*I*(a+c))+1/(exp(4*I*a)+2*exp(2*I*(a+c))
+exp(4*I*c))/b*ln(exp(I*(b*x+a))+1)*exp(4*I*c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.11 (sec) , antiderivative size = 413, normalized size of antiderivative = 413.00

$$\int \csc(a+bx)\tan^2(c+bx)\,dx = -\frac{\frac{\sqrt{2}\left(\left(\cos(-2a+2c)+1\right)\cos(bx+a)\sin(-2a+2c)+\left(\cos(-2a+2c)^2-1\right)\sin(bx+a)\right)\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(\cdots}{2\cos(bx+a)^2\cos(-2a+\cdots}\right.}{\sqrt{\cos(-2a+2c)+1}}}{\cdots}$$

input
```
integrate(csc(b*x+a)*tan(b*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(-2*a + 2*c) + (cos(-
2*a + 2*c)^2 - 1)*sin(b*x + a))*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*
cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) +
1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1)
- cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*
sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a + 2*c)
+ 1) - ((cos(-2*a + 2*c) - 1)*sin(b*x + a)*sin(-2*a + 2*c) - (cos(-2*a + 2
*c)^2 - 1)*cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + ((cos(-2*a + 2*c) -
 1)*sin(b*x + a)*sin(-2*a + 2*c) - (cos(-2*a + 2*c)^2 - 1)*cos(b*x + a))*l
og(-1/2*cos(b*x + a) + 1/2) + 4*cos(-2*a + 2*c) + 4)/((b*cos(-2*a + 2*c) +
 b)*sin(b*x + a)*sin(-2*a + 2*c) - (b*cos(-2*a + 2*c)^2 + 2*b*cos(-2*a + 2
*c) + b)*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\tan^2(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*tan(b*x+c)**2,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.51 (sec) , antiderivative size = 14864, normalized size of antiderivative = 14864.00

$$\int \csc(a+bx)\tan^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*tan(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(8*(cos(4*a)^2 + 4*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a +
2*c)^2 + 2*cos(4*a)*cos(4*c) + cos(4*c)^2 + sin(4*a)^2 + 4*(sin(4*a) + sin
(4*c))*sin(2*a + 2*c) + 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c
)^2)*cos(2*b*x + 2*a + 2*c)*cos(b*x + a + 2*c) + 8*(cos(4*a)^2 + 4*(cos(4*
a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)^2 + 2*cos(4*a)*cos(4*c) +
 cos(4*c)^2 + sin(4*a)^2 + 4*(sin(4*a) + sin(4*c))*sin(2*a + 2*c) + 4*sin(
2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)*cos(2*b*x + 4*c)*cos(b*x
+ a + 2*c) + 8*(cos(4*a)^2 + 4*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*co
s(2*a + 2*c)^2 + 2*cos(4*a)*cos(4*c) + cos(4*c)^2 + sin(4*a)^2 + 4*(sin(4*
a) + sin(4*c))*sin(2*a + 2*c) + 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) +
 sin(4*c)^2)*sin(2*b*x + 2*a + 2*c)*sin(b*x + a + 2*c) + 8*(cos(4*a)^2 + 4
*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)^2 + 2*cos(4*a)*co
s(4*c) + cos(4*c)^2 + sin(4*a)^2 + 4*(sin(4*a) + sin(4*c))*sin(2*a + 2*c)
+ 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)*sin(2*b*x + 4*c)*
sin(b*x + a + 2*c) - 4*(((cos(4*a) + cos(4*c))*cos(3*a + c) + 2*cos(3*a +
c)*cos(2*a + 2*c) - (cos(4*a) + 2*cos(2*a + 2*c) + cos(4*c))*cos(a + 3*c)
+ (sin(4*a) + sin(4*c))*sin(3*a + c) + 2*sin(3*a + c)*sin(2*a + 2*c) - (si
n(4*a) + 2*sin(2*a + 2*c) + sin(4*c))*sin(a + 3*c))*cos(2*b*x + 2*a + 2*c)
^2 + ((cos(4*a) + cos(4*c))*cos(3*a + c) + 2*cos(3*a + c)*cos(2*a + 2*c) -
 (cos(4*a) + 2*cos(2*a + 2*c) + cos(4*c))*cos(a + 3*c) + (sin(4*a) + si...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.69 (sec) , antiderivative size = 2953, normalized size of antiderivative = 2953.00

$$\int \csc(a+bx)\tan^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*tan(b*x+c)^2,x, algorithm="giac")
```

output
```
2*((tan(1/2*a)^5*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c)^5 - tan(1/2*a)^5*t
an(1/2*c)^3 + 2*tan(1/2*a)^4*tan(1/2*c)^4 - tan(1/2*a)^3*tan(1/2*c)^5 + ta
n(1/2*a)^5*tan(1/2*c)^2 + tan(1/2*a)^4*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2
*c)^4 - tan(1/2*a)^2*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c) + tan(1/2*a)^4
*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^4 - tan(1/2*a)*tan(1/2*c)^5 + 2*ta
n(1/2*a)^4*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*a)^4 + tan(1/2
*a)^3*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + tan(1/2*a)^3 +
 tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/
2*a)^2 + 2*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))
*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a
)*tan(1/2*a) - tan(1/2*b*x + 1/2*a)*tan(1/2*c) + tan(1/2*a)*tan(1/2*c) - t
an(1/2*b*x + 1/2*a) + tan(1/2*a) - tan(1/2*c) + 1))/(tan(1/2*a)^5*tan(1/2*
c)^5 - tan(1/2*a)^5*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c)^5 - 2*tan(1/2*a
)^5*tan(1/2*c)^3 + 9*tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^3*tan(1/2*c)
^5 + 2*tan(1/2*a)^5*tan(1/2*c)^2 - 10*tan(1/2*a)^4*tan(1/2*c)^3 + 10*tan(1
/2*a)^3*tan(1/2*c)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^5 + tan(1/2*a)^5*tan(1/2*
c) - 10*tan(1/2*a)^4*tan(1/2*c)^2 + 28*tan(1/2*a)^3*tan(1/2*c)^3 - 10*tan(
1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)*tan(1/2*c)^5 - tan(1/2*a)^5 + 9*tan(1/2
*a)^4*tan(1/2*c) - 28*tan(1/2*a)^3*tan(1/2*c)^2 + 28*tan(1/2*a)^2*tan(1/2*
c)^3 - 9*tan(1/2*a)*tan(1/2*c)^4 + tan(1/2*c)^5 + tan(1/2*a)^4 - 10*tan...
```

Mupad [B] (verification not implemented)

Time = 40.24 (sec) , antiderivative size = 14459, normalized size of antiderivative = 14459.00

$$\int \csc(a+bx)\tan^2(c+bx)\,dx = \text{Too large to display}$$

input
```
int(tan(c + b*x)^2/sin(a + b*x),x)
```

output
```
- ((2*(tan(c)^2 + 1))/(cos(a) + sin(a)*tan(c)) - (2*tan((b*x)/2)*tan(c)*(t
an(c)^2 + 1))/(cos(a) + sin(a)*tan(c)))/(b*(tan((b*x)/2)^2 + 2*tan((b*x)/2
)*tan(c) - 1)) - (atan((((sin(a) - cos(a)*tan(c))*(tan(c)^2 + 1)^(1/2)*((3
2*(cos(a)^2*sin(a)^4 - sin(a)^6*tan(c)^2 - 2*cos(a)^3*sin(a)^3*tan(c) + 6*
cos(a)*sin(a)^5*tan(c)^3 + cos(a)*sin(a)^5*tan(c)^5 - cos(a)^5*sin(a)*tan(
c)^5 + 4*cos(a)^2*sin(a)^4*tan(c)^2 + cos(a)^4*sin(a)^2*tan(c)^2 - 9*cos(a
)^3*sin(a)^3*tan(c)^3 - 9*cos(a)^2*sin(a)^4*tan(c)^4 + 6*cos(a)^4*sin(a)^2
*tan(c)^4 + 4*cos(a)^3*sin(a)^3*tan(c)^5 - 2*cos(a)^2*sin(a)^4*tan(c)^6 +
cos(a)^3*sin(a)^3*tan(c)^7))/(cos(a)^3 + sin(a)^3*tan(c)^3 + 3*cos(a)^2*si
n(a)*tan(c) + 3*cos(a)*sin(a)^2*tan(c)^2) + (32*tan((b*x)/2)*(2*cos(a)*sin
(a)^5 + 2*sin(a)^6*tan(c) + 2*cos(a)^3*sin(a)^3 + 4*sin(a)^6*tan(c)^3 + si
n(a)^6*tan(c)^5 - 4*cos(a)^2*sin(a)^4*tan(c) - 4*cos(a)^4*sin(a)^2*tan(c)
- 2*cos(a)*sin(a)^5*tan(c)^2 + 2*cos(a)^5*sin(a)*tan(c)^2 - 11*cos(a)*sin(
a)^5*tan(c)^4 + 5*cos(a)^5*sin(a)*tan(c)^4 - 2*cos(a)*sin(a)^5*tan(c)^6 +
4*cos(a)^5*sin(a)*tan(c)^6 + 7*cos(a)^3*sin(a)^3*tan(c)^2 - cos(a)^2*sin(a
)^4*tan(c)^3 - 10*cos(a)^4*sin(a)^2*tan(c)^3 + 4*cos(a)^3*sin(a)^3*tan(c)^
4 + 14*cos(a)^2*sin(a)^4*tan(c)^5 - 7*cos(a)^4*sin(a)^2*tan(c)^5 - 11*cos(
a)^3*sin(a)^3*tan(c)^6 + cos(a)^2*sin(a)^4*tan(c)^7 + 4*cos(a)^4*sin(a)^2*
tan(c)^7))/(cos(a)^3 + sin(a)^3*tan(c)^3 + 3*cos(a)^2*sin(a)*tan(c) + 3*co
s(a)*sin(a)^2*tan(c)^2) - ((sin(a) - cos(a)*tan(c))*(tan(c)^2 + 1)^(1/2...
```

Reduce [F]

$$\int \csc(a+bx)\tan^2(c+bx)\,dx = \int \csc(bx+a)\tan(bx+c)^2\,dx$$

input
```
int(csc(b*x+a)*tan(b*x+c)^2,x)
```

output
```
int(csc(a + b*x)*tan(b*x + c)**2,x)
```

3.245 $\int \csc(a+bx)\tan(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \csc(a+bx)\tan(c+bx)\,dx = \frac{\operatorname{arctanh}(\sin(c+bx))\sec(a-c)}{b} + \frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan(a-c)}{b} - \frac{\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan(a-c)}{b}$$

output
```
arctanh(sin(b*x+c))*sec(a-c)/b+ln(cos(1/2*a+1/2*b*x))*tan(a-c)/b-ln(sin(1/
2*a+1/2*b*x))*tan(a-c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.81

$$\int \csc(a+bx)\tan(c+bx)\,dx$$
$$= -\frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sec(a-c)}{b} + \frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan(a-c)}{b} - \frac{\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan(a-c)}{b}$$

input
```
Integrate[Csc[a + b*x]*Tan[c + b*x],x]
```

output
```
((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x
)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Sec[a - c])/b + (Log
[Cos[a/2 + (b*x)/2]]*Tan[a - c])/b - (Log[Sin[a/2 + (b*x)/2]]*Tan[a - c])/
b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\tan(bx+c)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \csc(a+bx)\tan(bx+c)dx$$

input
```
Int[Csc[a + b*x]*Tan[c + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.61

method	result
risch	$-\frac{2\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)e^{i(a+c)}}{(e^{2ia}+e^{2ic})b}-\frac{i\ln\left(e^{i(bx+a)}+1\right)e^{2ia}}{b(e^{2ia}+e^{2ic})}+\frac{i\ln\left(e^{i(bx+a)}+1\right)e^{2ic}}{b(e^{2ia}+e^{2ic})}+\frac{2\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)e^{i(a+c)}}{(e^{2ia}+e^{2ic})b}+\frac{i\ln(e}{b(}$

input
```
int(csc(b*x+a)*tan(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/(exp(2*I*a)+exp(2*I*c))/b*exp(I*(a+c)
)-I/b/(exp(2*I*a)+exp(2*I*c))*ln(exp(I*(b*x+a))+1)*exp(2*I*a)+I/b/(exp(2*I
*a)+exp(2*I*c))*ln(exp(I*(b*x+a))+1)*exp(2*I*c)+2*ln(exp(I*(b*x+a))+I*exp(
I*(a-c)))/(exp(2*I*a)+exp(2*I*c))/b*exp(I*(a+c))+I/b/(exp(2*I*a)+exp(2*I*c
))*ln(exp(I*(b*x+a))-1)*exp(2*I*a)-I/b/(exp(2*I*a)+exp(2*I*c))*ln(exp(I*(b
*x+a))-1)*exp(2*I*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(59) = 118.

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.46

$$\int \csc(a+bx)\tan(c+bx)\,dx = \frac{\sqrt{2}\sqrt{\cos(-2a+2c)+1}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\frac{2\sqrt{2}((\cos(-2a+2c)+1)\sin(bx+a)}{\sqrt{\cos(-2a+2}}}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos}\right.}{2(b}$$

input
```
integrate(csc(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*sqrt(cos(-2*a + 2*c) + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*
c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a +
2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c
) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*
x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1)) - log(1/2*cos(
b*x + a) + 1/2)*sin(-2*a + 2*c) + log(-1/2*cos(b*x + a) + 1/2)*sin(-2*a +
2*c))/(b*cos(-2*a + 2*c) + b)
```

Sympy [F]

$$\int \csc(a+bx)\tan(c+bx)\,dx = \int \tan(bx+c)\csc(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)*tan(b*x+c),x)
```

output
```
Integral(tan(b*x + c)*csc(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(59) = 118$.

Time = 0.18 (sec) , antiderivative size = 615, normalized size of antiderivative = 9.18

$$\int \csc(a+bx)\tan(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

output
```
(2*((sin(2*a) + sin(2*c))*cos(a + c) - (cos(2*a) + cos(2*c))*sin(a + c))*a
rctan2(2*(cos(b*x + 2*c)*cos(c) + sin(b*x + 2*c)*sin(c))/(cos(b*x + 2*c)^2
 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c
)*sin(c) + sin(c)^2), (cos(b*x + 2*c)^2 - cos(c)^2 + sin(b*x + 2*c)^2 - si
n(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x +
 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - (cos(2*a)^2 - cos(2*c)^2
+ sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) +
 (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(b*x) - si
n(a), cos(b*x) + cos(a)) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(cos
(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) +
sin(a)^2) - (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(cos(b*x)^2 - 2*cos
(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) - ((c
os(2*a) + cos(2*c))*cos(a + c) + (sin(2*a) + sin(2*c))*sin(a + c))*log((co
s(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2
*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)
*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)))
/(2*b*cos(2*a)*cos(2*c) + b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) + b*sin(2*c
)^2 + (cos(2*a)^2 + sin(2*a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(59) = 118$.

Time = 0.22 (sec) , antiderivative size = 404, normalized size of antiderivative = 6.03

$$\int \csc(a+bx)\tan(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

output
```
-((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(-t
an(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a
) - tan(1/2*b*x + 1/2*a)*tan(1/2*c) + tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x
+ 1/2*a) + tan(1/2*a) - tan(1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 - tan(
1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1) - (tan(1/2*a)^2*tan
(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(-tan(1/2*b*x + 1/2*a)
*tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*b*x + 1
/2*a)*tan(1/2*c) - tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*
a) - tan(1/2*c) - 1))/(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/
2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2
*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*a)))
/(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan
(1/2*c)^2 + 1))/b
```

Mupad [B] (verification not implemented)

Time = 59.38 (sec) , antiderivative size = 46071, normalized size of antiderivative = 687.63

$$\int \csc(a+bx)\tan(c+bx)\,dx = \text{Too large to display}$$

input
```
int(tan(c + b*x)/sin(a + b*x),x)
```

output
```
(atan((((8*cos(a + c)^4*cos(c)^2 + 8*cos(a + c)^4*sin(c)^2 - 16*cos(c)^6*s
in(a)^4 - 8*cos(a + c)*cos(c)^2*(-(2*cos(c)^2*sin(a)^2 - cos(a + c)^2 - 3*
sin(a + c)*cos(c)*sin(a) + cos(c)*sin(a)*sin(a - c))^3)^(1/2) + 4*cos(c)^4
*sin(a)^2*sin(a - c)^2 + 8*cos(a + c)*sin(c)^2*(-(2*cos(c)^2*sin(a)^2 - co
s(a + c)^2 - 3*sin(a + c)*cos(c)*sin(a) + cos(c)*sin(a)*sin(a - c))^3)^(1/
2) + 9*cos(a + c)^2*sin(a + c)^2*sin(c)^2 - 8*cos(a + c)^2*cos(c)^4*sin(a)
^2 + 36*sin(a + c)^2*cos(c)^4*sin(a)^2 + cos(a + c)^2*sin(c)^2*sin(a - c)^
2 + 8*cos(c)^4*sin(a)^4*sin(c)^2 - 6*cos(a + c)^2*sin(a + c)*sin(c)^2*sin(
a - c) - 24*cos(a + c)^3*cos(c)^2*sin(a)*sin(c) + 48*cos(a + c)*cos(c)^4*s
in(a)^3*sin(c) + 27*sin(a + c)^3*cos(c)*sin(a)*sin(c)^2 - 8*cos(c)^2*sin(a
)*sin(c)*(-(2*cos(c)^2*sin(a)^2 - cos(a + c)^2 - 3*sin(a + c)*cos(c)*sin(a
) + cos(c)*sin(a)*sin(a - c))^3)^(1/2) - 12*cos(a + c)^2*cos(c)^3*sin(a)*s
in(a - c) - 2*cos(c)^2*sin(a)^2*sin(c)^2*sin(a - c)^2 - 24*sin(a + c)*cos(
c)^4*sin(a)^2*sin(a - c) + 12*cos(a + c)^3*sin(a + c)*cos(c)*sin(c) - cos(
c)*sin(a)*sin(c)^2*sin(a - c)^3 - 12*sin(a + c)*cos(c)^3*sin(a)^3*sin(c)^2
 - 12*sin(a + c)*cos(c)*sin(c)*(-(2*cos(c)^2*sin(a)^2 - cos(a + c)^2 - 3*s
in(a + c)*cos(c)*sin(a) + cos(c)*sin(a)*sin(a - c))^3)^(1/2) - 20*cos(a +
c)^2*cos(c)^2*sin(a)^2*sin(c)^2 + 36*cos(a + c)^2*sin(a + c)*cos(c)^3*sin(
a) - 4*cos(a + c)^3*cos(c)*sin(c)*sin(a - c) + 4*cos(c)^3*sin(a)^3*sin(c)^
2*sin(a - c) - 18*sin(a + c)^2*cos(c)^2*sin(a)^2*sin(c)^2 + 4*cos(c)*si...
```

Reduce [F]

$$\int \csc(a + bx) \tan(c + bx) \, dx = \int \csc (bx + a) \tan (bx + c) \, dx$$

input
```
int(csc(b*x+a)*tan(b*x+c),x)
```

output
```
int(csc(a + b*x)*tan(b*x + c),x)
```

3.246 $\int \cot(c+bx)\csc(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \cot(c+bx)\csc(a+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 122.00

$$\begin{aligned}\int \cot(c+bx)\csc(a+bx)\,dx &= -\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\csc(a-c)}{b}\\&\quad+\frac{\cot(a-c)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\cot(a-c)\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\end{aligned}$$

input
```
Integrate[Cot[c + b*x]*Csc[a + b*x],x]
```

output
```
((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x
)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Csc[a - c])/b + (Cot
[a - c]*Log[Cos[a/2 + (b*x)/2]])/b - (Cot[a - c]*Log[Sin[a/2 + (b*x)/2]])/
b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\cot(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc(a+bx)\cot(bx+c)dx$$

input
```
Int[Cot[c + b*x]*Csc[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.13 (sec) , antiderivative size = 252, normalized size of antiderivative = 252.00

method	result
risch	$-\frac{2 i \ln\left(\mathrm{e}^{i(bx+a)}+\mathrm{e}^{i(a-c)}\right)\mathrm{e}^{i(a+c)}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}-\frac{i \ln\left(\mathrm{e}^{i(bx+a)}-1\right)\mathrm{e}^{2ia}}{\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)b}-\frac{i \ln\left(\mathrm{e}^{i(bx+a)}-1\right)\mathrm{e}^{2ic}}{\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)b}+\frac{2 i \ln\left(\mathrm{e}^{i(bx+a)}-\mathrm{e}^{i(a-c)}\right)\mathrm{e}^{i(a+c)}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}+\frac{i \ln(\mathrm{e}}{(\mathrm{e}}$

input
```
int(cot(b*x+c)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*I*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b/(exp(2*I*a)-exp(2*I*c))*exp(I*(a+c)
)-I/(exp(2*I*a)-exp(2*I*c))/b*ln(exp(I*(b*x+a))-1)*exp(2*I*a)-I/(exp(2*I*a
)-exp(2*I*c))/b*ln(exp(I*(b*x+a))-1)*exp(2*I*c)+2*I*ln(exp(I*(b*x+a))-exp(
I*(a-c)))/b/(exp(2*I*a)-exp(2*I*c))*exp(I*(a+c))+I/(exp(2*I*a)-exp(2*I*c))
/b*ln(exp(I*(b*x+a))+1)*exp(2*I*a)+I/(exp(2*I*a)-exp(2*I*c))/b*ln(exp(I*(b
*x+a))+1)*exp(2*I*c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 236.00

$$\int \cot(c+bx)\csc(a+bx)\,dx = \frac{\sqrt{2}\sqrt{\cos(-2a+2c)+1}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)+\frac{2\sqrt{2}((\cos(-2a+2c)+1)\cos(bx+a)}{\sqrt{\cos(-2a+2}}}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos}\right.}{}$$

input
```
integrate(cot(b*x+c)*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*sqrt(cos(-2*a + 2*c) + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*
c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a +
2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c
) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*
x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1)) - (cos(-2*a +
2*c) + 1)*log(1/2*cos(b*x + a) + 1/2) + (cos(-2*a + 2*c) + 1)*log(-1/2*cos
(b*x + a) + 1/2))/(b*sin(-2*a + 2*c))
```

Sympy [F]

$$\int \cot(c+bx)\csc(a+bx)\,dx = \int \cot(bx+c)\csc(a+bx)\,dx$$

input
```
integrate(cot(b*x+c)*csc(b*x+a),x)
```

output
```
Integral(cot(b*x + c)*csc(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.07 (sec) , antiderivative size = 527, normalized size of antiderivative = 527.00

$$\int \cot(c+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)*csc(b*x+a),x, algorithm="maxima")
```

output
```
-((cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(b*x) + s
in(a), cos(b*x) - cos(a)) - (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*
c)^2)*arctan2(sin(b*x) - sin(a), cos(b*x) + cos(a)) - 2*((cos(2*a) - cos(2
*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(b*x) + sin
(c), cos(b*x) - cos(c)) + 2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a)
- sin(2*c))*sin(a + c))*arctan2(sin(b*x) - sin(c), cos(b*x) + cos(c)) + (c
os(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) +
 cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (cos(2*c)*sin(2*a
) - cos(2*a)*sin(2*c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin
(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) - ((sin(2*a) - sin(2*c))*cos(a + c
) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) +
 cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + ((sin(2*a) - sin(
2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 - 2*co
s(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2))/(2*
b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*c)^2
- (cos(2*a)^2 + sin(2*a)^2)*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.15 (sec) , antiderivative size = 378, normalized size of antiderivative = 378.00

$$\int \cot(c+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)*csc(b*x+a),x, algorithm="giac")
```

output
```
1/2*((tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^3 + 4*tan(1/2*a)^2*tan(1/2*c)
 - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a))*log(abs(tan(1/2*b*x)*tan(1/2*a) -
 1))/(tan(1/2*a)^3*tan(1/2*c) - tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 -
 tan(1/2*a)*tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/
2*c) + tan(1/2*c)^3 + tan(1/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(t
an(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)
 - tan(1/2*c)^2) - (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a
)*tan(1/2*c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x) + tan(1/2*a)))/(tan(
1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c)) +
 (tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(tan
(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^
2 + tan(1/2*a) - tan(1/2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 57.48 (sec) , antiderivative size = 46101, normalized size of antiderivative = 46101.00

$$\int \cot(c+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
int(cot(c + b*x)/sin(a + b*x),x)
```

output
```
(atan(((-(16*sin(a)^4*sin(c)^6 - 8*sin(a + c)^4*sin(c)^2 - 8*sin(a + c)^4*
cos(c)^2 - 4*sin(a)^2*sin(c)^4*cos(a - c)^2 - 8*sin(a + c)*cos(c)^2*(-(2*s
in(a)^2*sin(c)^2 - sin(a + c)^2 + 3*cos(a + c)*sin(a)*sin(c) + sin(a)*sin(
c)*cos(a - c))^3)^(1/2) + 8*sin(a + c)*sin(c)^2*(-(2*sin(a)^2*sin(c)^2 - s
in(a + c)^2 + 3*cos(a + c)*sin(a)*sin(c) + sin(a)*sin(c)*cos(a - c))^3)^(1
/2) - 9*cos(a + c)^2*sin(a + c)^2*cos(c)^2 - 36*cos(a + c)^2*sin(a)^2*sin(
c)^4 + 8*sin(a + c)^2*sin(a)^2*sin(c)^4 - sin(a + c)^2*cos(c)^2*cos(a - c)
^2 - 8*cos(c)^2*sin(a)^4*sin(c)^4 + 4*cos(c)*sin(c)*cos(a - c)*(-(2*sin(a)
^2*sin(c)^2 - sin(a + c)^2 + 3*cos(a + c)*sin(a)*sin(c) + sin(a)*sin(c)*co
s(a - c))^3)^(1/2) + 27*cos(a + c)^3*cos(c)^2*sin(a)*sin(c) - 24*sin(a + c
)^3*cos(c)*sin(a)*sin(c)^2 + 48*sin(a + c)*cos(c)*sin(a)^3*sin(c)^4 + 2*co
s(c)^2*sin(a)^2*sin(c)^2*cos(a - c)^2 - 24*cos(a + c)*sin(a)^2*sin(c)^4*co
s(a - c) + 12*sin(a + c)^2*sin(a)*sin(c)^3*cos(a - c) - 8*cos(c)*sin(a)*si
n(c)^2*(-(2*sin(a)^2*sin(c)^2 - sin(a + c)^2 + 3*cos(a + c)*sin(a)*sin(c)
+ sin(a)*sin(c)*cos(a - c))^3)^(1/2) + cos(c)^2*sin(a)*sin(c)*cos(a - c)^3
 - 12*cos(a + c)*cos(c)^2*sin(a)^3*sin(c)^3 - 12*cos(a + c)*sin(a + c)^3*c
os(c)*sin(c) + 12*cos(a + c)*cos(c)*sin(c)*(-(2*sin(a)^2*sin(c)^2 - sin(a
+ c)^2 + 3*cos(a + c)*sin(a)*sin(c) + sin(a)*sin(c)*cos(a - c))^3)^(1/2) -
 4*cos(c)^2*sin(a)^3*sin(c)^3*cos(a - c) + 18*cos(a + c)^2*cos(c)^2*sin(a)
^2*sin(c)^2 - 4*sin(a + c)^3*cos(c)*sin(c)*cos(a - c) + 20*sin(a + c)^2...
```

Reduce [F]

$$\int \cot(c + bx)\csc(a + bx)\,dx = \int \cot(bx + c)\csc(bx + a)\,dx$$

input
```
int(cot(b*x+c)*csc(b*x+a),x)
```

output
```
int(cot(b*x + c)*csc(a + b*x),x)
```

3.247 $\int \cot^2(c+bx)\csc(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 1

$$\int \cot^2(c+bx)\csc(a+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.54 (sec) , antiderivative size = 82, normalized size of antiderivative = 82.00

$$\int \cot^2(c+bx)\csc(a+bx)\,dx$$
$$=\frac{2\text{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cot(a-c)\csc(a-c)-\csc(a-c)\csc(c+bx)+\cot^2(a-c)\left(-\log\left(c\right.\right.}{b}$$

input
```
Integrate[Cot[c + b*x]^2*Csc[a + b*x],x]
```

output
```
(2*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cot[a - c]*Csc[a - c] - Csc[a - c
]*Csc[c + b*x] + Cot[a - c]^2*(-Log[Cos[(a + b*x)/2]] + Log[Sin[(a + b*x)/
2]]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\cot^2(bx+c)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \csc(a+bx)\cot^2(bx+c)dx$$

input
```
Int[Cot[c + b*x]^2*Csc[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.16 (sec) , antiderivative size = 550, normalized size of antiderivative = 550.00

method	result
risch	$-\frac{4\,\mathrm{e}^{i(bx+3a+2c)}}{\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-1\right)\mathrm{e}^{4ia}}{\left(\mathrm{e}^{4ia}-2\,\mathrm{e}^{2i(a+c)}+\mathrm{e}^{4ic}\right)b}-\frac{2\ln\left(\mathrm{e}^{i(bx+a)}-1\right)\mathrm{e}^{2i(a+c)}}{b\left(\mathrm{e}^{4ia}-2\,\mathrm{e}^{2i(a+c)}+\mathrm{e}^{4ic}\right)}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-1\right)\mathrm{e}^{4ic}}{\left(\mathrm{e}^{4ia}-2\,\mathrm{e}^{2i(a+c)}+\mathrm{e}^{4ic}\right)b}+$

input
```
int(cot(b*x+c)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-4/(-exp(2*I*(b*x+a+c))+exp(2*I*a))/(exp(2*I*a)-exp(2*I*c))/b*exp(I*(b*x+3
*a+2*c))-1/(exp(4*I*a)-2*exp(2*I*(a+c))+exp(4*I*c))/b*ln(exp(I*(b*x+a))-1)
*exp(4*I*a)-2*ln(exp(I*(b*x+a))-1)/b/(exp(4*I*a)-2*exp(2*I*(a+c))+exp(4*I*
c))*exp(2*I*(a+c))-1/(exp(4*I*a)-2*exp(2*I*(a+c))+exp(4*I*c))/b*ln(exp(I*(
b*x+a))-1)*exp(4*I*c)+2*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b/(exp(4*I*a)-2*ex
p(2*I*(a+c))+exp(4*I*c))*exp(I*(3*a+c))+2*ln(exp(I*(b*x+a))-exp(I*(a-c)))/
b/(exp(4*I*a)-2*exp(2*I*(a+c))+exp(4*I*c))*exp(I*(a+3*c))+1/(exp(4*I*a)-2*
exp(2*I*(a+c))+exp(4*I*c))/b*ln(exp(I*(b*x+a))+1)*exp(4*I*a)+2*ln(exp(I*(b
*x+a))+1)/b/(exp(4*I*a)-2*exp(2*I*(a+c))+exp(4*I*c))*exp(2*I*(a+c))+1/(exp
(4*I*a)-2*exp(2*I*(a+c))+exp(4*I*c))/b*ln(exp(I*(b*x+a))+1)*exp(4*I*c)-2*l
n(exp(I*(b*x+a))+exp(I*(a-c)))/b/(exp(4*I*a)-2*exp(2*I*(a+c))+exp(4*I*c))*
exp(I*(3*a+c))-2*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b/(exp(4*I*a)-2*exp(2*I*(
a+c))+exp(4*I*c))*exp(I*(a+3*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.10 (sec) , antiderivative size = 433, normalized size of antiderivative = 433.00

$$\int \cot^2(c+bx)\csc(a+bx)\,dx =$$

$$-\frac{\frac{\sqrt{2}\left((\cos(-2\,a+2\,c)+1)\cos(bx+a)\sin(-2\,a+2\,c)+\left(\cos(-2\,a+2\,c)^2+2\,\cos(-2\,a+2\,c)+1\right)\sin(bx+a)\right)\log\left(\frac{2\,\cos(bx+a)^2\cos(-2\,a+2\,c)-2\,\cos(b}{2\,\cos}\right.}{\sqrt{\cos(-2\,a+2\,c)+1}}}{}$$

input
```
integrate(cot(b*x+c)^2*csc(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(-2*a + 2*c) + (cos(-
2*a + 2*c)^2 + 2*cos(-2*a + 2*c) + 1)*sin(b*x + a))*log((2*cos(b*x + a)^2*
cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*
((cos(-2*a + 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(c
os(-2*a + 2*c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*
c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/s
qrt(cos(-2*a + 2*c) + 1) - ((cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(-2*a +
2*c) + (cos(-2*a + 2*c)^2 + 2*cos(-2*a + 2*c) + 1)*sin(b*x + a))*log(1/2*c
os(b*x + a) + 1/2) + ((cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(-2*a + 2*c) +
 (cos(-2*a + 2*c)^2 + 2*cos(-2*a + 2*c) + 1)*sin(b*x + a))*log(-1/2*cos(b*
x + a) + 1/2) + 4*sin(-2*a + 2*c))/((b*cos(-2*a + 2*c) - b)*cos(b*x + a)*s
in(-2*a + 2*c) + (b*cos(-2*a + 2*c)^2 - b)*sin(b*x + a))
```

Sympy [F]

$$\int \cot^2(c+bx)\csc(a+bx)\,dx=\int \cot^2(bx+c)\csc(a+bx)\,dx$$

input
```
integrate(cot(b*x+c)**2*csc(b*x+a),x)
```

output
```
Integral(cot(b*x + c)**2*csc(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.53 (sec) , antiderivative size = 20524, normalized size of antiderivative = 20524.00

$$\int \cot^2(c+bx)\csc(a+bx)\,dx=\text{Too large to display}$$

input
```
integrate(cot(b*x+c)^2*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(8*(cos(4*a)^2 - 4*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a +
2*c)^2 + 2*cos(4*a)*cos(4*c) + cos(4*c)^2 + sin(4*a)^2 - 4*(sin(4*a) + sin
(4*c))*sin(2*a + 2*c) + 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c
)^2)*cos(2*b*x + 2*a + 2*c)*cos(b*x + a + 2*c) - 8*(cos(4*a)^2 - 4*(cos(4*
a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)^2 + 2*cos(4*a)*cos(4*c) +
 cos(4*c)^2 + sin(4*a)^2 - 4*(sin(4*a) + sin(4*c))*sin(2*a + 2*c) + 4*sin(
2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)*cos(2*b*x + 4*c)*cos(b*x
+ a + 2*c) + 8*(cos(4*a)^2 - 4*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*co
s(2*a + 2*c)^2 + 2*cos(4*a)*cos(4*c) + cos(4*c)^2 + sin(4*a)^2 - 4*(sin(4*
a) + sin(4*c))*sin(2*a + 2*c) + 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) +
 sin(4*c)^2)*sin(2*b*x + 2*a + 2*c)*sin(b*x + a + 2*c) - 8*(cos(4*a)^2 - 4
*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)^2 + 2*cos(4*a)*co
s(4*c) + cos(4*c)^2 + sin(4*a)^2 - 4*(sin(4*a) + sin(4*c))*sin(2*a + 2*c)
+ 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)*sin(2*b*x + 4*c)*
sin(b*x + a + 2*c) - 8*(((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a)
+ cos(4*c))*sin(2*a + 2*c))*cos(2*b*x + 2*a + 2*c)^2 + ((sin(4*a) + sin(4*
c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*cos(2*b*x + 4*c
)^2 + ((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*
a + 2*c))*sin(2*b*x + 2*a + 2*c)^2 + ((sin(4*a) + sin(4*c))*cos(2*a + 2*c)
 - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*sin(2*b*x + 4*c)^2 - 2*((cos(2...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.22 (sec) , antiderivative size = 1206, normalized size of antiderivative = 1206.00

$$\int \cot^2(c+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^2*csc(b*x+a),x, algorithm="giac")
```

output
```
-1/4*((tan(1/2*a)^5*tan(1/2*c)^4 - 2*tan(1/2*a)^5*tan(1/2*c)^2 + 8*tan(1/2
*a)^4*tan(1/2*c)^3 - 2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^5 - 8*tan(1/
2*a)^4*tan(1/2*c) + 20*tan(1/2*a)^3*tan(1/2*c)^2 - 8*tan(1/2*a)^2*tan(1/2*
c)^3 + tan(1/2*a)*tan(1/2*c)^4 - 2*tan(1/2*a)^3 + 8*tan(1/2*a)^2*tan(1/2*c
) - 2*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a))*log(abs(tan(1/2*b*x)*tan(1/2*a
) - 1))/(tan(1/2*a)^5*tan(1/2*c)^2 - 2*tan(1/2*a)^4*tan(1/2*c)^3 + tan(1/2
*a)^3*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c) - 4*tan(1/2*a)^3*tan(1/2*c)
^2 + 2*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3 - 2*tan(1/2*a)^2*tan(1/2*c
) + tan(1/2*a)*tan(1/2*c)^2) - (tan(1/2*a)^4*tan(1/2*c)^5 + 4*tan(1/2*a)^3
*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 4*tan(1/2*a)^3*tan(1/2*c)^2 + 4*
tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*c)^5 + 4*tan(1/2*a)*tan(1/2*c)^2 + tan(1
/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(tan(1/2*a)^4*tan(1/2*c)^3 -
2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^2*tan(1/2*c)^5 + 2*tan(1/2*a)^3*t
an(1/2*c)^2 - 4*tan(1/2*a)^2*tan(1/2*c)^3 + 2*tan(1/2*a)*tan(1/2*c)^4 + ta
n(1/2*a)^2*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3) - (tan(1
/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*a)^3*tan(1/
2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 8*tan(1/2*a)^3*tan(1
/2*c) + 20*tan(1/2*a)^2*tan(1/2*c)^2 - 8*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2
*c)^4 - 2*tan(1/2*a)^2 + 8*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*c)^2 + 1)*log
(abs(tan(1/2*b*x) + tan(1/2*a)))/(tan(1/2*a)^4*tan(1/2*c)^2 - 2*tan(1/2...
```

Mupad [B] (verification not implemented)

Time = 39.43 (sec) , antiderivative size = 14480, normalized size of antiderivative = 14480.00

$$\int \cot^2(c+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
int(cot(c + b*x)^2/sin(a + b*x),x)
```

output
```
((2*(cot(c)^2 + 1))/(cos(a) - cot(c)*sin(a)) + (2*tan((b*x)/2)*cot(c)*(cot
(c)^2 + 1))/(cos(a) - cot(c)*sin(a)))/(b*(2*tan((b*x)/2)*cot(c) - tan((b*x
)/2)^2 + 1)) + (atan((((sin(a) + cos(a)*cot(c))*(cot(c)^2 + 1)^(1/2)*((32*
(cos(a)^2*sin(a)^4 - cot(c)^2*sin(a)^6 + 2*cos(a)^3*cot(c)*sin(a)^3 - 6*co
s(a)*cot(c)^3*sin(a)^5 - cos(a)*cot(c)^5*sin(a)^5 + cos(a)^5*cot(c)^5*sin(
a) + 4*cos(a)^2*cot(c)^2*sin(a)^4 + cos(a)^4*cot(c)^2*sin(a)^2 + 9*cos(a)^
3*cot(c)^3*sin(a)^3 - 9*cos(a)^2*cot(c)^4*sin(a)^4 + 6*cos(a)^4*cot(c)^4*s
in(a)^2 - 4*cos(a)^3*cot(c)^5*sin(a)^3 - 2*cos(a)^2*cot(c)^6*sin(a)^4 - co
s(a)^3*cot(c)^7*sin(a)^3))/(cos(a)^3 - cot(c)^3*sin(a)^3 - 3*cos(a)^2*cot(
c)*sin(a) + 3*cos(a)*cot(c)^2*sin(a)^2) + (32*tan((b*x)/2)*(2*cos(a)*sin(a
)^5 - 2*cot(c)*sin(a)^6 + 2*cos(a)^3*sin(a)^3 - 4*cot(c)^3*sin(a)^6 - cot(
c)^5*sin(a)^6 + 4*cos(a)^2*cot(c)*sin(a)^4 + 4*cos(a)^4*cot(c)*sin(a)^2 -
2*cos(a)*cot(c)^2*sin(a)^5 + 2*cos(a)^5*cot(c)^2*sin(a) - 11*cos(a)*cot(c)
^4*sin(a)^5 + 5*cos(a)^5*cot(c)^4*sin(a) - 2*cos(a)*cot(c)^6*sin(a)^5 + 4*
cos(a)^5*cot(c)^6*sin(a) + 7*cos(a)^3*cot(c)^2*sin(a)^3 + cos(a)^2*cot(c)^
3*sin(a)^4 + 10*cos(a)^4*cot(c)^3*sin(a)^2 + 4*cos(a)^3*cot(c)^4*sin(a)^3
- 14*cos(a)^2*cot(c)^5*sin(a)^4 + 7*cos(a)^4*cot(c)^5*sin(a)^2 - 11*cos(a)
^3*cot(c)^6*sin(a)^3 - cos(a)^2*cot(c)^7*sin(a)^4 - 4*cos(a)^4*cot(c)^7*si
n(a)^2))/(cos(a)^3 - cot(c)^3*sin(a)^3 - 3*cos(a)^2*cot(c)*sin(a) + 3*cos(
a)*cot(c)^2*sin(a)^2) - ((sin(a) + cos(a)*cot(c))*(cot(c)^2 + 1)^(1/2)*...
```

Reduce [F]

$$\int \cot^2(c+bx)\csc(a+bx)\,dx = \int \cot(bx+c)^2\csc(bx+a)\,dx$$

input
```
int(cot(b*x+c)^2*csc(b*x+a),x)
```

output
```
int(cot(b*x + c)**2*csc(a + b*x),x)
```

3.248 $\int \csc^2(a+bx)\tan^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \csc^2(a+bx)\tan^2(c+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 1.42 (sec) , antiderivative size = 74, normalized size of antiderivative = 74.00

$$\int \csc^2(a+bx)\tan^2(c+bx)\,dx = \frac{\csc(a)\csc(a+bx)\sin(bx)\tan^2(a-c)+\sec^2(a-c)(\sec(c)\sec(c+bx)\sin(bx)+2(\log(\cos(c+bx))-\log}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Tan[c + b*x]^2,x]
```

output
```
(Csc[a]*Csc[a + b*x]*Sin[b*x]*Tan[a - c]^2 + Sec[a - c]^2*(Sec[c]*Sec[c +
b*x]*Sin[b*x] + 2*(Log[Cos[c + b*x]] - Log[Sin[a + b*x]])*Tan[a - c]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\tan^2(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc^2(a+bx)\tan^2(bx+c)dx$$

input
```
Int[Csc[a + b*x]^2*Tan[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.26 (sec) , antiderivative size = 384, normalized size of antiderivative = 384.00

method	result
risch	$\frac{2i\left(5\,e^{2i(bx+3a+c)}-2\,e^{2i(bx+2a+2c)}+e^{2i(bx+a+3c)}+e^{6ia}-6\,e^{2i(2a+c)}+e^{2i(a+2c)}\right)}{b\left(e^{2ia}+e^{2ic}\right)^2\left(e^{2i(bx+a)}-1\right)\left(e^{2i(bx+a+c)}+e^{2ia}\right)}-\frac{8i\ln\left(e^{2i(bx+a)}+e^{2i(a-c)}\right)e^{2i(2a+c)}}{\left(e^{6ia}+3\,e^{2i(2a+c)}+3\,e^{2i(a+2c)}+e^{6ic}\right)b}+\frac{8}{(e^{6}}$

input
```
int(csc(b*x+a)^2*tan(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2*I*(5*exp(2*I*(b*x+3*a+c))-2*exp(2*I*(b*x+2*a+2*c))+exp(2*I*(b*x+a+3*c))+
exp(6*I*a)-6*exp(2*I*(2*a+c))+exp(2*I*(a+2*c)))/b/(exp(2*I*a)+exp(2*I*c))^
2/(exp(2*I*(b*x+a))-1)/(exp(2*I*(b*x+a+c))+exp(2*I*a))-8*I*ln(exp(2*I*(b*x
+a))+exp(2*I*(a-c)))/(exp(6*I*a)+3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))+exp
(6*I*c))/b*exp(2*I*(2*a+c))+8*I*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))/(exp(6
*I*a)+3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))+exp(6*I*c))/b*exp(2*I*(a+2*c))
+8*I*ln(exp(2*I*(b*x+a))-1)/(exp(6*I*a)+3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*
c))+exp(6*I*c))/b*exp(2*I*(2*a+c))-8*I*ln(exp(2*I*(b*x+a))-1)/(exp(6*I*a)+
3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))+exp(6*I*c))/b*exp(2*I*(a+2*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.10 (sec) , antiderivative size = 352, normalized size of antiderivative = 352.00

$$\int \csc^2(a+bx)\tan^2(c+bx)\,dx =$$
$$-\frac{(\cos(-2a+2c)-3)\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\left(\cos(-2a+2c)^2-2\cos(-2a+2c\right.}{}$$

input
```
integrate(csc(b*x+a)^2*tan(b*x+c)^2,x, algorithm="fricas")
```

output
```
-((cos(-2*a + 2*c) - 3)*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - (cos(-
2*a + 2*c)^2 - 2*cos(-2*a + 2*c) - 3)*cos(b*x + a)^2 + 2*((cos(-2*a + 2*c)
 - 1)*cos(b*x + a)^2 - cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*
a + 2*c) + 1)*log(-1/4*cos(b*x + a)^2 + 1/4) - 2*((cos(-2*a + 2*c) - 1)*co
s(b*x + a)^2 - cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c)
 + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*
sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1)/(cos(-2*a + 2*c) + 1)) - 2*cos(-2*a
 + 2*c) - 2)/((b*cos(-2*a + 2*c)^2 + 2*b*cos(-2*a + 2*c) + b)*cos(b*x + a)
*sin(b*x + a) + ((b*cos(-2*a + 2*c) + b)*cos(b*x + a)^2 - b*cos(-2*a + 2*c
) - b)*sin(-2*a + 2*c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\tan^2(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*tan(b*x+c)**2,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 2.73 (sec) , antiderivative size = 120050, normalized size of antiderivative = 120050.00

$$\int \csc^2(a+bx)\tan^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*tan(b*x+c)^2,x, algorithm="maxima")
```

output
```
2*(72*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*
a + 2*c))*cos(4*a + 2*c)^2 + 72*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (c
os(4*a) + cos(4*c))*sin(2*a + 2*c))*cos(2*a + 4*c)^2 + 72*((sin(4*a) + sin
(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*sin(4*a + 2*
c)^2 + 72*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*si
n(2*a + 2*c))*sin(2*a + 4*c)^2 + 4*(((cos(6*a) + cos(6*c))*cos(4*a + 2*c)
+ 3*cos(4*a + 2*c)^2 - (cos(6*a) + cos(6*c))*cos(2*a + 4*c) - 3*cos(2*a +
4*c)^2 + (sin(6*a) + sin(6*c))*sin(4*a + 2*c) + 3*sin(4*a + 2*c)^2 - (sin(
6*a) + sin(6*c))*sin(2*a + 4*c) - 3*sin(2*a + 4*c)^2)*cos(4*b*x + 6*a + 2*
c)^2 + 4*((cos(6*a) + cos(6*c))*cos(4*a + 2*c) + 3*cos(4*a + 2*c)^2 - (cos
(6*a) + cos(6*c))*cos(2*a + 4*c) - 3*cos(2*a + 4*c)^2 + (sin(6*a) + sin(6*
c))*sin(4*a + 2*c) + 3*sin(4*a + 2*c)^2 - (sin(6*a) + sin(6*c))*sin(2*a +
4*c) - 3*sin(2*a + 4*c)^2)*cos(4*b*x + 4*a + 4*c)^2 + ((cos(6*a) + cos(6*c
))*cos(4*a + 2*c) + 3*cos(4*a + 2*c)^2 - (cos(6*a) + cos(6*c))*cos(2*a + 4
*c) - 3*cos(2*a + 4*c)^2 + (sin(6*a) + sin(6*c))*sin(4*a + 2*c) + 3*sin(4*
a + 2*c)^2 - (sin(6*a) + sin(6*c))*sin(2*a + 4*c) - 3*sin(2*a + 4*c)^2)*co
s(4*b*x + 2*a + 6*c)^2 + ((cos(6*a) + cos(6*c))*cos(4*a + 2*c) + 3*cos(4*a
 + 2*c)^2 - (cos(6*a) + cos(6*c))*cos(2*a + 4*c) - 3*cos(2*a + 4*c)^2 + (s
in(6*a) + sin(6*c))*sin(4*a + 2*c) + 3*sin(4*a + 2*c)^2 - (sin(6*a) + sin(
6*c))*sin(2*a + 4*c) - 3*sin(2*a + 4*c)^2)*cos(2*b*x + 6*a)^2 + ((cos(6...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.48 (sec) , antiderivative size = 3075, normalized size of antiderivative = 3075.00

$$\int \csc^2(a+bx)\tan^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*tan(b*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(8*(tan(1/2*a)^8*tan(1/2*c)^6 - 2*tan(1/2*a)^7*tan(1/2*c)^7 + tan(1/2*
a)^6*tan(1/2*c)^8 + 2*tan(1/2*a)^8*tan(1/2*c)^4 - 2*tan(1/2*a)^7*tan(1/2*c
)^5 - 2*tan(1/2*a)^5*tan(1/2*c)^7 + 2*tan(1/2*a)^4*tan(1/2*c)^8 + tan(1/2*
a)^8*tan(1/2*c)^2 + 2*tan(1/2*a)^7*tan(1/2*c)^3 - 2*tan(1/2*a)^6*tan(1/2*c
)^4 - 2*tan(1/2*a)^5*tan(1/2*c)^5 - 2*tan(1/2*a)^4*tan(1/2*c)^6 + 2*tan(1/
2*a)^3*tan(1/2*c)^7 + tan(1/2*a)^2*tan(1/2*c)^8 + 2*tan(1/2*a)^7*tan(1/2*c
) + 2*tan(1/2*a)^5*tan(1/2*c)^3 - 8*tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*
a)^3*tan(1/2*c)^5 + 2*tan(1/2*a)*tan(1/2*c)^7 + tan(1/2*a)^6 + 2*tan(1/2*a
)^5*tan(1/2*c) - 2*tan(1/2*a)^4*tan(1/2*c)^2 - 2*tan(1/2*a)^3*tan(1/2*c)^3
 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)*tan(1/2*c)^5 + tan(1/2*c)^6
+ 2*tan(1/2*a)^4 - 2*tan(1/2*a)^3*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c)^3 +
 2*tan(1/2*c)^4 + tan(1/2*a)^2 - 2*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2)*l
og(abs(-2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) + 2*tan(b*x + a)*tan(1/2*a)
*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 - 2*tan(b*x + a)*tan(1/2*a) + ta
n(1/2*a)^2 + 2*tan(b*x + a)*tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c) + tan(1/2
*c)^2 - 1))/(tan(1/2*a)^8*tan(1/2*c)^7 - tan(1/2*a)^7*tan(1/2*c)^8 - 3*tan
(1/2*a)^8*tan(1/2*c)^5 + 16*tan(1/2*a)^7*tan(1/2*c)^6 - 16*tan(1/2*a)^6*ta
n(1/2*c)^7 + 3*tan(1/2*a)^5*tan(1/2*c)^8 + 3*tan(1/2*a)^8*tan(1/2*c)^3 - 3
0*tan(1/2*a)^7*tan(1/2*c)^4 + 96*tan(1/2*a)^6*tan(1/2*c)^5 - 96*tan(1/2*a)
^5*tan(1/2*c)^6 + 30*tan(1/2*a)^4*tan(1/2*c)^7 - 3*tan(1/2*a)^3*tan(1/2...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\tan^2(c+bx)\,dx = \text{Hanged}$$

input
```
int(tan(c + b*x)^2/sin(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^2(a+bx)\tan^2(c+bx)\,dx = \int \csc\left(bx+a\right)^2\tan\left(bx+c\right)^2dx$$

input
```
int(csc(b*x+a)^2*tan(b*x+c)^2,x)
```

output
```
int(csc(a + b*x)**2*tan(b*x + c)**2,x)
```

3.249 $\int \csc^2(a+bx)\tan(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \csc^2(a+bx)\tan(c+bx)\,dx=-\frac{\log(-\cos(c+bx))\sec^2(a-c)}{b}+\frac{\log(\sin(a+bx))\sec^2(a-c)}{b}+\frac{\cot(a+bx)\tan(a-c)}{b}$$

output
```
-ln(-cos(b*x+c))*sec(a-c)^2/b+ln(sin(b*x+a))*sec(a-c)^2/b+cot(b*x+a)*tan(a
-c)/b
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int \csc^2(a+bx)\tan(c+bx)\,dx=\frac{\csc(a)\csc(a+bx)\sec^2(a-c)(-\cos(2a-2c-bx)+\cos(2a-2c+bx)+4(-\log(\cos(c+bx))+\log(\sin}{4b}$$

input
```
Integrate[Csc[a + b*x]^2*Tan[c + b*x],x]
```

output
```
(Csc[a]*Csc[a + b*x]*Sec[a - c]^2*(-Cos[2*a - 2*c - b*x] + Cos[2*a - 2*c +
 b*x] + 4*(-Log[Cos[c + b*x]] + Log[Sin[a + b*x]])*Sin[a]*Sin[a + b*x]))/(
4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\tan(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc^2(a+bx)\tan(bx+c)dx$$

input
```
Int[Csc[a + b*x]^2*Tan[c + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.00

method	result
risch	$\frac{2\,\mathrm{e}^{2ia}}{b(\mathrm{e}^{2ia}+\mathrm{e}^{2ic})(\mathrm{e}^{2i(bx+a)}-1)}-\frac{2\,\mathrm{e}^{2ic}}{b(\mathrm{e}^{2ia}+\mathrm{e}^{2ic})(\mathrm{e}^{2i(bx+a)}-1)}+\frac{4\ln(\mathrm{e}^{2i(bx+a)}-1)\mathrm{e}^{2i(a+c)}}{(\mathrm{e}^{4ia}+2\,\mathrm{e}^{2i(a+c)}+\mathrm{e}^{4ic})b}-\frac{4\ln(\mathrm{e}^{2i(bx+a)}+\mathrm{e}^{2i(a-c)})\mathrm{e}^{2i(a+c)}}{(\mathrm{e}^{4ia}+2\,\mathrm{e}^{2i(a+c)}+\mathrm{e}^{4ic})b}$

input
```
int(csc(b*x+a)^2*tan(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
2/b/(exp(2*I*a)+exp(2*I*c))/(exp(2*I*(b*x+a))-1)*exp(2*I*a)-2/b/(exp(2*I*a
)+exp(2*I*c))/(exp(2*I*(b*x+a))-1)*exp(2*I*c)+4*ln(exp(2*I*(b*x+a))-1)/(ex
p(4*I*a)+2*exp(2*I*(a+c))+exp(4*I*c))/b*exp(2*I*(a+c))-4*ln(exp(2*I*(b*x+a
))+exp(2*I*(a-c)))/(exp(4*I*a)+2*exp(2*I*(a+c))+exp(4*I*c))/b*exp(2*I*(a+c
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(58) = 116$.

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.31

$$\int \csc^2(a+bx)\tan(c+bx)\,dx = \frac{\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)\sin(bx+a) - \log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c) - 2\cos(bx+a)\sin(bx+a)\sin(-2a+2c) - \cos(-2a}{\cos(-2a+2c)+1}\right.}{(b\cos(-2a+2c)+b)\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*tan(b*x+c),x, algorithm="fricas")
```

output
```
(log(-1/4*cos(b*x + a)^2 + 1/4)*sin(b*x + a) - log((2*cos(b*x + a)^2*cos(-
2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c)
 + 1)/(cos(-2*a + 2*c) + 1))*sin(b*x + a) - cos(b*x + a)*sin(-2*a + 2*c))/
((b*cos(-2*a + 2*c) + b)*sin(b*x + a))
```

Sympy [F]

$$\int \csc^2(a+bx)\tan(c+bx)\,dx = \int \tan(bx+c)\csc^2(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*tan(b*x+c),x)
```

output
```
Integral(tan(b*x + c)*csc(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11681 vs. $2(58) = 116$.

Time = 0.21 (sec) , antiderivative size = 11681, normalized size of antiderivative = 201.40

$$\int \csc^2(a + bx) \tan(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*tan(b*x+c),x, algorithm="maxima")
```

output
```
-2*((cos(2*a)^2 + sin(2*a)^2)*cos(4*a)^2 + 4*(cos(2*a)^2 - cos(2*c)^2 + si
n(2*a)^2 - sin(2*c)^2)*cos(2*a + 2*c)^2 + (cos(2*a)^2 - cos(2*c)^2 + sin(2
*a)^2 - sin(2*c)^2)*cos(4*c)^2 - (cos(4*a)^2 + sin(4*a)^2)*cos(2*c)^2 + (c
os(2*a)^2 + sin(2*a)^2)*sin(4*a)^2 + 4*(cos(2*a)^2 - cos(2*c)^2 + sin(2*a)
^2 - sin(2*c)^2)*sin(2*a + 2*c)^2 + (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^2
- sin(2*c)^2)*sin(4*c)^2 - (cos(4*a)^2 + sin(4*a)^2)*sin(2*c)^2 + 2*(((sin
(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*c
os(2*b*x + 4*a)^2 + ((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + co
s(4*c))*sin(2*a + 2*c))*cos(2*b*x + 2*a + 2*c)^2 + ((sin(4*a) + sin(4*c))*
cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*sin(2*b*x + 4*a)^2
+ ((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a +
2*c))*sin(2*b*x + 2*a + 2*c)^2 - 2*((cos(2*a)*sin(4*a) + cos(2*c)*sin(4*a)
 + (cos(2*a) + cos(2*c))*sin(4*c))*cos(2*a + 2*c) - (cos(4*a)*cos(2*a) + (
cos(2*a) + cos(2*c))*cos(4*c) + cos(4*a)*cos(2*c))*sin(2*a + 2*c))*cos(2*b
*x + 4*a) + 2*(((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c
))*sin(2*a + 2*c))*cos(2*b*x + 4*a) - (cos(2*a)*sin(4*a) + cos(2*c)*sin(4*
a) + (cos(2*a) + cos(2*c))*sin(4*c))*cos(2*a + 2*c) + (cos(4*a)*cos(2*a) +
 (cos(2*a) + cos(2*c))*cos(4*c) + cos(4*a)*cos(2*c))*sin(2*a + 2*c))*cos(2
*b*x + 2*a + 2*c) + (2*cos(2*a)*cos(2*c)*sin(4*a) + cos(2*c)^2*sin(4*a) +
2*sin(4*a)*sin(2*a)*sin(2*c) + sin(4*a)*sin(2*c)^2 + (cos(2*a)^2 + sin(...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1312 vs. $2(58) = 116$.

Time = 0.20 (sec) , antiderivative size = 1312, normalized size of antiderivative = 22.62

$$\int \csc^2(a+bx)\tan(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*tan(b*x+c),x, algorithm="giac")
```

output
```
-((tan(1/2*a)^6*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^6 + 2*tan(1/2*a)^6*
tan(1/2*c)^3 - tan(1/2*a)^5*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c)^5 - 2*t
an(1/2*a)^3*tan(1/2*c)^6 + tan(1/2*a)^6*tan(1/2*c) + tan(1/2*a)^5*tan(1/2*
c)^2 + 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2
*a)^2*tan(1/2*c)^5 - tan(1/2*a)*tan(1/2*c)^6 + tan(1/2*a)^5 + tan(1/2*a)^4
*tan(1/2*c) + 2*tan(1/2*a)^3*tan(1/2*c)^2 - 2*tan(1/2*a)^2*tan(1/2*c)^3 -
tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*c)^5 + 2*tan(1/2*a)^3 - tan(1/2*a)^2*tan
(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - 2*tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*
c))*log(abs(2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/
2*a)*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a)
- tan(1/2*a)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan
(1/2*c)^2 + 1))/(tan(1/2*a)^6*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^6 - 2
*tan(1/2*a)^6*tan(1/2*c)^3 + 11*tan(1/2*a)^5*tan(1/2*c)^4 - 11*tan(1/2*a)^
4*tan(1/2*c)^5 + 2*tan(1/2*a)^3*tan(1/2*c)^6 + tan(1/2*a)^6*tan(1/2*c) - 1
1*tan(1/2*a)^5*tan(1/2*c)^2 + 38*tan(1/2*a)^4*tan(1/2*c)^3 - 38*tan(1/2*a)
^3*tan(1/2*c)^4 + 11*tan(1/2*a)^2*tan(1/2*c)^5 - tan(1/2*a)*tan(1/2*c)^6 +
 tan(1/2*a)^5 - 11*tan(1/2*a)^4*tan(1/2*c) + 38*tan(1/2*a)^3*tan(1/2*c)^2
- 38*tan(1/2*a)^2*tan(1/2*c)^3 + 11*tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*c)^5
 - 2*tan(1/2*a)^3 + 11*tan(1/2*a)^2*tan(1/2*c) - 11*tan(1/2*a)*tan(1/2*c)^
2 + 2*tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c)) - (tan(1/2*a)^4*tan(1/2*c...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\tan(c+bx)\,dx = \text{Hanged}$$

input
```
int(tan(c + b*x)/sin(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^2(a+bx)\tan(c+bx)\,dx = \int \csc(bx+a)^2\tan(bx+c)\,dx$$

input
```
int(csc(b*x+a)^2*tan(b*x+c),x)
```

output
```
int(csc(a + b*x)**2*tan(b*x + c),x)
```

3.250 $\int \cot(c+bx)\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \cot(c+bx)\csc^2(a+bx)\,dx = \frac{\cot(a-c)\cot(a+bx)}{b} - \frac{\csc^2(a-c)\log(\sin(a+bx))}{b} + \frac{\csc^2(a-c)\log(\sin(c+bx))}{b}$$

output
```
cot(a-c)*cot(b*x+a)/b-csc(a-c)^2*ln(sin(b*x+a))/b+csc(a-c)^2*ln(sin(b*x+c)
)/b
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

$$\int \cot(c+bx)\csc^2(a+bx)\,dx = \frac{\csc(a)\csc^2(a-c)\csc(a+bx)(-\cos(2a-2c-bx)+\cos(2a-2c+bx)+4(-\log(\sin(a+bx))+\log(\sin}{4b}$$

input
```
Integrate[Cot[c + b*x]*Csc[a + b*x]^2,x]
```

output
```
(Csc[a]*Csc[a - c]^2*Csc[a + b*x]*(-Cos[2*a - 2*c - b*x] + Cos[2*a - 2*c +
 b*x] + 4*(-Log[Sin[a + b*x]] + Log[Sin[c + b*x]])*Sin[a]*Sin[a + b*x]))/(
4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\cot(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \csc^2(a+bx)\cot(bx+c)dx$$

input
```
Int[Cot[c + b*x]*Csc[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.21

method	result
risch	$-\frac{2\,e^{2ia}}{b(e^{2ia}-e^{2ic})(e^{2i(bx+a)}-1)}-\frac{2\,e^{2ic}}{b(e^{2ia}-e^{2ic})(e^{2i(bx+a)}-1)}+\frac{4\ln(e^{2i(bx+a)}-1)e^{2i(a+c)}}{(e^{4ia}-2\,e^{2i(a+c)}+e^{4ic})b}-\frac{4\ln(e^{2i(bx+a)}-e^{2i(a-c)})e^{2i(a+c)}}{(e^{4ia}-2\,e^{2i(a+c)}+e^{4ic})b}$

input
```
int(cot(b*x+c)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b/(exp(2*I*a)-exp(2*I*c))/(exp(2*I*(b*x+a))-1)*exp(2*I*a)-2/b/(exp(2*I*
a)-exp(2*I*c))/(exp(2*I*(b*x+a))-1)*exp(2*I*c)+4*ln(exp(2*I*(b*x+a))-1)/(e
xp(4*I*a)-2*exp(2*I*(a+c))+exp(4*I*c))/b*exp(2*I*(a+c))-4*ln(exp(2*I*(b*x+
a))-exp(2*I*(a-c)))/(exp(4*I*a)-2*exp(2*I*(a+c))+exp(4*I*c))/b*exp(2*I*(a+
c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(56) = 112$.

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.43

$$\int \cot(c+bx)\csc^2(a+bx)\,dx$$
$$= \frac{\log\left(-\frac{1}{4}\cos\left(bx+a\right)^2+\frac{1}{4}\right)\sin\left(bx+a\right)-\log\left(-\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2}{\cos(-2a+2c)+1}\right.}{\left(b\cos\left(-2a+2c\right)-b\right)\sin\left(bx+a\right)}$$

input
```
integrate(cot(b*x+c)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
(log(-1/4*cos(b*x + a)^2 + 1/4)*sin(b*x + a) - log(-(2*cos(b*x + a)^2*cos(
-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c
) - 1)/(cos(-2*a + 2*c) + 1))*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))
/((b*cos(-2*a + 2*c) - b)*sin(b*x + a))
```

Sympy [F]

$$\int \cot(c+bx)\csc^2(a+bx)\,dx = \int \cot\left(bx+c\right)\csc^2\left(a+bx\right)\,dx$$

input
```
integrate(cot(b*x+c)*csc(b*x+a)**2,x)
```

output
```
Integral(cot(b*x + c)*csc(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14405 vs. $2(56) = 112$.

Time = 0.28 (sec) , antiderivative size = 14405, normalized size of antiderivative = 257.23

$$\int \cot(c+bx)\csc^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
2*((cos(2*a)^2 + sin(2*a)^2)*cos(4*a)^2 + 4*(cos(2*a)^2 - cos(2*c)^2 + sin
(2*a)^2 - sin(2*c)^2)*cos(2*a + 2*c)^2 + (cos(2*a)^2 - cos(2*c)^2 + sin(2*
a)^2 - sin(2*c)^2)*cos(4*c)^2 - (cos(4*a)^2 + sin(4*a)^2)*cos(2*c)^2 + (co
s(2*a)^2 + sin(2*a)^2)*sin(4*a)^2 + 4*(cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^
2 - sin(2*c)^2)*sin(2*a + 2*c)^2 + (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 -
 sin(2*c)^2)*sin(4*c)^2 - (cos(4*a)^2 + sin(4*a)^2)*sin(2*c)^2 + 2*(((sin(
4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*co
s(2*b*x + 4*a)^2 + ((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos
(4*c))*sin(2*a + 2*c))*cos(2*b*x + 2*a + 2*c)^2 + ((sin(4*a) + sin(4*c))*c
os(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*sin(2*b*x + 4*a)^2 +
 ((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2
*c))*sin(2*b*x + 2*a + 2*c)^2 - 2*((cos(2*a)*sin(4*a) - cos(2*c)*sin(4*a)
+ (cos(2*a) - cos(2*c))*sin(4*c))*cos(2*a + 2*c) - (cos(4*a)*cos(2*a) + (c
os(2*a) - cos(2*c))*cos(4*c) - cos(4*a)*cos(2*c))*sin(2*a + 2*c))*cos(2*b*
x + 4*a) - 2*(((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c)
)*sin(2*a + 2*c))*cos(2*b*x + 4*a) - (cos(2*a)*sin(4*a) - cos(2*c)*sin(4*a
) + (cos(2*a) - cos(2*c))*sin(4*c))*cos(2*a + 2*c) + (cos(4*a)*cos(2*a) +
(cos(2*a) - cos(2*c))*cos(4*c) - cos(4*a)*cos(2*c))*sin(2*a + 2*c))*cos(2*
b*x + 2*a + 2*c) - (2*cos(2*a)*cos(2*c)*sin(4*a) - cos(2*c)^2*sin(4*a) + 2
*sin(4*a)*sin(2*a)*sin(2*c) - sin(4*a)*sin(2*c)^2 - (cos(2*a)^2 + sin(2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1208 vs. $2(56) = 112$.

Time = 0.17 (sec) , antiderivative size = 1208, normalized size of antiderivative = 21.57

$$\int \cot(c + bx) \csc^2(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/4*((tan(1/2*a)^6*tan(1/2*c)^4 + 2*tan(1/2*a)^6*tan(1/2*c)^2 + tan(1/2*a
)^4*tan(1/2*c)^4 + tan(1/2*a)^6 + 2*tan(1/2*a)^4*tan(1/2*c)^2 - tan(1/2*a)
^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*c)^
4 - tan(1/2*a)^2 - 2*tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*a)^2 - tan
(b*x) - 2*tan(1/2*a)))/(tan(1/2*a)^6*tan(1/2*c)^2 - 2*tan(1/2*a)^5*tan(1/2
*c)^3 + tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^5*tan(1/2*c) - 5*tan(1/2*
a)^4*tan(1/2*c)^2 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^2*tan(1/2*c)^
4 + tan(1/2*a)^4 - 4*tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2
 - 2*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^2 + 2*tan(1/2*a)*tan(1/2*c) - ta
n(1/2*c)^2) - (tan(1/2*a)^4*tan(1/2*c)^6 + tan(1/2*a)^4*tan(1/2*c)^4 + 2*t
an(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(
1/2*c)^4 + tan(1/2*c)^6 - tan(1/2*a)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^2 + tan
(1/2*c)^4 - 2*tan(1/2*a)^2 - tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*c)
^2 - tan(b*x) - 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^3
*tan(1/2*c)^5 + tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^4*tan(1/2*c)^2 + 4*
tan(1/2*a)^3*tan(1/2*c)^3 - 5*tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)*tan
(1/2*c)^5 - 2*tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - 4*ta
n(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - tan(1/2*a)^2 + 2*tan(1/2*a)*tan(1/2
*c) - tan(1/2*c)^2) - (tan(b*x)*tan(1/2*a)^8*tan(1/2*c)^4 + 2*tan(b*x)*tan
(1/2*a)^8*tan(1/2*c)^2 + 2*tan(1/2*a)^8*tan(1/2*c)^3 - 4*tan(1/2*a)^7*t...
```

Mupad [F(-1)]

Timed out.

$$\int \cot(c+bx)\csc^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(cot(c + b*x)/sin(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cot(c+bx)\csc^2(a+bx)\,dx = \int \cot(bx+c)\csc(bx+a)^2\,dx$$

input
```
int(cot(b*x+c)*csc(b*x+a)^2,x)
```

output
```
int(cot(b*x + c)*csc(a + b*x)**2,x)
```

3.251 $\int \cot^2(c+bx)\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \cot^2(c+bx)\csc^2(a+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 6.43 (sec) , antiderivative size = 434, normalized size of antiderivative = 434.00

$$\int \cot^2(c+bx)\csc^2(a+bx)\,dx$$
$$= -\frac{2i\arctan(\tan(a+bx))\cot(a-c)\csc^2(a-c)}{b} + \frac{\cot(a-c)\csc^2(a-c)\log\left(\sin^2(a+bx)\right)}{b} - \frac{2\cot(a-c)\csc^2(a-c)\log(\sin(c+bx))}{b}$$
$$+ x\left(-2\cot(a-c)\csc(a)\csc(a-c)\csc(c) + \frac{2i\cos(a)\cos(c)}{(\cos(c)\sin(a)-\cos(a)\sin(c))^3} - \frac{\cos(a)\cos(c)\cot(a)}{(\cos(c)\sin(a)-\cos(a)\sin(c))^3} - \frac{\cos(c)\csc(a)}{(\cos(c)\sin(a)-\cos(a)\sin(c))^3} + \frac{\cos(c)\sin(a)}{(\cos(c)\sin(a)-\cos(a)\sin(c))^3} - \frac{2\cos(a)\sin(c)}{(\cos(c)\sin(a)-\cos(a)\sin(c))^3} - \frac{i\cos(a)\cot(a)\sin(c)}{(\cos(c)\sin(a)-\cos(a)\sin(c))^3} + \frac{i\csc(a)\sin(c)}{(\cos(c)\sin(a)-\cos(a)\sin(c))^3} + \frac{i\sin(a)\sin(c)}{(\cos(c)\sin(a)-\cos(a)\sin(c))^3} - \frac{\cos(a)\cos(c)\cot(c)}{(-\cos(c)\sin(a)+\cos(a)\sin(c))^3} - \frac{\cos(a)\csc(c)}{(-\cos(c)\sin(a)+\cos(a)\sin(c))^3} - \frac{2\cos(c)\sin(a)}{(-\cos(c)\sin(a)+\cos(a)\sin(c))^3} + \frac{\cos(a)\sin(c)}{(-\cos(c)\sin(a)+\cos(a)\sin(c))^3}\right) + \frac{\csc^2(a-c)\csc(c)\csc(c+bx)\sin(bx)}{b}$$
$$+ \frac{\csc(a)\csc^2(a-c)\csc(a+bx)(2\sin(bx)-\sin(2a-2c-bx)+\sin(2a-2c+bx))}{4b}$$

input
```
Integrate[Cot[c + b*x]^2*Csc[a + b*x]^2,x]
```

output
```
((-2*I)*ArcTan[Tan[a + b*x]]*Cot[a - c]*Csc[a - c]^2)/b + (Cot[a - c]*Csc[
a - c]^2*Log[Sin[a + b*x]^2])/b - (2*Cot[a - c]*Csc[a - c]^2*Log[Sin[c + b
*x]])/b + x*(-2*Cot[a - c]*Csc[a]*Csc[a - c]*Csc[c] + ((2*I)*Cos[a]*Cos[c]
)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^3 - (Cos[a]*Cos[c]*Cot[a])/(Cos[c]*Sin[a
] - Cos[a]*Sin[c])^3 - (Cos[c]*Csc[a])/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^3 +
 (Cos[c]*Sin[a])/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^3 - (2*Cos[a]*Sin[c])/(Co
s[c]*Sin[a] - Cos[a]*Sin[c])^3 - (I*Cos[a]*Cot[a]*Sin[c])/(Cos[c]*Sin[a] -
 Cos[a]*Sin[c])^3 + (I*Csc[a]*Sin[c])/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^3 +
(I*Sin[a]*Sin[c])/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^3 - (Cos[a]*Cos[c]*Cot[c
])/(-(Cos[c]*Sin[a]) + Cos[a]*Sin[c])^3 - (Cos[a]*Csc[c])/(-(Cos[c]*Sin[a]
) + Cos[a]*Sin[c])^3 - (2*Cos[c]*Sin[a])/(-(Cos[c]*Sin[a]) + Cos[a]*Sin[c]
)^3 + (Cos[a]*Sin[c])/(-(Cos[c]*Sin[a]) + Cos[a]*Sin[c])^3) + (Csc[a - c]^
2*Csc[c]*Csc[c + b*x]*Sin[b*x])/b + (Csc[a]*Csc[a - c]^2*Csc[a + b*x]*(2*S
in[b*x] - Sin[2*a - 2*c - b*x] + Sin[2*a - 2*c + b*x]))/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\cot^2(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc^2(a+bx)\cot^2(bx+c)dx$$

input
```
Int[Cot[c + b*x]^2*Csc[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.14 (sec) , antiderivative size = 402, normalized size of antiderivative = 402.00

method	result
risch	$\frac{2i\left(-5\,\mathrm{e}^{2i(bx+3a+c)}-2\,\mathrm{e}^{2i(bx+2a+2c)}-\mathrm{e}^{2i(bx+a+3c)}+\mathrm{e}^{6ia}+6\,\mathrm{e}^{2i(2a+c)}+\mathrm{e}^{2i(a+2c)}\right)}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)^2\left(\mathrm{e}^{2i(bx+a)}-1\right)\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{8i\ln\left(\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{2i(2a+c)}}{\left(\mathrm{e}^{6ia}-3\,\mathrm{e}^{2i(2a+c)}+3\,\mathrm{e}^{2i(a+2c)}-\mathrm{e}^{6ic}\right)b}+$

input
```
int(cot(b*x+c)^2*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*I*(-5*exp(2*I*(b*x+3*a+c))-2*exp(2*I*(b*x+2*a+2*c))-exp(2*I*(b*x+a+3*c))
+exp(6*I*a)+6*exp(2*I*(2*a+c))+exp(2*I*(a+2*c)))/b/(exp(2*I*a)-exp(2*I*c))
^2/(exp(2*I*(b*x+a))-1)/(-exp(2*I*(b*x+a+c))+exp(2*I*a))+8*I*ln(exp(2*I*(b
*x+a))-exp(2*I*(a-c)))/(exp(6*I*a)-3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))-e
xp(6*I*c))/b*exp(2*I*(2*a+c))+8*I*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))/(exp
(6*I*a)-3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))-exp(6*I*c))/b*exp(2*I*(a+2*c
))-8*I*ln(exp(2*I*(b*x+a))-1)/(exp(6*I*a)-3*exp(2*I*(2*a+c))+3*exp(2*I*(a+
2*c))-exp(6*I*c))/b*exp(2*I*(2*a+c))-8*I*ln(exp(2*I*(b*x+a))-1)/(exp(6*I*a
)-3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))-exp(6*I*c))/b*exp(2*I*(a+2*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 353.00

$$\int \cot^2(c+bx)\csc^2(a+bx)\,dx =$$

$$-\frac{(\cos(-2a+2c)+3)\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\left(\cos(-2a+2c)^2+2\cos(-2a+2\right.}{}$$

input
```
integrate(cot(b*x+c)^2*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
-((cos(-2*a + 2*c) + 3)*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - (cos(-
2*a + 2*c)^2 + 2*cos(-2*a + 2*c) - 3)*cos(b*x + a)^2 - 2*((cos(-2*a + 2*c)
 + 1)*cos(b*x + a)^2 - cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*
a + 2*c) - 1)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2*((cos(-2*a + 2*c) + 1)*co
s(b*x + a)^2 - cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c)
 - 1)*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)
*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1)/(cos(-2*a + 2*c) + 1)) + 2*cos(-2*
a + 2*c) - 2)/((b*cos(-2*a + 2*c)^2 - 2*b*cos(-2*a + 2*c) + b)*cos(b*x + a
)*sin(b*x + a) + ((b*cos(-2*a + 2*c) - b)*cos(b*x + a)^2 - b*cos(-2*a + 2*
c) + b)*sin(-2*a + 2*c))
```

Sympy [F(-1)]

Timed out.

$$\int \cot^2(c+bx)\csc^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cot(b*x+c)**2*csc(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 4.47 (sec) , antiderivative size = 151939, normalized size of antiderivative = 151939.00

$$\int \cot^2(c+bx)\csc^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^2*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*(72*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2
*a + 2*c))*cos(4*a + 2*c)^2 + 72*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (
cos(4*a) + cos(4*c))*sin(2*a + 2*c))*cos(2*a + 4*c)^2 + 72*((sin(4*a) + si
n(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*sin(4*a + 2
*c)^2 + 72*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*s
in(2*a + 2*c))*sin(2*a + 4*c)^2 - 4*(((cos(6*a) - cos(6*c))*cos(4*a + 2*c)
 - 3*cos(4*a + 2*c)^2 + (cos(6*a) - cos(6*c))*cos(2*a + 4*c) + 3*cos(2*a +
 4*c)^2 + (sin(6*a) - sin(6*c))*sin(4*a + 2*c) - 3*sin(4*a + 2*c)^2 + (sin
(6*a) - sin(6*c))*sin(2*a + 4*c) + 3*sin(2*a + 4*c)^2)*cos(4*b*x + 6*a + 2
*c)^2 + 4*((cos(6*a) - cos(6*c))*cos(4*a + 2*c) - 3*cos(4*a + 2*c)^2 + (co
s(6*a) - cos(6*c))*cos(2*a + 4*c) + 3*cos(2*a + 4*c)^2 + (sin(6*a) - sin(6
*c))*sin(4*a + 2*c) - 3*sin(4*a + 2*c)^2 + (sin(6*a) - sin(6*c))*sin(2*a +
 4*c) + 3*sin(2*a + 4*c)^2)*cos(4*b*x + 4*a + 4*c)^2 + ((cos(6*a) - cos(6*
c))*cos(4*a + 2*c) - 3*cos(4*a + 2*c)^2 + (cos(6*a) - cos(6*c))*cos(2*a +
4*c) + 3*cos(2*a + 4*c)^2 + (sin(6*a) - sin(6*c))*sin(4*a + 2*c) - 3*sin(4
*a + 2*c)^2 + (sin(6*a) - sin(6*c))*sin(2*a + 4*c) + 3*sin(2*a + 4*c)^2)*c
os(4*b*x + 2*a + 6*c)^2 + ((cos(6*a) - cos(6*c))*cos(4*a + 2*c) - 3*cos(4*
a + 2*c)^2 + (cos(6*a) - cos(6*c))*cos(2*a + 4*c) + 3*cos(2*a + 4*c)^2 + (
sin(6*a) - sin(6*c))*sin(4*a + 2*c) - 3*sin(4*a + 2*c)^2 + (sin(6*a) - sin
(6*c))*sin(2*a + 4*c) + 3*sin(2*a + 4*c)^2)*cos(2*b*x + 6*a)^2 + ((cos(...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.21 (sec) , antiderivative size = 2538, normalized size of antiderivative = 2538.00

$$\int \cot^2(c + bx) \csc^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^2*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*((tan(1/2*a)^8*tan(1/2*c)^6 + tan(1/2*a)^8*tan(1/2*c)^4 + 4*tan(1/2*a)
^7*tan(1/2*c)^5 - tan(1/2*a)^8*tan(1/2*c)^2 + 8*tan(1/2*a)^7*tan(1/2*c)^3
+ 4*tan(1/2*a)^5*tan(1/2*c)^5 - 2*tan(1/2*a)^4*tan(1/2*c)^6 - tan(1/2*a)^8
 + 4*tan(1/2*a)^7*tan(1/2*c) + 8*tan(1/2*a)^5*tan(1/2*c)^3 - 2*tan(1/2*a)^
4*tan(1/2*c)^4 - 4*tan(1/2*a)^3*tan(1/2*c)^5 + 4*tan(1/2*a)^5*tan(1/2*c) +
 2*tan(1/2*a)^4*tan(1/2*c)^2 - 8*tan(1/2*a)^3*tan(1/2*c)^3 - 4*tan(1/2*a)*
tan(1/2*c)^5 + tan(1/2*c)^6 + 2*tan(1/2*a)^4 - 4*tan(1/2*a)^3*tan(1/2*c) -
 8*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 4*tan(1/2*a)*tan(1/2*c) - tan(
1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*a)^2 - tan(b*x) - 2*tan(1/2*a)))/(t
an(1/2*a)^8*tan(1/2*c)^3 - 3*tan(1/2*a)^7*tan(1/2*c)^4 + 3*tan(1/2*a)^6*ta
n(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^6 + 3*tan(1/2*a)^7*tan(1/2*c)^2 - 10*
tan(1/2*a)^6*tan(1/2*c)^3 + 12*tan(1/2*a)^5*tan(1/2*c)^4 - 6*tan(1/2*a)^4*
tan(1/2*c)^5 + tan(1/2*a)^3*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c) - 12*
tan(1/2*a)^5*tan(1/2*c)^2 + 18*tan(1/2*a)^4*tan(1/2*c)^3 - 12*tan(1/2*a)^3
*tan(1/2*c)^4 + 3*tan(1/2*a)^2*tan(1/2*c)^5 + tan(1/2*a)^5 - 6*tan(1/2*a)^
4*tan(1/2*c) + 12*tan(1/2*a)^3*tan(1/2*c)^2 - 10*tan(1/2*a)^2*tan(1/2*c)^3
 + 3*tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*a)^3 + 3*tan(1/2*a)^2*tan(1/2*c) -
3*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3) - (tan(1/2*a)^6*tan(1/2*c)^8 + 4
*tan(1/2*a)^5*tan(1/2*c)^7 + tan(1/2*a)^4*tan(1/2*c)^8 - 2*tan(1/2*a)^6*ta
n(1/2*c)^4 + 4*tan(1/2*a)^5*tan(1/2*c)^5 + 8*tan(1/2*a)^3*tan(1/2*c)^7 ...
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+bx)\csc^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(cot(c + b*x)^2/sin(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cot^2(c+bx)\csc^2(a+bx)\,dx = \int \cot(bx+c)^2\csc(bx+a)^2\,dx$$

input
```
int(cot(b*x+c)^2*csc(b*x+a)^2,x)
```

output
```
int(cot(b*x + c)**2*csc(a + b*x)**2,x)
```

3.252 $\int \csc^3(a+bx)\tan^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \csc^3(a+bx)\tan^2(c+bx)\,dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 1.93 (sec) , antiderivative size = 185, normalized size of antiderivative = 185.00

$$\int \csc^3(a+bx)\tan^2(c+bx)\,dx = \frac{(-33+16\cos(2(a-c))+\cos(4(a-c)))\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)\right)\sec^4(a-c)+1}{}$$

input
```
Integrate[Csc[a + b*x]^3*Tan[c + b*x]^2,x]
```

output
```
((-33 + 16*Cos[2*(a - c)] + Cos[4*(a - c)])*(Log[Cos[(a + b*x)/2]] - Log[S
in[(a + b*x)/2]])*Sec[a - c]^4 + 16*Sec[a - c]^3*Sec[c + b*x] - 96*ArcTanh
[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sec[a - c]^3*Tan[a - c] - 64*Cos[a + (b*x)/
2]*Csc[a]*Csc[a + b*x]*Sec[a - c]^2*Sin[(b*x)/2]*Tan[a - c] + 2*(-Csc[(a +
 b*x)/2]^2 + Sec[(a + b*x)/2]^2)*Tan[a - c]^2)/(16*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\tan^2(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \csc^3(a+bx)\tan^2(bx+c)dx$$

input
```
Int[Csc[a + b*x]^3*Tan[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.59 (sec) , antiderivative size = 1267, normalized size of antiderivative = 1267.00

method	result	size
risch	Expression too large to display	1267

input
```
int(csc(b*x+a)^3*tan(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/(exp(2*I*a)+exp(2*I*c))^3/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a+c))+
exp(2*I*a))*(exp(I*(5*b*x+11*a+2*c))-33*exp(I*(5*b*x+9*a+4*c))+15*exp(I*(5
*b*x+7*a+6*c))+exp(I*(5*b*x+5*a+8*c))+exp(I*(3*b*x+11*a))-16*exp(I*(3*b*x+
9*a+2*c))+62*exp(I*(3*b*x+7*a+4*c))-16*exp(I*(3*b*x+5*a+6*c))+exp(I*(3*b*x
+3*a+8*c))+exp(I*(b*x+9*a))+15*exp(I*(b*x+7*a+2*c))-33*exp(I*(b*x+5*a+4*c)
)+exp(I*(b*x+3*a+6*c)))+1/2/(exp(8*I*a)+4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c)
)+4*exp(2*I*(a+3*c))+exp(8*I*c))/b*ln(exp(I*(b*x+a))+1)*exp(8*I*a)+8/(exp(
8*I*a)+4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))+4*exp(2*I*(a+3*c))+exp(8*I*c))/
b*ln(exp(I*(b*x+a))+1)*exp(2*I*(3*a+c))-33/(exp(8*I*a)+4*exp(2*I*(3*a+c))+
6*exp(4*I*(a+c))+4*exp(2*I*(a+3*c))+exp(8*I*c))/b*ln(exp(I*(b*x+a))+1)*exp
(4*I*(a+c))+8/(exp(8*I*a)+4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))+4*exp(2*I*(a
+3*c))+exp(8*I*c))/b*ln(exp(I*(b*x+a))+1)*exp(2*I*(a+3*c))+1/2/(exp(8*I*a)
+4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))+4*exp(2*I*(a+3*c))+exp(8*I*c))/b*ln(e
xp(I*(b*x+a))+1)*exp(8*I*c)-1/2/(exp(8*I*a)+4*exp(2*I*(3*a+c))+6*exp(4*I*(
a+c))+4*exp(2*I*(a+3*c))+exp(8*I*c))/b*ln(exp(I*(b*x+a))-1)*exp(8*I*a)-8/(
exp(8*I*a)+4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))+4*exp(2*I*(a+3*c))+exp(8*I*
c))/b*ln(exp(I*(b*x+a))-1)*exp(2*I*(3*a+c))+33/(exp(8*I*a)+4*exp(2*I*(3*a+
c))+6*exp(4*I*(a+c))+4*exp(2*I*(a+3*c))+exp(8*I*c))/b*ln(exp(I*(b*x+a))-1)
*exp(4*I*(a+c))-8/(exp(8*I*a)+4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))+4*exp(2*
I*(a+3*c))+exp(8*I*c))/b*ln(exp(I*(b*x+a))-1)*exp(2*I*(a+3*c))-1/2/(exp...
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.15 (sec) , antiderivative size = 892, normalized size of antiderivative = 892.00

$$\int \csc^3(a+bx)\tan^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*tan(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(2*(cos(-2*a + 2*c)^2 + 8*cos(-2*a + 2*c) + 7)*cos(b*x + a)*sin(b*x +
a)*sin(-2*a + 2*c) - 2*(cos(-2*a + 2*c)^3 + 9*cos(-2*a + 2*c)^2 - 9*cos(-2
*a + 2*c) - 17)*cos(b*x + a)^2 + 16*cos(-2*a + 2*c)^2 + 12*sqrt(2)*(((cos(
-2*a + 2*c)^2 - 1)*cos(b*x + a)^2 - cos(-2*a + 2*c)^2 + 1)*sin(b*x + a) +
((cos(-2*a + 2*c) + 1)*cos(b*x + a)^3 - (cos(-2*a + 2*c) + 1)*cos(b*x + a)
)*sin(-2*a + 2*c))*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*
sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x +
a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a +
2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*
sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a + 2*c) + 1) + ((cos(
-2*a + 2*c)^3 + 9*cos(-2*a + 2*c)^2 - 9*cos(-2*a + 2*c) - 17)*cos(b*x + a)
^3 - ((cos(-2*a + 2*c)^2 + 8*cos(-2*a + 2*c) - 17)*cos(b*x + a)^2 - cos(-2
*a + 2*c)^2 - 8*cos(-2*a + 2*c) + 17)*sin(b*x + a)*sin(-2*a + 2*c) - (cos(
-2*a + 2*c)^3 + 9*cos(-2*a + 2*c)^2 - 9*cos(-2*a + 2*c) - 17)*cos(b*x + a)
)*log(1/2*cos(b*x + a) + 1/2) - ((cos(-2*a + 2*c)^3 + 9*cos(-2*a + 2*c)^2
- 9*cos(-2*a + 2*c) - 17)*cos(b*x + a)^3 - ((cos(-2*a + 2*c)^2 + 8*cos(-2*
a + 2*c) - 17)*cos(b*x + a)^2 - cos(-2*a + 2*c)^2 - 8*cos(-2*a + 2*c) + 17
)*sin(b*x + a)*sin(-2*a + 2*c) - (cos(-2*a + 2*c)^3 + 9*cos(-2*a + 2*c)^2
- 9*cos(-2*a + 2*c) - 17)*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) - 16*
cos(-2*a + 2*c) - 32)/((b*cos(-2*a + 2*c)^3 + 3*b*cos(-2*a + 2*c)^2 + 3...
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\tan^2(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*tan(b*x+c)**2,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 72.45 (sec) , antiderivative size = 752466, normalized size of antiderivative = 752466.00

$$\int \csc^3(a+bx)\tan^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*tan(b*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(96*(((cos(8*a) + 4*cos(6*a + 2*c) + cos(8*c))*cos(5*a + 3*c) + 6*cos
(5*a + 3*c)*cos(4*a + 4*c) - (cos(8*a) + 4*cos(6*a + 2*c) + 6*cos(4*a + 4*
c) + cos(8*c))*cos(3*a + 5*c) + 4*(cos(5*a + 3*c) - cos(3*a + 5*c))*cos(2*
a + 6*c) + (sin(8*a) + 4*sin(6*a + 2*c) + sin(8*c))*sin(5*a + 3*c) + 6*sin
(5*a + 3*c)*sin(4*a + 4*c) - (sin(8*a) + 4*sin(6*a + 2*c) + 6*sin(4*a + 4*
c) + sin(8*c))*sin(3*a + 5*c) + 4*(sin(5*a + 3*c) - sin(3*a + 5*c))*sin(2*
a + 6*c))*cos(6*b*x + 10*a + 2*c)^2 + 9*((cos(8*a) + 4*cos(6*a + 2*c) + co
s(8*c))*cos(5*a + 3*c) + 6*cos(5*a + 3*c)*cos(4*a + 4*c) - (cos(8*a) + 4*c
os(6*a + 2*c) + 6*cos(4*a + 4*c) + cos(8*c))*cos(3*a + 5*c) + 4*(cos(5*a +
 3*c) - cos(3*a + 5*c))*cos(2*a + 6*c) + (sin(8*a) + 4*sin(6*a + 2*c) + si
n(8*c))*sin(5*a + 3*c) + 6*sin(5*a + 3*c)*sin(4*a + 4*c) - (sin(8*a) + 4*s
in(6*a + 2*c) + 6*sin(4*a + 4*c) + sin(8*c))*sin(3*a + 5*c) + 4*(sin(5*a +
 3*c) - sin(3*a + 5*c))*sin(2*a + 6*c))*cos(6*b*x + 8*a + 4*c)^2 + 9*((cos
(8*a) + 4*cos(6*a + 2*c) + cos(8*c))*cos(5*a + 3*c) + 6*cos(5*a + 3*c)*cos
(4*a + 4*c) - (cos(8*a) + 4*cos(6*a + 2*c) + 6*cos(4*a + 4*c) + cos(8*c))*
cos(3*a + 5*c) + 4*(cos(5*a + 3*c) - cos(3*a + 5*c))*cos(2*a + 6*c) + (sin
(8*a) + 4*sin(6*a + 2*c) + sin(8*c))*sin(5*a + 3*c) + 6*sin(5*a + 3*c)*sin
(4*a + 4*c) - (sin(8*a) + 4*sin(6*a + 2*c) + 6*sin(4*a + 4*c) + sin(8*c))*
sin(3*a + 5*c) + 4*(sin(5*a + 3*c) - sin(3*a + 5*c))*sin(2*a + 6*c))*cos(6
*b*x + 6*a + 6*c)^2 + ((cos(8*a) + 4*cos(6*a + 2*c) + cos(8*c))*cos(5*a...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.16 (sec) , antiderivative size = 13648, normalized size of antiderivative = 13648.00

$$\int \csc^3(a+bx)\tan^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*tan(b*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(12*(tan(1/2*a)^9*tan(1/2*c)^8 - tan(1/2*a)^8*tan(1/2*c)^9 - tan(1/2*a
)^9*tan(1/2*c)^7 + 2*tan(1/2*a)^8*tan(1/2*c)^8 - tan(1/2*a)^7*tan(1/2*c)^9
 + 3*tan(1/2*a)^9*tan(1/2*c)^6 - tan(1/2*a)^8*tan(1/2*c)^7 + tan(1/2*a)^7*
tan(1/2*c)^8 - 3*tan(1/2*a)^6*tan(1/2*c)^9 - 3*tan(1/2*a)^9*tan(1/2*c)^5 +
 5*tan(1/2*a)^8*tan(1/2*c)^6 - 4*tan(1/2*a)^7*tan(1/2*c)^7 + 5*tan(1/2*a)^
6*tan(1/2*c)^8 - 3*tan(1/2*a)^5*tan(1/2*c)^9 + 3*tan(1/2*a)^9*tan(1/2*c)^4
 + 3*tan(1/2*a)^8*tan(1/2*c)^5 + 4*tan(1/2*a)^7*tan(1/2*c)^6 - 4*tan(1/2*a
)^6*tan(1/2*c)^7 - 3*tan(1/2*a)^5*tan(1/2*c)^8 - 3*tan(1/2*a)^4*tan(1/2*c)
^9 - 3*tan(1/2*a)^9*tan(1/2*c)^3 + 3*tan(1/2*a)^8*tan(1/2*c)^4 - 6*tan(1/2
*a)^7*tan(1/2*c)^5 + 12*tan(1/2*a)^6*tan(1/2*c)^6 - 6*tan(1/2*a)^5*tan(1/2
*c)^7 + 3*tan(1/2*a)^4*tan(1/2*c)^8 - 3*tan(1/2*a)^3*tan(1/2*c)^9 + tan(1/
2*a)^9*tan(1/2*c)^2 + 5*tan(1/2*a)^8*tan(1/2*c)^3 + 6*tan(1/2*a)^7*tan(1/2
*c)^4 + 6*tan(1/2*a)^6*tan(1/2*c)^5 - 6*tan(1/2*a)^5*tan(1/2*c)^6 - 6*tan(
1/2*a)^4*tan(1/2*c)^7 - 5*tan(1/2*a)^3*tan(1/2*c)^8 - tan(1/2*a)^2*tan(1/2
*c)^9 - tan(1/2*a)^9*tan(1/2*c) - tan(1/2*a)^8*tan(1/2*c)^2 - 4*tan(1/2*a)
^7*tan(1/2*c)^3 + 6*tan(1/2*a)^6*tan(1/2*c)^4 + 6*tan(1/2*a)^4*tan(1/2*c)^
6 - 4*tan(1/2*a)^3*tan(1/2*c)^7 - tan(1/2*a)^2*tan(1/2*c)^8 - tan(1/2*a)*t
an(1/2*c)^9 + 2*tan(1/2*a)^8*tan(1/2*c) + 4*tan(1/2*a)^7*tan(1/2*c)^2 + 12
*tan(1/2*a)^6*tan(1/2*c)^3 - 12*tan(1/2*a)^3*tan(1/2*c)^6 - 4*tan(1/2*a)^2
*tan(1/2*c)^7 - 2*tan(1/2*a)*tan(1/2*c)^8 - tan(1/2*a)^8 - tan(1/2*a)^7...
```

Mupad [B] (verification not implemented)

Time = 50.20 (sec) , antiderivative size = 74535, normalized size of antiderivative = 74535.00

$$\int \csc^3(a+bx)\tan^2(c+bx)\,dx = \text{Too large to display}$$

input
```
int(tan(c + b*x)^2/sin(a + b*x)^3,x)
```

output
```
((tan((b*x)/2)*(9*sin(a)^5*tan(c) - 21*cos(a)*sin(a)^4 - 18*cos(a)^3*sin(a
)^2 + 2*cos(a)^5*tan(c)^2 + 12*sin(a)^5*tan(c)^3 + 2*sin(a)^5*tan(c)^5 + 8
*cos(a)^4*sin(a)*tan(c) + 24*cos(a)^2*sin(a)^3*tan(c) - 36*cos(a)*sin(a)^4
*tan(c)^2 + 12*cos(a)^4*sin(a)*tan(c)^3 - 16*cos(a)*sin(a)^4*tan(c)^4 - 37
*cos(a)^3*sin(a)^2*tan(c)^2 + 27*cos(a)^2*sin(a)^3*tan(c)^3 - 18*cos(a)^3*
sin(a)^2*tan(c)^4 + 2*cos(a)^2*sin(a)^3*tan(c)^5))/(sin(a)*(cos(a)^2 + sin
(a)^2)*(cos(a)^3 + sin(a)^3*tan(c)^3 + 3*cos(a)^2*sin(a)*tan(c) + 3*cos(a)
*sin(a)^2*tan(c)^2)) - (6*sin(a)^4 + 5*cos(a)^2*sin(a)^2 - cos(a)^4*tan(c)
^2 + 8*sin(a)^4*tan(c)^2 + 2*sin(a)^4*tan(c)^4 - 5*cos(a)*sin(a)^3*tan(c)
- 2*cos(a)^3*sin(a)*tan(c) - 4*cos(a)*sin(a)^3*tan(c)^3 - 5*cos(a)^3*sin(a
)*tan(c)^3 + 10*cos(a)^2*sin(a)^2*tan(c)^2 + 2*cos(a)^2*sin(a)^2*tan(c)^4)
/((cos(a)^2 + sin(a)^2)*(cos(a)^3 + sin(a)^3*tan(c)^3 + 3*cos(a)^2*sin(a)*
tan(c) + 3*cos(a)*sin(a)^2*tan(c)^2)) + (tan((b*x)/2)^5*(3*sin(a)^5*tan(c)
 - 3*cos(a)*sin(a)^4 - 2*cos(a)^3*sin(a)^2 + 2*cos(a)^5*tan(c)^2 + 4*sin(a
)^5*tan(c)^3 + 2*sin(a)^5*tan(c)^5 + 6*cos(a)^2*sin(a)^3*tan(c) - 6*cos(a)
*sin(a)^4*tan(c)^2 + 6*cos(a)^4*sin(a)*tan(c)^3 - 7*cos(a)^3*sin(a)^2*tan(
c)^2 + 9*cos(a)^2*sin(a)^3*tan(c)^3 + 2*cos(a)^2*sin(a)^3*tan(c)^5))/(sin(
a)*(cos(a)^2 + sin(a)^2)*(cos(a)^3 + sin(a)^3*tan(c)^3 + 3*cos(a)^2*sin(a)
*tan(c) + 3*cos(a)*sin(a)^2*tan(c)^2)) - (tan((b*x)/2)^4*(6*sin(a)^6 - 3*c
os(a)^2*sin(a)^4 - 6*cos(a)^4*sin(a)^2 + 2*cos(a)^6*tan(c)^2 + 6*sin(a)...
```

Reduce [F]

$$\int \csc^3(a+bx)\tan^2(c+bx)\,dx = \int \csc(bx+a)^3\tan(bx+c)^2\,dx$$

input
```
int(csc(b*x+a)^3*tan(b*x+c)^2,x)
```

output
```
int(csc(a + b*x)**3*tan(b*x + c)**2,x)
```

3.253 $\int \csc^3(a+bx)\tan(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 103

$$\int \csc^3(a+bx)\tan(c+bx)\,dx = -\frac{\csc(a+bx)\sec^2(a-c)}{b}+\frac{\operatorname{arctanh}(\sin(c+bx))\sec^3(a-c)}{b} - \frac{\operatorname{arctanh}(\cos(a+bx))\sec^3(a-c)(-9\sin(a-c)+\sin(3(-a+c)))}{8b} + \frac{\cot(a+bx)\csc(a+bx)\tan(a-c)}{2b}$$

output
```
-csc(b*x+a)*sec(a-c)^2/b+arctanh(sin(b*x+c))*sec(a-c)^3/b-1/8*arctanh(cos(
b*x+a))*sec(a-c)^3*(-9*sin(a-c)-sin(3*a-3*c))/b+1/2*cot(b*x+a)*csc(b*x+a)*
tan(a-c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.76 (sec) , antiderivative size = 322, normalized size of antiderivative = 3.13

$$\int \csc^3(a+bx)\tan(c+bx)\,dx$$
$$= -\frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)}{\frac{1}{4}b\cos(3a-3c)+\frac{3}{4}b\cos(a-c)}$$
$$+\frac{\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sec^3(a-c)(-\sin(3a-3c)-9\sin(a-c))}{8b}$$
$$+\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sec^3(a-c)(\sin(3a-3c)+9\sin(a-c))}{8b}$$
$$+\frac{\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\sec^2(a-c)\sin\left(\frac{bx}{2}\right)}{2b}-\frac{\sec\left(\frac{a}{2}\right)\sec^2(a-c)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{2b}$$
$$+\frac{\csc^2\left(\frac{a}{2}+\frac{bx}{2}\right)\tan(a-c)}{8b}-\frac{\sec^2\left(\frac{a}{2}+\frac{bx}{2}\right)\tan(a-c)}{8b}$$

input
```
Integrate[Csc[a + b*x]^3*Tan[c + b*x],x]
```

output
```
((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x
)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]])/((b*Cos[3*a - 3*c])
/4 + (3*b*Cos[a - c])/4) + (Log[Sin[a/2 + (b*x)/2]]*Sec[a - c]^3*(-Sin[3*a
 - 3*c] - 9*Sin[a - c]))/(8*b) + (Log[Cos[a/2 + (b*x)/2]]*Sec[a - c]^3*(Si
n[3*a - 3*c] + 9*Sin[a - c]))/(8*b) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*Sec[a -
 c]^2*Sin[(b*x)/2])/(2*b) - (Sec[a/2]*Sec[a - c]^2*Sec[a/2 + (b*x)/2]*Sin[
(b*x)/2])/(2*b) + (Csc[a/2 + (b*x)/2]^2*Tan[a - c])/(8*b) - (Sec[a/2 + (b*
x)/2]^2*Tan[a - c])/(8*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\tan(bx+c)\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \csc^3(a+bx)\tan(bx+c)dx$$

input
```
Int[Csc[a + b*x]^3*Tan[c + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 742, normalized size of antiderivative = 7.20

method	result
risch	$\frac{i\left(e^{i(3bx+7a)}-8\,e^{i(3bx+5a+2c)}-e^{i(3bx+3a+4c)}+e^{i(bx+5a)}+8\,e^{i(bx+3a+2c)}-e^{i(bx+a+4c)}\right)}{b\left(e^{2ia}+e^{2ic}\right)^2\left(e^{2i(bx+a)}-1\right)^2}+\frac{8\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)e^{3i(a+c)}}{\left(e^{6ia}+3\,e^{2i(2a+c)}+3\,e^{2i(a+2c)}+e^{6ic}\right)b}$

input
```
int(csc(b*x+a)^3*tan(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
I/b/(exp(2*I*a)+exp(2*I*c))^2/(exp(2*I*(b*x+a))-1)^2*(exp(I*(3*b*x+7*a))-8
*exp(I*(3*b*x+5*a+2*c))-exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+5*a))+8*exp(I*(b
*x+3*a+2*c))-exp(I*(b*x+a+4*c)))+8*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/(exp(
6*I*a)+3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))+exp(6*I*c))/b*exp(3*I*(a+c))-
1/2*I/b/(exp(6*I*a)+3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))+exp(6*I*c))*ln(e
xp(I*(b*x+a))+1)*exp(6*I*a)-9/2*I/b/(exp(6*I*a)+3*exp(2*I*(2*a+c))+3*exp(2
*I*(a+2*c))+exp(6*I*c))*ln(exp(I*(b*x+a))+1)*exp(2*I*(2*a+c))+9/2*I/b/(exp
(6*I*a)+3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))+exp(6*I*c))*ln(exp(I*(b*x+a)
)+1)*exp(2*I*(a+2*c))+1/2*I/b/(exp(6*I*a)+3*exp(2*I*(2*a+c))+3*exp(2*I*(a+
2*c))+exp(6*I*c))*ln(exp(I*(b*x+a))+1)*exp(6*I*c)+1/2*I/b/(exp(6*I*a)+3*ex
p(2*I*(2*a+c))+3*exp(2*I*(a+2*c))+exp(6*I*c))*ln(exp(I*(b*x+a))-1)*exp(6*I
*a)+9/2*I/b/(exp(6*I*a)+3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))+exp(6*I*c))*
ln(exp(I*(b*x+a))-1)*exp(2*I*(2*a+c))-9/2*I/b/(exp(6*I*a)+3*exp(2*I*(2*a+c
))+3*exp(2*I*(a+2*c))+exp(6*I*c))*ln(exp(I*(b*x+a))-1)*exp(2*I*(a+2*c))-1/
2*I/b/(exp(6*I*a)+3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))+exp(6*I*c))*ln(exp
(I*(b*x+a))-1)*exp(6*I*c)-8*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/(exp(6*I*a)+
3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))+exp(6*I*c))/b*exp(3*I*(a+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(99) = 198$.

Time = 0.12 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.08

$$\int \csc^3(a+bx)\tan(c+bx)\,dx$$

$$= \frac{2\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)\sin\left(-2\,a+2\,c\right)-\left(\left(\cos\left(-2\,a+2\,c\right)+5\right)\cos\left(bx+a\right)^{2}-\cos\left(-2\,a+\right.\right.}{}$$

input
```
integrate(csc(b*x+a)^3*tan(b*x+c),x, algorithm="fricas")
```

output
```
1/4*(2*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(-2*a + 2*c) - ((cos(-2*a + 2
*c) + 5)*cos(b*x + a)^2 - cos(-2*a + 2*c) - 5)*log(1/2*cos(b*x + a) + 1/2)
*sin(-2*a + 2*c) + ((cos(-2*a + 2*c) + 5)*cos(b*x + a)^2 - cos(-2*a + 2*c)
 - 5)*log(-1/2*cos(b*x + a) + 1/2)*sin(-2*a + 2*c) + 4*sqrt(2)*((cos(-2*a
+ 2*c) + 1)*cos(b*x + a)^2 - cos(-2*a + 2*c) - 1)*log((2*cos(b*x + a)^2*co
s(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((
cos(-2*a + 2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos
(-2*a + 2*c) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c)
 - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqr
t(cos(-2*a + 2*c) + 1) + 8*(cos(-2*a + 2*c) + 1)*sin(b*x + a))/((b*cos(-2*
a + 2*c)^2 + 2*b*cos(-2*a + 2*c) + b)*cos(b*x + a)^2 - b*cos(-2*a + 2*c)^2
 - 2*b*cos(-2*a + 2*c) - b)
```

Sympy [F]

$$\int \csc^3(a+bx)\tan(c+bx)\,dx = \int \tan(bx+c)\csc^3(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**3*tan(b*x+c),x)
```

output
```
Integral(tan(b*x + c)*csc(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116810 vs. $2(99) = 198$.

Time = 8.14 (sec) , antiderivative size = 116810, normalized size of antiderivative = 1134.08

$$\int \csc^3(a+bx)\tan(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*tan(b*x+c),x, algorithm="maxima")
```

output
```
1/2*(16*(((sin(6*a) + 3*sin(4*a + 2*c) + sin(6*c))*cos(3*a + 3*c) - (cos(6
*a) + 3*cos(4*a + 2*c) + cos(6*c))*sin(3*a + 3*c) - 3*cos(2*a + 4*c)*sin(3
*a + 3*c) + 3*cos(3*a + 3*c)*sin(2*a + 4*c))*cos(4*b*x + 8*a)^2 + 4*((sin(
6*a) + 3*sin(4*a + 2*c) + sin(6*c))*cos(3*a + 3*c) - (cos(6*a) + 3*cos(4*a
 + 2*c) + cos(6*c))*sin(3*a + 3*c) - 3*cos(2*a + 4*c)*sin(3*a + 3*c) + 3*c
os(3*a + 3*c)*sin(2*a + 4*c))*cos(4*b*x + 6*a + 2*c)^2 + ((sin(6*a) + 3*si
n(4*a + 2*c) + sin(6*c))*cos(3*a + 3*c) - (cos(6*a) + 3*cos(4*a + 2*c) + c
os(6*c))*sin(3*a + 3*c) - 3*cos(2*a + 4*c)*sin(3*a + 3*c) + 3*cos(3*a + 3*
c)*sin(2*a + 4*c))*cos(4*b*x + 4*a + 4*c)^2 + 4*((sin(6*a) + 3*sin(4*a + 2
*c) + sin(6*c))*cos(3*a + 3*c) - (cos(6*a) + 3*cos(4*a + 2*c) + cos(6*c))*
sin(3*a + 3*c) - 3*cos(2*a + 4*c)*sin(3*a + 3*c) + 3*cos(3*a + 3*c)*sin(2*
a + 4*c))*cos(2*b*x + 6*a)^2 + 16*((sin(6*a) + 3*sin(4*a + 2*c) + sin(6*c)
)*cos(3*a + 3*c) - (cos(6*a) + 3*cos(4*a + 2*c) + cos(6*c))*sin(3*a + 3*c)
 - 3*cos(2*a + 4*c)*sin(3*a + 3*c) + 3*cos(3*a + 3*c)*sin(2*a + 4*c))*cos(
2*b*x + 4*a + 2*c)^2 + 4*((sin(6*a) + 3*sin(4*a + 2*c) + sin(6*c))*cos(3*a
 + 3*c) - (cos(6*a) + 3*cos(4*a + 2*c) + cos(6*c))*sin(3*a + 3*c) - 3*cos(
2*a + 4*c)*sin(3*a + 3*c) + 3*cos(3*a + 3*c)*sin(2*a + 4*c))*cos(2*b*x + 2
*a + 4*c)^2 + ((sin(6*a) + 3*sin(4*a + 2*c) + sin(6*c))*cos(3*a + 3*c) - (
cos(6*a) + 3*cos(4*a + 2*c) + cos(6*c))*sin(3*a + 3*c) - 3*cos(2*a + 4*c)*
sin(3*a + 3*c) + 3*cos(3*a + 3*c)*sin(2*a + 4*c))*sin(4*b*x + 8*a)^2 + ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6012 vs. $2(99) = 198$.

Time = 0.43 (sec) , antiderivative size = 6012, normalized size of antiderivative = 58.37

$$\int \csc^3(a+bx)\tan(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*tan(b*x+c),x, algorithm="giac")
```

output
```
-1/4*(4*(tan(1/2*a)^7*tan(1/2*c)^7 - tan(1/2*a)^7*tan(1/2*c)^6 + tan(1/2*a
)^6*tan(1/2*c)^7 + 3*tan(1/2*a)^7*tan(1/2*c)^5 + tan(1/2*a)^6*tan(1/2*c)^6
 + 3*tan(1/2*a)^5*tan(1/2*c)^7 - 3*tan(1/2*a)^7*tan(1/2*c)^4 + 3*tan(1/2*a
)^6*tan(1/2*c)^5 - 3*tan(1/2*a)^5*tan(1/2*c)^6 + 3*tan(1/2*a)^4*tan(1/2*c)
^7 + 3*tan(1/2*a)^7*tan(1/2*c)^3 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 9*tan(1/2
*a)^5*tan(1/2*c)^5 + 3*tan(1/2*a)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^3*tan(1/2*
c)^7 - 3*tan(1/2*a)^7*tan(1/2*c)^2 + 3*tan(1/2*a)^6*tan(1/2*c)^3 - 9*tan(1
/2*a)^5*tan(1/2*c)^4 + 9*tan(1/2*a)^4*tan(1/2*c)^5 - 3*tan(1/2*a)^3*tan(1/
2*c)^6 + 3*tan(1/2*a)^2*tan(1/2*c)^7 + tan(1/2*a)^7*tan(1/2*c) + 3*tan(1/2
*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^5*tan(1/2*c)^3 + 9*tan(1/2*a)^4*tan(1/2*
c)^4 + 9*tan(1/2*a)^3*tan(1/2*c)^5 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2
*a)*tan(1/2*c)^7 - tan(1/2*a)^7 + tan(1/2*a)^6*tan(1/2*c) - 9*tan(1/2*a)^5
*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^3 - 9*tan(1/2*a)^3*tan(1/2*c)^4
+ 9*tan(1/2*a)^2*tan(1/2*c)^5 - tan(1/2*a)*tan(1/2*c)^6 + tan(1/2*c)^7 + t
an(1/2*a)^6 + 3*tan(1/2*a)^5*tan(1/2*c) + 9*tan(1/2*a)^4*tan(1/2*c)^2 + 9*
tan(1/2*a)^3*tan(1/2*c)^3 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + 3*tan(1/2*a)*tan
(1/2*c)^5 + tan(1/2*c)^6 - 3*tan(1/2*a)^5 + 3*tan(1/2*a)^4*tan(1/2*c) - 9*
tan(1/2*a)^3*tan(1/2*c)^2 + 9*tan(1/2*a)^2*tan(1/2*c)^3 - 3*tan(1/2*a)*tan
(1/2*c)^4 + 3*tan(1/2*c)^5 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^3*tan(1/2*c) +
9*tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*a)*tan(1/2*c)^3 + 3*tan(1/2*c)^...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\tan(c+bx)\,dx = \text{Hanged}$$

input
```
int(tan(c + b*x)/sin(a + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^3(a+bx)\tan(c+bx)\,dx=\int \csc(bx+a)^3\tan(bx+c)\,dx$$

input
```
int(csc(b*x+a)^3*tan(b*x+c),x)
```

output
```
int(csc(a + b*x)**3*tan(b*x + c),x)
```

3.254 $\int \cot(c+bx)\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 1

$$\int \cot(c+bx)\csc^3(a+bx)\,dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 6.66 (sec) , antiderivative size = 198, normalized size of antiderivative = 198.00

$$\int \cot(c+bx)\csc^3(a+bx)\,dx = \frac{-16\operatorname{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\csc^3(a-c)+\cot(a-c)\left(\csc^2\left(\frac{1}{2}(a+bx)\right)+9\csc^2(a-c)\left(\log\left(\text{co}\right.\right.\right.}{}$$

input
```
Integrate[Cot[c + b*x]*Csc[a + b*x]^3,x]
```

output
```
(-16*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Csc[a - c]^3 + Cot[a - c]*(Csc[
(a + b*x)/2]^2 + 9*Csc[a - c]^2*(Log[Cos[(a + b*x)/2]] - Log[Sin[(a + b*x)
/2]]) - Sec[(a + b*x)/2]^2) + Csc[a - c]^2*(-(Cos[3*(a - c)]*Csc[a - c]*(L
og[Cos[(a + b*x)/2]] - Log[Sin[(a + b*x)/2]])) + 4*(-(Csc[a/2]*Csc[(a + b*
x)/2]) + Sec[a/2]*Sec[(a + b*x)/2])*Sin[(b*x)/2]))/(8*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\cot(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \csc^3(a+bx)\cot(bx+c)dx$$

input
```
Int[Cot[c + b*x]*Csc[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.20 (sec) , antiderivative size = 762, normalized size of antiderivative = 762.00

method	result
risch	$-\frac{i\left(e^{i(3bx+7a)}+8\,e^{i(3bx+5a+2c)}-e^{i(3bx+3a+4c)}+e^{i(bx+5a)}-8\,e^{i(bx+3a+2c)}-e^{i(bx+a+4c)}\right)}{b\left(e^{2ia}-e^{2ic}\right)^{2}\left(e^{2i(bx+a)}-1\right)^{2}}+\frac{i\ln\left(e^{i(bx+a)}+1\right)e^{6ia}}{2\left(e^{6ia}-3\,e^{2i(2a+c)}+3\,e^{2i(a+2c)}-e^{6\cdots}\right.}$

input
```
int(cot(b*x+c)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-I/b/(exp(2*I*a)-exp(2*I*c))^2/(exp(2*I*(b*x+a))-1)^2*(exp(I*(3*b*x+7*a))+
8*exp(I*(3*b*x+5*a+2*c))-exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+5*a))-8*exp(I*(
b*x+3*a+2*c))-exp(I*(b*x+a+4*c)))+1/2*I/(exp(6*I*a)-3*exp(2*I*(2*a+c))+3*e
xp(2*I*(a+2*c))-exp(6*I*c))/b*ln(exp(I*(b*x+a))+1)*exp(6*I*a)-9/2*I/(exp(6
*I*a)-3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))-exp(6*I*c))/b*ln(exp(I*(b*x+a)
)+1)*exp(2*I*(2*a+c))-9/2*I/(exp(6*I*a)-3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*
c))-exp(6*I*c))/b*ln(exp(I*(b*x+a))+1)*exp(2*I*(a+2*c))+1/2*I/(exp(6*I*a)-
3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))-exp(6*I*c))/b*ln(exp(I*(b*x+a))+1)*e
xp(6*I*c)-8*I*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b/(exp(6*I*a)-3*exp(2*I*(2*a
+c))+3*exp(2*I*(a+2*c))-exp(6*I*c))*exp(3*I*(a+c))-1/2*I/(exp(6*I*a)-3*exp
(2*I*(2*a+c))+3*exp(2*I*(a+2*c))-exp(6*I*c))/b*ln(exp(I*(b*x+a))-1)*exp(6*
I*a)+9/2*I/(exp(6*I*a)-3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))-exp(6*I*c))/b
*ln(exp(I*(b*x+a))-1)*exp(2*I*(2*a+c))+9/2*I/(exp(6*I*a)-3*exp(2*I*(2*a+c)
)+3*exp(2*I*(a+2*c))-exp(6*I*c))/b*ln(exp(I*(b*x+a))-1)*exp(2*I*(a+2*c))-1
/2*I/(exp(6*I*a)-3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))-exp(6*I*c))/b*ln(ex
p(I*(b*x+a))-1)*exp(6*I*c)+8*I*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b/(exp(6*I*
a)-3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))-exp(6*I*c))*exp(3*I*(a+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.12 (sec) , antiderivative size = 429, normalized size of antiderivative = 429.00

$$\int \cot(c+bx)\csc^3(a+bx)\,dx$$

$$= \frac{2\left(\cos\left(-2\,a+2\,c\right)^2-1\right)\cos\left(bx+a\right)-\frac{4\,\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\cos(bx+a)^2-\cos(-2\,a+2\,c)-1\right)\log\left(\frac{2\,\cos(bx+a)^2\cos(-2\,a+2\,c)}{}\right.}{}}{}$$

input
```
integrate(cot(b*x+c)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(cos(-2*a + 2*c)^2 - 1)*cos(b*x + a) - 4*sqrt(2)*((cos(-2*a + 2*c)
+ 1)*cos(b*x + a)^2 - cos(-2*a + 2*c) - 1)*log((2*cos(b*x + a)^2*cos(-2*a
+ 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*
a + 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a +
 2*c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*co
s(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-
2*a + 2*c) + 1) - ((cos(-2*a + 2*c)^2 - 4*cos(-2*a + 2*c) - 5)*cos(b*x + a
)^2 - cos(-2*a + 2*c)^2 + 4*cos(-2*a + 2*c) + 5)*log(1/2*cos(b*x + a) + 1/
2) + ((cos(-2*a + 2*c)^2 - 4*cos(-2*a + 2*c) - 5)*cos(b*x + a)^2 - cos(-2*
a + 2*c)^2 + 4*cos(-2*a + 2*c) + 5)*log(-1/2*cos(b*x + a) + 1/2) + 8*sin(b
*x + a)*sin(-2*a + 2*c))/(((b*cos(-2*a + 2*c) - b)*cos(b*x + a)^2 - b*cos(
-2*a + 2*c) + b)*sin(-2*a + 2*c))
```

Sympy [F]

$$\int \cot(c+bx)\csc^3(a+bx)\,dx = \int \cot\left(bx+c\right)\csc^3\left(a+bx\right)dx$$

input
```
integrate(cot(b*x+c)*csc(b*x+a)**3,x)
```

output
```
Integral(cot(b*x + c)*csc(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 5.12 (sec) , antiderivative size = 137271, normalized size of antiderivative = 137271.00

$$\int \cot(c+bx)\csc^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(((cos(6*a)^2 - 6*(2*cos(6*a) - cos(6*c))*cos(4*a + 2*c) + 27*cos(4*a
+ 2*c)^2 - 6*(cos(6*a) - 2*cos(6*c))*cos(2*a + 4*c) - 27*cos(2*a + 4*c)^2
- cos(6*c)^2 + sin(6*a)^2 - 6*(2*sin(6*a) - sin(6*c))*sin(4*a + 2*c) + 27*
sin(4*a + 2*c)^2 - 6*(sin(6*a) - 2*sin(6*c))*sin(2*a + 4*c) - 27*sin(2*a +
 4*c)^2 - sin(6*c)^2)*cos(4*b*x + 8*a)^2 + 4*(cos(6*a)^2 - 6*(2*cos(6*a) -
 cos(6*c))*cos(4*a + 2*c) + 27*cos(4*a + 2*c)^2 - 6*(cos(6*a) - 2*cos(6*c)
)*cos(2*a + 4*c) - 27*cos(2*a + 4*c)^2 - cos(6*c)^2 + sin(6*a)^2 - 6*(2*si
n(6*a) - sin(6*c))*sin(4*a + 2*c) + 27*sin(4*a + 2*c)^2 - 6*(sin(6*a) - 2*
sin(6*c))*sin(2*a + 4*c) - 27*sin(2*a + 4*c)^2 - sin(6*c)^2)*cos(4*b*x + 6
*a + 2*c)^2 + (cos(6*a)^2 - 6*(2*cos(6*a) - cos(6*c))*cos(4*a + 2*c) + 27*
cos(4*a + 2*c)^2 - 6*(cos(6*a) - 2*cos(6*c))*cos(2*a + 4*c) - 27*cos(2*a +
 4*c)^2 - cos(6*c)^2 + sin(6*a)^2 - 6*(2*sin(6*a) - sin(6*c))*sin(4*a + 2*
c) + 27*sin(4*a + 2*c)^2 - 6*(sin(6*a) - 2*sin(6*c))*sin(2*a + 4*c) - 27*s
in(2*a + 4*c)^2 - sin(6*c)^2)*cos(4*b*x + 4*a + 4*c)^2 + 4*(cos(6*a)^2 - 6
*(2*cos(6*a) - cos(6*c))*cos(4*a + 2*c) + 27*cos(4*a + 2*c)^2 - 6*(cos(6*a
) - 2*cos(6*c))*cos(2*a + 4*c) - 27*cos(2*a + 4*c)^2 - cos(6*c)^2 + sin(6*
a)^2 - 6*(2*sin(6*a) - sin(6*c))*sin(4*a + 2*c) + 27*sin(4*a + 2*c)^2 - 6*
(sin(6*a) - 2*sin(6*c))*sin(2*a + 4*c) - 27*sin(2*a + 4*c)^2 - sin(6*c)^2)
*cos(2*b*x + 6*a)^2 + 16*(cos(6*a)^2 - 6*(2*cos(6*a) - cos(6*c))*cos(4*a +
 2*c) + 27*cos(4*a + 2*c)^2 - 6*(cos(6*a) - 2*cos(6*c))*cos(2*a + 4*c) ...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.34 (sec) , antiderivative size = 3565, normalized size of antiderivative = 3565.00

$$\int \cot(c + bx) \csc^3(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
1/16*(2*(tan(1/2*a)^7*tan(1/2*c)^6 + 3*tan(1/2*a)^7*tan(1/2*c)^4 + 3*tan(1
/2*a)^5*tan(1/2*c)^6 - 3*tan(1/2*a)^7*tan(1/2*c)^2 + 24*tan(1/2*a)^6*tan(1
/2*c)^3 - 27*tan(1/2*a)^5*tan(1/2*c)^4 + 24*tan(1/2*a)^4*tan(1/2*c)^5 - 3*
tan(1/2*a)^3*tan(1/2*c)^6 - tan(1/2*a)^7 + 27*tan(1/2*a)^5*tan(1/2*c)^2 -
32*tan(1/2*a)^4*tan(1/2*c)^3 + 27*tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)*t
an(1/2*c)^6 - 3*tan(1/2*a)^5 + 24*tan(1/2*a)^4*tan(1/2*c) - 27*tan(1/2*a)^
3*tan(1/2*c)^2 + 24*tan(1/2*a)^2*tan(1/2*c)^3 - 3*tan(1/2*a)*tan(1/2*c)^4
+ 3*tan(1/2*a)^3 + 3*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a))*log(abs(tan(1/2
*b*x)*tan(1/2*a) - 1))/(tan(1/2*a)^7*tan(1/2*c)^3 - 3*tan(1/2*a)^6*tan(1/2
*c)^4 + 3*tan(1/2*a)^5*tan(1/2*c)^5 - tan(1/2*a)^4*tan(1/2*c)^6 + 3*tan(1/
2*a)^6*tan(1/2*c)^2 - 9*tan(1/2*a)^5*tan(1/2*c)^3 + 9*tan(1/2*a)^4*tan(1/2
*c)^4 - 3*tan(1/2*a)^3*tan(1/2*c)^5 + 3*tan(1/2*a)^5*tan(1/2*c) - 9*tan(1/
2*a)^4*tan(1/2*c)^2 + 9*tan(1/2*a)^3*tan(1/2*c)^3 - 3*tan(1/2*a)^2*tan(1/2
*c)^4 + tan(1/2*a)^4 - 3*tan(1/2*a)^3*tan(1/2*c) + 3*tan(1/2*a)^2*tan(1/2*
c)^2 - tan(1/2*a)*tan(1/2*c)^3) - 2*(tan(1/2*a)^6*tan(1/2*c)^7 + 3*tan(1/2
*a)^6*tan(1/2*c)^5 + 3*tan(1/2*a)^4*tan(1/2*c)^7 + 3*tan(1/2*a)^6*tan(1/2*
c)^3 + 9*tan(1/2*a)^4*tan(1/2*c)^5 + 3*tan(1/2*a)^2*tan(1/2*c)^7 + tan(1/2
*a)^6*tan(1/2*c) + 9*tan(1/2*a)^4*tan(1/2*c)^3 + 9*tan(1/2*a)^2*tan(1/2*c)
^5 + tan(1/2*c)^7 + 3*tan(1/2*a)^4*tan(1/2*c) + 9*tan(1/2*a)^2*tan(1/2*c)^
3 + 3*tan(1/2*c)^5 + 3*tan(1/2*a)^2*tan(1/2*c) + 3*tan(1/2*c)^3 + tan(1...
```

Mupad [F(-1)]

Timed out.

$$\int \cot(c+bx)\csc^3(a+bx)\,dx = \text{Hanged}$$

input
```
int(cot(c + b*x)/sin(a + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cot(c+bx)\csc^3(a+bx)\,dx = \int \cot(bx+c)\csc(bx+a)^3\,dx$$

input
```
int(cot(b*x+c)*csc(b*x+a)^3,x)
```

output
```
int(cot(b*x + c)*csc(a + b*x)**3,x)
```

3.255 $\int \cot^2(c+bx)\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \cot^2(c+bx)\csc^3(a+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 4.34 (sec) , antiderivative size = 186, normalized size of antiderivative = 186.00

$$\int \cot^2(c+bx)\csc^3(a+bx)\,dx$$
$$=\frac{96\text{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cot(a-c)\csc^3(a-c)+\csc^3(a-c)\left(-16\csc(c+bx)+(-33-16\,\text{c}\right.}{}$$

input
```
Integrate[Cot[c + b*x]^2*Csc[a + b*x]^3,x]
```

output
```
(96*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cot[a - c]*Csc[a - c]^3 + Csc[a
- c]^3*(-16*Csc[c + b*x] + (-33 - 16*Cos[2*(a - c)] + Cos[4*(a - c)])*Csc[
a - c]*(Log[Cos[(a + b*x)/2]] - Log[Sin[(a + b*x)/2]])) - 2*Cot[a - c]^2*(
Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2) + 64*Cos[a + (b*x)/2]*Cot[a - c]*
Csc[a]*Csc[a - c]^2*Csc[a + b*x]*Sin[(b*x)/2])/(16*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\cot^2(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc^3(a+bx)\cot^2(bx+c)dx$$

input
```
Int[Cot[c + b*x]^2*Csc[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.27 (sec) , antiderivative size = 1263, normalized size of antiderivative = 1263.00

method	result	size
risch	Expression too large to display	1263

input
```
int(cot(b*x+c)^2*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b/(exp(2*I*a)-exp(2*I*c))^3/(exp(2*I*(b*x+a))-1)^2/(-exp(2*I*(b*x+a+c))
+exp(2*I*a))*(-exp(I*(5*b*x+11*a+2*c))-33*exp(I*(5*b*x+9*a+4*c))-15*exp(I*
(5*b*x+7*a+6*c))+exp(I*(5*b*x+5*a+8*c))+exp(I*(3*b*x+11*a))+16*exp(I*(3*b*
x+9*a+2*c))+62*exp(I*(3*b*x+7*a+4*c))+16*exp(I*(3*b*x+5*a+6*c))+exp(I*(3*b
*x+3*a+8*c))+exp(I*(b*x+9*a))-15*exp(I*(b*x+7*a+2*c))-33*exp(I*(b*x+5*a+4*
c))-exp(I*(b*x+3*a+6*c)))+24*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b/(exp(8*I*a)
-4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))-4*exp(2*I*(a+3*c))+exp(8*I*c))*exp(I*
(5*a+3*c))+24*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b/(exp(8*I*a)-4*exp(2*I*(3*a
+c))+6*exp(4*I*(a+c))-4*exp(2*I*(a+3*c))+exp(8*I*c))*exp(I*(3*a+5*c))-1/2/
(exp(8*I*a)-4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))-4*exp(2*I*(a+3*c))+exp(8*I
*c))/b*ln(exp(I*(b*x+a))-1)*exp(8*I*a)+8/(exp(8*I*a)-4*exp(2*I*(3*a+c))+6*
exp(4*I*(a+c))-4*exp(2*I*(a+3*c))+exp(8*I*c))/b*ln(exp(I*(b*x+a))-1)*exp(2
*I*(3*a+c))+33/(exp(8*I*a)-4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))-4*exp(2*I*(
a+3*c))+exp(8*I*c))/b*ln(exp(I*(b*x+a))-1)*exp(4*I*(a+c))+8/(exp(8*I*a)-4*
exp(2*I*(3*a+c))+6*exp(4*I*(a+c))-4*exp(2*I*(a+3*c))+exp(8*I*c))/b*ln(exp(
I*(b*x+a))-1)*exp(2*I*(a+3*c))-1/2/(exp(8*I*a)-4*exp(2*I*(3*a+c))+6*exp(4*
I*(a+c))-4*exp(2*I*(a+3*c))+exp(8*I*c))/b*ln(exp(I*(b*x+a))-1)*exp(8*I*c)+
1/2/(exp(8*I*a)-4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))-4*exp(2*I*(a+3*c))+exp
(8*I*c))/b*ln(exp(I*(b*x+a))+1)*exp(8*I*a)-8/(exp(8*I*a)-4*exp(2*I*(3*a+c)
)+6*exp(4*I*(a+c))-4*exp(2*I*(a+3*c))+exp(8*I*c))/b*ln(exp(I*(b*x+a))+1...
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.15 (sec) , antiderivative size = 905, normalized size of antiderivative = 905.00

$$\int \cot^2(c+bx)\csc^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^2*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(cos(-2*a + 2*c)^3 - 7*cos(-2*a + 2*c)^2 - cos(-2*a + 2*c) + 7)*cos
(b*x + a)*sin(b*x + a) - 12*sqrt(2)*(((cos(-2*a + 2*c)^2 + 2*cos(-2*a + 2*
c) + 1)*cos(b*x + a)^2 - cos(-2*a + 2*c)^2 - 2*cos(-2*a + 2*c) - 1)*sin(b*
x + a) + ((cos(-2*a + 2*c) + 1)*cos(b*x + a)^3 - (cos(-2*a + 2*c) + 1)*cos
(b*x + a))*sin(-2*a + 2*c))*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(
b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*c
os(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - co
s(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(
b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1)
 - (((cos(-2*a + 2*c)^3 - 7*cos(-2*a + 2*c)^2 - 25*cos(-2*a + 2*c) - 17)*c
os(b*x + a)^2 - cos(-2*a + 2*c)^3 + 7*cos(-2*a + 2*c)^2 + 25*cos(-2*a + 2*
c) + 17)*sin(b*x + a) + ((cos(-2*a + 2*c)^2 - 8*cos(-2*a + 2*c) - 17)*cos(
b*x + a)^3 - (cos(-2*a + 2*c)^2 - 8*cos(-2*a + 2*c) - 17)*cos(b*x + a))*si
n(-2*a + 2*c))*log(1/2*cos(b*x + a) + 1/2) + (((cos(-2*a + 2*c)^3 - 7*cos(
-2*a + 2*c)^2 - 25*cos(-2*a + 2*c) - 17)*cos(b*x + a)^2 - cos(-2*a + 2*c)^
3 + 7*cos(-2*a + 2*c)^2 + 25*cos(-2*a + 2*c) + 17)*sin(b*x + a) + ((cos(-2
*a + 2*c)^2 - 8*cos(-2*a + 2*c) - 17)*cos(b*x + a)^3 - (cos(-2*a + 2*c)^2
- 8*cos(-2*a + 2*c) - 17)*cos(b*x + a))*sin(-2*a + 2*c))*log(-1/2*cos(b*x
+ a) + 1/2) + 2*((cos(-2*a + 2*c)^2 - 8*cos(-2*a + 2*c) - 17)*cos(b*x + a)
^2 + 8*cos(-2*a + 2*c) + 16)*sin(-2*a + 2*c))/((b*cos(-2*a + 2*c)^3 - (...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^2(c+bx)\csc^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cot(b*x+c)**2*csc(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 52.89 (sec) , antiderivative size = 919799, normalized size of antiderivative = 919799.00

$$\int \cot^2(c + bx) \csc^3(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^2*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(24*(((sin(8*a) + sin(8*c))*cos(6*a + 2*c) + 6*(sin(8*a) - 5*sin(6*a +
 2*c) + sin(8*c))*cos(4*a + 4*c) + (sin(8*a) + 30*sin(4*a + 4*c) + sin(8*c
))*cos(2*a + 6*c) - (cos(8*a) + cos(8*c))*sin(6*a + 2*c) - 6*(cos(8*a) - 5
*cos(6*a + 2*c) + cos(8*c))*sin(4*a + 4*c) - (cos(8*a) + 30*cos(4*a + 4*c)
 + cos(8*c))*sin(2*a + 6*c))*cos(6*b*x + 10*a + 2*c)^2 + 9*((sin(8*a) + si
n(8*c))*cos(6*a + 2*c) + 6*(sin(8*a) - 5*sin(6*a + 2*c) + sin(8*c))*cos(4*
a + 4*c) + (sin(8*a) + 30*sin(4*a + 4*c) + sin(8*c))*cos(2*a + 6*c) - (cos
(8*a) + cos(8*c))*sin(6*a + 2*c) - 6*(cos(8*a) - 5*cos(6*a + 2*c) + cos(8*
c))*sin(4*a + 4*c) - (cos(8*a) + 30*cos(4*a + 4*c) + cos(8*c))*sin(2*a + 6
*c))*cos(6*b*x + 8*a + 4*c)^2 + 9*((sin(8*a) + sin(8*c))*cos(6*a + 2*c) +
6*(sin(8*a) - 5*sin(6*a + 2*c) + sin(8*c))*cos(4*a + 4*c) + (sin(8*a) + 30
*sin(4*a + 4*c) + sin(8*c))*cos(2*a + 6*c) - (cos(8*a) + cos(8*c))*sin(6*a
 + 2*c) - 6*(cos(8*a) - 5*cos(6*a + 2*c) + cos(8*c))*sin(4*a + 4*c) - (cos
(8*a) + 30*cos(4*a + 4*c) + cos(8*c))*sin(2*a + 6*c))*cos(6*b*x + 6*a + 6*
c)^2 + ((sin(8*a) + sin(8*c))*cos(6*a + 2*c) + 6*(sin(8*a) - 5*sin(6*a + 2
*c) + sin(8*c))*cos(4*a + 4*c) + (sin(8*a) + 30*sin(4*a + 4*c) + sin(8*c))
*cos(2*a + 6*c) - (cos(8*a) + cos(8*c))*sin(6*a + 2*c) - 6*(cos(8*a) - 5*c
os(6*a + 2*c) + cos(8*c))*sin(4*a + 4*c) - (cos(8*a) + 30*cos(4*a + 4*c) +
 cos(8*c))*sin(2*a + 6*c))*cos(6*b*x + 4*a + 8*c)^2 + ((sin(8*a) + sin(8*c
))*cos(6*a + 2*c) + 6*(sin(8*a) - 5*sin(6*a + 2*c) + sin(8*c))*cos(4*a ...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.82 (sec) , antiderivative size = 7181, normalized size of antiderivative = 7181.00

$$\int \cot^2(c+bx)\csc^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+c)^2*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/32*(2*(3*tan(1/2*a)^9*tan(1/2*c)^8 + 6*tan(1/2*a)^9*tan(1/2*c)^6 + 12*t
an(1/2*a)^8*tan(1/2*c)^7 + 6*tan(1/2*a)^7*tan(1/2*c)^8 - 2*tan(1/2*a)^9*ta
n(1/2*c)^4 + 44*tan(1/2*a)^8*tan(1/2*c)^5 + 44*tan(1/2*a)^6*tan(1/2*c)^7 -
 2*tan(1/2*a)^5*tan(1/2*c)^8 + 6*tan(1/2*a)^9*tan(1/2*c)^2 - 44*tan(1/2*a)
^8*tan(1/2*c)^3 + 212*tan(1/2*a)^7*tan(1/2*c)^4 - 180*tan(1/2*a)^6*tan(1/2
*c)^5 + 212*tan(1/2*a)^5*tan(1/2*c)^6 - 44*tan(1/2*a)^4*tan(1/2*c)^7 + 6*t
an(1/2*a)^3*tan(1/2*c)^8 + 3*tan(1/2*a)^9 - 12*tan(1/2*a)^8*tan(1/2*c) + 1
80*tan(1/2*a)^6*tan(1/2*c)^3 - 132*tan(1/2*a)^5*tan(1/2*c)^4 + 180*tan(1/2
*a)^4*tan(1/2*c)^5 - 12*tan(1/2*a)^2*tan(1/2*c)^7 + 3*tan(1/2*a)*tan(1/2*c
)^8 + 6*tan(1/2*a)^7 - 44*tan(1/2*a)^6*tan(1/2*c) + 212*tan(1/2*a)^5*tan(1
/2*c)^2 - 180*tan(1/2*a)^4*tan(1/2*c)^3 + 212*tan(1/2*a)^3*tan(1/2*c)^4 -
44*tan(1/2*a)^2*tan(1/2*c)^5 + 6*tan(1/2*a)*tan(1/2*c)^6 - 2*tan(1/2*a)^5
+ 44*tan(1/2*a)^4*tan(1/2*c) + 44*tan(1/2*a)^2*tan(1/2*c)^3 - 2*tan(1/2*a)
*tan(1/2*c)^4 + 6*tan(1/2*a)^3 + 12*tan(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*a)
*tan(1/2*c)^2 + 3*tan(1/2*a))*log(abs(tan(1/2*b*x)*tan(1/2*a) - 1))/(tan(1
/2*a)^9*tan(1/2*c)^4 - 4*tan(1/2*a)^8*tan(1/2*c)^5 + 6*tan(1/2*a)^7*tan(1/
2*c)^6 - 4*tan(1/2*a)^6*tan(1/2*c)^7 + tan(1/2*a)^5*tan(1/2*c)^8 + 4*tan(1
/2*a)^8*tan(1/2*c)^3 - 16*tan(1/2*a)^7*tan(1/2*c)^4 + 24*tan(1/2*a)^6*tan(
1/2*c)^5 - 16*tan(1/2*a)^5*tan(1/2*c)^6 + 4*tan(1/2*a)^4*tan(1/2*c)^7 + 6*
tan(1/2*a)^7*tan(1/2*c)^2 - 24*tan(1/2*a)^6*tan(1/2*c)^3 + 36*tan(1/2*a...
```

Mupad [B] (verification not implemented)

Time = 49.98 (sec) , antiderivative size = 74632, normalized size of antiderivative = 74632.00

$$\int \cot^2(c+bx)\csc^3(a+bx)\,dx = \text{Too large to display}$$

input
```
int(cot(c + b*x)^2/sin(a + b*x)^3,x)
```

output
```
((6*sin(a)^4 - cos(a)^4*cot(c)^2 + 5*cos(a)^2*sin(a)^2 + 8*cot(c)^2*sin(a)
^4 + 2*cot(c)^4*sin(a)^4 + 5*cos(a)*cot(c)*sin(a)^3 + 2*cos(a)^3*cot(c)*si
n(a) + 4*cos(a)*cot(c)^3*sin(a)^3 + 5*cos(a)^3*cot(c)^3*sin(a) + 10*cos(a)
^2*cot(c)^2*sin(a)^2 + 2*cos(a)^2*cot(c)^4*sin(a)^2)/((cos(a)^2 + sin(a)^2
)*(cos(a)^3 - cot(c)^3*sin(a)^3 - 3*cos(a)^2*cot(c)*sin(a) + 3*cos(a)*cot(
c)^2*sin(a)^2)) - (tan((b*x)/2)^4*(3*cos(a)^2*sin(a)^4 - 2*cos(a)^6*cot(c)
^2 - 6*sin(a)^6 + 6*cos(a)^4*sin(a)^2 - 6*cot(c)^2*sin(a)^6 - 2*cot(c)^4*s
in(a)^6 + 9*cos(a)*cot(c)*sin(a)^5 + 4*cos(a)^5*cot(c)*sin(a) + 20*cos(a)^
3*cot(c)*sin(a)^3 + 24*cos(a)*cot(c)^3*sin(a)^5 + 6*cos(a)^5*cot(c)^3*sin(
a) + 8*cos(a)*cot(c)^5*sin(a)^5 + 6*cos(a)^2*cot(c)^2*sin(a)^4 + 13*cos(a)
^4*cot(c)^2*sin(a)^2 + 33*cos(a)^3*cot(c)^3*sin(a)^3 + 8*cos(a)^2*cot(c)^4
*sin(a)^4 + 12*cos(a)^4*cot(c)^4*sin(a)^2 + 8*cos(a)^3*cot(c)^5*sin(a)^3))
/(sin(a)^2*(cos(a)^2 + sin(a)^2)*(cos(a)^3 - cot(c)^3*sin(a)^3 - 3*cos(a)^
2*cot(c)*sin(a) + 3*cos(a)*cot(c)^2*sin(a)^2)) + (2*tan((b*x)/2)^2*(3*cos(
a)^2*sin(a)^4 - cos(a)^6*cot(c)^2 - 6*sin(a)^6 + 7*cos(a)^4*sin(a)^2 - 9*c
ot(c)^2*sin(a)^6 - 2*cot(c)^4*sin(a)^6 + 12*cos(a)*cot(c)*sin(a)^5 + 2*cos
(a)^5*cot(c)*sin(a) + 17*cos(a)^3*cot(c)*sin(a)^3 + 14*cos(a)*cot(c)^3*sin
(a)^5 + 3*cos(a)^5*cot(c)^3*sin(a) + 4*cos(a)*cot(c)^5*sin(a)^5 + 18*cos(a
)^2*cot(c)^2*sin(a)^4 + 23*cos(a)^4*cot(c)^2*sin(a)^2 + 24*cos(a)^3*cot(c)
^3*sin(a)^3 + 13*cos(a)^2*cot(c)^4*sin(a)^4 + 18*cos(a)^4*cot(c)^4*sin(...
```

Reduce [F]

$$\int \cot^2(c+bx)\csc^3(a+bx)\,dx = \int \cot(bx+c)^2\csc(bx+a)^3\,dx$$

input
```
int(cot(b*x+c)^2*csc(b*x+a)^3,x)
```

output
```
int(cot(b*x + c)**2*csc(a + b*x)**3,x)
```

3.256 $\int \sin(a+bx)\tan(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 116

$$\int \sin(a+bx)\tan(c+dx)\,dx = \frac{i\cos(a+bx)}{b} - \frac{ie^{-i(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b} - \frac{ie^{i(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},-e^{2i(c+dx)}\right)}{b}$$

output
```
I*cos(b*x+a)/b-I*hypergeom([1, -1/2*b/d],[1-1/2*b/d],-exp(2*I*(d*x+c)))/b/
exp(I*(b*x+a))-I*exp(I*(b*x+a))*hypergeom([1, 1/2*b/d],[1+1/2*b/d],-exp(2*
I*(d*x+c)))/b
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\tan(c+dx)\,dx = -\frac{ie^{-i(a+bx)}\left(-1-e^{2i(a+bx)}+2\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)+2e^{2i(a+bx)}\,\text{Hypergeome}\right.}{2b}$$

input
```
Integrate[Sin[a + b*x]*Tan[c + d*x],x]
```

output
```
((-1/2*I)*(-1 - E^((2*I)*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 -
 b/(2*d), -E^((2*I)*(c + d*x))] + 2*E^((2*I)*(a + b*x))*Hypergeometric2F1[
1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))]))/(b*E^(I*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5068, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan(c+dx)\,dx$$

$$\Big\downarrow \text{5068}$$

$$\int \left(-\frac{e^{-i(a+bx)}}{1+e^{2i(c+dx)}}+\frac{e^{i(a+bx)}}{1+e^{2i(c+dx)}}+\frac{1}{2}e^{-i(a+bx)}-\frac{1}{2}e^{i(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ie^{-i(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b}-\frac{ie^{i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,-e^{2i(c+dx)}\right)}{b}+\frac{ie^{-i(a+bx)}}{2b}+\frac{ie^{i(a+bx)}}{2b}$$

input
```
Int[Sin[a + b*x]*Tan[c + d*x],x]
```

output
```
(I/2)/(b*E^(I*(a + b*x))) + ((I/2)*E^(I*(a + b*x)))/b - (I*Hypergeometric2
F1[1, -1/2*b/d, 1 - b/(2*d), -E^((2*I)*(c + d*x))])/(b*E^(I*(a + b*x))) -
(I*E^(I*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c
 + d*x))])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5068
```
Int[Sin[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(E^
(I*(a + b*x))*2) - E^(I*(a + b*x))/2 - 1/(E^(I*(a + b*x))*(1 + E^(2*I*(c +
d*x)))) + E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

Maple [F]

$$\int \sin (bx + a) \tan (dx + c) \, dx$$

input
```
int(sin(b*x+a)*tan(d*x+c),x)
```

output
```
int(sin(b*x+a)*tan(d*x+c),x)
```

Fricas [F]

$$\int \sin(a + bx) \tan(c + dx) \, dx = \int \sin (bx + a) \tan (dx + c) \, dx$$

input
```
integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="fricas")
```

output
```
integral(sin(b*x + a)*tan(d*x + c), x)
```

Sympy [F]

$$\int \sin(a+bx)\tan(c+dx)\,dx = \int \sin{(a+bx)}\tan{(c+dx)}\,dx$$

input
```
integrate(sin(b*x+a)*tan(d*x+c),x)
```

output
```
Integral(sin(a + b*x)*tan(c + d*x), x)
```

Maxima [F]

$$\int \sin(a+bx)\tan(c+dx)\,dx = \int \sin(bx+a)\tan(dx+c)\,dx$$

input
```
integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)*tan(d*x + c), x)
```

Giac [F]

$$\int \sin(a+bx)\tan(c+dx)\,dx = \int \sin(bx+a)\tan(dx+c)\,dx$$

input
```
integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*tan(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\tan(c+dx)\,dx = \int \sin\left(a+b\,x\right)\,\tan(c+d\,x)\,dx$$

input
```
int(sin(a + b*x)*tan(c + d*x),x)
```

output
```
int(sin(a + b*x)*tan(c + d*x), x)
```

Reduce [F]

$$\int \sin(a+bx)\tan(c+dx)\,dx = \int \sin\left(bx+a\right)\tan\left(dx+c\right)dx$$

input
```
int(sin(b*x+a)*tan(d*x+c),x)
```

output
```
int(sin(a + b*x)*tan(c + d*x),x)
```

3.257 $\int \cot(c+dx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 112

$$\begin{aligned}\int \cot(c+dx)\sin(a+bx)\,dx &= -\frac{i\cos(a+bx)}{b} + \frac{ie^{-i(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)}{b} \\ &\quad + \frac{ie^{i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},e^{2i(c+dx)}\right)}{b}\end{aligned}$$

output

```
-I*cos(b*x+a)/b+I*hypergeom([1, -1/2*b/d],[1-1/2*b/d],exp(2*I*(d*x+c)))/b/
exp(I*(b*x+a))+I*exp(I*(b*x+a))*hypergeom([1, 1/2*b/d],[1+1/2*b/d],exp(2*I
*(d*x+c)))/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 260 vs. 2(112) = 224.

Time = 1.55 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.32

$$\int \cot(c+dx)\sin(a+bx)\,dx = \frac{-\cos(a)\cos(bx)\cot(c) - \frac{ie^{-i(a-2c+bx)}\left(be^{2idx}\,\text{Hypergeometric2F1}\left(1,1-\frac{b}{2d},2-\frac{b}{2d},e^{2i(c+dx)}\right)-(b-2d)\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},\right.\right.}{(b-2d)(-1+e^{2ic})}}{}$$

input
```
Integrate[Cot[c + d*x]*Sin[a + b*x],x]
```

output
```
(-(Cos[a]*Cos[b*x]*Cot[c]) - (I*(b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 -
b/(2*d), 2 - b/(2*d), E^((2*I)*(c + d*x))] - (b - 2*d)*Hypergeometric2F1[1
, -1/2*b/d, 1 - b/(2*d), E^((2*I)*(c + d*x))]))/((b - 2*d)*E^(I*(a - 2*c +
 b*x))*(-1 + E^((2*I)*c))) - (I*E^(I*(a + 2*c + b*x))*(b*E^((2*I)*d*x)*Hyp
ergeometric2F1[1, 1 + b/(2*d), 2 + b/(2*d), E^((2*I)*(c + d*x))] - (b + 2*
d)*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x))]))/((b +
 2*d)*(-1 + E^((2*I)*c))) + Cot[c]*Sin[a]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{5070, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cot(c+dx)\,dx$$

$$\Big\downarrow \text{5070}$$

$$\int \left(\frac{e^{-i(a+bx)}}{1-e^{2i(c+dx)}}-\frac{e^{i(a+bx)}}{1-e^{2i(c+dx)}}-\frac{1}{2}e^{-i(a+bx)}+\frac{1}{2}e^{i(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ie^{-i(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)}{b}+\frac{ie^{i(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,e^{2i(c+dx)}\right)}{b}-\frac{ie^{-i(a+bx)}}{2b}-\frac{ie^{i(a+bx)}}{2b}$$

input
```
Int[Cot[c + d*x]*Sin[a + b*x],x]
```

output
```
(-1/2*I)/(b*E^(I*(a + b*x))) - ((I/2)*E^(I*(a + b*x)))/b + (I*Hypergeometr
ic2F1[1, -1/2*b/d, 1 - b/(2*d), E^((2*I)*(c + d*x))])/(b*E^(I*(a + b*x)))
+ (I*E^(I*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(
c + d*x))])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5070
```
Int[Cot[(c_.) + (d_.)*(x_)]*Sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Int[-E^((
-I)*(a + b*x))/2 + E^(I*(a + b*x))/2 + 1/(E^(I*(a + b*x))*(1 - E^(2*I*(c +
d*x)))) - E^(I*(a + b*x))/(1 - E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

Maple [F]

$$\int \cot (dx+c)\sin (bx+a)\,dx$$

input
```
int(cot(d*x+c)*sin(b*x+a),x)
```

output
```
int(cot(d*x+c)*sin(b*x+a),x)
```

Fricas [F]

$$\int \cot(c+dx)\sin(a+bx)\,dx=\int \cot (dx+c)\sin (bx+a)\;dx$$

input
```
integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
integral(cot(d*x + c)*sin(b*x + a), x)
```

Sympy [F]

$$\int \cot(c+dx)\sin(a+bx)\,dx = \int \sin(a+bx)\cot(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)*sin(b*x+a),x)
```

output
```
Integral(sin(a + b*x)*cot(c + d*x), x)
```

Maxima [F]

$$\int \cot(c+dx)\sin(a+bx)\,dx = \int \cot(dx+c)\sin(bx+a)\,dx$$

input
```
integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)*sin(b*x + a), x)
```

Giac [F]

$$\int \cot(c+dx)\sin(a+bx)\,dx = \int \cot(dx+c)\sin(bx+a)\,dx$$

input
```
integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(cot(d*x + c)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)\sin(a+bx)\,dx=\int \cot(c+d\,x)\;\sin\left(a+b\,x\right)\;dx$$

input
```
int(cot(c + d*x)*sin(a + b*x),x)
```

output
```
int(cot(c + d*x)*sin(a + b*x), x)
```

Reduce [F]

$$\int \cot(c+dx)\sin(a+bx)\,dx=\int \cot\left(dx+c\right)\sin\left(bx+a\right)dx$$

input
```
int(cot(d*x+c)*sin(b*x+a),x)
```

output
```
int(cot(c + d*x)*sin(a + b*x),x)
```

3.258 $\int \sin^2(a+bx)\tan^2(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 168

$$\begin{aligned}\int \sin^2(a+bx)\tan^2(c+dx)\,dx &= \frac{ie^{-2i(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{d},1-\frac{b}{d},-e^{2i(c+dx)}\right)}{2d} \\ &\quad+\frac{ie^{2i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{b}{d},\frac{b+d}{d},-e^{2i(c+dx)}\right)}{2d} \\ &\quad+\frac{\sec(c)\sec(c+dx)\sin(dx)\sin^2(a+bx)}{d} \\ &\quad+\frac{-2bx+\sin(2(a+bx))}{4b}+\frac{\cos(2a+2bx)\sec^2(c)}{2id-2d\tan(c)}\end{aligned}$$

output
```
1/2*I*hypergeom([1, -b/d],[1-b/d],-exp(2*I*(d*x+c)))/d/exp(2*I*(b*x+a))+1/
2*I*exp(2*I*(b*x+a))*hypergeom([1, b/d],[(b+d)/d],-exp(2*I*(d*x+c)))/d+sec
(c)*sec(d*x+c)*sin(d*x)*sin(b*x+a)^2/d+1/4*(-2*b*x+sin(2*b*x+2*a))/b+cos(2
*b*x+2*a)*sec(c)^2/(2*I*d-2*d*tan(c))
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.06

$$\int \sin^2(a+bx)\tan^2(c+dx)\,dx$$
$$= \frac{ie^{-2i(a+bx)}\left(-1-e^{4i(a+bx)}+\left(1+e^{2ic}\right)\text{Hypergeometric2F1}\left(1,-\frac{b}{d},1-\frac{b}{d},-e^{2i(c+dx)}\right)+e^{4i(a+bx)}\left(1+e^{2ic}\right)\right.}{2d\left(1+e^{2ic}\right)}$$
$$+\frac{\sec(c)\sec(c+dx)\sin(dx)\sin^2(a+bx)}{d}+\frac{-2bx+\sin(2(a+bx))}{4b}$$

input
```
Integrate[Sin[a + b*x]^2*Tan[c + d*x]^2,x]
```

output
```
((I/2)*(-1 - E^((4*I)*(a + b*x)) + (1 + E^((2*I)*c))*Hypergeometric2F1[1,
-(b/d), 1 - b/d, -E^((2*I)*(c + d*x))] + E^((4*I)*(a + b*x))*(1 + E^((2*I)
*c))*Hypergeometric2F1[1, b/d, (b + d)/d, -E^((2*I)*(c + d*x))]))/(d*E^((2
*I)*(a + b*x))*(1 + E^((2*I)*c))) + (Sec[c]*Sec[c + d*x]*Sin[d*x]*Sin[a +
b*x]^2)/d + (-2*b*x + Sin[2*(a + b*x)])/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\tan^2(c+dx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \sin^2(a+bx)\tan^2(c+dx)dx$$

input
```
Int[Sin[a + b*x]^2*Tan[c + d*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \sin(bx+a)^2 \tan(dx+c)^2 \, dx$$

input
```
int(sin(b*x+a)^2*tan(d*x+c)^2,x)
```

output
```
int(sin(b*x+a)^2*tan(d*x+c)^2,x)
```

Fricas [F]

$$\int \sin^2(a+bx)\tan^2(c+dx)\,dx = \int \sin(bx+a)^2 \tan(dx+c)^2 \,dx$$

input
```
integrate(sin(b*x+a)^2*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*tan(d*x + c)^2, x)
```

Sympy [F]

$$\int \sin^2(a+bx)\tan^2(c+dx)\,dx = \int \sin^2(a+bx)\tan^2(c+dx)\,dx$$

input
```
integrate(sin(b*x+a)**2*tan(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**2*tan(c + d*x)**2, x)
```

Maxima [F]

$$\int \sin^2(a+bx)\tan^2(c+dx)\,dx = \int \sin\left(bx+a\right)^2\tan\left(dx+c\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*(4*b*d*x*cos(2*(b + d)*x + 2*a + 2*c)^2 + 4*b*d*x*cos(2*b*x + 2*a)^2
+ 4*b*d*x*sin(2*(b + d)*x + 2*a + 2*c)^2 + 4*b*d*x*sin(2*b*x + 2*a)^2 - (4
*b + d)*cos(2*b*x + 2*a)*sin(4*b*x + 4*a) + (4*b + d)*cos(4*b*x + 4*a)*sin
(2*b*x + 2*a) - d*cos(2*d*x + 2*c)*sin(2*b*x + 2*a) + d*cos(2*b*x + 2*a)*s
in(2*d*x + 2*c) + (d*sin(2*(b + d)*x + 2*a + 2*c) + d*sin(2*b*x + 2*a))*co
s(2*(2*b + d)*x + 4*a + 2*c) + (8*b*d*x*cos(2*b*x + 2*a) - (4*b + d)*sin(4
*b*x + 4*a) + 8*b*sin(2*b*x + 2*a) + d*sin(2*d*x + 2*c))*cos(2*(b + d)*x +
 2*a + 2*c) + 8*(b*d*cos(2*(b + d)*x + 2*a + 2*c)^2 + 2*b*d*cos(2*(b + d)*
x + 2*a + 2*c)*cos(2*b*x + 2*a) + b*d*cos(2*b*x + 2*a)^2 + b*d*sin(2*(b +
d)*x + 2*a + 2*c)^2 + 2*b*d*sin(2*(b + d)*x + 2*a + 2*c)*sin(2*b*x + 2*a)
+ b*d*sin(2*b*x + 2*a)^2)*integrate((b*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) +
 b*sin(2*(b + d)*x + 2*a + 2*c)*sin(4*b*x + 4*a) + b*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) + (b*cos(4*b*x + 4*a) - b)*cos(2*(b + d)*x + 2*a + 2*c) - b*c
os(2*b*x + 2*a))/(d*cos(2*(b + d)*x + 2*a + 2*c)^2 + 2*d*cos(2*(b + d)*x +
 2*a + 2*c)*cos(2*b*x + 2*a) + d*cos(2*b*x + 2*a)^2 + d*sin(2*(b + d)*x +
2*a + 2*c)^2 + 2*d*sin(2*(b + d)*x + 2*a + 2*c)*sin(2*b*x + 2*a) + d*sin(2
*b*x + 2*a)^2), x) - (d*cos(2*(b + d)*x + 2*a + 2*c) + d*cos(2*b*x + 2*a))
*sin(2*(2*b + d)*x + 4*a + 2*c) + (8*b*d*x*sin(2*b*x + 2*a) + (4*b + d)*co
s(4*b*x + 4*a) - 8*b*cos(2*b*x + 2*a) - d*cos(2*d*x + 2*c) + 4*b - d)*sin(
2*(b + d)*x + 2*a + 2*c) + (4*b - d)*sin(2*b*x + 2*a))/(b*d*cos(2*(b + ...
```

Giac [F]

$$\int \sin^2(a+bx)\tan^2(c+dx)\,dx = \int \sin\left(bx+a\right)^2\tan\left(dx+c\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2*tan(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\tan^2(c+dx)\,dx = \int \sin(a+b\,x)^2\tan(c+d\,x)^2\,dx$$

input
```
int(sin(a + b*x)^2*tan(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)^2*tan(c + d*x)^2, x)
```

Reduce [F]

$$\int \sin^2(a+bx)\tan^2(c+dx)\,dx = \int \sin(bx+a)^2\tan(dx+c)^2\,dx$$

input
```
int(sin(b*x+a)^2*tan(d*x+c)^2,x)
```

output
```
int(sin(a + b*x)**2*tan(c + d*x)**2,x)
```

3.259 $\int \sin^2(a+bx)\tan(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 142

$$\int \sin^2(a+bx)\tan(c+dx)\,dx = \frac{ix}{2}+\frac{e^{-2i(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{d},1-\frac{b}{d},-e^{2i(c+dx)}\right)}{4b} - \frac{e^{2i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{b}{d},\frac{b+d}{d},-e^{2i(c+dx)}\right)}{4b} - \frac{\log\left(1+e^{2i(c+dx)}\right)}{2d}+\frac{i\cos(a+bx)\sin(a+bx)}{2b}$$

output
```
1/2*I*x+1/4*hypergeom([1, -b/d],[1-b/d],-exp(2*I*(d*x+c)))/b/exp(2*I*(b*x+
a))-1/4*exp(2*I*(b*x+a))*hypergeom([1, b/d],[(b+d)/d],-exp(2*I*(d*x+c)))/b
-1/2*ln(1+exp(2*I*(d*x+c)))/d+1/2*I*cos(b*x+a)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 5.77 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.10

$$\int \sin^2(a+bx)\tan(c+dx)\,dx =$$
$$-\frac{e^{-2i(a+bx)}\left(d-de^{4i(a+bx)}-4ibde^{2i(a+bx)}x-2d\,\text{Hypergeometric2F1}\left(1,-\frac{b}{d},1-\frac{b}{d},-e^{2i(c+dx)}\right)+2de^{4i(a}\right.}{8bd}$$

input
```
Integrate[Sin[a + b*x]^2*Tan[c + d*x],x]
```

output
```
-1/8*(d - d*E^((4*I)*(a + b*x)) - (4*I)*b*d*E^((2*I)*(a + b*x))*x - 2*d*Hy
pergeometric2F1[1, -(b/d), 1 - b/d, -E^((2*I)*(c + d*x))] + 2*d*E^((4*I)*(
a + b*x))*Hypergeometric2F1[1, b/d, (b + d)/d, -E^((2*I)*(c + d*x))] + 4*b
*E^((2*I)*(a + b*x))*Log[1 + E^((2*I)*(c + d*x))])/(b*d*E^((2*I)*(a + b*x)
))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\tan(c+dx)\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \sin^2(a+bx)\tan(c+dx)dx$$

input
```
Int[Sin[a + b*x]^2*Tan[c + d*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \sin\left(bx+a\right)^{2}\tan\left(dx+c\right)dx$$

input
```
int(sin(b*x+a)^2*tan(d*x+c),x)
```

output
```
int(sin(b*x+a)^2*tan(d*x+c),x)
```

Fricas [F]

$$\int \sin^2(a+bx)\tan(c+dx)\,dx=\int \sin\left(bx+a\right)^{2}\tan\left(dx+c\right)\,dx$$

input
```
integrate(sin(b*x+a)^2*tan(d*x+c),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*tan(d*x + c), x)
```

Sympy [F]

$$\int \sin^2(a+bx)\tan(c+dx)\,dx=\int \sin^{2}\left(a+bx\right)\tan\left(c+dx\right)dx$$

input
```
integrate(sin(b*x+a)**2*tan(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**2*tan(c + d*x), x)
```

Maxima [F]

$$\int \sin^2(a+bx)\tan(c+dx)\,dx = \int \sin{(bx+a)}^2\tan{(dx+c)}\,dx$$

input
```
integrate(sin(b*x+a)^2*tan(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2*tan(d*x + c), x)
```

Giac [F]

$$\int \sin^2(a+bx)\tan(c+dx)\,dx = \int \sin{(bx+a)}^2\tan{(dx+c)}\,dx$$

input
```
integrate(sin(b*x+a)^2*tan(d*x+c),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2*tan(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\tan(c+dx)\,dx = \int \sin{(a+b\,x)}^2\tan(c+d\,x)\,dx$$

input
```
int(sin(a + b*x)^2*tan(c + d*x),x)
```

output
```
int(sin(a + b*x)^2*tan(c + d*x), x)
```

Reduce [F]

$$\int \sin^2(a+bx)\tan(c+dx)\,dx = \int \sin\left(bx+a\right)^2\tan\left(dx+c\right)dx$$

input
```
int(sin(b*x+a)^2*tan(d*x+c),x)
```

output
```
int(sin(a + b*x)**2*tan(c + d*x),x)
```

3.260 $\int \cot(c+dx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 293

$$\begin{aligned}\int \cot(c+dx)\sin^2(a+bx)\,dx &= \frac{e^{-2i(a-c)-2i(b-d)x}\,\text{Hypergeometric2F1}\left(1,1-\frac{b}{d},2-\frac{b}{d},e^{2i(c+dx)}\right)}{4(b-d)\left(-1+e^{2ic}\right)}\\ &\quad-\frac{e^{-2i(a-c)-2ibx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{d},1-\frac{b}{d},e^{2i(c+dx)}\right)}{4b\left(-1+e^{2ic}\right)}\\ &\quad+\frac{e^{2i(a+c)+2ibx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{d},\frac{b+d}{d},e^{2i(c+dx)}\right)}{4b\left(-1+e^{2ic}\right)}\\ &\quad-\frac{e^{2i(a+c)+2i(b+d)x}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{d},2+\frac{b}{d},e^{2i(c+dx)}\right)}{4(b+d)\left(-1+e^{2ic}\right)}\\ &\quad+\frac{\log(\sin(c+dx))}{2d}-\frac{\cot(c)\sin(2a+2bx)}{4b}\end{aligned}$$

output
```
1/4*exp(-2*I*(a-c)-2*I*(b-d)*x)*hypergeom([1, 1-b/d],[2-b/d],exp(2*I*(d*x+
c)))/(b-d)/(-1+exp(2*I*c))-1/4*exp(-2*I*(a-c)-2*I*b*x)*hypergeom([1, -b/d]
,[1-b/d],exp(2*I*(d*x+c)))/b/(-1+exp(2*I*c))+1/4*exp(2*I*(a+c)+2*I*b*x)*hy
pergeom([1, b/d],[(b+d)/d],exp(2*I*(d*x+c)))/b/(-1+exp(2*I*c))-1/4*exp(2*I
*(a+c)+2*I*(b+d)*x)*hypergeom([1, (b+d)/d],[2+b/d],exp(2*I*(d*x+c)))/(b+d)
/(-1+exp(2*I*c))+1/2*ln(sin(d*x+c))/d-1/4*cot(c)*sin(2*b*x+2*a)/b
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.92

$$\int \cot(c+dx)\sin^2(a+bx)\,dx$$

$$= \frac{1}{4}\left(\frac{e^{-2i(a-c)}\left(\frac{e^{-2i(b-d)x}\,\text{Hypergeometric2F1}\left(1,1-\frac{b}{d},2-\frac{b}{d},e^{2i(c+dx)}\right)}{b-d}-\frac{e^{-2ibx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{d},1-\frac{b}{d},e^{2i(c+dx)}\right)}{b}\right)}{-1+e^{2ic}}\right.$$

$$+\frac{e^{2i(a+c)}\left((b+d)e^{2ibx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{d},\frac{b+d}{d},e^{2i(c+dx)}\right)-be^{2i(b+d)x}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{d},2\right.\right.}{b(b+d)\left(-1+e^{2ic}\right)}$$

$$\left.+\frac{2\log(\sin(c+dx))}{d}-\frac{\cos(2bx)\cot(c)\sin(2a)}{b}-\frac{\cos(2a)\cot(c)\sin(2bx)}{b}\right)$$

input
```
Integrate[Cot[c + d*x]*Sin[a + b*x]^2,x]
```

output
```
((Hypergeometric2F1[1, 1 - b/d, 2 - b/d, E^((2*I)*(c + d*x))]/((b - d)*E^(
(2*I)*(b - d)*x)) - Hypergeometric2F1[1, -(b/d), 1 - b/d, E^((2*I)*(c + d*
x))]/(b*E^((2*I)*b*x)))/(E^((2*I)*(a - c))*(-1 + E^((2*I)*c))) + (E^((2*I)
*(a + c))*((b + d)*E^((2*I)*b*x)*Hypergeometric2F1[1, b/d, (b + d)/d, E^((
2*I)*(c + d*x))] - b*E^((2*I)*(b + d)*x)*Hypergeometric2F1[1, (b + d)/d, 2
 + b/d, E^((2*I)*(c + d*x))]))/(b*(b + d)*(-1 + E^((2*I)*c))) + (2*Log[Sin
[c + d*x]])/d - (Cos[2*b*x]*Cot[c]*Sin[2*a])/b - (Cos[2*a]*Cot[c]*Sin[2*b*
x])/b)/4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cot(c+dx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin^2(a+bx)\cot(c+dx)dx$$

input
```
Int[Cot[c + d*x]*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cot\left(dx+c\right)\sin\left(bx+a\right)^{2}dx$$

input
```
int(cot(d*x+c)*sin(b*x+a)^2,x)
```

output
```
int(cot(d*x+c)*sin(b*x+a)^2,x)
```

Fricas [F]

$$\int \cot(c+dx)\sin^2(a+bx)\,dx = \int \cot\left(dx+c\right)\sin\left(bx+a\right)^{2}\,dx$$

input
```
integrate(cot(d*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*cot(d*x + c), x)
```

Sympy [F]

$$\int \cot(c+dx)\sin^2(a+bx)\,dx = \int \sin^2{(a+bx)}\cot{(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)*sin(b*x+a)**2,x)
```

output
```
Integral(sin(a + b*x)**2*cot(c + d*x), x)
```

Maxima [F]

$$\int \cot(c+dx)\sin^2(a+bx)\,dx = \int \cot{(dx+c)}\sin{(bx+a)}^2\,dx$$

input
```
integrate(cot(d*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int \cot(c+dx)\sin^2(a+bx)\,dx = \int \cot{(dx+c)}\sin{(bx+a)}^2\,dx$$

input
```
integrate(cot(d*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(cot(d*x + c)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)\sin^2(a+bx)\,dx = \int \cot(c+d\,x)\,\sin\left(a+b\,x\right)^2\,dx$$

input
```
int(cot(c + d*x)*sin(a + b*x)^2,x)
```

output
```
int(cot(c + d*x)*sin(a + b*x)^2, x)
```

Reduce [F]

$$\int \cot(c+dx)\sin^2(a+bx)\,dx = \int \cot\left(dx+c\right)\sin\left(bx+a\right)^2 dx$$

input
```
int(cot(d*x+c)*sin(b*x+a)^2,x)
```

output
```
int(cot(c + d*x)*sin(a + b*x)**2,x)
```

3.261 $\int \sec(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \sec(c+bx)\sin(a+bx)\,dx = -\frac{\cos(a-c)\log(\cos(c+bx))}{b} + x\sin(a-c)$$

output
```
-cos(a-c)*ln(cos(b*x+c))/b+x*sin(a-c)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sec(c+bx)\sin(a+bx)\,dx = -\frac{\cos(a-c)\log(\cos(c+bx))}{b} + x\sin(a-c)$$

input
```
Integrate[Sec[c + b*x]*Sin[a + b*x],x]
```

output
```
-((Cos[a - c]*Log[Cos[c + b*x]])/b) + x*Sin[a - c]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5091, 24, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \text{ 5091}$$

$$\cos(a-c)\int \tan(c+bx)dx+\sin(a-c)\int 1dx$$

$$\Big\downarrow \text{ 24}$$

$$\cos(a-c)\int \tan(c+bx)dx+x\sin(a-c)$$

$$\Big\downarrow \text{ 3042}$$

$$\cos(a-c)\int \tan(c+bx)dx+x\sin(a-c)$$

$$\Big\downarrow \text{ 3956}$$

$$x\sin(a-c)-\frac{\cos(a-c)\log(\cos(bx+c))}{b}$$

input
```
Int[Sec[c + b*x]*Sin[a + b*x],x]
```

output
```
-((Cos[a - c]*Log[Cos[c + b*x]])/b) + x*Sin[a - c]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.56

method	result
risch	$2i\cos\left(a-c\right)x-ix\,\mathrm{e}^{i(a-c)}+\frac{2i\cos(a-c)a}{b}-\frac{\ln\left(\mathrm{e}^{2i(bx+a)}+\mathrm{e}^{2i(a-c)}\right)\cos(a-c)}{b}$
default	$\frac{-\frac{(\cos(a)\cos(c)+\sin(a)\sin(c))\ln(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}{\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2}+\frac{\frac{(\cos(a)\cos(c)+\sin(a)\sin(c))\ln\Big(t}{2}}{\Big(co}}{b}$

input
```
int(sec(b*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*I*cos(a-c)*x-I*x*exp(I*(a-c))+2*I/b*cos(a-c)*a-ln(exp(2*I*(b*x+a))+exp(2
*I*(a-c)))/b*cos(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \sec(c+bx)\sin(a+bx)\,dx = -\frac{bx\sin(-a+c)+\cos(-a+c)\log(-\cos(bx+c))}{b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-(b*x*sin(-a + c) + cos(-a + c)*log(-cos(b*x + c)))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(20) = 40$.

Time = 4.84 (sec) , antiderivative size = 435, normalized size of antiderivative = 16.11

$$\int \sec(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a),x)
```

output
```
Piecewise((-x, Eq(c, pi/2)), (x, Eq(c, -pi/2)), (0, Eq(b, 0)), (-2*b*x*tan
(c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)
**2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - t
an(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b)
 - log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/2
)**2 + b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*t
an(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1)
- 1/(tan(c/2) + 1))/(b*tan(c/2)**2 + b), True))*cos(a) + Piecewise((-log(s
in(b*x))/b, Eq(c, pi/2)), (log(sin(b*x))/b, Eq(c, -pi/2)), (x/cos(c), Eq(b
, 0)), (-b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) + 2
*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) -
tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b) -
 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/(
b*tan(c/2)**2 + b), True))*sin(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(27) = 54$.

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.70

$$\int \sec(c+bx)\sin(a+bx)\,dx =$$
$$-\frac{2\,bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\cos\left(2\,bx\right)^{2}+2\,\cos\left(2\,bx\right)\cos\left(2\,c\right)+\cos\left(2\,c\right)^{2}+\sin\left(2\,bx\right)^{2}-2\,\mathrm{si}\right.}{2\,b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*b*x*sin(-a + c) + cos(-a + c)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(
2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(27) = 54$.

Time = 0.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 5.85

$$\int \sec(c+bx)\sin(a+bx)\,dx$$
$$=\frac{\frac{4\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)-\tan\left(\frac{1}{2}\,c\right)\right)(bx+c)}{\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)^{2}+\tan\left(\frac{1}{2}\,c\right)^{2}+1}+\frac{\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,a\right)^{2}+4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,c\right.}{\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)^{2}+\tan\left(\frac{1}{2}\,c\right)}}{2\,b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - t
an(1/2*c))*(b*x + c)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c
)^2 + 1) + (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/
2*c) - tan(1/2*c)^2 + 1)*log(tan(b*x + c)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^
2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.15

$$\int \sec(c+bx)\sin(a+bx)\,dx = x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}-\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)+x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}+\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)-\frac{\ln\left(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i+c\,1i}}{2}+\frac{e^{a\,1i-c\,1i}}{2}\right)}{b}$$

input
```
int(sin(a + b*x)/cos(c + b*x),x)
```

output
```
x*((exp(c*1i - a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2) + x*((exp(c*1i - a*1
i)*1i)/2 + (exp(a*1i - c*1i)*1i)/2) - (log(exp(a*2i - c*2i) + exp(a*2i + b
*x*2i))*(exp(c*1i - a*1i)/2 + exp(a*1i - c*1i)/2))/b
```

Reduce [F]

$$\int \sec(c+bx)\sin(a+bx)\,dx = \frac{\left(\int \frac{\sin(bx+a)}{\cos(bx+c)}dx\right)b-1}{b}$$

input
```
int(sec(b*x+c)*sin(b*x+a),x)
```

output
```
(int(sin(a + b*x)/cos(b*x + c),x)*b - 1)/b
```

3.262 $\int \sec^2(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \sec^2(c+bx)\sin(a+bx)\,dx=\frac{\cos(a-c)\sec(c+bx)}{b}+\frac{\operatorname{arctanh}(\sin(c+bx))\sin(a-c)}{b}$$

output
```
cos(a-c)*sec(b*x+c)/b+arctanh(sin(b*x+c))*sin(a-c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.59

$$\int \sec^2(c+bx)\sin(a+bx)\,dx = \frac{\cos(a-c)\sec(c+bx)}{b} - \frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b}$$

input
```
Integrate[Sec[c + b*x]^2*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x])/b - ((2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x
)/2]*Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*
Sin[c])]*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5091, 3042, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^2(bx+c)\,dx$$

$$\Big\downarrow \text{5091}$$

$$\sin(a-c)\int \sec(c+bx)dx+\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\cos(a-c)\int 1d\sec(c+bx)}{b}+\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{24}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\frac{\cos(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sin(a-c)\text{arctanh}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}$$

input
```
Int[Sec[c + b*x]^2*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.38

method	result
risch	$\frac{e^{i(bx+3a)}+e^{i(bx+a+2c)}}{b\left(e^{2i(bx+a+c)}+e^{2ia}\right)}+\frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(a-c)}{b}-\frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\sin(a-c)}{b}$
default	$\frac{4(-2\cos(a)\sin(c)+2\sin(a)\cos(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+8\cos(a)\cos(c)+8\sin(a)\sin}{\left(-4\cos(c)^2\sin(a)^2-4\cos(a)^2\cos(c)^2-4\sin(a)^2\sin(c)^2-4\sin(c)^2\cos(a)^2\right)\left(\cos(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+\sin(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-2\cos(c)\sin(\right.}$

input
```
int(sec(b*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))+
ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*sin(a-c)-ln(exp(I*(b*x+a))-I*exp(I*(a-
c)))/b*sin(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int \sec^2(c+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+c)\log(\sin(bx+c)+1)\sin(-a+c)-\cos(bx+c)\log(-\sin(bx+c)+1)\sin(-a+c)-2}{2\,b\cos(bx+c)}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cos(b*x + c)*log(sin(b*x + c) + 1)*sin(-a + c) - cos(b*x + c)*log(-s
in(b*x + c) + 1)*sin(-a + c) - 2*cos(-a + c))/(b*cos(b*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1448 vs. $2(27) = 54$.

Time = 98.58 (sec) , antiderivative size = 5545, normalized size of antiderivative = 163.09

$$\int \sec^2(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)**2*sin(b*x+a),x)
```

output
```
Piecewise((log(tan(b*x/2))/b, Eq(c, -pi/2) | Eq(c, pi/2)), (0, Eq(b, 0)),
(-2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)*
*3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/
2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(
tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**3/(b*ta
n(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b
*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 8*log(tan(b*x/2) - tan(c/2)/
(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*t
an(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*t
an(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) -
 1) - 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**
2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) -
 b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(ta
n(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*t
an(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2
*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3*
tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)*
*3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*log(tan
(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(c
/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(34) = 68$.

Time = 0.18 (sec) , antiderivative size = 387, normalized size of antiderivative = 11.38

$$\int \sec^2(c+bx)\sin(a+bx)\,dx$$

$$= \frac{2\left(\cos\left(bx+2a\right)+\cos\left(bx+2c\right)\right)\cos\left(2bx+a+2c\right)+2\cos\left(bx+2a\right)\cos\left(a\right)+2\cos\left(bx+2c\right)\cos\left(a\right)}{}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*(cos(b*x + 2*a) + cos(b*x + 2*c))*cos(2*b*x + a + 2*c) + 2*cos(b*x
+ 2*a)*cos(a) + 2*cos(b*x + 2*c)*cos(a) + (cos(2*b*x + a + 2*c)^2*sin(-a +
 c) + 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(2*b*x + a + 2*c)^2*s
in(-a + c) + 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) + (cos(a)^2 + sin(a
)^2)*sin(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*
c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c
)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x +
2*c)*sin(c) + sin(c)^2)) + 2*(sin(b*x + 2*a) + sin(b*x + 2*c))*sin(2*b*x +
 a + 2*c) + 2*sin(b*x + 2*a)*sin(a) + 2*sin(b*x + 2*c)*sin(a))/(b*cos(2*b*
x + a + 2*c)^2 + 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^
2 + 2*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(34) = 68$.

Time = 0.14 (sec) , antiderivative size = 248, normalized size of antiderivative = 7.29

$$\int \sec^2(c+bx)\sin(a+bx)\,dx = \frac{2\left(\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)+1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}-\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+t\right.}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+t}\right.}{b}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
2*((tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1
/2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan
(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(
1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) - 1))/(ta
n(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*a)^2
*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)
/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b
*x + 1/2*c)^2 - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 254, normalized size of antiderivative = 7.47

$$\int \sec^2(c+bx)\sin(a+bx)\,dx$$
$$= \frac{e^{a\,1i+b\,x\,1i}\left(e^{a\,2i-c\,2i}+1\right)}{b\left(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)} + \frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\left(e^{a\,2i}\,e^{-c\,2i}-1\right)1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\sqrt{-e^{a\,2i-c\,2i}}} - \frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\left(e^{a\,2i}\,e^{-c\,2i}-1\right)1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\sqrt{-e^{a\,2i-c\,2i}}}$$

input
```
int(sin(a + b*x)/cos(c + b*x)^2,x)
```

output
```
(exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) + 1))/(b*(exp(a*2i - c*2i) + exp(a*2
i + b*x*2i))) + (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i)
- (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*
2i))^(1/2))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log
(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) + (exp(a*2i)*exp(-c*
2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2
i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2))
```

Reduce [F]

$$\int \sec^2(c+bx)\sin(a+bx)\,dx = \int \sec(bx+c)^2\sin(bx+a)\,dx$$

input
```
int(sec(b*x+c)^2*sin(b*x+a),x)
```

output
```
int(sec(b*x + c)**2*sin(a + b*x),x)
```

3.263 $\int \sec^3(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sec^3(c+bx)\sin(a+bx)\,dx=\frac{\cos(a-c)\sec^2(c+bx)}{2b}+\frac{\sin(a-c)\tan(c+bx)}{b}$$

output
```
1/2*cos(a-c)*sec(b*x+c)^2/b+sin(a-c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \sec^3(c+bx)\sin(a+bx)\,dx=\frac{\sec(c)\sec^2(c+bx)(\cos(a)+\sin(a-c)\sin(c+2bx))}{2b}$$

input
```
Integrate[Sec[c + b*x]^3*Sin[a + b*x],x]
```

output
```
(Sec[c]*Sec[c + b*x]^2*(Cos[a] + Sin[a - c]*Sin[c + 2*b*x]))/(2*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5091, 3042, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^3(bx+c)\,dx$$

$$\Big\downarrow \text{5091}$$

$$\sin(a-c)\int \sec^2(c+bx)dx+\cos(a-c)\int \sec^2(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx+\cos(a-c)\int \sec(c+bx)^2\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\cos(a-c)\int \sec(c+bx)d\sec(c+bx)}{b}+\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{15}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx+\frac{\cos(a-c)\sec^2(bx+c)}{2b}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\cos(a-c)\sec^2(bx+c)}{2b}-\frac{\sin(a-c)\int 1d(-\tan(c+bx))}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(a-c)\tan(bx+c)}{b}+\frac{\cos(a-c)\sec^2(bx+c)}{2b}$$

input
```
Int[Sec[c + b*x]^3*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x]^2)/(2*b) + (Sin[a - c]*Tan[c + b*x])/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

method	result
parallelrisch	$\frac{1+\cos(2bx+2c)-2\cos(2bx+a+c)}{2b(1+\cos(2bx+2c))}$
risch	$\frac{2\,e^{i(2bx+5a+c)}+e^{i(5a-c)}-e^{i(3a+c)}}{\left(e^{2i(bx+a+c)}+e^{2ia}\right)^2 b}$
default	$\frac{-\frac{1}{(\sin(a)\cos(c)-\cos(a)\sin(c))^2(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}+\frac{}{2(\sin(a)\cos(c)-\cos(a)\sin(}}{b}$

input
```
int(sec(b*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(1+cos(2*b*x+2*c)-2*cos(2*b*x+a+c))/b/(1+cos(2*b*x+2*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \sec^3(c+bx)\sin(a+bx)\,dx = -\frac{2\cos(bx+c)\sin(bx+c)\sin(-a+c)-\cos(-a+c)}{2\,b\cos(bx+c)^2}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*cos(b*x + c)*sin(b*x + c)*sin(-a + c) - cos(-a + c))/(b*cos(b*x +
c)^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \sec^3(c+bx)\sin(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+c)**3*sin(b*x+a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 391, normalized size of antiderivative = 10.29

$$\int \sec^3(c+bx)\sin(a+bx)\,dx$$
$$= \frac{(2\cos(2bx+2a+2c)+\cos(2a)-\cos(2c))\cos(4bx+a+5c)+2(2\cos(2bx+2a+2c)+\cos(2a}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2+4b\cos(2}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
((2*cos(2*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(4*b*x + a + 5*c) + 2
*(2*cos(2*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(2*b*x + a + 3*c) + (
cos(2*a) - cos(2*c))*cos(a + c) + 2*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (2
*sin(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(4*b*x + a + 5*c) + 2*(2
*sin(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(2*b*x + a + 3*c) + (sin
(2*a) - sin(2*c))*sin(a + c) + 2*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos
(4*b*x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 + 4*b*cos(2*b*x + a + 3*c
)*cos(a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x +
 a + 3*c)^2 + 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(2*
b*cos(2*b*x + a + 3*c) + b*cos(a + c))*cos(4*b*x + a + 5*c) + 2*(2*b*sin(2
*b*x + a + 3*c) + b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(36) = 72$.

Time = 0.15 (sec) , antiderivative size = 174, normalized size of antiderivative = 4.58

$$\int \sec^3(c+bx)\sin(a+bx)\,dx$$
$$= \frac{\tan(bx+c)^2\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan(bx+c)^2\tan\left(\frac{1}{2}a\right)^2+4\tan(bx+c)^2\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+4\tan}{}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
1/2*(tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + c)^2*tan(1/2*a)^
2 + 4*tan(b*x + c)^2*tan(1/2*a)*tan(1/2*c) + 4*tan(b*x + c)*tan(1/2*a)^2*t
an(1/2*c) - tan(b*x + c)^2*tan(1/2*c)^2 - 4*tan(b*x + c)*tan(1/2*a)*tan(1/
2*c)^2 + tan(b*x + c)^2 + 4*tan(b*x + c)*tan(1/2*a) - 4*tan(b*x + c)*tan(1
/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+bx)\sin(a+bx)\,dx=\text{Hanged}$$

input
```
int(sin(a + b*x)/cos(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \sec^3(c+bx)\sin(a+bx)\,dx=\frac{\cos(bx+c)\cos(bx+a)-\sin(bx+c)\sin(bx+a)}{2b\left(\sin(bx+c)^2-1\right)}$$

input
```
int(sec(b*x+c)^3*sin(b*x+a),x)
```

output
```
(cos(b*x + c)*cos(a + b*x) - sin(b*x + c)*sin(a + b*x))/(2*b*(sin(b*x + c)
**2 - 1))
```

3.264 $\int \sec^4(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \frac{\cos(a-c)\sec^3(c+bx)}{3b} + \frac{\operatorname{arctanh}(\sin(c+bx))\sin(a-c)}{2b} + \frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b}$$

output
```
1/3*cos(a-c)*sec(b*x+c)^3/b+1/2*arctanh(sin(b*x+c))*sin(a-c)/b+1/2*sec(b*x
+c)*sin(a-c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \frac{12\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a-c)+\sec^3(c+bx)(4\cos(a-c)+3\sin(a-c)\sin(2(c+bx)))}{12b}$$

input
```
Integrate[Sec[c + b*x]^4*Sin[a + b*x],x]
```

output
```
(12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c] + Sec[c + b*x]^3*(4*C
os[a - c] + 3*Sin[a - c]*Sin[2*(c + b*x)]))/(12*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5091, 3042, 3086, 15, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^4(bx+c)\,dx$$

$$\Big\downarrow \text{5091}$$

$$\sin(a-c)\int \sec^3(c+bx)dx+\cos(a-c)\int \sec^3(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx+\cos(a-c)\int \sec(c+bx)^3\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\cos(a-c)\int \sec^2(c+bx)d\sec(c+bx)}{b}+\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{15}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx+\frac{\cos(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4255}$$

$$\sin(a-c)\left(\frac{1}{2}\int \sec(c+bx)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\cos(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\left(\frac{1}{2}\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\cos(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4257}$$

$$\sin(a-c)\left(\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\cos(a-c)\sec^3(bx+c)}{3b}$$

input
```
Int[Sec[c + b*x]^4*Sin[a + b*x],x]
```

output
```
(Cos[a - c]*Sec[c + b*x]^3)/(3*b) + Sin[a - c]*(ArcTanh[Sin[c + b*x]]/(2*b
) + (Sec[c + b*x]*Tan[c + b*x])/(2*b))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5091
```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Cos[v - w]   Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Simp[Sin[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.85

method	result
risch	$\frac{-3\,\mathrm{e}^{i(5bx+7a+4c)}+3\,\mathrm{e}^{i(5bx+5a+6c)}+8\,\mathrm{e}^{i(3bx+7a+2c)}+8\,\mathrm{e}^{i(3bx+5a+4c)}+3\,\mathrm{e}^{i(bx+7a)}-3\,\mathrm{e}^{i(bx+5a+2c)}}{6b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{3}}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a-c)}\right)\sin(a}{2b}$
default	Expression too large to display

input
```
int(sec(b*x+c)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))^3*(-3*exp(I*(5*b*x+7*a+4*c))+3*exp(I
*(5*b*x+5*a+6*c))+8*exp(I*(3*b*x+7*a+2*c))+8*exp(I*(3*b*x+5*a+4*c))+3*exp(
I*(b*x+7*a))-3*exp(I*(b*x+5*a+2*c)))+1/2*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))
/b*sin(a-c)-1/2*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = -\frac{3\,\cos\left(bx+c\right)^{3}\log\left(\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right)-3\,\cos\left(bx+c\right)^{3}\log\left(-\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right.}{12\,b\cos\left(bx+c\right)^{3}}$$

input
```
integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/12*(3*cos(b*x + c)^3*log(sin(b*x + c) + 1)*sin(-a + c) - 3*cos(b*x + c)
^3*log(-sin(b*x + c) + 1)*sin(-a + c) + 6*cos(b*x + c)*sin(b*x + c)*sin(-a
 + c) - 4*cos(-a + c))/(b*cos(b*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+c)**4*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1424 vs. $2(61) = 122$.

Time $= 0.22$ (sec) , antiderivative size $= 1424$, normalized size of antiderivative $= 21.25$

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/12*(2*(3*cos(5*b*x + 2*a + 4*c) - 3*cos(5*b*x + 6*c) - 8*cos(3*b*x + 2*
a + 2*c) - 8*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) + 3*cos(b*x + 2*c))*cos(6
*b*x + a + 6*c) + 6*(3*cos(4*b*x + a + 4*c) + 3*cos(2*b*x + a + 2*c) + cos
(a))*cos(5*b*x + 2*a + 4*c) - 6*(3*cos(4*b*x + a + 4*c) + 3*cos(2*b*x + a
+ 2*c) + cos(a))*cos(5*b*x + 6*c) - 6*(8*cos(3*b*x + 2*a + 2*c) + 8*cos(3*
b*x + 4*c) + 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) - 1
6*(3*cos(2*b*x + a + 2*c) + cos(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*cos(2*b
*x + a + 2*c) + cos(a))*cos(3*b*x + 4*c) - 18*(cos(b*x + 2*a) - cos(b*x +
2*c))*cos(2*b*x + a + 2*c) - 6*cos(b*x + 2*a)*cos(a) + 6*cos(b*x + 2*c)*co
s(a) - 3*(cos(6*b*x + a + 6*c)^2*sin(-a + c) + 9*cos(4*b*x + a + 4*c)^2*si
n(-a + c) + 9*cos(2*b*x + a + 2*c)^2*sin(-a + c) + 6*cos(2*b*x + a + 2*c)*
cos(a)*sin(-a + c) + sin(6*b*x + a + 6*c)^2*sin(-a + c) + 9*sin(4*b*x + a
+ 4*c)^2*sin(-a + c) + 9*sin(2*b*x + a + 2*c)^2*sin(-a + c) + 6*sin(2*b*x
+ a + 2*c)*sin(a)*sin(-a + c) + 2*(3*cos(4*b*x + a + 4*c)*sin(-a + c) + 3*
cos(2*b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(6*b*x + a + 6*c
) + 6*(3*cos(2*b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(4*b*x
+ a + 4*c) + 2*(3*sin(4*b*x + a + 4*c)*sin(-a + c) + 3*sin(2*b*x + a + 2*c
)*sin(-a + c) + sin(a)*sin(-a + c))*sin(6*b*x + a + 6*c) + 6*(3*sin(2*b*x
+ a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(4*b*x + a + 4*c) + (cos(a
)^2 + sin(a)^2)*sin(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(61) = 122$.

Time = 0.14 (sec) , antiderivative size = 495, normalized size of antiderivative = 7.39

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

output
```
1/3*(3*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - t
an(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 +
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 3*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a
)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) - 1
))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(3*ta
n(1/2*b*x + 1/2*c)^5*tan(1/2*a)^2*tan(1/2*c) - 3*tan(1/2*b*x + 1/2*c)^5*ta
n(1/2*a)*tan(1/2*c)^2 - 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2*tan(1/2*c)^2
 + 3*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a) + 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*
a)^2 - 3*tan(1/2*b*x + 1/2*c)^5*tan(1/2*c) - 12*tan(1/2*b*x + 1/2*c)^4*tan
(1/2*a)*tan(1/2*c) + 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*c)^2 - 3*tan(1/2*b*x
 + 1/2*c)^4 - 3*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c) + 3*tan(1/2*b
*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(1/
2*b*x + 1/2*c)*tan(1/2*a) + tan(1/2*a)^2 + 3*tan(1/2*b*x + 1/2*c)*tan(1/2*
c) - 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1)/((tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b*x + 1/2*c)^2 - 1)^3))/b
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/cos(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \int \sec(bx+c)^4 \sin(bx+a)\,dx$$

input
```
int(sec(b*x+c)^4*sin(b*x+a),x)
```

output
```
int(sec(b*x + c)**4*sin(a + b*x),x)
```

3.265 $\int \sec(c-bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \sec(c-bx)\sin(a+bx)\,dx=-\frac{\cos(a+c)\log(\cos(c-bx))}{b}+x\sin(a+c)$$

output
```
-cos(a+c)*ln(cos(b*x-c))/b+x*sin(a+c)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sec(c-bx)\sin(a+bx)\,dx=-\frac{\cos(a+c)\log(\cos(c-bx))}{b}+x\sin(a+c)$$

input
```
Integrate[Sec[c - b*x]*Sin[a + b*x],x]
```

output
```
-((Cos[a + c]*Log[Cos[c - b*x]])/b) + x*Sin[a + c]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec(c-bx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sin(a+bx)\sec(c-bx)dx$$

input
```
Int[Sec[c - b*x]*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

method	result
risch	$2i\cos\left(a+c\right)x-ix\,\mathrm{e}^{i(a+c)}+\frac{2i\cos(a+c)a}{b}-\frac{\ln\left(\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)\cos(a+c)}{b}$
default	$\frac{\frac{(-\cos(a)\cos(c)+\sin(a)\sin(c))\ln(\tan(bx+a)\sin(a)\cos(c)+\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)-\sin(a)\sin(c))}{\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2}+\frac{\frac{(\cos(a)\cos(c)-\sin(a)\sin(c))\ln\left(\mathrm{t}\right.}{2}}{\left(\mathrm{co}\right.}}{b}$

input
```
int(sec(b*x-c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*I*cos(a+c)*x-I*x*exp(I*(a+c))+2*I/b*cos(a+c)*a-ln(exp(2*I*(a+c))+exp(2*I
*(b*x+a)))/b*cos(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(25) = 50$.

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \sec(c-bx)\sin(a+bx)\,dx = \frac{bx\sin(a+c)-\cos(a+c)\log\left(\frac{2(\cos(bx+a)\cos(a+c)+\sin(bx+a)\sin(a+c))}{\cos(a+c)+1}\right)}{b}$$

input
```
integrate(sec(b*x-c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
(b*x*sin(a + c) - cos(a + c)*log(2*(cos(b*x + a)*cos(a + c) + sin(b*x + a)
*sin(a + c))/(cos(a + c) + 1)))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(20) = 40$.

Time = 4.94 (sec) , antiderivative size = 435, normalized size of antiderivative = 18.12

$$\int \sec(c-bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x-c)*sin(b*x+a),x)
```

output
```
Piecewise((x, Eq(c, pi/2)), (-x, Eq(c, -pi/2)), (0, Eq(b, 0)), (2*b*x*tan(
c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)*
*2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) + ta
n(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b)
- log(tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))/(b*tan(c/2)
**2 + b) + log(tan(b*x/2) - tan(c/2)/(tan(c/2) + 1) + 1/(tan(c/2) + 1))*ta
n(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - tan(c/2)/(tan(c/2) + 1) +
 1/(tan(c/2) + 1))/(b*tan(c/2)**2 + b), True))*cos(a) + Piecewise((log(sin
(b*x))/b, Eq(c, pi/2)), (-log(sin(b*x))/b, Eq(c, -pi/2)), (x/cos(c), Eq(b,
 0)), (-b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2*
log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2) + t
an(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b) +
2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) + 1) + 1/(tan(c/2) + 1))*tan(c/2)/(b
*tan(c/2)**2 + b), True))*sin(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(25) = 50$.

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.92

$$\int \sec(c-bx)\sin(a+bx)\,dx$$
$$= \frac{2\,bx\sin\left(a+c\right)-\cos\left(a+c\right)\log\left(\cos\left(2\,bx\right)^2+2\,\cos\left(2\,bx\right)\cos\left(2\,c\right)+\cos\left(2\,c\right)^2+\sin\left(2\,bx\right)^2+2\,\sin\left(2\,bx\right.\right.}{2\,b}$$

input
```
integrate(sec(b*x-c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*b*x*sin(a + c) - cos(a + c)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c
) + cos(2*c)^2 + sin(2*b*x)^2 + 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(25) = 50$.

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 6.83

$$\int \sec(c-bx)\sin(a+bx)\,dx =$$

$$-\frac{\frac{4\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)(bx-c)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}-\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)^2-4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan(\right.}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan(}}{2b}$$

input
```
integrate(sec(b*x-c)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(4*(tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) -
tan(1/2*c))*(b*x - c)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*
c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 - 4*tan(1/2*a)*tan(1
/2*c) - tan(1/2*c)^2 + 1)*log(tan(b*x - c)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.67

$$\int \sec(c-bx)\sin(a+bx)\,dx = x\left(\frac{e^{-a\,1i-c\,1i}\,1i}{2}-\frac{e^{a\,1i+c\,1i}\,1i}{2}\right) + x\left(\frac{e^{-a\,1i-c\,1i}\,1i}{2}+\frac{e^{a\,1i+c\,1i}\,1i}{2}\right) - \frac{\ln\left(e^{a\,2i+c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i-c\,1i}}{2}+\frac{e^{a\,1i+c\,1i}}{2}\right)}{b}$$

input
```
int(sin(a + b*x)/cos(c - b*x),x)
```

output
```
x*((exp(- a*1i - c*1i)*1i)/2 - (exp(a*1i + c*1i)*1i)/2) + x*((exp(- a*1i -
 c*1i)*1i)/2 + (exp(a*1i + c*1i)*1i)/2) - (log(exp(a*2i + c*2i) + exp(a*2i
 + b*x*2i))*(exp(- a*1i - c*1i)/2 + exp(a*1i + c*1i)/2))/b
```

Reduce [F]

$$\int \sec(c-bx)\sin(a+bx)\,dx=\frac{\left(\int \frac{\sin(bx+a)}{\cos(bx-c)}dx\right)b-1}{b}$$

input
```
int(sec(b*x-c)*sin(b*x+a),x)
```

output
```
(int(sin(a + b*x)/cos(b*x - c),x)*b - 1)/b
```

3.266 $\int \sec^2(c-bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \sec^2(c-bx)\sin(a+bx)\,dx=\frac{\cos(a+c)\sec(c-bx)}{b}-\frac{\operatorname{arctanh}(\sin(c-bx))\sin(a+c)}{b}$$

output
```
cos(a+c)*sec(b*x-c)/b+arctanh(sin(b*x-c))*sin(a+c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.55

$$\int \sec^2(c-bx)\sin(a+bx)\,dx$$
$$=\frac{\cos(a+c)\sec(c-bx)}{b}$$
$$+\frac{2\operatorname{arctanh}\left(\frac{(\cos(c)-i\sin(c))\left(-\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a+c)}{b}$$

input
```
Integrate[Sec[c - b*x]^2*Sin[a + b*x],x]
```

output
```
(Cos[a + c]*Sec[c - b*x])/b + (2*ArcTanh[((Cos[c] - I*Sin[c])*(-(Cos[(b*x)
/2]*Sin[c]) + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*
Sin[c])]*Sin[a + c])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^2(c-bx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin(a+bx)\sec^2(c-bx)dx$$

input
```
Int[Sec[c - b*x]^2*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.27

method	result
risch	$\frac{e^{i(bx+3a+2c)}+e^{i(bx+a)}}{b\left(e^{2i(a+c)}+e^{2i(bx+a)}\right)}+\frac{\ln\left(e^{i(bx+a)}+ie^{i(a+c)}\right)\sin(a+c)}{b}-\frac{\ln\left(e^{i(bx+a)}-ie^{i(a+c)}\right)\sin(a+c)}{b}$
default	$\frac{4(2\sin(a)\cos(c)+2\cos(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-8\sin(a)\sin(c)+8\cos(a)\cos(}{\left(-4\cos(c)^2\sin(a)^2-4\cos(a)^2\cos(c)^2-4\sin(a)^2\sin(c)^2-4\sin(c)^2\cos(a)^2\right)\left(\cos(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-\sin(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-2\sin(c)\cos(}$

input
```
int(sec(b*x-c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b/(exp(2*I*(a+c))+exp(2*I*(b*x+a)))*(exp(I*(b*x+3*a+2*c))+exp(I*(b*x+a))
)+ln(exp(I*(b*x+a))+I*exp(I*(a+c)))/b*sin(a+c)-ln(exp(I*(b*x+a))-I*exp(I*(
a+c)))/b*sin(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 5.30

$$\int \sec^2(c-bx)\sin(a+bx)\,dx$$
$$=\frac{\left(\cos(bx+a)\cos(a+c)\sin(a+c)-\left(\cos(a+c)^2-1\right)\sin(bx+a)\right)\log\left(\frac{2(\cos(a+c)\sin(bx+a)-\cos(bx+a)\sin(a-}{\cos(a+c)+1}\right.}{2\,(b\cos(bx}$$

input
```
integrate(sec(b*x-c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/2*((cos(b*x + a)*cos(a + c)*sin(a + c) - (cos(a + c)^2 - 1)*sin(b*x + a)
)*log(2*(cos(a + c)*sin(b*x + a) - cos(b*x + a)*sin(a + c) + 1)/(cos(a + c
) + 1)) - (cos(b*x + a)*cos(a + c)*sin(a + c) - (cos(a + c)^2 - 1)*sin(b*x
 + a))*log(-2*(cos(a + c)*sin(b*x + a) - cos(b*x + a)*sin(a + c) - 1)/(cos
(a + c) + 1)) + 2*cos(a + c))/(b*cos(b*x + a)*cos(a + c) + b*sin(b*x + a)*
sin(a + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1448 vs. $2(27) = 54$.

Time = 96.68 (sec) , antiderivative size = 5545, normalized size of antiderivative = 168.03

$$\int \sec^2(c-bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x-c)**2*sin(b*x+a),x)
```

output
```
Piecewise((log(tan(b*x/2))/b, Eq(c, -pi/2) | Eq(c, pi/2)), (0, Eq(b, 0)),
(-2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)*
*3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 + 4*b*tan(c/
2)**3*tan(b*x/2) + 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(
tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)**3/(b*ta
n(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 + 4*b*tan(c/2)**3*tan(b*x/2) + 4*b
*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 8*log(tan(b*x/2) + tan(c/2)/
(tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*t
an(b*x/2)**2 - b*tan(c/2)**4 + 4*b*tan(c/2)**3*tan(b*x/2) + 4*b*tan(c/2)*t
an(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) -
 1) + 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**
2 - b*tan(c/2)**4 + 4*b*tan(c/2)**3*tan(b*x/2) + 4*b*tan(c/2)*tan(b*x/2) -
 b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) + 1/(ta
n(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 + 4*b*t
an(c/2)**3*tan(b*x/2) + 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2
*log(tan(b*x/2) - tan(c/2)/(tan(c/2) + 1) + 1/(tan(c/2) + 1))*tan(c/2)**3*
tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 + 4*b*tan(c/2)*
*3*tan(b*x/2) + 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*log(tan
(b*x/2) - tan(c/2)/(tan(c/2) + 1) + 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(c
/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 + 4*b*tan(c/2)**3*tan(b*x/2) + 4*b...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(34) = 68$.

Time = 0.18 (sec) , antiderivative size = 365, normalized size of antiderivative = 11.06

$$\int \sec^2(c-bx)\sin(a+bx)\,dx$$
$$= \frac{2\left(\cos\left(bx+2a+2c\right)+\cos\left(bx\right)\right)\cos\left(2bx+a\right)+2\cos\left(bx+2a+2c\right)\cos\left(a+2c\right)+2\cos\left(bx\right)\cos\left(a+\right.}{}$$

input
```
integrate(sec(b*x-c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*(cos(b*x + 2*a + 2*c) + cos(b*x))*cos(2*b*x + a) + 2*cos(b*x + 2*a
+ 2*c)*cos(a + 2*c) + 2*cos(b*x)*cos(a + 2*c) - (cos(2*b*x + a)^2*sin(a +
c) + 2*cos(2*b*x + a)*cos(a + 2*c)*sin(a + c) + cos(a + 2*c)^2*sin(a + c)
+ sin(2*b*x + a)^2*sin(a + c) + 2*sin(2*b*x + a)*sin(a + 2*c)*sin(a + c) +
 sin(a + 2*c)^2*sin(a + c))*log((cos(b*x)^2 + cos(c)^2 - 2*cos(c)*sin(b*x)
 + sin(b*x)^2 + 2*cos(b*x)*sin(c) + sin(c)^2)/(cos(b*x)^2 + cos(c)^2 + 2*c
os(c)*sin(b*x) + sin(b*x)^2 - 2*cos(b*x)*sin(c) + sin(c)^2)) + 2*(sin(b*x
+ 2*a + 2*c) + sin(b*x))*sin(2*b*x + a) + 2*sin(b*x + 2*a + 2*c)*sin(a + 2
*c) + 2*sin(b*x)*sin(a + 2*c))/(b*cos(2*b*x + a)^2 + 2*b*cos(2*b*x + a)*co
s(a + 2*c) + b*cos(a + 2*c)^2 + b*sin(2*b*x + a)^2 + 2*b*sin(2*b*x + a)*si
n(a + 2*c) + b*sin(a + 2*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(34) = 68$.

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 7.55

$$\int \sec^2(c-bx)\sin(a+bx)\,dx =$$
$$-\frac{2\left(\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx-\frac{1}{2}c\right)+1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}-\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2\right.}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2}\right.}{b}$$

input
```
integrate(sec(b*x-c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-2*((tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) - tan(
1/2*c))*log(abs(tan(1/2*b*x - 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 + ta
n(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan
(1/2*c)^2 - tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x - 1/2*c) - 1))/(t
an(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + (tan(1/2*a)^
2*tan(1/2*c)^2 - tan(1/2*a)^2 - 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1
)/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*
b*x - 1/2*c)^2 - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 22.73 (sec) , antiderivative size = 254, normalized size of antiderivative = 7.70

$$\int \sec^2(c-bx)\sin(a+bx)\,dx$$
$$= \frac{e^{a\,1i+b\,x\,1i}\left(e^{a\,2i+c\,2i}+1\right)}{b\left(e^{a\,2i+c\,2i}+e^{a\,2i+b\,x\,2i}\right)} + \frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{c\,2i}\,1i-i\right)-\frac{e^{a\,2i}\,e^{c\,2i}\left(e^{a\,2i}\,e^{c\,2i}-1\right)1i}{\sqrt{-e^{a\,2i}\,e^{c\,2i}}}\right)\left(e^{a\,2i+c\,2i}-1\right)}{2\,b\sqrt{-e^{a\,2i+c\,2i}}} - \frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{c\,2i}\,1i-i\right)+\frac{e^{a\,2i}\,e^{c\,2i}\left(e^{a\,2i}\,e^{c\,2i}-1\right)1i}{\sqrt{-e^{a\,2i}\,e^{c\,2i}}}\right)\left(e^{a\,2i+c\,2i}-1\right)}{2\,b\sqrt{-e^{a\,2i+c\,2i}}}$$

input
```
int(sin(a + b*x)/cos(c - b*x)^2,x)
```

output
```
(exp(a*1i + b*x*1i)*(exp(a*2i + c*2i) + 1))/(b*(exp(a*2i + c*2i) + exp(a*2
i + b*x*2i))) + (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(c*2i)*1i - 1i) -
 (exp(a*2i)*exp(c*2i)*(exp(a*2i)*exp(c*2i) - 1)*1i)/(-exp(a*2i)*exp(c*2i))
^(1/2))*(exp(a*2i + c*2i) - 1))/(2*b*(-exp(a*2i + c*2i))^(1/2)) - (log(exp
(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(c*2i)*1i - 1i) + (exp(a*2i)*exp(c*2i)*(e
xp(a*2i)*exp(c*2i) - 1)*1i)/(-exp(a*2i)*exp(c*2i))^(1/2))*(exp(a*2i + c*2i
) - 1))/(2*b*(-exp(a*2i + c*2i))^(1/2))
```

Reduce [F]

$$\int \sec^2(c-bx)\sin(a+bx)\,dx = \int \sec\left(bx-c\right)^2\sin\left(bx+a\right)dx$$

input
```
int(sec(b*x-c)^2*sin(b*x+a),x)
```

output
```
int(sec(b*x - c)**2*sin(a + b*x),x)
```

3.267 $\int \sec^3(c-bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \sec^3(c-bx)\sin(a+bx)\,dx = \frac{\cos(a+c)\sec^2(c-bx)}{2b} - \frac{\sin(a+c)\tan(c-bx)}{b}$$

output
```
1/2*cos(a+c)*sec(b*x-c)^2/b+sin(a+c)*tan(b*x-c)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \sec^3(c-bx)\sin(a+bx)\,dx = \frac{\sec(c)\sec^2(c-bx)(\cos(a)-\sin(a+c)\sin(c-2bx))}{2b}$$

input
```
Integrate[Sec[c - b*x]^3*Sin[a + b*x],x]
```

output
```
(Sec[c]*Sec[c - b*x]^2*(Cos[a] - Sin[a + c]*Sin[c - 2*b*x]))/(2*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^3(c-bx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sin(a+bx)\sec^3(c-bx)dx$$

input
```
Int[Sec[c - b*x]^3*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{1+\cos(2bx-2c)-2\cos(2bx+a-c)}{2b(1+\cos(2bx-2c))}$
risch	$\frac{e^{5i(a+c)}+2\,e^{i(2bx+5a+3c)}-e^{3i(a+c)}}{\left(e^{2i(a+c)}+e^{2i(bx+a)}\right)^2 b}$
default	$\frac{-\frac{1}{(\sin(a)\cos(c)+\cos(a)\sin(c))^2(\tan(bx+a)\sin(a)\cos(c)+\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)-\sin(a)\sin(c))}-\frac{\cdots}{2(\sin(a)\cos(c)+\cos(a)\sin(\cdots}}{b}$

input
```
int(sec(b*x-c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(1+cos(2*b*x-2*c)-2*cos(2*b*x+a-c))/b/(1+cos(2*b*x-2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.41

$$\int \sec^3(c-bx)\sin(a+bx)\,dx = \frac{2\left(2\cos\left(a+c\right)^2-1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(a+c\right)+4\left(\cos\left(a+c\right)^3-\cos\left(a+c\right)\right)\cos\left(bx+a\right)^2-}{2\left(2\,b\cos\left(bx+a\right)\cos\left(a+c\right)\sin\left(bx+a\right)\sin\left(a+c\right)+\left(2\,b\cos\left(a+c\right)^2-b\right)\cos\left(bx+a\right)^2-}$$

input
```
integrate(sec(b*x-c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*(2*cos(a + c)^2 - 1)*cos(b*x + a)*sin(b*x + a)*sin(a + c) + 4*(cos(
a + c)^3 - cos(a + c))*cos(b*x + a)^2 - 2*cos(a + c)^3 + 3*cos(a + c))/(2*
b*cos(b*x + a)*cos(a + c)*sin(b*x + a)*sin(a + c) + (2*b*cos(a + c)^2 - b)
*cos(b*x + a)^2 - b*cos(a + c)^2 + b)
```

Sympy [F(-2)]

Exception generated.

$$\int \sec^3(c-bx)\sin(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x-c)**3*sin(b*x+a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 425, normalized size of antiderivative = 11.49

$$\int \sec^3(c-bx)\sin(a+bx)\,dx$$
$$= \frac{(2\cos(2bx+2a+3c)+\cos(2a+5c)-\cos(3c))\cos(4bx+a)+2(2\cos(2bx+a+2c)+\cos(a+}{b\cos(4bx+a)^2+4b\cos(2}$$

input
```
integrate(sec(b*x-c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
((2*cos(2*b*x + 2*a + 3*c) + cos(2*a + 5*c) - cos(3*c))*cos(4*b*x + a) + 2
*(2*cos(2*b*x + a + 2*c) + cos(a + 4*c))*cos(2*b*x + 2*a + 3*c) + 2*(cos(2
*a + 5*c) - cos(3*c))*cos(2*b*x + a + 2*c) + cos(2*a + 5*c)*cos(a + 4*c) -
 cos(a + 4*c)*cos(3*c) + (2*sin(2*b*x + 2*a + 3*c) + sin(2*a + 5*c) - sin(
3*c))*sin(4*b*x + a) + 2*(2*sin(2*b*x + a + 2*c) + sin(a + 4*c))*sin(2*b*x
 + 2*a + 3*c) + 2*(sin(2*a + 5*c) - sin(3*c))*sin(2*b*x + a + 2*c) + sin(2
*a + 5*c)*sin(a + 4*c) - sin(a + 4*c)*sin(3*c))/(b*cos(4*b*x + a)^2 + 4*b*
cos(2*b*x + a + 2*c)^2 + 4*b*cos(2*b*x + a + 2*c)*cos(a + 4*c) + b*cos(a +
 4*c)^2 + b*sin(4*b*x + a)^2 + 4*b*sin(2*b*x + a + 2*c)^2 + 4*b*sin(2*b*x
+ a + 2*c)*sin(a + 4*c) + b*sin(a + 4*c)^2 + 2*(2*b*cos(2*b*x + a + 2*c) +
 b*cos(a + 4*c))*cos(4*b*x + a) + 2*(2*b*sin(2*b*x + a + 2*c) + b*sin(a +
4*c))*sin(4*b*x + a))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 5.19

$$\int \sec^3(c-bx)\sin(a+bx)\,dx$$
$$= \frac{\tan(bx-c)^2\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan(bx-c)^2\tan\left(\frac{1}{2}a\right)^2-4\tan(bx-c)^2\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-4\tan}{}$$

input
```
integrate(sec(b*x-c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
1/2*(tan(b*x - c)^2*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x - c)^2*tan(1/2*a)^
2 - 4*tan(b*x - c)^2*tan(1/2*a)*tan(1/2*c) - 4*tan(b*x - c)*tan(1/2*a)^2*t
an(1/2*c) - tan(b*x - c)^2*tan(1/2*c)^2 - 4*tan(b*x - c)*tan(1/2*a)*tan(1/
2*c)^2 + tan(b*x - c)^2 + 4*tan(b*x - c)*tan(1/2*a) + 4*tan(b*x - c)*tan(1
/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c-bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/cos(c - b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \sec^3(c-bx)\sin(a+bx)\,dx = \frac{\cos(bx-c)\cos(bx+a)-\sin(bx-c)\sin(bx+a)}{2b\left(\sin(bx-c)^2-1\right)}$$

input
```
int(sec(b*x-c)^3*sin(b*x+a),x)
```

output
```
(cos(b*x - c)*cos(a + b*x) - sin(b*x - c)*sin(a + b*x))/(2*b*(sin(b*x - c)
**2 - 1))
```

3.268 $\int \sec^4(c-bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int \sec^4(c-bx)\sin(a+bx)\,dx = \frac{\cos(a+c)\sec^3(c-bx)}{3b} - \frac{\operatorname{arctanh}(\sin(c-bx))\sin(a+c)}{2b} - \frac{\sec(c-bx)\sin(a+c)\tan(c-bx)}{2b}$$

output
```
1/3*cos(a+c)*sec(b*x-c)^3/b+1/2*arctanh(sin(b*x-c))*sin(a+c)/b+1/2*sec(b*x
-c)*sin(a+c)*tan(b*x-c)/b
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \sec^4(c-bx)\sin(a+bx)\,dx = \frac{-12\operatorname{arctanh}\left(\sin(c)-\cos(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a+c)+\sec^3(c-bx)(4\cos(a+c)-3\sin(a+c)\sin(2(c-bx)}{12b}$$

input
```
Integrate[Sec[c - b*x]^4*Sin[a + b*x],x]
```

output
```
(-12*ArcTanh[Sin[c] - Cos[c]*Tan[(b*x)/2]]*Sin[a + c] + Sec[c - b*x]^3*(4*
Cos[a + c] - 3*Sin[a + c]*Sin[2*(c - b*x)]))/(12*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sec^4(c-bx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin(a+bx)\sec^4(c-bx)dx$$

input
```
Int[Sec[c - b*x]^4*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.78

method	result
risch	$\frac{3\,\mathrm{e}^{i(bx+7a+6c)}+8\,\mathrm{e}^{i(3bx+7a+4c)}-3\,\mathrm{e}^{i(bx+5a+4c)}-3\,\mathrm{e}^{i(5bx+7a+2c)}+8\,\mathrm{e}^{i(3bx+5a+2c)}+3\,\mathrm{e}^{5i(bx+a)}}{6b\left(\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)^{3}}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a+c)}\right)\sin(a+c}{2b}$
default	Expression too large to display

input
```
int(sec(b*x-c)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/b/(exp(2*I*(a+c))+exp(2*I*(b*x+a)))^3*(3*exp(I*(b*x+7*a+6*c))+8*exp(I*
(3*b*x+7*a+4*c))-3*exp(I*(b*x+5*a+4*c))-3*exp(I*(5*b*x+7*a+2*c))+8*exp(I*(
3*b*x+5*a+2*c))+3*exp(5*I*(b*x+a)))+1/2*ln(exp(I*(b*x+a))+I*exp(I*(a+c)))/
b*sin(a+c)-1/2*ln(exp(I*(b*x+a))-I*exp(I*(a+c)))/b*sin(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(63) = 126$.

Time = 0.10 (sec) , antiderivative size = 447, normalized size of antiderivative = 6.88

$$\int \sec^4(c - bx) \sin(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(sec(b*x-c)^4*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(6*(2*cos(a + c)^2 - 1)*cos(b*x + a)*sin(b*x + a)*sin(a + c) + 12*(co
s(a + c)^3 - cos(a + c))*cos(b*x + a)^2 - 6*cos(a + c)^3 + 3*((cos(a + c)^
4 - (4*cos(a + c)^4 - 5*cos(a + c)^2 + 1)*cos(b*x + a)^2 - 2*cos(a + c)^2
+ 1)*sin(b*x + a) + ((4*cos(a + c)^3 - 3*cos(a + c))*cos(b*x + a)^3 - 3*(c
os(a + c)^3 - cos(a + c))*cos(b*x + a))*sin(a + c))*log(2*(cos(a + c)*sin(
b*x + a) - cos(b*x + a)*sin(a + c) + 1)/(cos(a + c) + 1)) - 3*((cos(a + c)
^4 - (4*cos(a + c)^4 - 5*cos(a + c)^2 + 1)*cos(b*x + a)^2 - 2*cos(a + c)^2
 + 1)*sin(b*x + a) + ((4*cos(a + c)^3 - 3*cos(a + c))*cos(b*x + a)^3 - 3*(
cos(a + c)^3 - cos(a + c))*cos(b*x + a))*sin(a + c))*log(-2*(cos(a + c)*si
n(b*x + a) - cos(b*x + a)*sin(a + c) - 1)/(cos(a + c) + 1)) + 10*cos(a + c
))/((4*b*cos(a + c)^3 - 3*b*cos(a + c))*cos(b*x + a)^3 + ((4*b*cos(a + c)^
2 - b)*cos(b*x + a)^2 - b*cos(a + c)^2 + b)*sin(b*x + a)*sin(a + c) - 3*(b
*cos(a + c)^3 - b*cos(a + c))*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(c-bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x-c)**4*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1423 vs. $2(63) = 126$.

Time = 0.21 (sec) , antiderivative size = 1423, normalized size of antiderivative = 21.89

$$\int \sec^4(c-bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x-c)^4*sin(b*x+a),x, algorithm="maxima")
```

output

```
1/12*(2*(3*cos(5*b*x) - 3*cos(5*b*x + 2*a + 2*c) + 8*cos(3*b*x + 2*a + 4*c
) + 8*cos(3*b*x + 2*c) + 3*cos(b*x + 2*a + 6*c) - 3*cos(b*x + 4*c))*cos(6*
b*x + a) - 6*(3*cos(4*b*x + a + 2*c) + 3*cos(2*b*x + a + 4*c) + cos(a + 6*
c))*cos(5*b*x + 2*a + 2*c) + 6*(3*cos(5*b*x) + 8*cos(3*b*x + 2*a + 4*c) +
8*cos(3*b*x + 2*c) + 3*cos(b*x + 2*a + 6*c) - 3*cos(b*x + 4*c))*cos(4*b*x
+ a + 2*c) + 16*(3*cos(2*b*x + a + 4*c) + cos(a + 6*c))*cos(3*b*x + 2*a +
4*c) + 16*(3*cos(2*b*x + a + 4*c) + cos(a + 6*c))*cos(3*b*x + 2*c) + 18*(c
os(5*b*x) + cos(b*x + 2*a + 6*c) - cos(b*x + 4*c))*cos(2*b*x + a + 4*c) +
6*cos(5*b*x)*cos(a + 6*c) + 6*cos(b*x + 2*a + 6*c)*cos(a + 6*c) - 6*cos(b*
x + 4*c)*cos(a + 6*c) - 3*(cos(6*b*x + a)^2*sin(a + c) + 9*cos(4*b*x + a +
 2*c)^2*sin(a + c) + 9*cos(2*b*x + a + 4*c)^2*sin(a + c) + 6*cos(2*b*x + a
 + 4*c)*cos(a + 6*c)*sin(a + c) + cos(a + 6*c)^2*sin(a + c) + sin(6*b*x +
a)^2*sin(a + c) + 9*sin(4*b*x + a + 2*c)^2*sin(a + c) + 9*sin(2*b*x + a +
4*c)^2*sin(a + c) + 6*sin(2*b*x + a + 4*c)*sin(a + 6*c)*sin(a + c) + sin(a
 + 6*c)^2*sin(a + c) + 2*(3*cos(4*b*x + a + 2*c)*sin(a + c) + 3*cos(2*b*x
+ a + 4*c)*sin(a + c) + cos(a + 6*c)*sin(a + c))*cos(6*b*x + a) + 6*(3*cos
(2*b*x + a + 4*c)*sin(a + c) + cos(a + 6*c)*sin(a + c))*cos(4*b*x + a + 2*
c) + 2*(3*sin(4*b*x + a + 2*c)*sin(a + c) + 3*sin(2*b*x + a + 4*c)*sin(a +
 c) + sin(a + 6*c)*sin(a + c))*sin(6*b*x + a) + 6*(3*sin(2*b*x + a + 4*c)*
sin(a + c) + sin(a + 6*c)*sin(a + c))*sin(4*b*x + a + 2*c))*log((cos(b*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(63) = 126$.

Time = 0.16 (sec) , antiderivative size = 500, normalized size of antiderivative = 7.69

$$\int \sec^4(c - bx)\sin(a + bx)\,dx = \text{Too large to display}$$

input

```
integrate(sec(b*x-c)^4*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/3*(3*(tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) -
tan(1/2*c))*log(abs(tan(1/2*b*x - 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2
+ tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 3*(tan(1/2*a)^2*tan(1/2*c) + tan(1/2*
a)*tan(1/2*c)^2 - tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x - 1/2*c) -
1))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(3*t
an(1/2*b*x - 1/2*c)^5*tan(1/2*a)^2*tan(1/2*c) + 3*tan(1/2*b*x - 1/2*c)^5*t
an(1/2*a)*tan(1/2*c)^2 + 3*tan(1/2*b*x - 1/2*c)^4*tan(1/2*a)^2*tan(1/2*c)^
2 - 3*tan(1/2*b*x - 1/2*c)^5*tan(1/2*a) - 3*tan(1/2*b*x - 1/2*c)^4*tan(1/2
*a)^2 - 3*tan(1/2*b*x - 1/2*c)^5*tan(1/2*c) - 12*tan(1/2*b*x - 1/2*c)^4*ta
n(1/2*a)*tan(1/2*c) - 3*tan(1/2*b*x - 1/2*c)^4*tan(1/2*c)^2 + 3*tan(1/2*b*
x - 1/2*c)^4 - 3*tan(1/2*b*x - 1/2*c)*tan(1/2*a)^2*tan(1/2*c) - 3*tan(1/2*
b*x - 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1
/2*b*x - 1/2*c)*tan(1/2*a) - tan(1/2*a)^2 + 3*tan(1/2*b*x - 1/2*c)*tan(1/2
*c) - 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)/((tan(1/2*a)^2*tan(1/2*c
)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b*x - 1/2*c)^2 - 1)^3))/b
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(c-bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/cos(c - b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^4(c-bx)\sin(a+bx)\,dx = \int \sec(bx-c)^4 \sin(bx+a)\,dx$$

input
```
int(sec(b*x-c)^4*sin(b*x+a),x)
```

output
```
int(sec(b*x - c)**4*sin(a + b*x),x)
```

3.269 $\int \sec(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \sec(c+bx)\sin^2(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(c+bx))\cos^2(a-c)}{b}-\frac{\sin(2a-c+bx)}{b}$$

output
```
arctanh(sin(b*x+c))*cos(a-c)^2/b-sin(b*x+2*a-c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 129 vs. 2(36) = 72.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.58

$$\int \sec(c+bx)\sin^2(a+bx)\,dx=\frac{(-1-\cos(2a-2c))\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)-\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{2b}+\frac{(1+\cos(2a-2c))\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)+\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{2b}-\frac{\cos(bx)\sin(2a-c)}{b}-\frac{\cos(2a-c)\sin(bx)}{b}$$

input
```
Integrate[Sec[c + b*x]*Sin[a + b*x]^2,x]
```

output
```
((-1 - Cos[2*a - 2*c])*Log[Cos[c/2 + (b*x)/2] - Sin[c/2 + (b*x)/2]])/(2*b)
 + ((1 + Cos[2*a - 2*c])*Log[Cos[c/2 + (b*x)/2] + Sin[c/2 + (b*x)/2]])/(2*
b) - (Cos[b*x]*Sin[2*a - c])/b - (Cos[2*a - c]*Sin[b*x])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sin^2(a+bx)\sec(bx+c)dx$$

input
```
Int[Sec[c + b*x]*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.06

method	result
risch	$\frac{\ln(e^{i(bx+a)}+ie^{i(a-c)})}{2b}+\frac{\ln(e^{i(bx+a)}+ie^{i(a-c)})\cos(2a-2c)}{2b}-\frac{\ln(e^{i(bx+a)}-ie^{i(a-c)})}{2b}-\frac{\ln(e^{i(bx+a)}-ie^{i(a-c)})\cos(2a-2c)}{2b}-$
default	$\frac{\frac{2(-\cos(a)\cos(c)-\sin(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-2\sin(a)\cos(c)+2\cos(a)\sin(c)}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)}-\frac{8(\cos(a)\cos(c)+\sin(a)\sin(c))^2\arctan\left(\frac{2(c}{2}\right.}{\left(4\cos(c)^2\sin(a)^2+4\cos(a)^2\cos(c)^2+4\sin(a)^2\sin(c)^2+\right.}}{b}$

input
```
int(sec(b*x+c)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))+1/2/b*ln(exp(I*(b*x+a))+I*exp(I*(a
-c)))*cos(2*a-2*c)-1/2/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))-1/2/b*ln(exp(I*
(b*x+a))-I*exp(I*(a-c)))*cos(2*a-2*c)-sin(b*x+2*a-c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int \sec(c+bx)\sin^2(a+bx)\,dx$$
$$=\frac{\cos(-a+c)^2\log(\sin(bx+c)+1)-\cos(-a+c)^2\log(-\sin(bx+c)+1)+4\cos(bx+c)\cos(-a+c)}{2b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(cos(-a + c)^2*log(sin(b*x + c) + 1) - cos(-a + c)^2*log(-sin(b*x + c)
 + 1) + 4*cos(b*x + c)*cos(-a + c)*sin(-a + c) - 2*(2*cos(-a + c)^2 - 1)*s
in(b*x + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1581 vs. $2(27) = 54$.

Time = 19.00 (sec) , antiderivative size = 3645, normalized size of antiderivative = 101.25

$$\int \sec(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a)**2,x)
```

output
```
2*Piecewise((-sin(b*x)/b, Eq(c, pi/2)), (sin(b*x)/b, Eq(c, -pi/2)), (0, Eq
(b, 0)), (-2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*
tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2
*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*
log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**3/(
b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**
2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(
tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*ta
n(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)
**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) -
1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 +
2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2
*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3*
tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)*
*2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/
2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(c/2)**
4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(
c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1
) - 1/(tan(c/2) + 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2
+ b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*t...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(36) = 72$.

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.89

$$\int \sec(c+bx)\sin^2(a+bx)\,dx = -\frac{(\cos(-2a+2c)+1)\log\left(\frac{\cos(bx+2c)^2+\cos(c)^2-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2+2\cos(bx+2c)\sin(c)+\sin(c)^2}{\cos(bx+2c)^2+\cos(c)^2+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2-2\cos(bx+2c)\sin(c)+\sin(c)^2}\right)+4\sin}{4b}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*((cos(-2*a + 2*c) + 1)*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*si
n(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos
(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*
cos(b*x + 2*c)*sin(c) + sin(c)^2)) + 4*sin(b*x + 2*a - c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 961 vs. $2(36) = 72$.

Time = 0.15 (sec) , antiderivative size = 961, normalized size of antiderivative = 26.69

$$\int \sec(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
((tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*a)^3
*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 8*tan(1/2*a)^
3*tan(1/2*c) + 20*tan(1/2*a)^2*tan(1/2*c)^2 - 8*tan(1/2*a)*tan(1/2*c)^3 +
tan(1/2*c)^4 - 2*tan(1/2*a)^2 + 8*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*c)^2 +
 1)*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(
1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan
(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 +
1) - (tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*
a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 8*tan(1/2
*a)^3*tan(1/2*c) + 20*tan(1/2*a)^2*tan(1/2*c)^2 - 8*tan(1/2*a)*tan(1/2*c)^
3 + tan(1/2*c)^4 - 2*tan(1/2*a)^2 + 8*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*c)
^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c) - 1))/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*
tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4
*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^
2 + 1) - 2*(tan(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*b*x
 + 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^2 + 16*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3
*tan(1/2*c)^3 + 4*tan(1/2*a)^4*tan(1/2*c)^3 - 6*tan(1/2*b*x + 1/2*c)*tan(1
/2*a)^2*tan(1/2*c)^4 - 4*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*b*x + 1/2*c)*
tan(1/2*a)^4 - 16*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2
*a)^4*tan(1/2*c) + 36*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c)^2 + ...
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 217, normalized size of antiderivative = 6.03

$$\int \sec(c+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{e^{-a\,2i+c\,1i-b\,x\,1i}\,1i}{2\,b} + \frac{e^{a\,2i-c\,1i+b\,x\,1i}\,1i}{2\,b}$$
$$+ \frac{e^{-a\,2i+c\,2i}\,\ln\left(-\frac{\left(e^{a\,2i}\,e^{-c\,2i}+1\right)^2\,1i}{2} - \frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(2\,e^{a\,2i}\,e^{-c\,2i}+e^{a\,4i}\,e^{-c\,4i}+1\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}+1\right)^2}{4\,b}$$
$$- \frac{e^{-a\,2i+c\,2i}\,\ln\left(\frac{\left(e^{a\,2i}\,e^{-c\,2i}+1\right)^2\,1i}{2} - \frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(2\,e^{a\,2i}\,e^{-c\,2i}+e^{a\,4i}\,e^{-c\,4i}+1\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}+1\right)^2}{4\,b}$$

input
```
int(sin(a + b*x)^2/cos(c + b*x),x)
```

output
```
(exp(a*2i - c*1i + b*x*1i)*1i)/(2*b) - (exp(c*1i - a*2i - b*x*1i)*1i)/(2*b
) + (exp(c*2i - a*2i)*log(- ((exp(a*2i)*exp(-c*2i) + 1)^2*1i)/2 - (exp(c*1
i)*exp(b*x*1i)*(2*exp(a*2i)*exp(-c*2i) + exp(a*4i)*exp(-c*4i) + 1))/2)*(ex
p(a*2i - c*2i) + 1)^2)/(4*b) - (exp(c*2i - a*2i)*log(((exp(a*2i)*exp(-c*2i
) + 1)^2*1i)/2 - (exp(c*1i)*exp(b*x*1i)*(2*exp(a*2i)*exp(-c*2i) + exp(a*4i
)*exp(-c*4i) + 1))/2)*(exp(a*2i - c*2i) + 1)^2)/(4*b)
```

Reduce [F]

$$\int \sec(c+bx)\sin^2(a+bx)\,dx = \int \sec(bx+c)\sin(bx+a)^2\,dx$$

input
```
int(sec(b*x+c)*sin(b*x+a)^2,x)
```

output
```
int(sec(b*x + c)*sin(a + b*x)**2,x)
```

3.270 $\int \sec^2(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx = -x\cos(2(a-c)) - \frac{\log(\cos(c+bx))\sin(2(a-c))}{b} + \frac{\cos^2(a-c)\tan(c+bx)}{b}$$

output
```
-x*cos(2*a-2*c)-ln(cos(b*x+c))*sin(2*a-2*c)/b+cos(a-c)^2*tan(b*x+c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(50) = 100.

Time = 0.35 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.54

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx = -\frac{\sec(c)\sec(c+bx)(bx\cos(2a-4c-bx)+bx\cos(2a-2c-bx)+bx\cos(2a+bx)+bx\cos(2a-2c+}{}$$

input
```
Integrate[Sec[c + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
-1/4*(Sec[c]*Sec[c + b*x]*(b*x*Cos[2*a - 4*c - b*x] + b*x*Cos[2*a - 2*c -
b*x] + b*x*Cos[2*a + b*x] + b*x*Cos[2*a - 2*c + b*x] - 2*Sin[b*x] + Log[Co
s[c + b*x]]*Sin[2*a - 4*c - b*x] + Sin[2*a - 2*c - b*x] + Log[Cos[c + b*x]
]*Sin[2*a - 2*c - b*x] + Log[Cos[c + b*x]]*Sin[2*a + b*x] - Sin[2*a - 2*c
+ b*x] + Log[Cos[c + b*x]]*Sin[2*a - 2*c + b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec^2(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sin^2(a+bx)\sec^2(bx+c)dx$$

input
```
Int[Sec[c + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.34

method	result
risch	$-x\,\mathrm{e}^{2i(a-c)}+2i\sin\left(2a-2c\right)x+\frac{2i\sin(2a-2c)a}{b}+\frac{i\mathrm{e}^{2i(2a-c)}}{2b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{i\mathrm{e}^{2ia}}{b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{i\mathrm{e}^{2i}}{2b\left(\mathrm{e}^{2i(bx+a+}\right.}$
default	$-\frac{\left(2\cos(c)^3\sin(a)^2\cos(a)+2\cos(c)^2\sin(c)\sin(a)^3-4\cos(c)^2\sin(c)\cos(a)^2\sin(a)-4\cos(c)\sin(c)^2\cos(a)\sin(a)^2+2\cos(c)\sin(c)^2\cos(a)^3+2\sin(c)^{}\right.}{\left(\cos(c)^2+\sin(c)^2\right)^2\left(\cos(a)^2+\sin(a)^2\right)^2\left(\sin(a)\cos(c)-\cos\right.}$

input
```
int(sec(b*x+c)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-x*exp(2*I*(a-c))+2*I*sin(2*a-2*c)*x+2*I/b*sin(2*a-2*c)*a+1/2*I/b/(exp(2*I
*(b*x+a+c))+exp(2*I*a))*exp(2*I*(2*a-c))+I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a
))*exp(2*I*a)+1/2*I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))*exp(2*I*c)-ln(exp(2*
I*(b*x+a))+exp(2*I*(a-c)))/b*sin(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.64

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx$$
$$=\frac{2\cos\left(bx+c\right)\cos\left(-a+c\right)\log\left(-\cos\left(bx+c\right)\right)\sin\left(-a+c\right)+\cos\left(-a+c\right)^2\sin\left(bx+c\right)-\left(2\,bx\cos\left(-a\right.\right.}{b\cos\left(bx+c\right)}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
(2*cos(b*x + c)*cos(-a + c)*log(-cos(b*x + c))*sin(-a + c) + cos(-a + c)^2
*sin(b*x + c) - (2*b*x*cos(-a + c)^2 - b*x)*cos(b*x + c))/(b*cos(b*x + c))
```

Sympy [F(-2)]

Exception generated.

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+c)**2*sin(b*x+a)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(50) = 100$.

Time = 0.06 (sec) , antiderivative size = 534, normalized size of antiderivative = 10.68

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(b*cos(2*a + 2*c)*cos(4*c) + b*sin(2*a + 2*c)*sin(4*c))*x + (2*b*x
*cos(4*c) + sin(4*a) + 2*sin(2*a + 2*c) + sin(4*c))*cos(2*b*x + 2*a + 4*c)
 + 2*(b*x*cos(2*b*x + 2*a + 4*c) + b*x*cos(2*a + 2*c))*cos(2*b*x + 6*c) +
(sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(2*b*x + 2*a + 4*c)^2*sin(-2*a
+ 2*c) + 2*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c)*sin(-2*a + 2*c) + cos(2*a
 + 2*c)^2*sin(-2*a + 2*c) + sin(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c) + 2*s
in(2*b*x + 2*a + 4*c)*sin(2*a + 2*c)*sin(-2*a + 2*c) + sin(2*a + 2*c)^2*si
n(-2*a + 2*c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin
(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2) + (2*b*x*sin(4*c) - cos(4*
a) - 2*cos(2*a + 2*c) - cos(4*c))*sin(2*b*x + 2*a + 4*c) + 2*(b*x*sin(2*b*
x + 2*a + 4*c) + b*x*sin(2*a + 2*c))*sin(2*b*x + 6*c) - (cos(4*a) + cos(4*
c))*sin(2*a + 2*c))/(b*cos(2*b*x + 2*a + 4*c)^2 + 2*b*cos(2*b*x + 2*a + 4*
c)*cos(2*a + 2*c) + b*cos(2*a + 2*c)^2 + b*sin(2*b*x + 2*a + 4*c)^2 + 2*b*
sin(2*b*x + 2*a + 4*c)*sin(2*a + 2*c) + b*sin(2*a + 2*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(50) = 100$.

Time = 0.14 (sec) , antiderivative size = 760, normalized size of antiderivative = 15.20

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-((tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*a)^4*tan(1/2*c)^2 + 16*tan(1/2*a)
^3*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 16*tan(1/2*
a)^3*tan(1/2*c) + 36*tan(1/2*a)^2*tan(1/2*c)^2 - 16*tan(1/2*a)*tan(1/2*c)^
3 + tan(1/2*c)^4 - 6*tan(1/2*a)^2 + 16*tan(1/2*a)*tan(1/2*c) - 6*tan(1/2*c
)^2 + 1)*(b*x + c)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^
2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4
*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*ta
n(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2
*c)^4 - tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)
^2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))*log(tan(b*x + c)^2 + 1)/(tan(
1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1
/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*ta
n(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - (tan(b*x + c)*tan(1/2*a)^4*tan(1/2*c)^4
 - 2*tan(b*x + c)*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(b*x + c)*tan(1/2*a)^3*
tan(1/2*c)^3 - 2*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^4 + tan(b*x + c)*tan
(1/2*a)^4 - 8*tan(b*x + c)*tan(1/2*a)^3*tan(1/2*c) + 20*tan(b*x + c)*tan(1
/2*a)^2*tan(1/2*c)^2 - 8*tan(b*x + c)*tan(1/2*a)*tan(1/2*c)^3 + tan(b*x +
c)*tan(1/2*c)^4 - 2*tan(b*x + c)*tan(1/2*a)^2 + 8*tan(b*x + c)*tan(1/2*a)*
tan(1/2*c) - 2*tan(b*x + c)*tan(1/2*c)^2 + tan(b*x + c))/(tan(1/2*a)^4*...
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.92

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx$$
$$= -x\left(\cos\left(2a-2c\right)-\sin\left(2a-2c\right)\,1\mathrm{i}\right)+\frac{\left(2\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}-c\,4\mathrm{i}}+1\right)\,1\mathrm{i}}{2\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}$$
$$-\frac{\mathrm{e}^{-a\,4\mathrm{i}+c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\right)\,\left(2\,b\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-2\,b\,\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$

input
```
int(sin(a + b*x)^2/cos(c + b*x)^2,x)
```

output
```
((2*exp(a*2i - c*2i) + exp(a*4i - c*4i) + 1)*1i)/(2*b*(exp(a*2i - c*2i) +
exp(a*2i + b*x*2i))) - x*(cos(2*a - 2*c) - sin(2*a - 2*c)*1i) - (exp(c*4i
- a*4i)*log(exp(a*2i)*exp(b*x*2i) + exp(a*2i)*exp(-c*2i))*(2*b*exp(a*2i -
c*2i) - 2*b*exp(a*6i - c*6i))*1i)/(4*b^2)
```

Reduce [F]

$$\int \sec^2(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
int(sec(b*x+c)^2*sin(b*x+a)^2,x)
```

output

```
(7*cos(b*x + c)*cos(a + b*x)*sin(a + b*x) + 96*cos(b*x + c)*int(tan((b*x +
 c)/2)**2/(tan((b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)**4
*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan((a
+ b*x)/2)**4 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x + c)
/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b + 96*cos(b*
x + c)*int(tan((a + b*x)/2)**2/(tan((b*x + c)/2)**4*tan((a + b*x)/2)**4 +
2*tan((b*x + c)/2)**4*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b
*x + c)/2)**2*tan((a + b*x)/2)**4 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)
**2 - 2*tan((b*x + c)/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2
+ 1),x)*b - 128*cos(b*x + c)*int((tan((b*x + c)/2)*tan((a + b*x)/2))/(tan(
(b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)**4*tan((a + b*x)/
2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan((a + b*x)/2)**4 -
4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x + c)/2)**2 + tan((a
 + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b - 32*cos(b*x + c)*int(1/(t
an((b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)**4*tan((a + b*
x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan((a + b*x)/2)**4
 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x + c)/2)**2 + tan
((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b + 16*cos(b*x + c)*sin(a
 + b*x) + 9*cos(b*x + c)*a + 9*cos(b*x + c)*b*x - 8*cos(a + b*x)*sin(b*x +
 c) + 8*cos(a + b*x)*sin(a + b*x) + 4*sin(b*x + c)*sin(a + b*x)**2 - 8*...
```

3.271 $\int \sec^3(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(c+bx))\cos^2(a-c)}{2b} - \frac{\operatorname{arctanh}(\sin(c+bx))\cos(2(a-c))}{b} + \frac{\sec(c+bx)\sin(2(a-c))}{b} + \frac{\cos^2(a-c)\sec(c+bx)\tan(c+bx)}{2b}$$

output
```
1/2*arctanh(sin(b*x+c))*cos(a-c)^2/b-arctanh(sin(b*x+c))*cos(2*a-2*c)/b+se
c(b*x+c)*sin(2*a-2*c)/b+1/2*cos(a-c)^2*sec(b*x+c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.88

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx$$
$$= \frac{-\log\left(\cos\left(\frac{1}{2}(c+bx)\right)-\sin\left(\frac{1}{2}(c+bx)\right)\right)+3\cos(2(a-c))\left(\log\left(\cos\left(\frac{1}{2}(c+bx)\right)-\sin\left(\frac{1}{2}(c+bx)\right)\right)-\mathrm{lo}\right.}{}$$

input
```
Integrate[Sec[c + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(-Log[Cos[(c + b*x)/2] - Sin[(c + b*x)/2]] + 3*Cos[2*(a - c)]*(Log[Cos[(c
+ b*x)/2] - Sin[(c + b*x)/2]] - Log[Cos[(c + b*x)/2] + Sin[(c + b*x)/2]])
+ Log[Cos[(c + b*x)/2] + Sin[(c + b*x)/2]] - 4*Sec[c]*Sin[2*(a - c)] + 4*S
ec[c + b*x]*Sin[2*(a - c)] + 2*Cos[a - c]^2*Sec[c + b*x]*Tan[c + b*x])/(4*
b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec^3(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sin^2(a+bx)\sec^3(bx+c)dx$$

input
```
Int[Sec[c + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.84

method	result
risch	$-\frac{i\left(5\,\mathrm{e}^{i(3bx+6a+c)}+2\,\mathrm{e}^{i(3bx+4a+3c)}-3\,\mathrm{e}^{i(3bx+2a+5c)}+3\,\mathrm{e}^{i(bx+6a-c)}-2\,\mathrm{e}^{i(bx+4a+c)}-5\,\mathrm{e}^{i(bx+2a+3c)}\right)}{4\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^2 b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a-c)}\right)}{4b}$
default	Expression too large to display

input
```
int(sec(b*x+c)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/(exp(2*I*(b*x+a+c))+exp(2*I*a))^2/b*(5*exp(I*(3*b*x+6*a+c))+2*exp(I
*(3*b*x+4*a+3*c))-3*exp(I*(3*b*x+2*a+5*c))+3*exp(I*(b*x+6*a-c))-2*exp(I*(b
*x+4*a+c))-5*exp(I*(b*x+2*a+3*c)))-1/4/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))
+3/4/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))*cos(2*a-2*c)+1/4/b*ln(exp(I*(b*x+
a))+I*exp(I*(a-c)))-3/4/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))*cos(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\left(3\,\cos\left(-a+c\right)^2-2\right)\cos\left(bx+c\right)^2\log\left(\sin\left(bx+c\right)+1\right)-\left(3\,\cos\left(-a+c\right)^2-2\right)\cos\left(bx+c\right)^2\log\left(-\mathrm{s}\right.}{4\,b\cos\left(bx+c\right)}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*((3*cos(-a + c)^2 - 2)*cos(b*x + c)^2*log(sin(b*x + c) + 1) - (3*cos(
-a + c)^2 - 2)*cos(b*x + c)^2*log(-sin(b*x + c) + 1) - 2*cos(-a + c)^2*sin
(b*x + c) + 8*cos(b*x + c)*cos(-a + c)*sin(-a + c))/(b*cos(b*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+c)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. $2(84) = 168$.

Time $= 0.24$ (sec) , antiderivative size $= 1263$, normalized size of antiderivative $= 14.35$

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/8*(2*(5*sin(3*b*x + 4*a + 2*c) + 2*sin(3*b*x + 2*a + 4*c) - 3*sin(3*b*x
+ 6*c) + 3*sin(b*x + 4*a) - 2*sin(b*x + 2*a + 2*c) - 5*sin(b*x + 4*c))*cos
(4*b*x + 2*a + 5*c) - 10*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + c))*cos(3*b
*x + 4*a + 2*c) - 4*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + c))*cos(3*b*x +
2*a + 4*c) + 6*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + c))*cos(3*b*x + 6*c)
+ 4*(3*sin(b*x + 4*a) - 2*sin(b*x + 2*a + 2*c) - 5*sin(b*x + 4*c))*cos(2*b
*x + 2*a + 3*c) + ((3*cos(-2*a + 2*c) - 1)*cos(4*b*x + 2*a + 5*c)^2 + 4*(3
*cos(-2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c)^2 + (3*cos(-2*a + 2*c) - 1)*s
in(4*b*x + 2*a + 5*c)^2 + 4*(3*cos(-2*a + 2*c) - 1)*sin(2*b*x + 2*a + 3*c)
^2 + 2*(2*(3*cos(-2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c) + 3*cos(2*a + c)*
cos(-2*a + 2*c) - cos(2*a + c))*cos(4*b*x + 2*a + 5*c) + 4*(3*cos(2*a + c)
*cos(-2*a + 2*c) - cos(2*a + c))*cos(2*b*x + 2*a + 3*c) - cos(2*a + c)^2 +
 3*(cos(2*a + c)^2 + sin(2*a + c)^2)*cos(-2*a + 2*c) + 2*(2*(3*cos(-2*a +
2*c) - 1)*sin(2*b*x + 2*a + 3*c) + 3*cos(-2*a + 2*c)*sin(2*a + c) - sin(2*
a + c))*sin(4*b*x + 2*a + 5*c) + 4*(3*cos(-2*a + 2*c)*sin(2*a + c) - sin(2
*a + c))*sin(2*b*x + 2*a + 3*c) - sin(2*a + c)^2)*log((cos(b*x + 2*c)^2 +
cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*s
in(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) +
 sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - 2*(5*cos(3*b*x
+ 4*a + 2*c) + 2*cos(3*b*x + 2*a + 4*c) - 3*cos(3*b*x + 6*c) + 3*cos(b*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1502 vs. $2(84) = 168$.

Time = 0.19 (sec) , antiderivative size = 1502, normalized size of antiderivative = 17.07

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*((tan(1/2*a)^4*tan(1/2*c)^4 - 10*tan(1/2*a)^4*tan(1/2*c)^2 + 24*tan(1
/2*a)^3*tan(1/2*c)^3 - 10*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 24*ta
n(1/2*a)^3*tan(1/2*c) + 52*tan(1/2*a)^2*tan(1/2*c)^2 - 24*tan(1/2*a)*tan(1
/2*c)^3 + tan(1/2*c)^4 - 10*tan(1/2*a)^2 + 24*tan(1/2*a)*tan(1/2*c) - 10*t
an(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^4*tan(1/2*
c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2
*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*ta
n(1/2*c)^2 + 1) - (tan(1/2*a)^4*tan(1/2*c)^4 - 10*tan(1/2*a)^4*tan(1/2*c)^
2 + 24*tan(1/2*a)^3*tan(1/2*c)^3 - 10*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*
a)^4 - 24*tan(1/2*a)^3*tan(1/2*c) + 52*tan(1/2*a)^2*tan(1/2*c)^2 - 24*tan(
1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 10*tan(1/2*a)^2 + 24*tan(1/2*a)*tan(1
/2*c) - 10*tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c) - 1))/(tan(1/2*a
)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)
^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2
*a)^2 + 2*tan(1/2*c)^2 + 1) - 2*(tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1
/2*c)^4 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*b
*x + 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^3 - 8*tan(1/2*b*x + 1/2*c)^2*tan(1/2
*a)^4*tan(1/2*c)^3 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^2*tan(1/2*c)^4 +
8*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*b*x + 1/2*c)*
tan(1/2*a)^4*tan(1/2*c)^4 + tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4 - 8*tan...
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^2/cos(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^3(c+bx)\sin^2(a+bx)\,dx = \int \sec(bx+c)^3 \sin(bx+a)^2\,dx$$

input
```
int(sec(b*x+c)^3*sin(b*x+a)^2,x)
```

output
```
int(sec(b*x + c)**3*sin(a + b*x)**2,x)
```

3.272 $\int \sec^4(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \sec^4(c+bx)\sin^2(a+bx)\,dx = \frac{\sec^2(c+bx)\sin(2(a-c))}{2b} + \frac{\cos^2(a-c)\tan(c+bx)}{b} - \frac{\cos(2(a-c))\tan(c+bx)}{b} + \frac{\cos^2(a-c)\tan^3(c+bx)}{3b}$$

output
```
1/2*sec(b*x+c)^2*sin(2*a-2*c)/b+cos(a-c)^2*tan(b*x+c)/b-cos(2*a-2*c)*tan(b
*x+c)/b+1/3*cos(a-c)^2*tan(b*x+c)^3/b
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \sec^4(c+bx)\sin^2(a+bx)\,dx$$
$$= \frac{\sec(c)\sec^3(c+bx)(3\sin(bx)+\sin(2a-4c-3bx)+3\sin(2a-2c-bx)+3\sin(2a+bx)-\sin(2a+3bx}{12b}$$

input
```
Integrate[Sec[c + b*x]^4*Sin[a + b*x]^2,x]
```

output
```
(Sec[c]*Sec[c + b*x]^3*(3*Sin[b*x] + Sin[2*a - 4*c - 3*b*x] + 3*Sin[2*a -
2*c - b*x] + 3*Sin[2*a + b*x] - Sin[2*a + 3*b*x] + Sin[2*c + 3*b*x]))/(12*
b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec^4(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin^2(a+bx)\sec^4(bx+c)dx$$

input
```
Int[Sec[c + b*x]^4*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 5.70 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{3\sin(bx+c)-2\sin(3bx+2a+c)+\sin(3bx+3c)}{3b(\cos(3bx+3c)+3\cos(bx+c))}$
risch	$-\frac{2i\left(3\,\mathrm{e}^{2i(2bx+4a+c)}+3\,\mathrm{e}^{2i(bx+4a)}-3\,\mathrm{e}^{2i(bx+3a+c)}+\mathrm{e}^{2i(4a-c)}-\mathrm{e}^{6ia}+\mathrm{e}^{2i(2a+c)}\right)}{3\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^3 b}$
default	$\frac{2\cos(a)\cos(c)+2\sin(a)\sin(c)}{2(\sin(a)\cos(c)-\cos(a)\sin(c))^3(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))^2}-\frac{}{3(\sin(a)\cos(c)-\cos(a)\sin(}$

input
```
int(sec(b*x+c)^4*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*sin(b*x+c)-2*sin(3*b*x+2*a+c)+sin(3*b*x+3*c))/b/(cos(3*b*x+3*c)+3*c
os(b*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \sec^4(c+bx)\sin^2(a+bx)\,dx = -\frac{3\cos(bx+c)\cos(-a+c)\sin(-a+c)+\left(\left(4\cos(-a+c)^2-3\right)\cos(bx+c)^2-\cos(-a+c)^2\right)\sin(b}{3b\cos(bx+c)^3}$$

input
```
integrate(sec(b*x+c)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*cos(b*x + c)*cos(-a + c)*sin(-a + c) + ((4*cos(-a + c)^2 - 3)*cos(
b*x + c)^2 - cos(-a + c)^2)*sin(b*x + c))/(b*cos(b*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+c)**4*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(80) = 160$.

Time = 0.04 (sec) , antiderivative size = 894, normalized size of antiderivative = 10.64

$$\int \sec^4(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
2/3*((3*sin(4*b*x + 4*a + 4*c) + 3*sin(2*b*x + 4*a + 2*c) - 3*sin(2*b*x +
2*a + 4*c) + sin(4*a) - sin(2*a + 2*c) + sin(4*c))*cos(6*b*x + 2*a + 8*c)
- 3*(3*sin(2*b*x + 2*a + 4*c) + sin(2*a + 2*c))*cos(4*b*x + 4*a + 4*c) + 3
*(3*sin(4*b*x + 4*a + 4*c) + 3*sin(2*b*x + 4*a + 2*c) - 3*sin(2*b*x + 2*a
+ 4*c) + sin(4*a) - sin(2*a + 2*c) + sin(4*c))*cos(4*b*x + 2*a + 6*c) + 3*
(3*sin(2*b*x + 4*a + 2*c) + sin(4*a) + sin(4*c))*cos(2*b*x + 2*a + 4*c) +
(sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (3*cos(4*b*x + 4*a + 4*c) + 3*cos(2
*b*x + 4*a + 2*c) - 3*cos(2*b*x + 2*a + 4*c) + cos(4*a) - cos(2*a + 2*c) +
 cos(4*c))*sin(6*b*x + 2*a + 8*c) + 3*(3*cos(2*b*x + 2*a + 4*c) + cos(2*a
+ 2*c))*sin(4*b*x + 4*a + 4*c) - 3*(3*cos(4*b*x + 4*a + 4*c) + 3*cos(2*b*x
 + 4*a + 2*c) - 3*cos(2*b*x + 2*a + 4*c) + cos(4*a) - cos(2*a + 2*c) + cos
(4*c))*sin(4*b*x + 2*a + 6*c) + 3*cos(2*a + 2*c)*sin(2*b*x + 4*a + 2*c) -
3*(3*cos(2*b*x + 4*a + 2*c) + cos(4*a) + cos(4*c))*sin(2*b*x + 2*a + 4*c)
- (cos(4*a) + cos(4*c))*sin(2*a + 2*c) - 3*cos(2*b*x + 4*a + 2*c)*sin(2*a
+ 2*c))/(b*cos(6*b*x + 2*a + 8*c)^2 + 9*b*cos(4*b*x + 2*a + 6*c)^2 + 9*b*c
os(2*b*x + 2*a + 4*c)^2 + 6*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c) + b*co
s(2*a + 2*c)^2 + b*sin(6*b*x + 2*a + 8*c)^2 + 9*b*sin(4*b*x + 2*a + 6*c)^2
 + 9*b*sin(2*b*x + 2*a + 4*c)^2 + 6*b*sin(2*b*x + 2*a + 4*c)*sin(2*a + 2*c
) + b*sin(2*a + 2*c)^2 + 2*(3*b*cos(4*b*x + 2*a + 6*c) + 3*b*cos(2*b*x + 2
*a + 4*c) + b*cos(2*a + 2*c))*cos(6*b*x + 2*a + 8*c) + 6*(3*b*cos(2*b*x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(80) = 160$.

Time = 0.14 (sec) , antiderivative size = 718, normalized size of antiderivative = 8.55

$$\int \sec^4(c + bx) \sin^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+c)^4*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/3*(tan(b*x + c)^3*tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(b*x + c)^3*tan(1/2*a
)^4*tan(1/2*c)^2 + 8*tan(b*x + c)^3*tan(1/2*a)^3*tan(1/2*c)^3 + 6*tan(b*x
+ c)^2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(b*x + c)^3*tan(1/2*a)^2*tan(1/2*c
)^4 - 6*tan(b*x + c)^2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(b*x + c)^3*tan(1/2*
a)^4 - 8*tan(b*x + c)^3*tan(1/2*a)^3*tan(1/2*c) - 6*tan(b*x + c)^2*tan(1/2
*a)^4*tan(1/2*c) + 20*tan(b*x + c)^3*tan(1/2*a)^2*tan(1/2*c)^2 + 36*tan(b*
x + c)^2*tan(1/2*a)^3*tan(1/2*c)^2 + 12*tan(b*x + c)*tan(1/2*a)^4*tan(1/2*
c)^2 - 8*tan(b*x + c)^3*tan(1/2*a)*tan(1/2*c)^3 - 36*tan(b*x + c)^2*tan(1/
2*a)^2*tan(1/2*c)^3 - 24*tan(b*x + c)*tan(1/2*a)^3*tan(1/2*c)^3 + tan(b*x
+ c)^3*tan(1/2*c)^4 + 6*tan(b*x + c)^2*tan(1/2*a)*tan(1/2*c)^4 + 12*tan(b*
x + c)*tan(1/2*a)^2*tan(1/2*c)^4 - 2*tan(b*x + c)^3*tan(1/2*a)^2 - 6*tan(b
*x + c)^2*tan(1/2*a)^3 + 8*tan(b*x + c)^3*tan(1/2*a)*tan(1/2*c) + 36*tan(b
*x + c)^2*tan(1/2*a)^2*tan(1/2*c) + 24*tan(b*x + c)*tan(1/2*a)^3*tan(1/2*c
) - 2*tan(b*x + c)^3*tan(1/2*c)^2 - 36*tan(b*x + c)^2*tan(1/2*a)*tan(1/2*c
)^2 - 48*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(b*x + c)^2*tan(1/2
*c)^3 + 24*tan(b*x + c)*tan(1/2*a)*tan(1/2*c)^3 + tan(b*x + c)^3 + 6*tan(b
*x + c)^2*tan(1/2*a) + 12*tan(b*x + c)*tan(1/2*a)^2 - 6*tan(b*x + c)^2*tan
(1/2*c) - 24*tan(b*x + c)*tan(1/2*a)*tan(1/2*c) + 12*tan(b*x + c)*tan(1/2*
c)^2)/((tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/
2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(...
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+bx)\sin^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^2/cos(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \sec^4(c+bx)\sin^2(a+bx)\,dx = \frac{\cos(bx+c)\cos(bx+a)\sin(bx+a)+\sin(bx+c)^3-\sin(bx+c)\sin(bx+a)^2-\sin(bx+c)}{3\cos(bx+c)\,b\left(\sin(bx+c)^2-1\right)}$$

input
```
int(sec(b*x+c)^4*sin(b*x+a)^2,x)
```

output
```
(cos(b*x + c)*cos(a + b*x)*sin(a + b*x) + sin(b*x + c)**3 - sin(b*x + c)*s
in(a + b*x)**2 - sin(b*x + c))/(3*cos(b*x + c)*b*(sin(b*x + c)**2 - 1))
```

3.273 $\int \sec(c-bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \sec(c-bx)\sin^2(a+bx)\,dx=-\frac{\operatorname{arctanh}(\sin(c-bx))\cos^2(a+c)}{b}-\frac{\sin(2a+c+bx)}{b}$$

output
```
arctanh(sin(b*x-c))*cos(a+c)^2/b-sin(b*x+2*a+c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 76 vs. 2(34) = 68.

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int \sec(c-bx)\sin^2(a+bx)\,dx$$
$$=\frac{\cos^2(a+c)\left(\log\left(\cos\left(\frac{1}{2}(c-bx)\right)-\sin\left(\frac{1}{2}(c-bx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c-bx)\right)+\sin\left(\frac{1}{2}(c-bx)\right)\right)\right)-\sin(2a}{b}$$

input
```
Integrate[Sec[c - b*x]*Sin[a + b*x]^2,x]
```

output
```
(Cos[a + c]^2*(Log[Cos[(c - b*x)/2] - Sin[(c - b*x)/2]] - Log[Cos[(c - b*x
)/2] + Sin[(c - b*x)/2]]) - Sin[2*a + c + b*x])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec(c-bx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \sin^2(a+bx)\sec(c-bx)dx$$

input
```
Int[Sec[c - b*x]*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 4.00

method	result
risch	$-\frac{\ln\left(e^{i(bx+a)}-ie^{i(a+c)}\right)}{2b}-\frac{\ln\left(e^{i(bx+a)}-ie^{i(a+c)}\right)\cos(2a+2c)}{2b}+\frac{\ln\left(e^{i(bx+a)}+ie^{i(a+c)}\right)}{2b}+\frac{\ln\left(e^{i(bx+a)}+ie^{i(a+c)}\right)\cos(2a+2c)}{2b}$
default	$\frac{\frac{2(-\cos(a)\cos(c)+\sin(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-2\sin(a)\cos(c)-2\cos(a)\sin(c)}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)}-\frac{8(\cos(a)\cos(c)-\sin(a)\sin(c))^2\arctan\left(\frac{2(c}{2}\right.}{\left(4\cos(c)^2\sin(a)^2+4\cos(a)^2\cos(c)^2+4\sin(a)^2\sin(c)^2+\right.}}{b}$

input
```
int(sec(b*x-c)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*ln(exp(I*(b*x+a))-I*exp(I*(a+c)))-1/2/b*ln(exp(I*(b*x+a))-I*exp(I*(
a+c)))*cos(2*a+2*c)+1/2/b*ln(exp(I*(b*x+a))+I*exp(I*(a+c)))+1/2/b*ln(exp(I
*(b*x+a))+I*exp(I*(a+c)))*cos(2*a+2*c)-sin(b*x+2*a+c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(34) = 68.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.44

$$\int \sec(c-bx)\sin^2(a+bx)\,dx = \frac{\cos(a+c)^2\log\left(\frac{2(\cos(a+c)\sin(bx+a)-\cos(bx+a)\sin(a+c)+1)}{\cos(a+c)+1}\right)-\cos(a+c)^2\log\left(-\frac{2(\cos(a+c)\sin(bx+a)-\cos(bx+a)\sin(a}{\cos(a+c)+1}\right.}{2b}$$

input
```
integrate(sec(b*x-c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(cos(a + c)^2*log(2*(cos(a + c)*sin(b*x + a) - cos(b*x + a)*sin(a + c)
 + 1)/(cos(a + c) + 1)) - cos(a + c)^2*log(-2*(cos(a + c)*sin(b*x + a) - c
os(b*x + a)*sin(a + c) - 1)/(cos(a + c) + 1)) - 2*cos(a + c)*sin(b*x + a)
- 2*cos(b*x + a)*sin(a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1581 vs. 2(27) = 54.

Time = 20.15 (sec) , antiderivative size = 3645, normalized size of antiderivative = 107.21

$$\int \sec(c-bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x-c)*sin(b*x+a)**2,x)
```

output
```
2*Piecewise((sin(b*x)/b, Eq(c, pi/2)), (-sin(b*x)/b, Eq(c, -pi/2)), (0, Eq
(b, 0)), (-2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))*
tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2
*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*
log(tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)**3/(
b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**
2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) + tan(c/2)/(
tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*ta
n(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)
**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) +
1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 +
2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2
*log(tan(b*x/2) - tan(c/2)/(tan(c/2) + 1) + 1/(tan(c/2) + 1))*tan(c/2)**3*
tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)*
*2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/
2) - tan(c/2)/(tan(c/2) + 1) + 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(c/2)**
4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(
c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) + 1
) + 1/(tan(c/2) + 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2
+ b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*t...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(34) = 68$.

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.12

$$\int \sec(c-bx)\sin^2(a+bx)\,dx = -\frac{(\cos(2a+2c)+1)\log\left(\frac{\cos(bx)^2+\cos(c)^2-2\cos(c)\sin(bx)+\sin(bx)^2+2\cos(bx)\sin(c)+\sin(c)^2}{\cos(bx)^2+\cos(c)^2+2\cos(c)\sin(bx)+\sin(bx)^2-2\cos(bx)\sin(c)+\sin(c)^2}\right)+4\sin(bx+2a+c)}{4b}$$

input
```
integrate(sec(b*x-c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*((cos(2*a + 2*c) + 1)*log((cos(b*x)^2 + cos(c)^2 - 2*cos(c)*sin(b*x)
+ sin(b*x)^2 + 2*cos(b*x)*sin(c) + sin(c)^2)/(cos(b*x)^2 + cos(c)^2 + 2*co
s(c)*sin(b*x) + sin(b*x)^2 - 2*cos(b*x)*sin(c) + sin(c)^2)) + 4*sin(b*x +
2*a + c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 961 vs. $2(34) = 68$.

Time = 0.16 (sec) , antiderivative size = 961, normalized size of antiderivative = 28.26

$$\int \sec(c - bx) \sin^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(sec(b*x-c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
((tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^4*tan(1/2*c)^2 - 8*tan(1/2*a)^3
*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 8*tan(1/2*a)^
3*tan(1/2*c) + 20*tan(1/2*a)^2*tan(1/2*c)^2 + 8*tan(1/2*a)*tan(1/2*c)^3 +
tan(1/2*c)^4 - 2*tan(1/2*a)^2 - 8*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*c)^2 +
 1)*log(abs(tan(1/2*b*x - 1/2*c) + 1))/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(
1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan
(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 +
1) - (tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^4*tan(1/2*c)^2 - 8*tan(1/2*
a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 8*tan(1/2
*a)^3*tan(1/2*c) + 20*tan(1/2*a)^2*tan(1/2*c)^2 + 8*tan(1/2*a)*tan(1/2*c)^
3 + tan(1/2*c)^4 - 2*tan(1/2*a)^2 - 8*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*c)
^2 + 1)*log(abs(tan(1/2*b*x - 1/2*c) - 1))/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*
tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4
*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^
2 + 1) - 2*(tan(1/2*b*x - 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*b*x
 - 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^2 - 16*tan(1/2*b*x - 1/2*c)*tan(1/2*a)^3
*tan(1/2*c)^3 - 4*tan(1/2*a)^4*tan(1/2*c)^3 - 6*tan(1/2*b*x - 1/2*c)*tan(1
/2*a)^2*tan(1/2*c)^4 - 4*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*b*x - 1/2*c)*
tan(1/2*a)^4 + 16*tan(1/2*b*x - 1/2*c)*tan(1/2*a)^3*tan(1/2*c) + 4*tan(1/2
*a)^4*tan(1/2*c) + 36*tan(1/2*b*x - 1/2*c)*tan(1/2*a)^2*tan(1/2*c)^2 + ...
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 217, normalized size of antiderivative = 6.38

$$\int \sec(c-bx)\sin^2(a+bx)\,dx$$
$$= -\frac{e^{-a\,2i-c\,1i-b\,x\,1i}\,1i}{2\,b}+\frac{e^{a\,2i+c\,1i+b\,x\,1i}\,1i}{2\,b}$$
$$+\frac{e^{-a\,2i-c\,2i}\,\ln\left(-\frac{\left(e^{a\,2i}\,e^{c\,2i}+1\right)^2\,1i}{2}-\frac{e^{-c\,1i}\,e^{b\,x\,1i}\,\left(2\,e^{a\,2i}\,e^{c\,2i}+e^{a\,4i}\,e^{c\,4i}+1\right)}{2}\right)\,\left(e^{a\,2i+c\,2i}+1\right)^2}{4\,b}$$
$$-\frac{e^{-a\,2i-c\,2i}\,\ln\left(\frac{\left(e^{a\,2i}\,e^{c\,2i}+1\right)^2\,1i}{2}-\frac{e^{-c\,1i}\,e^{b\,x\,1i}\,\left(2\,e^{a\,2i}\,e^{c\,2i}+e^{a\,4i}\,e^{c\,4i}+1\right)}{2}\right)\,\left(e^{a\,2i+c\,2i}+1\right)^2}{4\,b}$$

input
```
int(sin(a + b*x)^2/cos(c - b*x),x)
```

output
```
(exp(a*2i + c*1i + b*x*1i)*1i)/(2*b) - (exp(- a*2i - c*1i - b*x*1i)*1i)/(2
*b) + (exp(- a*2i - c*2i)*log(- ((exp(a*2i)*exp(c*2i) + 1)^2*1i)/2 - (exp(
-c*1i)*exp(b*x*1i)*(2*exp(a*2i)*exp(c*2i) + exp(a*4i)*exp(c*4i) + 1))/2)*(
exp(a*2i + c*2i) + 1)^2)/(4*b) - (exp(- a*2i - c*2i)*log(((exp(a*2i)*exp(c
*2i) + 1)^2*1i)/2 - (exp(-c*1i)*exp(b*x*1i)*(2*exp(a*2i)*exp(c*2i) + exp(a
*4i)*exp(c*4i) + 1))/2)*(exp(a*2i + c*2i) + 1)^2)/(4*b)
```

Reduce [F]

$$\int \sec(c-bx)\sin^2(a+bx)\,dx=\int \sec\left(bx-c\right)\sin\left(bx+a\right)^2dx$$

input
```
int(sec(b*x-c)*sin(b*x+a)^2,x)
```

output
```
int(sec(b*x - c)*sin(a + b*x)**2,x)
```

3.274 $\int \sec^2(c-bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \sec^2(c-bx)\sin^2(a+bx)\,dx=-x\cos(2(a+c))-\frac{\log(\cos(c-bx))\sin(2(a+c))}{b}-\frac{\cos^2(a+c)\tan(c-bx)}{b}$$

output
```
-x*cos(2*a+2*c)-ln(cos(b*x-c))*sin(2*a+2*c)/b+cos(a+c)^2*tan(b*x-c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. $2(47) = 94$.

Time = 0.43 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.87

$$\int \sec^2(c-bx)\sin^2(a+bx)\,dx= -\frac{\sec(c)\sec(c-bx)(bx\cos(2a+2c-bx)+bx\cos(2a+4c-bx)+bx\cos(2a+bx)+bx\cos(2a+2c+$$

input
```
Integrate[Sec[c - b*x]^2*Sin[a + b*x]^2,x]
```

output
```
-1/4*(Sec[c]*Sec[c - b*x]*(b*x*Cos[2*a + 2*c - b*x] + b*x*Cos[2*a + 4*c -
b*x] + b*x*Cos[2*a + b*x] + b*x*Cos[2*a + 2*c + b*x] - 2*Sin[b*x] + Sin[2*
a + 2*c - b*x] + Log[Cos[c - b*x]]*Sin[2*a + 2*c - b*x] + Log[Cos[c - b*x]
]*Sin[2*a + 4*c - b*x] + Log[Cos[c - b*x]]*Sin[2*a + b*x] - Sin[2*a + 2*c
+ b*x] + Log[Cos[c - b*x]]*Sin[2*a + 2*c + b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec^2(c-bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin^2(a+bx)\sec^2(c-bx)dx$$

input
```
Int[Sec[c - b*x]^2*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.38

method	result
risch	$-x\,e^{2i(a+c)}+2i\sin(2a+2c)\,x+\frac{2i\sin(2a+2c)a}{b}+\frac{ie^{4i(a+c)}}{2b\left(e^{2i(a+c)}+e^{2i(bx+a)}\right)}+\frac{ie^{2i(a+c)}}{b\left(e^{2i(a+c)}+e^{2i(bx+a)}\right)}+\frac{}{2b\left(e^{2i(a+\right.}}$
default	$-\frac{\left(2\sin(c)^3\sin(a)\cos(a)^2+4\cos(c)\sin(c)^2\cos(a)\sin(a)^2-2\cos(c)\sin(c)^2\cos(a)^3+2\cos(c)^2\sin(c)\sin(a)^3-4\cos(c)^2\sin(c)\cos(a)^2\sin(a)-2\cos(c)\right.}{\left(\cos(c)^2+\sin(c)^2\right)^2\left(\cos(a)^2+\sin(a)^2\right)^2\left(-\sin(a)\cos(c)-\right.}$

input
```
int(sec(b*x-c)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-x*exp(2*I*(a+c))+2*I*sin(2*a+2*c)*x+2*I/b*sin(2*a+2*c)*a+1/2*I/b/(exp(2*I
*(a+c))+exp(2*I*(b*x+a)))*exp(4*I*(a+c))+I/b/(exp(2*I*(a+c))+exp(2*I*(b*x+
a)))*exp(2*I*(a+c))+1/2*I/b/(exp(2*I*(a+c))+exp(2*I*(b*x+a)))-ln(exp(2*I*(
a+c))+exp(2*I*(b*x+a)))/b*sin(2*a+2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(52) = 104$.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.91

$$\int \sec^2(c-bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\cos(bx+a)\cos(a+c)^2\sin(a+c)+\left(2\,bx\cos(a+c)^3-bx\cos(a+c)\right)\cos(bx+a)+2\left(\cos(bx+a)\right.}{}$$

input
```
integrate(sec(b*x-c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(cos(b*x + a)*cos(a + c)^2*sin(a + c) + (2*b*x*cos(a + c)^3 - b*x*cos(a +
 c))*cos(b*x + a) + 2*(cos(b*x + a)*cos(a + c)^2*sin(a + c) - (cos(a + c)^
3 - cos(a + c))*sin(b*x + a))*log(2*(cos(b*x + a)*cos(a + c) + sin(b*x + a
)*sin(a + c))/(cos(a + c) + 1)) - (cos(a + c)^3 - (2*b*x*cos(a + c)^2 - b*
x)*sin(a + c))*sin(b*x + a))/(b*cos(b*x + a)*cos(a + c) + b*sin(b*x + a)*s
in(a + c))
```

Sympy [F(-2)]

Exception generated.

$$\int \sec^2(c-bx)\sin^2(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x-c)**2*sin(b*x+a)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(52) = 104$.

Time = 0.06 (sec) , antiderivative size = 542, normalized size of antiderivative = 11.53

$$\int \sec^2(c-bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x-c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*b*x*cos(2*b*x)*cos(2*a + 4*c) + 2*b*x*sin(2*b*x)*sin(2*a + 4*c) +
2*(b*cos(2*a + 4*c)*cos(2*c) + b*sin(2*a + 4*c)*sin(2*c))*x + (2*b*x*cos(2
*b*x) + 2*b*x*cos(2*c) + sin(4*a + 6*c) + 2*sin(2*a + 4*c) + sin(2*c))*cos
(2*b*x + 2*a + 2*c) + (cos(2*b*x + 2*a + 2*c)^2*sin(2*a + 2*c) + 2*cos(2*b
*x + 2*a + 2*c)*cos(2*a + 4*c)*sin(2*a + 2*c) + cos(2*a + 4*c)^2*sin(2*a +
 2*c) + sin(2*b*x + 2*a + 2*c)^2*sin(2*a + 2*c) + 2*sin(2*b*x + 2*a + 2*c)
*sin(2*a + 4*c)*sin(2*a + 2*c) + sin(2*a + 4*c)^2*sin(2*a + 2*c))*log(cos(
2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 + 2*sin(2*b*x
)*sin(2*c) + sin(2*c)^2) + (2*b*x*sin(2*b*x) + 2*b*x*sin(2*c) - cos(4*a +
6*c) - 2*cos(2*a + 4*c) - cos(2*c))*sin(2*b*x + 2*a + 2*c) + cos(2*a + 4*c
)*sin(4*a + 6*c) - cos(4*a + 6*c)*sin(2*a + 4*c) - cos(2*c)*sin(2*a + 4*c)
 + cos(2*a + 4*c)*sin(2*c))/(b*cos(2*b*x + 2*a + 2*c)^2 + 2*b*cos(2*b*x +
2*a + 2*c)*cos(2*a + 4*c) + b*cos(2*a + 4*c)^2 + b*sin(2*b*x + 2*a + 2*c)^
2 + 2*b*sin(2*b*x + 2*a + 2*c)*sin(2*a + 4*c) + b*sin(2*a + 4*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(52) = 104$.

Time = 0.15 (sec) , antiderivative size = 790, normalized size of antiderivative = 16.81

$$\int \sec^2(c-bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x-c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-((tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*a)^4*tan(1/2*c)^2 - 16*tan(1/2*a)
^3*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 16*tan(1/2*
a)^3*tan(1/2*c) + 36*tan(1/2*a)^2*tan(1/2*c)^2 + 16*tan(1/2*a)*tan(1/2*c)^
3 + tan(1/2*c)^4 - 6*tan(1/2*a)^2 - 16*tan(1/2*a)*tan(1/2*c) - 6*tan(1/2*c
)^2 + 1)*(b*x - c)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^
2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^4
*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) - 6*ta
n(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2
*c)^4 + tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*a)*tan(1/2*c)
^2 + tan(1/2*c)^3 - tan(1/2*a) - tan(1/2*c))*log(tan(b*x - c)^2 + 1)/(tan(
1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1
/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*ta
n(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - (tan(b*x - c)*tan(1/2*a)^4*tan(1/2*c)^4
 - 2*tan(b*x - c)*tan(1/2*a)^4*tan(1/2*c)^2 - 8*tan(b*x - c)*tan(1/2*a)^3*
tan(1/2*c)^3 - 2*tan(b*x - c)*tan(1/2*a)^2*tan(1/2*c)^4 + tan(b*x - c)*tan
(1/2*a)^4 + 8*tan(b*x - c)*tan(1/2*a)^3*tan(1/2*c) + 20*tan(b*x - c)*tan(1
/2*a)^2*tan(1/2*c)^2 + 8*tan(b*x - c)*tan(1/2*a)*tan(1/2*c)^3 + tan(b*x -
c)*tan(1/2*c)^4 - 2*tan(b*x - c)*tan(1/2*a)^2 - 8*tan(b*x - c)*tan(1/2*a)*
tan(1/2*c) - 2*tan(b*x - c)*tan(1/2*c)^2 + tan(b*x - c))/(tan(1/2*a)^4*...
```

Mupad [B] (verification not implemented)

Time = 18.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.11

$$\int \sec^2(c-bx)\sin^2(a+bx)\,dx$$
$$= -x\left(\cos\left(2\,a+2\,c\right)-\sin\left(2\,a+2\,c\right)\,1\mathrm{i}\right)+\frac{\left(2\,\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}+c\,4\mathrm{i}}+1\right)\,1\mathrm{i}}{2\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}$$
$$-\frac{\mathrm{e}^{-a\,4\mathrm{i}-c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}\right)\,\left(2\,b\,\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}-2\,b\,\mathrm{e}^{a\,6\mathrm{i}+c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$

input
```
int(sin(a + b*x)^2/cos(c - b*x)^2,x)
```

output
```
((2*exp(a*2i + c*2i) + exp(a*4i + c*4i) + 1)*1i)/(2*b*(exp(a*2i + c*2i) +
exp(a*2i + b*x*2i))) - x*(cos(2*a + 2*c) - sin(2*a + 2*c)*1i) - (exp(- a*4
i - c*4i)*log(exp(a*2i)*exp(b*x*2i) + exp(a*2i)*exp(c*2i))*(2*b*exp(a*2i +
 c*2i) - 2*b*exp(a*6i + c*6i))*1i)/(4*b^2)
```

Reduce [F]

$$\int \sec^2(c-bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
int(sec(b*x-c)^2*sin(b*x+a)^2,x)
```

output

```
(7*cos(b*x - c)*cos(a + b*x)*sin(a + b*x) + 96*cos(b*x - c)*int(tan((b*x -
 c)/2)**2/(tan((b*x - c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x - c)/2)**4
*tan((a + b*x)/2)**2 + tan((b*x - c)/2)**4 - 2*tan((b*x - c)/2)**2*tan((a
+ b*x)/2)**4 - 4*tan((b*x - c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x - c)
/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b + 96*cos(b*
x - c)*int(tan((a + b*x)/2)**2/(tan((b*x - c)/2)**4*tan((a + b*x)/2)**4 +
2*tan((b*x - c)/2)**4*tan((a + b*x)/2)**2 + tan((b*x - c)/2)**4 - 2*tan((b
*x - c)/2)**2*tan((a + b*x)/2)**4 - 4*tan((b*x - c)/2)**2*tan((a + b*x)/2)
**2 - 2*tan((b*x - c)/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2
+ 1),x)*b - 128*cos(b*x - c)*int((tan((b*x - c)/2)*tan((a + b*x)/2))/(tan(
(b*x - c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x - c)/2)**4*tan((a + b*x)/
2)**2 + tan((b*x - c)/2)**4 - 2*tan((b*x - c)/2)**2*tan((a + b*x)/2)**4 -
4*tan((b*x - c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x - c)/2)**2 + tan((a
 + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b - 32*cos(b*x - c)*int(1/(t
an((b*x - c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x - c)/2)**4*tan((a + b*
x)/2)**2 + tan((b*x - c)/2)**4 - 2*tan((b*x - c)/2)**2*tan((a + b*x)/2)**4
 - 4*tan((b*x - c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x - c)/2)**2 + tan
((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b + 16*cos(b*x - c)*sin(a
 + b*x) + 9*cos(b*x - c)*a + 9*cos(b*x - c)*b*x - 8*cos(a + b*x)*sin(b*x -
 c) + 8*cos(a + b*x)*sin(a + b*x) + 4*sin(b*x - c)*sin(a + b*x)**2 - 8*...
```

3.275 $\int \sec^3(c-bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 84

$$\int \sec^3(c-bx)\sin^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\sin(c-bx))\cos^2(a+c)}{2b} + \frac{\operatorname{arctanh}(\sin(c-bx))\cos(2(a+c))}{b} + \frac{\sec(c-bx)\sin(2(a+c))}{b} - \frac{\cos^2(a+c)\sec(c-bx)\tan(c-bx)}{2b}$$

output
```
1/2*arctanh(sin(b*x-c))*cos(a+c)^2/b-arctanh(sin(b*x-c))*cos(2*a+2*c)/b+se
c(b*x-c)*sin(2*a+2*c)/b+1/2*cos(a+c)^2*sec(b*x-c)*tan(b*x-c)/b
```

Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.00

$$\int \sec^3(c-bx)\sin^2(a+bx)\,dx = -\frac{-\log\left(\cos\left(\frac{1}{2}(c-bx)\right)-\sin\left(\frac{1}{2}(c-bx)\right)\right)+3\cos(2(a+c))\left(\log\left(\cos\left(\frac{1}{2}(c-bx)\right)-\sin\left(\frac{1}{2}(c-bx)\right)\right)-\right.}{}$$

input
```
Integrate[Sec[c - b*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/4*(-Log[Cos[(c - b*x)/2] - Sin[(c - b*x)/2]] + 3*Cos[2*(a + c)]*(Log[Co
s[(c - b*x)/2] - Sin[(c - b*x)/2]] - Log[Cos[(c - b*x)/2] + Sin[(c - b*x)/
2]]) + Log[Cos[(c - b*x)/2] + Sin[(c - b*x)/2]] + 4*Sec[c]*Sin[2*(a + c)]
- 4*Sec[c - b*x]*Sin[2*(a + c)] + 2*Cos[a + c]^2*Sec[c - b*x]*Tan[c - b*x]
)/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec^3(c-bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin^2(a+bx)\sec^3(c-bx)dx$$

input
```
Int[Sec[c - b*x]^3*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.32 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.86

method	result
risch	$-\frac{i\left(3\,\mathrm{e}^{i(bx+6a+5c)}+5\,\mathrm{e}^{3i(bx+2a+c)}-2\,\mathrm{e}^{i(bx+4a+3c)}+2\,\mathrm{e}^{i(3bx+4a+c)}-5\,\mathrm{e}^{i(bx+2a+c)}-3\,\mathrm{e}^{i(3bx+2a-c)}\right)}{4\left(\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)^{2}b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a+c)}\right)}{4b}-$
default	Expression too large to display

input
```
int(sec(b*x-c)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/(exp(2*I*(a+c))+exp(2*I*(b*x+a)))^2/b*(3*exp(I*(b*x+6*a+5*c))+5*exp
(3*I*(b*x+2*a+c))-2*exp(I*(b*x+4*a+3*c))+2*exp(I*(3*b*x+4*a+c))-5*exp(I*(b
*x+2*a+c))-3*exp(I*(3*b*x+2*a-c)))+1/4/b*ln(exp(I*(b*x+a))+I*exp(I*(a+c)))
-3/4/b*ln(exp(I*(b*x+a))+I*exp(I*(a+c)))*cos(2*a+2*c)-1/4/b*ln(exp(I*(b*x+
a))-I*exp(I*(a+c)))+3/4/b*ln(exp(I*(b*x+a))-I*exp(I*(a+c)))*cos(2*a+2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(90) = 180.

Time = 0.09 (sec) , antiderivative size = 335, normalized size of antiderivative = 3.99

$$\int \sec^3(c-bx)\sin^2(a+bx)\,dx$$
$$=\frac{6\,\cos\left(bx+a\right)\cos\left(a+c\right)^2\sin\left(a+c\right)+\left(3\,\cos\left(a+c\right)^4-2\,\left(3\,\cos\left(a+c\right)^3-2\,\cos\left(a+c\right)\right)\cos\left(bx+a\right)\mathrm{s}\right.}{}$$

input
```
integrate(sec(b*x-c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(6*cos(b*x + a)*cos(a + c)^2*sin(a + c) + (3*cos(a + c)^4 - 2*(3*cos(a
 + c)^3 - 2*cos(a + c))*cos(b*x + a)*sin(b*x + a)*sin(a + c) - (6*cos(a +
c)^4 - 7*cos(a + c)^2 + 2)*cos(b*x + a)^2 - 5*cos(a + c)^2 + 2)*log(2*(cos
(a + c)*sin(b*x + a) - cos(b*x + a)*sin(a + c) + 1)/(cos(a + c) + 1)) - (3
*cos(a + c)^4 - 2*(3*cos(a + c)^3 - 2*cos(a + c))*cos(b*x + a)*sin(b*x + a
)*sin(a + c) - (6*cos(a + c)^4 - 7*cos(a + c)^2 + 2)*cos(b*x + a)^2 - 5*co
s(a + c)^2 + 2)*log(-2*(cos(a + c)*sin(b*x + a) - cos(b*x + a)*sin(a + c)
- 1)/(cos(a + c) + 1)) - 2*(3*cos(a + c)^3 - 4*cos(a + c))*sin(b*x + a))/(
2*b*cos(b*x + a)*cos(a + c)*sin(b*x + a)*sin(a + c) + (2*b*cos(a + c)^2 -
b)*cos(b*x + a)^2 - b*cos(a + c)^2 + b)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(c-bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x-c)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1217 vs. $2(90) = 180$.

Time $= 0.21$ (sec) , antiderivative size $= 1217$, normalized size of antiderivative $= 14.49$

$$\int \sec^3(c-bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x-c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*(2*(3*sin(3*b*x) - 5*sin(3*b*x + 4*a + 4*c) - 2*sin(3*b*x + 2*a + 2*c
) - 3*sin(b*x + 4*a + 6*c) + 2*sin(b*x + 2*a + 4*c) + 5*sin(b*x + 2*c))*co
s(4*b*x + 2*a + c) + 10*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + 5*c))*cos(3*
b*x + 4*a + 4*c) + 4*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + 5*c))*cos(3*b*x
 + 2*a + 2*c) + 4*(3*sin(3*b*x) - 3*sin(b*x + 4*a + 6*c) + 2*sin(b*x + 2*a
 + 4*c) + 5*sin(b*x + 2*c))*cos(2*b*x + 2*a + 3*c) - ((3*cos(2*a + 2*c) -
1)*cos(4*b*x + 2*a + c)^2 + 4*(3*cos(2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c
)^2 + 4*(3*cos(2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c)*cos(2*a + 5*c) + (3*
cos(2*a + 2*c) - 1)*cos(2*a + 5*c)^2 + (3*cos(2*a + 2*c) - 1)*sin(4*b*x +
2*a + c)^2 + 4*(3*cos(2*a + 2*c) - 1)*sin(2*b*x + 2*a + 3*c)^2 + 4*(3*cos(
2*a + 2*c) - 1)*sin(2*b*x + 2*a + 3*c)*sin(2*a + 5*c) + (3*cos(2*a + 2*c)
- 1)*sin(2*a + 5*c)^2 + 2*(2*(3*cos(2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c)
 + (3*cos(2*a + 2*c) - 1)*cos(2*a + 5*c))*cos(4*b*x + 2*a + c) + 2*(2*(3*c
os(2*a + 2*c) - 1)*sin(2*b*x + 2*a + 3*c) + (3*cos(2*a + 2*c) - 1)*sin(2*a
 + 5*c))*sin(4*b*x + 2*a + c))*log((cos(b*x)^2 + cos(c)^2 - 2*cos(c)*sin(b
*x) + sin(b*x)^2 + 2*cos(b*x)*sin(c) + sin(c)^2)/(cos(b*x)^2 + cos(c)^2 +
2*cos(c)*sin(b*x) + sin(b*x)^2 - 2*cos(b*x)*sin(c) + sin(c)^2)) + 6*cos(2*
a + 5*c)*sin(3*b*x) - 2*(3*cos(3*b*x) - 5*cos(3*b*x + 4*a + 4*c) - 2*cos(3
*b*x + 2*a + 2*c) - 3*cos(b*x + 4*a + 6*c) + 2*cos(b*x + 2*a + 4*c) + 5*co
s(b*x + 2*c))*sin(4*b*x + 2*a + c) - 10*(2*cos(2*b*x + 2*a + 3*c) + cos...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1502 vs. $2(90) = 180$.

Time = 0.17 (sec) , antiderivative size = 1502, normalized size of antiderivative = 17.88

$$\int \sec^3(c - bx) \sin^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(sec(b*x-c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*((tan(1/2*a)^4*tan(1/2*c)^4 - 10*tan(1/2*a)^4*tan(1/2*c)^2 - 24*tan(1
/2*a)^3*tan(1/2*c)^3 - 10*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 24*ta
n(1/2*a)^3*tan(1/2*c) + 52*tan(1/2*a)^2*tan(1/2*c)^2 + 24*tan(1/2*a)*tan(1
/2*c)^3 + tan(1/2*c)^4 - 10*tan(1/2*a)^2 - 24*tan(1/2*a)*tan(1/2*c) - 10*t
an(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x - 1/2*c) + 1))/(tan(1/2*a)^4*tan(1/2*
c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2
*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*ta
n(1/2*c)^2 + 1) - (tan(1/2*a)^4*tan(1/2*c)^4 - 10*tan(1/2*a)^4*tan(1/2*c)^
2 - 24*tan(1/2*a)^3*tan(1/2*c)^3 - 10*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*
a)^4 + 24*tan(1/2*a)^3*tan(1/2*c) + 52*tan(1/2*a)^2*tan(1/2*c)^2 + 24*tan(
1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 10*tan(1/2*a)^2 - 24*tan(1/2*a)*tan(1
/2*c) - 10*tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x - 1/2*c) - 1))/(tan(1/2*a
)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)
^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2
*a)^2 + 2*tan(1/2*c)^2 + 1) - 2*(tan(1/2*b*x - 1/2*c)^3*tan(1/2*a)^4*tan(1
/2*c)^4 - 2*tan(1/2*b*x - 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^2 - 8*tan(1/2*b
*x - 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^3 + 8*tan(1/2*b*x - 1/2*c)^2*tan(1/2
*a)^4*tan(1/2*c)^3 - 2*tan(1/2*b*x - 1/2*c)^3*tan(1/2*a)^2*tan(1/2*c)^4 +
8*tan(1/2*b*x - 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*b*x - 1/2*c)*
tan(1/2*a)^4*tan(1/2*c)^4 + tan(1/2*b*x - 1/2*c)^3*tan(1/2*a)^4 + 8*tan...
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c - bx)\sin^2(a + bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^2/cos(c - b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^3(c - bx)\sin^2(a + bx)\, dx = \int \sec(bx - c)^3 \sin(bx + a)^2\, dx$$

input
```
int(sec(b*x-c)^3*sin(b*x+a)^2,x)
```

output
```
int(sec(b*x - c)**3*sin(a + b*x)**2,x)
```

3.276 $\int \sec^4(c-bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \sec^4(c-bx)\sin^2(a+bx)\,dx=\frac{\sec^2(c-bx)\sin(2(a+c))}{2b}-\frac{\cos^2(a+c)\tan(c-bx)}{b}+\frac{\cos(2(a+c))\tan(c-bx)}{b}-\frac{\cos^2(a+c)\tan^3(c-bx)}{3b}$$

output
```
1/2*sec(b*x-c)^2*sin(2*a+2*c)/b+cos(a+c)^2*tan(b*x-c)/b-cos(2*a+2*c)*tan(b
*x-c)/b+1/3*cos(a+c)^2*tan(b*x-c)^3/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \sec^4(c-bx)\sin^2(a+bx)\,dx$$
$$=\frac{\sec(c)\sec^3(c-bx)(3\sin(bx)-\sin(2c-3bx)+\sin(2a+4c-3bx)+3\sin(2a+2c-bx)+3\sin(2a+bx}{12b}$$

input
```
Integrate[Sec[c - b*x]^4*Sin[a + b*x]^2,x]
```

output
```
(Sec[c]*Sec[c - b*x]^3*(3*Sin[b*x] - Sin[2*c - 3*b*x] + Sin[2*a + 4*c - 3*
b*x] + 3*Sin[2*a + 2*c - b*x] + 3*Sin[2*a + b*x] - Sin[2*a + 3*b*x]))/(12*
b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sec^4(c-bx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin^2(a+bx)\sec^4(c-bx)dx$$

input
```
Int[Sec[c - b*x]^4*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 5.48 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{3\sin(bx-c)+\sin(3bx-3c)-2\sin(3bx+2a-c)}{3b(\cos(3bx-3c)+3\cos(bx-c))}$
risch	$-\frac{2i\left(e^{8i(a+c)}+3\,e^{2i(bx+4a+3c)}-e^{6i(a+c)}+3\,e^{4i(bx+2a+c)}-3\,e^{2i(bx+3a+2c)}+e^{4i(a+c)}\right)}{3\left(e^{2i(a+c)}+e^{2i(bx+a)}\right)^3 b}$
default	$-\frac{1}{(\sin(a)\cos(c)+\cos(a)\sin(c))^3(\tan(bx+a)\sin(a)\cos(c)+\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)-\sin(a)\sin(c))}-\frac{}{2(\sin(a)\cos(c)+\cos(a)\sin(c}$

input
```
int(sec(b*x-c)^4*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*sin(b*x-c)+sin(3*b*x-3*c)-2*sin(3*b*x+2*a-c))/b/(cos(3*b*x-3*c)+3*c
os(b*x-c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(84) = 168.

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.78

$$\int \sec^4(c-bx)\sin^2(a+bx)\,dx$$
$$= \frac{\left(4\cos\left(a+c\right)^5-\left(16\cos\left(a+c\right)^5-24\cos\left(a+c\right)^3+9\cos\left(a+c\right)\right)\cos\left(bx+a\right)^2-9\cos\left(a+c\right)^3+6\cos\cdots\right.}{3\left(\left(4\,b\cos\left(a+c\right)^3-3\,b\cos\left(a+c\right)\right)\cos\left(bx+a\right)^3+\left(\left(4\,b\cos\left(a+c\right)^2\cdots\right.\right.}$$

input
```
integrate(sec(b*x-c)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*((4*cos(a + c)^5 - (16*cos(a + c)^5 - 24*cos(a + c)^3 + 9*cos(a + c))*
cos(b*x + a)^2 - 9*cos(a + c)^3 + 6*cos(a + c))*sin(b*x + a) + ((16*cos(a
+ c)^4 - 16*cos(a + c)^2 + 3)*cos(b*x + a)^3 - 3*(4*cos(a + c)^4 - 5*cos(a
 + c)^2 + 1)*cos(b*x + a))*sin(a + c))/((4*b*cos(a + c)^3 - 3*b*cos(a + c)
)*cos(b*x + a)^3 + ((4*b*cos(a + c)^2 - b)*cos(b*x + a)^2 - b*cos(a + c)^2
 + b)*sin(b*x + a)*sin(a + c) - 3*(b*cos(a + c)^3 - b*cos(a + c))*cos(b*x
+ a))
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(c-bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x-c)**4*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 927 vs. $2(84) = 168$.

Time = 0.05 (sec) , antiderivative size = 927, normalized size of antiderivative = 11.59

$$\int \sec^4(c - bx) \sin^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(sec(b*x-c)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
2/3*((3*sin(4*b*x + 4*a + 4*c) + 3*sin(2*b*x + 4*a + 6*c) - 3*sin(2*b*x +
2*a + 4*c) + sin(4*a + 8*c) - sin(2*a + 6*c) + sin(4*c))*cos(6*b*x + 2*a)
- 3*(3*sin(4*b*x + 2*a + 2*c) + 3*sin(2*b*x + 2*a + 4*c) + sin(2*a + 6*c))
*cos(4*b*x + 4*a + 4*c) + 3*(3*sin(2*b*x + 4*a + 6*c) - 3*sin(2*b*x + 2*a
+ 4*c) + sin(4*a + 8*c) - sin(2*a + 6*c) + sin(4*c))*cos(4*b*x + 2*a + 2*c
) - 3*(3*sin(2*b*x + 2*a + 4*c) + sin(2*a + 6*c))*cos(2*b*x + 4*a + 6*c) +
 3*(sin(4*a + 8*c) + sin(4*c))*cos(2*b*x + 2*a + 4*c) - (3*cos(4*b*x + 4*a
 + 4*c) + 3*cos(2*b*x + 4*a + 6*c) - 3*cos(2*b*x + 2*a + 4*c) + cos(4*a +
8*c) - cos(2*a + 6*c) + cos(4*c))*sin(6*b*x + 2*a) + 3*(3*cos(4*b*x + 2*a
+ 2*c) + 3*cos(2*b*x + 2*a + 4*c) + cos(2*a + 6*c))*sin(4*b*x + 4*a + 4*c)
 - 3*(3*cos(2*b*x + 4*a + 6*c) - 3*cos(2*b*x + 2*a + 4*c) + cos(4*a + 8*c)
 - cos(2*a + 6*c) + cos(4*c))*sin(4*b*x + 2*a + 2*c) + 3*(3*cos(2*b*x + 2*
a + 4*c) + cos(2*a + 6*c))*sin(2*b*x + 4*a + 6*c) - 3*(cos(4*a + 8*c) + co
s(4*c))*sin(2*b*x + 2*a + 4*c) + cos(2*a + 6*c)*sin(4*a + 8*c) - cos(4*a +
 8*c)*sin(2*a + 6*c) - cos(4*c)*sin(2*a + 6*c) + cos(2*a + 6*c)*sin(4*c))/
(b*cos(6*b*x + 2*a)^2 + 9*b*cos(4*b*x + 2*a + 2*c)^2 + 9*b*cos(2*b*x + 2*a
 + 4*c)^2 + 6*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 6*c) + b*cos(2*a + 6*c)^2
 + b*sin(6*b*x + 2*a)^2 + 9*b*sin(4*b*x + 2*a + 2*c)^2 + 9*b*sin(2*b*x + 2
*a + 4*c)^2 + 6*b*sin(2*b*x + 2*a + 4*c)*sin(2*a + 6*c) + b*sin(2*a + 6*c)
^2 + 2*(3*b*cos(4*b*x + 2*a + 2*c) + 3*b*cos(2*b*x + 2*a + 4*c) + b*cos...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(84) = 168$.

Time = 0.15 (sec) , antiderivative size = 786, normalized size of antiderivative = 9.82

$$\int \sec^4(c - bx) \sin^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(sec(b*x-c)^4*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/3*(tan(b*x - c)^3*tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(b*x - c)^3*tan(1/2*a
)^4*tan(1/2*c)^2 - 8*tan(b*x - c)^3*tan(1/2*a)^3*tan(1/2*c)^3 - 6*tan(b*x
- c)^2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(b*x - c)^3*tan(1/2*a)^2*tan(1/2*c
)^4 - 6*tan(b*x - c)^2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(b*x - c)^3*tan(1/2*
a)^4 + 8*tan(b*x - c)^3*tan(1/2*a)^3*tan(1/2*c) + 6*tan(b*x - c)^2*tan(1/2
*a)^4*tan(1/2*c) + 20*tan(b*x - c)^3*tan(1/2*a)^2*tan(1/2*c)^2 + 36*tan(b*
x - c)^2*tan(1/2*a)^3*tan(1/2*c)^2 + 12*tan(b*x - c)*tan(1/2*a)^4*tan(1/2*
c)^2 + 8*tan(b*x - c)^3*tan(1/2*a)*tan(1/2*c)^3 + 36*tan(b*x - c)^2*tan(1/
2*a)^2*tan(1/2*c)^3 + 24*tan(b*x - c)*tan(1/2*a)^3*tan(1/2*c)^3 + tan(b*x
- c)^3*tan(1/2*c)^4 + 6*tan(b*x - c)^2*tan(1/2*a)*tan(1/2*c)^4 + 12*tan(b*
x - c)*tan(1/2*a)^2*tan(1/2*c)^4 - 2*tan(b*x - c)^3*tan(1/2*a)^2 - 6*tan(b
*x - c)^2*tan(1/2*a)^3 - 8*tan(b*x - c)^3*tan(1/2*a)*tan(1/2*c) - 36*tan(b
*x - c)^2*tan(1/2*a)^2*tan(1/2*c) - 24*tan(b*x - c)*tan(1/2*a)^3*tan(1/2*c
) - 2*tan(b*x - c)^3*tan(1/2*c)^2 - 36*tan(b*x - c)^2*tan(1/2*a)*tan(1/2*c
)^2 - 48*tan(b*x - c)*tan(1/2*a)^2*tan(1/2*c)^2 - 6*tan(b*x - c)^2*tan(1/2
*c)^3 - 24*tan(b*x - c)*tan(1/2*a)*tan(1/2*c)^3 + tan(b*x - c)^3 + 6*tan(b
*x - c)^2*tan(1/2*a) + 12*tan(b*x - c)*tan(1/2*a)^2 + 6*tan(b*x - c)^2*tan
(1/2*c) + 24*tan(b*x - c)*tan(1/2*a)*tan(1/2*c) + 12*tan(b*x - c)*tan(1/2*
c)^2)/((tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/
2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(...
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(c-bx)\sin^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^2/cos(c - b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int \sec^4(c-bx)\sin^2(a+bx)\,dx = \frac{\cos(bx-c)\cos(bx+a)\sin(bx+a)+\sin(bx-c)^3-\sin(bx-c)\sin(bx+a)^2-\sin(bx-c)}{3\cos(bx-c)\,b\left(\sin(bx-c)^2-1\right)}$$

input
```
int(sec(b*x-c)^4*sin(b*x+a)^2,x)
```

output
```
(cos(b*x - c)*cos(a + b*x)*sin(a + b*x) + sin(b*x - c)**3 - sin(b*x - c)*s
in(a + b*x)**2 - sin(b*x - c))/(3*cos(b*x - c)*b*(sin(b*x - c)**2 - 1))
```

3.277 $\int \csc(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \csc(c+bx)\sin(a+bx)\,dx = x\cos(a-c)+\frac{\log(\sin(c+bx))\sin(a-c)}{b}$$

output
```
x*cos(a-c)+ln(sin(b*x+c))*sin(a-c)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc(c+bx)\sin(a+bx)\,dx = x\cos(a-c)+\frac{\log(\sin(c+bx))\sin(a-c)}{b}$$

input
```
Integrate[Csc[c + b*x]*Sin[a + b*x],x]
```

output
```
x*Cos[a - c] + (Log[Sin[c + b*x]]*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5093, 24, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \text{5093}$$

$$\sin(a-c)\int \cot(c+bx)dx+\cos(a-c)\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\sin(a-c)\int \cot(c+bx)dx+x\cos(a-c)$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int -\tan\left(c+bx+\frac{\pi}{2}\right)dx+x\cos(a-c)$$

$$\Big\downarrow \text{25}$$

$$x\cos(a-c)-\sin(a-c)\int \tan\left(\frac{1}{2}(2c+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\sin(a-c)\log(-\sin(bx+c))}{b}+x\cos(a-c)$$

input
```
Int[Csc[c + b*x]*Sin[a + b*x],x]
```

output
```
x*Cos[a - c] + (Log[-Sin[c + b*x]]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

method	result
risch	$-2i\sin\left(a-c\right)x+x\,\mathrm{e}^{i(a-c)}-\frac{2i\sin(a-c)a}{b}+\frac{\ln\left(\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{2i(a-c)}\right)\sin(a-c)}{b}$
default	$\frac{\frac{\frac{(-\sin(a)\cos(c)+\cos(a)\sin(c))\ln\left(\tan(bx+a)^2+1\right)}{2}+(\cos(a)\cos(c)+\sin(a)\sin(c))\arctan(\tan(bx+a))}{\left(\cos(c)^2+\sin(c)^2\right)\left(\cos(a)^2+\sin(a)^2\right)}+\frac{(\sin(a)\cos(c)-\cos(a)\sin(c))\ln(\tan(bx+a)\,\mathrm{co}\cdots}{\cos(a)^2\cos(c)^2+\sin(c)\cdots}}{b}$

input
```
int(csc(b*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*I*sin(a-c)*x+x*exp(I*(a-c))-2*I/b*sin(a-c)*a+ln(exp(2*I*(b*x+a))-exp(2*
I*(a-c)))/b*sin(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \csc(c+bx)\sin(a+bx)\,dx = \frac{bx\cos(-a+c)-\log\left(\frac{1}{2}\sin(bx+c)\right)\sin(-a+c)}{b}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
(b*x*cos(-a + c) - log(1/2*sin(b*x + c))*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(20) = 40.

Time = 4.26 (sec) , antiderivative size = 333, normalized size of antiderivative = 12.81

$$\int \csc(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x, Eq(c, 0)), (0, Eq(b, 0)), (-b*x*ta
n(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2*log(tan(c/2) +
 tan(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2))
*tan(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c
/2)**2 + b), True))*cos(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (log(
sin(b*x))/b, Eq(c, 0)), (x/sin(c), Eq(b, 0)), (2*b*x*tan(c/2)/(b*tan(c/2)*
*2 + b) - log(tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) + log
(tan(c/2) + tan(b*x/2))/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - 1/tan(c/2))
*tan(c/2)**2/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - 1/tan(c/2))/(b*tan(c/2
)**2 + b) + log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(t
an(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b), True))*sin(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(26) = 52.

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.15

$$\int \csc(c+bx)\sin(a+bx)\,dx$$
$$= \frac{2\,bx\cos\left(-a+c\right) - \log\left(\cos\left(bx\right)^2 + 2\,\cos\left(bx\right)\cos\left(c\right) + \cos\left(c\right)^2 + \sin\left(bx\right)^2 - 2\,\sin\left(bx\right)\sin\left(c\right) + \sin\left(c\right)^2\right)}{}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*b*x*cos(-a + c) - log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + s
in(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2)*sin(-a + c) - log(cos(b*x)^2 - 2
*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*s
in(-a + c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(26) = 52.

Time = 0.14 (sec) , antiderivative size = 236, normalized size of antiderivative = 9.08

$$\int \csc(c+bx)\sin(a+bx)\,dx$$
$$= \frac{\frac{\left(\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2 - \tan\left(\frac{1}{2}\,a\right)^2 + 4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right) - \tan\left(\frac{1}{2}\,c\right)^2 + 1\right)(bx+c)}{\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + \tan\left(\frac{1}{2}\,a\right)^2 + \tan\left(\frac{1}{2}\,c\right)^2 + 1} - \frac{2\left(\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right) - \tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^2 + \tan\left(\frac{1}{2}\,a\right) - \tan\right.}{\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + \tan\left(\frac{1}{2}\,a\right)^2 + \tan}}{b}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

output
```
((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan
(1/2*c)^2 + 1)*(b*x + c)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1
/2*c)^2 + 1) - 2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(
1/2*a) - tan(1/2*c))*log(tan(1/2*b*x + 1/2*c)^2 + 1)/(tan(1/2*a)^2*tan(1/2
*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^2*tan(1/2*c) - ta
n(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2
*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.27

$$\int \csc(c+bx)\sin(a+bx)\,dx = x\left(\frac{e^{-a\,1i+c\,1i}}{2}-\frac{e^{a\,1i-c\,1i}}{2}\right)+x\left(\frac{e^{-a\,1i+c\,1i}}{2}+\frac{e^{a\,1i-c\,1i}}{2}\right) + \frac{\ln\left(-e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}-\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)}{b}$$

input
```
int(sin(a + b*x)/sin(c + b*x),x)
```

output
```
x*(exp(c*1i - a*1i)/2 - exp(a*1i - c*1i)/2) + x*(exp(c*1i - a*1i)/2 + exp(
a*1i - c*1i)/2) + (log(exp(a*2i + b*x*2i) - exp(a*2i - c*2i))*((exp(c*1i -
 a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2))/b
```

Reduce [F]

$$\int \csc(c+bx)\sin(a+bx)\,dx = \int \csc(bx+c)\sin(bx+a)\,dx$$

input
```
int(csc(b*x+c)*sin(b*x+a),x)
```

output
```
int(csc(b*x + c)*sin(a + b*x),x)
```

3.278 $\int \csc^2(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \csc^2(c+bx)\sin(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(c+bx))\cos(a-c)}{b}-\frac{\csc(c+bx)\sin(a-c)}{b}$$

output
```
-arctanh(cos(b*x+c))*cos(a-c)/b-csc(b*x+c)*sin(a-c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\begin{aligned}&\int \csc^2(c+bx)\sin(a+bx)\,dx\\&=-\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b}\\&\quad-\frac{\csc(c+bx)\sin(a-c)}{b}\end{aligned}$$

input
```
Integrate[Csc[c + b*x]^2*Sin[a + b*x],x]
```

output
```
((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x
)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Csc
[c + b*x]*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5093, 3042, 25, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \text{5093}$$

$$\cos(a-c)\int \csc(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\cos(a-c)\int \csc(c+bx)dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc(c+bx)dx-\frac{\sin(a-c)\int 1d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$\cos(a-c)\int \csc(c+bx)dx-\frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\cos(a-c)\text{arctanh}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}$$

input
```
Int[Csc[c + b*x]^2*Sin[a + b*x],x]
```

output
```
-((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) - (Csc[c + b*x]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.00 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.19

method	result
risch	$\frac{e^{i(bx+3a)}-e^{i(bx+a+2c)}}{b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos(a-c)}{b}+\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos(a-c)}{b}$
default	$\frac{4(-2\cos(a)\cos(c)-2\sin(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+8\sin(a)\cos(c)-8\cos(a)\sin}{\left(-4\cos(c)^2\sin(a)^2-4\cos(a)^2\cos(c)^2-4\sin(a)^2\sin(c)^2-4\sin(c)^2\cos(a)^2\right)\left(\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+2\tan\left(\frac{a}{2}+\frac{b}{}\right.\right.}$

input
```
int(csc(b*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))*(exp(I*(b*x+3*a))-exp(I*(b*x+a+2*c)))
-ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)+ln(exp(I*(b*x+a))-exp(I*(a-c))
)/b*cos(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.97

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = -\frac{\cos(-a+c)\log\left(\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)-\cos(-a+c)\log\left(-\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c}{2b\sin(bx+c)}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cos(-a + c)*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - cos(-a + c)*l
og(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - 2*sin(-a + c))/(b*sin(b*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(29) = 58$.

Time = 62.54 (sec) , antiderivative size = 3264, normalized size of antiderivative = 90.67

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)**2*sin(b*x+a),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), (log(tan(b*x/2))/b, Eq(c, 0)), (0, Eq(
b, 0)), (-log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4
*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan
(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(c
/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)*
*2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2))
 + log(tan(c/2) + tan(b*x/2))*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*ta
n(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan
(c/2) - b*tan(b*x/2)) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**2*tan(b*x/2
)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3
+ b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(c/2) + t
an(b*x/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**
3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) -
b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x
/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)*
*2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(b*x/2)/(b
*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*
tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/t
an(c/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*
tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) -...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(36) = 72$.

Time = 0.06 (sec) , antiderivative size = 454, normalized size of antiderivative = 12.61

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*(cos(b*x + 2*a) - cos(b*x + 2*c))*cos(2*b*x + a + 2*c) - 2*cos(b*x
 + 2*a)*cos(a) + 2*cos(b*x + 2*c)*cos(a) + (cos(2*b*x + a + 2*c)^2*cos(-a
+ c) - 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(2*b*x +
 a + 2*c)^2 - 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(
a)^2)*cos(-a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x
)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*b*x + a + 2*c)^2*cos(-a + c)
- 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(2*b*x + a +
2*c)^2 - 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)
*cos(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 +
 2*sin(b*x)*sin(c) + sin(c)^2) + 2*(sin(b*x + 2*a) - sin(b*x + 2*c))*sin(2
*b*x + a + 2*c) - 2*sin(b*x + 2*a)*sin(a) + 2*sin(b*x + 2*c)*sin(a))/(b*co
s(2*b*x + a + 2*c)^2 - 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a +
 2*c)^2 - 2*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(36) = 72$.

Time = 0.15 (sec) , antiderivative size = 349, normalized size of antiderivative = 9.69

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan
(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 +
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan
(1/2*c) - tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*b*x + 1/2
*c)*tan(1/2*a) - tan(1/2*b*x + 1/2*c)*tan(1/2*c))/(tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2
*tan(1/2*c)^2 - tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2 + 4*tan(1/2*b*x + 1/2*c)
*tan(1/2*a)*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c) - tan(1/2*b*x + 1/2*c)*ta
n(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*b*x + 1/2*c) + tan(1/2*a) -
 tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1
)*tan(1/2*b*x + 1/2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 252, normalized size of antiderivative = 7.00

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = -\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\left(e^{a\,2i}\,e^{-c\,2i}+1\right)1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}} + \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\left(e^{a\,2i}\,e^{-c\,2i}+1\right)1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}} + \frac{e^{a\,1i+b\,x\,1i}\left(e^{a\,2i-c\,2i}-1\right)}{b\,\left(e^{a\,2i-c\,2i}-e^{a\,2i+b\,x\,2i}\right)}$$

input
```
int(sin(a + b*x)/sin(c + b*x)^2,x)
```

output
```
(log((exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-
c*2i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i))*(exp(
a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log(- exp(a*1i)*exp(b*x
*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i) - (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp
(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) + 1))/(2*
b*exp(a*2i - c*2i)^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) - 1))/(b
*(exp(a*2i - c*2i) - exp(a*2i + b*x*2i)))
```

Reduce [F]

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = \int \csc(bx+c)^2 \sin(bx+a)\,dx$$

input
```
int(csc(b*x+c)^2*sin(b*x+a),x)
```

output
```
int(csc(b*x + c)**2*sin(a + b*x),x)
```

3.279 $\int \csc^3(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \csc^3(c+bx)\sin(a+bx)\,dx=-\frac{\cos(a-c)\cot(c+bx)}{b}-\frac{\csc^2(c+bx)\sin(a-c)}{2b}$$

output
```
-cos(a-c)*cot(b*x+c)/b-1/2*csc(b*x+c)^2*sin(a-c)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \csc^3(c+bx)\sin(a+bx)\,dx=\frac{(\cos(a)-\cos(a-c)\cos(c+2bx))\csc(c)\csc^2(c+bx)}{2b}$$

input
```
Integrate[Csc[c + b*x]^3*Sin[a + b*x],x]
```

output
```
((Cos[a] - Cos[a - c]*Cos[c + 2*b*x])*Csc[c]*Csc[c + b*x]^2)/(2*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5093, 3042, 25, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^3(bx+c)\,dx$$

$$\Big\downarrow \text{5093}$$

$$\cos(a-c)\int \csc^2(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc^2(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)^2dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^2\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\cos(a-c)\int \csc(c+bx)^2dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc(c+bx)^2dx-\frac{\sin(a-c)\int \csc(c+bx)d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(a-c)\int \csc(c+bx)^2dx-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\cos(a-c)\int 1d\cot(c+bx)}{b}-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\cos(a-c)\cot(bx+c)}{b}-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

input
```
Int[Csc[c + b*x]^3*Sin[a + b*x],x]
```

output
```
-((Cos[a - c]*Cot[c + b*x])/b) - (Csc[c + b*x]^2*Sin[a - c])/(2*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

method	result
risch	$\frac{i\left(-2\,\mathrm{e}^{i(2bx+5a+c)}+\mathrm{e}^{i(5a-c)}+\mathrm{e}^{i(3a+c)}\right)}{\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{2}b}$
parallelrisch	$-\frac{\csc\left(\frac{bx}{2}+\frac{c}{2}\right)\left(\sin(bx+a)\left(-\frac{\sec\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}}{2}+1\right)\csc\left(\frac{bx}{2}+\frac{c}{2}\right)+\sec\left(\frac{bx}{2}+\frac{c}{2}\right)\cos(bx+a)\right)}{4b}$
default	$\frac{-\frac{\sin(a)\cos(c)-\cos(a)\sin(c)}{2(\cos(a)\cos(c)+\sin(a)\sin(c))^{2}(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))^{2}}-\frac{\cdots}{(\cos(a)\cos(c)+\sin(a)\sin \cdots}}{b}$

input
```
int(csc(b*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
I/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^2/b*(-2*exp(I*(2*b*x+5*a+c))+exp(I*(5*a
-c))+exp(I*(3*a+c)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \csc^3(c+bx)\sin(a+bx)\,dx=\frac{2\,\cos\left(bx+c\right)\cos\left(-a+c\right)\sin\left(bx+c\right)-\sin\left(-a+c\right)}{2\left(b\cos\left(bx+c\right)^{2}-b\right)}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c) - sin(-a + c))/(b*cos(b*x + c
)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**3*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(37) = 74$.

Time = 0.04 (sec) , antiderivative size = 399, normalized size of antiderivative = 10.23

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \frac{(2\sin(2bx+2a+2c)-\sin(2a)-\sin(2c))\cos(4bx+a+5c)-2(2\sin(2bx+2a+2c)-\sin(2a)}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2-4b\cos(2}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
((2*sin(2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(4*b*x + a + 5*c) - 2
*(2*sin(2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(2*b*x + a + 3*c) - (
sin(2*a) + sin(2*c))*cos(a + c) - (2*cos(2*b*x + 2*a + 2*c) - cos(2*a) - c
os(2*c))*sin(4*b*x + a + 5*c) + 2*cos(a + c)*sin(2*b*x + 2*a + 2*c) + 2*(2
*cos(2*b*x + 2*a + 2*c) - cos(2*a) - cos(2*c))*sin(2*b*x + a + 3*c) + (cos
(2*a) + cos(2*c))*sin(a + c) - 2*cos(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos
(4*b*x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 - 4*b*cos(2*b*x + a + 3*c
)*cos(a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x +
 a + 3*c)^2 - 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(2*
b*cos(2*b*x + a + 3*c) - b*cos(a + c))*cos(4*b*x + a + 5*c) - 2*(2*b*sin(2
*b*x + a + 3*c) - b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(37) = 74$.

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.72

$$\int \csc^3(c+bx)\sin(a+bx)\,dx =$$

$$-\frac{\tan\left(bx+c\right)\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2 - \tan\left(bx+c\right)\tan\left(\frac{1}{2}\,a\right)^2 + 4\,\tan\left(bx+c\right)\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right) + \tan\left(\right.}{\left(\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + \tan\left(\frac{1}{2}\,a\right)^2\right.}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-(tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + c)*tan(1/2*a)^2 + 4*t
an(b*x + c)*tan(1/2*a)*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c) - tan(b*x + c)
*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^2 + tan(b*x + c) + tan(1/2*a) - tan(
1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b*t
an(b*x + c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)/sin(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \frac{-\cos(bx+c)\sin(bx+a)-\cos(bx+a)\sin(bx+c)}{2\sin(bx+c)^2 b}$$

input
```
int(csc(b*x+c)^3*sin(b*x+a),x)
```

output
```
( - (cos(b*x + c)*sin(a + b*x) + cos(a + b*x)*sin(b*x + c)))/(2*sin(b*x +
c)**2*b)
```

3.280 $\int \csc^4(c+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} - \frac{\csc^3(c+bx)\sin(a-c)}{3b}$$

output
```
-1/2*arctanh(cos(b*x+c))*cos(a-c)/b-1/2*cos(a-c)*cot(b*x+c)*csc(b*x+c)/b-1
/3*csc(b*x+c)^3*sin(a-c)/b
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = -\frac{6\operatorname{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+3\cos(a-c)\cot(c+bx)\csc(c+bx)+2\csc^3(c+bx)\,\mathrm{s}}{6b}$$

input
```
Integrate[Csc[c + b*x]^4*Sin[a + b*x],x]
```

output
```
-1/6*(6*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + 3*Cos[a - c]*Co
t[c + b*x]*Csc[c + b*x] + 2*Csc[c + b*x]^3*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {5093, 3042, 25, 3086, 15, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^4(bx+c)\,dx$$

$$\Big\downarrow \text{5093}$$

$$\cos(a-c)\int \csc^3(c+bx)dx+\sin(a-c)\int \cot(c+bx)\csc^3(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)^3dx+\sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^3\tan\left(c+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\cos(a-c)\int \csc(c+bx)^3dx-\sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^3\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc(c+bx)^3dx-\frac{\sin(a-c)\int \csc^2(c+bx)d\csc(c+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(a-c)\int \csc(c+bx)^3dx-\frac{\sin(a-c)\csc^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4255}$$

$$\cos(a-c)\left(\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\sin(a-c)\csc^3(bx+c)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\left(\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\sin(a-c)\csc^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4257}$$

$$\cos(a-c)\left(-\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\sin(a-c)\csc^3(bx+c)}{3b}$$

input
```
Int[Csc[c + b*x]^4*Sin[a + b*x],x]
```

output
```
Cos[a - c]*(-1/2*ArcTanh[Cos[c + b*x]]/b - (Cot[c + b*x]*Csc[c + b*x])/(2*
b)) - (Csc[c + b*x]^3*Sin[a - c])/(3*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.00 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.82

method	result
risch	$\frac{-3\,\mathrm{e}^{i(5bx+7a+4c)}-3\,\mathrm{e}^{i(5bx+5a+6c)}-8\,\mathrm{e}^{i(3bx+7a+2c)}+8\,\mathrm{e}^{i(3bx+5a+4c)}+3\,\mathrm{e}^{i(bx+7a)}+3\,\mathrm{e}^{i(bx+5a+2c)}}{6b\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{3}} - \frac{\ln\left(\mathrm{e}^{i(bx+a)}+\mathrm{e}^{i(a-c)}\right)\cos(a$
default	Expression too large to display

input
```
int(csc(b*x+c)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^3*(-3*exp(I*(5*b*x+7*a+4*c))-3*exp(
I*(5*b*x+5*a+6*c))-8*exp(I*(3*b*x+7*a+2*c))+8*exp(I*(3*b*x+5*a+4*c))+3*exp
(I*(b*x+7*a))+3*exp(I*(b*x+5*a+2*c)))-1/2*ln(exp(I*(b*x+a))+exp(I*(a-c)))/
b*cos(a-c)+1/2*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*cos(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(61) = 122.

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.10

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \frac{6\,\cos(bx+c)\cos(-a+c)\sin(bx+c) - 3\left(\cos(bx+c)^2\cos(-a+c) - \cos(-a+c)\right)\log\left(\frac{1}{2}\,\cos(bx+c\right.}{12\left(b\cos\right.}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(6*cos(b*x + c)*cos(-a + c)*sin(b*x + c) - 3*(cos(b*x + c)^2*cos(-a +
 c) - cos(-a + c))*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) + 3*(cos(b*x +
 c)^2*cos(-a + c) - cos(-a + c))*log(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c)
 - 4*sin(-a + c))/((b*cos(b*x + c)^2 - b)*sin(b*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**4*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1773 vs. $2(61) = 122$.

Time = 0.09 (sec) , antiderivative size = 1773, normalized size of antiderivative = 26.46

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/12*(2*(3*cos(5*b*x + 2*a + 4*c) + 3*cos(5*b*x + 6*c) + 8*cos(3*b*x + 2*a
 + 2*c) - 8*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(6*
b*x + a + 6*c) - 6*(3*cos(4*b*x + a + 4*c) - 3*cos(2*b*x + a + 2*c) + cos(
a))*cos(5*b*x + 2*a + 4*c) - 6*(3*cos(4*b*x + a + 4*c) - 3*cos(2*b*x + a +
 2*c) + cos(a))*cos(5*b*x + 6*c) - 6*(8*cos(3*b*x + 2*a + 2*c) - 8*cos(3*b
*x + 4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) + 16
*(3*cos(2*b*x + a + 2*c) - cos(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*cos(2*b*
x + a + 2*c) - cos(a))*cos(3*b*x + 4*c) - 18*(cos(b*x + 2*a) + cos(b*x + 2
*c))*cos(2*b*x + a + 2*c) + 6*cos(b*x + 2*a)*cos(a) + 6*cos(b*x + 2*c)*cos
(a) - 3*(cos(6*b*x + a + 6*c)^2*cos(-a + c) + 9*cos(4*b*x + a + 4*c)^2*cos
(-a + c) + 9*cos(2*b*x + a + 2*c)^2*cos(-a + c) - 6*cos(2*b*x + a + 2*c)*c
os(a)*cos(-a + c) + cos(-a + c)*sin(6*b*x + a + 6*c)^2 + 9*cos(-a + c)*sin
(4*b*x + a + 4*c)^2 + 9*cos(-a + c)*sin(2*b*x + a + 2*c)^2 - 6*cos(-a + c)
*sin(2*b*x + a + 2*c)*sin(a) - 2*(3*cos(4*b*x + a + 4*c)*cos(-a + c) - 3*c
os(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(-a + c))*cos(6*b*x + a + 6*c)
 - 6*(3*cos(2*b*x + a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(4*b*x +
 a + 4*c) + (cos(a)^2 + sin(a)^2)*cos(-a + c) - 2*(3*cos(-a + c)*sin(4*b*x
 + a + 4*c) - 3*cos(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)*sin(a))*sin
(6*b*x + a + 6*c) - 6*(3*cos(-a + c)*sin(2*b*x + a + 2*c) - cos(-a + c)*si
n(a))*sin(4*b*x + a + 4*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2221 vs. $2(61) = 122$.

Time = 0.15 (sec) , antiderivative size = 2221, normalized size of antiderivative = 33.15

$$\int \csc^4(c + bx) \sin(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

output
```
1/24*(12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*
c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(1/
2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (2*tan(1/2*b*x + 1/2*c)^3*tan(
1/2*a)^6*tan(1/2*c)^5 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^6
 - 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^6 + 4*tan(1/2*b*x + 1/
2*c)^3*tan(1/2*a)^6*tan(1/2*c)^3 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5*t
an(1/2*c)^4 - 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^4 + 2*tan(1
/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^5 - 12*tan(1/2*b*x + 1/2*c)^2*ta
n(1/2*a)^5*tan(1/2*c)^5 + 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^6*tan(1/2*c)^5
 - 4*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^6 - 3*tan(1/2*b*x + 1/
2*c)^2*tan(1/2*a)^4*tan(1/2*c)^6 - 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^5*tan
(1/2*c)^6 + 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^6*tan(1/2*c) + 2*tan(1/2*b
*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^2 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2
*a)^6*tan(1/2*c)^2 + 4*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^3 -
24*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^5*tan(1/2*c)^3 + 12*tan(1/2*b*x + 1/2
*c)*tan(1/2*a)^6*tan(1/2*c)^3 - 4*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^3*tan(
1/2*c)^4 - 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*
b*x + 1/2*c)*tan(1/2*a)^5*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*
a)^2*tan(1/2*c)^5 - 24*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c)^5 +
6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^5 - 2*tan(1/2*b*x + 1/2*...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+bx)\sin(a+bx)\,dx=\text{Hanged}$$

input
```
int(sin(a + b*x)/sin(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^4(c+bx)\sin(a+bx)\,dx$$

$$= \frac{-2\cos(bx+c)\cos(bx+a)\sin(bx+c) - 4\cos(bx+c)\sin(bx+c)^2\sin(bx+a) - 2\cos(bx+c)\sin(bx}{}$$

input
```
int(csc(b*x+c)^4*sin(b*x+a),x)
```

output
```
( - 2*cos(b*x + c)*cos(a + b*x)*sin(b*x + c) - 4*cos(b*x + c)*sin(b*x + c)
**2*sin(a + b*x) - 2*cos(b*x + c)*sin(b*x + c) - 4*cos(b*x + c)*sin(a + b*
x) - 4*cos(a + b*x)*sin(b*x + c) - 6*int(tan((b*x + c)/2)/(tan((a + b*x)/2
)**2 + 1),x)*sin(b*x + c)**3*b - 2*int(1/(tan((b*x + c)/2)**3*tan((a + b*x
)/2)**2 + tan((b*x + c)/2)**3),x)*sin(b*x + c)**3*b + 4*log(tan((b*x + c)/
2)**2 + 1)*sin(b*x + c)**3 - 2*log(tan((b*x + c)/2))*sin(b*x + c)**3 + 3*s
in(b*x + c)**3 + 2*sin(b*x + c)**2*sin(a + b*x) - 2*sin(b*x + c))/(12*sin(
b*x + c)**3*b)
```

3.281 $\int \csc(c-bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \csc(c-bx)\sin(a+bx)\,dx = -x\cos(a+c) - \frac{\log(\sin(c-bx))\sin(a+c)}{b}$$

output
```
-x*cos(a+c)-ln(-sin(b*x-c))*sin(a+c)/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \csc(c-bx)\sin(a+bx)\,dx = -\frac{bx\cos(a+c)+\log(\sin(c-bx))\sin(a+c)}{b}$$

input
```
Integrate[Csc[c - b*x]*Sin[a + b*x],x]
```

output
```
-((b*x*Cos[a + c] + Log[Sin[c - b*x]]*Sin[a + c])/b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc(c-bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin(a+bx)\csc(c-bx)dx$$

input
```
Int[Csc[c - b*x]*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

method	result
risch	$2i\sin\left(a+c\right)x-x\,\mathrm{e}^{i(a+c)}+\frac{2i\sin(a+c)a}{b}-\frac{\ln\left(-\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)\sin(a+c)}{b}$
default	$-\frac{\frac{(\sin(a)\cos(c)+\cos(a)\sin(c))\ln(\tan(bx+a)\cos(a)\cos(c)-\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)-\cos(a)\sin(c))}{\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2}+\frac{\frac{(-\sin(a)\cos(c)-\cos(a)\sin(c))\ln}{2}}{(}}{b}$

input
```
int(-csc(b*x-c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*I*sin(a+c)*x-x*exp(I*(a+c))+2*I/b*sin(a+c)*a-ln(-exp(2*I*(a+c))+exp(2*I*
(b*x+a)))/b*sin(a+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.08

$$\int \csc(c-bx)\sin(a+bx)\,dx$$
$$= -\frac{bx\cos\left(a+c\right)+\log\left(\frac{\cos(a+c)\sin(bx+a)-\cos(bx+a)\sin(a+c)}{\cos(a+c)+1}\right)\sin\left(a+c\right)}{b}$$

input
```
integrate(-csc(b*x-c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-(b*x*cos(a + c) + log((cos(a + c)*sin(b*x + a) - cos(b*x + a)*sin(a + c))
/(cos(a + c) + 1))*sin(a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(24) = 48.

Time = 4.74 (sec) , antiderivative size = 337, normalized size of antiderivative = 13.48

$$\int \csc(c-bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(-csc(b*x-c)*sin(b*x+a),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), (-x, Eq(c, 0)), (0, Eq(b, 0)), (b*x*ta
n(c/2)**2/(b*tan(c/2)**2 + b) - b*x/(b*tan(c/2)**2 + b) - 2*log(-tan(c/2)
+ tan(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) + 1/tan(c/2)
)*tan(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(
c/2)**2 + b), True))*cos(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (-lo
g(sin(b*x))/b, Eq(c, 0)), (x/sin(c), Eq(b, 0)), (2*b*x*tan(c/2)/(b*tan(c/2
)**2 + b) + log(-tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) -
log(-tan(c/2) + tan(b*x/2))/(b*tan(c/2)**2 + b) + log(tan(b*x/2) + 1/tan(c
/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + 1/tan(c/2))/(b*tan
(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) + l
og(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b), True))*sin(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(28) = 56$.

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.00

$$\int \csc(c-bx)\sin(a+bx)\,dx =$$

$$-\frac{2\,bx\cos\left(a+c\right)+\log\left(\cos\left(bx\right)^{2}+2\,\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^{2}+\sin\left(bx\right)^{2}+2\,\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^{2}}{}$$

input
```
integrate(-csc(b*x-c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*b*x*cos(a + c) + log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + s
in(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*sin(a + c) + log(cos(b*x)^2 - 2*
cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2)*si
n(a + c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(28) = 56$.

Time = 0.13 (sec) , antiderivative size = 241, normalized size of antiderivative = 9.64

$$\int \csc(c-bx)\sin(a+bx)\,dx = -\frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)^2-4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2+1\right)(bx-c)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}+\frac{2\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)-t\cdots\right.}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\cdots}}{b}$$

input
```
integrate(-csc(b*x-c)*sin(b*x+a),x, algorithm="giac")
```

output
```
-((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 - 4*tan(1/2*a)*tan(1/2*c) - ta
n(1/2*c)^2 + 1)*(b*x - c)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(
1/2*c)^2 + 1) + 2*(tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan
(1/2*a) - tan(1/2*c))*log(tan(1/2*b*x - 1/2*c)^2 + 1)/(tan(1/2*a)^2*tan(1/
2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^2*tan(1/2*c) + t
an(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x - 1/
2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.56

$$\int \csc(c-bx)\sin(a+bx)\,dx = -x\left(\frac{e^{-a\,1i-c\,1i}}{2}-\frac{e^{a\,1i+c\,1i}}{2}\right)-x\left(\frac{e^{-a\,1i-c\,1i}}{2}+\frac{e^{a\,1i+c\,1i}}{2}\right) - \frac{\ln\left(-e^{a\,2i+c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i-c\,1i}\,1i}{2}-\frac{e^{a\,1i+c\,1i}\,1i}{2}\right)}{b}$$

input
```
int(sin(a + b*x)/sin(c - b*x),x)
```

output
```
- x*(exp(- a*1i - c*1i)/2 - exp(a*1i + c*1i)/2) - x*(exp(- a*1i - c*1i)/2
+ exp(a*1i + c*1i)/2) - (log(exp(a*2i + b*x*2i) - exp(a*2i + c*2i))*((exp(
- a*1i - c*1i)*1i)/2 - (exp(a*1i + c*1i)*1i)/2))/b
```

Reduce [F]

$$\int \csc(c - bx) \sin(a + bx) \, dx = -\left(\int \csc(bx - c) \sin(bx + a) \, dx \right)$$

input
```
int(-csc(b*x-c)*sin(b*x+a),x)
```

output
```
- int(csc(b*x - c)*sin(a + b*x),x)
```

3.282 $\int \csc^2(c-bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \csc^2(c-bx)\sin(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c-bx))\cos(a+c)}{b}+\frac{\csc(c-bx)\sin(a+c)}{b}$$

output
```
-arctanh(cos(b*x-c))*cos(a+c)/b-csc(b*x-c)*sin(a+c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.58

$$\int \csc^2(c-bx)\sin(a+bx)\,dx = -\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)+\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a+c)}{b}+\frac{\csc(c-bx)\sin(a+c)}{b}$$

input
```
Integrate[Csc[c - b*x]^2*Sin[a + b*x],x]
```

output
```
((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] + Sin[c]*Sin[(b*x
)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a + c])/b + (Csc
[c - b*x]*Sin[a + c])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^2(c-bx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin(a+bx)\csc^2(c-bx)dx$$

input
```
Int[Csc[c - b*x]^2*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.27

method	result
risch	$\frac{e^{i(bx+3a+2c)}-e^{i(bx+a)}}{b\left(e^{2i(a+c)}-e^{2i(bx+a)}\right)}-\frac{\ln\left(e^{i(a+c)}+e^{i(bx+a)}\right)\cos(a+c)}{b}+\frac{\ln\left(-e^{i(a+c)}+e^{i(bx+a)}\right)\cos(a+c)}{b}$
default	$\frac{4(-2\cos(a)\cos(c)+2\sin(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+8\sin(a)\cos(c)+8\cos(a)\sin}{\left(-4\cos(c)^2\sin(a)^2-4\cos(a)^2\cos(c)^2-4\sin(a)^2\sin(c)^2-4\sin(c)^2\cos(a)^2\right)\left(\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+2\tan\left(\frac{a}{2}+\frac{b}{2}\right.\right.}$

input
```
int(csc(b*x-c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b/(exp(2*I*(a+c))-exp(2*I*(b*x+a)))*(exp(I*(b*x+3*a+2*c))-exp(I*(b*x+a))
)-ln(exp(I*(a+c))+exp(I*(b*x+a)))/b*cos(a+c)+ln(-exp(I*(a+c))+exp(I*(b*x+a
)))/b*cos(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 5.12

$$\int \csc^2(c-bx)\sin(a+bx)\,dx =$$

$$-\frac{\left(\cos(a+c)^2\sin(bx+a)-\cos(bx+a)\cos(a+c)\sin(a+c)\right)\log\left(\frac{\cos(bx+a)\cos(a+c)+\sin(bx+a)\sin(a+c)+1}{\cos(a+c)+1}\right)}{2\left(b\cos(a+c)\sin\right.}$$

input
```
integrate(csc(b*x-c)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*((cos(a + c)^2*sin(b*x + a) - cos(b*x + a)*cos(a + c)*sin(a + c))*log
((cos(b*x + a)*cos(a + c) + sin(b*x + a)*sin(a + c) + 1)/(cos(a + c) + 1))
 - (cos(a + c)^2*sin(b*x + a) - cos(b*x + a)*cos(a + c)*sin(a + c))*log(-(
cos(b*x + a)*cos(a + c) + sin(b*x + a)*sin(a + c) - 1)/(cos(a + c) + 1)) +
 2*sin(a + c))/(b*cos(a + c)*sin(b*x + a) - b*cos(b*x + a)*sin(a + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(29) = 58$.

Time = 62.34 (sec) , antiderivative size = 3264, normalized size of antiderivative = 98.91

$$\int \csc^2(c-bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x-c)**2*sin(b*x+a),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), (log(tan(b*x/2))/b, Eq(c, 0)), (0, Eq(
b, 0)), (log(-tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(-b*tan(c/2)**
4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*ta
n(b*x/2)**2 - b*tan(c/2) + b*tan(b*x/2)) - log(-tan(c/2) + tan(b*x/2))*tan
(c/2)**3*tan(b*x/2)**2/(-b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/
2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) + b*tan(b*x/
2)) + log(-tan(c/2) + tan(b*x/2))*tan(c/2)**3/(-b*tan(c/2)**4*tan(b*x/2) +
 b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 -
b*tan(c/2) + b*tan(b*x/2)) - 2*log(-tan(c/2) + tan(b*x/2))*tan(c/2)**2*tan
(b*x/2)/(-b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c
/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) + b*tan(b*x/2)) + log(-tan(
c/2) + tan(b*x/2))*tan(c/2)*tan(b*x/2)**2/(-b*tan(c/2)**4*tan(b*x/2) + b*t
an(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*ta
n(c/2) + b*tan(b*x/2)) - log(-tan(c/2) + tan(b*x/2))*tan(c/2)/(-b*tan(c/2)
**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*
tan(b*x/2)**2 - b*tan(c/2) + b*tan(b*x/2)) + log(-tan(c/2) + tan(b*x/2))*t
an(b*x/2)/(-b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan
(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) + b*tan(b*x/2)) - log(tan
(b*x/2) + 1/tan(c/2))*tan(c/2)**4*tan(b*x/2)/(-b*tan(c/2)**4*tan(b*x/2) +
b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(36) = 72$.

Time = 0.05 (sec) , antiderivative size = 461, normalized size of antiderivative = 13.97

$$\int \csc^2(c - bx)\sin(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x-c)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*(cos(b*x + 2*a + 2*c) - cos(b*x))*cos(2*b*x + a) - 2*cos(b*x + 2*a
 + 2*c)*cos(a + 2*c) + 2*cos(b*x)*cos(a + 2*c) + (cos(2*b*x + a)^2*cos(a +
 c) - 2*cos(2*b*x + a)*cos(a + 2*c)*cos(a + c) + cos(a + 2*c)^2*cos(a + c)
 + cos(a + c)*sin(2*b*x + a)^2 - 2*cos(a + c)*sin(2*b*x + a)*sin(a + 2*c)
+ cos(a + c)*sin(a + 2*c)^2)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2
 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*b*x + a)^2*cos(a +
c) - 2*cos(2*b*x + a)*cos(a + 2*c)*cos(a + c) + cos(a + 2*c)^2*cos(a + c)
+ cos(a + c)*sin(2*b*x + a)^2 - 2*cos(a + c)*sin(2*b*x + a)*sin(a + 2*c) +
 cos(a + c)*sin(a + 2*c)^2)*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2
+ sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + 2*(sin(b*x + 2*a + 2*c) - s
in(b*x))*sin(2*b*x + a) - 2*sin(b*x + 2*a + 2*c)*sin(a + 2*c) + 2*sin(b*x)
*sin(a + 2*c))/(b*cos(2*b*x + a)^2 - 2*b*cos(2*b*x + a)*cos(a + 2*c) + b*c
os(a + 2*c)^2 + b*sin(2*b*x + a)^2 - 2*b*sin(2*b*x + a)*sin(a + 2*c) + b*s
in(a + 2*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(36) = 72$.

Time = 0.14 (sec) , antiderivative size = 347, normalized size of antiderivative = 10.52

$$\int \csc^2(c - bx)\sin(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x-c)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 - 4*tan(1/2*a)*tan(1/2*c) - tan
(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x - 1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 +
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + (tan(1/2*b*x - 1/2*c)*tan(1/2*a)^2*tan
(1/2*c) + tan(1/2*b*x - 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*b*x - 1/2
*c)*tan(1/2*a) - tan(1/2*b*x - 1/2*c)*tan(1/2*c))/(tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x - 1/2*c)*tan(1/2*a)^2
*tan(1/2*c)^2 - tan(1/2*b*x - 1/2*c)*tan(1/2*a)^2 - 4*tan(1/2*b*x - 1/2*c)
*tan(1/2*a)*tan(1/2*c) - tan(1/2*a)^2*tan(1/2*c) - tan(1/2*b*x - 1/2*c)*ta
n(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*b*x - 1/2*c) + tan(1/2*a) +
 tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1
)*tan(1/2*b*x - 1/2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 22.00 (sec) , antiderivative size = 252, normalized size of antiderivative = 7.64

$$\int \csc^2(c-bx)\sin(a+bx)\,dx = -\frac{\ln\left(-\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)-\frac{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}+1\right)\,1\mathrm{i}}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}}}\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}+1\right)}{2\,b\,\sqrt{\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}}} + \frac{\ln\left(-\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)+\frac{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}+1\right)\,1\mathrm{i}}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}}}\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}+1\right)}{2\,b\,\sqrt{\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}}} + \frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}-1\right)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}$$

input
```
int(sin(a + b*x)/sin(c - b*x)^2,x)
```

output
```
(log((exp(a*2i)*exp(c*2i)*(exp(a*2i)*exp(c*2i) + 1)*1i)/(exp(a*2i)*exp(c*2
i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(c*2i)*1i + 1i))*(exp(a*2i
 + c*2i) + 1))/(2*b*exp(a*2i + c*2i)^(1/2)) - (log(- exp(a*1i)*exp(b*x*1i)
*(exp(a*2i)*exp(c*2i)*1i + 1i) - (exp(a*2i)*exp(c*2i)*(exp(a*2i)*exp(c*2i)
 + 1)*1i)/(exp(a*2i)*exp(c*2i))^(1/2))*(exp(a*2i + c*2i) + 1))/(2*b*exp(a*
2i + c*2i)^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i + c*2i) - 1))/(b*(exp(a*
2i + c*2i) - exp(a*2i + b*x*2i)))
```

Reduce [F]

$$\int \csc^2(c-bx)\sin(a+bx)\,dx=\int \csc\left(bx-c\right)^2\sin\left(bx+a\right)dx$$

input
```
int(csc(b*x-c)^2*sin(b*x+a),x)
```

output
```
int(csc(b*x - c)**2*sin(a + b*x),x)
```

3.283 $\int \csc^3(c-bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \csc^3(c-bx)\sin(a+bx)\,dx=-\frac{\cos(a+c)\cot(c-bx)}{b}+\frac{\csc^2(c-bx)\sin(a+c)}{2b}$$

output
```
cos(a+c)*cot(b*x-c)/b+1/2*csc(b*x-c)^2*sin(a+c)/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \csc^3(c-bx)\sin(a+bx)\,dx=\frac{(\cos(a)-\cos(a+c)\cos(c-2bx))\csc(c)\csc^2(c-bx)}{2b}$$

input
```
Integrate[Csc[c - b*x]^3*Sin[a + b*x],x]
```

output
```
((Cos[a] - Cos[a + c]*Cos[c - 2*b*x])*Csc[c]*Csc[c - b*x]^2)/(2*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^3(c-bx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sin(a+bx)\csc^3(c-bx)dx$$

input
```
Int[Csc[c - b*x]^3*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

method	result
risch	$-\frac{i\left(\mathrm{e}^{5i(a+c)}-2\,\mathrm{e}^{i(2bx+5a+3c)}+\mathrm{e}^{3i(a+c)}\right)}{\left(\mathrm{e}^{2i(a+c)}-\mathrm{e}^{2i(bx+a)}\right)^2 b}$
parallelrisch	$\frac{\csc\left(\frac{bx}{2}-\frac{c}{2}\right)\left(\sin(bx+a)\left(-\frac{\sec\left(\frac{bx}{2}-\frac{c}{2}\right)^2}{2}+1\right)\csc\left(\frac{bx}{2}-\frac{c}{2}\right)+\sec\left(\frac{bx}{2}-\frac{c}{2}\right)\cos(bx+a)\right)}{4b}$
default	$-\frac{\frac{1}{(-\cos(a)\cos(c)+\sin(a)\sin(c))^2(-\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)+\cos(a)\sin(c)+\sin(a)\cos(c))}+\frac{2(-\cos(a)\cos(c)+\sin}{}}{b}$

input
```
int(-csc(b*x-c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-I/(exp(2*I*(a+c))-exp(2*I*(b*x+a)))^2/b*(exp(5*I*(a+c))-2*exp(I*(2*b*x+5*
a+3*c))+exp(3*I*(a+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.27

$$\int \csc^3(c-bx)\sin(a+bx)\,dx = -\frac{2\left(2\cos\left(a+c\right)^3-\cos\left(a+c\right)\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-\left(4\cos\left(bx+a\right)^2\cos\left(a+c\right)^2-2\cos\left(a+c\right)\right.}{2\left(2\,b\cos\left(bx+a\right)\cos\left(a+c\right)\sin\left(bx+a\right)\sin\left(a+c\right)+\left(2\,b\cos\left(a+c\right)^2-b\right)\cos\left(bx+a\right)^2-b\cos\right.}$$

input
```
integrate(-csc(b*x-c)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*(2*cos(a + c)^3 - cos(a + c))*cos(b*x + a)*sin(b*x + a) - (4*cos(b
*x + a)^2*cos(a + c)^2 - 2*cos(a + c)^2 - 1)*sin(a + c))/(2*b*cos(b*x + a)
*cos(a + c)*sin(b*x + a)*sin(a + c) + (2*b*cos(a + c)^2 - b)*cos(b*x + a)^
2 - b*cos(a + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(c-bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(-csc(b*x-c)**3*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 433, normalized size of antiderivative = 11.70

$$\int \csc^3(c-bx)\sin(a+bx)\,dx =$$
$$-\frac{(2\sin(2bx+2a+3c)-\sin(2a+5c)-\sin(3c))\cos(4bx+a)+2(2\sin(2bx+a+2c)-\sin(a+}{b\cos(4bx+a)^2+4b\cos}$$

input
```
integrate(-csc(b*x-c)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-((2*sin(2*b*x + 2*a + 3*c) - sin(2*a + 5*c) - sin(3*c))*cos(4*b*x + a) +
2*(2*sin(2*b*x + a + 2*c) - sin(a + 4*c))*cos(2*b*x + 2*a + 3*c) + 2*(sin(
2*a + 5*c) + sin(3*c))*cos(2*b*x + a + 2*c) - (2*cos(2*b*x + 2*a + 3*c) -
cos(2*a + 5*c) - cos(3*c))*sin(4*b*x + a) - 2*(2*cos(2*b*x + a + 2*c) - co
s(a + 4*c))*sin(2*b*x + 2*a + 3*c) - 2*(cos(2*a + 5*c) + cos(3*c))*sin(2*b
*x + a + 2*c) - cos(a + 4*c)*sin(2*a + 5*c) + cos(2*a + 5*c)*sin(a + 4*c)
+ cos(3*c)*sin(a + 4*c) - cos(a + 4*c)*sin(3*c))/(b*cos(4*b*x + a)^2 + 4*b
*cos(2*b*x + a + 2*c)^2 - 4*b*cos(2*b*x + a + 2*c)*cos(a + 4*c) + b*cos(a
+ 4*c)^2 + b*sin(4*b*x + a)^2 + 4*b*sin(2*b*x + a + 2*c)^2 - 4*b*sin(2*b*x
 + a + 2*c)*sin(a + 4*c) + b*sin(a + 4*c)^2 - 2*(2*b*cos(2*b*x + a + 2*c)
- b*cos(a + 4*c))*cos(4*b*x + a) - 2*(2*b*sin(2*b*x + a + 2*c) - b*sin(a +
 4*c))*sin(4*b*x + a))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(36) = 72$.

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.19

$$\int \csc^3(c-bx)\sin(a+bx)\,dx$$
$$=\frac{\tan(bx-c)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan(bx-c)\tan\left(\frac{1}{2}a\right)^2-4\tan(bx-c)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right.}{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\right.}$$

input
```
integrate(-csc(b*x-c)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
(tan(b*x - c)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x - c)*tan(1/2*a)^2 - 4*ta
n(b*x - c)*tan(1/2*a)*tan(1/2*c) - tan(1/2*a)^2*tan(1/2*c) - tan(b*x - c)*
tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^2 + tan(b*x - c) + tan(1/2*a) + tan(1
/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b*ta
n(b*x - c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c-bx)\sin(a+bx)\,dx=\text{Hanged}$$

input
```
int(sin(a + b*x)/sin(c - b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \csc^3(c-bx)\sin(a+bx)\,dx=\frac{\cos(bx-c)\sin(bx+a)+\cos(bx+a)\sin(bx-c)}{2\sin(bx-c)^2 b}$$

input
```
int(-csc(b*x-c)^3*sin(b*x+a),x)
```

output
```
(cos(b*x - c)*sin(a + b*x) + cos(a + b*x)*sin(b*x - c))/(2*sin(b*x - c)**2
*b)
```

3.284 $\int \csc^4(c-bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int \csc^4(c-bx)\sin(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c-bx))\cos(a+c)}{2b} - \frac{\cos(a+c)\cot(c-bx)\csc(c-bx)}{2b} + \frac{\csc^3(c-bx)\sin(a+c)}{3b}$$

output
```
-1/2*arctanh(cos(b*x-c))*cos(a+c)/b-1/2*cos(a+c)*cot(b*x-c)*csc(b*x-c)/b-1
/3*csc(b*x-c)^3*sin(a+c)/b
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \csc^4(c-bx)\sin(a+bx)\,dx = \frac{-12\operatorname{arctanh}\left(\cos(c)+\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a+c)+\csc^3(c-bx)(4\sin(a+c)-3\cos(a+c)\sin(2(c-bx)}{12b}$$

input
```
Integrate[Csc[c - b*x]^4*Sin[a + b*x],x]
```

output
```
(-12*ArcTanh[Cos[c] + Sin[c]*Tan[(b*x)/2]]*Cos[a + c] + Csc[c - b*x]^3*(4*
Sin[a + c] - 3*Cos[a + c]*Sin[2*(c - b*x)]))/(12*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^4(c-bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin(a+bx)\csc^4(c-bx)dx$$

input
```
Int[Csc[c - b*x]^4*Sin[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.42 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.75

method	result
risch	$\frac{3\,\mathrm{e}^{i(bx+7a+6c)}-8\,\mathrm{e}^{i(3bx+7a+4c)}+3\,\mathrm{e}^{i(bx+5a+4c)}-3\,\mathrm{e}^{i(5bx+7a+2c)}+8\,\mathrm{e}^{i(3bx+5a+2c)}-3\,\mathrm{e}^{5i(bx+a)}}{6b\left(\mathrm{e}^{2i(a+c)}-\mathrm{e}^{2i(bx+a)}\right)^3}-\frac{\ln\left(\mathrm{e}^{i(a+c)}+\mathrm{e}^{i(bx+a)}\right)\cos(a+c)}{2b}$
default	Expression too large to display

input
```
int(csc(b*x-c)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/b/(exp(2*I*(a+c))-exp(2*I*(b*x+a)))^3*(3*exp(I*(b*x+7*a+6*c))-8*exp(I*
(3*b*x+7*a+4*c))+3*exp(I*(b*x+5*a+4*c))-3*exp(I*(5*b*x+7*a+2*c))+8*exp(I*(
3*b*x+5*a+2*c))-3*exp(5*I*(b*x+a)))-1/2*ln(exp(I*(a+c))+exp(I*(b*x+a)))/b*
cos(a+c)+1/2*ln(-exp(I*(a+c))+exp(I*(b*x+a)))/b*cos(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(63) = 126.

Time = 0.10 (sec) , antiderivative size = 398, normalized size of antiderivative = 6.12

$$\int \csc^4(c-bx)\sin(a+bx)\,dx =$$

$$-\frac{6\left(2\cos\left(a+c\right)^3-\cos\left(a+c\right)\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+3\left(\left(\cos\left(a+c\right)^4-\left(4\cos\left(a+c\right)^4-3\cos\left(a\right.\right.\right.\right.}{}$$

input
```
integrate(csc(b*x-c)^4*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/12*(6*(2*cos(a + c)^3 - cos(a + c))*cos(b*x + a)*sin(b*x + a) + 3*((cos
(a + c)^4 - (4*cos(a + c)^4 - 3*cos(a + c)^2)*cos(b*x + a)^2)*sin(b*x + a)
 + ((4*cos(a + c)^3 - cos(a + c))*cos(b*x + a)^3 - 3*cos(b*x + a)*cos(a +
c)^3)*sin(a + c))*log((cos(b*x + a)*cos(a + c) + sin(b*x + a)*sin(a + c) +
 1)/(cos(a + c) + 1)) - 3*((cos(a + c)^4 - (4*cos(a + c)^4 - 3*cos(a + c)^
2)*cos(b*x + a)^2)*sin(b*x + a) + ((4*cos(a + c)^3 - cos(a + c))*cos(b*x +
 a)^3 - 3*cos(b*x + a)*cos(a + c)^3)*sin(a + c))*log(-(cos(b*x + a)*cos(a
+ c) + sin(b*x + a)*sin(a + c) - 1)/(cos(a + c) + 1)) - 2*(6*cos(b*x + a)^
2*cos(a + c)^2 - 3*cos(a + c)^2 - 2)*sin(a + c))/((b*cos(a + c)^3 - (4*b*c
os(a + c)^3 - 3*b*cos(a + c))*cos(b*x + a)^2)*sin(b*x + a) + ((4*b*cos(a +
 c)^2 - b)*cos(b*x + a)^3 - 3*b*cos(b*x + a)*cos(a + c)^2)*sin(a + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(c-bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x-c)**4*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(63) = 126$.

Time = 0.09 (sec) , antiderivative size = 1798, normalized size of antiderivative = 27.66

$$\int \csc^4(c-bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x-c)^4*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/12*(2*(3*cos(5*b*x) + 3*cos(5*b*x + 2*a + 2*c) + 8*cos(3*b*x + 2*a + 4*c
) - 8*cos(3*b*x + 2*c) - 3*cos(b*x + 2*a + 6*c) - 3*cos(b*x + 4*c))*cos(6*
b*x + a) - 6*(3*cos(4*b*x + a + 2*c) - 3*cos(2*b*x + a + 4*c) + cos(a + 6*
c))*cos(5*b*x + 2*a + 2*c) - 6*(3*cos(5*b*x) + 8*cos(3*b*x + 2*a + 4*c) -
8*cos(3*b*x + 2*c) - 3*cos(b*x + 2*a + 6*c) - 3*cos(b*x + 4*c))*cos(4*b*x
+ a + 2*c) + 16*(3*cos(2*b*x + a + 4*c) - cos(a + 6*c))*cos(3*b*x + 2*a +
4*c) - 16*(3*cos(2*b*x + a + 4*c) - cos(a + 6*c))*cos(3*b*x + 2*c) + 18*(c
os(5*b*x) - cos(b*x + 2*a + 6*c) - cos(b*x + 4*c))*cos(2*b*x + a + 4*c) -
6*cos(5*b*x)*cos(a + 6*c) + 6*cos(b*x + 2*a + 6*c)*cos(a + 6*c) + 6*cos(b*
x + 4*c)*cos(a + 6*c) - 3*(cos(6*b*x + a)^2*cos(a + c) + 9*cos(4*b*x + a +
 2*c)^2*cos(a + c) + 9*cos(2*b*x + a + 4*c)^2*cos(a + c) - 6*cos(2*b*x + a
 + 4*c)*cos(a + 6*c)*cos(a + c) + cos(a + 6*c)^2*cos(a + c) + cos(a + c)*s
in(6*b*x + a)^2 + 9*cos(a + c)*sin(4*b*x + a + 2*c)^2 + 9*cos(a + c)*sin(2
*b*x + a + 4*c)^2 - 6*cos(a + c)*sin(2*b*x + a + 4*c)*sin(a + 6*c) + cos(a
 + c)*sin(a + 6*c)^2 - 2*(3*cos(4*b*x + a + 2*c)*cos(a + c) - 3*cos(2*b*x
+ a + 4*c)*cos(a + c) + cos(a + 6*c)*cos(a + c))*cos(6*b*x + a) - 6*(3*cos
(2*b*x + a + 4*c)*cos(a + c) - cos(a + 6*c)*cos(a + c))*cos(4*b*x + a + 2*
c) - 2*(3*cos(a + c)*sin(4*b*x + a + 2*c) - 3*cos(a + c)*sin(2*b*x + a + 4
*c) + cos(a + c)*sin(a + 6*c))*sin(6*b*x + a) - 6*(3*cos(a + c)*sin(2*b*x
+ a + 4*c) - cos(a + c)*sin(a + 6*c))*sin(4*b*x + a + 2*c))*log(cos(b*x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2220 vs. $2(63) = 126$.

Time = 0.18 (sec) , antiderivative size = 2220, normalized size of antiderivative = 34.15

$$\int \csc^4(c - bx)\sin(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x-c)^4*sin(b*x+a),x, algorithm="giac")
```

output
```
1/24*(12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 - 4*tan(1/2*a)*tan(1/2*
c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x - 1/2*c)))/(tan(1/2*a)^2*tan(1/
2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + (2*tan(1/2*b*x - 1/2*c)^3*tan(
1/2*a)^6*tan(1/2*c)^5 + 2*tan(1/2*b*x - 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^6
 + 3*tan(1/2*b*x - 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^6 + 4*tan(1/2*b*x - 1/
2*c)^3*tan(1/2*a)^6*tan(1/2*c)^3 + 2*tan(1/2*b*x - 1/2*c)^3*tan(1/2*a)^5*t
an(1/2*c)^4 + 3*tan(1/2*b*x - 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^4 + 2*tan(1
/2*b*x - 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^5 - 12*tan(1/2*b*x - 1/2*c)^2*ta
n(1/2*a)^5*tan(1/2*c)^5 + 6*tan(1/2*b*x - 1/2*c)*tan(1/2*a)^6*tan(1/2*c)^5
 + 4*tan(1/2*b*x - 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^6 + 3*tan(1/2*b*x - 1/
2*c)^2*tan(1/2*a)^4*tan(1/2*c)^6 + 6*tan(1/2*b*x - 1/2*c)*tan(1/2*a)^5*tan
(1/2*c)^6 + 2*tan(1/2*b*x - 1/2*c)^3*tan(1/2*a)^6*tan(1/2*c) - 2*tan(1/2*b
*x - 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^2 - 3*tan(1/2*b*x - 1/2*c)^2*tan(1/2
*a)^6*tan(1/2*c)^2 + 4*tan(1/2*b*x - 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^3 -
24*tan(1/2*b*x - 1/2*c)^2*tan(1/2*a)^5*tan(1/2*c)^3 + 12*tan(1/2*b*x - 1/2
*c)*tan(1/2*a)^6*tan(1/2*c)^3 + 4*tan(1/2*b*x - 1/2*c)^3*tan(1/2*a)^3*tan(
1/2*c)^4 + 3*tan(1/2*b*x - 1/2*c)^2*tan(1/2*a)^4*tan(1/2*c)^4 + 6*tan(1/2*
b*x - 1/2*c)*tan(1/2*a)^5*tan(1/2*c)^4 - 2*tan(1/2*b*x - 1/2*c)^3*tan(1/2*
a)^2*tan(1/2*c)^5 - 24*tan(1/2*b*x - 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c)^5 +
6*tan(1/2*b*x - 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^5 + 2*tan(1/2*b*x - 1/2*...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(c-bx)\sin(a+bx)\,dx=\text{Hanged}$$

input
```
int(sin(a + b*x)/sin(c - b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^4(c-bx)\sin(a+bx)\,dx$$

$$= \frac{-2\cos(bx-c)\cos(bx+a)\sin(bx-c)-4\cos(bx-c)\sin(bx-c)^2\sin(bx+a)-2\cos(bx-c)\sin(bx}{}$$

input
```
int(csc(b*x-c)^4*sin(b*x+a),x)
```

output
```
( - 2*cos(b*x - c)*cos(a + b*x)*sin(b*x - c) - 4*cos(b*x - c)*sin(b*x - c)
**2*sin(a + b*x) - 2*cos(b*x - c)*sin(b*x - c) - 4*cos(b*x - c)*sin(a + b*
x) - 4*cos(a + b*x)*sin(b*x - c) - 6*int(tan((b*x - c)/2)/(tan((a + b*x)/2
)**2 + 1),x)*sin(b*x - c)**3*b - 2*int(1/(tan((b*x - c)/2)**3*tan((a + b*x
)/2)**2 + tan((b*x - c)/2)**3),x)*sin(b*x - c)**3*b + 4*log(tan((b*x - c)/
2)**2 + 1)*sin(b*x - c)**3 - 2*log(tan((b*x - c)/2))*sin(b*x - c)**3 + 3*s
in(b*x - c)**3 + 2*sin(b*x - c)**2*sin(a + b*x) - 2*sin(b*x - c))/(12*sin(
b*x - c)**3*b)
```

3.285 $\int \csc(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \csc(c+bx)\sin^2(a+bx)\,dx=-\frac{\cos(2a-c+bx)}{b}-\frac{\operatorname{arctanh}(\cos(c+bx))\sin^2(a-c)}{b}$$

output
```
-cos(b*x+2*a-c)/b-arctanh(cos(b*x+c))*sin(a-c)^2/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int \csc(c+bx)\sin^2(a+bx)\,dx=\frac{-\cos(2a-c+bx)+\left(-\log\left(\cos\left(\frac{1}{2}(c+bx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+bx)\right)\right)\right)\sin^2(a-c)}{b}$$

input
```
Integrate[Csc[c + b*x]*Sin[a + b*x]^2,x]
```

output
```
(-Cos[2*a - c + b*x] + (-Log[Cos[(c + b*x)/2]] + Log[Sin[(c + b*x)/2]])*Si
n[a - c]^2)/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \sin^2(a+bx)\csc(bx+c)dx$$

input
```
Int[Csc[c + b*x]*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.73

method	result
risch	$-\frac{\ln\left(\mathrm{e}^{i(bx+a)}+\mathrm{e}^{i(a-c)}\right)}{2b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+\mathrm{e}^{i(a-c)}\right)\cos(2a-2c)}{2b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}-\mathrm{e}^{i(a-c)}\right)}{2b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-\mathrm{e}^{i(a-c)}\right)\cos(2a-2c)}{2b}-$
default	$\frac{\frac{8(\sin(a)\cos(c)-\cos(a)\sin(c))^2\arctan\left(\frac{2(\sin(a)\cos(c)-\cos(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2\cos(a)\cos(c)+2\sin(a)\sin(c)}{2\sqrt{-\cos(c)^2\sin(a)^2-\cos(a)^2\cos(c)^2-\sin(a)^2\sin(c)^2-\sin(c)^2\cos(a)^2}}\right)}{\left(4\cos(c)^2\sin(a)^2+4\cos(a)^2\cos(c)^2+4\sin(a)^2\sin(c)^2+4\sin(c)^2\cos(a)^2\right)\sqrt{-\cos(c)^2\sin(a)^2-\cos(a)^2\cos(c)^2-\sin(a)^2\sin(c)^2-\sin(c)^2\cos(a)^2}}-}{b}$

input
```
int(csc(b*x+c)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))+1/2/b*ln(exp(I*(b*x+a))+exp(I*(a-c)
))*cos(2*a-2*c)+1/2/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))-1/2/b*ln(exp(I*(b*x+
a))-exp(I*(a-c)))*cos(2*a-2*c)-cos(b*x+2*a-c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(37) = 74.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.46

$$\int \csc(c+bx)\sin^2(a+bx)\,dx = -\frac{4\cos(-a+c)\sin(bx+c)\sin(-a+c)+2\left(2\cos(-a+c)^2-1\right)\cos(bx+c)-\left(\cos(-a+c)^2-1\right)\mathrm{l}}{2\,b}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(4*cos(-a + c)*sin(b*x + c)*sin(-a + c) + 2*(2*cos(-a + c)^2 - 1)*cos
(b*x + c) - (cos(-a + c)^2 - 1)*log(1/2*cos(b*x + c) + 1/2) + (cos(-a + c)
^2 - 1)*log(-1/2*cos(b*x + c) + 1/2))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. 2(29) = 58.

Time = 10.03 (sec) , antiderivative size = 3215, normalized size of antiderivative = 86.89

$$\int \csc(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a)**2,x)
```

output

```
2*Piecewise((0, Eq(b, 0) & Eq(c, 0)), (sin(b*x)/b, Eq(c, 0)), (0, Eq(b, 0)
), (2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*
tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/
2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**3/(b
*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2
 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(c/2) + tan(b*x/2))*t
an(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*t
an(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(
tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)*
*4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b
) - 2*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**
4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(
c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**
3/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2
)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - 1/tan(c
/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 +
 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) +
2*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*t
an(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2
)**2 + b) - 2*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*t...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(37) = 74$.

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.19

$$\int \csc(c+bx)\sin^2(a+bx)\,dx$$
$$= \frac{(\cos(-2a+2c)-1)\log\left(\cos(bx)^2+2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2-2\sin(bx)\sin(c)+\sin(c\right.}{}$$

input

```
integrate(csc(b*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/4*((cos(-2*a + 2*c) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 +
 sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(-2*a + 2*c) - 1)*log(co
s(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) +
 sin(c)^2) - 4*cos(b*x + 2*a - c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(37) = 74$.

Time = 0.14 (sec) , antiderivative size = 688, normalized size of antiderivative = 18.59

$$\int \csc(c + bx)\sin^2(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
2*(2*(tan(1/2*a)^4*tan(1/2*c)^2 - 2*tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)
^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2*a)^2*tan(1/2*c)^2
+ 2*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2 - 2*tan(1/2*a)*tan(1/2*c) + tan
(1/2*c)^2)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*t
an(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*
tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2
 + 1) + (4*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^3 - 4*tan(1/2*b*x
+ 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c)^4 - 4*tan(1/2
*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c) + 24*tan(1/2*b*x + 1/2*c)*tan(1/2*a)
^3*tan(1/2*c)^2 + 6*tan(1/2*a)^4*tan(1/2*c)^2 - 24*tan(1/2*b*x + 1/2*c)*ta
n(1/2*a)^2*tan(1/2*c)^3 - 16*tan(1/2*a)^3*tan(1/2*c)^3 + 4*tan(1/2*b*x + 1
/2*c)*tan(1/2*a)*tan(1/2*c)^4 + 6*tan(1/2*a)^2*tan(1/2*c)^4 - 4*tan(1/2*b*
x + 1/2*c)*tan(1/2*a)^3 - tan(1/2*a)^4 + 24*tan(1/2*b*x + 1/2*c)*tan(1/2*a
)^2*tan(1/2*c) + 16*tan(1/2*a)^3*tan(1/2*c) - 24*tan(1/2*b*x + 1/2*c)*tan(
1/2*a)*tan(1/2*c)^2 - 36*tan(1/2*a)^2*tan(1/2*c)^2 + 4*tan(1/2*b*x + 1/2*c
)*tan(1/2*c)^3 + 16*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + 4*tan(1/2*b*x
 + 1/2*c)*tan(1/2*a) + 6*tan(1/2*a)^2 - 4*tan(1/2*b*x + 1/2*c)*tan(1/2*c)
- 16*tan(1/2*a)*tan(1/2*c) + 6*tan(1/2*c)^2 - 1)/((tan(1/2*a)^4*tan(1/2*c)
^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a
)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*t...
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 223, normalized size of antiderivative = 6.03

$$\int \csc(c+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{e^{-a\,2i+c\,1i-b\,x\,1i}}{2\,b} - \frac{e^{a\,2i-c\,1i+b\,x\,1i}}{2\,b}$$
$$-\frac{e^{-a\,2i+c\,2i}\,\ln\left(-\frac{\left(e^{a\,2i}\,e^{-c\,2i}-1\right)^2\,1i}{2}+\frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(-e^{a\,2i}\,e^{-c\,2i}\,2i+e^{a\,4i}\,e^{-c\,4i}\,1i+1i\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}-1\right)^2}{4\,b}$$
$$+\frac{e^{-a\,2i+c\,2i}\,\ln\left(\frac{\left(e^{a\,2i}\,e^{-c\,2i}-1\right)^2\,1i}{2}+\frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(-e^{a\,2i}\,e^{-c\,2i}\,2i+e^{a\,4i}\,e^{-c\,4i}\,1i+1i\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}-1\right)^2}{4\,b}$$

input
```
int(sin(a + b*x)^2/sin(c + b*x),x)
```

output
```
(exp(c*2i - a*2i)*log(((exp(a*2i)*exp(-c*2i) - 1)^2*1i)/2 + (exp(c*1i)*exp
(b*x*1i)*(exp(a*4i)*exp(-c*4i)*1i - exp(a*2i)*exp(-c*2i)*2i + 1i))/2)*(exp
(a*2i - c*2i) - 1)^2)/(4*b) - exp(a*2i - c*1i + b*x*1i)/(2*b) - (exp(c*2i
- a*2i)*log((exp(c*1i)*exp(b*x*1i)*(exp(a*4i)*exp(-c*4i)*1i - exp(a*2i)*ex
p(-c*2i)*2i + 1i))/2 - ((exp(a*2i)*exp(-c*2i) - 1)^2*1i)/2)*(exp(a*2i - c*
2i) - 1)^2)/(4*b) - exp(c*1i - a*2i - b*x*1i)/(2*b)
```

Reduce [F]

$$\int \csc(c+bx)\sin^2(a+bx)\,dx = \int \csc(bx+c)\sin(bx+a)^2\,dx$$

input
```
int(csc(b*x+c)*sin(b*x+a)^2,x)
```

output
```
int(csc(b*x + c)*sin(a + b*x)**2,x)
```

3.286 $\int \csc^2(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx = x\cos(2(a-c)) - \frac{\cot(c+bx)\sin^2(a-c)}{b} + \frac{\log(\sin(c+bx))\sin(2(a-c))}{b}$$

output
```
x*cos(2*a-2*c)-cot(b*x+c)*sin(a-c)^2/b+ln(sin(b*x+c))*sin(2*a-2*c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(49) = 98.

Time = 0.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.69

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx = -\frac{\csc(c)\csc(c+bx)(bx\cos(2a-4c-bx) - bx\cos(2a-2c-bx) + bx\cos(2a+bx) - bx\cos(2a-2c+\cdots}{}$$

input
```
Integrate[Csc[c + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
-1/4*(Csc[c]*Csc[c + b*x]*(b*x*Cos[2*a - 4*c - b*x] - b*x*Cos[2*a - 2*c -
b*x] + b*x*Cos[2*a + b*x] - b*x*Cos[2*a - 2*c + b*x] - 2*Sin[b*x] + Log[Si
n[c + b*x]]*Sin[2*a - 4*c - b*x] - Sin[2*a - 2*c - b*x] - Log[Sin[c + b*x]
]*Sin[2*a - 2*c - b*x] + Log[Sin[c + b*x]]*Sin[2*a + b*x] + Sin[2*a - 2*c
+ b*x] - Log[Sin[c + b*x]]*Sin[2*a - 2*c + b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin^2(a+bx)\csc^2(bx+c)dx$$

input
```
Int[Csc[c + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.00 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.53

method	result
risch	$x\,\mathrm{e}^{2i(a-c)}-2i\sin(2a-2c)\,x-\frac{2i\sin(2a-2c)a}{b}-\frac{i\mathrm{e}^{2i(2a-c)}}{2b(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia})}+\frac{i\mathrm{e}^{2ia}}{b(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia})}-\frac{i\mathrm{e}}{2b(-\mathrm{e}^{2i(bx}}$
default	$\frac{\frac{(2\cos(a)^2\cos(c)\sin(c)-2\cos(c)^2\cos(a)\sin(a)+2\cos(a)\sin(a)\sin(c)^2-2\sin(a)^2\cos(c)\sin(c))\ln(\tan(bx+a)^2+1)}{2}+(\cos(a)^2\cos(c)^2-\sin(c)^2\cos(a}{(\cos(c)^2+\sin(c)^2)^2(\cos(a)^2+\sin(a)^2)^2}$

input
```
int(csc(b*x+c)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*exp(2*I*(a-c))-2*I*sin(2*a-2*c)*x-2*I/b*sin(2*a-2*c)*a-1/2*I/b/(-exp(2*I
*(b*x+a+c))+exp(2*I*a))*exp(2*I*(2*a-c))+I/b/(-exp(2*I*(b*x+a+c))+exp(2*I*
a))*exp(2*I*a)-1/2*I/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))*exp(2*I*c)+ln(exp(
2*I*(b*x+a))-exp(2*I*(a-c)))/b*sin(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx = -\frac{2\cos(-a+c)\log\left(\frac{1}{2}\sin(bx+c)\right)\sin(bx+c)\sin(-a+c)-\left(\cos(-a+c)^2-1\right)\cos(bx+c)-(2\,bx}{b\sin(bx+c)}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*cos(-a + c)*log(1/2*sin(b*x + c))*sin(b*x + c)*sin(-a + c) - (cos(-a +
 c)^2 - 1)*cos(b*x + c) - (2*b*x*cos(-a + c)^2 - b*x)*sin(b*x + c))/(b*sin
(b*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**2*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(49) = 98$.

Time = 0.07 (sec) , antiderivative size = 711, normalized size of antiderivative = 14.51

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(b*cos(2*a + 2*c)*cos(4*c) + b*sin(2*a + 2*c)*sin(4*c))*x - (2*b*x*
cos(4*c) + sin(4*a) - 2*sin(2*a + 2*c) + sin(4*c))*cos(2*b*x + 2*a + 4*c)
+ 2*(b*x*cos(2*b*x + 2*a + 4*c) - b*x*cos(2*a + 2*c))*cos(2*b*x + 6*c) + (
sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(2*b*x + 2*a + 4*c)^2*sin(-2*a +
 2*c) - 2*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c)*sin(-2*a + 2*c) + cos(2*a
+ 2*c)^2*sin(-2*a + 2*c) + sin(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c) - 2*si
n(2*b*x + 2*a + 4*c)*sin(2*a + 2*c)*sin(-2*a + 2*c) + sin(2*a + 2*c)^2*sin
(-2*a + 2*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 -
 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c)
 - 2*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c)*sin(-2*a + 2*c) + cos(2*a + 2*c
)^2*sin(-2*a + 2*c) + sin(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c) - 2*sin(2*b
*x + 2*a + 4*c)*sin(2*a + 2*c)*sin(-2*a + 2*c) + sin(2*a + 2*c)^2*sin(-2*a
 + 2*c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*si
n(b*x)*sin(c) + sin(c)^2) - (2*b*x*sin(4*c) - cos(4*a) + 2*cos(2*a + 2*c)
- cos(4*c))*sin(2*b*x + 2*a + 4*c) + 2*(b*x*sin(2*b*x + 2*a + 4*c) - b*x*s
in(2*a + 2*c))*sin(2*b*x + 6*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))/(b
*cos(2*b*x + 2*a + 4*c)^2 - 2*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c) + b*
cos(2*a + 2*c)^2 + b*sin(2*b*x + 2*a + 4*c)^2 - 2*b*sin(2*b*x + 2*a + 4*c)
*sin(2*a + 2*c) + b*sin(2*a + 2*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(49) = 98$.

Time = 0.16 (sec) , antiderivative size = 1402, normalized size of antiderivative = 28.61

$$\int \csc^2(c + bx) \sin^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
((tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*a)^4*tan(1/2*c)^2 + 16*tan(1/2*a)^
3*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 16*tan(1/2*a
)^3*tan(1/2*c) + 36*tan(1/2*a)^2*tan(1/2*c)^2 - 16*tan(1/2*a)*tan(1/2*c)^3
 + tan(1/2*c)^4 - 6*tan(1/2*a)^2 + 16*tan(1/2*a)*tan(1/2*c) - 6*tan(1/2*c)
^2 + 1)*(b*x + c)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2
 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^
2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 4*(tan(1/2*a)^4*
tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*tan
(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*
c)^4 - tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)^
2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))*log(tan(1/2*b*x + 1/2*c)^2 + 1
)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^
2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4
 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) + 4*(tan(1/2*a)^4*tan(1/2*c)^3 - t
an(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*tan(1/2*a)^3*tan(1/
2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*a
)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3
 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^4*t
an(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 +
tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a...
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.02

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx$$
$$= x\left(\cos\left(2a-2c\right)-\sin\left(2a-2c\right)\,1\mathrm{i}\right)-\frac{\left(1+\mathrm{e}^{a\,4\mathrm{i}-c\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}\right)\,1\mathrm{i}}{2\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}$$
$$+\frac{\mathrm{e}^{-a\,4\mathrm{i}+c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\right)\,\left(2\,b\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-2\,b\,\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$

input
```
int(sin(a + b*x)^2/sin(c + b*x)^2,x)
```

output
```
x*(cos(2*a - 2*c) - sin(2*a - 2*c)*1i) - ((exp(a*4i - c*4i) - 2*exp(a*2i -
 c*2i) + 1)*1i)/(2*b*(exp(a*2i - c*2i) - exp(a*2i + b*x*2i))) + (exp(c*4i
- a*4i)*log(exp(a*2i)*exp(b*x*2i) - exp(a*2i)*exp(-c*2i))*(2*b*exp(a*2i -
c*2i) - 2*b*exp(a*6i - c*6i))*1i)/(4*b^2)
```

Reduce [F]

$$\int \csc^2(c+bx)\sin^2(a+bx)\,dx = \int \csc(bx+c)^2 \sin(bx+a)^2\,dx$$

input
```
int(csc(b*x+c)^2*sin(b*x+a)^2,x)
```

output
```
int(csc(b*x + c)**2*sin(a + b*x)**2,x)
```

3.287 $\int \csc^3(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+bx))\cos(2(a-c))}{b} - \frac{\operatorname{arctanh}(\cos(c+bx))\sin^2(a-c)}{2b} - \frac{\cot(c+bx)\csc(c+bx)\sin^2(a-c)}{2b} - \frac{\csc(c+bx)\sin(2(a-c))}{b}$$

output
```
-arctanh(cos(b*x+c))*cos(2*a-2*c)/b-1/2*arctanh(cos(b*x+c))*sin(a-c)^2/b-1
/2*cot(b*x+c)*csc(b*x+c)*sin(a-c)^2/b-csc(b*x+c)*sin(2*a-2*c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 247 vs. 2(89) = 178.

Time = 2.74 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.78

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx = \frac{\left(\cos\left(2a-2c-\frac{bx}{2}\right)-\cos\left(2a-2c+\frac{bx}{2}\right)\right)\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{bx}{2}\right)}{4b} + \frac{(-1+\cos(2a-2c))\csc^2\left(\frac{c}{2}+\frac{bx}{2}\right)}{16b} + \frac{(-1-3\cos(2a-2c))\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{4b} + \frac{(1+3\cos(2a-2c))\log\left(\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{4b} + \frac{\left(-\cos\left(2a-2c-\frac{bx}{2}\right)+\cos\left(2a-2c+\frac{bx}{2}\right)\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{bx}{2}\right)}{4b} + \frac{(1-\cos(2a-2c))\sec^2\left(\frac{c}{2}+\frac{bx}{2}\right)}{16b}$$

input
```
Integrate[Csc[c + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
((Cos[2*a - 2*c - (b*x)/2] - Cos[2*a - 2*c + (b*x)/2])*Csc[c/2]*Csc[c/2 +
(b*x)/2])/(4*b) + ((-1 + Cos[2*a - 2*c])*Csc[c/2 + (b*x)/2]^2)/(16*b) + ((
-1 - 3*Cos[2*a - 2*c])*Log[Cos[c/2 + (b*x)/2]])/(4*b) + ((1 + 3*Cos[2*a -
2*c])*Log[Sin[c/2 + (b*x)/2]])/(4*b) + ((-Cos[2*a - 2*c - (b*x)/2] + Cos[2
*a - 2*c + (b*x)/2])*Sec[c/2]*Sec[c/2 + (b*x)/2])/(4*b) + ((1 - Cos[2*a -
2*c])*Sec[c/2 + (b*x)/2]^2)/(16*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^3(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin^2(a+bx)\csc^3(bx+c)dx$$

input
```
Int[Csc[c + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.00 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.73

method	result
risch	$\frac{-5\,e^{i(3bx+6a+c)}+2\,e^{i(3bx+4a+3c)}+3\,e^{i(3bx+2a+5c)}+3\,e^{i(bx+6a-c)}+2\,e^{i(bx+4a+c)}-5\,e^{i(bx+2a+3c)}}{4\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^2 b} - \frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)}{4b} - 3$
default	Expression too large to display

input
```
int(csc(b*x+c)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^2/b*(-5*exp(I*(3*b*x+6*a+c))+2*exp(I*
(3*b*x+4*a+3*c))+3*exp(I*(3*b*x+2*a+5*c))+3*exp(I*(b*x+6*a-c))+2*exp(I*(b*
x+4*a+c))-5*exp(I*(b*x+2*a+3*c)))-1/4/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))-3/
4/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))*cos(2*a-2*c)+1/4/b*ln(exp(I*(b*x+a))-e
xp(I*(a-c)))+3/4/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))*cos(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx = -\frac{8\cos(-a+c)\sin(bx+c)\sin(-a+c)+2\left(\cos(-a+c)^2-1\right)\cos(bx+c)+\left(\left(3\cos(-a+c)^2-1\right)\right.}{}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(8*cos(-a + c)*sin(b*x + c)*sin(-a + c) + 2*(cos(-a + c)^2 - 1)*cos(b
*x + c) + ((3*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - 3*cos(-a + c)^2 + 1)*log
(1/2*cos(b*x + c) + 1/2) - ((3*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - 3*cos(-
a + c)^2 + 1)*log(-1/2*cos(b*x + c) + 1/2))/(b*cos(b*x + c)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1595 vs. $2(85) = 170$.

Time $= 0.09$ (sec) , antiderivative size $= 1595$, normalized size of antiderivative $= 17.92$

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*(2*(5*cos(3*b*x + 4*a + 2*c) - 2*cos(3*b*x + 2*a + 4*c) - 3*cos(3*b*x
 + 6*c) - 3*cos(b*x + 4*a) - 2*cos(b*x + 2*a + 2*c) + 5*cos(b*x + 4*c))*co
s(4*b*x + 2*a + 5*c) - 10*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + c))*cos(3*
b*x + 4*a + 2*c) + 4*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + c))*cos(3*b*x +
 2*a + 4*c) + 6*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + c))*cos(3*b*x + 6*c)
 + 4*(3*cos(b*x + 4*a) + 2*cos(b*x + 2*a + 2*c) - 5*cos(b*x + 4*c))*cos(2*
b*x + 2*a + 3*c) - 6*cos(b*x + 4*a)*cos(2*a + c) - 4*cos(b*x + 2*a + 2*c)*
cos(2*a + c) + 10*cos(b*x + 4*c)*cos(2*a + c) + ((3*cos(-2*a + 2*c) + 1)*c
os(4*b*x + 2*a + 5*c)^2 + 4*(3*cos(-2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c)
^2 + (3*cos(-2*a + 2*c) + 1)*sin(4*b*x + 2*a + 5*c)^2 + 4*(3*cos(-2*a + 2*
c) + 1)*sin(2*b*x + 2*a + 3*c)^2 - 2*(2*(3*cos(-2*a + 2*c) + 1)*cos(2*b*x
+ 2*a + 3*c) - 3*cos(2*a + c)*cos(-2*a + 2*c) - cos(2*a + c))*cos(4*b*x +
2*a + 5*c) - 4*(3*cos(2*a + c)*cos(-2*a + 2*c) + cos(2*a + c))*cos(2*b*x +
 2*a + 3*c) + cos(2*a + c)^2 + 3*(cos(2*a + c)^2 + sin(2*a + c)^2)*cos(-2*
a + 2*c) - 2*(2*(3*cos(-2*a + 2*c) + 1)*sin(2*b*x + 2*a + 3*c) - 3*cos(-2*
a + 2*c)*sin(2*a + c) - sin(2*a + c))*sin(4*b*x + 2*a + 5*c) - 4*(3*cos(-2
*a + 2*c)*sin(2*a + c) + sin(2*a + c))*sin(2*b*x + 2*a + 3*c) + sin(2*a +
c)^2)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b
*x)*sin(c) + sin(c)^2) - ((3*cos(-2*a + 2*c) + 1)*cos(4*b*x + 2*a + 5*c)^2
 + 4*(3*cos(-2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c)^2 + (3*cos(-2*a + 2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2867 vs. $2(85) = 170$.

Time = 0.18 (sec) , antiderivative size = 2867, normalized size of antiderivative = 32.21

$$\int \csc^3(c + bx) \sin^2(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(2*(tan(1/2*a)^4*tan(1/2*c)^4 - 4*tan(1/2*a)^4*tan(1/2*c)^2 + 12*tan(1
/2*a)^3*tan(1/2*c)^3 - 4*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 12*tan
(1/2*a)^3*tan(1/2*c) + 28*tan(1/2*a)^2*tan(1/2*c)^2 - 12*tan(1/2*a)*tan(1/
2*c)^3 + tan(1/2*c)^4 - 4*tan(1/2*a)^2 + 12*tan(1/2*a)*tan(1/2*c) - 4*tan(
1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^4*tan(1/2*c)^4 +
2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 +
 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c
)^2 + 1) + (tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^8*tan(1/2*c)^6 - 2*tan(1/2*b
*x + 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c)^7 - 4*tan(1/2*b*x + 1/2*c)*tan(1/2*a
)^8*tan(1/2*c)^7 + tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^8 + 4*ta
n(1/2*b*x + 1/2*c)*tan(1/2*a)^7*tan(1/2*c)^8 + 2*tan(1/2*b*x + 1/2*c)^2*ta
n(1/2*a)^8*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c)
^5 - 4*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^8*tan(1/2*c)^5 - 16*tan(1/2*b*x + 1
/2*c)*tan(1/2*a)^7*tan(1/2*c)^6 - 2*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^5*ta
n(1/2*c)^7 + 16*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^6*tan(1/2*c)^7 + 2*tan(1/2
*b*x + 1/2*c)^2*tan(1/2*a)^4*tan(1/2*c)^8 + 4*tan(1/2*b*x + 1/2*c)*tan(1/2
*a)^5*tan(1/2*c)^8 + tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^8*tan(1/2*c)^2 + 2*
tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c)^3 + 4*tan(1/2*b*x + 1/2*c)*
tan(1/2*a)^8*tan(1/2*c)^3 - 2*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*
c)^4 - 40*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^7*tan(1/2*c)^4 - 2*tan(1/2*b*...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^2/sin(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^3(c+bx)\sin^2(a+bx)\,dx=\int \csc(bx+c)^3\sin(bx+a)^2\,dx$$

input
```
int(csc(b*x+c)^3*sin(b*x+a)^2,x)
```

output
```
int(csc(b*x + c)**3*sin(a + b*x)**2,x)
```

3.288 $\int \csc^4(c+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx = -\frac{\cos(2(a-c))\cot(c+bx)}{b}-\frac{\cot(c+bx)\sin^2(a-c)}{b} -\frac{\cot^3(c+bx)\sin^2(a-c)}{3b}-\frac{\csc^2(c+bx)\sin(2(a-c))}{2b}$$

output
```
-cos(2*a-2*c)*cot(b*x+c)/b-cot(b*x+c)*sin(a-c)^2/b-1/3*cot(b*x+c)^3*sin(a-
c)^2/b-1/2*csc(b*x+c)^2*sin(2*a-2*c)/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx = -\frac{\csc(c)\csc^3(c+bx)(-3\sin(bx)-\sin(2a-4c-3bx)+3\sin(2a-2c-bx)-3\sin(2a+bx)+\sin(2a-}{12b}$$

input
```
Integrate[Csc[c + b*x]^4*Sin[a + b*x]^2,x]
```

output
```
-1/12*(Csc[c]*Csc[c + b*x]^3*(-3*Sin[b*x] - Sin[2*a - 4*c - 3*b*x] + 3*Sin
[2*a - 2*c - b*x] - 3*Sin[2*a + b*x] + Sin[2*a + 3*b*x] + Sin[2*c + 3*b*x]
))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^4(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sin^2(a+bx)\csc^4(bx+c)dx$$

input
```
Int[Csc[c + b*x]^4*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{\sec\left(\frac{bx}{2}+\frac{c}{2}\right)^{3}\csc\left(\frac{bx}{2}+\frac{c}{2}\right)^{3}\left(-3\cos(bx+c)+2\cos(3bx+2a+c)+\cos(3bx+3c)\right)}{96b}$
risch	$\frac{2i\left(3\,\mathrm{e}^{2i(2bx+4a+c)}-3\,\mathrm{e}^{2i(bx+4a)}-3\,\mathrm{e}^{2i(bx+3a+c)}+\mathrm{e}^{2i(4a-c)}+\mathrm{e}^{6ia}+\mathrm{e}^{2i(2a+c)}\right)}{3\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{3}b}$
default	$-\frac{-2\cos(a)\sin(c)+2\sin(a)\cos(c)}{2(\cos(a)\cos(c)+\sin(a)\sin(c))^{3}(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))^{2}}-\frac{}{3(\cos(a)\cos(c)+\sin(a)\sin\ldots}$

input
```
int(csc(b*x+c)^4*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/96*sec(1/2*b*x+1/2*c)^3*csc(1/2*b*x+1/2*c)^3*(-3*cos(b*x+c)+2*cos(3*b*x+
2*a+c)+cos(3*b*x+3*c))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx = -\frac{\left(4\,\cos\left(-a+c\right)^2-1\right)\cos\left(bx+c\right)^3-3\,\cos\left(bx+c\right)\cos\left(-a+c\right)^2+3\,\cos\left(-a+c\right)\sin\left(bx+c\right)\sin\left(-\right.}{3\left(b\cos\left(bx+c\right)^2-b\right)\sin\left(bx+c\right)}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3*((4*cos(-a + c)^2 - 1)*cos(b*x + c)^3 - 3*cos(b*x + c)*cos(-a + c)^2
+ 3*cos(-a + c)*sin(b*x + c)*sin(-a + c))/((b*cos(b*x + c)^2 - b)*sin(b*x
+ c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+c)**4*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(81) = 162$.

Time = 0.04 (sec) , antiderivative size = 900, normalized size of antiderivative = 10.59

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
2/3*((3*sin(4*b*x + 4*a + 4*c) - 3*sin(2*b*x + 4*a + 2*c) - 3*sin(2*b*x +
2*a + 4*c) + sin(4*a) + sin(2*a + 2*c) + sin(4*c))*cos(6*b*x + 2*a + 8*c)
- 3*(3*sin(2*b*x + 2*a + 4*c) - sin(2*a + 2*c))*cos(4*b*x + 4*a + 4*c) - 3
*(3*sin(4*b*x + 4*a + 4*c) - 3*sin(2*b*x + 4*a + 2*c) - 3*sin(2*b*x + 2*a
+ 4*c) + sin(4*a) + sin(2*a + 2*c) + sin(4*c))*cos(4*b*x + 2*a + 6*c) - 3*
(3*sin(2*b*x + 4*a + 2*c) - sin(4*a) - sin(4*c))*cos(2*b*x + 2*a + 4*c) -
(sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (3*cos(4*b*x + 4*a + 4*c) - 3*cos(2
*b*x + 4*a + 2*c) - 3*cos(2*b*x + 2*a + 4*c) + cos(4*a) + cos(2*a + 2*c) +
 cos(4*c))*sin(6*b*x + 2*a + 8*c) + 3*(3*cos(2*b*x + 2*a + 4*c) - cos(2*a
+ 2*c))*sin(4*b*x + 4*a + 4*c) + 3*(3*cos(4*b*x + 4*a + 4*c) - 3*cos(2*b*x
 + 4*a + 2*c) - 3*cos(2*b*x + 2*a + 4*c) + cos(4*a) + cos(2*a + 2*c) + cos
(4*c))*sin(4*b*x + 2*a + 6*c) + 3*cos(2*a + 2*c)*sin(2*b*x + 4*a + 2*c) +
3*(3*cos(2*b*x + 4*a + 2*c) - cos(4*a) - cos(4*c))*sin(2*b*x + 2*a + 4*c)
+ (cos(4*a) + cos(4*c))*sin(2*a + 2*c) - 3*cos(2*b*x + 4*a + 2*c)*sin(2*a
+ 2*c))/(b*cos(6*b*x + 2*a + 8*c)^2 + 9*b*cos(4*b*x + 2*a + 6*c)^2 + 9*b*c
os(2*b*x + 2*a + 4*c)^2 - 6*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c) + b*co
s(2*a + 2*c)^2 + b*sin(6*b*x + 2*a + 8*c)^2 + 9*b*sin(4*b*x + 2*a + 6*c)^2
 + 9*b*sin(2*b*x + 2*a + 4*c)^2 - 6*b*sin(2*b*x + 2*a + 4*c)*sin(2*a + 2*c
) + b*sin(2*a + 2*c)^2 - 2*(3*b*cos(4*b*x + 2*a + 6*c) - 3*b*cos(2*b*x + 2
*a + 4*c) + b*cos(2*a + 2*c))*cos(6*b*x + 2*a + 8*c) - 6*(3*b*cos(2*b*x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(81) = 162$.

Time = 0.13 (sec) , antiderivative size = 653, normalized size of antiderivative = 7.68

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+c)^4*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/3*(3*tan(b*x + c)^2*tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(b*x + c)^2*tan(1/
2*a)^4*tan(1/2*c)^2 + 24*tan(b*x + c)^2*tan(1/2*a)^3*tan(1/2*c)^3 + 6*tan(
b*x + c)*tan(1/2*a)^4*tan(1/2*c)^3 - 6*tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2
*c)^4 - 6*tan(b*x + c)*tan(1/2*a)^3*tan(1/2*c)^4 + 3*tan(b*x + c)^2*tan(1/
2*a)^4 - 24*tan(b*x + c)^2*tan(1/2*a)^3*tan(1/2*c) - 6*tan(b*x + c)*tan(1/
2*a)^4*tan(1/2*c) + 60*tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 36*tan(b
*x + c)*tan(1/2*a)^3*tan(1/2*c)^2 + 4*tan(1/2*a)^4*tan(1/2*c)^2 - 24*tan(b
*x + c)^2*tan(1/2*a)*tan(1/2*c)^3 - 36*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c
)^3 - 8*tan(1/2*a)^3*tan(1/2*c)^3 + 3*tan(b*x + c)^2*tan(1/2*c)^4 + 6*tan(
b*x + c)*tan(1/2*a)*tan(1/2*c)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^4 - 6*tan(b*x
 + c)^2*tan(1/2*a)^2 - 6*tan(b*x + c)*tan(1/2*a)^3 + 24*tan(b*x + c)^2*tan
(1/2*a)*tan(1/2*c) + 36*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c) + 8*tan(1/2*a
)^3*tan(1/2*c) - 6*tan(b*x + c)^2*tan(1/2*c)^2 - 36*tan(b*x + c)*tan(1/2*a
)*tan(1/2*c)^2 - 16*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(b*x + c)*tan(1/2*c)^
3 + 8*tan(1/2*a)*tan(1/2*c)^3 + 3*tan(b*x + c)^2 + 6*tan(b*x + c)*tan(1/2*
a) + 4*tan(1/2*a)^2 - 6*tan(b*x + c)*tan(1/2*c) - 8*tan(1/2*a)*tan(1/2*c)
+ 4*tan(1/2*c)^2)/((tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^
2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1)*b*tan(b*x + c)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^2/sin(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int \csc^4(c+bx)\sin^2(a+bx)\,dx$$
$$=\frac{-\cos(bx+c)\sin(bx+c)^2-\cos(bx+c)\sin(bx+a)^2-\cos(bx+a)\sin(bx+c)\sin(bx+a)}{3\sin(bx+c)^3 b}$$

input
```
int(csc(b*x+c)^4*sin(b*x+a)^2,x)
```

output
```
( - (cos(b*x + c)*sin(b*x + c)**2 + cos(b*x + c)*sin(a + b*x)**2 + cos(a +
 b*x)*sin(b*x + c)*sin(a + b*x)))/(3*sin(b*x + c)**3*b)
```

3.289 $\int \csc(c-bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \csc(c-bx)\sin^2(a+bx)\,dx=\frac{\cos(2a+c+bx)}{b}+\frac{\operatorname{arctanh}(\cos(c-bx))\sin^2(a+c)}{b}$$

output
```
cos(b*x+2*a+c)/b+arctanh(cos(b*x-c))*sin(a+c)^2/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.50

$$\int \csc(c-bx)\sin^2(a+bx)\,dx =\frac{\cos(2a+c+bx)+\left(\log\left(\cos\left(\frac{1}{2}(c-bx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c-bx)\right)\right)\right)\sin^2(a+c)}{b}$$

input
```
Integrate[Csc[c - b*x]*Sin[a + b*x]^2,x]
```

output
```
(Cos[2*a + c + b*x] + (Log[Cos[(c - b*x)/2]] - Log[Sin[(c - b*x)/2]])*Sin[
a + c]^2)/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc(c-bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin^2(a+bx)\csc(c-bx)dx$$

input
```
Int[Csc[c - b*x]*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.97

method	result
risch	$\frac{\ln\left(e^{i(a+c)}+e^{i(bx+a)}\right)}{2b} - \frac{\ln\left(e^{i(a+c)}+e^{i(bx+a)}\right)\cos(2a+2c)}{2b} - \frac{\ln\left(-e^{i(a+c)}+e^{i(bx+a)}\right)}{2b} + \frac{\ln\left(-e^{i(a+c)}+e^{i(bx+a)}\right)\cos(2a+2c)}{2b} +$
default	$-\frac{\frac{2(\sin(a)\cos(c)+\cos(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2\sin(a)\sin(c)-2\cos(a)\cos(c)}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)}+\frac{8(\sin(a)\cos(c)+\cos(a)\sin(c))^2\arctan\left(\right.}{\left(4\cos(c)^2\sin(a)^2+4\cos(a)^2\cos(c)^2+4\sin(a)^2\sin(c)\right.}}{b}$

input
```
int(-csc(b*x-c)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/b*ln(exp(I*(a+c))+exp(I*(b*x+a)))-1/2/b*ln(exp(I*(a+c))+exp(I*(b*x+a))
)*cos(2*a+2*c)-1/2/b*ln(-exp(I*(a+c))+exp(I*(b*x+a)))+1/2/b*ln(-exp(I*(a+c
))+exp(I*(b*x+a)))*cos(2*a+2*c)+cos(b*x+2*a+c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(33) = 66$.

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.69

$$\int \csc(c - bx)\sin^2(a + bx)\,dx = \frac{2\cos(bx + a)\cos(a + c) - \left(\cos(a + c)^2 - 1\right)\log\left(\frac{\cos(bx+a)\cos(a+c)+\sin(bx+a)\sin(a+c)+1}{\cos(a+c)+1}\right) + \left(\cos(a + c)^2 - \right.}{2b}$$

input
```
integrate(-csc(b*x-c)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + a)*cos(a + c) - (cos(a + c)^2 - 1)*log((cos(b*x + a)*cos(
a + c) + sin(b*x + a)*sin(a + c) + 1)/(cos(a + c) + 1)) + (cos(a + c)^2 -
1)*log(-(cos(b*x + a)*cos(a + c) + sin(b*x + a)*sin(a + c) - 1)/(cos(a + c
) + 1)) - 2*sin(b*x + a)*sin(a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(27) = 54$.

Time = 10.61 (sec) , antiderivative size = 3216, normalized size of antiderivative = 100.50

$$\int \csc(c - bx)\sin^2(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(-csc(b*x-c)*sin(b*x+a)**2,x)
```

output
```
2*Piecewise((0, Eq(b, 0) & Eq(c, 0)), (-sin(b*x)/b, Eq(c, 0)), (0, Eq(b, 0
)), (2*log(-tan(c/2) + tan(b*x/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**
4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(
c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(-tan(c/2) + tan(b*x/2))*tan(c/2)**3
/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)
**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(-tan(c/2) + tan(b*x/2
))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2
*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*
log(-tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(
c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**
2 + b) - 2*log(tan(b*x/2) + 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c
/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b
*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + 1/tan(c/2))*tan(c
/2)**3/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(
b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) + 1/
tan(c/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)
**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 +
b) + 2*log(tan(b*x/2) + 1/tan(c/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2
+ b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(
b*x/2)**2 + b) + 2*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2)**2 ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(33) = 66$.

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.62

$$\int \csc(c-bx)\sin^2(a+bx)\,dx =$$
$$-\frac{(\cos(2a+2c)-1)\log\left(\cos(bx)^2+2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2+2\sin(bx)\sin(c)+\sin(c\ldots\right.}{}$$

input
```
integrate(-csc(b*x-c)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*((cos(2*a + 2*c) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 +
 sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*a + 2*c) - 1)*log(cos
(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) +
sin(c)^2) - 4*cos(b*x + 2*a + c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(33) = 66$.

Time = 0.13 (sec) , antiderivative size = 684, normalized size of antiderivative = 21.38

$$\int \csc(c - bx) \sin^2(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate(-csc(b*x-c)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-2*(2*(tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a
)^2*tan(1/2*c)^4 - 2*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2*a)^2*tan(1/2*c)^2
 - 2*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2 + 2*tan(1/2*a)*tan(1/2*c) + ta
n(1/2*c)^2)*log(abs(tan(1/2*b*x - 1/2*c)))/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*
tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4
*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^
2 + 1) - (4*tan(1/2*b*x - 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^3 + 4*tan(1/2*b*x
 - 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c)^4 - 4*tan(1/
2*b*x - 1/2*c)*tan(1/2*a)^4*tan(1/2*c) - 24*tan(1/2*b*x - 1/2*c)*tan(1/2*a
)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^4*tan(1/2*c)^2 - 24*tan(1/2*b*x - 1/2*c)*t
an(1/2*a)^2*tan(1/2*c)^3 - 16*tan(1/2*a)^3*tan(1/2*c)^3 - 4*tan(1/2*b*x -
1/2*c)*tan(1/2*a)*tan(1/2*c)^4 - 6*tan(1/2*a)^2*tan(1/2*c)^4 + 4*tan(1/2*b
*x - 1/2*c)*tan(1/2*a)^3 + tan(1/2*a)^4 + 24*tan(1/2*b*x - 1/2*c)*tan(1/2*
a)^2*tan(1/2*c) + 16*tan(1/2*a)^3*tan(1/2*c) + 24*tan(1/2*b*x - 1/2*c)*tan
(1/2*a)*tan(1/2*c)^2 + 36*tan(1/2*a)^2*tan(1/2*c)^2 + 4*tan(1/2*b*x - 1/2*
c)*tan(1/2*c)^3 + 16*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 4*tan(1/2*b*
x - 1/2*c)*tan(1/2*a) - 6*tan(1/2*a)^2 - 4*tan(1/2*b*x - 1/2*c)*tan(1/2*c)
 - 16*tan(1/2*a)*tan(1/2*c) - 6*tan(1/2*c)^2 + 1)/((tan(1/2*a)^4*tan(1/2*c
)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*
a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*...
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 223, normalized size of antiderivative = 6.97

$$\int \csc(c-bx)\sin^2(a+bx)\,dx = \frac{e^{-a\,2i-c\,1i-b\,x\,1i}}{2\,b}+\frac{e^{a\,2i+c\,1i+b\,x\,1i}}{2\,b} - \frac{e^{-a\,2i-c\,2i}\,\ln\left(-\frac{\left(e^{a\,2i}\,e^{c\,2i}-1\right)^2\,1i}{2}-\frac{e^{-c\,1i}\,e^{b\,x\,1i}\,\left(-e^{a\,2i}\,e^{c\,2i}\,2i+e^{a\,4i}\,e^{c\,4i}\,1i+1i\right)}{2}\right)\,\left(e^{a\,2i+c\,2i}-1\right)^2}{4\,b} + \frac{e^{-a\,2i-c\,2i}\,\ln\left(\frac{\left(e^{a\,2i}\,e^{c\,2i}-1\right)^2\,1i}{2}-\frac{e^{-c\,1i}\,e^{b\,x\,1i}\,\left(-e^{a\,2i}\,e^{c\,2i}\,2i+e^{a\,4i}\,e^{c\,4i}\,1i+1i\right)}{2}\right)\,\left(e^{a\,2i+c\,2i}-1\right)^2}{4\,b}$$

input
```
int(sin(a + b*x)^2/sin(c - b*x),x)
```

output
```
exp(- a*2i - c*1i - b*x*1i)/(2*b) + exp(a*2i + c*1i + b*x*1i)/(2*b) - (exp
(- a*2i - c*2i)*log(- ((exp(a*2i)*exp(c*2i) - 1)^2*1i)/2 - (exp(-c*1i)*exp
(b*x*1i)*(exp(a*4i)*exp(c*4i)*1i - exp(a*2i)*exp(c*2i)*2i + 1i))/2)*(exp(a
*2i + c*2i) - 1)^2)/(4*b) + (exp(- a*2i - c*2i)*log(((exp(a*2i)*exp(c*2i)
- 1)^2*1i)/2 - (exp(-c*1i)*exp(b*x*1i)*(exp(a*4i)*exp(c*4i)*1i - exp(a*2i)
*exp(c*2i)*2i + 1i))/2)*(exp(a*2i + c*2i) - 1)^2)/(4*b)
```

Reduce [F]

$$\int \csc(c-bx)\sin^2(a+bx)\,dx = -\left(\int \csc\left(bx-c\right)\sin\left(bx+a\right)^2 dx\right)$$

input
```
int(-csc(b*x-c)*sin(b*x+a)^2,x)
```

output
```
- int(csc(b*x - c)*sin(a + b*x)**2,x)
```

3.290 $\int \csc^2(c-bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \csc^2(c-bx)\sin^2(a+bx)\,dx = x\cos(2(a+c)) + \frac{\cot(c-bx)\sin^2(a+c)}{b} + \frac{\log(\sin(c-bx))\sin(2(a+c))}{b}$$

output
```
x*cos(2*a+2*c)-cot(b*x-c)*sin(a+c)^2/b+ln(-sin(b*x-c))*sin(2*a+2*c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 186 vs. 2(44) = 88.

Time = 0.43 (sec) , antiderivative size = 186, normalized size of antiderivative = 4.23

$$\int \csc^2(c-bx)\sin^2(a+bx)\,dx = \frac{\csc(c)\csc(c-bx)(bx\cos(2a+2c-bx) - bx\cos(2a+4c-bx) - bx\cos(2a+bx) + bx\cos(2a+2c+bx)}{}$$

input
```
Integrate[Csc[c - b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(Csc[c]*Csc[c - b*x]*(b*x*Cos[2*a + 2*c - b*x] - b*x*Cos[2*a + 4*c - b*x]
- b*x*Cos[2*a + b*x] + b*x*Cos[2*a + 2*c + b*x] + 2*Sin[b*x] + Sin[2*a + 2
*c - b*x] + Log[Sin[c - b*x]]*Sin[2*a + 2*c - b*x] - Log[Sin[c - b*x]]*Sin
[2*a + 4*c - b*x] - Log[Sin[c - b*x]]*Sin[2*a + b*x] - Sin[2*a + 2*c + b*x
] + Log[Sin[c - b*x]]*Sin[2*a + 2*c + b*x]))/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^2(c-bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin^2(a+bx)\csc^2(c-bx)dx$$

input
```
Int[Csc[c - b*x]^2*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.75

method	result
risch	$x\,\mathrm{e}^{2i(a+c)}-2i\sin(2a+2c)\,x-\frac{2i\sin(2a+2c)a}{b}-\frac{i\mathrm{e}^{4i(a+c)}}{2b(\mathrm{e}^{2i(a+c)}-\mathrm{e}^{2i(bx+a)})}+\frac{i\mathrm{e}^{2i(a+c)}}{b(\mathrm{e}^{2i(a+c)}-\mathrm{e}^{2i(bx+a)})}-\frac{}{2b(\mathrm{e}^{2i(a+c)}}$
default	$\frac{\left(2\cos(c)^{3}\sin(a)\cos(a)^{2}+2\cos(c)^{2}\sin(c)\cos(a)^{3}-4\cos(c)^{2}\sin(c)\sin(a)^{2}\cos(a)-4\cos(c)\sin(c)^{2}\sin(a)\cos(a)^{2}+2\cos(c)\sin(c)^{2}\sin(a)^{3}+2\sin(c)^{3}\right.}{\left(\cos(c)^{2}+\sin(c)^{2}\right)^{2}\left(\cos(a)^{2}+\sin(a)^{2}\right)^{2}(\cos(a)\cos(c)-\sin(}$

input
```
int(csc(b*x-c)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*exp(2*I*(a+c))-2*I*sin(2*a+2*c)*x-2*I/b*sin(2*a+2*c)*a-1/2*I/b/(exp(2*I*
(a+c))-exp(2*I*(b*x+a)))*exp(4*I*(a+c))+I/b/(exp(2*I*(a+c))-exp(2*I*(b*x+a
)))*exp(2*I*(a+c))-1/2*I/b/(exp(2*I*(a+c))-exp(2*I*(b*x+a)))+ln(-exp(2*I*(
a+c))+exp(2*I*(b*x+a)))/b*sin(2*a+2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(53) = 106$.

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 4.36

$$\int \csc^2(c-bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\left(2\,bx\cos\left(a+c\right)^{2}-bx\right)\cos\left(bx+a\right)\sin\left(a+c\right)-\left(\cos\left(a+c\right)^{3}-\cos\left(a+c\right)\right)\cos\left(bx+a\right)-2\left(\cos\left(a+\right.\right.}{}$$

input
```
integrate(csc(b*x-c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-((2*b*x*cos(a + c)^2 - b*x)*cos(b*x + a)*sin(a + c) - (cos(a + c)^3 - cos
(a + c))*cos(b*x + a) - 2*(cos(a + c)^2*sin(b*x + a)*sin(a + c) + (cos(a +
 c)^3 - cos(a + c))*cos(b*x + a))*log((cos(a + c)*sin(b*x + a) - cos(b*x +
 a)*sin(a + c))/(cos(a + c) + 1)) - (2*b*x*cos(a + c)^3 - b*x*cos(a + c) +
 (cos(a + c)^2 - 1)*sin(a + c))*sin(b*x + a))/(b*cos(a + c)*sin(b*x + a) -
 b*cos(b*x + a)*sin(a + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(c-bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x-c)**2*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(53) = 106$.

Time = 0.07 (sec) , antiderivative size = 718, normalized size of antiderivative = 16.32

$$\int \csc^2(c-bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x-c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*b*x*cos(2*b*x)*cos(2*a + 4*c) + 2*b*x*sin(2*b*x)*sin(2*a + 4*c) -
2*(b*cos(2*a + 4*c)*cos(2*c) + b*sin(2*a + 4*c)*sin(2*c))*x - (2*b*x*cos(2
*b*x) - 2*b*x*cos(2*c) - sin(4*a + 6*c) + 2*sin(2*a + 4*c) - sin(2*c))*cos
(2*b*x + 2*a + 2*c) - (cos(2*b*x + 2*a + 2*c)^2*sin(2*a + 2*c) - 2*cos(2*b
*x + 2*a + 2*c)*cos(2*a + 4*c)*sin(2*a + 2*c) + cos(2*a + 4*c)^2*sin(2*a +
 2*c) + sin(2*b*x + 2*a + 2*c)^2*sin(2*a + 2*c) - 2*sin(2*b*x + 2*a + 2*c)
*sin(2*a + 4*c)*sin(2*a + 2*c) + sin(2*a + 4*c)^2*sin(2*a + 2*c))*log(cos(
b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + s
in(c)^2) - (cos(2*b*x + 2*a + 2*c)^2*sin(2*a + 2*c) - 2*cos(2*b*x + 2*a +
2*c)*cos(2*a + 4*c)*sin(2*a + 2*c) + cos(2*a + 4*c)^2*sin(2*a + 2*c) + sin
(2*b*x + 2*a + 2*c)^2*sin(2*a + 2*c) - 2*sin(2*b*x + 2*a + 2*c)*sin(2*a +
4*c)*sin(2*a + 2*c) + sin(2*a + 4*c)^2*sin(2*a + 2*c))*log(cos(b*x)^2 - 2*
cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) -
(2*b*x*sin(2*b*x) - 2*b*x*sin(2*c) + cos(4*a + 6*c) - 2*cos(2*a + 4*c) + c
os(2*c))*sin(2*b*x + 2*a + 2*c) - cos(2*a + 4*c)*sin(4*a + 6*c) + cos(4*a
+ 6*c)*sin(2*a + 4*c) + cos(2*c)*sin(2*a + 4*c) - cos(2*a + 4*c)*sin(2*c))
/(b*cos(2*b*x + 2*a + 2*c)^2 - 2*b*cos(2*b*x + 2*a + 2*c)*cos(2*a + 4*c) +
 b*cos(2*a + 4*c)^2 + b*sin(2*b*x + 2*a + 2*c)^2 - 2*b*sin(2*b*x + 2*a + 2
*c)*sin(2*a + 4*c) + b*sin(2*a + 4*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1410 vs. $2(53) = 106$.

Time = 0.18 (sec) , antiderivative size = 1410, normalized size of antiderivative = 32.05

$$\int \csc^2(c-bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x-c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
((tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*a)^4*tan(1/2*c)^2 - 16*tan(1/2*a)^
3*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 16*tan(1/2*a
)^3*tan(1/2*c) + 36*tan(1/2*a)^2*tan(1/2*c)^2 + 16*tan(1/2*a)*tan(1/2*c)^3
 + tan(1/2*c)^4 - 6*tan(1/2*a)^2 - 16*tan(1/2*a)*tan(1/2*c) - 6*tan(1/2*c)
^2 + 1)*(b*x - c)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2
 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^
2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) + 4*(tan(1/2*a)^4*
tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) - 6*tan
(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*
c)^4 + tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*a)*tan(1/2*c)^
2 + tan(1/2*c)^3 - tan(1/2*a) - tan(1/2*c))*log(tan(1/2*b*x - 1/2*c)^2 + 1
)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^
2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4
 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 4*(tan(1/2*a)^4*tan(1/2*c)^3 + t
an(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) - 6*tan(1/2*a)^3*tan(1/
2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 + tan(1/2*a
)^3 + 6*tan(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3
 - tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x - 1/2*c)))/(tan(1/2*a)^4*t
an(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 +
tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a...
```

Mupad [B] (verification not implemented)

Time = 18.60 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.36

$$\int \csc^2(c-bx)\sin^2(a+bx)\,dx$$
$$= x\,(\cos(2\,a+2\,c)-\sin(2\,a+2\,c)\,1\mathrm{i})-\frac{\left(1+\mathrm{e}^{a\,4\mathrm{i}+c\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}\right)\,1\mathrm{i}}{2\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}$$
$$+\frac{\mathrm{e}^{-a\,4\mathrm{i}-c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}\right)\,\left(2\,b\,\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}-2\,b\,\mathrm{e}^{a\,6\mathrm{i}+c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$

input
```
int(sin(a + b*x)^2/sin(c - b*x)^2,x)
```

output
```
x*(cos(2*a + 2*c) - sin(2*a + 2*c)*1i) - ((exp(a*4i + c*4i) - 2*exp(a*2i +
 c*2i) + 1)*1i)/(2*b*(exp(a*2i + c*2i) - exp(a*2i + b*x*2i))) + (exp(- a*4
i - c*4i)*log(exp(a*2i)*exp(b*x*2i) - exp(a*2i)*exp(c*2i))*(2*b*exp(a*2i +
 c*2i) - 2*b*exp(a*6i + c*6i))*1i)/(4*b^2)
```

Reduce [F]

$$\int \csc^2(c-bx)\sin^2(a+bx)\,dx=\int \csc\,(bx-c)^2\sin\,(bx+a)^2\,dx$$

input
```
int(csc(b*x-c)^2*sin(b*x+a)^2,x)
```

output
```
int(csc(b*x - c)**2*sin(a + b*x)**2,x)
```

3.291 $\int \csc^3(c-bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \csc^3(c-bx)\sin^2(a+bx)\,dx = \frac{\operatorname{arctanh}(\cos(c-bx))\cos(2(a+c))}{b} + \frac{\operatorname{arctanh}(\cos(c-bx))\sin^2(a+c)}{2b} + \frac{\cot(c-bx)\csc(c-bx)\sin^2(a+c)}{2b} - \frac{\csc(c-bx)\sin(2(a+c))}{b}$$

output
```
arctanh(cos(b*x-c))*cos(2*a+2*c)/b+1/2*arctanh(cos(b*x-c))*sin(a+c)^2/b+1/
2*cot(b*x-c)*csc(b*x-c)*sin(a+c)^2/b+csc(b*x-c)*sin(2*a+2*c)/b
```

Mathematica [A] (verified)

Time = 5.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.91

$$\int \csc^3(c-bx)\sin^2(a+bx)\,dx$$

$$= \frac{4(1+3\cos(2(a+c)))\log\left(\cos\left(\frac{1}{2}(c-bx)\right)\right) - 4(1+3\cos(2(a+c)))\log\left(\sin\left(\frac{1}{2}(c-bx)\right)\right) + 2\csc^2\left(\frac{1}{2}(c-\right.}{}$$

input
```
Integrate[Csc[c - b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(4*(1 + 3*Cos[2*(a + c)])*Log[Cos[(c - b*x)/2]] - 4*(1 + 3*Cos[2*(a + c)])
*Log[Sin[(c - b*x)/2]] + 2*Csc[(c - b*x)/2]^2*Sin[a + c]^2 - 2*Sec[(c - b*
x)/2]^2*Sin[a + c]^2 - 8*Csc[c/2]*Csc[(c - b*x)/2]*Sin[2*(a + c)]*Sin[(b*x
)/2] + 8*Sec[c/2]*Sec[(c - b*x)/2]*Sin[2*(a + c)]*Sin[(b*x)/2])/(16*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^3(c-bx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sin^2(a+bx)\csc^3(c-bx)dx$$

input
```
Int[Csc[c - b*x]^3*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.74

method	result
risch	$-\frac{3\,\mathrm{e}^{i(bx+6a+5c)}-5\,\mathrm{e}^{3i(bx+2a+c)}+2\,\mathrm{e}^{i(bx+4a+3c)}+2\,\mathrm{e}^{i(3bx+4a+c)}-5\,\mathrm{e}^{i(bx+2a+c)}+3\,\mathrm{e}^{i(3bx+2a-c)}}{4\left(\mathrm{e}^{2i(a+c)}-\mathrm{e}^{2i(bx+a)}\right)^{2}b}+\frac{\ln\left(\mathrm{e}^{i(a+c)}+\mathrm{e}^{i(bx+a)}\right)}{4b}+$ 3l
default	Expression too large to display

input
```
int(-csc(b*x-c)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(exp(2*I*(a+c))-exp(2*I*(b*x+a)))^2/b*(3*exp(I*(b*x+6*a+5*c))-5*exp(3
*I*(b*x+2*a+c))+2*exp(I*(b*x+4*a+3*c))+2*exp(I*(3*b*x+4*a+c))-5*exp(I*(b*x
+2*a+c))+3*exp(I*(3*b*x+2*a-c)))+1/4/b*ln(exp(I*(a+c))+exp(I*(b*x+a)))+3/4
/b*ln(exp(I*(a+c))+exp(I*(b*x+a)))*cos(2*a+2*c)-1/4/b*ln(-exp(I*(a+c))+exp
(I*(b*x+a)))-3/4/b*ln(-exp(I*(a+c))+exp(I*(b*x+a)))*cos(2*a+2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(89) = 178.

Time = 0.09 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.89

$$\int \csc^3(c-bx)\sin^2(a+bx)\,dx =$$

$$-\frac{2\left(3\cos\left(a+c\right)^{2}+1\right)\sin\left(bx+a\right)\sin\left(a+c\right)+6\left(\cos\left(a+c\right)^{3}-\cos\left(a+c\right)\right)\cos\left(bx+a\right)+\left(3\cos\left(a-\right.\right.}{}$$

input
```
integrate(-csc(b*x-c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(3*cos(a + c)^2 + 1)*sin(b*x + a)*sin(a + c) + 6*(cos(a + c)^3 - c
os(a + c))*cos(b*x + a) + (3*cos(a + c)^4 - 2*(3*cos(a + c)^3 - cos(a + c)
)*cos(b*x + a)*sin(b*x + a)*sin(a + c) - (6*cos(a + c)^4 - 5*cos(a + c)^2
+ 1)*cos(b*x + a)^2 - cos(a + c)^2)*log((cos(b*x + a)*cos(a + c) + sin(b*x
 + a)*sin(a + c) + 1)/(cos(a + c) + 1)) - (3*cos(a + c)^4 - 2*(3*cos(a + c
)^3 - cos(a + c))*cos(b*x + a)*sin(b*x + a)*sin(a + c) - (6*cos(a + c)^4 -
 5*cos(a + c)^2 + 1)*cos(b*x + a)^2 - cos(a + c)^2)*log(-(cos(b*x + a)*cos
(a + c) + sin(b*x + a)*sin(a + c) - 1)/(cos(a + c) + 1)))/(2*b*cos(b*x + a
)*cos(a + c)*sin(b*x + a)*sin(a + c) + (2*b*cos(a + c)^2 - b)*cos(b*x + a)
^2 - b*cos(a + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(c-bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(-csc(b*x-c)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1571 vs. $2(89) = 178$.

Time = 0.10 (sec) , antiderivative size = 1571, normalized size of antiderivative = 18.48

$$\int \csc^3(c-bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(-csc(b*x-c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*(2*(3*cos(3*b*x) - 5*cos(3*b*x + 4*a + 4*c) + 2*cos(3*b*x + 2*a + 2*c
) + 3*cos(b*x + 4*a + 6*c) + 2*cos(b*x + 2*a + 4*c) - 5*cos(b*x + 2*c))*co
s(4*b*x + 2*a + c) + 10*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + 5*c))*cos(3*
b*x + 4*a + 4*c) - 4*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + 5*c))*cos(3*b*x
 + 2*a + 2*c) - 4*(3*cos(3*b*x) + 3*cos(b*x + 4*a + 6*c) + 2*cos(b*x + 2*a
 + 4*c) - 5*cos(b*x + 2*c))*cos(2*b*x + 2*a + 3*c) + 6*cos(3*b*x)*cos(2*a
+ 5*c) + 6*cos(b*x + 4*a + 6*c)*cos(2*a + 5*c) + 4*cos(b*x + 2*a + 4*c)*co
s(2*a + 5*c) - 10*cos(b*x + 2*c)*cos(2*a + 5*c) - ((3*cos(2*a + 2*c) + 1)*
cos(4*b*x + 2*a + c)^2 + 4*(3*cos(2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c)^2
 - 4*(3*cos(2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c)*cos(2*a + 5*c) + (3*cos
(2*a + 2*c) + 1)*cos(2*a + 5*c)^2 + (3*cos(2*a + 2*c) + 1)*sin(4*b*x + 2*a
 + c)^2 + 4*(3*cos(2*a + 2*c) + 1)*sin(2*b*x + 2*a + 3*c)^2 - 4*(3*cos(2*a
 + 2*c) + 1)*sin(2*b*x + 2*a + 3*c)*sin(2*a + 5*c) + (3*cos(2*a + 2*c) + 1
)*sin(2*a + 5*c)^2 - 2*(2*(3*cos(2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c) -
(3*cos(2*a + 2*c) + 1)*cos(2*a + 5*c))*cos(4*b*x + 2*a + c) - 2*(2*(3*cos(
2*a + 2*c) + 1)*sin(2*b*x + 2*a + 3*c) - (3*cos(2*a + 2*c) + 1)*sin(2*a +
5*c))*sin(4*b*x + 2*a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2
+ sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) + ((3*cos(2*a + 2*c) + 1)*cos
(4*b*x + 2*a + c)^2 + 4*(3*cos(2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c)^2 -
4*(3*cos(2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c)*cos(2*a + 5*c) + (3*cos...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2867 vs. $2(89) = 178$.

Time = 0.17 (sec) , antiderivative size = 2867, normalized size of antiderivative = 33.73

$$\int \csc^3(c - bx)\sin^2(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(-csc(b*x-c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*(tan(1/2*a)^4*tan(1/2*c)^4 - 4*tan(1/2*a)^4*tan(1/2*c)^2 - 12*tan(
1/2*a)^3*tan(1/2*c)^3 - 4*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 12*ta
n(1/2*a)^3*tan(1/2*c) + 28*tan(1/2*a)^2*tan(1/2*c)^2 + 12*tan(1/2*a)*tan(1
/2*c)^3 + tan(1/2*c)^4 - 4*tan(1/2*a)^2 - 12*tan(1/2*a)*tan(1/2*c) - 4*tan
(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x - 1/2*c)))/(tan(1/2*a)^4*tan(1/2*c)^4 +
 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4
+ 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*
c)^2 + 1) + (tan(1/2*b*x - 1/2*c)^2*tan(1/2*a)^8*tan(1/2*c)^6 + 2*tan(1/2*
b*x - 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c)^7 + 4*tan(1/2*b*x - 1/2*c)*tan(1/2*
a)^8*tan(1/2*c)^7 + tan(1/2*b*x - 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^8 + 4*t
an(1/2*b*x - 1/2*c)*tan(1/2*a)^7*tan(1/2*c)^8 + 2*tan(1/2*b*x - 1/2*c)^2*t
an(1/2*a)^8*tan(1/2*c)^4 + 2*tan(1/2*b*x - 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c
)^5 + 4*tan(1/2*b*x - 1/2*c)*tan(1/2*a)^8*tan(1/2*c)^5 - 16*tan(1/2*b*x -
1/2*c)*tan(1/2*a)^7*tan(1/2*c)^6 + 2*tan(1/2*b*x - 1/2*c)^2*tan(1/2*a)^5*t
an(1/2*c)^7 - 16*tan(1/2*b*x - 1/2*c)*tan(1/2*a)^6*tan(1/2*c)^7 + 2*tan(1/
2*b*x - 1/2*c)^2*tan(1/2*a)^4*tan(1/2*c)^8 + 4*tan(1/2*b*x - 1/2*c)*tan(1/
2*a)^5*tan(1/2*c)^8 + tan(1/2*b*x - 1/2*c)^2*tan(1/2*a)^8*tan(1/2*c)^2 - 2
*tan(1/2*b*x - 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c)^3 - 4*tan(1/2*b*x - 1/2*c)
*tan(1/2*a)^8*tan(1/2*c)^3 - 2*tan(1/2*b*x - 1/2*c)^2*tan(1/2*a)^6*tan(1/2
*c)^4 - 40*tan(1/2*b*x - 1/2*c)*tan(1/2*a)^7*tan(1/2*c)^4 + 2*tan(1/2*b...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c-bx)\sin^2(a+bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^2/sin(c - b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \csc^3(c-bx)\sin^2(a+bx)\,dx = -\left(\int \csc(bx-c)^3\sin(bx+a)^2\,dx\right)$$

input
```
int(-csc(b*x-c)^3*sin(b*x+a)^2,x)
```

output
```
- int(csc(b*x - c)**3*sin(a + b*x)**2,x)
```

3.292 $\int \csc^4(c-bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 79

$$\int \csc^4(c-bx)\sin^2(a+bx)\,dx=\frac{\cos(2(a+c))\cot(c-bx)}{b}+\frac{\cot(c-bx)\sin^2(a+c)}{b}+\frac{\cot^3(c-bx)\sin^2(a+c)}{3b}-\frac{\csc^2(c-bx)\sin(2(a+c))}{2b}$$

output
```
-cos(2*a+2*c)*cot(b*x-c)/b-cot(b*x-c)*sin(a+c)^2/b-1/3*cot(b*x-c)^3*sin(a+
c)^2/b-1/2*csc(b*x-c)^2*sin(2*a+2*c)/b
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \csc^4(c-bx)\sin^2(a+bx)\,dx$$
$$=\frac{\csc(c)\csc^3(c-bx)(3\sin(bx)+\sin(2c-3bx)+\sin(2a+4c-3bx)-3\sin(2a+2c-bx)+3\sin(2a+bx}{12b}$$

input
```
Integrate[Csc[c - b*x]^4*Sin[a + b*x]^2,x]
```

output
```
(Csc[c]*Csc[c - b*x]^3*(3*Sin[b*x] + Sin[2*c - 3*b*x] + Sin[2*a + 4*c - 3*
b*x] - 3*Sin[2*a + 2*c - b*x] + 3*Sin[2*a + b*x] - Sin[2*a + 3*b*x]))/(12*
b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^4(c-bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin^2(a+bx)\csc^4(c-bx)dx$$

input
```
Int[Csc[c - b*x]^4*Sin[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 3.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{\sec\left(\frac{bx}{2}-\frac{c}{2}\right)^{3}\csc\left(\frac{bx}{2}-\frac{c}{2}\right)^{3}\left(\cos(3bx-3c)-3\cos(bx-c)+2\cos(3bx+2a-c)\right)}{96b}$
risch	$\frac{2i\left(e^{8i(a+c)}-3\,e^{2i(bx+4a+3c)}+e^{6i(a+c)}+3\,e^{4i(bx+2a+c)}-3\,e^{2i(bx+3a+2c)}+e^{4i(a+c)}\right)}{3\left(e^{2i(a+c)}-e^{2i(bx+a)}\right)^{3}b}$
default	$\frac{2\sin(a)\cos(c)+2\cos(a)\sin(c)}{2(-\cos(a)\cos(c)+\sin(a)\sin(c))^{3}(-\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)+\cos(a)\sin(c)+\sin(a)\cos(c))^{2}}-\frac{}{3(-\cos(a)\cos(c)+\sin(}$

input
```
int(csc(b*x-c)^4*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/96*sec(1/2*b*x-1/2*c)^3*csc(1/2*b*x-1/2*c)^3*(cos(3*b*x-3*c)-3*cos(b*x-c
)+2*cos(3*b*x+2*a-c))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(85) = 170.

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.59

$$\int \csc^4(c-bx)\sin^2(a+bx)\,dx = \frac{(16\cos(a+c)^5-16\cos(a+c)^3+3\cos(a+c))\cos(bx+a)^3-(4\cos(a+c)^4-(16\cos(a+c)^4-8}{3((b\cos(a+c)^3-(4b\cos(a+c)^3-3b\cos(a+c))\cos(bx+a)^2)\sin}$$

input
```
integrate(csc(b*x-c)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*((16*cos(a + c)^5 - 16*cos(a + c)^3 + 3*cos(a + c))*cos(b*x + a)^3 - (
4*cos(a + c)^4 - (16*cos(a + c)^4 - 8*cos(a + c)^2 + 1)*cos(b*x + a)^2 + c
os(a + c)^2 + 1)*sin(b*x + a)*sin(a + c) - 3*(4*cos(a + c)^5 - 3*cos(a + c
)^3)*cos(b*x + a))/((b*cos(a + c)^3 - (4*b*cos(a + c)^3 - 3*b*cos(a + c))*
cos(b*x + a)^2)*sin(b*x + a) + ((4*b*cos(a + c)^2 - b)*cos(b*x + a)^3 - 3*
b*cos(b*x + a)*cos(a + c)^2)*sin(a + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(c-bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x-c)**4*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(85) = 170$.

Time = 0.05 (sec) , antiderivative size = 937, normalized size of antiderivative = 11.86

$$\int \csc^4(c-bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x-c)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
2/3*((3*sin(4*b*x + 4*a + 4*c) - 3*sin(2*b*x + 4*a + 6*c) - 3*sin(2*b*x +
2*a + 4*c) + sin(4*a + 8*c) + sin(2*a + 6*c) + sin(4*c))*cos(6*b*x + 2*a)
+ 3*(3*sin(4*b*x + 2*a + 2*c) - 3*sin(2*b*x + 2*a + 4*c) + sin(2*a + 6*c))
*cos(4*b*x + 4*a + 4*c) + 3*(3*sin(2*b*x + 4*a + 6*c) + 3*sin(2*b*x + 2*a
+ 4*c) - sin(4*a + 8*c) - sin(2*a + 6*c) - sin(4*c))*cos(4*b*x + 2*a + 2*c
) + 3*(3*sin(2*b*x + 2*a + 4*c) - sin(2*a + 6*c))*cos(2*b*x + 4*a + 6*c) +
 3*(sin(4*a + 8*c) + sin(4*c))*cos(2*b*x + 2*a + 4*c) - (3*cos(4*b*x + 4*a
 + 4*c) - 3*cos(2*b*x + 4*a + 6*c) - 3*cos(2*b*x + 2*a + 4*c) + cos(4*a +
8*c) + cos(2*a + 6*c) + cos(4*c))*sin(6*b*x + 2*a) - 3*(3*cos(4*b*x + 2*a
+ 2*c) - 3*cos(2*b*x + 2*a + 4*c) + cos(2*a + 6*c))*sin(4*b*x + 4*a + 4*c)
 - 3*(3*cos(2*b*x + 4*a + 6*c) + 3*cos(2*b*x + 2*a + 4*c) - cos(4*a + 8*c)
 - cos(2*a + 6*c) - cos(4*c))*sin(4*b*x + 2*a + 2*c) - 3*(3*cos(2*b*x + 2*
a + 4*c) - cos(2*a + 6*c))*sin(2*b*x + 4*a + 6*c) - 3*(cos(4*a + 8*c) + co
s(4*c))*sin(2*b*x + 2*a + 4*c) - cos(2*a + 6*c)*sin(4*a + 8*c) + cos(4*a +
 8*c)*sin(2*a + 6*c) + cos(4*c)*sin(2*a + 6*c) - cos(2*a + 6*c)*sin(4*c))/
(b*cos(6*b*x + 2*a)^2 + 9*b*cos(4*b*x + 2*a + 2*c)^2 + 9*b*cos(2*b*x + 2*a
 + 4*c)^2 - 6*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 6*c) + b*cos(2*a + 6*c)^2
 + b*sin(6*b*x + 2*a)^2 + 9*b*sin(4*b*x + 2*a + 2*c)^2 + 9*b*sin(2*b*x + 2
*a + 4*c)^2 - 6*b*sin(2*b*x + 2*a + 4*c)*sin(2*a + 6*c) + b*sin(2*a + 6*c)
^2 - 2*(3*b*cos(4*b*x + 2*a + 2*c) - 3*b*cos(2*b*x + 2*a + 4*c) + b*cos...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(85) = 170$.

Time = 0.15 (sec) , antiderivative size = 705, normalized size of antiderivative = 8.92

$$\int \csc^4(c - bx)\sin^2(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x-c)^4*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/3*(3*tan(b*x - c)^2*tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(b*x - c)^2*tan(1/
2*a)^4*tan(1/2*c)^2 - 24*tan(b*x - c)^2*tan(1/2*a)^3*tan(1/2*c)^3 - 6*tan(
b*x - c)*tan(1/2*a)^4*tan(1/2*c)^3 - 6*tan(b*x - c)^2*tan(1/2*a)^2*tan(1/2
*c)^4 - 6*tan(b*x - c)*tan(1/2*a)^3*tan(1/2*c)^4 + 3*tan(b*x - c)^2*tan(1/
2*a)^4 + 24*tan(b*x - c)^2*tan(1/2*a)^3*tan(1/2*c) + 6*tan(b*x - c)*tan(1/
2*a)^4*tan(1/2*c) + 60*tan(b*x - c)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 36*tan(b
*x - c)*tan(1/2*a)^3*tan(1/2*c)^2 + 4*tan(1/2*a)^4*tan(1/2*c)^2 + 24*tan(b
*x - c)^2*tan(1/2*a)*tan(1/2*c)^3 + 36*tan(b*x - c)*tan(1/2*a)^2*tan(1/2*c
)^3 + 8*tan(1/2*a)^3*tan(1/2*c)^3 + 3*tan(b*x - c)^2*tan(1/2*c)^4 + 6*tan(
b*x - c)*tan(1/2*a)*tan(1/2*c)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^4 - 6*tan(b*x
 - c)^2*tan(1/2*a)^2 - 6*tan(b*x - c)*tan(1/2*a)^3 - 24*tan(b*x - c)^2*tan
(1/2*a)*tan(1/2*c) - 36*tan(b*x - c)*tan(1/2*a)^2*tan(1/2*c) - 8*tan(1/2*a
)^3*tan(1/2*c) - 6*tan(b*x - c)^2*tan(1/2*c)^2 - 36*tan(b*x - c)*tan(1/2*a
)*tan(1/2*c)^2 - 16*tan(1/2*a)^2*tan(1/2*c)^2 - 6*tan(b*x - c)*tan(1/2*c)^
3 - 8*tan(1/2*a)*tan(1/2*c)^3 + 3*tan(b*x - c)^2 + 6*tan(b*x - c)*tan(1/2*
a) + 4*tan(1/2*a)^2 + 6*tan(b*x - c)*tan(1/2*c) + 8*tan(1/2*a)*tan(1/2*c)
+ 4*tan(1/2*c)^2)/((tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^
2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1)*b*tan(b*x - c)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(c - bx)\sin^2(a + bx)\,dx = \text{Hanged}$$

input
```
int(sin(a + b*x)^2/sin(c - b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \csc^4(c-bx)\sin^2(a+bx)\,dx$$
$$= \frac{-\cos(bx-c)\sin(bx-c)^2-\cos(bx-c)\sin(bx+a)^2-\cos(bx+a)\sin(bx-c)\sin(bx+a)}{3\sin(bx-c)^3 b}$$

input
```
int(csc(b*x-c)^4*sin(b*x+a)^2,x)
```

output
```
( - (cos(b*x - c)*sin(b*x - c)**2 + cos(b*x - c)*sin(a + b*x)**2 + cos(a +
 b*x)*sin(b*x - c)*sin(a + b*x)))/(3*sin(b*x - c)**3*b)
```

3.293 $\int \cos(a+bx)\cos(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{1}{2}x\cos(a-c)+\frac{\sin(a+c+2bx)}{4b}$$

output
```
1/2*x*cos(a-c)+1/4*sin(2*b*x+a+c)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{2bx\cos(a-c)+\sin(a+c+2bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]*Cos[c + b*x],x]
```

output
```
(2*b*x*Cos[a - c] + Sin[a + c + 2*b*x])/(4*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos(bx+c)\,dx$$

$$\Big\downarrow \text{ 5081}$$

$$\int \left(\frac{1}{2}\cos(a+2bx+c)+\frac{1}{2}\cos(a-c)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sin(a+2bx+c)}{4b}+\frac{1}{2}x\cos(a-c)$$

input
```
Int[Cos[a + b*x]*Cos[c + b*x],x]
```

output
```
(x*Cos[a - c])/2 + Sin[a + c + 2*b*x]/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cos(a-c)}{2}+\frac{\sin(2bx+a+c)}{4b}$
risch	$\frac{x\cos(a-c)}{2}+\frac{\sin(2bx+a+c)}{4b}$
parallelrisch	$\frac{2x\cos(a-c)b-\sin(a-c)+\sin(2bx+a+c)}{4b}$
orering	$x\cos(bx+a)\cos(bx+c)-\frac{-\sin(bx+a)b\cos(bx+c)-b\cos(bx+a)\sin(bx+c)}{4b^2}+\frac{x(-2\cos(bx+a)\cos(bx+c)b^2+2}{4b^2}$
norman	$\frac{\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}+\frac{x}{2}-\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{2}-\frac{x\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2}{2}+2x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)+\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2}{2}-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan}{b}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(1+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(cos(b*x+a)*cos(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*cos(a-c)+1/4*sin(2*b*x+a+c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(23) = 46$.

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \cos(a+bx)\cos(c+bx)\,dx$$
$$=\frac{bx\cos(-a+c)+\cos(bx+c)\cos(-a+c)\sin(bx+c)-\cos(bx+c)^2\sin(-a+c)}{2b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="fricas")
```

output
```
1/2*(b*x*cos(-a + c) + cos(b*x + c)*cos(-a + c)*sin(b*x + c) - cos(b*x + c
)^2*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cos(a+bx)\cos(c+bx)\,dx = \begin{cases} \frac{x\sin(a+bx)\sin(bx+c)}{2} + \frac{x\cos(a+bx)\cos(bx+c)}{2} + \frac{\sin(a+bx)\cos(bx+c)}{2b} & \text{for } b \neq 0 \\ x\cos(a)\cos(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c),x)
```

output
```
Piecewise((x*sin(a + b*x)*sin(b*x + c)/2 + x*cos(a + b*x)*cos(b*x + c)/2 +
 sin(a + b*x)*cos(b*x + c)/(2*b), Ne(b, 0)), (x*cos(a)*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{1}{2}\,x\cos(-a+c) + \frac{\sin(2\,bx+a+c)}{4\,b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="maxima")
```

output
```
1/2*x*cos(-a + c) + 1/4*sin(2*b*x + a + c)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{1}{2}\,x\cos\left(a-c\right) + \frac{\sin\left(2\,bx+a+c\right)}{4\,b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="giac")
```

output
```
1/2*x*cos(a - c) + 1/4*sin(2*b*x + a + c)/b
```

Mupad [B] (verification not implemented)

Time = 17.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \cos(a+bx)\cos(c+bx)\,dx = \begin{cases} x\,\cos\left(a\right)\,\cos\left(c\right) & \text{if } b=0 \\ \frac{x\,\cos(a-c)}{2}+\frac{\sin(a+c+2\,b\,x)}{4\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int(cos(a + b*x)*cos(c + b*x),x)
```

output
```
piecewise(b == 0, x*cos(a)*cos(c), b ~= 0, (x*cos(a - c))/2 + sin(a + c +
2*b*x)/(4*b))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{\cos\left(bx+c\right)\cos\left(bx+a\right)bx+\cos\left(bx+c\right)\sin\left(bx+a\right)+\sin\left(bx+c\right)\sin\left(bx+a\right)bx}{2b}$$

input
```
int(cos(b*x+a)*cos(b*x+c),x)
```

output
```
(cos(b*x + c)*cos(a + b*x)*b*x + cos(b*x + c)*sin(a + b*x) + sin(b*x + c)*
sin(a + b*x)*b*x)/(2*b)
```

3.294 $\int \cos(c-bx)\cos(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \cos(c-bx)\cos(a+bx)\,dx = \frac{1}{2}x\cos(a+c) + \frac{\sin(a-c+2bx)}{4b}$$

output
```
1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \cos(c-bx)\cos(a+bx)\,dx = \frac{2bx\cos(a+c)+\sin(a-c+2bx)}{4b}$$

input
```
Integrate[Cos[c - b*x]*Cos[a + b*x],x]
```

output
```
(2*b*x*Cos[a + c] + Sin[a - c + 2*b*x])/(4*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos(c-bx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{2}\cos(a+2bx-c)+\frac{1}{2}\cos(a+c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a+2bx-c)}{4b}+\frac{1}{2}x\cos(a+c)$$

input
```
Int[Cos[c - b*x]*Cos[a + b*x],x]
```

output
```
(x*Cos[a + c])/2 + Sin[a - c + 2*b*x]/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cos(a+c)}{2}+\frac{\sin(2bx+a-c)}{4b}$
risch	$\frac{x\cos(a+c)}{2}+\frac{\sin(2bx+a-c)}{4b}$
parallelrisch	$\frac{2x\cos(a+c)b+\sin(2bx+a-c)-\sin(a+c)}{4b}$
orering	$x\cos\left(bx-c\right)\cos\left(bx+a\right)-\frac{-b\cos(bx-c)\sin(bx+a)-\sin(bx-c)b\cos(bx+a)}{4b^2}+\frac{x(2\sin(bx+a)\sin(bx-c)b^2-2\cos$
norman	$\frac{\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}+\frac{x}{2}-\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{2}-\frac{x\tan\left(\frac{bx}{2}-\frac{c}{2}\right)^2}{2}+2x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}-\frac{c}{2}\right)+\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan\left(\frac{bx}{2}-\frac{c}{2}\right)^2}{2}-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan}{b}}{\left(1+\tan\left(\frac{bx}{2}-\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)}$

input
```
int(cos(b*x-c)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int\cos(c-bx)\cos(a+bx)\,dx$$
$$=\frac{bx\cos\left(a+c\right)+\cos\left(bx+a\right)\cos\left(a+c\right)\sin\left(bx+a\right)-\cos\left(bx+a\right)^2\sin\left(a+c\right)}{2\,b}$$

input
```
integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b*x*cos(a + c) + cos(b*x + a)*cos(a + c)*sin(b*x + a) - cos(b*x + a)^
2*sin(a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cos(c-bx)\cos(a+bx)\,dx = \begin{cases} \frac{x\sin(a+bx)\sin(bx-c)}{2} + \frac{x\cos(a+bx)\cos(bx-c)}{2} + \frac{\sin(bx-c)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\cos(a)\cos(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x-c)*cos(b*x+a),x)
```

output
```
Piecewise((x*sin(a + b*x)*sin(b*x - c)/2 + x*cos(a + b*x)*cos(b*x - c)/2 +
 sin(b*x - c)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*cos(a)*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cos(c-bx)\cos(a+bx)\,dx = \frac{1}{2}x\cos(a+c) + \frac{\sin(2bx+a-c)}{4b}$$

input
```
integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cos(c-bx)\cos(a+bx)\,dx = \frac{1}{2}x\cos(a+c) + \frac{\sin(2bx+a-c)}{4b}$$

input
```
integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="giac")
```

output
```
1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

Mupad [B] (verification not implemented)

Time = 16.82 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \cos(c-bx)\cos(a+bx)\,dx = \begin{cases} x\cos(a)\cos(c) & \text{if } b=0 \\ \frac{\sin(a-c+2bx)}{4b} + \frac{x\cos(a+c)}{2} & \text{if } b\neq 0 \end{cases}$$

input
```
int(cos(a + b*x)*cos(c - b*x),x)
```

output
```
piecewise(b == 0, x*cos(a)*cos(c), b ~= 0, sin(a - c + 2*b*x)/(4*b) + (x*c
os(a + c))/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \cos(c-bx)\cos(a+bx)\,dx = \frac{\cos(bx-c)\cos(bx+a)bx + \cos(bx-c)\sin(bx+a) + \sin(bx-c)\sin(bx+a)bx}{2b}$$

input
```
int(cos(b*x-c)*cos(b*x+a),x)
```

output
```
(cos(b*x - c)*cos(a + b*x)*b*x + cos(b*x - c)*sin(a + b*x) + sin(b*x - c)*
sin(a + b*x)*b*x)/(2*b)
```

3.295 $\int \cos(a+bx)\cos^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \cos(a+bx)\cos^3(c+bx)\,dx = \frac{3}{8}x\cos(a-c) - \frac{\sin(a-3c-2bx)}{16b} + \frac{3\sin(a+c+2bx)}{16b} + \frac{\sin(a+3c+4bx)}{32b}$$

output
```
3/8*x*cos(a-c)-1/16*sin(-2*b*x+a-3*c)/b+3/16*sin(2*b*x+a+c)/b+1/32*sin(4*b
*x+a+3*c)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \cos(a+bx)\cos^3(c+bx)\,dx = \frac{12bx\cos(a-c) - 2\sin(a-3c-2bx) + 6\sin(a+c+2bx) + \sin(a+3c+4bx)}{32b}$$

input
```
Integrate[Cos[a + b*x]*Cos[c + b*x]^3,x]
```

output
```
(12*b*x*Cos[a - c] - 2*Sin[a - 3*c - 2*b*x] + 6*Sin[a + c + 2*b*x] + Sin[a
 + 3*c + 4*b*x])/(32*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos^3(bx+c)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{8}\cos(a-2bx-3c)+\frac{3}{8}\cos(a+2bx+c)+\frac{1}{8}\cos(a+4bx+3c)+\frac{3}{8}\cos(a-c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(a-2bx-3c)}{16b}+\frac{3\sin(a+2bx+c)}{16b}+\frac{\sin(a+4bx+3c)}{32b}+\frac{3}{8}x\cos(a-c)$$

input
```
Int[Cos[a + b*x]*Cos[c + b*x]^3,x]
```

output
```
(3*x*Cos[a - c])/8 - Sin[a - 3*c - 2*b*x]/(16*b) + (3*Sin[a + c + 2*b*x])/
(16*b) + Sin[a + 3*c + 4*b*x]/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result
default	$\frac{3x\cos(a-c)}{8}-\frac{\sin(-2bx+a-3c)}{16b}+\frac{3\sin(2bx+a+c)}{16b}+\frac{\sin(4bx+a+3c)}{32b}$
risch	$\frac{3x\cos(a-c)}{8}-\frac{\sin(-2bx+a-3c)}{16b}+\frac{3\sin(2bx+a+c)}{16b}+\frac{\sin(4bx+a+3c)}{32b}$
parallelrisch	$\frac{12x\cos(a-c)b-2\sin(-2bx+a-3c)+6\sin(2bx+a+c)+\sin(4bx+a+3c)-21\sin(a-c)}{32b}$
orering	$x\cos(bx+a)\cos(bx+c)^{3}-\frac{5\left(-b\sin(bx+a)\cos(bx+c)^{3}-3\cos(bx+a)\cos(bx+c)^{2}b\sin(bx+c)\right)}{16b^{2}}+\frac{5x\left(-4\cos(b\right.}{}$

input
```
int(cos(b*x+a)*cos(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
3/8*x*cos(a-c)-1/16*sin(-2*b*x+a-3*c)/b+3/16*sin(2*b*x+a+c)/b+1/32*sin(4*b
*x+a+3*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int \cos(a+bx)\cos^3(c+bx)\,dx = -\frac{2\cos(bx+c)^4\sin(-a+c)-3\,bx\cos(-a+c)-\left(2\cos(bx+c)^3\cos(-a+c)+3\cos(bx+c)\cos(-\right.}{8\,b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*cos(b*x + c)^4*sin(-a + c) - 3*b*x*cos(-a + c) - (2*cos(b*x + c)^3
*cos(-a + c) + 3*cos(b*x + c)*cos(-a + c))*sin(b*x + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(54) = 108$.

Time = 0.83 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.82

$$\int \cos(a+bx)\cos^3(c+bx)\,dx$$
$$= \begin{cases} \frac{3x\sin{(a+bx)}\sin^3{(bx+c)}}{8} + \frac{3x\sin{(a+bx)}\sin{(bx+c)}\cos^2{(bx+c)}}{8} + \frac{3x\sin^2{(bx+c)}\cos{(a+bx)}\cos{(bx+c)}}{8} + \frac{3x\cos{(a+bx)}\cos^3{(bx+c)}}{8} \\ x\cos{(a)}\cos^3{(c)} \end{cases}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c)**3,x)
```

output
```
Piecewise((3*x*sin(a + b*x)*sin(b*x + c)**3/8 + 3*x*sin(a + b*x)*sin(b*x +
 c)*cos(b*x + c)**2/8 + 3*x*sin(b*x + c)**2*cos(a + b*x)*cos(b*x + c)/8 +
3*x*cos(a + b*x)*cos(b*x + c)**3/8 + 3*sin(a + b*x)*sin(b*x + c)**2*cos(b*
x + c)/(4*b) + 5*sin(a + b*x)*cos(b*x + c)**3/(8*b) - 3*sin(b*x + c)**3*co
s(a + b*x)/(8*b), Ne(b, 0)), (x*cos(a)*cos(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \cos(a+bx)\cos^3(c+bx)\,dx$$
$$= \frac{12\,bx\cos{(-a+c)} + \sin{(4\,bx+a+3\,c)} + 6\,\sin{(2\,bx+a+c)} + 2\,\sin{(2\,bx-a+3\,c)}}{32\,b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c)^3,x, algorithm="maxima")
```

output
```
1/32*(12*b*x*cos(-a + c) + sin(4*b*x + a + 3*c) + 6*sin(2*b*x + a + c) + 2
*sin(2*b*x - a + 3*c))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\cos^3(c+bx)\,dx = \frac{3}{8}\,x\cos{(a-c)} + \frac{\sin{(4\,bx+a+3\,c)}}{32\,b} + \frac{3\,\sin{(2\,bx+a+c)}}{16\,b} - \frac{\sin{(-2\,bx+a-3\,c)}}{16\,b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c)^3,x, algorithm="giac")
```

output
```
3/8*x*cos(a - c) + 1/32*sin(4*b*x + a + 3*c)/b + 3/16*sin(2*b*x + a + c)/b
 - 1/16*sin(-2*b*x + a - 3*c)/b
```

Mupad [B] (verification not implemented)

Time = 18.86 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \cos(a+bx)\cos^3(c+bx)\,dx = \frac{\frac{\sin(a+3\,c+4\,b\,x)}{32} + \frac{\sin(3\,c-a+2\,b\,x)}{16} + \frac{3\,\sin(a+c+2\,b\,x)}{16}}{b} + \frac{3\,x\,\cos{(a-c)}}{8}$$

input
```
int(cos(a + b*x)*cos(c + b*x)^3,x)
```

output
```
(sin(a + 3*c + 4*b*x)/32 + sin(3*c - a + 2*b*x)/16 + (3*sin(a + c + 2*b*x)
)/16)/b + (3*x*cos(a - c))/8
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.69

$$\int \cos(a+bx)\cos^3(c+bx)\,dx$$
$$= \frac{3\cos(bx+c)\cos(bx+a)\,bx+\cos(bx+c)\sin(bx+c)^2\sin(bx+a)-4\cos(bx+c)\sin(bx+a)-3\cos}{8b}$$

input
```
int(cos(b*x+a)*cos(b*x+c)^3,x)
```

output
```
(3*cos(b*x + c)*cos(a + b*x)*b*x + cos(b*x + c)*sin(b*x + c)**2*sin(a + b*
x) - 4*cos(b*x + c)*sin(a + b*x) - 3*cos(a + b*x)*sin(b*x + c)**3 + 9*cos(
a + b*x)*sin(b*x + c) + 3*sin(b*x + c)*sin(a + b*x)*b*x)/(8*b)
```

3.296 $\int \cos(a+bx)\cos^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \cos(a+bx)\cos^2(c+bx)\,dx=-\frac{\sin(a-2c-bx)}{4b}+\frac{\sin(a+bx)}{2b}+\frac{\sin(a+2c+3bx)}{12b}$$

output
```
-1/4*sin(-b*x+a-2*c)/b+1/2*sin(b*x+a)/b+1/12*sin(3*b*x+a+2*c)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \cos(a+bx)\cos^2(c+bx)\,dx=\frac{-3\sin(a-2c-bx)+6\sin(a+bx)+\sin(a+2c+3bx)}{12b}$$

input
```
Integrate[Cos[a + b*x]*Cos[c + b*x]^2,x]
```

output
```
(-3*Sin[a - 2*c - b*x] + 6*Sin[a + b*x] + Sin[a + 2*c + 3*b*x])/(12*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos^2(bx+c)\,dx$$

$$\Big\downarrow \text{ 5081}$$

$$\int \left(\frac{1}{4}\cos(a-bx-2c)+\frac{1}{4}\cos(a+3bx+2c)+\frac{1}{2}\cos(a+bx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sin(a-bx-2c)}{4b}+\frac{\sin(a+3bx+2c)}{12b}+\frac{\sin(a+bx)}{2b}$$

input
```
Int[Cos[a + b*x]*Cos[c + b*x]^2,x]
```

output
```
-1/4*Sin[a - 2*c - b*x]/b + Sin[a + b*x]/(2*b) + Sin[a + 2*c + 3*b*x]/(12*
b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\sin(-bx+a-2c)}{4b}+\frac{\sin(bx+a)}{2b}+\frac{\sin(3bx+a+2c)}{12b}$
risch	$-\frac{\sin(-bx+a-2c)}{4b}+\frac{\sin(bx+a)}{2b}+\frac{\sin(3bx+a+2c)}{12b}$
orering	$-\frac{10\left(-b\sin(bx+a)\cos(bx+c)^2-2\cos(bx+a)\cos(bx+c)b\sin(bx+c)\right)}{9b^2}-\frac{7b^3\sin(bx+a)\cos(bx+c)^2+14b^3\cos(bx+a)\cos(bx-}{9b^4}$
norman	$\frac{\frac{4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2}{b}+\frac{8\tan\left(\frac{bx}{2}+\frac{c}{2}\right)}{3b}-\frac{2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{3b}+\frac{2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^4}{b}-\frac{8\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan\left(\frac{bx}{2}+\frac{c}{2}\right)}{3b}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(1+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\right)^2}$
parallelrisch	$\frac{2\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-\frac{2\tan\left(\frac{bx}{2}+\frac{c}{2}\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{3}+4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2-2\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^3+\frac{4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{3}+\frac{2\text{t}}{}}{b\left(1+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)}$

input
```
int(cos(b*x+a)*cos(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*sin(-b*x+a-2*c)/b+1/2*sin(b*x+a)/b+1/12*sin(3*b*x+a+2*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int\cos(a+bx)\cos^2(c+bx)\,dx = -\frac{\cos(bx+c)^3\sin(-a+c)-\left(\cos(bx+c)^2\cos(-a+c)+2\cos(-a+c)\right)\sin(bx+c)}{3\,b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c)^2,x, algorithm="fricas")
```

output
```
-1/3*(cos(b*x + c)^3*sin(-a + c) - (cos(b*x + c)^2*cos(-a + c) + 2*cos(-a
+ c))*sin(b*x + c))/b
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \cos(a+bx)\cos^2(c+bx)\,dx = \begin{cases} \frac{2\sin(a+bx)\sin^2(bx+c)}{3b} + \frac{\sin(a+bx)\cos^2(bx+c)}{3b} + \frac{2\sin(bx+c)\cos(a+bx)\cos(bx+c)}{3b} & \text{for } b \neq 0 \\ x\cos(a)\cos^2(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c)**2,x)
```

output
```
Piecewise((2*sin(a + b*x)*sin(b*x + c)**2/(3*b) + sin(a + b*x)*cos(b*x + c
)**2/(3*b) + 2*sin(b*x + c)*cos(a + b*x)*cos(b*x + c)/(3*b), Ne(b, 0)), (x
*cos(a)*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \cos(a+bx)\cos^2(c+bx)\,dx = \frac{\sin(3bx+a+2c)+6\sin(bx+a)+3\sin(bx-a+2c)}{12b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/12*(sin(3*b*x + a + 2*c) + 6*sin(b*x + a) + 3*sin(b*x - a + 2*c))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \cos(a+bx)\cos^2(c+bx)\,dx = \frac{\sin(3bx+a+2c)}{12b} + \frac{\sin(bx+a)}{2b} - \frac{\sin(-bx+a-2c)}{4b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c)^2,x, algorithm="giac")
```

output
```
1/12*sin(3*b*x + a + 2*c)/b + 1/2*sin(b*x + a)/b - 1/4*sin(-b*x + a - 2*c)
/b
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \cos(a+bx)\cos^2(c+bx)\,dx = \frac{\sin\left(a+2\,c+3\,b\,x\right)+6\,\sin\left(a+b\,x\right)+3\,\sin\left(2\,c-a+b\,x\right)}{12\,b}$$

input
```
int(cos(a + b*x)*cos(c + b*x)^2,x)
```

output
```
(sin(a + 2*c + 3*b*x) + 6*sin(a + b*x) + 3*sin(2*c - a + b*x))/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

$$\int \cos(a+bx)\cos^2(c+bx)\,dx = \frac{2\cos(bx+c)\cos(bx+a)\sin(bx+c)+\cos(bx+c)\sin(bx+a)-\cos(bx+a)\sin(bx+c)+\sin(bx+c}{3b}$$

input
```
int(cos(b*x+a)*cos(b*x+c)^2,x)
```

output
```
(2*cos(b*x + c)*cos(a + b*x)*sin(b*x + c) + cos(b*x + c)*sin(a + b*x) - co
s(a + b*x)*sin(b*x + c) + sin(b*x + c)**2*sin(a + b*x) + sin(a + b*x))/(3*
b)
```

3.297 $\int \cos(a+bx)\cos(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{1}{2}x\cos(a-c) + \frac{\sin(a+c+2bx)}{4b}$$

output
```
1/2*x*cos(a-c)+1/4*sin(2*b*x+a+c)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{2bx\cos(a-c)+\sin(a+c+2bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]*Cos[c + b*x],x]
```

output
```
(2*b*x*Cos[a - c] + Sin[a + c + 2*b*x])/(4*b)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos(bx+c)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{2}\cos(a+2bx+c)+\frac{1}{2}\cos(a-c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a+2bx+c)}{4b}+\frac{1}{2}x\cos(a-c)$$

input
```
Int[Cos[a + b*x]*Cos[c + b*x],x]
```

output
```
(x*Cos[a - c])/2 + Sin[a + c + 2*b*x]/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cos(a-c)}{2}+\frac{\sin(2bx+a+c)}{4b}$
risch	$\frac{x\cos(a-c)}{2}+\frac{\sin(2bx+a+c)}{4b}$
parallelrisch	$\frac{2x\cos(a-c)b-\sin(a-c)+\sin(2bx+a+c)}{4b}$
orering	$x\cos\left(bx+a\right)\cos\left(bx+c\right)-\frac{-\sin(bx+a)b\cos(bx+c)-b\cos(bx+a)\sin(bx+c)}{4b^2}+\frac{x(-2\cos(bx+a)\cos(bx+c)b^2+2}{4b^2}$
norman	$\frac{\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}+\frac{x}{2}-\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{2}-\frac{x\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2}{2}+2x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)+\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2}{2}-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\text{ta}}{b}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(1+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(cos(b*x+a)*cos(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*cos(a-c)+1/4*sin(2*b*x+a+c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(23) = 46$.

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int\cos(a+bx)\cos(c+bx)\,dx$$
$$=\frac{bx\cos\left(-a+c\right)+\cos\left(bx+c\right)\cos\left(-a+c\right)\sin\left(bx+c\right)-\cos\left(bx+c\right)^2\sin\left(-a+c\right)}{2\,b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="fricas")
```

output
```
1/2*(b*x*cos(-a + c) + cos(b*x + c)*cos(-a + c)*sin(b*x + c) - cos(b*x + c
)^2*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cos(a+bx)\cos(c+bx)\,dx = \begin{cases} \frac{x\sin(a+bx)\sin(bx+c)}{2} + \frac{x\cos(a+bx)\cos(bx+c)}{2} + \frac{\sin(a+bx)\cos(bx+c)}{2b} & \text{for } b \neq 0 \\ x\cos(a)\cos(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c),x)
```

output
```
Piecewise((x*sin(a + b*x)*sin(b*x + c)/2 + x*cos(a + b*x)*cos(b*x + c)/2 +
 sin(a + b*x)*cos(b*x + c)/(2*b), Ne(b, 0)), (x*cos(a)*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{1}{2}\,x\cos(-a+c) + \frac{\sin(2\,bx+a+c)}{4\,b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="maxima")
```

output
```
1/2*x*cos(-a + c) + 1/4*sin(2*b*x + a + c)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{1}{2}x\cos(a-c) + \frac{\sin(2bx+a+c)}{4b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="giac")
```

output
```
1/2*x*cos(a - c) + 1/4*sin(2*b*x + a + c)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \cos(a+bx)\cos(c+bx)\,dx = \begin{cases} x\cos(a)\cos(c) & \text{if } b=0 \\ \frac{x\cos(a-c)}{2} + \frac{\sin(a+c+2bx)}{4b} & \text{if } b\neq 0 \end{cases}$$

input
```
int(cos(a + b*x)*cos(c + b*x),x)
```

output
```
piecewise(b == 0, x*cos(a)*cos(c), b ~= 0, (x*cos(a - c))/2 + sin(a + c +
2*b*x)/(4*b))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{\cos(bx+c)\cos(bx+a)bx + \cos(bx+c)\sin(bx+a) + \sin(bx+c)\sin(bx+a)bx}{2b}$$

input
```
int(cos(b*x+a)*cos(b*x+c),x)
```

output
```
(cos(b*x + c)*cos(a + b*x)*b*x + cos(b*x + c)*sin(a + b*x) + sin(b*x + c)*
sin(a + b*x)*b*x)/(2*b)
```

3.298 $\int \cos(a+bx)\sec(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \cos(a+bx)\sec(c+bx)\,dx = x\cos(a-c) + \frac{\log(\cos(c+bx))\sin(a-c)}{b}$$

output
```
x*cos(a-c)+ln(cos(b*x+c))*sin(a-c)/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sec(c+bx)\,dx = x\cos(a-c) + \frac{\log(\cos(c+bx))\sin(a-c)}{b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + b*x],x]
```

output
```
x*Cos[a - c] + (Log[Cos[c + b*x]]*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5094, 24, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \text{5094}$$

$$\cos(a-c)\int 1dx-\sin(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{24}$$

$$x\cos(a-c)-\sin(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$x\cos(a-c)-\sin(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\sin(a-c)\log(\cos(bx+c))}{b}+x\cos(a-c)$$

input
```
Int[Cos[a + b*x]*Sec[c + b*x],x]
```

output
```
x*Cos[a - c] + (Log[Cos[c + b*x]]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5094
```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Simp[-Sin[v - w]   Int[Tan[w]*Sec[w
]^(n - 1), x], x] + Simp[Cos[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n,
0] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

method	result
risch	$-2i \sin \left(a-c\right) x+x \,{\mathrm e}^{i \left(a-c\right)}-\frac{2 i \sin \left(a-c\right) a}{b}+\frac{\ln \left({\mathrm e}^{2 i \left(b x+a\right)}+{\mathrm e}^{2 i \left(a-c\right)}\right) \sin \left(a-c\right)}{b}$
default	$\frac{\frac{\left(\sin \left(a\right) \cos \left(c\right)-\cos \left(a\right) \sin \left(c\right)\right) \ln \left(\tan \left(b x+a\right) \sin \left(a\right) \cos \left(c\right)-\tan \left(b x+a\right) \cos \left(a\right) \sin \left(c\right)+\cos \left(a\right) \cos \left(c\right)+\sin \left(a\right) \sin \left(c\right)\right)}{\cos \left(a\right)^{2} \cos \left(c\right)^{2}+\sin \left(c\right)^{2} \cos \left(a\right)^{2}+\cos \left(c\right)^{2} \sin \left(a\right)^{2}+\sin \left(a\right)^{2} \sin \left(c\right)^{2}}+\frac{\frac{\left(-\sin \left(a\right) \cos \left(c\right)+\cos \left(a\right) \sin \left(c\right)\right) \ln \left(\mathrm{t}\right.}{2}}{\left(\mathrm{cos}\right.}}{b}$

input
```
int(cos(b*x+a)*sec(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2*I*sin(a-c)*x+x*exp(I*(a-c))-2*I/b*sin(a-c)*a+ln(exp(2*I*(b*x+a))+exp(2*
I*(a-c)))/b*sin(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \cos(a+bx)\sec(c+bx)\,dx = \frac{bx\cos(-a+c) - \log(-\cos(bx+c))\sin(-a+c)}{b}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="fricas")
```

output
```
(b*x*cos(-a + c) - log(-cos(b*x + c))*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(20) = 40.

Time = 4.63 (sec) , antiderivative size = 435, normalized size of antiderivative = 16.73

$$\int \cos(a+bx)\sec(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c),x)
```

output
```
-Piecewise((-x, Eq(c, pi/2)), (x, Eq(c, -pi/2)), (0, Eq(b, 0)), (-2*b*x*ta
n(c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2
)**2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) -
tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b
) - log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/
2)**2 + b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*
tan(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1)
 - 1/(tan(c/2) + 1))/(b*tan(c/2)**2 + b), True))*sin(a) + Piecewise((-log(
sin(b*x))/b, Eq(c, pi/2)), (log(sin(b*x))/b, Eq(c, -pi/2)), (x/cos(c), Eq(
b, 0)), (-b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) +
2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) -
 tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b)
- 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/
(b*tan(c/2)**2 + b), True))*cos(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(26) = 52$.

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.85

$$\int \cos(a+bx)\sec(c+bx)\,dx$$
$$= \frac{2\,bx\cos\left(-a+c\right) - \log\left(\cos\left(2\,bx\right)^2 + 2\,\cos\left(2\,bx\right)\cos\left(2\,c\right) + \cos\left(2\,c\right)^2 + \sin\left(2\,bx\right)^2 - 2\,\sin\left(2\,bx\right)\sin\left(2\,c\right)\right.}{2\,b}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="maxima")
```

output
```
1/2*(2*b*x*cos(-a + c) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*
c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2)*sin(-a + c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(26) = 52$.

Time = 0.18 (sec) , antiderivative size = 440, normalized size of antiderivative = 16.92

$$\int \cos(a+bx)\sec(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="giac")
```

output
```
((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan
(1/2*c)^2 + 1)*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1
/2*c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/
2*a) - tan(1/2*c))*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^2 + ta
n(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^4*tan(1/2*c)^2 - 2*tan(1/2*
a)^3*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)
- 4*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2 -
 2*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2)*log(abs(2*tan(b*x + a)*tan(1/2*a)
^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a)^2*tan(
1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)^2 - 2*tan(b*x + a)*tan(1
/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1))/(tan(1/2*a)^4*tan(1/2
*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c) - tan(1/2*a)*t
an(1/2*c)^4 + tan(1/2*a)^3 - tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 17.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.19

$$\int \cos(a+bx)\sec(c+bx)\,dx = x\left(\frac{e^{-a\,1i+c\,1i}}{2}-\frac{e^{a\,1i-c\,1i}}{2}\right)+x\left(\frac{e^{-a\,1i+c\,1i}}{2}+\frac{e^{a\,1i-c\,1i}}{2}\right) + \frac{\ln\left(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}-\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)}{b}$$

input
```
int(cos(a + b*x)/cos(c + b*x),x)
```

output
```
x*(exp(c*1i - a*1i)/2 - exp(a*1i - c*1i)/2) + x*(exp(c*1i - a*1i)/2 + exp(
a*1i - c*1i)/2) + (log(exp(a*2i - c*2i) + exp(a*2i + b*x*2i))*((exp(c*1i -
 a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2))/b
```

Reduce [F]

$$\int \cos(a+bx)\sec(c+bx)\,dx$$
$$= \frac{-4\left(\int \frac{1}{\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}dx\right)b+\log\left(\tan\left(\frac{bx}{2}+\frac{c}{2}\right)-1\right)-\log\left(\tan\left(\frac{bx}{2}+\frac{c}{2}\right)\right.}{b}$$

input
```
int(cos(b*x+a)*sec(b*x+c),x)
```

output
```
( - 4*int(1/(tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**2
 - tan((a + b*x)/2)**2 - 1),x)*b + log(tan((b*x + c)/2) - 1) - log(tan((b*
x + c)/2) + 1) - sin(a + b*x) - a - b*x)/b
```

3.299 $\int \cos(a+bx)\sec^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \cos(a+bx)\sec^2(c+bx)\,dx=\frac{\operatorname{arctanh}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}$$

output
```
arctanh(sin(b*x+c))*cos(a-c)/b-sec(b*x+c)*sin(a-c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.54

$$\int \cos(a+bx)\sec^2(c+bx)\,dx =-\frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + b*x]^2,x]
```

output
```
((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x
)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Sec
[c + b*x]*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5094, 3042, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^2(bx+c)\,dx$$

$$\Big\downarrow \text{5094}$$

$$\cos(a-c)\int \sec(c+bx)dx - \sin(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx - \sin(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx - \frac{\sin(a-c)\int 1d\sec(c+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx - \frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(a-c)\text{arctanh}(\sin(bx+c))}{b} - \frac{\sin(a-c)\sec(bx+c)}{b}$$

input
```
Int[Cos[a + b*x]*Sec[c + b*x]^2,x]
```

output
```
(ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - (Sec[c + b*x]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5094
```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Simp[-Sin[v - w]   Int[Tan[w]*Sec[w
]^(n - 1), x], x] + Simp[Cos[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n,
0] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.40

method	result
risch	$\frac{i\left(\mathrm{e}^{i(bx+3a)}-\mathrm{e}^{i(bx+a+2c)}\right)}{b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a-c)}\right)\cos(a-c)}{b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a-c)}\right)\cos(a-c)}{b}$
default	$-\frac{2\left(-\frac{\left(\cos(c)^2\sin(a)^2-2\cos(a)\cos(c)\sin(a)\sin(c)+\sin(c)^2\cos(a)^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)(\cos(a)\cos(c)+\sin(a)\sin(c))}+\frac{-\sin(a)\cos(c)+\cos(a)}{\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2}\right.}{\cos(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+\sin(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-2\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-\cos(a)\cos(c)-s}$

input
```
int(cos(b*x+a)*sec(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))*(exp(I*(b*x+3*a))-exp(I*(b*x+a+2*c)))-
ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*cos(a-c)+ln(exp(I*(b*x+a))+I*exp(I*(a-
c)))/b*cos(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \cos(a+bx)\sec^2(c+bx)\,dx = \frac{\cos(bx+c)\cos(-a+c)\log(\sin(bx+c)+1) - \cos(bx+c)\cos(-a+c)\log(-\sin(bx+c)+1) + 2\,\mathrm{si}}{2\,b\cos(bx+c)}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(cos(b*x + c)*cos(-a + c)*log(sin(b*x + c) + 1) - cos(b*x + c)*cos(-a
+ c)*log(-sin(b*x + c) + 1) + 2*sin(-a + c))/(b*cos(b*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4095 vs. 2(27) = 54.

Time = 97.09 (sec) , antiderivative size = 5545, normalized size of antiderivative = 158.43

$$\int \cos(a+bx)\sec^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)**2,x)
```

output
```
-Piecewise((log(tan(b*x/2))/b, Eq(c, -pi/2) | Eq(c, pi/2)), (0, Eq(b, 0)),
 (-2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)
**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c
/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log
(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**3/(b*t
an(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*
b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 8*log(tan(b*x/2) - tan(c/2)
/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*
tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*
tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2)
- 1) - 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)*
*2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2)
- b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(t
an(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*
tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) +
2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3
*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)
**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*log(ta
n(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(
c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(35) = 70$.

Time = 0.18 (sec) , antiderivative size = 391, normalized size of antiderivative = 11.17

$$\int \cos(a+bx)\sec^2(c+bx)\,dx =$$

$$-\frac{2\left(\sin\left(bx+2a\right)-\sin\left(bx+2c\right)\right)\cos\left(2bx+a+2c\right)+\left(\cos\left(2bx+a+2c\right)^2\cos\left(-a+c\right)+2\cos\left(2b\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(sin(b*x + 2*a) - sin(b*x + 2*c))*cos(2*b*x + a + 2*c) + (cos(2*b*
x + a + 2*c)^2*cos(-a + c) + 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + c
os(-a + c)*sin(2*b*x + a + 2*c)^2 + 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin
(a) + (cos(a)^2 + sin(a)^2)*cos(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2
- 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + s
in(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x
+ 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - 2*(cos(b*x + 2*a) - cos(
b*x + 2*c))*sin(2*b*x + a + 2*c) + 2*cos(a)*sin(b*x + 2*a) - 2*cos(a)*sin(
b*x + 2*c) - 2*cos(b*x + 2*a)*sin(a) + 2*cos(b*x + 2*c)*sin(a))/(b*cos(2*b
*x + a + 2*c)^2 + 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)
^2 + 2*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1341 vs. $2(35) = 70$.

Time = 0.21 (sec) , antiderivative size = 1341, normalized size of antiderivative = 38.31

$$\int \cos(a + bx) \sec^2(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="giac")
```

output
```
-((tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)^2*ta
n(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1
/2*a)*tan(1/2*c)^3 + tan(1/2*a)^3 - 5*tan(1/2*a)^2*tan(1/2*c) + 5*tan(1/2*
a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) -
tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*
tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*b*x + 1/
2*a)*tan(1/2*c) + tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a
) - tan(1/2*c) + 1))/(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^
2 + tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c) + tan(1/2*a)^2*tan
(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 + tan(1/2*a)^2*tan(1/2*
c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a)^2 + tan(1/2*a)*ta
n(1/2*c) + tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1) - (tan(1/2*a)^3*tan
(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/
2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3
- tan(1/2*a)^3 + 5*tan(1/2*a)^2*tan(1/2*c) - 5*tan(1/2*a)*tan(1/2*c)^2 + t
an(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + tan(
1/2*a) - tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*
c) - tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*b*x + 1/2*a)*tan(1/2*c) - t
an(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan(1/2*c) - 1)
)/(tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2...
```

Mupad [B] (verification not implemented)

Time = 21.93 (sec) , antiderivative size = 246, normalized size of antiderivative = 7.03

$$\int \cos(a+bx)\sec^2(c+bx)\,dx$$
$$= \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,(e^{a\,2i}\,e^{-c\,2i}+1)-\frac{e^{a\,2i}\,e^{-c\,2i}\,(e^{a\,2i}\,e^{-c\,2i}+1)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)(e^{a\,2i-c\,2i}+1)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$
$$-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,(e^{a\,2i}\,e^{-c\,2i}+1)+\frac{e^{a\,2i}\,e^{-c\,2i}\,(e^{a\,2i}\,e^{-c\,2i}+1)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)(e^{a\,2i-c\,2i}+1)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$
$$+\frac{e^{a\,1i+b\,x\,1i}\,(e^{a\,2i-c\,2i}-1)\,1i}{b\,(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i})}$$

input
```
int(cos(a + b*x)/cos(c + b*x)^2,x)
```

output
```
(log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1) - (exp(a*2i)*exp(-
c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*
2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log((exp(a*2i)*exp(-c*2i)
*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*e
xp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*
2i - c*2i)^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) - 1)*1i)/(b*(exp
(a*2i - c*2i) + exp(a*2i + b*x*2i)))
```

Reduce [F]

$$\int \cos(a+bx)\sec^2(c+bx)\,dx$$
$$= \frac{-\cos(bx+c)\left(\int \frac{\sin(bx+c)^2}{\sin(bx+c)^2-1}dx\right)b-\cos(bx+c)\left(\int \frac{\cos(bx+a)\sin(bx+c)^2}{\sin(bx+c)^2-1}dx\right)b+\cos(bx+c)\sin(bx+a)+}{\cos(bx+c)\,b}$$

input
```
int(cos(b*x+a)*sec(b*x+c)^2,x)
```

output
```
( - cos(b*x + c)*int(sin(b*x + c)**2/(sin(b*x + c)**2 - 1),x)*b - cos(b*x
+ c)*int((cos(a + b*x)*sin(b*x + c)**2)/(sin(b*x + c)**2 - 1),x)*b + cos(b
*x + c)*sin(a + b*x) + 2*cos(b*x + c)*a + cos(b*x + c)*b*x - sin(b*x + c))
/(cos(b*x + c)*b)
```

3.300 $\int \cos(a+bx)\sec^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = -\frac{\sec^2(c+bx)\sin(a-c)}{2b}+\frac{\cos(a-c)\tan(c+bx)}{b}$$

output
```
-1/2*sec(b*x+c)^2*sin(a-c)/b+cos(a-c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = -\frac{\sec(c)\sec^2(c+bx)(\sin(a)-\cos(a-c)\sin(c+2bx))}{2b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + b*x]^3,x]
```

output
```
-1/2*(Sec[c]*Sec[c + b*x]^2*(Sin[a] - Cos[a - c]*Sin[c + 2*b*x]))/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5094, 3042, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^3(bx+c)\,dx$$

$$\Big\downarrow \text{5094}$$

$$\cos(a-c)\int \sec^2(c+bx)dx-\sin(a-c)\int \sec^2(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx-\sin(a-c)\int \sec(c+bx)^2\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx-\frac{\sin(a-c)\int \sec(c+bx)d\sec(c+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx-\frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\cos(a-c)\int 1d(-\tan(c+bx))}{b}-\frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\cos(a-c)\tan(bx+c)}{b}-\frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

input
```
Int[Cos[a + b*x]*Sec[c + b*x]^3,x]
```

output
```
-1/2*(Sec[c + b*x]^2*Sin[a - c])/b + (Cos[a - c]*Tan[c + b*x])/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5094
```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Simp[-Sin[v - w]   Int[Tan[w]*Sec[w
]^(n - 1), x], x] + Simp[Cos[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n,
0] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{\sin(2bx+a+c)}{b(1+\cos(2bx+2c))}$	26
default	$-\frac{1}{2b(\sin(a)\cos(c)-\cos(a)\sin(c))(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))^{2}}$	56
risch	$\frac{i\left(2\,\mathrm{e}^{i(2bx+5a+c)}+\mathrm{e}^{i(5a-c)}+\mathrm{e}^{i(3a+c)}\right)}{\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{2}b}$	61

input
```
int(cos(b*x+a)*sec(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*sin(2*b*x+a+c)/(1+cos(2*b*x+2*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = \frac{2\,\cos(bx+c)\cos(-a+c)\sin(bx+c)+\sin(-a+c)}{2\,b\cos(bx+c)^2}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c) + sin(-a + c))/(b*cos(b*x + c
)^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 382, normalized size of antiderivative = 10.05

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = -\frac{(2\sin(2bx+2a+2c)+\sin(2a)+\sin(2c))\cos(4bx+a+5c)+2(2\sin(2bx+2a+2c)+\sin(2}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2+4b\cos}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="maxima")
```

output
```
-((2*sin(2*b*x + 2*a + 2*c) + sin(2*a) + sin(2*c))*cos(4*b*x + a + 5*c) +
2*(2*sin(2*b*x + 2*a + 2*c) + sin(2*a) + sin(2*c))*cos(2*b*x + a + 3*c) +
(sin(2*a) + sin(2*c))*cos(a + c) - (2*cos(2*b*x + 2*a + 2*c) + cos(2*a) +
cos(2*c))*sin(4*b*x + a + 5*c) + 2*cos(a + c)*sin(2*b*x + 2*a + 2*c) - 2*(
2*cos(2*b*x + 2*a + 2*c) + cos(2*a) + cos(2*c))*sin(2*b*x + a + 3*c) - (co
s(2*a) + cos(2*c))*sin(a + c) - 2*cos(2*b*x + 2*a + 2*c)*sin(a + c))/(b*co
s(4*b*x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 + 4*b*cos(2*b*x + a + 3*
c)*cos(a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x
+ a + 3*c)^2 + 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(2
*b*cos(2*b*x + a + 3*c) + b*cos(a + c))*cos(4*b*x + a + 5*c) + 2*(2*b*sin(
2*b*x + a + 3*c) + b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(36) = 72$.

Time = 0.17 (sec) , antiderivative size = 315, normalized size of antiderivative = 8.29

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = -\frac{\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^2+9\,\mathrm{t}}{4\left(2\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-2\tan(bx+a)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}\right.\right.}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*
a)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c
)^4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*
c)^2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan
(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2
+ 1)/((2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*
tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan
(1/2*a)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*
c)^2 + 1)^2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a
) - tan(1/2*c))*b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/cos(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = \frac{-\cos(bx+c)\sin(bx+a)-\cos(bx+a)\sin(bx+c)}{2b\left(\sin(bx+c)^2-1\right)}$$

input
```
int(cos(b*x+a)*sec(b*x+c)^3,x)
```

output
```
( - (cos(b*x + c)*sin(a + b*x) + cos(a + b*x)*sin(b*x + c)))/(2*b*(sin(b*x
 + c)**2 - 1))
```

3.301 $\int \cos(a+bx)\sec^4(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \cos(a+bx)\sec^4(c+bx)\,dx = \frac{\operatorname{arctanh}(\sin(c+bx))\cos(a-c)}{2b} - \frac{\sec^3(c+bx)\sin(a-c)}{3b} + \frac{\cos(a-c)\sec(c+bx)\tan(c+bx)}{2b}$$

output
```
1/2*arctanh(sin(b*x+c))*cos(a-c)/b-1/3*sec(b*x+c)^3*sin(a-c)/b+1/2*cos(a-c
)*sec(b*x+c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \cos(a+bx)\sec^4(c+bx)\,dx = \frac{12\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+\sec^3(c+bx)(-4\sin(a-c)+3\cos(a-c)\sin(2(c+bx)}{12b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + b*x]^4,x]
```

output
```
(12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Cos[a - c] + Sec[c + b*x]^3*(-4*
Sin[a - c] + 3*Cos[a - c]*Sin[2*(c + b*x)]))/(12*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5094, 3042, 3086, 15, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^4(bx+c)\,dx$$

$$\Big\downarrow \text{5094}$$

$$\cos(a-c)\int \sec^3(c+bx)dx-\sin(a-c)\int \sec^3(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx-\sin(a-c)\int \sec(c+bx)^3\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx-\frac{\sin(a-c)\int \sec^2(c+bx)d\sec(c+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx-\frac{\sin(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4255}$$

$$\cos(a-c)\left(\frac{1}{2}\int \sec(c+bx)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)-\frac{\sin(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\left(\frac{1}{2}\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)-\frac{\sin(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4257}$$

$$\cos(a-c)\left(\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)-\frac{\sin(a-c)\sec^3(bx+c)}{3b}$$

input
```
Int[Cos[a + b*x]*Sec[c + b*x]^4,x]
```

output
```
-1/3*(Sec[c + b*x]^3*Sin[a - c])/b + Cos[a - c]*(ArcTanh[Sin[c + b*x]]/(2*
b) + (Sec[c + b*x]*Tan[c + b*x])/(2*b))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5094
```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Simp[-Sin[v - w]   Int[Tan[w]*Sec[w
]^(n - 1), x], x] + Simp[Cos[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n,
0] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.39 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.87

method	result
risch	$\frac{i\left(-3\,{\mathrm e}^{i(5bx+7a+4c)}-3\,{\mathrm e}^{i(5bx+5a+6c)}+8\,{\mathrm e}^{i(3bx+7a+2c)}-8\,{\mathrm e}^{i(3bx+5a+4c)}+3\,{\mathrm e}^{i(bx+7a)}+3\,{\mathrm e}^{i(bx+5a+2c)}\right)}{6b\left({\mathrm e}^{2i(bx+a+c)}+{\mathrm e}^{2ia}\right)^{3}}+\frac{\ln\left({\mathrm e}^{i(bx+a)}+i{\mathrm e}^{i(a-c)}\right)\cos}{2b}$
default	Expression too large to display

input
```
int(cos(b*x+a)*sec(b*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))^3*(-3*exp(I*(5*b*x+7*a+4*c))-3*exp
(I*(5*b*x+5*a+6*c))+8*exp(I*(3*b*x+7*a+2*c))-8*exp(I*(3*b*x+5*a+4*c))+3*ex
p(I*(b*x+7*a))+3*exp(I*(b*x+5*a+2*c)))+1/2*ln(exp(I*(b*x+a))+I*exp(I*(a-c)
))/b*cos(a-c)-1/2*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*cos(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \cos(a+bx)\sec^4(c+bx)\,dx$$
$$=\frac{3\,\cos\left(bx+c\right)^{3}\cos\left(-a+c\right)\log\left(\sin\left(bx+c\right)+1\right)-3\,\cos\left(bx+c\right)^{3}\cos\left(-a+c\right)\log\left(-\sin\left(bx+c\right)+1\right)+}{12\,b\cos\left(bx+c\right)^{3}}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^4,x, algorithm="fricas")
```

output
```
1/12*(3*cos(b*x + c)^3*cos(-a + c)*log(sin(b*x + c) + 1) - 3*cos(b*x + c)^
3*cos(-a + c)*log(-sin(b*x + c) + 1) + 6*cos(b*x + c)*cos(-a + c)*sin(b*x
+ c) + 4*sin(-a + c))/(b*cos(b*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^4(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. $2(61) = 122$.

Time = 0.20 (sec) , antiderivative size = 1420, normalized size of antiderivative = 21.19

$$\int \cos(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^4,x, algorithm="maxima")
```

output
```
1/12*(2*(3*sin(5*b*x + 2*a + 4*c) + 3*sin(5*b*x + 6*c) - 8*sin(3*b*x + 2*a
 + 2*c) + 8*sin(3*b*x + 4*c) - 3*sin(b*x + 2*a) - 3*sin(b*x + 2*c))*cos(6*
b*x + a + 6*c) - 6*(3*sin(4*b*x + a + 4*c) + 3*sin(2*b*x + a + 2*c) + sin(
a))*cos(5*b*x + 2*a + 4*c) - 6*(3*sin(4*b*x + a + 4*c) + 3*sin(2*b*x + a +
 2*c) + sin(a))*cos(5*b*x + 6*c) - 6*(8*sin(3*b*x + 2*a + 2*c) - 8*sin(3*b
*x + 4*c) + 3*sin(b*x + 2*a) + 3*sin(b*x + 2*c))*cos(4*b*x + a + 4*c) + 16
*(3*sin(2*b*x + a + 2*c) + sin(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*sin(2*b*
x + a + 2*c) + sin(a))*cos(3*b*x + 4*c) - 18*(sin(b*x + 2*a) + sin(b*x + 2
*c))*cos(2*b*x + a + 2*c) - 3*(cos(6*b*x + a + 6*c)^2*cos(-a + c) + 9*cos(
4*b*x + a + 4*c)^2*cos(-a + c) + 9*cos(2*b*x + a + 2*c)^2*cos(-a + c) + 6*
cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(6*b*x + a + 6*c)
^2 + 9*cos(-a + c)*sin(4*b*x + a + 4*c)^2 + 9*cos(-a + c)*sin(2*b*x + a +
2*c)^2 + 6*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + 2*(3*cos(4*b*x + a +
4*c)*cos(-a + c) + 3*cos(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(-a + c)
)*cos(6*b*x + a + 6*c) + 6*(3*cos(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*co
s(-a + c))*cos(4*b*x + a + 4*c) + (cos(a)^2 + sin(a)^2)*cos(-a + c) + 2*(3
*cos(-a + c)*sin(4*b*x + a + 4*c) + 3*cos(-a + c)*sin(2*b*x + a + 2*c) + c
os(-a + c)*sin(a))*sin(6*b*x + a + 6*c) + 6*(3*cos(-a + c)*sin(2*b*x + a +
 2*c) + cos(-a + c)*sin(a))*sin(4*b*x + a + 4*c))*log((cos(b*x + 2*c)^2 +
cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12158 vs. $2(61) = 122$.

Time = 1.00 (sec) , antiderivative size = 12158, normalized size of antiderivative = 181.46

$$\int \cos(a + bx) \sec^4(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^4,x, algorithm="giac")
```

output
```
-1/6*(3*(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a
)^2*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 -
 tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^3 - 5*tan(1/2*a)^2*tan(1/2*c) + 5*ta
n(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2
*c) - tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1
/2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*b*
x + 1/2*a)*tan(1/2*c) + tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan
(1/2*a) - tan(1/2*c) + 1))/(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1
/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c) + tan(1/2*a)
^2*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 + tan(1/2*a)^2*ta
n(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a)^2 + tan(1/2
*a)*tan(1/2*c) + tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1) - 3*(tan(1/2*
a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3
- tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1
/2*c)^3 - tan(1/2*a)^3 + 5*tan(1/2*a)^2*tan(1/2*c) - 5*tan(1/2*a)*tan(1/2*
c)^2 + tan(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^
2 + tan(1/2*a) - tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*
tan(1/2*c) - tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*b*x + 1/2*a)*tan(1/
2*c) - tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan(1/2
*c) - 1))/(tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(...
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^4(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/cos(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
int(cos(b*x+a)*sec(b*x+c)^4,x)
```

output
```
( - 4*cos(b*x + c)**2*sin(a + b*x) + 8*cos(b*x + c)*int(cos(a + b*x)/(sin(
b*x + c)**4 - 2*sin(b*x + c)**2 + 1),x)*sin(b*x + c)**2*b - 8*cos(b*x + c)
*int(cos(a + b*x)/(sin(b*x + c)**4 - 2*sin(b*x + c)**2 + 1),x)*b - cos(b*x
 + c)*int((cos(a + b*x)*sin(b*x + c)**4)/(sin(b*x + c)**4 - 2*sin(b*x + c)
**2 + 1),x)*sin(b*x + c)**2*b + cos(b*x + c)*int((cos(a + b*x)*sin(b*x + c
)**4)/(sin(b*x + c)**4 - 2*sin(b*x + c)**2 + 1),x)*b - 4*cos(b*x + c)*int(
(cos(a + b*x)*sin(b*x + c)**2)/(sin(b*x + c)**4 - 2*sin(b*x + c)**2 + 1),x
)*sin(b*x + c)**2*b + 4*cos(b*x + c)*int((cos(a + b*x)*sin(b*x + c)**2)/(s
in(b*x + c)**4 - 2*sin(b*x + c)**2 + 1),x)*b - 2*cos(b*x + c)*log(sin(b*x
+ c) - 1)*sin(b*x + c)**2 + 2*cos(b*x + c)*log(sin(b*x + c) - 1) + 2*cos(b
*x + c)*log(sin(b*x + c) + 1)*sin(b*x + c)**2 - 2*cos(b*x + c)*log(sin(b*x
 + c) + 1) + 4*cos(b*x + c)*log(tan((b*x + c)/2) - 1)*sin(b*x + c)**2 - 4*
cos(b*x + c)*log(tan((b*x + c)/2) - 1) - 4*cos(b*x + c)*log(tan((b*x + c)/
2) + 1)*sin(b*x + c)**2 + 4*cos(b*x + c)*log(tan((b*x + c)/2) + 1) + cos(b
*x + c)*sin(b*x + c)**2*sin(a + b*x) + cos(b*x + c)*sin(b*x + c)**2*a - 3*
cos(b*x + c)*sin(a + b*x) - cos(b*x + c)*a - 4*cos(a + b*x)*sin(b*x + c) -
 4*sin(b*x + c)**2*sin(a + b*x) + 4*sin(a + b*x))/(15*cos(b*x + c)*b*(sin(
b*x + c)**2 - 1))
```

3.302 $\int \cos(a+bx)\sec^5(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \cos(a+bx)\sec^5(c+bx)\,dx = -\frac{\sec^4(c+bx)\sin(a-c)}{4b}+\frac{\cos(a-c)\tan(c+bx)}{b}+\frac{\cos(a-c)\tan^3(c+bx)}{3b}$$

output
```
-1/4*sec(b*x+c)^4*sin(a-c)/b+cos(a-c)*tan(b*x+c)/b+1/3*cos(a-c)*tan(b*x+c)
^3/b
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \cos(a+bx)\sec^5(c+bx)\,dx = \frac{\sec(c)\sec^4(c+bx)(-6\sin(a)+2\cos(a-c)(4\sin(c+2bx)+\sin(3c+4bx)))}{24b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + b*x]^5,x]
```

output
```
(Sec[c]*Sec[c + b*x]^4*(-6*Sin[a] + 2*Cos[a - c]*(4*Sin[c + 2*b*x] + Sin[3
*c + 4*b*x])))/(24*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5094, 3042, 3086, 15, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^5(bx+c)\,dx$$

$$\Big\downarrow \text{5094}$$

$$\cos(a-c)\int \sec^4(c+bx)dx-\sin(a-c)\int \sec^4(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^4dx-\sin(a-c)\int \sec(c+bx)^4\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^4dx-\frac{\sin(a-c)\int \sec^3(c+bx)d\sec(c+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^4dx-\frac{\sin(a-c)\sec^4(bx+c)}{4b}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\cos(a-c)\int \left(\tan^2(c+bx)+1\right)d(-\tan(c+bx))}{b}-\frac{\sin(a-c)\sec^4(bx+c)}{4b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(a-c)\left(-\frac{1}{3}\tan^3(bx+c)-\tan(bx+c)\right)}{b}-\frac{\sin(a-c)\sec^4(bx+c)}{4b}$$

input
```
Int[Cos[a + b*x]*Sec[c + b*x]^5,x]
```

output
```
-1/4*(Sec[c + b*x]^4*Sin[a - c])/b - (Cos[a - c]*(-Tan[c + b*x] - Tan[c +
b*x]^3/3))/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5094
```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Simp[-Sin[v - w]   Int[Tan[w]*Sec[w
]^(n - 1), x], x] + Simp[Cos[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n,
0] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.61

method	result
risch	$\frac{2i\left(6\,\mathrm{e}^{i(4bx+9a+3c)}+4\,\mathrm{e}^{i(2bx+9a+c)}+4\,\mathrm{e}^{i(2bx+7a+3c)}+\mathrm{e}^{i(9a-c)}+\mathrm{e}^{i(7a+c)}\right)}{3\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{4}b}$
default	$-\frac{-2\cos(a)\cos(c)-2\sin(a)\sin(c)}{3(\sin(a)\cos(c)-\cos(a)\sin(c))^{3}(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))^{3}}-\frac{\ldots}{2(\sin(a)\cos(c)-\cos(a)\sin\ldots}$
parallelrisch	$\frac{-5\tan(\frac{bx}{2}+\frac{c}{2})^{8}\tan(\frac{a}{2}+\frac{bx}{2})+(-7\tan(\frac{a}{2}+\frac{bx}{2})^{2}+7)\tan(\frac{bx}{2}+\frac{c}{2})^{7}+6\tan(\frac{bx}{2}+\frac{c}{2})^{6}\tan(\frac{a}{2}+\frac{bx}{2})+(-5\tan(\frac{a}{2}+\frac{bx}{2})^{2}+5\ldots}{6b(\tan(\frac{bx}{2}+\frac{c}{2})-1)^{4}(\ldots}$

input
```
int(cos(b*x+a)*sec(b*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
2/3*I/(exp(2*I*(b*x+a+c))+exp(2*I*a))^4/b*(6*exp(I*(4*b*x+9*a+3*c))+4*exp(
I*(2*b*x+9*a+c))+4*exp(I*(2*b*x+7*a+3*c))+exp(I*(9*a-c))+exp(I*(7*a+c)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \cos(a+bx)\sec^5(c+bx)\,dx$$
$$=\frac{4\left(2\,\cos\left(bx+c\right)^{3}\cos\left(-a+c\right)+\cos\left(bx+c\right)\cos\left(-a+c\right)\right)\sin\left(bx+c\right)+3\,\sin\left(-a+c\right)}{12\,b\cos\left(bx+c\right)^{4}}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^5,x, algorithm="fricas")
```

output
```
1/12*(4*(2*cos(b*x + c)^3*cos(-a + c) + cos(b*x + c)*cos(-a + c))*sin(b*x
+ c) + 3*sin(-a + c))/(b*cos(b*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^5(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(55) = 110$.

Time = 0.05 (sec) , antiderivative size = 1056, normalized size of antiderivative = 17.90

$$\int \cos(a+bx)\sec^5(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^5,x, algorithm="maxima")
```

output
```
-2/3*((6*sin(4*b*x + 2*a + 4*c) + 4*sin(2*b*x + 2*a + 2*c) + 4*sin(2*b*x +
 4*c) + sin(2*a) + sin(2*c))*cos(8*b*x + a + 9*c) + 4*(6*sin(4*b*x + 2*a +
 4*c) + 4*sin(2*b*x + 2*a + 2*c) + 4*sin(2*b*x + 4*c) + sin(2*a) + sin(2*c
))*cos(6*b*x + a + 7*c) - 6*(4*sin(2*b*x + a + 3*c) + sin(a + c))*cos(4*b*
x + 2*a + 4*c) + 6*(6*sin(4*b*x + 2*a + 4*c) + 4*sin(2*b*x + 2*a + 2*c) +
4*sin(2*b*x + 4*c) + sin(2*a) + sin(2*c))*cos(4*b*x + a + 5*c) + 4*(4*sin(
2*b*x + 2*a + 2*c) + sin(2*a) + sin(2*c))*cos(2*b*x + a + 3*c) - 4*(4*sin(
2*b*x + a + 3*c) + sin(a + c))*cos(2*b*x + 4*c) + (sin(2*a) + sin(2*c))*co
s(a + c) - (6*cos(4*b*x + 2*a + 4*c) + 4*cos(2*b*x + 2*a + 2*c) + 4*cos(2*
b*x + 4*c) + cos(2*a) + cos(2*c))*sin(8*b*x + a + 9*c) - 4*(6*cos(4*b*x +
2*a + 4*c) + 4*cos(2*b*x + 2*a + 2*c) + 4*cos(2*b*x + 4*c) + cos(2*a) + co
s(2*c))*sin(6*b*x + a + 7*c) + 6*(4*cos(2*b*x + a + 3*c) + cos(a + c))*sin
(4*b*x + 2*a + 4*c) - 6*(6*cos(4*b*x + 2*a + 4*c) + 4*cos(2*b*x + 2*a + 2*
c) + 4*cos(2*b*x + 4*c) + cos(2*a) + cos(2*c))*sin(4*b*x + a + 5*c) + 4*co
s(a + c)*sin(2*b*x + 2*a + 2*c) - 4*(4*cos(2*b*x + 2*a + 2*c) + cos(2*a) +
 cos(2*c))*sin(2*b*x + a + 3*c) + 4*(4*cos(2*b*x + a + 3*c) + cos(a + c))*
sin(2*b*x + 4*c) - (cos(2*a) + cos(2*c))*sin(a + c) - 4*cos(2*b*x + 2*a +
2*c)*sin(a + c))/(b*cos(8*b*x + a + 9*c)^2 + 16*b*cos(6*b*x + a + 7*c)^2 +
 36*b*cos(4*b*x + a + 5*c)^2 + 16*b*cos(2*b*x + a + 3*c)^2 + 8*b*cos(2*b*x
 + a + 3*c)*cos(a + c) + b*cos(a + c)^2 + b*sin(8*b*x + a + 9*c)^2 + 16...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5419 vs. $2(55) = 110$.

Time = 0.28 (sec) , antiderivative size = 5419, normalized size of antiderivative = 91.85

$$\int \cos(a + bx) \sec^5(c + bx)\, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^5,x, algorithm="giac")
```

output
```
-1/96*(24*tan(b*x + a)^2*tan(1/2*a)^14*tan(1/2*c)^12 - 48*tan(b*x + a)^2*t
an(1/2*a)^13*tan(1/2*c)^13 + 8*tan(b*x + a)*tan(1/2*a)^14*tan(1/2*c)^13 +
24*tan(b*x + a)^2*tan(1/2*a)^12*tan(1/2*c)^14 - 8*tan(b*x + a)*tan(1/2*a)^
13*tan(1/2*c)^14 + tan(1/2*a)^14*tan(1/2*c)^14 + 120*tan(b*x + a)^2*tan(1/
2*a)^14*tan(1/2*c)^10 - 192*tan(b*x + a)^2*tan(1/2*a)^13*tan(1/2*c)^11 + 3
2*tan(b*x + a)*tan(1/2*a)^14*tan(1/2*c)^11 + 144*tan(b*x + a)^2*tan(1/2*a)
^12*tan(1/2*c)^12 + 8*tan(b*x + a)*tan(1/2*a)^13*tan(1/2*c)^12 + 15*tan(1/
2*a)^14*tan(1/2*c)^12 - 192*tan(b*x + a)^2*tan(1/2*a)^11*tan(1/2*c)^13 - 8
*tan(b*x + a)*tan(1/2*a)^12*tan(1/2*c)^13 - 16*tan(1/2*a)^13*tan(1/2*c)^13
 + 120*tan(b*x + a)^2*tan(1/2*a)^10*tan(1/2*c)^14 - 32*tan(b*x + a)*tan(1/
2*a)^11*tan(1/2*c)^14 + 15*tan(1/2*a)^12*tan(1/2*c)^14 + 240*tan(b*x + a)^
2*tan(1/2*a)^14*tan(1/2*c)^8 - 240*tan(b*x + a)^2*tan(1/2*a)^13*tan(1/2*c)
^9 + 40*tan(b*x + a)*tan(1/2*a)^14*tan(1/2*c)^9 + 384*tan(b*x + a)^2*tan(1
/2*a)^12*tan(1/2*c)^10 + 152*tan(b*x + a)*tan(1/2*a)^13*tan(1/2*c)^10 + 61
*tan(1/2*a)^14*tan(1/2*c)^10 - 768*tan(b*x + a)^2*tan(1/2*a)^11*tan(1/2*c)
^11 - 32*tan(b*x + a)*tan(1/2*a)^12*tan(1/2*c)^11 - 64*tan(1/2*a)^13*tan(1
/2*c)^11 + 384*tan(b*x + a)^2*tan(1/2*a)^10*tan(1/2*c)^12 + 32*tan(b*x + a
)*tan(1/2*a)^11*tan(1/2*c)^12 + 97*tan(1/2*a)^12*tan(1/2*c)^12 - 240*tan(b
*x + a)^2*tan(1/2*a)^9*tan(1/2*c)^13 - 152*tan(b*x + a)*tan(1/2*a)^10*tan(
1/2*c)^13 - 64*tan(1/2*a)^11*tan(1/2*c)^13 + 240*tan(b*x + a)^2*tan(1/2...
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^5(c+bx)\,dx=\text{Hanged}$$

input
```
int(cos(a + b*x)/cos(c + b*x)^5,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.59

$$\int \cos(a+bx)\sec^5(c+bx)\,dx$$
$$=\frac{-4\cos(bx+c)\sin(bx+c)^2\sin(bx+a)+5\cos(bx+c)\sin(bx+a)-4\cos(bx+a)\sin(bx+c)^3+7\,\mathrm{co}}{12b\left(\sin(bx+c)^4-2\sin(bx+c)^2+1\right)}$$

input
```
int(cos(b*x+a)*sec(b*x+c)^5,x)
```

output
```
( - 4*cos(b*x + c)*sin(b*x + c)**2*sin(a + b*x) + 5*cos(b*x + c)*sin(a + b
*x) - 4*cos(a + b*x)*sin(b*x + c)**3 + 7*cos(a + b*x)*sin(b*x + c))/(12*b*
(sin(b*x + c)**4 - 2*sin(b*x + c)**2 + 1))
```

3.303 $\int \cos(a+bx)\sec^6(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int \cos(a+bx)\sec^6(c+bx)\,dx = \frac{3\operatorname{arctanh}(\sin(c+bx))\cos(a-c)}{8b} - \frac{\sec^5(c+bx)\sin(a-c)}{5b} + \frac{3\cos(a-c)\sec(c+bx)\tan(c+bx)}{8b} + \frac{\cos(a-c)\sec^3(c+bx)\tan(c+bx)}{4b}$$

output
```
3/8*arctanh(sin(b*x+c))*cos(a-c)/b-1/5*sec(b*x+c)^5*sin(a-c)/b+3/8*cos(a-c
)*sec(b*x+c)*tan(b*x+c)/b+1/4*cos(a-c)*sec(b*x+c)^3*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \cos(a+bx)\sec^6(c+bx)\,dx = \frac{480\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+2\sec^5(c+bx)(-64\sin(a-c)+5\cos(a-c)(14\sin(2(}{640b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + b*x]^6,x]
```

output
```
(480*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Cos[a - c] + 2*Sec[c + b*x]^5*(
-64*Sin[a - c] + 5*Cos[a - c]*(14*Sin[2*(c + b*x)] + 3*Sin[4*(c + b*x)])))
/(640*b)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5094, 3042, 3086, 15, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^6(bx+c)\,dx$$

$$\Big\downarrow \text{5094}$$

$$\cos(a-c)\int \sec^5(c+bx)dx-\sin(a-c)\int \sec^5(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^5dx-\sin(a-c)\int \sec(c+bx)^5\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^5dx-\frac{\sin(a-c)\int \sec^4(c+bx)d\sec(c+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^5dx-\frac{\sin(a-c)\sec^5(bx+c)}{5b}$$

$$\Big\downarrow \text{4255}$$

$$\cos(a-c)\left(\frac{3}{4}\int \sec^3(c+bx)dx+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\sec^5(bx+c)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\left(\frac{3}{4}\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\sec^5(bx+c)}{5b}$$

$$\Big\downarrow \text{ 4255}$$

$$\cos(a-c)\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+bx)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\sec^5(bx+c)}{5b}$$

$$\Big\downarrow \text{ 3042}$$

$$\cos(a-c)\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\sec^5(bx+c)}{5b}$$

$$\Big\downarrow \text{ 4257}$$

$$\cos(a-c)\left(\frac{3}{4}\left(\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)+\frac{\tan(bx+c)\sec^3(bx+c)}{4b}\right)-\frac{\sin(a-c)\sec^5(bx+c)}{5b}$$

input

```
Int[Cos[a + b*x]*Sec[c + b*x]^6,x]
```

output

```
-1/5*(Sec[c + b*x]^5*Sin[a - c])/b + Cos[a - c]*((Sec[c + b*x]^3*Tan[c + b
*x])/(4*b) + (3*(ArcTanh[Sin[c + b*x]]/(2*b) + (Sec[c + b*x]*Tan[c + b*x])
/(2*b)))/4)
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5094
```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Simp[-Sin[v - w]   Int[Tan[w]*Sec[w
]^(n - 1), x], x] + Simp[Cos[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n,
0] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 68.34 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.77

method	result
risch	$\frac{i\left(-15\,e^{i(9bx+11a+8c)}-15\,e^{i(9bx+9a+10c)}-70\,e^{i(7bx+11a+6c)}-70\,e^{i(7bx+9a+8c)}+128\,e^{i(5bx+11a+4c)}-128\,e^{i(5bx+9a+6c)}+70\,e^{i(3bx+}\right.}{40b\left(e^{2i(bx+a+c)}+e^{2ia}\right)^{5}}$
default	Expression too large to display

input
```
int(cos(b*x+a)*sec(b*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/40*I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))^5*(-15*exp(I*(9*b*x+11*a+8*c))-15
*exp(I*(9*b*x+9*a+10*c))-70*exp(I*(7*b*x+11*a+6*c))-70*exp(I*(7*b*x+9*a+8*
c))+128*exp(I*(5*b*x+11*a+4*c))-128*exp(I*(5*b*x+9*a+6*c))+70*exp(I*(3*b*x
+11*a+2*c))+70*exp(I*(3*b*x+9*a+4*c))+15*exp(I*(b*x+11*a))+15*exp(I*(b*x+9
*a+2*c)))-3/8*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*cos(a-c)+3/8*ln(exp(I*(b
*x+a))+I*exp(I*(a-c)))/b*cos(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.20

$$\int \cos(a+bx)\sec^6(c+bx)\,dx$$
$$= \frac{15\,\cos\left(bx+c\right)^5\cos\left(-a+c\right)\log\left(\sin\left(bx+c\right)+1\right)-15\,\cos\left(bx+c\right)^5\cos\left(-a+c\right)\log\left(-\sin\left(bx+c\right)+1\right.}{80\,b\cos\left(bx\right.}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^6,x, algorithm="fricas")
```

output
```
1/80*(15*cos(b*x + c)^5*cos(-a + c)*log(sin(b*x + c) + 1) - 15*cos(b*x + c
)^5*cos(-a + c)*log(-sin(b*x + c) + 1) + 10*(3*cos(b*x + c)^3*cos(-a + c)
+ 2*cos(b*x + c)*cos(-a + c))*sin(b*x + c) + 16*sin(-a + c))/(b*cos(b*x +
c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^6(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)**6,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3092 vs. $2(86) = 172$.

Time = 0.29 (sec) , antiderivative size = 3092, normalized size of antiderivative = 32.89

$$\int \cos(a + bx) \sec^6(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^6,x, algorithm="maxima")
```

output
```
1/80*(2*(15*sin(9*b*x + 2*a + 8*c) + 15*sin(9*b*x + 10*c) + 70*sin(7*b*x +
 2*a + 6*c) + 70*sin(7*b*x + 8*c) - 128*sin(5*b*x + 2*a + 4*c) + 128*sin(5
*b*x + 6*c) - 70*sin(3*b*x + 2*a + 2*c) - 70*sin(3*b*x + 4*c) - 15*sin(b*x
 + 2*a) - 15*sin(b*x + 2*c))*cos(10*b*x + a + 10*c) - 30*(5*sin(8*b*x + a
+ 8*c) + 10*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x +
 a + 2*c) + sin(a))*cos(9*b*x + 2*a + 8*c) - 30*(5*sin(8*b*x + a + 8*c) +
10*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c)
 + sin(a))*cos(9*b*x + 10*c) + 10*(70*sin(7*b*x + 2*a + 6*c) + 70*sin(7*b*
x + 8*c) - 128*sin(5*b*x + 2*a + 4*c) + 128*sin(5*b*x + 6*c) - 70*sin(3*b*
x + 2*a + 2*c) - 70*sin(3*b*x + 4*c) - 15*sin(b*x + 2*a) - 15*sin(b*x + 2*
c))*cos(8*b*x + a + 8*c) - 140*(10*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a
 + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*cos(7*b*x + 2*a + 6*c) - 140*(1
0*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c)
+ sin(a))*cos(7*b*x + 8*c) - 20*(128*sin(5*b*x + 2*a + 4*c) - 128*sin(5*b*
x + 6*c) + 70*sin(3*b*x + 2*a + 2*c) + 70*sin(3*b*x + 4*c) + 15*sin(b*x +
2*a) + 15*sin(b*x + 2*c))*cos(6*b*x + a + 6*c) + 256*(10*sin(4*b*x + a + 4
*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*cos(5*b*x + 2*a + 4*c) - 256*(10*si
n(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*cos(5*b*x + 6*c) - 1
00*(14*sin(3*b*x + 2*a + 2*c) + 14*sin(3*b*x + 4*c) + 3*sin(b*x + 2*a) + 3
*sin(b*x + 2*c))*cos(4*b*x + a + 4*c) + 140*(5*sin(2*b*x + a + 2*c) + s...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52256 vs. $2(86) = 172$.

Time = 16.10 (sec) , antiderivative size = 52256, normalized size of antiderivative = 555.91

$$\int \cos(a + bx) \sec^6(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^6,x, algorithm="giac")
```

output
```
-1/40*(15*(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2
*a)^2*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2
 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^3 - 5*tan(1/2*a)^2*tan(1/2*c) + 5*
tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1
/2*c) - tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x +
 1/2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*
b*x + 1/2*a)*tan(1/2*c) + tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + t
an(1/2*a) - tan(1/2*c) + 1))/(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan
(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c) + tan(1/2*
a)^2*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 + tan(1/2*a)^2*
tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a)^2 + tan(1
/2*a)*tan(1/2*c) + tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1) - 15*(tan(1
/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)
^3 - tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*ta
n(1/2*c)^3 - tan(1/2*a)^3 + 5*tan(1/2*a)^2*tan(1/2*c) - 5*tan(1/2*a)*tan(1
/2*c)^2 + tan(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) - tan(1/2*
c)^2 + tan(1/2*a) - tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*
a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*b*x + 1/2*a)*tan
(1/2*c) - tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan(
1/2*c) - 1))/(tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - t...
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^6(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/cos(c + b*x)^6,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos(a+bx)\sec^6(c+bx)\,dx = \text{Too large to display}$$

input
```
int(cos(b*x+a)*sec(b*x+c)^6,x)
```

output

```
( - 56*cos(b*x + c)**2*sin(b*x + c)**2*sin(a + b*x) + 60*cos(b*x + c)**2*s
in(a + b*x) - 128*cos(b*x + c)*int(cos(a + b*x)/(sin(b*x + c)**6 - 3*sin(b
*x + c)**4 + 3*sin(b*x + c)**2 - 1),x)*sin(b*x + c)**4*b + 256*cos(b*x + c
)*int(cos(a + b*x)/(sin(b*x + c)**6 - 3*sin(b*x + c)**4 + 3*sin(b*x + c)**
2 - 1),x)*sin(b*x + c)**2*b - 128*cos(b*x + c)*int(cos(a + b*x)/(sin(b*x +
 c)**6 - 3*sin(b*x + c)**4 + 3*sin(b*x + c)**2 - 1),x)*b - 2*cos(b*x + c)*
int((cos(a + b*x)*sin(b*x + c)**6)/(sin(b*x + c)**6 - 3*sin(b*x + c)**4 +
3*sin(b*x + c)**2 - 1),x)*sin(b*x + c)**4*b + 4*cos(b*x + c)*int((cos(a +
b*x)*sin(b*x + c)**6)/(sin(b*x + c)**6 - 3*sin(b*x + c)**4 + 3*sin(b*x + c
)**2 - 1),x)*sin(b*x + c)**2*b - 2*cos(b*x + c)*int((cos(a + b*x)*sin(b*x
+ c)**6)/(sin(b*x + c)**6 - 3*sin(b*x + c)**4 + 3*sin(b*x + c)**2 - 1),x)*
b - 24*cos(b*x + c)*int((cos(a + b*x)*sin(b*x + c)**4)/(sin(b*x + c)**6 -
3*sin(b*x + c)**4 + 3*sin(b*x + c)**2 - 1),x)*sin(b*x + c)**4*b + 48*cos(b
*x + c)*int((cos(a + b*x)*sin(b*x + c)**4)/(sin(b*x + c)**6 - 3*sin(b*x +
c)**4 + 3*sin(b*x + c)**2 - 1),x)*sin(b*x + c)**2*b - 24*cos(b*x + c)*int(
(cos(a + b*x)*sin(b*x + c)**4)/(sin(b*x + c)**6 - 3*sin(b*x + c)**4 + 3*si
n(b*x + c)**2 - 1),x)*b + 144*cos(b*x + c)*int((cos(a + b*x)*sin(b*x + c)*
*2)/(sin(b*x + c)**6 - 3*sin(b*x + c)**4 + 3*sin(b*x + c)**2 - 1),x)*sin(b
*x + c)**4*b - 288*cos(b*x + c)*int((cos(a + b*x)*sin(b*x + c)**2)/(sin(b*
x + c)**6 - 3*sin(b*x + c)**4 + 3*sin(b*x + c)**2 - 1),x)*sin(b*x + c)*...
```

3.304 $\int \cos^2(a+bx)\cos^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 103

$$\int \cos^2(a+bx)\cos^3(c+bx)\,dx = -\frac{\sin(2a-3c-bx)}{16b}+\frac{3\sin(2a-c+bx)}{16b}+\frac{3\sin(c+bx)}{8b}+\frac{\sin(2a+c+3bx)}{16b}+\frac{\sin(3c+3bx)}{24b}+\frac{\sin(2a+3c+5bx)}{80b}$$

output
```
-1/16*sin(-b*x+2*a-3*c)/b+3/16*sin(b*x+2*a-c)/b+3/8*sin(b*x+c)/b+1/16*sin(
3*b*x+2*a+c)/b+1/24*sin(3*b*x+3*c)/b+1/80*sin(5*b*x+2*a+3*c)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

$$\int \cos^2(a+bx)\cos^3(c+bx)\,dx = \frac{-15\sin(2a-3c-bx)+45\sin(2a-c+bx)+90\sin(c+bx)+10\sin(3(c+bx))+15\sin(2a+c+3bx)}{240b}$$

input
```
Integrate[Cos[a + b*x]^2*Cos[c + b*x]^3,x]
```

output
```
(-15*Sin[2*a - 3*c - b*x] + 45*Sin[2*a - c + b*x] + 90*Sin[c + b*x] + 10*S
in[3*(c + b*x)] + 15*Sin[2*a + c + 3*b*x] + 3*Sin[2*a + 3*c + 5*b*x])/(240
*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cos^3(bx+c)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{16}\cos(2a-bx-3c)+\frac{3}{16}\cos(2a+bx-c)+\frac{3}{16}\cos(2a+3bx+c)+\frac{1}{16}\cos(2a+5bx+3c)+\frac{3}{8}\cos(bx+c$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(2a-bx-3c)}{16b}+\frac{3\sin(2a+bx-c)}{16b}+\frac{\sin(2a+3bx+c)}{16b}+\frac{\sin(2a+5bx+3c)}{80b}+\frac{3\sin(bx+c)}{8b}+\frac{\sin(3bx+3c)}{24b}$$

input
```
Int[Cos[a + b*x]^2*Cos[c + b*x]^3,x]
```

output
```
-1/16*Sin[2*a - 3*c - b*x]/b + (3*Sin[2*a - c + b*x])/(16*b) + (3*Sin[c +
b*x])/(8*b) + Sin[2*a + c + 3*b*x]/(16*b) + Sin[3*c + 3*b*x]/(24*b) + Sin[
2*a + 3*c + 5*b*x]/(80*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 3.83 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\sin(-bx+2a-3c)}{16b}+\frac{3\sin(bx+2a-c)}{16b}+\frac{3\sin(bx+c)}{8b}+\frac{\sin(3bx+2a+c)}{16b}+\frac{\sin(3bx+3c)}{24b}+\frac{\sin(5bx+2a+3c)}{80b}$
risch	$-\frac{\sin(-bx+2a-3c)}{16b}+\frac{3\sin(bx+2a-c)}{16b}+\frac{3\sin(bx+c)}{8b}+\frac{\sin(3bx+2a+c)}{16b}+\frac{\sin(3bx+3c)}{24b}+\frac{\sin(5bx+2a+3c)}{80b}$
parallelrisch	$\frac{-20\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^{6}\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+\left(30\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}+10\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^{5}+\left(60\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}-80\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^{4}}{}$
orering	$-\frac{259\left(-2\cos(bx+c)^{3}\sin(bx+a)b\cos(bx+a)-3\cos(bx+a)^{2}\cos(bx+c)^{2}b\sin(bx+c)\right)}{225b^{2}}-\frac{7\left(-36\cos(bx+c)\sin(bx+a)b^{3}\sin(b\right.}{}$

input
```
int(cos(b*x+a)^2*cos(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*sin(-b*x+2*a-3*c)/b+3/16*sin(b*x+2*a-c)/b+3/8*sin(b*x+c)/b+1/16*sin(
3*b*x+2*a+c)/b+1/24*sin(3*b*x+3*c)/b+1/80*sin(5*b*x+2*a+3*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\cos^3(c+bx)\,dx =$$
$$-\frac{6\cos(bx+c)^{5}\cos(-a+c)\sin(-a+c)-\left(3\left(2\cos(-a+c)^{2}-1\right)\cos(bx+c)^{4}+\left(3\cos(-a+c)^{2}-\right.\right.}{15\,b}$$

input
```
integrate(cos(b*x+a)^2*cos(b*x+c)^3,x, algorithm="fricas")
```

output
```
-1/15*(6*cos(b*x + c)^5*cos(-a + c)*sin(-a + c) - (3*(2*cos(-a + c)^2 - 1)
*cos(b*x + c)^4 + (3*cos(-a + c)^2 + 1)*cos(b*x + c)^2 + 6*cos(-a + c)^2 +
 2)*sin(b*x + c))/b
```

Sympy [A] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.68

$$\int \cos^2(a+bx)\cos^3(c+bx)\,dx$$

$$= \begin{cases} \frac{8\sin^2(a+bx)\sin^3(bx+c)}{15b} + \frac{2\sin^2(a+bx)\sin(bx+c)\cos^2(bx+c)}{5b} + \frac{4\sin(a+bx)\sin^2(bx+c)\cos(a+bx)\cos(bx+c)}{5b} + \frac{2\sin(a+bx)\cos(a}{5b} \\ x\cos^2(a)\cos^3(c) \end{cases}$$

input
```
integrate(cos(b*x+a)**2*cos(b*x+c)**3,x)
```

output
```
Piecewise((8*sin(a + b*x)**2*sin(b*x + c)**3/(15*b) + 2*sin(a + b*x)**2*si
n(b*x + c)*cos(b*x + c)**2/(5*b) + 4*sin(a + b*x)*sin(b*x + c)**2*cos(a +
b*x)*cos(b*x + c)/(5*b) + 2*sin(a + b*x)*cos(a + b*x)*cos(b*x + c)**3/(5*b
) + 2*sin(b*x + c)**3*cos(a + b*x)**2/(15*b) + 3*sin(b*x + c)*cos(a + b*x)
**2*cos(b*x + c)**2/(5*b), Ne(b, 0)), (x*cos(a)**2*cos(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \cos^2(a+bx)\cos^3(c+bx)\,dx$$

$$= \frac{3\sin(5bx+2a+3c) + 15\sin(3bx+2a+c) + 10\sin(3bx+3c) + 45\sin(bx+2a-c) + 15\sin(bx}{240b}$$

input
```
integrate(cos(b*x+a)^2*cos(b*x+c)^3,x, algorithm="maxima")
```

output
```
1/240*(3*sin(5*b*x + 2*a + 3*c) + 15*sin(3*b*x + 2*a + c) + 10*sin(3*b*x +
 3*c) + 45*sin(b*x + 2*a - c) + 15*sin(b*x - 2*a + 3*c) + 90*sin(b*x + c))
/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\cos^3(c+bx)\,dx = \frac{\sin(5\,bx+2\,a+3\,c)}{80\,b} + \frac{\sin(3\,bx+2\,a+c)}{16\,b} + \frac{\sin(3\,bx+3\,c)}{24\,b} + \frac{3\,\sin(bx+2\,a-c)}{16\,b} + \frac{3\,\sin(bx+c)}{8\,b} - \frac{\sin(-bx+2\,a-3\,c)}{16\,b}$$

input
```
integrate(cos(b*x+a)^2*cos(b*x+c)^3,x, algorithm="giac")
```

output
```
1/80*sin(5*b*x + 2*a + 3*c)/b + 1/16*sin(3*b*x + 2*a + c)/b + 1/24*sin(3*b
*x + 3*c)/b + 3/16*sin(b*x + 2*a - c)/b + 3/8*sin(b*x + c)/b - 1/16*sin(-b
*x + 2*a - 3*c)/b
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \cos^2(a+bx)\cos^3(c+bx)\,dx = \frac{15\,\sin(2\,a+c+3\,b\,x)+90\,\sin(c+b\,x)+45\,\sin(2\,a-c+b\,x)+15\,\sin(3\,c-2\,a+b\,x)+3\,\sin(2\,a+}{240\,b}$$

input
```
int(cos(a + b*x)^2*cos(c + b*x)^3,x)
```

output
```
(15*sin(2*a + c + 3*b*x) + 90*sin(c + b*x) + 45*sin(2*a - c + b*x) + 15*si
n(3*c - 2*a + b*x) + 3*sin(2*a + 3*c + 5*b*x) + 10*sin(3*c + 3*b*x))/(240*
b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.30

$$\int \cos^2(a+bx)\cos^3(c+bx)\,dx$$

$$= \frac{6\cos(bx+c)\cos(bx+a)\sin(bx+c)^2\sin(bx+a)+6\cos(bx+c)\cos(bx+a)\sin(bx+a)+30\cos(bx$$

input
```
int(cos(b*x+a)^2*cos(b*x+c)^3,x)
```

output
```
(6*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x) + 6*cos(b*x + c)
*cos(a + b*x)*sin(a + b*x) + 30*cos(b*x + c)*sin(a + b*x) - 30*cos(a + b*x
)*sin(b*x + c) + 9*sin(b*x + c)**3*sin(a + b*x)**2 - 7*sin(b*x + c)**3 - 3
*sin(b*x + c)*sin(a + b*x)**2 + 9*sin(b*x + c))/(15*b)
```

3.305 $\int \cos^2(a+bx)\cos^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \cos^2(a+bx)\cos^2(c+bx)\,dx = \frac{1}{8}x(2+\cos(2(a-c)))+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2bx)}{8b}+\frac{\sin(2(a+c)+4bx)}{32b}$$

output
```
1/8*x*(2+cos(2*a-2*c))+1/8*sin(2*b*x+2*a)/b+1/8*sin(2*b*x+2*c)/b+1/32*sin(
4*b*x+2*a+2*c)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \cos^2(a+bx)\cos^2(c+bx)\,dx = \frac{8bx+4bx\cos(2(a-c))+4\sin(2(a+bx))+4\sin(2(c+bx))+\sin(2(a+c+2bx))}{32b}$$

input
```
Integrate[Cos[a + b*x]^2*Cos[c + b*x]^2,x]
```

output
```
(8*b*x + 4*b*x*Cos[2*(a - c)] + 4*Sin[2*(a + b*x)] + 4*Sin[2*(c + b*x)] +
Sin[2*(a + c + 2*b*x)])/(32*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cos^2(bx+c)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{8}\cos(2(a+c)+4bx)+\frac{1}{4}\cos(2a+2bx)+\frac{1}{8}\cos(2a-2c)+\frac{1}{4}\cos(2bx+2c)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(2(a+c)+4bx)}{32b}+\frac{\sin(2a+2bx)}{8b}+\frac{1}{8}x(\cos(2(a-c))+2)+\frac{\sin(2bx+2c)}{8b}$$

input
```
Int[Cos[a + b*x]^2*Cos[c + b*x]^2,x]
```

output
```
(x*(2 + Cos[2*(a - c)]))/8 + Sin[2*a + 2*b*x]/(8*b) + Sin[2*c + 2*b*x]/(8*
b) + Sin[2*(a + c) + 4*b*x]/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

method	result
default	$\frac{x}{4}+\frac{x\cos(2a-2c)}{8}+\frac{\sin(2bx+2a)}{8b}+\frac{\sin(2bx+2c)}{8b}+\frac{\sin(4bx+2a+2c)}{32b}$
risch	$\frac{x}{4}+\frac{x\cos(2a-2c)}{8}+\frac{\sin(2bx+2a)}{8b}+\frac{\sin(2bx+2c)}{8b}+\frac{\sin(4bx+2a+2c)}{32b}$
parallelrisch	$\frac{8bx+4bx\cos(2a-2c)+\sin(4bx+2a+2c)-5\sin(2a-2c)+4\sin(2bx+2a)+4\sin(2bx+2c)}{32b}$
orering	$x\cos\left(bx+a\right)^{2}\cos\left(bx+c\right)^{2}-\frac{5\left(-2\cos(bx+a)\cos(bx+c)^{2}b\sin(bx+a)-2\cos(bx+a)^{2}\cos(bx+c)b\sin(bx+c)\right)}{16b^{2}}+$

input
```
int(cos(b*x+a)^2*cos(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x+1/8*x*cos(2*a-2*c)+1/8*sin(2*b*x+2*a)/b+1/8*sin(2*b*x+2*c)/b+1/32*si
n(4*b*x+2*a+2*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int\cos^2(a+bx)\cos^2(c+bx)\,dx = -\frac{4\cos\left(bx+c\right)^{4}\cos\left(-a+c\right)\sin\left(-a+c\right)-2\,bx\cos\left(-a+c\right)^{2}-bx-\left(2\left(2\cos\left(-a+c\right)^{2}-1\right)\cos\left(bx\right.\right.}{8\,b}$$

input
```
integrate(cos(b*x+a)^2*cos(b*x+c)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*cos(b*x + c)^4*cos(-a + c)*sin(-a + c) - 2*b*x*cos(-a + c)^2 - b*x
 - (2*(2*cos(-a + c)^2 - 1)*cos(b*x + c)^3 + (2*cos(-a + c)^2 + 1)*cos(b*x
 + c))*sin(b*x + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(56) = 112$.

Time = 0.84 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.09

$$\int \cos^2(a+bx)\cos^2(c+bx)\,dx$$
$$= \begin{cases} \frac{3x\sin^2(a+bx)\sin^2(bx+c)}{8} + \frac{x\sin^2(a+bx)\cos^2(bx+c)}{8} + \frac{x\sin(a+bx)\sin(bx+c)\cos(a+bx)\cos(bx+c)}{2} + \frac{x\sin^2(bx+c)\cos^2(a+bx)}{8} + \\ x\cos^2(a)\cos^2(c) \end{cases}$$

input
```
integrate(cos(b*x+a)**2*cos(b*x+c)**2,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**2*sin(b*x + c)**2/8 + x*sin(a + b*x)**2*cos(b
*x + c)**2/8 + x*sin(a + b*x)*sin(b*x + c)*cos(a + b*x)*cos(b*x + c)/2 + x
*sin(b*x + c)**2*cos(a + b*x)**2/8 + 3*x*cos(a + b*x)**2*cos(b*x + c)**2/8
 + sin(a + b*x)**2*sin(b*x + c)*cos(b*x + c)/(2*b) - sin(a + b*x)*sin(b*x
+ c)**2*cos(a + b*x)/(8*b) + 5*sin(a + b*x)*cos(a + b*x)*cos(b*x + c)**2/(
8*b), Ne(b, 0)), (x*cos(a)**2*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \cos^2(a+bx)\cos^2(c+bx)\,dx$$
$$= \frac{4\,(b\cos(-2\,a+2\,c)+2\,b)x+\sin(4\,bx+2\,a+2\,c)+4\,\sin(2\,bx+2\,a)+4\,\sin(2\,bx+2\,c)}{32\,b}$$

input
```
integrate(cos(b*x+a)^2*cos(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/32*(4*(b*cos(-2*a + 2*c) + 2*b)*x + sin(4*b*x + 2*a + 2*c) + 4*sin(2*b*x
 + 2*a) + 4*sin(2*b*x + 2*c))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \cos^2(a+bx)\cos^2(c+bx)\,dx = \frac{1}{8}\,x\cos\left(2\,a-2\,c\right)+\frac{1}{4}\,x+\frac{\sin\left(4\,bx+2\,a+2\,c\right)}{32\,b}+\frac{\sin\left(2\,bx+2\,a\right)}{8\,b}+\frac{\sin\left(2\,bx+2\,c\right)}{8\,b}$$

input
```
integrate(cos(b*x+a)^2*cos(b*x+c)^2,x, algorithm="giac")
```

output
```
1/8*x*cos(2*a - 2*c) + 1/4*x + 1/32*sin(4*b*x + 2*a + 2*c)/b + 1/8*sin(2*b
*x + 2*a)/b + 1/8*sin(2*b*x + 2*c)/b
```

Mupad [B] (verification not implemented)

Time = 19.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int \cos^2(a+bx)\cos^2(c+bx)\,dx = \frac{\frac{\sin(2\,a+2\,c+4\,b\,x)}{4}+\sin\left(2\,a+2\,b\,x\right)+\sin\left(2\,c+2\,b\,x\right)+2\,b\,x+b\,x\,\cos\left(2\,a-2\,c\right)}{8\,b}$$

input
```
int(cos(a + b*x)^2*cos(c + b*x)^2,x)
```

output
```
(sin(2*a + 2*c + 4*b*x)/4 + sin(2*a + 2*b*x) + sin(2*c + 2*b*x) + 2*b*x +
b*x*cos(2*a - 2*c))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.76

$$\int \cos^2(a+bx)\cos^2(c+bx)\,dx$$

$$= \frac{12\cos(bx+c)\cos(bx+a)\sin(bx+c)\sin(bx+a)\,bx-2\cos(bx+c)\sin(bx+c)\sin(bx+a)^2+7\cos(\ldots}{}$$

input
```
int(cos(b*x+a)^2*cos(b*x+c)^2,x)
```

output
```
(12*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)*sin(a + b*x)*b*x - 2*cos(b*x +
c)*sin(b*x + c)*sin(a + b*x)**2 + 7*cos(b*x + c)*sin(b*x + c) - 8*cos(b*x
+ c)*sin(a + b*x) - 4*cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x) + 8*cos(a
+ b*x)*sin(b*x + c) + 8*cos(a + b*x)*sin(a + b*x) + 12*sin(b*x + c)**2*sin
(a + b*x)**2*b*x - 6*sin(b*x + c)**2*b*x - 6*sin(a + b*x)**2*b*x + 9*b*x)/
(24*b)
```

3.306 $\int \cos^2(a+bx)\cos(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \cos^2(a+bx)\cos(c+bx)\,dx=\frac{\sin(2a-c+bx)}{4b}+\frac{\sin(c+bx)}{2b}+\frac{\sin(2a+c+3bx)}{12b}$$

output
```
1/4*sin(b*x+2*a-c)/b+1/2*sin(b*x+c)/b+1/12*sin(3*b*x+2*a+c)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \cos^2(a+bx)\cos(c+bx)\,dx=\frac{3\sin(2a-c+bx)+6\sin(c+bx)+\sin(2a+c+3bx)}{12b}$$

input
```
Integrate[Cos[a + b*x]^2*Cos[c + b*x],x]
```

output
```
(3*Sin[2*a - c + b*x] + 6*Sin[c + b*x] + Sin[2*a + c + 3*b*x])/(12*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cos(bx+c)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{4}\cos(2a+bx-c)+\frac{1}{4}\cos(2a+3bx+c)+\frac{1}{2}\cos(bx+c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(2a+bx-c)}{4b}+\frac{\sin(2a+3bx+c)}{12b}+\frac{\sin(bx+c)}{2b}$$

input
```
Int[Cos[a + b*x]^2*Cos[c + b*x],x]
```

output
```
Sin[2*a - c + b*x]/(4*b) + Sin[c + b*x]/(2*b) + Sin[2*a + c + 3*b*x]/(12*b
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result
default	$\frac{\sin(bx+2a-c)}{4b}+\frac{\sin(bx+c)}{2b}+\frac{\sin(3bx+2a+c)}{12b}$
risch	$\frac{\sin(bx+2a-c)}{4b}+\frac{\sin(bx+c)}{2b}+\frac{\sin(3bx+2a+c)}{12b}$
parallelrisch	$\frac{6\sin(bx+c)+8\sin(a-c)+3\sin(bx+2a-c)+\sin(3bx+2a+c)}{12b}$
orering	$-\frac{10\left(-2\cos(bx+a)\cos(bx+c)b\sin(bx+a)-\cos(bx+a)^{2}b\sin(bx+c)\right)}{9b^{2}}-\frac{14b^{3}\sin(bx+a)\cos(bx+c)\cos(bx+a)-6b^{3}\sin(bx+c}{9b^{4}}$
norman	$\frac{\frac{2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}\tan\left(\frac{bx}{2}+\frac{c}{2}\right)}{b}+\frac{4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}\tan\left(\frac{bx}{2}+\frac{c}{2}\right)}{b}-\frac{2\tan\left(\frac{bx}{2}+\frac{c}{2}\right)}{3b}+\frac{8\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{3b}-\frac{8\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}}{3b}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}\right)^{2}\left(1+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}\right)}$

input
```
int(cos(b*x+a)^2*cos(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(b*x+2*a-c)/b+1/2*sin(b*x+c)/b+1/12*sin(3*b*x+2*a+c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \cos^2(a+bx)\cos(c+bx)\,dx =$$
$$-\frac{2\cos(bx+c)^3\cos(-a+c)\sin(-a+c)-\left(\left(2\cos(-a+c)^2-1\right)\cos(bx+c)^2+\cos(-a+c)^2+1\right)s}{3b}$$

input
```
integrate(cos(b*x+a)^2*cos(b*x+c),x, algorithm="fricas")
```

output
```
-1/3*(2*cos(b*x + c)^3*cos(-a + c)*sin(-a + c) - ((2*cos(-a + c)^2 - 1)*co
s(b*x + c)^2 + cos(-a + c)^2 + 1)*sin(b*x + c))/b
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int \cos^2(a+bx)\cos(c+bx)\,dx = \begin{cases} \frac{2\sin^2(a+bx)\sin(bx+c)}{3b} + \frac{2\sin(a+bx)\cos(a+bx)\cos(bx+c)}{3b} + \frac{\sin(bx+c)\cos^2(a+bx)}{3b} & \text{for } b\neq 0 \\ x\cos^2(a)\cos(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**2*cos(b*x+c),x)
```

output
```
Piecewise((2*sin(a + b*x)**2*sin(b*x + c)/(3*b) + 2*sin(a + b*x)*cos(a + b
*x)*cos(b*x + c)/(3*b) + sin(b*x + c)*cos(a + b*x)**2/(3*b), Ne(b, 0)), (x
*cos(a)**2*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \cos^2(a+bx)\cos(c+bx)\,dx = \frac{\sin(3\,bx+2\,a+c)+3\,\sin(bx+2\,a-c)+6\,\sin(bx+c)}{12\,b}$$

input
```
integrate(cos(b*x+a)^2*cos(b*x+c),x, algorithm="maxima")
```

output
```
1/12*(sin(3*b*x + 2*a + c) + 3*sin(b*x + 2*a - c) + 6*sin(b*x + c))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\cos(c+bx)\,dx = \frac{\sin(3\,bx+2\,a+c)}{12\,b} + \frac{\sin(bx+2\,a-c)}{4\,b} + \frac{\sin(bx+c)}{2\,b}$$

input
```
integrate(cos(b*x+a)^2*cos(b*x+c),x, algorithm="giac")
```

output
```
1/12*sin(3*b*x + 2*a + c)/b + 1/4*sin(b*x + 2*a - c)/b + 1/2*sin(b*x + c)/
b
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \cos^2(a+bx)\cos(c+bx)\,dx = \frac{\sin(2a+c+3bx)+6\sin(c+bx)+3\sin(2a-c+bx)}{12b}$$

input
```
int(cos(a + b*x)^2*cos(c + b*x),x)
```

output
```
(sin(2*a + c + 3*b*x) + 6*sin(c + b*x) + 3*sin(2*a - c + b*x))/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \cos^2(a+bx)\cos(c+bx)\,dx = \frac{2\cos(bx+c)\cos(bx+a)\sin(bx+a)+2\cos(bx+c)\sin(bx+a)-2\cos(bx+a)\sin(bx+c)+\sin(bx}{3b}$$

input
```
int(cos(b*x+a)^2*cos(b*x+c),x)
```

output
```
(2*cos(b*x + c)*cos(a + b*x)*sin(a + b*x) + 2*cos(b*x + c)*sin(a + b*x) -
2*cos(a + b*x)*sin(b*x + c) + sin(b*x + c)*sin(a + b*x)**2 + sin(b*x + c))
/(3*b)
```

3.307 $\int \cos^2(a+bx)\sec(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \cos^2(a+bx)\sec(c+bx)\,dx=\frac{\operatorname{arctanh}(\sin(c+bx))\sin^2(a-c)}{b}+\frac{\sin(2a-c+bx)}{b}$$

output
```
arctanh(sin(b*x+c))*sin(a-c)^2/b+sin(b*x+2*a-c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 127 vs. 2(35) = 70.

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.63

$$\begin{aligned}\int \cos^2(a+bx)\sec(c+bx)\,dx=&\frac{(-1+\cos(2a-2c))\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)-\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{2b}\\&+\frac{(1-\cos(2a-2c))\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)+\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{2b}\\&+\frac{\cos(bx)\sin(2a-c)}{b}+\frac{\cos(2a-c)\sin(bx)}{b}\end{aligned}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c + b*x],x]
```

output
```
((-1 + Cos[2*a - 2*c])*Log[Cos[c/2 + (b*x)/2] - Sin[c/2 + (b*x)/2]])/(2*b)
 + ((1 - Cos[2*a - 2*c])*Log[Cos[c/2 + (b*x)/2] + Sin[c/2 + (b*x)/2]])/(2*
b) + (Cos[b*x]*Sin[2*a - c])/b + (Cos[2*a - c]*Sin[b*x])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos^2(a+bx)\sec(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 4.14

method	result
risch	$-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a-c)}\right)}{2b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a-c)}\right)\cos(2a-2c)}{2b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a-c)}\right)}{2b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a-c)}\right)\cos(2a-2c)}{2b}$
default	$\frac{-\frac{2\left((-\cos(a)\cos(c)-\sin(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin(a)\cos(c)+\cos(a)\sin(c)\right)}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)}+\frac{2\left(-\cos(c)^2\sin(a)^2+2\cos(a)\cos(c)\sin(a)\sin(c)-\sin(c\right.}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(c\right.}}{b}$

input
```
int(cos(b*x+a)^2*sec(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))+1/2/b*ln(exp(I*(b*x+a))-I*exp(I*(
a-c)))*cos(2*a-2*c)+1/2/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))-1/2/b*ln(exp(I
*(b*x+a))+I*exp(I*(a-c)))*cos(2*a-2*c)+sin(b*x+2*a-c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.54

$$\int \cos^2(a+bx)\sec(c+bx)\,dx = -\frac{4\cos(bx+c)\cos(-a+c)\sin(-a+c)+\left(\cos(-a+c)^2-1\right)\log(\sin(bx+c)+1)-\left(\cos(-a+c)^2\right.}{2b}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(4*cos(b*x + c)*cos(-a + c)*sin(-a + c) + (cos(-a + c)^2 - 1)*log(sin
(b*x + c) + 1) - (cos(-a + c)^2 - 1)*log(-sin(b*x + c) + 1) - 2*(2*cos(-a
+ c)^2 - 1)*sin(b*x + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(27) = 54$.

Time = 19.02 (sec) , antiderivative size = 3645, normalized size of antiderivative = 104.14

$$\int \cos^2(a + bx) \sec(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**2*sec(b*x+c),x)
```

output
```
-2*Piecewise((-sin(b*x)/b, Eq(c, pi/2)), (sin(b*x)/b, Eq(c, -pi/2)), (0, E
q(b, 0)), (-2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))
*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 +
2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2
*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**3/
(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)*
*2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/
(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*t
an(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2
)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) -
 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 +
 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) +
2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3
*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)
**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x
/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(c/2)*
*4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan
(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) +
1) - 1/(tan(c/2) + 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2
 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(35) = 70$.

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 4.00

$$\int \cos^2(a+bx)\sec(c+bx)\,dx$$
$$= \frac{(\cos(-2a+2c)-1)\log\left(\frac{\cos(bx+2c)^2+\cos(c)^2-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2+2\cos(bx+2c)\sin(c)+\sin(c)^2}{\cos(bx+2c)^2+\cos(c)^2+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2-2\cos(bx+2c)\sin(c)+\sin(c)^2}\right)+4\sin(bx}{4b}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c),x, algorithm="maxima")
```

output
```
1/4*((cos(-2*a + 2*c) - 1)*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin
(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(
b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*c
os(b*x + 2*c)*sin(c) + sin(c)^2)) + 4*sin(b*x + 2*a - c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1708 vs. $2(35) = 70$.

Time = 0.22 (sec) , antiderivative size = 1708, normalized size of antiderivative = 48.80

$$\int \cos^2(a+bx)\sec(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c),x, algorithm="giac")
```

output
```
-2*(2*(tan(1/2*a)^5*tan(1/2*c)^3 - 2*tan(1/2*a)^4*tan(1/2*c)^4 + tan(1/2*a
)^3*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^2 + 3*tan(1/2*a)^4*tan(1/2*c)^3
 - 3*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^2*tan(1/2*c)^5 + 3*tan(1/2*a)^
4*tan(1/2*c)^2 - 6*tan(1/2*a)^3*tan(1/2*c)^3 + 3*tan(1/2*a)^2*tan(1/2*c)^4
 - 2*tan(1/2*a)^4*tan(1/2*c) + 6*tan(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^
2*tan(1/2*c)^3 + 2*tan(1/2*a)*tan(1/2*c)^4 + 3*tan(1/2*a)^3*tan(1/2*c) - 6
*tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 + 3*
tan(1/2*a)^2*tan(1/2*c) - 3*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1
/2*a)^2 - 2*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2)*log(abs(-tan(1/2*b*x + 1
/2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*b*
x + 1/2*a)*tan(1/2*c) + tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan
(1/2*a) - tan(1/2*c) + 1))/(tan(1/2*a)^5*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1
/2*c)^4 + tan(1/2*a)^4*tan(1/2*c)^5 + 2*tan(1/2*a)^5*tan(1/2*c)^3 + tan(1/
2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)^5 - 2*tan(1/2*a)^5*tan(1/2
*c)^2 + 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(1/2*a)^3*tan(1/2*c)^4 + 2*tan(
1/2*a)^2*tan(1/2*c)^5 + tan(1/2*a)^5*tan(1/2*c) + 2*tan(1/2*a)^4*tan(1/2*c
)^2 + 4*tan(1/2*a)^3*tan(1/2*c)^3 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*
a)*tan(1/2*c)^5 - tan(1/2*a)^5 + tan(1/2*a)^4*tan(1/2*c) - 4*tan(1/2*a)^3*
tan(1/2*c)^2 + 4*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 + tan
(1/2*c)^5 + tan(1/2*a)^4 + 2*tan(1/2*a)^3*tan(1/2*c) + 4*tan(1/2*a)^2*t...
```

Mupad [B] (verification not implemented)

Time = 19.14 (sec) , antiderivative size = 217, normalized size of antiderivative = 6.20

$$\int \cos^2(a+bx)\sec(c+bx)\,dx = \frac{e^{-a\,2i+c\,1i-b\,x\,1i}\,1i}{2\,b} - \frac{e^{a\,2i-c\,1i+b\,x\,1i}\,1i}{2\,b} + \frac{e^{-a\,2i+c\,2i}\,\ln\left(-\frac{\left(e^{a\,2i}\,e^{-c\,2i}-1\right)^2\,1i}{2} + \frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(1+e^{a\,4i}\,e^{-c\,4i}-2\,e^{a\,2i}\,e^{-c\,2i}\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}-1\right)^2}{4\,b} - \frac{e^{-a\,2i+c\,2i}\,\ln\left(\frac{\left(e^{a\,2i}\,e^{-c\,2i}-1\right)^2\,1i}{2} + \frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(1+e^{a\,4i}\,e^{-c\,4i}-2\,e^{a\,2i}\,e^{-c\,2i}\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}-1\right)^2}{4\,b}$$

input
```
int(cos(a + b*x)^2/cos(c + b*x),x)
```

output
```
(exp(c*1i - a*2i - b*x*1i)*1i)/(2*b) - (exp(a*2i - c*1i + b*x*1i)*1i)/(2*b
) + (exp(c*2i - a*2i)*log((exp(c*1i)*exp(b*x*1i)*(exp(a*4i)*exp(-c*4i) - 2
*exp(a*2i)*exp(-c*2i) + 1))/2 - ((exp(a*2i)*exp(-c*2i) - 1)^2*1i)/2)*(exp(
a*2i - c*2i) - 1)^2)/(4*b) - (exp(c*2i - a*2i)*log(((exp(a*2i)*exp(-c*2i)
- 1)^2*1i)/2 + (exp(c*1i)*exp(b*x*1i)*(exp(a*4i)*exp(-c*4i) - 2*exp(a*2i)*
exp(-c*2i) + 1))/2)*(exp(a*2i - c*2i) - 1)^2)/(4*b)
```

Reduce [F]

$$\int \cos^2(a+bx)\sec(c+bx)\,dx=\int \cos\left(bx+a\right)^2\sec\left(bx+c\right)dx$$

input
```
int(cos(b*x+a)^2*sec(b*x+c),x)
```

output
```
int(cos(a + b*x)**2*sec(b*x + c),x)
```

3.308 $\int \cos^2(a+bx)\sec^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \cos^2(a+bx)\sec^2(c+bx)\,dx = x\cos(2(a-c)) + \frac{\log(\cos(c+bx))\sin(2(a-c))}{b} + \frac{\sin^2(a-c)\tan(c+bx)}{b}$$

output
```
x*cos(2*a-2*c)+ln(cos(b*x+c))*sin(2*a-2*c)/b+sin(a-c)^2*tan(b*x+c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(48) = 96.

Time = 0.41 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.69

$$\int \cos^2(a+bx)\sec^2(c+bx)\,dx$$
$$= \frac{\sec(c)\sec(c+bx)(bx\cos(2a-4c-bx)+bx\cos(2a-2c-bx)+bx\cos(2a+bx)+bx\cos(2a-2c+bx)}{}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c + b*x]^2,x]
```

output
```
(Sec[c]*Sec[c + b*x]*(b*x*Cos[2*a - 4*c - b*x] + b*x*Cos[2*a - 2*c - b*x]
+ b*x*Cos[2*a + b*x] + b*x*Cos[2*a - 2*c + b*x] + 2*Sin[b*x] + Log[Cos[c +
 b*x]]*Sin[2*a - 4*c - b*x] + Sin[2*a - 2*c - b*x] + Log[Cos[c + b*x]]*Sin
[2*a - 2*c - b*x] + Log[Cos[c + b*x]]*Sin[2*a + b*x] - Sin[2*a - 2*c + b*x
] + Log[Cos[c + b*x]]*Sin[2*a - 2*c + b*x]))/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec^2(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \cos^2(a+bx)\sec^2(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.76 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.44

method	result
risch	$x\,\mathrm{e}^{2i(a-c)}-2i\sin\left(2a-2c\right)x-\frac{2i\sin(2a-2c)a}{b}-\frac{i\mathrm{e}^{2i(2a-c)}}{2b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{i\mathrm{e}^{2ia}}{b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}-\frac{i\mathrm{e}^{2ic}}{2b\left(\mathrm{e}^{2i(bx+a+c)}\right.}$
default	$-\frac{\cos(c)^2\sin(a)^2-2\cos(a)\cos(c)\sin(a)\sin(c)+\sin(c)^2\cos(a)^2}{\left(\cos(a)^2+\sin(a)^2\right)\left(\cos(c)^2+\sin(c)^2\right)(\sin(a)\cos(c)-\cos(a)\sin(c))(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}$

input
```
int(cos(b*x+a)^2*sec(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
x*exp(2*I*(a-c))-2*I*sin(2*a-2*c)*x-2*I/b*sin(2*a-2*c)*a-1/2*I/b/(exp(2*I*
(b*x+a+c))+exp(2*I*a))*exp(2*I*(2*a-c))+I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a)
)*exp(2*I*a)-1/2*I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))*exp(2*I*c)+ln(exp(2*I
*(b*x+a))+exp(2*I*(a-c)))/b*sin(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.77

$$\int \cos^2(a+bx)\sec^2(c+bx)\,dx =$$
$$-\frac{2\cos\left(bx+c\right)\cos\left(-a+c\right)\log\left(-\cos\left(bx+c\right)\right)\sin\left(-a+c\right)-\left(2\,bx\cos\left(-a+c\right)^2-bx\right)\cos\left(bx+c\right)+}{b\cos\left(bx+c\right)}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^2,x, algorithm="fricas")
```

output
```
-(2*cos(b*x + c)*cos(-a + c)*log(-cos(b*x + c))*sin(-a + c) - (2*b*x*cos(-
a + c)^2 - b*x)*cos(b*x + c) + (cos(-a + c)^2 - 1)*sin(b*x + c))/(b*cos(b*
x + c))
```

Sympy [F(-2)]

Exception generated.

$$\int \cos^2(a + bx) \sec^2(c + bx) \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(cos(b*x+a)**2*sec(b*x+c)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(48) = 96$.

Time = 0.06 (sec) , antiderivative size = 534, normalized size of antiderivative = 11.12

$$\int \cos^2(a + bx) \sec^2(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(b*cos(2*a + 2*c)*cos(4*c) + b*sin(2*a + 2*c)*sin(4*c))*x + (2*b*x*
cos(4*c) + sin(4*a) - 2*sin(2*a + 2*c) + sin(4*c))*cos(2*b*x + 2*a + 4*c)
+ 2*(b*x*cos(2*b*x + 2*a + 4*c) + b*x*cos(2*a + 2*c))*cos(2*b*x + 6*c) + (
sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(2*b*x + 2*a + 4*c)^2*sin(-2*a +
 2*c) + 2*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c)*sin(-2*a + 2*c) + cos(2*a
+ 2*c)^2*sin(-2*a + 2*c) + sin(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c) + 2*si
n(2*b*x + 2*a + 4*c)*sin(2*a + 2*c)*sin(-2*a + 2*c) + sin(2*a + 2*c)^2*sin
(-2*a + 2*c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(
2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2) + (2*b*x*sin(4*c) - cos(4*a
) + 2*cos(2*a + 2*c) - cos(4*c))*sin(2*b*x + 2*a + 4*c) + 2*(b*x*sin(2*b*x
 + 2*a + 4*c) + b*x*sin(2*a + 2*c))*sin(2*b*x + 6*c) - (cos(4*a) + cos(4*c
))*sin(2*a + 2*c))/(b*cos(2*b*x + 2*a + 4*c)^2 + 2*b*cos(2*b*x + 2*a + 4*c
)*cos(2*a + 2*c) + b*cos(2*a + 2*c)^2 + b*sin(2*b*x + 2*a + 4*c)^2 + 2*b*s
in(2*b*x + 2*a + 4*c)*sin(2*a + 2*c) + b*sin(2*a + 2*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1954 vs. $2(48) = 96$.

Time = 0.21 (sec) , antiderivative size = 1954, normalized size of antiderivative = 40.71

$$\int \cos^2(a+bx)\sec^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^2,x, algorithm="giac")
```

output
```
((tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*a)^4*tan(1/2*c)^2 + 16*tan(1/2*a)^
3*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 16*tan(1/2*a
)^3*tan(1/2*c) + 36*tan(1/2*a)^2*tan(1/2*c)^2 - 16*tan(1/2*a)*tan(1/2*c)^3
 + tan(1/2*c)^4 - 6*tan(1/2*a)^2 + 16*tan(1/2*a)*tan(1/2*c) - 6*tan(1/2*c)
^2 + 1)*(b*x + a)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2
 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^
2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4*
tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*tan
(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*
c)^4 - tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)^
2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))*log(tan(b*x + a)^2 + 1)/(tan(1
/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/
2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan
(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) + 4*(tan(1/2*a)^6*tan(1/2*c)^4 - 2*tan(1/2
*a)^5*tan(1/2*c)^5 + tan(1/2*a)^4*tan(1/2*c)^6 - tan(1/2*a)^6*tan(1/2*c)^2
 + 8*tan(1/2*a)^5*tan(1/2*c)^3 - 14*tan(1/2*a)^4*tan(1/2*c)^4 + 8*tan(1/2*
a)^3*tan(1/2*c)^5 - tan(1/2*a)^2*tan(1/2*c)^6 - 2*tan(1/2*a)^5*tan(1/2*c)
+ 14*tan(1/2*a)^4*tan(1/2*c)^2 - 24*tan(1/2*a)^3*tan(1/2*c)^3 + 14*tan(1/2
*a)^2*tan(1/2*c)^4 - 2*tan(1/2*a)*tan(1/2*c)^5 - tan(1/2*a)^4 + 8*tan(1/2*
a)^3*tan(1/2*c) - 14*tan(1/2*a)^2*tan(1/2*c)^2 + 8*tan(1/2*a)*tan(1/2*c...
```

Mupad [B] (verification not implemented)

Time = 19.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.02

$$\int \cos^2(a+bx)\sec^2(c+bx)\,dx$$
$$= x\left(\cos\left(2\,a-2\,c\right)-\sin\left(2\,a-2\,c\right)\,1\mathrm{i}\right)-\frac{\left(1+\mathrm{e}^{a\,4\mathrm{i}-c\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}\right)\,1\mathrm{i}}{2\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}$$
$$+\frac{\mathrm{e}^{-a\,4\mathrm{i}+c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\right)\,\left(2\,b\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-2\,b\,\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$

input
```
int(cos(a + b*x)^2/cos(c + b*x)^2,x)
```

output
```
x*(cos(2*a - 2*c) - sin(2*a - 2*c)*1i) - ((exp(a*4i - c*4i) - 2*exp(a*2i -
 c*2i) + 1)*1i)/(2*b*(exp(a*2i - c*2i) + exp(a*2i + b*x*2i))) + (exp(c*4i
- a*4i)*log(exp(a*2i)*exp(b*x*2i) + exp(a*2i)*exp(-c*2i))*(2*b*exp(a*2i -
c*2i) - 2*b*exp(a*6i - c*6i))*1i)/(4*b^2)
```

Reduce [F]

$$\int \cos^2(a+bx)\sec^2(c+bx)\,dx = \text{Too large to display}$$

input
```
int(cos(b*x+a)^2*sec(b*x+c)^2,x)
```

output

```
( - 7*cos(b*x + c)*cos(a + b*x)*sin(a + b*x) - 96*cos(b*x + c)*int(tan((b*
x + c)/2)**2/(tan((b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)
**4*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan(
(a + b*x)/2)**4 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x +
 c)/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b - 96*cos
(b*x + c)*int(tan((a + b*x)/2)**2/(tan((b*x + c)/2)**4*tan((a + b*x)/2)**4
 + 2*tan((b*x + c)/2)**4*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan
((b*x + c)/2)**2*tan((a + b*x)/2)**4 - 4*tan((b*x + c)/2)**2*tan((a + b*x)
/2)**2 - 2*tan((b*x + c)/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)*
*2 + 1),x)*b + 128*cos(b*x + c)*int((tan((b*x + c)/2)*tan((a + b*x)/2))/(t
an((b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)**4*tan((a + b*
x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan((a + b*x)/2)**4
 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x + c)/2)**2 + tan
((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b + 32*cos(b*x + c)*int(1
/(tan((b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)**4*tan((a +
 b*x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan((a + b*x)/2)
**4 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x + c)/2)**2 +
tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b - 16*cos(b*x + c)*si
n(a + b*x) - 9*cos(b*x + c)*a - 9*cos(b*x + c)*b*x + 8*cos(a + b*x)*sin(b*
x + c) - 8*cos(a + b*x)*sin(a + b*x) - 4*sin(b*x + c)*sin(a + b*x)**2 +...
```

3.309 $\int \cos^2(a+bx)\sec^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx = \frac{\operatorname{arctanh}(\sin(c+bx))\cos(2(a-c))}{b} + \frac{\operatorname{arctanh}(\sin(c+bx))\sin^2(a-c)}{2b} - \frac{\sec(c+bx)\sin(2(a-c))}{b} + \frac{\sec(c+bx)\sin^2(a-c)\tan(c+bx)}{2b}$$

output
```
arctanh(sin(b*x+c))*cos(2*a-2*c)/b+1/2*arctanh(sin(b*x+c))*sin(a-c)^2/b-se
c(b*x+c)*sin(2*a-2*c)/b+1/2*sec(b*x+c)*sin(a-c)^2*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.88

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx = \frac{-\log\left(\cos\left(\frac{1}{2}(c+bx)\right)-\sin\left(\frac{1}{2}(c+bx)\right)\right)-3\cos(2(a-c))\left(\log\left(\cos\left(\frac{1}{2}(c+bx)\right)-\sin\left(\frac{1}{2}(c+bx)\right)\right)-\text{lo}\right.}{}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c + b*x]^3,x]
```

output
```
(-Log[Cos[(c + b*x)/2] - Sin[(c + b*x)/2]] - 3*Cos[2*(a - c)]*(Log[Cos[(c
+ b*x)/2] - Sin[(c + b*x)/2]] - Log[Cos[(c + b*x)/2] + Sin[(c + b*x)/2]])
+ Log[Cos[(c + b*x)/2] + Sin[(c + b*x)/2]] + 4*Sec[c]*Sin[2*(a - c)] - 4*S
ec[c + b*x]*Sin[2*(a - c)] + 2*Sec[c + b*x]*Sin[a - c]^2*Tan[c + b*x])/(4*
b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec^3(bx+c)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \cos^2(a+bx)\sec^3(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.94 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.84

method	result
risch	$\frac{i\left(5\,\mathrm{e}^{i(3bx+6a+c)}-2\,\mathrm{e}^{i(3bx+4a+3c)}-3\,\mathrm{e}^{i(3bx+2a+5c)}+3\,\mathrm{e}^{i(bx+6a-c)}+2\,\mathrm{e}^{i(bx+4a+c)}-5\,\mathrm{e}^{i(bx+2a+3c)}\right)}{4\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^2 b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a-c)}\right)}{4b}-$
default	Expression too large to display

input
```
int(cos(b*x+a)^2*sec(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/(exp(2*I*(b*x+a+c))+exp(2*I*a))^2/b*(5*exp(I*(3*b*x+6*a+c))-2*exp(I*
(3*b*x+4*a+3*c))-3*exp(I*(3*b*x+2*a+5*c))+3*exp(I*(b*x+6*a-c))+2*exp(I*(b*
x+4*a+c))-5*exp(I*(b*x+2*a+3*c)))-1/4/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))-
3/4/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))*cos(2*a-2*c)+1/4/b*ln(exp(I*(b*x+a
))+I*exp(I*(a-c)))+3/4/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))*cos(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx$$
$$=\frac{\left(3\,\cos\left(-a+c\right)^2-1\right)\cos\left(bx+c\right)^2\log\left(\sin\left(bx+c\right)+1\right)-\left(3\,\cos\left(-a+c\right)^2-1\right)\cos\left(bx+c\right)^2\log\left(-\sin\right.}{4\,b\cos\left(bx+c\right.}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*((3*cos(-a + c)^2 - 1)*cos(b*x + c)^2*log(sin(b*x + c) + 1) - (3*cos(-
a + c)^2 - 1)*cos(b*x + c)^2*log(-sin(b*x + c) + 1) + 8*cos(b*x + c)*cos(-
a + c)*sin(-a + c) - 2*(cos(-a + c)^2 - 1)*sin(b*x + c))/(b*cos(b*x + c)^2
)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sec(b*x+c)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1251 vs. 2(84) = 168.

Time = 0.21 (sec) , antiderivative size = 1251, normalized size of antiderivative = 14.22

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^3,x, algorithm="maxima")
```

output
```
-1/8*(2*(5*sin(3*b*x + 4*a + 2*c) - 2*sin(3*b*x + 2*a + 4*c) - 3*sin(3*b*x
 + 6*c) + 3*sin(b*x + 4*a) + 2*sin(b*x + 2*a + 2*c) - 5*sin(b*x + 4*c))*co
s(4*b*x + 2*a + 5*c) - 10*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + c))*cos(3*
b*x + 4*a + 2*c) + 4*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + c))*cos(3*b*x +
 2*a + 4*c) + 6*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + c))*cos(3*b*x + 6*c)
 + 4*(3*sin(b*x + 4*a) + 2*sin(b*x + 2*a + 2*c) - 5*sin(b*x + 4*c))*cos(2*
b*x + 2*a + 3*c) + ((3*cos(-2*a + 2*c) + 1)*cos(4*b*x + 2*a + 5*c)^2 + 4*(
3*cos(-2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c)^2 + (3*cos(-2*a + 2*c) + 1)*
sin(4*b*x + 2*a + 5*c)^2 + 4*(3*cos(-2*a + 2*c) + 1)*sin(2*b*x + 2*a + 3*c
)^2 + 2*(2*(3*cos(-2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c) + 3*cos(2*a + c)
*cos(-2*a + 2*c) + cos(2*a + c))*cos(4*b*x + 2*a + 5*c) + 4*(3*cos(2*a + c
)*cos(-2*a + 2*c) + cos(2*a + c))*cos(2*b*x + 2*a + 3*c) + cos(2*a + c)^2
+ 3*(cos(2*a + c)^2 + sin(2*a + c)^2)*cos(-2*a + 2*c) + 2*(2*(3*cos(-2*a +
 2*c) + 1)*sin(2*b*x + 2*a + 3*c) + 3*cos(-2*a + 2*c)*sin(2*a + c) + sin(2
*a + c))*sin(4*b*x + 2*a + 5*c) + 4*(3*cos(-2*a + 2*c)*sin(2*a + c) + sin(
2*a + c))*sin(2*b*x + 2*a + 3*c) + sin(2*a + c)^2)*log((cos(b*x + 2*c)^2 +
 cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*
sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c)
+ sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - 2*(5*cos(3*b*x
 + 4*a + 2*c) - 2*cos(3*b*x + 2*a + 4*c) - 3*cos(3*b*x + 6*c) + 3*cos(b...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6958 vs. $2(84) = 168$.

Time = 0.44 (sec) , antiderivative size = 6958, normalized size of antiderivative = 79.07

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^3,x, algorithm="giac")
```

output
```
-((tan(1/2*a)^5*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^4 + tan(1/2*a)^4*ta
n(1/2*c)^5 - 4*tan(1/2*a)^5*tan(1/2*c)^3 + 13*tan(1/2*a)^4*tan(1/2*c)^4 -
4*tan(1/2*a)^3*tan(1/2*c)^5 + 4*tan(1/2*a)^5*tan(1/2*c)^2 - 16*tan(1/2*a)^
4*tan(1/2*c)^3 + 16*tan(1/2*a)^3*tan(1/2*c)^4 - 4*tan(1/2*a)^2*tan(1/2*c)^
5 + tan(1/2*a)^5*tan(1/2*c) - 16*tan(1/2*a)^4*tan(1/2*c)^2 + 40*tan(1/2*a)
^3*tan(1/2*c)^3 - 16*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)*tan(1/2*c)^5 -
 tan(1/2*a)^5 + 13*tan(1/2*a)^4*tan(1/2*c) - 40*tan(1/2*a)^3*tan(1/2*c)^2
+ 40*tan(1/2*a)^2*tan(1/2*c)^3 - 13*tan(1/2*a)*tan(1/2*c)^4 + tan(1/2*c)^5
 + tan(1/2*a)^4 - 16*tan(1/2*a)^3*tan(1/2*c) + 40*tan(1/2*a)^2*tan(1/2*c)^
2 - 16*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 + 4*tan(1/2*a)^3 - 16*tan(1/
2*a)^2*tan(1/2*c) + 16*tan(1/2*a)*tan(1/2*c)^2 - 4*tan(1/2*c)^3 - 4*tan(1/
2*a)^2 + 13*tan(1/2*a)*tan(1/2*c) - 4*tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*
c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x
+ 1/2*a)*tan(1/2*a) - tan(1/2*b*x + 1/2*a)*tan(1/2*c) + tan(1/2*a)*tan(1/2
*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan(1/2*c) + 1))/(tan(1/2*a)^5*t
an(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c)^5 + 2*ta
n(1/2*a)^5*tan(1/2*c)^3 + tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1
/2*c)^5 - 2*tan(1/2*a)^5*tan(1/2*c)^2 + 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*ta
n(1/2*a)^3*tan(1/2*c)^4 + 2*tan(1/2*a)^2*tan(1/2*c)^5 + tan(1/2*a)^5*tan(1
/2*c) + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 4*tan(1/2*a)^3*tan(1/2*c)^3 + 2*t...
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)^2/cos(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx = \int \cos(bx+a)^2 \sec(bx+c)^3\,dx$$

input
```
int(cos(b*x+a)^2*sec(b*x+c)^3,x)
```

output
```
int(cos(a + b*x)**2*sec(b*x + c)**3,x)
```

3.310 $\int \cos^2(a+bx)\sec^4(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx = -\frac{\sec^2(c+bx)\sin(2(a-c))}{2b}+\frac{\cos(2(a-c))\tan(c+bx)}{b}+\frac{\sin^2(a-c)\tan(c+bx)}{b}+\frac{\sin^2(a-c)\tan^3(c+bx)}{3b}$$

output
```
-1/2*sec(b*x+c)^2*sin(2*a-2*c)/b+cos(2*a-2*c)*tan(b*x+c)/b+sin(a-c)^2*tan(
b*x+c)/b+1/3*sin(a-c)^2*tan(b*x+c)^3/b
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx$$
$$= \frac{\sec(c)\sec^3(c+bx)(3\sin(bx)-\sin(2a-4c-3bx)-3\sin(2a-2c-bx)-3\sin(2a+bx)+\sin(2a+3bx}{12b}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c + b*x]^4,x]
```

output
```
(Sec[c]*Sec[c + b*x]^3*(3*Sin[b*x] - Sin[2*a - 4*c - 3*b*x] - 3*Sin[2*a -
2*c - b*x] - 3*Sin[2*a + b*x] + Sin[2*a + 3*b*x] + Sin[2*c + 3*b*x]))/(12*
b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec^4(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \cos^2(a+bx)\sec^4(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c + b*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 5.39 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{1}{3b(\sin(a)\cos(c)-\cos(a)\sin(c))(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))^3}$	56
parallelrisch	$\frac{3\sin(bx+c)+2\sin(3bx+2a+c)+\sin(3bx+3c)}{3b(\cos(3bx+3c)+3\cos(bx+c))}$	56
risch	$\frac{2i\left(3\,e^{2i(2bx+4a+c)}+3\,e^{2i(bx+4a)}+3\,e^{2i(bx+3a+c)}+e^{2i(4a-c)}+e^{6ia}+e^{2i(2a+c)}\right)}{3\left(e^{2i(bx+a+c)}+e^{2ia}\right)^3 b}$	93

input
```
int(cos(b*x+a)^2*sec(b*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(sin(a)*cos(c)-cos(a)*sin(c))/(tan(b*x+a)*sin(a)*cos(c)-tan(b*x+a)*
cos(a)*sin(c)+cos(a)*cos(c)+sin(a)*sin(c))^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx = \frac{3\cos(bx+c)\cos(-a+c)\sin(-a+c)+\left((4\cos(-a+c)^2-1\right)\cos(bx+c)^2-\cos(-a+c)^2+1\right)\sin(}{3\,b\cos(bx+c)^3}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(3*cos(b*x + c)*cos(-a + c)*sin(-a + c) + ((4*cos(-a + c)^2 - 1)*cos(b
*x + c)^2 - cos(-a + c)^2 + 1)*sin(b*x + c))/(b*cos(b*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sec(b*x+c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(79) = 158$.

Time = 0.05 (sec) , antiderivative size = 886, normalized size of antiderivative = 10.67

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^4,x, algorithm="maxima")
```

output
```
-2/3*((3*sin(4*b*x + 4*a + 4*c) + 3*sin(2*b*x + 4*a + 2*c) + 3*sin(2*b*x +
 2*a + 4*c) + sin(4*a) + sin(2*a + 2*c) + sin(4*c))*cos(6*b*x + 2*a + 8*c)
 - 3*(3*sin(2*b*x + 2*a + 4*c) + sin(2*a + 2*c))*cos(4*b*x + 4*a + 4*c) +
3*(3*sin(4*b*x + 4*a + 4*c) + 3*sin(2*b*x + 4*a + 2*c) + 3*sin(2*b*x + 2*a
 + 4*c) + sin(4*a) + sin(2*a + 2*c) + sin(4*c))*cos(4*b*x + 2*a + 6*c) + 3
*(3*sin(2*b*x + 4*a + 2*c) + sin(4*a) + sin(4*c))*cos(2*b*x + 2*a + 4*c) +
 (sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (3*cos(4*b*x + 4*a + 4*c) + 3*cos(
2*b*x + 4*a + 2*c) + 3*cos(2*b*x + 2*a + 4*c) + cos(4*a) + cos(2*a + 2*c)
+ cos(4*c))*sin(6*b*x + 2*a + 8*c) + 3*(3*cos(2*b*x + 2*a + 4*c) + cos(2*a
 + 2*c))*sin(4*b*x + 4*a + 4*c) - 3*(3*cos(4*b*x + 4*a + 4*c) + 3*cos(2*b*
x + 4*a + 2*c) + 3*cos(2*b*x + 2*a + 4*c) + cos(4*a) + cos(2*a + 2*c) + co
s(4*c))*sin(4*b*x + 2*a + 6*c) + 3*cos(2*a + 2*c)*sin(2*b*x + 4*a + 2*c) -
 3*(3*cos(2*b*x + 4*a + 2*c) + cos(4*a) + cos(4*c))*sin(2*b*x + 2*a + 4*c)
 - (cos(4*a) + cos(4*c))*sin(2*a + 2*c) - 3*cos(2*b*x + 4*a + 2*c)*sin(2*a
 + 2*c))/(b*cos(6*b*x + 2*a + 8*c)^2 + 9*b*cos(4*b*x + 2*a + 6*c)^2 + 9*b*
cos(2*b*x + 2*a + 4*c)^2 + 6*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c) + b*c
os(2*a + 2*c)^2 + b*sin(6*b*x + 2*a + 8*c)^2 + 9*b*sin(4*b*x + 2*a + 6*c)^
2 + 9*b*sin(2*b*x + 2*a + 4*c)^2 + 6*b*sin(2*b*x + 2*a + 4*c)*sin(2*a + 2*
c) + b*sin(2*a + 2*c)^2 + 2*(3*b*cos(4*b*x + 2*a + 6*c) + 3*b*cos(2*b*x +
2*a + 4*c) + b*cos(2*a + 2*c))*cos(6*b*x + 2*a + 8*c) + 6*(3*b*cos(2*b*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(79) = 158$.

Time = 0.19 (sec) , antiderivative size = 429, normalized size of antiderivative = 5.17

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^4,x, algorithm="giac")
```

output
```
-1/6*(tan(1/2*a)^8*tan(1/2*c)^8 + 4*tan(1/2*a)^8*tan(1/2*c)^6 + 4*tan(1/2*
a)^6*tan(1/2*c)^8 + 6*tan(1/2*a)^8*tan(1/2*c)^4 + 16*tan(1/2*a)^6*tan(1/2*
c)^6 + 6*tan(1/2*a)^4*tan(1/2*c)^8 + 4*tan(1/2*a)^8*tan(1/2*c)^2 + 24*tan(
1/2*a)^6*tan(1/2*c)^4 + 24*tan(1/2*a)^4*tan(1/2*c)^6 + 4*tan(1/2*a)^2*tan(
1/2*c)^8 + tan(1/2*a)^8 + 16*tan(1/2*a)^6*tan(1/2*c)^2 + 36*tan(1/2*a)^4*t
an(1/2*c)^4 + 16*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*c)^8 + 4*tan(1/2*a)^6
 + 24*tan(1/2*a)^4*tan(1/2*c)^2 + 24*tan(1/2*a)^2*tan(1/2*c)^4 + 4*tan(1/2
*c)^6 + 6*tan(1/2*a)^4 + 16*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(1/2*c)^4 + 4
*tan(1/2*a)^2 + 4*tan(1/2*c)^2 + 1)/((2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*
c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 +
2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*t
an(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)^3*(tan(1/2*a)^2*tan(1/2*c) - tan(
1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)^2/cos(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx = \frac{-\cos(bx+c)\cos(bx+a)\sin(bx+a)+\sin(bx+c)^3+\sin(bx+c)\sin(bx+a)^2-2\sin(bx+c)}{3\cos(bx+c)\,b\left(\sin(bx+c)^2-1\right)}$$

input
```
int(cos(b*x+a)^2*sec(b*x+c)^4,x)
```

output
```
( - cos(b*x + c)*cos(a + b*x)*sin(a + b*x) + sin(b*x + c)**3 + sin(b*x + c
)*sin(a + b*x)**2 - 2*sin(b*x + c))/(3*cos(b*x + c)*b*(sin(b*x + c)**2 - 1
))
```

3.311 $\int \cos^3(a+bx)\cos^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 126

$$\int \cos^3(a+bx)\cos^3(c+bx)\,dx = \frac{1}{32}x(9\cos(a-c)+\cos(3(a-c))) - \frac{3\sin(a-3c-2bx)}{64b} + \frac{3\sin(3a-c+2bx)}{64b} + \frac{9\sin(a+c+2bx)}{64b} + \frac{3\sin(3a+c+4bx)}{128b} + \frac{3\sin(a+3c+4bx)}{128b} + \frac{\sin(3(a+c)+6bx)}{192b}$$

output

```
1/32*x*(9*cos(a-c)+cos(3*a-3*c))-3/64*sin(-2*b*x+a-3*c)/b+3/64*sin(2*b*x+3
*a-c)/b+9/64*sin(2*b*x+a+c)/b+3/128*sin(4*b*x+3*a+c)/b+3/128*sin(4*b*x+a+3
*c)/b+1/192*sin(6*b*x+3*a+3*c)/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81

$$\int \cos^3(a+bx)\cos^3(c+bx)\,dx = \frac{108bx\cos(a-c)+12bx\cos(3(a-c))-18\sin(a-3c-2bx)+18\sin(3a-c+2bx)+54\sin(a+c+2b}{384b}$$

input
```
Integrate[Cos[a + b*x]^3*Cos[c + b*x]^3,x]
```

output
```
(108*b*x*Cos[a - c] + 12*b*x*Cos[3*(a - c)] - 18*Sin[a - 3*c - 2*b*x] + 18
*Sin[3*a - c + 2*b*x] + 54*Sin[a + c + 2*b*x] + 2*Sin[3*(a + c + 2*b*x)] +
 9*Sin[3*a + c + 4*b*x] + 9*Sin[a + 3*c + 4*b*x])/(384*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\cos^3(bx+c)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{3}{32}\cos(a-2bx-3c)+\frac{3}{32}\cos(3a+2bx-c)+\frac{9}{32}\cos(a+2bx+c)+\frac{3}{32}\cos(3a+4bx+c)+\frac{3}{32}\cos(a+4b\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sin(a-2bx-3c)}{64b}+\frac{3\sin(3a+2bx-c)}{64b}+\frac{9\sin(a+2bx+c)}{64b}+\frac{3\sin(3a+4bx+c)}{128b}+\frac{3\sin(a+4bx+3c)}{128b}+\frac{\sin(3(a+c)+6bx)}{192b}+\frac{1}{32}x(9\cos(a-c)+\cos(3(a-c)))$$

input
```
Int[Cos[a + b*x]^3*Cos[c + b*x]^3,x]
```

output
```
(x*(9*Cos[a - c] + Cos[3*(a - c)]))/32 - (3*Sin[a - 3*c - 2*b*x])/(64*b) +
 (3*Sin[3*a - c + 2*b*x])/(64*b) + (9*Sin[a + c + 2*b*x])/(64*b) + (3*Sin[
3*a + c + 4*b*x])/(128*b) + (3*Sin[a + 3*c + 4*b*x])/(128*b) + Sin[3*(a +
c) + 6*b*x]/(192*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 7.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

method	result
default	$\frac{9x\cos(a-c)}{32}+\frac{x\cos(3a-3c)}{32}-\frac{3\sin(-2bx+a-3c)}{64b}+\frac{9\sin(2bx+a+c)}{64b}+\frac{3\sin(2bx+3a-c)}{64b}+\frac{3\sin(4bx+a+3c)}{128b}+$
risch	$\frac{9x\cos(a-c)}{32}+\frac{x\cos(3a-3c)}{32}-\frac{3\sin(-2bx+a-3c)}{64b}+\frac{9\sin(2bx+a+c)}{64b}+\frac{3\sin(2bx+3a-c)}{64b}+\frac{3\sin(4bx+a+3c)}{128b}+$
parallelrisch	$\frac{-29\sin(3a-3c)+12bx\cos(3a-3c)+108x\cos(a-c)b+18\sin(2bx+3a-c)+9\sin(4bx+3a+c)+2\sin(6bx+3a+3c)+54\sin(2bx+}{384b}$
orering	Expression too large to display

input
```
int(cos(b*x+a)^3*cos(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
9/32*x*cos(a-c)+1/32*x*cos(3*a-3*c)-3/64*sin(-2*b*x+a-3*c)/b+9/64*sin(2*b*
x+a+c)/b+3/64*sin(2*b*x+3*a-c)/b+3/128*sin(4*b*x+a+3*c)/b+3/128*sin(4*b*x+
3*a+c)/b+1/192*sin(6*b*x+3*a+3*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.37

$$\int \cos^3(a+bx)\cos^3(c+bx)\,dx$$
$$= \frac{6\,bx\cos{(-a+c)}^3 + 9\,bx\cos{(-a+c)} + \left(8\left(4\,\cos{(-a+c)}^3 - 3\,\cos{(-a+c)}\right)\cos{(bx+c)}^5 + 2\left(2\,\cos{(-\right.}}{}$$

input
```
integrate(cos(b*x+a)^3*cos(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/48*(6*b*x*cos(-a + c)^3 + 9*b*x*cos(-a + c) + (8*(4*cos(-a + c)^3 - 3*co
s(-a + c))*cos(b*x + c)^5 + 2*(2*cos(-a + c)^3 + 3*cos(-a + c))*cos(b*x +
c)^3 + 3*(2*cos(-a + c)^3 + 3*cos(-a + c))*cos(b*x + c))*sin(b*x + c) - 4*
(2*(4*cos(-a + c)^2 - 1)*cos(b*x + c)^6 - 3*(cos(-a + c)^2 - 1)*cos(b*x +
c)^4)*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(117) = 234.

Time = 4.97 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.22

$$\int \cos^3(a+bx)\cos^3(c+bx)\,dx$$
$$= \begin{cases} \frac{5x\sin^3{(a+bx)}\sin^3{(bx+c)}}{16} + \frac{3x\sin^3{(a+bx)}\sin{(bx+c)}\cos^2{(bx+c)}}{16} + \frac{9x\sin^2{(a+bx)}\sin^2{(bx+c)}\cos{(a+bx)}\cos{(bx+c)}}{16} + \frac{3x\sin^2{(a+bx}}{} \\ x\cos^3{(a)}\cos^3{(c)} \end{cases}$$

input
```
integrate(cos(b*x+a)**3*cos(b*x+c)**3,x)
```

output
```
Piecewise((5*x*sin(a + b*x)**3*sin(b*x + c)**3/16 + 3*x*sin(a + b*x)**3*si
n(b*x + c)*cos(b*x + c)**2/16 + 9*x*sin(a + b*x)**2*sin(b*x + c)**2*cos(a
+ b*x)*cos(b*x + c)/16 + 3*x*sin(a + b*x)**2*cos(a + b*x)*cos(b*x + c)**3/
16 + 3*x*sin(a + b*x)*sin(b*x + c)**3*cos(a + b*x)**2/16 + 9*x*sin(a + b*x
)*sin(b*x + c)*cos(a + b*x)**2*cos(b*x + c)**2/16 + 3*x*sin(b*x + c)**2*co
s(a + b*x)**3*cos(b*x + c)/16 + 5*x*cos(a + b*x)**3*cos(b*x + c)**3/16 - 3
*sin(a + b*x)**3*sin(b*x + c)**2*cos(b*x + c)/(16*b) - 5*sin(a + b*x)**3*c
os(b*x + c)**3/(16*b) + sin(a + b*x)**2*sin(b*x + c)**3*cos(a + b*x)/(2*b)
 + 3*sin(a + b*x)**2*sin(b*x + c)*cos(a + b*x)*cos(b*x + c)**2/(4*b) + 19*
sin(b*x + c)**3*cos(a + b*x)**3/(48*b) + 11*sin(b*x + c)*cos(a + b*x)**3*c
os(b*x + c)**2/(16*b), Ne(b, 0)), (x*cos(a)**3*cos(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \cos^3(a+bx)\cos^3(c+bx)\,dx = \frac{12\,(9\,b\cos(-a+c)+b\cos(-3\,a+3\,c))x+2\,\sin(6\,bx+3\,a+3\,c)+9\,\sin(4\,bx+3\,a+c)+9\,\sin(4\,bx}{384\,b}$$

input
```
integrate(cos(b*x+a)^3*cos(b*x+c)^3,x, algorithm="maxima")
```

output
```
1/384*(12*(9*b*cos(-a + c) + b*cos(-3*a + 3*c))*x + 2*sin(6*b*x + 3*a + 3*
c) + 9*sin(4*b*x + 3*a + c) + 9*sin(4*b*x + a + 3*c) + 18*sin(2*b*x + 3*a
- c) + 54*sin(2*b*x + a + c) + 18*sin(2*b*x - a + 3*c))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \cos^3(a+bx)\cos^3(c+bx)\,dx &= \frac{1}{32}\,x\cos(3\,a-3\,c)+\frac{9}{32}\,x\cos(a-c)\\&\quad+\frac{\sin(6\,bx+3\,a+3\,c)}{192\,b}+\frac{3\,\sin(4\,bx+3\,a+c)}{128\,b}\\&\quad+\frac{3\,\sin(4\,bx+a+3\,c)}{128\,b}+\frac{3\,\sin(2\,bx+3\,a-c)}{64\,b}\\&\quad+\frac{9\,\sin(2\,bx+a+c)}{64\,b}-\frac{3\,\sin(-2\,bx+a-3\,c)}{64\,b}\end{aligned}$$

input
```
integrate(cos(b*x+a)^3*cos(b*x+c)^3,x, algorithm="giac")
```

output
```
1/32*x*cos(3*a - 3*c) + 9/32*x*cos(a - c) + 1/192*sin(6*b*x + 3*a + 3*c)/b
 + 3/128*sin(4*b*x + 3*a + c)/b + 3/128*sin(4*b*x + a + 3*c)/b + 3/64*sin(
2*b*x + 3*a - c)/b + 9/64*sin(2*b*x + a + c)/b - 3/64*sin(-2*b*x + a - 3*c
)/b
```

Mupad [B] (verification not implemented)

Time = 21.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \cos^3(a+bx)\cos^3(c+bx)\,dx$$
$$= \frac{\frac{9\sin(a+3c+4bx)}{8}+\frac{9\sin(3a+c+4bx)}{8}+\frac{9\sin(3c-a+2bx)}{4}+\frac{9\sin(3a-c+2bx)}{4}+\frac{\sin(3a+3c+6bx)}{4}+\frac{27\sin(a+c+2bx)}{4}+\frac{27b}{}}{48\,b}$$

input
```
int(cos(a + b*x)^3*cos(c + b*x)^3,x)
```

output
```
((9*sin(a + 3*c + 4*b*x))/8 + (9*sin(3*a + c + 4*b*x))/8 + (9*sin(3*c - a
+ 2*b*x))/4 + (9*sin(3*a - c + 2*b*x))/4 + sin(3*a + 3*c + 6*b*x)/4 + (27*
sin(a + c + 2*b*x))/4 + (27*b*x*cos(a - c))/2 + (3*b*x*cos(3*a - 3*c))/2)/
(48*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.30

$$\int \cos^3(a+bx)\cos^3(c+bx)\,dx$$
$$= \frac{24\cos(bx+c)\cos(bx+a)\sin(bx+c)^2\sin(bx+a)^2bx-6\cos(bx+c)\cos(bx+a)\sin(bx+c)^2bx-6}{}$$

input
```
int(cos(b*x+a)^3*cos(b*x+c)^3,x)
```

output
```
(24*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x)**2*b*x - 6*cos(
b*x + c)*cos(a + b*x)*sin(b*x + c)**2*b*x - 6*cos(b*x + c)*cos(a + b*x)*si
n(a + b*x)**2*b*x + 15*cos(b*x + c)*cos(a + b*x)*b*x + 6*cos(b*x + c)*sin(
b*x + c)**2*sin(a + b*x)**3 - 15*cos(b*x + c)*sin(a + b*x)**3 + 2*cos(a +
b*x)*sin(b*x + c)**3*sin(a + b*x)**2 - 14*cos(a + b*x)*sin(b*x + c)**3 + 3
*cos(a + b*x)*sin(b*x + c)*sin(a + b*x)**2 + 33*cos(a + b*x)*sin(b*x + c)
+ 24*sin(b*x + c)**3*sin(a + b*x)**3*b*x - 18*sin(b*x + c)**3*sin(a + b*x)
*b*x - 18*sin(b*x + c)*sin(a + b*x)**3*b*x + 27*sin(b*x + c)*sin(a + b*x)*
b*x)/(48*b)
```

3.312 $\int \cos^3(a+bx)\cos^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \cos^3(a+bx)\cos^2(c+bx)\,dx = -\frac{3\sin(a-2c-bx)}{16b}+\frac{3\sin(a+bx)}{8b}+\frac{\sin(3a-2c+bx)}{16b}+\frac{\sin(3a+3bx)}{24b}+\frac{\sin(a+2c+3bx)}{16b}+\frac{\sin(3a+2c+5bx)}{80b}$$

output
```
-3/16*sin(-b*x+a-2*c)/b+3/8*sin(b*x+a)/b+1/16*sin(b*x+3*a-2*c)/b+1/24*sin(
3*b*x+3*a)/b+1/16*sin(3*b*x+a+2*c)/b+1/80*sin(5*b*x+3*a+2*c)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \cos^3(a+bx)\cos^2(c+bx)\,dx = \frac{-45\sin(a-2c-bx)+90\sin(a+bx)+10\sin(3(a+bx))+15\sin(3a-2c+bx)+15\sin(a+2c+3bx}{240b}$$

input
```
Integrate[Cos[a + b*x]^3*Cos[c + b*x]^2,x]
```

output
```
(-45*Sin[a - 2*c - b*x] + 90*Sin[a + b*x] + 10*Sin[3*(a + b*x)] + 15*Sin[3
*a - 2*c + b*x] + 15*Sin[a + 2*c + 3*b*x] + 3*Sin[3*a + 2*c + 5*b*x])/(240
*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\cos^2(bx+c)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{3}{16}\cos(a-bx-2c)+\frac{1}{16}\cos(3a+bx-2c)+\frac{3}{16}\cos(a+3bx+2c)+\frac{1}{16}\cos(3a+5bx+2c)+\frac{3}{8}\cos(a+bx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sin(a-bx-2c)}{16b}+\frac{\sin(3a+bx-2c)}{16b}+\frac{\sin(a+3bx+2c)}{16b}+\frac{\sin(3a+5bx+2c)}{80b}+\frac{3\sin(a+bx)}{8b}+\frac{\sin(3a+3bx)}{24b}$$

input
```
Int[Cos[a + b*x]^3*Cos[c + b*x]^2,x]
```

output
```
(-3*Sin[a - 2*c - b*x])/(16*b) + (3*Sin[a + b*x])/(8*b) + Sin[3*a - 2*c +
b*x]/(16*b) + Sin[3*a + 3*b*x]/(24*b) + Sin[a + 2*c + 3*b*x]/(16*b) + Sin[
3*a + 2*c + 5*b*x]/(80*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

method	result
default	$-\frac{3\sin(-bx+a-2c)}{16b}+\frac{3\sin(bx+a)}{8b}+\frac{\sin(bx+3a-2c)}{16b}+\frac{\sin(3bx+3a)}{24b}+\frac{\sin(3bx+a+2c)}{16b}+\frac{\sin(5bx+3a+2c)}{80b}$
risch	$-\frac{3\sin(-bx+a-2c)}{16b}+\frac{3\sin(bx+a)}{8b}+\frac{\sin(bx+3a-2c)}{16b}+\frac{\sin(3bx+3a)}{24b}+\frac{\sin(3bx+a+2c)}{16b}+\frac{\sin(5bx+3a+2c)}{80b}$
parallelrisch	$\frac{15\sin(bx+3a-2c)+10\sin(3bx+3a)+3\sin(5bx+3a+2c)+112\sin(2a-2c)-45\sin(-bx+a-2c)+15\sin(3bx+a+2c)+90\sin(bx}{240b}$
orering	$-\frac{259\left(-3\cos(bx+a)^2\cos(bx+c)^2b\sin(bx+a)-2\cos(bx+a)^3\cos(bx+c)b\sin(bx+c)\right)}{225b^2}-\frac{7\left(-6b^3\sin(bx+a)^3\cos(bx+c)^2-36\right.}{}$

input
```
int(cos(b*x+a)^3*cos(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/16*sin(-b*x+a-2*c)/b+3/8*sin(b*x+a)/b+1/16*sin(b*x+3*a-2*c)/b+1/24*sin(
3*b*x+3*a)/b+1/16*sin(3*b*x+a+2*c)/b+1/80*sin(5*b*x+3*a+2*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.37

$$\int \cos^3(a+bx)\cos^2(c+bx)\,dx$$
$$=\frac{\left(3\left(4\cos(-a+c)^3-3\cos(-a+c)\right)\cos(bx+c)^4+\left(\cos(-a+c)^3+3\cos(-a+c)\right)\cos(bx+c)^2+2}{}$$

input
```
integrate(cos(b*x+a)^3*cos(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/15*((3*(4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^4 + (cos(-a + c)^3
 + 3*cos(-a + c))*cos(b*x + c)^2 + 2*cos(-a + c)^3 + 6*cos(-a + c))*sin(b*
x + c) - (3*(4*cos(-a + c)^2 - 1)*cos(b*x + c)^5 - 5*(cos(-a + c)^2 - 1)*c
os(b*x + c)^3)*sin(-a + c))/b
```

Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.71

$$\int \cos^3(a+bx)\cos^2(c+bx)\,dx$$
$$=\begin{cases} \frac{8\sin^3(a+bx)\sin^2(bx+c)}{15b}+\frac{2\sin^3(a+bx)\cos^2(bx+c)}{15b}+\frac{4\sin^2(a+bx)\sin(bx+c)\cos(a+bx)\cos(bx+c)}{5b}+\frac{2\sin(a+bx)\sin^2(bx+c)\cos^2}{5b} \\ x\cos^3(a)\cos^2(c) \end{cases}$$

input
```
integrate(cos(b*x+a)**3*cos(b*x+c)**2,x)
```

output
```
Piecewise((8*sin(a + b*x)**3*sin(b*x + c)**2/(15*b) + 2*sin(a + b*x)**3*co
s(b*x + c)**2/(15*b) + 4*sin(a + b*x)**2*sin(b*x + c)*cos(a + b*x)*cos(b*x
 + c)/(5*b) + 2*sin(a + b*x)*sin(b*x + c)**2*cos(a + b*x)**2/(5*b) + 3*sin
(a + b*x)*cos(a + b*x)**2*cos(b*x + c)**2/(5*b) + 2*sin(b*x + c)*cos(a + b
*x)**3*cos(b*x + c)/(5*b), Ne(b, 0)), (x*cos(a)**3*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \cos^3(a+bx)\cos^2(c+bx)\,dx$$
$$=\frac{3\sin(5bx+3a+2c)+10\sin(3bx+3a)+15\sin(3bx+a+2c)+15\sin(bx+3a-2c)+90\sin(}{240b}$$

input
```
integrate(cos(b*x+a)^3*cos(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/240*(3*sin(5*b*x + 3*a + 2*c) + 10*sin(3*b*x + 3*a) + 15*sin(3*b*x + a +
 2*c) + 15*sin(b*x + 3*a - 2*c) + 90*sin(b*x + a) + 45*sin(b*x - a + 2*c))
/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \cos^3(a+bx)\cos^2(c+bx)\,dx = \frac{\sin(5\,bx+3\,a+2\,c)}{80\,b} + \frac{\sin(3\,bx+3\,a)}{24\,b} + \frac{\sin(3\,bx+a+2\,c)}{16\,b} + \frac{\sin(bx+3\,a-2\,c)}{16\,b} + \frac{3\,\sin(bx+a)}{8\,b} - \frac{3\,\sin(-bx+a-2\,c)}{16\,b}$$

input
```
integrate(cos(b*x+a)^3*cos(b*x+c)^2,x, algorithm="giac")
```

output
```
1/80*sin(5*b*x + 3*a + 2*c)/b + 1/24*sin(3*b*x + 3*a)/b + 1/16*sin(3*b*x +
 a + 2*c)/b + 1/16*sin(b*x + 3*a - 2*c)/b + 3/8*sin(b*x + a)/b - 3/16*sin(
-b*x + a - 2*c)/b
```

Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \cos^3(a+bx)\cos^2(c+bx)\,dx = \frac{15\,\sin(a+2\,c+3\,b\,x)+90\,\sin(a+b\,x)+45\,\sin(2\,c-a+b\,x)+15\,\sin(3\,a-2\,c+b\,x)+3\,\sin(3\,a+}{240\,b}$$

input
```
int(cos(a + b*x)^3*cos(c + b*x)^2,x)
```

output
```
(15*sin(a + 2*c + 3*b*x) + 90*sin(a + b*x) + 45*sin(2*c - a + b*x) + 15*si
n(3*a - 2*c + b*x) + 3*sin(3*a + 2*c + 5*b*x) + 10*sin(3*a + 3*b*x))/(240*
b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.03

$$\int \cos^3(a+bx)\cos^2(c+bx)\,dx$$
$$= \frac{6\cos(bx+c)\cos(bx+a)\sin(bx+c)\sin(bx+a)^2 + 6\cos(bx+c)\cos(bx+a)\sin(bx+c) - 2\cos(bx+$$

input
```
int(cos(b*x+a)^3*cos(b*x+c)^2,x)
```

output
```
(6*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)*sin(a + b*x)**2 + 6*cos(b*x + c)
*cos(a + b*x)*sin(b*x + c) - 2*cos(b*x + c)*sin(b*x + c)*sin(a + b*x)**2 +
 cos(b*x + c)*sin(b*x + c) + 10*cos(b*x + c)*sin(a + b*x) + 2*cos(a + b*x)
*sin(b*x + c)**2*sin(a + b*x) - 10*cos(a + b*x)*sin(b*x + c) - cos(a + b*x
)*sin(a + b*x) + 9*sin(b*x + c)**2*sin(a + b*x)**3 - 3*sin(b*x + c)**2*sin
(a + b*x) - 7*sin(a + b*x)**3 + 9*sin(a + b*x))/(15*b)
```

3.313 $\int \cos^3(a+bx)\cos(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \cos^3(a+bx)\cos(c+bx)\,dx = \frac{3}{8}x\cos(a-c)+\frac{\sin(3a-c+2bx)}{16b}+\frac{3\sin(a+c+2bx)}{16b}+\frac{\sin(3a+c+4bx)}{32b}$$

output
```
3/8*x*cos(a-c)+1/16*sin(2*b*x+3*a-c)/b+3/16*sin(2*b*x+a+c)/b+1/32*sin(4*b*
x+3*a+c)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \cos^3(a+bx)\cos(c+bx)\,dx = \frac{12bx\cos(a-c)+2\sin(3a-c+2bx)+6\sin(a+c+2bx)+\sin(3a+c+4bx)}{32b}$$

input
```
Integrate[Cos[a + b*x]^3*Cos[c + b*x],x]
```

output
```
(12*b*x*Cos[a - c] + 2*Sin[3*a - c + 2*b*x] + 6*Sin[a + c + 2*b*x] + Sin[3
*a + c + 4*b*x])/(32*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\cos(bx+c)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{8}\cos(3a+2bx-c)+\frac{3}{8}\cos(a+2bx+c)+\frac{1}{8}\cos(3a+4bx+c)+\frac{3}{8}\cos(a-c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(3a+2bx-c)}{16b}+\frac{3\sin(a+2bx+c)}{16b}+\frac{\sin(3a+4bx+c)}{32b}+\frac{3}{8}x\cos(a-c)$$

input
```
Int[Cos[a + b*x]^3*Cos[c + b*x],x]
```

output
```
(3*x*Cos[a - c])/8 + Sin[3*a - c + 2*b*x]/(16*b) + (3*Sin[a + c + 2*b*x])/
(16*b) + Sin[3*a + c + 4*b*x]/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result
default	$\frac{3x\cos(a-c)}{8}+\frac{\sin(2bx+3a-c)}{16b}+\frac{3\sin(2bx+a+c)}{16b}+\frac{\sin(4bx+3a+c)}{32b}$
risch	$\frac{3x\cos(a-c)}{8}+\frac{\sin(2bx+3a-c)}{16b}+\frac{3\sin(2bx+a+c)}{16b}+\frac{\sin(4bx+3a+c)}{32b}$
parallelrisch	$\frac{12x\cos(a-c)b-15\sin(a-c)+2\sin(2bx+3a-c)+6\sin(2bx+a+c)+\sin(4bx+3a+c)}{32b}$
orering	$x\cos(bx+a)^{3}\cos(bx+c)-\frac{5\left(-3\cos(bx+a)^{2}\cos(bx+c)b\sin(bx+a)-\cos(bx+a)^{3}b\sin(bx+c)\right)}{16b^{2}}+\frac{5x\left(6\cos(bx\right.}{}$

input
```
int(cos(b*x+a)^3*cos(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
3/8*x*cos(a-c)+1/16*sin(2*b*x+3*a-c)/b+3/16*sin(2*b*x+a+c)/b+1/32*sin(4*b*
x+3*a+c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(55) = 110.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

$$\int \cos^3(a+bx)\cos(c+bx)\,dx$$
$$=\frac{3\,bx\cos(-a+c)+\left(2\left(4\cos(-a+c)^{3}-3\cos(-a+c)\right)\cos(bx+c)^{3}+3\cos(bx+c)\cos(-a+c)\right)\sin}{8\,b}$$

input
```
integrate(cos(b*x+a)^3*cos(b*x+c),x, algorithm="fricas")
```

output
```
1/8*(3*b*x*cos(-a + c) + (2*(4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)
^3 + 3*cos(b*x + c)*cos(-a + c))*sin(b*x + c) - 2*((4*cos(-a + c)^2 - 1)*c
os(b*x + c)^4 - 2*(cos(-a + c)^2 - 1)*cos(b*x + c)^2)*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(54) = 108.

Time = 0.88 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.73

$$\int \cos^3(a+bx)\cos(c+bx)\,dx$$
$$= \begin{cases} \frac{3x\sin^3(a+bx)\sin(bx+c)}{8} + \frac{3x\sin^2(a+bx)\cos(a+bx)\cos(bx+c)}{8} + \frac{3x\sin(a+bx)\sin(bx+c)\cos^2(a+bx)}{8} + \frac{3x\cos^3(a+bx)\cos(bx+c)}{8} \\ x\cos^3(a)\cos(c) \end{cases}$$

input
```
integrate(cos(b*x+a)**3*cos(b*x+c),x)
```

output
```
Piecewise((3*x*sin(a + b*x)**3*sin(b*x + c)/8 + 3*x*sin(a + b*x)**2*cos(a
+ b*x)*cos(b*x + c)/8 + 3*x*sin(a + b*x)*sin(b*x + c)*cos(a + b*x)**2/8 +
3*x*cos(a + b*x)**3*cos(b*x + c)/8 - 3*sin(a + b*x)**3*cos(b*x + c)/(8*b)
+ 3*sin(a + b*x)**2*sin(b*x + c)*cos(a + b*x)/(4*b) + 5*sin(b*x + c)*cos(a
 + b*x)**3/(8*b), Ne(b, 0)), (x*cos(a)**3*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \cos^3(a+bx)\cos(c+bx)\,dx$$
$$= \frac{12\,bx\cos(-a+c) + \sin(4\,bx+3\,a+c) + 2\,\sin(2\,bx+3\,a-c) + 6\,\sin(2\,bx+a+c)}{32\,b}$$

input
```
integrate(cos(b*x+a)^3*cos(b*x+c),x, algorithm="maxima")
```

output
```
1/32*(12*b*x*cos(-a + c) + sin(4*b*x + 3*a + c) + 2*sin(2*b*x + 3*a - c) +
 6*sin(2*b*x + a + c))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\cos(c+bx)\,dx = \frac{3}{8}\,x\cos\left(a-c\right) + \frac{\sin\left(4\,bx+3\,a+c\right)}{32\,b} + \frac{\sin\left(2\,bx+3\,a-c\right)}{16\,b} + \frac{3\,\sin\left(2\,bx+a+c\right)}{16\,b}$$

input
```
integrate(cos(b*x+a)^3*cos(b*x+c),x, algorithm="giac")
```

output
```
3/8*x*cos(a - c) + 1/32*sin(4*b*x + 3*a + c)/b + 1/16*sin(2*b*x + 3*a - c)
/b + 3/16*sin(2*b*x + a + c)/b
```

Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \cos^3(a+bx)\cos(c+bx)\,dx = \frac{\frac{\sin(3\,a+c+4\,b\,x)}{32}+\frac{\sin(3\,a-c+2\,b\,x)}{16}+\frac{3\,\sin(a+c+2\,b\,x)}{16}}{b} + \frac{3\,x\,\cos\left(a-c\right)}{8}$$

input
```
int(cos(a + b*x)^3*cos(c + b*x),x)
```

output
```
(sin(3*a + c + 4*b*x)/32 + sin(3*a - c + 2*b*x)/16 + (3*sin(a + c + 2*b*x)
)/16)/b + (3*x*cos(a - c))/8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.41

$$\int \cos^3(a+bx)\cos(c+bx)\,dx$$
$$= \frac{3\cos(bx+c)\cos(bx+a)\,bx-3\cos(bx+c)\sin(bx+a)^3+\cos(bx+a)\sin(bx+c)\sin(bx+a)^2+5\,\mathrm{co}}{8b}$$

input
```
int(cos(b*x+a)^3*cos(b*x+c),x)
```

output
```
(3*cos(b*x + c)*cos(a + b*x)*b*x - 3*cos(b*x + c)*sin(a + b*x)**3 + cos(a
+ b*x)*sin(b*x + c)*sin(a + b*x)**2 + 5*cos(a + b*x)*sin(b*x + c) + 3*sin(
b*x + c)*sin(a + b*x)*b*x)/(8*b)
```

3.314 $\int \cos^3(a+bx)\sec(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \cos^3(a+bx)\sec(c+bx)\,dx = \frac{3}{4}x\cos(a-c)-\frac{1}{4}x\cos(3(a-c))+\frac{\log(\cos(c+bx))\sin^3(a-c)}{b}+\frac{\sin(3a-c+2bx)}{4b}$$

output
```
3/4*x*cos(a-c)-1/4*x*cos(3*a-3*c)+ln(cos(b*x+c))*sin(a-c)^3/b+1/4*sin(2*b*
x+3*a-c)/b
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17

$$\int \cos^3(a+bx)\sec(c+bx)\,dx = \frac{3bx\cos(a-c)-bx\cos(3(a-c))+3\log(\cos(c+bx))\sin(a-c)-\log(\cos(c+bx))\sin(3(a-c))+\sin(3}{4b}$$

input
```
Integrate[Cos[a + b*x]^3*Sec[c + b*x],x]
```

output
```
(3*b*x*Cos[a - c] - b*x*Cos[3*(a - c)] + 3*Log[Cos[c + b*x]]*Sin[a - c] -
Log[Cos[c + b*x]]*Sin[3*(a - c)] + Sin[3*a - c + 2*b*x])/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \cos^3(a+bx)\sec(bx+c)dx$$

input
```
Int[Cos[a + b*x]^3*Sec[c + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.49

method	result
risch	$-\frac{x\,e^{3i(a-c)}}{4}+\frac{3x\,e^{i(a-c)}}{4}-\frac{3i\sin(a-c)x}{2}+\frac{i\sin(3a-3c)x}{2}-\frac{3i\sin(a-c)a}{2b}+\frac{i\sin(3a-3c)a}{2b}+\frac{3\ln\left(e^{2i(bx+a)}+e^{2i(a-c)}\right)\text{si}}{4b}$
default	Expression too large to display

input
```
int(cos(b*x+a)^3*sec(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x*exp(3*I*(a-c))+3/4*x*exp(I*(a-c))-3/2*I*sin(a-c)*x+1/2*I*sin(3*a-3*
c)*x-3/2*I/b*sin(a-c)*a+1/2*I/b*sin(3*a-3*c)*a+3/4/b*ln(exp(2*I*(b*x+a))+e
xp(2*I*(a-c)))*sin(a-c)-1/4/b*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*sin(3*a-
3*c)+1/4*sin(2*b*x+3*a-c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(57) = 114$.

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.83

$$\int \cos^3(a+bx)\sec(c+bx)\,dx = -\frac{2\,bx\cos\left(-a+c\right)^3 + \left(4\,\cos\left(-a+c\right)^2-1\right)\cos\left(bx+c\right)^2\sin\left(-a+c\right) - 3\,bx\cos\left(-a+c\right) - \left(4\,\cos\left(-a\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sec(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(2*b*x*cos(-a + c)^3 + (4*cos(-a + c)^2 - 1)*cos(b*x + c)^2*sin(-a +
c) - 3*b*x*cos(-a + c) - (4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)*si
n(b*x + c) - 2*(cos(-a + c)^2 - 1)*log(-cos(b*x + c))*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6686 vs. $2(0) = 0$.

Time = 68.92 (sec) , antiderivative size = 34286, normalized size of antiderivative = 544.22

$$\int \cos^3(a+bx)\sec(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**3*sec(b*x+c),x)
```

output
```
3*Piecewise((-sin(b*x)**2/(2*b), Eq(c, pi/2)), (sin(b*x)**2/(2*b), Eq(c, -
pi/2)), (0, Eq(b, 0)), (-b*x*tan(c/2)**6*tan(b*x/2)**4/(2*b*tan(c/2)**6*ta
n(b*x/2)**4 + 4*b*tan(c/2)**6*tan(b*x/2)**2 + 2*b*tan(c/2)**6 + 6*b*tan(c/
2)**4*tan(b*x/2)**4 + 12*b*tan(c/2)**4*tan(b*x/2)**2 + 6*b*tan(c/2)**4 + 6
*b*tan(c/2)**2*tan(b*x/2)**4 + 12*b*tan(c/2)**2*tan(b*x/2)**2 + 6*b*tan(c/
2)**2 + 2*b*tan(b*x/2)**4 + 4*b*tan(b*x/2)**2 + 2*b) - 2*b*x*tan(c/2)**6*t
an(b*x/2)**2/(2*b*tan(c/2)**6*tan(b*x/2)**4 + 4*b*tan(c/2)**6*tan(b*x/2)**
2 + 2*b*tan(c/2)**6 + 6*b*tan(c/2)**4*tan(b*x/2)**4 + 12*b*tan(c/2)**4*tan
(b*x/2)**2 + 6*b*tan(c/2)**4 + 6*b*tan(c/2)**2*tan(b*x/2)**4 + 12*b*tan(c/
2)**2*tan(b*x/2)**2 + 6*b*tan(c/2)**2 + 2*b*tan(b*x/2)**4 + 4*b*tan(b*x/2)
**2 + 2*b) - b*x*tan(c/2)**6/(2*b*tan(c/2)**6*tan(b*x/2)**4 + 4*b*tan(c/2)
**6*tan(b*x/2)**2 + 2*b*tan(c/2)**6 + 6*b*tan(c/2)**4*tan(b*x/2)**4 + 12*b
*tan(c/2)**4*tan(b*x/2)**2 + 6*b*tan(c/2)**4 + 6*b*tan(c/2)**2*tan(b*x/2)*
*4 + 12*b*tan(c/2)**2*tan(b*x/2)**2 + 6*b*tan(c/2)**2 + 2*b*tan(b*x/2)**4
+ 4*b*tan(b*x/2)**2 + 2*b) + 7*b*x*tan(c/2)**4*tan(b*x/2)**4/(2*b*tan(c/2)
**6*tan(b*x/2)**4 + 4*b*tan(c/2)**6*tan(b*x/2)**2 + 2*b*tan(c/2)**6 + 6*b*
tan(c/2)**4*tan(b*x/2)**4 + 12*b*tan(c/2)**4*tan(b*x/2)**2 + 6*b*tan(c/2)*
*4 + 6*b*tan(c/2)**2*tan(b*x/2)**4 + 12*b*tan(c/2)**2*tan(b*x/2)**2 + 6*b*
tan(c/2)**2 + 2*b*tan(b*x/2)**4 + 4*b*tan(b*x/2)**2 + 2*b) + 14*b*x*tan(c/
2)**4*tan(b*x/2)**2/(2*b*tan(c/2)**6*tan(b*x/2)**4 + 4*b*tan(c/2)**6*ta...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(57) = 114$.

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.83

$$\int \cos^3(a+bx)\sec(c+bx)\,dx$$
$$= \frac{2\,(3\,b\cos\left(-a+c\right)-b\cos\left(-3\,a+3\,c\right))x-(3\,\sin\left(-a+c\right)-\sin\left(-3\,a+3\,c\right))\log\left(\cos\left(2\,bx\right)^2+2\,\cos\left(2\right.\right.}{8\,b}$$

input
```
integrate(cos(b*x+a)^3*sec(b*x+c),x, algorithm="maxima")
```

output
```
1/8*(2*(3*b*cos(-a + c) - b*cos(-3*a + 3*c))*x - (3*sin(-a + c) - sin(-3*a
 + 3*c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x
)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2) + 2*sin(2*b*x + 3*a - c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2907 vs. $2(57) = 114$.

Time = 0.18 (sec) , antiderivative size = 2907, normalized size of antiderivative = 46.14

$$\int \cos^3(a+bx)\sec(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sec(b*x+c),x, algorithm="giac")
```

output
```
1/2*((tan(1/2*a)^6*tan(1/2*c)^6 + 9*tan(1/2*a)^6*tan(1/2*c)^4 - 12*tan(1/2
*a)^5*tan(1/2*c)^5 + 9*tan(1/2*a)^4*tan(1/2*c)^6 - 9*tan(1/2*a)^6*tan(1/2*
c)^2 + 72*tan(1/2*a)^5*tan(1/2*c)^3 - 111*tan(1/2*a)^4*tan(1/2*c)^4 + 72*t
an(1/2*a)^3*tan(1/2*c)^5 - 9*tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^6 - 12
*tan(1/2*a)^5*tan(1/2*c) + 111*tan(1/2*a)^4*tan(1/2*c)^2 - 176*tan(1/2*a)^
3*tan(1/2*c)^3 + 111*tan(1/2*a)^2*tan(1/2*c)^4 - 12*tan(1/2*a)*tan(1/2*c)^
5 - tan(1/2*c)^6 - 9*tan(1/2*a)^4 + 72*tan(1/2*a)^3*tan(1/2*c) - 111*tan(1
/2*a)^2*tan(1/2*c)^2 + 72*tan(1/2*a)*tan(1/2*c)^3 - 9*tan(1/2*c)^4 + 9*tan
(1/2*a)^2 - 12*tan(1/2*a)*tan(1/2*c) + 9*tan(1/2*c)^2 + 1)*(b*x + a)/(tan(
1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a)^4*tan(1
/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^4 + 3*ta
n(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^2 + 9*t
an(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2*a)^2*
tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1) - 8*(
tan(1/2*a)^6*tan(1/2*c)^3 - 3*tan(1/2*a)^5*tan(1/2*c)^4 + 3*tan(1/2*a)^4*t
an(1/2*c)^5 - tan(1/2*a)^3*tan(1/2*c)^6 + 3*tan(1/2*a)^5*tan(1/2*c)^2 - 9*
tan(1/2*a)^4*tan(1/2*c)^3 + 9*tan(1/2*a)^3*tan(1/2*c)^4 - 3*tan(1/2*a)^2*t
an(1/2*c)^5 + 3*tan(1/2*a)^4*tan(1/2*c) - 9*tan(1/2*a)^3*tan(1/2*c)^2 + 9*
tan(1/2*a)^2*tan(1/2*c)^3 - 3*tan(1/2*a)*tan(1/2*c)^4 + tan(1/2*a)^3 - 3*t
an(1/2*a)^2*tan(1/2*c) + 3*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3)*log(...
```

Mupad [B] (verification not implemented)

Time = 18.70 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.67

$$\int \cos^3(a+bx)\sec(c+bx)\,dx$$
$$= \frac{e^{-a\,3i+c\,1i-b\,x\,2i}\,1i}{8\,b} - \frac{e^{a\,3i-c\,1i+b\,x\,2i}\,1i}{8\,b} + \frac{x\,e^{-a\,3i+c\,1i}\,(3\,e^{a\,2i}-e^{c\,2i})}{4} - \frac{e^{-a\,6i+c\,6i}\,\ln\left(e^{a\,2i}\,e^{b\,x\,2i}+e^{a\,2i}\,e^{-c\,2i}\right)\,(b\,e^{a\,3i-c\,3i}\,8i-b\,e^{a\,5i-c\,5i}\,24i+b\,e^{a\,7i-c\,7i}\,24i-b\,e^{a\,9i-c\,9i}\,8i)}{64\,b^2}$$

input

```
int(cos(a + b*x)^3/cos(c + b*x),x)
```

output

```
(exp(c*1i - a*3i - b*x*2i)*1i)/(8*b) - (exp(a*3i - c*1i + b*x*2i)*1i)/(8*b
) + (x*exp(c*1i - a*3i)*(3*exp(a*2i) - exp(c*2i)))/4 - (exp(c*6i - a*6i)*l
og(exp(a*2i)*exp(b*x*2i) + exp(a*2i)*exp(-c*2i))*(b*exp(a*3i - c*3i)*8i -
b*exp(a*5i - c*5i)*24i + b*exp(a*7i - c*7i)*24i - b*exp(a*9i - c*9i)*8i))/
(64*b^2)
```

Reduce [F]

$$\int \cos^3(a+bx)\sec(c+bx)\,dx = \int \cos(bx+a)^3\sec(bx+c)\,dx$$

input

```
int(cos(b*x+a)^3*sec(b*x+c),x)
```

output

```
int(cos(a + b*x)**3*sec(b*x + c),x)
```

3.315 $\int \cos^3(a+bx)\sec^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \cos^3(a+bx)\sec^2(c+bx)\,dx = \frac{\cos(bx)\sin(3a-2c)}{b} + \frac{3\operatorname{arctanh}(\sin(c+bx))\cos(a-c)\sin^2(a-c)}{b} - \frac{\sec(c+bx)\sin^3(a-c)}{b} + \frac{\cos(3a-2c)\sin(bx)}{b}$$

output

```
cos(b*x)*sin(3*a-2*c)/b+3*arctanh(sin(b*x+c))*cos(a-c)*sin(a-c)^2/b-sec(b*
x+c)*sin(a-c)^3/b+cos(3*a-2*c)*sin(b*x)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.68

$$\int \cos^3(a+bx)\sec^2(c+bx)\,dx = \frac{\cos(bx)\sin(3a-2c)}{b} - \frac{6i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)\sin^2(a-c)}{b} - \frac{\sec(c+bx)\sin^3(a-c)}{b} + \frac{\cos(3a-2c)\sin(bx)}{b}$$

input
```
Integrate[Cos[a + b*x]^3*Sec[c + b*x]^2,x]
```

output
```
(Cos[b*x]*Sin[3*a - 2*c])/b - ((6*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x
)/2]*Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*
Sin[c])]*Cos[a - c]*Sin[a - c]^2)/b - (Sec[c + b*x]*Sin[a - c]^3)/b + (Cos
[3*a - 2*c]*Sin[b*x])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\sec^2(bx+c)\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \cos^3(a+bx)\sec^2(bx+c)dx$$

input
```
Int[Cos[a + b*x]^3*Sec[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.82 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.05

method	result
risch	$-\frac{i\left(e^{i(bx+5a-2c)}-3\,e^{i(bx+3a)}+3\,e^{i(bx+a+2c)}-e^{-i(-bx+a-4c)}\right)}{4\left(e^{2i(bx+a+c)}+e^{2ia}\right)b}+\frac{3\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\cos(a-c)}{4b}-\frac{3\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)}{4b}$
default	Expression too large to display

input
```
int(cos(b*x+a)^3*sec(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/(exp(2*I*(b*x+a+c))+exp(2*I*a))/b*(exp(I*(b*x+5*a-2*c))-3*exp(I*(b*
x+3*a))+3*exp(I*(b*x+a+2*c))-exp(-I*(-b*x+a-4*c)))+3/4*ln(exp(I*(b*x+a))+I
*exp(I*(a-c)))/b*cos(a-c)-3/4*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*cos(3*a-
3*c)-3/4*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*cos(a-c)+3/4*ln(exp(I*(b*x+a)
)-I*exp(I*(a-c)))/b*cos(3*a-3*c)+sin(b*x+3*a-2*c)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.00

$$\int \cos^3(a+bx)\sec^2(c+bx)\,dx =$$

$$-\frac{3\left(\cos\left(-a+c\right)^3-\cos\left(-a+c\right)\right)\cos\left(bx+c\right)\log\left(\sin\left(bx+c\right)+1\right)-3\left(\cos\left(-a+c\right)^3-\cos\left(-a+c\right)\right)}{}$$

input
```
integrate(cos(b*x+a)^3*sec(b*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(3*(cos(-a + c)^3 - cos(-a + c))*cos(b*x + c)*log(sin(b*x + c) + 1) -
 3*(cos(-a + c)^3 - cos(-a + c))*cos(b*x + c)*log(-sin(b*x + c) + 1) - 2*(
4*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)*sin(b*x + c) + 2*((4*cos(-a
+ c)^2 - 1)*cos(b*x + c)^2 + cos(-a + c)^2 - 1)*sin(-a + c))/(b*cos(b*x +
c))
```

Sympy [F(-2)]

Exception generated.

$$\int \cos^3(a+bx)\sec^2(c+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(cos(b*x+a)**3*sec(b*x+c)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 883 vs. $2(78) = 156$.

Time = 0.22 (sec) , antiderivative size = 883, normalized size of antiderivative = 11.32

$$\int \cos^3(a+bx)\sec^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sec(b*x+c)^2,x, algorithm="maxima")
```

output

```
-1/8*(4*(sin(3*b*x + 3*a + 4*c) + sin(b*x + 3*a + 2*c))*cos(4*b*x + 6*a +
2*c) - 2*(3*sin(2*b*x + 6*a) - 3*sin(2*b*x + 4*a + 2*c) + 3*sin(2*b*x + 2*
a + 4*c) - 3*sin(2*b*x + 6*c) - 2*sin(4*c))*cos(3*b*x + 3*a + 4*c) + 3*((c
os(-a + c) - cos(-3*a + 3*c))*cos(3*b*x + 3*a + 4*c)^2 + 2*(cos(-a + c) -
cos(-3*a + 3*c))*cos(3*b*x + 3*a + 4*c)*cos(b*x + 3*a + 2*c) + (cos(-a + c
) - cos(-3*a + 3*c))*cos(b*x + 3*a + 2*c)^2 + (cos(-a + c) - cos(-3*a + 3*
c))*sin(3*b*x + 3*a + 4*c)^2 + 2*(cos(-a + c) - cos(-3*a + 3*c))*sin(3*b*x
 + 3*a + 4*c)*sin(b*x + 3*a + 2*c) + (cos(-a + c) - cos(-3*a + 3*c))*sin(b
*x + 3*a + 2*c)^2)*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2
*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*
c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x +
 2*c)*sin(c) + sin(c)^2)) - 4*(cos(3*b*x + 3*a + 4*c) + cos(b*x + 3*a + 2*
c))*sin(4*b*x + 6*a + 2*c) + 2*(3*cos(2*b*x + 6*a) - 3*cos(2*b*x + 4*a + 2
*c) + 3*cos(2*b*x + 2*a + 4*c) - 3*cos(2*b*x + 6*c) - 2*cos(4*c))*sin(3*b*
x + 3*a + 4*c) - 6*cos(b*x + 3*a + 2*c)*sin(2*b*x + 6*a) + 6*cos(b*x + 3*a
 + 2*c)*sin(2*b*x + 4*a + 2*c) - 6*cos(b*x + 3*a + 2*c)*sin(2*b*x + 2*a +
4*c) + 6*cos(b*x + 3*a + 2*c)*sin(2*b*x + 6*c) + 6*cos(2*b*x + 6*a)*sin(b*
x + 3*a + 2*c) - 6*cos(2*b*x + 4*a + 2*c)*sin(b*x + 3*a + 2*c) + 6*cos(2*b
*x + 2*a + 4*c)*sin(b*x + 3*a + 2*c) - 6*cos(2*b*x + 6*c)*sin(b*x + 3*a +
2*c) - 4*cos(4*c)*sin(b*x + 3*a + 2*c) + 4*cos(b*x + 3*a + 2*c)*sin(4*c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6607 vs. $2(78) = 156$.

Time = 0.64 (sec) , antiderivative size = 6607, normalized size of antiderivative = 84.71

$$\int \cos^3(a+bx)\sec^2(c+bx)\,dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)^3*sec(b*x+c)^2,x, algorithm="giac")
```

output
```
2*(6*(tan(1/2*a)^7*tan(1/2*c)^5 - 2*tan(1/2*a)^6*tan(1/2*c)^6 + tan(1/2*a)
^5*tan(1/2*c)^7 + tan(1/2*a)^7*tan(1/2*c)^4 - 3*tan(1/2*a)^6*tan(1/2*c)^5
+ 3*tan(1/2*a)^5*tan(1/2*c)^6 - tan(1/2*a)^4*tan(1/2*c)^7 - tan(1/2*a)^7*t
an(1/2*c)^3 + 9*tan(1/2*a)^6*tan(1/2*c)^4 - 16*tan(1/2*a)^5*tan(1/2*c)^5 +
 9*tan(1/2*a)^4*tan(1/2*c)^6 - tan(1/2*a)^3*tan(1/2*c)^7 - tan(1/2*a)^7*ta
n(1/2*c)^2 + 9*tan(1/2*a)^6*tan(1/2*c)^3 - 22*tan(1/2*a)^5*tan(1/2*c)^4 +
22*tan(1/2*a)^4*tan(1/2*c)^5 - 9*tan(1/2*a)^3*tan(1/2*c)^6 + tan(1/2*a)^2*
tan(1/2*c)^7 - 3*tan(1/2*a)^6*tan(1/2*c)^2 + 22*tan(1/2*a)^5*tan(1/2*c)^3
- 38*tan(1/2*a)^4*tan(1/2*c)^4 + 22*tan(1/2*a)^3*tan(1/2*c)^5 - 3*tan(1/2*
a)^2*tan(1/2*c)^6 - 2*tan(1/2*a)^6*tan(1/2*c) + 16*tan(1/2*a)^5*tan(1/2*c)
^2 - 38*tan(1/2*a)^4*tan(1/2*c)^3 + 38*tan(1/2*a)^3*tan(1/2*c)^4 - 16*tan(
1/2*a)^2*tan(1/2*c)^5 + 2*tan(1/2*a)*tan(1/2*c)^6 - 3*tan(1/2*a)^5*tan(1/2
*c) + 22*tan(1/2*a)^4*tan(1/2*c)^2 - 38*tan(1/2*a)^3*tan(1/2*c)^3 + 22*tan
(1/2*a)^2*tan(1/2*c)^4 - 3*tan(1/2*a)*tan(1/2*c)^5 - tan(1/2*a)^5 + 9*tan(
1/2*a)^4*tan(1/2*c) - 22*tan(1/2*a)^3*tan(1/2*c)^2 + 22*tan(1/2*a)^2*tan(1
/2*c)^3 - 9*tan(1/2*a)*tan(1/2*c)^4 + tan(1/2*c)^5 - tan(1/2*a)^4 + 9*tan(
1/2*a)^3*tan(1/2*c) - 16*tan(1/2*a)^2*tan(1/2*c)^2 + 9*tan(1/2*a)*tan(1/2*
c)^3 - tan(1/2*c)^4 + tan(1/2*a)^3 - 3*tan(1/2*a)^2*tan(1/2*c) + 3*tan(1/2
*a)*tan(1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*a)^2 - 2*tan(1/2*a)*tan(1/2*c) +
 tan(1/2*c)^2)*log(abs(tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) + tan...
```

Mupad [B] (verification not implemented)

Time = 24.67 (sec) , antiderivative size = 399, normalized size of antiderivative = 5.12

$$\int \cos^3(a+bx)\sec^2(c+bx)\,dx = \text{Too large to display}$$

input
```
int(cos(a + b*x)^3/cos(c + b*x)^2,x)
```

output
```
(exp(c*2i - a*3i - b*x*1i)*1i)/(2*b) - (exp(a*3i - c*2i + b*x*1i)*1i)/(2*b
) + (exp(c*2i - a*1i + b*x*1i)*(3*exp(a*2i - c*2i) - 3*exp(a*4i - c*4i) +
exp(a*6i - c*6i) - 1))/(4*b*(exp(a*2i - c*2i)*1i + exp(a*2i + b*x*2i)*1i))
 + (3*log(- (3*exp(a*1i)*exp(b*x*1i)*(sin(2*a - 2*c)*1i - sin(2*a - 2*c)*e
xp(a*2i)*exp(-c*2i)*1i))/2 - (sin(2*a - 2*c)*exp(a*2i)*exp(-c*2i)*(exp(a*2
i)*exp(-c*2i) - 1)*3i)/(2*(-exp(a*2i)*exp(-c*2i))^(1/2)))*sin(2*a - 2*c)*(
exp(a*2i - c*2i) - 1))/(4*b*(-exp(a*2i - c*2i))^(1/2)) - (3*log((sin(2*a -
 2*c)*exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*3i)/(2*(-exp(a*2i)*e
xp(-c*2i))^(1/2)) - (3*exp(a*1i)*exp(b*x*1i)*(sin(2*a - 2*c)*1i - sin(2*a
- 2*c)*exp(a*2i)*exp(-c*2i)*1i))/2)*sin(2*a - 2*c)*(exp(a*2i - c*2i) - 1))
/(4*b*(-exp(a*2i - c*2i))^(1/2))
```

Reduce [F]

$$\int \cos^3(a+bx)\sec^2(c+bx)\,dx = \int \cos(bx+a)^3 \sec(bx+c)^2\,dx$$

input
```
int(cos(b*x+a)^3*sec(b*x+c)^2,x)
```

output
```
int(cos(a + b*x)**3*sec(b*x + c)**2,x)
```

3.316 $\int \cos^3(a+bx)\sec^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \cos^3(a+bx)\sec^3(c+bx)\,dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.60 (sec) , antiderivative size = 291, normalized size of antiderivative = 291.00

$$\int \cos^3(a+bx)\sec^3(c+bx)\,dx$$
$$= \frac{\sec(c)\sec^2(c+bx)(4bx\cos(3a-4c)+4bx\cos(3a-2c)+2bx\cos(3a-6c-2bx)+2bx\cos(3a-4c-2$$

input
```
Integrate[Cos[a + b*x]^3*Sec[c + b*x]^3,x]
```

output
```
(Sec[c]*Sec[c + b*x]^2*(4*b*x*Cos[3*a - 4*c] + 4*b*x*Cos[3*a - 2*c] + 2*b*
x*Cos[3*a - 6*c - 2*b*x] + 2*b*x*Cos[3*a - 4*c - 2*b*x] + 2*b*x*Cos[3*a +
2*b*x] + 2*b*x*Cos[3*a - 2*c + 2*b*x] - 6*Sin[a] - 2*Sin[3*a - 4*c] + 4*Lo
g[Cos[c + b*x]]*Sin[3*a - 4*c] + 4*Sin[3*a - 2*c] + 4*Log[Cos[c + b*x]]*Si
n[3*a - 2*c] + 2*Log[Cos[c + b*x]]*Sin[3*a - 6*c - 2*b*x] + 3*Sin[3*a - 4*
c - 2*b*x] + 2*Log[Cos[c + b*x]]*Sin[3*a - 4*c - 2*b*x] + 3*Sin[a + 2*b*x]
 + 2*Log[Cos[c + b*x]]*Sin[3*a + 2*b*x] - 3*Sin[3*a - 2*c + 2*b*x] + 2*Log
[Cos[c + b*x]]*Sin[3*a - 2*c + 2*b*x] - 3*Sin[a - 2*(c + b*x)]))/(16*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\sec^3(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \cos^3(a+bx)\sec^3(bx+c)dx$$

input
```
Int[Cos[a + b*x]^3*Sec[c + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 15.97 (sec) , antiderivative size = 192, normalized size of antiderivative = 192.00

method	result
risch	$x\,e^{3i(a-c)}-2i\sin(3a-3c)\,x-\frac{2i\sin(3a-3c)a}{b}-\frac{i\left(4\,e^{i(2bx+7a-c)}-6\,e^{i(2bx+5a+c)}+2\,e^{i(2bx+a+5c)}+3\,e^{i(7a-3c)}-3\,e^{i(}\right.}{4\left(e^{2i(bx+a+c)}+e^{2ia}\right)^2 b}$
default	Expression too large to display

input
```
int(cos(b*x+a)^3*sec(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
x*exp(3*I*(a-c))-2*I*sin(3*a-3*c)*x-2*I/b*sin(3*a-3*c)*a-1/4*I/(exp(2*I*(b
*x+a+c))+exp(2*I*a))^2/b*(4*exp(I*(2*b*x+7*a-c))-6*exp(I*(2*b*x+5*a+c))+2*
exp(I*(2*b*x+a+5*c))+3*exp(I*(7*a-3*c))-3*exp(I*(5*a-c))-3*exp(I*(3*a+c))+
3*exp(I*(a+3*c)))+1/b*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*sin(3*a-3*c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 132.00

$$\int \cos^3(a+bx)\sec^3(c+bx)\,dx =$$
$$-\frac{2\left(4\cos(-a+c)^2-1\right)\cos(bx+c)^2\log\left(-\cos(bx+c)\right)\sin(-a+c)-2\left(4\,bx\cos(-a+c)^3-3\,bx\,\mathrm{co}\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sec(b*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(4*cos(-a + c)^2 - 1)*cos(b*x + c)^2*log(-cos(b*x + c))*sin(-a + c
) - 2*(4*b*x*cos(-a + c)^3 - 3*b*x*cos(-a + c))*cos(b*x + c)^2 + 6*(cos(-a
 + c)^3 - cos(-a + c))*cos(b*x + c)*sin(b*x + c) + (cos(-a + c)^2 - 1)*sin
(-a + c))/(b*cos(b*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sec^3(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*sec(b*x+c)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.09 (sec) , antiderivative size = 1401, normalized size of antiderivative = 1401.00

$$\int \cos^3(a+bx)\sec^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sec(b*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*(4*(b*cos(3*a + 3*c)*cos(6*c) + b*sin(3*a + 3*c)*sin(6*c))*x + (8*b*x*
cos(2*b*x + 8*c) + 4*b*x*cos(6*c) + 4*sin(2*b*x + 6*a + 2*c) - 6*sin(2*b*x
 + 4*a + 4*c) + 2*sin(2*b*x + 8*c) + 3*sin(6*a) - 3*sin(4*a + 2*c) - 3*sin
(2*a + 4*c) + 3*sin(6*c))*cos(4*b*x + 3*a + 7*c) + 4*(b*x*cos(4*b*x + 3*a
+ 7*c) + 2*b*x*cos(2*b*x + 3*a + 5*c) + b*x*cos(3*a + 3*c))*cos(4*b*x + 10
*c) + 2*(4*b*x*cos(6*c) + 4*sin(2*b*x + 6*a + 2*c) - 6*sin(2*b*x + 4*a + 4
*c) + 3*sin(6*a) - 3*sin(4*a + 2*c) - 3*sin(2*a + 4*c) + 3*sin(6*c))*cos(2
*b*x + 3*a + 5*c) + 2*(8*b*x*cos(2*b*x + 3*a + 5*c) + 4*b*x*cos(3*a + 3*c)
 - 2*sin(2*b*x + 3*a + 5*c) - sin(3*a + 3*c))*cos(2*b*x + 8*c) + 3*(sin(6*
a) - sin(4*a + 2*c) + sin(6*c))*cos(3*a + 3*c) - 2*(cos(4*b*x + 3*a + 7*c)
^2*sin(-3*a + 3*c) + 4*cos(2*b*x + 3*a + 5*c)^2*sin(-3*a + 3*c) + 4*cos(2*
b*x + 3*a + 5*c)*cos(3*a + 3*c)*sin(-3*a + 3*c) + cos(3*a + 3*c)^2*sin(-3*
a + 3*c) + sin(4*b*x + 3*a + 7*c)^2*sin(-3*a + 3*c) + 4*sin(2*b*x + 3*a +
5*c)^2*sin(-3*a + 3*c) + 4*sin(2*b*x + 3*a + 5*c)*sin(3*a + 3*c)*sin(-3*a
+ 3*c) + sin(3*a + 3*c)^2*sin(-3*a + 3*c) + 2*(2*cos(2*b*x + 3*a + 5*c)*si
n(-3*a + 3*c) + cos(3*a + 3*c)*sin(-3*a + 3*c))*cos(4*b*x + 3*a + 7*c) + 2
*(2*sin(2*b*x + 3*a + 5*c)*sin(-3*a + 3*c) + sin(3*a + 3*c)*sin(-3*a + 3*c
))*sin(4*b*x + 3*a + 7*c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(
2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2) + (8*b*x*sin(2
*b*x + 8*c) + 4*b*x*sin(6*c) - 4*cos(2*b*x + 6*a + 2*c) + 6*cos(2*b*x +...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.30 (sec) , antiderivative size = 5096, normalized size of antiderivative = 5096.00

$$\int \cos^3(a + bx) \sec^3(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sec(b*x+c)^3,x, algorithm="giac")
```

output
```
((tan(1/2*a)^6*tan(1/2*c)^6 - 15*tan(1/2*a)^6*tan(1/2*c)^4 + 36*tan(1/2*a)
^5*tan(1/2*c)^5 - 15*tan(1/2*a)^4*tan(1/2*c)^6 + 15*tan(1/2*a)^6*tan(1/2*c
)^2 - 120*tan(1/2*a)^5*tan(1/2*c)^3 + 225*tan(1/2*a)^4*tan(1/2*c)^4 - 120*
tan(1/2*a)^3*tan(1/2*c)^5 + 15*tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^6 +
36*tan(1/2*a)^5*tan(1/2*c) - 225*tan(1/2*a)^4*tan(1/2*c)^2 + 400*tan(1/2*a
)^3*tan(1/2*c)^3 - 225*tan(1/2*a)^2*tan(1/2*c)^4 + 36*tan(1/2*a)*tan(1/2*c
)^5 - tan(1/2*c)^6 + 15*tan(1/2*a)^4 - 120*tan(1/2*a)^3*tan(1/2*c) + 225*t
an(1/2*a)^2*tan(1/2*c)^2 - 120*tan(1/2*a)*tan(1/2*c)^3 + 15*tan(1/2*c)^4 -
 15*tan(1/2*a)^2 + 36*tan(1/2*a)*tan(1/2*c) - 15*tan(1/2*c)^2 + 1)*(b*x +
a)/(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a)
^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^
4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)
^2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1
/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 +
1) - (3*tan(1/2*a)^6*tan(1/2*c)^5 - 3*tan(1/2*a)^5*tan(1/2*c)^6 - 10*tan(1
/2*a)^6*tan(1/2*c)^3 + 45*tan(1/2*a)^5*tan(1/2*c)^4 - 45*tan(1/2*a)^4*tan(
1/2*c)^5 + 10*tan(1/2*a)^3*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c) - 45*t
an(1/2*a)^5*tan(1/2*c)^2 + 150*tan(1/2*a)^4*tan(1/2*c)^3 - 150*tan(1/2*a)^
3*tan(1/2*c)^4 + 45*tan(1/2*a)^2*tan(1/2*c)^5 - 3*tan(1/2*a)*tan(1/2*c)^6
+ 3*tan(1/2*a)^5 - 45*tan(1/2*a)^4*tan(1/2*c) + 150*tan(1/2*a)^3*tan(1/...
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sec^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)^3/cos(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos^3(a + bx)\sec^3(c + bx)\,dx = \text{too large to display}$$

input
```
int(cos(b*x+a)^3*sec(b*x+c)^3,x)
```

output
```
(36*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)*sin(a + b*x)**2 - 144*cos(b*x +
 c)*cos(a + b*x)*sin(b*x + c) + 108*cos(b*x + c)*cos(a + b*x)*sin(a + b*x)
 + 108*cos(b*x + c)*sin(b*x + c)*sin(a + b*x)**2 - 144*cos(b*x + c)*sin(b*
x + c) - 54*cos(b*x + c)*sin(a + b*x)**3 + 63*cos(b*x + c)*sin(a + b*x) +
27*cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x) + 54*cos(a + b*x)*sin(b*x + c
)*sin(a + b*x)**2 - 261*cos(a + b*x)*sin(b*x + c) + 81*cos(a + b*x)*sin(a
+ b*x) + 4320*int(tan((b*x + c)/2)**4/(tan((b*x + c)/2)**6*tan((a + b*x)/2
)**6 + 3*tan((b*x + c)/2)**6*tan((a + b*x)/2)**4 + 3*tan((b*x + c)/2)**6*t
an((a + b*x)/2)**2 + tan((b*x + c)/2)**6 - 3*tan((b*x + c)/2)**4*tan((a +
b*x)/2)**6 - 9*tan((b*x + c)/2)**4*tan((a + b*x)/2)**4 - 9*tan((b*x + c)/2
)**4*tan((a + b*x)/2)**2 - 3*tan((b*x + c)/2)**4 + 3*tan((b*x + c)/2)**2*t
an((a + b*x)/2)**6 + 9*tan((b*x + c)/2)**2*tan((a + b*x)/2)**4 + 9*tan((b*
x + c)/2)**2*tan((a + b*x)/2)**2 + 3*tan((b*x + c)/2)**2 - tan((a + b*x)/2
)**6 - 3*tan((a + b*x)/2)**4 - 3*tan((a + b*x)/2)**2 - 1),x)*sin(b*x + c)*
*2*b - 4320*int(tan((b*x + c)/2)**4/(tan((b*x + c)/2)**6*tan((a + b*x)/2)*
*6 + 3*tan((b*x + c)/2)**6*tan((a + b*x)/2)**4 + 3*tan((b*x + c)/2)**6*tan
((a + b*x)/2)**2 + tan((b*x + c)/2)**6 - 3*tan((b*x + c)/2)**4*tan((a + b*
x)/2)**6 - 9*tan((b*x + c)/2)**4*tan((a + b*x)/2)**4 - 9*tan((b*x + c)/2)*
*4*tan((a + b*x)/2)**2 - 3*tan((b*x + c)/2)**4 + 3*tan((b*x + c)/2)**2*tan
((a + b*x)/2)**6 + 9*tan((b*x + c)/2)**2*tan((a + b*x)/2)**4 + 9*tan((b...
```

3.317 $\int \cos^3(a+bx)\sec^4(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \cos^3(a+bx)\sec^4(c+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.55 (sec) , antiderivative size = 105, normalized size of antiderivative = 105.00

$$\int \cos^3(a+bx)\sec^4(c+bx)\,dx$$
$$=\frac{12\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\left(3\cos(a-c)+5\cos(3(a-c))\right)-(32+40\cos(2(a-c))+15\cos(2}{48b}$$

input
```
Integrate[Cos[a + b*x]^3*Sec[c + b*x]^4,x]
```

output
```
(12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*(3*Cos[a - c] + 5*Cos[3*(a - c)]
) - (32 + 40*Cos[2*(a - c)] + 15*Cos[2*(a - 2*c - b*x)] + 33*Cos[2*(a + b*
x)] + 24*Cos[2*(c + b*x)])*Sec[c + b*x]^3*Sin[a - c])/(48*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\sec^4(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \cos^3(a+bx)\sec^4(bx+c)dx$$

input
```
Int[Cos[a + b*x]^3*Sec[c + b*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 40.12 (sec) , antiderivative size = 357, normalized size of antiderivative = 357.00

method	result
risch	$\frac{i\left(33\,e^{i(5bx+9a+2c)}-9\,e^{i(5bx+7a+4c)}-9\,e^{i(5bx+5a+6c)}-15\,e^{i(5bx+3a+8c)}+40\,e^{3i(bx+3a)}+24\,e^{i(3bx+7a+2c)}-24\,e^{i(3bx+5a+4c)}-40\,e^{3i}\right.}{24\left(e^{2i(bx+a+c)}+e^{2ia}\right)^3 b}$
default	Expression too large to display

input
```
int(cos(b*x+a)^3*sec(b*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/24*I/(exp(2*I*(b*x+a+c))+exp(2*I*a))^3/b*(33*exp(I*(5*b*x+9*a+2*c))-9*ex
p(I*(5*b*x+7*a+4*c))-9*exp(I*(5*b*x+5*a+6*c))-15*exp(I*(5*b*x+3*a+8*c))+40
*exp(3*I*(b*x+3*a))+24*exp(I*(3*b*x+7*a+2*c))-24*exp(I*(3*b*x+5*a+4*c))-40
*exp(3*I*(b*x+a+2*c))+15*exp(I*(b*x+9*a-2*c))+9*exp(I*(b*x+7*a))+9*exp(I*(
b*x+5*a+2*c))-33*exp(I*(b*x+3*a+4*c)))+3/8*ln(exp(I*(b*x+a))+I*exp(I*(a-c)
))/b*cos(a-c)+5/8*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*cos(3*a-3*c)-3/8*ln(
exp(I*(b*x+a))-I*exp(I*(a-c)))/b*cos(a-c)-5/8*ln(exp(I*(b*x+a))-I*exp(I*(a
-c)))/b*cos(3*a-3*c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 165.00

$$\int \cos^3(a+bx)\sec^4(c+bx)\,dx$$
$$= \frac{3\left(5\cos(-a+c)^3-3\cos(-a+c)\right)\cos(bx+c)^3\log(\sin(bx+c)+1)-3\left(5\cos(-a+c)^3-3\cos(-a\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sec(b*x+c)^4,x, algorithm="fricas")
```

output
```
1/12*(3*(5*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^3*log(sin(b*x + c)
+ 1) - 3*(5*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^3*log(-sin(b*x + c
) + 1) - 18*(cos(-a + c)^3 - cos(-a + c))*cos(b*x + c)*sin(b*x + c) + 4*(3
*(4*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)*sin(-a + c))/(b
*cos(b*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sec^4(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*sec(b*x+c)**4,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.31 (sec) , antiderivative size = 2778, normalized size of antiderivative = 2778.00

$$\int \cos^3(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sec(b*x+c)^4,x, algorithm="maxima")
```

output
```
-1/48*(2*(33*sin(5*b*x + 6*a + 4*c) - 9*sin(5*b*x + 4*a + 6*c) - 9*sin(5*b
*x + 2*a + 8*c) - 15*sin(5*b*x + 10*c) + 40*sin(3*b*x + 6*a + 2*c) + 24*si
n(3*b*x + 4*a + 4*c) - 24*sin(3*b*x + 2*a + 6*c) - 40*sin(3*b*x + 8*c) + 1
5*sin(b*x + 6*a) + 9*sin(b*x + 4*a + 2*c) + 9*sin(b*x + 2*a + 4*c) - 33*si
n(b*x + 6*c))*cos(6*b*x + 3*a + 8*c) - 66*(3*sin(4*b*x + 3*a + 6*c) + 3*si
n(2*b*x + 3*a + 4*c) + sin(3*a + 2*c))*cos(5*b*x + 6*a + 4*c) + 18*(3*sin(
4*b*x + 3*a + 6*c) + 3*sin(2*b*x + 3*a + 4*c) + sin(3*a + 2*c))*cos(5*b*x
+ 4*a + 6*c) + 18*(3*sin(4*b*x + 3*a + 6*c) + 3*sin(2*b*x + 3*a + 4*c) + s
in(3*a + 2*c))*cos(5*b*x + 2*a + 8*c) + 30*(3*sin(4*b*x + 3*a + 6*c) + 3*s
in(2*b*x + 3*a + 4*c) + sin(3*a + 2*c))*cos(5*b*x + 10*c) + 6*(40*sin(3*b*
x + 6*a + 2*c) + 24*sin(3*b*x + 4*a + 4*c) - 24*sin(3*b*x + 2*a + 6*c) - 4
0*sin(3*b*x + 8*c) + 15*sin(b*x + 6*a) + 9*sin(b*x + 4*a + 2*c) + 9*sin(b*
x + 2*a + 4*c) - 33*sin(b*x + 6*c))*cos(4*b*x + 3*a + 6*c) - 80*(3*sin(2*b
*x + 3*a + 4*c) + sin(3*a + 2*c))*cos(3*b*x + 6*a + 2*c) - 48*(3*sin(2*b*x
 + 3*a + 4*c) + sin(3*a + 2*c))*cos(3*b*x + 4*a + 4*c) + 48*(3*sin(2*b*x +
 3*a + 4*c) + sin(3*a + 2*c))*cos(3*b*x + 2*a + 6*c) + 80*(3*sin(2*b*x + 3
*a + 4*c) + sin(3*a + 2*c))*cos(3*b*x + 8*c) + 18*(5*sin(b*x + 6*a) + 3*si
n(b*x + 4*a + 2*c) + 3*sin(b*x + 2*a + 4*c) - 11*sin(b*x + 6*c))*cos(2*b*x
 + 3*a + 4*c) + 3*((3*cos(-a + c) + 5*cos(-3*a + 3*c))*cos(6*b*x + 3*a + 8
*c)^2 + 9*(3*cos(-a + c) + 5*cos(-3*a + 3*c))*cos(4*b*x + 3*a + 6*c)^2 ...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.07 (sec) , antiderivative size = 22836, normalized size of antiderivative = 22836.00

$$\int \cos^3(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sec(b*x+c)^4,x, algorithm="giac")
```

output
```
-1/3*(3*(tan(1/2*a)^7*tan(1/2*c)^7 - tan(1/2*a)^7*tan(1/2*c)^6 + tan(1/2*a
)^6*tan(1/2*c)^7 - 9*tan(1/2*a)^7*tan(1/2*c)^5 + 25*tan(1/2*a)^6*tan(1/2*c
)^6 - 9*tan(1/2*a)^5*tan(1/2*c)^7 + 9*tan(1/2*a)^7*tan(1/2*c)^4 - 33*tan(1
/2*a)^6*tan(1/2*c)^5 + 33*tan(1/2*a)^5*tan(1/2*c)^6 - 9*tan(1/2*a)^4*tan(1
/2*c)^7 + 9*tan(1/2*a)^7*tan(1/2*c)^3 - 81*tan(1/2*a)^6*tan(1/2*c)^4 + 165
*tan(1/2*a)^5*tan(1/2*c)^5 - 81*tan(1/2*a)^4*tan(1/2*c)^6 + 9*tan(1/2*a)^3
*tan(1/2*c)^7 - 9*tan(1/2*a)^7*tan(1/2*c)^2 + 81*tan(1/2*a)^6*tan(1/2*c)^3
 - 213*tan(1/2*a)^5*tan(1/2*c)^4 + 213*tan(1/2*a)^4*tan(1/2*c)^5 - 81*tan(
1/2*a)^3*tan(1/2*c)^6 + 9*tan(1/2*a)^2*tan(1/2*c)^7 - tan(1/2*a)^7*tan(1/2
*c) + 33*tan(1/2*a)^6*tan(1/2*c)^2 - 213*tan(1/2*a)^5*tan(1/2*c)^3 + 397*t
an(1/2*a)^4*tan(1/2*c)^4 - 213*tan(1/2*a)^3*tan(1/2*c)^5 + 33*tan(1/2*a)^2
*tan(1/2*c)^6 - tan(1/2*a)*tan(1/2*c)^7 + tan(1/2*a)^7 - 25*tan(1/2*a)^6*t
an(1/2*c) + 165*tan(1/2*a)^5*tan(1/2*c)^2 - 397*tan(1/2*a)^4*tan(1/2*c)^3
+ 397*tan(1/2*a)^3*tan(1/2*c)^4 - 165*tan(1/2*a)^2*tan(1/2*c)^5 + 25*tan(1
/2*a)*tan(1/2*c)^6 - tan(1/2*c)^7 - tan(1/2*a)^6 + 33*tan(1/2*a)^5*tan(1/2
*c) - 213*tan(1/2*a)^4*tan(1/2*c)^2 + 397*tan(1/2*a)^3*tan(1/2*c)^3 - 213*
tan(1/2*a)^2*tan(1/2*c)^4 + 33*tan(1/2*a)*tan(1/2*c)^5 - tan(1/2*c)^6 - 9*
tan(1/2*a)^5 + 81*tan(1/2*a)^4*tan(1/2*c) - 213*tan(1/2*a)^3*tan(1/2*c)^2
+ 213*tan(1/2*a)^2*tan(1/2*c)^3 - 81*tan(1/2*a)*tan(1/2*c)^4 + 9*tan(1/2*c
)^5 + 9*tan(1/2*a)^4 - 81*tan(1/2*a)^3*tan(1/2*c) + 165*tan(1/2*a)^2*ta...
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sec^4(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)^3/cos(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos^3(a+bx)\sec^4(c+bx)\,dx = \text{too large to display}$$

input
```
int(cos(b*x+a)^3*sec(b*x+c)^4,x)
```

output
```
( - 63*cos(b*x + c)*cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x) + 192*cos(b*
x + c)*cos(a + b*x)*sin(b*x + c)*sin(a + b*x)**2 - 768*cos(b*x + c)*cos(a
+ b*x)*sin(b*x + c) + 309*cos(b*x + c)*cos(a + b*x)*sin(a + b*x) - 4992*co
s(b*x + c)*int(tan((b*x + c)/2)**6/(tan((b*x + c)/2)**8*tan((a + b*x)/2)**
6 + 3*tan((b*x + c)/2)**8*tan((a + b*x)/2)**4 + 3*tan((b*x + c)/2)**8*tan(
(a + b*x)/2)**2 + tan((b*x + c)/2)**8 - 4*tan((b*x + c)/2)**6*tan((a + b*x
)/2)**6 - 12*tan((b*x + c)/2)**6*tan((a + b*x)/2)**4 - 12*tan((b*x + c)/2)
**6*tan((a + b*x)/2)**2 - 4*tan((b*x + c)/2)**6 + 6*tan((b*x + c)/2)**4*ta
n((a + b*x)/2)**6 + 18*tan((b*x + c)/2)**4*tan((a + b*x)/2)**4 + 18*tan((b
*x + c)/2)**4*tan((a + b*x)/2)**2 + 6*tan((b*x + c)/2)**4 - 4*tan((b*x + c
)/2)**2*tan((a + b*x)/2)**6 - 12*tan((b*x + c)/2)**2*tan((a + b*x)/2)**4 -
 12*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 4*tan((b*x + c)/2)**2 + tan(
(a + b*x)/2)**6 + 3*tan((a + b*x)/2)**4 + 3*tan((a + b*x)/2)**2 + 1),x)*si
n(b*x + c)**2*b + 4992*cos(b*x + c)*int(tan((b*x + c)/2)**6/(tan((b*x + c)
/2)**8*tan((a + b*x)/2)**6 + 3*tan((b*x + c)/2)**8*tan((a + b*x)/2)**4 + 3
*tan((b*x + c)/2)**8*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**8 - 4*tan((b*
x + c)/2)**6*tan((a + b*x)/2)**6 - 12*tan((b*x + c)/2)**6*tan((a + b*x)/2)
**4 - 12*tan((b*x + c)/2)**6*tan((a + b*x)/2)**2 - 4*tan((b*x + c)/2)**6 +
 6*tan((b*x + c)/2)**4*tan((a + b*x)/2)**6 + 18*tan((b*x + c)/2)**4*tan((a
 + b*x)/2)**4 + 18*tan((b*x + c)/2)**4*tan((a + b*x)/2)**2 + 6*tan((b*x...
```

3.318 $\int \sec(a+bx)\sec(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \sec(a+bx)\sec(c+bx)\,dx = -\frac{\csc(a-c)\log(\cos(a+bx))}{b}+\frac{\csc(a-c)\log(\cos(c+bx))}{b}$$

output
```
-csc(a-c)*ln(cos(b*x+a))/b+csc(a-c)*ln(cos(b*x+c))/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \sec(a+bx)\sec(c+bx)\,dx = -\frac{\csc(a-c)(\log(\cos(a+bx))-\log(\cos(c+bx)))}{b}$$

input
```
Integrate[Sec[a + b*x]*Sec[c + b*x],x]
```

output
```
-((Csc[a - c]*(Log[Cos[a + b*x]] - Log[Cos[c + b*x]]))/b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5121, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \text{5121}$$

$$\csc(a-c)\int \tan(a+bx)dx-\csc(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a-c)\int \tan(a+bx)dx-\csc(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\csc(a-c)\log(\cos(bx+c))}{b}-\frac{\csc(a-c)\log(\cos(a+bx))}{b}$$

input
```
Int[Sec[a + b*x]*Sec[c + b*x],x]
```

output
```
-((Csc[a - c]*Log[Cos[a + b*x]])/b) + (Csc[a - c]*Log[Cos[c + b*x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5121
```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Csc[
(b*c - a*d)/d]   Int[Tan[a + b*x], x], x] + Simp[Csc[(b*c - a*d)/b]   Int[T
an[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b
*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\ln(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}{b(\sin(a)\cos(c)-\cos(a)\sin(c))}$	54
risch	$-\frac{2i\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)\mathrm{e}^{i(a+c)}}{\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)b}+\frac{2i\ln\left(\mathrm{e}^{2i(bx+a)}+\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{i(a+c)}}{\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)b}$	90

input
```
int(sec(b*x+a)*sec(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b/(sin(a)*cos(c)-cos(a)*sin(c))*ln(tan(b*x+a)*sin(a)*cos(c)-tan(b*x+a)*c
os(a)*sin(c)+cos(a)*cos(c)+sin(a)*sin(c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(36) = 72.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.97

$$\int \sec(a+bx)\sec(c+bx)\,dx = -\frac{\log\left(\cos\left(bx+c\right)^2\right)-\log\left(\frac{4\left(2\,\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(2\,\cos(-a+c)^2-1\right)\cos(bx+c)^2-\cos(-a+c)^2+1\right)}{\cos(-a+c)^2+2\,\cos(-a+c)+1}\right)}{2\,b\sin\left(-a+c\right)}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(log(cos(b*x + c)^2) - log(4*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)
*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)/(
cos(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*sin(-a + c))
```

Sympy [F]

$$\int \sec(a+bx)\sec(c+bx)\,dx = \int \sec(a+bx)\sec(bx+c)\,dx$$

input
```
integrate(sec(b*x+a)*sec(b*x+c),x)
```

output
```
Integral(sec(a + b*x)*sec(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(36) = 72.

Time = 0.05 (sec) , antiderivative size = 349, normalized size of antiderivative = 9.69

$$\int \sec(a+bx)\sec(c+bx)\,dx =$$
$$-\frac{2\left((\cos(2a)-\cos(2c))\cos(a+c)+(\sin(2a)-\sin(2c))\sin(a+c)\right)\arctan\left(\sin(2bx)-\sin(2a),\cos\right.}{}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="maxima")
```

output
```
-(2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*
arctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - 2*((cos(2*a) - cos
(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(2*b*x) -
 sin(2*c), cos(2*b*x) + cos(2*c)) - ((sin(2*a) - sin(2*c))*cos(a + c) - (c
os(2*a) - cos(2*c))*sin(a + c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) +
 cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + ((sin(2
*a) - sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(2*b
*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*s
in(2*c) + sin(2*c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a
)*sin(2*c) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(36) = 72$.

Time = 0.17 (sec) , antiderivative size = 171, normalized size of antiderivative = 4.75

$$\int \sec(a+bx)\sec(c+bx)\,dx$$
$$= \frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1\right)\log\left(\left|2\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-2\tan\left(bx\right.\right.\right.}{2\left(\tan\left(\frac{1}{2}\right.\right.}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="giac")
```

output
```
1/2*(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(
2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2
*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)
^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 +
1))/((tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan
(1/2*c))*b)
```

Mupad [B] (verification not implemented)

Time = 25.03 (sec) , antiderivative size = 249, normalized size of antiderivative = 6.92

$$\int \sec(a+bx)\sec(c+bx)\,dx$$
$$= \frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}\left(\ln\left(-\frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}\,(4\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,4\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}})}{b\,(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}-1)}+\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\,4\mathrm{i}\right)-\ln\left(-\right.\right.}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-1\right)}$$

input
```
int(1/(cos(a + b*x)*cos(c + b*x)),x)
```

output
```
(2*(-exp(a*2i - c*2i))^(1/2)*(log(exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2
i)*4i - (2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(4*b*exp(a*2i)*exp(-c*2i) + 2*b*e
xp(a*2i)*exp(b*x*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))/(b*(exp(a*2i
)*exp(-c*2i) - 1))) - log(exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2i)*4i -
(2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*2i)
*exp(b*x*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))/(b - b*exp(a*2i)*exp
(-c*2i)))))/(b*(exp(a*2i - c*2i) - 1))
```

Reduce [F]

$$\int \sec(a+bx)\sec(c+bx)\,dx$$
$$= \frac{4\left(\int \frac{1}{\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1}dx\right)b+\log\left(\tan\left(\frac{bx}{2}+\frac{c}{2}\right)-1\right)-\log\left(\tan\left(\frac{bx}{2}+\frac{c}{2}\right)+\right.}{b}$$

input
```
int(sec(b*x+a)*sec(b*x+c),x)
```

output
```
(4*int(1/(tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - tan((b*x + c)/2)**2 -
tan((a + b*x)/2)**2 + 1),x)*b + log(tan((b*x + c)/2) - 1) - log(tan((b*x +
 c)/2) + 1) + log(tan((a + b*x)/2) - 1) - log(tan((a + b*x)/2) + 1) - b*x)
/b
```

3.319 $\int \sec(a+bx)\sec^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \sec(a+bx)\sec^2(c+bx)\,dx = -\frac{\operatorname{arctanh}(\sin(c+bx))\cot(a-c)\csc(a-c)}{b} + \frac{\operatorname{arctanh}(\sin(a+bx))\csc^2(a-c)}{b} - \frac{\csc(a-c)\sec(c+bx)}{b}$$

output
```
-arctanh(sin(b*x+c))*cot(a-c)*csc(a-c)/b+arctanh(sin(b*x+a))*csc(a-c)^2/b-
csc(a-c)*sec(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int \sec(a+bx)\sec^2(c+bx)\,dx = -\frac{\csc(a-c)\left(2\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\cot(a-c)+\csc(a-c)\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(\right.\right.\right.\right.}{b}$$

input
```
Integrate[Sec[a + b*x]*Sec[c + b*x]^2,x]
```

output
```
-((Csc[a - c]*(2*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Cot[a - c] + Csc[a
- c]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] + Si
n[(a + b*x)/2]]) + Sec[c + b*x]))/b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\sec^2(bx+c)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \sec(a+bx)\sec^2(bx+c)dx$$

input
```
Int[Sec[a + b*x]*Sec[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(61) = 122$.

Time = 1.51 (sec) , antiderivative size = 356, normalized size of antiderivative = 5.84

method	result
default	$-\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-1\right)}{(\sin(a)\cos(c)-\cos(a)\sin(c))^2}+\frac{2\left(\frac{\left(\cos(c)^2\sin(a)^2-2\cos(a)\cos(c)\sin(a)\sin(c)+\sin(c)^2\cos(a)^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\cos(a)\cos(c)+\sin(a)\sin(c)}+\sin(a)\cos(c)-c\right.}{\cos(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+\sin(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-2\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right.}$
risch	$-\frac{4ie^{i(bx+3a+2c)}}{\left(e^{2i(bx+a+c)}+e^{2ia}\right)\left(e^{2ia}-e^{2ic}\right)b}-\frac{4\ln\left(e^{i(bx+a)}+i\right)e^{2i(a+c)}}{\left(e^{4ia}-2e^{2i(a+c)}+e^{4ic}\right)b}-\frac{2\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)e^{i(3a+c)}}{\left(e^{4ia}-2e^{2i(a+c)}+e^{4ic}\right)b}-\frac{2\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)}{\left(e^{4ia}-2e^{2i(a+c)}+e^{4}\right.}$

input
```
int(sec(b*x+a)*sec(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/(sin(a)*cos(c)-cos(a)*sin(c))^2*ln(tan(1/2*a+1/2*b*x)-1)+2/(sin(a)
*cos(c)-cos(a)*sin(c))^2*(((cos(c)^2*sin(a)^2-2*cos(a)*cos(c)*sin(a)*sin(c
)+sin(c)^2*cos(a)^2)/(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2*a+1/2*b*x)+sin(
a)*cos(c)-cos(a)*sin(c))/(cos(c)*cos(a)*tan(1/2*a+1/2*b*x)^2+sin(c)*sin(a)
*tan(1/2*a+1/2*b*x)^2-2*cos(c)*sin(a)*tan(1/2*a+1/2*b*x)+2*sin(c)*cos(a)*t
an(1/2*a+1/2*b*x)-cos(a)*cos(c)-sin(a)*sin(c))+(cos(a)*cos(c)+sin(a)*sin(c
))/(-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-sin(c)^2*cos(a)
^2)^(1/2)*arctan(1/2*(2*(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2*a+1/2*b*x)-2
*sin(a)*cos(c)+2*cos(a)*sin(c))/(-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(
a)^2*sin(c)^2-sin(c)^2*cos(a)^2)^(1/2)))+1/(sin(a)*cos(c)-cos(a)*sin(c))^2
*ln(tan(1/2*a+1/2*b*x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(61) = 122$.

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.97

$$\int \sec(a+bx)\sec^2(c+bx)\,dx$$
$$=\frac{\cos(bx+c)\cos(-a+c)\log(\sin(bx+c)+1)-\cos(bx+c)\cos(-a+c)\log(-\sin(bx+c)+1)-\cos(\ldots}{2\,(b\ldots}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(cos(b*x + c)*cos(-a + c)*log(sin(b*x + c) + 1) - cos(b*x + c)*cos(-a
+ c)*log(-sin(b*x + c) + 1) - cos(b*x + c)*log(2*(cos(-a + c)*sin(b*x + c)
 - cos(b*x + c)*sin(-a + c) + 1)/(cos(-a + c) + 1)) + cos(b*x + c)*log(-2*
(cos(-a + c)*sin(b*x + c) - cos(b*x + c)*sin(-a + c) - 1)/(cos(-a + c) + 1
)) - 2*sin(-a + c))/((b*cos(-a + c)^2 - b)*cos(b*x + c))
```

Sympy [F]

$$\int \sec(a+bx)\sec^2(c+bx)\,dx = \int \sec(a+bx)\sec^2(bx+c)\,dx$$

input
```
integrate(sec(b*x+a)*sec(b*x+c)**2,x)
```

output
```
Integral(sec(a + b*x)*sec(b*x + c)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12393 vs. 2(61) = 122.

Time = 0.34 (sec) , antiderivative size = 12393, normalized size of antiderivative = 203.16

$$\int \sec(a+bx)\sec^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c)^2,x, algorithm="maxima")
```

output
```
-(4*(cos(4*a)^2 - 4*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c
)^2 + 2*cos(4*a)*cos(4*c) + cos(4*c)^2 + sin(4*a)^2 - 4*(sin(4*a) + sin(4*
c))*sin(2*a + 2*c) + 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2
)*cos(b*x + a + 2*c)*sin(2*b*x + 2*a + 2*c) - 4*(cos(4*a)^2 - 4*(cos(4*a)
+ cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)^2 + 2*cos(4*a)*cos(4*c) + co
s(4*c)^2 + sin(4*a)^2 - 4*(sin(4*a) + sin(4*c))*sin(2*a + 2*c) + 4*sin(2*a
 + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)*cos(b*x + a + 2*c)*sin(2*b*x
 + 4*c) - 4*(cos(4*a)^2 - 4*(cos(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2
*a + 2*c)^2 + 2*cos(4*a)*cos(4*c) + cos(4*c)^2 + sin(4*a)^2 - 4*(sin(4*a)
+ sin(4*c))*sin(2*a + 2*c) + 4*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + si
n(4*c)^2)*cos(2*b*x + 2*a + 2*c)*sin(b*x + a + 2*c) + 4*(cos(4*a)^2 - 4*(c
os(4*a) + cos(4*c))*cos(2*a + 2*c) + 4*cos(2*a + 2*c)^2 + 2*cos(4*a)*cos(4
*c) + cos(4*c)^2 + sin(4*a)^2 - 4*(sin(4*a) + sin(4*c))*sin(2*a + 2*c) + 4
*sin(2*a + 2*c)^2 + 2*sin(4*a)*sin(4*c) + sin(4*c)^2)*cos(2*b*x + 4*c)*sin
(b*x + a + 2*c) + 4*(((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + c
os(4*c))*sin(2*a + 2*c))*cos(2*b*x + 2*a + 2*c)^2 + ((sin(4*a) + sin(4*c))
*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*cos(2*b*x + 4*c)^2
 + ((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a +
 2*c))*sin(2*b*x + 2*a + 2*c)^2 + ((sin(4*a) + sin(4*c))*cos(2*a + 2*c) -
(cos(4*a) + cos(4*c))*sin(2*a + 2*c))*sin(2*b*x + 4*c)^2 + 2*((cos(2*a)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2905 vs. $2(61) = 122$.

Time = 0.36 (sec) , antiderivative size = 2905, normalized size of antiderivative = 47.62

$$\int \sec(a + bx)\sec^2(c + bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c)^2,x, algorithm="giac")
```

output
```
1/4*((tan(1/2*a)^5*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^4 + tan(1/2*a)^4
*tan(1/2*c)^5 + 5*tan(1/2*a)^4*tan(1/2*c)^4 - 4*tan(1/2*a)^4*tan(1/2*c)^3
+ 4*tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^5*tan(1/2*c) + 4*tan(1/2*a)^4*t
an(1/2*c)^2 + 4*tan(1/2*a)^3*tan(1/2*c)^3 + 4*tan(1/2*a)^2*tan(1/2*c)^4 -
tan(1/2*a)*tan(1/2*c)^5 + tan(1/2*a)^5 - 5*tan(1/2*a)^4*tan(1/2*c) + 4*tan
(1/2*a)^3*tan(1/2*c)^2 - 4*tan(1/2*a)^2*tan(1/2*c)^3 + 5*tan(1/2*a)*tan(1/
2*c)^4 - tan(1/2*c)^5 - tan(1/2*a)^4 + 4*tan(1/2*a)^3*tan(1/2*c) + 4*tan(1
/2*a)^2*tan(1/2*c)^2 + 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/
2*a)^2*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^2 + 5*tan(1/2*a)*tan(1/2*c) -
tan(1/2*a) + tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(
1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*b*x + 1/2*a)*tan(1/2*c)
 + tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan(1/2*c)
+ 1))/(tan(1/2*a)^5*tan(1/2*c)^3 - 2*tan(1/2*a)^4*tan(1/2*c)^4 + tan(1/2*a
)^3*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^2 + 3*tan(1/2*a)^4*tan(1/2*c)^3
 - 3*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^2*tan(1/2*c)^5 + 3*tan(1/2*a)^
4*tan(1/2*c)^2 - 6*tan(1/2*a)^3*tan(1/2*c)^3 + 3*tan(1/2*a)^2*tan(1/2*c)^4
 - 2*tan(1/2*a)^4*tan(1/2*c) + 6*tan(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^
2*tan(1/2*c)^3 + 2*tan(1/2*a)*tan(1/2*c)^4 + 3*tan(1/2*a)^3*tan(1/2*c) - 6
*tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 + 3*
tan(1/2*a)^2*tan(1/2*c) - 3*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + ta...
```

Mupad [F(-1)]

Timed out.

$$\int \sec(a+bx)\sec^2(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)*cos(c + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec(a+bx)\sec^2(c+bx)\,dx$$
$$= \frac{-\cos(bx+c)\left(\int \frac{\sin(bx+c)^2}{\sin(bx+c)^2-1}dx\right)b-\cos(bx+c)\left(\int \frac{\sin(bx+c)^2}{\cos(bx+a)\sin(bx+c)^2-\cos(bx+a)}dx\right)b-\cos(bx+c)\log(\mathrm{ta}}{\cos(bx+c)\,b}$$

input
```
int(sec(b*x+a)*sec(b*x+c)^2,x)
```

output
```
( - cos(b*x + c)*int(sin(b*x + c)**2/(sin(b*x + c)**2 - 1),x)*b - cos(b*x
+ c)*int(sin(b*x + c)**2/(cos(a + b*x)*sin(b*x + c)**2 - cos(a + b*x)),x)*
b - cos(b*x + c)*log(tan((a + b*x)/2) - 1) + cos(b*x + c)*log(tan((a + b*x
)/2) + 1) + cos(b*x + c)*b*x - sin(b*x + c))/(cos(b*x + c)*b)
```

3.320 $\int \sec(a+bx)\sec^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 84

$$\int \sec(a+bx)\sec^3(c+bx)\,dx = -\frac{\csc^3(a-c)\log(\cos(a+bx))}{b} + \frac{\csc^3(a-c)\log(\cos(c+bx))}{b} - \frac{\csc(a-c)\sec^2(c+bx)}{2b} - \frac{\cot(a-c)\csc(a-c)\tan(c+bx)}{b}$$

output
```
-csc(a-c)^3*ln(cos(b*x+a))/b+csc(a-c)^3*ln(cos(b*x+c))/b-1/2*csc(a-c)*sec(
b*x+c)^2/b-cot(a-c)*csc(a-c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.82

$$\int \sec(a+bx)\sec^3(c+bx)\,dx$$
$$= \frac{\csc^3(a-c)(2\cos(2a-3c) - \cos(2a-3c-2bx) + \cos(2a-c+2bx) - 2\cos(c+2bx)\log(\cos(a+bx))}{}$$

input
```
Integrate[Sec[a + b*x]*Sec[c + b*x]^3,x]
```

output
```
(Csc[a - c]^3*(2*Cos[2*a - 3*c] - Cos[2*a - 3*c - 2*b*x] + Cos[2*a - c + 2
*b*x] - 2*Cos[c + 2*b*x]*Log[Cos[a + b*x]] - 2*Cos[3*c + 2*b*x]*Log[Cos[a
+ b*x]] + 2*Cos[c + 2*b*x]*Log[Cos[c + b*x]] + 2*Cos[3*c + 2*b*x]*Log[Cos[
c + b*x]] + Cos[c]*(-2 - 4*Log[Cos[a + b*x]] + 4*Log[Cos[c + b*x]]))*Sec[c
]*Sec[c + b*x]^2)/(8*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\sec^3(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec(a+bx)\sec^3(bx+c)dx$$

input
```
Int[Sec[a + b*x]*Sec[c + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.36 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.51

method	result
risch	$\frac{4i\left(2\,e^{i(2bx+5a+5c)}+e^{i(7a+c)}+e^{i(5a+3c)}\right)}{\left(e^{2i(bx+a+c)}+e^{2ia}\right)^2\left(e^{2ia}-e^{2ic}\right)^2 b}+\frac{8i\ln\left(e^{2i(bx+a)}+1\right)e^{3i(a+c)}}{\left(e^{6ia}-3\,e^{2i(2a+c)}+3\,e^{2i(a+2c)}-e^{6ic}\right)b}-\frac{8i\ln\left(e^{2i(bx+a)}+e^{2i(a-c)}\right)e^{3i(a+c)}}{\left(e^{6ia}-3\,e^{2i(2a+c)}+3\,e^{2i(a+2c)}-e^{6ic}\right)b}$
default	$-\frac{-2\cos(a)\cos(c)-2\sin(a)\sin(c)}{(\sin(a)\cos(c)-\cos(a)\sin(c))^3(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}+\frac{\ln(\tan(bx+a)\sin(a)\cos(c)-\tan(bx}{(\sin(a)\,c}$

input
```
int(sec(b*x+a)*sec(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
4*I/(exp(2*I*(b*x+a+c))+exp(2*I*a))^2/(exp(2*I*a)-exp(2*I*c))^2/b*(2*exp(I
*(2*b*x+5*a+5*c))+exp(I*(7*a+c))+exp(I*(5*a+3*c)))+8*I*ln(exp(2*I*(b*x+a))
+1)/(exp(6*I*a)-3*exp(2*I*(2*a+c))+3*exp(2*I*(a+2*c))-exp(6*I*c))/b*exp(3*
I*(a+c))-8*I*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))/(exp(6*I*a)-3*exp(2*I*(2*
a+c))+3*exp(2*I*(a+2*c))-exp(6*I*c))/b*exp(3*I*(a+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(82) = 164$.

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.14

$$\int \sec(a+bx)\sec^3(c+bx)\,dx$$

$$=\frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\cos(bx+c)^2\log\left(\cos(bx+c)^2\right)-\cos(bx+c)^2\log}{2\left(b\cos(-a+c)^2-b\right)\cos(}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) + cos(b*x + c)^2*
log(cos(b*x + c)^2) - cos(b*x + c)^2*log(4*(2*cos(b*x + c)*cos(-a + c)*sin
(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)
^2 + 1)/(cos(-a + c)^2 + 2*cos(-a + c) + 1)) + cos(-a + c)^2 - 1)/((b*cos(
-a + c)^2 - b)*cos(b*x + c)^2*sin(-a + c))
```

Sympy [F]

$$\int \sec(a+bx)\sec^3(c+bx)\,dx = \int \sec{(a+bx)}\sec^3{(bx+c)}\,dx$$

input
```
integrate(sec(b*x+a)*sec(b*x+c)**3,x)
```

output
```
Integral(sec(a + b*x)*sec(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69828 vs. $2(82) = 164$.

Time $= 0.90$ (sec) , antiderivative size $= 69828$, normalized size of antiderivative $= 831.29$

$$\int \sec(a+bx)\sec^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c)^3,x, algorithm="maxima")
```

output
```
4*(9*((sin(4*a) + sin(4*c))*cos(3*a + c) - (cos(4*a) + cos(4*c))*sin(3*a +
 c) + 2*cos(2*a + 2*c)*sin(3*a + c) - 2*cos(3*a + c)*sin(2*a + 2*c))*cos(4
*a + 2*c)^2 + 9*((sin(4*a) + sin(4*c))*cos(3*a + c) + (sin(4*a) - 2*sin(2*
a + 2*c) + sin(4*c))*cos(a + 3*c) - (cos(4*a) + cos(4*c))*sin(3*a + c) + 2
*cos(2*a + 2*c)*sin(3*a + c) - 2*cos(3*a + c)*sin(2*a + 2*c) - (cos(4*a) -
 2*cos(2*a + 2*c) + cos(4*c))*sin(a + 3*c))*cos(2*a + 4*c)^2 + 9*((sin(4*a
) + sin(4*c))*cos(3*a + c) - (cos(4*a) + cos(4*c))*sin(3*a + c) + 2*cos(2*
a + 2*c)*sin(3*a + c) - 2*cos(3*a + c)*sin(2*a + 2*c))*sin(4*a + 2*c)^2 +
2*(cos(6*a)^2 - 2*cos(6*a)*cos(6*c) + cos(6*c)^2 + sin(6*a)^2 - 2*sin(6*a)
*sin(6*c) + sin(6*c)^2)*cos(2*a + 2*c)*sin(3*a + c) + 9*((sin(4*a) + sin(4
*c))*cos(3*a + c) + (sin(4*a) - 2*sin(2*a + 2*c) + sin(4*c))*cos(a + 3*c)
- (cos(4*a) + cos(4*c))*sin(3*a + c) + 2*cos(2*a + 2*c)*sin(3*a + c) - 2*c
os(3*a + c)*sin(2*a + 2*c) - (cos(4*a) - 2*cos(2*a + 2*c) + cos(4*c))*sin(
a + 3*c))*sin(2*a + 4*c)^2 - 2*(cos(6*a)^2 - 2*cos(6*a)*cos(6*c) + cos(6*c
)^2 + sin(6*a)^2 - 2*sin(6*a)*sin(6*c) + sin(6*c)^2)*cos(3*a + c)*sin(2*a
+ 2*c) - 2*(((cos(6*a) - 3*cos(4*a + 2*c) - cos(6*c))*cos(3*a + 3*c) + 3*c
os(3*a + 3*c)*cos(2*a + 4*c) + (sin(6*a) - 3*sin(4*a + 2*c) - sin(6*c))*si
n(3*a + 3*c) + 3*sin(3*a + 3*c)*sin(2*a + 4*c))*cos(4*b*x + 4*a + 4*c)^2 +
 4*((cos(6*a) - 3*cos(4*a + 2*c) - cos(6*c))*cos(3*a + 3*c) + 3*cos(3*a +
3*c)*cos(2*a + 4*c) + (sin(6*a) - 3*sin(4*a + 2*c) - sin(6*c))*sin(3*a ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2285 vs. $2(82) = 164$.

Time = 0.24 (sec) , antiderivative size = 2285, normalized size of antiderivative = 27.20

$$\int \sec(a + bx) \sec^3(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c)^3,x, algorithm="giac")
```

output
```
1/8*((tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*
a)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c
)^4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*
c)^2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan
(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2
+ 1)*log(abs(2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1
/2*a)*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a)
 - tan(1/2*a)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - ta
n(1/2*c)^2 + 1))/(tan(1/2*a)^6*tan(1/2*c)^3 - 3*tan(1/2*a)^5*tan(1/2*c)^4
+ 3*tan(1/2*a)^4*tan(1/2*c)^5 - tan(1/2*a)^3*tan(1/2*c)^6 + 3*tan(1/2*a)^5
*tan(1/2*c)^2 - 9*tan(1/2*a)^4*tan(1/2*c)^3 + 9*tan(1/2*a)^3*tan(1/2*c)^4
- 3*tan(1/2*a)^2*tan(1/2*c)^5 + 3*tan(1/2*a)^4*tan(1/2*c) - 9*tan(1/2*a)^3
*tan(1/2*c)^2 + 9*tan(1/2*a)^2*tan(1/2*c)^3 - 3*tan(1/2*a)*tan(1/2*c)^4 +
tan(1/2*a)^3 - 3*tan(1/2*a)^2*tan(1/2*c) + 3*tan(1/2*a)*tan(1/2*c)^2 - tan
(1/2*c)^3) - 2*(3*tan(b*x + a)^2*tan(1/2*a)^8*tan(1/2*c)^7 - 3*tan(b*x + a
)^2*tan(1/2*a)^7*tan(1/2*c)^8 + tan(b*x + a)*tan(1/2*a)^8*tan(1/2*c)^8 + 9
*tan(b*x + a)^2*tan(1/2*a)^8*tan(1/2*c)^5 - 6*tan(b*x + a)^2*tan(1/2*a)^7*
tan(1/2*c)^6 + 2*tan(b*x + a)*tan(1/2*a)^8*tan(1/2*c)^6 + 6*tan(b*x + a)^2
*tan(1/2*a)^6*tan(1/2*c)^7 + 4*tan(b*x + a)*tan(1/2*a)^7*tan(1/2*c)^7 + ta
n(1/2*a)^8*tan(1/2*c)^7 - 9*tan(b*x + a)^2*tan(1/2*a)^5*tan(1/2*c)^8 + ...
```

Mupad [F(-1)]

Timed out.

$$\int \sec(a+bx)\sec^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)*cos(c + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec(a+bx)\sec^3(c+bx)\,dx = \text{Too large to display}$$

input

```
int(sec(b*x+a)*sec(b*x+c)^3,x)
```

output

```
(2*int(sin(b*x + c)**2/(sin(b*x + c)**2 - 1),x)*sin(b*x + c)**2*b - 2*int(
sin(b*x + c)**2/(sin(b*x + c)**2 - 1),x)*b + 2*int(sin(b*x + c)**2/(cos(a
+ b*x)*sin(b*x + c)**2 - cos(a + b*x)),x)*sin(b*x + c)**2*b - 2*int(sin(b*
x + c)**2/(cos(a + b*x)*sin(b*x + c)**2 - cos(a + b*x)),x)*b - 2*int(1/(si
n(b*x + c)**2 - 1),x)*sin(b*x + c)**2*b + 2*int(1/(sin(b*x + c)**2 - 1),x)
*b - 2*int(1/(cos(b*x + c)*cos(a + b*x)*sin(b*x + c)**2 - cos(b*x + c)*cos
(a + b*x)),x)*sin(b*x + c)**2*b + 2*int(1/(cos(b*x + c)*cos(a + b*x)*sin(b
*x + c)**2 - cos(b*x + c)*cos(a + b*x)),x)*b - 2*int(1/(cos(b*x + c)*sin(b
*x + c)**2 - cos(b*x + c)),x)*sin(b*x + c)**2*b + 2*int(1/(cos(b*x + c)*si
n(b*x + c)**2 - cos(b*x + c)),x)*b - 2*int(1/(cos(a + b*x)*sin(b*x + c)**2
 - cos(a + b*x)),x)*sin(b*x + c)**2*b + 2*int(1/(cos(a + b*x)*sin(b*x + c)
**2 - cos(a + b*x)),x)*b + log(tan((b*x + c)/2) - 1)*sin(b*x + c)**2 - log
(tan((b*x + c)/2) - 1) - log(tan((b*x + c)/2) + 1)*sin(b*x + c)**2 + log(t
an((b*x + c)/2) + 1) + 2*log(tan((a + b*x)/2) - 1)*sin(b*x + c)**2 - 2*log
(tan((a + b*x)/2) - 1) - 2*log(tan((a + b*x)/2) + 1)*sin(b*x + c)**2 + 2*l
og(tan((a + b*x)/2) + 1) - 2*sin(b*x + c)**2*b*x + sin(b*x + c) + 2*b*x)/(
2*b*(sin(b*x + c)**2 - 1))
```

3.321 $\int \sec^2(a+bx)\sec^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \sec^2(a+bx)\sec^2(c+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 1.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 87.00

$$\int \sec^2(a+bx)\sec^2(c+bx)\,dx$$
$$=\frac{\csc^2(a-c)\left(2i\arctan(\tan(c+bx))\cot(a-c)-\cot(a-c)\left(2ibx-2\log(\cos(a+bx))+\log\left(\cos^2(c+bx\right.\right.\right.}{b}$$

input
```
Integrate[Sec[a + b*x]^2*Sec[c + b*x]^2,x]
```

output
```
(Csc[a - c]^2*((2*I)*ArcTan[Tan[c + b*x]]*Cot[a - c] - Cot[a - c]*((2*I)*b
*x - 2*Log[Cos[a + b*x]] + Log[Cos[c + b*x]^2]) + (Sec[a]*Sec[a + b*x] + S
ec[c]*Sec[c + b*x])*Sin[b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\sec^2(bx+c)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sec^2(a+bx)\sec^2(bx+c)dx$$

input
```
Int[Sec[a + b*x]^2*Sec[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 3.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 180.00

method	result
default	$\frac{\frac{\tan(bx+a)}{(\sin(a)\cos(c)-\cos(a)\sin(c))^2}-\frac{(2\cos(a)\cos(c)+2\sin(a)\sin(c))\ln(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}{(\sin(a)\cos(c)-\cos(a)\sin(c))^3}-}{b}$
risch	$-\frac{8i\left(e^{2i(bx+3a+c)}+e^{2i(bx+2a+2c)}+2\,e^{2i(2a+c)}\right)}{\left(e^{2i(bx+a)}+1\right)\left(e^{2i(bx+a+c)}+e^{2ia}\right)\left(-e^{2ia}+e^{2ic}\right)^2 b}-\frac{8i\ln\left(e^{2i(bx+a)}+1\right)e^{2i(2a+c)}}{\left(e^{6ia}-3\,e^{2i(2a+c)}+3\,e^{2i(a+2c)}-e^{6ic}\right)b}-\frac{8i\ln\left(e^{2i(bx+a)}+1\right)e^{2i(a+}}{\left(e^{6ia}-3\,e^{2i(2a+c)}+3\,e^{2i(a+2c)}\right.}$

input
```
int(sec(b*x+a)^2*sec(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(tan(b*x+a)/(sin(a)*cos(c)-cos(a)*sin(c))^2-(2*cos(a)*cos(c)+2*sin(a)*
sin(c))/(sin(a)*cos(c)-cos(a)*sin(c))^3*ln(tan(b*x+a)*sin(a)*cos(c)-tan(b*
x+a)*cos(a)*sin(c)+cos(a)*cos(c)+sin(a)*sin(c))-1/(sin(a)*cos(c)-cos(a)*si
n(c))^3*(cos(a)^2*cos(c)^2+sin(c)^2*cos(a)^2+cos(c)^2*sin(a)^2+sin(a)^2*si
n(c)^2)/(tan(b*x+a)*sin(a)*cos(c)-tan(b*x+a)*cos(a)*sin(c)+cos(a)*cos(c)+s
in(a)*sin(c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.10 (sec) , antiderivative size = 315, normalized size of antiderivative = 315.00

$$\int \sec^2(a+bx)\sec^2(c+bx)\,dx =$$

$$-\frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+2\left(\cos(-a+c)^2-1\right)\cos(bx+c)^2-\cos(-a+ }{}$$

input
```
integrate(sec(b*x+a)^2*sec(b*x+c)^2,x, algorithm="fricas")
```

output
```
-(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) + 2*(cos(-a + c)^2 -
 1)*cos(b*x + c)^2 - cos(-a + c)^2 + (cos(b*x + c)^2*cos(-a + c)^2 + cos(b
*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c))*log(cos(b*x + c)^2) - (cos(b
*x + c)^2*cos(-a + c)^2 + cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c
))*log(4*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a + c) + (2*cos(-a
+ c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)/(cos(-a + c)^2 + 2*cos(-a
+ c) + 1)) + 1)/((b*cos(-a + c)^3 - b*cos(-a + c))*cos(b*x + c)^2*sin(-a +
 c) - (b*cos(-a + c)^4 - 2*b*cos(-a + c)^2 + b)*cos(b*x + c)*sin(b*x + c))
```

Sympy [F]

$$\int \sec^2(a+bx)\sec^2(c+bx)\,dx = \int \sec^2(a+bx)\sec^2(bx+c)\,dx$$

input
```
integrate(sec(b*x+a)**2*sec(b*x+c)**2,x)
```

output
```
Integral(sec(a + b*x)**2*sec(b*x + c)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.60 (sec) , antiderivative size = 91340, normalized size of antiderivative = 91340.00

$$\int \sec^2(a+bx)\sec^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)^2*sec(b*x+c)^2,x, algorithm="maxima")
```

output
```
-4*(36*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2
*a + 2*c))*cos(4*a + 2*c)^2 + 36*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (
cos(4*a) + cos(4*c))*sin(2*a + 2*c))*cos(2*a + 4*c)^2 + 36*((sin(4*a) + si
n(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))*sin(4*a + 2
*c)^2 + 36*((sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(4*a) + cos(4*c))*s
in(2*a + 2*c))*sin(2*a + 4*c)^2 - 2*(((cos(6*a) - cos(6*c))*cos(4*a + 2*c)
 - 3*cos(4*a + 2*c)^2 + (cos(6*a) - cos(6*c))*cos(2*a + 4*c) + 3*cos(2*a +
 4*c)^2 + (sin(6*a) - sin(6*c))*sin(4*a + 2*c) - 3*sin(4*a + 2*c)^2 + (sin
(6*a) - sin(6*c))*sin(2*a + 4*c) + 3*sin(2*a + 4*c)^2)*cos(4*b*x + 6*a + 2
*c)^2 + 4*((cos(6*a) - cos(6*c))*cos(4*a + 2*c) - 3*cos(4*a + 2*c)^2 + (co
s(6*a) - cos(6*c))*cos(2*a + 4*c) + 3*cos(2*a + 4*c)^2 + (sin(6*a) - sin(6
*c))*sin(4*a + 2*c) - 3*sin(4*a + 2*c)^2 + (sin(6*a) - sin(6*c))*sin(2*a +
 4*c) + 3*sin(2*a + 4*c)^2)*cos(4*b*x + 4*a + 4*c)^2 + ((cos(6*a) - cos(6*
c))*cos(4*a + 2*c) - 3*cos(4*a + 2*c)^2 + (cos(6*a) - cos(6*c))*cos(2*a +
4*c) + 3*cos(2*a + 4*c)^2 + (sin(6*a) - sin(6*c))*sin(4*a + 2*c) - 3*sin(4
*a + 2*c)^2 + (sin(6*a) - sin(6*c))*sin(2*a + 4*c) + 3*sin(2*a + 4*c)^2)*c
os(4*b*x + 2*a + 6*c)^2 + ((cos(6*a) - cos(6*c))*cos(4*a + 2*c) - 3*cos(4*
a + 2*c)^2 + (cos(6*a) - cos(6*c))*cos(2*a + 4*c) + 3*cos(2*a + 4*c)^2 + (
sin(6*a) - sin(6*c))*sin(4*a + 2*c) - 3*sin(4*a + 2*c)^2 + (sin(6*a) - sin
(6*c))*sin(2*a + 4*c) + 3*sin(2*a + 4*c)^2)*cos(2*b*x + 6*a)^2 + ((cos(...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.20 (sec) , antiderivative size = 2375, normalized size of antiderivative = 2375.00

$$\int \sec^2(a+bx)\sec^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)^2*sec(b*x+c)^2,x, algorithm="giac")
```

output
```
-1/8*(2*(tan(1/2*a)^6*tan(1/2*c)^6 + tan(1/2*a)^6*tan(1/2*c)^4 + 4*tan(1/2
*a)^5*tan(1/2*c)^5 + tan(1/2*a)^4*tan(1/2*c)^6 - tan(1/2*a)^6*tan(1/2*c)^2
 + 8*tan(1/2*a)^5*tan(1/2*c)^3 + tan(1/2*a)^4*tan(1/2*c)^4 + 8*tan(1/2*a)^
3*tan(1/2*c)^5 - tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*a)^6 + 4*tan(1/2*a)^5
*tan(1/2*c) - tan(1/2*a)^4*tan(1/2*c)^2 + 16*tan(1/2*a)^3*tan(1/2*c)^3 - t
an(1/2*a)^2*tan(1/2*c)^4 + 4*tan(1/2*a)*tan(1/2*c)^5 - tan(1/2*c)^6 - tan(
1/2*a)^4 + 8*tan(1/2*a)^3*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c)^2 + 8*tan(1
/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c)
 + tan(1/2*c)^2 + 1)*log(abs(2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*ta
n(b*x + a)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x
 + a)*tan(1/2*a) - tan(1/2*a)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)
*tan(1/2*c) - tan(1/2*c)^2 + 1))/(tan(1/2*a)^6*tan(1/2*c)^3 - 3*tan(1/2*a)
^5*tan(1/2*c)^4 + 3*tan(1/2*a)^4*tan(1/2*c)^5 - tan(1/2*a)^3*tan(1/2*c)^6
+ 3*tan(1/2*a)^5*tan(1/2*c)^2 - 9*tan(1/2*a)^4*tan(1/2*c)^3 + 9*tan(1/2*a)
^3*tan(1/2*c)^4 - 3*tan(1/2*a)^2*tan(1/2*c)^5 + 3*tan(1/2*a)^4*tan(1/2*c)
- 9*tan(1/2*a)^3*tan(1/2*c)^2 + 9*tan(1/2*a)^2*tan(1/2*c)^3 - 3*tan(1/2*a)
*tan(1/2*c)^4 + tan(1/2*a)^3 - 3*tan(1/2*a)^2*tan(1/2*c) + 3*tan(1/2*a)*ta
n(1/2*c)^2 - tan(1/2*c)^3) - 2*(tan(b*x + a)*tan(1/2*a)^4*tan(1/2*c)^4 + 2
*tan(b*x + a)*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a)^2*tan(
1/2*c)^4 + tan(b*x + a)*tan(1/2*a)^4 + 4*tan(b*x + a)*tan(1/2*a)^2*tan(...
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(a+bx)\sec^2(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)^2*cos(c + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^2(a+bx)\sec^2(c+bx)\,dx$$
$$=\frac{\cos\left(bx+c\right)\cos\left(bx+a\right)\left(\int \frac{\sin(bx+c)^2\sin(bx+a)^2}{\sin(bx+c)^2\sin(bx+a)^2-\sin(bx+c)^2-\sin(bx+a)^2+1}dx\right)b-\cos\left(bx+c\right)\cos\left(bx+a\right)bx+}{\cos\left(bx+c\right)\cos\left(bx+a\right)b}$$

input
```
int(sec(b*x+a)^2*sec(b*x+c)^2,x)
```

output
```
(cos(b*x + c)*cos(a + b*x)*int((sin(b*x + c)**2*sin(a + b*x)**2)/(sin(b*x
+ c)**2*sin(a + b*x)**2 - sin(b*x + c)**2 - sin(a + b*x)**2 + 1),x)*b - co
s(b*x + c)*cos(a + b*x)*b*x + cos(b*x + c)*sin(a + b*x) + cos(a + b*x)*sin
(b*x + c))/(cos(b*x + c)*cos(a + b*x)*b)
```

3.322 $\int \sec^2(a+bx)\sec^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \sec^2(a+bx)\sec^3(c+bx)\,dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 8.71 (sec) , antiderivative size = 491, normalized size of antiderivative = 491.00

$$\int \sec^2(a+bx)\sec^3(c+bx)\,dx$$
$$= \frac{6i\arctan\left(\frac{(i\cos(a)+\sin(a))\left(\cos\left(\frac{bx}{2}\right)\sin(a)+\cos(a)\sin\left(\frac{bx}{2}\right)\right)}{\cos(a)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(a)}\right)\cos(a-c)}{\frac{3b}{8}+\frac{1}{8}b\cos(4a-4c)-\frac{1}{2}b\cos(2a-2c)} - \frac{3(3+\cos(2a-2c))\csc^4(a-c)\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)-\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{4b} + \frac{3(3+\cos(2a-2c))\csc^4(a-c)\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)+\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{4b} + \frac{\csc^3(a-c)\sec(a+bx)}{b} + \frac{\csc^2(a-c)}{4b\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)-\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)^2} - \frac{\csc^3(a-c)\left(\sin\left(a-c-\frac{bx}{2}\right)-\sin\left(a-c+\frac{bx}{2}\right)\right)}{b\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)-\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)} - \frac{\csc^2(a-c)}{4b\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)+\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)^2} - \frac{\csc^3(a-c)\left(-\sin\left(a-c-\frac{bx}{2}\right)+\sin\left(a-c+\frac{bx}{2}\right)\right)}{b\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)+\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}$$

input
```
Integrate[Sec[a + b*x]^2*Sec[c + b*x]^3,x]
```

output
```
((6*I)*ArcTan[((I*Cos[a] + Sin[a])*(Cos[(b*x)/2]*Sin[a] + Cos[a]*Sin[(b*x)
/2]))/(Cos[a]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[a])]*Cos[a - c])/((3*b)/8
+ (b*Cos[4*a - 4*c])/8 - (b*Cos[2*a - 2*c])/2) - (3*(3 + Cos[2*a - 2*c])*C
sc[a - c]^4*Log[Cos[c/2 + (b*x)/2] - Sin[c/2 + (b*x)/2]])/(4*b) + (3*(3 +
Cos[2*a - 2*c])*Csc[a - c]^4*Log[Cos[c/2 + (b*x)/2] + Sin[c/2 + (b*x)/2]])
/(4*b) + (Csc[a - c]^3*Sec[a + b*x])/b + Csc[a - c]^2/(4*b*(Cos[c/2 + (b*x
)/2] - Sin[c/2 + (b*x)/2])^2) - (Csc[a - c]^3*(Sin[a - c - (b*x)/2] - Sin[
a - c + (b*x)/2]))/(b*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (b*x)/2] - Sin[c/2
+ (b*x)/2])) - Csc[a - c]^2/(4*b*(Cos[c/2 + (b*x)/2] + Sin[c/2 + (b*x)/2])
^2) - (Csc[a - c]^3*(-Sin[a - c - (b*x)/2] + Sin[a - c + (b*x)/2]))/(b*(Co
s[c/2] + Sin[c/2])*(Cos[c/2 + (b*x)/2] + Sin[c/2 + (b*x)/2]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\sec^3(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sec^2(a+bx)\sec^3(bx+c)dx$$

input
```
Int[Sec[a + b*x]^2*Sec[c + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 7.40 (sec) , antiderivative size = 943, normalized size of antiderivative = 943.00

method	result	size
risch	Expression too large to display	943
default	Expression too large to display	1071

input
```
int(sec(b*x+a)^2*sec(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-4*I/(exp(2*I*(b*x+a))+1)/(exp(2*I*(b*x+a+c))+exp(2*I*a))^2/(exp(2*I*a)-ex
p(2*I*c))^3/b*(3*exp(5*I*(b*x+2*a+c))+9*exp(I*(5*b*x+8*a+7*c))+5*exp(I*(3*
b*x+10*a+3*c))+14*exp(I*(3*b*x+8*a+5*c))+5*exp(I*(3*b*x+6*a+7*c))+9*exp(I*
(b*x+8*a+3*c))+3*exp(I*(b*x+6*a+5*c)))-24*ln(exp(I*(b*x+a))+I)/(exp(8*I*a)
-4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))-4*exp(2*I*(a+3*c))+exp(8*I*c))/b*exp(
I*(5*a+3*c))-24*ln(exp(I*(b*x+a))+I)/(exp(8*I*a)-4*exp(2*I*(3*a+c))+6*exp(
4*I*(a+c))-4*exp(2*I*(a+3*c))+exp(8*I*c))/b*exp(I*(3*a+5*c))-6*ln(exp(I*(b
*x+a))-I*exp(I*(a-c)))/(exp(8*I*a)-4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))-4*e
xp(2*I*(a+3*c))+exp(8*I*c))/b*exp(2*I*(3*a+c))-36*ln(exp(I*(b*x+a))-I*exp(
I*(a-c)))/(exp(8*I*a)-4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))-4*exp(2*I*(a+3*c
))+exp(8*I*c))/b*exp(4*I*(a+c))-6*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/(exp(8
*I*a)-4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))-4*exp(2*I*(a+3*c))+exp(8*I*c))/b
*exp(2*I*(a+3*c))+6*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/(exp(8*I*a)-4*exp(2*
I*(3*a+c))+6*exp(4*I*(a+c))-4*exp(2*I*(a+3*c))+exp(8*I*c))/b*exp(2*I*(3*a+
c))+36*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/(exp(8*I*a)-4*exp(2*I*(3*a+c))+6*
exp(4*I*(a+c))-4*exp(2*I*(a+3*c))+exp(8*I*c))/b*exp(4*I*(a+c))+6*ln(exp(I*
(b*x+a))+I*exp(I*(a-c)))/(exp(8*I*a)-4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))-4
*exp(2*I*(a+3*c))+exp(8*I*c))/b*exp(2*I*(a+3*c))+24*ln(exp(I*(b*x+a))-I)/(
exp(8*I*a)-4*exp(2*I*(3*a+c))+6*exp(4*I*(a+c))-4*exp(2*I*(a+3*c))+exp(8*I*
c))/b*exp(I*(5*a+3*c))+24*ln(exp(I*(b*x+a))-I)/(exp(8*I*a)-4*exp(2*I*(3...
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.15 (sec) , antiderivative size = 472, normalized size of antiderivative = 472.00

$$\int \sec^2(a+bx)\sec^3(c+bx)\,dx$$
$$= \frac{6\left(\cos(-a+c)^3-\cos(-a+c)\right)\cos(bx+c)\sin(bx+c)-6\left(\cos(bx+c)^3\cos(-a+c)^2+\cos(bx+c)^2\right.}{}$$

input
```
integrate(sec(b*x+a)^2*sec(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(6*(cos(-a + c)^3 - cos(-a + c))*cos(b*x + c)*sin(b*x + c) - 6*(cos(b*
x + c)^3*cos(-a + c)^2 + cos(b*x + c)^2*cos(-a + c)*sin(b*x + c)*sin(-a +
c))*log(2*(cos(-a + c)*sin(b*x + c) - cos(b*x + c)*sin(-a + c) + 1)/(cos(-
a + c) + 1)) + 6*(cos(b*x + c)^3*cos(-a + c)^2 + cos(b*x + c)^2*cos(-a + c
)*sin(b*x + c)*sin(-a + c))*log(-2*(cos(-a + c)*sin(b*x + c) - cos(b*x + c
)*sin(-a + c) - 1)/(cos(-a + c) + 1)) + 3*((cos(-a + c)^2 + 1)*cos(b*x + c
)^2*sin(b*x + c)*sin(-a + c) + (cos(-a + c)^3 + cos(-a + c))*cos(b*x + c)^
3)*log(sin(b*x + c) + 1) - 3*((cos(-a + c)^2 + 1)*cos(b*x + c)^2*sin(b*x +
 c)*sin(-a + c) + (cos(-a + c)^3 + cos(-a + c))*cos(b*x + c)^3)*log(-sin(b
*x + c) + 1) - 2*(3*(cos(-a + c)^2 + 1)*cos(b*x + c)^2 + cos(-a + c)^2 - 1
)*sin(-a + c))/((b*cos(-a + c)^4 - 2*b*cos(-a + c)^2 + b)*cos(b*x + c)^2*s
in(b*x + c)*sin(-a + c) + (b*cos(-a + c)^5 - 2*b*cos(-a + c)^3 + b*cos(-a
+ c))*cos(b*x + c)^3)
```

Sympy [F]

$$\int \sec^2(a+bx)\sec^3(c+bx)\,dx = \int \sec^2{(a+bx)}\sec^3{(bx+c)}\,dx$$

input
```
integrate(sec(b*x+a)**2*sec(b*x+c)**3,x)
```

output
```
Integral(sec(a + b*x)**2*sec(b*x + c)**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 12.63 (sec) , antiderivative size = 504922, normalized size of antiderivative = 504922.00

$$\int \sec^2(a+bx)\sec^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)^2*sec(b*x+c)^3,x, algorithm="maxima")
```

output
```
-(24*(((sin(8*a) - 4*sin(6*a + 2*c) + sin(8*c))*cos(5*a + 3*c) + (sin(8*a)
 - 4*sin(6*a + 2*c) + 6*sin(4*a + 4*c) + sin(8*c))*cos(3*a + 5*c) + 4*(sin
(5*a + 3*c) + sin(3*a + 5*c))*cos(2*a + 6*c) - (cos(8*a) - 4*cos(6*a + 2*c
) + cos(8*c))*sin(5*a + 3*c) - 6*cos(4*a + 4*c)*sin(5*a + 3*c) + 6*cos(5*a
 + 3*c)*sin(4*a + 4*c) - (cos(8*a) - 4*cos(6*a + 2*c) + 6*cos(4*a + 4*c) +
 cos(8*c))*sin(3*a + 5*c) - 4*(cos(5*a + 3*c) + cos(3*a + 5*c))*sin(2*a +
6*c))*cos(6*b*x + 8*a + 4*c)^2 + 9*((sin(8*a) - 4*sin(6*a + 2*c) + sin(8*c
))*cos(5*a + 3*c) + (sin(8*a) - 4*sin(6*a + 2*c) + 6*sin(4*a + 4*c) + sin(
8*c))*cos(3*a + 5*c) + 4*(sin(5*a + 3*c) + sin(3*a + 5*c))*cos(2*a + 6*c)
- (cos(8*a) - 4*cos(6*a + 2*c) + cos(8*c))*sin(5*a + 3*c) - 6*cos(4*a + 4*
c)*sin(5*a + 3*c) + 6*cos(5*a + 3*c)*sin(4*a + 4*c) - (cos(8*a) - 4*cos(6*
a + 2*c) + 6*cos(4*a + 4*c) + cos(8*c))*sin(3*a + 5*c) - 4*(cos(5*a + 3*c)
 + cos(3*a + 5*c))*sin(2*a + 6*c))*cos(6*b*x + 6*a + 6*c)^2 + 9*((sin(8*a)
 - 4*sin(6*a + 2*c) + sin(8*c))*cos(5*a + 3*c) + (sin(8*a) - 4*sin(6*a + 2
*c) + 6*sin(4*a + 4*c) + sin(8*c))*cos(3*a + 5*c) + 4*(sin(5*a + 3*c) + si
n(3*a + 5*c))*cos(2*a + 6*c) - (cos(8*a) - 4*cos(6*a + 2*c) + cos(8*c))*si
n(5*a + 3*c) - 6*cos(4*a + 4*c)*sin(5*a + 3*c) + 6*cos(5*a + 3*c)*sin(4*a
+ 4*c) - (cos(8*a) - 4*cos(6*a + 2*c) + 6*cos(4*a + 4*c) + cos(8*c))*sin(3
*a + 5*c) - 4*(cos(5*a + 3*c) + cos(3*a + 5*c))*sin(2*a + 6*c))*cos(6*b*x
+ 4*a + 8*c)^2 + ((sin(8*a) - 4*sin(6*a + 2*c) + sin(8*c))*cos(5*a + 3*...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 2.87 (sec) , antiderivative size = 15002, normalized size of antiderivative = 15002.00

$$\int \sec^2(a+bx)\sec^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)^2*sec(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/16*(3*(tan(1/2*a)^9*tan(1/2*c)^9 - tan(1/2*a)^9*tan(1/2*c)^8 + tan(1/2*
a)^8*tan(1/2*c)^9 + 2*tan(1/2*a)^9*tan(1/2*c)^7 + 5*tan(1/2*a)^8*tan(1/2*c
)^8 + 2*tan(1/2*a)^7*tan(1/2*c)^9 - 2*tan(1/2*a)^9*tan(1/2*c)^6 - 2*tan(1/
2*a)^8*tan(1/2*c)^7 + 2*tan(1/2*a)^7*tan(1/2*c)^8 + 2*tan(1/2*a)^6*tan(1/2
*c)^9 + 2*tan(1/2*a)^9*tan(1/2*c)^5 + 6*tan(1/2*a)^8*tan(1/2*c)^6 + 20*tan
(1/2*a)^7*tan(1/2*c)^7 + 6*tan(1/2*a)^6*tan(1/2*c)^8 + 2*tan(1/2*a)^5*tan(
1/2*c)^9 - 2*tan(1/2*a)^9*tan(1/2*c)^4 - 2*tan(1/2*a)^8*tan(1/2*c)^5 - 12*
tan(1/2*a)^7*tan(1/2*c)^6 + 12*tan(1/2*a)^6*tan(1/2*c)^7 + 2*tan(1/2*a)^5*
tan(1/2*c)^8 + 2*tan(1/2*a)^4*tan(1/2*c)^9 + 2*tan(1/2*a)^9*tan(1/2*c)^3 -
 2*tan(1/2*a)^8*tan(1/2*c)^4 + 32*tan(1/2*a)^7*tan(1/2*c)^5 + 20*tan(1/2*a
)^6*tan(1/2*c)^6 + 32*tan(1/2*a)^5*tan(1/2*c)^7 - 2*tan(1/2*a)^4*tan(1/2*c
)^8 + 2*tan(1/2*a)^3*tan(1/2*c)^9 - 2*tan(1/2*a)^9*tan(1/2*c)^2 + 6*tan(1/
2*a)^8*tan(1/2*c)^3 - 32*tan(1/2*a)^7*tan(1/2*c)^4 + 24*tan(1/2*a)^6*tan(1
/2*c)^5 - 24*tan(1/2*a)^5*tan(1/2*c)^6 + 32*tan(1/2*a)^4*tan(1/2*c)^7 - 6*
tan(1/2*a)^3*tan(1/2*c)^8 + 2*tan(1/2*a)^2*tan(1/2*c)^9 + tan(1/2*a)^9*tan
(1/2*c) - 2*tan(1/2*a)^8*tan(1/2*c)^2 + 12*tan(1/2*a)^7*tan(1/2*c)^3 + 24*
tan(1/2*a)^6*tan(1/2*c)^4 + 56*tan(1/2*a)^5*tan(1/2*c)^5 + 24*tan(1/2*a)^4
*tan(1/2*c)^6 + 12*tan(1/2*a)^3*tan(1/2*c)^7 - 2*tan(1/2*a)^2*tan(1/2*c)^8
 + tan(1/2*a)*tan(1/2*c)^9 - tan(1/2*a)^9 + 5*tan(1/2*a)^8*tan(1/2*c) - 20
*tan(1/2*a)^7*tan(1/2*c)^2 + 20*tan(1/2*a)^6*tan(1/2*c)^3 - 56*tan(1/2*...
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(a+bx)\sec^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)^2*cos(c + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^2(a+bx)\sec^3(c+bx)\,dx = \text{too large to display}$$

input
```
int(sec(b*x+a)^2*sec(b*x+c)^3,x)
```

output
```
(32*cos(b*x + c)*sin(a + b*x) + 20*cos(a + b*x)*int(cos(a + b*x)/(cos(b*x
+ c)*sin(b*x + c)**2*sin(a + b*x)**2 - cos(b*x + c)*sin(b*x + c)**2 - cos(
b*x + c)*sin(a + b*x)**2 + cos(b*x + c)),x)*sin(b*x + c)**2*b - 20*cos(a +
 b*x)*int(cos(a + b*x)/(cos(b*x + c)*sin(b*x + c)**2*sin(a + b*x)**2 - cos
(b*x + c)*sin(b*x + c)**2 - cos(b*x + c)*sin(a + b*x)**2 + cos(b*x + c)),x
)*b + 20*cos(a + b*x)*int(cos(a + b*x)/(sin(b*x + c)**2*sin(a + b*x)**2 -
sin(b*x + c)**2 - sin(a + b*x)**2 + 1),x)*sin(b*x + c)**2*b - 20*cos(a + b
*x)*int(cos(a + b*x)/(sin(b*x + c)**2*sin(a + b*x)**2 - sin(b*x + c)**2 -
sin(a + b*x)**2 + 1),x)*b + 12*cos(a + b*x)*int(sin(b*x + c)**2/(sin(b*x +
 c)**2 - 1),x)*sin(b*x + c)**2*b - 12*cos(a + b*x)*int(sin(b*x + c)**2/(si
n(b*x + c)**2 - 1),x)*b - 12*cos(a + b*x)*int(sin(b*x + c)**2/(cos(b*x + c
)*cos(a + b*x)*sin(b*x + c)**2 - cos(b*x + c)*cos(a + b*x)),x)*sin(b*x + c
)**2*b + 12*cos(a + b*x)*int(sin(b*x + c)**2/(cos(b*x + c)*cos(a + b*x)*si
n(b*x + c)**2 - cos(b*x + c)*cos(a + b*x)),x)*b + 20*cos(a + b*x)*int(sin(
b*x + c)**2/(cos(b*x + c)*sin(b*x + c)**2*sin(a + b*x)**2 - cos(b*x + c)*s
in(b*x + c)**2 - cos(b*x + c)*sin(a + b*x)**2 + cos(b*x + c)),x)*sin(b*x +
 c)**2*b - 20*cos(a + b*x)*int(sin(b*x + c)**2/(cos(b*x + c)*sin(b*x + c)*
*2*sin(a + b*x)**2 - cos(b*x + c)*sin(b*x + c)**2 - cos(b*x + c)*sin(a + b
*x)**2 + cos(b*x + c)),x)*b - 12*cos(a + b*x)*int(sin(b*x + c)**2/(cos(b*x
 + c)*sin(b*x + c)**2 - cos(b*x + c)),x)*sin(b*x + c)**2*b + 12*cos(a +...
```

3.323 $\int \sec^2(a+bx)\sec^4(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \sec^2(a+bx)\sec^4(c+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 6.49 (sec) , antiderivative size = 966, normalized size of antiderivative = 966.00

$$\int \sec^2(a+bx)\sec^4(c+bx)\,dx=\text{Too large to display}$$

input
```
Integrate[Sec[a + b*x]^2*Sec[c + b*x]^4,x]
```

output
```
((-4*I)*ArcTan[Tan[a + b*x]]*Cot[a - c]*Csc[a - c]^4)/b + ((4*I)*ArcTan[Ta
n[c + b*x]]*Cot[a - c]*Csc[a - c]^4)/b + (2*Cot[a - c]*Csc[a - c]^4*Log[Co
s[a + b*x]^2])/b - (2*Cot[a - c]*Csc[a - c]^4*Log[Cos[c + b*x]^2])/b + (Cs
c[a - c]^4*Sec[a]*Sec[c]*Sec[a + b*x]*Sec[c + b*x]^3*(-15*Sin[2*a] + 3*Sin
[2*a - 4*c] + 3*Sin[4*a - 4*c] + 9*Sin[2*a - 2*c] - 18*Sin[2*c] + 25*Sin[2
*b*x] - Sin[2*a - 4*c - 4*b*x] + 2*Sin[2*a - 4*c - 2*b*x] - Sin[4*a - 4*c
- 2*b*x] - 7*Sin[2*a - 2*c - 2*b*x] + 16*Sin[2*a + 2*b*x] + 3*Sin[2*a - 2*
c + 2*b*x] + 3*Sin[4*a - 2*c + 2*b*x] + 7*Sin[2*c + 2*b*x] - 6*Sin[2*a + 2
*c + 2*b*x] - 6*Sin[4*c + 2*b*x] + Sin[2*a + 4*b*x] + Sin[4*a + 4*b*x] + 1
0*Sin[2*c + 4*b*x] + 7*Sin[2*a + 2*c + 4*b*x] + 4*Sin[4*c + 4*b*x]))/(48*b
) + x*(((2*I)*Cos[a]*Cos[c])/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^5 + ((2*I)*Co
s[c]*Sec[a])/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^5 + (4*Cos[c]*Sin[a])/(Cos[c]
*Sin[a] - Cos[a]*Sin[c])^5 - (2*Cos[a]*Sin[c])/(Cos[c]*Sin[a] - Cos[a]*Sin
[c])^5 + (2*Sec[a]*Sin[c])/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^5 + ((4*I)*Sin[
a]*Sin[c])/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^5 + ((2*I)*Cos[a]*Cos[c])/(-(Co
s[c]*Sin[a]) + Cos[a]*Sin[c])^5 + ((2*I)*Cos[a]*Sec[c])/(-(Cos[c]*Sin[a])
+ Cos[a]*Sin[c])^5 - (2*Cos[c]*Sin[a])/(-(Cos[c]*Sin[a]) + Cos[a]*Sin[c])^
5 + (2*Sec[c]*Sin[a])/(-(Cos[c]*Sin[a]) + Cos[a]*Sin[c])^5 + (4*Cos[a]*Sin
[c])/(-(Cos[c]*Sin[a]) + Cos[a]*Sin[c])^5 + ((4*I)*Sin[a]*Sin[c])/(-(Cos[c
]*Sin[a]) + Cos[a]*Sin[c])^5 + ((-1 + Cos[2*c] + I*Sin[2*c])*(Cos[2*a] ...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\sec^4(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sec^2(a+bx)\sec^4(bx+c)dx$$

input
```
Int[Sec[a + b*x]^2*Sec[c + b*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 18.17 (sec) , antiderivative size = 506, normalized size of antiderivative = 506.00

method	result
default	Expression too large to display
risch	$\frac{32i\left(6\,\mathrm{e}^{2i(3bx+6a+4c)}+6\,\mathrm{e}^{2i(3bx+5a+5c)}+15\,\mathrm{e}^{2i(2bx+6a+3c)}+18\,\mathrm{e}^{2i(2bx+5a+4c)}+3\,\mathrm{e}^{2i(2bx+4a+5c)}+\mathrm{e}^{2i(bx+7a+c)}+7\,\mathrm{e}^{2i(bx+6a+2c)}+\cdots\right.}{3\left(\mathrm{e}^{2i(bx+a)}+1\right)\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{3}\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)^{4}b}$

input
```
int(sec(b*x+a)^2*sec(b*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(tan(b*x+a)/(sin(a)*cos(c)-cos(a)*sin(c))^4-(2*cos(c)^2*sin(a)^2+6*cos
(a)^2*cos(c)^2+8*cos(a)*cos(c)*sin(a)*sin(c)+6*sin(a)^2*sin(c)^2+2*sin(c)^
2*cos(a)^2)/(sin(a)*cos(c)-cos(a)*sin(c))^5/(tan(b*x+a)*sin(a)*cos(c)-tan(
b*x+a)*cos(a)*sin(c)+cos(a)*cos(c)+sin(a)*sin(c))-(4*cos(a)*cos(c)+4*sin(a
)*sin(c))/(sin(a)*cos(c)-cos(a)*sin(c))^5*ln(tan(b*x+a)*sin(a)*cos(c)-tan(
b*x+a)*cos(a)*sin(c)+cos(a)*cos(c)+sin(a)*sin(c))-1/3/(sin(a)*cos(c)-cos(a
)*sin(c))^5*(sin(a)^4*cos(c)^4+2*cos(a)^2*sin(a)^2*cos(c)^4+cos(a)^4*cos(c
)^4+2*sin(a)^4*cos(c)^2*sin(c)^2+4*cos(a)^2*sin(a)^2*cos(c)^2*sin(c)^2+2*c
os(a)^4*cos(c)^2*sin(c)^2+sin(a)^4*sin(c)^4+2*cos(a)^2*sin(a)^2*sin(c)^4+c
os(a)^4*sin(c)^4)/(tan(b*x+a)*sin(a)*cos(c)-tan(b*x+a)*cos(a)*sin(c)+cos(a
)*cos(c)+sin(a)*sin(c))^3+1/2*(4*cos(c)^3*sin(a)^2*cos(a)+4*cos(c)^3*cos(a
)^3+4*cos(c)^2*sin(c)*sin(a)^3+4*cos(c)^2*sin(c)*cos(a)^2*sin(a)+4*cos(c)*
sin(c)^2*cos(a)*sin(a)^2+4*cos(c)*sin(c)^2*cos(a)^3+4*sin(c)^3*sin(a)^3+4*
sin(c)^3*sin(a)*cos(a)^2)/(sin(a)*cos(c)-cos(a)*sin(c))^5/(tan(b*x+a)*sin(
a)*cos(c)-tan(b*x+a)*cos(a)*sin(c)+cos(a)*cos(c)+sin(a)*sin(c))^2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.11 (sec) , antiderivative size = 429, normalized size of antiderivative = 429.00

$$\int \sec^2(a+bx)\sec^4(c+bx)\,dx$$

$$= \frac{4\left(\cos(-a+c)^4+\cos(-a+c)^2-2\right)\cos(bx+c)^4+\cos(-a+c)^4-2\left(\cos(-a+c)^4+\cos(-a+c)^2-\right.}{}$$

input
```
integrate(sec(b*x+a)^2*sec(b*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(4*(cos(-a + c)^4 + cos(-a + c)^2 - 2)*cos(b*x + c)^4 + cos(-a + c)^4
- 2*(cos(-a + c)^4 + cos(-a + c)^2 - 2)*cos(b*x + c)^2 + 2*(2*(cos(-a + c)
^3 + 2*cos(-a + c))*cos(b*x + c)^3 + (cos(-a + c)^3 - cos(-a + c))*cos(b*x
 + c))*sin(b*x + c)*sin(-a + c) - 2*cos(-a + c)^2 + 6*(cos(b*x + c)^4*cos(
-a + c)^2 + cos(b*x + c)^3*cos(-a + c)*sin(b*x + c)*sin(-a + c))*log(cos(b
*x + c)^2) - 6*(cos(b*x + c)^4*cos(-a + c)^2 + cos(b*x + c)^3*cos(-a + c)*
sin(b*x + c)*sin(-a + c))*log(4*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*s
in(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)/(co
s(-a + c)^2 + 2*cos(-a + c) + 1)) + 1)/((b*cos(-a + c)^5 - 2*b*cos(-a + c)
^3 + b*cos(-a + c))*cos(b*x + c)^4*sin(-a + c) - (b*cos(-a + c)^6 - 3*b*co
s(-a + c)^4 + 3*b*cos(-a + c)^2 - b)*cos(b*x + c)^3*sin(b*x + c))
```

Sympy [F]

$$\int \sec^2(a+bx)\sec^4(c+bx)\,dx = \int \sec^2(a+bx)\sec^4(bx+c)\,dx$$

input
```
integrate(sec(b*x+a)**2*sec(b*x+c)**4,x)
```

output
```
Integral(sec(a + b*x)**2*sec(b*x + c)**4, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 34.92 (sec) , antiderivative size = 1342628, normalized size of antiderivative = 1342628.00

$$\int \sec^2(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)^2*sec(b*x+c)^4,x, algorithm="maxima")
```

output
```
32/3*(25*((sin(8*a) + sin(8*c))*cos(6*a + 2*c) - (cos(8*a) + cos(8*c))*sin
(6*a + 2*c))*cos(8*a + 2*c)^2 + 100*((sin(8*a) + sin(8*c))*cos(6*a + 2*c)
+ 2*(5*sin(8*a) - 23*sin(6*a + 2*c) + 5*sin(8*c))*cos(4*a + 4*c) - (cos(8*
a) + cos(8*c))*sin(6*a + 2*c) - 2*(5*cos(8*a) - 23*cos(6*a + 2*c) + 5*cos(
8*c))*sin(4*a + 4*c))*cos(6*a + 4*c)^2 + 100*((sin(8*a) + sin(8*c))*cos(6*
a + 2*c) + 2*(5*sin(8*a) - 23*sin(6*a + 2*c) + 5*sin(8*c))*cos(4*a + 4*c)
+ (sin(8*a) + 46*sin(4*a + 4*c) + sin(8*c))*cos(2*a + 6*c) - (cos(8*a) + c
os(8*c))*sin(6*a + 2*c) - 2*(5*cos(8*a) - 23*cos(6*a + 2*c) + 5*cos(8*c))*
sin(4*a + 4*c) - (cos(8*a) + 46*cos(4*a + 4*c) + cos(8*c))*sin(2*a + 6*c))
*cos(4*a + 6*c)^2 + 25*((sin(8*a) + sin(8*c))*cos(6*a + 2*c) + 2*(5*sin(8*
a) - 23*sin(6*a + 2*c) + 5*sin(8*c))*cos(4*a + 4*c) + (sin(8*a) + 46*sin(4
*a + 4*c) + sin(8*c))*cos(2*a + 6*c) - (cos(8*a) + cos(8*c))*sin(6*a + 2*c
) - 2*(5*cos(8*a) - 23*cos(6*a + 2*c) + 5*cos(8*c))*sin(4*a + 4*c) - (cos(
8*a) + 46*cos(4*a + 4*c) + cos(8*c))*sin(2*a + 6*c))*cos(2*a + 8*c)^2 + 25
*((sin(8*a) + sin(8*c))*cos(6*a + 2*c) - (cos(8*a) + cos(8*c))*sin(6*a + 2
*c))*sin(8*a + 2*c)^2 + 100*((sin(8*a) + sin(8*c))*cos(6*a + 2*c) + 2*(5*s
in(8*a) - 23*sin(6*a + 2*c) + 5*sin(8*c))*cos(4*a + 4*c) - (cos(8*a) + cos
(8*c))*sin(6*a + 2*c) - 2*(5*cos(8*a) - 23*cos(6*a + 2*c) + 5*cos(8*c))*si
n(4*a + 4*c))*sin(6*a + 4*c)^2 + 100*((sin(8*a) + sin(8*c))*cos(6*a + 2*c)
 + 2*(5*sin(8*a) - 23*sin(6*a + 2*c) + 5*sin(8*c))*cos(4*a + 4*c) + (si...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.43 (sec) , antiderivative size = 13226, normalized size of antiderivative = 13226.00

$$\int \sec^2(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)^2*sec(b*x+c)^4,x, algorithm="giac")
```

output
```
-1/96*(12*(tan(1/2*a)^10*tan(1/2*c)^10 + 3*tan(1/2*a)^10*tan(1/2*c)^8 + 4*
tan(1/2*a)^9*tan(1/2*c)^9 + 3*tan(1/2*a)^8*tan(1/2*c)^10 + 2*tan(1/2*a)^10
*tan(1/2*c)^6 + 16*tan(1/2*a)^9*tan(1/2*c)^7 + 9*tan(1/2*a)^8*tan(1/2*c)^8
 + 16*tan(1/2*a)^7*tan(1/2*c)^9 + 2*tan(1/2*a)^6*tan(1/2*c)^10 - 2*tan(1/2
*a)^10*tan(1/2*c)^4 + 24*tan(1/2*a)^9*tan(1/2*c)^5 + 6*tan(1/2*a)^8*tan(1/
2*c)^6 + 64*tan(1/2*a)^7*tan(1/2*c)^7 + 6*tan(1/2*a)^6*tan(1/2*c)^8 + 24*t
an(1/2*a)^5*tan(1/2*c)^9 - 2*tan(1/2*a)^4*tan(1/2*c)^10 - 3*tan(1/2*a)^10*
tan(1/2*c)^2 + 16*tan(1/2*a)^9*tan(1/2*c)^3 - 6*tan(1/2*a)^8*tan(1/2*c)^4
+ 96*tan(1/2*a)^7*tan(1/2*c)^5 + 4*tan(1/2*a)^6*tan(1/2*c)^6 + 96*tan(1/2*
a)^5*tan(1/2*c)^7 - 6*tan(1/2*a)^4*tan(1/2*c)^8 + 16*tan(1/2*a)^3*tan(1/2*
c)^9 - 3*tan(1/2*a)^2*tan(1/2*c)^10 - tan(1/2*a)^10 + 4*tan(1/2*a)^9*tan(1
/2*c) - 9*tan(1/2*a)^8*tan(1/2*c)^2 + 64*tan(1/2*a)^7*tan(1/2*c)^3 - 4*tan
(1/2*a)^6*tan(1/2*c)^4 + 144*tan(1/2*a)^5*tan(1/2*c)^5 - 4*tan(1/2*a)^4*ta
n(1/2*c)^6 + 64*tan(1/2*a)^3*tan(1/2*c)^7 - 9*tan(1/2*a)^2*tan(1/2*c)^8 +
4*tan(1/2*a)*tan(1/2*c)^9 - tan(1/2*c)^10 - 3*tan(1/2*a)^8 + 16*tan(1/2*a)
^7*tan(1/2*c) - 6*tan(1/2*a)^6*tan(1/2*c)^2 + 96*tan(1/2*a)^5*tan(1/2*c)^3
 + 4*tan(1/2*a)^4*tan(1/2*c)^4 + 96*tan(1/2*a)^3*tan(1/2*c)^5 - 6*tan(1/2*
a)^2*tan(1/2*c)^6 + 16*tan(1/2*a)*tan(1/2*c)^7 - 3*tan(1/2*c)^8 - 2*tan(1/
2*a)^6 + 24*tan(1/2*a)^5*tan(1/2*c) + 6*tan(1/2*a)^4*tan(1/2*c)^2 + 64*tan
(1/2*a)^3*tan(1/2*c)^3 + 6*tan(1/2*a)^2*tan(1/2*c)^4 + 24*tan(1/2*a)*ta...
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(a+bx)\sec^4(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)^2*cos(c + b*x)^4),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^2(a+bx)\sec^4(c+bx)\,dx = \int \sec(bx+a)^2\sec(bx+c)^4\,dx$$

input
```
int(sec(b*x+a)^2*sec(b*x+c)^4,x)
```

output
```
int(sec(b*x+a)^2*sec(b*x+c)^4,x)
```

3.324 $\int \sec^3(a+bx)\sec^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \sec^3(a+bx)\sec^3(c+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 6.72 (sec) , antiderivative size = 1733, normalized size of antiderivative = 1733.00

$$\int \sec^3(a+bx)\sec^3(c+bx)\,dx=\text{Too large to display}$$

input
```
Integrate[Sec[a + b*x]^3*Sec[c + b*x]^3,x]
```

output
```
((2*I)*ArcTan[Tan[a + b*x]]*(2 + Cos[2*a - 2*c])*Csc[a - c]^5)/b - ((2*I)*
ArcTan[Tan[c + b*x]]*(2 + Cos[2*a - 2*c])*Csc[a - c]^5)/b - ((2 + Cos[2*a
- 2*c])*Csc[a - c]^5*Log[Cos[a + b*x]^2])/b + ((2 + Cos[2*a - 2*c])*Csc[a
- c]^5*Log[Cos[c + b*x]^2])/b + (Csc[a/2 - c/2]^3*Sec[a/2 - c/2]^3*Sec[a +
 b*x]^2)/(16*b) - (Csc[a/2 - c/2]^3*Sec[a/2 - c/2]^3*Sec[c + b*x]^2)/(16*b
) - (3*Csc[a/2 - c/2]^4*Sec[a/2 - c/2]^4*Sec[a + b*x]*(-Sin[a - c - b*x] +
 Sin[a - c + b*x]))/(32*b*(Cos[a/2] - Sin[a/2])*(Cos[a/2] + Sin[a/2])) - (
3*Csc[a/2 - c/2]^4*Sec[a/2 - c/2]^4*Sec[c + b*x]*(-Sin[a - c - b*x] + Sin[
a - c + b*x]))/(32*b*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])) + x*((-4
*I)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^5 - (I*Cos[c]^2)/(Cos[c]*Sin[a] - Cos[
a]*Sin[c])^5 - (I*Cos[a]^2*Cos[c]^2)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^5 - (
3*Cos[a]*Cos[c]^2*Sin[a])/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^5 + ((3*I)*Cos[c
]^2*Sin[a]^2)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^5 - (2*Cos[c]*Sin[c])/(Cos[c
]*Sin[a] - Cos[a]*Sin[c])^5 + (2*Cos[a]^2*Cos[c]*Sin[c])/(Cos[c]*Sin[a] -
Cos[a]*Sin[c])^5 - ((6*I)*Cos[a]*Cos[c]*Sin[a]*Sin[c])/(Cos[c]*Sin[a] - Co
s[a]*Sin[c])^5 - (6*Cos[c]*Sin[a]^2*Sin[c])/(Cos[c]*Sin[a] - Cos[a]*Sin[c]
)^5 + (I*Sin[c]^2)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^5 + (I*Cos[a]^2*Sin[c]^
2)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^5 + (3*Cos[a]*Sin[a]*Sin[c]^2)/(Cos[c]*
Sin[a] - Cos[a]*Sin[c])^5 - ((3*I)*Sin[a]^2*Sin[c]^2)/(Cos[c]*Sin[a] - Cos
[a]*Sin[c])^5 - (4*I)/(-(Cos[c]*Sin[a]) + Cos[a]*Sin[c])^5 - (I*Cos[a]^...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)\sec^3(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sec^3(a+bx)\sec^3(bx+c)dx$$

input
```
Int[Sec[a + b*x]^3*Sec[c + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 17.78 (sec) , antiderivative size = 492, normalized size of antiderivative = 492.00

method	result
default	$\frac{\frac{\frac{\tan(bx+a)^2\sin(a)\cos(c)}{2}-\frac{\tan(bx+a)^2\cos(a)\sin(c)}{2}-3\tan(bx+a)\cos(a)\cos(c)-3\tan(bx+a)\sin(a)\sin(c)}{(\sin(a)\cos(c)-\cos(a)\sin(c))^4}+\frac{-\sin(a)^4\cos(c)^4-2\cos(a)^2\sin(a)^2\cos}{}}{}$
risch	$-\frac{16i\left(2\,e^{i(6bx+13a+5c)}+8\,e^{i(6bx+11a+7c)}+2\,e^{3i(2bx+3a+3c)}+3\,e^{i(4bx+13a+3c)}+15\,e^{i(4bx+11a+5c)}+15\,e^{i(4bx+9a+7c)}+3\,e^{i(4bx+7a+}\right.}{\left(e^{2i(bx+a+c)}+e^{2ia}\right)^2\left(e^{2i(bx+a)}+1\right)^2\left(-e^{2ia}+e^{2}\right.}$

input
```
int(sec(b*x+a)^3*sec(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/(sin(a)*cos(c)-cos(a)*sin(c))^4*(1/2*tan(b*x+a)^2*sin(a)*cos(c)-1/2
*tan(b*x+a)^2*cos(a)*sin(c)-3*tan(b*x+a)*cos(a)*cos(c)-3*tan(b*x+a)*sin(a)
*sin(c))+1/2*(-sin(a)^4*cos(c)^4-2*cos(a)^2*sin(a)^2*cos(c)^4-cos(a)^4*cos
(c)^4-2*sin(a)^4*cos(c)^2*sin(c)^2-4*cos(a)^2*sin(a)^2*cos(c)^2*sin(c)^2-2
*cos(a)^4*cos(c)^2*sin(c)^2-sin(a)^4*sin(c)^4-2*cos(a)^2*sin(a)^2*sin(c)^4
-cos(a)^4*sin(c)^4)/(sin(a)*cos(c)-cos(a)*sin(c))^5/(tan(b*x+a)*sin(a)*cos
(c)-tan(b*x+a)*cos(a)*sin(c)+cos(a)*cos(c)+sin(a)*sin(c))^2-(-4*cos(c)^3*s
in(a)^2*cos(a)-4*cos(c)^3*cos(a)^3-4*cos(c)^2*sin(c)*sin(a)^3-4*cos(c)^2*s
in(c)*cos(a)^2*sin(a)-4*cos(c)*sin(c)^2*cos(a)*sin(a)^2-4*cos(c)*sin(c)^2*
cos(a)^3-4*sin(c)^3*sin(a)^3-4*sin(c)^3*sin(a)*cos(a)^2)/(sin(a)*cos(c)-co
s(a)*sin(c))^5/(tan(b*x+a)*sin(a)*cos(c)-tan(b*x+a)*cos(a)*sin(c)+cos(a)*c
os(c)+sin(a)*sin(c))-(-2*cos(c)^2*sin(a)^2-6*cos(a)^2*cos(c)^2-8*cos(a)*co
s(c)*sin(a)*sin(c)-6*sin(a)^2*sin(c)^2-2*sin(c)^2*cos(a)^2)/(sin(a)*cos(c)
-cos(a)*sin(c))^5*ln(tan(b*x+a)*sin(a)*cos(c)-tan(b*x+a)*cos(a)*sin(c)+cos
(a)*cos(c)+sin(a)*sin(c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.13 (sec) , antiderivative size = 597, normalized size of antiderivative = 597.00

$$\int \sec^3(a+bx)\sec^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)^3*sec(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(24*(cos(-a + c)^4 - cos(-a + c)^2)*cos(b*x + c)^4 - cos(-a + c)^4 - 2
*(8*cos(-a + c)^4 - 7*cos(-a + c)^2 - 1)*cos(b*x + c)^2 + 4*(3*(2*cos(-a +
 c)^3 - cos(-a + c))*cos(b*x + c)^3 - (cos(-a + c)^3 - cos(-a + c))*cos(b*
x + c))*sin(b*x + c)*sin(-a + c) + 2*cos(-a + c)^2 + 2*(2*(2*cos(-a + c)^3
 + cos(-a + c))*cos(b*x + c)^3*sin(b*x + c)*sin(-a + c) + (4*cos(-a + c)^4
 - 1)*cos(b*x + c)^4 - (2*cos(-a + c)^4 - cos(-a + c)^2 - 1)*cos(b*x + c)^
2)*log(cos(b*x + c)^2) - 2*(2*(2*cos(-a + c)^3 + cos(-a + c))*cos(b*x + c)
^3*sin(b*x + c)*sin(-a + c) + (4*cos(-a + c)^4 - 1)*cos(b*x + c)^4 - (2*co
s(-a + c)^4 - cos(-a + c)^2 - 1)*cos(b*x + c)^2)*log(4*(2*cos(b*x + c)*cos
(-a + c)*sin(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 -
 cos(-a + c)^2 + 1)/(cos(-a + c)^2 + 2*cos(-a + c) + 1)) - 1)/(2*(b*cos(-a
 + c)^7 - 3*b*cos(-a + c)^5 + 3*b*cos(-a + c)^3 - b*cos(-a + c))*cos(b*x +
 c)^3*sin(b*x + c) - ((2*b*cos(-a + c)^6 - 5*b*cos(-a + c)^4 + 4*b*cos(-a
+ c)^2 - b)*cos(b*x + c)^4 - (b*cos(-a + c)^6 - 3*b*cos(-a + c)^4 + 3*b*co
s(-a + c)^2 - b)*cos(b*x + c)^2)*sin(-a + c))
```

Sympy [F]

$$\int \sec^3(a+bx)\sec^3(c+bx)\,dx = \int \sec^3{(a+bx)}\sec^3{(bx+c)}\,dx$$

input
```
integrate(sec(b*x+a)**3*sec(b*x+c)**3,x)
```

output
```
Integral(sec(a + b*x)**3*sec(b*x + c)**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 61.56 (sec) , antiderivative size = 1929031, normalized size of antiderivative = 1929031.00

$$\int \sec^3(a+bx)\sec^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)^3*sec(b*x+c)^3,x, algorithm="maxima")
```

output
```
-16*(600*((sin(8*a) - 4*sin(6*a + 2*c) + sin(8*c))*cos(5*a + 3*c) - (cos(8
*a) - 4*cos(6*a + 2*c) + cos(8*c))*sin(5*a + 3*c) - 6*cos(4*a + 4*c)*sin(5
*a + 3*c) + 6*cos(5*a + 3*c)*sin(4*a + 4*c))*cos(6*a + 4*c)^2 + 600*((sin(
8*a) - 4*sin(6*a + 2*c) + sin(8*c))*cos(5*a + 3*c) + (sin(8*a) - 4*sin(6*a
 + 2*c) + 6*sin(4*a + 4*c) + sin(8*c))*cos(3*a + 5*c) + 4*(sin(5*a + 3*c)
+ sin(3*a + 5*c))*cos(2*a + 6*c) - (cos(8*a) - 4*cos(6*a + 2*c) + cos(8*c)
)*sin(5*a + 3*c) - 6*cos(4*a + 4*c)*sin(5*a + 3*c) + 6*cos(5*a + 3*c)*sin(
4*a + 4*c) - (cos(8*a) - 4*cos(6*a + 2*c) + 6*cos(4*a + 4*c) + cos(8*c))*s
in(3*a + 5*c) - 4*(cos(5*a + 3*c) + cos(3*a + 5*c))*sin(2*a + 6*c))*cos(4*
a + 6*c)^2 + 150*((sin(8*a) - 4*sin(6*a + 2*c) + sin(8*c))*cos(5*a + 3*c)
+ (sin(8*a) - 4*sin(6*a + 2*c) + 6*sin(4*a + 4*c) + sin(8*c))*cos(3*a + 5*
c) + 4*(sin(5*a + 3*c) + sin(3*a + 5*c))*cos(2*a + 6*c) - (cos(8*a) - 4*co
s(6*a + 2*c) + cos(8*c))*sin(5*a + 3*c) - 6*cos(4*a + 4*c)*sin(5*a + 3*c)
+ 6*cos(5*a + 3*c)*sin(4*a + 4*c) - (cos(8*a) - 4*cos(6*a + 2*c) + 6*cos(4
*a + 4*c) + cos(8*c))*sin(3*a + 5*c) - 4*(cos(5*a + 3*c) + cos(3*a + 5*c))
*sin(2*a + 6*c))*cos(2*a + 8*c)^2 + 600*((sin(8*a) - 4*sin(6*a + 2*c) + si
n(8*c))*cos(5*a + 3*c) - (cos(8*a) - 4*cos(6*a + 2*c) + cos(8*c))*sin(5*a
+ 3*c) - 6*cos(4*a + 4*c)*sin(5*a + 3*c) + 6*cos(5*a + 3*c)*sin(4*a + 4*c)
)*sin(6*a + 4*c)^2 - 36*(cos(10*a)^2 - 10*(cos(10*a) - cos(10*c))*cos(8*a
+ 2*c) + 25*cos(8*a + 2*c)^2 - 2*cos(10*a)*cos(10*c) + cos(10*c)^2 + si...
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.36 (sec) , antiderivative size = 10230, normalized size of antiderivative = 10230.00

$$\int \sec^3(a+bx)\sec^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)^3*sec(b*x+c)^3,x, algorithm="giac")
```

output
```
1/64*(4*(3*tan(1/2*a)^10*tan(1/2*c)^10 + 7*tan(1/2*a)^10*tan(1/2*c)^8 + 16
*tan(1/2*a)^9*tan(1/2*c)^9 + 7*tan(1/2*a)^8*tan(1/2*c)^10 + 6*tan(1/2*a)^1
0*tan(1/2*c)^6 + 32*tan(1/2*a)^9*tan(1/2*c)^7 + 59*tan(1/2*a)^8*tan(1/2*c)
^8 + 32*tan(1/2*a)^7*tan(1/2*c)^9 + 6*tan(1/2*a)^6*tan(1/2*c)^10 + 6*tan(1
/2*a)^10*tan(1/2*c)^4 + 142*tan(1/2*a)^8*tan(1/2*c)^6 + 64*tan(1/2*a)^7*ta
n(1/2*c)^7 + 142*tan(1/2*a)^6*tan(1/2*c)^8 + 6*tan(1/2*a)^4*tan(1/2*c)^10
 + 7*tan(1/2*a)^10*tan(1/2*c)^2 - 32*tan(1/2*a)^9*tan(1/2*c)^3 + 142*tan(1/
2*a)^8*tan(1/2*c)^4 + 396*tan(1/2*a)^6*tan(1/2*c)^6 + 142*tan(1/2*a)^4*tan
(1/2*c)^8 - 32*tan(1/2*a)^3*tan(1/2*c)^9 + 7*tan(1/2*a)^2*tan(1/2*c)^10 +
3*tan(1/2*a)^10 - 16*tan(1/2*a)^9*tan(1/2*c) + 59*tan(1/2*a)^8*tan(1/2*c)^
2 - 64*tan(1/2*a)^7*tan(1/2*c)^3 + 396*tan(1/2*a)^6*tan(1/2*c)^4 + 396*tan
(1/2*a)^4*tan(1/2*c)^6 - 64*tan(1/2*a)^3*tan(1/2*c)^7 + 59*tan(1/2*a)^2*ta
n(1/2*c)^8 - 16*tan(1/2*a)*tan(1/2*c)^9 + 3*tan(1/2*c)^10 + 7*tan(1/2*a)^8
 - 32*tan(1/2*a)^7*tan(1/2*c) + 142*tan(1/2*a)^6*tan(1/2*c)^2 + 396*tan(1/
2*a)^4*tan(1/2*c)^4 + 142*tan(1/2*a)^2*tan(1/2*c)^6 - 32*tan(1/2*a)*tan(1/
2*c)^7 + 7*tan(1/2*c)^8 + 6*tan(1/2*a)^6 + 142*tan(1/2*a)^4*tan(1/2*c)^2 +
 64*tan(1/2*a)^3*tan(1/2*c)^3 + 142*tan(1/2*a)^2*tan(1/2*c)^4 + 6*tan(1/2*
c)^6 + 6*tan(1/2*a)^4 + 32*tan(1/2*a)^3*tan(1/2*c) + 59*tan(1/2*a)^2*tan(1
/2*c)^2 + 32*tan(1/2*a)*tan(1/2*c)^3 + 6*tan(1/2*c)^4 + 7*tan(1/2*a)^2 + 1
6*tan(1/2*a)*tan(1/2*c) + 7*tan(1/2*c)^2 + 3)*log(abs(2*tan(b*x + a)*ta...
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(a+bx)\sec^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)^3*cos(c + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^3(a+bx)\sec^3(c+bx)\,dx = \text{too large to display}$$

input
```
int(sec(b*x+a)^3*sec(b*x+c)^3,x)
```

output

```
( - 8*cos(b*x + c)**2*cos(a + b*x)**2*sin(a + b*x) - 5*cos(b*x + c)**2*cos
(a + b*x)*sin(a + b*x) - 14*cos(b*x + c)**2*sin(a + b*x)**3 + 14*cos(b*x +
 c)**2*sin(a + b*x) + 42*cos(b*x + c)*cos(a + b*x)**2*sin(b*x + c)**2*sin(
a + b*x) - 11*cos(b*x + c)*cos(a + b*x)**2*sin(b*x + c) - 42*cos(b*x + c)*
cos(a + b*x)**2*sin(a + b*x) + 8*cos(b*x + c)*cos(a + b*x)*int(cos(b*x + c
)/(cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x)**2 - cos(a + b*x)*sin(b*x + c
)**2 - cos(a + b*x)*sin(a + b*x)**2 + cos(a + b*x)),x)*sin(b*x + c)**2*sin
(a + b*x)**2*b - 8*cos(b*x + c)*cos(a + b*x)*int(cos(b*x + c)/(cos(a + b*x
)*sin(b*x + c)**2*sin(a + b*x)**2 - cos(a + b*x)*sin(b*x + c)**2 - cos(a +
 b*x)*sin(a + b*x)**2 + cos(a + b*x)),x)*sin(b*x + c)**2*b - 8*cos(b*x + c
)*cos(a + b*x)*int(cos(b*x + c)/(cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x)
**2 - cos(a + b*x)*sin(b*x + c)**2 - cos(a + b*x)*sin(a + b*x)**2 + cos(a
+ b*x)),x)*sin(a + b*x)**2*b + 8*cos(b*x + c)*cos(a + b*x)*int(cos(b*x + c
)/(cos(a + b*x)*sin(b*x + c)**2*sin(a + b*x)**2 - cos(a + b*x)*sin(b*x + c
)**2 - cos(a + b*x)*sin(a + b*x)**2 + cos(a + b*x)),x)*b + 8*cos(b*x + c)*
cos(a + b*x)*int(cos(b*x + c)/(sin(b*x + c)**2*sin(a + b*x)**2 - sin(b*x +
 c)**2 - sin(a + b*x)**2 + 1),x)*sin(b*x + c)**2*sin(a + b*x)**2*b - 8*cos
(b*x + c)*cos(a + b*x)*int(cos(b*x + c)/(sin(b*x + c)**2*sin(a + b*x)**2 -
 sin(b*x + c)**2 - sin(a + b*x)**2 + 1),x)*sin(b*x + c)**2*b - 8*cos(b*x +
 c)*cos(a + b*x)*int(cos(b*x + c)/(sin(b*x + c)**2*sin(a + b*x)**2 - si...
```

3.325 $\int \sec^3(a+bx)\sec^4(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \sec^3(a+bx)\sec^4(c+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 14.86 (sec) , antiderivative size = 243, normalized size of antiderivative = 243.00

$$\int \sec^3(a+bx)\sec^4(c+bx)\,dx=$$
$$-\frac{\csc^5(a-c)\left(240\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)(15\cos(a-c)+\cos(3(a-c)))\csc(a-c)+240(5-\right.}{}$$

input
```
Integrate[Sec[a + b*x]^3*Sec[c + b*x]^4,x]
```

output
```
-1/192*(Csc[a - c]^5*(240*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*(15*Cos[a
- c] + Cos[3*(a - c)])*Csc[a - c] + 240*(5 + 3*Cos[2*(a - c)])*Csc[a - c]*
(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] + Sin[(a
+ b*x)/2]]) + (349 + 338*Cos[2*(a - c)] + 33*Cos[4*(a - c)] + 120*Cos[2*(a
 - 2*c - b*x)] + 400*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)] + 400*Cos[2*(c
 + b*x)] + 75*Cos[4*(c + b*x)] + 40*Cos[4*a - 2*c + 2*b*x] + 150*Cos[2*(a
+ c + 2*b*x)])*Sec[a + b*x]^2*Sec[c + b*x]^3))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)\sec^4(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sec^3(a+bx)\sec^4(bx+c)dx$$

input
```
Int[Sec[a + b*x]^3*Sec[c + b*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 35.21 (sec) , antiderivative size = 1761, normalized size of antiderivative = 1761.00

method	result	size
risch	Expression too large to display	1761
default	Expression too large to display	3107

input
```
int(sec(b*x+a)^3*sec(b*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-8/3*I/(exp(2*I*(b*x+a))+1)^2/(exp(2*I*(b*x+a+c))+exp(2*I*a))^3/(exp(2*I*a
)-exp(2*I*c))^5/b*(15*exp(I*(9*b*x+17*a+8*c))+150*exp(I*(9*b*x+15*a+10*c))
+75*exp(I*(9*b*x+13*a+12*c))+40*exp(I*(7*b*x+17*a+6*c))+400*exp(I*(7*b*x+1
5*a+8*c))+400*exp(I*(7*b*x+13*a+10*c))+120*exp(I*(7*b*x+11*a+12*c))+33*exp
(I*(5*b*x+17*a+4*c))+338*exp(I*(5*b*x+15*a+6*c))+698*exp(I*(5*b*x+13*a+8*c
))+338*exp(I*(5*b*x+11*a+10*c))+33*exp(I*(5*b*x+9*a+12*c))+120*exp(I*(3*b*
x+15*a+4*c))+400*exp(I*(3*b*x+13*a+6*c))+400*exp(I*(3*b*x+11*a+8*c))+40*ex
p(I*(3*b*x+9*a+10*c))+75*exp(I*(b*x+13*a+4*c))+150*exp(I*(b*x+11*a+6*c))+1
5*exp(I*(b*x+9*a+8*c)))-120*ln(exp(I*(b*x+a))+I)/(exp(12*I*a)-6*exp(2*I*(5
*a+c))+15*exp(4*I*(2*a+c))-20*exp(6*I*(a+c))+15*exp(4*I*(a+2*c))-6*exp(2*I
*(a+5*c))+exp(12*I*c))/b*exp(4*I*(2*a+c))-400*ln(exp(I*(b*x+a))+I)/(exp(12
*I*a)-6*exp(2*I*(5*a+c))+15*exp(4*I*(2*a+c))-20*exp(6*I*(a+c))+15*exp(4*I*
(a+2*c))-6*exp(2*I*(a+5*c))+exp(12*I*c))/b*exp(6*I*(a+c))-120*ln(exp(I*(b*
x+a))+I)/(exp(12*I*a)-6*exp(2*I*(5*a+c))+15*exp(4*I*(2*a+c))-20*exp(6*I*(a
+c))+15*exp(4*I*(a+2*c))-6*exp(2*I*(a+5*c))+exp(12*I*c))/b*exp(4*I*(a+2*c)
)-20*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/(exp(12*I*a)-6*exp(2*I*(5*a+c))+15*
exp(4*I*(2*a+c))-20*exp(6*I*(a+c))+15*exp(4*I*(a+2*c))-6*exp(2*I*(a+5*c))+
exp(12*I*c))/b*exp(3*I*(3*a+c))-300*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/(exp
(12*I*a)-6*exp(2*I*(5*a+c))+15*exp(4*I*(2*a+c))-20*exp(6*I*(a+c))+15*exp(4
*I*(a+2*c))-6*exp(2*I*(a+5*c))+exp(12*I*c))/b*exp(I*(7*a+5*c))-300*ln(e...
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.24 (sec) , antiderivative size = 890, normalized size of antiderivative = 890.00

$$\int \sec^3(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)^3*sec(b*x+c)^4,x, algorithm="fricas")
```

output
```
-1/12*(15*(2*(3*cos(-a + c)^3 + cos(-a + c))*cos(b*x + c)^4*sin(b*x + c)*s
in(-a + c) + (6*cos(-a + c)^4 - cos(-a + c)^2 - 1)*cos(b*x + c)^5 - (3*cos
(-a + c)^4 - 2*cos(-a + c)^2 - 1)*cos(b*x + c)^3)*log(2*(cos(-a + c)*sin(b
*x + c) - cos(b*x + c)*sin(-a + c) + 1)/(cos(-a + c) + 1)) - 15*(2*(3*cos(
-a + c)^3 + cos(-a + c))*cos(b*x + c)^4*sin(b*x + c)*sin(-a + c) + (6*cos(
-a + c)^4 - cos(-a + c)^2 - 1)*cos(b*x + c)^5 - (3*cos(-a + c)^4 - 2*cos(-
a + c)^2 - 1)*cos(b*x + c)^3)*log(-2*(cos(-a + c)*sin(b*x + c) - cos(b*x +
 c)*sin(-a + c) - 1)/(cos(-a + c) + 1)) - 15*(2*(cos(-a + c)^4 + 3*cos(-a
+ c)^2)*cos(b*x + c)^4*sin(b*x + c)*sin(-a + c) + (2*cos(-a + c)^5 + 5*cos
(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^5 - (cos(-a + c)^5 + 2*cos(-a + c
)^3 - 3*cos(-a + c))*cos(b*x + c)^3)*log(sin(b*x + c) + 1) + 15*(2*(cos(-a
 + c)^4 + 3*cos(-a + c)^2)*cos(b*x + c)^4*sin(b*x + c)*sin(-a + c) + (2*co
s(-a + c)^5 + 5*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^5 - (cos(-a +
c)^5 + 2*cos(-a + c)^3 - 3*cos(-a + c))*cos(b*x + c)^3)*log(-sin(b*x + c)
+ 1) - 10*(6*(cos(-a + c)^5 + cos(-a + c)^3 - 2*cos(-a + c))*cos(b*x + c)^
3 + (cos(-a + c)^5 - 2*cos(-a + c)^3 + cos(-a + c))*cos(b*x + c))*sin(b*x
+ c) + 2*(15*(2*cos(-a + c)^4 + 3*cos(-a + c)^2 - 1)*cos(b*x + c)^4 + 2*co
s(-a + c)^4 - 10*(cos(-a + c)^4 - 1)*cos(b*x + c)^2 - 4*cos(-a + c)^2 + 2)
*sin(-a + c))/(2*(b*cos(-a + c)^7 - 3*b*cos(-a + c)^5 + 3*b*cos(-a + c)^3
- b*cos(-a + c))*cos(b*x + c)^4*sin(b*x + c)*sin(-a + c) + (2*b*cos(-a ...
```

Sympy [F]

$$\int \sec^3(a+bx)\sec^4(c+bx)\,dx = \int \sec^3(a+bx)\sec^4(bx+c)\,dx$$

input
```
integrate(sec(b*x+a)**3*sec(b*x+c)**4,x)
```

output
```
Integral(sec(a + b*x)**3*sec(b*x + c)**4, x)
```

Maxima [F(-1)]

Timed out.

$$\int \sec^3(a+bx)\sec^4(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+a)^3*sec(b*x+c)^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 41.48 (sec) , antiderivative size = 53926, normalized size of antiderivative = 53926.00

$$\int \sec^3(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)^3*sec(b*x+c)^4,x, algorithm="giac")
```

output
```
1/96*(15*(tan(1/2*a)^13*tan(1/2*c)^13 - tan(1/2*a)^13*tan(1/2*c)^12 + tan(
1/2*a)^12*tan(1/2*c)^13 + 3*tan(1/2*a)^13*tan(1/2*c)^11 + 7*tan(1/2*a)^12*
tan(1/2*c)^12 + 3*tan(1/2*a)^11*tan(1/2*c)^13 - 3*tan(1/2*a)^13*tan(1/2*c)
^10 - 3*tan(1/2*a)^12*tan(1/2*c)^11 + 3*tan(1/2*a)^11*tan(1/2*c)^12 + 3*ta
n(1/2*a)^10*tan(1/2*c)^13 + 3*tan(1/2*a)^13*tan(1/2*c)^9 + 21*tan(1/2*a)^1
2*tan(1/2*c)^10 + 30*tan(1/2*a)^11*tan(1/2*c)^11 + 21*tan(1/2*a)^10*tan(1/
2*c)^12 + 3*tan(1/2*a)^9*tan(1/2*c)^13 - 3*tan(1/2*a)^13*tan(1/2*c)^8 - 15
*tan(1/2*a)^12*tan(1/2*c)^9 - 6*tan(1/2*a)^11*tan(1/2*c)^10 + 6*tan(1/2*a)
^10*tan(1/2*c)^11 + 15*tan(1/2*a)^9*tan(1/2*c)^12 + 3*tan(1/2*a)^8*tan(1/2
*c)^13 + 27*tan(1/2*a)^12*tan(1/2*c)^8 + 57*tan(1/2*a)^11*tan(1/2*c)^9 + 1
18*tan(1/2*a)^10*tan(1/2*c)^10 + 57*tan(1/2*a)^9*tan(1/2*c)^11 + 27*tan(1/
2*a)^8*tan(1/2*c)^12 - 24*tan(1/2*a)^12*tan(1/2*c)^7 - 15*tan(1/2*a)^11*ta
n(1/2*c)^8 - 55*tan(1/2*a)^10*tan(1/2*c)^9 + 55*tan(1/2*a)^9*tan(1/2*c)^10
 + 15*tan(1/2*a)^8*tan(1/2*c)^11 + 24*tan(1/2*a)^7*tan(1/2*c)^12 - 3*tan(1
/2*a)^13*tan(1/2*c)^5 + 24*tan(1/2*a)^12*tan(1/2*c)^6 + 24*tan(1/2*a)^11*t
an(1/2*c)^7 + 231*tan(1/2*a)^10*tan(1/2*c)^8 + 163*tan(1/2*a)^9*tan(1/2*c)
^9 + 231*tan(1/2*a)^8*tan(1/2*c)^10 + 24*tan(1/2*a)^7*tan(1/2*c)^11 + 24*t
an(1/2*a)^6*tan(1/2*c)^12 - 3*tan(1/2*a)^5*tan(1/2*c)^13 + 3*tan(1/2*a)^13
*tan(1/2*c)^4 - 27*tan(1/2*a)^12*tan(1/2*c)^5 + 24*tan(1/2*a)^11*tan(1/2*c
)^6 - 192*tan(1/2*a)^10*tan(1/2*c)^7 + 123*tan(1/2*a)^9*tan(1/2*c)^8 - ...
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(a+bx)\sec^4(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)^3*cos(c + b*x)^4),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^3(a+bx)\sec^4(c+bx)\,dx = \text{too large to display}$$

input
```
int(sec(b*x+a)^3*sec(b*x+c)^4,x)
```

output
```
(60*cos(b*x + c)**2*cos(a + b*x)**2*sin(b*x + c) - 30*cos(b*x + c)**2*cos(
a + b*x)*sin(b*x + c)*sin(a + b*x)**2 + 60*cos(b*x + c)**2*cos(a + b*x)*si
n(b*x + c) + 24*cos(b*x + c)**2*sin(a + b*x)**3 - 24*cos(b*x + c)**2*sin(a
 + b*x) - 84*cos(b*x + c)*cos(a + b*x)**2*sin(b*x + c)**2*sin(a + b*x) + 1
20*cos(b*x + c)*cos(a + b*x)**2*sin(b*x + c) + 42*cos(b*x + c)*cos(a + b*x
)**2*sin(a + b*x) + 24*cos(b*x + c)*cos(a + b*x)*int(cos(a + b*x)/(cos(b*x
 + c)*sin(b*x + c)**2*sin(a + b*x)**2 - cos(b*x + c)*sin(b*x + c)**2 - cos
(b*x + c)*sin(a + b*x)**2 + cos(b*x + c)),x)*sin(b*x + c)**2*sin(a + b*x)*
*2*b - 24*cos(b*x + c)*cos(a + b*x)*int(cos(a + b*x)/(cos(b*x + c)*sin(b*x
 + c)**2*sin(a + b*x)**2 - cos(b*x + c)*sin(b*x + c)**2 - cos(b*x + c)*sin
(a + b*x)**2 + cos(b*x + c)),x)*sin(b*x + c)**2*b - 24*cos(b*x + c)*cos(a
+ b*x)*int(cos(a + b*x)/(cos(b*x + c)*sin(b*x + c)**2*sin(a + b*x)**2 - co
s(b*x + c)*sin(b*x + c)**2 - cos(b*x + c)*sin(a + b*x)**2 + cos(b*x + c)),
x)*sin(a + b*x)**2*b + 24*cos(b*x + c)*cos(a + b*x)*int(cos(a + b*x)/(cos(
b*x + c)*sin(b*x + c)**2*sin(a + b*x)**2 - cos(b*x + c)*sin(b*x + c)**2 -
cos(b*x + c)*sin(a + b*x)**2 + cos(b*x + c)),x)*b + 24*cos(b*x + c)*cos(a
+ b*x)*int(cos(a + b*x)/(sin(b*x + c)**2*sin(a + b*x)**2 - sin(b*x + c)**2
 - sin(a + b*x)**2 + 1),x)*sin(b*x + c)**2*sin(a + b*x)**2*b - 24*cos(b*x
+ c)*cos(a + b*x)*int(cos(a + b*x)/(sin(b*x + c)**2*sin(a + b*x)**2 - sin(
b*x + c)**2 - sin(a + b*x)**2 + 1),x)*sin(b*x + c)**2*b - 24*cos(b*x + ...
```

3.326 $\int \sec^3(a+bx)\sec^5(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \sec^3(a+bx)\sec^5(c+bx)\,dx=0$$

output
```
0
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 6.88 (sec) , antiderivative size = 1972, normalized size of antiderivative = 1972.00

$$\int \sec^3(a+bx)\sec^5(c+bx)\,dx=\text{Too large to display}$$

input
```
Integrate[Sec[a + b*x]^3*Sec[c + b*x]^5,x]
```

output
```
((3*I)*ArcTan[Tan[a + b*x]]*(3 + 2*Cos[2*a - 2*c])*Csc[a - c]^7)/b - ((3*I
)*ArcTan[Tan[c + b*x]]*(3 + 2*Cos[2*a - 2*c])*Csc[a - c]^7)/b - (3*(3 + 2*
Cos[2*a - 2*c])*Csc[a - c]^7*Log[Cos[a + b*x]^2])/(2*b) + (3*(3 + 2*Cos[2*
a - 2*c])*Csc[a - c]^7*Log[Cos[c + b*x]^2])/(2*b) + (Csc[a - c]^6*Sec[a]*S
ec[c]*Sec[a + b*x]^2*Sec[c + b*x]^4*(220*Sin[2*a] - 4*Sin[4*a - 6*c] - 4*S
in[6*a - 6*c] - 76*Sin[2*a - 4*c] - 36*Sin[4*a - 4*c] - 76*Sin[2*a - 2*c]
+ 52*Sin[4*a - 2*c] + 248*Sin[2*c] - 263*Sin[2*b*x] + Sin[2*a - 6*c - 6*b*
x] + 10*Sin[2*a - 6*c - 4*b*x] + 2*Sin[4*a - 6*c - 4*b*x] + 50*Sin[2*a - 4
*c - 4*b*x] - 18*Sin[2*a - 6*c - 2*b*x] - 7*Sin[4*a - 6*c - 2*b*x] + Sin[6
*a - 6*c - 2*b*x] + 3*Sin[2*a - 4*c - 2*b*x] + 19*Sin[4*a - 4*c - 2*b*x] +
 171*Sin[2*a - 2*c - 2*b*x] - 141*Sin[2*a + 2*b*x] + 42*Sin[4*a + 2*b*x] -
 6*Sin[4*a - 4*c + 2*b*x] - 6*Sin[6*a - 4*c + 2*b*x] - 140*Sin[2*a - 2*c +
 2*b*x] - 80*Sin[4*a - 2*c + 2*b*x] - 77*Sin[2*c + 2*b*x] + 144*Sin[2*a +
2*c + 2*b*x] + 96*Sin[4*c + 2*b*x] - 106*Sin[2*a + 4*b*x] - 66*Sin[4*a + 4
*b*x] - 4*Sin[4*a - 2*c + 4*b*x] - 4*Sin[6*a - 2*c + 4*b*x] - 138*Sin[2*c
+ 4*b*x] - 106*Sin[2*a + 2*c + 4*b*x] + 12*Sin[4*a + 2*c + 4*b*x] - 54*Sin
[4*c + 4*b*x] + 36*Sin[2*a + 4*c + 4*b*x] + 12*Sin[6*c + 4*b*x] - Sin[4*a
+ 6*b*x] - Sin[6*a + 6*b*x] - 29*Sin[2*a + 2*c + 6*b*x] - 19*Sin[4*a + 2*c
 + 6*b*x] - 29*Sin[4*c + 6*b*x] - 29*Sin[2*a + 4*c + 6*b*x] - 11*Sin[6*c +
 6*b*x]))/(256*b) + x*((-9*I)/(Cos[c]*Sin[a] - Cos[a]*Sin[c])^7 - ((3*I...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)\sec^5(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sec^3(a+bx)\sec^5(bx+c)dx$$

input
```
Int[Sec[a + b*x]^3*Sec[c + b*x]^5,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 68.42 (sec) , antiderivative size = 1201, normalized size of antiderivative = 1201.00

method	result	size
default	Expression too large to display	1201
risch	Expression too large to display	1214

input
```
int(sec(b*x+a)^3*sec(b*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/(-sin(a)*cos(c)+cos(a)*sin(c))^6*(1/2*tan(b*x+a)^2*cos(a)*sin(c)-1
/2*tan(b*x+a)^2*sin(a)*cos(c)+5*tan(b*x+a)*sin(a)*sin(c)+5*tan(b*x+a)*cos(
a)*cos(c))-1/3*(-6*sin(c)^5*sin(a)^5-12*sin(c)^5*sin(a)^3*cos(a)^2-6*sin(c
)^5*sin(a)*cos(a)^4-6*cos(c)*sin(c)^4*sin(a)^4*cos(a)-12*sin(c)^4*cos(c)*s
in(a)^2*cos(a)^3-6*sin(c)^4*cos(c)*cos(a)^5-12*sin(c)^3*cos(c)^2*sin(a)^5-
24*cos(c)^2*sin(c)^3*sin(a)^3*cos(a)^2-12*sin(c)^3*cos(c)^2*sin(a)*cos(a)^
4-12*sin(c)^2*cos(c)^3*sin(a)^4*cos(a)-24*cos(c)^3*sin(c)^2*sin(a)^2*cos(a
)^3-12*sin(c)^2*cos(c)^3*cos(a)^5-6*sin(c)*cos(c)^4*sin(a)^5-12*sin(c)*cos
(c)^4*sin(a)^3*cos(a)^2-6*cos(c)^4*sin(c)*sin(a)*cos(a)^4-6*cos(c)^5*sin(a
)^4*cos(a)-12*cos(c)^5*sin(a)^2*cos(a)^3-6*cos(c)^5*cos(a)^5)/(-sin(a)*cos
(c)+cos(a)*sin(c))^6/(sin(a)*cos(c)-cos(a)*sin(c))/(tan(b*x+a)*sin(a)*cos(
c)-tan(b*x+a)*cos(a)*sin(c)+cos(a)*cos(c)+sin(a)*sin(c))^3+1/2*(-15*sin(a)
^4*sin(c)^4-18*cos(a)^2*sin(a)^2*sin(c)^4-3*cos(a)^4*sin(c)^4-24*cos(a)*si
n(a)^3*cos(c)*sin(c)^3-24*cos(a)^3*sin(a)*cos(c)*sin(c)^3-18*sin(a)^4*cos(
c)^2*sin(c)^2-36*cos(a)^2*sin(a)^2*cos(c)^2*sin(c)^2-18*cos(a)^4*cos(c)^2*
sin(c)^2-24*cos(a)*sin(a)^3*cos(c)^3*sin(c)-24*cos(a)^3*cos(c)^3*sin(a)*si
n(c)-3*sin(a)^4*cos(c)^4-18*cos(a)^2*sin(a)^2*cos(c)^4-15*cos(a)^4*cos(c)^
4)/(-sin(a)*cos(c)+cos(a)*sin(c))^6/(sin(a)*cos(c)-cos(a)*sin(c))/(tan(b*x
+a)*sin(a)*cos(c)-tan(b*x+a)*cos(a)*sin(c)+cos(a)*cos(c)+sin(a)*sin(c))^2+
1/4*(-sin(c)^6*sin(a)^6-3*sin(c)^6*sin(a)^4*cos(a)^2-3*sin(c)^6*sin(a)^...
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.17 (sec) , antiderivative size = 761, normalized size of antiderivative = 761.00

$$\int \sec^3(a+bx)\sec^5(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)^3*sec(b*x+c)^5,x, algorithm="fricas")
```

output
```
-1/4*(8*(2*cos(-a + c)^6 + 11*cos(-a + c)^4 - 13*cos(-a + c)^2)*cos(b*x +
c)^6 + cos(-a + c)^6 - 2*(4*cos(-a + c)^6 + 34*cos(-a + c)^4 - 35*cos(-a +
 c)^2 - 3)*cos(b*x + c)^4 - 3*cos(-a + c)^4 - (2*cos(-a + c)^6 - cos(-a +
c)^4 - 4*cos(-a + c)^2 + 3)*cos(b*x + c)^2 + 2*(2*(4*cos(-a + c)^5 + 24*co
s(-a + c)^3 - 13*cos(-a + c))*cos(b*x + c)^5 - 10*(cos(-a + c)^3 - cos(-a
+ c))*cos(b*x + c)^3 + (cos(-a + c)^5 - 2*cos(-a + c)^3 + cos(-a + c))*cos
(b*x + c))*sin(b*x + c)*sin(-a + c) + 3*cos(-a + c)^2 + 6*(2*(4*cos(-a + c
)^3 + cos(-a + c))*cos(b*x + c)^5*sin(b*x + c)*sin(-a + c) + (8*cos(-a + c
)^4 - 2*cos(-a + c)^2 - 1)*cos(b*x + c)^6 - (4*cos(-a + c)^4 - 3*cos(-a +
c)^2 - 1)*cos(b*x + c)^4)*log(cos(b*x + c)^2) - 6*(2*(4*cos(-a + c)^3 + co
s(-a + c))*cos(b*x + c)^5*sin(b*x + c)*sin(-a + c) + (8*cos(-a + c)^4 - 2*
cos(-a + c)^2 - 1)*cos(b*x + c)^6 - (4*cos(-a + c)^4 - 3*cos(-a + c)^2 - 1
)*cos(b*x + c)^4)*log(4*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*sin(-a +
c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)/(cos(-a + c
)^2 + 2*cos(-a + c) + 1)) - 1)/(2*(b*cos(-a + c)^9 - 4*b*cos(-a + c)^7 + 6
*b*cos(-a + c)^5 - 4*b*cos(-a + c)^3 + b*cos(-a + c))*cos(b*x + c)^5*sin(b
*x + c) - ((2*b*cos(-a + c)^8 - 7*b*cos(-a + c)^6 + 9*b*cos(-a + c)^4 - 5*
b*cos(-a + c)^2 + b)*cos(b*x + c)^6 - (b*cos(-a + c)^8 - 4*b*cos(-a + c)^6
 + 6*b*cos(-a + c)^4 - 4*b*cos(-a + c)^2 + b)*cos(b*x + c)^4)*sin(-a + c))
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(a+bx)\sec^5(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+a)**3*sec(b*x+c)**5,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec^3(a+bx)\sec^5(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+a)^3*sec(b*x+c)^5,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.72 (sec) , antiderivative size = 34934, normalized size of antiderivative = 34934.00

$$\int \sec^3(a+bx)\sec^5(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)^3*sec(b*x+c)^5,x, algorithm="giac")
```

output
```
1/128*(3*(5*tan(1/2*a)^14*tan(1/2*c)^14 + 19*tan(1/2*a)^14*tan(1/2*c)^12 +
 32*tan(1/2*a)^13*tan(1/2*c)^13 + 19*tan(1/2*a)^12*tan(1/2*c)^14 + 25*tan(
1/2*a)^14*tan(1/2*c)^10 + 128*tan(1/2*a)^13*tan(1/2*c)^11 + 149*tan(1/2*a)
^12*tan(1/2*c)^12 + 128*tan(1/2*a)^11*tan(1/2*c)^13 + 25*tan(1/2*a)^10*tan
(1/2*c)^14 + 15*tan(1/2*a)^14*tan(1/2*c)^8 + 160*tan(1/2*a)^13*tan(1/2*c)^
9 + 479*tan(1/2*a)^12*tan(1/2*c)^10 + 512*tan(1/2*a)^11*tan(1/2*c)^11 + 47
9*tan(1/2*a)^10*tan(1/2*c)^12 + 160*tan(1/2*a)^9*tan(1/2*c)^13 + 15*tan(1/
2*a)^8*tan(1/2*c)^14 + 15*tan(1/2*a)^14*tan(1/2*c)^6 + 825*tan(1/2*a)^12*t
an(1/2*c)^8 + 640*tan(1/2*a)^11*tan(1/2*c)^9 + 2045*tan(1/2*a)^10*tan(1/2*
c)^10 + 640*tan(1/2*a)^9*tan(1/2*c)^11 + 825*tan(1/2*a)^8*tan(1/2*c)^12 +
15*tan(1/2*a)^6*tan(1/2*c)^14 + 25*tan(1/2*a)^14*tan(1/2*c)^4 - 160*tan(1/
2*a)^13*tan(1/2*c)^5 + 825*tan(1/2*a)^12*tan(1/2*c)^6 + 3915*tan(1/2*a)^10
*tan(1/2*c)^8 + 800*tan(1/2*a)^9*tan(1/2*c)^9 + 3915*tan(1/2*a)^8*tan(1/2*
c)^10 + 825*tan(1/2*a)^6*tan(1/2*c)^12 - 160*tan(1/2*a)^5*tan(1/2*c)^13 +
25*tan(1/2*a)^4*tan(1/2*c)^14 + 19*tan(1/2*a)^14*tan(1/2*c)^2 - 128*tan(1/
2*a)^13*tan(1/2*c)^3 + 479*tan(1/2*a)^12*tan(1/2*c)^4 - 640*tan(1/2*a)^11*
tan(1/2*c)^5 + 3915*tan(1/2*a)^10*tan(1/2*c)^6 + 7725*tan(1/2*a)^8*tan(1/2
*c)^8 + 3915*tan(1/2*a)^6*tan(1/2*c)^10 - 640*tan(1/2*a)^5*tan(1/2*c)^11 +
 479*tan(1/2*a)^4*tan(1/2*c)^12 - 128*tan(1/2*a)^3*tan(1/2*c)^13 + 19*tan(
1/2*a)^2*tan(1/2*c)^14 + 5*tan(1/2*a)^14 - 32*tan(1/2*a)^13*tan(1/2*c) ...
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(a+bx)\sec^5(c+bx)\,dx = \text{Hanged}$$

input
```
int(1/(cos(a + b*x)^3*cos(c + b*x)^5),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \sec^3(a+bx)\sec^5(c+bx)\,dx = \text{too large to display}$$

input
```
int(sec(b*x+a)^3*sec(b*x+c)^5,x)
```

output
```
( - 3024*cos(b*x + c)**2*cos(a + b*x)**2*sin(b*x + c)**2*sin(a + b*x) + 28
56*cos(b*x + c)**2*cos(a + b*x)**2*sin(a + b*x) + 70400*cos(b*x + c)**2*co
s(a + b*x)*sin(b*x + c)**3*sin(a + b*x)**2 - 70400*cos(b*x + c)**2*cos(a +
 b*x)*sin(b*x + c)**3 - 1857*cos(b*x + c)**2*cos(a + b*x)*sin(b*x + c)**2*
sin(a + b*x) + 70400*cos(b*x + c)**2*cos(a + b*x)*sin(b*x + c)*sin(a + b*x
)**2 - 70400*cos(b*x + c)**2*cos(a + b*x)*sin(b*x + c) + 1884*cos(b*x + c)
**2*cos(a + b*x)*sin(a + b*x) - 175310*cos(b*x + c)**2*sin(b*x + c)**2*sin
(a + b*x)**3 + 175310*cos(b*x + c)**2*sin(b*x + c)**2*sin(a + b*x) + 14080
0*cos(b*x + c)**2*sin(b*x + c)*sin(a + b*x)**2 - 140800*cos(b*x + c)**2*si
n(b*x + c) + 187570*cos(b*x + c)**2*sin(a + b*x)**3 - 187570*cos(b*x + c)*
*2*sin(a + b*x) + 13566*cos(b*x + c)*cos(a + b*x)**2*sin(b*x + c)**4*sin(a
 + b*x) - 10191*cos(b*x + c)*cos(a + b*x)**2*sin(b*x + c)**3 - 27132*cos(b
*x + c)*cos(a + b*x)**2*sin(b*x + c)**2*sin(a + b*x) + 9102*cos(b*x + c)*c
os(a + b*x)**2*sin(b*x + c) + 13566*cos(b*x + c)*cos(a + b*x)**2*sin(a + b
*x) - 8580*cos(b*x + c)*cos(a + b*x)*int(cos(b*x + c)/(cos(a + b*x)*sin(b*
x + c)**4*sin(a + b*x)**2 - cos(a + b*x)*sin(b*x + c)**4 - 2*cos(a + b*x)*
sin(b*x + c)**2*sin(a + b*x)**2 + 2*cos(a + b*x)*sin(b*x + c)**2 + cos(a +
 b*x)*sin(a + b*x)**2 - cos(a + b*x)),x)*sin(b*x + c)**4*sin(a + b*x)**2*b
 + 8580*cos(b*x + c)*cos(a + b*x)*int(cos(b*x + c)/(cos(a + b*x)*sin(b*x +
 c)**4*sin(a + b*x)**2 - cos(a + b*x)*sin(b*x + c)**4 - 2*cos(a + b*x)*...
```

3.327 $\int \cos(a+bx)\cos^3(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \cos(a+bx)\cos^3(c+dx)\,dx = \frac{\sin(a-3c+(b-3d)x)}{8(b-3d)} + \frac{3\sin(a-c+(b-d)x)}{8(b-d)} + \frac{3\sin(a+c+(b+d)x)}{8(b+d)} + \frac{\sin(a+3c+(b+3d)x)}{8(b+3d)}$$

output
```
sin(a-3*c+(b-3*d)*x)/(8*b-24*d)+3*sin(a-c+(b-d)*x)/(8*b-8*d)+3*sin(a+c+(b+
d)*x)/(8*b+8*d)+sin(a+3*c+(b+3*d)*x)/(8*b+24*d)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

$$\int \cos(a+bx)\cos^3(c+dx)\,dx = \frac{1}{8}\left(\frac{\sin(a-3c+bx-3dx)}{b-3d} + \frac{3\sin(a-c+bx-dx)}{b-d} + \frac{\sin(a+3c+bx+3dx)}{b+3d} + \frac{3\sin(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Cos[a + b*x]*Cos[c + d*x]^3,x]
```

output
```
(Sin[a - 3*c + b*x - 3*d*x]/(b - 3*d) + (3*Sin[a - c + b*x - d*x])/(b - d)
 + Sin[a + 3*c + b*x + 3*d*x]/(b + 3*d) + (3*Sin[a + c + (b + d)*x])/(b +
d))/8
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{8}\cos(a+x(b-3d)-3c)+\frac{3}{8}\cos(a+x(b-d)-c)+\frac{3}{8}\cos(a+x(b+d)+c)+\frac{1}{8}\cos(a+x(b+3d)+3c)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}+\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

input
```
Int[Cos[a + b*x]*Cos[c + d*x]^3,x]
```

output
```
Sin[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sin[a - c + (b - d)*x])/(8*(
b - d)) + (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[a + 3*c + (b + 3*d)
*x]/(8*(b + 3*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 3.94 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sin(a-3c+(b-3d)x)}{8b-24d}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}+\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(a+3c+(b+3d)x)}{8b+24d}$
risch	$\frac{\sin(bx-3dx+a-3c)b}{8(b-3d)(b+3d)}+\frac{3\sin(bx-3dx+a-3c)d}{8(b-3d)(b+3d)}+\frac{3\sin(bx-dx+a-c)b}{8(b-d)(b+d)}+\frac{3\sin(bx-dx+a-c)d}{8(b-d)(b+d)}+\frac{3\sin(bx+dx+a+c)b}{8(b-d)(b+d)}$
parallelrisch	$-2b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)(b^2-7d^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+6d\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-1\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)(b^2-3d^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+6b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)($
orering	Expression too large to display

input
```
int(cos(b*x+a)*cos(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8/(b-d)*sin(a-c+(b-d)*x)+3/8/(b+d)*sin(
a+c+(b+d)*x)+1/8/(b+3*d)*sin(a+3*c+(b+3*d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\int \cos(a+bx)\cos^3(c+dx)\,dx =$$

$$-\frac{\left(6\,bd^2\cos\left(dx+c\right)-\left(b^3-bd^2\right)\cos\left(dx+c\right)^3\right)\sin\left(bx+a\right)-3\left(2\,d^3\cos\left(bx+a\right)-\left(b^2d-d^3\right)\cos\left(bx+\right.\right.}{b^4-10\,b^2d^2+9\,d^4}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="fricas")
```

output
```
-((6*b*d^2*cos(d*x + c) - (b^3 - b*d^2)*cos(d*x + c)^3)*sin(b*x + a) - 3*(
2*d^3*cos(b*x + a) - (b^2*d - d^3)*cos(b*x + a)*cos(d*x + c)^2)*sin(d*x +
c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(76) = 152$.

Time = 1.94 (sec) , antiderivative size = 918, normalized size of antiderivative = 10.09

$$\int \cos(a+bx)\cos^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)**3,x)
```

output
```
Piecewise((x*cos(a)*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x)*sin
(c + d*x)**3/8 - 3*x*sin(a - 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*s
in(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/8 + x*cos(a - 3*d*x)*cos(c + d*
x)**3/8 - 3*sin(a - 3*d*x)*cos(c + d*x)**3/(8*d) - sin(c + d*x)**3*cos(a -
 3*d*x)/(24*d) - sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b,
-3*d)), (-3*x*sin(a - d*x)*sin(c + d*x)**3/8 - 3*x*sin(a - d*x)*sin(c + d*
x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/8 + 3
*x*cos(a - d*x)*cos(c + d*x)**3/8 + sin(a - d*x)*cos(c + d*x)**3/(8*d) + 3
*sin(c + d*x)**3*cos(a - d*x)/(8*d) + 3*sin(c + d*x)*cos(a - d*x)*cos(c +
d*x)**2/(4*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**3/8 + 3*x*sin(a
 + d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a + d*x)*
cos(c + d*x)/8 + 3*x*cos(a + d*x)*cos(c + d*x)**3/8 - sin(a + d*x)*cos(c +
 d*x)**3/(8*d) + 3*sin(c + d*x)**3*cos(a + d*x)/(8*d) + 3*sin(c + d*x)*cos
(a + d*x)*cos(c + d*x)**2/(4*d), Eq(b, d)), (-x*sin(a + 3*d*x)*sin(c + d*x
)**3/8 + 3*x*sin(a + 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*sin(c + d
*x)**2*cos(a + 3*d*x)*cos(c + d*x)/8 + x*cos(a + 3*d*x)*cos(c + d*x)**3/8
+ 3*sin(a + 3*d*x)*cos(c + d*x)**3/(8*d) - sin(c + d*x)**3*cos(a + 3*d*x)/
(24*d) - sin(c + d*x)*cos(a + 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b, 3*d)), (
b**3*sin(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*
d*sin(c + d*x)*cos(a + b*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(83) = 166$.

Time = 0.09 (sec) , antiderivative size = 914, normalized size of antiderivative = 10.04

$$\int \cos(a + bx)\cos^3(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/16*((b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))
*cos((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin
(3*c) + 3*d^3*sin(3*c))*cos((b + 3*d)*x + a) + 3*(b^3*sin(3*c) - b^2*d*sin
(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x + a + 4*c) - 3*(b
^3*sin(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b +
 d)*x + a - 2*c) - 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9
*d^3*sin(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*sin(3*c) + b^2*d*sin(3*c
) - 9*b*d^2*sin(3*c) - 9*d^3*sin(3*c))*cos(-(b - d)*x - a - 2*c) - (b^3*si
n(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))*cos(-(b - 3*d
)*x - a + 6*c) + (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3
*sin(3*c))*cos(-(b - 3*d)*x - a) - (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^
2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*cos(3*c) -
3*b^2*d*cos(3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a) -
 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*sin
((b + d)*x + a + 4*c) - 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c
) + 9*d^3*cos(3*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos
(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 3*(
b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b
 - d)*x - a - 2*c) + (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3
*d^3*cos(3*c))*sin(-(b - 3*d)*x - a + 6*c) + (b^3*cos(3*c) + 3*b^2*d*co...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\int \cos(a+bx)\cos^3(c+dx)\,dx = \frac{\sin(bx+3\,dx+a+3\,c)}{8\,(b+3\,d)} + \frac{3\,\sin(bx+dx+a+c)}{8\,(b+d)} + \frac{3\,\sin(bx-dx+a-c)}{8\,(b-d)} + \frac{\sin(bx-3\,dx+a-3\,c)}{8\,(b-3\,d)}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/8*sin(b*x + d*x + a + c)/(b +
 d) + 3/8*sin(b*x - d*x + a - c)/(b - d) + 1/8*sin(b*x - 3*d*x + a - 3*c)/
(b - 3*d)
```

Mupad [B] (verification not implemented)

Time = 21.03 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.44

$$\int \cos(a+bx)\cos^3(c+dx)\,dx = \mathrm{e}^{a\,1\mathrm{i}-c\,3\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,3\mathrm{i}}\left(\frac{b+3\,d}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}} - \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(b-3\,d)}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}\right) + \mathrm{e}^{a\,1\mathrm{i}+c\,3\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,3\mathrm{i}}\left(\frac{b-3\,d}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}} - \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(b+3\,d)}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}\right) + \mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{3\,b+3\,d}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}} - \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(3\,b-3\,d)}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}\right) + \mathrm{e}^{a\,1\mathrm{i}+c\,1\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,1\mathrm{i}}\left(\frac{3\,b-3\,d}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}} - \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(3\,b+3\,d)}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}\right)$$

input
```
int(cos(a + b*x)*cos(c + d*x)^3,x)
```

output
```
exp(a*1i - c*3i + b*x*1i - d*x*3i)*((b + 3*d)/(b^2*16i - d^2*144i) - (exp(
- a*2i - b*x*2i)*(b - 3*d))/(b^2*16i - d^2*144i)) + exp(a*1i + c*3i + b*x*
1i + d*x*3i)*((b - 3*d)/(b^2*16i - d^2*144i) - (exp(- a*2i - b*x*2i)*(b +
3*d))/(b^2*16i - d^2*144i)) + exp(a*1i - c*1i + b*x*1i - d*x*1i)*((3*b + 3
*d)/(b^2*16i - d^2*16i) - (exp(- a*2i - b*x*2i)*(3*b - 3*d))/(b^2*16i - d^
2*16i)) + exp(a*1i + c*1i + b*x*1i + d*x*1i)*((3*b - 3*d)/(b^2*16i - d^2*1
6i) - (exp(- a*2i - b*x*2i)*(3*b + 3*d))/(b^2*16i - d^2*16i))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.97

$$\int \cos(a+bx)\cos^3(c+dx)\,dx$$
$$= \frac{3\cos(bx+a)\sin(dx+c)^3 b^2d - 3\cos(bx+a)\sin(dx+c)^3 d^3 - 3\cos(bx+a)\sin(dx+c)b^2d + 9\cos(b}{}$$

input
```
int(cos(b*x+a)*cos(d*x+c)^3,x)
```

output
```
(3*cos(a + b*x)*sin(c + d*x)**3*b**2*d - 3*cos(a + b*x)*sin(c + d*x)**3*d*
*3 - 3*cos(a + b*x)*sin(c + d*x)*b**2*d + 9*cos(a + b*x)*sin(c + d*x)*d**3
 - cos(c + d*x)*sin(a + b*x)*sin(c + d*x)**2*b**3 + cos(c + d*x)*sin(a + b
*x)*sin(c + d*x)**2*b*d**2 + cos(c + d*x)*sin(a + b*x)*b**3 - 7*cos(c + d*
x)*sin(a + b*x)*b*d**2)/(b**4 - 10*b**2*d**2 + 9*d**4)
```

3.328 $\int \cos(a+bx)\cos^2(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \cos(a+bx)\cos^2(c+dx)\,dx = \frac{\sin(a+bx)}{2b} + \frac{\sin(a-2c+(b-2d)x)}{4(b-2d)} + \frac{\sin(a+2c+(b+2d)x)}{4(b+2d)}$$

output
```
1/2*sin(b*x+a)/b+sin(a-2*c+(b-2*d)*x)/(4*b-8*d)+sin(a+2*c+(b+2*d)*x)/(4*b+
8*d)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \cos(a+bx)\cos^2(c+dx)\,dx = \frac{1}{4}\left(\frac{2\cos(bx)\sin(a)}{b} + \frac{2\cos(a)\sin(bx)}{b} + \frac{\sin(a-2c+bx-2dx)}{b-2d} + \frac{\sin(a+2c+bx+2dx)}{b+2d}\right)$$

input
```
Integrate[Cos[a + b*x]*Cos[c + d*x]^2,x]
```

output
```
((2*Cos[b*x]*Sin[a])/b + (2*Cos[a]*Sin[b*x])/b + Sin[a - 2*c + b*x - 2*d*x
]/(b - 2*d) + Sin[a + 2*c + b*x + 2*d*x]/(b + 2*d))/4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos^2(c+dx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{4}\cos(a+x(b-2d)-2c)+\frac{1}{4}\cos(a+x(b+2d)+2c)+\frac{1}{2}\cos(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\sin(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\sin(a+bx)}{2b}$$

input
```
Int[Cos[a + b*x]*Cos[c + d*x]^2,x]
```

output
```
Sin[a + b*x]/(2*b) + Sin[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Sin[a + 2*
c + (b + 2*d)*x]/(4*(b + 2*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sin(bx+a)}{2b}+\frac{\sin(a-2c+(b-2d)x)}{4b-8d}+\frac{\sin(a+2c+(b+2d)x)}{4b+8d}$
parallelrisch	$\frac{b(b+2d)\sin(a-2c+(b-2d)x)+2\left(\frac{b\sin(a+2c+(b+2d)x)}{2}+\sin(bx+a)(b+2d)\right)(b-2d)}{4b^{3}-16b\,d^{2}}$
risch	$\frac{\sin(bx+a)}{2b}+\frac{\sin(bx-2dx+a-2c)b}{4(b-2d)(b+2d)}+\frac{\sin(bx-2dx+a-2c)d}{2(b-2d)(b+2d)}+\frac{\sin(bx+2dx+a+2c)b}{4(b-2d)(b+2d)}-\frac{\sin(bx+2dx+a+2c)d}{2(b-2d)(b+2d)}$
norman	$\frac{-\frac{4d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^{2}-4d^{2}}+\frac{4d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{b^{2}-4d^{2}}+\frac{4d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^{2}-4d^{2}}-\frac{4d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{b^{2}-4d^{2}}+\frac{2\left(b^{2}-2d^{2}\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\left(b^{2}-4d^{2}\right)b}+2\left(b^{2}\right.}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}$
orering	$-\frac{\left(3b^{4}+16d^{4}\right)\left(-b\sin(bx+a)\cos(dx+c)^{2}-2\cos(bx+a)\cos(dx+c)d\sin(dx+c)\right)}{b^{2}\left(b^{4}-8b^{2}d^{2}+16d^{4}\right)}-\frac{\left(3b^{2}+8d^{2}\right)\left(b^{3}\sin(bx+a)\cos(dx+c)^{2}+6\right.}{}$

input
```
int(cos(b*x+a)*cos(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(b*x+a)/b+1/4/(b-2*d)*sin(a-2*c+(b-2*d)*x)+1/4/(b+2*d)*sin(a+2*c+(b
+2*d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int\cos(a+bx)\cos^{2}(c+dx)\,dx$$
$$=-\frac{2\,bd\cos\left(bx+a\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(b^{2}\cos\left(dx+c\right)^{2}-2\,d^{2}\right)\sin\left(bx+a\right)}{b^{3}-4\,bd^{2}}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(2*b*d*cos(b*x + a)*cos(d*x + c)*sin(d*x + c) - (b^2*cos(d*x + c)^2 - 2*d
^2)*sin(b*x + a))/(b^3 - 4*b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(49) = 98$.

Time = 0.70 (sec) , antiderivative size = 405, normalized size of antiderivative = 6.53

$$\int \cos(a+bx)\cos^2(c+dx)\,dx$$

$$= \begin{cases} x\cos(a)\cos^2(c) \\ \left(\frac{x\sin^2(c+dx)}{2} + \frac{x\cos^2(c+dx)}{2} + \frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\cos(a) \\ -\frac{x\sin(a-2dx)\sin(c+dx)\cos(c+dx)}{2} - \frac{x\sin^2(c+dx)\cos(a-2dx)}{4} + \frac{x\cos(a-2dx)\cos^2(c+dx)}{4} - \frac{\sin(a-2dx)\cos^2(c+dx)}{2d} - \frac{\sin(c+ \\ \frac{x\sin(a+2dx)\sin(c+dx)\cos(c+dx)}{2} - \frac{x\sin^2(c+dx)\cos(a+2dx)}{4} + \frac{x\cos(a+2dx)\cos^2(c+dx)}{4} + \frac{\sin(a+2dx)\cos^2(c+dx)}{2d} - \frac{\sin(c+dx \\ \frac{b^2\sin(a+bx)\cos^2(c+dx)}{b^3-4bd^2} - \frac{2bd\sin(c+dx)\cos(a+bx)\cos(c+dx)}{b^3-4bd^2} - \frac{2d^2\sin(a+bx)\sin^2(c+dx)}{b^3-4bd^2} - \frac{2d^2\sin(a+bx)\cos^2(c+dx)}{b^3-4bd^2} \end{cases}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)**2,x)
```

output
```
Piecewise((x*cos(a)*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*cos(a), Eq(b, 0)
), (-x*sin(a - 2*d*x)*sin(c + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)**2*cos(
a - 2*d*x)/4 + x*cos(a - 2*d*x)*cos(c + d*x)**2/4 - sin(a - 2*d*x)*cos(c +
 d*x)**2/(2*d) - sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/(4*d), Eq(b, -2*
d)), (x*sin(a + 2*d*x)*sin(c + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)**2*cos
(a + 2*d*x)/4 + x*cos(a + 2*d*x)*cos(c + d*x)**2/4 + sin(a + 2*d*x)*cos(c
+ d*x)**2/(2*d) - sin(c + d*x)*cos(a + 2*d*x)*cos(c + d*x)/(4*d), Eq(b, 2*
d)), (b**2*sin(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*sin(c +
d*x)*cos(a + b*x)*cos(c + d*x)/(b**3 - 4*b*d**2) - 2*d**2*sin(a + b*x)*sin
(c + d*x)**2/(b**3 - 4*b*d**2) - 2*d**2*sin(a + b*x)*cos(c + d*x)**2/(b**3
 - 4*b*d**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(56) = 112$.

Time = 0.06 (sec) , antiderivative size = 416, normalized size of antiderivative = 6.71

$$\int \cos(a+bx)\cos^2(c+dx)\,dx = -\frac{(b^2\sin(2c) - 2bd\sin(2c))\cos((b+2d)x+a+4c) - (b^2\sin(2c) - 2bd\sin(2c))\cos((b+2d)x+a}{}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*((b^2*sin(2*c) - 2*b*d*sin(2*c))*cos((b + 2*d)*x + a + 4*c) - (b^2*si
n(2*c) - 2*b*d*sin(2*c))*cos((b + 2*d)*x + a) - (b^2*sin(2*c) + 2*b*d*sin(
2*c))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*cos(-(
b - 2*d)*x - a) + 2*(b^2*sin(2*c) - 4*d^2*sin(2*c))*cos(b*x + a + 2*c) - 2
*(b^2*sin(2*c) - 4*d^2*sin(2*c))*cos(b*x + a - 2*c) - (b^2*cos(2*c) - 2*b*
d*cos(2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*cos(2*c) - 2*b*d*cos(2*c))*s
in((b + 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*sin(-(b - 2*d)*x - a
 + 4*c) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*sin(-(b - 2*d)*x - a) - 2*(b^2*c
os(2*c) - 4*d^2*cos(2*c))*sin(b*x + a + 2*c) - 2*(b^2*cos(2*c) - 4*d^2*cos
(2*c))*sin(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c
)^2 + b*sin(2*c)^2)*d^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \cos(a+bx)\cos^2(c+dx)\,dx = \frac{\sin(bx+2dx+a+2c)}{4(b+2d)} + \frac{\sin(bx-2dx+a-2c)}{4(b-2d)} + \frac{\sin(bx+a)}{2b}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*sin(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*sin(b*x - 2*d*x + a - 2*c)/
(b - 2*d) + 1/2*sin(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 19.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.58

$$\int \cos(a+bx)\cos^2(c+dx)\,dx = \frac{\sin(a+b\,x)}{2\,b} - \frac{d\,(2\,b\,\sin(a-2\,c+b\,x-2\,d\,x)-2\,b\,\sin(a+2\,c+b\,x+2\,d\,x))+b^2\,\sin(a-2\,c+b\,x-2\,d\,x)+b^2\,\mathrm{si}}{16\,b\,d^2-4\,b^3}$$

input
```
int(cos(a + b*x)*cos(c + d*x)^2,x)
```

output
```
sin(a + b*x)/(2*b) - (d*(2*b*sin(a - 2*c + b*x - 2*d*x) - 2*b*sin(a + 2*c
+ b*x + 2*d*x)) + b^2*sin(a - 2*c + b*x - 2*d*x) + b^2*sin(a + 2*c + b*x +
 2*d*x))/(16*b*d^2 - 4*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26

$$\int \cos(a+bx)\cos^2(c+dx)\,dx = \frac{-2\cos(bx+a)\cos(dx+c)\sin(dx+c)\,bd-\sin(bx+a)\sin(dx+c)^2\,b^2+\sin(bx+a)\,b^2-2\sin(bx+}{b\,(b^2-4d^2)}$$

input
```
int(cos(b*x+a)*cos(d*x+c)^2,x)
```

output
```
( - 2*cos(a + b*x)*cos(c + d*x)*sin(c + d*x)*b*d - sin(a + b*x)*sin(c + d*
x)**2*b**2 + sin(a + b*x)*b**2 - 2*sin(a + b*x)*d**2)/(b*(b**2 - 4*d**2))
```

3.329 $\int \cos(a+bx)\cos(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \cos(a+bx)\cos(c+dx)\,dx=\frac{\sin(a-c+(b-d)x)}{2(b-d)}+\frac{\sin(a+c+(b+d)x)}{2(b+d)}$$

output
```
sin(a-c+(b-d)*x)/(2*b-2*d)+sin(a+c+(b+d)*x)/(2*b+2*d)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\cos(c+dx)\,dx=\frac{\sin(a-c+(b-d)x)}{2(b-d)}+\frac{\sin(a+c+(b+d)x)}{2(b+d)}$$

input
```
Integrate[Cos[a + b*x]*Cos[c + d*x],x]
```

output
```
Sin[a - c + (b - d)*x]/(2*(b - d)) + Sin[a + c + (b + d)*x]/(2*(b + d))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos(c+dx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{2}\cos(a+x(b-d)-c)+\frac{1}{2}\cos(a+x(b+d)+c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}+\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

input
```
Int[Cos[a + b*x]*Cos[c + d*x],x]
```

output
```
Sin[a - c + (b - d)*x]/(2*(b - d)) + Sin[a + c + (b + d)*x]/(2*(b + d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result
default	$\frac{\sin(a-c+(b-d)x)}{2b-2d}+\frac{\sin(a+c+(b+d)x)}{2b+2d}$
parallelrisch	$\frac{(b+d)\sin(a-c+(b-d)x)+\sin(a+c+(b+d)x)(b-d)}{2b^2-2d^2}$
risch	$\frac{\sin(bx-dx+a-c)b}{2(b-d)(b+d)}+\frac{\sin(bx-dx+a-c)d}{2(b-d)(b+d)}+\frac{\sin(bx+dx+a+c)b}{2(b-d)(b+d)}-\frac{\sin(bx+dx+a+c)d}{2(b-d)(b+d)}$
norman	$\frac{\frac{2b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b^2-d^2}-\frac{2d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-d^2}-\frac{2b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{b^2-d^2}+\frac{2d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-d^2}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
orering	$-\frac{2(b^2+d^2)(-\sin(bx+a)\cos(dx+c)b-\cos(bx+a)\sin(dx+c)d)}{b^4-2b^2d^2+d^4}-\frac{3b^2\cos(bx+a)\sin(dx+c)d+\cos(dx+c)b^3\sin(bx+a)+3b\,\mathrm{si}}{b^4-2b^2d^2+}$

input
```
int(cos(b*x+a)*cos(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/(b-d)*sin(a-c+(b-d)*x)+1/2/(b+d)*sin(a+c+(b+d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int\cos(a+bx)\cos(c+dx)\,dx=\frac{b\cos(dx+c)\sin(bx+a)-d\cos(bx+a)\sin(dx+c)}{b^2-d^2}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="fricas")
```

output
```
(b*cos(d*x + c)*sin(b*x + a) - d*cos(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(32) = 64$.

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.56

$$\int \cos(a+bx)\cos(c+dx)\,dx = \begin{cases} x\cos(a)\cos(c) & \text{for } b=0 \wedge d=0 \\ -\frac{x\sin(a-dx)\sin(c+dx)}{2} + \frac{x\cos(a-dx)\cos(c+dx)}{2} - \frac{\sin(a-dx)\cos(c+dx)}{2d} & \text{for } b=-d \\ \frac{x\sin(a+dx)\sin(c+dx)}{2} + \frac{x\cos(a+dx)\cos(c+dx)}{2} + \frac{\sin(a+dx)\cos(c+dx)}{2d} & \text{for } b=d \\ \frac{b\sin(a+bx)\cos(c+dx)}{b^2-d^2} - \frac{d\sin(c+dx)\cos(a+bx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c),x)
```

output
```
Piecewise((x*cos(a)*cos(c), Eq(b, 0) & Eq(d, 0)), (-x*sin(a - d*x)*sin(c +
 d*x)/2 + x*cos(a - d*x)*cos(c + d*x)/2 - sin(a - d*x)*cos(c + d*x)/(2*d),
 Eq(b, -d)), (x*sin(a + d*x)*sin(c + d*x)/2 + x*cos(a + d*x)*cos(c + d*x)/
2 + sin(a + d*x)*cos(c + d*x)/(2*d), Eq(b, d)), (b*sin(a + b*x)*cos(c + d*
x)/(b**2 - d**2) - d*sin(c + d*x)*cos(a + b*x)/(b**2 - d**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \cos(a+bx)\cos(c+dx)\,dx = \frac{\sin(bx+dx+a+c)}{2(b+d)} - \frac{\sin(-bx+dx-a+c)}{2(b-d)}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="maxima")
```

output
```
1/2*sin(b*x + d*x + a + c)/(b + d) - 1/2*sin(-b*x + d*x - a + c)/(b - d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \cos(a+bx)\cos(c+dx)\,dx = \frac{\sin(bx+dx+a+c)}{2(b+d)} + \frac{\sin(bx-dx+a-c)}{2(b-d)}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="giac")
```

output
```
1/2*sin(b*x + d*x + a + c)/(b + d) + 1/2*sin(b*x - d*x + a - c)/(b - d)
```

Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.95

$$\int \cos(a+bx)\cos(c+dx)\,dx = \frac{b\left(\frac{\sin(a+c+bx+dx)}{2}+\frac{\sin(a-c+bx-dx)}{2}\right)}{b^2-d^2} - \frac{d\left(\frac{\sin(a+c+bx+dx)}{2}-\frac{\sin(a-c+bx-dx)}{2}\right)}{b^2-d^2}$$

input
```
int(cos(a + b*x)*cos(c + d*x),x)
```

output
```
(b*(sin(a + c + b*x + d*x)/2 + sin(a - c + b*x - d*x)/2))/(b^2 - d^2) - (d
*(sin(a + c + b*x + d*x)/2 - sin(a - c + b*x - d*x)/2))/(b^2 - d^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \cos(a+bx)\cos(c+dx)\,dx = \frac{-\cos(bx+a)\sin(dx+c)\,d+\cos(dx+c)\sin(bx+a)\,b}{b^2-d^2}$$

input
```
int(cos(b*x+a)*cos(d*x+c),x)
```

output
```
( - cos(a + b*x)*sin(c + d*x)*d + cos(c + d*x)*sin(a + b*x)*b)/(b**2 - d**
2)
```

3.330 $\int \cos(a+bx)\sec(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 142

$$\int \cos(a+bx)\sec(c+dx)\,dx = \frac{ie^{-ia-ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2d},\frac{1}{2}\left(3-\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{b-d} - \frac{ie^{ia+ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{b+d}$$

output
```
I*exp(-I*a-I*b*x+I*(d*x+c))*hypergeom([1, -1/2*(b-d)/d],[3/2-1/2*b/d],-exp
(2*I*(d*x+c)))/(b-d)-I*exp(I*a+I*b*x+I*(d*x+c))*hypergeom([1, 1/2*(b+d)/d]
,[3/2+1/2*b/d],-exp(2*I*(d*x+c)))/(b+d)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93

$$\int \cos(a+bx)\sec(c+dx)\,dx = \frac{ie^{-i(a-c+(b-d)x)}\left((b+d)\,\text{Hypergeometric2F1}\left(1,\frac{-b+d}{2d},\frac{3}{2}-\frac{b}{2d},-e^{2i(c+dx)}\right)-(b-d)e^{2i(a+bx)}\,\text{Hypergeomet}\right.}{(b-d)(b+d)}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + d*x],x]
```

output
```
(I*((b + d)*Hypergeometric2F1[1, (-b + d)/(2*d), 3/2 - b/(2*d), -E^((2*I)*
(c + d*x))] - (b - d)*E^((2*I)*(a + b*x))*Hypergeometric2F1[1, (b + d)/(2*
d), (3 + b/d)/2, -E^((2*I)*(c + d*x))]))/((b - d)*(b + d)*E^(I*(a - c + (b
 - d)*x)))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \cos(a+bx)\sec(c+dx)dx$$

input
```
Int[Cos[a + b*x]*Sec[c + d*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos (bx + a) \sec (dx + c) \, dx$$

input
```
int(cos(b*x+a)*sec(d*x+c),x)
```

output
```
int(cos(b*x+a)*sec(d*x+c),x)
```

Fricas [F]

$$\int \cos(a + bx) \sec(c + dx) \, dx = \int \cos (bx + a) \sec (dx + c) \, dx$$

input
```
integrate(cos(b*x+a)*sec(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*sec(d*x + c), x)
```

Sympy [F]

$$\int \cos(a + bx) \sec(c + dx) \, dx = \int \cos (a + bx) \sec (c + dx) \, dx$$

input
```
integrate(cos(b*x+a)*sec(d*x+c),x)
```

output
```
Integral(cos(a + b*x)*sec(c + d*x), x)
```

Maxima [F]

$$\int \cos(a+bx)\sec(c+dx)\,dx = \int \cos(bx+a)\sec(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*sec(d*x + c), x)
```

Giac [F]

$$\int \cos(a+bx)\sec(c+dx)\,dx = \int \cos(bx+a)\sec(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)*sec(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec(c+dx)\,dx = \int \frac{\cos(a+b\,x)}{\cos(c+d\,x)}\,dx$$

input
```
int(cos(a + b*x)/cos(c + d*x),x)
```

output
```
int(cos(a + b*x)/cos(c + d*x), x)
```

Reduce [F]

$$\int \cos(a+bx)\sec(c+dx)\,dx$$
$$= \frac{-4\left(\int \frac{1}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}dx\right)bd+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{bd}$$

input
```
int(cos(b*x+a)*sec(d*x+c),x)
```

output
```
( - 4*int(1/(tan((a + b*x)/2)**2*tan((c + d*x)/2)**2 - tan((a + b*x)/2)**2
 + tan((c + d*x)/2)**2 - 1),x)*b*d + log(tan((c + d*x)/2) - 1)*b - log(tan
((c + d*x)/2) + 1)*b - sin(a + b*x)*d - b*d*x)/(b*d)
```

3.331 $\int \cos(a+bx)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 139

$$\int \cos(a+bx)\sec^2(c+dx)\,dx = \frac{2ie^{-ia-ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{b}{2d},2-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b-2d} - \frac{2ie^{ia+ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{b}{2d},2+\frac{b}{2d},-e^{2i(c+dx)}\right)}{b+2d}$$

output
```
2*I*exp(-I*a-I*b*x+2*I*(d*x+c))*hypergeom([2, 1-1/2*b/d],[2-1/2*b/d],-exp(
2*I*(d*x+c)))/(b-2*d)-2*I*exp(I*a+I*b*x+2*I*(d*x+c))*hypergeom([2, 1+1/2*b
/d],[2+1/2*b/d],-exp(2*I*(d*x+c)))/(b+2*d)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.19

$$\int \cos(a+bx)\sec^2(c+dx)\,dx = \frac{e^{-i(a+bx)}\left(-i(1+e^{2ic})\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right) - ie^{2i(a+bx)}(1+e^{2ic})\,\text{Hypergeome}\right.}{d\left(1+e^{2ic}\right)}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + d*x]^2,x]
```

output
```
((-I)*(1 + E^((2*I)*c))*Hypergeometric2F1[1, -1/2*b/d, 1 - b/(2*d), -E^((2
*I)*(c + d*x))] - I*E^((2*I)*(a + b*x))*(1 + E^((2*I)*c))*Hypergeometric2F
1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))] + (1 + E^((2*I)*(a + b*x)
))*Cos[c]*Sec[c + d*x]*(I*Cos[d*x] + Sin[d*x]))/(d*E^(I*(a + b*x))*(1 + E^
((2*I)*c)))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^2(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos(a+bx)\sec^2(c+dx)dx$$

input
```
Int[Cos[a + b*x]*Sec[c + d*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos \left(b x+a\right) \sec \left(d x+c\right)^{2} d x$$

input
```
int(cos(b*x+a)*sec(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int \cos(a+bx)\sec^2(c+dx)\,dx=\int \cos \left(bx+a\right) \sec \left(dx+c\right)^{2}\,dx$$

input
```
integrate(cos(b*x+a)*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*sec(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^2(c+dx)\,dx=\text{Timed out}$$

input
```
integrate(cos(b*x+a)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(a+bx)\sec^2(c+dx)\,dx = \int \cos(bx+a)\sec(dx+c)^2\,dx$$

input
```
integrate(cos(b*x+a)*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
((d*cos((b + 2*d)*x + a + 2*c)^2 + 2*d*cos((b + 2*d)*x + a + 2*c)*cos(b*x
+ a) + d*cos(b*x + a)^2 + d*sin((b + 2*d)*x + a + 2*c)^2 + 2*d*sin((b + 2*
d)*x + a + 2*c)*sin(b*x + a) + d*sin(b*x + a)^2)*integrate((b*cos(2*b*x +
2*a)*cos(b*x + a) + b*sin((b + 2*d)*x + a + 2*c)*sin(2*b*x + 2*a) + b*sin(
2*b*x + 2*a)*sin(b*x + a) + (b*cos(2*b*x + 2*a) - b)*cos((b + 2*d)*x + a +
 2*c) - b*cos(b*x + a))/(d*cos((b + 2*d)*x + a + 2*c)^2 + 2*d*cos((b + 2*d
)*x + a + 2*c)*cos(b*x + a) + d*cos(b*x + a)^2 + d*sin((b + 2*d)*x + a + 2
*c)^2 + 2*d*sin((b + 2*d)*x + a + 2*c)*sin(b*x + a) + d*sin(b*x + a)^2), x
) + (cos(2*b*x + 2*a) + 1)*sin((b + 2*d)*x + a + 2*c) - cos((b + 2*d)*x +
a + 2*c)*sin(2*b*x + 2*a) - cos(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*
a)*sin(b*x + a) + sin(b*x + a))/(d*cos((b + 2*d)*x + a + 2*c)^2 + 2*d*cos(
(b + 2*d)*x + a + 2*c)*cos(b*x + a) + d*cos(b*x + a)^2 + d*sin((b + 2*d)*x
 + a + 2*c)^2 + 2*d*sin((b + 2*d)*x + a + 2*c)*sin(b*x + a) + d*sin(b*x +
a)^2)
```

Giac [F]

$$\int \cos(a+bx)\sec^2(c+dx)\,dx = \int \cos(bx+a)\sec(dx+c)^2\,dx$$

input
```
integrate(cos(b*x+a)*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^2(c+dx)\,dx = \int \frac{\cos(a+b\,x)}{\cos(c+d\,x)^2}\,dx$$

input
```
int(cos(a + b*x)/cos(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \cos(a+bx)\sec^2(c+dx)\,dx = \text{too large to display}$$

input
```
int(cos(b*x+a)*sec(d*x+c)^2,x)
```

output
```
(2*cos(a + b*x)*sin(c + d*x)*tan((a + b*x)/2)**2*tan((c + d*x)/2)**2*b*d**
2 - 2*cos(a + b*x)*sin(c + d*x)*tan((a + b*x)/2)**2*b*d**2 + 2*cos(a + b*x
)*sin(c + d*x)*tan((c + d*x)/2)**2*b*d**2 - 2*cos(a + b*x)*sin(c + d*x)*b*
d**2 + 8*cos(c + d*x)*int(tan((c + d*x)/2)**2/(tan((a + b*x)/2)**2*tan((c
+ d*x)/2)**4 - 2*tan((a + b*x)/2)**2*tan((c + d*x)/2)**2 + tan((a + b*x)/2
)**2 + tan((c + d*x)/2)**4 - 2*tan((c + d*x)/2)**2 + 1),x)*tan((a + b*x)/2
)**2*tan((c + d*x)/2)**2*b**3*d + 16*cos(c + d*x)*int(tan((c + d*x)/2)**2/
(tan((a + b*x)/2)**2*tan((c + d*x)/2)**4 - 2*tan((a + b*x)/2)**2*tan((c +
d*x)/2)**2 + tan((a + b*x)/2)**2 + tan((c + d*x)/2)**4 - 2*tan((c + d*x)/2
)**2 + 1),x)*tan((a + b*x)/2)**2*tan((c + d*x)/2)**2*b*d**3 - 8*cos(c + d*
x)*int(tan((c + d*x)/2)**2/(tan((a + b*x)/2)**2*tan((c + d*x)/2)**4 - 2*ta
n((a + b*x)/2)**2*tan((c + d*x)/2)**2 + tan((a + b*x)/2)**2 + tan((c + d*x
)/2)**4 - 2*tan((c + d*x)/2)**2 + 1),x)*tan((a + b*x)/2)**2*b**3*d - 16*co
s(c + d*x)*int(tan((c + d*x)/2)**2/(tan((a + b*x)/2)**2*tan((c + d*x)/2)**
4 - 2*tan((a + b*x)/2)**2*tan((c + d*x)/2)**2 + tan((a + b*x)/2)**2 + tan(
(c + d*x)/2)**4 - 2*tan((c + d*x)/2)**2 + 1),x)*tan((a + b*x)/2)**2*b*d**3
 + 8*cos(c + d*x)*int(tan((c + d*x)/2)**2/(tan((a + b*x)/2)**2*tan((c + d*
x)/2)**4 - 2*tan((a + b*x)/2)**2*tan((c + d*x)/2)**2 + tan((a + b*x)/2)**2
 + tan((c + d*x)/2)**4 - 2*tan((c + d*x)/2)**2 + 1),x)*tan((c + d*x)/2)**2
*b**3*d + 16*cos(c + d*x)*int(tan((c + d*x)/2)**2/(tan((a + b*x)/2)**2*...
```

3.332 $\int \cos(a+bx)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 145

$$\int \cos(a+bx)\sec^3(c+dx)\,dx = \frac{4ie^{-ia-ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{b}{d}\right),\frac{1}{2}\left(5-\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{b-3d} - \frac{4ie^{ia+ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{b}{d}\right),\frac{1}{2}\left(5+\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{b+3d}$$

output
```
4*I*exp(-I*a-I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2-1/2*b/d],[5/2-1/2*b/d],-
exp(2*I*(d*x+c)))/(b-3*d)-4*I*exp(I*a+I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2
+1/2*b/d],[5/2+1/2*b/d],-exp(2*I*(d*x+c)))/(b+3*d)
```

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.19

$$\int \cos(a+bx)\sec^3(c+dx)\,dx = \frac{-2i(b+d)e^{-i(a-c+(b-d)x)}\,\text{Hypergeometric2F1}\left(1,\frac{-b+d}{2d},\frac{3}{2}-\frac{b}{2d},-e^{2i(c+dx)}\right)+2i(b-d)e^{i(a+c+(b+d)x)}\,\text{Hype}}{}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + d*x]^3,x]
```

output
```
(((-2*I)*(b + d)*Hypergeometric2F1[1, (-b + d)/(2*d), 3/2 - b/(2*d), -E^((
2*I)*(c + d*x))])/E^(I*(a - c + (b - d)*x)) + (2*I)*(b - d)*E^(I*(a + c +
(b + d)*x))*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, -E^((2*I)*(c
+ d*x))] + Sec[c + d*x]^2*((b - d)*Sin[a - c + b*x - d*x] + (b + d)*Sin[a
+ c + (b + d)*x]))/(4*d^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^3(c+dx)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \cos(a+bx)\sec^3(c+dx)dx$$

input
```
Int[Cos[a + b*x]*Sec[c + d*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos (bx+a) \sec (dx+c)^3 \, dx$$

input
```
int(cos(b*x+a)*sec(d*x+c)^3,x)
```

output
```
int(cos(b*x+a)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int \cos(a+bx) \sec^3(c+dx) \, dx = \int \cos (bx+a) \sec (dx+c)^3 \, dx$$

input
```
integrate(cos(b*x+a)*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*sec(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx) \sec^3(c+dx) \, dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(a+bx)\sec^3(c+dx)\,dx = \int \cos(bx+a)\sec(dx+c)^3\,dx$$

input
```
integrate(cos(b*x+a)*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*((b - d)*cos(b*x + a)*sin((2*b + d)*x + 2*a + c) - (b - d)*cos((2*b +
d)*x + 2*a + c)*sin(b*x + a) + (b - d)*cos(3*d*x + 3*c)*sin(b*x + a) + (b
+ d)*cos(d*x + c)*sin(b*x + a) - (b - d)*cos(b*x + a)*sin(3*d*x + 3*c) - (
b + d)*cos(b*x + a)*sin(d*x + c) - (2*(b + d)*sin((b + 2*d)*x + a + 2*c) +
 (b + d)*sin(b*x + a))*cos((2*b + 3*d)*x + 2*a + 3*c) + ((b + d)*sin((2*b
+ 3*d)*x + 2*a + 3*c) + (b - d)*sin((2*b + d)*x + 2*a + c) - (b - d)*sin(3
*d*x + 3*c) - (b + d)*sin(d*x + c))*cos((b + 4*d)*x + a + 4*c) + 2*((b - d
)*sin((2*b + d)*x + 2*a + c) - (b - d)*sin(3*d*x + 3*c) - (b + d)*sin(d*x
+ c))*cos((b + 2*d)*x + a + 2*c) + 2*(d^2*cos((b + 4*d)*x + a + 4*c)^2 + 4
*d^2*cos((b + 2*d)*x + a + 2*c)^2 + 4*d^2*cos((b + 2*d)*x + a + 2*c)*cos(b
*x + a) + d^2*cos(b*x + a)^2 + d^2*sin((b + 4*d)*x + a + 4*c)^2 + 4*d^2*si
n((b + 2*d)*x + a + 2*c)^2 + 4*d^2*sin((b + 2*d)*x + a + 2*c)*sin(b*x + a)
 + d^2*sin(b*x + a)^2 + 2*(2*d^2*cos((b + 2*d)*x + a + 2*c) + d^2*cos(b*x
+ a))*cos((b + 4*d)*x + a + 4*c) + 2*(2*d^2*sin((b + 2*d)*x + a + 2*c) + d
^2*sin(b*x + a))*sin((b + 4*d)*x + a + 4*c))*integrate(-1/2*((b^2 - d^2)*c
os((2*b + d)*x + 2*a + c)*cos(b*x + a) + (b^2 - d^2)*cos(b*x + a)*cos(d*x
+ c) + (b^2 - d^2)*sin((2*b + d)*x + 2*a + c)*sin(b*x + a) + (b^2 - d^2)*s
in(b*x + a)*sin(d*x + c) + ((b^2 - d^2)*cos((2*b + d)*x + 2*a + c) + (b^2
- d^2)*cos(d*x + c))*cos((b + 2*d)*x + a + 2*c) + ((b^2 - d^2)*sin((2*b +
d)*x + 2*a + c) + (b^2 - d^2)*sin(d*x + c))*sin((b + 2*d)*x + a + 2*c))...
```

Giac [F]

$$\int \cos(a+bx)\sec^3(c+dx)\,dx = \int \cos(bx+a)\sec(dx+c)^3\,dx$$

input
```
integrate(cos(b*x+a)*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^3(c+dx)\,dx = \int \frac{\cos(a+b\,x)}{\cos(c+d\,x)^3}\,dx$$

input
```
int(cos(a + b*x)/cos(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \cos(a+bx)\sec^3(c+dx)\,dx = \text{too large to display}$$

input
```
int(cos(b*x+a)*sec(d*x+c)^3,x)
```

output
```
( - 12*cos(a + b*x)*cos(c + d*x)*sin(c + d*x)*b**3*d**2 + 12*cos(a + b*x)*
cos(c + d*x)*sin(c + d*x)*b*d**4 - 18*cos(a + b*x)*sin(c + d*x)*b**3*d**2
- 12*cos(a + b*x)*sin(c + d*x)*b*d**4 + 6*cos(c + d*x)*sin(a + b*x)*b**4*d
 - 36*cos(c + d*x)*sin(a + b*x)*b**2*d**3 + 6*cos(c + d*x)*sin(c + d*x)*b*
*5 - 18*cos(c + d*x)*sin(c + d*x)*b**3*d**2 + 12*cos(c + d*x)*sin(c + d*x)
*b*d**4 - 192*int((tan((a + b*x)/2)*tan((c + d*x)/2))/(tan((a + b*x)/2)**2
*tan((c + d*x)/2)**6*b**4 + 15*tan((a + b*x)/2)**2*tan((c + d*x)/2)**6*b**
2*d**2 + 14*tan((a + b*x)/2)**2*tan((c + d*x)/2)**6*d**4 - 3*tan((a + b*x)
/2)**2*tan((c + d*x)/2)**4*b**4 - 45*tan((a + b*x)/2)**2*tan((c + d*x)/2)*
*4*b**2*d**2 - 42*tan((a + b*x)/2)**2*tan((c + d*x)/2)**4*d**4 + 3*tan((a
+ b*x)/2)**2*tan((c + d*x)/2)**2*b**4 + 45*tan((a + b*x)/2)**2*tan((c + d*
x)/2)**2*b**2*d**2 + 42*tan((a + b*x)/2)**2*tan((c + d*x)/2)**2*d**4 - tan
((a + b*x)/2)**2*b**4 - 15*tan((a + b*x)/2)**2*b**2*d**2 - 14*tan((a + b*x
)/2)**2*d**4 + tan((c + d*x)/2)**6*b**4 + 15*tan((c + d*x)/2)**6*b**2*d**2
 + 14*tan((c + d*x)/2)**6*d**4 - 3*tan((c + d*x)/2)**4*b**4 - 45*tan((c +
d*x)/2)**4*b**2*d**2 - 42*tan((c + d*x)/2)**4*d**4 + 3*tan((c + d*x)/2)**2
*b**4 + 45*tan((c + d*x)/2)**2*b**2*d**2 + 42*tan((c + d*x)/2)**2*d**4 - b
**4 - 15*b**2*d**2 - 14*d**4),x)*sin(c + d*x)**2*b**8*d**2 - 2688*int((tan
((a + b*x)/2)*tan((c + d*x)/2))/(tan((a + b*x)/2)**2*tan((c + d*x)/2)**6*b
**4 + 15*tan((a + b*x)/2)**2*tan((c + d*x)/2)**6*b**2*d**2 + 14*tan((a ...
```

3.333 $\int \cos^2(a+bx)\cos^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx = \frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)} + \frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)} + \frac{3\sin(c+dx)}{8d} + \frac{\sin(3c+3dx)}{24d} + \frac{3\sin(2a+c+(2b+d)x)}{16(2b+d)} + \frac{\sin(2a+3c+(2b+3d)x)}{16(2b+3d)}$$

output
```
sin(2*a-3*c+(2*b-3*d)*x)/(32*b-48*d)+3*sin(2*a-c+(2*b-d)*x)/(32*b-16*d)+3/
8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d+3*sin(2*a+c+(2*b+d)*x)/(32*b+16*d)+si
n(2*a+3*c+(2*b+3*d)*x)/(32*b+48*d)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx = \frac{1}{48}\left(\frac{18\cos(dx)\sin(c)}{d}+\frac{2\cos(3dx)\sin(3c)}{d}+\frac{18\cos(c)\sin(dx)}{d}+\frac{2\cos(3c)\sin(3dx)}{d}+\frac{3\sin(2a-3c+2bx-3dx)}{2b-3d}+\frac{9\sin(2a-c+2bx-dx)}{2b-d}+\frac{9\sin(2a+c+2bx+dx)}{2b+d}+\frac{3\sin(2a+3c+2bx+3dx)}{2b+3d}\right)$$

input
```
Integrate[Cos[a + b*x]^2*Cos[c + d*x]^3,x]
```

output
```
((18*Cos[d*x]*Sin[c])/d + (2*Cos[3*d*x]*Sin[3*c])/d + (18*Cos[c]*Sin[d*x])
/d + (2*Cos[3*c]*Sin[3*d*x])/d + (3*Sin[2*a - 3*c + 2*b*x - 3*d*x])/(2*b -
 3*d) + (9*Sin[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*Sin[2*a + c + 2*b*x
+ d*x])/(2*b + d) + (3*Sin[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d))/48
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \text{ 5081}$$

$$\int \left(\frac{1}{16}\cos(2a+x(2b-3d)-3c)+\frac{3}{16}\cos(2a+x(2b-d)-c)+\frac{3}{16}\cos(2a+x(2b+d)+c)+\frac{1}{16}\cos(2a+x(2$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sin(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sin(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sin(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3\sin(c+dx)}{8d}+\frac{\sin(3c+3dx)}{24d}$$

input
```
Int[Cos[a + b*x]^2*Cos[c + d*x]^3,x]
```

output
```
Sin[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sin[2*a - c + (2*b -
d)*x])/(16*(2*b - d)) + (3*Sin[c + d*x])/(8*d) + Sin[3*c + 3*d*x]/(24*d) +
 (3*Sin[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sin[2*a + 3*c + (2*b + 3*
d)*x]/(16*(2*b + 3*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 12.38 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

method	result
default	$\frac{3\sin(dx+c)}{8d}+\frac{\sin(3dx+3c)}{24d}+\frac{\sin(2a-3c+(2b-3d)x)}{32b-48d}+\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(2a+c+(2b+d)x)}{16(2b+d)}+\frac{\sin(2a+3c}{32b}$
parallelrisch	$\frac{(24d\,b^3+36b^2d^2-6d^3b-9d^4)\sin(2a-3c+(2b-3d)x)+72\left(d\left(b+\frac{3d}{2}\right)\left(b+\frac{d}{2}\right)\sin(2a-c+(2b-d)x)+\left(b-\frac{d}{2}\right)\left(\frac{d\left(b+\frac{d}{2}\right)\sin(2a+3c}{3}\right.\right.}{768b^4d-1920b^2d^3+432d^5}$
risch	$\frac{3\sin(dx+c)b^2}{2d(2b-d)(2b+d)}-\frac{3d\sin(dx+c)}{8(2b-d)(2b+d)}+\frac{\sin(2bx-3dx+2a-3c)b}{8(2b-3d)(2b+3d)}+\frac{3d\sin(2bx-3dx+2a-3c)}{16(2b-3d)(2b+3d)}+\frac{3\sin(2bx-dx+2a-c)b}{8(2b-d)(2b+d)}+$
orering	Expression too large to display

input
```
int(cos(b*x+a)^2*cos(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
3/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d+1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-
3*d)+3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)+3/16/(2*b+d)*sin(2*a+c+(2*b+d)*x)+1
/16*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.13

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx =$$

$$-\frac{6\left(6\,bd^3\cos\left(bx+a\right)\cos\left(dx+c\right)-\left(4\,b^3d-bd^3\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^3\right)\sin\left(bx+a\right)-\left(18\,d^4\cos\left(b\right.\right.}{3\left(16\,b^4d-40\right.}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/3*(6*(6*b*d^3*cos(b*x + a)*cos(d*x + c) - (4*b^3*d - b*d^3)*cos(b*x + a
)*cos(d*x + c)^3)*sin(b*x + a) - (18*d^4*cos(b*x + a)^2 + 16*b^4 - 40*b^2*
d^2 + (8*b^4 - 2*b^2*d^2 - 9*(4*b^2*d^2 - d^4)*cos(b*x + a)^2)*cos(d*x + c
)^2)*sin(d*x + c))/(16*b^4*d - 40*b^2*d^3 + 9*d^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2004 vs. 2(116) = 232.

Time = 5.60 (sec) , antiderivative size = 2004, normalized size of antiderivative = 13.92

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**2*cos(d*x+c)**3,x)
```

output
```
Piecewise((x*cos(a)**2*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - 3*d*x
/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 - x*sin(a - 3*d*x/2)**2*cos(c + d*x
)**3/16 + x*sin(a - 3*d*x/2)*sin(c + d*x)**3*cos(a - 3*d*x/2)/8 - 3*x*sin(
a - 3*d*x/2)*sin(c + d*x)*cos(a - 3*d*x/2)*cos(c + d*x)**2/8 - 3*x*sin(c +
 d*x)**2*cos(a - 3*d*x/2)**2*cos(c + d*x)/16 + x*cos(a - 3*d*x/2)**2*cos(c
 + d*x)**3/16 + 11*sin(a - 3*d*x/2)**2*sin(c + d*x)**3/(48*d) + sin(a - 3*
d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*sin(a - 3*d*x/2)*sin(c + d*x)
**2*cos(a - 3*d*x/2)*cos(c + d*x)/(4*d) - 5*sin(a - 3*d*x/2)*cos(a - 3*d*x
/2)*cos(c + d*x)**3/(8*d) + 7*sin(c + d*x)**3*cos(a - 3*d*x/2)**2/(16*d),
Eq(b, -3*d/2)), (-3*x*sin(a - d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 -
3*x*sin(a - d*x/2)**2*cos(c + d*x)**3/16 - 3*x*sin(a - d*x/2)*sin(c + d*x)
**3*cos(a - d*x/2)/8 - 3*x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)*cos(
c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a - d*x/2)**2*cos(c + d*x)/16 + 3*
x*cos(a - d*x/2)**2*cos(c + d*x)**3/16 + 49*sin(a - d*x/2)**2*sin(c + d*x)
**3/(48*d) + sin(a - d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d - 7*sin(a -
d*x/2)*sin(c + d*x)**2*cos(a - d*x/2)*cos(c + d*x)/(4*d) - 13*sin(a - d*x/
2)*cos(a - d*x/2)*cos(c + d*x)**3/(8*d) - 17*sin(c + d*x)**3*cos(a - d*x/2
)**2/(48*d), Eq(b, -d/2)), (-3*x*sin(a + d*x/2)**2*sin(c + d*x)**2*cos(c +
 d*x)/16 - 3*x*sin(a + d*x/2)**2*cos(c + d*x)**3/16 + 3*x*sin(a + d*x/2)*s
in(c + d*x)**3*cos(a + d*x/2)/8 + 3*x*sin(a + d*x/2)*sin(c + d*x)*cos(a...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(132) = 264$.

Time = 0.11 (sec) , antiderivative size = 1362, normalized size of antiderivative = 9.46

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/96*(3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^
4*sin(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*
d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*cos((2*b + 3*d)*x + 2*a)
 + 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*si
n(3*c))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin
(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c))*cos((2*b + d)*x + 2*a - 2*c) -
 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(
3*c))*cos(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(
3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) -
 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(
3*c))*cos(-(2*b - 3*d)*x - 2*a + 6*c) + 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*s
in(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(-(2*b - 3*d)*x - 2*a) - 2
*(16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x) + 2*(
16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x + 6*c) +
 18*(16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x + 4*c
) - 18*(16*b^4*sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x -
2*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^
4*cos(3*c))*sin((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*
d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((2*b + 3*d)*x + 2*a)
 - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx = \frac{\sin(2\,bx+3\,dx+2\,a+3\,c)}{16\,(2\,b+3\,d)} + \frac{3\,\sin(2\,bx+dx+2\,a+c)}{16\,(2\,b+d)} + \frac{3\,\sin(2\,bx-dx+2\,a-c)}{16\,(2\,b-d)} + \frac{\sin(2\,bx-3\,dx+2\,a-3\,c)}{16\,(2\,b-3\,d)} + \frac{\sin(3\,dx+3\,c)}{24\,d} + \frac{3\,\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="giac")
```

output
```
1/16*sin(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/16*sin(2*b*x + d*x + 2
*a + c)/(2*b + d) + 3/16*sin(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/16*sin(2
*b*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*sin(3*d*x + 3*c)/d + 3/8*sin(
d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 21.04 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.44

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx = -e^{a\,2i-c\,1i+b\,x\,2i-d\,x\,1i}\left(\frac{e^{-a\,2i-b\,x\,2i}\,(24\,b^2-6\,d^2)}{b^2\,d\,128i-d^3\,32i} - \frac{3\,d\,(2\,b+d)}{b^2\,d\,128i-d^3\,32i} + \frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b-d)}{b^2\,d\,128i-d^3\,32i}\right) + e^{a\,2i+c\,1i+b\,x\,2i+d\,x\,1i}\left(\frac{3\,d\,(2\,b-d)}{b^2\,d\,128i-d^3\,32i} + \frac{e^{-a\,2i-b\,x\,2i}\,(24\,b^2-6\,d^2)}{b^2\,d\,128i-d^3\,32i} - \frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b+d)}{b^2\,d\,128i-d^3\,32i}\right) - e^{a\,2i-c\,3i+b\,x\,2i-d\,x\,3i}\left(-\frac{3\,d\,(2\,b+3\,d)}{b^2\,d\,384i-d^3\,864i} + \frac{e^{-a\,2i-b\,x\,2i}\,(8\,b^2-18\,d^2)}{b^2\,d\,384i-d^3\,864i} + \frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b-3\,d)}{b^2\,d\,384i-d^3\,864i}\right) + e^{a\,2i+c\,3i+b\,x\,2i+d\,x\,3i}\left(\frac{3\,d\,(2\,b-3\,d)}{b^2\,d\,384i-d^3\,864i} + \frac{e^{-a\,2i-b\,x\,2i}\,(8\,b^2-18\,d^2)}{b^2\,d\,384i-d^3\,864i} - \frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b+3\,d)}{b^2\,d\,384i-d^3\,864i}\right)$$

input
```
int(cos(a + b*x)^2*cos(c + d*x)^3,x)
```

output
```
exp(a*2i + c*1i + b*x*2i + d*x*1i)*((3*d*(2*b - d))/(b^2*d*128i - d^3*32i)
 + (exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(b^2*d*128i - d^3*32i) - (3*d*e
xp(- a*4i - b*x*4i)*(2*b + d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*1i +
 b*x*2i - d*x*1i)*((exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(b^2*d*128i - d
^3*32i) - (3*d*(2*b + d))/(b^2*d*128i - d^3*32i) + (3*d*exp(- a*4i - b*x*4
i)*(2*b - d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*3i + b*x*2i - d*x*3i)
*((exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(b^2*d*384i - d^3*864i) - (3*d*(
2*b + 3*d))/(b^2*d*384i - d^3*864i) + (3*d*exp(- a*4i - b*x*4i)*(2*b - 3*d
))/(b^2*d*384i - d^3*864i)) + exp(a*2i + c*3i + b*x*2i + d*x*3i)*((3*d*(2*
b - 3*d))/(b^2*d*384i - d^3*864i) + (exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2)
)/(b^2*d*384i - d^3*864i) - (3*d*exp(- a*4i - b*x*4i)*(2*b + 3*d))/(b^2*d*
384i - d^3*864i))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.10

$$\int \cos^2(a+bx)\cos^3(c+dx)\,dx$$
$$= \frac{-24\cos(bx+a)\cos(dx+c)\sin(bx+a)\sin(dx+c)^2 b^3d + 6\cos(bx+a)\cos(dx+c)\sin(bx+a)\sin(c}{}$$

input
```
int(cos(b*x+a)^2*cos(d*x+c)^3,x)
```

output
```
( - 24*cos(a + b*x)*cos(c + d*x)*sin(a + b*x)*sin(c + d*x)**2*b**3*d + 6*c
os(a + b*x)*cos(c + d*x)*sin(a + b*x)*sin(c + d*x)**2*b*d**3 + 24*cos(a +
b*x)*cos(c + d*x)*sin(a + b*x)*b**3*d - 42*cos(a + b*x)*cos(c + d*x)*sin(a
 + b*x)*b*d**3 - 36*sin(a + b*x)**2*sin(c + d*x)**3*b**2*d**2 + 9*sin(a +
b*x)**2*sin(c + d*x)**3*d**4 + 36*sin(a + b*x)**2*sin(c + d*x)*b**2*d**2 -
 27*sin(a + b*x)**2*sin(c + d*x)*d**4 - 8*sin(c + d*x)**3*b**4 + 38*sin(c
+ d*x)**3*b**2*d**2 - 9*sin(c + d*x)**3*d**4 + 24*sin(c + d*x)*b**4 - 78*s
in(c + d*x)*b**2*d**2 + 27*sin(c + d*x)*d**4)/(3*d*(16*b**4 - 40*b**2*d**2
 + 9*d**4))
```

3.334 $\int \cos^2(a+bx)\cos^2(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx = \frac{x}{4}+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)} + \frac{\sin(2c+2dx)}{8d}+\frac{\sin(2(a+c)+2(b+d)x)}{16(b+d)}$$

output
```
1/4*x+1/8*sin(2*b*x+2*a)/b+sin(2*a-2*c+2*(b-d)*x)/(16*b-16*d)+1/8*sin(2*d*
x+2*c)/d+sin(2*a+2*c+2*(b+d)*x)/(16*b+16*d)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx = \frac{2d(b^2-d^2)\sin(2(a+bx))+bd(b+d)\sin(2(a-c+(b-d)x))+b(b-d)(2(b+d)\sin(2(c+dx))+d(4}{16b(b-d)d(b+d)}$$

input
```
Integrate[Cos[a + b*x]^2*Cos[c + d*x]^2,x]
```

output
```
(2*d*(b^2 - d^2)*Sin[2*(a + b*x)] + b*d*(b + d)*Sin[2*(a - c + (b - d)*x)]
 + b*(b - d)*(2*(b + d)*Sin[2*(c + d*x)] + d*(4*(b + d)*x + Sin[2*(a + c +
 (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{8}\cos(2(a-c)+2x(b-d))+\frac{1}{8}\cos(2(a+c)+2x(b+d))+\frac{1}{4}\cos(2a+2bx)+\frac{1}{4}\cos(2c+2dx)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

input
```
Int[Cos[a + b*x]^2*Cos[c + d*x]^2,x]
```

output
```
x/4 + Sin[2*a + 2*b*x]/(8*b) + Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) +
 Sin[2*c + 2*d*x]/(8*d) + Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
default	$\frac{x}{4}+\frac{\sin(2bx+2a)}{8b}+\frac{\sin(2dx+2c)}{8d}+\frac{\sin((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sin((2b+2d)x+2a+2c)}{16b+16d}$
parallelrisch	$\frac{bd(b+d)\sin((2b-2d)x+2a-2c)+4(b-d)\left(\frac{bd\sin((2b+2d)x+2a+2c)}{4}+\left(\frac{d\sin(2bx+2a)}{2}+b\left(dx+\frac{\sin(2dx+2c)}{2}\right)\right)(b+d)\right)}{16d\,b^3-16d^3b}$
risch	$\frac{x}{4}+\frac{\sin(2bx+2a)}{8b}+\frac{\sin(2dx+2c)b^2}{8(b-d)d(b+d)}-\frac{d\sin(2dx+2c)}{8(b-d)(b+d)}+\frac{\sin(2bx-2dx+2a-2c)b}{16(b-d)(b+d)}+\frac{d\sin(2bx-2dx+2a-2c)}{16(b-d)(b+d)}+\frac{\sin(2b}{16}$
orering	Expression too large to display

input
```
int(cos(b*x+a)^2*cos(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x+1/8*sin(2*b*x+2*a)/b+1/8*sin(2*d*x+2*c)/d+1/8/(2*b-2*d)*sin((2*b-2*d
)*x+2*a-2*c)+1/8/(2*b+2*d)*sin((2*b+2*d)*x+2*a+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx =$$
$$-\frac{\left(2\,bd^2\cos\left(bx+a\right)^2-b^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-(b^3d-bd^3)x-\left(2\,b^2d\cos\left(bx+a\right)\cos\left(dx+c\right)^2-\right.}{4\left(b^3d-bd^3\right)}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/4*((2*b*d^2*cos(b*x + a)^2 - b^3)*cos(d*x + c)*sin(d*x + c) - (b^3*d -
b*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - d^3*cos(b*x + a))*sin(b*
x + a))/(b^3*d - b*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. 2(76) = 152.

Time = 1.64 (sec) , antiderivative size = 1027, normalized size of antiderivative = 11.67

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**2*cos(d*x+c)**2,x)
```

output
```
Piecewise((x*cos(a)**2*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**
2/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*cos(a)**2, Eq
(b, 0)), (3*x*sin(a - d*x)**2*sin(c + d*x)**2/8 + x*sin(a - d*x)**2*cos(c
+ d*x)**2/8 - x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + x*
sin(c + d*x)**2*cos(a - d*x)**2/8 + 3*x*cos(a - d*x)**2*cos(c + d*x)**2/8
+ sin(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(2*d) + sin(a - d*x)*sin(c + d
*x)**2*cos(a - d*x)/(8*d) - 5*sin(a - d*x)*cos(a - d*x)*cos(c + d*x)**2/(8
*d), Eq(b, -d)), (3*x*sin(a + d*x)**2*sin(c + d*x)**2/8 + x*sin(a + d*x)**
2*cos(c + d*x)**2/8 + x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x
)/2 + x*sin(c + d*x)**2*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)**2*cos(c + d*
x)**2/8 + 3*sin(a + d*x)*sin(c + d*x)**2*cos(a + d*x)/(8*d) + sin(a + d*x)
*cos(a + d*x)*cos(c + d*x)**2/(8*d) + sin(c + d*x)*cos(a + d*x)**2*cos(c +
 d*x)/(2*d), Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(
a + b*x)*cos(a + b*x)/(2*b))*cos(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)*
*2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c
+ d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2
/(4*b**3*d - 4*b*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*
d - 4*b*d**3) + b**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d -
 4*b*d**3) + b**3*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*
b*d**3) + 2*b**2*d*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**3*d ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(78) = 156$.

Time = 0.06 (sec) , antiderivative size = 620, normalized size of antiderivative = 7.05

$$\int \cos^2(a + bx)\cos^2(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c
)^2)*d)*x + (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c)
 - (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) - (b^2*d*sin(2
*c) + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) + (b^2*d*sin(2*c) + b*
d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*
cos(2*b*x + 2*a + 2*c) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a
 - 2*c) - 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) + 2*(b^3*sin(2*c) -
 b*d^2*sin(2*c))*cos(2*d*x + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(
2*(b + d)*x + 2*a + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)
*x + 2*a) + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c
) + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) - 2*(b^2*d*c
os(2*c) - d^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) - 2*(b^2*d*cos(2*c) - d^3*c
os(2*c))*sin(2*b*x + 2*a - 2*c) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*
d*x) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2
+ b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \cos^2(a + bx)\cos^2(c + dx)\,dx = {} & \frac{1}{4}\,x + \frac{\sin\left(2\,bx + 2\,dx + 2\,a + 2\,c\right)}{16\,(b + d)} \\ & + \frac{\sin\left(2\,bx - 2\,dx + 2\,a - 2\,c\right)}{16\,(b - d)} \\ & + \frac{\sin\left(2\,bx + 2\,a\right)}{8\,b} + \frac{\sin\left(2\,dx + 2\,c\right)}{8\,d}\end{aligned}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*x + 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/16*sin(2*b*x - 2*d
*x + 2*a - 2*c)/(b - d) + 1/8*sin(2*b*x + 2*a)/b + 1/8*sin(2*d*x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 20.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.01

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx = \frac{2\,b^3\,\sin\left(2\,c+2\,d\,x\right)-2\,d^3\,\sin\left(2\,a+2\,b\,x\right)+b\,d^2\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)-b\,d^2\,\sin\left(2\,a+2\,c+2\,b$$

input
```
int(cos(a + b*x)^2*cos(c + d*x)^2,x)
```

output
```
(2*b^3*sin(2*c + 2*d*x) - 2*d^3*sin(2*a + 2*b*x) + b*d^2*sin(2*a - 2*c + 2
*b*x - 2*d*x) - b*d^2*sin(2*a + 2*c + 2*b*x + 2*d*x) + b^2*d*sin(2*a - 2*c
 + 2*b*x - 2*d*x) + b^2*d*sin(2*a + 2*c + 2*b*x + 2*d*x) + 2*b^2*d*sin(2*a
 + 2*b*x) - 2*b*d^2*sin(2*c + 2*d*x) - 4*b*d^3*x + 4*b^3*d*x)/(16*b*d*(b^2
 - d^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.75

$$\int \cos^2(a+bx)\cos^2(c+dx)\,dx = \frac{-2\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(dx+c\right)^2b^2d+2\cos\left(bx+a\right)\sin\left(bx+a\right)b^2d-\cos\left(bx+a\right)\sin\left(bx+a\right)$$

input
```
int(cos(b*x+a)^2*cos(d*x+c)^2,x)
```

output
```
( - 2*cos(a + b*x)*sin(a + b*x)*sin(c + d*x)**2*b**2*d + 2*cos(a + b*x)*si
n(a + b*x)*b**2*d - cos(a + b*x)*sin(a + b*x)*d**3 + 2*cos(c + d*x)*sin(a
+ b*x)**2*sin(c + d*x)*b*d**2 + cos(c + d*x)*sin(c + d*x)*b**3 - 2*cos(c +
 d*x)*sin(c + d*x)*b*d**2 + b**3*d*x - b*d**3*x)/(4*b*d*(b**2 - d**2))
```

3.335 $\int \cos^2(a+bx)\cos(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \cos^2(a+bx)\cos(c+dx)\,dx = \frac{\sin(2a-c+(2b-d)x)}{4(2b-d)} + \frac{\sin(c+dx)}{2d} + \frac{\sin(2a+c+(2b+d)x)}{4(2b+d)}$$

output
```
sin(2*a-c+(2*b-d)*x)/(8*b-4*d)+1/2*sin(d*x+c)/d+sin(2*a+c+(2*b+d)*x)/(8*b+
4*d)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int \cos^2(a+bx)\cos(c+dx)\,dx = \frac{1}{4}\left(\frac{2\cos(dx)\sin(c)}{d} + \frac{2\cos(c)\sin(dx)}{d} + \frac{\sin(2a-c+2bx-dx)}{2b-d} + \frac{\sin(2a+c+2bx+dx)}{2b+d}\right)$$

input
```
Integrate[Cos[a + b*x]^2*Cos[c + d*x],x]
```

output
```
((2*Cos[d*x]*Sin[c])/d + (2*Cos[c]*Sin[d*x])/d + Sin[2*a - c + 2*b*x - d*x
]/(2*b - d) + Sin[2*a + c + 2*b*x + d*x]/(2*b + d))/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cos(c+dx)\,dx$$

$$\Big\downarrow \text{ 5081}$$

$$\int \left(\frac{1}{4}\cos(2a+x(2b-d)-c)+\frac{1}{4}\cos(2a+x(2b+d)+c)+\frac{1}{2}\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sin(2a+x(2b-d)-c)}{4(2b-d)}+\frac{\sin(2a+x(2b+d)+c)}{4(2b+d)}+\frac{\sin(c+dx)}{2d}$$

input
```
Int[Cos[a + b*x]^2*Cos[c + d*x],x]
```

output
```
Sin[2*a - c + (2*b - d)*x]/(4*(2*b - d)) + Sin[c + d*x]/(2*d) + Sin[2*a +
c + (2*b + d)*x]/(4*(2*b + d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

method	result
default	$\frac{\sin(2a-c+(2b-d)x)}{8b-4d}+\frac{\sin(dx+c)}{2d}+\frac{\sin(2a+c+(2b+d)x)}{8b+4d}$
parallelrisch	$\frac{(2bd+d^2)\sin(2a-c+(2b-d)x)+(2bd-d^2)\sin(2a+c+(2b+d)x)+(8b^2-2d^2)\sin(dx+c)}{16b^2d-4d^3}$
risch	$\frac{2\sin(dx+c)b^2}{d(2b-d)(2b+d)}-\frac{d\sin(dx+c)}{2(2b-d)(2b+d)}+\frac{\sin(2bx-dx+2a-c)b}{2(2b-d)(2b+d)}+\frac{d\sin(2bx-dx+2a-c)}{4(2b-d)(2b+d)}+\frac{\sin(2bx+dx+2a+c)b}{2(2b-d)(2b+d)}-\frac{d\sin(2}{4(2}$
norman	$\frac{\frac{4b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{4b^2-d^2}-\frac{4b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{4b^2-d^2}-\frac{4b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4b^2-d^2}+\frac{4b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4b^2-d^2}+\frac{2(2b^2-d^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d(4b^2-d^2)}+\frac{2(2b^2-}{}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
orering	$-\frac{(16b^4+3d^4)\left(-2\cos(bx+a)\cos(dx+c)b\sin(bx+a)-\cos(bx+a)^2d\sin(dx+c)\right)}{d^2(16b^4-8b^2d^2+d^4)}-\frac{(8b^2+3d^2)\left(8b^3\sin(bx+a)\cos(dx+c)\cos\right.}{}$

input
```
int(cos(b*x+a)^2*cos(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(d*x+c)/d+1/4*sin(2*a-c+(2*b-d)*x)/(2*b-d)+1/4/(2*b+d)*sin(2*a+c+(2
*b+d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \cos^2(a+bx)\cos(c+dx)\,dx = \frac{2\,bd\cos(bx+a)\cos(dx+c)\sin(bx+a)-\left(d^2\cos(bx+a)^2-2\,b^2\right)\sin(dx+c)}{4\,b^2d-d^3}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c),x, algorithm="fricas")
```

output
```
(2*b*d*cos(b*x + a)*cos(d*x + c)*sin(b*x + a) - (d^2*cos(b*x + a)^2 - 2*b^
2)*sin(d*x + c))/(4*b^2*d - d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(49) = 98$.

Time = 0.75 (sec) , antiderivative size = 410, normalized size of antiderivative = 6.03

$$\int \cos^2(a+bx)\cos(c+dx)\,dx$$

$$= \begin{cases} x\cos^2(a)\cos(c) \\ -\frac{x\sin^2\left(a-\frac{dx}{2}\right)\cos(c+dx)}{4} - \frac{x\sin\left(a-\frac{dx}{2}\right)\sin(c+dx)\cos\left(a-\frac{dx}{2}\right)}{2} + \frac{x\cos^2\left(a-\frac{dx}{2}\right)\cos(c+dx)}{4} + \frac{\sin^2\left(a-\frac{dx}{2}\right)\sin(c+dx)}{d} - 3\sin \\ -\frac{x\sin^2\left(a+\frac{dx}{2}\right)\cos(c+dx)}{4} + \frac{x\sin\left(a+\frac{dx}{2}\right)\sin(c+dx)\cos\left(a+\frac{dx}{2}\right)}{2} + \frac{x\cos^2\left(a+\frac{dx}{2}\right)\cos(c+dx)}{4} - \frac{\sin\left(a+\frac{dx}{2}\right)\cos\left(a+\frac{dx}{2}\right)\cos(c+a}{2d} \\ \left(\frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} + \frac{\sin(a+bx)\cos(a+bx)}{2b}\right)\cos(c) \\ \frac{2b^2\sin^2(a+bx)\sin(c+dx)}{4b^2d-d^3} + \frac{2b^2\sin(c+dx)\cos^2(a+bx)}{4b^2d-d^3} + \frac{2bd\sin(a+bx)\cos(a+bx)\cos(c+dx)}{4b^2d-d^3} - \frac{d^2\sin(c+dx)\cos^2(a+bx)}{4b^2d-d^3} \end{cases}$$

input
```
integrate(cos(b*x+a)**2*cos(d*x+c),x)
```

output
```
Piecewise((x*cos(a)**2*cos(c), Eq(b, 0) & Eq(d, 0)), (-x*sin(a - d*x/2)**2
*cos(c + d*x)/4 - x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)/2 + x*cos(a
 - d*x/2)**2*cos(c + d*x)/4 + sin(a - d*x/2)**2*sin(c + d*x)/d - 3*sin(a -
 d*x/2)*cos(a - d*x/2)*cos(c + d*x)/(2*d), Eq(b, -d/2)), (-x*sin(a + d*x/2
)**2*cos(c + d*x)/4 + x*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)/2 + x*c
os(a + d*x/2)**2*cos(c + d*x)/4 - sin(a + d*x/2)*cos(a + d*x/2)*cos(c + d*
x)/(2*d) + sin(c + d*x)*cos(a + d*x/2)**2/d, Eq(b, d/2)), ((x*sin(a + b*x)
**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*x)*cos(a + b*x)/(2*b))*cos(c), Eq(
d, 0)), (2*b**2*sin(a + b*x)**2*sin(c + d*x)/(4*b**2*d - d**3) + 2*b**2*si
n(c + d*x)*cos(a + b*x)**2/(4*b**2*d - d**3) + 2*b*d*sin(a + b*x)*cos(a +
b*x)*cos(c + d*x)/(4*b**2*d - d**3) - d**2*sin(c + d*x)*cos(a + b*x)**2/(4
*b**2*d - d**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(62) = 124$.

Time = 0.06 (sec) , antiderivative size = 371, normalized size of antiderivative = 5.46

$$\int \cos^2(a+bx)\cos(c+dx)\,dx = \frac{(2\,bd\sin(c) - d^2\sin(c))\cos((2\,b+d)x + 2\,a + 2\,c) - (2\,bd\sin(c) - d^2\sin(c))\cos((2\,b+d)x + 2\,a) - (2$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c),x, algorithm="maxima")
```

output
```
1/8*((2*b*d*sin(c) - d^2*sin(c))*cos((2*b + d)*x + 2*a + 2*c) - (2*b*d*sin
(c) - d^2*sin(c))*cos((2*b + d)*x + 2*a) - (2*b*d*sin(c) + d^2*sin(c))*cos
(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*sin(c) + d^2*sin(c))*cos(-(2*b - d)*x
- 2*a) + 2*(4*b^2*sin(c) - d^2*sin(c))*cos(d*x + 2*c) - 2*(4*b^2*sin(c) -
d^2*sin(c))*cos(d*x) - (2*b*d*cos(c) - d^2*cos(c))*sin((2*b + d)*x + 2*a +
 2*c) - (2*b*d*cos(c) - d^2*cos(c))*sin((2*b + d)*x + 2*a) + (2*b*d*cos(c)
 + d^2*cos(c))*sin(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*cos(c) + d^2*cos(c))
*sin(-(2*b - d)*x - 2*a) - 2*(4*b^2*cos(c) - d^2*cos(c))*sin(d*x + 2*c) -
2*(4*b^2*cos(c) - d^2*cos(c))*sin(d*x))/((cos(c)^2 + sin(c)^2)*d^3 - 4*(b^
2*cos(c)^2 + b^2*sin(c)^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \cos^2(a+bx)\cos(c+dx)\,dx = \frac{\sin(2\,bx + dx + 2\,a + c)}{4\,(2\,b+d)} + \frac{\sin(2\,bx - dx + 2\,a - c)}{4\,(2\,b-d)} + \frac{\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c),x, algorithm="giac")
```

output
```
1/4*sin(2*b*x + d*x + 2*a + c)/(2*b + d) + 1/4*sin(2*b*x - d*x + 2*a - c)/
(2*b - d) + 1/2*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 19.61 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int \cos^2(a+bx)\cos(c+dx)\,dx$$
$$= \frac{b\,(2\,d\,\sin\left(2\,a+c+2\,b\,x+d\,x\right)+2\,d\,\sin\left(2\,a-c+2\,b\,x-d\,x\right))-d^2\,\sin\left(2\,a+c+2\,b\,x+d\,x\right)+d^2\,\sin}{16\,b^2\,d-4\,d^3}$$
$$+\frac{\sin\left(c+d\,x\right)}{2\,d}$$

input
```
int(cos(a + b*x)^2*cos(c + d*x),x)
```

output
```
(b*(2*d*sin(2*a + c + 2*b*x + d*x) + 2*d*sin(2*a - c + 2*b*x - d*x)) - d^2
*sin(2*a + c + 2*b*x + d*x) + d^2*sin(2*a - c + 2*b*x - d*x))/(16*b^2*d -
4*d^3) + sin(c + d*x)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \cos^2(a+bx)\cos(c+dx)\,dx$$
$$= \frac{2\cos\left(bx+a\right)\cos\left(dx+c\right)\sin\left(bx+a\right)bd+\sin\left(bx+a\right)^2\sin\left(dx+c\right)d^2+2\sin\left(dx+c\right)b^2-\sin\left(dx+c\right)}{d\left(4b^2-d^2\right)}$$

input
```
int(cos(b*x+a)^2*cos(d*x+c),x)
```

output
```
(2*cos(a + b*x)*cos(c + d*x)*sin(a + b*x)*b*d + sin(a + b*x)**2*sin(c + d*
x)*d**2 + 2*sin(c + d*x)*b**2 - sin(c + d*x)*d**2)/(d*(4*b**2 - d**2))
```

3.336 $\int \cos^2(a+bx)\sec(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 157

$$\int \cos^2(a+bx)\sec(c+dx)\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{ie^{-2ia-2ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{b}{d},\frac{3}{2}-\frac{b}{d},-e^{2i(c+dx)}\right)}{2(2b-d)} - \frac{ie^{2ia+2ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{b}{d},\frac{3}{2}+\frac{b}{d},-e^{2i(c+dx)}\right)}{2(2b+d)}$$

output

```
1/2*arctanh(sin(d*x+c))/d+1/2*I*exp(-2*I*a-2*I*b*x+I*(d*x+c))*hypergeom([1
, 1/2-b/d],[3/2-b/d],-exp(2*I*(d*x+c)))/(2*b-d)-1/2*I*exp(2*I*a+2*I*b*x+I*
(d*x+c))*hypergeom([1, 1/2+b/d],[3/2+b/d],-exp(2*I*(d*x+c)))/(2*b+d)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99

$$\int \cos^2(a+bx)\sec(c+dx)\,dx = \frac{1}{2}\left(-\frac{2i\arctan\left(e^{i(c+dx)}\right)}{d} + \frac{ie^{-i(2a-c+2bx-dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{b}{d},\frac{3}{2}-\frac{b}{d},-e^{2i(c+dx)}\right)}{2b-d} - \frac{ie^{i(2a+c+2bx+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{b}{d},\frac{3}{2}+\frac{b}{d},-e^{2i(c+dx)}\right)}{2b+d}\right)$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c + d*x],x]
```

output
```
(((-2*I)*ArcTan[E^(I*(c + d*x))])/d + (I*Hypergeometric2F1[1, 1/2 - b/d, 3
/2 - b/d, -E^((2*I)*(c + d*x))])/((2*b - d)*E^(I*(2*a - c + 2*b*x - d*x)))
 - (I*E^(I*(2*a + c + 2*b*x + d*x))*Hypergeometric2F1[1, 1/2 + b/d, 3/2 +
b/d, -E^((2*I)*(c + d*x))])/(2*b + d))/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos^2(a+bx)\sec(c+dx)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c + d*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos(bx+a)^2 \sec(dx+c)\,dx$$

input
```
int(cos(b*x+a)^2*sec(d*x+c),x)
```

output
```
int(cos(b*x+a)^2*sec(d*x+c),x)
```

Fricas [F]

$$\int \cos^2(a+bx)\sec(c+dx)\,dx = \int \cos(bx+a)^2 \sec(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)^2*sec(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*sec(d*x + c), x)
```

Sympy [F]

$$\int \cos^2(a+bx)\sec(c+dx)\,dx = \int \cos^2(a+bx)\sec(c+dx)\,dx$$

input
```
integrate(cos(b*x+a)**2*sec(d*x+c),x)
```

output
```
Integral(cos(a + b*x)**2*sec(c + d*x), x)
```

Maxima [F]

$$\int \cos^2(a+bx)\sec(c+dx)\,dx = \int \cos(bx+a)^2\sec(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)^2*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2*sec(d*x + c), x)
```

Giac [F]

$$\int \cos^2(a+bx)\sec(c+dx)\,dx = \int \cos(bx+a)^2\sec(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)^2*sec(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec(c+dx)\,dx = \int \frac{\cos(a+b\,x)^2}{\cos(c+d\,x)}\,dx$$

input
```
int(cos(a + b*x)^2/cos(c + d*x),x)
```

output
```
int(cos(a + b*x)^2/cos(c + d*x), x)
```

Reduce [F]

$$\int \cos^2(a+bx)\sec(c+dx)\,dx=\int \cos\left(bx+a\right)^2\sec\left(dx+c\right)dx$$

input
```
int(cos(b*x+a)^2*sec(d*x+c),x)
```

output
```
int(cos(a + b*x)**2*sec(c + d*x),x)
```

3.337 $\int \cos^2(a+bx)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \cos^2(a+bx)\sec^2(c+dx)\,dx = \frac{ie^{-2ia-2ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{b}{d},2-\frac{b}{d},-e^{2i(c+dx)}\right)}{2(b-d)} - \frac{ie^{2ia+2ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,\frac{b+d}{d},2+\frac{b}{d},-e^{2i(c+dx)}\right)}{2(b+d)} + \frac{\tan(c+dx)}{2d}$$

output
```
1/2*I*exp(-2*I*a-2*I*b*x+2*I*(d*x+c))*hypergeom([2, 1-b/d],[2-b/d],-exp(2*
I*(d*x+c)))/(b-d)-1/2*I*exp(2*I*a+2*I*b*x+2*I*(d*x+c))*hypergeom([2, (b+d)
/d],[2+b/d],-exp(2*I*(d*x+c)))/(b+d)+1/2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \cos^2(a+bx)\sec^2(c+dx)\,dx = -\frac{ie^{-2i(a+bx)}\left(-1-e^{4i(a+bx)}+\left(1+e^{2ic}\right)\text{Hypergeometric2F1}\left(1,-\frac{b}{d},1-\frac{b}{d},-e^{2i(c+dx)}\right)+e^{4i(a+bx)}\left(1+e^{2}\right.\right.}{2d\left(1+e^{2ic}\right)} + \frac{\cos^2(a+bx)\sec(c)\sec(c+dx)\sin(dx)}{d}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c + d*x]^2,x]
```

output
```
((-1/2*I)*(-1 - E^((4*I)*(a + b*x)) + (1 + E^((2*I)*c))*Hypergeometric2F1[
1, -(b/d), 1 - b/d, -E^((2*I)*(c + d*x))] + E^((4*I)*(a + b*x))*(1 + E^((2
*I)*c))*Hypergeometric2F1[1, b/d, (b + d)/d, -E^((2*I)*(c + d*x))]))/(d*E^
((2*I)*(a + b*x))*(1 + E^((2*I)*c))) + (Cos[a + b*x]^2*Sec[c]*Sec[c + d*x]
*Sin[d*x])/d
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec^2(c+dx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \cos^2(a+bx)\sec^2(c+dx)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c + d*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos (bx+a)^2 \sec (dx+c)^2 \, dx$$

input
```
int(cos(b*x+a)^2*sec(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)^2*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int \cos^2(a+bx) \sec^2(c+dx) \, dx = \int \cos (bx+a)^2 \sec (dx+c)^2 \, dx$$

input
```
integrate(cos(b*x+a)^2*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*sec(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx) \sec^2(c+dx) \, dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(a+bx)\sec^2(c+dx)\,dx = \int \cos\left(bx+a\right)^2\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*((sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(2*(b + d)*x + 2*a + 2*c)
 - 2*(d*cos(2*(b + d)*x + 2*a + 2*c)^2 + 2*d*cos(2*(b + d)*x + 2*a + 2*c)*
cos(2*b*x + 2*a) + d*cos(2*b*x + 2*a)^2 + d*sin(2*(b + d)*x + 2*a + 2*c)^2
 + 2*d*sin(2*(b + d)*x + 2*a + 2*c)*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^
2)*integrate((b*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + b*sin(2*(b + d)*x + 2*
a + 2*c)*sin(4*b*x + 4*a) + b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b*cos(4
*b*x + 4*a) - b)*cos(2*(b + d)*x + 2*a + 2*c) - b*cos(2*b*x + 2*a))/(d*cos
(2*(b + d)*x + 2*a + 2*c)^2 + 2*d*cos(2*(b + d)*x + 2*a + 2*c)*cos(2*b*x +
 2*a) + d*cos(2*b*x + 2*a)^2 + d*sin(2*(b + d)*x + 2*a + 2*c)^2 + 2*d*sin(
2*(b + d)*x + 2*a + 2*c)*sin(2*b*x + 2*a) + d*sin(2*b*x + 2*a)^2), x) - (c
os(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) + 1)*sin(2*(b + d)*x + 2*a + 2*c) + c
os(2*b*x + 2*a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin
(2*b*x + 2*a))/(d*cos(2*(b + d)*x + 2*a + 2*c)^2 + 2*d*cos(2*(b + d)*x + 2
*a + 2*c)*cos(2*b*x + 2*a) + d*cos(2*b*x + 2*a)^2 + d*sin(2*(b + d)*x + 2*
a + 2*c)^2 + 2*d*sin(2*(b + d)*x + 2*a + 2*c)*sin(2*b*x + 2*a) + d*sin(2*b
*x + 2*a)^2)
```

Giac [F]

$$\int \cos^2(a+bx)\sec^2(c+dx)\,dx = \int \cos\left(bx+a\right)^2\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^2(c+dx)\,dx=\int \frac{\cos\left(a+b\,x\right)^2}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int(cos(a + b*x)^2/cos(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)^2/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \cos^2(a+bx)\sec^2(c+dx)\,dx=\int \cos\left(bx+a\right)^2\sec\left(dx+c\right)^2dx$$

input
```
int(cos(b*x+a)^2*sec(d*x+c)^2,x)
```

output
```
int(cos(a + b*x)**2*sec(c + d*x)**2,x)
```

3.338 $\int \cos^2(a+bx)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 174

$$\int \cos^2(a+bx)\sec^3(c+dx)\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{4d} + \frac{2ie^{-2ia-2ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3}{2}-\frac{b}{d},\frac{5}{2}-\frac{b}{d},-e^{2i(c+dx)}\right)}{2b-3d} - \frac{2ie^{2ia+2ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3}{2}+\frac{b}{d},\frac{5}{2}+\frac{b}{d},-e^{2i(c+dx)}\right)}{2b+3d} + \frac{\sec(c+dx)\tan(c+dx)}{4d}$$

output
```
1/4*arctanh(sin(d*x+c))/d+2*I*exp(-2*I*a-2*I*b*x+3*I*(d*x+c))*hypergeom([3
, 3/2-b/d],[5/2-b/d],-exp(2*I*(d*x+c)))/(2*b-3*d)-2*I*exp(2*I*a+2*I*b*x+3*
I*(d*x+c))*hypergeom([3, 3/2+b/d],[5/2+b/d],-exp(2*I*(d*x+c)))/(2*b+3*d)+1
/4*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 6.37 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \cos^2(a+bx)\sec^3(c+dx)\,dx$$
$$= \frac{-4id\arctan\left(e^{i(c+dx)}\right)-2i(2b+d)e^{-i(2a-c+2bx-dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{b}{d},\frac{3}{2}-\frac{b}{d},-e^{2i(c+dx)}\right)+2i$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c + d*x]^3,x]
```

output
```
((-4*I)*d*ArcTan[E^(I*(c + d*x))] - ((2*I)*(2*b + d)*Hypergeometric2F1[1,
1/2 - b/d, 3/2 - b/d, -E^((2*I)*(c + d*x))])/E^(I*(2*a - c + 2*b*x - d*x))
 + (2*I)*(2*b - d)*E^(I*(2*a + c + (2*b + d)*x))*Hypergeometric2F1[1, 1/2
+ b/d, 3/2 + b/d, -E^((2*I)*(c + d*x))] + 2*Cos[a + b*x]*Sec[c + d*x]^2*((
2*b - d)*Sin[a - c + b*x - d*x] + (2*b + d)*Sin[a + c + (b + d)*x]))/(8*d^
2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec^3(c+dx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \cos^2(a+bx)\sec^3(c+dx)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c + d*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos(bx+a)^2 \sec(dx+c)^3\,dx$$

input
```
int(cos(b*x+a)^2*sec(d*x+c)^3,x)
```

output
```
int(cos(b*x+a)^2*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int \cos^2(a+bx)\sec^3(c+dx)\,dx = \int \cos(bx+a)^2 \sec(dx+c)^3\,dx$$

input
```
integrate(cos(b*x+a)^2*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*sec(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(a+bx)\sec^3(c+dx)\,dx = \int \cos\left(bx+a\right)^2\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(cos(b*x+a)^2*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*((2*b - d)*cos(2*b*x + 2*a)*sin((4*b + d)*x + 4*a + c) - 2*d*cos(2*b*x
 + 2*a)*sin((2*b + d)*x + 2*a + c) - (2*b - d)*cos((4*b + d)*x + 4*a + c)*
sin(2*b*x + 2*a) + 2*d*cos((2*b + d)*x + 2*a + c)*sin(2*b*x + 2*a) + (2*b
- d)*cos(3*d*x + 3*c)*sin(2*b*x + 2*a) + (2*b + d)*cos(d*x + c)*sin(2*b*x
+ 2*a) - (2*b - d)*cos(2*b*x + 2*a)*sin(3*d*x + 3*c) - (2*b + d)*cos(2*b*x
 + 2*a)*sin(d*x + c) - (2*(2*b + d)*sin(2*(b + d)*x + 2*a + 2*c) + (2*b +
d)*sin(2*b*x + 2*a))*cos((4*b + 3*d)*x + 4*a + 3*c) - 2*(2*d*sin(2*(b + d)
*x + 2*a + 2*c) + d*sin(2*b*x + 2*a))*cos((2*b + 3*d)*x + 2*a + 3*c) + ((2
*b + d)*sin((4*b + 3*d)*x + 4*a + 3*c) + (2*b - d)*sin((4*b + d)*x + 4*a +
 c) + 2*d*sin((2*b + 3*d)*x + 2*a + 3*c) - 2*d*sin((2*b + d)*x + 2*a + c)
- (2*b - d)*sin(3*d*x + 3*c) - (2*b + d)*sin(d*x + c))*cos(2*(b + 2*d)*x +
 2*a + 4*c) + 2*((2*b - d)*sin((4*b + d)*x + 4*a + c) - 2*d*sin((2*b + d)*
x + 2*a + c) - (2*b - d)*sin(3*d*x + 3*c) - (2*b + d)*sin(d*x + c))*cos(2*
(b + d)*x + 2*a + 2*c) + 4*(d^2*cos(2*(b + 2*d)*x + 2*a + 4*c)^2 + 4*d^2*c
os(2*(b + d)*x + 2*a + 2*c)^2 + 4*d^2*cos(2*(b + d)*x + 2*a + 2*c)*cos(2*b
*x + 2*a) + d^2*cos(2*b*x + 2*a)^2 + d^2*sin(2*(b + 2*d)*x + 2*a + 4*c)^2
+ 4*d^2*sin(2*(b + d)*x + 2*a + 2*c)^2 + 4*d^2*sin(2*(b + d)*x + 2*a + 2*c
)*sin(2*b*x + 2*a) + d^2*sin(2*b*x + 2*a)^2 + 2*(2*d^2*cos(2*(b + d)*x + 2
*a + 2*c) + d^2*cos(2*b*x + 2*a))*cos(2*(b + 2*d)*x + 2*a + 4*c) + 2*(2*d^
2*sin(2*(b + d)*x + 2*a + 2*c) + d^2*sin(2*b*x + 2*a))*sin(2*(b + 2*d)*...
```

Giac [F]

$$\int \cos^2(a+bx)\sec^3(c+dx)\,dx = \int \cos\left(bx+a\right)^2\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(cos(b*x+a)^2*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^3(c+dx)\,dx=\int \frac{\cos\left(a+b\,x\right)^2}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int(cos(a + b*x)^2/cos(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)^2/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \cos^2(a+bx)\sec^3(c+dx)\,dx=\int \cos\left(bx+a\right)^2\sec\left(dx+c\right)^3dx$$

input
```
int(cos(b*x+a)^2*sec(d*x+c)^3,x)
```

output
```
int(cos(a + b*x)**2*sec(c + d*x)**3,x)
```

3.339 $\int \cos^2(a+bx)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 157

$$\begin{aligned}\int \cos^2(a+bx)\sec^4(c+dx)\,dx &= \frac{2ie^{-2ia-2ibx+4i(c+dx)}\,\text{Hypergeometric2F1}\left(4,2-\frac{b}{d},3-\frac{b}{d},-e^{2i(c+dx)}\right)}{b-2d} \\ &\quad - \frac{2ie^{2ia+2ibx+4i(c+dx)}\,\text{Hypergeometric2F1}\left(4,2+\frac{b}{d},3+\frac{b}{d},-e^{2i(c+dx)}\right)}{b+2d} \\ &\quad + \frac{\tan(c+dx)}{2d} + \frac{\tan^3(c+dx)}{6d}\end{aligned}$$

output
```
2*I*exp(-2*I*a-2*I*b*x+4*I*(d*x+c))*hypergeom([4, 2-b/d],[3-b/d],-exp(2*I*
(d*x+c)))/(b-2*d)-2*I*exp(2*I*a+2*I*b*x+4*I*(d*x+c))*hypergeom([4, 2+b/d],
[3+b/d],-exp(2*I*(d*x+c)))/(b+2*d)+1/2*tan(d*x+c)/d+1/6*tan(d*x+c)^3/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 429 vs. 2(157) = 314.

Time = 1.64 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.73

$$\int \cos^2(a+bx)\sec^4(c+dx)\,dx = \frac{i(b^2-d^2)\,e^{-2i(a+bx)}\left(-1-e^{4i(a+bx)}+\left(1+e^{2ic}\right)\text{Hypergeometric2F1}\left(1,-\frac{b}{d},1-\frac{b}{d},-e^{2i(c+dx)}\right)+e^{4i(a+bx)}\right.}{3d^3\left(1+e^{2ic}\right)} + \frac{\sec(c)\sec^3(c+dx)\left(6d^2\sin(dx)+(b^2-d^2)\sin(2a-2c+2bx-3dx)+2b^2\sin(2a+2bx-dx)+bd\,\text{si}\right.}{}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c + d*x]^4,x]
```

output
```
((I/3)*(b^2 - d^2)*(-1 - E^((4*I)*(a + b*x)) + (1 + E^((2*I)*c))*Hypergeom
etric2F1[1, -(b/d), 1 - b/d, -E^((2*I)*(c + d*x))] + E^((4*I)*(a + b*x))*(
1 + E^((2*I)*c))*Hypergeometric2F1[1, b/d, (b + d)/d, -E^((2*I)*(c + d*x))
]))/(d^3*E^((2*I)*(a + b*x))*(1 + E^((2*I)*c))) + (Sec[c]*Sec[c + d*x]^3*(
6*d^2*Sin[d*x] + (b^2 - d^2)*Sin[2*a - 2*c + 2*b*x - 3*d*x] + 2*b^2*Sin[2*
a + 2*b*x - d*x] + b*d*Sin[2*a + 2*b*x - d*x] - 3*d^2*Sin[2*a + 2*b*x - d*
x] - b^2*Sin[2*a - 2*c + 2*b*x - d*x] + b*d*Sin[2*a - 2*c + 2*b*x - d*x] -
 2*b^2*Sin[2*a + 2*b*x + d*x] + b*d*Sin[2*a + 2*b*x + d*x] + 3*d^2*Sin[2*a
 + 2*b*x + d*x] + b^2*Sin[2*a + 2*c + 2*b*x + d*x] + b*d*Sin[2*a + 2*c + 2
*b*x + d*x] + 2*d^2*Sin[2*c + 3*d*x] - b^2*Sin[2*a + 2*c + 2*b*x + 3*d*x]
+ d^2*Sin[2*a + 2*c + 2*b*x + 3*d*x]))/(24*d^3)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec^4(c+dx)\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \cos^2(a+bx)\sec^4(c+dx)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c + d*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos(bx+a)^2 \sec(dx+c)^4 \, dx$$

input
```
int(cos(b*x+a)^2*sec(d*x+c)^4,x)
```

output
```
int(cos(b*x+a)^2*sec(d*x+c)^4,x)
```

Fricas [F]

$$\int \cos^2(a+bx)\sec^4(c+dx)\,dx = \int \cos(bx+a)^2 \sec(dx+c)^4 \, dx$$

input
```
integrate(cos(b*x+a)^2*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*sec(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(a+bx)\sec^4(c+dx)\,dx = \int \cos\left(bx+a\right)^2\sec\left(dx+c\right)^4\,dx$$

input
```
integrate(cos(b*x+a)^2*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/3*((b^2 - d^2)*cos(2*b*x + 2*a)*sin(4*b*x + 4*a) - (b^2 - d^2)*cos(4*b*x
 + 4*a)*sin(2*b*x + 2*a) - (b^2 - b*d)*cos(4*d*x + 4*c)*sin(2*b*x + 2*a) -
 (2*b^2 - b*d - 3*d^2)*cos(2*d*x + 2*c)*sin(2*b*x + 2*a) + (b^2 - b*d)*cos
(2*b*x + 2*a)*sin(4*d*x + 4*c) + (2*b^2 - b*d - 3*d^2)*cos(2*b*x + 2*a)*si
n(2*d*x + 2*c) - (3*(2*b^2 + b*d - 3*d^2)*sin(2*(b + d)*x + 2*a + 2*c) + (
2*b^2 + b*d - 3*d^2)*sin(2*b*x + 2*a))*cos(2*(2*b + d)*x + 4*a + 2*c) - (6
*d^2*sin(2*(b + d)*x + 2*a + 2*c) + 2*d^2*sin(2*b*x + 2*a) - (2*b^2 + b*d
- 3*d^2)*sin(2*(2*b + d)*x + 4*a + 2*c) - (b^2 + b*d)*sin(4*(b + d)*x + 4*
a + 4*c) - (b^2 - d^2)*sin(4*b*x + 4*a) - (b^2 - b*d)*sin(4*d*x + 4*c) - (
2*b^2 - b*d - 3*d^2)*sin(2*d*x + 2*c))*cos(2*(b + 3*d)*x + 2*a + 6*c) - 3*
(6*d^2*sin(2*(b + d)*x + 2*a + 2*c) + 2*d^2*sin(2*b*x + 2*a) - (2*b^2 + b*
d - 3*d^2)*sin(2*(2*b + d)*x + 4*a + 2*c) - (b^2 - d^2)*sin(4*b*x + 4*a) -
 (b^2 - b*d)*sin(4*d*x + 4*c) - (2*b^2 - b*d - 3*d^2)*sin(2*d*x + 2*c))*co
s(2*(b + 2*d)*x + 2*a + 4*c) - (3*(b^2 + b*d)*sin(2*(b + 2*d)*x + 2*a + 4*
c) + 3*(b^2 + b*d)*sin(2*(b + d)*x + 2*a + 2*c) + (b^2 + b*d)*sin(2*b*x +
2*a))*cos(4*(b + d)*x + 4*a + 4*c) + 3*((b^2 - d^2)*sin(4*b*x + 4*a) + (b^
2 - b*d)*sin(4*d*x + 4*c) + (2*b^2 - b*d - 3*d^2)*sin(2*d*x + 2*c))*cos(2*
(b + d)*x + 2*a + 2*c) + 3*(d^3*cos(2*(b + 3*d)*x + 2*a + 6*c)^2 + 9*d^3*c
os(2*(b + 2*d)*x + 2*a + 4*c)^2 + 9*d^3*cos(2*(b + d)*x + 2*a + 2*c)^2 + 6
*d^3*cos(2*(b + d)*x + 2*a + 2*c)*cos(2*b*x + 2*a) + d^3*cos(2*b*x + 2*...
```

Giac [F]

$$\int \cos^2(a+bx)\sec^4(c+dx)\,dx=\int \cos(bx+a)^2\sec(dx+c)^4\,dx$$

input
```
integrate(cos(b*x+a)^2*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^4(c+dx)\,dx=\int \frac{\cos\left(a+b\,x\right)^2}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int(cos(a + b*x)^2/cos(c + d*x)^4,x)
```

output
```
int(cos(a + b*x)^2/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int \cos^2(a+bx)\sec^4(c+dx)\,dx=\int \cos\left(bx+a\right)^2\sec\left(dx+c\right)^4dx$$

input
```
int(cos(b*x+a)^2*sec(d*x+c)^4,x)
```

output
```
int(cos(a + b*x)**2*sec(c + d*x)**4,x)
```

3.340 $\int \cos^3(a+bx)\cos^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 195

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx = \frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}+\frac{\sin(3(a-c)+3(b-d)x)}{96(b-d)}+\frac{3\sin(3a-c+(3b-d)x)}{32(3b-d)}+\frac{9\sin(a+c+(b+d)x)}{32(b+d)}+\frac{\sin(3(a+c)+3(b+d)x)}{96(b+d)}+\frac{3\sin(3a+c+(3b+d)x)}{32(3b+d)}+\frac{3\sin(a+3c+(b+3d)x)}{32(b+3d)}$$

output
```
3*sin(a-3*c+(b-3*d)*x)/(32*b-96*d)+9*sin(a-c+(b-d)*x)/(32*b-32*d)+sin(3*a-
3*c+3*(b-d)*x)/(96*b-96*d)+3*sin(3*a-c+(3*b-d)*x)/(96*b-32*d)+9*sin(a+c+(b
+d)*x)/(32*b+32*d)+sin(3*a+3*c+3*(b+d)*x)/(96*b+96*d)+3*sin(3*a+c+(3*b+d)*
x)/(96*b+32*d)+3*sin(a+3*c+(b+3*d)*x)/(32*b+96*d)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx = \frac{1}{96}\left(\frac{9\sin(a-3c+bx-3dx)}{b-3d} + \frac{27\sin(a-c+bx-dx)}{b-d} + \frac{\sin(3(a-c+bx-dx))}{b-d} + \frac{9\sin(3a-c+3bx-dx)}{3b-d} + \frac{9\sin(3a+c+3bx+dx)}{3b+d} + \frac{9\sin(a+3c+bx+3dx)}{b+3d} + \frac{27\sin(a+c+(b+d)x)}{b+d} + \frac{\sin(3(a+c+(b+d)x))}{b+d}\right)$$

input
```
Integrate[Cos[a + b*x]^3*Cos[c + d*x]^3,x]
```

output
```
((9*Sin[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sin[a - c + b*x - d*x])/(b
 - d) + Sin[3*(a - c + b*x - d*x)]/(b - d) + (9*Sin[3*a - c + 3*b*x - d*x]
)/(3*b - d) + (9*Sin[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sin[a + 3*c +
b*x + 3*d*x])/(b + 3*d) + (27*Sin[a + c + (b + d)*x])/(b + d) + Sin[3*(a +
 c + (b + d)*x)]/(b + d))/96
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \text{ 5081}$$

$$\int \left(\frac{3}{32}\cos(a+x(b-3d)-3c)+\frac{9}{32}\cos(a+x(b-d)-c)+\frac{1}{32}\cos(3(a-c)+3x(b-d))+\frac{3}{32}\cos(3a+x(3b-\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sin(a+x(b+d)+c)}{32(b+d)}+\frac{\sin(3(a+c)+3x(b+d))}{96(b+d)}+\frac{3\sin(3a+x(3b+d)+c)}{32(3b+d)}+\frac{3\sin(a+x(b+3d)+3c)}{32(b+3d)}$$

input
```
Int[Cos[a + b*x]^3*Cos[c + d*x]^3,x]
```

output
```
(3*Sin[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sin[a - c + (b - d)*x])
/(32*(b - d)) + Sin[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sin[3*a - c
 + (3*b - d)*x])/(32*(3*b - d)) + (9*Sin[a + c + (b + d)*x])/(32*(b + d))
+ Sin[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sin[3*a + c + (3*b + d)*x
])/(32*(3*b + d)) + (3*Sin[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 44.81 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.97

method	result
default	$\frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}+\frac{9\sin(a+c+(b+d)x)}{32(b+d)}+\frac{3\sin(a+3c+(b+3d)x)}{32(b+3d)}+\frac{\sin((3b-3d)x+3a-3c)}{96b-96d}$
parallelrisch	$\frac{\frac{9\left(b+\frac{d}{3}\right)(b-3d)(b+3d)(b-d)(b+d)\sin(3a-c+(3b-d)x)}{32}+\frac{9\left(\frac{\left(b+\frac{d}{3}\right)(b-3d)(b+3d)(b+d)\sin((3b-3d)x+3a-3c)}{3}+\frac{\left(b+\frac{d}{3}\right)(b-3d)(b+3d)(b-d}{3}\right.}{}}{}$
risch	Expression too large to display
orering	Expression too large to display

input
```
int(cos(b*x+a)^3*cos(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32/(b-d)*sin(a-c+(b-d)*x)+9/32/(b+d)*s
in(a+c+(b+d)*x)+3/32/(b+3*d)*sin(a+3*c+(b+3*d)*x)+1/32/(3*b-3*d)*sin((3*b-
3*d)*x+3*a-3*c)+3/32/(3*b-d)*sin(3*a-c+(3*b-d)*x)+3/32/(3*b+d)*sin(3*a+c+(
3*b+d)*x)+1/32/(3*b+3*d)*sin((3*b+3*d)*x+3*a+3*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.23

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx = \frac{\left(\left(18\,b^5-2\,b^3d^2+\left(9\,b^5-82\,b^3d^2+9\,bd^4\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)^3-6\left(20\,b^3d^2+\left(b^3d^2-9\,bd^4\right)\cos\left(b\right.\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/3*(((18*b^5 - 2*b^3*d^2 + (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cos(b*x + a)^2)
*cos(d*x + c)^3 - 6*(20*b^3*d^2 + (b^3*d^2 - 9*b*d^4)*cos(b*x + a)^2)*cos(
d*x + c))*sin(b*x + a) + (120*b^2*d^3*cos(b*x + a) + 2*(b^2*d^3 - 9*d^5)*c
os(b*x + a)^3 - ((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cos(b*x + a)^3 + 6*(9*b^4*
d - b^2*d^3)*cos(b*x + a))*cos(d*x + c)^2)*sin(d*x + c))/(9*b^6 - 91*b^4*d
^2 + 91*b^2*d^4 - 9*d^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3577 vs. $2(172) = 344$.

Time = 17.60 (sec) , antiderivative size = 3577, normalized size of antiderivative = 18.34

$$\int \cos^3(a + bx)\cos^3(c + dx)\, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**3*cos(d*x+c)**3,x)
```

output
```
Piecewise((x*cos(a)**3*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - 3*d*x
)**3*sin(c + d*x)**3/32 - 9*x*sin(a - 3*d*x)**3*sin(c + d*x)*cos(c + d*x)*
*2/32 - 9*x*sin(a - 3*d*x)**2*sin(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/
32 + 3*x*sin(a - 3*d*x)**2*cos(a - 3*d*x)*cos(c + d*x)**3/32 + 3*x*sin(a -
 3*d*x)*sin(c + d*x)**3*cos(a - 3*d*x)**2/32 - 9*x*sin(a - 3*d*x)*sin(c +
d*x)*cos(a - 3*d*x)**2*cos(c + d*x)**2/32 - 9*x*sin(c + d*x)**2*cos(a - 3*
d*x)**3*cos(c + d*x)/32 + 3*x*cos(a - 3*d*x)**3*cos(c + d*x)**3/32 - 3*sin
(a - 3*d*x)**3*sin(c + d*x)**2*cos(c + d*x)/(320*d) - sin(a - 3*d*x)**3*co
s(c + d*x)**3/(4*d) - 11*sin(a - 3*d*x)**2*sin(c + d*x)**3*cos(a - 3*d*x)/
(320*d) - 3*sin(a - 3*d*x)**2*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/
(20*d) - 117*sin(a - 3*d*x)*cos(a - 3*d*x)**2*cos(c + d*x)**3/(320*d) - si
n(c + d*x)**3*cos(a - 3*d*x)**3/(30*d) - 61*sin(c + d*x)*cos(a - 3*d*x)**3
*cos(c + d*x)**2/(320*d), Eq(b, -3*d)), (-5*x*sin(a - d*x)**3*sin(c + d*x)
**3/16 - 3*x*sin(a - d*x)**3*sin(c + d*x)*cos(c + d*x)**2/16 + 9*x*sin(a -
 d*x)**2*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/16 + 3*x*sin(a - d*x)**
2*cos(a - d*x)*cos(c + d*x)**3/16 - 3*x*sin(a - d*x)*sin(c + d*x)**3*cos(a
 - d*x)**2/16 - 9*x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)
**2/16 + 3*x*sin(c + d*x)**2*cos(a - d*x)**3*cos(c + d*x)/16 + 5*x*cos(a -
 d*x)**3*cos(c + d*x)**3/16 - 5*sin(a - d*x)**3*sin(c + d*x)**2*cos(c + d*
x)/(16*d) - sin(a - d*x)**3*cos(c + d*x)**3/(48*d) + 3*sin(a - d*x)**2*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2614 vs. $2(179) = 358$.

Time = 0.21 (sec) , antiderivative size = 2614, normalized size of antiderivative = 13.41

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/192*(9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a +
 4*c) - 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a -
 2*c) - 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a
+ 4*c) + 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2
*d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a
 - 2*c) + 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30
*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a +
 6*c) - 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b
^2*d^3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a) +
(9*b^5*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(
3*c) + 9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a + 6*c) - (
9*b^5*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3
*c) + 9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a) + 27*(9*b^
5*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c)
+ 9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((b + d)*x + a + 4*c) - 27*(9*b^5*
sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) +
9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5...
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx = \frac{\sin(3\,bx+3\,dx+3\,a+3\,c)}{96\,(b+d)} + \frac{3\,\sin(3\,bx+dx+3\,a+c)}{32\,(3\,b+d)} + \frac{3\,\sin(3\,bx-dx+3\,a-c)}{32\,(3\,b-d)} + \frac{\sin(3\,bx-3\,dx+3\,a-3\,c)}{96\,(b-d)} + \frac{3\,\sin(bx+3\,dx+a+3\,c)}{32\,(b+3\,d)} + \frac{9\,\sin(bx+dx+a+c)}{32\,(b+d)} + \frac{9\,\sin(bx-dx+a-c)}{32\,(b-d)} + \frac{3\,\sin(bx-3\,dx+a-3\,c)}{32\,(b-3\,d)}$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="giac")
```

output
```
1/96*sin(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*sin(3*b*x + d*x + 3*a +
 c)/(3*b + d) + 3/32*sin(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*sin(3*b*x
 - 3*d*x + 3*a - 3*c)/(b - d) + 3/32*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d)
+ 9/32*sin(b*x + d*x + a + c)/(b + d) + 9/32*sin(b*x - d*x + a - c)/(b - d
) + 3/32*sin(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

Mupad [B] (verification not implemented)

Time = 25.52 (sec) , antiderivative size = 999, normalized size of antiderivative = 5.12

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx = \text{Too large to display}$$

input
```
int(cos(a + b*x)^3*cos(c + d*x)^3,x)
```

output
```
- exp(a*3i - c*1i + b*x*3i - d*x*1i)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/
(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*6i - b*x*6i)*(9*b*d^2 + 3*b
^2*d - 9*b^3 - 3*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i) + (exp(- a*2i -
 b*x*2i)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*
d^2*640i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(
b^4*576i + d^4*64i - b^2*d^2*640i)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*(
(9*b*d^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(b^4*576i + d^4*64i - b^2*d^2*640i) -
(exp(- a*6i - b*x*6i)*(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3))/(b^4*576i + d^4
*64i - b^2*d^2*640i) + (exp(- a*2i - b*x*2i)*(9*b*d^2 + 81*b^2*d - 81*b^3
- 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*4i - b*x*4i)*(9*b
*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i)) -
exp(a*3i - c*3i + b*x*3i - d*x*3i)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(b^4*1
92i + d^4*1728i - b^2*d^2*1920i) - (exp(- a*6i - b*x*6i)*(9*b*d^2 + b^2*d
- b^3 - 9*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i) + (exp(- a*2i - b*x
*2i)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(b^4*192i + d^4*1728i - b^2*d^
2*1920i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(b
^4*192i + d^4*1728i - b^2*d^2*1920i)) - exp(a*3i + c*3i + b*x*3i + d*x*3i)
*((9*b*d^2 + b^2*d - b^3 - 9*d^3)/(b^4*192i + d^4*1728i - b^2*d^2*1920i) -
 (exp(- a*6i - b*x*6i)*(9*b*d^2 - b^2*d - b^3 + 9*d^3))/(b^4*192i + d^4*17
28i - b^2*d^2*1920i) + (exp(- a*2i - b*x*2i)*(9*b*d^2 + 27*b^2*d - 9*b^...
```

Reduce [F]

$$\int \cos^3(a+bx)\cos^3(c+dx)\,dx=\int \cos(bx+a)^3\cos(dx+c)^3\,dx$$

input
```
int(cos(b*x+a)^3*cos(d*x+c)^3,x)
```

output
```
int(cos(b*x+a)^3*cos(d*x+c)^3,x)
```

3.341 $\int \cos^3(a+bx)\cos^2(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 138

$$\int \cos^3(a+bx)\cos^2(c+dx)\,dx = \frac{3\sin(a+bx)}{8b}+\frac{\sin(3a+3bx)}{24b}+\frac{3\sin(a-2c+(b-2d)x)}{16(b-2d)}+\frac{\sin(3a-2c+(3b-2d)x)}{16(3b-2d)}+\frac{3\sin(a+2c+(b+2d)x)}{16(b+2d)}+\frac{\sin(3a+2c+(3b+2d)x)}{16(3b+2d)}$$

output
```
3/8*sin(b*x+a)/b+1/24*sin(3*b*x+3*a)/b+3*sin(a-2*c+(b-2*d)*x)/(16*b-32*d)+
sin(3*a-2*c+(3*b-2*d)*x)/(48*b-32*d)+3*sin(a+2*c+(b+2*d)*x)/(16*b+32*d)+si
n(3*a+2*c+(3*b+2*d)*x)/(48*b+32*d)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \cos^3(a+bx)\cos^2(c+dx)\,dx = \frac{1}{48}\left(\frac{18\cos(bx)\sin(a)}{b}+\frac{2\cos(3bx)\sin(3a)}{b}+\frac{18\cos(a)\sin(bx)}{b}+\frac{2\cos(3a)\sin(3bx)}{b}+\frac{9\sin(a-2c+bx-2dx)}{b-2d}+\frac{3\sin(3a-2c+3bx-2dx)}{3b-2d}+\frac{9\sin(a+2c+bx+2dx)}{b+2d}+\frac{3\sin(3a+2c+3bx+2dx)}{3b+2d}\right)$$

input
```
Integrate[Cos[a + b*x]^3*Cos[c + d*x]^2,x]
```

output
```
((18*Cos[b*x]*Sin[a])/b + (2*Cos[3*b*x]*Sin[3*a])/b + (18*Cos[a]*Sin[b*x])
/b + (2*Cos[3*a]*Sin[3*b*x])/b + (9*Sin[a - 2*c + b*x - 2*d*x])/(b - 2*d)
+ (3*Sin[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d) + (9*Sin[a + 2*c + b*x +
2*d*x])/(b + 2*d) + (3*Sin[3*a + 2*c + 3*b*x + 2*d*x])/(3*b + 2*d))/48
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\cos^2(c+dx)\,dx$$

$$\Big\downarrow \; 5081$$

$$\int \left(\frac{3}{16}\cos(a+x(b-2d)-2c)+\frac{1}{16}\cos(3a+x(3b-2d)-2c)+\frac{3}{16}\cos(a+x(b+2d)+2c)+\frac{1}{16}\cos(3a+x(3$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sin(a+x(b-2d)-2c)}{16(b-2d)}+\frac{\sin(3a+x(3b-2d)-2c)}{16(3b-2d)}+\frac{3\sin(a+x(b+2d)+2c)}{16(b+2d)}+\frac{\sin(3a+x(3b+2d)+2c)}{16(3b+2d)}+\frac{3\sin(a+bx)}{8b}+\frac{\sin(3a+3bx)}{24b}$$

input
```
Int[Cos[a + b*x]^3*Cos[c + d*x]^2,x]
```

output
```
(3*Sin[a + b*x])/(8*b) + Sin[3*a + 3*b*x]/(24*b) + (3*Sin[a - 2*c + (b - 2
*d)*x])/(16*(b - 2*d)) + Sin[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d)) +
 (3*Sin[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) + Sin[3*a + 2*c + (3*b + 2*
d)*x]/(16*(3*b + 2*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 13.52 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

method	result
default	$\frac{3\sin(bx+a)}{8b}+\frac{\sin(3bx+3a)}{24b}+\frac{3\sin(a-2c+(b-2d)x)}{16(b-2d)}+\frac{3\sin(a+2c+(b+2d)x)}{16(b+2d)}+\frac{\sin(3a-2c+(3b-2d)x)}{48b-32d}+$
parallelrisch	$9(b+2d)(b-2d)\left(b+\frac{2d}{3}\right)b\sin(3a-2c+(3b-2d)x)+9\left(\left(b^3-4bd^2\right)\sin(3a+2c+(3b+2d)x)+18\left(b+\frac{2d}{3}\right)\left(\frac{b(b+2d)\sin(a-2c+(b-2d)}{2}\right.\right.$ / $432b^5-1920b^3d^2+768bd^4$
risch	$\frac{3\sin(bx+a)}{8b}+\frac{27\sin(bx-2dx+a-2c)b^3}{16(b-2d)(3b-2d)(3b+2d)(b+2d)}+\frac{27\sin(bx-2dx+a-2c)b^2d}{8(b-2d)(3b-2d)(3b+2d)(b+2d)}-\frac{3\sin(bx-2dx+a-2c)bd^2}{4(b-2d)(3b-2d)(3b+2d)(b+2d)}$
orering	Expression too large to display

input
```
int(cos(b*x+a)^3*cos(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/8*sin(b*x+a)/b+1/24*sin(3*b*x+3*a)/b+3/16/(b-2*d)*sin(a-2*c+(b-2*d)*x)+3
/16/(b+2*d)*sin(a+2*c+(b+2*d)*x)+1/16/(3*b-2*d)*sin(3*a-2*c+(3*b-2*d)*x)+1
/16/(3*b+2*d)*sin(3*a+2*c+(3*b+2*d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.12

$$\int \cos^3(a+bx)\cos^2(c+dx)\,dx = -\frac{6\left(6\,b^3d\cos\left(bx+a\right)+\left(b^3d-4\,bd^3\right)\cos\left(bx+a\right)^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(40\,b^2d^2-16\,d^4+2\,(b^2d\right.}{3\,(9\,b^5-40\,b^3d^2+1}$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/3*(6*(6*b^3*d*cos(b*x + a) + (b^3*d - 4*b*d^3)*cos(b*x + a)^3)*cos(d*x
+ c)*sin(d*x + c) + (40*b^2*d^2 - 16*d^4 + 2*(b^2*d^2 - 4*d^4)*cos(b*x + a
)^2 - 9*(2*b^4 + (b^4 - 4*b^2*d^2)*cos(b*x + a)^2)*cos(d*x + c)^2)*sin(b*x
 + a))/(9*b^5 - 40*b^3*d^2 + 16*b*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2020 vs. 2(116) = 232.

Time = 5.68 (sec) , antiderivative size = 2020, normalized size of antiderivative = 14.64

$$\int \cos^3(a+bx)\cos^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**3*cos(d*x+c)**2,x)
```

output
```
Piecewise((x*cos(a)**3*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**
2/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*cos(a)**3, Eq
(b, 0)), (-3*x*sin(a - 2*d*x)**3*sin(c + d*x)*cos(c + d*x)/8 - 3*x*sin(a -
 2*d*x)**2*sin(c + d*x)**2*cos(a - 2*d*x)/16 + 3*x*sin(a - 2*d*x)**2*cos(a
 - 2*d*x)*cos(c + d*x)**2/16 - 3*x*sin(a - 2*d*x)*sin(c + d*x)*cos(a - 2*d
*x)**2*cos(c + d*x)/8 - 3*x*sin(c + d*x)**2*cos(a - 2*d*x)**3/16 + 3*x*cos
(a - 2*d*x)**3*cos(c + d*x)**2/16 - sin(a - 2*d*x)**3*sin(c + d*x)**2/(96*
d) - 31*sin(a - 2*d*x)**3*cos(c + d*x)**2/(96*d) - sin(a - 2*d*x)**2*sin(c
 + d*x)*cos(a - 2*d*x)*cos(c + d*x)/(8*d) - sin(a - 2*d*x)*cos(a - 2*d*x)*
*2*cos(c + d*x)**2/(2*d) - 3*sin(c + d*x)*cos(a - 2*d*x)**3*cos(c + d*x)/(
16*d), Eq(b, -2*d)), (x*sin(a - 2*d*x/3)**3*sin(c + d*x)*cos(c + d*x)/8 +
3*x*sin(a - 2*d*x/3)**2*sin(c + d*x)**2*cos(a - 2*d*x/3)/16 - 3*x*sin(a -
2*d*x/3)**2*cos(a - 2*d*x/3)*cos(c + d*x)**2/16 - 3*x*sin(a - 2*d*x/3)*sin
(c + d*x)*cos(a - 2*d*x/3)**2*cos(c + d*x)/8 - x*sin(c + d*x)**2*cos(a - 2
*d*x/3)**3/16 + x*cos(a - 2*d*x/3)**3*cos(c + d*x)**2/16 - 27*sin(a - 2*d*
x/3)**3*sin(c + d*x)**2/(32*d) - 5*sin(a - 2*d*x/3)**3*cos(c + d*x)**2/(32
*d) + 15*sin(a - 2*d*x/3)**2*sin(c + d*x)*cos(a - 2*d*x/3)*cos(c + d*x)/(8
*d) - 3*sin(a - 2*d*x/3)*cos(a - 2*d*x/3)**2*cos(c + d*x)**2/(2*d) - sin(c
 + d*x)*cos(a - 2*d*x/3)**3*cos(c + d*x)/(16*d), Eq(b, -2*d/3)), (-x*sin(a
 + 2*d*x/3)**3*sin(c + d*x)*cos(c + d*x)/8 + 3*x*sin(a + 2*d*x/3)**2*si...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(126) = 252$.

Time = 0.10 (sec) , antiderivative size = 1360, normalized size of antiderivative = 9.86

$$\int \cos^3(a+bx)\cos^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/96*(3*(3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) + 8*b*d^
3*sin(2*c))*cos((3*b + 2*d)*x + 3*a + 4*c) - 3*(3*b^4*sin(2*c) - 2*b^3*d*s
in(2*c) - 12*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*cos((3*b + 2*d)*x + 3*a)
 - 3*(3*b^4*sin(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) - 8*b*d^3*si
n(2*c))*cos(-(3*b - 2*d)*x - 3*a + 4*c) + 3*(3*b^4*sin(2*c) + 2*b^3*d*sin(
2*c) - 12*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*cos(-(3*b - 2*d)*x - 3*a) +
 9*(9*b^4*sin(2*c) - 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) + 8*b*d^3*sin(
2*c))*cos((b + 2*d)*x + a + 4*c) - 9*(9*b^4*sin(2*c) - 18*b^3*d*sin(2*c) -
 4*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*cos((b + 2*d)*x + a) - 9*(9*b^4*si
n(2*c) + 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*cos(-(
b - 2*d)*x - a + 4*c) + 9*(9*b^4*sin(2*c) + 18*b^3*d*sin(2*c) - 4*b^2*d^2*
sin(2*c) - 8*b*d^3*sin(2*c))*cos(-(b - 2*d)*x - a) + 2*(9*b^4*sin(2*c) - 4
0*b^2*d^2*sin(2*c) + 16*d^4*sin(2*c))*cos(3*b*x + 3*a + 2*c) - 2*(9*b^4*si
n(2*c) - 40*b^2*d^2*sin(2*c) + 16*d^4*sin(2*c))*cos(3*b*x + 3*a - 2*c) + 1
8*(9*b^4*sin(2*c) - 40*b^2*d^2*sin(2*c) + 16*d^4*sin(2*c))*cos(b*x + a + 2
*c) - 18*(9*b^4*sin(2*c) - 40*b^2*d^2*sin(2*c) + 16*d^4*sin(2*c))*cos(b*x
+ a - 2*c) - 3*(3*b^4*cos(2*c) - 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) +
8*b*d^3*cos(2*c))*sin((3*b + 2*d)*x + 3*a + 4*c) - 3*(3*b^4*cos(2*c) - 2*b
^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*sin((3*b + 2*d)*x
+ 3*a) + 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) - 8...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int \cos^3(a+bx)\cos^2(c+dx)\,dx = \frac{\sin(3\,bx+2\,dx+3\,a+2\,c)}{16\,(3\,b+2\,d)} + \frac{\sin(3\,bx-2\,dx+3\,a-2\,c)}{16\,(3\,b-2\,d)} + \frac{\sin(3\,bx+3\,a)}{24\,b} + \frac{3\,\sin(bx+2\,dx+a+2\,c)}{16\,(b+2\,d)} + \frac{3\,\sin(bx-2\,dx+a-2\,c)}{16\,(b-2\,d)} + \frac{3\,\sin(bx+a)}{8\,b}$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^2,x, algorithm="giac")
```

output
```
1/16*sin(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) + 1/16*sin(3*b*x - 2*d*x +
 3*a - 2*c)/(3*b - 2*d) + 1/24*sin(3*b*x + 3*a)/b + 3/16*sin(b*x + 2*d*x +
 a + 2*c)/(b + 2*d) + 3/16*sin(b*x - 2*d*x + a - 2*c)/(b - 2*d) + 3/8*sin(
b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 23.11 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.17

$$\int \cos^3(a+bx)\cos^2(c+dx)\,dx$$
$$= \frac{81\,b^4\,\sin\left(a-2\,c+b\,x-2\,d\,x\right)+81\,b^4\,\sin\left(a+2\,c+b\,x+2\,d\,x\right)+162\,b^4\,\sin\left(a+b\,x\right)+288\,d^4\,\sin\left(a+\right.}{}$$

input
```
int(cos(a + b*x)^3*cos(c + d*x)^2,x)
```

output
```
(81*b^4*sin(a - 2*c + b*x - 2*d*x) + 81*b^4*sin(a + 2*c + b*x + 2*d*x) + 1
62*b^4*sin(a + b*x) + 288*d^4*sin(a + b*x) + 9*b^4*sin(3*a - 2*c + 3*b*x -
 2*d*x) + 9*b^4*sin(3*a + 2*c + 3*b*x + 2*d*x) + 18*b^4*sin(3*a + 3*b*x) +
 32*d^4*sin(3*a + 3*b*x) - 24*b*d^3*sin(3*a - 2*c + 3*b*x - 2*d*x) + 24*b*
d^3*sin(3*a + 2*c + 3*b*x + 2*d*x) + 6*b^3*d*sin(3*a - 2*c + 3*b*x - 2*d*x
) - 6*b^3*d*sin(3*a + 2*c + 3*b*x + 2*d*x) - 36*b^2*d^2*sin(a - 2*c + b*x
- 2*d*x) - 36*b^2*d^2*sin(a + 2*c + b*x + 2*d*x) - 720*b^2*d^2*sin(a + b*x
) - 36*b^2*d^2*sin(3*a - 2*c + 3*b*x - 2*d*x) - 36*b^2*d^2*sin(3*a + 2*c +
 3*b*x + 2*d*x) - 80*b^2*d^2*sin(3*a + 3*b*x) - 72*b*d^3*sin(a - 2*c + b*x
 - 2*d*x) + 72*b*d^3*sin(a + 2*c + b*x + 2*d*x) + 162*b^3*d*sin(a - 2*c +
b*x - 2*d*x) - 162*b^3*d*sin(a + 2*c + b*x + 2*d*x))/(48*(16*b*d^4 + 9*b^5
 - 40*b^3*d^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.20

$$\int \cos^3(a+bx)\cos^2(c+dx)\,dx$$
$$= \frac{6\cos(bx+a)\cos(dx+c)\sin(bx+a)^2\sin(dx+c)\,b^3d - 24\cos(bx+a)\cos(dx+c)\sin(bx+a)^2\sin(dx$$

input
```
int(cos(b*x+a)^3*cos(d*x+c)^2,x)
```

output
```
(6*cos(a + b*x)*cos(c + d*x)*sin(a + b*x)**2*sin(c + d*x)*b**3*d - 24*cos(
a + b*x)*cos(c + d*x)*sin(a + b*x)**2*sin(c + d*x)*b*d**3 - 42*cos(a + b*x
)*cos(c + d*x)*sin(c + d*x)*b**3*d + 24*cos(a + b*x)*cos(c + d*x)*sin(c +
d*x)*b*d**3 + 9*sin(a + b*x)**3*sin(c + d*x)**2*b**4 - 36*sin(a + b*x)**3*
sin(c + d*x)**2*b**2*d**2 - 9*sin(a + b*x)**3*b**4 + 38*sin(a + b*x)**3*b*
*2*d**2 - 8*sin(a + b*x)**3*d**4 - 27*sin(a + b*x)*sin(c + d*x)**2*b**4 +
36*sin(a + b*x)*sin(c + d*x)**2*b**2*d**2 + 27*sin(a + b*x)*b**4 - 78*sin(
a + b*x)*b**2*d**2 + 24*sin(a + b*x)*d**4)/(3*b*(9*b**4 - 40*b**2*d**2 + 1
6*d**4))
```

3.342 $\int \cos^3(a+bx)\cos(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \cos^3(a+bx)\cos(c+dx)\,dx = \frac{3\sin(a-c+(b-d)x)}{8(b-d)} + \frac{\sin(3a-c+(3b-d)x)}{8(3b-d)} + \frac{3\sin(a+c+(b+d)x)}{8(b+d)} + \frac{\sin(3a+c+(3b+d)x)}{8(3b+d)}$$

output
```
3*sin(a-c+(b-d)*x)/(8*b-8*d)+sin(3*a-c+(3*b-d)*x)/(24*b-8*d)+3*sin(a+c+(b+
d)*x)/(8*b+8*d)+sin(3*a+c+(3*b+d)*x)/(24*b+8*d)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \cos^3(a+bx)\cos(c+dx)\,dx = \frac{1}{8}\left(\frac{3\sin(a-c+bx-dx)}{b-d} + \frac{\sin(3a-c+3bx-dx)}{3b-d} + \frac{\sin(3a+c+3bx+dx)}{3b+d} + \frac{3\sin(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Cos[a + b*x]^3*Cos[c + d*x],x]
```

output
```
((3*Sin[a - c + b*x - d*x])/(b - d) + Sin[3*a - c + 3*b*x - d*x]/(3*b - d)
 + Sin[3*a + c + 3*b*x + d*x]/(3*b + d) + (3*Sin[a + c + (b + d)*x])/(b +
d))/8
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\cos(c+dx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{3}{8}\cos(a+x(b-d)-c)+\frac{1}{8}\cos(3a+x(3b-d)-c)+\frac{3}{8}\cos(a+x(b+d)+c)+\frac{1}{8}\cos(3a+x(3b+d)+c)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sin(a+x(b-d)-c)}{8(b-d)}+\frac{\sin(3a+x(3b-d)-c)}{8(3b-d)}+\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(3a+x(3b+d)+c)}{8(3b+d)}$$

input
```
Int[Cos[a + b*x]^3*Cos[c + d*x],x]
```

output
```
(3*Sin[a - c + (b - d)*x])/(8*(b - d)) + Sin[3*a - c + (3*b - d)*x]/(8*(3*
b - d)) + (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[3*a + c + (3*b + d)
*x]/(8*(3*b + d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
default	$\frac{3\sin(a-c+(b-d)x)}{8(b-d)}+\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(3a-c+(3b-d)x)}{24b-8d}+\frac{\sin(3a+c+(3b+d)x)}{24b+8d}$
parallelrisch	$\frac{3\left(b+\frac{d}{3}\right)(b-d)(b+d)\sin(3a-c+(3b-d)x)+27\left(\left(b+\frac{d}{3}\right)(b+d)\sin(a-c+(b-d)x)+\left(\left(\frac{b}{9}+\frac{d}{9}\right)\sin(3a+c+(3b+d)x)+\left(b+\frac{d}{3}\right)\sin(\right.\right.}{72b^4-80b^2d^2+8d^4}$
risch	$\frac{27\sin(bx-dx+a-c)b^3}{8(-3b+d)(-b+d)(3b+d)(b+d)}+\frac{27\sin(bx-dx+a-c)b^2d}{8(-3b+d)(-b+d)(3b+d)(b+d)}-\frac{3\sin(bx-dx+a-c)b\,d^2}{8(-3b+d)(-b+d)(3b+d)(b+d)}-\frac{3\sin(bx-dx\cdot}{8(-3b+d)(-b+d)}$
orering	Expression too large to display

input
```
int(cos(b*x+a)^3*cos(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
3/8/(b-d)*sin(a-c+(b-d)*x)+3/8/(b+d)*sin(a+c+(b+d)*x)+1/8/(3*b-d)*sin(3*a-
c+(3*b-d)*x)+1/8/(3*b+d)*sin(3*a+c+(3*b+d)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int \cos^3(a+bx)\cos(c+dx)\,dx$$
$$=\frac{3\left(2\,b^3+(b^3-bd^2)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)\sin\left(bx+a\right)-\left(6\,b^2d\cos\left(bx+a\right)+(b^2d-d^3)\cos\left(bx+a\right)\right.}{9\,b^4-10\,b^2d^2+d^4}$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c),x, algorithm="fricas")
```

output
```
(3*(2*b^3 + (b^3 - b*d^2)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a) - (6*b
^2*d*cos(b*x + a) + (b^2*d - d^3)*cos(b*x + a)^3)*sin(d*x + c))/(9*b^4 - 1
0*b^2*d^2 + d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(76) = 152$.

Time = 1.98 (sec) , antiderivative size = 937, normalized size of antiderivative = 9.66

$$\int \cos^3(a + bx)\cos(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**3*cos(d*x+c),x)
```

output
```
Piecewise((x*cos(a)**3*cos(c), Eq(b, 0) & Eq(d, 0)), (-3*x*sin(a - d*x)**3
*sin(c + d*x)/8 + 3*x*sin(a - d*x)**2*cos(a - d*x)*cos(c + d*x)/8 - 3*x*si
n(a - d*x)*sin(c + d*x)*cos(a - d*x)**2/8 + 3*x*cos(a - d*x)**3*cos(c + d*
x)/8 + 3*sin(a - d*x)**3*cos(c + d*x)/(8*d) + 3*sin(a - d*x)**2*sin(c + d*
x)*cos(a - d*x)/(4*d) + 5*sin(c + d*x)*cos(a - d*x)**3/(8*d), Eq(b, -d)),
(x*sin(a - d*x/3)**3*sin(c + d*x)/8 - 3*x*sin(a - d*x/3)**2*cos(a - d*x/3)
*cos(c + d*x)/8 - 3*x*sin(a - d*x/3)*sin(c + d*x)*cos(a - d*x/3)**2/8 + x*
cos(a - d*x/3)**3*cos(c + d*x)/8 + 3*sin(a - d*x/3)**3*cos(c + d*x)/(8*d)
+ 3*sin(a - d*x/3)**2*sin(c + d*x)*cos(a - d*x/3)/(4*d) + 7*sin(c + d*x)*c
os(a - d*x/3)**3/(8*d), Eq(b, -d/3)), (-x*sin(a + d*x/3)**3*sin(c + d*x)/8
 - 3*x*sin(a + d*x/3)**2*cos(a + d*x/3)*cos(c + d*x)/8 + 3*x*sin(a + d*x/3
)*sin(c + d*x)*cos(a + d*x/3)**2/8 + x*cos(a + d*x/3)**3*cos(c + d*x)/8 -
3*sin(a + d*x/3)**3*cos(c + d*x)/(8*d) + 3*sin(a + d*x/3)**2*sin(c + d*x)*
cos(a + d*x/3)/(4*d) + 7*sin(c + d*x)*cos(a + d*x/3)**3/(8*d), Eq(b, d/3))
, (3*x*sin(a + d*x)**3*sin(c + d*x)/8 + 3*x*sin(a + d*x)**2*cos(a + d*x)*c
os(c + d*x)/8 + 3*x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)**2/8 + 3*x*cos(
a + d*x)**3*cos(c + d*x)/8 + 3*sin(a + d*x)**3*cos(c + d*x)/(8*d) + 3*sin(
a + d*x)*cos(a + d*x)**2*cos(c + d*x)/(4*d) - sin(c + d*x)*cos(a + d*x)**3
/(8*d), Eq(b, d)), (6*b**3*sin(a + b*x)**3*cos(c + d*x)/(9*b**4 - 10*b**2*
d**2 + d**4) + 9*b**3*sin(a + b*x)*cos(a + b*x)**2*cos(c + d*x)/(9*b**4...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(89) = 178$.

Time = 0.08 (sec) , antiderivative size = 787, normalized size of antiderivative = 8.11

$$\int \cos^3(a+bx)\cos(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c),x, algorithm="maxima")
```

output
```
-1/16*((3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c) + d^3*sin(c))*cos((3*
b + d)*x + 3*a + 2*c) - (3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c) + d^
3*sin(c))*cos((3*b + d)*x + 3*a) - (3*b^3*sin(c) + b^2*d*sin(c) - 3*b*d^2*
sin(c) - d^3*sin(c))*cos(-(3*b - d)*x - 3*a + 2*c) + (3*b^3*sin(c) + b^2*d
*sin(c) - 3*b*d^2*sin(c) - d^3*sin(c))*cos(-(3*b - d)*x - 3*a) + 3*(9*b^3*
sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*cos((b + d)*x + a + 2
*c) - 3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*cos((b
 + d)*x + a) - 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*sin(c) - d^3*sin(c
))*cos(-(b - d)*x - a + 2*c) + 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*si
n(c) - d^3*sin(c))*cos(-(b - d)*x - a) - (3*b^3*cos(c) - b^2*d*cos(c) - 3*
b*d^2*cos(c) + d^3*cos(c))*sin((3*b + d)*x + 3*a + 2*c) - (3*b^3*cos(c) -
b^2*d*cos(c) - 3*b*d^2*cos(c) + d^3*cos(c))*sin((3*b + d)*x + 3*a) + (3*b^
3*cos(c) + b^2*d*cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*sin(-(3*b - d)*x -
3*a + 2*c) + (3*b^3*cos(c) + b^2*d*cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*s
in(-(3*b - d)*x - 3*a) - 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) +
 d^3*cos(c))*sin((b + d)*x + a + 2*c) - 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) -
 b*d^2*cos(c) + d^3*cos(c))*sin((b + d)*x + a) + 3*(9*b^3*cos(c) + 9*b^2*d
*cos(c) - b*d^2*cos(c) - d^3*cos(c))*sin(-(b - d)*x - a + 2*c) + 3*(9*b^3*
cos(c) + 9*b^2*d*cos(c) - b*d^2*cos(c) - d^3*cos(c))*sin(-(b - d)*x - a))/
(9*b^4*cos(c)^2 + 9*b^4*sin(c)^2 + (cos(c)^2 + sin(c)^2)*d^4 - 10*(b^2*...
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \cos^3(a+bx)\cos(c+dx)\,dx = \frac{\sin(3\,bx+dx+3\,a+c)}{8\,(3\,b+d)}+\frac{\sin(3\,bx-dx+3\,a-c)}{8\,(3\,b-d)} + \frac{3\,\sin(bx+dx+a+c)}{8\,(b+d)}+\frac{3\,\sin(bx-dx+a-c)}{8\,(b-d)}$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c),x, algorithm="giac")
```

output
```
1/8*sin(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/8*sin(3*b*x - d*x + 3*a - c)/
(3*b - d) + 3/8*sin(b*x + d*x + a + c)/(b + d) + 3/8*sin(b*x - d*x + a - c
)/(b - d)
```

Mupad [B] (verification not implemented)

Time = 22.20 (sec) , antiderivative size = 495, normalized size of antiderivative = 5.10

$$\int \cos^3(a+bx)\cos(c+dx)\,dx = -\mathrm{e}^{a\,3\mathrm{i}-c\,1\mathrm{i}+b\,x\,3\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{-3\,b^3-b^2\,d+3\,b\,d^2+d^3}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}} - \frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-3\,b^3+b^2\,d+3\,b\,d^2-d^3)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}} + \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}} - \frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}}\right) - \mathrm{e}^{a\,3\mathrm{i}+c\,1\mathrm{i}+b\,x\,3\mathrm{i}+d\,x\,1\mathrm{i}}\left(\frac{-3\,b^3+b^2\,d+3\,b\,d^2-d^3}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}} - \frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,(-3\,b^3-b^2\,d+3\,b\,d^2+d^3)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}} + \frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}} - \frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}}\right)$$

input
```
int(cos(a + b*x)^3*cos(c + d*x),x)
```

output
```
- exp(a*3i - c*1i + b*x*3i - d*x*1i)*((3*b*d^2 - b^2*d - 3*b^3 + d^3)/(b^4
*144i + d^4*16i - b^2*d^2*160i) - (exp(- a*6i - b*x*6i)*(3*b*d^2 + b^2*d -
 3*b^3 - d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i) + (exp(- a*2i - b*x*2i)
*(3*b*d^2 - 27*b^2*d - 27*b^3 + 3*d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i
) - (exp(- a*4i - b*x*4i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(b^4*144i
 + d^4*16i - b^2*d^2*160i)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((3*b*d^2
 + b^2*d - 3*b^3 - d^3)/(b^4*144i + d^4*16i - b^2*d^2*160i) - (exp(- a*6i
- b*x*6i)*(3*b*d^2 - b^2*d - 3*b^3 + d^3))/(b^4*144i + d^4*16i - b^2*d^2*1
60i) + (exp(- a*2i - b*x*2i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(b^4*1
44i + d^4*16i - b^2*d^2*160i) - (exp(- a*4i - b*x*4i)*(3*b*d^2 - 27*b^2*d
- 27*b^3 + 3*d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.85

$$\int \cos^3(a+bx)\cos(c+dx)\,dx$$
$$= \frac{\cos(bx+a)\sin(bx+a)^2\sin(dx+c)\,b^2d - \cos(bx+a)\sin(bx+a)^2\sin(dx+c)\,d^3 - 7\cos(bx+a)\sin(\cdots}{\cdots}$$

input
```
int(cos(b*x+a)^3*cos(d*x+c),x)
```

output
```
(cos(a + b*x)*sin(a + b*x)**2*sin(c + d*x)*b**2*d - cos(a + b*x)*sin(a + b
*x)**2*sin(c + d*x)*d**3 - 7*cos(a + b*x)*sin(c + d*x)*b**2*d + cos(a + b*
x)*sin(c + d*x)*d**3 - 3*cos(c + d*x)*sin(a + b*x)**3*b**3 + 3*cos(c + d*x
)*sin(a + b*x)**3*b*d**2 + 9*cos(c + d*x)*sin(a + b*x)*b**3 - 3*cos(c + d*
x)*sin(a + b*x)*b*d**2)/(9*b**4 - 10*b**2*d**2 + d**4)
```

3.343 $\int \cos^3(a+bx)\sec(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 298

$$\begin{aligned}\int \cos^3(a+bx)\sec(c+dx)\,dx &= \frac{3ie^{-ia-ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2d},\frac{1}{2}\left(3-\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{4(b-d)} \\ &+ \frac{ie^{-3ia-3ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,-\frac{3b-d}{2d},\frac{3}{2}\left(1-\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{4(3b-d)} \\ &- \frac{3ie^{ia+ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{4(b+d)} \\ &- \frac{ie^{3ia+3ibx+i(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{3b+d}{2d},\frac{3(b+d)}{2d},-e^{2i(c+dx)}\right)}{4(3b+d)}\end{aligned}$$

output
```
3/4*I*exp(-I*a-I*b*x+I*(d*x+c))*hypergeom([1, -1/2*(b-d)/d],[3/2-1/2*b/d],
-exp(2*I*(d*x+c)))/(b-d)+1/4*I*exp(-3*I*a-3*I*b*x+I*(d*x+c))*hypergeom([1,
 -1/2*(3*b-d)/d],[3/2-3/2*b/d],-exp(2*I*(d*x+c)))/(3*b-d)-3/4*I*exp(I*a+I*
b*x+I*(d*x+c))*hypergeom([1, 1/2*(b+d)/d],[3/2+1/2*b/d],-exp(2*I*(d*x+c)))
/(b+d)-1/4*I*exp(3*I*a+3*I*b*x+I*(d*x+c))*hypergeom([1, 1/2*(3*b+d)/d],[3/
2*(b+d)/d],-exp(2*I*(d*x+c)))/(3*b+d)
```

Mathematica [A] (verified)

Time = 7.60 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.05

$$\int \cos^3(a+bx)\sec(c+dx)\,dx$$
$$= \frac{e^{-3i(a+bx)}\left(-\text{Hypergeometric2F1}\left(1,-\frac{3b}{d},1-\frac{3b}{d},-ie^{i(c+dx)}\right)+\text{Hypergeometric2F1}\left(1,-\frac{3b}{d},1-\frac{3b}{d},ie^{i(c+dx)}\right)\ldots\right.}{\ldots}$$

input
```
Integrate[Cos[a + b*x]^3*Sec[c + d*x],x]
```

output
```
(-Hypergeometric2F1[1, (-3*b)/d, 1 - (3*b)/d, (-I)*E^(I*(c + d*x))] + Hype
rgeometric2F1[1, (-3*b)/d, 1 - (3*b)/d, I*E^(I*(c + d*x))] + E^((2*I)*(a +
 b*x))*(-9*Hypergeometric2F1[1, -(b/d), 1 - b/d, (-I)*E^(I*(c + d*x))] + 9
*Hypergeometric2F1[1, -(b/d), 1 - b/d, I*E^(I*(c + d*x))] + E^((2*I)*(a +
b*x))*(9*Hypergeometric2F1[1, b/d, (b + d)/d, (-I)*E^(I*(c + d*x))] - 9*Hy
pergeometric2F1[1, b/d, (b + d)/d, I*E^(I*(c + d*x))] + E^((2*I)*(a + b*x)
)*(Hypergeometric2F1[1, (3*b)/d, 1 + (3*b)/d, (-I)*E^(I*(c + d*x))] - Hype
rgeometric2F1[1, (3*b)/d, 1 + (3*b)/d, I*E^(I*(c + d*x))]))))/(24*b*E^((3*
I)*(a + b*x)))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\sec(c+dx)\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \cos^3(a+bx)\sec(c+dx)dx$$

input
```
Int[Cos[a + b*x]^3*Sec[c + d*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos \left(b x+a\right)^{3} \sec \left(d x+c\right)dx$$

input
```
int(cos(b*x+a)^3*sec(d*x+c),x)
```

output
```
int(cos(b*x+a)^3*sec(d*x+c),x)
```

Fricas [F]

$$\int \cos^3(a+bx)\sec(c+dx)\,dx=\int \cos \left(b x+a\right)^{3} \sec \left(d x+c\right)\,dx$$

input
```
integrate(cos(b*x+a)^3*sec(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^3*sec(d*x + c), x)
```

Sympy [F]

$$\int \cos^3(a+bx)\sec(c+dx)\,dx=\int \cos^{3}\left(a+bx\right)\sec\left(c+dx\right)dx$$

input
```
integrate(cos(b*x+a)**3*sec(d*x+c),x)
```

output
```
Integral(cos(a + b*x)**3*sec(c + d*x), x)
```

Maxima [F]

$$\int \cos^3(a+bx)\sec(c+dx)\,dx = \int \cos(bx+a)^3\sec(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)^3*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3*sec(d*x + c), x)
```

Giac [F]

$$\int \cos^3(a+bx)\sec(c+dx)\,dx = \int \cos(bx+a)^3\sec(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)^3*sec(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sec(c+dx)\,dx = \int \frac{\cos(a+b\,x)^3}{\cos(c+d\,x)}\,dx$$

input
```
int(cos(a + b*x)^3/cos(c + d*x),x)
```

output
```
int(cos(a + b*x)^3/cos(c + d*x), x)
```

Reduce [F]

$$\int \cos^3(a+bx)\sec(c+dx)\,dx = \int \cos(bx+a)^3 \sec(dx+c)\,dx$$

input
```
int(cos(b*x+a)^3*sec(d*x+c),x)
```

output
```
int(cos(a + b*x)**3*sec(c + d*x),x)
```

3.344 $\int \cos^3(a+bx)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 289

$$\begin{aligned}\int \cos^3(a+bx)\sec^2(c+dx)\,dx &= \frac{ie^{-3ia-3ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{3b}{2d},2-\frac{3b}{2d},-e^{2i(c+dx)}\right)}{2(3b-2d)} \\ &+ \frac{3ie^{-ia-ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{b}{2d},2-\frac{b}{2d},-e^{2i(c+dx)}\right)}{2(b-2d)} \\ &- \frac{3ie^{ia+ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{b}{2d},2+\frac{b}{2d},-e^{2i(c+dx)}\right)}{2(b+2d)} \\ &- \frac{ie^{3ia+3ibx+2i(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{3b}{2d},2+\frac{3b}{2d},-e^{2i(c+dx)}\right)}{2(3b+2d)}\end{aligned}$$

output
```
1/2*I*exp(-3*I*a-3*I*b*x+2*I*(d*x+c))*hypergeom([2, 1-3/2*b/d],[2-3/2*b/d]
,-exp(2*I*(d*x+c)))/(3*b-2*d)+3/2*I*exp(-I*a-I*b*x+2*I*(d*x+c))*hypergeom(
[2, 1-1/2*b/d],[2-1/2*b/d],-exp(2*I*(d*x+c)))/(b-2*d)-3/2*I*exp(I*a+I*b*x+
2*I*(d*x+c))*hypergeom([2, 1+1/2*b/d],[2+1/2*b/d],-exp(2*I*(d*x+c)))/(b+2*
d)-1/2*I*exp(3*I*a+3*I*b*x+2*I*(d*x+c))*hypergeom([2, 1+3/2*b/d],[2+3/2*b/
d],-exp(2*I*(d*x+c)))/(3*b+2*d)
```

Mathematica [A] (verified)

Time = 5.76 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.33

$$\int \cos^3(a+bx)\sec^2(c+dx)\,dx =$$
$$-\frac{ie^{-3i(a+bx)}\left(-1-3e^{2i(a+bx)}-3e^{4i(a+bx)}-e^{6i(a+bx)}+(1+e^{2ic})\,\text{Hypergeometric2F1}\left(1,-\frac{3b}{2d},1-\frac{3b}{2d},-e\right.\right.}{}$$
$$+\frac{\cos^3(a+bx)\sec(c)\sec(c+dx)\sin(dx)}{d}$$

input
```
Integrate[Cos[a + b*x]^3*Sec[c + d*x]^2,x]
```

output
```
((-1/4*I)*(-1 - 3*E^((2*I)*(a + b*x)) - 3*E^((4*I)*(a + b*x)) - E^((6*I)*(
a + b*x)) + (1 + E^((2*I)*c))*Hypergeometric2F1[1, (-3*b)/(2*d), 1 - (3*b)
/(2*d), -E^((2*I)*(c + d*x))] + 3*E^((2*I)*(a + b*x))*(1 + E^((2*I)*c))*Hy
pergeometric2F1[1, -1/2*b/d, 1 - b/(2*d), -E^((2*I)*(c + d*x))] + 3*E^((4*
I)*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*
x))] + 3*E^((2*I)*(2*a + c + 2*b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(
2*d), -E^((2*I)*(c + d*x))] + E^((6*I)*(a + b*x))*Hypergeometric2F1[1, (3*
b)/(2*d), 1 + (3*b)/(2*d), -E^((2*I)*(c + d*x))] + E^((2*I)*(3*a + c + 3*b
*x))*Hypergeometric2F1[1, (3*b)/(2*d), 1 + (3*b)/(2*d), -E^((2*I)*(c + d*x
))]))/(d*E^((3*I)*(a + b*x))*(1 + E^((2*I)*c))) + (Cos[a + b*x]^3*Sec[c]*S
ec[c + d*x]*Sin[d*x])/d
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\sec^2(c+dx)\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \cos^3(a+bx)\sec^2(c+dx)dx$$

input
```
Int[Cos[a + b*x]^3*Sec[c + d*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos(bx+a)^3 \sec(dx+c)^2\,dx$$

input
```
int(cos(b*x+a)^3*sec(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)^3*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int \cos^3(a+bx)\sec^2(c+dx)\,dx = \int \cos(bx+a)^3 \sec(dx+c)^2\,dx$$

input
```
integrate(cos(b*x+a)^3*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^3*sec(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(a+bx)\sec^2(c+dx)\,dx = \int \cos\left(bx+a\right)^3\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(b*x+a)^3*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*((sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*cos((3*
b + 2*d)*x + 3*a + 2*c) - 4*(d*cos((3*b + 2*d)*x + 3*a + 2*c)^2 + 2*d*cos(
(3*b + 2*d)*x + 3*a + 2*c)*cos(3*b*x + 3*a) + d*cos(3*b*x + 3*a)^2 + d*sin
((3*b + 2*d)*x + 3*a + 2*c)^2 + 2*d*sin((3*b + 2*d)*x + 3*a + 2*c)*sin(3*b
*x + 3*a) + d*sin(3*b*x + 3*a)^2)*integrate(3/4*(b*cos(6*b*x + 6*a)*cos(3*
b*x + 3*a) + b*cos(4*b*x + 4*a)*cos(3*b*x + 3*a) + b*sin(6*b*x + 6*a)*sin(
3*b*x + 3*a) + b*sin(4*b*x + 4*a)*sin(3*b*x + 3*a) - b*sin(3*b*x + 3*a)*si
n(2*b*x + 2*a) + (b*cos(6*b*x + 6*a) + b*cos(4*b*x + 4*a) - b*cos(2*b*x +
2*a) - b)*cos((3*b + 2*d)*x + 3*a + 2*c) - (b*cos(2*b*x + 2*a) + b)*cos(3*
b*x + 3*a) + (b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a)
)*sin((3*b + 2*d)*x + 3*a + 2*c))/(d*cos((3*b + 2*d)*x + 3*a + 2*c)^2 + 2*
d*cos((3*b + 2*d)*x + 3*a + 2*c)*cos(3*b*x + 3*a) + d*cos(3*b*x + 3*a)^2 +
 d*sin((3*b + 2*d)*x + 3*a + 2*c)^2 + 2*d*sin((3*b + 2*d)*x + 3*a + 2*c)*s
in(3*b*x + 3*a) + d*sin(3*b*x + 3*a)^2), x) - (cos(6*b*x + 6*a) + 3*cos(4*
b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*sin((3*b + 2*d)*x + 3*a + 2*c) + cos(
3*b*x + 3*a)*sin(6*b*x + 6*a) + 3*cos(3*b*x + 3*a)*sin(4*b*x + 4*a) - (3*c
os(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a) - cos(6*b*x + 6*a)*sin(3*b*x + 3*a)
- 3*cos(4*b*x + 4*a)*sin(3*b*x + 3*a) + 3*cos(3*b*x + 3*a)*sin(2*b*x + 2*a
))/(d*cos((3*b + 2*d)*x + 3*a + 2*c)^2 + 2*d*cos((3*b + 2*d)*x + 3*a + 2*c
)*cos(3*b*x + 3*a) + d*cos(3*b*x + 3*a)^2 + d*sin((3*b + 2*d)*x + 3*a +...
```

Giac [F]

$$\int \cos^3(a+bx)\sec^2(c+dx)\,dx=\int \cos(bx+a)^3\sec(dx+c)^2\,dx$$

input
```
integrate(cos(b*x+a)^3*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sec^2(c+dx)\,dx=\int \frac{\cos\left(a+b\,x\right)^3}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int(cos(a + b*x)^3/cos(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)^3/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \cos^3(a+bx)\sec^2(c+dx)\,dx=\int \cos\left(bx+a\right)^3\sec\left(dx+c\right)^2dx$$

input
```
int(cos(b*x+a)^3*sec(d*x+c)^2,x)
```

output
```
int(cos(a + b*x)**3*sec(c + d*x)**2,x)
```

3.345 $\int \cos^3(a+bx)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 291

$$\begin{aligned}\int \cos^3(a+bx)\sec^3(c+dx)\,dx &= \frac{ie^{-3ia-3ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3}{2}\left(1-\frac{b}{d}\right),\frac{1}{2}\left(5-\frac{3b}{d}\right),-e^{2i(c+dx)}\right)}{3(b-d)}\\&\quad+\frac{3ie^{-ia-ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{b}{d}\right),\frac{1}{2}\left(5-\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{b-3d}\\&\quad-\frac{3ie^{ia+ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{b}{d}\right),\frac{1}{2}\left(5+\frac{b}{d}\right),-e^{2i(c+dx)}\right)}{b+3d}\\&\quad-\frac{ie^{3ia+3ibx+3i(c+dx)}\,\text{Hypergeometric2F1}\left(3,\frac{3(b+d)}{2d},\frac{1}{2}\left(5+\frac{3b}{d}\right),-e^{2i(c+dx)}\right)}{3(b+d)}\end{aligned}$$

output
```
1/3*I*exp(-3*I*a-3*I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2-3/2*b/d],[5/2-3/2*
b/d],-exp(2*I*(d*x+c)))/(b-d)+3*I*exp(-I*a-I*b*x+3*I*(d*x+c))*hypergeom([3
, 3/2-1/2*b/d],[5/2-1/2*b/d],-exp(2*I*(d*x+c)))/(b-3*d)-3*I*exp(I*a+I*b*x+
3*I*(d*x+c))*hypergeom([3, 3/2+1/2*b/d],[5/2+1/2*b/d],-exp(2*I*(d*x+c)))/(
b+3*d)-1/3*I*exp(3*I*a+3*I*b*x+3*I*(d*x+c))*hypergeom([3, 3/2*(b+d)/d],[5/
2+3/2*b/d],-exp(2*I*(d*x+c)))/(b+d)
```

Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.03

$$\int \cos^3(a+bx)\sec^3(c+dx)\,dx$$
$$= \frac{-2ie^{-i(3a-c+3bx-dx)}\left((3b+d)\,\text{Hypergeometric2F1}\left(1,\frac{-3b+d}{2d},\frac{3}{2}-\frac{3b}{2d},-e^{2i(c+dx)}\right)+3(b+d)e^{2i(a+bx)}\,\text{Hyper}\right.}{}$$

input
```
Integrate[Cos[a + b*x]^3*Sec[c + d*x]^3,x]
```

output
```
(((-2*I)*((3*b + d)*Hypergeometric2F1[1, (-3*b + d)/(2*d), 3/2 - (3*b)/(2*
d), -E^((2*I)*(c + d*x))] + 3*(b + d)*E^((2*I)*(a + b*x))*Hypergeometric2F
1[1, (-b + d)/(2*d), 3/2 - b/(2*d), -E^((2*I)*(c + d*x))] - 3*(b - d)*E^((
4*I)*(a + b*x))*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, -E^((2*I)
*(c + d*x))] - (3*b - d)*E^((6*I)*(a + b*x))*Hypergeometric2F1[1, (3*b + d
)/(2*d), (3*(b + d))/(2*d), -E^((2*I)*(c + d*x))]))/E^(I*(3*a - c + 3*b*x
- d*x)) + 4*Cos[a + b*x]^2*Sec[c + d*x]^2*((3*b - d)*Sin[a - c + b*x - d*x
] + (3*b + d)*Sin[a + c + (b + d)*x]))/(16*d^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\sec^3(c+dx)\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \cos^3(a+bx)\sec^3(c+dx)dx$$

input
```
Int[Cos[a + b*x]^3*Sec[c + d*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos (bx+a)^3 \sec (dx+c)^3 \, dx$$

input
```
int(cos(b*x+a)^3*sec(d*x+c)^3,x)
```

output
```
int(cos(b*x+a)^3*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int \cos^3(a+bx) \sec^3(c+dx) \, dx = \int \cos (bx+a)^3 \sec (dx+c)^3 \, dx$$

input
```
integrate(cos(b*x+a)^3*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^3*sec(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx) \sec^3(c+dx) \, dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(a+bx)\sec^3(c+dx)\,dx = \int \cos(bx+a)^3\sec(dx+c)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/8*((3*b - d)*cos(3*b*x + 3*a)*sin((6*b + d)*x + 6*a + c) + 3*(b - d)*cos
(3*b*x + 3*a)*sin((4*b + d)*x + 4*a + c) - 3*(b + d)*cos(3*b*x + 3*a)*sin(
(2*b + d)*x + 2*a + c) - (3*b - d)*cos((6*b + d)*x + 6*a + c)*sin(3*b*x +
3*a) - 3*(b - d)*cos((4*b + d)*x + 4*a + c)*sin(3*b*x + 3*a) + 3*(b + d)*c
os((2*b + d)*x + 2*a + c)*sin(3*b*x + 3*a) + (3*b - d)*cos(3*d*x + 3*c)*si
n(3*b*x + 3*a) + (3*b + d)*cos(d*x + c)*sin(3*b*x + 3*a) - (3*b - d)*cos(3
*b*x + 3*a)*sin(3*d*x + 3*c) - (3*b + d)*cos(3*b*x + 3*a)*sin(d*x + c) - 3
*(2*(b + d)*sin((3*b + 2*d)*x + 3*a + 2*c) + (b + d)*sin(3*b*x + 3*a))*cos
((4*b + 3*d)*x + 4*a + 3*c) + ((3*b - d)*sin((6*b + d)*x + 6*a + c) + 3*(b
 + d)*sin((4*b + 3*d)*x + 4*a + 3*c) + 3*(b - d)*sin((4*b + d)*x + 4*a + c
) - 3*(b - d)*sin((2*b + 3*d)*x + 2*a + 3*c) + (3*b + d)*sin(3*(2*b + d)*x
 + 6*a + 3*c) - 3*(b + d)*sin((2*b + d)*x + 2*a + c) - (3*b - d)*sin(3*d*x
 + 3*c) - (3*b + d)*sin(d*x + c))*cos((3*b + 4*d)*x + 3*a + 4*c) + 2*((3*b
 - d)*sin((6*b + d)*x + 6*a + c) + 3*(b - d)*sin((4*b + d)*x + 4*a + c) -
3*(b + d)*sin((2*b + d)*x + 2*a + c) - (3*b - d)*sin(3*d*x + 3*c) - (3*b +
 d)*sin(d*x + c))*cos((3*b + 2*d)*x + 3*a + 2*c) + 3*(2*(b - d)*sin((3*b +
 2*d)*x + 3*a + 2*c) + (b - d)*sin(3*b*x + 3*a))*cos((2*b + 3*d)*x + 2*a +
 3*c) - (2*(3*b + d)*sin((3*b + 2*d)*x + 3*a + 2*c) + (3*b + d)*sin(3*b*x
+ 3*a))*cos(3*(2*b + d)*x + 6*a + 3*c) + 8*(d^2*cos((3*b + 4*d)*x + 3*a +
4*c)^2 + 4*d^2*cos((3*b + 2*d)*x + 3*a + 2*c)^2 + 4*d^2*cos((3*b + 2*d)...
```

Giac [F]

$$\int \cos^3(a+bx)\sec^3(c+dx)\,dx = \int \cos(bx+a)^3\sec(dx+c)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sec^3(c+dx)\,dx=\int \frac{\cos\left(a+b\,x\right)^3}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int(cos(a + b*x)^3/cos(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)^3/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \cos^3(a+bx)\sec^3(c+dx)\,dx=\int \cos\left(bx+a\right)^3\sec\left(dx+c\right)^3dx$$

input
```
int(cos(b*x+a)^3*sec(d*x+c)^3,x)
```

output
```
int(cos(a + b*x)**3*sec(c + d*x)**3,x)
```

3.346 $\int \cos^3(a+bx)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 281

$$
\begin{aligned}
\int \cos^3(a+bx)\sec^4(c+dx)\,dx
&= \frac{2ie^{-3ia-3ibx+4i(c+dx)}\,\text{Hypergeometric2F1}\left(4,2-\frac{3b}{2d},3-\frac{3b}{2d},-e^{2i(c+dx)}\right)}{3b-4d} \\
&\quad + \frac{6ie^{-ia-ibx+4i(c+dx)}\,\text{Hypergeometric2F1}\left(4,2-\frac{b}{2d},3-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b-4d} \\
&\quad - \frac{6ie^{ia+ibx+4i(c+dx)}\,\text{Hypergeometric2F1}\left(4,2+\frac{b}{2d},3+\frac{b}{2d},-e^{2i(c+dx)}\right)}{b+4d} \\
&\quad - \frac{2ie^{3ia+3ibx+4i(c+dx)}\,\text{Hypergeometric2F1}\left(4,2+\frac{3b}{2d},3+\frac{3b}{2d},-e^{2i(c+dx)}\right)}{3b+4d}
\end{aligned}
$$

output

```
2*I*exp(-3*I*a-3*I*b*x+4*I*(d*x+c))*hypergeom([4, 2-3/2*b/d],[3-3/2*b/d],-
exp(2*I*(d*x+c)))/(3*b-4*d)+6*I*exp(-I*a-I*b*x+4*I*(d*x+c))*hypergeom([4,
2-1/2*b/d],[3-1/2*b/d],-exp(2*I*(d*x+c)))/(b-4*d)-6*I*exp(I*a+I*b*x+4*I*(d
*x+c))*hypergeom([4, 2+1/2*b/d],[3+1/2*b/d],-exp(2*I*(d*x+c)))/(b+4*d)-2*I
*exp(3*I*a+3*I*b*x+4*I*(d*x+c))*hypergeom([4, 2+3/2*b/d],[3+3/2*b/d],-exp(
2*I*(d*x+c)))/(3*b+4*d)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1347 vs. 2(281) = 562.

Time = 9.25 (sec) , antiderivative size = 1347, normalized size of antiderivative = 4.79

$$\int \cos^3(a+bx)\sec^4(c+dx)\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[a + b*x]^3*Sec[c + d*x]^4,x]
```

output
```
(9*b^3*(((-I)*E^(I*(3*a + c + 3*b*x + 2*d*x))*Hypergeometric2F1[1, 1 + (3*
b)/(2*d), 2 + (3*b)/(2*d), -E^((2*I)*(c + d*x))])/(3*b + 2*d) + ((I/3)*Hyp
ergeometric2F1[1, (-3*b)/(2*d), 1 - (3*b)/(2*d), -E^((2*I)*(c + d*x))])/(b
*E^(I*(3*a - c + 3*b*x))))*Sec[c])/(16*d^3) - (b*(((-I)*E^(I*(3*a + c + 3*
b*x + 2*d*x))*Hypergeometric2F1[1, 1 + (3*b)/(2*d), 2 + (3*b)/(2*d), -E^((
2*I)*(c + d*x))])/(3*b + 2*d) + ((I/3)*Hypergeometric2F1[1, (-3*b)/(2*d),
1 - (3*b)/(2*d), -E^((2*I)*(c + d*x))])/(b*E^(I*(3*a - c + 3*b*x))))*Sec[c
])/(4*d) + (b^3*(((-I)*E^(I*(a + c + b*x + 2*d*x))*Hypergeometric2F1[1, 1
+ b/(2*d), 2 + b/(2*d), -E^((2*I)*(c + d*x))])/(b + 2*d) + (I*Hypergeometr
ic2F1[1, -1/2*b/d, 1 - b/(2*d), -E^((2*I)*(c + d*x))])/(b*E^(I*(a - c + b*
x))))*Sec[c])/(16*d^3) - (b*(((-I)*E^(I*(a + c + b*x + 2*d*x))*Hypergeomet
ric2F1[1, 1 + b/(2*d), 2 + b/(2*d), -E^((2*I)*(c + d*x))])/(b + 2*d) + (I*
Hypergeometric2F1[1, -1/2*b/d, 1 - b/(2*d), -E^((2*I)*(c + d*x))])/(b*E^(I
*(a - c + b*x))))*Sec[c])/(4*d) - ((I/16)*b^2*(b*Hypergeometric2F1[1, 1 -
b/(2*d), 2 - b/(2*d), -E^((2*I)*(c + d*x))] - (b - 2*d)*E^((2*I)*(a + (b -
 d)*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))])*
Sec[c])/((b - 2*d)*d^3*E^(I*(a - c + (b - 2*d)*x))) + ((I/4)*(b*Hypergeome
tric2F1[1, 1 - b/(2*d), 2 - b/(2*d), -E^((2*I)*(c + d*x))] - (b - 2*d)*E^(
(2*I)*(a + (b - d)*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I
)*(c + d*x))])*Sec[c])/((b - 2*d)*d*E^(I*(a - c + (b - 2*d)*x))) + (((3...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\sec^4(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \cos^3(a+bx)\sec^4(c+dx)dx$$

input
```
Int[Cos[a + b*x]^3*Sec[c + d*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \cos(bx+a)^3\sec(dx+c)^4\,dx$$

input
```
int(cos(b*x+a)^3*sec(d*x+c)^4,x)
```

output
```
int(cos(b*x+a)^3*sec(d*x+c)^4,x)
```

Fricas [F]

$$\int \cos^3(a+bx)\sec^4(c+dx)\,dx = \int \cos\left(bx+a\right)^3\sec\left(dx+c\right)^4\,dx$$

input
```
integrate(cos(b*x+a)^3*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^3*sec(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(a+bx)\sec^4(c+dx)\,dx = \int \cos\left(bx+a\right)^3\sec\left(dx+c\right)^4\,dx$$

input
```
integrate(cos(b*x+a)^3*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/24*(6*(b^2 - b*d - 6*d^2)*cos(3*b*x + 3*a)*sin(2*(b + d)*x + 2*a + 2*c)
+ (9*b^2 - 4*d^2)*cos(3*b*x + 3*a)*sin(6*b*x + 6*a) + 3*(b^2 - 4*d^2)*cos(
3*b*x + 3*a)*sin(4*b*x + 4*a) - 6*(b^2 - b*d - 6*d^2)*cos(2*(b + d)*x + 2*
a + 2*c)*sin(3*b*x + 3*a) - (9*b^2 - 4*d^2)*cos(6*b*x + 6*a)*sin(3*b*x + 3
*a) - 3*(b^2 - 4*d^2)*cos(4*b*x + 4*a)*sin(3*b*x + 3*a) - 3*(3*b^2 - 2*b*d
)*cos(4*d*x + 4*c)*sin(3*b*x + 3*a) - 6*(3*b^2 - b*d - 2*d^2)*cos(2*d*x +
2*c)*sin(3*b*x + 3*a) + 3*(b^2 - 4*d^2)*cos(3*b*x + 3*a)*sin(2*b*x + 2*a)
+ 3*(3*b^2 - 2*b*d)*cos(3*b*x + 3*a)*sin(4*d*x + 4*c) + 6*(3*b^2 - b*d - 2
*d^2)*cos(3*b*x + 3*a)*sin(2*d*x + 2*c) + 3*(6*(3*b^2 + b*d - 2*d^2)*sin(2
*(3*b + d)*x + 6*a + 2*c) + 6*(b^2 + b*d - 6*d^2)*sin(2*(2*b + d)*x + 4*a
+ 2*c) + 3*(b^2 - 2*b*d)*sin(2*(b + 2*d)*x + 2*a + 4*c) + 6*(b^2 - b*d - 6
*d^2)*sin(2*(b + d)*x + 2*a + 2*c) + (9*b^2 - 4*d^2)*sin(6*b*x + 6*a) + 3*
(b^2 - 4*d^2)*sin(4*b*x + 4*a) + 3*(b^2 - 4*d^2)*sin(2*b*x + 2*a) + 3*(3*b
^2 - 2*b*d)*sin(4*d*x + 4*c) + 6*(3*b^2 - b*d - 2*d^2)*sin(2*d*x + 2*c))*c
os((3*b + 4*d)*x + 3*a + 4*c) - 3*(3*(3*b^2 + 2*b*d)*sin((3*b + 4*d)*x + 3
*a + 4*c) + 3*(3*b^2 + 2*b*d)*sin((3*b + 2*d)*x + 3*a + 2*c) + (3*b^2 + 2*
b*d)*sin(3*b*x + 3*a))*cos(2*(3*b + 2*d)*x + 6*a + 4*c) + 3*(6*(b^2 - b*d
- 6*d^2)*sin(2*(b + d)*x + 2*a + 2*c) + (9*b^2 - 4*d^2)*sin(6*b*x + 6*a) +
 3*(b^2 - 4*d^2)*sin(4*b*x + 4*a) + 3*(b^2 - 4*d^2)*sin(2*b*x + 2*a) + 3*(
3*b^2 - 2*b*d)*sin(4*d*x + 4*c) + 6*(3*b^2 - b*d - 2*d^2)*sin(2*d*x + 2...
```

Giac [F]

$$\int \cos^3(a+bx)\sec^4(c+dx)\,dx=\int \cos\left(bx+a\right)^3\sec\left(dx+c\right)^4\,dx$$

input
```
integrate(cos(b*x+a)^3*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sec^4(c+dx)\,dx=\int \frac{\cos\left(a+b\,x\right)^3}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int(cos(a + b*x)^3/cos(c + d*x)^4,x)
```

output
```
int(cos(a + b*x)^3/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int \cos^3(a+bx)\sec^4(c+dx)\,dx=\int \cos\left(bx+a\right)^3\sec\left(dx+c\right)^4dx$$

input
```
int(cos(b*x+a)^3*sec(d*x+c)^4,x)
```

output
```
int(cos(b*x+a)^3*sec(d*x+c)^4,x)
```

3.347 $\int \sec(a+bx)\sec(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \sec(a+bx)\sec(c+dx)\,dx=\text{Int}(\sec(a+bx)\sec(c+dx),x)$$

output
```
Defer(Int)(sec(b*x+a)*sec(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 9.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sec(a+bx)\sec(c+dx)\,dx=\int \sec(a+bx)\sec(c+dx)\,dx$$

input
```
Integrate[Sec[a + b*x]*Sec[c + d*x],x]
```

output
```
Integrate[Sec[a + b*x]*Sec[c + d*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\sec(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec(a+bx)\sec(c+dx)dx$$

input
```
Int[Sec[a + b*x]*Sec[c + d*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sec(bx+a)\sec(dx+c)\,dx$$

input
```
int(sec(b*x+a)*sec(d*x+c),x)
```

output
```
int(sec(b*x+a)*sec(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sec(a+bx)\sec(c+dx)\,dx=\int \sec\left(bx+a\right)\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(b*x+a)*sec(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*sec(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \sec(a+bx)\sec(c+dx)\,dx=\int \sec\left(a+bx\right)\sec\left(c+dx\right)dx$$

input
```
integrate(sec(b*x+a)*sec(d*x+c),x)
```

output
```
Integral(sec(a + b*x)*sec(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sec(a+bx)\sec(c+dx)\,dx = \int \sec\left(bx+a\right)\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(b*x+a)*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)*sec(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sec(a+bx)\sec(c+dx)\,dx = \int \sec\left(bx+a\right)\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(b*x+a)*sec(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sec(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 21.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \sec(a+bx)\sec(c+dx)\,dx = \int \frac{1}{\cos\left(a+b\,x\right)\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(1/(cos(a + b*x)*cos(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)*cos(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 130, normalized size of antiderivative = 10.00

$$\int \sec(a+bx)\sec(c+dx)\,dx$$
$$= \frac{4\left(\int \frac{1}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}dx\right)bd+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)d-\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{bd}$$

input
```
int(sec(b*x+a)*sec(d*x+c),x)
```

output
```
(4*int(1/(tan((a + b*x)/2)**2*tan((c + d*x)/2)**2 - tan((a + b*x)/2)**2 -
tan((c + d*x)/2)**2 + 1),x)*b*d + log(tan((a + b*x)/2) - 1)*d - log(tan((a
 + b*x)/2) + 1)*d + log(tan((c + d*x)/2) - 1)*b - log(tan((c + d*x)/2) + 1
)*b - b*d*x)/(b*d)
```

3.348 $\int \sec(a+bx)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \sec(a+bx)\sec^2(c+dx)\,dx = \mathrm{Int}\left(\sec(a+bx)\sec^2(c+dx),x\right)$$

output
```
Defer(Int)(sec(b*x+a)*sec(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 15.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \sec(a+bx)\sec^2(c+dx)\,dx = \int \sec(a+bx)\sec^2(c+dx)\,dx$$

input
```
Integrate[Sec[a + b*x]*Sec[c + d*x]^2,x]
```

output
```
Integrate[Sec[a + b*x]*Sec[c + d*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\sec^2(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec(a+bx)\sec^2(c+dx)dx$$

input
```
Int[Sec[a + b*x]*Sec[c + d*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec(bx+a)\sec(dx+c)^2\,dx$$

input
```
int(sec(b*x+a)*sec(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)*sec(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \sec(a+bx)\sec^2(c+dx)\,dx = \int \sec\left(bx+a\right)\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(b*x+a)*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*sec(d*x + c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\sec^2(c+dx)\,dx = \int \sec\left(a+bx\right)\sec^2\left(c+dx\right)dx$$

input
```
integrate(sec(b*x+a)*sec(d*x+c)**2,x)
```

output
```
Integral(sec(a + b*x)*sec(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 1488, normalized size of antiderivative = 99.20

$$\int \sec(a+bx)\sec^2(c+dx)\,dx = \int \sec(bx+a)\sec(dx+c)^2\,dx$$

input
```
integrate(sec(b*x+a)*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
-((d*cos(2*(b + d)*x + 2*a + 2*c)^2 + d*cos(2*b*x + 2*a)^2 + d*cos(2*d*x +
 2*c)^2 + d*sin(2*(b + d)*x + 2*a + 2*c)^2 + d*sin(2*b*x + 2*a)^2 + 2*d*si
n(2*b*x + 2*a)*sin(2*d*x + 2*c) + d*sin(2*d*x + 2*c)^2 + 2*(d*cos(2*b*x +
2*a) + d*cos(2*d*x + 2*c) + d)*cos(2*(b + d)*x + 2*a + 2*c) + 2*d*cos(2*b*
x + 2*a) + 2*(d*cos(2*b*x + 2*a) + d)*cos(2*d*x + 2*c) + 2*(d*sin(2*b*x +
2*a) + d*sin(2*d*x + 2*c))*sin(2*(b + d)*x + 2*a + 2*c) + d)*integrate(-4*
(2*b*cos(2*b*x + 2*a)*cos(b*x + a) - 2*b*sin(3*b*x + 3*a)*sin(2*b*x + 2*a)
 + 2*b*sin(2*b*x + 2*a)*sin(b*x + a) - (b*cos(3*b*x + 3*a) - b*cos(b*x + a
))*cos(2*(2*b + d)*x + 4*a + 2*c) - 2*(b*cos(3*b*x + 3*a) - b*cos(b*x + a)
)*cos(2*(b + d)*x + 2*a + 2*c) - (b*cos(3*b*x + 3*a) - b*cos(b*x + a))*cos
(4*b*x + 4*a) - (2*b*cos(2*b*x + 2*a) + b)*cos(3*b*x + 3*a) + b*cos(b*x +
a) - (b*cos(3*b*x + 3*a) - b*cos(b*x + a))*cos(2*d*x + 2*c) - (b*sin(3*b*x
 + 3*a) - b*sin(b*x + a))*sin(2*(2*b + d)*x + 4*a + 2*c) - 2*(b*sin(3*b*x
+ 3*a) - b*sin(b*x + a))*sin(2*(b + d)*x + 2*a + 2*c) - (b*sin(3*b*x + 3*a
) - b*sin(b*x + a))*sin(4*b*x + 4*a) - (b*sin(3*b*x + 3*a) - b*sin(b*x + a
))*sin(2*d*x + 2*c))/(d*cos(2*(2*b + d)*x + 4*a + 2*c)^2 + 4*d*cos(2*(b +
d)*x + 2*a + 2*c)^2 + d*cos(4*b*x + 4*a)^2 + 4*d*cos(2*b*x + 2*a)^2 + d*co
s(2*d*x + 2*c)^2 + d*sin(2*(2*b + d)*x + 4*a + 2*c)^2 + 4*d*sin(2*(b + d)*
x + 2*a + 2*c)^2 + d*sin(4*b*x + 4*a)^2 + 4*d*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) + 4*d*sin(2*b*x + 2*a)^2 + d*sin(2*d*x + 2*c)^2 + 2*(2*d*cos(2*(b...
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \sec(a+bx)\sec^2(c+dx)\,dx = \int \sec(bx+a)\sec(dx+c)^2\,dx$$

input
```
integrate(sec(b*x+a)*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sec(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 22.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \sec(a+bx)\sec^2(c+dx)\,dx = \int \frac{1}{\cos(a+b\,x)\,\cos(c+d\,x)^2}\,dx$$

input
```
int(1/(cos(a + b*x)*cos(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)*cos(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \sec(a+bx)\sec^2(c+dx)\,dx = \int \sec(bx+a)\sec(dx+c)^2\,dx$$

input
```
int(sec(b*x+a)*sec(d*x+c)^2,x)
```

output
```
int(sec(a + b*x)*sec(c + d*x)**2,x)
```

3.349 $\int \sec(a+bx)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \sec(a+bx)\sec^3(c+dx)\,dx = \operatorname{Int}\left(\sec(a+bx)\sec^3(c+dx),x\right)$$

output
```
Defer(Int)(sec(b*x+a)*sec(d*x+c)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 10.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \sec(a+bx)\sec^3(c+dx)\,dx = \int \sec(a+bx)\sec^3(c+dx)\,dx$$

input
```
Integrate[Sec[a + b*x]*Sec[c + d*x]^3,x]
```

output
```
Integrate[Sec[a + b*x]*Sec[c + d*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\sec^3(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sec(a+bx)\sec^3(c+dx)dx$$

input
```
Int[Sec[a + b*x]*Sec[c + d*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec (bx+a)\sec (dx+c)^3\,dx$$

input
```
int(sec(b*x+a)*sec(d*x+c)^3,x)
```

output
```
int(sec(b*x+a)*sec(d*x+c)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \sec(a+bx)\sec^3(c+dx)\,dx = \int \sec\left(bx+a\right)\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(b*x+a)*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*sec(d*x + c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 4.94 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\sec^3(c+dx)\,dx = \int \sec\left(a+bx\right)\sec^3\left(c+dx\right)dx$$

input
```
integrate(sec(b*x+a)*sec(d*x+c)**3,x)
```

output
```
Integral(sec(a + b*x)*sec(c + d*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 5.45 (sec) , antiderivative size = 6267, normalized size of antiderivative = 417.80

$$\int \sec(a+bx)\sec^3(c+dx)\,dx = \int \sec(bx+a)\sec(dx+c)^3\,dx$$

input
```
integrate(sec(b*x+a)*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
(2*((b + d)*sin(4*b*x + 4*a) + 2*(b + d)*sin(2*b*x + 2*a) + (b + d)*sin(4*
d*x + 4*c) + 2*(b + d)*sin(2*d*x + 2*c))*cos((3*b + d)*x + 3*a + c) - 4*((
b + d)*sin((3*b + d)*x + 3*a + c) - (b - d)*sin((b + d)*x + a + c))*cos(2*
(2*b + d)*x + 4*a + 2*c) - 2*(2*(b + d)*sin(2*(2*b + d)*x + 4*a + 2*c) + 4
*(b + d)*sin(2*(b + d)*x + 2*a + 2*c) + (b + d)*sin(4*b*x + 4*a) + 2*(b +
d)*sin(2*b*x + 2*a) + (b + d)*sin(4*d*x + 4*c) + 2*(b + d)*sin(2*d*x + 2*c
))*cos((b + 3*d)*x + a + 3*c) - 4*((b + d)*sin((3*b + d)*x + 3*a + c) - (b
 + d)*sin((b + 3*d)*x + a + 3*c) + (b - d)*sin(3*(b + d)*x + 3*a + 3*c) -
(b - d)*sin((b + d)*x + a + c))*cos(2*(b + 2*d)*x + 2*a + 4*c) - 2*((b + d
)*sin((3*b + d)*x + 3*a + c) - (b + d)*sin((b + 3*d)*x + a + 3*c) + (b - d
)*sin(3*(b + d)*x + 3*a + 3*c) - (b - d)*sin((b + d)*x + a + c))*cos(4*(b
+ d)*x + 4*a + 4*c) + 2*(2*(b - d)*sin(2*(2*b + d)*x + 4*a + 2*c) + 4*(b -
 d)*sin(2*(b + d)*x + 2*a + 2*c) + (b - d)*sin(4*b*x + 4*a) + 2*(b - d)*si
n(2*b*x + 2*a) + (b - d)*sin(4*d*x + 4*c) + 2*(b - d)*sin(2*d*x + 2*c))*co
s(3*(b + d)*x + 3*a + 3*c) - 8*((b + d)*sin((3*b + d)*x + 3*a + c) - (b -
d)*sin((b + d)*x + a + c))*cos(2*(b + d)*x + 2*a + 2*c) - 2*((b - d)*sin(4
*b*x + 4*a) + 2*(b - d)*sin(2*b*x + 2*a) + (b - d)*sin(4*d*x + 4*c) + 2*(b
 - d)*sin(2*d*x + 2*c))*cos((b + d)*x + a + c) + (4*d^2*cos(2*(2*b + d)*x
+ 4*a + 2*c)^2 + 4*d^2*cos(2*(b + 2*d)*x + 2*a + 4*c)^2 + d^2*cos(4*(b + d
)*x + 4*a + 4*c)^2 + 16*d^2*cos(2*(b + d)*x + 2*a + 2*c)^2 + d^2*cos(4*...
```

Giac [N/A]

Not integrable

Time = 17.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \sec(a+bx)\sec^3(c+dx)\,dx=\int \sec(bx+a)\sec(dx+c)^3\,dx$$

input
```
integrate(sec(b*x+a)*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sec(d*x + c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 21.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \sec(a+bx)\sec^3(c+dx)\,dx=\int \frac{1}{\cos(a+b\,x)\,\cos(c+d\,x)^3}\,dx$$

input
```
int(1/(cos(a + b*x)*cos(c + d*x)^3),x)
```

output
```
int(1/(cos(a + b*x)*cos(c + d*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \sec(a+bx)\sec^3(c+dx)\,dx=\int \sec(bx+a)\sec(dx+c)^3\,dx$$

input
```
int(sec(b*x+a)*sec(d*x+c)^3,x)
```

output
```
int(sec(a + b*x)*sec(c + d*x)**3,x)
```

3.350 $\int \sec^2(a+bx)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \sec^2(a+bx)\sec^2(c+dx)\,dx = \mathrm{Int}\left(\sec^2(a+bx)\sec^2(c+dx),x\right)$$

output
```
Defer(Int)(sec(b*x+a)^2*sec(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 21.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec^2(a+bx)\sec^2(c+dx)\,dx = \int \sec^2(a+bx)\sec^2(c+dx)\,dx$$

input
```
Integrate[Sec[a + b*x]^2*Sec[c + d*x]^2,x]
```

output
```
Integrate[Sec[a + b*x]^2*Sec[c + d*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\sec^2(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec^2(a+bx)\sec^2(c+dx)dx$$

input
```
Int[Sec[a + b*x]^2*Sec[c + d*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec(bx+a)^2 \sec(dx+c)^2\,dx$$

input
```
int(sec(b*x+a)^2*sec(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)^2*sec(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec^2(a+bx)\sec^2(c+dx)\,dx = \int \sec\left(bx+a\right)^2\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(b*x+a)^2*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2*sec(d*x + c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 5.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\sec^2(c+dx)\,dx = \int \sec^2\left(a+bx\right)\sec^2\left(c+dx\right)dx$$

input
```
integrate(sec(b*x+a)**2*sec(d*x+c)**2,x)
```

output
```
Integral(sec(a + b*x)**2*sec(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 2684, normalized size of antiderivative = 157.88

$$\int \sec^2(a+bx)\sec^2(c+dx)\,dx = \int \sec(bx+a)^2\sec(dx+c)^2\,dx$$

input
```
integrate(sec(b*x+a)^2*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
-((d*cos(2*(2*b + d)*x + 4*a + 2*c)^2 + 4*d*cos(2*(b + d)*x + 2*a + 2*c)^2
 + d*cos(4*b*x + 4*a)^2 + 4*d*cos(2*b*x + 2*a)^2 + d*cos(2*d*x + 2*c)^2 +
d*sin(2*(2*b + d)*x + 4*a + 2*c)^2 + 4*d*sin(2*(b + d)*x + 2*a + 2*c)^2 +
d*sin(4*b*x + 4*a)^2 + 4*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*d*sin(2*b
*x + 2*a)^2 + d*sin(2*d*x + 2*c)^2 + 2*(2*d*cos(2*(b + d)*x + 2*a + 2*c) +
 d*cos(4*b*x + 4*a) + 2*d*cos(2*b*x + 2*a) + d*cos(2*d*x + 2*c) + d)*cos(2
*(2*b + d)*x + 4*a + 2*c) + 4*(d*cos(4*b*x + 4*a) + 2*d*cos(2*b*x + 2*a) +
 d*cos(2*d*x + 2*c) + d)*cos(2*(b + d)*x + 2*a + 2*c) + 2*(2*d*cos(2*b*x +
 2*a) + d)*cos(4*b*x + 4*a) + 4*d*cos(2*b*x + 2*a) + 2*(d*cos(4*b*x + 4*a)
 + 2*d*cos(2*b*x + 2*a) + d)*cos(2*d*x + 2*c) + 2*(2*d*sin(2*(b + d)*x + 2
*a + 2*c) + d*sin(4*b*x + 4*a) + 2*d*sin(2*b*x + 2*a) + d*sin(2*d*x + 2*c)
)*sin(2*(2*b + d)*x + 4*a + 2*c) + 4*(d*sin(4*b*x + 4*a) + 2*d*sin(2*b*x +
 2*a) + d*sin(2*d*x + 2*c))*sin(2*(b + d)*x + 2*a + 2*c) + 2*(d*sin(4*b*x
+ 4*a) + 2*d*sin(2*b*x + 2*a))*sin(2*d*x + 2*c) + d)*integrate(16*(3*b*cos
(4*b*x + 4*a)^2 - 3*b*cos(2*b*x + 2*a)^2 + 3*b*sin(4*b*x + 4*a)^2 - 3*b*si
n(2*b*x + 2*a)^2 + (b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a))*cos(2*(3*b +
d)*x + 6*a + 2*c) + 3*(b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a))*cos(2*(2*b
 + d)*x + 4*a + 2*c) + 3*(b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a))*cos(2*(
b + d)*x + 2*a + 2*c) + (b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a))*cos(6*b*
x + 6*a) + b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) + (b*cos(4*b*x + 4*a...
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec^2(a+bx)\sec^2(c+dx)\,dx = \int \sec(bx+a)^2\sec(dx+c)^2\,dx$$

input
```
integrate(sec(b*x+a)^2*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^2*sec(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 20.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec^2(a+bx)\sec^2(c+dx)\,dx = \int \frac{1}{\cos(a+b\,x)^2\cos(c+d\,x)^2}\,dx$$

input
```
int(1/(cos(a + b*x)^2*cos(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^2*cos(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 8.47

$$\int \sec^2(a+bx)\sec^2(c+dx)\,dx$$
$$= \frac{\cos(bx+a)\cos(dx+c)\left(\int \frac{\sin(bx+a)^2\sin(dx+c)^2}{\sin(bx+a)^2\sin(dx+c)^2-\sin(bx+a)^2-\sin(dx+c)^2+1}dx\right)bd-\cos(bx+a)\cos(dx+c)\,bdx}{\cos(bx+a)\cos(dx+c)\,bd}$$

input
```
int(sec(b*x+a)^2*sec(d*x+c)^2,x)
```

output
```
(cos(a + b*x)*cos(c + d*x)*int((sin(a + b*x)**2*sin(c + d*x)**2)/(sin(a +
b*x)**2*sin(c + d*x)**2 - sin(a + b*x)**2 - sin(c + d*x)**2 + 1),x)*b*d -
cos(a + b*x)*cos(c + d*x)*b*d*x + cos(a + b*x)*sin(c + d*x)*b + cos(c + d*
x)*sin(a + b*x)*d)/(cos(a + b*x)*cos(c + d*x)*b*d)
```

3.351 $\int \sec^2(a+bx)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \sec^2(a+bx)\sec^3(c+dx)\,dx = \text{Int}\left(\sec^2(a+bx)\sec^3(c+dx),x\right)$$

output
```
Defer(Int)(sec(b*x+a)^2*sec(d*x+c)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 13.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec^2(a+bx)\sec^3(c+dx)\,dx = \int \sec^2(a+bx)\sec^3(c+dx)\,dx$$

input
```
Integrate[Sec[a + b*x]^2*Sec[c + d*x]^3,x]
```

output
```
Integrate[Sec[a + b*x]^2*Sec[c + d*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\sec^3(c+dx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \sec^2(a+bx)\sec^3(c+dx)dx$$

input
```
Int[Sec[a + b*x]^2*Sec[c + d*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec (bx+a)^2 \sec (dx+c)^3\,dx$$

input
```
int(sec(b*x+a)^2*sec(d*x+c)^3,x)
```

output
```
int(sec(b*x+a)^2*sec(d*x+c)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec^2(a+bx)\sec^3(c+dx)\,dx=\int \sec\left(bx+a\right)^2\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(b*x+a)^2*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2*sec(d*x + c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 15.62 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\sec^3(c+dx)\,dx=\int \sec^2\left(a+bx\right)\sec^3\left(c+dx\right)dx$$

input
```
integrate(sec(b*x+a)**2*sec(d*x+c)**3,x)
```

output
```
Integral(sec(a + b*x)**2*sec(c + d*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 12.41 (sec) , antiderivative size = 10269, normalized size of antiderivative = 604.06

$$\int \sec^2(a+bx)\sec^3(c+dx)\,dx = \int \sec\left(bx+a\right)^2 \sec\left(dx+c\right)^3\,dx$$

input

```
integrate(sec(b*x+a)^2*sec(d*x+c)^3,x, algorithm="maxima")
```

output

```
(4*(2*(2*b - d)*sin(2*(3*b + d)*x + 6*a + 2*c) + 6*(2*b - d)*sin(2*(2*b +
d)*x + 4*a + 2*c) + 6*(2*b - d)*sin(2*(b + d)*x + 2*a + 2*c) + (2*b - d)*s
in(6*b*x + 6*a) + 3*(2*b - d)*sin(4*b*x + 4*a) + 3*(2*b - d)*sin(2*b*x + 2
*a) + (2*b - d)*sin(4*d*x + 4*c) + 2*(2*b - d)*sin(2*d*x + 2*c))*cos((4*b
+ 3*d)*x + 4*a + 3*c) + 4*((2*b + d)*sin(6*b*x + 6*a) + 3*(2*b + d)*sin(4*
b*x + 4*a) + 3*(2*b + d)*sin(2*b*x + 2*a) + (2*b + d)*sin(4*d*x + 4*c) + 2
*(2*b + d)*sin(2*d*x + 2*c))*cos((4*b + d)*x + 4*a + c) - 4*((2*b - d)*sin
((4*b + 3*d)*x + 4*a + 3*c) + (2*b + d)*sin((4*b + d)*x + 4*a + c) - (2*b
+ d)*sin((2*b + 3*d)*x + 2*a + 3*c) - (2*b - d)*sin((2*b + d)*x + 2*a + c)
)*cos(2*(3*b + 2*d)*x + 6*a + 4*c) - 8*((2*b + d)*sin((4*b + d)*x + 4*a +
c) - (2*b - d)*sin((2*b + d)*x + 2*a + c))*cos(2*(3*b + d)*x + 6*a + 2*c)
- 4*(2*(2*b + d)*sin(2*(3*b + d)*x + 6*a + 2*c) + 6*(2*b + d)*sin(2*(2*b +
 d)*x + 4*a + 2*c) + 6*(2*b + d)*sin(2*(b + d)*x + 2*a + 2*c) + (2*b + d)*
sin(6*b*x + 6*a) + 3*(2*b + d)*sin(4*b*x + 4*a) + 3*(2*b + d)*sin(2*b*x +
2*a) + (2*b + d)*sin(4*d*x + 4*c) + 2*(2*b + d)*sin(2*d*x + 2*c))*cos((2*b
 + 3*d)*x + 2*a + 3*c) - 24*((2*b + d)*sin((4*b + d)*x + 4*a + c) - (2*b -
 d)*sin((2*b + d)*x + 2*a + c))*cos(2*(2*b + d)*x + 4*a + 2*c) - 4*((2*b -
 d)*sin(6*b*x + 6*a) + 3*(2*b - d)*sin(4*b*x + 4*a) + 3*(2*b - d)*sin(2*b*
x + 2*a) + (2*b - d)*sin(4*d*x + 4*c) + 2*(2*b - d)*sin(2*d*x + 2*c))*cos(
(2*b + d)*x + 2*a + c) - 12*((2*b - d)*sin((4*b + 3*d)*x + 4*a + 3*c) +...
```

Giac [N/A]

Not integrable

Time = 40.99 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec^2(a+bx)\sec^3(c+dx)\,dx = \int \sec(bx+a)^2\sec(dx+c)^3\,dx$$

input
```
integrate(sec(b*x+a)^2*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^2*sec(d*x + c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 20.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec^2(a+bx)\sec^3(c+dx)\,dx = \int \frac{1}{\cos(a+b\,x)^2\cos(c+d\,x)^3}\,dx$$

input
```
int(1/(cos(a + b*x)^2*cos(c + d*x)^3),x)
```

output
```
int(1/(cos(a + b*x)^2*cos(c + d*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 7625, normalized size of antiderivative = 448.53

$$\int \sec^2(a+bx)\sec^3(c+dx)\,dx = \text{Too large to display}$$

input
```
int(sec(b*x+a)^2*sec(d*x+c)^3,x)
```

output
```
( - 4*cos(a + b*x)*cos(c + d*x)**2*sin(c + d*x)*b**5 + 4*cos(a + b*x)*cos(
c + d*x)**2*sin(c + d*x)*b*d**4 - 8*cos(a + b*x)*cos(c + d*x)*int(cos(a +
b*x)/(cos(c + d*x)*sin(a + b*x)**2*sin(c + d*x)**2 - cos(c + d*x)*sin(a +
b*x)**2 - cos(c + d*x)*sin(c + d*x)**2 + cos(c + d*x)),x)*sin(c + d*x)**2*
b**3*d**3 + 16*cos(a + b*x)*cos(c + d*x)*int(cos(a + b*x)/(cos(c + d*x)*si
n(a + b*x)**2*sin(c + d*x)**2 - cos(c + d*x)*sin(a + b*x)**2 - cos(c + d*x
)*sin(c + d*x)**2 + cos(c + d*x)),x)*sin(c + d*x)**2*b*d**5 + 8*cos(a + b*
x)*cos(c + d*x)*int(cos(a + b*x)/(cos(c + d*x)*sin(a + b*x)**2*sin(c + d*x
)**2 - cos(c + d*x)*sin(a + b*x)**2 - cos(c + d*x)*sin(c + d*x)**2 + cos(c
 + d*x)),x)*b**3*d**3 - 16*cos(a + b*x)*cos(c + d*x)*int(cos(a + b*x)/(cos
(c + d*x)*sin(a + b*x)**2*sin(c + d*x)**2 - cos(c + d*x)*sin(a + b*x)**2 -
 cos(c + d*x)*sin(c + d*x)**2 + cos(c + d*x)),x)*b*d**5 - 8*cos(a + b*x)*c
os(c + d*x)*int(cos(a + b*x)/(sin(a + b*x)**2*sin(c + d*x)**2 - sin(a + b*
x)**2 - sin(c + d*x)**2 + 1),x)*sin(c + d*x)**2*b**3*d**3 + 16*cos(a + b*x
)*cos(c + d*x)*int(cos(a + b*x)/(sin(a + b*x)**2*sin(c + d*x)**2 - sin(a +
 b*x)**2 - sin(c + d*x)**2 + 1),x)*sin(c + d*x)**2*b*d**5 + 8*cos(a + b*x)
*cos(c + d*x)*int(cos(a + b*x)/(sin(a + b*x)**2*sin(c + d*x)**2 - sin(a +
b*x)**2 - sin(c + d*x)**2 + 1),x)*b**3*d**3 - 16*cos(a + b*x)*cos(c + d*x)
*int(cos(a + b*x)/(sin(a + b*x)**2*sin(c + d*x)**2 - sin(a + b*x)**2 - sin
(c + d*x)**2 + 1),x)*b*d**5 + 8*cos(a + b*x)*cos(c + d*x)*int(sin(a + b...
```

3.352 $\int \sec^2(a+bx)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \sec^2(a+bx)\sec^4(c+dx)\,dx = \text{Int}\big(\sec^2(a+bx)\sec^4(c+dx),x\big)$$

output
```
Defer(Int)(sec(b*x+a)^2*sec(d*x+c)^4,x)
```

Mathematica [N/A]

Not integrable

Time = 11.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec^2(a+bx)\sec^4(c+dx)\,dx = \int \sec^2(a+bx)\sec^4(c+dx)\,dx$$

input
```
Integrate[Sec[a + b*x]^2*Sec[c + d*x]^4,x]
```

output
```
Integrate[Sec[a + b*x]^2*Sec[c + d*x]^4, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\sec^4(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec^2(a+bx)\sec^4(c+dx)dx$$

input
```
Int[Sec[a + b*x]^2*Sec[c + d*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec(bx+a)^2 \sec(dx+c)^4\,dx$$

input
```
int(sec(b*x+a)^2*sec(d*x+c)^4,x)
```

output
```
int(sec(b*x+a)^2*sec(d*x+c)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec^2(a+bx)\sec^4(c+dx)\,dx = \int \sec(bx+a)^2\sec(dx+c)^4\,dx$$

input
```
integrate(sec(b*x+a)^2*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2*sec(d*x + c)^4, x)
```

Sympy [N/A]

Not integrable

Time = 40.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\sec^4(c+dx)\,dx = \int \sec^2(a+bx)\sec^4(c+dx)\,dx$$

input
```
integrate(sec(b*x+a)**2*sec(d*x+c)**4,x)
```

output
```
Integral(sec(a + b*x)**2*sec(c + d*x)**4, x)
```

Maxima [N/A]

Not integrable

Time = 28.06 (sec) , antiderivative size = 24945, normalized size of antiderivative = 1467.35

$$\int \sec^2(a+bx)\sec^4(c+dx)\,dx = \int \sec(bx+a)^2\sec(dx+c)^4\,dx$$

input
```
integrate(sec(b*x+a)^2*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/3*(32*(11*b^2 + d^2)*cos(6*b*x + 6*a)*sin(4*b*x + 4*a) - 32*(11*b^2 + d
^2)*cos(4*b*x + 4*a)*sin(2*b*x + 2*a) + 16*(4*b^2*sin(4*(b + d)*x + 4*a +
4*c) - (b^2 - b*d)*sin(2*(3*b + 2*d)*x + 6*a + 4*c) - (2*b^2 - b*d - 3*d^2
)*sin(2*(3*b + d)*x + 6*a + 2*c) + 2*(4*b^2 + 3*d^2)*sin(2*(2*b + d)*x + 4
*a + 2*c) - (b^2 + b*d)*sin(2*(b + 2*d)*x + 2*a + 4*c) - (2*b^2 + b*d - 3*
d^2)*sin(2*(b + d)*x + 2*a + 2*c) - (b^2 - d^2)*sin(6*b*x + 6*a) + 2*(2*b^
2 + d^2)*sin(4*b*x + 4*a) - (b^2 - d^2)*sin(2*b*x + 2*a))*cos(2*(4*b + 3*d
)*x + 8*a + 6*c) - 48*((2*b^2 - b*d - 3*d^2)*sin(2*(3*b + d)*x + 6*a + 2*c
) - 2*(4*b^2 + 3*d^2)*sin(2*(2*b + d)*x + 4*a + 2*c) + (2*b^2 + b*d - 3*d^
2)*sin(2*(b + d)*x + 2*a + 2*c) + (b^2 - d^2)*sin(6*b*x + 6*a) - 2*(2*b^2
+ d^2)*sin(4*b*x + 4*a) + (b^2 - d^2)*sin(2*b*x + 2*a))*cos(2*(4*b + d)*x
+ 8*a + 2*c) - 16*(24*b*d*sin(2*(b + 2*d)*x + 2*a + 4*c) - 3*(b^2 - b*d)*s
in(2*(4*b + d)*x + 8*a + 2*c) + 12*(b^2 - 3*d^2)*sin(2*(3*b + d)*x + 6*a +
 2*c) - 6*(19*b^2 - 3*b*d + 12*d^2)*sin(2*(2*b + d)*x + 4*a + 2*c) - 6*(11
*b^2 - 3*b*d)*sin(4*(b + d)*x + 4*a + 4*c) + 12*(b^2 + 2*b*d - 3*d^2)*sin(
2*(b + d)*x + 2*a + 2*c) - (b^2 - b*d)*sin(8*b*x + 8*a) + 4*(2*b^2 + b*d -
 3*d^2)*sin(6*b*x + 6*a) - 6*(9*b^2 - b*d + 4*d^2)*sin(4*b*x + 4*a) + 4*(2
*b^2 + b*d - 3*d^2)*sin(2*b*x + 2*a) - (b^2 - b*d)*sin(6*d*x + 6*c) - 3*(b
^2 - b*d)*sin(4*d*x + 4*c) - 3*(b^2 - b*d)*sin(2*d*x + 2*c))*cos(2*(3*b +
2*d)*x + 6*a + 4*c) - 16*(24*b*d*sin(2*(b + d)*x + 2*a + 2*c) - 6*(22*b...
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec^2(a+bx)\sec^4(c+dx)\,dx = \int \sec(bx+a)^2\sec(dx+c)^4\,dx$$

input
```
integrate(sec(b*x+a)^2*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^2*sec(d*x + c)^4, x)
```

Mupad [N/A]

Not integrable

Time = 21.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec^2(a+bx)\sec^4(c+dx)\,dx = \int \frac{1}{\cos(a+b\,x)^2\cos(c+d\,x)^4}\,dx$$

input
```
int(1/(cos(a + b*x)^2*cos(c + d*x)^4),x)
```

output
```
int(1/(cos(a + b*x)^2*cos(c + d*x)^4), x)
```

Reduce [N/A]

Not integrable

Time = 5.45 (sec) , antiderivative size = 12086, normalized size of antiderivative = 710.94

$$\int \sec^2(a+bx)\sec^4(c+dx)\,dx = \text{Too large to display}$$

input
```
int(sec(b*x+a)^2*sec(d*x+c)^4,x)
```

output
```
( - 256*cos(a + b*x)*cos(c + d*x)*int(tan((a + b*x)/2)**2/(tan((a + b*x)/2
)**4*tan((c + d*x)/2)**8 - 4*tan((a + b*x)/2)**4*tan((c + d*x)/2)**6 + 6*t
an((a + b*x)/2)**4*tan((c + d*x)/2)**4 - 4*tan((a + b*x)/2)**4*tan((c + d*
x)/2)**2 + tan((a + b*x)/2)**4 - 2*tan((a + b*x)/2)**2*tan((c + d*x)/2)**8
 + 8*tan((a + b*x)/2)**2*tan((c + d*x)/2)**6 - 12*tan((a + b*x)/2)**2*tan(
(c + d*x)/2)**4 + 8*tan((a + b*x)/2)**2*tan((c + d*x)/2)**2 - 2*tan((a + b
*x)/2)**2 + tan((c + d*x)/2)**8 - 4*tan((c + d*x)/2)**6 + 6*tan((c + d*x)/
2)**4 - 4*tan((c + d*x)/2)**2 + 1),x)*sin(c + d*x)**2*b**5*d + 2176*cos(a
+ b*x)*cos(c + d*x)*int(tan((a + b*x)/2)**2/(tan((a + b*x)/2)**4*tan((c +
d*x)/2)**8 - 4*tan((a + b*x)/2)**4*tan((c + d*x)/2)**6 + 6*tan((a + b*x)/2
)**4*tan((c + d*x)/2)**4 - 4*tan((a + b*x)/2)**4*tan((c + d*x)/2)**2 + tan
((a + b*x)/2)**4 - 2*tan((a + b*x)/2)**2*tan((c + d*x)/2)**8 + 8*tan((a +
b*x)/2)**2*tan((c + d*x)/2)**6 - 12*tan((a + b*x)/2)**2*tan((c + d*x)/2)**
4 + 8*tan((a + b*x)/2)**2*tan((c + d*x)/2)**2 - 2*tan((a + b*x)/2)**2 + ta
n((c + d*x)/2)**8 - 4*tan((c + d*x)/2)**6 + 6*tan((c + d*x)/2)**4 - 4*tan(
(c + d*x)/2)**2 + 1),x)*sin(c + d*x)**2*b**3*d**3 + 576*cos(a + b*x)*cos(c
 + d*x)*int(tan((a + b*x)/2)**2/(tan((a + b*x)/2)**4*tan((c + d*x)/2)**8 -
 4*tan((a + b*x)/2)**4*tan((c + d*x)/2)**6 + 6*tan((a + b*x)/2)**4*tan((c
+ d*x)/2)**4 - 4*tan((a + b*x)/2)**4*tan((c + d*x)/2)**2 + tan((a + b*x)/2
)**4 - 2*tan((a + b*x)/2)**2*tan((c + d*x)/2)**8 + 8*tan((a + b*x)/2)**...
```

3.353 $\int \cos^3(a+bx)\sqrt{\cos(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 485

$$\int \cos^3(a+bx)\sqrt{\cos(c+dx)}\,dx =$$

$$-\frac{3ie^{\frac{1}{2}i(2a-c)+\frac{1}{2}i(2b-d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{2b}{d}\right),\frac{1}{4}\left(3+\frac{2b}{d}\right),-e^{2i(c+dx)}\right)}{4(2b-d)\sqrt{1+e^{2i(c+dx)}}}$$

$$-\frac{ie^{\frac{1}{2}i(6a-c)+\frac{1}{2}i(6b-d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{6b}{d}\right),\frac{3(2b+d)}{4d},-e^{2i(c+dx)}\right)}{4(6b-d)\sqrt{1+e^{2i(c+dx)}}}$$

$$+\frac{3ie^{-\frac{1}{2}i(2a+c)-\frac{1}{2}i(2b+d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2b+d}{4d},\frac{1}{4}\left(3-\frac{2b}{d}\right),-e^{2i(c+dx)}\right)}{4(2b+d)\sqrt{1+e^{2i(c+dx)}}}$$

$$+\frac{ie^{-\frac{1}{2}i(6a+c)-\frac{1}{2}i(6b+d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{6b+d}{4d},\frac{3}{4}\left(1-\frac{2b}{d}\right),-e^{2i(c+dx)}\right)}{4(6b+d)\sqrt{1+e^{2i(c+dx)}}}$$

output
```
-3/4*I*exp(1/2*I*(2*a-c)+1/2*I*(2*b-d)*x+1/2*I*(d*x+c))*cos(d*x+c)^(1/2)*h
ypergeom([-1/2, -1/4+1/2*b/d],[3/4+1/2*b/d],-exp(2*I*(d*x+c)))/(2*b-d)/(1+
exp(2*I*(d*x+c)))^(1/2)-1/4*I*exp(1/2*I*(6*a-c)+1/2*I*(6*b-d)*x+1/2*I*(d*x
+c))*cos(d*x+c)^(1/2)*hypergeom([-1/2, -1/4+3/2*b/d],[3/4*(2*b+d)/d],-exp(
2*I*(d*x+c)))/(6*b-d)/(1+exp(2*I*(d*x+c)))^(1/2)+3/4*I*exp(-1/2*I*(2*a+c)-
1/2*I*(2*b+d)*x+1/2*I*(d*x+c))*cos(d*x+c)^(1/2)*hypergeom([-1/2, -1/4*(2*b
+d)/d],[3/4-1/2*b/d],-exp(2*I*(d*x+c)))/(2*b+d)/(1+exp(2*I*(d*x+c)))^(1/2)
+1/4*I*exp(-1/2*I*(6*a+c)-1/2*I*(6*b+d)*x+1/2*I*(d*x+c))*cos(d*x+c)^(1/2)*
hypergeom([-1/2, -1/4*(6*b+d)/d],[3/4-3/2*b/d],-exp(2*I*(d*x+c)))/(6*b+d)/
(1+exp(2*I*(d*x+c)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 16601 vs. 2(485) = 970.

Time = 42.73 (sec) , antiderivative size = 16601, normalized size of antiderivative = 34.23

$$\int \cos^3(a+bx)\sqrt{\cos(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[a + b*x]^3*Sqrt[Cos[c + d*x]],x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5065, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\sqrt{\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{5065}$$

$$\frac{\int \left(3e^{-ia-ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}+3e^{ia+ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}+e^{-3ia-3ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}+e^{3ia+3ibx}\right.}{8\sqrt{2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6i\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(\frac{2b}{d}-1\right),\frac{1}{4}\left(\frac{2b}{d}+3\right),-e^{2i(c+dx)}\right)\exp\left(\frac{1}{2}i(2a-c)+\frac{1}{2}ix(2b-d)+\frac{1}{2}i(c+dx)\right)}{(2b-d)\sqrt{1+e^{2ic+2idx}}}-\frac{2i\sqrt{e^{-i(c+dx}}}{}$$

input
```
Int[Cos[a + b*x]^3*Sqrt[Cos[c + d*x]],x]
```

output
```
(((-6*I)*E^((I/2)*(2*a - c) + (I/2)*(2*b - d)*x + (I/2)*(c + d*x))*Sqrt[E^
((-I)*(c + d*x)) + E^(I*(c + d*x))]*Hypergeometric2F1[-1/2, (-1 + (2*b)/d)
/4, (3 + (2*b)/d)/4, -E^((2*I)*(c + d*x))])/((2*b - d)*Sqrt[1 + E^((2*I)*c
 + (2*I)*d*x)]) - ((2*I)*E^((I/2)*(6*a - c) + (I/2)*(6*b - d)*x + (I/2)*(c
 + d*x))*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))]*Hypergeometric2F1[-1/2
, (-1 + (6*b)/d)/4, (3*(2*b + d))/(4*d), -E^((2*I)*(c + d*x))])/((6*b - d)
*Sqrt[1 + E^((2*I)*c + (2*I)*d*x)]) + ((6*I)*E^((-1/2*I)*(2*a + c) - (I/2)
*(2*b + d)*x + (I/2)*(c + d*x))*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))]
*Hypergeometric2F1[-1/2, -1/4*(2*b + d)/d, (3 - (2*b)/d)/4, -E^((2*I)*(c +
 d*x))])/((2*b + d)*Sqrt[1 + E^((2*I)*c + (2*I)*d*x)]) + ((2*I)*E^((-1/2*I
)*(6*a + c) - (I/2)*(6*b + d)*x + (I/2)*(c + d*x))*Sqrt[E^((-I)*(c + d*x))
 + E^(I*(c + d*x))]*Hypergeometric2F1[-1/2, -1/4*(6*b + d)/d, (3*(1 - (2*b
)/d))/4, -E^((2*I)*(c + d*x))])/((6*b + d)*Sqrt[1 + E^((2*I)*c + (2*I)*d*x
)]))/(8*Sqrt[2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5065
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Cos[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (E^((-I)*(a + b*x)) + E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \cos \left(bx + a\right)^{3} \sqrt{\cos \left(dx + c\right)}dx$$

input
```
int(cos(b*x+a)^3*cos(d*x+c)^(1/2),x)
```

output
```
int(cos(b*x+a)^3*cos(d*x+c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \cos^3(a + bx)\sqrt{\cos(c + dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a + bx)\sqrt{\cos(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(a+bx)\sqrt{\cos(c+dx)}\,dx = \int \cos(bx+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \cos^3(a+bx)\sqrt{\cos(c+dx)}\,dx = \int \cos(bx+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3*sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sqrt{\cos(c+dx)}\,dx = \int \cos(a+b\,x)^3\sqrt{\cos(c+d\,x)}\,dx$$

input
```
int(cos(a + b*x)^3*cos(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^3*cos(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \cos^3(a+bx)\sqrt{\cos(c+dx)}\,dx=\int \sqrt{\cos(dx+c)}\,\cos(bx+a)^3\,dx$$

input
```
int(cos(b*x+a)^3*cos(d*x+c)^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(a + b*x)**3,x)
```

3.354 $\int \cos^2(a+bx)\sqrt{\cos(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 258

$$\int \cos^2(a+bx)\sqrt{\cos(c+dx)}\,dx = \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{ie^{-\frac{1}{2}i(4a+c)-\frac{1}{2}i(4b+d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1-\frac{4b}{d}\right),\frac{1}{4}\left(3-\frac{4b}{d}\right),-e^{2i(c+dx}\right.}{2(4b+d)\sqrt{1+e^{2i(c+dx)}}} - \frac{ie^{\frac{1}{2}i(4a-c)+\frac{1}{2}i(4b-d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{4b}{d}\right),\frac{1}{4}\left(3+\frac{4b}{d}\right),-e^{2i(c+dx)}\right.}{2(4b-d)\sqrt{1+e^{2i(c+dx)}}}$$

output
```
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+1/2*I*exp(-1/2*I*(4*a+c)-1/2*I*(4*
b+d)*x+1/2*I*(d*x+c))*cos(d*x+c)^(1/2)*hypergeom([-1/2, -1/4-b/d],[3/4-b/d
],-exp(2*I*(d*x+c)))/(4*b+d)/(1+exp(2*I*(d*x+c)))^(1/2)-1/2*I*exp(1/2*I*(4
*a-c)+1/2*I*(4*b-d)*x+1/2*I*(d*x+c))*cos(d*x+c)^(1/2)*hypergeom([-1/2, -1/
4+b/d],[3/4+b/d],-exp(2*I*(d*x+c)))/(4*b-d)/(1+exp(2*I*(d*x+c)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3372 vs. 2(258) = 516.

Time = 13.85 (sec) , antiderivative size = 3372, normalized size of antiderivative = 13.07

$$\int \cos^2(a+bx)\sqrt{\cos(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[a + b*x]^2*Sqrt[Cos[c + d*x]],x]
```

output
```
(2*Sqrt[2]*b^2*Csc[c]*(3*(1 + E^((2*I)*(c + d*x))) + E^((2*I)*(c + d*x))*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4
, 7/4, -E^((2*I)*(c + d*x))]))/(3*d*E^(I*(2*c + d*x))*Sqrt[(1 + E^((2*I)*(
c + d*x)))/E^(I*(c + d*x))]*(4*b*Cos[c] - I*d*Sin[c])*(4*b*Cos[c] + I*d*Si
n[c])) - (d*Csc[c]*(3*(1 + E^((2*I)*(c + d*x))) + E^((2*I)*(c + d*x))*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7
/4, -E^((2*I)*(c + d*x))]))/(12*Sqrt[2]*E^(I*(2*c + d*x))*Sqrt[(1 + E^((2*
I)*(c + d*x)))/E^(I*(c + d*x))]*(4*b*Cos[c] - I*d*Sin[c])*(4*b*Cos[c] + I*
d*Sin[c])) - (2*Sqrt[2]*b^2*Csc[c]*(-3*E^((4*I)*c)*(1 + E^((2*I)*(c + d*x)
)) + E^((2*I)*d*x)*(-1 + E^((6*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperg
eometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(3*d*E^(I*(2*c + d*x))*
Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(4*b*Cos[c] - I*d*Sin[c])*
(4*b*Cos[c] + I*d*Sin[c])) + (d*Csc[c]*(-3*E^((4*I)*c)*(1 + E^((2*I)*(c +
d*x))) + E^((2*I)*d*x)*(-1 + E^((6*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hy
pergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(12*Sqrt[2]*E^(I*(2
*c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(4*b*Cos[c] - I
*d*Sin[c])*(4*b*Cos[c] + I*d*Sin[c])) + (d^2*Sqrt[(1 + E^((2*I)*(c + d*x))
)/E^(I*(c + d*x))]*Csc[c]*(-((4*b - d)*E^((2*I)*d*x)*Hypergeometric2F1[1,
5/4 - b/d, 7/4 - b/d, -E^((2*I)*(c + d*x))]) - (4*b - 3*d)*E^((4*I)*(a + b
*x))*Hypergeometric2F1[1, 1/4 + b/d, 3/4 + b/d, -E^((2*I)*(c + d*x))]))...
```

Rubi [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 646 vs. 2(258) = 516.

Time = 1.16 (sec) , antiderivative size = 646, normalized size of antiderivative = 2.50, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5065, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sqrt{\cos(c+dx)}\,dx$$

$$\Big\downarrow \ 5065$$

$$\frac{\int \left(e^{-2ia-2ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}+e^{2ia+2ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}+2\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}\right)dx}{4\sqrt{2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{2i\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-\frac{4b}{d}-1\right),\frac{1}{4}\left(3-\frac{4b}{d}\right),-e^{2i(c+dx)}\right)\exp\left(-\frac{1}{2}i(4a+c)-\frac{1}{2}ix(4b+d)+\frac{1}{2}i(c+dx)\right)}{(4b+d)\sqrt{1+e^{2ic+2idx}}} - \frac{2i\sqrt{e^{-i(c+}}}{}$$

input
```
Int[Cos[a + b*x]^2*Sqrt[Cos[c + d*x]],x]
```

output
```
(((4*I)*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))])/d - ((8*I)*E^(I*(c + d
*x))*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))])/(d*(1 + E^(I*(c + d*x))))
 + ((8*I)*Sqrt[E^(I*(c + d*x))]*(1 + E^(I*(c + d*x)))*Sqrt[E^((-I)*(c + d*
x)) + E^(I*(c + d*x))]*Sqrt[(1 + E^((2*I)*(c + d*x)))/(1 + E^(I*(c + d*x))
)^2]*EllipticE[2*ArcTan[Sqrt[E^(I*(c + d*x))]], 1/2])/(d*(1 + E^((2*I)*(c
+ d*x)))) - ((4*I)*Sqrt[E^(I*(c + d*x))]*(1 + E^(I*(c + d*x)))*Sqrt[E^((-I
)*(c + d*x)) + E^(I*(c + d*x))]*Sqrt[(1 + E^((2*I)*(c + d*x)))/(1 + E^(I*(
c + d*x)))^2]*EllipticF[2*ArcTan[Sqrt[E^(I*(c + d*x))]], 1/2])/(d*(1 + E^(
(2*I)*(c + d*x)))) + ((2*I)*E^((-1/2*I)*(4*a + c) - (I/2)*(4*b + d)*x + (I
/2)*(c + d*x))*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))]*Hypergeometric2F
1[-1/2, (-1 - (4*b)/d)/4, (3 - (4*b)/d)/4, -E^((2*I)*(c + d*x))])/((4*b +
d)*Sqrt[1 + E^((2*I)*c + (2*I)*d*x)]) - ((2*I)*E^((I/2)*(4*a - c) + (I/2)*
(4*b - d)*x + (I/2)*(c + d*x))*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))]*
Hypergeometric2F1[-1/2, (-1 + (4*b)/d)/4, (3 + (4*b)/d)/4, -E^((2*I)*(c +
d*x))])/((4*b - d)*Sqrt[1 + E^((2*I)*c + (2*I)*d*x)]))/(4*Sqrt[2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5065
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Cos[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (E^((-I)*(a + b*x)) + E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \cos (bx+a)^2 \sqrt{\cos (dx+c)}dx$$

input
```
int(cos(b*x+a)^2*cos(d*x+c)^(1/2),x)
```

output
```
int(cos(b*x+a)^2*cos(d*x+c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \cos^2(a+bx)\sqrt{\cos(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \cos^2(a+bx)\sqrt{\cos(c+dx)}\,dx = \int \cos^2{(a+bx)}\sqrt{\cos{(c+dx)}}\,dx$$

input
```
integrate(cos(b*x+a)**2*cos(d*x+c)**(1/2),x)
```

output
```
Integral(cos(a + b*x)**2*sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \cos^2(a+bx)\sqrt{\cos(c+dx)}\,dx = \int \cos{(bx+a)}^2\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \cos^2(a+bx)\sqrt{\cos(c+dx)}\,dx = \int \cos(bx+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sqrt{\cos(c+dx)}\,dx = \int \cos(a+b\,x)^2\sqrt{\cos(c+d\,x)}\,dx$$

input
```
int(cos(a + b*x)^2*cos(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2*cos(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \cos^2(a+bx)\sqrt{\cos(c+dx)}\,dx = \int \sqrt{\cos(dx+c)}\,\cos(bx+a)^2\,dx$$

input
```
int(cos(b*x+a)^2*cos(d*x+c)^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(a + b*x)**2,x)
```

3.355 $\int \cos(a+bx)\sqrt{\cos(c+dx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 239

$$\int \cos(a+bx)\sqrt{\cos(c+dx)}\,dx = -\frac{ie^{\frac{1}{2}i(2a-c)+\frac{1}{2}i(2b-d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{2b}{d}\right),\frac{1}{4}\left(3+\frac{2b}{d}\right),-e^{2i(c+dx)}\right)}{(2b-d)\sqrt{1+e^{2i(c+dx)}}} + \frac{ie^{-\frac{1}{2}i(2a+c)-\frac{1}{2}i(2b+d)x+\frac{1}{2}i(c+dx)}\sqrt{\cos(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2b+d}{4d},\frac{1}{4}\left(3-\frac{2b}{d}\right),-e^{2i(c+dx)}\right)}{(2b+d)\sqrt{1+e^{2i(c+dx)}}}$$

output

```
-I*exp(1/2*I*(2*a-c)+1/2*I*(2*b-d)*x+1/2*I*(d*x+c))*cos(d*x+c)^(1/2)*hyper
geom([-1/2, -1/4+1/2*b/d],[3/4+1/2*b/d],-exp(2*I*(d*x+c)))/(2*b-d)/(1+exp(
2*I*(d*x+c)))^(1/2)+I*exp(-1/2*I*(2*a+c)-1/2*I*(2*b+d)*x+1/2*I*(d*x+c))*co
s(d*x+c)^(1/2)*hypergeom([-1/2, -1/4*(2*b+d)/d],[3/4-1/2*b/d],-exp(2*I*(d*
x+c)))/(2*b+d)/(1+exp(2*I*(d*x+c)))^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 614 vs. $2(239) = 478$.

Time = 4.20 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.57

$$\int \cos(a+bx)\sqrt{\cos(c+dx)}\,dx =$$

$$-\frac{i\sqrt{2}de^{-\frac{1}{2}i(2a-2c+4bx+3dx)}\sqrt{1+e^{2i(c+dx)}}\Big(-\Big((8b^3+12b^2d-2bd^2-3d^3)\,e^{\frac{1}{2}i(2b+5d)x}(d(-1+e^{2ic})+2b(1+\cdots}{\cdots}$$

$$+\frac{\cos(c)\sqrt{\cos(c+dx)}(-2d\cos(a+bx)\sin(c)+4b\cos(c)\sin(a+bx))}{4b^2\cos^2(c)+d^2\sin^2(c)}$$

input
```
Integrate[Cos[a + b*x]*Sqrt[Cos[c + d*x]],x]
```

output
```
((-I)*Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*(-((8*b^3 + 12*b^2*d - 2*b*d
^2 - 3*d^3)*E^((I/2)*(2*b + 5*d)*x)*(d*(-1 + E^((2*I)*c)) + 2*b*(1 + E^((2
*I)*c)))*Hypergeometric2F1[1/2, 3/4 - b/(2*d), 7/4 - b/(2*d), -E^((2*I)*(c
 + d*x))]) + (2*b - 3*d)*((4*b^2 + 8*b*d + 3*d^2)*E^((I/2)*(4*a + (6*b + d
)*x))*(d - d*E^((2*I)*c) + 2*b*(1 + E^((2*I)*c)))*Hypergeometric2F1[1/2, -
1/4 + b/(2*d), 3/4 + b/(2*d), -E^((2*I)*(c + d*x))] - (2*b - d)*((2*b + d)
*E^((I/2)*(4*a + 6*b*x + 5*d*x))*(d - d*E^((2*I)*c) + 2*b*(1 + E^((2*I)*c)
))*Hypergeometric2F1[1/2, 3/4 + b/(2*d), 7/4 + b/(2*d), -E^((2*I)*(c + d*x
))] - (2*b + 3*d)*E^((I/2)*(2*b + d)*x)*(d*(-1 + E^((2*I)*c)) + 2*b*(1 + E
^((2*I)*c)))*Hypergeometric2F1[1/2, -1/4*(2*b + d)/d, 3/4 - b/(2*d), -E^((
2*I)*(c + d*x))]))))/((16*b^4 - 40*b^2*d^2 + 9*d^4)*E^((I/2)*(2*a - 2*c +
4*b*x + 3*d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(d - d*E^(
(2*I)*c) + 2*b*(1 + E^((2*I)*c)))*(d*(-1 + E^((2*I)*c)) + 2*b*(1 + E^((2*I
)*c)))) + (Cos[c]*Sqrt[Cos[c + d*x]]*(-2*d*Cos[a + b*x]*Sin[c] + 4*b*Cos[c
]*Sin[a + b*x]))/(4*b^2*Cos[c]^2 + d^2*Sin[c]^2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5065, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sqrt{\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{5065}$$

$$\frac{\int \left(e^{-ia-ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}+e^{ia+ibx}\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}\right)dx}{2\sqrt{2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2i\sqrt{e^{-i(c+dx)}+e^{i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2b+d}{4d},\frac{1}{4}\left(3-\frac{2b}{d}\right),-e^{2i(c+dx)}\right)\exp\left(-\frac{1}{2}i(2a+c)-\frac{1}{2}ix(2b+d)+\frac{1}{2}i(c+dx)\right)}{(2b+d)\sqrt{1+e^{2ic+2idx}}}-\frac{2i\sqrt{e^{-i(c+dx)}+}}{}}{2\sqrt{2}}$$

input
```
Int[Cos[a + b*x]*Sqrt[Cos[c + d*x]],x]
```

output
```
(((-2*I)*E^((I/2)*(2*a - c) + (I/2)*(2*b - d)*x + (I/2)*(c + d*x))*Sqrt[E^
((-I)*(c + d*x)) + E^(I*(c + d*x))]*Hypergeometric2F1[-1/2, (-1 + (2*b)/d)
/4, (3 + (2*b)/d)/4, -E^((2*I)*(c + d*x))])/((2*b - d)*Sqrt[1 + E^((2*I)*c
 + (2*I)*d*x)]) + ((2*I)*E^((-1/2*I)*(2*a + c) - (I/2)*(2*b + d)*x + (I/2)
*(c + d*x))*Sqrt[E^((-I)*(c + d*x)) + E^(I*(c + d*x))]*Hypergeometric2F1[-
1/2, -1/4*(2*b + d)/d, (3 - (2*b)/d)/4, -E^((2*I)*(c + d*x))])/((2*b + d)*
Sqrt[1 + E^((2*I)*c + (2*I)*d*x)]))/(2*Sqrt[2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5065
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Cos[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (E^((-I)*(a + b*x)) + E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \cos(bx+a)\sqrt{\cos(dx+c)}dx$$

input
```
int(cos(b*x+a)*cos(d*x+c)^(1/2),x)
```

output
```
int(cos(b*x+a)*cos(d*x+c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \cos(a+bx)\sqrt{\cos(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \cos(a+bx)\sqrt{\cos(c+dx)}\,dx = \int \cos{(a+bx)}\sqrt{\cos{(c+dx)}}\,dx$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)**(1/2),x)
```

output
```
Integral(cos(a + b*x)*sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \cos(a+bx)\sqrt{\cos(c+dx)}\,dx = \int \cos{(bx+a)}\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \cos(a+bx)\sqrt{\cos(c+dx)}\,dx = \int \cos{(bx+a)}\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sqrt{\cos(c+dx)}\,dx = \int \cos\left(a+b\,x\right)\,\sqrt{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(a + b*x)*cos(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)*cos(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \cos(a+bx)\sqrt{\cos(c+dx)}\,dx = \int \sqrt{\cos\left(dx+c\right)}\,\cos\left(bx+a\right)dx$$

input
```
int(cos(b*x+a)*cos(d*x+c)^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x))*cos(a + b*x),x)
```

3.356 $\int \sqrt{\cos(c+dx)}\sec(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \sqrt{\cos(c+dx)}\sec(a+bx)\,dx = \text{Int}\left(\sqrt{\cos(c+dx)}\sec(a+bx),x\right)$$

output
```
Defer(Int)(cos(d*x+c)^(1/2)*sec(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 10.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sqrt{\cos(c+dx)}\sec(a+bx)\,dx = \int \sqrt{\cos(c+dx)}\sec(a+bx)\,dx$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sec[a + b*x],x]
```

output
```
Integrate[Sqrt[Cos[c + d*x]]*Sec[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\sqrt{\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec(a+bx)\sqrt{\cos(c+dx)}dx$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sec[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{\cos(dx+c)}\,\sec(bx+a)\,dx$$

input
```
int(cos(d*x+c)^(1/2)*sec(b*x+a),x)
```

output
```
int(cos(d*x+c)^(1/2)*sec(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\sec(a+bx)\,dx=\int \sqrt{\cos\left(dx+c\right)}\sec\left(bx+a\right)\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*sec(b*x+a),x, algorithm="fricas")
```

output
```
integral(sqrt(cos(d*x + c))*sec(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\sec(a+bx)\,dx=\int \sqrt{\cos\left(c+dx\right)}\sec\left(a+bx\right)dx$$

input
```
integrate(cos(d*x+c)**(1/2)*sec(b*x+a),x)
```

output
```
Integral(sqrt(cos(c + d*x))*sec(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\sec(a+bx)\,dx = \int \sqrt{\cos(dx+c)}\sec(bx+a)\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*sec(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))*sec(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\sec(a+bx)\,dx = \int \sqrt{\cos(dx+c)}\sec(bx+a)\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))*sec(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 19.73 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sqrt{\cos(c+dx)}\sec(a+bx)\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{\cos(a+b\,x)}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/cos(a + b*x),x)
```

output
```
int(cos(c + d*x)^(1/2)/cos(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \sqrt{\cos(c+dx)}\sec(a+bx)\,dx=\int \sqrt{\cos\left(dx+c\right)}\,\sec\left(bx+a\right)dx$$

input
```
int(cos(d*x+c)^(1/2)*sec(b*x+a),x)
```

output
```
int(sqrt(cos(c + d*x))*sec(a + b*x),x)
```

3.357 $\int \sqrt{\cos(c+dx)}\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \sqrt{\cos(c+dx)}\sec^2(a+bx)\,dx = \text{Int}\left(\sqrt{\cos(c+dx)}\sec^2(a+bx),x\right)$$

output
```
Defer(Int)(cos(d*x+c)^(1/2)*sec(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 20.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sqrt{\cos(c+dx)}\sec^2(a+bx)\,dx = \int \sqrt{\cos(c+dx)}\sec^2(a+bx)\,dx$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sec[a + b*x]^2,x]
```

output
```
Integrate[Sqrt[Cos[c + d*x]]*Sec[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\sqrt{\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec^2(a+bx)\sqrt{\cos(c+dx)}dx$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \sqrt{\cos\left(dx+c\right)}\,\sec\left(bx+a\right)^2\,dx$$

input
```
int(cos(d*x+c)^(1/2)*sec(b*x+a)^2,x)
```

output
```
int(cos(d*x+c)^(1/2)*sec(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\sec^2(a+bx)\,dx=\int \sqrt{\cos\left(dx+c\right)}\sec\left(bx+a\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(cos(d*x + c))*sec(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\sec^2(a+bx)\,dx=\int \sqrt{\cos\left(c+dx\right)}\sec^2\left(a+bx\right)dx$$

input
```
integrate(cos(d*x+c)**(1/2)*sec(b*x+a)**2,x)
```

output
```
Integral(sqrt(cos(c + d*x))*sec(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\sec^2(a+bx)\,dx = \int \sqrt{\cos(dx+c)}\sec(bx+a)^2\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))*sec(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\sec^2(a+bx)\,dx = \int \sqrt{\cos(dx+c)}\sec(bx+a)^2\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))*sec(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 19.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\sec^2(a+bx)\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{\cos(a+b\,x)^2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/cos(a + b*x)^2,x)
```

output
```
int(cos(c + d*x)^(1/2)/cos(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \sqrt{\cos(c+dx)}\sec^2(a+bx)\,dx = \int \sqrt{\cos\left(dx+c\right)}\,\sec\left(bx+a\right)^2dx$$

input
```
int(cos(d*x+c)^(1/2)*sec(b*x+a)^2,x)
```

output
```
int(sqrt(cos(c + d*x))*sec(a + b*x)**2,x)
```

3.358 $\int \cos^3(a+bx)\cos^q(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 481

$$\int \cos^3(a+bx)\cos^q(c+dx)\,dx = -\frac{ie^{i(3a-cq)+i(3b-dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{3b}{d}-q\right),-q,\frac{1}{2}\left(2+\frac{3b}{d}\right.\right.}{8(3b-dq)}$$
$$-\frac{3ie^{i(a-cq)+i(b-dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(-q,\frac{b-dq}{2d},\frac{1}{2}\left(2+\frac{b}{d}-q\right),-\right.}{8(b-dq)}$$
$$+\frac{3ie^{-i(a+cq)-i(b+dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(-q,-\frac{b+dq}{2d},-\frac{b-d(2-q)}{2d},-\right.}{8(b+dq)}$$
$$+\frac{ie^{-i(3a+cq)-i(3b+dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(-q,-\frac{3b+dq}{2d},\frac{1}{2}\left(2-\frac{3b}{d}-\right.\right.}{8(3b+dq)}$$

output
```
-1/8*I*exp(I*(-c*q+3*a)+I*(-d*q+3*b)*x+I*q*(d*x+c))*cos(d*x+c)^q*hypergeom
([-q, 3/2*b/d-1/2*q],[1+3/2*b/d-1/2*q],-exp(2*I*(d*x+c)))/((1+exp(2*I*(d*x
+c)))^q)/(-d*q+3*b)-3/8*I*exp(I*(-c*q+a)+I*(-d*q+b)*x+I*q*(d*x+c))*cos(d*x
+c)^q*hypergeom([-q, 1/2*(-d*q+b)/d],[1+1/2*b/d-1/2*q],-exp(2*I*(d*x+c)))/
((1+exp(2*I*(d*x+c)))^q)/(-d*q+b)+3/8*I*exp(-I*(c*q+a)-I*(d*q+b)*x+I*q*(d*
x+c))*cos(d*x+c)^q*hypergeom([-q, -1/2*(d*q+b)/d],[-1/2*(b-d*(2-q))/d],-ex
p(2*I*(d*x+c)))/((1+exp(2*I*(d*x+c)))^q)/(d*q+b)+1/8*I*exp(-I*(c*q+3*a)-I*
(d*q+3*b)*x+I*q*(d*x+c))*cos(d*x+c)^q*hypergeom([-q, -1/2*(d*q+3*b)/d],[1-
3/2*b/d-1/2*q],-exp(2*I*(d*x+c)))/((1+exp(2*I*(d*x+c)))^q)/(d*q+3*b)
```

Mathematica [F]

$$\int \cos^3(a+bx)\cos^q(c+dx)\,dx = \int \cos^3(a+bx)\cos^q(c+dx)\,dx$$

input
```
Integrate[Cos[a + b*x]^3*Cos[c + d*x]^q,x]
```

output
```
Integrate[Cos[a + b*x]^3*Cos[c + d*x]^q, x]
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5065, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\cos^q(c+dx)\,dx$$

$$\Big\downarrow \text{5065}$$

$$2^{-q-3}\int \left(3e^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q+3e^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q+e^{-3ia-3ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q\right.$$

$$\Big\downarrow \text{2009}$$

$$2^{-q-3}\left(-\frac{i\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q\left(1+e^{2ic+2idx}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{3b}{d}-q\right),-q,\frac{1}{2}\left(\frac{3b}{d}-q+2\right),-e^{2i(c+dx)}\right)}{3b-dq}\right.$$

input
```
Int[Cos[a + b*x]^3*Cos[c + d*x]^q,x]
```

output
```
2^(-3 - q)*(((-I)*E^(I*(3*a - c*q) + I*(3*b - d*q)*x + I*q*(c + d*x))*(E^(
(-I)*(c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[((3*b)/d - q)/2, -q
, (2 + (3*b)/d - q)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x
))^q*(3*b - d*q)) - ((3*I)*E^(I*(a - c*q) + I*(b - d*q)*x + I*q*(c + d*x))
*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, (b - d*q)/
(2*d), (2 + b/d - q)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*
x))^q*(b - d*q)) + ((3*I)*E^((-I)*(a + c*q) - I*(b + d*q)*x + I*q*(c + d*x
))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, -1/2*(b
+ d*q)/d, 1 - (b + d*q)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (
2*I)*d*x))^q*(b + d*q)) + (I*E^((-I)*(3*a + c*q) - I*(3*b + d*q)*x + I*q*(
c + d*x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, -
1/2*(3*b + d*q)/d, (2 - (3*b)/d - q)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2
*I)*c + (2*I)*d*x))^q*(3*b + d*q)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5065
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Cos[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (E^((-I)*(a + b*x)) + E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \cos(bx+a)^3 \cos(dx+c)^q \, dx$$

input
```
int(cos(b*x+a)^3*cos(d*x+c)^q,x)
```

output
```
int(cos(b*x+a)^3*cos(d*x+c)^q,x)
```

Fricas [F]

$$\int \cos^3(a+bx)\cos^q(c+dx)\,dx = \int \cos(dx+c)^q \cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^q,x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^q*cos(b*x + a)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\cos^q(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*cos(d*x+c)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(a+bx)\cos^q(c+dx)\,dx = \int \cos(dx+c)^q \cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^q,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^q*cos(b*x + a)^3, x)
```

Giac [F]

$$\int \cos^3(a+bx)\cos^q(c+dx)\,dx = \int \cos(dx+c)^q \cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*cos(d*x+c)^q,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^q*cos(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\cos^q(c+dx)\,dx = \int \cos(a+b\,x)^3 \cos(c+d\,x)^q\,dx$$

input
```
int(cos(a + b*x)^3*cos(c + d*x)^q,x)
```

output
```
int(cos(a + b*x)^3*cos(c + d*x)^q, x)
```

Reduce [F]

$$\int \cos^3(a+bx)\cos^q(c+dx)\,dx = \int \cos(dx+c)^q \cos(bx+a)^3\,dx$$

input
```
int(cos(b*x+a)^3*cos(d*x+c)^q,x)
```

output
```
int(cos(c + d*x)**q*cos(a + b*x)**3,x)
```

3.359 $\int \cos^2(a+bx)\cos^q(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 314

$$\int \cos^2(a+bx)\cos^q(c+dx)\,dx = \frac{ie^{-i(2a+cq)-i(2b+dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2b}{d}-q\right),-q,\frac{1}{2}\left(2-\frac{2b}{d}\right.\right.}{4(2b+dq)} - \frac{ie^{i(2a-cq)+i(2b-dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{2b}{d}-q\right),-q,\frac{1}{2}\left(2+\frac{2b}{d}\right.\right.}{4(2b-dq)} - \frac{\cos^{1+q}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+q}{2},\frac{3+q}{2},\cos^2(c+dx)\right)\sin(c+dx)}{2d(1+q)\sqrt{\sin^2(c+dx)}}$$

output

```
1/4*I*exp(-I*(c*q+2*a)-I*(d*q+2*b)*x+I*q*(d*x+c))*cos(d*x+c)^q*hypergeom([
-q, -b/d-1/2*q],[1-b/d-1/2*q],-exp(2*I*(d*x+c)))/((1+exp(2*I*(d*x+c)))^q)/
(d*q+2*b)-1/4*I*exp(I*(-c*q+2*a)+I*(-d*q+2*b)*x+I*q*(d*x+c))*cos(d*x+c)^q*
hypergeom([-q, b/d-1/2*q],[1+b/d-1/2*q],-exp(2*I*(d*x+c)))/((1+exp(2*I*(d*
x+c)))^q)/(-d*q+2*b)-1/2*cos(d*x+c)^(1+q)*hypergeom([1/2, 1/2+1/2*q],[3/2+
1/2*q],cos(d*x+c)^2)*sin(d*x+c)/d/(1+q)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.77

$$\int \cos^2(a+bx)\cos^q(c+dx)\,dx$$
$$= \frac{i2^{-2-q}e^{-2i(a+bx)+i(c+dx)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{1+q}\left(dq(-2b+dq)\,\text{Hypergeometric2F1}\left(1,1-\frac{b}{d}+\frac{q}{2},1-\right.\right.}{}$$

input
```
Integrate[Cos[a + b*x]^2*Cos[c + d*x]^q,x]
```

output
```
(I*2^(-2 - q)*E^((-2*I)*(a + b*x) + I*(c + d*x))*((1 + E^((2*I)*(c + d*x))
)/E^(I*(c + d*x)))^(1 + q)*(d*q*(-2*b + d*q)*Hypergeometric2F1[1, 1 - b/d
+ q/2, 1 - b/d - q/2, -E^((2*I)*(c + d*x))] + E^((2*I)*(a + b*x))*(2*b + d
*q)*(d*E^((2*I)*(a + b*x))*q*Hypergeometric2F1[1, 1 + b/d + q/2, 1 + b/d -
 q/2, -E^((2*I)*(c + d*x))] + 2*(-2*b + d*q)*Hypergeometric2F1[1, (2 + q)/
2, 1 - q/2, -E^((2*I)*(c + d*x))])))/(-4*b^2*d*q + d^3*q^3)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5065, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cos^q(c+dx)\,dx$$
$$\Big\downarrow \text{5065}$$
$$2^{-q-2}\int \left(e^{-2ia-2ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q+e^{2ia+2ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q+2\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q\right)dx$$
$$\Big\downarrow \text{2009}$$

$$2^{-q-2}\left(\frac{i\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q\left(1+e^{2ic+2idx}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2b}{d}-q\right),-q,\frac{1}{2}\left(-\frac{2b}{d}-q+2\right),-e^{2i(c+\right.}{2b+dq}\right.$$

input
```
Int[Cos[a + b*x]^2*Cos[c + d*x]^q,x]
```

output
```
2^(-2 - q)*((I*E^((-I)*(2*a + c*q) - I*(2*b + d*q)*x + I*q*(c + d*x))*(E^(
(-I)*(c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[((-2*b)/d - q)/2, -
q, (2 - (2*b)/d - q)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*
x))^q*(2*b + d*q)) - (I*E^(I*(2*a - c*q) + I*(2*b - d*q)*x + I*q*(c + d*x)
)*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[((2*b)/d - q)
/2, -q, (2 + (2*b)/d - q)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*
I)*d*x))^q*(2*b - d*q)) + ((2*I)*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^q*
Hypergeometric2F1[-q, -1/2*q, 1 - q/2, -E^((2*I)*(c + d*x))])/(d*(1 + E^((
2*I)*(c + d*x)))^q*q))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5065
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Cos[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (E^((-I)*(a + b*x)) + E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \cos(bx+a)^2 \cos(dx+c)^q \, dx$$

input
```
int(cos(b*x+a)^2*cos(d*x+c)^q,x)
```

output
```
int(cos(b*x+a)^2*cos(d*x+c)^q,x)
```

Fricas [F]

$$\int \cos^2(a+bx)\cos^q(c+dx)\,dx = \int \cos(dx+c)^q \cos(bx+a)^2 \,dx$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^q,x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^q*cos(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\cos^q(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*cos(d*x+c)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(a+bx)\cos^q(c+dx)\,dx = \int \cos(dx+c)^q \cos(bx+a)^2 \,dx$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^q,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^q*cos(b*x + a)^2, x)
```

Giac [F]

$$\int \cos^2(a+bx)\cos^q(c+dx)\,dx = \int \cos(dx+c)^q \cos(bx+a)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*cos(d*x+c)^q,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^q*cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\cos^q(c+dx)\,dx = \int \cos(a+b\,x)^2 \cos(c+d\,x)^q\,dx$$

input
```
int(cos(a + b*x)^2*cos(c + d*x)^q,x)
```

output
```
int(cos(a + b*x)^2*cos(c + d*x)^q, x)
```

Reduce [F]

$$\int \cos^2(a+bx)\cos^q(c+dx)\,dx = \int \cos(dx+c)^q \cos(bx+a)^2\,dx$$

input
```
int(cos(b*x+a)^2*cos(d*x+c)^q,x)
```

output
```
int(cos(c + d*x)**q*cos(a + b*x)**2,x)
```

3.360 $\int \cos(a+bx)\cos^q(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 234

$$\int \cos(a+bx)\cos^q(c+dx)\,dx = -\frac{ie^{i(a-cq)+i(b-dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(-q,\frac{b-dq}{2d},\frac{1}{2}\left(2+\frac{b}{d}-q\right),-e\right.}{2(b-dq)} + \frac{ie^{-i(a+cq)-i(b+dq)x+iq(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-q}\cos^q(c+dx)\,\text{Hypergeometric2F1}\left(-q,-\frac{b+dq}{2d},-\frac{b-d(2-q)}{2d},-e\right.}{2(b+dq)}$$

output
```
-1/2*I*exp(I*(-c*q+a)+I*(-d*q+b)*x+I*q*(d*x+c))*cos(d*x+c)^q*hypergeom([-q
, 1/2*(-d*q+b)/d],[1+1/2*b/d-1/2*q],-exp(2*I*(d*x+c)))/((1+exp(2*I*(d*x+c)
))^q)/(-d*q+b)+1/2*I*exp(-I*(c*q+a)-I*(d*q+b)*x+I*q*(d*x+c))*cos(d*x+c)^q*
hypergeom([-q, -1/2*(d*q+b)/d],[-1/2*(b-d*(2-q))/d],-exp(2*I*(d*x+c)))/((1
+exp(2*I*(d*x+c)))^q)/(d*q+b)
```

Mathematica [A] (warning: unable to verify)

Time = 1.43 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.79

$$\int \cos(a+bx)\cos^q(c+dx)\,dx = -\frac{i2^{-1-q}e^{-i(a-c+(b-d)x)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{1+q}\left((-b+dq)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{b}{d}+q\right),-\right.\right.}{(b-dq)}$$

input
```
Integrate[Cos[a + b*x]*Cos[c + d*x]^q,x]
```

output
```
((-I)*2^(-1 - q)*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^(1 + q)*((-b
+ d*q)*Hypergeometric2F1[1, (2 - b/d + q)/2, -1/2*(b + d*(-2 + q))/d, -E^(
(2*I)*(c + d*x))] + E^((2*I)*(a + b*x))*(b + d*q)*Hypergeometric2F1[1, (b
+ d*(2 + q))/(2*d), (2 + b/d - q)/2, -E^((2*I)*(c + d*x))]))/(E^(I*(a - c
+ (b - d)*x))*(b - d*q)*(b + d*q))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5065, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos^q(c+dx)\,dx$$

$$\Big\downarrow \ 5065$$

$$2^{-q-1}\int \left(e^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q+e^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q\right)dx$$

$$\Big\downarrow \ 2009$$

$$2^{-q-1}\left(\frac{i\left(1+e^{2ic+2idx}\right)^{-q}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^q\text{Hypergeometric2F1}\left(-q,-\frac{b+dq}{2d},1-\frac{b+dq}{2d},-e^{2i(c+dx)}\right)\exp(-i(\right.}{b+dq}\right.$$

input
```
Int[Cos[a + b*x]*Cos[c + d*x]^q,x]
```

output
```
2^(-1 - q)*(((-I)*E^(I*(a - c*q) + I*(b - d*q)*x + I*q*(c + d*x))*(E^((-I)
*(c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, (b - d*q)/(2*d), (2
 + b/d - q)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^q*(b
- d*q)) + (I*E^((-I)*(a + c*q) - I*(b + d*q)*x + I*q*(c + d*x))*(E^((-I)*(
c + d*x)) + E^(I*(c + d*x)))^q*Hypergeometric2F1[-q, -1/2*(b + d*q)/d, 1 -
 (b + d*q)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^q*
(b + d*q)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5065
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Cos[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (E^((-I)*(a + b*x)) + E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Maple [F]

$$\int \cos(bx+a)\cos(dx+c)^q\,dx$$

input
```
int(cos(b*x+a)*cos(d*x+c)^q,x)
```

output
```
int(cos(b*x+a)*cos(d*x+c)^q,x)
```

Fricas [F]

$$\int \cos(a + bx)\cos^q(c + dx)\,dx = \int \cos(dx + c)^q \cos(bx + a)\,dx$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^q,x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^q*cos(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a + bx)\cos^q(c + dx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(a + bx)\cos^q(c + dx)\,dx = \int \cos(dx + c)^q \cos(bx + a)\,dx$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^q,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^q*cos(b*x + a), x)
```

Giac [F]

$$\int \cos(a+bx)\cos^q(c+dx)\,dx = \int \cos(dx+c)^q\cos(bx+a)\,dx$$

input
```
integrate(cos(b*x+a)*cos(d*x+c)^q,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^q*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\cos^q(c+dx)\,dx = \int \cos(a+b\,x)\,\cos(c+d\,x)^q\,dx$$

input
```
int(cos(a + b*x)*cos(c + d*x)^q,x)
```

output
```
int(cos(a + b*x)*cos(c + d*x)^q, x)
```

Reduce [F]

$$\int \cos(a+bx)\cos^q(c+dx)\,dx = \int \cos(dx+c)^q\cos(bx+a)\,dx$$

input
```
int(cos(b*x+a)*cos(d*x+c)^q,x)
```

output
```
int(cos(c + d*x)**q*cos(a + b*x),x)
```

3.361 $\int \cos^q(c+dx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos^q(c+dx)\sec(a+bx)\,dx = \text{Int}(\cos^q(c+dx)\sec(a+bx),x)$$

output
```
Defer(Int)(cos(d*x+c)^q*sec(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos^q(c+dx)\sec(a+bx)\,dx = \int \cos^q(c+dx)\sec(a+bx)\,dx$$

input
```
Integrate[Cos[c + d*x]^q*Sec[a + b*x],x]
```

output
```
Integrate[Cos[c + d*x]^q*Sec[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\cos^q(c+dx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sec(a+bx)\cos^q(c+dx)dx$$

input
```
Int[Cos[c + d*x]^q*Sec[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos\left(dx+c\right)^{q} \sec\left(bx+a\right)dx$$

input
```
int(cos(d*x+c)^q*sec(b*x+a),x)
```

output
```
int(cos(d*x+c)^q*sec(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos^q(c+dx)\sec(a+bx)\,dx=\int \cos\left(dx+c\right)^{q}\sec\left(bx+a\right)\,dx$$

input
```
integrate(cos(d*x+c)^q*sec(b*x+a),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^q*sec(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos^q(c+dx)\sec(a+bx)\,dx=\int \cos^{q}\left(c+dx\right)\sec\left(a+bx\right)dx$$

input
```
integrate(cos(d*x+c)**q*sec(b*x+a),x)
```

output
```
Integral(cos(c + d*x)**q*sec(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos^q(c+dx)\sec(a+bx)\,dx = \int \cos(dx+c)^q \sec(bx+a)\,dx$$

input
```
integrate(cos(d*x+c)^q*sec(b*x+a),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^q*sec(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos^q(c+dx)\sec(a+bx)\,dx = \int \cos(dx+c)^q \sec(bx+a)\,dx$$

input
```
integrate(cos(d*x+c)^q*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^q*sec(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 19.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cos^q(c+dx)\sec(a+bx)\,dx = \int \frac{\cos(c+d\,x)^q}{\cos(a+b\,x)}\,dx$$

input
```
int(cos(c + d*x)^q/cos(a + b*x),x)
```

output
```
int(cos(c + d*x)^q/cos(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos^q(c+dx)\sec(a+bx)\,dx=\int \cos\left(dx+c\right)^q\sec\left(bx+a\right)dx$$

input
```
int(cos(d*x+c)^q*sec(b*x+a),x)
```

output
```
int(cos(c + d*x)**q*sec(a + b*x),x)
```

3.362 $\int \cos^q(c+dx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \cos^q(c+dx)\sec^2(a+bx)\,dx = \mathrm{Int}\left(\cos^q(c+dx)\sec^2(a+bx),x\right)$$

output
```
Defer(Int)(cos(d*x+c)^q*sec(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cos^q(c+dx)\sec^2(a+bx)\,dx = \int \cos^q(c+dx)\sec^2(a+bx)\,dx$$

input
```
Integrate[Cos[c + d*x]^q*Sec[a + b*x]^2,x]
```

output
```
Integrate[Cos[c + d*x]^q*Sec[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\cos^q(c+dx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sec^2(a+bx)\cos^q(c+dx)dx$$

input
```
Int[Cos[c + d*x]^q*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(dx+c)^q \sec(bx+a)^2\,dx$$

input
```
int(cos(d*x+c)^q*sec(b*x+a)^2,x)
```

output
```
int(cos(d*x+c)^q*sec(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cos^q(c+dx)\sec^2(a+bx)\,dx=\int \cos\left(dx+c\right)^q\sec\left(bx+a\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^q*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^q*sec(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 12.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos^q(c+dx)\sec^2(a+bx)\,dx=\int \cos^q\left(c+dx\right)\sec^2\left(a+bx\right)dx$$

input
```
integrate(cos(d*x+c)**q*sec(b*x+a)**2,x)
```

output
```
Integral(cos(c + d*x)**q*sec(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cos^q(c+dx)\sec^2(a+bx)\,dx = \int \cos(dx+c)^q \sec(bx+a)^2\,dx$$

input
```
integrate(cos(d*x+c)^q*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^q*sec(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cos^q(c+dx)\sec^2(a+bx)\,dx = \int \cos(dx+c)^q \sec(bx+a)^2\,dx$$

input
```
integrate(cos(d*x+c)^q*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^q*sec(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 19.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cos^q(c+dx)\sec^2(a+bx)\,dx = \int \frac{\cos(c+d\,x)^q}{\cos(a+b\,x)^2}\,dx$$

input
```
int(cos(c + d*x)^q/cos(a + b*x)^2,x)
```

output
```
int(cos(c + d*x)^q/cos(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cos^q(c+dx)\sec^2(a+bx)\,dx = \int \cos(dx+c)^q \sec(bx+a)^2\,dx$$

input
```
int(cos(d*x+c)^q*sec(b*x+a)^2,x)
```

output
```
int(cos(c + d*x)**q*sec(a + b*x)**2,x)
```

3.363 $\int \cos(a+bx)\tan^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \cos(a+bx)\tan^3(c+bx)\,dx = \frac{\cos(a+bx)}{b}+\frac{\cos(a-c)\sec(c+bx)}{b}+\frac{3\operatorname{arctanh}(\sin(c+bx))\sin(a-c)}{2b}-\frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b}$$

output
```
cos(b*x+a)/b+cos(a-c)*sec(b*x+c)/b+3/2*arctanh(sin(b*x+c))*sin(a-c)/b-1/2*
sec(b*x+c)*sin(a-c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \cos(a+bx)\tan^3(c+bx)\,dx = \frac{(2\cos(a-2c-bx)+5\cos(a+bx)+\cos(a+2c+3bx))\sec^2(c+bx)+12\operatorname{arctanh}(\sin(c)+\cos(c)\tan(}{4b}$$

input
```
Integrate[Cos[a + b*x]*Tan[c + b*x]^3,x]
```

output
```
((2*Cos[a - 2*c - b*x] + 5*Cos[a + b*x] + Cos[a + 2*c + 3*b*x])*Sec[c + b*
x]^2 + 12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c])/(4*b)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {5090, 3042, 3091, 3042, 4257, 5087, 3042, 3086, 24, 5090, 3042, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\tan^3(bx+c)\,dx$$

$$\Big\downarrow \text{5090}$$

$$\int \sin(a+bx)\tan^2(c+bx)dx-\sin(a-c)\int \sec(c+bx)\tan^2(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\tan^2(c+bx)dx-\sin(a-c)\int \sec(c+bx)\tan(c+bx)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\int \sin(a+bx)\tan^2(c+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{1}{2}\int \sec(c+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\tan^2(c+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{1}{2}\int \csc\left(c+bx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{4257}$$

$$\int \sin(a+bx)\tan^2(c+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \text{5087}$$

$$-\int \cos(a+bx)\tan(c+bx)dx+\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \cos(a+bx)\tan(c+bx)dx+\cos(a-c)\int \sec(c+bx)\tan(c+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \text{3086}$$

$$-\int \cos(a+bx)\tan(c+bx)dx+\frac{\cos(a-c)\int 1d\sec(c+bx)}{b}-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)$$

$$\Big\downarrow \text{24}$$

$$-\int \cos(a+bx)\tan(c+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\cos(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{5090}$$

$$\sin(a-c)\int \sec(c+bx)dx-\int \sin(a+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\cos(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\int \sin(a+bx)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\cos(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\cos(a-c)\sec(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sin(a-c)\text{arctanh}(\sin(bx+c))}{b}-\sin(a-c)\left(\frac{\tan(bx+c)\sec(bx+c)}{2b}-\frac{\text{arctanh}(\sin(bx+c))}{2b}\right)+\frac{\cos(a-c)\sec(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*Tan[c + b*x]^3,x]
```

output
```
Cos[a + b*x]/b + (Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[
a - c])/b - Sin[a - c]*(-1/2*ArcTanh[Sin[c + b*x]]/b + (Sec[c + b*x]*Tan[c
 + b*x])/(2*b))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5087
```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5090
```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Sim
p[Sin[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.51

method	result
risch	$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}+\frac{3\,e^{i(3bx+5a+2c)}+e^{i(3bx+3a+4c)}+e^{i(bx+5a)}+3\,e^{i(bx+3a+2c)}}{2b\left(e^{2i(bx+a+c)}+e^{2ia}\right)^{2}}+\frac{3\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(a-c)}{2b}-\frac{3}{}$

input
```
int(cos(b*x+a)*tan(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/b*exp(I*(b*x+a))+1/2/b*exp(-I*(b*x+a))+1/2/b/(exp(2*I*(b*x+a+c))+exp(2
*I*a))^2*(3*exp(I*(3*b*x+5*a+2*c))+exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+5*a))
+3*exp(I*(b*x+3*a+2*c)))+3/2*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*sin(a-c)-
3/2*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(68) = 136.

Time = 0.10 (sec) , antiderivative size = 366, normalized size of antiderivative = 5.08

$$\int \cos(a+bx)\tan^3(c+bx)\,dx$$

$$=\frac{16\,\cos\left(bx+a\right)^{3}\cos\left(-2\,a+2\,c\right)-4\left(4\,\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right)-4\left(\cos\left(-2\,a+2\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/8*(16*cos(b*x + a)^3*cos(-2*a + 2*c) - 4*(4*cos(b*x + a)^2 + 1)*sin(b*x
+ a)*sin(-2*a + 2*c) - 4*(cos(-2*a + 2*c) - 5)*cos(b*x + a) + 3*sqrt(2)*(2
*(cos(-2*a + 2*c)^2 - 1)*cos(b*x + a)*sin(b*x + a) + (2*cos(b*x + a)^2*cos
(-2*a + 2*c) - cos(-2*a + 2*c) + 1)*sin(-2*a + 2*c))*log(-(2*cos(b*x + a)^
2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2
)*((cos(-2*a + 2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt
(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a +
2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))
/sqrt(cos(-2*a + 2*c) + 1))/(2*b*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos(
b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) + b)
```

Sympy [F]

$$\int \cos(a+bx)\tan^3(c+bx)\,dx = \int \cos{\left(a+bx \right)}\tan^3{\left(bx+c \right)}\,dx$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)**3,x)
```

output
```
Integral(cos(a + b*x)*tan(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(68) = 136$.

Time $= 0.18$ (sec) , antiderivative size $= 1027$, normalized size of antiderivative $= 14.26$

$$\int \cos(a+bx)\tan^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*(2*(cos(5*b*x + a + 4*c) + 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*cos(
6*b*x + 2*a + 4*c) + 2*(5*cos(4*b*x + 2*a + 2*c) + 2*cos(4*b*x + 4*c) + 2*
cos(2*b*x + 2*a) + 5*cos(2*b*x + 2*c) + 1)*cos(5*b*x + a + 4*c) + 10*(2*co
s(3*b*x + a + 2*c) + cos(b*x + a))*cos(4*b*x + 2*a + 2*c) + 4*(2*cos(3*b*x
 + a + 2*c) + cos(b*x + a))*cos(4*b*x + 4*c) + 4*(2*cos(2*b*x + 2*a) + 5*c
os(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 4*cos(2*b*x + 2*a)*cos(b*x + a
) + 10*cos(2*b*x + 2*c)*cos(b*x + a) + 3*(cos(5*b*x + a + 4*c)^2*sin(-a +
c) + 4*cos(3*b*x + a + 2*c)^2*sin(-a + c) + 4*cos(3*b*x + a + 2*c)*cos(b*x
 + a)*sin(-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*si
n(-a + c) + 4*sin(3*b*x + a + 2*c)^2*sin(-a + c) + 4*sin(3*b*x + a + 2*c)*
sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c) + 2*(2*cos(3*b*x + a
 + 2*c)*sin(-a + c) + cos(b*x + a)*sin(-a + c))*cos(5*b*x + a + 4*c) + 2*(
2*sin(3*b*x + a + 2*c)*sin(-a + c) + sin(b*x + a)*sin(-a + c))*sin(5*b*x +
 a + 4*c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + si
n(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + c
os(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*si
n(c) + sin(c)^2)) + 2*(sin(5*b*x + a + 4*c) + 2*sin(3*b*x + a + 2*c) + sin
(b*x + a))*sin(6*b*x + 2*a + 4*c) + 2*(5*sin(4*b*x + 2*a + 2*c) + 2*sin(4*
b*x + 4*c) + 2*sin(2*b*x + 2*a) + 5*sin(2*b*x + 2*c))*sin(5*b*x + a + 4*c)
 + 10*(2*sin(3*b*x + a + 2*c) + sin(b*x + a))*sin(4*b*x + 2*a + 2*c) + ...
```

Giac [F]

$$\int \cos(a+bx)\tan^3(c+bx)\,dx=\int \cos\left(bx+a\right)\tan\left(bx+c\right)^3\,dx$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*tan(b*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\tan^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)*tan(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos(a+bx)\tan^3(c+bx)\,dx = \frac{-2\left(\int \frac{\sin(bx+c)^3}{\cos(bx+c)\sin(bx+c)^2-\cos(bx+c)}dx\right)\sin(bx+c)^2 b+2\left(\int \frac{\sin(bx+c)^3}{\cos(bx+c)\sin(bx+c)^2-\cos(bx+c)}dx\right)b-2\left(\int \frac{\cos(b}{\cos(bx+c)\,s}\right.}{}$$

input
```
int(cos(b*x+a)*tan(b*x+c)^3,x)
```

output
```
( - 2*int(sin(b*x + c)**3/(cos(b*x + c)*sin(b*x + c)**2 - cos(b*x + c)),x)
*sin(b*x + c)**2*b + 2*int(sin(b*x + c)**3/(cos(b*x + c)*sin(b*x + c)**2 -
 cos(b*x + c)),x)*b - 2*int((cos(a + b*x)*sin(b*x + c)**3)/(cos(b*x + c)*s
in(b*x + c)**2 - cos(b*x + c)),x)*sin(b*x + c)**2*b + 2*int((cos(a + b*x)*
sin(b*x + c)**3)/(cos(b*x + c)*sin(b*x + c)**2 - cos(b*x + c)),x)*b + 2*lo
g(cos(b*x + c))*sin(b*x + c)**2 - 2*log(cos(b*x + c)) + 4*log(tan((b*x + c
)/2)**2 + 1)*sin(b*x + c)**2 - 4*log(tan((b*x + c)/2)**2 + 1) - 4*log(tan(
(b*x + c)/2) - 1)*sin(b*x + c)**2 + 4*log(tan((b*x + c)/2) - 1) - 4*log(ta
n((b*x + c)/2) + 1)*sin(b*x + c)**2 + 4*log(tan((b*x + c)/2) + 1) - 3*sin(
b*x + c)**2 + 4)/(2*b*(sin(b*x + c)**2 - 1))
```

3.364 $\int \cos(a+bx)\tan^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \cos(a+bx)\tan^2(c+bx)\,dx=\frac{\operatorname{arctanh}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}-\frac{\sin(a+bx)}{b}$$

output
```
arctanh(sin(b*x+c))*cos(a-c)/b-sec(b*x+c)*sin(a-c)/b-sin(b*x+a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.41

$$\int \cos(a+bx)\tan^2(c+bx)\,dx=-\frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b}-\frac{\cos(bx)\sin(a)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}-\frac{\cos(a)\sin(bx)}{b}$$

input
```
Integrate[Cos[a + b*x]*Tan[c + b*x]^2,x]
```

output
```
((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x
)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Cos
[b*x]*Sin[a])/b - (Sec[c + b*x]*Sin[a - c])/b - (Cos[a]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {5090, 3042, 3086, 24, 5087, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\tan^2(bx+c)\,dx$$

$$\Big\downarrow \text{ 5090}$$

$$\int \sin(a+bx)\tan(c+bx)dx-\sin(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)\tan(c+bx)dx-\sin(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{ 3086}$$

$$\int \sin(a+bx)\tan(c+bx)dx-\frac{\sin(a-c)\int 1d\sec(c+bx)}{b}$$

$$\Big\downarrow \text{ 24}$$

$$\int \sin(a+bx)\tan(c+bx)dx-\frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{ 5087}$$

$$\cos(a-c)\int \sec(c+bx)dx-\int \cos(a+bx)dx-\frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\int \sin\left(a+bx+\frac{\pi}{2}\right)dx-\frac{\sin(a-c)\sec(bx+c)}{b}$$

$\big\downarrow$ 3117

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\frac{\sin(a-c)\sec(bx+c)}{b}-\frac{\sin(a+bx)}{b}$$

$\big\downarrow$ 4257

$$\frac{\cos(a-c)\text{arctanh}(\sin(bx+c))}{b}-\frac{\sin(a-c)\sec(bx+c)}{b}-\frac{\sin(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*Tan[c + b*x]^2,x]
```

output
```
(ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - (Sec[c + b*x]*Sin[a - c])/b - Sin[a
 + b*x]/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5087
```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5090
```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Sim
p[Sin[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.24

method	result
risch	$\frac{i e^{i(bx+a)}}{2b} - \frac{i e^{-i(bx+a)}}{2b} - \frac{i\left(-e^{i(bx+3a)}+e^{i(bx+a+2c)}\right)}{b\left(e^{2i(bx+a+c)}+e^{2ia}\right)} - \frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\cos(a-c)}{b} + \frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\cos(a-c)}{b}$

input
```
int(cos(b*x+a)*tan(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b*exp(I*(b*x+a))-1/2*I/b*exp(-I*(b*x+a))-I/b/(exp(2*I*(b*x+a+c))+exp
(2*I*a))*(-exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))-ln(exp(I*(b*x+a))-I*exp(I*
(a-c)))/b*cos(a-c)+ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*cos(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(46) = 92.

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 6.87

$$\int \cos(a+bx)\tan^2(c+bx)\,dx$$

$$= \frac{4\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+\frac{\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\sin(bx+a)\sin(-2\,a+2\,c)-\left(\cos(-2\,a+2\,c)^2+2\,\cos\right.\right.}{}}{4\left(b\sin\right.}$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*((cos(-2*
a + 2*c) + 1)*sin(b*x + a)*sin(-2*a + 2*c) - (cos(-2*a + 2*c)^2 + 2*cos(-2
*a + 2*c) + 1)*cos(b*x + a))*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos
(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*
sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - c
os(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin
(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a + 2*c) + 1
) + 4*(cos(b*x + a)^2 - 2)*sin(-2*a + 2*c))/(b*sin(b*x + a)*sin(-2*a + 2*c
) - (b*cos(-2*a + 2*c) + b)*cos(b*x + a))
```

Sympy [F]

$$\int \cos(a+bx)\tan^2(c+bx)\,dx = \int \cos(a+bx)\tan^2(bx+c)\,dx$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)*tan(b*x + c)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(46) = 92$.

Time = 0.18 (sec) , antiderivative size = 526, normalized size of antiderivative = 11.43

$$\int \cos(a+bx)\tan^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*((sin(3*b*x + a + 2*c) + sin(b*x + a))*cos(4*b*x + 2*a + 2*c) - 3*(sin
(2*b*x + 2*a) - sin(2*b*x + 2*c))*cos(3*b*x + a + 2*c) - (cos(3*b*x + a +
2*c)^2*cos(-a + c) + 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos(-a + c) + cos
(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b*x + a + 2*c)^2 + 2*cos(-a +
c)*sin(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2)*log((co
s(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2
*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)
*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2))
- (cos(3*b*x + a + 2*c) + cos(b*x + a))*sin(4*b*x + 2*a + 2*c) + (3*cos(2*
b*x + 2*a) - 3*cos(2*b*x + 2*c) - 1)*sin(3*b*x + a + 2*c) - 3*cos(b*x + a)
*sin(2*b*x + 2*a) + 3*cos(b*x + a)*sin(2*b*x + 2*c) + 3*cos(2*b*x + 2*a)*s
in(b*x + a) - 3*cos(2*b*x + 2*c)*sin(b*x + a) - sin(b*x + a))/(b*cos(3*b*x
 + a + 2*c)^2 + 2*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 +
 b*sin(3*b*x + a + 2*c)^2 + 2*b*sin(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(
b*x + a)^2)
```

Giac [F]

$$\int \cos(a+bx)\tan^2(c+bx)\,dx = \int \cos(bx+a)\tan(bx+c)^2\,dx$$

input
```
integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*tan(b*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 25.11 (sec) , antiderivative size = 285, normalized size of antiderivative = 6.20

$$\int \cos(a+bx)\tan^2(c+bx)\,dx$$
$$= -\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b} + \frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b} - \frac{e^{a\,1i+b\,x\,1i}\,(e^{a\,2i-c\,2i}-1)}{b\,(e^{a\,2i-c\,2i}\,1i+e^{a\,2i+b\,x\,2i}\,1i)} + \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,(e^{a\,2i}\,e^{-c\,2i}+1) - \frac{e^{a\,2i}\,e^{-c\,2i}\,(e^{a\,2i}\,e^{-c\,2i}+1)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)(e^{a\,2i-c\,2i}+1)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}} - \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,(e^{a\,2i}\,e^{-c\,2i}+1) + \frac{e^{a\,2i}\,e^{-c\,2i}\,(e^{a\,2i}\,e^{-c\,2i}+1)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)(e^{a\,2i-c\,2i}+1)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

input
```
int(cos(a + b*x)*tan(c + b*x)^2,x)
```

output
```
(exp(a*1i + b*x*1i)*1i)/(2*b) - (exp(- a*1i - b*x*1i)*1i)/(2*b) - (exp(a*1
i + b*x*1i)*(exp(a*2i - c*2i) - 1))/(b*(exp(a*2i - c*2i)*1i + exp(a*2i + b
*x*2i)*1i)) + (log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1) - (e
xp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^
(1/2))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log((exp(a*
2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2)
 - exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1))*(exp(a*2i - c*2i) + 1
))/(2*b*exp(a*2i - c*2i)^(1/2))
```

Reduce [F]

$$\int \cos(a+bx)\tan^2(c+bx)\,dx$$
$$= \frac{-\cos(bx+c)\left(\int \frac{\sin(bx+c)^2}{\sin(bx+c)^2-1}dx\right)b - \cos(bx+c)\left(\int \frac{\cos(bx+a)\sin(bx+c)^2}{\sin(bx+c)^2-1}dx\right)b + 2\cos(bx+c)\,a + \cos(bx+}{\cos(bx+c)\,b}$$

input
```
int(cos(b*x+a)*tan(b*x+c)^2,x)
```

output

```
( - cos(b*x + c)*int(sin(b*x + c)**2/(sin(b*x + c)**2 - 1),x)*b - cos(b*x
+ c)*int((cos(a + b*x)*sin(b*x + c)**2)/(sin(b*x + c)**2 - 1),x)*b + 2*cos
(b*x + c)*a + cos(b*x + c)*b*x - sin(b*x + c))/(cos(b*x + c)*b)
```

3.365 $\int \cos(a+bx)\tan(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \cos(a+bx)\tan(c+bx)\,dx = -\frac{\cos(a+bx)}{b} - \frac{\operatorname{arctanh}(\sin(c+bx))\sin(a-c)}{b}$$

output
```
-cos(b*x+a)/b-arctanh(sin(b*x+c))*sin(a-c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.10

$$\int \cos(a+bx)\tan(c+bx)\,dx = -\frac{\cos(a)\cos(bx)}{b} + \frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b} + \frac{\sin(a)\sin(bx)}{b}$$

input
```
Integrate[Cos[a + b*x]*Tan[c + b*x],x]
```

output
```
-((Cos[a]*Cos[b*x])/b) + ((2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*
Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c
])]*Sin[a - c])/b + (Sin[a]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5090, 3042, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\tan(bx+c)\,dx$$

$$\Big\downarrow \text{5090}$$

$$\int \sin(a+bx)dx - \sin(a-c)\int \sec(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)dx - \sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3118}$$

$$-\sin(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx - \frac{\cos(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\sin(a-c)\text{arctanh}(\sin(bx+c))}{b} - \frac{\cos(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*Tan[c + b*x],x]
```

output
```
-(Cos[a + b*x]/b) - (ArcTanh[Sin[c + b*x]]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5090
```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Sim
p[Sin[v - w]   Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.23

method	result	size
risch	$-\frac{e^{i(bx+a)}}{2b}-\frac{e^{-i(bx+a)}}{2b}-\frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(a-c)}{b}+\frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\sin(a-c)}{b}$	97

input
```
int(cos(b*x+a)*tan(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*exp(I*(b*x+a))-1/2/b*exp(-I*(b*x+a))-ln(exp(I*(b*x+a))+I*exp(I*(a-c
)))/b*sin(a-c)+ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 6.53

$$\int \cos(a+bx)\tan(c+bx)\,dx = \frac{\frac{\sqrt{2}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\frac{2\sqrt{2}((\cos(-2a+2c)+1)\sin(bx+a)+\cos(bx+a)\sin(-2a+2c))}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)-3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)+1}\right)}{\sqrt{\cos(-2a+2c)+1}}}{4b}$$

input
```
integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*
x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a) + c
os(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) -
 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2
*a + 2*c) - cos(-2*a + 2*c) + 1))*sin(-2*a + 2*c)/sqrt(cos(-2*a + 2*c) + 1
) - 4*cos(b*x + a))/b
```

Sympy [F]

$$\int \cos(a+bx)\tan(c+bx)\,dx = \int \cos(a+bx)\tan(bx+c)\,dx$$

input
```
integrate(cos(b*x+a)*tan(b*x+c),x)
```

output
```
Integral(cos(a + b*x)*tan(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(30) = 60$.

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.37

$$\int \cos(a+bx)\tan(c+bx)\,dx = -\frac{\log\left(\frac{\cos(bx+2c)^2+\cos(c)^2-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2+2\cos(bx+2c)\sin(c)+\sin(c)^2}{\cos(bx+2c)^2+\cos(c)^2+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2-2\cos(bx+2c)\sin(c)+\sin(c)^2}\right)\sin(-a+c)+2\cos(bx+a)}{2b}$$

input
```
integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

output
```
-1/2*(log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x
 + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)
^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c)
+ sin(c)^2))*sin(-a + c) + 2*cos(b*x + a))/b
```

Giac [F]

$$\int \cos(a+bx)\tan(c+bx)\,dx = \int \cos(bx+a)\tan(bx+c)\,dx$$

input
```
integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*tan(b*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 23.44 (sec) , antiderivative size = 237, normalized size of antiderivative = 7.90

$$\int \cos(a+bx)\tan(c+bx)\,dx$$
$$= -\frac{e^{-a\,1i-b\,x\,1i}}{2\,b}-\frac{e^{a\,1i+b\,x\,1i}}{2\,b}+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

input
```
int(cos(a + b*x)*tan(c + b*x),x)
```

output
```
(log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) - (exp(a*2i)*e
xp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(e
xp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - exp(a*1i + b*x*1i)
/(2*b) - exp(- a*1i - b*x*1i)/(2*b) - (log((exp(a*2i)*exp(-c*2i)*(exp(a*2i
)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1i
)*(exp(a*2i)*exp(-c*2i)*1i - 1i))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i
- c*2i))^(1/2))
```

Reduce [F]

$$\int \cos(a+bx)\tan(c+bx)\,dx=\int \cos(bx+a)\tan(bx+c)\,dx$$

input
```
int(cos(b*x+a)*tan(b*x+c),x)
```

output
```
int(cos(a + b*x)*tan(b*x + c),x)
```

3.366 $\int \cos(a+bx)\cot(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \cos(a+bx)\cot(c+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+bx))\cos(a-c)}{b}+\frac{\cos(a+bx)}{b}$$

output
```
-arctanh(cos(b*x+c))*cos(a-c)/b+cos(b*x+a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.24

$$\int \cos(a+bx)\cot(c+bx)\,dx = -\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b}+\frac{\cos(a)\cos(bx)}{b}-\frac{\sin(a)\sin(bx)}{b}$$

input
```
Integrate[Cos[a + b*x]*Cot[c + b*x],x]
```

output
```
((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x
)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b + (Cos
[a]*Cos[b*x])/b - (Sin[a]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5088, 3042, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot(bx+c)\,dx$$

$$\Big\downarrow \text{5088}$$

$$\cos(a-c)\int \csc(c+bx)dx - \int \sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc(c+bx)dx - \int \sin(a+bx)dx$$

$$\Big\downarrow \text{3118}$$

$$\cos(a-c)\int \csc(c+bx)dx + \frac{\cos(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(a+bx)}{b} - \frac{\cos(a-c)\text{arctanh}(\cos(bx+c))}{b}$$

input
```
Int[Cos[a + b*x]*Cot[c + b*x],x]
```

output
```
-((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) + Cos[a + b*x]/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5088
```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

method	result	size
risch	$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}+\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos(a-c)}{b}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos(a-c)}{b}$	93

input
```
int(cos(b*x+a)*cot(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*exp(I*(b*x+a))+1/2/b*exp(-I*(b*x+a))+ln(exp(I*(b*x+a))-exp(I*(a-c)))
/b*cos(a-c)-ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 6.55

$$\int \cos(a+bx)\cot(c+bx)\,dx = \frac{\sqrt{2}\sqrt{\cos(-2a+2c)+1}\log\left(-\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\frac{2\sqrt{2}((\cos(-2a+2c)+1)\cos(bx-}{\sqrt{\cos(-2a}}}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-}\right.}{4b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*sqrt(cos(-2*a + 2*c) + 1)*log(-(2*cos(b*x + a)^2*cos(-2*a + 2
*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a +
 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*
c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b
*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1)) + 4*cos(b*x +
 a))/b
```

Sympy [F]

$$\int \cos(a+bx)\cot(c+bx)\,dx = \int \cos(a+bx)\cot(bx+c)\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+c),x)
```

output
```
Integral(cos(a + b*x)*cot(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(29) = 58$.

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.62

$$\int \cos(a+bx)\cot(c+bx)\,dx = -\frac{\cos(-a+c)\log\left(\cos(bx)^2 + 2\cos(bx)\cos(c) + \cos(c)^2 + \sin(bx)^2 - 2\sin(bx)\sin(c) + \sin(c)^2\right) - c}{}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="maxima")
```

output
```
-1/2*(cos(-a + c)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)
^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - cos(-a + c)*log(cos(b*x)^2 - 2*cos(b*
x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*cos(
b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(29) = 58$.

Time = 0.14 (sec) , antiderivative size = 234, normalized size of antiderivative = 8.07

$$\int \cos(a+bx)\cot(c+bx)\,dx = -\frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3 - \tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right) + 4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}c\right)^3 + \tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right) - 1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3 + \tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right) + \tan\left(\frac{1}{2}c\right)^3 + \tan\left(\frac{1}{2}c\right)} - \frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\right.\right.}{}}{b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="giac")
```

output
```
-((tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a)*tan(
1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1)
)/(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + ta
n(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1
/2*c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^
2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(2*tan(1/2*b*x)*tan(
1/2*a) + tan(1/2*a)^2 - 1)/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 23.86 (sec) , antiderivative size = 231, normalized size of antiderivative = 7.97

$$\int \cos(a+bx)\cot(c+bx)\,dx$$
$$= \frac{e^{-a\,1i-b\,x\,1i}}{2\,b} + \frac{e^{a\,1i+b\,x\,1i}}{2\,b} - \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right) - \frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}} + \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right) + \frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

input
```
int(cos(a + b*x)*cot(c + b*x),x)
```

output
```
exp(- a*1i - b*x*1i)/(2*b) + exp(a*1i + b*x*1i)/(2*b) - (log(- exp(a*1i)*e
xp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i) - (exp(a*2i)*exp(-c*2i)*(exp(a*2
i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) + 1
))/(2*b*exp(a*2i - c*2i)^(1/2)) + (log((exp(a*2i)*exp(-c*2i)*(exp(a*2i)*ex
p(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(ex
p(a*2i)*exp(-c*2i)*1i + 1i))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)
^(1/2))
```

Reduce [F]

$$\int \cos(a+bx)\cot(c+bx)\,dx = \int \cos(bx+a)\cot(bx+c)\,dx$$

input
```
int(cos(b*x+a)*cot(b*x+c),x)
```

output
```
int(cos(a + b*x)*cot(b*x + c),x)
```

3.367 $\int \cos(a+bx)\cot^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \cos(a+bx)\cot^2(c+bx)\,dx = -\frac{\cos(a-c)\csc(c+bx)}{b} + \frac{\operatorname{arctanh}(\cos(c+bx))\sin(a-c)}{b} - \frac{\sin(a+bx)}{b}$$

output
```
-cos(a-c)*csc(b*x+c)/b+arctanh(cos(b*x+c))*sin(a-c)/b-sin(b*x+a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.43

$$\int \cos(a+bx)\cot^2(c+bx)\,dx = -\frac{\cos(a-c)\csc(c+bx)}{b} - \frac{\cos(bx)\sin(a)}{b} + \frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b} - \frac{\cos(a)\sin(bx)}{b}$$

input
```
Integrate[Cos[a + b*x]*Cot[c + b*x]^2,x]
```

output
```
-((Cos[a - c]*Csc[c + b*x])/b) - (Cos[b*x]*Sin[a])/b + ((2*I)*ArcTan[((Cos
[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos
[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Sin[a - c])/b - (Cos[a]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5088, 3042, 25, 3086, 24, 5089, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot^2(bx+c)\,dx$$

$$\Big\downarrow \text{5088}$$

$$\cos(a-c)\int \cot(c+bx)\csc(c+bx)dx-\int \cot(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx-\int \cot(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cot(c+bx)\sin(a+bx)dx-\cos(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(a-c)\int 1d\csc(c+bx)}{b}-\int \cot(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{24}$$

$$-\int \cot(c+bx)\sin(a+bx)dx-\frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{5089}$$

$$-\sin(a-c)\int \csc(c+bx)dx-\int \cos(a+bx)dx-\frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\sin(a-c)\int \csc(c+bx)dx-\int \sin\left(a+bx+\frac{\pi}{2}\right)dx-\frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{3117}$$

$$-\sin(a-c)\int \csc(c+bx)dx-\frac{\cos(a-c)\csc(bx+c)}{b}-\frac{\sin(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sin(a-c)\text{arctanh}(\cos(bx+c))}{b}-\frac{\cos(a-c)\csc(bx+c)}{b}-\frac{\sin(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*Cot[c + b*x]^2,x]
```

output
```
-((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b - Si
n[a + b*x]/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5088
```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5089
```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Sim
p[Sin[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.15

method	result
risch	$\frac{i\,\mathrm{e}^{i(bx+a)}}{2b}-\frac{i\,\mathrm{e}^{-i(bx+a)}}{2b}+\frac{i\left(\mathrm{e}^{i(bx+3a)}+\mathrm{e}^{i(bx+a+2c)}\right)}{b\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-\mathrm{e}^{i(a-c)}\right)\sin(a-c)}{b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+\mathrm{e}^{i(a-c)}\right)\sin(a-c)}{b}$

input
```
int(cos(b*x+a)*cot(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b*exp(I*(b*x+a))-1/2*I/b*exp(-I*(b*x+a))+I/b/(-exp(2*I*(b*x+a+c))+ex
p(2*I*a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))-ln(exp(I*(b*x+a))-exp(I*(a
-c)))/b*sin(a-c)+ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(46) = 92$.

Time = 0.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 6.87

$$\int \cos(a + bx) \cot^2(c + bx) \, dx$$

$$= \frac{4\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)^2 - 4\,\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right) + \frac{\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\sin\cdots\right.}{\cdots}}{4\left(\cdots\right.}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)^2 - 4*cos(b*x + a)*sin(b*x + a)*
sin(-2*a + 2*c) + sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a)*sin(-2*a + 2
*c) - (cos(-2*a + 2*c)^2 - 1)*cos(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*
a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-
2*a + 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a
 + 2*c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*
cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos
(-2*a + 2*c) + 1) - 8*cos(-2*a + 2*c) - 8)/(b*cos(b*x + a)*sin(-2*a + 2*c)
 + (b*cos(-2*a + 2*c) + b)*sin(b*x + a))
```

Sympy [F]

$$\int \cos(a + bx) \cot^2(c + bx) \, dx = \int \cos\left(a + bx\right)\cot^2\left(bx + c\right) dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)*cot(b*x + c)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(46) = 92$.

Time = 0.06 (sec) , antiderivative size = 613, normalized size of antiderivative = 13.33

$$\int \cos(a + bx)\cot^2(c + bx)\, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*((sin(3*b*x + a + 2*c) - sin(b*x + a))*cos(4*b*x + 2*a + 2*c) + 3*(sin
(2*b*x + 2*a) + sin(2*b*x + 2*c))*cos(3*b*x + a + 2*c) - (cos(3*b*x + a +
2*c)^2*sin(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a + c) + cos
(b*x + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c)^2*sin(-a + c) - 2*sin(3*b*x
 + a + 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c))*log(cos
(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) +
sin(c)^2) + (cos(3*b*x + a + 2*c)^2*sin(-a + c) - 2*cos(3*b*x + a + 2*c)*c
os(b*x + a)*sin(-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c
)^2*sin(-a + c) - 2*sin(3*b*x + a + 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*
x + a)^2*sin(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(
b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(3*b*x + a + 2*c) - cos(b*x +
 a))*sin(4*b*x + 2*a + 2*c) - (3*cos(2*b*x + 2*a) + 3*cos(2*b*x + 2*c) - 1
)*sin(3*b*x + a + 2*c) - 3*cos(b*x + a)*sin(2*b*x + 2*a) - 3*cos(b*x + a)*
sin(2*b*x + 2*c) + 3*cos(2*b*x + 2*a)*sin(b*x + a) + 3*cos(2*b*x + 2*c)*si
n(b*x + a) - sin(b*x + a))/(b*cos(3*b*x + a + 2*c)^2 - 2*b*cos(3*b*x + a +
 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + a + 2*c)^2 - 2*b*sin
(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(46) = 92$.

Time = 0.16 (sec) , antiderivative size = 627, normalized size of antiderivative = 13.63

$$\int \cos(a + bx)\cot^2(c + bx)\, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(4*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*t
an(1/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(tan(1/2*a
)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1/2*c)) -
4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/
2*c))*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan
(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^4
- 6*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^2 + 4*tan(1/2*b*x)^3*tan(1/2*a)
*tan(1/2*c)^3 - 6*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)^
3*tan(1/2*c)^4 + tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*b*x)^3*t
an(1/2*a)^2 - 4*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c) + 6*tan(1/2*b*x)^2*ta
n(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*b*x)^3*tan(1/2*c)^2 + 2*tan(1/2*b*x)*tan
(1/2*a)^2*tan(1/2*c)^2 + 6*tan(1/2*b*x)^2*tan(1/2*c)^3 + 12*tan(1/2*b*x)*t
an(1/2*a)*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)*tan(1/
2*c)^4 - tan(1/2*b*x)^3 + tan(1/2*b*x)*tan(1/2*a)^2 - 6*tan(1/2*b*x)^2*tan
(1/2*c) - 12*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c) + 2*tan(1/2*a)^2*tan(1/2*c
) - 2*tan(1/2*b*x)*tan(1/2*c)^2 - 16*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*c
)^3 - tan(1/2*b*x) - 2*tan(1/2*c))/((tan(1/2*b*x)^4*tan(1/2*c) + tan(1/2*b
*x)^3*tan(1/2*c)^2 - tan(1/2*b*x)^3 + tan(1/2*b*x)*tan(1/2*c)^2 - tan(1/2*
b*x) - tan(1/2*c))*(tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c))))/b
```

Mupad [B] (verification not implemented)

Time = 23.54 (sec) , antiderivative size = 289, normalized size of antiderivative = 6.28

$$\int \cos(a+bx)\cot^2(c+bx)\,dx$$
$$= -\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b}-\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}+1\right)}{b\,\left(e^{a\,2i-c\,2i}\,1i-e^{a\,2i+b\,x\,2i}\,1i\right)}$$
$$-\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$
$$+\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

input
```
int(cos(a + b*x)*cot(c + b*x)^2,x)
```

output
```
(exp(a*1i + b*x*1i)*1i)/(2*b) - (exp(- a*1i - b*x*1i)*1i)/(2*b) - (exp(a*1
i + b*x*1i)*(exp(a*2i - c*2i) + 1))/(b*(exp(a*2i - c*2i)*1i - exp(a*2i + b
*x*2i)*1i)) - (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) - (exp
(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(
1/2))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) + (log(exp(a
*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) + (exp(a*2i)*exp(-c*2i)*(exp(a
*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i)
- 1))/(2*b*(-exp(a*2i - c*2i))^(1/2))
```

Reduce [F]

$$\int \cos(a+bx)\cot^2(c+bx)\,dx = \int \cos(bx+a)\cot(bx+c)^2\,dx$$

input
```
int(cos(b*x+a)*cot(b*x+c)^2,x)
```

output
```
int(cos(a + b*x)*cot(b*x + c)**2,x)
```

3.368 $\int \cos(a+bx)\cot^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \cos(a+bx)\cot^3(c+bx)\,dx = \frac{3\text{arctanh}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a+bx)}{b} - \frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} + \frac{\csc(c+bx)\sin(a-c)}{b}$$

output
```
3/2*arctanh(cos(b*x+c))*cos(a-c)/b-cos(b*x+a)/b-1/2*cos(a-c)*cot(b*x+c)*cs
c(b*x+c)/b+csc(b*x+c)*sin(a-c)/b
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \cos(a+bx)\cot^3(c+bx)\,dx = \frac{12\text{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+(2\cos(a-2c-bx)-5\cos(a+bx)+\cos(a+2c+3bx}{4b}$$

input
```
Integrate[Cos[a + b*x]*Cot[c + b*x]^3,x]
```

output
```
(12*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + (2*Cos[a - 2*c - b*
x] - 5*Cos[a + b*x] + Cos[a + 2*c + 3*b*x])*Csc[c + b*x]^2)/(4*b)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.933, Rules used = {5088, 3042, 3091, 3042, 4257, 5089, 3042, 25, 3086, 24, 5088, 3042, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot^3(bx+c)\,dx$$

$$\Big\downarrow \text{5088}$$

$$\cos(a-c)\int \cot^2(c+bx)\csc(c+bx)dx-\int \cot^2(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)^2dx-\int \cot^2(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3091}$$

$$\cos(a-c)\left(-\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\int \cot^2(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\left(-\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\int \cot^2(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{4257}$$

$$\cos(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\int \cot^2(c+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{5089}$$

$$-\int \cos(a+bx)\cot(c+bx)dx-\sin(a-c)\int \cot(c+bx)\csc(c+bx)dx+\cos(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \cos(a+bx)\cot(c+bx)dx - \sin(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx + \cos(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b} - \frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(a+bx)\cot(c+bx)dx + \sin(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx + \cos(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b} - \frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3086}$$

$$-\int \cos(a+bx)\cot(c+bx)dx + \frac{\sin(a-c)\int 1d\csc(c+bx)}{b} + \cos(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b} - \frac{\cot(bx+c)\csc(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{24}$$

$$-\int \cos(a+bx)\cot(c+bx)dx + \cos(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b} - \frac{\cot(bx+c)\csc(bx+c)}{2b}\right) + \frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{5088}$$

$$-\cos(a-c)\int \csc(c+bx)dx + \int \sin(a+bx)dx + \cos(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b} - \frac{\cot(bx+c)\csc(bx+c)}{2b}\right) + \frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\cos(a-c)\int \csc(c+bx)dx + \int \sin(a+bx)dx + \cos(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b} - \frac{\cot(bx+c)\csc(bx+c)}{2b}\right) + \frac{\sin(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{3118}$$

$$-\cos(a-c)\int \csc(c+bx)dx + \cos(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b} - \frac{\cot(bx+c)\csc(bx+c)}{2b}\right) + \frac{\sin(a-c)\csc(bx+c)}{b} - \frac{\cos(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(a-c)\text{arctanh}(\cos(bx+c))}{b}+\cos(a-c)\left(\frac{\text{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)+\frac{\sin(a-c)\csc(bx+c)}{b}-\frac{\cos(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*Cot[c + b*x]^3,x]
```

output
```
(ArcTanh[Cos[c + b*x]]*Cos[a - c])/b - Cos[a + b*x]/b + Cos[a - c]*(ArcTan
h[Cos[c + b*x]]/(2*b) - (Cot[c + b*x]*Csc[c + b*x])/(2*b)) + (Csc[c + b*x]
*Sin[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5088
```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Si
mp[Cos[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
 w, x] && NeQ[w, v]
```

rule 5089
```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Sim
p[Sin[v - w]   Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.45

method	result
risch	$-\frac{e^{i(bx+a)}}{2b}-\frac{e^{-i(bx+a)}}{2b}-\frac{-3\,e^{i(3bx+5a+2c)}+e^{i(3bx+3a+4c)}+e^{i(bx+5a)}-3\,e^{i(bx+3a+2c)}}{2b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^{2}}+\frac{3\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos(a-c)}{2b}$ -

input
```
int(cos(b*x+a)*cot(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*exp(I*(b*x+a))-1/2/b*exp(-I*(b*x+a))-1/2/b/(-exp(2*I*(b*x+a+c))+exp
(2*I*a))^2*(-3*exp(I*(3*b*x+5*a+2*c))+exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+5*
a))-3*exp(I*(b*x+3*a+2*c)))+3/2*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)
-3/2*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*cos(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(69) = 138$.

Time = 0.10 (sec) , antiderivative size = 385, normalized size of antiderivative = 5.27

$$\int \cos(a + bx)\cot^3(c + bx)\,dx =$$

$$-\frac{16\cos(bx+a)^3\cos(-2a+2c) - 4\left(4\cos(bx+a)^2+1\right)\sin(bx+a)\sin(-2a+2c) - 4\left(\cos(-2a\right.}{}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="fricas")
```

output
```
-1/8*(16*cos(b*x + a)^3*cos(-2*a + 2*c) - 4*(4*cos(b*x + a)^2 + 1)*sin(b*x
 + a)*sin(-2*a + 2*c) - 4*(cos(-2*a + 2*c) + 5)*cos(b*x + a) + 3*sqrt(2)*(
2*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*(cos
(-2*a + 2*c)^2 + cos(-2*a + 2*c))*cos(b*x + a)^2 + cos(-2*a + 2*c)^2 + 2*c
os(-2*a + 2*c) + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)
*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x +
 a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a +
 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)
*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1))/(2*b*c
os(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2
*c) - b*cos(-2*a + 2*c) - b)
```

Sympy [F]

$$\int \cos(a + bx)\cot^3(c + bx)\,dx = \int \cos(a + bx)\cot^3(bx + c)\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)**3,x)
```

output
```
Integral(cos(a + b*x)*cot(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. $2(69) = 138$.

Time = 0.07 (sec) , antiderivative size = 1254, normalized size of antiderivative = 17.18

$$\int \cos(a+bx)\cot^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*(cos(5*b*x + a + 4*c) - 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*cos
(6*b*x + 2*a + 4*c) - 2*(5*cos(4*b*x + 2*a + 2*c) - 2*cos(4*b*x + 4*c) - 2
*cos(2*b*x + 2*a) + 5*cos(2*b*x + 2*c) - 1)*cos(5*b*x + a + 4*c) + 10*(2*c
os(3*b*x + a + 2*c) - cos(b*x + a))*cos(4*b*x + 2*a + 2*c) - 4*(2*cos(3*b*
x + a + 2*c) - cos(b*x + a))*cos(4*b*x + 4*c) - 4*(2*cos(2*b*x + 2*a) - 5*
cos(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 4*cos(2*b*x + 2*a)*cos(b*x +
a) - 10*cos(2*b*x + 2*c)*cos(b*x + a) - 3*(cos(5*b*x + a + 4*c)^2*cos(-a +
 c) + 4*cos(3*b*x + a + 2*c)^2*cos(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*
x + a)*cos(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x +
a + 4*c)^2 + 4*cos(-a + c)*sin(3*b*x + a + 2*c)^2 - 4*cos(-a + c)*sin(3*b*
x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2 - 2*(2*cos(3*b*x +
a + 2*c)*cos(-a + c) - cos(b*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) - 2*
(2*cos(-a + c)*sin(3*b*x + a + 2*c) - cos(-a + c)*sin(b*x + a))*sin(5*b*x
+ a + 4*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2
*sin(b*x)*sin(c) + sin(c)^2) + 3*(cos(5*b*x + a + 4*c)^2*cos(-a + c) + 4*c
os(3*b*x + a + 2*c)^2*cos(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*co
s(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x + a + 4*c)^
2 + 4*cos(-a + c)*sin(3*b*x + a + 2*c)^2 - 4*cos(-a + c)*sin(3*b*x + a + 2
*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2 - 2*(2*cos(3*b*x + a + 2*c)*
cos(-a + c) - cos(b*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*cos...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(69) = 138$.

Time = 0.21 (sec) , antiderivative size = 963, normalized size of antiderivative = 13.19

$$\int \cos(a + bx) \cot^3(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="giac")
```

output
```
1/8*(12*(tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a
)*tan(1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c
) - 1))/(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^
3 + tan(1/2*c)) - 12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2
*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(ta
n(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 16*(2*tan(1/2
*b*x)*tan(1/2*a) + tan(1/2*a)^2 - 1)/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 +
 1)) + (2*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^7 + tan(1/2*b*x)^2*tan(1/
2*a)^2*tan(1/2*c)^8 + 6*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^5 + 2*tan(1
/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^6 - 2*tan(1/2*b*x)^3*tan(1/2*c)^7 - 4*ta
n(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^7 - 2*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2
*c)^7 - tan(1/2*b*x)^2*tan(1/2*c)^8 - 6*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/
2*c)^3 + 16*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^4 - 22*tan(1/2*b*x)^2*tan
(1/2*a)^2*tan(1/2*c)^4 - 6*tan(1/2*b*x)^3*tan(1/2*c)^5 + 20*tan(1/2*b*x)^2
*tan(1/2*a)*tan(1/2*c)^5 - 14*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^5 - 2*t
an(1/2*b*x)^2*tan(1/2*c)^6 + 16*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^6 + 2*t
an(1/2*a)^2*tan(1/2*c)^6 + 2*tan(1/2*b*x)*tan(1/2*c)^7 - 2*tan(1/2*b*x)^3*
tan(1/2*a)^2*tan(1/2*c) + 2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 6*t
an(1/2*b*x)^3*tan(1/2*c)^3 - 20*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^3 + 1
4*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^3 + 22*tan(1/2*b*x)^2*tan(1/2*c)...
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\cot^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)*cot(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos(a+bx)\cot^3(c+bx)\,dx$$
$$= \frac{2\cos(bx+c)\cos(bx+a)+2\cos(bx+c)+6\left(\int \frac{\tan\left(\frac{bx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1}dx\right)\sin(bx+c)^2 b+2\left(\int \frac{}{\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^3\tan\left(\right.}\right.}{}$$

input
```
int(cos(b*x+a)*cot(b*x+c)^3,x)
```

output
```
(2*cos(b*x + c)*cos(a + b*x) + 2*cos(b*x + c) + 6*int(tan((b*x + c)/2)/(ta
n((a + b*x)/2)**2 + 1),x)*sin(b*x + c)**2*b + 2*int(1/(tan((b*x + c)/2)**3
*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**3),x)*sin(b*x + c)**2*b - 4*log(t
an((b*x + c)/2)**2 + 1)*sin(b*x + c)**2 + 2*log(tan((b*x + c)/2))*sin(b*x
+ c)**2 - sin(b*x + c)**2 - 2*sin(b*x + c)*sin(a + b*x) + 2)/(4*sin(b*x +
c)**2*b)
```

3.369 $\int \cos(a+bx)\tan(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 111

$$\int \cos(a+bx)\tan(c+dx)\,dx = -\frac{e^{-i(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b} + \frac{e^{i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},-e^{2i(c+dx)}\right)}{b} - \frac{i\sin(a+bx)}{b}$$

output
```
-hypergeom([1, -1/2*b/d],[1-1/2*b/d],-exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))+e
xp(I*(b*x+a))*hypergeom([1, 1/2*b/d],[1+1/2*b/d],-exp(2*I*(d*x+c)))/b-I*si
n(b*x+a)/b
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.03

$$\int \cos(a+bx)\tan(c+dx)\,dx = \frac{e^{-i(a+bx)}\left(1-e^{2i(a+bx)}-2\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)+2e^{2i(a+bx)}\,\text{Hypergeometric2}\right.}{2b}$$

input
```
Integrate[Cos[a + b*x]*Tan[c + d*x],x]
```

output
```
(1 - E^((2*I)*(a + b*x)) - 2*Hypergeometric2F1[1, -1/2*b/d, 1 - b/(2*d), -
E^((2*I)*(c + d*x))] + 2*E^((2*I)*(a + b*x))*Hypergeometric2F1[1, b/(2*d),
 1 + b/(2*d), -E^((2*I)*(c + d*x))])/(2*b*E^(I*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{5071, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\tan(c+dx)\,dx$$

$$\Big\downarrow \text{ 5071}$$

$$\int \left(\frac{ie^{-i(a+bx)}}{1+e^{2i(c+dx)}}+\frac{ie^{i(a+bx)}}{1+e^{2i(c+dx)}}-\frac{1}{2}ie^{-i(a+bx)}-\frac{1}{2}ie^{i(a+bx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{e^{-i(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b}+\frac{e^{i(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,-e^{2i(c+dx)}\right)}{b}+\frac{e^{-i(a+bx)}}{2b}-\frac{e^{i(a+bx)}}{2b}$$

input
```
Int[Cos[a + b*x]*Tan[c + d*x],x]
```

output
```
1/(2*b*E^(I*(a + b*x))) - E^(I*(a + b*x))/(2*b) - Hypergeometric2F1[1, -1/
2*b/d, 1 - b/(2*d), -E^((2*I)*(c + d*x))]/(b*E^(I*(a + b*x))) + (E^(I*(a +
 b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5071
```
Int[Cos[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[(-I)*
(1/(E^(I*(a + b*x))*2)) - I*(E^(I*(a + b*x))/2) + I*(1/(E^(I*(a + b*x))*(1
+ E^(2*I*(c + d*x))))) + I*(E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x)))), x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [F]

$$\int \cos(bx+a)\tan(dx+c)\,dx$$

input
```
int(cos(b*x+a)*tan(d*x+c),x)
```

output
```
int(cos(b*x+a)*tan(d*x+c),x)
```

Fricas [F]

$$\int \cos(a+bx)\tan(c+dx)\,dx = \int \cos(bx+a)\tan(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*tan(d*x + c), x)
```

Sympy [F]

$$\int \cos(a+bx)\tan(c+dx)\,dx = \int \cos(a+bx)\tan(c+dx)\,dx$$

input
```
integrate(cos(b*x+a)*tan(d*x+c),x)
```

output
```
Integral(cos(a + b*x)*tan(c + d*x), x)
```

Maxima [F]

$$\int \cos(a+bx)\tan(c+dx)\,dx = \int \cos(bx+a)\tan(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*tan(d*x + c), x)
```

Giac [F]

$$\int \cos(a+bx)\tan(c+dx)\,dx = \int \cos(bx+a)\tan(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*tan(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\tan(c+dx)\,dx = \int \cos\left(a+b\,x\right)\,\tan(c+d\,x)\,dx$$

input
```
int(cos(a + b*x)*tan(c + d*x),x)
```

output
```
int(cos(a + b*x)*tan(c + d*x), x)
```

Reduce [F]

$$\int \cos(a+bx)\tan(c+dx)\,dx = \int \cos\left(bx+a\right)\tan\left(dx+c\right)dx$$

input
```
int(cos(b*x+a)*tan(d*x+c),x)
```

output
```
int(cos(a + b*x)*tan(c + d*x),x)
```

3.370 $\int \cos(a+bx)\cot(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 107

$$\int \cos(a+bx)\cot(c+dx)\,dx = \frac{e^{-i(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)}{b} - \frac{e^{i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},e^{2i(c+dx)}\right)}{b} + \frac{i\sin(a+bx)}{b}$$

output
```
hypergeom([1, -1/2*b/d],[1-1/2*b/d],exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))-exp
(I*(b*x+a))*hypergeom([1, 1/2*b/d],[1+1/2*b/d],exp(2*I*(d*x+c)))/b+I*sin(b
*x+a)/b
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

$$\int \cos(a+bx)\cot(c+dx)\,dx = \frac{e^{-i(a+bx)}\left(-1+e^{2i(a+bx)}+2\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)-2e^{2i(a+bx)}\,\text{Hypergeometric2I}\right.}{2b}$$

input
```
Integrate[Cos[a + b*x]*Cot[c + d*x],x]
```

output
```
(-1 + E^((2*I)*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 - b/(2*d),
E^((2*I)*(c + d*x))] - 2*E^((2*I)*(a + b*x))*Hypergeometric2F1[1, b/(2*d),
 1 + b/(2*d), E^((2*I)*(c + d*x))])/(2*b*E^(I*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5069, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot(c+dx)\,dx$$

$$\Big\downarrow \text{5069}$$

$$\int \left(-\frac{ie^{-i(a+bx)}}{1-e^{2i(c+dx)}}-\frac{ie^{i(a+bx)}}{1-e^{2i(c+dx)}}+\frac{1}{2}ie^{-i(a+bx)}+\frac{1}{2}ie^{i(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{-i(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)}{b}-\frac{e^{i(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,e^{2i(c+dx)}\right)}{b}-\frac{e^{-i(a+bx)}}{2b}+\frac{e^{i(a+bx)}}{2b}$$

input
```
Int[Cos[a + b*x]*Cot[c + d*x],x]
```

output
```
-1/2*1/(b*E^(I*(a + b*x))) + E^(I*(a + b*x))/(2*b) + Hypergeometric2F1[1,
-1/2*b/d, 1 - b/(2*d), E^((2*I)*(c + d*x))]/(b*E^(I*(a + b*x))) - (E^(I*(a
 + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x))])/
b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5069
```
Int[Cos[(a_.) + (b_.)*(x_)]*Cot[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[I*(1/
(E^(I*(a + b*x))*2)) + I*(E^(I*(a + b*x))/2) - I*(1/(E^(I*(a + b*x))*(1 - E
^(2*I*(c + d*x))))) - I*(E^(I*(a + b*x))/(1 - E^(2*I*(c + d*x)))), x] /; Fr
eeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [F]

$$\int \cos(bx+a)\cot(dx+c)\,dx$$

input
```
int(cos(b*x+a)*cot(d*x+c),x)
```

output
```
int(cos(b*x+a)*cot(d*x+c),x)
```

Fricas [F]

$$\int \cos(a+bx)\cot(c+dx)\,dx = \int \cos(bx+a)\cot(dx+c)\,dx$$

input
```
integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*cot(d*x + c), x)
```

Sympy [F]

$$\int \cos(a+bx)\cot(c+dx)\,dx = \int \cos{(a+bx)}\cot{(c+dx)}\,dx$$

input
```
integrate(cos(b*x+a)*cot(d*x+c),x)
```

output
```
Integral(cos(a + b*x)*cot(c + d*x), x)
```

Maxima [F]

$$\int \cos(a+bx)\cot(c+dx)\,dx = \int \cos{(bx+a)}\cot{(dx+c)}\;dx$$

input
```
integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*cot(d*x + c), x)
```

Giac [F]

$$\int \cos(a+bx)\cot(c+dx)\,dx = \int \cos{(bx+a)}\cot{(dx+c)}\;dx$$

input
```
integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*cot(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\cot(c+dx)\,dx = \int \cos\left(a+b\,x\right)\,\cot(c+d\,x)\,dx$$

input
```
int(cos(a + b*x)*cot(c + d*x),x)
```

output
```
int(cos(a + b*x)*cot(c + d*x), x)
```

Reduce [F]

$$\int \cos(a+bx)\cot(c+dx)\,dx = \int \cos\left(bx+a\right)\cot\left(dx+c\right)dx$$

input
```
int(cos(b*x+a)*cot(d*x+c),x)
```

output
```
int(cos(a + b*x)*cot(c + d*x),x)
```

3.371 $\int \cos(a+bx)\sec(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \cos(a+bx)\sec(c+bx)\,dx = x\cos(a-c)+\frac{\log(\cos(c+bx))\sin(a-c)}{b}$$

output
```
x*cos(a-c)+ln(cos(b*x+c))*sin(a-c)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sec(c+bx)\,dx = x\cos(a-c)+\frac{\log(\cos(c+bx))\sin(a-c)}{b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + b*x],x]
```

output
```
x*Cos[a - c] + (Log[Cos[c + b*x]]*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5094, 24, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \text{ 5094}$$

$$\cos(a-c)\int 1dx-\sin(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{ 24}$$

$$x\cos(a-c)-\sin(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$x\cos(a-c)-\sin(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\sin(a-c)\log(\cos(bx+c))}{b}+x\cos(a-c)$$

input
```
Int[Cos[a + b*x]*Sec[c + b*x],x]
```

output
```
x*Cos[a - c] + (Log[Cos[c + b*x]]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5094
```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Simp[-Sin[v - w]   Int[Tan[w]*Sec[w
]^(n - 1), x], x] + Simp[Cos[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n,
0] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

method	result
risch	$-2i\sin\left(a-c\right)x+x\,\mathrm{e}^{i(a-c)}-\frac{2i\sin(a-c)a}{b}+\frac{\ln\left(\mathrm{e}^{2i(bx+a)}+\mathrm{e}^{2i(a-c)}\right)\sin(a-c)}{b}$
default	$\frac{\frac{(\sin(a)\cos(c)-\cos(a)\sin(c))\ln(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}{\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2}+\frac{\frac{(-\sin(a)\cos(c)+\cos(a)\sin(c))\ln\left(\mathrm{t}\right.}{2}}{\left(\cos\right.}}{b}$

input
```
int(cos(b*x+a)*sec(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2*I*sin(a-c)*x-2*I/b*sin(a-c)*a+x*exp(I*(a-c))+ln(exp(2*I*(b*x+a))+exp(2*
I*(a-c)))/b*sin(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \cos(a+bx)\sec(c+bx)\,dx = \frac{bx\cos(-a+c)-\log(-\cos(bx+c))\sin(-a+c)}{b}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="fricas")
```

output
```
(b*x*cos(-a + c) - log(-cos(b*x + c))*sin(-a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(20) = 40.

Time = 4.80 (sec) , antiderivative size = 435, normalized size of antiderivative = 16.73

$$\int \cos(a+bx)\sec(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c),x)
```

output
```
-Piecewise((-x, Eq(c, pi/2)), (x, Eq(c, -pi/2)), (0, Eq(b, 0)), (-2*b*x*ta
n(c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2
)**2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) -
tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b
) - log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/
2)**2 + b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*
tan(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1)
 - 1/(tan(c/2) + 1))/(b*tan(c/2)**2 + b), True))*sin(a) + Piecewise((-log(
sin(b*x))/b, Eq(c, pi/2)), (log(sin(b*x))/b, Eq(c, -pi/2)), (x/cos(c), Eq(
b, 0)), (-b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) +
2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) -
 tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b)
- 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/
(b*tan(c/2)**2 + b), True))*cos(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(26) = 52$.

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.85

$$\int \cos(a+bx)\sec(c+bx)\,dx$$
$$= \frac{2\,bx\cos\left(-a+c\right)-\log\left(\cos\left(2\,bx\right)^2+2\,\cos\left(2\,bx\right)\cos\left(2\,c\right)+\cos\left(2\,c\right)^2+\sin\left(2\,bx\right)^2-2\,\sin\left(2\,bx\right)\sin\left(2\,c\right.\right.}{2\,b}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="maxima")
```

output
```
1/2*(2*b*x*cos(-a + c) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*
c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2)*sin(-a + c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(26) = 52$.

Time = 0.16 (sec) , antiderivative size = 440, normalized size of antiderivative = 16.92

$$\int \cos(a+bx)\sec(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="giac")
```

output
```
((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan
(1/2*c)^2 + 1)*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1
/2*c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/
2*a) - tan(1/2*c))*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^2 + ta
n(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^4*tan(1/2*c)^2 - 2*tan(1/2*
a)^3*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)
- 4*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2 -
 2*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2)*log(abs(2*tan(b*x + a)*tan(1/2*a)
^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a)^2*tan(
1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)^2 - 2*tan(b*x + a)*tan(1
/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1))/(tan(1/2*a)^4*tan(1/2
*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c) - tan(1/2*a)*t
an(1/2*c)^4 + tan(1/2*a)^3 - tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.19

$$\int \cos(a+bx)\sec(c+bx)\,dx = x\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}}{2}-\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}}{2}\right)+x\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}}{2}\right) + \frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}{b}$$

input
```
int(cos(a + b*x)/cos(c + b*x),x)
```

output
```
x*(exp(c*1i - a*1i)/2 - exp(a*1i - c*1i)/2) + x*(exp(c*1i - a*1i)/2 + exp(
a*1i - c*1i)/2) + (log(exp(a*2i - c*2i) + exp(a*2i + b*x*2i))*((exp(c*1i -
 a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2))/b
```

Reduce [F]

$$\int \cos(a+bx)\sec(c+bx)\,dx$$
$$= \frac{-4\left(\int \frac{1}{\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}dx\right)b+\log\left(\tan\left(\frac{bx}{2}+\frac{c}{2}\right)-1\right)-\log\left(\tan\left(\frac{bx}{2}+\frac{c}{2}\right)\right.}{b}$$

input
```
int(cos(b*x+a)*sec(b*x+c),x)
```

output
```
( - 4*int(1/(tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**2
 - tan((a + b*x)/2)**2 - 1),x)*b + log(tan((b*x + c)/2) - 1) - log(tan((b*
x + c)/2) + 1) - sin(a + b*x) - a - b*x)/b
```

3.372 $\int \cos(a+bx)\sec^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \cos(a+bx)\sec^2(c+bx)\,dx=\frac{\operatorname{arctanh}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}$$

output
```
arctanh(sin(b*x+c))*cos(a-c)/b-sec(b*x+c)*sin(a-c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.54

$$\int \cos(a+bx)\sec^2(c+bx)\,dx = -\frac{2i\arctan\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + b*x]^2,x]
```

output
```
((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x
)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Sec
[c + b*x]*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5094, 3042, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^2(bx+c)\,dx$$

$$\Big\downarrow \ 5094$$

$$\cos(a-c)\int \sec(c+bx)dx-\sin(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \ 3042$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\sin(a-c)\int \sec(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \ 3086$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\frac{\sin(a-c)\int 1d\sec(c+bx)}{b}$$

$$\Big\downarrow \ 24$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)dx-\frac{\sin(a-c)\sec(bx+c)}{b}$$

$$\Big\downarrow \ 4257$$

$$\frac{\cos(a-c)\text{arctanh}(\sin(bx+c))}{b}-\frac{\sin(a-c)\sec(bx+c)}{b}$$

input
```
Int[Cos[a + b*x]*Sec[c + b*x]^2,x]
```

output
```
(ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - (Sec[c + b*x]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5094
```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Simp[-Sin[v - w]   Int[Tan[w]*Sec[w
]^(n - 1), x], x] + Simp[Cos[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n,
0] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.40

method	result
risch	$\frac{i\left(\mathrm{e}^{i(bx+3a)}-\mathrm{e}^{i(bx+a+2c)}\right)}{b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a-c)}\right)\cos(a-c)}{b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a-c)}\right)\cos(a-c)}{b}$
default	$-\frac{2\left(-\frac{\left(\cos(c)^2\sin(a)^2-2\cos(a)\cos(c)\sin(a)\sin(c)+\sin(c)^2\cos(a)^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)(\cos(a)\cos(c)+\sin(a)\sin(c))}+\frac{-\sin(a)\cos(c)+\cos(a)}{\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2}\right.}{\cos(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+\sin(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-2\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-\cos(a)\cos(c)-}$

input
```
int(cos(b*x+a)*sec(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))*(exp(I*(b*x+3*a))-exp(I*(b*x+a+2*c)))-
ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*cos(a-c)+ln(exp(I*(b*x+a))+I*exp(I*(a-
c)))/b*cos(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \cos(a+bx)\sec^2(c+bx)\,dx$$
$$=\frac{\cos(bx+c)\cos(-a+c)\log(\sin(bx+c)+1)-\cos(bx+c)\cos(-a+c)\log(-\sin(bx+c)+1)+2\,\mathrm{si}}{2\,b\cos(bx+c)}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(cos(b*x + c)*cos(-a + c)*log(sin(b*x + c) + 1) - cos(b*x + c)*cos(-a
+ c)*log(-sin(b*x + c) + 1) + 2*sin(-a + c))/(b*cos(b*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4095 vs. $2(27) = 54$.

Time = 97.14 (sec) , antiderivative size = 5545, normalized size of antiderivative = 158.43

$$\int \cos(a+bx)\sec^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)**2,x)
```

output
```
-Piecewise((log(tan(b*x/2))/b, Eq(c, -pi/2) | Eq(c, pi/2)), (0, Eq(b, 0)),
 (-2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)
**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c
/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log
(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**3/(b*t
an(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*
b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 8*log(tan(b*x/2) - tan(c/2)
/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*
tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*
tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2)
- 1) - 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)*
*2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2)
- b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(t
an(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*
tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) +
2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3
*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)
**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*log(ta
n(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(
c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(35) = 70$.

Time = 0.17 (sec) , antiderivative size = 391, normalized size of antiderivative = 11.17

$$\int \cos(a+bx)\sec^2(c+bx)\,dx =$$

$$-\frac{2\left(\sin\left(bx+2a\right)-\sin\left(bx+2c\right)\right)\cos\left(2bx+a+2c\right)+\left(\cos\left(2bx+a+2c\right)^2\cos\left(-a+c\right)+2\cos\left(2b\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(sin(b*x + 2*a) - sin(b*x + 2*c))*cos(2*b*x + a + 2*c) + (cos(2*b*
x + a + 2*c)^2*cos(-a + c) + 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + c
os(-a + c)*sin(2*b*x + a + 2*c)^2 + 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin
(a) + (cos(a)^2 + sin(a)^2)*cos(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2
- 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + s
in(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x
+ 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - 2*(cos(b*x + 2*a) - cos(
b*x + 2*c))*sin(2*b*x + a + 2*c) + 2*cos(a)*sin(b*x + 2*a) - 2*cos(a)*sin(
b*x + 2*c) - 2*cos(b*x + 2*a)*sin(a) + 2*cos(b*x + 2*c)*sin(a))/(b*cos(2*b
*x + a + 2*c)^2 + 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)
^2 + 2*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1341 vs. $2(35) = 70$.

Time = 0.21 (sec) , antiderivative size = 1341, normalized size of antiderivative = 38.31

$$\int \cos(a + bx)\sec^2(c + bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="giac")
```

output
```
-((tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)^2*ta
n(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1
/2*a)*tan(1/2*c)^3 + tan(1/2*a)^3 - 5*tan(1/2*a)^2*tan(1/2*c) + 5*tan(1/2*
a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) -
tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*
tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*b*x + 1/
2*a)*tan(1/2*c) + tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a
) - tan(1/2*c) + 1))/(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^
2 + tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c) + tan(1/2*a)^2*tan
(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 + tan(1/2*a)^2*tan(1/2*
c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a)^2 + tan(1/2*a)*ta
n(1/2*c) + tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1) - (tan(1/2*a)^3*tan
(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/
2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3
- tan(1/2*a)^3 + 5*tan(1/2*a)^2*tan(1/2*c) - 5*tan(1/2*a)*tan(1/2*c)^2 + t
an(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + tan(
1/2*a) - tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*
c) - tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*b*x + 1/2*a)*tan(1/2*c) - t
an(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan(1/2*c) - 1)
)/(tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2...
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 246, normalized size of antiderivative = 7.03

$$\int \cos(a+bx)\sec^2(c+bx)\,dx$$
$$= \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,(e^{a\,2i}\,e^{-c\,2i}+1)-\frac{e^{a\,2i}\,e^{-c\,2i}\,(e^{a\,2i}\,e^{-c\,2i}+1)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,(e^{a\,2i-c\,2i}+1)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$
$$-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,(e^{a\,2i}\,e^{-c\,2i}+1)+\frac{e^{a\,2i}\,e^{-c\,2i}\,(e^{a\,2i}\,e^{-c\,2i}+1)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,(e^{a\,2i-c\,2i}+1)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$
$$+\frac{e^{a\,1i+b\,x\,1i}\,(e^{a\,2i-c\,2i}-1)\,1i}{b\,(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i})}$$

input
```
int(cos(a + b*x)/cos(c + b*x)^2,x)
```

output
```
(log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1) - (exp(a*2i)*exp(-
c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*
2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log((exp(a*2i)*exp(-c*2i)
*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*e
xp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*
2i - c*2i)^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) - 1)*1i)/(b*(exp
(a*2i - c*2i) + exp(a*2i + b*x*2i)))
```

Reduce [F]

$$\int \cos(a+bx)\sec^2(c+bx)\,dx$$
$$= \frac{-\cos(bx+c)\left(\int \frac{\sin(bx+c)^2}{\sin(bx+c)^2-1}dx\right)b-\cos(bx+c)\left(\int \frac{\cos(bx+a)\sin(bx+c)^2}{\sin(bx+c)^2-1}dx\right)b+\cos(bx+c)\sin(bx+a)+}{\cos(bx+c)\,b}$$

input
```
int(cos(b*x+a)*sec(b*x+c)^2,x)
```

output
```
( - cos(b*x + c)*int(sin(b*x + c)**2/(sin(b*x + c)**2 - 1),x)*b - cos(b*x
+ c)*int((cos(a + b*x)*sin(b*x + c)**2)/(sin(b*x + c)**2 - 1),x)*b + cos(b
*x + c)*sin(a + b*x) + 2*cos(b*x + c)*a + cos(b*x + c)*b*x - sin(b*x + c))
/(cos(b*x + c)*b)
```

3.373 $\int \cos(a+bx)\sec^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = -\frac{\sec^2(c+bx)\sin(a-c)}{2b}+\frac{\cos(a-c)\tan(c+bx)}{b}$$

output
```
-1/2*sec(b*x+c)^2*sin(a-c)/b+cos(a-c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = -\frac{\sec(c)\sec^2(c+bx)(\sin(a)-\cos(a-c)\sin(c+2bx))}{2b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + b*x]^3,x]
```

output
```
-1/2*(Sec[c]*Sec[c + b*x]^2*(Sin[a] - Cos[a - c]*Sin[c + 2*b*x]))/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5094, 3042, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^3(bx+c)\,dx$$

$$\Big\downarrow \text{ 5094}$$

$$\cos(a-c)\int \sec^2(c+bx)dx-\sin(a-c)\int \sec^2(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx-\sin(a-c)\int \sec(c+bx)^2\tan(c+bx)dx$$

$$\Big\downarrow \text{ 3086}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx-\frac{\sin(a-c)\int \sec(c+bx)d\sec(c+bx)}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^2dx-\frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{\cos(a-c)\int 1d(-\tan(c+bx))}{b}-\frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\cos(a-c)\tan(bx+c)}{b}-\frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

input
```
Int[Cos[a + b*x]*Sec[c + b*x]^3,x]
```

output
```
-1/2*(Sec[c + b*x]^2*Sin[a - c])/b + (Cos[a - c]*Tan[c + b*x])/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5094
```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Simp[-Sin[v - w]   Int[Tan[w]*Sec[w
]^(n - 1), x], x] + Simp[Cos[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n,
0] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{\sin(2bx+a+c)}{b(1+\cos(2bx+2c))}$	26
default	$-\frac{1}{2b(\sin(a)\cos(c)-\cos(a)\sin(c))(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))^2}$	56
risch	$\frac{i\left(2\,e^{i(2bx+5a+c)}+e^{i(5a-c)}+e^{i(3a+c)}\right)}{\left(e^{2i(bx+a+c)}+e^{2ia}\right)^2 b}$	61

input
```
int(cos(b*x+a)*sec(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*sin(2*b*x+a+c)/(1+cos(2*b*x+2*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = \frac{2\,\cos\left(bx+c\right)\cos\left(-a+c\right)\sin\left(bx+c\right)+\sin\left(-a+c\right)}{2\,b\cos\left(bx+c\right)^2}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c) + sin(-a + c))/(b*cos(b*x + c
)^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 382, normalized size of antiderivative = 10.05

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = -\frac{(2\sin(2bx+2a+2c)+\sin(2a)+\sin(2c))\cos(4bx+a+5c)+2(2\sin(2bx+2a+2c)+\sin(2}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2+4b\cos}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="maxima")
```

output
```
-((2*sin(2*b*x + 2*a + 2*c) + sin(2*a) + sin(2*c))*cos(4*b*x + a + 5*c) +
2*(2*sin(2*b*x + 2*a + 2*c) + sin(2*a) + sin(2*c))*cos(2*b*x + a + 3*c) +
(sin(2*a) + sin(2*c))*cos(a + c) - (2*cos(2*b*x + 2*a + 2*c) + cos(2*a) +
cos(2*c))*sin(4*b*x + a + 5*c) + 2*cos(a + c)*sin(2*b*x + 2*a + 2*c) - 2*(
2*cos(2*b*x + 2*a + 2*c) + cos(2*a) + cos(2*c))*sin(2*b*x + a + 3*c) - (co
s(2*a) + cos(2*c))*sin(a + c) - 2*cos(2*b*x + 2*a + 2*c)*sin(a + c))/(b*co
s(4*b*x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 + 4*b*cos(2*b*x + a + 3*
c)*cos(a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x
+ a + 3*c)^2 + 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(2
*b*cos(2*b*x + a + 3*c) + b*cos(a + c))*cos(4*b*x + a + 5*c) + 2*(2*b*sin(
2*b*x + a + 3*c) + b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(36) = 72$.

Time = 0.16 (sec) , antiderivative size = 315, normalized size of antiderivative = 8.29

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = -\frac{\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^2+9\,\mathrm{t}}{4\left(2\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-2\tan(bx+a)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}\right.\right.}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*
a)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c
)^4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*
c)^2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan
(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2
+ 1)/((2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*
tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan
(1/2*a)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*
c)^2 + 1)^2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a
) - tan(1/2*c))*b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/cos(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = \frac{-\cos(bx+c)\sin(bx+a)-\cos(bx+a)\sin(bx+c)}{2b\left(\sin(bx+c)^2-1\right)}$$

input
```
int(cos(b*x+a)*sec(b*x+c)^3,x)
```

output
```
( - (cos(b*x + c)*sin(a + b*x) + cos(a + b*x)*sin(b*x + c)))/(2*b*(sin(b*x
 + c)**2 - 1))
```

3.374 $\int \cos(a+bx)\sec^4(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \cos(a+bx)\sec^4(c+bx)\,dx = \frac{\operatorname{arctanh}(\sin(c+bx))\cos(a-c)}{2b} - \frac{\sec^3(c+bx)\sin(a-c)}{3b} + \frac{\cos(a-c)\sec(c+bx)\tan(c+bx)}{2b}$$

output
```
1/2*arctanh(sin(b*x+c))*cos(a-c)/b-1/3*sec(b*x+c)^3*sin(a-c)/b+1/2*cos(a-c
)*sec(b*x+c)*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \cos(a+bx)\sec^4(c+bx)\,dx = \frac{12\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+\sec^3(c+bx)(-4\sin(a-c)+3\cos(a-c)\sin(2(c+bx)}{12b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c + b*x]^4,x]
```

output
```
(12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Cos[a - c] + Sec[c + b*x]^3*(-4*
Sin[a - c] + 3*Cos[a - c]*Sin[2*(c + b*x)]))/(12*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5094, 3042, 3086, 15, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^4(bx+c)\,dx$$

$$\Big\downarrow \text{5094}$$

$$\cos(a-c)\int \sec^3(c+bx)dx-\sin(a-c)\int \sec^3(c+bx)\tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx-\sin(a-c)\int \sec(c+bx)^3\tan(c+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx-\frac{\sin(a-c)\int \sec^2(c+bx)d\sec(c+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(a-c)\int \csc\left(c+bx+\frac{\pi}{2}\right)^3dx-\frac{\sin(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4255}$$

$$\cos(a-c)\left(\frac{1}{2}\int \sec(c+bx)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)-\frac{\sin(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\left(\frac{1}{2}\int \csc\left(c+bx+\frac{\pi}{2}\right)dx+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)-\frac{\sin(a-c)\sec^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4257}$$

$$\cos(a-c)\left(\frac{\operatorname{arctanh}(\sin(bx+c))}{2b}+\frac{\tan(bx+c)\sec(bx+c)}{2b}\right)-\frac{\sin(a-c)\sec^3(bx+c)}{3b}$$

input
```
Int[Cos[a + b*x]*Sec[c + b*x]^4,x]
```

output
```
-1/3*(Sec[c + b*x]^3*Sin[a - c])/b + Cos[a - c]*(ArcTanh[Sin[c + b*x]]/(2*
b) + (Sec[c + b*x]*Tan[c + b*x])/(2*b))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5094
```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Simp[-Sin[v - w]   Int[Tan[w]*Sec[w
]^(n - 1), x], x] + Simp[Cos[v - w]   Int[Sec[w]^(n - 1), x], x] /; GtQ[n,
0] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.33 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.87

method	result
risch	$\frac{i\left(-3\,\mathrm{e}^{i(5bx+7a+4c)}-3\,\mathrm{e}^{i(5bx+5a+6c)}+8\,\mathrm{e}^{i(3bx+7a+2c)}-8\,\mathrm{e}^{i(3bx+5a+4c)}+3\,\mathrm{e}^{i(bx+7a)}+3\,\mathrm{e}^{i(bx+5a+2c)}\right)}{6b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{3}}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a-c)}\right)\,\mathrm{co}}{2b}$
default	Expression too large to display

input
```
int(cos(b*x+a)*sec(b*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))^3*(-3*exp(I*(5*b*x+7*a+4*c))-3*exp
(I*(5*b*x+5*a+6*c))+8*exp(I*(3*b*x+7*a+2*c))-8*exp(I*(3*b*x+5*a+4*c))+3*ex
p(I*(b*x+7*a))+3*exp(I*(b*x+5*a+2*c)))+1/2*ln(exp(I*(b*x+a))+I*exp(I*(a-c)
))/b*cos(a-c)-1/2*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*cos(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \cos(a+bx)\sec^4(c+bx)\,dx$$
$$=\frac{3\,\cos\left(bx+c\right)^{3}\cos\left(-a+c\right)\log\left(\sin\left(bx+c\right)+1\right)-3\,\cos\left(bx+c\right)^{3}\cos\left(-a+c\right)\log\left(-\sin\left(bx+c\right)+1\right)}{12\,b\cos\left(bx+c\right)^{3}}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^4,x, algorithm="fricas")
```

output
```
1/12*(3*cos(b*x + c)^3*cos(-a + c)*log(sin(b*x + c) + 1) - 3*cos(b*x + c)^
3*cos(-a + c)*log(-sin(b*x + c) + 1) + 6*cos(b*x + c)*cos(-a + c)*sin(b*x
+ c) + 4*sin(-a + c))/(b*cos(b*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^4(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. $2(61) = 122$.

Time = 0.21 (sec) , antiderivative size = 1420, normalized size of antiderivative = 21.19

$$\int \cos(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^4,x, algorithm="maxima")
```

output
```
1/12*(2*(3*sin(5*b*x + 2*a + 4*c) + 3*sin(5*b*x + 6*c) - 8*sin(3*b*x + 2*a
 + 2*c) + 8*sin(3*b*x + 4*c) - 3*sin(b*x + 2*a) - 3*sin(b*x + 2*c))*cos(6*
b*x + a + 6*c) - 6*(3*sin(4*b*x + a + 4*c) + 3*sin(2*b*x + a + 2*c) + sin(
a))*cos(5*b*x + 2*a + 4*c) - 6*(3*sin(4*b*x + a + 4*c) + 3*sin(2*b*x + a +
 2*c) + sin(a))*cos(5*b*x + 6*c) - 6*(8*sin(3*b*x + 2*a + 2*c) - 8*sin(3*b
*x + 4*c) + 3*sin(b*x + 2*a) + 3*sin(b*x + 2*c))*cos(4*b*x + a + 4*c) + 16
*(3*sin(2*b*x + a + 2*c) + sin(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*sin(2*b*
x + a + 2*c) + sin(a))*cos(3*b*x + 4*c) - 18*(sin(b*x + 2*a) + sin(b*x + 2
*c))*cos(2*b*x + a + 2*c) - 3*(cos(6*b*x + a + 6*c)^2*cos(-a + c) + 9*cos(
4*b*x + a + 4*c)^2*cos(-a + c) + 9*cos(2*b*x + a + 2*c)^2*cos(-a + c) + 6*
cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(6*b*x + a + 6*c)
^2 + 9*cos(-a + c)*sin(4*b*x + a + 4*c)^2 + 9*cos(-a + c)*sin(2*b*x + a +
2*c)^2 + 6*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + 2*(3*cos(4*b*x + a +
4*c)*cos(-a + c) + 3*cos(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(-a + c)
)*cos(6*b*x + a + 6*c) + 6*(3*cos(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*co
s(-a + c))*cos(4*b*x + a + 4*c) + (cos(a)^2 + sin(a)^2)*cos(-a + c) + 2*(3
*cos(-a + c)*sin(4*b*x + a + 4*c) + 3*cos(-a + c)*sin(2*b*x + a + 2*c) + c
os(-a + c)*sin(a))*sin(6*b*x + a + 6*c) + 6*(3*cos(-a + c)*sin(2*b*x + a +
 2*c) + cos(-a + c)*sin(a))*sin(4*b*x + a + 4*c))*log((cos(b*x + 2*c)^2 +
cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12158 vs. $2(61) = 122$.

Time = 1.04 (sec) , antiderivative size = 12158, normalized size of antiderivative = 181.46

$$\int \cos(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x+c)^4,x, algorithm="giac")
```

output
```
-1/6*(3*(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a
)^2*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 -
 tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^3 - 5*tan(1/2*a)^2*tan(1/2*c) + 5*ta
n(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2
*c) - tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1
/2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*b*
x + 1/2*a)*tan(1/2*c) + tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan
(1/2*a) - tan(1/2*c) + 1))/(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1
/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c) + tan(1/2*a)
^2*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 + tan(1/2*a)^2*ta
n(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a)^2 + tan(1/2
*a)*tan(1/2*c) + tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1) - 3*(tan(1/2*
a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3
- tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1
/2*c)^3 - tan(1/2*a)^3 + 5*tan(1/2*a)^2*tan(1/2*c) - 5*tan(1/2*a)*tan(1/2*
c)^2 + tan(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^
2 + tan(1/2*a) - tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*
tan(1/2*c) - tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*b*x + 1/2*a)*tan(1/
2*c) - tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan(1/2
*c) - 1))/(tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(...
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^4(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/cos(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
int(cos(b*x+a)*sec(b*x+c)^4,x)
```

output
```
( - 4*cos(b*x + c)**2*sin(a + b*x) + 8*cos(b*x + c)*int(cos(a + b*x)/(sin(
b*x + c)**4 - 2*sin(b*x + c)**2 + 1),x)*sin(b*x + c)**2*b - 8*cos(b*x + c)
*int(cos(a + b*x)/(sin(b*x + c)**4 - 2*sin(b*x + c)**2 + 1),x)*b - cos(b*x
 + c)*int((cos(a + b*x)*sin(b*x + c)**4)/(sin(b*x + c)**4 - 2*sin(b*x + c)
**2 + 1),x)*sin(b*x + c)**2*b + cos(b*x + c)*int((cos(a + b*x)*sin(b*x + c
)**4)/(sin(b*x + c)**4 - 2*sin(b*x + c)**2 + 1),x)*b - 4*cos(b*x + c)*int(
(cos(a + b*x)*sin(b*x + c)**2)/(sin(b*x + c)**4 - 2*sin(b*x + c)**2 + 1),x
)*sin(b*x + c)**2*b + 4*cos(b*x + c)*int((cos(a + b*x)*sin(b*x + c)**2)/(s
in(b*x + c)**4 - 2*sin(b*x + c)**2 + 1),x)*b - 2*cos(b*x + c)*log(sin(b*x
+ c) - 1)*sin(b*x + c)**2 + 2*cos(b*x + c)*log(sin(b*x + c) - 1) + 2*cos(b
*x + c)*log(sin(b*x + c) + 1)*sin(b*x + c)**2 - 2*cos(b*x + c)*log(sin(b*x
 + c) + 1) + 4*cos(b*x + c)*log(tan((b*x + c)/2) - 1)*sin(b*x + c)**2 - 4*
cos(b*x + c)*log(tan((b*x + c)/2) - 1) - 4*cos(b*x + c)*log(tan((b*x + c)/
2) + 1)*sin(b*x + c)**2 + 4*cos(b*x + c)*log(tan((b*x + c)/2) + 1) + cos(b
*x + c)*sin(b*x + c)**2*sin(a + b*x) + cos(b*x + c)*sin(b*x + c)**2*a - 3*
cos(b*x + c)*sin(a + b*x) - cos(b*x + c)*a - 4*cos(a + b*x)*sin(b*x + c) -
 4*sin(b*x + c)**2*sin(a + b*x) + 4*sin(a + b*x))/(15*cos(b*x + c)*b*(sin(
b*x + c)**2 - 1))
```

3.375 $\int \cos(a+bx)\sec(c-bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \cos(a+bx)\sec(c-bx)\,dx = x\cos(a+c)+\frac{\log(\cos(c-bx))\sin(a+c)}{b}$$

output
```
x*cos(a+c)+ln(cos(b*x-c))*sin(a+c)/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sec(c-bx)\,dx = x\cos(a+c)+\frac{\log(\cos(c-bx))\sin(a+c)}{b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c - b*x],x]
```

output
```
x*Cos[a + c] + (Log[Cos[c - b*x]]*Sin[a + c])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec(c-bx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \cos(a+bx)\sec(c-bx)dx$$

input
```
Int[Cos[a + b*x]*Sec[c - b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

method	result
risch	$-2i\sin(a+c)\,x + x\,\mathrm{e}^{i(a+c)} - \frac{2i\sin(a+c)a}{b} + \frac{\ln\left(\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)\sin(a+c)}{b}$
default	$\frac{\frac{(\sin(a)\cos(c)+\cos(a)\sin(c))\ln(\tan(bx+a)\sin(a)\cos(c)+\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)-\sin(a)\sin(c))}{\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2} + \frac{\frac{(-\sin(a)\cos(c)-\cos(a)\sin(c))\ln\left(t\cdots\right.}{2}}{\left(\cos\cdots\right.}}{b}$

input
```
int(cos(b*x+a)*sec(b*x-c),x,method=_RETURNVERBOSE)
```

output
```
-2*I*sin(a+c)*x+x*exp(I*(a+c))-2*I/b*sin(a+c)*a+ln(exp(2*I*(a+c))+exp(2*I*
(b*x+a)))/b*sin(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(24) = 48$.

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \cos(a+bx)\sec(c-bx)\,dx = \frac{bx\cos(a+c) + \log\left(\frac{2\,(\cos(bx+a)\cos(a+c)+\sin(bx+a)\sin(a+c))}{\cos(a+c)+1}\right)\sin(a+c)}{b}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c),x, algorithm="fricas")
```

output
```
(b*x*cos(a + c) + log(2*(cos(b*x + a)*cos(a + c) + sin(b*x + a)*sin(a + c)
)/(cos(a + c) + 1))*sin(a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(20) = 40$.

Time = 4.61 (sec) , antiderivative size = 435, normalized size of antiderivative = 18.91

$$\int \cos(a+bx)\sec(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c),x)
```

output
```
-Piecewise((x, Eq(c, pi/2)), (-x, Eq(c, -pi/2)), (0, Eq(b, 0)), (2*b*x*tan
(c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)
**2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) + t
an(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b)
 - log(tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))/(b*tan(c/2
)**2 + b) + log(tan(b*x/2) - tan(c/2)/(tan(c/2) + 1) + 1/(tan(c/2) + 1))*t
an(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - tan(c/2)/(tan(c/2) + 1)
+ 1/(tan(c/2) + 1))/(b*tan(c/2)**2 + b), True))*sin(a) + Piecewise((log(si
n(b*x))/b, Eq(c, pi/2)), (-log(sin(b*x))/b, Eq(c, -pi/2)), (x/cos(c), Eq(b
, 0)), (-b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2
*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2) +
tan(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b) +
 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) + 1) + 1/(tan(c/2) + 1))*tan(c/2)/(
b*tan(c/2)**2 + b), True))*cos(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(24) = 48$.

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.00

$$\int \cos(a + bx) \sec(c - bx) \, dx$$
$$= \frac{2\, bx \cos\left(a + c\right) + \log\left(\cos\left(2\, bx\right)^2 + 2\, \cos\left(2\, bx\right) \cos\left(2\, c\right) + \cos\left(2\, c\right)^2 + \sin\left(2\, bx\right)^2 + 2\, \sin\left(2\, bx\right) \sin\left(2\, c\right) -}{2\, b}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c),x, algorithm="maxima")
```

output
```
1/2*(2*b*x*cos(a + c) + log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c
)^2 + sin(2*b*x)^2 + 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2)*sin(a + c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(24) = 48$.

Time = 0.17 (sec) , antiderivative size = 439, normalized size of antiderivative = 19.09

$$\int \cos(a+bx)\sec(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c),x, algorithm="giac")
```

output
```
((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 - 4*tan(1/2*a)*tan(1/2*c) - tan
(1/2*c)^2 + 1)*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1
/2*c)^2 + 1) + (tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/
2*a) - tan(1/2*c))*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^2 + ta
n(1/2*a)^2 + tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*
a)^3*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c)^4 - 2*tan(1/2*a)^3*tan(1/2*c)
- 4*tan(1/2*a)^2*tan(1/2*c)^2 - 2*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2 +
 2*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2)*log(abs(2*tan(b*x + a)*tan(1/2*a)
^2*tan(1/2*c) + 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(
1/2*c)^2 - 2*tan(b*x + a)*tan(1/2*a) + tan(1/2*a)^2 - 2*tan(b*x + a)*tan(1
/2*c) + 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1))/(tan(1/2*a)^4*tan(1/2
*c)^3 + tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c) + tan(1/2*a)*t
an(1/2*c)^4 - tan(1/2*a)^3 - tan(1/2*c)^3 - tan(1/2*a) - tan(1/2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.74

$$\int \cos(a+bx)\sec(c-bx)\,dx = x\left(\frac{e^{-a\,1i-c\,1i}}{2}-\frac{e^{a\,1i+c\,1i}}{2}\right)+x\left(\frac{e^{-a\,1i-c\,1i}}{2}+\frac{e^{a\,1i+c\,1i}}{2}\right) + \frac{\ln\left(e^{a\,2i+c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i-c\,1i}\,1i}{2}-\frac{e^{a\,1i+c\,1i}\,1i}{2}\right)}{b}$$

input
```
int(cos(a + b*x)/cos(c - b*x),x)
```

output
```
x*(exp(- a*1i - c*1i)/2 - exp(a*1i + c*1i)/2) + x*(exp(- a*1i - c*1i)/2 +
exp(a*1i + c*1i)/2) + (log(exp(a*2i + c*2i) + exp(a*2i + b*x*2i))*((exp(-
a*1i - c*1i)*1i)/2 - (exp(a*1i + c*1i)*1i)/2))/b
```

Reduce [F]

$$\int \cos(a+bx)\sec(c-bx)\,dx$$
$$=\frac{-4\left(\int \frac{1}{\tan\left(\frac{bx}{2}-\frac{c}{2}\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+\tan\left(\frac{bx}{2}-\frac{c}{2}\right)^2-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}dx\right)b+\log\left(\tan\left(\frac{bx}{2}-\frac{c}{2}\right)-1\right)-\log\left(\tan\left(\frac{bx}{2}-\frac{c}{2}\right)\right.}{b}$$

input
```
int(cos(b*x+a)*sec(b*x-c),x)
```

output
```
( - 4*int(1/(tan((b*x - c)/2)**2*tan((a + b*x)/2)**2 + tan((b*x - c)/2)**2
 - tan((a + b*x)/2)**2 - 1),x)*b + log(tan((b*x - c)/2) - 1) - log(tan((b*
x - c)/2) + 1) - sin(a + b*x) - a - b*x)/b
```

3.376 $\int \cos(a+bx)\sec^2(c-bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \cos(a+bx)\sec^2(c-bx)\,dx=-\frac{\operatorname{arctanh}(\sin(c-bx))\cos(a+c)}{b}-\frac{\sec(c-bx)\sin(a+c)}{b}$$

output
```
arctanh(sin(b*x-c))*cos(a+c)/b-sec(b*x-c)*sin(a+c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 159, normalized size of antiderivative = 4.68

$$\int \cos(a+bx)\sec^2(c-bx)\,dx =$$

$$-\frac{2i\arctan\left(\frac{\cos\left(\frac{bx}{2}\right)-\cos^2(c)\cos\left(\frac{bx}{2}\right)-2i\cos(c)\cos\left(\frac{bx}{2}\right)\sin(c)+\cos\left(\frac{bx}{2}\right)\sin^2(c)+i\sin\left(\frac{bx}{2}\right)+i\cos^2(c)\sin\left(\frac{bx}{2}\right)-2\cos(c)\sin(c)\sin\left(\frac{bx}{2}\right)}{2\cos(c)\cos\left(\frac{bx}{2}\right)+2i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)}{b}$$

$$-\frac{\sec(c-bx)\sin(a+c)}{b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c - b*x]^2,x]
```

output
```
((-2*I)*ArcTan[(Cos[(b*x)/2] - Cos[c]^2*Cos[(b*x)/2] - (2*I)*Cos[c]*Cos[(b
*x)/2]*Sin[c] + Cos[(b*x)/2]*Sin[c]^2 + I*Sin[(b*x)/2] + I*Cos[c]^2*Sin[(b
*x)/2] - 2*Cos[c]*Sin[c]*Sin[(b*x)/2] - I*Sin[c]^2*Sin[(b*x)/2])/(2*Cos[c]
*Cos[(b*x)/2] + (2*I)*Cos[(b*x)/2]*Sin[c])]*Cos[a + c])/b - (Sec[c - b*x]*
Sin[a + c])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^2(c-bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \cos(a+bx)\sec^2(c-bx)dx$$

input
```
Int[Cos[a + b*x]*Sec[c - b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.29

method	result
risch	$\frac{i\left(\mathrm{e}^{i(bx+3a+2c)}-\mathrm{e}^{i(bx+a)}\right)}{b\left(\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a+c)}\right)\cos(a+c)}{b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a+c)}\right)\cos(a+c)}{b}$
default	$-\frac{2\left(-\frac{\left(\cos(c)^2\sin(a)^2+2\cos(a)\cos(c)\sin(a)\sin(c)+\sin(c)^2\cos(a)^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)(\cos(a)\cos(c)-\sin(a)\sin(c))}-\frac{\sin(a)\cos(c)+\cos(a)\sin}{\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2}\right.}{\cos(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-\sin(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-2\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-2\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-\cos(a)\cos(c)+s}$

input
```
int(cos(b*x+a)*sec(b*x-c)^2,x,method=_RETURNVERBOSE)
```

output
```
I/b/(exp(2*I*(a+c))+exp(2*I*(b*x+a)))*(exp(I*(b*x+3*a+2*c))-exp(I*(b*x+a))
)+ln(exp(I*(b*x+a))+I*exp(I*(a+c)))/b*cos(a+c)-ln(exp(I*(b*x+a))-I*exp(I*(
a+c)))/b*cos(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 4.97

$$\int \cos(a+bx)\sec^2(c-bx)\,dx$$
$$= \frac{\left(\cos(bx+a)\cos(a+c)^2+\cos(a+c)\sin(bx+a)\sin(a+c)\right)\log\left(\frac{2(\cos(a+c)\sin(bx+a)-\cos(bx+a)\sin(a+c)+1)}{\cos(a+c)+1}\right)}{2\left(b\cos(bx+a)\right.}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c)^2,x, algorithm="fricas")
```

output
```
1/2*((cos(b*x + a)*cos(a + c)^2 + cos(a + c)*sin(b*x + a)*sin(a + c))*log(
2*(cos(a + c)*sin(b*x + a) - cos(b*x + a)*sin(a + c) + 1)/(cos(a + c) + 1)
) - (cos(b*x + a)*cos(a + c)^2 + cos(a + c)*sin(b*x + a)*sin(a + c))*log(-
2*(cos(a + c)*sin(b*x + a) - cos(b*x + a)*sin(a + c) - 1)/(cos(a + c) + 1)
) - 2*sin(a + c))/(b*cos(b*x + a)*cos(a + c) + b*sin(b*x + a)*sin(a + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4095 vs. $2(27) = 54$.

Time = 95.50 (sec) , antiderivative size = 5545, normalized size of antiderivative = 163.09

$$\int \cos(a+bx)\sec^2(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c)**2,x)
```

output
```
-Piecewise((log(tan(b*x/2))/b, Eq(c, -pi/2) | Eq(c, pi/2)), (0, Eq(b, 0)),
 (-2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)
**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 + 4*b*tan(c
/2)**3*tan(b*x/2) + 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log
(tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)**3/(b*t
an(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 + 4*b*tan(c/2)**3*tan(b*x/2) + 4*
b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 8*log(tan(b*x/2) + tan(c/2)
/(tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*
tan(b*x/2)**2 - b*tan(c/2)**4 + 4*b*tan(c/2)**3*tan(b*x/2) + 4*b*tan(c/2)*
tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2)
- 1) + 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)*
*2 - b*tan(c/2)**4 + 4*b*tan(c/2)**3*tan(b*x/2) + 4*b*tan(c/2)*tan(b*x/2)
- b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) + 1/(t
an(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 + 4*b*
tan(c/2)**3*tan(b*x/2) + 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) +
2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) + 1) + 1/(tan(c/2) + 1))*tan(c/2)**3
*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 + 4*b*tan(c/2)
**3*tan(b*x/2) + 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*log(ta
n(b*x/2) - tan(c/2)/(tan(c/2) + 1) + 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(
c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 + 4*b*tan(c/2)**3*tan(b*x/2) + 4*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(35) = 70$.

Time = 0.17 (sec) , antiderivative size = 368, normalized size of antiderivative = 10.82

$$\int \cos(a+bx)\sec^2(c-bx)\,dx =$$

$$-\frac{2\left(\sin\left(bx+2\,a+2\,c\right)-\sin\left(bx\right)\right)\cos\left(2\,bx+a\right)+\left(\cos\left(2\,bx+a\right)^2\cos\left(a+c\right)+2\,\cos\left(2\,bx+a\right)\cos\left(a\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(sin(b*x + 2*a + 2*c) - sin(b*x))*cos(2*b*x + a) + (cos(2*b*x + a)
^2*cos(a + c) + 2*cos(2*b*x + a)*cos(a + 2*c)*cos(a + c) + cos(a + 2*c)^2*
cos(a + c) + cos(a + c)*sin(2*b*x + a)^2 + 2*cos(a + c)*sin(2*b*x + a)*sin
(a + 2*c) + cos(a + c)*sin(a + 2*c)^2)*log((cos(b*x)^2 + cos(c)^2 - 2*cos(
c)*sin(b*x) + sin(b*x)^2 + 2*cos(b*x)*sin(c) + sin(c)^2)/(cos(b*x)^2 + cos
(c)^2 + 2*cos(c)*sin(b*x) + sin(b*x)^2 - 2*cos(b*x)*sin(c) + sin(c)^2)) -
2*(cos(b*x + 2*a + 2*c) - cos(b*x))*sin(2*b*x + a) + 2*cos(a + 2*c)*sin(b*
x + 2*a + 2*c) - 2*cos(a + 2*c)*sin(b*x) - 2*cos(b*x + 2*a + 2*c)*sin(a +
2*c) + 2*cos(b*x)*sin(a + 2*c))/(b*cos(2*b*x + a)^2 + 2*b*cos(2*b*x + a)*c
os(a + 2*c) + b*cos(a + 2*c)^2 + b*sin(2*b*x + a)^2 + 2*b*sin(2*b*x + a)*s
in(a + 2*c) + b*sin(a + 2*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1342 vs. $2(35) = 70$.

Time = 0.22 (sec) , antiderivative size = 1342, normalized size of antiderivative = 39.47

$$\int \cos(a+bx)\sec^2(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c)^2,x, algorithm="giac")
```

output
```
((tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan
(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c) - 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/
2*a)*tan(1/2*c)^3 + tan(1/2*a)^3 + 5*tan(1/2*a)^2*tan(1/2*c) + 5*tan(1/2*a
)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) + t
an(1/2*c)^2 - tan(1/2*a) - tan(1/2*c) - 1)*log(abs(-tan(1/2*b*x + 1/2*a)*t
an(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*b*x + 1/2
*a)*tan(1/2*c) - tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a) - tan(1/2*a)
 - tan(1/2*c) + 1))/(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2
 - tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c) - tan(1/2*a)^2*tan(
1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 - tan(1/2*a)^2*tan(1/2*c
) - tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a)^2 + tan(1/2*a)*tan
(1/2*c) - tan(1/2*c)^2 - tan(1/2*a) - tan(1/2*c) - 1) - (tan(1/2*a)^3*tan(
1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2
*a)^3*tan(1/2*c) - 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 -
 tan(1/2*a)^3 - 5*tan(1/2*a)^2*tan(1/2*c) - 5*tan(1/2*a)*tan(1/2*c)^2 - ta
n(1/2*c)^3 + tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 + tan(1
/2*a) + tan(1/2*c) - 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c
) - tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*b*x + 1/2*a)*tan(1/2*c) + ta
n(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a) - tan(1/2*a) - tan(1/2*c) - 1))
/(tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)^2*...
```

Mupad [B] (verification not implemented)

Time = 26.59 (sec) , antiderivative size = 246, normalized size of antiderivative = 7.24

$$\int \cos(a+bx)\sec^2(c-bx)\,dx = \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{c\,2i}+1\right)-\frac{e^{a\,2i}\,e^{c\,2i}\,\left(e^{a\,2i}\,e^{c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{c\,2i}}}\right)\,\left(e^{a\,2i+c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i+c\,2i}}} - \frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{c\,2i}+1\right)+\frac{e^{a\,2i}\,e^{c\,2i}\,\left(e^{a\,2i}\,e^{c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{c\,2i}}}\right)\,\left(e^{a\,2i+c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i+c\,2i}}} + \frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i+c\,2i}-1\right)\,1i}{b\,\left(e^{a\,2i+c\,2i}+e^{a\,2i+b\,x\,2i}\right)}$$

input
```
int(cos(a + b*x)/cos(c - b*x)^2,x)
```

output
```
(log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(c*2i) + 1) - (exp(a*2i)*exp(c*
2i)*(exp(a*2i)*exp(c*2i) + 1)*1i)/(exp(a*2i)*exp(c*2i))^(1/2))*(exp(a*2i +
 c*2i) + 1))/(2*b*exp(a*2i + c*2i)^(1/2)) - (log((exp(a*2i)*exp(c*2i)*(exp
(a*2i)*exp(c*2i) + 1)*1i)/(exp(a*2i)*exp(c*2i))^(1/2) - exp(a*1i)*exp(b*x*
1i)*(exp(a*2i)*exp(c*2i) + 1))*(exp(a*2i + c*2i) + 1))/(2*b*exp(a*2i + c*2
i)^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i + c*2i) - 1)*1i)/(b*(exp(a*2i +
c*2i) + exp(a*2i + b*x*2i)))
```

Reduce [F]

$$\int \cos(a+bx)\sec^2(c-bx)\,dx$$
$$= \frac{-\cos(bx-c)\left(\int \frac{\sin(bx-c)^2}{\sin(bx-c)^2-1}dx\right)b-\cos(bx-c)\left(\int \frac{\cos(bx+a)\sin(bx-c)^2}{\sin(bx-c)^2-1}dx\right)b+\cos(bx-c)\sin(bx+a)+}{\cos(bx-c)\,b}$$

input
```
int(cos(b*x+a)*sec(b*x-c)^2,x)
```

output
```
( - cos(b*x - c)*int(sin(b*x - c)**2/(sin(b*x - c)**2 - 1),x)*b - cos(b*x
- c)*int((cos(a + b*x)*sin(b*x - c)**2)/(sin(b*x - c)**2 - 1),x)*b + cos(b
*x - c)*sin(a + b*x) + 2*cos(b*x - c)*a + cos(b*x - c)*b*x - sin(b*x - c))
/(cos(b*x - c)*b)
```

3.377 $\int \cos(a+bx)\sec^3(c-bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \cos(a+bx)\sec^3(c-bx)\,dx = -\frac{\sec^2(c-bx)\sin(a+c)}{2b} - \frac{\cos(a+c)\tan(c-bx)}{b}$$

output
```
-1/2*sec(b*x-c)^2*sin(a+c)/b+cos(a+c)*tan(b*x-c)/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \cos(a+bx)\sec^3(c-bx)\,dx = -\frac{\sec(c)\sec^2(c-bx)(\sin(a)+\cos(a+c)\sin(c-2bx))}{2b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c - b*x]^3,x]
```

output
```
-1/2*(Sec[c]*Sec[c - b*x]^2*(Sin[a] + Cos[a + c]*Sin[c - 2*b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^3(c-bx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos(a+bx)\sec^3(c-bx)dx$$

input
```
Int[Cos[a + b*x]*Sec[c - b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$\frac{\sin(2bx+a-c)}{b(1+\cos(2bx-2c))}$	28
default	$-\frac{1}{2b(\sin(a)\cos(c)+\cos(a)\sin(c))(\tan(bx+a)\sin(a)\cos(c)+\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)-\sin(a)\sin(c))^2}$	55
risch	$\frac{i\left(e^{5i(a+c)}+2\,e^{i(2bx+5a+3c)}+e^{3i(a+c)}\right)}{\left(e^{2i(a+c)}+e^{2i(bx+a)}\right)^2 b}$	58

input
```
int(cos(b*x+a)*sec(b*x-c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*sin(2*b*x+a-c)/(1+cos(2*b*x-2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(36) = 72.

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.30

$$\int \cos(a+bx)\sec^3(c-bx)\,dx$$
$$= \frac{2\left(2\cos\left(a+c\right)^3-\cos\left(a+c\right)\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-\left(4\cos\left(bx+a\right)^2\cos\left(a+c\right)^2-2\cos\left(a+c\right)^2\right.}{2\left(2\,b\cos\left(bx+a\right)\cos\left(a+c\right)\sin\left(bx+a\right)\sin\left(a+c\right)+\left(2\,b\cos\left(a+c\right)^2-b\right)\cos\left(bx+a\right)^2-b\cos\left(a\right.\right.}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c)^3,x, algorithm="fricas")
```

output
```
1/2*(2*(2*cos(a + c)^3 - cos(a + c))*cos(b*x + a)*sin(b*x + a) - (4*cos(b*
x + a)^2*cos(a + c)^2 - 2*cos(a + c)^2 + 1)*sin(a + c))/(2*b*cos(b*x + a)*
cos(a + c)*sin(b*x + a)*sin(a + c) + (2*b*cos(a + c)^2 - b)*cos(b*x + a)^2
 - b*cos(a + c)^2 + b)
```

Sympy [F(-2)]

Exception generated.

$$\int \cos(a+bx)\sec^3(c-bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(36) = 72$.

Time = 0.06 (sec) , antiderivative size = 419, normalized size of antiderivative = 11.32

$$\int \cos(a + bx)\sec^3(c - bx)\, dx = -\frac{(2\sin(2bx + 2a + 3c) + \sin(2a + 5c) + \sin(3c))\cos(4bx + a) - 2(2\sin(2bx + a + 2c) + \sin(a +}{b\cos(4bx + a)^2 + 4b\cos}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c)^3,x, algorithm="maxima")
```

output
```
-((2*sin(2*b*x + 2*a + 3*c) + sin(2*a + 5*c) + sin(3*c))*cos(4*b*x + a) -
2*(2*sin(2*b*x + a + 2*c) + sin(a + 4*c))*cos(2*b*x + 2*a + 3*c) + 2*(sin(
2*a + 5*c) + sin(3*c))*cos(2*b*x + a + 2*c) - (2*cos(2*b*x + 2*a + 3*c) +
cos(2*a + 5*c) + cos(3*c))*sin(4*b*x + a) + 2*(2*cos(2*b*x + a + 2*c) + co
s(a + 4*c))*sin(2*b*x + 2*a + 3*c) - 2*(cos(2*a + 5*c) + cos(3*c))*sin(2*b
*x + a + 2*c) + cos(a + 4*c)*sin(2*a + 5*c) - cos(2*a + 5*c)*sin(a + 4*c)
- cos(3*c)*sin(a + 4*c) + cos(a + 4*c)*sin(3*c))/(b*cos(4*b*x + a)^2 + 4*b
*cos(2*b*x + a + 2*c)^2 + 4*b*cos(2*b*x + a + 2*c)*cos(a + 4*c) + b*cos(a
+ 4*c)^2 + b*sin(4*b*x + a)^2 + 4*b*sin(2*b*x + a + 2*c)^2 + 4*b*sin(2*b*x
 + a + 2*c)*sin(a + 4*c) + b*sin(a + 4*c)^2 + 2*(2*b*cos(2*b*x + a + 2*c)
+ b*cos(a + 4*c))*cos(4*b*x + a) + 2*(2*b*sin(2*b*x + a + 2*c) + b*sin(a +
 4*c))*sin(4*b*x + a))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(36) = 72$.

Time = 0.15 (sec) , antiderivative size = 313, normalized size of antiderivative = 8.46

$$\int \cos(a + bx)\sec^3(c - bx)\, dx = \frac{\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^6 + 3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^4 + 3\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^6 + 3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^2 + 9\tan}{4\left(2\tan(bx + a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right) + 2\tan(bx + a)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}a\right)^2\right.}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c)^3,x, algorithm="giac")
```

output
```
1/4*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a
)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)
^4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c
)^2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(
1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 +
 1)/((2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) + 2*tan(b*x + a)*tan(1/2*a)*t
an(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 - 2*tan(b*x + a)*tan(1/2*a) + tan(
1/2*a)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c
)^2 - 1)^2*(tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)
 - tan(1/2*c))*b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^3(c-bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/cos(c - b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int \cos(a+bx)\sec^3(c-bx)\,dx = \frac{-\cos(bx-c)\sin(bx+a)-\cos(bx+a)\sin(bx-c)}{2b\left(\sin(bx-c)^2-1\right)}$$

input
```
int(cos(b*x+a)*sec(b*x-c)^3,x)
```

output
```
( - (cos(b*x - c)*sin(a + b*x) + cos(a + b*x)*sin(b*x - c)))/(2*b*(sin(b*x
 - c)**2 - 1))
```

3.378 $\int \cos(a+bx)\sec^4(c-bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int \cos(a+bx)\sec^4(c-bx)\,dx = -\frac{\operatorname{arctanh}(\sin(c-bx))\cos(a+c)}{2b} - \frac{\sec^3(c-bx)\sin(a+c)}{3b} - \frac{\cos(a+c)\sec(c-bx)\tan(c-bx)}{2b}$$

output
```
1/2*arctanh(sin(b*x-c))*cos(a+c)/b-1/3*sec(b*x-c)^3*sin(a+c)/b+1/2*cos(a+c
)*sec(b*x-c)*tan(b*x-c)/b
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \cos(a+bx)\sec^4(c-bx)\,dx = -\frac{12\operatorname{arctanh}\left(\sin(c)-\cos(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a+c)+\sec^3(c-bx)(4\sin(a+c)+3\cos(a+c)\sin(2(c-bx}{12b}$$

input
```
Integrate[Cos[a + b*x]*Sec[c - b*x]^4,x]
```

output
```
-1/12*(12*ArcTanh[Sin[c] - Cos[c]*Tan[(b*x)/2]]*Cos[a + c] + Sec[c - b*x]^
3*(4*Sin[a + c] + 3*Cos[a + c]*Sin[2*(c - b*x)]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sec^4(c-bx)\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \cos(a+bx)\sec^4(c-bx)dx$$

input
```
Int[Cos[a + b*x]*Sec[c - b*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.80

method	result
risch	$\frac{i\left(3\,\mathrm{e}^{i(bx+7a+6c)}+8\,\mathrm{e}^{i(3bx+7a+4c)}+3\,\mathrm{e}^{i(bx+5a+4c)}-3\,\mathrm{e}^{i(5bx+7a+2c)}-8\,\mathrm{e}^{i(3bx+5a+2c)}-3\,\mathrm{e}^{5i(bx+a)}\right)}{6b\left(\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)^3}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a+c)}\right)\cos(a}{2b}$
default	Expression too large to display

input
```
int(cos(b*x+a)*sec(b*x-c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*I/b/(exp(2*I*(a+c))+exp(2*I*(b*x+a)))^3*(3*exp(I*(b*x+7*a+6*c))+8*exp(
I*(3*b*x+7*a+4*c))+3*exp(I*(b*x+5*a+4*c))-3*exp(I*(5*b*x+7*a+2*c))-8*exp(I
*(3*b*x+5*a+2*c))-3*exp(5*I*(b*x+a)))-1/2*ln(exp(I*(b*x+a))-I*exp(I*(a+c))
)/b*cos(a+c)+1/2*ln(exp(I*(b*x+a))+I*exp(I*(a+c)))/b*cos(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(63) = 126$.

Time = 0.10 (sec) , antiderivative size = 433, normalized size of antiderivative = 6.66

$$\int \cos(a+bx)\sec^4(c-bx)\,dx$$
$$= \frac{6\left(2\cos(a+c)^3-\cos(a+c)\right)\cos(bx+a)\sin(bx+a)+3\left(\left(4\cos(a+c)^4-3\cos(a+c)^2\right)\cos(bx+c\right.}{}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c)^4,x, algorithm="fricas")
```

output
```
1/12*(6*(2*cos(a + c)^3 - cos(a + c))*cos(b*x + a)*sin(b*x + a) + 3*((4*co
s(a + c)^4 - 3*cos(a + c)^2)*cos(b*x + a)^3 + ((4*cos(a + c)^3 - cos(a + c
))*cos(b*x + a)^2 - cos(a + c)^3 + cos(a + c))*sin(b*x + a)*sin(a + c) - 3
*(cos(a + c)^4 - cos(a + c)^2)*cos(b*x + a))*log(2*(cos(a + c)*sin(b*x + a
) - cos(b*x + a)*sin(a + c) + 1)/(cos(a + c) + 1)) - 3*((4*cos(a + c)^4 -
3*cos(a + c)^2)*cos(b*x + a)^3 + ((4*cos(a + c)^3 - cos(a + c))*cos(b*x +
a)^2 - cos(a + c)^3 + cos(a + c))*sin(b*x + a)*sin(a + c) - 3*(cos(a + c)^
4 - cos(a + c)^2)*cos(b*x + a))*log(-2*(cos(a + c)*sin(b*x + a) - cos(b*x
+ a)*sin(a + c) - 1)/(cos(a + c) + 1)) - 2*(6*cos(b*x + a)^2*cos(a + c)^2
- 3*cos(a + c)^2 + 2)*sin(a + c))/((4*b*cos(a + c)^3 - 3*b*cos(a + c))*cos
(b*x + a)^3 + ((4*b*cos(a + c)^2 - b)*cos(b*x + a)^2 - b*cos(a + c)^2 + b)
*sin(b*x + a)*sin(a + c) - 3*(b*cos(a + c)^3 - b*cos(a + c))*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^4(c-bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1427 vs. $2(63) = 126$.

Time = 0.20 (sec) , antiderivative size = 1427, normalized size of antiderivative = 21.95

$$\int \cos(a+bx)\sec^4(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c)^4,x, algorithm="maxima")
```

output
```
1/12*(2*(3*sin(5*b*x) + 3*sin(5*b*x + 2*a + 2*c) - 8*sin(3*b*x + 2*a + 4*c
) + 8*sin(3*b*x + 2*c) - 3*sin(b*x + 2*a + 6*c) - 3*sin(b*x + 4*c))*cos(6*
b*x + a) - 6*(3*sin(4*b*x + a + 2*c) + 3*sin(2*b*x + a + 4*c) + sin(a + 6*
c))*cos(5*b*x + 2*a + 2*c) + 6*(3*sin(5*b*x) - 8*sin(3*b*x + 2*a + 4*c) +
8*sin(3*b*x + 2*c) - 3*sin(b*x + 2*a + 6*c) - 3*sin(b*x + 4*c))*cos(4*b*x
+ a + 2*c) + 16*(3*sin(2*b*x + a + 4*c) + sin(a + 6*c))*cos(3*b*x + 2*a +
4*c) - 16*(3*sin(2*b*x + a + 4*c) + sin(a + 6*c))*cos(3*b*x + 2*c) + 18*(s
in(5*b*x) - sin(b*x + 2*a + 6*c) - sin(b*x + 4*c))*cos(2*b*x + a + 4*c) -
3*(cos(6*b*x + a)^2*cos(a + c) + 9*cos(4*b*x + a + 2*c)^2*cos(a + c) + 9*c
os(2*b*x + a + 4*c)^2*cos(a + c) + 6*cos(2*b*x + a + 4*c)*cos(a + 6*c)*cos
(a + c) + cos(a + 6*c)^2*cos(a + c) + cos(a + c)*sin(6*b*x + a)^2 + 9*cos(
a + c)*sin(4*b*x + a + 2*c)^2 + 9*cos(a + c)*sin(2*b*x + a + 4*c)^2 + 6*co
s(a + c)*sin(2*b*x + a + 4*c)*sin(a + 6*c) + cos(a + c)*sin(a + 6*c)^2 + 2
*(3*cos(4*b*x + a + 2*c)*cos(a + c) + 3*cos(2*b*x + a + 4*c)*cos(a + c) +
cos(a + 6*c)*cos(a + c))*cos(6*b*x + a) + 6*(3*cos(2*b*x + a + 4*c)*cos(a
+ c) + cos(a + 6*c)*cos(a + c))*cos(4*b*x + a + 2*c) + 2*(3*cos(a + c)*sin
(4*b*x + a + 2*c) + 3*cos(a + c)*sin(2*b*x + a + 4*c) + cos(a + c)*sin(a +
 6*c))*sin(6*b*x + a) + 6*(3*cos(a + c)*sin(2*b*x + a + 4*c) + cos(a + c)*
sin(a + 6*c))*sin(4*b*x + a + 2*c))*log((cos(b*x)^2 + cos(c)^2 - 2*cos(c)*
sin(b*x) + sin(b*x)^2 + 2*cos(b*x)*sin(c) + sin(c)^2)/(cos(b*x)^2 + cos...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12160 vs. $2(63) = 126$.

Time = 0.95 (sec) , antiderivative size = 12160, normalized size of antiderivative = 187.08

$$\int \cos(a + bx) \sec^4(c - bx)\, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sec(b*x-c)^4,x, algorithm="giac")
```

output
```
1/6*(3*(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)
^2*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c) - 5*tan(1/2*a)^2*tan(1/2*c)^2 -
tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^3 + 5*tan(1/2*a)^2*tan(1/2*c) + 5*tan
(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*
c) + tan(1/2*c)^2 - tan(1/2*a) - tan(1/2*c) - 1)*log(abs(-tan(1/2*b*x + 1/
2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*b*x
 + 1/2*a)*tan(1/2*c) - tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a) - tan(
1/2*a) - tan(1/2*c) + 1))/(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/
2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c) - tan(1/2*a)^
2*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 - tan(1/2*a)^2*tan
(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a)^2 + tan(1/2*
a)*tan(1/2*c) - tan(1/2*c)^2 - tan(1/2*a) - tan(1/2*c) - 1) - 3*(tan(1/2*a
)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^3 -
 tan(1/2*a)^3*tan(1/2*c) - 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/
2*c)^3 - tan(1/2*a)^3 - 5*tan(1/2*a)^2*tan(1/2*c) - 5*tan(1/2*a)*tan(1/2*c
)^2 - tan(1/2*c)^3 + tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2
 + tan(1/2*a) + tan(1/2*c) - 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*t
an(1/2*c) - tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*b*x + 1/2*a)*tan(1/2
*c) + tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a) - tan(1/2*a) - tan(1/2*
c) - 1))/(tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 + tan(1...
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sec^4(c-bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/cos(c - b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos(a+bx)\sec^4(c-bx)\,dx = \text{Too large to display}$$

input
```
int(cos(b*x+a)*sec(b*x-c)^4,x)
```

output
```
( - 4*cos(b*x - c)**2*sin(a + b*x) + 8*cos(b*x - c)*int(cos(a + b*x)/(sin(
b*x - c)**4 - 2*sin(b*x - c)**2 + 1),x)*sin(b*x - c)**2*b - 8*cos(b*x - c)
*int(cos(a + b*x)/(sin(b*x - c)**4 - 2*sin(b*x - c)**2 + 1),x)*b - cos(b*x
 - c)*int((cos(a + b*x)*sin(b*x - c)**4)/(sin(b*x - c)**4 - 2*sin(b*x - c)
**2 + 1),x)*sin(b*x - c)**2*b + cos(b*x - c)*int((cos(a + b*x)*sin(b*x - c
)**4)/(sin(b*x - c)**4 - 2*sin(b*x - c)**2 + 1),x)*b - 4*cos(b*x - c)*int(
(cos(a + b*x)*sin(b*x - c)**2)/(sin(b*x - c)**4 - 2*sin(b*x - c)**2 + 1),x
)*sin(b*x - c)**2*b + 4*cos(b*x - c)*int((cos(a + b*x)*sin(b*x - c)**2)/(s
in(b*x - c)**4 - 2*sin(b*x - c)**2 + 1),x)*b - 2*cos(b*x - c)*log(sin(b*x
- c) - 1)*sin(b*x - c)**2 + 2*cos(b*x - c)*log(sin(b*x - c) - 1) + 2*cos(b
*x - c)*log(sin(b*x - c) + 1)*sin(b*x - c)**2 - 2*cos(b*x - c)*log(sin(b*x
 - c) + 1) + 4*cos(b*x - c)*log(tan((b*x - c)/2) - 1)*sin(b*x - c)**2 - 4*
cos(b*x - c)*log(tan((b*x - c)/2) - 1) - 4*cos(b*x - c)*log(tan((b*x - c)/
2) + 1)*sin(b*x - c)**2 + 4*cos(b*x - c)*log(tan((b*x - c)/2) + 1) + cos(b
*x - c)*sin(b*x - c)**2*sin(a + b*x) + cos(b*x - c)*sin(b*x - c)**2*a - 3*
cos(b*x - c)*sin(a + b*x) - cos(b*x - c)*a - 4*cos(a + b*x)*sin(b*x - c) -
 4*sin(b*x - c)**2*sin(a + b*x) + 4*sin(a + b*x))/(15*cos(b*x - c)*b*(sin(
b*x - c)**2 - 1))
```

3.379 $\int \cos^2(a+bx)\sec(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \cos^2(a+bx)\sec(c+bx)\,dx=\frac{\operatorname{arctanh}(\sin(c+bx))\sin^2(a-c)}{b}+\frac{\sin(2a-c+bx)}{b}$$

output
```
arctanh(sin(b*x+c))*sin(a-c)^2/b+sin(b*x+2*a-c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 127 vs. 2(35) = 70.

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.63

$$\int \cos^2(a+bx)\sec(c+bx)\,dx=\frac{(-1+\cos(2a-2c))\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)-\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{2b}+\frac{(1-\cos(2a-2c))\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)+\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{2b}+\frac{\cos(bx)\sin(2a-c)}{b}+\frac{\cos(2a-c)\sin(bx)}{b}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c + b*x],x]
```

output
```
((-1 + Cos[2*a - 2*c])*Log[Cos[c/2 + (b*x)/2] - Sin[c/2 + (b*x)/2]])/(2*b)
 + ((1 - Cos[2*a - 2*c])*Log[Cos[c/2 + (b*x)/2] + Sin[c/2 + (b*x)/2]])/(2*
b) + (Cos[b*x]*Sin[2*a - c])/b + (Cos[2*a - c]*Sin[b*x])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec(bx+c)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \cos^2(a+bx)\sec(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 4.14

method	result
risch	$-\frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)}{2b}+\frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\cos(2a-2c)}{2b}+\frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)}{2b}-\frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\cos(2a-2c)}{2b}$
default	$\frac{-\frac{2\left((-\cos(a)\cos(c)-\sin(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin(a)\cos(c)+\cos(a)\sin(c)\right)}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)}+\frac{2\left(-\cos(c)^2\sin(a)^2+2\cos(a)\cos(c)\sin(a)\sin(c)-\sin(c\right.}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(c\right.}}{b}$

input
```
int(cos(b*x+a)^2*sec(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))+1/2/b*ln(exp(I*(b*x+a))-I*exp(I*(
a-c)))*cos(2*a-2*c)+1/2/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))-1/2/b*ln(exp(I
*(b*x+a))+I*exp(I*(a-c)))*cos(2*a-2*c)+sin(b*x+2*a-c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.54

$$\int \cos^2(a+bx)\sec(c+bx)\,dx = -\frac{4\cos(bx+c)\cos(-a+c)\sin(-a+c)+\left(\cos(-a+c)^2-1\right)\log(\sin(bx+c)+1)-\left(\cos(-a+c)^2\right.}{2b}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(4*cos(b*x + c)*cos(-a + c)*sin(-a + c) + (cos(-a + c)^2 - 1)*log(sin
(b*x + c) + 1) - (cos(-a + c)^2 - 1)*log(-sin(b*x + c) + 1) - 2*(2*cos(-a
+ c)^2 - 1)*sin(b*x + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(27) = 54$.

Time = 19.36 (sec) , antiderivative size = 3645, normalized size of antiderivative = 104.14

$$\int \cos^2(a + bx) \sec(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**2*sec(b*x+c),x)
```

output
```
-2*Piecewise((-sin(b*x)/b, Eq(c, pi/2)), (sin(b*x)/b, Eq(c, -pi/2)), (0, E
q(b, 0)), (-2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))
*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 +
2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2
*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**3/
(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)*
*2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/
(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*t
an(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2
)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) -
 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 +
 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) +
2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3
*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)
**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x
/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(c/2)*
*4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan
(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) +
1) - 1/(tan(c/2) + 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2
 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(35) = 70$.

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 4.00

$$\int \cos^2(a+bx)\sec(c+bx)\,dx$$
$$= \frac{(\cos(-2a+2c)-1)\log\left(\frac{\cos(bx+2c)^2+\cos(c)^2-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2+2\cos(bx+2c)\sin(c)+\sin(c)^2}{\cos(bx+2c)^2+\cos(c)^2+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2-2\cos(bx+2c)\sin(c)+\sin(c)^2}\right)+4\sin(bx}{4b}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c),x, algorithm="maxima")
```

output
```
1/4*((cos(-2*a + 2*c) - 1)*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin
(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(
b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*c
os(b*x + 2*c)*sin(c) + sin(c)^2)) + 4*sin(b*x + 2*a - c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1708 vs. $2(35) = 70$.

Time = 0.23 (sec) , antiderivative size = 1708, normalized size of antiderivative = 48.80

$$\int \cos^2(a+bx)\sec(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c),x, algorithm="giac")
```

output
```
-2*(2*(tan(1/2*a)^5*tan(1/2*c)^3 - 2*tan(1/2*a)^4*tan(1/2*c)^4 + tan(1/2*a
)^3*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^2 + 3*tan(1/2*a)^4*tan(1/2*c)^3
 - 3*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^2*tan(1/2*c)^5 + 3*tan(1/2*a)^
4*tan(1/2*c)^2 - 6*tan(1/2*a)^3*tan(1/2*c)^3 + 3*tan(1/2*a)^2*tan(1/2*c)^4
 - 2*tan(1/2*a)^4*tan(1/2*c) + 6*tan(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^
2*tan(1/2*c)^3 + 2*tan(1/2*a)*tan(1/2*c)^4 + 3*tan(1/2*a)^3*tan(1/2*c) - 6
*tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 + 3*
tan(1/2*a)^2*tan(1/2*c) - 3*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1
/2*a)^2 - 2*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2)*log(abs(-tan(1/2*b*x + 1
/2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*b*
x + 1/2*a)*tan(1/2*c) + tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan
(1/2*a) - tan(1/2*c) + 1))/(tan(1/2*a)^5*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1
/2*c)^4 + tan(1/2*a)^4*tan(1/2*c)^5 + 2*tan(1/2*a)^5*tan(1/2*c)^3 + tan(1/
2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)^5 - 2*tan(1/2*a)^5*tan(1/2
*c)^2 + 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(1/2*a)^3*tan(1/2*c)^4 + 2*tan(
1/2*a)^2*tan(1/2*c)^5 + tan(1/2*a)^5*tan(1/2*c) + 2*tan(1/2*a)^4*tan(1/2*c
)^2 + 4*tan(1/2*a)^3*tan(1/2*c)^3 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*
a)*tan(1/2*c)^5 - tan(1/2*a)^5 + tan(1/2*a)^4*tan(1/2*c) - 4*tan(1/2*a)^3*
tan(1/2*c)^2 + 4*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 + tan
(1/2*c)^5 + tan(1/2*a)^4 + 2*tan(1/2*a)^3*tan(1/2*c) + 4*tan(1/2*a)^2*t...
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 217, normalized size of antiderivative = 6.20

$$\int \cos^2(a+bx)\sec(c+bx)\,dx = \frac{e^{-a\,2i+c\,1i-b\,x\,1i}\,1i}{2\,b} - \frac{e^{a\,2i-c\,1i+b\,x\,1i}\,1i}{2\,b} + \frac{e^{-a\,2i+c\,2i}\,\ln\left(-\frac{\left(e^{a\,2i}\,e^{-c\,2i}-1\right)^2\,1i}{2}+\frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(1+e^{a\,4i}\,e^{-c\,4i}-2\,e^{a\,2i}\,e^{-c\,2i}\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}-1\right)^2}{4\,b} - \frac{e^{-a\,2i+c\,2i}\,\ln\left(\frac{\left(e^{a\,2i}\,e^{-c\,2i}-1\right)^2\,1i}{2}+\frac{e^{c\,1i}\,e^{b\,x\,1i}\,\left(1+e^{a\,4i}\,e^{-c\,4i}-2\,e^{a\,2i}\,e^{-c\,2i}\right)}{2}\right)\,\left(e^{a\,2i-c\,2i}-1\right)^2}{4\,b}$$

input
```
int(cos(a + b*x)^2/cos(c + b*x),x)
```

output
```
(exp(c*1i - a*2i - b*x*1i)*1i)/(2*b) - (exp(a*2i - c*1i + b*x*1i)*1i)/(2*b
) + (exp(c*2i - a*2i)*log((exp(c*1i)*exp(b*x*1i)*(exp(a*4i)*exp(-c*4i) - 2
*exp(a*2i)*exp(-c*2i) + 1))/2 - ((exp(a*2i)*exp(-c*2i) - 1)^2*1i)/2)*(exp(
a*2i - c*2i) - 1)^2)/(4*b) - (exp(c*2i - a*2i)*log(((exp(a*2i)*exp(-c*2i)
- 1)^2*1i)/2 + (exp(c*1i)*exp(b*x*1i)*(exp(a*4i)*exp(-c*4i) - 2*exp(a*2i)*
exp(-c*2i) + 1))/2)*(exp(a*2i - c*2i) - 1)^2)/(4*b)
```

Reduce [F]

$$\int \cos^2(a+bx)\sec(c+bx)\,dx = \int \cos(bx+a)^2 \sec(bx+c)\,dx$$

input
```
int(cos(b*x+a)^2*sec(b*x+c),x)
```

output
```
int(cos(a + b*x)**2*sec(b*x + c),x)
```

3.380 $\int \cos^2(a+bx)\sec^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \cos^2(a+bx)\sec^2(c+bx)\,dx = x\cos(2(a-c)) + \frac{\log(\cos(c+bx))\sin(2(a-c))}{b} + \frac{\sin^2(a-c)\tan(c+bx)}{b}$$

output
```
x*cos(2*a-2*c)+ln(cos(b*x+c))*sin(2*a-2*c)/b+sin(a-c)^2*tan(b*x+c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(48) = 96$.

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.69

$$\int \cos^2(a+bx)\sec^2(c+bx)\,dx = \frac{\sec(c)\sec(c+bx)(bx\cos(2a-4c-bx) + bx\cos(2a-2c-bx) + bx\cos(2a+bx) + bx\cos(2a-2c+bx)}{}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c + b*x]^2,x]
```

output
```
(Sec[c]*Sec[c + b*x]*(b*x*Cos[2*a - 4*c - b*x] + b*x*Cos[2*a - 2*c - b*x]
+ b*x*Cos[2*a + b*x] + b*x*Cos[2*a - 2*c + b*x] + 2*Sin[b*x] + Log[Cos[c +
 b*x]]*Sin[2*a - 4*c - b*x] + Sin[2*a - 2*c - b*x] + Log[Cos[c + b*x]]*Sin
[2*a - 2*c - b*x] + Log[Cos[c + b*x]]*Sin[2*a + b*x] - Sin[2*a - 2*c + b*x
] + Log[Cos[c + b*x]]*Sin[2*a - 2*c + b*x]))/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec^2(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos^2(a+bx)\sec^2(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.06 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.44

method	result
risch	$x\,\mathrm{e}^{2i(a-c)}-2i\sin\left(2a-2c\right)x-\frac{2i\sin(2a-2c)a}{b}-\frac{i\mathrm{e}^{2i(2a-c)}}{2b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}+\frac{i\mathrm{e}^{2ia}}{b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)}-\frac{i\mathrm{e}^{2ic}}{2b\left(\mathrm{e}^{2i(bx+a+c)}\right.}$
default	$-\frac{\cos(c)^{2}\sin(a)^{2}-2\cos(a)\cos(c)\sin(a)\sin(c)+\sin(c)^{2}\cos(a)^{2}}{\left(\cos(a)^{2}+\sin(a)^{2}\right)\left(\cos(c)^{2}+\sin(c)^{2}\right)(\sin(a)\cos(c)-\cos(a)\sin(c))(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}$

input
```
int(cos(b*x+a)^2*sec(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
x*exp(2*I*(a-c))-2*I*sin(2*a-2*c)*x-2*I/b*sin(2*a-2*c)*a-1/2*I/b/(exp(2*I*
(b*x+a+c))+exp(2*I*a))*exp(2*I*(2*a-c))+I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a)
)*exp(2*I*a)-1/2*I/b/(exp(2*I*(b*x+a+c))+exp(2*I*a))*exp(2*I*c)+ln(exp(2*I
*(b*x+a))+exp(2*I*(a-c)))/b*sin(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.77

$$\int \cos^2(a+bx)\sec^2(c+bx)\,dx =$$

$$-\frac{2\,\cos\left(bx+c\right)\cos\left(-a+c\right)\log\left(-\cos\left(bx+c\right)\right)\sin\left(-a+c\right)-\left(2\,bx\cos\left(-a+c\right)^{2}-bx\right)\cos\left(bx+c\right)+}{b\cos\left(bx+c\right)}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^2,x, algorithm="fricas")
```

output
```
-(2*cos(b*x + c)*cos(-a + c)*log(-cos(b*x + c))*sin(-a + c) - (2*b*x*cos(-
a + c)^2 - b*x)*cos(b*x + c) + (cos(-a + c)^2 - 1)*sin(b*x + c))/(b*cos(b*
x + c))
```

Sympy [F(-2)]

Exception generated.

$$\int \cos^2(a+bx)\sec^2(c+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(cos(b*x+a)**2*sec(b*x+c)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(48) = 96$.

Time = 0.06 (sec) , antiderivative size = 534, normalized size of antiderivative = 11.12

$$\int \cos^2(a+bx)\sec^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(b*cos(2*a + 2*c)*cos(4*c) + b*sin(2*a + 2*c)*sin(4*c))*x + (2*b*x*
cos(4*c) + sin(4*a) - 2*sin(2*a + 2*c) + sin(4*c))*cos(2*b*x + 2*a + 4*c)
+ 2*(b*x*cos(2*b*x + 2*a + 4*c) + b*x*cos(2*a + 2*c))*cos(2*b*x + 6*c) + (
sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(2*b*x + 2*a + 4*c)^2*sin(-2*a +
 2*c) + 2*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c)*sin(-2*a + 2*c) + cos(2*a
+ 2*c)^2*sin(-2*a + 2*c) + sin(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c) + 2*si
n(2*b*x + 2*a + 4*c)*sin(2*a + 2*c)*sin(-2*a + 2*c) + sin(2*a + 2*c)^2*sin
(-2*a + 2*c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(
2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2) + (2*b*x*sin(4*c) - cos(4*a
) + 2*cos(2*a + 2*c) - cos(4*c))*sin(2*b*x + 2*a + 4*c) + 2*(b*x*sin(2*b*x
 + 2*a + 4*c) + b*x*sin(2*a + 2*c))*sin(2*b*x + 6*c) - (cos(4*a) + cos(4*c
))*sin(2*a + 2*c))/(b*cos(2*b*x + 2*a + 4*c)^2 + 2*b*cos(2*b*x + 2*a + 4*c
)*cos(2*a + 2*c) + b*cos(2*a + 2*c)^2 + b*sin(2*b*x + 2*a + 4*c)^2 + 2*b*s
in(2*b*x + 2*a + 4*c)*sin(2*a + 2*c) + b*sin(2*a + 2*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1954 vs. $2(48) = 96$.

Time = 0.22 (sec) , antiderivative size = 1954, normalized size of antiderivative = 40.71

$$\int \cos^2(a + bx) \sec^2(c + bx) \, dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)^2*sec(b*x+c)^2,x, algorithm="giac")
```

output

```
((tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*a)^4*tan(1/2*c)^2 + 16*tan(1/2*a)^
3*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 16*tan(1/2*a
)^3*tan(1/2*c) + 36*tan(1/2*a)^2*tan(1/2*c)^2 - 16*tan(1/2*a)*tan(1/2*c)^3
 + tan(1/2*c)^4 - 6*tan(1/2*a)^2 + 16*tan(1/2*a)*tan(1/2*c) - 6*tan(1/2*c)
^2 + 1)*(b*x + a)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2
 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^
2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4*
tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*tan
(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*
c)^4 - tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)^
2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))*log(tan(b*x + a)^2 + 1)/(tan(1
/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/
2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan
(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) + 4*(tan(1/2*a)^6*tan(1/2*c)^4 - 2*tan(1/2
*a)^5*tan(1/2*c)^5 + tan(1/2*a)^4*tan(1/2*c)^6 - tan(1/2*a)^6*tan(1/2*c)^2
 + 8*tan(1/2*a)^5*tan(1/2*c)^3 - 14*tan(1/2*a)^4*tan(1/2*c)^4 + 8*tan(1/2*
a)^3*tan(1/2*c)^5 - tan(1/2*a)^2*tan(1/2*c)^6 - 2*tan(1/2*a)^5*tan(1/2*c)
+ 14*tan(1/2*a)^4*tan(1/2*c)^2 - 24*tan(1/2*a)^3*tan(1/2*c)^3 + 14*tan(1/2
*a)^2*tan(1/2*c)^4 - 2*tan(1/2*a)*tan(1/2*c)^5 - tan(1/2*a)^4 + 8*tan(1/2*
a)^3*tan(1/2*c) - 14*tan(1/2*a)^2*tan(1/2*c)^2 + 8*tan(1/2*a)*tan(1/2*c...
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.02

$$\int \cos^2(a+bx)\sec^2(c+bx)\,dx = x\,(\cos(2\,a-2\,c)-\sin(2\,a-2\,c)\,1\mathrm{i})-\frac{\left(1+\mathrm{e}^{a\,4\mathrm{i}-c\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}\right)\,1\mathrm{i}}{2\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)} + \frac{\mathrm{e}^{-a\,4\mathrm{i}+c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\right)\,\left(2\,b\,\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-2\,b\,\mathrm{e}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$

input
```
int(cos(a + b*x)^2/cos(c + b*x)^2,x)
```

output
```
x*(cos(2*a - 2*c) - sin(2*a - 2*c)*1i) - ((exp(a*4i - c*4i) - 2*exp(a*2i -
 c*2i) + 1)*1i)/(2*b*(exp(a*2i - c*2i) + exp(a*2i + b*x*2i))) + (exp(c*4i
- a*4i)*log(exp(a*2i)*exp(b*x*2i) + exp(a*2i)*exp(-c*2i))*(2*b*exp(a*2i -
c*2i) - 2*b*exp(a*6i - c*6i))*1i)/(4*b^2)
```

Reduce [F]

$$\int \cos^2(a+bx)\sec^2(c+bx)\,dx = \text{Too large to display}$$

input
```
int(cos(b*x+a)^2*sec(b*x+c)^2,x)
```

output

```
( - 7*cos(b*x + c)*cos(a + b*x)*sin(a + b*x) - 96*cos(b*x + c)*int(tan((b*
x + c)/2)**2/(tan((b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)
**4*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan(
(a + b*x)/2)**4 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x +
 c)/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b - 96*cos
(b*x + c)*int(tan((a + b*x)/2)**2/(tan((b*x + c)/2)**4*tan((a + b*x)/2)**4
 + 2*tan((b*x + c)/2)**4*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan
((b*x + c)/2)**2*tan((a + b*x)/2)**4 - 4*tan((b*x + c)/2)**2*tan((a + b*x)
/2)**2 - 2*tan((b*x + c)/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)*
*2 + 1),x)*b + 128*cos(b*x + c)*int((tan((b*x + c)/2)*tan((a + b*x)/2))/(t
an((b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)**4*tan((a + b*
x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan((a + b*x)/2)**4
 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x + c)/2)**2 + tan
((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b + 32*cos(b*x + c)*int(1
/(tan((b*x + c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x + c)/2)**4*tan((a +
 b*x)/2)**2 + tan((b*x + c)/2)**4 - 2*tan((b*x + c)/2)**2*tan((a + b*x)/2)
**4 - 4*tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x + c)/2)**2 +
tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b - 16*cos(b*x + c)*si
n(a + b*x) - 9*cos(b*x + c)*a - 9*cos(b*x + c)*b*x + 8*cos(a + b*x)*sin(b*
x + c) - 8*cos(a + b*x)*sin(a + b*x) - 4*sin(b*x + c)*sin(a + b*x)**2 +...
```

3.381 $\int \cos^2(a+bx)\sec^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx=\frac{\operatorname{arctanh}(\sin(c+bx))\cos(2(a-c))}{b}+\frac{\operatorname{arctanh}(\sin(c+bx))\sin^2(a-c)}{2b}-\frac{\sec(c+bx)\sin(2(a-c))}{b}+\frac{\sec(c+bx)\sin^2(a-c)\tan(c+bx)}{2b}$$

output
```
arctanh(sin(b*x+c))*cos(2*a-2*c)/b+1/2*arctanh(sin(b*x+c))*sin(a-c)^2/b-se
c(b*x+c)*sin(2*a-2*c)/b+1/2*sec(b*x+c)*sin(a-c)^2*tan(b*x+c)/b
```

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.88

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx$$
$$=\frac{-\log\left(\cos\left(\frac{1}{2}(c+bx)\right)-\sin\left(\frac{1}{2}(c+bx)\right)\right)-3\cos(2(a-c))\left(\log\left(\cos\left(\frac{1}{2}(c+bx)\right)-\sin\left(\frac{1}{2}(c+bx)\right)\right)-\text{lo}\right.}{}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c + b*x]^3,x]
```

output
```
(-Log[Cos[(c + b*x)/2] - Sin[(c + b*x)/2]] - 3*Cos[2*(a - c)]*(Log[Cos[(c
+ b*x)/2] - Sin[(c + b*x)/2]] - Log[Cos[(c + b*x)/2] + Sin[(c + b*x)/2]])
+ Log[Cos[(c + b*x)/2] + Sin[(c + b*x)/2]] + 4*Sec[c]*Sin[2*(a - c)] - 4*S
ec[c + b*x]*Sin[2*(a - c)] + 2*Sec[c + b*x]*Sin[a - c]^2*Tan[c + b*x])/(4*
b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec^3(bx+c)\,dx$$
$$\Big\downarrow \ 7299$$
$$\int \cos^2(a+bx)\sec^3(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.96 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.84

method	result
risch	$\frac{i\left(5\,\mathrm{e}^{i(3bx+6a+c)}-2\,\mathrm{e}^{i(3bx+4a+3c)}-3\,\mathrm{e}^{i(3bx+2a+5c)}+3\,\mathrm{e}^{i(bx+6a-c)}+2\,\mathrm{e}^{i(bx+4a+c)}-5\,\mathrm{e}^{i(bx+2a+3c)}\right)}{4\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{2}b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a-c)}\right)}{4b}-$
default	Expression too large to display

input
```
int(cos(b*x+a)^2*sec(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/(exp(2*I*(b*x+a+c))+exp(2*I*a))^2/b*(5*exp(I*(3*b*x+6*a+c))-2*exp(I*
(3*b*x+4*a+3*c))-3*exp(I*(3*b*x+2*a+5*c))+3*exp(I*(b*x+6*a-c))+2*exp(I*(b*
x+4*a+c))-5*exp(I*(b*x+2*a+3*c)))-1/4/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))-
3/4/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))*cos(2*a-2*c)+1/4/b*ln(exp(I*(b*x+a
))+I*exp(I*(a-c)))+3/4/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))*cos(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx$$
$$=\frac{\left(3\,\cos\left(-a+c\right)^2-1\right)\cos\left(bx+c\right)^2\log\left(\sin\left(bx+c\right)+1\right)-\left(3\,\cos\left(-a+c\right)^2-1\right)\cos\left(bx+c\right)^2\log\left(-\sin\right.}{4\,b\cos\left(bx+c\right.}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*((3*cos(-a + c)^2 - 1)*cos(b*x + c)^2*log(sin(b*x + c) + 1) - (3*cos(-
a + c)^2 - 1)*cos(b*x + c)^2*log(-sin(b*x + c) + 1) + 8*cos(b*x + c)*cos(-
a + c)*sin(-a + c) - 2*(cos(-a + c)^2 - 1)*sin(b*x + c))/(b*cos(b*x + c)^2
)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sec(b*x+c)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1251 vs. $2(84) = 168$.

Time = 0.21 (sec) , antiderivative size = 1251, normalized size of antiderivative = 14.22

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^3,x, algorithm="maxima")
```

output
```
-1/8*(2*(5*sin(3*b*x + 4*a + 2*c) - 2*sin(3*b*x + 2*a + 4*c) - 3*sin(3*b*x
 + 6*c) + 3*sin(b*x + 4*a) + 2*sin(b*x + 2*a + 2*c) - 5*sin(b*x + 4*c))*co
s(4*b*x + 2*a + 5*c) - 10*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + c))*cos(3*
b*x + 4*a + 2*c) + 4*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + c))*cos(3*b*x +
 2*a + 4*c) + 6*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + c))*cos(3*b*x + 6*c)
 + 4*(3*sin(b*x + 4*a) + 2*sin(b*x + 2*a + 2*c) - 5*sin(b*x + 4*c))*cos(2*
b*x + 2*a + 3*c) + ((3*cos(-2*a + 2*c) + 1)*cos(4*b*x + 2*a + 5*c)^2 + 4*(
3*cos(-2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c)^2 + (3*cos(-2*a + 2*c) + 1)*
sin(4*b*x + 2*a + 5*c)^2 + 4*(3*cos(-2*a + 2*c) + 1)*sin(2*b*x + 2*a + 3*c
)^2 + 2*(2*(3*cos(-2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c) + 3*cos(2*a + c)
*cos(-2*a + 2*c) + cos(2*a + c))*cos(4*b*x + 2*a + 5*c) + 4*(3*cos(2*a + c
)*cos(-2*a + 2*c) + cos(2*a + c))*cos(2*b*x + 2*a + 3*c) + cos(2*a + c)^2
+ 3*(cos(2*a + c)^2 + sin(2*a + c)^2)*cos(-2*a + 2*c) + 2*(2*(3*cos(-2*a +
 2*c) + 1)*sin(2*b*x + 2*a + 3*c) + 3*cos(-2*a + 2*c)*sin(2*a + c) + sin(2
*a + c))*sin(4*b*x + 2*a + 5*c) + 4*(3*cos(-2*a + 2*c)*sin(2*a + c) + sin(
2*a + c))*sin(2*b*x + 2*a + 3*c) + sin(2*a + c)^2)*log((cos(b*x + 2*c)^2 +
 cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*
sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c)
+ sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - 2*(5*cos(3*b*x
 + 4*a + 2*c) - 2*cos(3*b*x + 2*a + 4*c) - 3*cos(3*b*x + 6*c) + 3*cos(b...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6958 vs. $2(84) = 168$.

Time = 0.45 (sec) , antiderivative size = 6958, normalized size of antiderivative = 79.07

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^3,x, algorithm="giac")
```

output
```
-((tan(1/2*a)^5*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^4 + tan(1/2*a)^4*ta
n(1/2*c)^5 - 4*tan(1/2*a)^5*tan(1/2*c)^3 + 13*tan(1/2*a)^4*tan(1/2*c)^4 -
4*tan(1/2*a)^3*tan(1/2*c)^5 + 4*tan(1/2*a)^5*tan(1/2*c)^2 - 16*tan(1/2*a)^
4*tan(1/2*c)^3 + 16*tan(1/2*a)^3*tan(1/2*c)^4 - 4*tan(1/2*a)^2*tan(1/2*c)^
5 + tan(1/2*a)^5*tan(1/2*c) - 16*tan(1/2*a)^4*tan(1/2*c)^2 + 40*tan(1/2*a)
^3*tan(1/2*c)^3 - 16*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)*tan(1/2*c)^5 -
 tan(1/2*a)^5 + 13*tan(1/2*a)^4*tan(1/2*c) - 40*tan(1/2*a)^3*tan(1/2*c)^2
+ 40*tan(1/2*a)^2*tan(1/2*c)^3 - 13*tan(1/2*a)*tan(1/2*c)^4 + tan(1/2*c)^5
 + tan(1/2*a)^4 - 16*tan(1/2*a)^3*tan(1/2*c) + 40*tan(1/2*a)^2*tan(1/2*c)^
2 - 16*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 + 4*tan(1/2*a)^3 - 16*tan(1/
2*a)^2*tan(1/2*c) + 16*tan(1/2*a)*tan(1/2*c)^2 - 4*tan(1/2*c)^3 - 4*tan(1/
2*a)^2 + 13*tan(1/2*a)*tan(1/2*c) - 4*tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*
c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x
+ 1/2*a)*tan(1/2*a) - tan(1/2*b*x + 1/2*a)*tan(1/2*c) + tan(1/2*a)*tan(1/2
*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan(1/2*c) + 1))/(tan(1/2*a)^5*t
an(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c)^5 + 2*ta
n(1/2*a)^5*tan(1/2*c)^3 + tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1
/2*c)^5 - 2*tan(1/2*a)^5*tan(1/2*c)^2 + 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*ta
n(1/2*a)^3*tan(1/2*c)^4 + 2*tan(1/2*a)^2*tan(1/2*c)^5 + tan(1/2*a)^5*tan(1
/2*c) + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 4*tan(1/2*a)^3*tan(1/2*c)^3 + 2*t...
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)^2/cos(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos^2(a+bx)\sec^3(c+bx)\,dx = \int \cos(bx+a)^2\sec(bx+c)^3\,dx$$

input
```
int(cos(b*x+a)^2*sec(b*x+c)^3,x)
```

output
```
int(cos(a + b*x)**2*sec(b*x + c)**3,x)
```

3.382 $\int \cos^2(a+bx)\sec^4(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx=-\frac{\sec^2(c+bx)\sin(2(a-c))}{2b}+\frac{\cos(2(a-c))\tan(c+bx)}{b}+\frac{\sin^2(a-c)\tan(c+bx)}{b}+\frac{\sin^2(a-c)\tan^3(c+bx)}{3b}$$

output
```
-1/2*sec(b*x+c)^2*sin(2*a-2*c)/b+cos(2*a-2*c)*tan(b*x+c)/b+sin(a-c)^2*tan(
b*x+c)/b+1/3*sin(a-c)^2*tan(b*x+c)^3/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx =\frac{\sec(c)\sec^3(c+bx)(3\sin(bx)-\sin(2a-4c-3bx)-3\sin(2a-2c-bx)-3\sin(2a+bx)+\sin(2a+3bx}{12b}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c + b*x]^4,x]
```

output
```
(Sec[c]*Sec[c + b*x]^3*(3*Sin[b*x] - Sin[2*a - 4*c - 3*b*x] - 3*Sin[2*a -
2*c - b*x] - 3*Sin[2*a + b*x] + Sin[2*a + 3*b*x] + Sin[2*c + 3*b*x]))/(12*
b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec^4(bx+c)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos^2(a+bx)\sec^4(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c + b*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 5.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{1}{3b(\sin(a)\cos(c)-\cos(a)\sin(c))(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))^3}$	56
parallelrisch	$\frac{3\sin(bx+c)+2\sin(3bx+2a+c)+\sin(3bx+3c)}{3b(\cos(3bx+3c)+3\cos(bx+c))}$	56
risch	$\frac{2i\left(3\,e^{2i(2bx+4a+c)}+3\,e^{2i(bx+4a)}+3\,e^{2i(bx+3a+c)}+e^{2i(4a-c)}+e^{6ia}+e^{2i(2a+c)}\right)}{3\left(e^{2i(bx+a+c)}+e^{2ia}\right)^3 b}$	93

input
```
int(cos(b*x+a)^2*sec(b*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(sin(a)*cos(c)-cos(a)*sin(c))/(tan(b*x+a)*sin(a)*cos(c)-tan(b*x+a)*
cos(a)*sin(c)+cos(a)*cos(c)+sin(a)*sin(c))^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx = \frac{3\cos(bx+c)\cos(-a+c)\sin(-a+c)+\left(\left(4\cos(-a+c)^2-1\right)\cos(bx+c)^2-\cos(-a+c)^2+1\right)\sin(}{3\,b\cos(bx+c)^3}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(3*cos(b*x + c)*cos(-a + c)*sin(-a + c) + ((4*cos(-a + c)^2 - 1)*cos(b
*x + c)^2 - cos(-a + c)^2 + 1)*sin(b*x + c))/(b*cos(b*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sec(b*x+c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(79) = 158$.

Time = 0.04 (sec) , antiderivative size = 886, normalized size of antiderivative = 10.67

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^4,x, algorithm="maxima")
```

output
```
-2/3*((3*sin(4*b*x + 4*a + 4*c) + 3*sin(2*b*x + 4*a + 2*c) + 3*sin(2*b*x +
 2*a + 4*c) + sin(4*a) + sin(2*a + 2*c) + sin(4*c))*cos(6*b*x + 2*a + 8*c)
 - 3*(3*sin(2*b*x + 2*a + 4*c) + sin(2*a + 2*c))*cos(4*b*x + 4*a + 4*c) +
3*(3*sin(4*b*x + 4*a + 4*c) + 3*sin(2*b*x + 4*a + 2*c) + 3*sin(2*b*x + 2*a
 + 4*c) + sin(4*a) + sin(2*a + 2*c) + sin(4*c))*cos(4*b*x + 2*a + 6*c) + 3
*(3*sin(2*b*x + 4*a + 2*c) + sin(4*a) + sin(4*c))*cos(2*b*x + 2*a + 4*c) +
 (sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (3*cos(4*b*x + 4*a + 4*c) + 3*cos(
2*b*x + 4*a + 2*c) + 3*cos(2*b*x + 2*a + 4*c) + cos(4*a) + cos(2*a + 2*c)
+ cos(4*c))*sin(6*b*x + 2*a + 8*c) + 3*(3*cos(2*b*x + 2*a + 4*c) + cos(2*a
 + 2*c))*sin(4*b*x + 4*a + 4*c) - 3*(3*cos(4*b*x + 4*a + 4*c) + 3*cos(2*b*
x + 4*a + 2*c) + 3*cos(2*b*x + 2*a + 4*c) + cos(4*a) + cos(2*a + 2*c) + co
s(4*c))*sin(4*b*x + 2*a + 6*c) + 3*cos(2*a + 2*c)*sin(2*b*x + 4*a + 2*c) -
 3*(3*cos(2*b*x + 4*a + 2*c) + cos(4*a) + cos(4*c))*sin(2*b*x + 2*a + 4*c)
 - (cos(4*a) + cos(4*c))*sin(2*a + 2*c) - 3*cos(2*b*x + 4*a + 2*c)*sin(2*a
 + 2*c))/(b*cos(6*b*x + 2*a + 8*c)^2 + 9*b*cos(4*b*x + 2*a + 6*c)^2 + 9*b*
cos(2*b*x + 2*a + 4*c)^2 + 6*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c) + b*c
os(2*a + 2*c)^2 + b*sin(6*b*x + 2*a + 8*c)^2 + 9*b*sin(4*b*x + 2*a + 6*c)^
2 + 9*b*sin(2*b*x + 2*a + 4*c)^2 + 6*b*sin(2*b*x + 2*a + 4*c)*sin(2*a + 2*
c) + b*sin(2*a + 2*c)^2 + 2*(3*b*cos(4*b*x + 2*a + 6*c) + 3*b*cos(2*b*x +
2*a + 4*c) + b*cos(2*a + 2*c))*cos(6*b*x + 2*a + 8*c) + 6*(3*b*cos(2*b*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(79) = 158$.

Time = 0.19 (sec) , antiderivative size = 429, normalized size of antiderivative = 5.17

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x+c)^4,x, algorithm="giac")
```

output
```
-1/6*(tan(1/2*a)^8*tan(1/2*c)^8 + 4*tan(1/2*a)^8*tan(1/2*c)^6 + 4*tan(1/2*
a)^6*tan(1/2*c)^8 + 6*tan(1/2*a)^8*tan(1/2*c)^4 + 16*tan(1/2*a)^6*tan(1/2*
c)^6 + 6*tan(1/2*a)^4*tan(1/2*c)^8 + 4*tan(1/2*a)^8*tan(1/2*c)^2 + 24*tan(
1/2*a)^6*tan(1/2*c)^4 + 24*tan(1/2*a)^4*tan(1/2*c)^6 + 4*tan(1/2*a)^2*tan(
1/2*c)^8 + tan(1/2*a)^8 + 16*tan(1/2*a)^6*tan(1/2*c)^2 + 36*tan(1/2*a)^4*t
an(1/2*c)^4 + 16*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*c)^8 + 4*tan(1/2*a)^6
 + 24*tan(1/2*a)^4*tan(1/2*c)^2 + 24*tan(1/2*a)^2*tan(1/2*c)^4 + 4*tan(1/2
*c)^6 + 6*tan(1/2*a)^4 + 16*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(1/2*c)^4 + 4
*tan(1/2*a)^2 + 4*tan(1/2*c)^2 + 1)/((2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*
c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 +
2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*t
an(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)^3*(tan(1/2*a)^2*tan(1/2*c) - tan(
1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)^2/cos(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\sec^4(c+bx)\,dx$$
$$=\frac{-\cos(bx+c)\cos(bx+a)\sin(bx+a)+\sin(bx+c)^3+\sin(bx+c)\sin(bx+a)^2-2\sin(bx+c)}{3\cos(bx+c)\,b\left(\sin(bx+c)^2-1\right)}$$

input
```
int(cos(b*x+a)^2*sec(b*x+c)^4,x)
```

output
```
( - cos(b*x + c)*cos(a + b*x)*sin(a + b*x) + sin(b*x + c)**3 + sin(b*x + c
)*sin(a + b*x)**2 - 2*sin(b*x + c))/(3*cos(b*x + c)*b*(sin(b*x + c)**2 - 1
))
```

3.383 $\int \cos^2(a+bx)\sec(c-bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \cos^2(a+bx)\sec(c-bx)\,dx=-\frac{\operatorname{arctanh}(\sin(c-bx))\sin^2(a+c)}{b}+\frac{\sin(2a+c+bx)}{b}$$

output
```
arctanh(sin(b*x-c))*sin(a+c)^2/b+sin(b*x+2*a+c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 74 vs. $2(33)=66$.

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int \cos^2(a+bx)\sec(c-bx)\,dx$$
$$=\frac{\left(\log\left(\cos\left(\frac{1}{2}(c-bx)\right)-\sin\left(\frac{1}{2}(c-bx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c-bx)\right)+\sin\left(\frac{1}{2}(c-bx)\right)\right)\right)\sin^2(a+c)+\sin(2a}{b}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c - b*x],x]
```

output
```
((Log[Cos[(c - b*x)/2] - Sin[(c - b*x)/2]] - Log[Cos[(c - b*x)/2] + Sin[(c
 - b*x)/2]])*Sin[a + c]^2 + Sin[2*a + c + b*x])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec(c-bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \cos^2(a+bx)\sec(c-bx)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c - b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 4.09

method	result
risch	$-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a+c)}\right)}{2b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a+c)}\right)\cos(2a+2c)}{2b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a+c)}\right)}{2b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a+c)}\right)\cos(2a+2c)}{2b}$
default	$\frac{\frac{2\left(-\sin(c)^2\cos(a)^2-2\cos(a)\cos(c)\sin(a)\sin(c)-\cos(c)^2\sin(a)^2\right)\arctan\left(\frac{2(\cos(a)\cos(c)-\sin(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-2\sin(a)\cos(c)-2\cos(a)\sin(}{2\sqrt{-\cos(c)^2\sin(a)^2-\cos(a)^2\cos(c)^2-\sin(a)^2\sin(c)^2-\sin(c)^2\cos(a)^2}}\right.}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)\sqrt{-\cos(c)^2\sin(a)^2-\cos(a)^2\cos(c)^2-\sin(a)^2\sin(c)^2-\sin(c)^2\cos(a)^2}}}{b}$

input
```
int(cos(b*x+a)^2*sec(b*x-c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*ln(exp(I*(b*x+a))-I*exp(I*(a+c)))+1/2/b*ln(exp(I*(b*x+a))-I*exp(I*(
a+c)))*cos(2*a+2*c)+1/2/b*ln(exp(I*(b*x+a))+I*exp(I*(a+c)))-1/2/b*ln(exp(I
*(b*x+a))+I*exp(I*(a+c)))*cos(2*a+2*c)+sin(b*x+2*a+c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(33) = 66$.

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.67

$$\int \cos^2(a+bx)\sec(c-bx)\,dx =$$
$$-\frac{\left(\cos\left(a+c\right)^2-1\right)\log\left(\frac{2\left(\cos(a+c)\sin(bx+a)-\cos(bx+a)\sin(a+c)+1\right)}{\cos(a+c)+1}\right)-\left(\cos\left(a+c\right)^2-1\right)\log\left(-\frac{2\left(\cos(a+c)\sin(bx\cdots\right.}{c\cdots}\cdots\right.}{2b}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x-c),x, algorithm="fricas")
```

output
```
-1/2*((cos(a + c)^2 - 1)*log(2*(cos(a + c)*sin(b*x + a) - cos(b*x + a)*sin
(a + c) + 1)/(cos(a + c) + 1)) - (cos(a + c)^2 - 1)*log(-2*(cos(a + c)*sin
(b*x + a) - cos(b*x + a)*sin(a + c) - 1)/(cos(a + c) + 1)) - 2*cos(a + c)*
sin(b*x + a) - 2*cos(b*x + a)*sin(a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(27) = 54$.

Time = 19.43 (sec) , antiderivative size = 3645, normalized size of antiderivative = 110.45

$$\int \cos^2(a+bx)\sec(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**2*sec(b*x-c),x)
```

output
```
-2*Piecewise((sin(b*x)/b, Eq(c, pi/2)), (-sin(b*x)/b, Eq(c, -pi/2)), (0, E
q(b, 0)), (-2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))
*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 +
2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2
*log(tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)**3/
(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)*
*2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) + tan(c/2)/
(tan(c/2) - 1) + 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*t
an(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2
)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) - 1) +
 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 +
 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) +
2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) + 1) + 1/(tan(c/2) + 1))*tan(c/2)**3
*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)
**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x
/2) - tan(c/2)/(tan(c/2) + 1) + 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(c/2)*
*4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan
(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) +
1) + 1/(tan(c/2) + 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2
 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(33) = 66$.

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.21

$$\int \cos^2(a+bx)\sec(c-bx)\,dx = \frac{(\cos(2a+2c)-1)\log\left(\frac{\cos(bx)^2+\cos(c)^2-2\cos(c)\sin(bx)+\sin(bx)^2+2\cos(bx)\sin(c)+\sin(c)^2}{\cos(bx)^2+\cos(c)^2+2\cos(c)\sin(bx)+\sin(bx)^2-2\cos(bx)\sin(c)+\sin(c)^2}\right)+4\sin(bx+2a+c)}{4b}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x-c),x, algorithm="maxima")
```

output
```
1/4*((cos(2*a + 2*c) - 1)*log((cos(b*x)^2 + cos(c)^2 - 2*cos(c)*sin(b*x) +
 sin(b*x)^2 + 2*cos(b*x)*sin(c) + sin(c)^2)/(cos(b*x)^2 + cos(c)^2 + 2*cos
(c)*sin(b*x) + sin(b*x)^2 - 2*cos(b*x)*sin(c) + sin(c)^2)) + 4*sin(b*x + 2
*a + c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1725 vs. $2(33) = 66$.

Time = 0.24 (sec) , antiderivative size = 1725, normalized size of antiderivative = 52.27

$$\int \cos^2(a+bx)\sec(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x-c),x, algorithm="giac")
```

output
```
2*(2*(tan(1/2*a)^5*tan(1/2*c)^3 + 2*tan(1/2*a)^4*tan(1/2*c)^4 + tan(1/2*a)
^3*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^2 - 3*tan(1/2*a)^4*tan(1/2*c)^3
- 3*tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^2*tan(1/2*c)^5 - 3*tan(1/2*a)^4
*tan(1/2*c)^2 - 6*tan(1/2*a)^3*tan(1/2*c)^3 - 3*tan(1/2*a)^2*tan(1/2*c)^4
+ 2*tan(1/2*a)^4*tan(1/2*c) + 6*tan(1/2*a)^3*tan(1/2*c)^2 + 6*tan(1/2*a)^2
*tan(1/2*c)^3 + 2*tan(1/2*a)*tan(1/2*c)^4 + 3*tan(1/2*a)^3*tan(1/2*c) + 6*
tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 - 3*t
an(1/2*a)^2*tan(1/2*c) - 3*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/
2*a)^2 - 2*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2)*log(abs(-tan(1/2*b*x + 1/
2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*b*x
 + 1/2*a)*tan(1/2*c) - tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a) - tan(
1/2*a) - tan(1/2*c) + 1))/(tan(1/2*a)^5*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/
2*c)^4 - tan(1/2*a)^4*tan(1/2*c)^5 + 2*tan(1/2*a)^5*tan(1/2*c)^3 - tan(1/2
*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)^5 - 2*tan(1/2*a)^5*tan(1/2*
c)^2 - 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(1/2*a)^3*tan(1/2*c)^4 - 2*tan(1
/2*a)^2*tan(1/2*c)^5 + tan(1/2*a)^5*tan(1/2*c) - 2*tan(1/2*a)^4*tan(1/2*c)
^2 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a
)*tan(1/2*c)^5 - tan(1/2*a)^5 - tan(1/2*a)^4*tan(1/2*c) - 4*tan(1/2*a)^3*t
an(1/2*c)^2 - 4*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 - tan(
1/2*c)^5 - tan(1/2*a)^4 + 2*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2*a)^2*ta...
```

Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 217, normalized size of antiderivative = 6.58

$$\int \cos^2(a+bx)\sec(c-bx)\,dx = \frac{e^{-a\,2i-c\,1i-b\,x\,1i}\,1i}{2\,b} - \frac{e^{a\,2i+c\,1i+b\,x\,1i}\,1i}{2\,b} + \frac{e^{-a\,2i-c\,2i}\,\ln\left(-\frac{\left(e^{a\,2i}\,e^{c\,2i}-1\right)^2\,1i}{2}+\frac{e^{-c\,1i}\,e^{b\,x\,1i}\,\left(1+e^{a\,4i}\,e^{c\,4i}-2\,e^{a\,2i}\,e^{c\,2i}\right)}{2}\right)\,\left(e^{a\,2i+c\,2i}-1\right)^2}{4\,b} - \frac{e^{-a\,2i-c\,2i}\,\ln\left(\frac{\left(e^{a\,2i}\,e^{c\,2i}-1\right)^2\,1i}{2}+\frac{e^{-c\,1i}\,e^{b\,x\,1i}\,\left(1+e^{a\,4i}\,e^{c\,4i}-2\,e^{a\,2i}\,e^{c\,2i}\right)}{2}\right)\,\left(e^{a\,2i+c\,2i}-1\right)^2}{4\,b}$$

input
```
int(cos(a + b*x)^2/cos(c - b*x),x)
```

output
```
(exp(- a*2i - c*1i - b*x*1i)*1i)/(2*b) - (exp(a*2i + c*1i + b*x*1i)*1i)/(2
*b) + (exp(- a*2i - c*2i)*log((exp(-c*1i)*exp(b*x*1i)*(exp(a*4i)*exp(c*4i)
 - 2*exp(a*2i)*exp(c*2i) + 1))/2 - ((exp(a*2i)*exp(c*2i) - 1)^2*1i)/2)*(ex
p(a*2i + c*2i) - 1)^2)/(4*b) - (exp(- a*2i - c*2i)*log(((exp(a*2i)*exp(c*2
i) - 1)^2*1i)/2 + (exp(-c*1i)*exp(b*x*1i)*(exp(a*4i)*exp(c*4i) - 2*exp(a*2
i)*exp(c*2i) + 1))/2)*(exp(a*2i + c*2i) - 1)^2)/(4*b)
```

Reduce [F]

$$\int \cos^2(a+bx)\sec(c-bx)\,dx = \int \cos(bx+a)^2\sec(bx-c)\,dx$$

input
```
int(cos(b*x+a)^2*sec(b*x-c),x)
```

output
```
int(cos(a + b*x)**2*sec(b*x - c),x)
```

3.384 $\int \cos^2(a+bx)\sec^2(c-bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 45

$$\int \cos^2(a+bx)\sec^2(c-bx)\,dx = x\cos(2(a+c)) + \frac{\log(\cos(c-bx))\sin(2(a+c))}{b} - \frac{\sin^2(a+c)\tan(c-bx)}{b}$$

output
```
x*cos(2*a+2*c)+ln(cos(b*x-c))*sin(2*a+2*c)/b+sin(a+c)^2*tan(b*x-c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(45) = 90.

Time = 0.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.04

$$\int \cos^2(a+bx)\sec^2(c-bx)\,dx = \frac{\sec(c)\sec(c-bx)(bx\cos(2a+2c-bx)+bx\cos(2a+4c-bx)+bx\cos(2a+bx)+bx\cos(2a+2c+bx)}{}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c - b*x]^2,x]
```

output
```
(Sec[c]*Sec[c - b*x]*(b*x*Cos[2*a + 2*c - b*x] + b*x*Cos[2*a + 4*c - b*x]
+ b*x*Cos[2*a + b*x] + b*x*Cos[2*a + 2*c + b*x] + 2*Sin[b*x] + Sin[2*a + 2
*c - b*x] + Log[Cos[c - b*x]]*Sin[2*a + 2*c - b*x] + Log[Cos[c - b*x]]*Sin
[2*a + 4*c - b*x] + Log[Cos[c - b*x]]*Sin[2*a + b*x] - Sin[2*a + 2*c + b*x
] + Log[Cos[c - b*x]]*Sin[2*a + 2*c + b*x]))/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec^2(c-bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \cos^2(a+bx)\sec^2(c-bx)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c - b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.49

method	result
risch	$x\,\mathrm{e}^{2i(a+c)}-2i\sin(2a+2c)\,x-\frac{2i\sin(2a+2c)a}{b}-\frac{i\mathrm{e}^{4i(a+c)}}{2b(\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)})}+\frac{i\mathrm{e}^{2i(a+c)}}{b(\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)})}-\frac{}{2b(\mathrm{e}^{2i(a+c)}}$
default	$\frac{\frac{\left(-2\cos(c)^2\cos(a)\sin(a)-2\cos(a)^2\cos(c)\sin(c)+2\sin(a)^2\cos(c)\sin(c)+2\cos(a)\sin(a)\sin(c)^2\right)\ln\left(\tan(bx+a)^2+1\right)}{2}+\left(-\sin(c)^2\cos(a)^2+\cos(a)^2\right.}{\left(\cos(c)^2+\sin(c)^2\right)^2\left(\cos(a)^2+\sin(a)^2\right)^2}$

input
```
int(cos(b*x+a)^2*sec(b*x-c)^2,x,method=_RETURNVERBOSE)
```

output
```
x*exp(2*I*(a+c))-2*I*sin(2*a+2*c)*x-2*I/b*sin(2*a+2*c)*a-1/2*I/b/(exp(2*I*
(a+c))+exp(2*I*(b*x+a)))*exp(4*I*(a+c))+I/b/(exp(2*I*(a+c))+exp(2*I*(b*x+a
)))*exp(2*I*(a+c))-1/2*I/b/(exp(2*I*(a+c))+exp(2*I*(b*x+a)))+ln(exp(2*I*(a
+c))+exp(2*I*(b*x+a)))/b*sin(2*a+2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(50) = 100$.

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 4.24

$$\int \cos^2(a+bx)\sec^2(c-bx)\,dx$$
$$= \frac{\left(\cos\left(a+c\right)^2-1\right)\cos\left(bx+a\right)\sin\left(a+c\right)+\left(2\,bx\cos\left(a+c\right)^3-bx\cos\left(a+c\right)\right)\cos\left(bx+a\right)+2\left(\cos\left(bx\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x-c)^2,x, algorithm="fricas")
```

output
```
((cos(a + c)^2 - 1)*cos(b*x + a)*sin(a + c) + (2*b*x*cos(a + c)^3 - b*x*co
s(a + c))*cos(b*x + a) + 2*(cos(b*x + a)*cos(a + c)^2*sin(a + c) - (cos(a
+ c)^3 - cos(a + c))*sin(b*x + a))*log(2*(cos(b*x + a)*cos(a + c) + sin(b*
x + a)*sin(a + c))/(cos(a + c) + 1)) - (cos(a + c)^3 - (2*b*x*cos(a + c)^2
 - b*x)*sin(a + c) - cos(a + c))*sin(b*x + a))/(b*cos(b*x + a)*cos(a + c)
+ b*sin(b*x + a)*sin(a + c))
```

Sympy [F(-2)]

Exception generated.

$$\int \cos^2(a+bx)\sec^2(c-bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(cos(b*x+a)**2*sec(b*x-c)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(50) = 100$.

Time = 0.06 (sec) , antiderivative size = 542, normalized size of antiderivative = 12.04

$$\int \cos^2(a+bx)\sec^2(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x-c)^2,x, algorithm="maxima")
```

output
```
1/2*(2*b*x*cos(2*b*x)*cos(2*a + 4*c) + 2*b*x*sin(2*b*x)*sin(2*a + 4*c) + 2
*(b*cos(2*a + 4*c)*cos(2*c) + b*sin(2*a + 4*c)*sin(2*c))*x + (2*b*x*cos(2*
b*x) + 2*b*x*cos(2*c) + sin(4*a + 6*c) - 2*sin(2*a + 4*c) + sin(2*c))*cos(
2*b*x + 2*a + 2*c) + (cos(2*b*x + 2*a + 2*c)^2*sin(2*a + 2*c) + 2*cos(2*b*
x + 2*a + 2*c)*cos(2*a + 4*c)*sin(2*a + 2*c) + cos(2*a + 4*c)^2*sin(2*a +
2*c) + sin(2*b*x + 2*a + 2*c)^2*sin(2*a + 2*c) + 2*sin(2*b*x + 2*a + 2*c)*
sin(2*a + 4*c)*sin(2*a + 2*c) + sin(2*a + 4*c)^2*sin(2*a + 2*c))*log(cos(2
*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 + 2*sin(2*b*x)
*sin(2*c) + sin(2*c)^2) + (2*b*x*sin(2*b*x) + 2*b*x*sin(2*c) - cos(4*a + 6
*c) + 2*cos(2*a + 4*c) - cos(2*c))*sin(2*b*x + 2*a + 2*c) + cos(2*a + 4*c)
*sin(4*a + 6*c) - cos(4*a + 6*c)*sin(2*a + 4*c) - cos(2*c)*sin(2*a + 4*c)
+ cos(2*a + 4*c)*sin(2*c))/(b*cos(2*b*x + 2*a + 2*c)^2 + 2*b*cos(2*b*x + 2
*a + 2*c)*cos(2*a + 4*c) + b*cos(2*a + 4*c)^2 + b*sin(2*b*x + 2*a + 2*c)^2
 + 2*b*sin(2*b*x + 2*a + 2*c)*sin(2*a + 4*c) + b*sin(2*a + 4*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1950 vs. $2(50) = 100$.

Time = 0.19 (sec) , antiderivative size = 1950, normalized size of antiderivative = 43.33

$$\int \cos^2(a+bx)\sec^2(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x-c)^2,x, algorithm="giac")
```

output
```
((tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*a)^4*tan(1/2*c)^2 - 16*tan(1/2*a)^
3*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 16*tan(1/2*a
)^3*tan(1/2*c) + 36*tan(1/2*a)^2*tan(1/2*c)^2 + 16*tan(1/2*a)*tan(1/2*c)^3
 + tan(1/2*c)^4 - 6*tan(1/2*a)^2 - 16*tan(1/2*a)*tan(1/2*c) - 6*tan(1/2*c)
^2 + 1)*(b*x + a)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2
 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^
2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^4*
tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) - 6*tan
(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*
c)^4 + tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*a)*tan(1/2*c)^
2 + tan(1/2*c)^3 - tan(1/2*a) - tan(1/2*c))*log(tan(b*x + a)^2 + 1)/(tan(1
/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/
2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*tan
(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 4*(tan(1/2*a)^6*tan(1/2*c)^4 + 2*tan(1/2
*a)^5*tan(1/2*c)^5 + tan(1/2*a)^4*tan(1/2*c)^6 - tan(1/2*a)^6*tan(1/2*c)^2
 - 8*tan(1/2*a)^5*tan(1/2*c)^3 - 14*tan(1/2*a)^4*tan(1/2*c)^4 - 8*tan(1/2*
a)^3*tan(1/2*c)^5 - tan(1/2*a)^2*tan(1/2*c)^6 + 2*tan(1/2*a)^5*tan(1/2*c)
+ 14*tan(1/2*a)^4*tan(1/2*c)^2 + 24*tan(1/2*a)^3*tan(1/2*c)^3 + 14*tan(1/2
*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)*tan(1/2*c)^5 - tan(1/2*a)^4 - 8*tan(1/2*
a)^3*tan(1/2*c) - 14*tan(1/2*a)^2*tan(1/2*c)^2 - 8*tan(1/2*a)*tan(1/2*c...
```

Mupad [B] (verification not implemented)

Time = 22.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.22

$$\int \cos^2(a+bx)\sec^2(c-bx)\,dx$$
$$= x\left(\cos\left(2a+2c\right)-\sin\left(2a+2c\right)\,1\mathrm{i}\right)-\frac{\left(1+\mathrm{e}^{a\,4\mathrm{i}+c\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}\right)\,1\mathrm{i}}{2\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}$$
$$+\frac{\mathrm{e}^{-a\,4\mathrm{i}-c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}\right)\,\left(2\,b\,\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}-2\,b\,\mathrm{e}^{a\,6\mathrm{i}+c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$

input
```
int(cos(a + b*x)^2/cos(c - b*x)^2,x)
```

output
```
x*(cos(2*a + 2*c) - sin(2*a + 2*c)*1i) - ((exp(a*4i + c*4i) - 2*exp(a*2i +
 c*2i) + 1)*1i)/(2*b*(exp(a*2i + c*2i) + exp(a*2i + b*x*2i))) + (exp(- a*4
i - c*4i)*log(exp(a*2i)*exp(b*x*2i) + exp(a*2i)*exp(c*2i))*(2*b*exp(a*2i +
 c*2i) - 2*b*exp(a*6i + c*6i))*1i)/(4*b^2)
```

Reduce [F]

$$\int \cos^2(a+bx)\sec^2(c-bx)\,dx = \text{Too large to display}$$

input
```
int(cos(b*x+a)^2*sec(b*x-c)^2,x)
```

output
```
( - 7*cos(b*x - c)*cos(a + b*x)*sin(a + b*x) - 96*cos(b*x - c)*int(tan((b*
x - c)/2)**2/(tan((b*x - c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x - c)/2)
**4*tan((a + b*x)/2)**2 + tan((b*x - c)/2)**4 - 2*tan((b*x - c)/2)**2*tan(
(a + b*x)/2)**4 - 4*tan((b*x - c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x -
 c)/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b - 96*cos
(b*x - c)*int(tan((a + b*x)/2)**2/(tan((b*x - c)/2)**4*tan((a + b*x)/2)**4
 + 2*tan((b*x - c)/2)**4*tan((a + b*x)/2)**2 + tan((b*x - c)/2)**4 - 2*tan
((b*x - c)/2)**2*tan((a + b*x)/2)**4 - 4*tan((b*x - c)/2)**2*tan((a + b*x)
/2)**2 - 2*tan((b*x - c)/2)**2 + tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)*
*2 + 1),x)*b + 128*cos(b*x - c)*int((tan((b*x - c)/2)*tan((a + b*x)/2))/(t
an((b*x - c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x - c)/2)**4*tan((a + b*
x)/2)**2 + tan((b*x - c)/2)**4 - 2*tan((b*x - c)/2)**2*tan((a + b*x)/2)**4
 - 4*tan((b*x - c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x - c)/2)**2 + tan
((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b + 32*cos(b*x - c)*int(1
/(tan((b*x - c)/2)**4*tan((a + b*x)/2)**4 + 2*tan((b*x - c)/2)**4*tan((a +
 b*x)/2)**2 + tan((b*x - c)/2)**4 - 2*tan((b*x - c)/2)**2*tan((a + b*x)/2)
**4 - 4*tan((b*x - c)/2)**2*tan((a + b*x)/2)**2 - 2*tan((b*x - c)/2)**2 +
tan((a + b*x)/2)**4 + 2*tan((a + b*x)/2)**2 + 1),x)*b - 16*cos(b*x - c)*si
n(a + b*x) - 9*cos(b*x - c)*a - 9*cos(b*x - c)*b*x + 8*cos(a + b*x)*sin(b*
x - c) - 8*cos(a + b*x)*sin(a + b*x) - 4*sin(b*x - c)*sin(a + b*x)**2 +...
```

3.385 $\int \cos^2(a+bx)\sec^3(c-bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \cos^2(a+bx)\sec^3(c-bx)\,dx = -\frac{\operatorname{arctanh}(\sin(c-bx))\cos(2(a+c))}{b} - \frac{\operatorname{arctanh}(\sin(c-bx))\sin^2(a+c)}{2b} - \frac{\sec(c-bx)\sin(2(a+c))}{b} - \frac{\sec(c-bx)\sin^2(a+c)\tan(c-bx)}{2b}$$

output
```
arctanh(sin(b*x-c))*cos(2*a+2*c)/b+1/2*arctanh(sin(b*x-c))*sin(a+c)^2/b-se
c(b*x-c)*sin(2*a+2*c)/b+1/2*sec(b*x-c)*sin(a+c)^2*tan(b*x-c)/b
```

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.95

$$\int \cos^2(a+bx)\sec^3(c-bx)\,dx = \frac{\log\left(\cos\left(\frac{1}{2}(c-bx)\right)-\sin\left(\frac{1}{2}(c-bx)\right)\right)+3\cos(2(a+c))\left(\log\left(\cos\left(\frac{1}{2}(c-bx)\right)-\sin\left(\frac{1}{2}(c-bx)\right)\right)-\log\right.}{}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c - b*x]^3,x]
```

output
```
(Log[Cos[(c - b*x)/2] - Sin[(c - b*x)/2]] + 3*Cos[2*(a + c)]*(Log[Cos[(c -
 b*x)/2] - Sin[(c - b*x)/2]] - Log[Cos[(c - b*x)/2] + Sin[(c - b*x)/2]]) -
 Log[Cos[(c - b*x)/2] + Sin[(c - b*x)/2]] + 4*Sec[c]*Sin[2*(a + c)] - 4*Se
c[c - b*x]*Sin[2*(a + c)] - 2*Sec[c - b*x]*Sin[a + c]^2*Tan[c - b*x])/(4*b
)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec^3(c-bx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \cos^2(a+bx)\sec^3(c-bx)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c - b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.95 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.79

method	result
risch	$\frac{i\left(3\,\mathrm{e}^{i(bx+6a+5c)}+5\,\mathrm{e}^{3i(bx+2a+c)}+2\,\mathrm{e}^{i(bx+4a+3c)}-2\,\mathrm{e}^{i(3bx+4a+c)}-5\,\mathrm{e}^{i(bx+2a+c)}-3\,\mathrm{e}^{i(3bx+2a-c)}\right)}{4\left(\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)^{2}b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a+c)}\right)}{4b}-3$
default	Expression too large to display

input
```
int(cos(b*x+a)^2*sec(b*x-c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/(exp(2*I*(a+c))+exp(2*I*(b*x+a)))^2/b*(3*exp(I*(b*x+6*a+5*c))+5*exp(
3*I*(b*x+2*a+c))+2*exp(I*(b*x+4*a+3*c))-2*exp(I*(3*b*x+4*a+c))-5*exp(I*(b*
x+2*a+c))-3*exp(I*(3*b*x+2*a-c)))-1/4/b*ln(exp(I*(b*x+a))-I*exp(I*(a+c)))-
3/4/b*ln(exp(I*(b*x+a))-I*exp(I*(a+c)))*cos(2*a+2*c)+1/4/b*ln(exp(I*(b*x+a
))+I*exp(I*(a+c)))+3/4/b*ln(exp(I*(b*x+a))+I*exp(I*(a+c)))*cos(2*a+2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(90) = 180.

Time = 0.09 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.92

$$\int \cos^2(a+bx)\sec^3(c-bx)\,dx =$$

$$-\frac{2\left(3\cos\left(a+c\right)^{2}+1\right)\cos\left(bx+a\right)\sin\left(a+c\right)+\left(3\cos\left(a+c\right)^{4}-2\left(3\cos\left(a+c\right)^{3}-\cos\left(a+c\right)\right)\cos\left(b\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x-c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*(3*cos(a + c)^2 + 1)*cos(b*x + a)*sin(a + c) + (3*cos(a + c)^4 - 2
*(3*cos(a + c)^3 - cos(a + c))*cos(b*x + a)*sin(b*x + a)*sin(a + c) - (6*c
os(a + c)^4 - 5*cos(a + c)^2 + 1)*cos(b*x + a)^2 - 4*cos(a + c)^2 + 1)*log
(2*(cos(a + c)*sin(b*x + a) - cos(b*x + a)*sin(a + c) + 1)/(cos(a + c) + 1
)) - (3*cos(a + c)^4 - 2*(3*cos(a + c)^3 - cos(a + c))*cos(b*x + a)*sin(b*
x + a)*sin(a + c) - (6*cos(a + c)^4 - 5*cos(a + c)^2 + 1)*cos(b*x + a)^2 -
 4*cos(a + c)^2 + 1)*log(-2*(cos(a + c)*sin(b*x + a) - cos(b*x + a)*sin(a
+ c) - 1)/(cos(a + c) + 1)) - 6*(cos(a + c)^3 - cos(a + c))*sin(b*x + a))/
(2*b*cos(b*x + a)*cos(a + c)*sin(b*x + a)*sin(a + c) + (2*b*cos(a + c)^2 -
 b)*cos(b*x + a)^2 - b*cos(a + c)^2 + b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^3(c-bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sec(b*x-c)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1217 vs. $2(90) = 180$.

Time = 0.21 (sec) , antiderivative size = 1217, normalized size of antiderivative = 14.15

$$\int \cos^2(a+bx)\sec^3(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x-c)^3,x, algorithm="maxima")
```

output
```
1/8*(2*(3*sin(3*b*x) - 5*sin(3*b*x + 4*a + 4*c) + 2*sin(3*b*x + 2*a + 2*c)
 - 3*sin(b*x + 4*a + 6*c) - 2*sin(b*x + 2*a + 4*c) + 5*sin(b*x + 2*c))*cos
(4*b*x + 2*a + c) + 10*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + 5*c))*cos(3*b
*x + 4*a + 4*c) - 4*(2*sin(2*b*x + 2*a + 3*c) + sin(2*a + 5*c))*cos(3*b*x
+ 2*a + 2*c) + 4*(3*sin(3*b*x) - 3*sin(b*x + 4*a + 6*c) - 2*sin(b*x + 2*a
+ 4*c) + 5*sin(b*x + 2*c))*cos(2*b*x + 2*a + 3*c) - ((3*cos(2*a + 2*c) + 1
)*cos(4*b*x + 2*a + c)^2 + 4*(3*cos(2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c)
^2 + 4*(3*cos(2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c)*cos(2*a + 5*c) + (3*c
os(2*a + 2*c) + 1)*cos(2*a + 5*c)^2 + (3*cos(2*a + 2*c) + 1)*sin(4*b*x + 2
*a + c)^2 + 4*(3*cos(2*a + 2*c) + 1)*sin(2*b*x + 2*a + 3*c)^2 + 4*(3*cos(2
*a + 2*c) + 1)*sin(2*b*x + 2*a + 3*c)*sin(2*a + 5*c) + (3*cos(2*a + 2*c) +
 1)*sin(2*a + 5*c)^2 + 2*(2*(3*cos(2*a + 2*c) + 1)*cos(2*b*x + 2*a + 3*c)
+ (3*cos(2*a + 2*c) + 1)*cos(2*a + 5*c))*cos(4*b*x + 2*a + c) + 2*(2*(3*co
s(2*a + 2*c) + 1)*sin(2*b*x + 2*a + 3*c) + (3*cos(2*a + 2*c) + 1)*sin(2*a
+ 5*c))*sin(4*b*x + 2*a + c))*log((cos(b*x)^2 + cos(c)^2 - 2*cos(c)*sin(b*
x) + sin(b*x)^2 + 2*cos(b*x)*sin(c) + sin(c)^2)/(cos(b*x)^2 + cos(c)^2 + 2
*cos(c)*sin(b*x) + sin(b*x)^2 - 2*cos(b*x)*sin(c) + sin(c)^2)) + 6*cos(2*a
 + 5*c)*sin(3*b*x) - 2*(3*cos(3*b*x) - 5*cos(3*b*x + 4*a + 4*c) + 2*cos(3*
b*x + 2*a + 2*c) - 3*cos(b*x + 4*a + 6*c) - 2*cos(b*x + 2*a + 4*c) + 5*cos
(b*x + 2*c))*sin(4*b*x + 2*a + c) - 10*(2*cos(2*b*x + 2*a + 3*c) + cos(...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6975 vs. $2(90) = 180$.

Time = 0.44 (sec) , antiderivative size = 6975, normalized size of antiderivative = 81.10

$$\int \cos^2(a+bx)\sec^3(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x-c)^3,x, algorithm="giac")
```

output
```
((tan(1/2*a)^5*tan(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^4 - tan(1/2*a)^4*tan
(1/2*c)^5 - 4*tan(1/2*a)^5*tan(1/2*c)^3 - 13*tan(1/2*a)^4*tan(1/2*c)^4 - 4
*tan(1/2*a)^3*tan(1/2*c)^5 + 4*tan(1/2*a)^5*tan(1/2*c)^2 + 16*tan(1/2*a)^4
*tan(1/2*c)^3 + 16*tan(1/2*a)^3*tan(1/2*c)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^5
 + tan(1/2*a)^5*tan(1/2*c) + 16*tan(1/2*a)^4*tan(1/2*c)^2 + 40*tan(1/2*a)^
3*tan(1/2*c)^3 + 16*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)*tan(1/2*c)^5 -
tan(1/2*a)^5 - 13*tan(1/2*a)^4*tan(1/2*c) - 40*tan(1/2*a)^3*tan(1/2*c)^2 -
 40*tan(1/2*a)^2*tan(1/2*c)^3 - 13*tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*c)^5
- tan(1/2*a)^4 - 16*tan(1/2*a)^3*tan(1/2*c) - 40*tan(1/2*a)^2*tan(1/2*c)^2
 - 16*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + 4*tan(1/2*a)^3 + 16*tan(1/2
*a)^2*tan(1/2*c) + 16*tan(1/2*a)*tan(1/2*c)^2 + 4*tan(1/2*c)^3 + 4*tan(1/2
*a)^2 + 13*tan(1/2*a)*tan(1/2*c) + 4*tan(1/2*c)^2 - tan(1/2*a) - tan(1/2*c
) - 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x +
 1/2*a)*tan(1/2*a) + tan(1/2*b*x + 1/2*a)*tan(1/2*c) - tan(1/2*a)*tan(1/2*
c) + tan(1/2*b*x + 1/2*a) - tan(1/2*a) - tan(1/2*c) + 1))/(tan(1/2*a)^5*ta
n(1/2*c)^5 - tan(1/2*a)^5*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c)^5 + 2*tan
(1/2*a)^5*tan(1/2*c)^3 - tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/
2*c)^5 - 2*tan(1/2*a)^5*tan(1/2*c)^2 - 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan
(1/2*a)^3*tan(1/2*c)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^5 + tan(1/2*a)^5*tan(1/
2*c) - 2*tan(1/2*a)^4*tan(1/2*c)^2 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - 2*ta...
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^3(c-bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)^2/cos(c - b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos^2(a+bx)\sec^3(c-bx)\,dx = \int \cos{(bx+a)}^2 \sec{(bx-c)}^3\,dx$$

input
```
int(cos(b*x+a)^2*sec(b*x-c)^3,x)
```

output
```
int(cos(a + b*x)**2*sec(b*x - c)**3,x)
```

3.386 $\int \cos^2(a+bx)\sec^4(c-bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \cos^2(a+bx)\sec^4(c-bx)\,dx = -\frac{\sec^2(c-bx)\sin(2(a+c))}{2b} - \frac{\cos(2(a+c))\tan(c-bx)}{b} - \frac{\sin^2(a+c)\tan(c-bx)}{b} - \frac{\sin^2(a+c)\tan^3(c-bx)}{3b}$$

output
```
-1/2*sec(b*x-c)^2*sin(2*a+2*c)/b+cos(2*a+2*c)*tan(b*x-c)/b+sin(a+c)^2*tan(
b*x-c)/b+1/3*sin(a+c)^2*tan(b*x-c)^3/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\sec^4(c-bx)\,dx = -\frac{\sec(c)\sec^3(c-bx)(-3\sin(bx)+\sin(2c-3bx)+\sin(2a+4c-3bx)+3\sin(2a+2c-bx)+3\sin(2a}{12b}$$

input
```
Integrate[Cos[a + b*x]^2*Sec[c - b*x]^4,x]
```

output
```
-1/12*(Sec[c]*Sec[c - b*x]^3*(-3*Sin[b*x] + Sin[2*c - 3*b*x] + Sin[2*a + 4
*c - 3*b*x] + 3*Sin[2*a + 2*c - b*x] + 3*Sin[2*a + b*x] - Sin[2*a + 3*b*x]
))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sec^4(c-bx)\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \cos^2(a+bx)\sec^4(c-bx)dx$$

input
```
Int[Cos[a + b*x]^2*Sec[c - b*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 5.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{1}{3b(\sin(a)\cos(c)+\cos(a)\sin(c))(\tan(bx+a)\sin(a)\cos(c)+\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)-\sin(a)\sin(c))^3}$	55
parallelrisch	$\frac{3\sin(bx-c)+\sin(3bx-3c)+2\sin(3bx+2a-c)}{3b(\cos(3bx-3c)+3\cos(bx-c))}$	62
risch	$\frac{2i\left(e^{8i(a+c)}+3\,e^{2i(bx+4a+3c)}+e^{6i(a+c)}+3\,e^{4i(bx+2a+c)}+3\,e^{2i(bx+3a+2c)}+e^{4i(a+c)}\right)}{3\left(e^{2i(a+c)}+e^{2i(bx+a)}\right)^3 b}$	94

input
```
int(cos(b*x+a)^2*sec(b*x-c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(sin(a)*cos(c)+cos(a)*sin(c))/(tan(b*x+a)*sin(a)*cos(c)+tan(b*x+a)*
cos(a)*sin(c)+cos(a)*cos(c)-sin(a)*sin(c))^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(83) = 166$.

Time = 0.10 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.73

$$\int \cos^2(a+bx)\sec^4(c-bx)\,dx = -\frac{\left(4\cos\left(a+c\right)^5-\left(16\cos\left(a+c\right)^5-16\cos\left(a+c\right)^3+3\cos\left(a+c\right)\right)\cos\left(bx+a\right)^2-7\cos\left(a+c\right)^3+3}{3\left(\left(4\,b\cos\left(a+c\right)^3-3\,b\cos\left(a+c\right)\right)\cos\left(bx+a\right)^3+\left(\left(4\,b\cos\left(a+c\right)^2\right.\right.\right.}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x-c)^4,x, algorithm="fricas")
```

output
```
-1/3*((4*cos(a + c)^5 - (16*cos(a + c)^5 - 16*cos(a + c)^3 + 3*cos(a + c))
*cos(b*x + a)^2 - 7*cos(a + c)^3 + 3*cos(a + c))*sin(b*x + a) + ((16*cos(a
 + c)^4 - 8*cos(a + c)^2 + 1)*cos(b*x + a)^3 - 3*(4*cos(a + c)^4 - 3*cos(a
 + c)^2)*cos(b*x + a))*sin(a + c))/((4*b*cos(a + c)^3 - 3*b*cos(a + c))*co
s(b*x + a)^3 + ((4*b*cos(a + c)^2 - b)*cos(b*x + a)^2 - b*cos(a + c)^2 + b
)*sin(b*x + a)*sin(a + c) - 3*(b*cos(a + c)^3 - b*cos(a + c))*cos(b*x + a)
)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sec^4(c-bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sec(b*x-c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(83) = 166$.

Time = 0.05 (sec) , antiderivative size = 919, normalized size of antiderivative = 11.35

$$\int \cos^2(a+bx)\sec^4(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x-c)^4,x, algorithm="maxima")
```

output
```
-2/3*((3*sin(4*b*x + 4*a + 4*c) + 3*sin(2*b*x + 4*a + 6*c) + 3*sin(2*b*x +
 2*a + 4*c) + sin(4*a + 8*c) + sin(2*a + 6*c) + sin(4*c))*cos(6*b*x + 2*a)
 - 3*(3*sin(4*b*x + 2*a + 2*c) + 3*sin(2*b*x + 2*a + 4*c) + sin(2*a + 6*c)
)*cos(4*b*x + 4*a + 4*c) + 3*(3*sin(2*b*x + 4*a + 6*c) + 3*sin(2*b*x + 2*a
 + 4*c) + sin(4*a + 8*c) + sin(2*a + 6*c) + sin(4*c))*cos(4*b*x + 2*a + 2*
c) - 3*(3*sin(2*b*x + 2*a + 4*c) + sin(2*a + 6*c))*cos(2*b*x + 4*a + 6*c)
+ 3*(sin(4*a + 8*c) + sin(4*c))*cos(2*b*x + 2*a + 4*c) - (3*cos(4*b*x + 4*
a + 4*c) + 3*cos(2*b*x + 4*a + 6*c) + 3*cos(2*b*x + 2*a + 4*c) + cos(4*a +
 8*c) + cos(2*a + 6*c) + cos(4*c))*sin(6*b*x + 2*a) + 3*(3*cos(4*b*x + 2*a
 + 2*c) + 3*cos(2*b*x + 2*a + 4*c) + cos(2*a + 6*c))*sin(4*b*x + 4*a + 4*c
) - 3*(3*cos(2*b*x + 4*a + 6*c) + 3*cos(2*b*x + 2*a + 4*c) + cos(4*a + 8*c
) + cos(2*a + 6*c) + cos(4*c))*sin(4*b*x + 2*a + 2*c) + 3*(3*cos(2*b*x + 2
*a + 4*c) + cos(2*a + 6*c))*sin(2*b*x + 4*a + 6*c) - 3*(cos(4*a + 8*c) + c
os(4*c))*sin(2*b*x + 2*a + 4*c) + cos(2*a + 6*c)*sin(4*a + 8*c) - cos(4*a
+ 8*c)*sin(2*a + 6*c) - cos(4*c)*sin(2*a + 6*c) + cos(2*a + 6*c)*sin(4*c))
/(b*cos(6*b*x + 2*a)^2 + 9*b*cos(4*b*x + 2*a + 2*c)^2 + 9*b*cos(2*b*x + 2*
a + 4*c)^2 + 6*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 6*c) + b*cos(2*a + 6*c)^
2 + b*sin(6*b*x + 2*a)^2 + 9*b*sin(4*b*x + 2*a + 2*c)^2 + 9*b*sin(2*b*x +
2*a + 4*c)^2 + 6*b*sin(2*b*x + 2*a + 4*c)*sin(2*a + 6*c) + b*sin(2*a + 6*c
)^2 + 2*(3*b*cos(4*b*x + 2*a + 2*c) + 3*b*cos(2*b*x + 2*a + 4*c) + b*co...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(83) = 166$.

Time = 0.16 (sec) , antiderivative size = 427, normalized size of antiderivative = 5.27

$$\int \cos^2(a + bx) \sec^4(c - bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sec(b*x-c)^4,x, algorithm="giac")
```

output
```
-1/6*(tan(1/2*a)^8*tan(1/2*c)^8 + 4*tan(1/2*a)^8*tan(1/2*c)^6 + 4*tan(1/2*
a)^6*tan(1/2*c)^8 + 6*tan(1/2*a)^8*tan(1/2*c)^4 + 16*tan(1/2*a)^6*tan(1/2*
c)^6 + 6*tan(1/2*a)^4*tan(1/2*c)^8 + 4*tan(1/2*a)^8*tan(1/2*c)^2 + 24*tan(
1/2*a)^6*tan(1/2*c)^4 + 24*tan(1/2*a)^4*tan(1/2*c)^6 + 4*tan(1/2*a)^2*tan(
1/2*c)^8 + tan(1/2*a)^8 + 16*tan(1/2*a)^6*tan(1/2*c)^2 + 36*tan(1/2*a)^4*t
an(1/2*c)^4 + 16*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*c)^8 + 4*tan(1/2*a)^6
 + 24*tan(1/2*a)^4*tan(1/2*c)^2 + 24*tan(1/2*a)^2*tan(1/2*c)^4 + 4*tan(1/2
*c)^6 + 6*tan(1/2*a)^4 + 16*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(1/2*c)^4 + 4
*tan(1/2*a)^2 + 4*tan(1/2*c)^2 + 1)/((2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*
c) + 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 -
2*tan(b*x + a)*tan(1/2*a) + tan(1/2*a)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*t
an(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1)^3*(tan(1/2*a)^2*tan(1/2*c) + tan(
1/2*a)*tan(1/2*c)^2 - tan(1/2*a) - tan(1/2*c))*b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a + bx) \sec^4(c - bx) \, dx = \text{Hanged}$$

input
```
int(cos(a + b*x)^2/cos(c - b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int \cos^2(a+bx)\sec^4(c-bx)\,dx$$
$$= \frac{-\cos(bx-c)\cos(bx+a)\sin(bx+a)+\sin(bx-c)^3+\sin(bx-c)\sin(bx+a)^2-2\sin(bx-c)}{3\cos(bx-c)\,b\left(\sin(bx-c)^2-1\right)}$$

input
```
int(cos(b*x+a)^2*sec(b*x-c)^4,x)
```

output
```
( - cos(b*x - c)*cos(a + b*x)*sin(a + b*x) + sin(b*x - c)**3 + sin(b*x - c
)*sin(a + b*x)**2 - 2*sin(b*x - c))/(3*cos(b*x - c)*b*(sin(b*x - c)**2 - 1
))
```

3.387 $\int \cos(a+bx)\csc(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \cos(a+bx)\csc(c+bx)\,dx = \frac{\cos(a-c)\log(\sin(c+bx))}{b} - x\sin(a-c)$$

output
```
cos(a-c)*ln(sin(b*x+c))/b-x*sin(a-c)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cos(a+bx)\csc(c+bx)\,dx$$
$$= \frac{-2i\arctan(\tan(c+bx))\cos(a-c) + \cos(a-c)\left(2ibx + \log\left(\sin^2(c+bx)\right)\right) - 2bx\sin(a-c)}{2b}$$

input
```
Integrate[Cos[a + b*x]*Csc[c + b*x],x]
```

output
```
((-2*I)*ArcTan[Tan[c + b*x]]*Cos[a - c] + Cos[a - c]*((2*I)*b*x + Log[Sin[
c + b*x]^2]) - 2*b*x*Sin[a - c])/(2*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5092, 24, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \text{5092}$$

$$\cos(a-c)\int \cot(c+bx)dx-\sin(a-c)\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\cos(a-c)\int \cot(c+bx)dx-x\sin(a-c)$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int -\tan\left(c+bx+\frac{\pi}{2}\right)dx-x\sin(a-c)$$

$$\Big\downarrow \text{25}$$

$$-\cos(a-c)\int \tan\left(\frac{1}{2}(2c+\pi)+bx\right)dx-x\sin(a-c)$$

$$\Big\downarrow \text{3956}$$

$$\frac{\cos(a-c)\log(-\sin(bx+c))}{b}-x\sin(a-c)$$

input
```
Int[Cos[a + b*x]*Csc[c + b*x],x]
```

output
```
(Cos[a - c]*Log[-Sin[c + b*x]])/b - x*Sin[a - c]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5092
```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Simp[Cos[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] - Simp[Sin[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

method	result
risch	$-2i\cos\left(a-c\right)x+ix\,\mathrm{e}^{i(a-c)}-\frac{2i\cos(a-c)a}{b}+\frac{\ln\left(\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{2i(a-c)}\right)\cos(a-c)}{b}$
default	$\frac{\frac{\frac{(-\cos(a)\cos(c)-\sin(a)\sin(c))\ln\left(\tan(bx+a)^2+1\right)}{2}+(-\sin(a)\cos(c)+\cos(a)\sin(c))\arctan(\tan(bx+a))}{\left(\cos(c)^2+\sin(c)^2\right)\left(\cos(a)^2+\sin(a)^2\right)}+\frac{(\cos(a)\cos(c)+\sin(a)\sin(c))\ln(\tan(bx+a)}{\cos(a)^2\cos(c)^2+\sin}}{b}$

input
```
int(cos(b*x+a)*csc(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2*I*cos(a-c)*x+I*x*exp(I*(a-c))-2*I/b*cos(a-c)*a+ln(exp(2*I*(b*x+a))-exp(
2*I*(a-c)))/b*cos(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \cos(a+bx)\csc(c+bx)\,dx = \frac{bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\frac{1}{2}\sin\left(bx+c\right)\right)}{b}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c),x, algorithm="fricas")
```

output
```
(b*x*sin(-a + c) + cos(-a + c)*log(1/2*sin(b*x + c)))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(20) = 40$.

Time = 4.03 (sec) , antiderivative size = 333, normalized size of antiderivative = 12.33

$$\int \cos(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c),x)
```

output
```
-Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x, Eq(c, 0)), (0, Eq(b, 0)), (-b*x*t
an(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2*log(tan(c/2)
+ tan(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2)
)*tan(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(
c/2)**2 + b), True))*sin(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (log
(sin(b*x))/b, Eq(c, 0)), (x/sin(c), Eq(b, 0)), (2*b*x*tan(c/2)/(b*tan(c/2)
**2 + b) - log(tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) + lo
g(tan(c/2) + tan(b*x/2))/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - 1/tan(c/2)
)*tan(c/2)**2/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - 1/tan(c/2))/(b*tan(c/
2)**2 + b) + log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(
tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b), True))*cos(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(27) = 54.

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.93

$$\int \cos(a+bx)\csc(c+bx)\,dx$$
$$= \frac{2\,bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\cos\left(bx\right)^{2}+2\,\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^{2}+\sin\left(bx\right)^{2}-2\,\sin\left(bx\right)\sin\left(\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c),x, algorithm="maxima")
```

output
```
1/2*(2*b*x*sin(-a + c) + cos(-a + c)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) +
cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + cos(-a + c)*log(co
s(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) +
 sin(c)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(27) = 54.

Time = 0.16 (sec) , antiderivative size = 482, normalized size of antiderivative = 17.85

$$\int \cos(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c),x, algorithm="giac")
```

output
```
-1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) -
tan(1/2*c))*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*
c)^2 + 1) + (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1
/2*c) - tan(1/2*c)^2 + 1)*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4*tan(1/2*c)^4 - 2*t
an(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^2*ta
n(1/2*c)^4 + tan(1/2*a)^4 - 8*tan(1/2*a)^3*tan(1/2*c) + 20*tan(1/2*a)^2*ta
n(1/2*c)^2 - 8*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 2*tan(1/2*a)^2 + 8
*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)*tan(1/2*
a)^2*tan(1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*
tan(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan
(1/2*a)*tan(1/2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1
/2*a)^4*tan(1/2*c)^4 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^4 + 4*tan(
1/2*a)^3*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + 4*tan(1/2
*a)*tan(1/2*c) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 115, normalized size of antiderivative = 4.26

$$\int \cos(a+bx)\csc(c+bx)\,dx = -x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}-\frac{e^{a\,1i-c\,1i}\,1i}{2}\right) - x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}+\frac{e^{a\,1i-c\,1i}\,1i}{2}\right) + \frac{\ln\left(-e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i+c\,1i}}{2}+\frac{e^{a\,1i-c\,1i}}{2}\right)}{b}$$

input
```
int(cos(a + b*x)/sin(c + b*x),x)
```

output
```
(log(exp(a*2i + b*x*2i) - exp(a*2i - c*2i))*(exp(c*1i - a*1i)/2 + exp(a*1i
 - c*1i)/2))/b - x*((exp(c*1i - a*1i)*1i)/2 + (exp(a*1i - c*1i)*1i)/2) - x
*((exp(c*1i - a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2)
```

Reduce [F]

$$\int \cos(a+bx)\csc(c+bx)\,dx = \int \cos(bx+a)\csc(bx+c)\,dx$$

input
```
int(cos(b*x+a)*csc(b*x+c),x)
```

output
```
int(cos(a + b*x)*csc(b*x + c),x)
```

3.388 $\int \cos(a+bx)\csc^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \cos(a+bx)\csc^2(c+bx)\,dx = -\frac{\cos(a-c)\csc(c+bx)}{b} + \frac{\operatorname{arctanh}(\cos(c+bx))\sin(a-c)}{b}$$

output
```
-cos(a-c)*csc(b*x+c)/b+arctanh(cos(b*x+c))*sin(a-c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.57

$$\begin{aligned}&\int \cos(a+bx)\csc^2(c+bx)\,dx\\&= -\frac{\cos(a-c)\csc(c+bx)}{b}\\&\quad+\frac{2i\arctan\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b}\end{aligned}$$

input
```
Integrate[Cos[a + b*x]*Csc[c + b*x]^2,x]
```

output
```
-((Cos[a - c]*Csc[c + b*x])/b) + ((2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c
]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/
2]*Sin[c])]*Sin[a - c])/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5092, 3042, 25, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^2(bx+c)\,dx$$

$$\Big\downarrow \text{5092}$$

$$\cos(a-c)\int \cot(c+bx)\csc(c+bx)dx-\sin(a-c)\int \csc(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)\tan\left(c+bx-\frac{\pi}{2}\right)dx-\sin(a-c)\int \csc(c+bx)dx$$

$$\Big\downarrow \text{25}$$

$$-\sin(a-c)\int \csc(c+bx)dx-\cos(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(a-c)\int 1d\csc(c+bx)}{b}-\sin(a-c)\int \csc(c+bx)dx$$

$$\Big\downarrow \text{24}$$

$$-\sin(a-c)\int \csc(c+bx)dx-\frac{\cos(a-c)\csc(bx+c)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sin(a-c)\text{arctanh}(\cos(bx+c))}{b}-\frac{\cos(a-c)\csc(bx+c)}{b}$$

input
```
Int[Cos[a + b*x]*Csc[c + b*x]^2,x]
```

output
```
-((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5092
```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Simp[Cos[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] - Simp[Sin[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.29

method	result
risch	$\frac{i\left(e^{i(bx+3a)}+e^{i(bx+a+2c)}\right)}{b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)}+\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\sin(a-c)}{b}-\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\sin(a-c)}{b}$
default	$-\frac{2\left(-\frac{\left(\cos(a)^2\cos(c)^2+2\cos(a)\cos(c)\sin(a)\sin(c)+\sin(a)^2\sin(c)^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)(\sin(a)\cos(c)-\cos(a)\sin(c))}+\frac{\cos(a)\cos(c)+\sin(a)\sin}{\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2}\right.}{\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\cos(a)\cos(c)+2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\sin(a)\sin(c)-\sin(a)\cos(c)+c}$

input
```
int(cos(b*x+a)*csc(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
I/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))
+ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin(a-c)-ln(exp(I*(b*x+a))-exp(I*(a-c))
)/b*sin(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(35) = 70.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.03

$$\int \cos(a+bx)\csc^2(c+bx)\,dx = -\frac{\log\left(\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)\sin(-a+c)-\log\left(-\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)\sin(-a+c)}{2\,b\sin(bx+c)}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) - log(-1/2*cos(
b*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) + 2*cos(-a + c))/(b*sin(b*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1572 vs. $2(27) = 54$.

Time = 60.66 (sec) , antiderivative size = 3264, normalized size of antiderivative = 93.26

$$\int \cos(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)*csc(b*x+c)**2,x)
```

output

```
-Piecewise((0, Eq(b, 0) & Eq(c, 0)), (log(tan(b*x/2))/b, Eq(c, 0)), (0, Eq
(b, 0)), (-log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**
4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*ta
n(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(
c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)
**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)
) + log(tan(c/2) + tan(b*x/2))*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*t
an(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*ta
n(c/2) - b*tan(b*x/2)) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**2*tan(b*x/
2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3
 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(c/2) +
tan(b*x/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)*
*3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) -
 b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*
x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)
**2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(b*x/2)/(
b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b
*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/
tan(c/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3
*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(35) = 70$.

Time = 0.06 (sec) , antiderivative size = 450, normalized size of antiderivative = 12.86

$$\int \cos(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(sin(b*x + 2*a) + sin(b*x + 2*c))*cos(2*b*x + a + 2*c) - (cos(2*b*x
 + a + 2*c)^2*sin(-a + c) - 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + si
n(2*b*x + a + 2*c)^2*sin(-a + c) - 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a +
c) + (cos(a)^2 + sin(a)^2)*sin(-a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c)
 + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + (cos(2*b*x + a
+ 2*c)^2*sin(-a + c) - 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(2*b
*x + a + 2*c)^2*sin(-a + c) - 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) +
(cos(a)^2 + sin(a)^2)*sin(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + co
s(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*(cos(b*x + 2*a) +
cos(b*x + 2*c))*sin(2*b*x + a + 2*c) - 2*cos(a)*sin(b*x + 2*a) - 2*cos(a)*
sin(b*x + 2*c) + 2*cos(b*x + 2*a)*sin(a) + 2*cos(b*x + 2*c)*sin(a))/(b*cos
(2*b*x + a + 2*c)^2 - 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a +
2*c)^2 - 2*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(35) = 70$.

Time = 0.24 (sec) , antiderivative size = 893, normalized size of antiderivative = 25.51

$$\int \cos(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) -
tan(1/2*c))*log(abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan
(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2
*a) - 2*tan(1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c) + 4*tan(1/2*a)*ta
n(1/2*c) - 2*tan(1/2*c)^2)/abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2
*c) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(
1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) - 2*tan(1/2*b*x + 1/2*a)*tan(
1/2*c) + 4*tan(1/2*a)*tan(1/2*c) + 2))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/
2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)
^4 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*b*x + 1/
2*a)*tan(1/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(1/2*b
*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(
1/2*b*x + 1/2*a)*tan(1/2*a)^4 - 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/
2*c) + 2*tan(1/2*a)^4*tan(1/2*c) + 20*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*ta
n(1/2*c)^2 - 12*tan(1/2*a)^3*tan(1/2*c)^2 - 8*tan(1/2*b*x + 1/2*a)*tan(1/2
*a)*tan(1/2*c)^3 + 12*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*b*x + 1/2*a)*tan
(1/2*c)^4 - 2*tan(1/2*a)*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^
2 + 2*tan(1/2*a)^3 + 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - 12*tan
(1/2*a)^2*tan(1/2*c) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c)^2 + 12*tan(1/2*a)
*tan(1/2*c)^2 - 2*tan(1/2*c)^3 + tan(1/2*b*x + 1/2*a) - 2*tan(1/2*a) + ...
```

Mupad [B] (verification not implemented)

Time = 24.74 (sec) , antiderivative size = 252, normalized size of antiderivative = 7.20

$$\int \cos(a+bx)\csc^2(c+bx)\,dx = -\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}} + \frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}} + \frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}+1\right)\,1i}{b\,\left(e^{a\,2i-c\,2i}-e^{a\,2i+b\,x\,2i}\right)}$$

input
```
int(cos(a + b*x)/sin(c + b*x)^2,x)
```

output
```
(log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) + (exp(a*2i)*exp(-c*
2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2
i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log(exp(a*1i)*exp(b*x*1
i)*(exp(a*2i)*exp(-c*2i) - 1) - (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i
) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) - 1))/(2*b*(-e
xp(a*2i - c*2i))^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) + 1)*1i)/(
b*(exp(a*2i - c*2i) - exp(a*2i + b*x*2i)))
```

Reduce [F]

$$\int \cos(a+bx)\csc^2(c+bx)\,dx=\int \cos(bx+a)\csc(bx+c)^2\,dx$$

input
```
int(cos(b*x+a)*csc(b*x+c)^2,x)
```

output
```
int(cos(a + b*x)*csc(b*x + c)**2,x)
```

3.389 $\int \cos(a+bx)\csc^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = -\frac{\cos(a-c)\csc^2(c+bx)}{2b}+\frac{\cot(c+bx)\sin(a-c)}{b}$$

output
```
-1/2*cos(a-c)*csc(b*x+c)^2/b+cot(b*x+c)*sin(a-c)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = -\frac{\csc(c)\csc^2(c+bx)(\sin(a)-\cos(c+2bx)\sin(a-c))}{2b}$$

input
```
Integrate[Cos[a + b*x]*Csc[c + b*x]^3,x]
```

output
```
-1/2*(Csc[c]*Csc[c + b*x]^2*(Sin[a] - Cos[c + 2*b*x]*Sin[a - c]))/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5092, 3042, 25, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^3(bx+c)\,dx$$

$$\Big\downarrow \text{5092}$$

$$\cos(a-c)\int \cot(c+bx)\csc^2(c+bx)dx-\sin(a-c)\int \csc^2(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^2\tan\left(c+bx-\frac{\pi}{2}\right)dx-\sin(a-c)\int \csc(c+bx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\sin(a-c)\int \csc(c+bx)^2dx-\cos(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(a-c)\int \csc(c+bx)d\csc(c+bx)}{b}-\sin(a-c)\int \csc(c+bx)^2dx$$

$$\Big\downarrow \text{15}$$

$$-\sin(a-c)\int \csc(c+bx)^2dx-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sin(a-c)\int 1d\cot(c+bx)}{b}-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(a-c)\cot(bx+c)}{b}-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

input
```
Int[Cos[a + b*x]*Csc[c + b*x]^3,x]
```

output
```
-1/2*(Cos[a - c]*Csc[c + b*x]^2)/b + (Cot[c + b*x]*Sin[a - c])/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5092
```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Simp[Cos[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] - Simp[Sin[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$-\frac{\sec\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}\csc\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}\cos(2bx+a+c)}{8b}$	36
default	$-\frac{1}{2b(\cos(a)\cos(c)+\sin(a)\sin(c))(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))^{2}}$	55
risch	$-\frac{-2\,\mathrm{e}^{i(2bx+5a+c)}+\mathrm{e}^{i(5a-c)}-\mathrm{e}^{i(3a+c)}}{\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{2}b}$	64

input
```
int(cos(b*x+a)*csc(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/b*sec(1/2*b*x+1/2*c)^2*csc(1/2*b*x+1/2*c)^2*cos(2*b*x+a+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \frac{2\cos(bx+c)\sin(bx+c)\sin(-a+c)+\cos(-a+c)}{2\left(b\cos(bx+c)^{2}-b\right)}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + c)*sin(b*x + c)*sin(-a + c) + cos(-a + c))/(b*cos(b*x + c
)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(36) = 72.

Time = 0.04 (sec) , antiderivative size = 395, normalized size of antiderivative = 10.39

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \frac{(2\cos(2bx+2a+2c)-\cos(2a)+\cos(2c))\cos(4bx+a+5c)-2(2\cos(2bx+2a+2c)-\cos(2a}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2-4b\cos(2}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^3,x, algorithm="maxima")
```

output
```
((2*cos(2*b*x + 2*a + 2*c) - cos(2*a) + cos(2*c))*cos(4*b*x + a + 5*c) - 2
*(2*cos(2*b*x + 2*a + 2*c) - cos(2*a) + cos(2*c))*cos(2*b*x + a + 3*c) - (
cos(2*a) - cos(2*c))*cos(a + c) + 2*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (2
*sin(2*b*x + 2*a + 2*c) - sin(2*a) + sin(2*c))*sin(4*b*x + a + 5*c) - 2*(2
*sin(2*b*x + 2*a + 2*c) - sin(2*a) + sin(2*c))*sin(2*b*x + a + 3*c) - (sin
(2*a) - sin(2*c))*sin(a + c) + 2*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos
(4*b*x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 - 4*b*cos(2*b*x + a + 3*c
)*cos(a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x +
 a + 3*c)^2 - 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(2*
b*cos(2*b*x + a + 3*c) - b*cos(a + c))*cos(4*b*x + a + 5*c) - 2*(2*b*sin(2
*b*x + a + 3*c) - b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(36) = 72.

Time = 0.16 (sec) , antiderivative size = 327, normalized size of antiderivative = 8.61

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = -\frac{\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^2+9\,\mathrm{t}}{2\left(\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2+4\tan(bx+a)\tan\left(\frac{1}{2}a\right)\tan\left(\right.\right.}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/2*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*
a)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c
)^4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*
c)^2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan
(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2
+ 1)/((tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2
+ 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c) - tan(b
*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^2 + tan(b*x + a) - 2*tan(1/
2*a) + 2*tan(1/2*c))^2*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1
/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/sin(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \frac{-\cos(bx+c)\cos(bx+a)+\sin(bx+c)\sin(bx+a)}{2\sin(bx+c)^2 b}$$

input
```
int(cos(b*x+a)*csc(b*x+c)^3,x)
```

output
```
( - cos(b*x + c)*cos(a + b*x) + sin(b*x + c)*sin(a + b*x))/(2*sin(b*x + c)
**2*b)
```

3.390 $\int \cos(a+bx)\csc^4(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \cos(a+bx)\csc^4(c+bx)\,dx=-\frac{\cos(a-c)\csc^3(c+bx)}{3b}+\frac{1}{2}\left(\frac{\operatorname{arctanh}(\cos(c+bx))}{b}+\frac{\cot(c+bx)\csc(c+bx)}{b}\right)\sin(a-c)$$

output
```
-1/3*cos(a-c)*csc(b*x+c)^3/b+1/2*(arctanh(cos(b*x+c))/b+cot(b*x+c)*csc(b*x
+c)/b)*sin(a-c)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \cos(a+bx)\csc^4(c+bx)\,dx$$
$$=\frac{12\operatorname{arctanh}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a-c)+\csc^3(c+bx)(-4\cos(a-c)+3\sin(a-c)\sin(2(c+bx)}{12b}$$

input
```
Integrate[Cos[a + b*x]*Csc[c + b*x]^4,x]
```

output
```
(12*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Sin[a - c] + Csc[c + b*x]^3*(-4*
Cos[a - c] + 3*Sin[a - c]*Sin[2*(c + b*x)]))/(12*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {5092, 3042, 25, 3086, 15, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^4(bx+c)\,dx$$

$$\Big\downarrow \text{5092}$$

$$\cos(a-c)\int \cot(c+bx)\csc^3(c+bx)dx-\sin(a-c)\int \csc^3(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\int -\sec\left(c+bx-\frac{\pi}{2}\right)^3\tan\left(c+bx-\frac{\pi}{2}\right)dx-\sin(a-c)\int \csc(c+bx)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\sin(a-c)\int \csc(c+bx)^3dx-\cos(a-c)\int \sec\left(\frac{1}{2}(2c-\pi)+bx\right)^3\tan\left(\frac{1}{2}(2c-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(a-c)\int \csc^2(c+bx)d\csc(c+bx)}{b}-\sin(a-c)\int \csc(c+bx)^3dx$$

$$\Big\downarrow \text{15}$$

$$-\sin(a-c)\int \csc(c+bx)^3dx-\frac{\cos(a-c)\csc^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4255}$$

$$-\sin(a-c)\left(\frac{1}{2}\int \csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cos(a-c)\csc^3(bx+c)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\sin(a-c)\left(\frac{1}{2}\int\csc(c+bx)dx-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cos(a-c)\csc^3(bx+c)}{3b}$$

$$\Big\downarrow \text{4257}$$

$$-\sin(a-c)\left(-\frac{\operatorname{arctanh}(\cos(bx+c))}{2b}-\frac{\cot(bx+c)\csc(bx+c)}{2b}\right)-\frac{\cos(a-c)\csc^3(bx+c)}{3b}$$

input
```
Int[Cos[a + b*x]*Csc[c + b*x]^4,x]
```

output
```
-1/3*(Cos[a - c]*Csc[c + b*x]^3)/b - (-1/2*ArcTanh[Cos[c + b*x]]/b - (Cot[
c + b*x]*Csc[c + b*x])/(2*b))*Sin[a - c]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5092
```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Simp[Cos[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] - Simp[Sin[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.49 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.17

method	result
risch	$\frac{i\left(-3\,\mathrm{e}^{i(5bx+7a+4c)}+3\,\mathrm{e}^{i(5bx+5a+6c)}-8\,\mathrm{e}^{i(3bx+7a+2c)}-8\,\mathrm{e}^{i(3bx+5a+4c)}+3\,\mathrm{e}^{i(bx+7a)}-3\,\mathrm{e}^{i(bx+5a+2c)}\right)}{6b\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{3}}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-\mathrm{e}^{i(a-c)}\right)\sin}{2b}$
default	Expression too large to display

input
```
int(cos(b*x+a)*csc(b*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*I/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^3*(-3*exp(I*(5*b*x+7*a+4*c))+3*ex
p(I*(5*b*x+5*a+6*c))-8*exp(I*(3*b*x+7*a+2*c))-8*exp(I*(3*b*x+5*a+4*c))+3*e
xp(I*(b*x+7*a))-3*exp(I*(b*x+5*a+2*c)))-1/2*ln(exp(I*(b*x+a))-exp(I*(a-c))
)/b*sin(a-c)+1/2*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin(a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(56) = 112.

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.08

$$\int \cos(a+bx)\csc^4(c+bx)\,dx = -\frac{3\left(\cos\left(bx+c\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-3\left(\cos\left(bx+c\right)^2-1\right)\log\left(-\frac{1}{2}\right.}{12\left(b\cos\left(bx+c\right)^2\right.}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^4,x, algorithm="fricas")
```

output
```
-1/12*(3*(cos(b*x + c)^2 - 1)*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c)*sin
(-a + c) - 3*(cos(b*x + c)^2 - 1)*log(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c
)*sin(-a + c) - 6*cos(b*x + c)*sin(b*x + c)*sin(-a + c) - 4*cos(-a + c))/(
(b*cos(b*x + c)^2 - b)*sin(b*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^4(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1777 vs. $2(56) = 112$.

Time = 0.10 (sec) , antiderivative size = 1777, normalized size of antiderivative = 29.62

$$\int \cos(a+bx)\csc^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^4,x, algorithm="maxima")
```

output
```
-1/12*(2*(3*sin(5*b*x + 2*a + 4*c) - 3*sin(5*b*x + 6*c) + 8*sin(3*b*x + 2*
a + 2*c) + 8*sin(3*b*x + 4*c) - 3*sin(b*x + 2*a) + 3*sin(b*x + 2*c))*cos(6
*b*x + a + 6*c) + 6*(3*sin(4*b*x + a + 4*c) - 3*sin(2*b*x + a + 2*c) + sin
(a))*cos(5*b*x + 2*a + 4*c) - 6*(3*sin(4*b*x + a + 4*c) - 3*sin(2*b*x + a
+ 2*c) + sin(a))*cos(5*b*x + 6*c) - 6*(8*sin(3*b*x + 2*a + 2*c) + 8*sin(3*
b*x + 4*c) - 3*sin(b*x + 2*a) + 3*sin(b*x + 2*c))*cos(4*b*x + a + 4*c) - 1
6*(3*sin(2*b*x + a + 2*c) - sin(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*sin(2*b
*x + a + 2*c) - sin(a))*cos(3*b*x + 4*c) - 18*(sin(b*x + 2*a) - sin(b*x +
2*c))*cos(2*b*x + a + 2*c) + 3*(cos(6*b*x + a + 6*c)^2*sin(-a + c) + 9*cos
(4*b*x + a + 4*c)^2*sin(-a + c) + 9*cos(2*b*x + a + 2*c)^2*sin(-a + c) - 6
*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(6*b*x + a + 6*c)^2*sin(-a +
 c) + 9*sin(4*b*x + a + 4*c)^2*sin(-a + c) + 9*sin(2*b*x + a + 2*c)^2*sin(
-a + c) - 6*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) - 2*(3*cos(4*b*x + a +
 4*c)*sin(-a + c) - 3*cos(2*b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c
))*cos(6*b*x + a + 6*c) - 6*(3*cos(2*b*x + a + 2*c)*sin(-a + c) - cos(a)*s
in(-a + c))*cos(4*b*x + a + 4*c) - 2*(3*sin(4*b*x + a + 4*c)*sin(-a + c) -
 3*sin(2*b*x + a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(6*b*x + a +
6*c) - 6*(3*sin(2*b*x + a + 2*c)*sin(-a + c) - sin(a)*sin(-a + c))*sin(4*b
*x + a + 4*c) + (cos(a)^2 + sin(a)^2)*sin(-a + c))*log(cos(b*x)^2 + 2*cos(
b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - 3...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11407 vs. $2(56) = 112$.

Time = 1.12 (sec) , antiderivative size = 11407, normalized size of antiderivative = 190.12

$$\int \cos(a + bx) \csc^4(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x+c)^4,x, algorithm="giac")
```

output
```
-1/24*(24*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a)
- tan(1/2*c))*log(abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) - 2*t
an(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1
/2*a) - 2*tan(1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c) + 4*tan(1/2*a)*
tan(1/2*c) - 2*tan(1/2*c)^2)/abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1
/2*c) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*a)^2*ta
n(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) - 2*tan(1/2*b*x + 1/2*a)*ta
n(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) + 2))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(
1/2*a)^2 + tan(1/2*c)^2 + 1) - (3*tan(1/2*b*x + 1/2*a)^5*tan(1/2*a)^12*tan
(1/2*c)^10 - 6*tan(1/2*b*x + 1/2*a)^5*tan(1/2*a)^11*tan(1/2*c)^11 + 3*tan(
1/2*b*x + 1/2*a)^4*tan(1/2*a)^12*tan(1/2*c)^11 + 3*tan(1/2*b*x + 1/2*a)^5*
tan(1/2*a)^10*tan(1/2*c)^12 - 3*tan(1/2*b*x + 1/2*a)^4*tan(1/2*a)^11*tan(1
/2*c)^12 + tan(1/2*b*x + 1/2*a)^3*tan(1/2*a)^12*tan(1/2*c)^12 + 12*tan(1/2
*b*x + 1/2*a)^5*tan(1/2*a)^12*tan(1/2*c)^8 - 18*tan(1/2*b*x + 1/2*a)^5*tan
(1/2*a)^11*tan(1/2*c)^9 + 9*tan(1/2*b*x + 1/2*a)^4*tan(1/2*a)^12*tan(1/2*c
)^9 + 12*tan(1/2*b*x + 1/2*a)^5*tan(1/2*a)^10*tan(1/2*c)^10 + 6*tan(1/2*b*
x + 1/2*a)^4*tan(1/2*a)^11*tan(1/2*c)^10 - 2*tan(1/2*b*x + 1/2*a)^3*tan(1/
2*a)^12*tan(1/2*c)^10 - 18*tan(1/2*b*x + 1/2*a)^5*tan(1/2*a)^9*tan(1/2*c)^
11 - 6*tan(1/2*b*x + 1/2*a)^4*tan(1/2*a)^10*tan(1/2*c)^11 + 16*tan(1/2*b*x
 + 1/2*a)^3*tan(1/2*a)^11*tan(1/2*c)^11 - 3*tan(1/2*b*x + 1/2*a)^2*tan(...
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^4(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/sin(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos(a+bx)\csc^4(c+bx)\,dx$$

$$= \frac{-24\cos(bx+c)\cos(bx+a)+20\cos(bx+c)\sin(bx+c)^2+4\cos(bx+c)\sin(bx+c)\sin(bx+a)-8\cdots}{}$$

input
```
int(cos(b*x+a)*csc(b*x+c)^4,x)
```

output
```
( - 24*cos(b*x + c)*cos(a + b*x) + 20*cos(b*x + c)*sin(b*x + c)**2 + 4*cos
(b*x + c)*sin(b*x + c)*sin(a + b*x) - 8*cos(b*x + c) + 4*cos(a + b*x)*sin(
b*x + c)**2 - 8*cos(a + b*x) - 12*int((tan((b*x + c)/2)**2*tan((a + b*x)/2
)**2)/(tan((a + b*x)/2)**2 + 1),x)*sin(b*x + c)**3*b - 6*int(1/(tan((b*x +
 c)/2)**4*tan((a + b*x)/2)**2 + tan((b*x + c)/2)**4),x)*sin(b*x + c)**3*b
+ 9*sin(b*x + c)**3*sin(a + b*x) + 24*sin(b*x + c)**3*tan((b*x + c)/2) - 3
*sin(b*x + c)**3*b*x + 12*sin(b*x + c)*sin(a + b*x) - 8)/(48*sin(b*x + c)*
*3*b)
```

3.391 $\int \cos(a+bx)\csc(c-bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \cos(a+bx)\csc(c-bx)\,dx = -\frac{\cos(a+c)\log(\sin(c-bx))}{b} + x\sin(a+c)$$

output
```
-cos(a+c)*ln(-sin(b*x-c))/b+x*sin(a+c)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.29

$$\int \cos(a+bx)\csc(c-bx)\,dx$$
$$= \frac{2i\arctan(\tan(c+bx))\cos(a+c) + \cos(a+c)\left(-2ibx - \log\left(\sin^2(c-bx)\right)\right) + 2bx\sin(a+c)}{2b}$$

input
```
Integrate[Cos[a + b*x]*Csc[c - b*x],x]
```

output
```
((2*I)*ArcTan[Tan[c + b*x]]*Cos[a + c] + Cos[a + c]*((-2*I)*b*x - Log[Sin[
c - b*x]^2]) + 2*b*x*Sin[a + c])/(2*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc(c-bx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \cos(a+bx)\csc(c-bx)dx$$

input
```
Int[Cos[a + b*x]*Csc[c - b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

method	result
risch	$2i\cos\left(a+c\right)x-ix\,\mathrm{e}^{i(a+c)}+\frac{2i\cos(a+c)a}{b}-\frac{\ln\left(-\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)\cos(a+c)}{b}$
default	$-\frac{\frac{(\cos(a)\cos(c)-\sin(a)\sin(c))\ln(\tan(bx+a)\cos(a)\cos(c)-\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)-\cos(a)\sin(c))}{\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2}+\frac{(-\cos(a)\cos(c)+\sin(a)\sin(c))\,\mathrm{l}}{2}}{b}$

input
```
int(-cos(b*x+a)*csc(b*x-c),x,method=_RETURNVERBOSE)
```

output
```
2*I*cos(a+c)*x-I*x*exp(I*(a+c))+2*I/b*cos(a+c)*a-ln(-exp(2*I*(a+c))+exp(2*
I*(b*x+a)))/b*cos(a+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \cos(a+bx)\csc(c-bx)\,dx$$
$$=\frac{bx\sin(a+c)-\cos(a+c)\log\left(\frac{\cos(a+c)\sin(bx+a)-\cos(bx+a)\sin(a+c)}{\cos(a+c)+1}\right)}{b}$$

input
```
integrate(-cos(b*x+a)*csc(b*x-c),x, algorithm="fricas")
```

output
```
(b*x*sin(a + c) - cos(a + c)*log((cos(a + c)*sin(b*x + a) - cos(b*x + a)*s
in(a + c))/(cos(a + c) + 1)))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(22) = 44.

Time = 4.80 (sec) , antiderivative size = 337, normalized size of antiderivative = 14.04

$$\int \cos(a+bx)\csc(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(-cos(b*x+a)*csc(b*x-c),x)
```

output
```
-Piecewise((0, Eq(b, 0) & Eq(c, 0)), (-x, Eq(c, 0)), (0, Eq(b, 0)), (b*x*t
an(c/2)**2/(b*tan(c/2)**2 + b) - b*x/(b*tan(c/2)**2 + b) - 2*log(-tan(c/2)
 + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) + 1/tan(c/2
))*tan(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan
(c/2)**2 + b), True))*sin(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (-l
og(sin(b*x))/b, Eq(c, 0)), (x/sin(c), Eq(b, 0)), (2*b*x*tan(c/2)/(b*tan(c/
2)**2 + b) + log(-tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) -
 log(-tan(c/2) + tan(b*x/2))/(b*tan(c/2)**2 + b) + log(tan(b*x/2) + 1/tan(
c/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + 1/tan(c/2))/(b*ta
n(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) +
log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b), True))*cos(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(27) = 54$.

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.25

$$\int \cos(a+bx)\csc(c-bx)\,dx$$
$$= \frac{2\,bx\sin\left(a+c\right) - \cos\left(a+c\right)\log\left(\cos\left(bx\right)^2 + 2\,\cos\left(bx\right)\cos\left(c\right) + \cos\left(c\right)^2 + \sin\left(bx\right)^2 + 2\,\sin\left(bx\right)\sin\left(c\right) + \right.}{}$$

input
```
integrate(-cos(b*x+a)*csc(b*x-c),x, algorithm="maxima")
```

output
```
1/2*(2*b*x*sin(a + c) - cos(a + c)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + co
s(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - cos(a + c)*log(cos(b
*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + si
n(c)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(27) = 54$.

Time = 0.17 (sec) , antiderivative size = 484, normalized size of antiderivative = 20.17

$$\int \cos(a+bx)\csc(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(-cos(b*x+a)*csc(b*x-c),x, algorithm="giac")
```

output
```
-1/2*(4*(tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) -
tan(1/2*c))*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*
c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 - 4*tan(1/2*a)*tan(1
/2*c) - tan(1/2*c)^2 + 1)*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^4*tan(1/2*c)^4 - 2*t
an(1/2*a)^4*tan(1/2*c)^2 - 8*tan(1/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^2*ta
n(1/2*c)^4 + tan(1/2*a)^4 + 8*tan(1/2*a)^3*tan(1/2*c) + 20*tan(1/2*a)^2*ta
n(1/2*c)^2 + 8*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 2*tan(1/2*a)^2 - 8
*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)*tan(1/2*
a)^2*tan(1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2 - 4*tan(b*x + a)*tan(1/2*a)*
tan(1/2*c) + 2*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan
(1/2*a)*tan(1/2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) - 2*tan(1/2*c)))/(tan(1
/2*a)^4*tan(1/2*c)^4 - 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^4 - 4*tan(
1/2*a)^3*tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2
*a)*tan(1/2*c) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.75

$$\int \cos(a+bx)\csc(c-bx)\,dx = x\left(\frac{e^{-a\,1i-c\,1i}\,1i}{2}-\frac{e^{a\,1i+c\,1i}\,1i}{2}\right) + x\left(\frac{e^{-a\,1i-c\,1i}\,1i}{2}+\frac{e^{a\,1i+c\,1i}\,1i}{2}\right) - \frac{\ln\left(-e^{a\,2i+c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i-c\,1i}}{2}+\frac{e^{a\,1i+c\,1i}}{2}\right)}{b}$$

input
```
int(cos(a + b*x)/sin(c - b*x),x)
```

output
```
x*((exp(- a*1i - c*1i)*1i)/2 - (exp(a*1i + c*1i)*1i)/2) + x*((exp(- a*1i -
 c*1i)*1i)/2 + (exp(a*1i + c*1i)*1i)/2) - (log(exp(a*2i + b*x*2i) - exp(a*
2i + c*2i))*(exp(- a*1i - c*1i)/2 + exp(a*1i + c*1i)/2))/b
```

Reduce [F]

$$\int \cos(a+bx)\csc(c-bx)\,dx = -\left(\int \cos\left(bx+a\right)\csc\left(bx-c\right)dx\right)$$

input
```
int(-cos(b*x+a)*csc(b*x-c),x)
```

output
```
 - int(cos(a + b*x)*csc(b*x - c),x)
```

3.392 $\int \cos(a+bx)\csc^2(c-bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \cos(a+bx)\csc^2(c-bx)\,dx = \frac{\cos(a+c)\csc(c-bx)}{b} + \frac{\operatorname{arctanh}(\cos(c-bx))\sin(a+c)}{b}$$

output
```
-cos(a+c)*csc(b*x-c)/b+arctanh(cos(b*x-c))*sin(a+c)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 4.94

$$\int \cos(a+bx)\csc^2(c-bx)\,dx = \frac{\cos(a+c)\csc(c-bx)}{b} + \frac{2i\arctan\left(\frac{\cos\left(\frac{bx}{2}\right)+\cos^2(c)\cos\left(\frac{bx}{2}\right)+2i\cos(c)\cos\left(\frac{bx}{2}\right)\sin(c)-\cos\left(\frac{bx}{2}\right)\sin^2(c)+i\sin\left(\frac{bx}{2}\right)-i\cos^2(c)\sin\left(\frac{bx}{2}\right)+2\cos(c)\sin(c)\sin\left(\frac{bx}{2}\right)}{2i\cos(c)\cos\left(\frac{bx}{2}\right)-2\cos\left(\frac{bx}{2}\right)\sin(c)}$$

input
```
Integrate[Cos[a + b*x]*Csc[c - b*x]^2,x]
```

output
```
(Cos[a + c]*Csc[c - b*x])/b + ((2*I)*ArcTan[(Cos[(b*x)/2] + Cos[c]^2*Cos[(
b*x)/2] + (2*I)*Cos[c]*Cos[(b*x)/2]*Sin[c] - Cos[(b*x)/2]*Sin[c]^2 + I*Sin
[(b*x)/2] - I*Cos[c]^2*Sin[(b*x)/2] + 2*Cos[c]*Sin[c]*Sin[(b*x)/2] + I*Sin
[c]^2*Sin[(b*x)/2])/((2*I)*Cos[c]*Cos[(b*x)/2] - 2*Cos[(b*x)/2]*Sin[c])]*S
in[a + c])/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^2(c-bx)\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \cos(a+bx)\csc^2(c-bx)dx$$

input
```
Int[Cos[a + b*x]*Csc[c - b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.38

method	result
risch	$\frac{i\left(e^{i(bx+3a+2c)}+e^{i(bx+a)}\right)}{b\left(e^{2i(a+c)}-e^{2i(bx+a)}\right)}+\frac{\ln\left(e^{i(a+c)}+e^{i(bx+a)}\right)\sin(a+c)}{b}-\frac{\ln\left(-e^{i(a+c)}+e^{i(bx+a)}\right)\sin(a+c)}{b}$
default	$-\frac{2\left(-\frac{\left(\sin(a)^2\sin(c)^2-2\cos(a)\cos(c)\sin(a)\sin(c)+\cos(a)^2\cos(c)^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{2\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)(\sin(a)\cos(c)+\cos(a)\sin(c))}+\frac{\cos(a)\cos(c)-\sin(}{2\cos(a)^2\cos(c)^2+2\cos(c)^2\sin(a)^2+2\sin}\right.}{\frac{\cos(c)\sin(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{2}+\frac{\sin(c)\cos(a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{2}+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\cos(a)\cos(c)-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\sin(a)\sin(c)-\frac{\sin(a)\cos(c)}{2}}$

input
```
int(cos(b*x+a)*csc(b*x-c)^2,x,method=_RETURNVERBOSE)
```

output
```
I/b/(exp(2*I*(a+c))-exp(2*I*(b*x+a)))*(exp(I*(b*x+3*a+2*c))+exp(I*(b*x+a))
)+ln(exp(I*(a+c))+exp(I*(b*x+a)))/b*sin(a+c)-ln(-exp(I*(a+c))+exp(I*(b*x+a
)))/b*sin(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(35) = 70$.

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 5.34

$$\int \cos(a+bx)\csc^2(c-bx)\,dx$$
$$=\frac{\left(\cos\left(a+c\right)\sin\left(bx+a\right)\sin\left(a+c\right)+\left(\cos\left(a+c\right)^2-1\right)\cos\left(bx+a\right)\right)\log\left(\frac{\cos(bx+a)\cos(a+c)+\sin(bx+a)\sin(a+c)}{\cos(a+c)+1}\right.}{2\left(b\cos\left(a+\right.\right.}$$

input
```
integrate(cos(b*x+a)*csc(b*x-c)^2,x, algorithm="fricas")
```

output
```
1/2*((cos(a + c)*sin(b*x + a)*sin(a + c) + (cos(a + c)^2 - 1)*cos(b*x + a)
)*log((cos(b*x + a)*cos(a + c) + sin(b*x + a)*sin(a + c) + 1)/(cos(a + c)
+ 1)) - (cos(a + c)*sin(b*x + a)*sin(a + c) + (cos(a + c)^2 - 1)*cos(b*x +
 a))*log(-(cos(b*x + a)*cos(a + c) + sin(b*x + a)*sin(a + c) - 1)/(cos(a +
 c) + 1)) - 2*cos(a + c))/(b*cos(a + c)*sin(b*x + a) - b*cos(b*x + a)*sin(
a + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1572 vs. $2(27) = 54$.

Time = 59.68 (sec) , antiderivative size = 3264, normalized size of antiderivative = 102.00

$$\int \cos(a+bx)\csc^2(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x-c)**2,x)
```

output
```
-Piecewise((0, Eq(b, 0) & Eq(c, 0)), (log(tan(b*x/2))/b, Eq(c, 0)), (0, Eq
(b, 0)), (log(-tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(-b*tan(c/2)*
*4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*t
an(b*x/2)**2 - b*tan(c/2) + b*tan(b*x/2)) - log(-tan(c/2) + tan(b*x/2))*ta
n(c/2)**3*tan(b*x/2)**2/(-b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x
/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) + b*tan(b*x
/2)) + log(-tan(c/2) + tan(b*x/2))*tan(c/2)**3/(-b*tan(c/2)**4*tan(b*x/2)
+ b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 -
 b*tan(c/2) + b*tan(b*x/2)) - 2*log(-tan(c/2) + tan(b*x/2))*tan(c/2)**2*ta
n(b*x/2)/(-b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(
c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) + b*tan(b*x/2)) + log(-tan
(c/2) + tan(b*x/2))*tan(c/2)*tan(b*x/2)**2/(-b*tan(c/2)**4*tan(b*x/2) + b*
tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*t
an(c/2) + b*tan(b*x/2)) - log(-tan(c/2) + tan(b*x/2))*tan(c/2)/(-b*tan(c/2
)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)
*tan(b*x/2)**2 - b*tan(c/2) + b*tan(b*x/2)) + log(-tan(c/2) + tan(b*x/2))*
tan(b*x/2)/(-b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*ta
n(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) + b*tan(b*x/2)) - log(ta
n(b*x/2) + 1/tan(c/2))*tan(c/2)**4*tan(b*x/2)/(-b*tan(c/2)**4*tan(b*x/2) +
 b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(35) = 70$.

Time = 0.06 (sec) , antiderivative size = 457, normalized size of antiderivative = 14.28

$$\int \cos(a+bx)\csc^2(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x-c)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(sin(b*x + 2*a + 2*c) + sin(b*x))*cos(2*b*x + a) + (cos(2*b*x + a)^
2*sin(a + c) - 2*cos(2*b*x + a)*cos(a + 2*c)*sin(a + c) + cos(a + 2*c)^2*s
in(a + c) + sin(2*b*x + a)^2*sin(a + c) - 2*sin(2*b*x + a)*sin(a + 2*c)*si
n(a + c) + sin(a + 2*c)^2*sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) +
 cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*b*x + a)^2
*sin(a + c) - 2*cos(2*b*x + a)*cos(a + 2*c)*sin(a + c) + cos(a + 2*c)^2*si
n(a + c) + sin(2*b*x + a)^2*sin(a + c) - 2*sin(2*b*x + a)*sin(a + 2*c)*sin
(a + c) + sin(a + 2*c)^2*sin(a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) +
cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - 2*(cos(b*x + 2*a +
 2*c) + cos(b*x))*sin(2*b*x + a) - 2*cos(a + 2*c)*sin(b*x + 2*a + 2*c) - 2
*cos(a + 2*c)*sin(b*x) + 2*cos(b*x + 2*a + 2*c)*sin(a + 2*c) + 2*cos(b*x)*
sin(a + 2*c))/(b*cos(2*b*x + a)^2 - 2*b*cos(2*b*x + a)*cos(a + 2*c) + b*co
s(a + 2*c)^2 + b*sin(2*b*x + a)^2 - 2*b*sin(2*b*x + a)*sin(a + 2*c) + b*si
n(a + 2*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 982 vs. $2(35) = 70$.

Time = 0.19 (sec) , antiderivative size = 982, normalized size of antiderivative = 30.69

$$\int \cos(a+bx)\csc^2(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x-c)^2,x, algorithm="giac")
```

output
```
1/2*(4*(tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^3 - 2*tan(1/2*
a)^2*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) + tan(1/2*c))*log
(abs(tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + t
an(1/2*a) + tan(1/2*c)))/(tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2
*c) - tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^2 +
 tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 - 1) - 4*(tan(1/2*a)^3*tan(1/2*c) +
2*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^2 - 2*t
an(1/2*a)*tan(1/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x + 1/2*a)*tan(1/2*
a) + tan(1/2*b*x + 1/2*a)*tan(1/2*c) - tan(1/2*a)*tan(1/2*c) + 1))/(tan(1/
2*a)^3*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3 + tan(1/2*a
)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a) + tan
(1/2*c)) + (tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*b*x
 + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)^2 - 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^3*
tan(1/2*c)^3 + 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(1/2*b*x + 1/2*a)*tan(1/
2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*b*x + 1/2*a)*t
an(1/2*a)^4 + 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/2*c) - 2*tan(1/2*a
)^4*tan(1/2*c) + 20*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^2 - 12*ta
n(1/2*a)^3*tan(1/2*c)^2 + 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^3 -
 12*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*b*x + 1/2*a)*tan(1/2*c)^4 - 2*tan(
1/2*a)*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2 + 2*tan(1/2*a...
```

Mupad [B] (verification not implemented)

Time = 24.68 (sec) , antiderivative size = 252, normalized size of antiderivative = 7.88

$$\int \cos(a+bx)\csc^2(c-bx)\,dx = -\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{c\,2i}-1\right)-\frac{e^{a\,2i}\,e^{c\,2i}\,\left(e^{a\,2i}\,e^{c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{c\,2i}}}\right)\,\left(e^{a\,2i+c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i+c\,2i}}} + \frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{c\,2i}-1\right)+\frac{e^{a\,2i}\,e^{c\,2i}\,\left(e^{a\,2i}\,e^{c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{c\,2i}}}\right)\,\left(e^{a\,2i+c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i+c\,2i}}} + \frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i+c\,2i}+1\right)\,1i}{b\,\left(e^{a\,2i+c\,2i}-e^{a\,2i+b\,x\,2i}\right)}$$

input
```
int(cos(a + b*x)/sin(c - b*x)^2,x)
```

output
```
(log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(c*2i) - 1) + (exp(a*2i)*exp(c*2i
)*(exp(a*2i)*exp(c*2i) - 1)*1i)/(-exp(a*2i)*exp(c*2i))^(1/2))*(exp(a*2i +
c*2i) - 1))/(2*b*(-exp(a*2i + c*2i))^(1/2)) - (log(exp(a*1i)*exp(b*x*1i)*(
exp(a*2i)*exp(c*2i) - 1) - (exp(a*2i)*exp(c*2i)*(exp(a*2i)*exp(c*2i) - 1)*
1i)/(-exp(a*2i)*exp(c*2i))^(1/2))*(exp(a*2i + c*2i) - 1))/(2*b*(-exp(a*2i
+ c*2i))^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i + c*2i) + 1)*1i)/(b*(exp(a
*2i + c*2i) - exp(a*2i + b*x*2i)))
```

Reduce [F]

$$\int \cos(a+bx)\csc^2(c-bx)\,dx = \int \cos(bx+a)\csc(bx-c)^2\,dx$$

input
```
int(cos(b*x+a)*csc(b*x-c)^2,x)
```

output
```
int(cos(a + b*x)*csc(b*x - c)**2,x)
```

3.393 $\int \cos(a+bx)\csc^3(c-bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \cos(a+bx)\csc^3(c-bx)\,dx=\frac{\cos(a+c)\csc^2(c-bx)}{2b}+\frac{\cot(c-bx)\sin(a+c)}{b}$$

output
```
1/2*cos(a+c)*csc(b*x-c)^2/b-cot(b*x-c)*sin(a+c)/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \cos(a+bx)\csc^3(c-bx)\,dx=-\frac{\csc(c)\csc^2(c-bx)(\sin(a)-\cos(c-2bx)\sin(a+c))}{2b}$$

input
```
Integrate[Cos[a + b*x]*Csc[c - b*x]^3,x]
```

output
```
-1/2*(Csc[c]*Csc[c - b*x]^2*(Sin[a] - Cos[c - 2*b*x]*Sin[a + c]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^3(c-bx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos(a+bx)\csc^3(c-bx)dx$$

input
```
Int[Cos[a + b*x]*Csc[c - b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

method	result	size
parallelrisch	$\frac{\sec\left(\frac{bx}{2}-\frac{c}{2}\right)^2\csc\left(\frac{bx}{2}-\frac{c}{2}\right)^2\cos(2bx+a-c)}{8b}$	38
default	$\frac{1}{2b(\cos(a)\cos(c)-\sin(a)\sin(c))(\tan(bx+a)\cos(a)\cos(c)-\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)-\cos(a)\sin(c))^2}$	58
risch	$\frac{e^{5i(a+c)}-2\,e^{i(2bx+5a+3c)}-e^{3i(a+c)}}{\left(e^{2i(a+c)}-e^{2i(bx+a)}\right)^2 b}$	60

input
```
int(-cos(b*x+a)*csc(b*x-c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*sec(1/2*b*x-1/2*c)^2*csc(1/2*b*x-1/2*c)^2*cos(2*b*x+a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(37) = 74.

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.42

$$\int \cos(a+bx)\csc^3(c-bx)\,dx$$
$$= \frac{2\left(2\cos(a+c)^2-1\right)\cos(bx+a)\sin(bx+a)\sin(a+c)+4\left(\cos(a+c)^3-\cos(a+c)\right)\cos(bx+a)^2-}{2\left(2b\cos(bx+a)\cos(a+c)\sin(bx+a)\sin(a+c)+\left(2b\cos(a+c)^2-b\right)\cos(bx+a)^2-\right.}$$

input
```
integrate(-cos(b*x+a)*csc(b*x-c)^3,x, algorithm="fricas")
```

output
```
1/2*(2*(2*cos(a + c)^2 - 1)*cos(b*x + a)*sin(b*x + a)*sin(a + c) + 4*(cos(
a + c)^3 - cos(a + c))*cos(b*x + a)^2 - 2*cos(a + c)^3 + cos(a + c))/(2*b*
cos(b*x + a)*cos(a + c)*sin(b*x + a)*sin(a + c) + (2*b*cos(a + c)^2 - b)*c
os(b*x + a)^2 - b*cos(a + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^3(c-bx)\,dx = \text{Timed out}$$

input
```
integrate(-cos(b*x+a)*csc(b*x-c)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(37) = 74$.

Time = 0.04 (sec) , antiderivative size = 432, normalized size of antiderivative = 12.00

$$\int \cos(a+bx)\csc^3(c-bx)\,dx = -\frac{(2\cos(2bx+2a+3c)-\cos(2a+5c)+\cos(3c))\cos(4bx+a)-2(2\cos(2bx+a+2c)-\cos(a}{b\cos(4bx+a)^2+4b\cos}$$

input
```
integrate(-cos(b*x+a)*csc(b*x-c)^3,x, algorithm="maxima")
```

output
```
-((2*cos(2*b*x + 2*a + 3*c) - cos(2*a + 5*c) + cos(3*c))*cos(4*b*x + a) -
2*(2*cos(2*b*x + a + 2*c) - cos(a + 4*c))*cos(2*b*x + 2*a + 3*c) + 2*(cos(
2*a + 5*c) - cos(3*c))*cos(2*b*x + a + 2*c) - cos(2*a + 5*c)*cos(a + 4*c)
+ cos(a + 4*c)*cos(3*c) + (2*sin(2*b*x + 2*a + 3*c) - sin(2*a + 5*c) + sin
(3*c))*sin(4*b*x + a) - 2*(2*sin(2*b*x + a + 2*c) - sin(a + 4*c))*sin(2*b*
x + 2*a + 3*c) + 2*(sin(2*a + 5*c) - sin(3*c))*sin(2*b*x + a + 2*c) - sin(
2*a + 5*c)*sin(a + 4*c) + sin(a + 4*c)*sin(3*c))/(b*cos(4*b*x + a)^2 + 4*b
*cos(2*b*x + a + 2*c)^2 - 4*b*cos(2*b*x + a + 2*c)*cos(a + 4*c) + b*cos(a
+ 4*c)^2 + b*sin(4*b*x + a)^2 + 4*b*sin(2*b*x + a + 2*c)^2 - 4*b*sin(2*b*x
 + a + 2*c)*sin(a + 4*c) + b*sin(a + 4*c)^2 - 2*(2*b*cos(2*b*x + a + 2*c)
- b*cos(a + 4*c))*cos(4*b*x + a) - 2*(2*b*sin(2*b*x + a + 2*c) - b*sin(a +
 4*c))*sin(4*b*x + a))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(37) = 74$.

Time = 0.16 (sec) , antiderivative size = 327, normalized size of antiderivative = 9.08

$$\int \cos(a+bx)\csc^3(c-bx)\,dx = \frac{\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^2+9\tan}{2\left(\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2-4\tan(bx+a)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right.\right.}$$

input
```
integrate(-cos(b*x+a)*csc(b*x-c)^3,x, algorithm="giac")
```

output
```
1/2*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a
)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)
^4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c
)^2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(
1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 +
 1)/((tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2 -
 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) + 2*tan(1/2*a)^2*tan(1/2*c) - tan(b*
x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^2 + tan(b*x + a) - 2*tan(1/2
*a) - 2*tan(1/2*c))^2*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 - 4*tan(1/
2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^3(c-bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/sin(c - b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int \cos(a+bx)\csc^3(c-bx)\,dx = \frac{\cos(bx-c)\cos(bx+a)-\sin(bx-c)\sin(bx+a)}{2\sin(bx-c)^2 b}$$

input
```
int(-cos(b*x+a)*csc(b*x-c)^3,x)
```

output
```
(cos(b*x - c)*cos(a + b*x) - sin(b*x - c)*sin(a + b*x))/(2*sin(b*x - c)**2
*b)
```

3.394 $\int \cos(a+bx)\csc^4(c-bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 60

$$\int \cos(a+bx)\csc^4(c-bx)\,dx=\frac{\cos(a+c)\csc^3(c-bx)}{3b}+\frac{1}{2}\left(\frac{\operatorname{arctanh}(\cos(c-bx))}{b}+\frac{\cot(c-bx)\csc(c-bx)}{b}\right)\sin(a+c)$$

output
```
-1/3*cos(a+c)*csc(b*x-c)^3/b+1/2*(arctanh(cos(b*x-c))/b+cot(b*x-c)*csc(b*x
-c)/b)*sin(a+c)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\csc^4(c-bx)\,dx = \frac{12\operatorname{arctanh}\left(\cos(c)+\sin(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a+c)+\csc^3(c-bx)(4\cos(a+c)+3\sin(a+c)\sin(2(c-bx)))}{12b}$$

input
```
Integrate[Cos[a + b*x]*Csc[c - b*x]^4,x]
```

output
```
(12*ArcTanh[Cos[c] + Sin[c]*Tan[(b*x)/2]]*Sin[a + c] + Csc[c - b*x]^3*(4*C
os[a + c] + 3*Sin[a + c]*Sin[2*(c - b*x)]))/(12*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^4(c-bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \cos(a+bx)\csc^4(c-bx)dx$$

input
```
Int[Cos[a + b*x]*Csc[c - b*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.69 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.00

method	result
risch	$\frac{i\left(3\,\mathrm{e}^{i(bx+7a+6c)}-8\,\mathrm{e}^{i(3bx+7a+4c)}-3\,\mathrm{e}^{i(bx+5a+4c)}-3\,\mathrm{e}^{i(5bx+7a+2c)}-8\,\mathrm{e}^{i(3bx+5a+2c)}+3\,\mathrm{e}^{5i(bx+a)}\right)}{6b\left(\mathrm{e}^{2i(a+c)}-\mathrm{e}^{2i(bx+a)}\right)^{3}}+\frac{\ln\left(\mathrm{e}^{i(a+c)}+\mathrm{e}^{i(bx+a)}\right)\sin(a-}{2b}$
default	Expression too large to display

input
```
int(cos(b*x+a)*csc(b*x-c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*I/b/(exp(2*I*(a+c))-exp(2*I*(b*x+a)))^3*(3*exp(I*(b*x+7*a+6*c))-8*exp(
I*(3*b*x+7*a+4*c))-3*exp(I*(b*x+5*a+4*c))-3*exp(I*(5*b*x+7*a+2*c))-8*exp(I
*(3*b*x+5*a+2*c))+3*exp(5*I*(b*x+a)))+1/2*ln(exp(I*(a+c))+exp(I*(b*x+a)))/
b*sin(a+c)-1/2*ln(-exp(I*(a+c))+exp(I*(b*x+a)))/b*sin(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(60) = 120$.

Time = 0.11 (sec) , antiderivative size = 420, normalized size of antiderivative = 7.00

$$\int \cos(a+bx)\csc^4(c-bx)\,dx$$
$$= \frac{6\left(2\cos\left(a+c\right)^2-1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(a+c\right)+12\left(\cos\left(a+c\right)^3-\cos\left(a+c\right)\right)\cos\left(bx+a\right)^2}{}$$

input
```
integrate(cos(b*x+a)*csc(b*x-c)^4,x, algorithm="fricas")
```

output
```
1/12*(6*(2*cos(a + c)^2 - 1)*cos(b*x + a)*sin(b*x + a)*sin(a + c) + 12*(co
s(a + c)^3 - cos(a + c))*cos(b*x + a)^2 - 6*cos(a + c)^3 - 3*((4*cos(a + c
)^4 - 5*cos(a + c)^2 + 1)*cos(b*x + a)^3 + ((4*cos(a + c)^3 - 3*cos(a + c)
)*cos(b*x + a)^2 - cos(a + c)^3)*sin(b*x + a)*sin(a + c) - 3*(cos(a + c)^4
 - cos(a + c)^2)*cos(b*x + a))*log((cos(b*x + a)*cos(a + c) + sin(b*x + a)
*sin(a + c) + 1)/(cos(a + c) + 1)) + 3*((4*cos(a + c)^4 - 5*cos(a + c)^2 +
 1)*cos(b*x + a)^3 + ((4*cos(a + c)^3 - 3*cos(a + c))*cos(b*x + a)^2 - cos
(a + c)^3)*sin(b*x + a)*sin(a + c) - 3*(cos(a + c)^4 - cos(a + c)^2)*cos(b
*x + a))*log(-(cos(b*x + a)*cos(a + c) + sin(b*x + a)*sin(a + c) - 1)/(cos
(a + c) + 1)) + 2*cos(a + c))/((b*cos(a + c)^3 - (4*b*cos(a + c)^3 - 3*b*c
os(a + c))*cos(b*x + a)^2)*sin(b*x + a) + ((4*b*cos(a + c)^2 - b)*cos(b*x
+ a)^3 - 3*b*cos(b*x + a)*cos(a + c)^2)*sin(a + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^4(c-bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*csc(b*x-c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1794 vs. $2(60) = 120$.

Time = 0.09 (sec) , antiderivative size = 1794, normalized size of antiderivative = 29.90

$$\int \cos(a+bx)\csc^4(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x-c)^4,x, algorithm="maxima")
```

output
```
1/12*(2*(3*sin(5*b*x) - 3*sin(5*b*x + 2*a + 2*c) - 8*sin(3*b*x + 2*a + 4*c
) - 8*sin(3*b*x + 2*c) + 3*sin(b*x + 2*a + 6*c) - 3*sin(b*x + 4*c))*cos(6*
b*x + a) - 6*(3*sin(4*b*x + a + 2*c) - 3*sin(2*b*x + a + 4*c) + sin(a + 6*
c))*cos(5*b*x + 2*a + 2*c) - 6*(3*sin(5*b*x) - 8*sin(3*b*x + 2*a + 4*c) -
8*sin(3*b*x + 2*c) + 3*sin(b*x + 2*a + 6*c) - 3*sin(b*x + 4*c))*cos(4*b*x
+ a + 2*c) + 16*(3*sin(2*b*x + a + 4*c) - sin(a + 6*c))*cos(3*b*x + 2*a +
4*c) + 16*(3*sin(2*b*x + a + 4*c) - sin(a + 6*c))*cos(3*b*x + 2*c) + 18*(s
in(5*b*x) + sin(b*x + 2*a + 6*c) - sin(b*x + 4*c))*cos(2*b*x + a + 4*c) +
3*(cos(6*b*x + a)^2*sin(a + c) + 9*cos(4*b*x + a + 2*c)^2*sin(a + c) + 9*c
os(2*b*x + a + 4*c)^2*sin(a + c) - 6*cos(2*b*x + a + 4*c)*cos(a + 6*c)*sin
(a + c) + cos(a + 6*c)^2*sin(a + c) + sin(6*b*x + a)^2*sin(a + c) + 9*sin(
4*b*x + a + 2*c)^2*sin(a + c) + 9*sin(2*b*x + a + 4*c)^2*sin(a + c) - 6*si
n(2*b*x + a + 4*c)*sin(a + 6*c)*sin(a + c) + sin(a + 6*c)^2*sin(a + c) - 2
*(3*cos(4*b*x + a + 2*c)*sin(a + c) - 3*cos(2*b*x + a + 4*c)*sin(a + c) +
cos(a + 6*c)*sin(a + c))*cos(6*b*x + a) - 6*(3*cos(2*b*x + a + 4*c)*sin(a
+ c) - cos(a + 6*c)*sin(a + c))*cos(4*b*x + a + 2*c) - 2*(3*sin(4*b*x + a
+ 2*c)*sin(a + c) - 3*sin(2*b*x + a + 4*c)*sin(a + c) + sin(a + 6*c)*sin(a
 + c))*sin(6*b*x + a) - 6*(3*sin(2*b*x + a + 4*c)*sin(a + c) - sin(a + 6*c
)*sin(a + c))*sin(4*b*x + a + 2*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + c
os(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 3*(cos(6*b*x + a...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11496 vs. $2(60) = 120$.

Time = 1.06 (sec) , antiderivative size = 11496, normalized size of antiderivative = 191.60

$$\int \cos(a + bx) \csc^4(c - bx) \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(b*x-c)^4,x, algorithm="giac")
```

output
```
1/24*(24*(tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^3 - 2*tan(1/
2*a)^2*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) + tan(1/2*c))*l
og(abs(tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) +
 tan(1/2*a) + tan(1/2*c)))/(tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1
/2*c) - tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^2
 + tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 - 1) - 24*(tan(1/2*a)^3*tan(1/2*c)
 + 2*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^2 -
2*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x + 1/2*a)*tan(1
/2*a) + tan(1/2*b*x + 1/2*a)*tan(1/2*c) - tan(1/2*a)*tan(1/2*c) + 1))/(tan
(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3 + tan(1/
2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a) +
tan(1/2*c)) + (3*tan(1/2*b*x + 1/2*a)^5*tan(1/2*a)^12*tan(1/2*c)^10 + 6*ta
n(1/2*b*x + 1/2*a)^5*tan(1/2*a)^11*tan(1/2*c)^11 - 3*tan(1/2*b*x + 1/2*a)^
4*tan(1/2*a)^12*tan(1/2*c)^11 + 3*tan(1/2*b*x + 1/2*a)^5*tan(1/2*a)^10*tan
(1/2*c)^12 - 3*tan(1/2*b*x + 1/2*a)^4*tan(1/2*a)^11*tan(1/2*c)^12 + tan(1/
2*b*x + 1/2*a)^3*tan(1/2*a)^12*tan(1/2*c)^12 + 12*tan(1/2*b*x + 1/2*a)^5*t
an(1/2*a)^12*tan(1/2*c)^8 + 18*tan(1/2*b*x + 1/2*a)^5*tan(1/2*a)^11*tan(1/
2*c)^9 - 9*tan(1/2*b*x + 1/2*a)^4*tan(1/2*a)^12*tan(1/2*c)^9 + 12*tan(1/2*
b*x + 1/2*a)^5*tan(1/2*a)^10*tan(1/2*c)^10 + 6*tan(1/2*b*x + 1/2*a)^4*tan(
1/2*a)^11*tan(1/2*c)^10 - 2*tan(1/2*b*x + 1/2*a)^3*tan(1/2*a)^12*tan(1/...
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^4(c-bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)/sin(c - b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos(a+bx)\csc^4(c-bx)\,dx$$

$$= \frac{-24\cos(bx-c)\cos(bx+a)+20\cos(bx-c)\sin(bx-c)^2+4\cos(bx-c)\sin(bx-c)\sin(bx+a)-8}{}$$

input
```
int(cos(b*x+a)*csc(b*x-c)^4,x)
```

output
```
( - 24*cos(b*x - c)*cos(a + b*x) + 20*cos(b*x - c)*sin(b*x - c)**2 + 4*cos
(b*x - c)*sin(b*x - c)*sin(a + b*x) - 8*cos(b*x - c) + 4*cos(a + b*x)*sin(
b*x - c)**2 - 8*cos(a + b*x) - 12*int((tan((b*x - c)/2)**2*tan((a + b*x)/2
)**2)/(tan((a + b*x)/2)**2 + 1),x)*sin(b*x - c)**3*b - 6*int(1/(tan((b*x -
 c)/2)**4*tan((a + b*x)/2)**2 + tan((b*x - c)/2)**4),x)*sin(b*x - c)**3*b
+ 9*sin(b*x - c)**3*sin(a + b*x) + 24*sin(b*x - c)**3*tan((b*x - c)/2) - 3
*sin(b*x - c)**3*b*x + 12*sin(b*x - c)*sin(a + b*x) - 8)/(48*sin(b*x - c)*
*3*b)
```

3.395 $\int \cos^2(a+bx)\csc(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \cos^2(a+bx)\csc(c+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+bx))\cos^2(a-c)}{b}+\frac{\cos(2a-c+bx)}{b}$$

output
```
-arctanh(cos(b*x+c))*cos(a-c)^2/b+cos(b*x+2*a-c)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int \cos^2(a+bx)\csc(c+bx)\,dx = \frac{\cos(2a-c+bx)+\cos^2(a-c)\left(-\log\left(\cos\left(\frac{1}{2}(c+bx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+bx)\right)\right)\right)}{b}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c + b*x],x]
```

output
```
(Cos[2*a - c + b*x] + Cos[a - c]^2*(-Log[Cos[(c + b*x)/2]] + Log[Sin[(c +
b*x)/2]]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \cos^2(a+bx)\csc(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Csc[c + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.81

method	result
risch	$\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)}{2b}+\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos(2a-2c)}{2b}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)}{2b}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos(2a-2c)}{2b}+\text{co}$
default	$\frac{-\frac{2\left((\sin(a)\cos(c)-\cos(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin(a)\sin(c)-\cos(a)\cos(c)\right)}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)}+\frac{2\left(-\sin(a)^2\sin(c)^2-2\cos(a)\cos(c)\sin(a)\sin(c)-\cos(\right.}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(\right.}}{b}$

input
```
int(cos(b*x+a)^2*csc(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))+1/2/b*ln(exp(I*(b*x+a))-exp(I*(a-c))
)*cos(2*a-2*c)-1/2/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))-1/2/b*ln(exp(I*(b*x+a
))+exp(I*(a-c)))*cos(2*a-2*c)+cos(b*x+2*a-c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.42

$$\int \cos^2(a+bx)\csc(c+bx)\,dx = -\frac{\cos(-a+c)^2\log\left(\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)-\cos(-a+c)^2\log\left(-\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)-4\cos(-a+c)\sin}{2b}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(cos(-a + c)^2*log(1/2*cos(b*x + c) + 1/2) - cos(-a + c)^2*log(-1/2*c
os(b*x + c) + 1/2) - 4*cos(-a + c)*sin(b*x + c)*sin(-a + c) - 2*(2*cos(-a
+ c)^2 - 1)*cos(b*x + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1459 vs. $2(27) = 54$.

Time = 10.02 (sec) , antiderivative size = 3215, normalized size of antiderivative = 89.31

$$\int \cos^2(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**2*csc(b*x+c),x)
```

output
```
-2*Piecewise((0, Eq(b, 0) & Eq(c, 0)), (sin(b*x)/b, Eq(c, 0)), (0, Eq(b, 0
)), (2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4
*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c
/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**3/(
b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**
2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(c/2) + tan(b*x/2))*
tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*
tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log
(tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)
**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 +
b) - 2*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)*
*4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan
(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)*
*3/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/
2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - 1/tan(
c/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4
+ 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) +
 2*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*
tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/
2)**2 + b) - 2*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(36) = 72$.

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.28

$$\int \cos^2(a+bx)\csc(c+bx)\,dx =$$
$$-\frac{(\cos(-2a+2c)+1)\log\left(\cos(bx)^2+2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2-2\sin(bx)\sin(c)+\sin$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c),x, algorithm="maxima")
```

output
```
-1/4*((cos(-2*a + 2*c) + 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2
+ sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(-2*a + 2*c) + 1)*log(c
os(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c)
+ sin(c)^2) - 4*cos(b*x + 2*a - c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 527, normalized size of antiderivative = 14.64

$$\int \cos^2(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c),x, algorithm="giac")
```

output
```
((tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*a)^3
*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 8*tan(1/2*a)^
3*tan(1/2*c) + 20*tan(1/2*a)^2*tan(1/2*c)^2 - 8*tan(1/2*a)*tan(1/2*c)^3 +
tan(1/2*c)^4 - 2*tan(1/2*a)^2 + 8*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*c)^2 +
 1)*log(abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(1/2*b*x
 + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) - 2*
tan(1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)
 - 2*tan(1/2*c)^2)/abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) - 2*
tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^2
 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c) +
 4*tan(1/2*a)*tan(1/2*c) + 2))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 +
 tan(1/2*c)^2 + 1)^2 - 2*(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) -
 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^
2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*a)^2 - 2*tan(1/2*b*x + 1/2
*a)*tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1)/((tan(1/2*a)^
2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b*x + 1/2*a)^2
+ 1)))/b
```

Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 223, normalized size of antiderivative = 6.19

$$\int \cos^2(a+bx)\csc(c+bx)\,dx$$
$$= \frac{e^{-a\,2i+c\,1i-b\,x\,1i}}{2\,b} + \frac{e^{a\,2i-c\,1i+b\,x\,1i}}{2\,b} - \frac{e^{-a\,2i+c\,2i}\ln\left(-\frac{\left(e^{a\,2i}\,e^{-c\,2i}+1\right)^2 1i}{2} - \frac{e^{c\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{-c\,2i}\,2i+e^{a\,4i}\,e^{-c\,4i}\,1i+1i\right)}{2}\right)\left(e^{a\,2i-c\,2i}+1\right)^2}{4\,b} + \frac{e^{-a\,2i+c\,2i}\ln\left(\frac{\left(e^{a\,2i}\,e^{-c\,2i}+1\right)^2 1i}{2} - \frac{e^{c\,1i}\,e^{b\,x\,1i}\left(e^{a\,2i}\,e^{-c\,2i}\,2i+e^{a\,4i}\,e^{-c\,4i}\,1i+1i\right)}{2}\right)\left(e^{a\,2i-c\,2i}+1\right)^2}{4\,b}$$

input
```
int(cos(a + b*x)^2/sin(c + b*x),x)
```

output
```
exp(c*1i - a*2i - b*x*1i)/(2*b) + exp(a*2i - c*1i + b*x*1i)/(2*b) - (exp(c
*2i - a*2i)*log(- ((exp(a*2i)*exp(-c*2i) + 1)^2*1i)/2 - (exp(c*1i)*exp(b*x
*1i)*(exp(a*2i)*exp(-c*2i)*2i + exp(a*4i)*exp(-c*4i)*1i + 1i))/2)*(exp(a*2
i - c*2i) + 1)^2)/(4*b) + (exp(c*2i - a*2i)*log(((exp(a*2i)*exp(-c*2i) + 1
)^2*1i)/2 - (exp(c*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*2i + exp(a*4i)*ex
p(-c*4i)*1i + 1i))/2)*(exp(a*2i - c*2i) + 1)^2)/(4*b)
```

Reduce [F]

$$\int \cos^2(a+bx)\csc(c+bx)\,dx = \int \cos(bx+a)^2 \csc(bx+c)\,dx$$

input
```
int(cos(b*x+a)^2*csc(b*x+c),x)
```

output
```
int(cos(a + b*x)**2*csc(b*x + c),x)
```

3.396 $\int \cos^2(a+bx)\csc^2(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = -x\cos(2(a-c)) - \frac{\cos^2(a-c)\cot(c+bx)}{b} - \frac{\log(\sin(c+bx))\sin(2(a-c))}{b}$$

output
```
-x*cos(2*a-2*c)-cos(a-c)^2*cot(b*x+c)/b-ln(sin(b*x+c))*sin(2*a-2*c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(51) = 102.

Time = 0.34 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.55

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = \frac{\csc(c)\csc(c+bx)(bx\cos(2a-4c-bx) - bx\cos(2a-2c-bx) + bx\cos(2a+bx) - bx\cos(2a-2c+bx}{}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c + b*x]^2,x]
```

output
```
(Csc[c]*Csc[c + b*x]*(b*x*Cos[2*a - 4*c - b*x] - b*x*Cos[2*a - 2*c - b*x]
+ b*x*Cos[2*a + b*x] - b*x*Cos[2*a - 2*c + b*x] + 2*Sin[b*x] + Log[Sin[c +
 b*x]]*Sin[2*a - 4*c - b*x] - Sin[2*a - 2*c - b*x] - Log[Sin[c + b*x]]*Sin
[2*a - 2*c - b*x] + Log[Sin[c + b*x]]*Sin[2*a + b*x] + Sin[2*a - 2*c + b*x
] - Log[Sin[c + b*x]]*Sin[2*a - 2*c + b*x]))/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^2(bx+c)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \cos^2(a+bx)\csc^2(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Csc[c + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.43

method	result
risch	$-x\,e^{2i(a-c)}+2i\sin\left(2a-2c\right)x+\frac{2i\sin(2a-2c)a}{b}+\frac{i e^{2i(2a-c)}}{2b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)}+\frac{i e^{2ia}}{b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)}+\frac{}{2b\left(-e^{2i(\right.}}$
default	$\frac{\frac{\left(-2\cos(a)^2\cos(c)\sin(c)+2\cos(c)^2\cos(a)\sin(a)-2\cos(a)\sin(a)\sin(c)^2+2\sin(a)^2\cos(c)\sin(c)\right)\ln\left(\tan(bx+a)^2+1\right)}{2}+\left(-\cos(a)^2\cos(c)^2+\cos(c)^2 s\right.}{\left(\cos(c)^2+\sin(c)^2\right)^2\left(\cos(a)^2+\sin(a)^2\right)^2}$

input
```
int(cos(b*x+a)^2*csc(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-x*exp(2*I*(a-c))+2*I*sin(2*a-2*c)*x+2*I/b*sin(2*a-2*c)*a+1/2*I/b/(-exp(2*
I*(b*x+a+c))+exp(2*I*a))*exp(2*I*(2*a-c))+I/b/(-exp(2*I*(b*x+a+c))+exp(2*I
*a))*exp(2*I*a)+1/2*I/b/(-exp(2*I*(b*x+a+c))+exp(2*I*a))*exp(2*I*c)-ln(exp
(2*I*(b*x+a))-exp(2*I*(a-c)))/b*sin(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx$$
$$=\frac{2\,\cos\left(-a+c\right)\log\left(\frac{1}{2}\,\sin\left(bx+c\right)\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-\cos\left(bx+c\right)\cos\left(-a+c\right)^2-\left(2\,bx\cos\left(-a\right.\right.}{b\sin\left(bx+c\right)}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^2,x, algorithm="fricas")
```

output
```
(2*cos(-a + c)*log(1/2*sin(b*x + c))*sin(b*x + c)*sin(-a + c) - cos(b*x +
c)*cos(-a + c)^2 - (2*b*x*cos(-a + c)^2 - b*x)*sin(b*x + c))/(b*sin(b*x +
c))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*csc(b*x+c)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(51) = 102$.

Time = 0.07 (sec) , antiderivative size = 711, normalized size of antiderivative = 13.94

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(b*cos(2*a + 2*c)*cos(4*c) + b*sin(2*a + 2*c)*sin(4*c))*x - (2*b*x
*cos(4*c) + sin(4*a) + 2*sin(2*a + 2*c) + sin(4*c))*cos(2*b*x + 2*a + 4*c)
 + 2*(b*x*cos(2*b*x + 2*a + 4*c) - b*x*cos(2*a + 2*c))*cos(2*b*x + 6*c) +
(sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (cos(2*b*x + 2*a + 4*c)^2*sin(-2*a
+ 2*c) - 2*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c)*sin(-2*a + 2*c) + cos(2*a
 + 2*c)^2*sin(-2*a + 2*c) + sin(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c) - 2*s
in(2*b*x + 2*a + 4*c)*sin(2*a + 2*c)*sin(-2*a + 2*c) + sin(2*a + 2*c)^2*si
n(-2*a + 2*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2
- 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c
) - 2*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c)*sin(-2*a + 2*c) + cos(2*a + 2*
c)^2*sin(-2*a + 2*c) + sin(2*b*x + 2*a + 4*c)^2*sin(-2*a + 2*c) - 2*sin(2*
b*x + 2*a + 4*c)*sin(2*a + 2*c)*sin(-2*a + 2*c) + sin(2*a + 2*c)^2*sin(-2*
a + 2*c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*s
in(b*x)*sin(c) + sin(c)^2) - (2*b*x*sin(4*c) - cos(4*a) - 2*cos(2*a + 2*c)
 - cos(4*c))*sin(2*b*x + 2*a + 4*c) + 2*(b*x*sin(2*b*x + 2*a + 4*c) - b*x*
sin(2*a + 2*c))*sin(2*b*x + 6*c) - (cos(4*a) + cos(4*c))*sin(2*a + 2*c))/(
b*cos(2*b*x + 2*a + 4*c)^2 - 2*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c) + b
*cos(2*a + 2*c)^2 + b*sin(2*b*x + 2*a + 4*c)^2 - 2*b*sin(2*b*x + 2*a + 4*c
)*sin(2*a + 2*c) + b*sin(2*a + 2*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2042 vs. $2(51) = 102$.

Time = 0.20 (sec) , antiderivative size = 2042, normalized size of antiderivative = 40.04

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^2,x, algorithm="giac")
```

output
```
-((tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*a)^4*tan(1/2*c)^2 + 16*tan(1/2*a)
^3*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 16*tan(1/2*
a)^3*tan(1/2*c) + 36*tan(1/2*a)^2*tan(1/2*c)^2 - 16*tan(1/2*a)*tan(1/2*c)^
3 + tan(1/2*c)^4 - 6*tan(1/2*a)^2 + 16*tan(1/2*a)*tan(1/2*c) - 6*tan(1/2*c
)^2 + 1)*(b*x + a)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^
2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4
*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*ta
n(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2
*c)^4 - tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)
^2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))*log(tan(b*x + a)^2 + 1)/(tan(
1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1
/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*ta
n(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) + 4*(tan(1/2*a)^6*tan(1/2*c)^5 - tan(1/2*
a)^5*tan(1/2*c)^6 - 2*tan(1/2*a)^6*tan(1/2*c)^3 + 11*tan(1/2*a)^5*tan(1/2*
c)^4 - 11*tan(1/2*a)^4*tan(1/2*c)^5 + 2*tan(1/2*a)^3*tan(1/2*c)^6 + tan(1/
2*a)^6*tan(1/2*c) - 11*tan(1/2*a)^5*tan(1/2*c)^2 + 38*tan(1/2*a)^4*tan(1/2
*c)^3 - 38*tan(1/2*a)^3*tan(1/2*c)^4 + 11*tan(1/2*a)^2*tan(1/2*c)^5 - tan(
1/2*a)*tan(1/2*c)^6 + tan(1/2*a)^5 - 11*tan(1/2*a)^4*tan(1/2*c) + 38*tan(1
/2*a)^3*tan(1/2*c)^2 - 38*tan(1/2*a)^2*tan(1/2*c)^3 + 11*tan(1/2*a)*tan...
```

Mupad [B] (verification not implemented)

Time = 20.99 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.92

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = -x\left(\cos\left(2a-2c\right)-\sin\left(2a-2c\right)\,1\mathrm{i}\right)+\frac{\left(2\,{\mathrm{e}}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+{\mathrm{e}}^{a\,4\mathrm{i}-c\,4\mathrm{i}}+1\right)\,1\mathrm{i}}{2\,b\,\left({\mathrm{e}}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-{\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)} - \frac{{\mathrm{e}}^{-a\,4\mathrm{i}+c\,4\mathrm{i}}\,\ln\left({\mathrm{e}}^{a\,2\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2\mathrm{i}}-{\mathrm{e}}^{a\,2\mathrm{i}}\,{\mathrm{e}}^{-c\,2\mathrm{i}}\right)\,\left(2\,b\,{\mathrm{e}}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-2\,b\,{\mathrm{e}}^{a\,6\mathrm{i}-c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$

input
```
int(cos(a + b*x)^2/sin(c + b*x)^2,x)
```

output
```
((2*exp(a*2i - c*2i) + exp(a*4i - c*4i) + 1)*1i)/(2*b*(exp(a*2i - c*2i) -
exp(a*2i + b*x*2i))) - x*(cos(2*a - 2*c) - sin(2*a - 2*c)*1i) - (exp(c*4i
- a*4i)*log(exp(a*2i)*exp(b*x*2i) - exp(a*2i)*exp(-c*2i))*(2*b*exp(a*2i -
c*2i) - 2*b*exp(a*6i - c*6i))*1i)/(4*b^2)
```

Reduce [F]

$$\int \cos^2(a+bx)\csc^2(c+bx)\,dx = \int \cos(bx+a)^2 \csc(bx+c)^2\,dx$$

input
```
int(cos(b*x+a)^2*csc(b*x+c)^2,x)
```

output
```
int(cos(a + b*x)**2*csc(b*x + c)**2,x)
```

3.397 $\int \cos^2(a+bx)\csc^3(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(c+bx))\cos^2(a-c)}{2b} + \frac{\operatorname{arctanh}(\cos(c+bx))\cos(2(a-c))}{b} - \frac{\cos^2(a-c)\cot(c+bx)\csc(c+bx)}{2b} + \frac{\csc(c+bx)\sin(2(a-c))}{b}$$

output
```
-1/2*arctanh(cos(b*x+c))*cos(a-c)^2/b+arctanh(cos(b*x+c))*cos(2*a-2*c)/b-1
/2*cos(a-c)^2*cot(b*x+c)*csc(b*x+c)/b+csc(b*x+c)*sin(2*a-2*c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 247 vs. 2(87) = 174.

Time = 2.73 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.84

$$
\begin{aligned}
&\int \cos^2(a+bx)\csc^3(c+bx)\,dx \\
&= \frac{\left(-\cos\left(2a-2c-\frac{bx}{2}\right)+\cos\left(2a-2c+\frac{bx}{2}\right)\right)\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{bx}{2}\right)}{4b} \\
&+\frac{(-1-\cos(2a-2c))\csc^2\left(\frac{c}{2}+\frac{bx}{2}\right)}{16b}+\frac{(-1+3\cos(2a-2c))\log\left(\cos\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{4b} \\
&+\frac{(1-3\cos(2a-2c))\log\left(\sin\left(\frac{c}{2}+\frac{bx}{2}\right)\right)}{4b} \\
&+\frac{\left(\cos\left(2a-2c-\frac{bx}{2}\right)-\cos\left(2a-2c+\frac{bx}{2}\right)\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{bx}{2}\right)}{4b} \\
&+\frac{(1+\cos(2a-2c))\sec^2\left(\frac{c}{2}+\frac{bx}{2}\right)}{16b}
\end{aligned}
$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c + b*x]^3,x]
```

output
```
((-Cos[2*a - 2*c - (b*x)/2] + Cos[2*a - 2*c + (b*x)/2])*Csc[c/2]*Csc[c/2 +
 (b*x)/2])/(4*b) + ((-1 - Cos[2*a - 2*c])*Csc[c/2 + (b*x)/2]^2)/(16*b) + (
(-1 + 3*Cos[2*a - 2*c])*Log[Cos[c/2 + (b*x)/2]])/(4*b) + ((1 - 3*Cos[2*a -
 2*c])*Log[Sin[c/2 + (b*x)/2]])/(4*b) + ((Cos[2*a - 2*c - (b*x)/2] - Cos[2
*a - 2*c + (b*x)/2])*Sec[c/2]*Sec[c/2 + (b*x)/2])/(4*b) + ((1 + Cos[2*a -
2*c])*Sec[c/2 + (b*x)/2]^2)/(16*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$
\int \cos^2(a+bx)\csc^3(bx+c)\,dx
\Big\downarrow 7299
\int \cos^2(a+bx)\csc^3(bx+c)dx
$$

input
```
Int[Cos[a + b*x]^2*Csc[c + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.14 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.79

method	result
risch	$-\frac{-5\,\mathrm{e}^{i(3bx+6a+c)}-2\,\mathrm{e}^{i(3bx+4a+3c)}+3\,\mathrm{e}^{i(3bx+2a+5c)}+3\,\mathrm{e}^{i(bx+6a-c)}-2\,\mathrm{e}^{i(bx+4a+c)}-5\,\mathrm{e}^{i(bx+2a+3c)}}{4\left(-\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)^{2}b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}-\mathrm{e}^{i(a-c)}\right)}{4b}-$
default	Expression too large to display

input
```
int(cos(b*x+a)^2*csc(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(-exp(2*I*(b*x+a+c))+exp(2*I*a))^2/b*(-5*exp(I*(3*b*x+6*a+c))-2*exp(I
*(3*b*x+4*a+3*c))+3*exp(I*(3*b*x+2*a+5*c))+3*exp(I*(b*x+6*a-c))-2*exp(I*(b
*x+4*a+c))-5*exp(I*(b*x+2*a+3*c)))+1/4/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))-3
/4/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))*cos(2*a-2*c)-1/4/b*ln(exp(I*(b*x+a))+
exp(I*(a-c)))+3/4/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))*cos(2*a-2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.68

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx$$
$$=\frac{2\cos(bx+c)\cos(-a+c)^{2}+8\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(\left(3\cos(-a+c)^{2}-2\right)\cos(bx+\right.}{}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(2*cos(b*x + c)*cos(-a + c)^2 + 8*cos(-a + c)*sin(b*x + c)*sin(-a + c)
 + ((3*cos(-a + c)^2 - 2)*cos(b*x + c)^2 - 3*cos(-a + c)^2 + 2)*log(1/2*co
s(b*x + c) + 1/2) - ((3*cos(-a + c)^2 - 2)*cos(b*x + c)^2 - 3*cos(-a + c)^
2 + 2)*log(-1/2*cos(b*x + c) + 1/2))/(b*cos(b*x + c)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*csc(b*x+c)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(83) = 166$.

Time $= 0.09$ (sec) , antiderivative size $= 1603$, normalized size of antiderivative $= 18.43$

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^3,x, algorithm="maxima")
```

output
```
1/8*(2*(5*cos(3*b*x + 4*a + 2*c) + 2*cos(3*b*x + 2*a + 4*c) - 3*cos(3*b*x
+ 6*c) - 3*cos(b*x + 4*a) + 2*cos(b*x + 2*a + 2*c) + 5*cos(b*x + 4*c))*cos
(4*b*x + 2*a + 5*c) - 10*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + c))*cos(3*b
*x + 4*a + 2*c) - 4*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + c))*cos(3*b*x +
2*a + 4*c) + 6*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + c))*cos(3*b*x + 6*c)
+ 4*(3*cos(b*x + 4*a) - 2*cos(b*x + 2*a + 2*c) - 5*cos(b*x + 4*c))*cos(2*b
*x + 2*a + 3*c) - 6*cos(b*x + 4*a)*cos(2*a + c) + 4*cos(b*x + 2*a + 2*c)*c
os(2*a + c) + 10*cos(b*x + 4*c)*cos(2*a + c) + ((3*cos(-2*a + 2*c) - 1)*co
s(4*b*x + 2*a + 5*c)^2 + 4*(3*cos(-2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c)^
2 + (3*cos(-2*a + 2*c) - 1)*sin(4*b*x + 2*a + 5*c)^2 + 4*(3*cos(-2*a + 2*c
) - 1)*sin(2*b*x + 2*a + 3*c)^2 - 2*(2*(3*cos(-2*a + 2*c) - 1)*cos(2*b*x +
 2*a + 3*c) - 3*cos(2*a + c)*cos(-2*a + 2*c) + cos(2*a + c))*cos(4*b*x + 2
*a + 5*c) - 4*(3*cos(2*a + c)*cos(-2*a + 2*c) - cos(2*a + c))*cos(2*b*x +
2*a + 3*c) - cos(2*a + c)^2 + 3*(cos(2*a + c)^2 + sin(2*a + c)^2)*cos(-2*a
 + 2*c) - 2*(2*(3*cos(-2*a + 2*c) - 1)*sin(2*b*x + 2*a + 3*c) - 3*cos(-2*a
 + 2*c)*sin(2*a + c) + sin(2*a + c))*sin(4*b*x + 2*a + 5*c) - 4*(3*cos(-2*
a + 2*c)*sin(2*a + c) - sin(2*a + c))*sin(2*b*x + 2*a + 3*c) - sin(2*a + c
)^2)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*
x)*sin(c) + sin(c)^2) - ((3*cos(-2*a + 2*c) - 1)*cos(4*b*x + 2*a + 5*c)^2
+ 4*(3*cos(-2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c)^2 + (3*cos(-2*a + 2*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5628 vs. $2(83) = 166$.

Time = 0.44 (sec) , antiderivative size = 5628, normalized size of antiderivative = 64.69

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/8*(4*(tan(1/2*a)^4*tan(1/2*c)^4 - 10*tan(1/2*a)^4*tan(1/2*c)^2 + 24*tan
(1/2*a)^3*tan(1/2*c)^3 - 10*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 - 24*
tan(1/2*a)^3*tan(1/2*c) + 52*tan(1/2*a)^2*tan(1/2*c)^2 - 24*tan(1/2*a)*tan
(1/2*c)^3 + tan(1/2*c)^4 - 10*tan(1/2*a)^2 + 24*tan(1/2*a)*tan(1/2*c) - 10
*tan(1/2*c)^2 + 1)*log(abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)
- 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*
tan(1/2*a) - 2*tan(1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c) + 4*tan(1/
2*a)*tan(1/2*c) - 2*tan(1/2*c)^2)/abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*
tan(1/2*c) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*a)
^2*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) - 2*tan(1/2*b*x + 1/2*
a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) + 2))/(tan(1/2*a)^2*tan(1/2*c)^2 +
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1)^2 + (2*tan(1/2*b*x + 1/2*a)^3*tan(1/2*a)
^10*tan(1/2*c)^9 - 2*tan(1/2*b*x + 1/2*a)^3*tan(1/2*a)^9*tan(1/2*c)^10 + t
an(1/2*b*x + 1/2*a)^2*tan(1/2*a)^10*tan(1/2*c)^10 + 12*tan(1/2*b*x + 1/2*a
)^3*tan(1/2*a)^10*tan(1/2*c)^7 - 18*tan(1/2*b*x + 1/2*a)^3*tan(1/2*a)^9*ta
n(1/2*c)^8 + 9*tan(1/2*b*x + 1/2*a)^2*tan(1/2*a)^10*tan(1/2*c)^8 + 18*tan(
1/2*b*x + 1/2*a)^3*tan(1/2*a)^8*tan(1/2*c)^9 - 8*tan(1/2*b*x + 1/2*a)^2*ta
n(1/2*a)^9*tan(1/2*c)^9 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^10*tan(1/2*c)^
9 - 12*tan(1/2*b*x + 1/2*a)^3*tan(1/2*a)^7*tan(1/2*c)^10 + 9*tan(1/2*b*x +
 1/2*a)^2*tan(1/2*a)^8*tan(1/2*c)^10 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a...
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx=\text{Hanged}$$

input
```
int(cos(a + b*x)^2/sin(c + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos^2(a+bx)\csc^3(c+bx)\,dx = \int \cos(bx+a)^2 \csc(bx+c)^3\,dx$$

input
```
int(cos(b*x+a)^2*csc(b*x+c)^3,x)
```

output
```
int(cos(a + b*x)**2*csc(b*x + c)**3,x)
```

3.398 $\int \cos^2(a+bx)\csc^4(c+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \cos^2(a+bx)\csc^4(c+bx)\,dx = -\frac{\cos^2(a-c)\cot(c+bx)}{b}+\frac{\cos(2(a-c))\cot(c+bx)}{b}-\frac{\cos^2(a-c)\cot^3(c+bx)}{3b}+\frac{\csc^2(c+bx)\sin(2(a-c))}{2b}$$

output
```
-cos(a-c)^2*cot(b*x+c)/b+cos(2*a-2*c)*cot(b*x+c)/b-1/3*cos(a-c)^2*cot(b*x+
c)^3/b+1/2*csc(b*x+c)^2*sin(2*a-2*c)/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \cos^2(a+bx)\csc^4(c+bx)\,dx$$
$$= \frac{\csc(c)\csc^3(c+bx)(3\sin(bx)-\sin(2a-4c-3bx)+3\sin(2a-2c-bx)-3\sin(2a+bx)+\sin(2a+3bx}{12b}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c + b*x]^4,x]
```

output
```
(Csc[c]*Csc[c + b*x]^3*(3*Sin[b*x] - Sin[2*a - 4*c - 3*b*x] + 3*Sin[2*a -
2*c - b*x] - 3*Sin[2*a + b*x] + Sin[2*a + 3*b*x] - Sin[2*c + 3*b*x]))/(12*
b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^4(bx+c)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \cos^2(a+bx)\csc^4(bx+c)dx$$

input
```
Int[Cos[a + b*x]^2*Csc[c + b*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 3.82 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{1}{3b(\cos(a)\cos(c)+\sin(a)\sin(c))(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))^3}$	55
parallelrisch	$\frac{\sec\left(\frac{bx}{2}+\frac{c}{2}\right)^3\csc\left(\frac{bx}{2}+\frac{c}{2}\right)^3(-3\cos(bx+c)-2\cos(3bx+2a+c)+\cos(3bx+3c))}{96b}$	58
risch	$-\frac{2i\left(3\,e^{2i(2bx+4a+c)}-3\,e^{2i(bx+4a)}+3\,e^{2i(bx+3a+c)}+e^{2i(4a-c)}-e^{6ia}+e^{2i(2a+c)}\right)}{3\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^3b}$	97

input
```
int(cos(b*x+a)^2*csc(b*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(cos(a)*cos(c)+sin(a)*sin(c))/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*
sin(a)*sin(c)-sin(a)*cos(c)+cos(a)*sin(c))^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \cos^2(a+bx)\csc^4(c+bx)\,dx = \frac{\left(4\cos\left(-a+c\right)^2-3\right)\cos\left(bx+c\right)^3+3\cos\left(-a+c\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-3\left(\cos\left(-a+c\right)^2-1\right)\cos}{3\left(b\cos\left(bx+c\right)^2-b\right)\sin\left(bx+c\right)}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*((4*cos(-a + c)^2 - 3)*cos(b*x + c)^3 + 3*cos(-a + c)*sin(b*x + c)*sin
(-a + c) - 3*(cos(-a + c)^2 - 1)*cos(b*x + c))/((b*cos(b*x + c)^2 - b)*sin
(b*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^4(c+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*csc(b*x+c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 908 vs. $2(80) = 160$.

Time = 0.05 (sec) , antiderivative size = 908, normalized size of antiderivative = 10.81

$$\int \cos^2(a+bx)\csc^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^4,x, algorithm="maxima")
```

output
```
-2/3*((3*sin(4*b*x + 4*a + 4*c) - 3*sin(2*b*x + 4*a + 2*c) + 3*sin(2*b*x +
 2*a + 4*c) + sin(4*a) - sin(2*a + 2*c) + sin(4*c))*cos(6*b*x + 2*a + 8*c)
 - 3*(3*sin(2*b*x + 2*a + 4*c) - sin(2*a + 2*c))*cos(4*b*x + 4*a + 4*c) -
3*(3*sin(4*b*x + 4*a + 4*c) - 3*sin(2*b*x + 4*a + 2*c) + 3*sin(2*b*x + 2*a
 + 4*c) + sin(4*a) - sin(2*a + 2*c) + sin(4*c))*cos(4*b*x + 2*a + 6*c) - 3
*(3*sin(2*b*x + 4*a + 2*c) - sin(4*a) - sin(4*c))*cos(2*b*x + 2*a + 4*c) -
 (sin(4*a) + sin(4*c))*cos(2*a + 2*c) - (3*cos(4*b*x + 4*a + 4*c) - 3*cos(
2*b*x + 4*a + 2*c) + 3*cos(2*b*x + 2*a + 4*c) + cos(4*a) - cos(2*a + 2*c)
+ cos(4*c))*sin(6*b*x + 2*a + 8*c) + 3*(3*cos(2*b*x + 2*a + 4*c) - cos(2*a
 + 2*c))*sin(4*b*x + 4*a + 4*c) + 3*(3*cos(4*b*x + 4*a + 4*c) - 3*cos(2*b*
x + 4*a + 2*c) + 3*cos(2*b*x + 2*a + 4*c) + cos(4*a) - cos(2*a + 2*c) + co
s(4*c))*sin(4*b*x + 2*a + 6*c) + 3*cos(2*a + 2*c)*sin(2*b*x + 4*a + 2*c) +
 3*(3*cos(2*b*x + 4*a + 2*c) - cos(4*a) - cos(4*c))*sin(2*b*x + 2*a + 4*c)
 + (cos(4*a) + cos(4*c))*sin(2*a + 2*c) - 3*cos(2*b*x + 4*a + 2*c)*sin(2*a
 + 2*c))/(b*cos(6*b*x + 2*a + 8*c)^2 + 9*b*cos(4*b*x + 2*a + 6*c)^2 + 9*b*
cos(2*b*x + 2*a + 4*c)^2 - 6*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 2*c) + b*c
os(2*a + 2*c)^2 + b*sin(6*b*x + 2*a + 8*c)^2 + 9*b*sin(4*b*x + 2*a + 6*c)^
2 + 9*b*sin(2*b*x + 2*a + 4*c)^2 - 6*b*sin(2*b*x + 2*a + 4*c)*sin(2*a + 2*
c) + b*sin(2*a + 2*c)^2 - 2*(3*b*cos(4*b*x + 2*a + 6*c) - 3*b*cos(2*b*x +
2*a + 4*c) + b*cos(2*a + 2*c))*cos(6*b*x + 2*a + 8*c) - 6*(3*b*cos(2*b*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(80) = 160$.

Time = 0.18 (sec) , antiderivative size = 441, normalized size of antiderivative = 5.25

$$\int \cos^2(a+bx)\csc^4(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+c)^4,x, algorithm="giac")
```

output
```
-1/3*(tan(1/2*a)^8*tan(1/2*c)^8 + 4*tan(1/2*a)^8*tan(1/2*c)^6 + 4*tan(1/2*
a)^6*tan(1/2*c)^8 + 6*tan(1/2*a)^8*tan(1/2*c)^4 + 16*tan(1/2*a)^6*tan(1/2*
c)^6 + 6*tan(1/2*a)^4*tan(1/2*c)^8 + 4*tan(1/2*a)^8*tan(1/2*c)^2 + 24*tan(
1/2*a)^6*tan(1/2*c)^4 + 24*tan(1/2*a)^4*tan(1/2*c)^6 + 4*tan(1/2*a)^2*tan(
1/2*c)^8 + tan(1/2*a)^8 + 16*tan(1/2*a)^6*tan(1/2*c)^2 + 36*tan(1/2*a)^4*t
an(1/2*c)^4 + 16*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*c)^8 + 4*tan(1/2*a)^6
 + 24*tan(1/2*a)^4*tan(1/2*c)^2 + 24*tan(1/2*a)^2*tan(1/2*c)^4 + 4*tan(1/2
*c)^6 + 6*tan(1/2*a)^4 + 16*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(1/2*c)^4 + 4
*tan(1/2*a)^2 + 4*tan(1/2*c)^2 + 1)/((tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)
^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2*
tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2
*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c))^3*(tan(1/2*a)^2*tan(1/
2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^4(c+bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)^2/sin(c + b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \cos^2(a+bx)\csc^4(c+bx)\,dx$$
$$= \frac{-\cos(bx+c)\sin(bx+c)^2+\cos(bx+c)\sin(bx+a)^2-\cos(bx+c)+\cos(bx+a)\sin(bx+c)\sin(bx}{3\sin(bx+c)^3 b}$$

input
```
int(cos(b*x+a)^2*csc(b*x+c)^4,x)
```

output
```
( - cos(b*x + c)*sin(b*x + c)**2 + cos(b*x + c)*sin(a + b*x)**2 - cos(b*x
+ c) + cos(a + b*x)*sin(b*x + c)*sin(a + b*x))/(3*sin(b*x + c)**3*b)
```

3.399 $\int \cos^2(a+bx)\csc(c-bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \cos^2(a+bx)\csc(c-bx)\,dx=\frac{\operatorname{arctanh}(\cos(c-bx))\cos^2(a+c)}{b}-\frac{\cos(2a+c+bx)}{b}$$

output
```
arctanh(cos(b*x-c))*cos(a+c)^2/b-cos(b*x+2*a+c)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

$$\int \cos^2(a+bx)\csc(c-bx)\,dx =\frac{-\cos(2a+c+bx)+\cos^2(a+c)\left(\log\left(\cos\left(\frac{1}{2}(c-bx)\right)\right)-\log\left(-\sin\left(\frac{1}{2}(c-bx)\right)\right)\right)}{b}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c - b*x],x]
```

output
```
(-Cos[2*a + c + b*x] + Cos[a + c]^2*(Log[Cos[(c - b*x)/2]] - Log[-Sin[(c -
 b*x)/2]]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc(c-bx)\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \cos^2(a+bx)\csc(c-bx)dx$$

input
```
Int[Cos[a + b*x]^2*Csc[c - b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.88

method	result
risch	$-\frac{\ln\left(-e^{i(a+c)}+e^{i(bx+a)}\right)}{2b}-\frac{\ln\left(-e^{i(a+c)}+e^{i(bx+a)}\right)\cos(2a+2c)}{2b}+\frac{\ln\left(e^{i(a+c)}+e^{i(bx+a)}\right)}{2b}+\frac{\ln\left(e^{i(a+c)}+e^{i(bx+a)}\right)\cos(2a+2c)}{2b}$
default	$-\frac{-\frac{2\left((\sin(a)\cos(c)+\cos(a)\sin(c))\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin(a)\sin(c)-\cos(a)\cos(c)\right)}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin(a)^2+\sin(a)^2\sin(c)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)}+\frac{2\left(\sin(a)^2\sin(c)^2-2\cos(a)\cos(c)\sin(a)\sin(c)+\cos(\right.}{\left(\cos(a)^2\cos(c)^2+\sin(c)^2\cos(a)^2+\cos(c)^2\sin\right.}}{b}$

input
```
int(-cos(b*x+a)^2*csc(b*x-c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*ln(-exp(I*(a+c))+exp(I*(b*x+a)))-1/2/b*ln(-exp(I*(a+c))+exp(I*(b*x+
a)))*cos(2*a+2*c)+1/2/b*ln(exp(I*(a+c))+exp(I*(b*x+a)))+1/2/b*ln(exp(I*(a+
c))+exp(I*(b*x+a)))*cos(2*a+2*c)-cos(b*x+2*a+c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.45

$$\int \cos^2(a+bx)\csc(c-bx)\,dx$$
$$= \frac{\cos(a+c)^2\log\left(\frac{\cos(bx+a)\cos(a+c)+\sin(bx+a)\sin(a+c)+1}{\cos(a+c)+1}\right) - \cos(a+c)^2\log\left(-\frac{\cos(bx+a)\cos(a+c)+\sin(bx+a)\sin(a+c)-}{\cos(a+c)+1}\right.}{2b}$$

input
```
integrate(-cos(b*x+a)^2*csc(b*x-c),x, algorithm="fricas")
```

output
```
1/2*(cos(a + c)^2*log((cos(b*x + a)*cos(a + c) + sin(b*x + a)*sin(a + c) +
 1)/(cos(a + c) + 1)) - cos(a + c)^2*log(-(cos(b*x + a)*cos(a + c) + sin(b
*x + a)*sin(a + c) - 1)/(cos(a + c) + 1)) - 2*cos(b*x + a)*cos(a + c) + 2*
sin(b*x + a)*sin(a + c))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1460 vs. $2(27) = 54$.

Time = 10.58 (sec) , antiderivative size = 3216, normalized size of antiderivative = 97.45

$$\int \cos^2(a+bx)\csc(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(-cos(b*x+a)**2*csc(b*x-c),x)
```

output
```
-2*Piecewise((0, Eq(b, 0) & Eq(c, 0)), (-sin(b*x)/b, Eq(c, 0)), (0, Eq(b,
0)), (2*log(-tan(c/2) + tan(b*x/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)*
*4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan
(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(-tan(c/2) + tan(b*x/2))*tan(c/2)**
3/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2
)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(-tan(c/2) + tan(b*x/
2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 +
2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2
*log(-tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan
(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)*
*2 + b) - 2*log(tan(b*x/2) + 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(
c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*
b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + 1/tan(c/2))*tan(
c/2)**3/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan
(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) + 1
/tan(c/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2
)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan(b*x/2)**2 +
 b) + 2*log(tan(b*x/2) + 1/tan(c/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2
 + b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 + 2*b*tan(c/2)**2 + b*tan
(b*x/2)**2 + b) + 2*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2)**2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(34) = 68$.

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.52

$$\int \cos^2(a+bx)\csc(c-bx)\,dx$$
$$= \frac{(\cos(2a+2c)+1)\log\left(\cos(bx)^2+2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2+2\sin(bx)\sin(c)+\sin(c)^2\right)}{}$$

input
```
integrate(-cos(b*x+a)^2*csc(b*x-c),x, algorithm="maxima")
```

output
```
1/4*((cos(2*a + 2*c) + 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 +
sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*a + 2*c) + 1)*log(cos(
b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + s
in(c)^2) - 4*cos(b*x + 2*a + c))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(34) = 68$.

Time = 0.21 (sec) , antiderivative size = 1043, normalized size of antiderivative = 31.61

$$\int \cos^2(a + bx) \csc(c - bx) \, dx = \text{Too large to display}$$

input
```
integrate(-cos(b*x+a)^2*csc(b*x-c),x, algorithm="giac")
```

output
```
-((tan(1/2*a)^5*tan(1/2*c)^5 - 2*tan(1/2*a)^5*tan(1/2*c)^3 - 9*tan(1/2*a)^
4*tan(1/2*c)^4 - 2*tan(1/2*a)^3*tan(1/2*c)^5 + tan(1/2*a)^5*tan(1/2*c) + 1
0*tan(1/2*a)^4*tan(1/2*c)^2 + 28*tan(1/2*a)^3*tan(1/2*c)^3 + 10*tan(1/2*a)
^2*tan(1/2*c)^4 + tan(1/2*a)*tan(1/2*c)^5 - tan(1/2*a)^4 - 10*tan(1/2*a)^3
*tan(1/2*c) - 28*tan(1/2*a)^2*tan(1/2*c)^2 - 10*tan(1/2*a)*tan(1/2*c)^3 -
tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 9*tan(1/2*a)*tan(1/2*c) + 2*tan(1/2*c)^2 -
 1)*log(abs(tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2
*a) + tan(1/2*a) + tan(1/2*c)))/(tan(1/2*a)^5*tan(1/2*c)^5 + 2*tan(1/2*a)^
5*tan(1/2*c)^3 - tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)^5 +
 tan(1/2*a)^5*tan(1/2*c) - 2*tan(1/2*a)^4*tan(1/2*c)^2 + 4*tan(1/2*a)^3*ta
n(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)*tan(1/2*c)^5 - tan(1
/2*a)^4 + 2*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(
1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 - 2*tan(1/2*a)^2 + tan(1/2*a)*tan(1/2*c
) - 2*tan(1/2*c)^2 - 1) - (tan(1/2*a)^5*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/
2*c)^5 - 2*tan(1/2*a)^5*tan(1/2*c)^2 - 10*tan(1/2*a)^4*tan(1/2*c)^3 - 10*t
an(1/2*a)^3*tan(1/2*c)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^5 + tan(1/2*a)^5 + 9*
tan(1/2*a)^4*tan(1/2*c) + 28*tan(1/2*a)^3*tan(1/2*c)^2 + 28*tan(1/2*a)^2*t
an(1/2*c)^3 + 9*tan(1/2*a)*tan(1/2*c)^4 + tan(1/2*c)^5 - 2*tan(1/2*a)^3 -
10*tan(1/2*a)^2*tan(1/2*c) - 10*tan(1/2*a)*tan(1/2*c)^2 - 2*tan(1/2*c)^3 +
 tan(1/2*a) + tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan...
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 223, normalized size of antiderivative = 6.76

$$\int \cos^2(a+bx)\csc(c-bx)\,dx$$
$$= -\frac{e^{-a\,2i-c\,1i-b\,x\,1i}}{2\,b}-\frac{e^{a\,2i+c\,1i+b\,x\,1i}}{2\,b}$$
$$-\frac{e^{-a\,2i-c\,2i}\ln\left(-\frac{\left(e^{a\,2i}\,e^{c\,2i}+1\right)^2\,1i}{2}+\frac{e^{-c\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{c\,2i}\,2i+e^{a\,4i}\,e^{c\,4i}\,1i+1i\right)}{2}\right)\left(e^{a\,2i+c\,2i}+1\right)^2}{4\,b}$$
$$+\frac{e^{-a\,2i-c\,2i}\ln\left(\frac{\left(e^{a\,2i}\,e^{c\,2i}+1\right)^2\,1i}{2}+\frac{e^{-c\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{c\,2i}\,2i+e^{a\,4i}\,e^{c\,4i}\,1i+1i\right)}{2}\right)\left(e^{a\,2i+c\,2i}+1\right)^2}{4\,b}$$

input
```
int(cos(a + b*x)^2/sin(c - b*x),x)
```

output
```
(exp(- a*2i - c*2i)*log(((exp(a*2i)*exp(c*2i) + 1)^2*1i)/2 + (exp(-c*1i)*e
xp(b*x*1i)*(exp(a*2i)*exp(c*2i)*2i + exp(a*4i)*exp(c*4i)*1i + 1i))/2)*(exp
(a*2i + c*2i) + 1)^2)/(4*b) - exp(a*2i + c*1i + b*x*1i)/(2*b) - (exp(- a*2
i - c*2i)*log((exp(-c*1i)*exp(b*x*1i)*(exp(a*2i)*exp(c*2i)*2i + exp(a*4i)*
exp(c*4i)*1i + 1i))/2 - ((exp(a*2i)*exp(c*2i) + 1)^2*1i)/2)*(exp(a*2i + c*
2i) + 1)^2)/(4*b) - exp(- a*2i - c*1i - b*x*1i)/(2*b)
```

Reduce [F]

$$\int \cos^2(a+bx)\csc(c-bx)\,dx = -\left(\int \cos(bx+a)^2\csc(bx-c)\,dx\right)$$

input
```
int(-cos(b*x+a)^2*csc(b*x-c),x)
```

output
```
- int(cos(a + b*x)**2*csc(b*x - c),x)
```

3.400 $\int \cos^2(a+bx)\csc^2(c-bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \cos^2(a+bx)\csc^2(c-bx)\,dx = -x\cos(2(a+c)) + \frac{\cos^2(a+c)\cot(c-bx)}{b} - \frac{\log(\sin(c-bx))\sin(2(a+c))}{b}$$

output
```
-x*cos(2*a+2*c)-cos(a+c)^2*cot(b*x-c)/b-ln(-sin(b*x-c))*sin(2*a+2*c)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 194 vs. $2(46) = 92$.

Time = 0.44 (sec) , antiderivative size = 194, normalized size of antiderivative = 4.22

$$\int \cos^2(a+bx)\csc^2(c-bx)\,dx = \frac{\csc(c)\csc(c-bx)(-bx\cos(2a+2c-bx)+bx\cos(2a+4c-bx)+bx\cos(2a+bx)-bx\cos(2a+2c+b$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c - b*x]^2,x]
```

output
```
(Csc[c]*Csc[c - b*x]*(-(b*x*Cos[2*a + 2*c - b*x]) + b*x*Cos[2*a + 4*c - b*
x] + b*x*Cos[2*a + b*x] - b*x*Cos[2*a + 2*c + b*x] + 2*Sin[b*x] - Sin[2*a
+ 2*c - b*x] - Log[-Sin[c - b*x]]*Sin[2*a + 2*c - b*x] + Log[-Sin[c - b*x]
]*Sin[2*a + 4*c - b*x] + Log[-Sin[c - b*x]]*Sin[2*a + b*x] + Sin[2*a + 2*c
 + b*x] - Log[-Sin[c - b*x]]*Sin[2*a + 2*c + b*x]))/(4*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^2(c-bx)\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \cos^2(a+bx)\csc^2(c-bx)dx$$

input
```
Int[Cos[a + b*x]^2*Csc[c - b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.63

method	result
risch	$-x\,\mathrm{e}^{2i(a+c)}+2i\sin\left(2a+2c\right)x+\frac{2i\sin(2a+2c)a}{b}+\frac{i\mathrm{e}^{4i(a+c)}}{2b\left(\mathrm{e}^{2i(a+c)}-\mathrm{e}^{2i(bx+a)}\right)}+\frac{i\mathrm{e}^{2i(a+c)}}{b\left(\mathrm{e}^{2i(a+c)}-\mathrm{e}^{2i(bx+a)}\right)}+\frac{}{2b\left(\mathrm{e}^{2i(a+\right.}}$
default	$\frac{\frac{\left(-2\cos(c)^{2}\sin(c)\cos(a)^{3}-2\cos(c)^{3}\sin(a)\cos(a)^{2}+4\cos(c)\sin(c)^{2}\sin(a)\cos(a)^{2}+4\cos(c)^{2}\sin(c)\sin(a)^{2}\cos(a)-2\sin(c)^{3}\sin(a)^{2}\cos(a)-2\cos(c)\right.}{\left(\cos(c)^{2}+\sin(c)^{2}\right)^{2}\left(\cos(a)^{2}+\sin(a)^{2}\right)^{2}\left(\cos(a)\cos(c)-\sin(\right.}}{}$

input
```
int(cos(b*x+a)^2*csc(b*x-c)^2,x,method=_RETURNVERBOSE)
```

output
```
-x*exp(2*I*(a+c))+2*I*sin(2*a+2*c)*x+2*I/b*sin(2*a+2*c)*a+1/2*I/b/(exp(2*I
*(a+c))-exp(2*I*(b*x+a)))*exp(4*I*(a+c))+I/b/(exp(2*I*(a+c))-exp(2*I*(b*x+
a)))*exp(2*I*(a+c))+1/2*I/b/(exp(2*I*(a+c))-exp(2*I*(b*x+a)))-ln(-exp(2*I*
(a+c))+exp(2*I*(b*x+a)))/b*sin(2*a+2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(55) = 110$.

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.96

$$\int \cos^2(a+bx)\csc^2(c-bx)\,dx =$$

$$-\frac{\cos\left(bx+a\right)\cos\left(a+c\right)^{3}-\left(2\,bx\cos\left(a+c\right)^{2}-bx\right)\cos\left(bx+a\right)\sin\left(a+c\right)+2\left(\cos\left(a+c\right)^{2}\sin\left(bx+a\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x-c)^2,x, algorithm="fricas")
```

output
```
-(cos(b*x + a)*cos(a + c)^3 - (2*b*x*cos(a + c)^2 - b*x)*cos(b*x + a)*sin(
a + c) + 2*(cos(a + c)^2*sin(b*x + a)*sin(a + c) + (cos(a + c)^3 - cos(a +
 c))*cos(b*x + a))*log((cos(a + c)*sin(b*x + a) - cos(b*x + a)*sin(a + c))
/(cos(a + c) + 1)) + (2*b*x*cos(a + c)^3 - b*x*cos(a + c) + cos(a + c)^2*s
in(a + c))*sin(b*x + a))/(b*cos(a + c)*sin(b*x + a) - b*cos(b*x + a)*sin(a
 + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^2(c-bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*csc(b*x-c)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(55) = 110$.

Time = 0.07 (sec) , antiderivative size = 718, normalized size of antiderivative = 15.61

$$\int \cos^2(a+bx)\csc^2(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x-c)^2,x, algorithm="maxima")
```

output
```
1/2*(2*b*x*cos(2*b*x)*cos(2*a + 4*c) + 2*b*x*sin(2*b*x)*sin(2*a + 4*c) - 2
*(b*cos(2*a + 4*c)*cos(2*c) + b*sin(2*a + 4*c)*sin(2*c))*x - (2*b*x*cos(2*
b*x) - 2*b*x*cos(2*c) - sin(4*a + 6*c) - 2*sin(2*a + 4*c) - sin(2*c))*cos(
2*b*x + 2*a + 2*c) - (cos(2*b*x + 2*a + 2*c)^2*sin(2*a + 2*c) - 2*cos(2*b*
x + 2*a + 2*c)*cos(2*a + 4*c)*sin(2*a + 2*c) + cos(2*a + 4*c)^2*sin(2*a +
2*c) + sin(2*b*x + 2*a + 2*c)^2*sin(2*a + 2*c) - 2*sin(2*b*x + 2*a + 2*c)*
sin(2*a + 4*c)*sin(2*a + 2*c) + sin(2*a + 4*c)^2*sin(2*a + 2*c))*log(cos(b
*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + si
n(c)^2) - (cos(2*b*x + 2*a + 2*c)^2*sin(2*a + 2*c) - 2*cos(2*b*x + 2*a + 2
*c)*cos(2*a + 4*c)*sin(2*a + 2*c) + cos(2*a + 4*c)^2*sin(2*a + 2*c) + sin(
2*b*x + 2*a + 2*c)^2*sin(2*a + 2*c) - 2*sin(2*b*x + 2*a + 2*c)*sin(2*a + 4
*c)*sin(2*a + 2*c) + sin(2*a + 4*c)^2*sin(2*a + 2*c))*log(cos(b*x)^2 - 2*c
os(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) - (
2*b*x*sin(2*b*x) - 2*b*x*sin(2*c) + cos(4*a + 6*c) + 2*cos(2*a + 4*c) + co
s(2*c))*sin(2*b*x + 2*a + 2*c) - cos(2*a + 4*c)*sin(4*a + 6*c) + cos(4*a +
 6*c)*sin(2*a + 4*c) + cos(2*c)*sin(2*a + 4*c) - cos(2*a + 4*c)*sin(2*c))/
(b*cos(2*b*x + 2*a + 2*c)^2 - 2*b*cos(2*b*x + 2*a + 2*c)*cos(2*a + 4*c) +
b*cos(2*a + 4*c)^2 + b*sin(2*b*x + 2*a + 2*c)^2 - 2*b*sin(2*b*x + 2*a + 2*
c)*sin(2*a + 4*c) + b*sin(2*a + 4*c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2046 vs. $2(55) = 110$.

Time = 0.20 (sec) , antiderivative size = 2046, normalized size of antiderivative = 44.48

$$\int \cos^2(a+bx)\csc^2(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x-c)^2,x, algorithm="giac")
```

output
```
-((tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*a)^4*tan(1/2*c)^2 - 16*tan(1/2*a)
^3*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 16*tan(1/2*
a)^3*tan(1/2*c) + 36*tan(1/2*a)^2*tan(1/2*c)^2 + 16*tan(1/2*a)*tan(1/2*c)^
3 + tan(1/2*c)^4 - 6*tan(1/2*a)^2 - 16*tan(1/2*a)*tan(1/2*c) - 6*tan(1/2*c
)^2 + 1)*(b*x + a)/(tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^
2 + 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*c)^4 + 2*tan(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^4
*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) - 6*ta
n(1/2*a)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2
*c)^4 + tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*a)*tan(1/2*c)
^2 + tan(1/2*c)^3 - tan(1/2*a) - tan(1/2*c))*log(tan(b*x + a)^2 + 1)/(tan(
1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^4*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1
/2*c)^4 + tan(1/2*a)^4 + 4*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^4 + 2*ta
n(1/2*a)^2 + 2*tan(1/2*c)^2 + 1) - 4*(tan(1/2*a)^6*tan(1/2*c)^5 + tan(1/2*
a)^5*tan(1/2*c)^6 - 2*tan(1/2*a)^6*tan(1/2*c)^3 - 11*tan(1/2*a)^5*tan(1/2*
c)^4 - 11*tan(1/2*a)^4*tan(1/2*c)^5 - 2*tan(1/2*a)^3*tan(1/2*c)^6 + tan(1/
2*a)^6*tan(1/2*c) + 11*tan(1/2*a)^5*tan(1/2*c)^2 + 38*tan(1/2*a)^4*tan(1/2
*c)^3 + 38*tan(1/2*a)^3*tan(1/2*c)^4 + 11*tan(1/2*a)^2*tan(1/2*c)^5 + tan(
1/2*a)*tan(1/2*c)^6 - tan(1/2*a)^5 - 11*tan(1/2*a)^4*tan(1/2*c) - 38*tan(1
/2*a)^3*tan(1/2*c)^2 - 38*tan(1/2*a)^2*tan(1/2*c)^3 - 11*tan(1/2*a)*tan...
```

Mupad [B] (verification not implemented)

Time = 20.73 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.24

$$\int \cos^2(a+bx)\csc^2(c-bx)\,dx = -x\left(\cos\left(2a+2c\right)-\sin\left(2a+2c\right)\,1\mathrm{i}\right)+\frac{\left(2\,\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}+c\,4\mathrm{i}}+1\right)\,1\mathrm{i}}{2\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)} - \frac{\mathrm{e}^{-a\,4\mathrm{i}-c\,4\mathrm{i}}\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}\right)\,\left(2\,b\,\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}-2\,b\,\mathrm{e}^{a\,6\mathrm{i}+c\,6\mathrm{i}}\right)\,1\mathrm{i}}{4\,b^2}$$

input
```
int(cos(a + b*x)^2/sin(c - b*x)^2,x)
```

output
```
((2*exp(a*2i + c*2i) + exp(a*4i + c*4i) + 1)*1i)/(2*b*(exp(a*2i + c*2i) -
exp(a*2i + b*x*2i))) - x*(cos(2*a + 2*c) - sin(2*a + 2*c)*1i) - (exp(- a*4
i - c*4i)*log(exp(a*2i)*exp(b*x*2i) - exp(a*2i)*exp(c*2i))*(2*b*exp(a*2i +
 c*2i) - 2*b*exp(a*6i + c*6i))*1i)/(4*b^2)
```

Reduce [F]

$$\int \cos^2(a+bx)\csc^2(c-bx)\,dx=\int \cos(bx+a)^2\csc(bx-c)^2\,dx$$

input
```
int(cos(b*x+a)^2*csc(b*x-c)^2,x)
```

output
```
int(cos(a + b*x)**2*csc(b*x - c)**2,x)
```

3.401 $\int \cos^2(a+bx)\csc^3(c-bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \cos^2(a+bx)\csc^3(c-bx)\,dx = \frac{\operatorname{arctanh}(\cos(c-bx))\cos^2(a+c)}{2b} - \frac{\operatorname{arctanh}(\cos(c-bx))\cos(2(a+c))}{b} + \frac{\cos^2(a+c)\cot(c-bx)\csc(c-bx)}{2b} + \frac{\csc(c-bx)\sin(2(a+c))}{b}$$

output
```
1/2*arctanh(cos(b*x-c))*cos(a+c)^2/b-arctanh(cos(b*x-c))*cos(2*a+2*c)/b+1/
2*cos(a+c)^2*cot(b*x-c)*csc(b*x-c)/b-csc(b*x-c)*sin(2*a+2*c)/b
```

Mathematica [A] (verified)

Time = 5.97 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.93

$$\int \cos^2(a+bx)\csc^3(c-bx)\,dx = \frac{2\cos^2(a+c)\csc^2\left(\frac{1}{2}(c-bx)\right)+4(1-3\cos(2(a+c)))\log\left(\cos\left(\frac{1}{2}(c-bx)\right)\right)+4(-1+3\cos(2(a+c)))\log \cdots}{\cdots}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c - b*x]^3,x]
```

output
```
(2*Cos[a + c]^2*Csc[(c - b*x)/2]^2 + 4*(1 - 3*Cos[2*(a + c)])*Log[Cos[(c -
 b*x)/2]] + 4*(-1 + 3*Cos[2*(a + c)])*Log[-Sin[(c - b*x)/2]] - 2*Cos[a + c
]^2*Sec[(c - b*x)/2]^2 + 8*Csc[c/2]*Csc[(c - b*x)/2]*Sin[2*(a + c)]*Sin[(b
*x)/2] - 8*Sec[c/2]*Sec[(c - b*x)/2]*Sin[2*(a + c)]*Sin[(b*x)/2])/(16*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^3(c-bx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos^2(a+bx)\csc^3(c-bx)dx$$

input
```
Int[Cos[a + b*x]^2*Csc[c - b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.14 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.74

method	result
risch	$\frac{3\,e^{i(bx+6a+5c)}-5\,e^{3i(bx+2a+c)}-2\,e^{i(bx+4a+3c)}-2\,e^{i(3bx+4a+c)}-5\,e^{i(bx+2a+c)}+3\,e^{i(3bx+2a-c)}}{4\left(e^{2i(a+c)}-e^{2i(bx+a)}\right)^2 b}-\frac{\ln\left(-e^{i(a+c)}+e^{i(bx+a)}\right)}{4b}+\frac{3\ln}{}$
default	Expression too large to display

input
```
int(-cos(b*x+a)^2*csc(b*x-c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/(exp(2*I*(a+c))-exp(2*I*(b*x+a)))^2/b*(3*exp(I*(b*x+6*a+5*c))-5*exp(3*
I*(b*x+2*a+c))-2*exp(I*(b*x+4*a+3*c))-2*exp(I*(3*b*x+4*a+c))-5*exp(I*(b*x+
2*a+c))+3*exp(I*(3*b*x+2*a-c)))-1/4/b*ln(-exp(I*(a+c))+exp(I*(b*x+a)))+3/4
/b*ln(-exp(I*(a+c))+exp(I*(b*x+a)))*cos(2*a+2*c)+1/4/b*ln(exp(I*(a+c))+exp
(I*(b*x+a)))-3/4/b*ln(exp(I*(a+c))+exp(I*(b*x+a)))*cos(2*a+2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(91) = 182.

Time = 0.09 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.87

$$\int \cos^2(a+bx)\csc^3(c-bx)\,dx$$
$$=\frac{6\cos(a+c)^2\sin(bx+a)\sin(a+c)+2\left(3\cos(a+c)^3-4\cos(a+c)\right)\cos(bx+a)+\left(3\cos(a+c)^4-\right.}{}$$

input
```
integrate(-cos(b*x+a)^2*csc(b*x-c)^3,x, algorithm="fricas")
```

output
```
1/4*(6*cos(a + c)^2*sin(b*x + a)*sin(a + c) + 2*(3*cos(a + c)^3 - 4*cos(a
+ c))*cos(b*x + a) + (3*cos(a + c)^4 - 2*(3*cos(a + c)^3 - 2*cos(a + c))*c
os(b*x + a)*sin(b*x + a)*sin(a + c) - (6*cos(a + c)^4 - 7*cos(a + c)^2 + 2
)*cos(b*x + a)^2 - 2*cos(a + c)^2)*log((cos(b*x + a)*cos(a + c) + sin(b*x
+ a)*sin(a + c) + 1)/(cos(a + c) + 1)) - (3*cos(a + c)^4 - 2*(3*cos(a + c)
^3 - 2*cos(a + c))*cos(b*x + a)*sin(b*x + a)*sin(a + c) - (6*cos(a + c)^4
- 7*cos(a + c)^2 + 2)*cos(b*x + a)^2 - 2*cos(a + c)^2)*log(-(cos(b*x + a)*
cos(a + c) + sin(b*x + a)*sin(a + c) - 1)/(cos(a + c) + 1)))/(2*b*cos(b*x
+ a)*cos(a + c)*sin(b*x + a)*sin(a + c) + (2*b*cos(a + c)^2 - b)*cos(b*x +
 a)^2 - b*cos(a + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^3(c-bx)\,dx = \text{Timed out}$$

input
```
integrate(-cos(b*x+a)**2*csc(b*x-c)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1571 vs. $2(91) = 182$.

Time = 0.10 (sec) , antiderivative size = 1571, normalized size of antiderivative = 18.48

$$\int \cos^2(a+bx)\csc^3(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(-cos(b*x+a)^2*csc(b*x-c)^3,x, algorithm="maxima")
```

output
```
1/8*(2*(3*cos(3*b*x) - 5*cos(3*b*x + 4*a + 4*c) - 2*cos(3*b*x + 2*a + 2*c)
 + 3*cos(b*x + 4*a + 6*c) - 2*cos(b*x + 2*a + 4*c) - 5*cos(b*x + 2*c))*cos
(4*b*x + 2*a + c) + 10*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + 5*c))*cos(3*b
*x + 4*a + 4*c) + 4*(2*cos(2*b*x + 2*a + 3*c) - cos(2*a + 5*c))*cos(3*b*x
+ 2*a + 2*c) - 4*(3*cos(3*b*x) + 3*cos(b*x + 4*a + 6*c) - 2*cos(b*x + 2*a
+ 4*c) - 5*cos(b*x + 2*c))*cos(2*b*x + 2*a + 3*c) + 6*cos(3*b*x)*cos(2*a +
 5*c) + 6*cos(b*x + 4*a + 6*c)*cos(2*a + 5*c) - 4*cos(b*x + 2*a + 4*c)*cos
(2*a + 5*c) - 10*cos(b*x + 2*c)*cos(2*a + 5*c) - ((3*cos(2*a + 2*c) - 1)*c
os(4*b*x + 2*a + c)^2 + 4*(3*cos(2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c)^2
- 4*(3*cos(2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c)*cos(2*a + 5*c) + (3*cos(
2*a + 2*c) - 1)*cos(2*a + 5*c)^2 + (3*cos(2*a + 2*c) - 1)*sin(4*b*x + 2*a
+ c)^2 + 4*(3*cos(2*a + 2*c) - 1)*sin(2*b*x + 2*a + 3*c)^2 - 4*(3*cos(2*a
+ 2*c) - 1)*sin(2*b*x + 2*a + 3*c)*sin(2*a + 5*c) + (3*cos(2*a + 2*c) - 1)
*sin(2*a + 5*c)^2 - 2*(2*(3*cos(2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c) - (
3*cos(2*a + 2*c) - 1)*cos(2*a + 5*c))*cos(4*b*x + 2*a + c) - 2*(2*(3*cos(2
*a + 2*c) - 1)*sin(2*b*x + 2*a + 3*c) - (3*cos(2*a + 2*c) - 1)*sin(2*a + 5
*c))*sin(4*b*x + 2*a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 +
 sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) + ((3*cos(2*a + 2*c) - 1)*cos(
4*b*x + 2*a + c)^2 + 4*(3*cos(2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c)^2 - 4
*(3*cos(2*a + 2*c) - 1)*cos(2*b*x + 2*a + 3*c)*cos(2*a + 5*c) + (3*cos(...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6142 vs. 2(91) = 182.

Time = 0.39 (sec) , antiderivative size = 6142, normalized size of antiderivative = 72.26

$$\int \cos^2(a+bx)\csc^3(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(-cos(b*x+a)^2*csc(b*x-c)^3,x, algorithm="giac")
```

output
```
1/8*(4*(tan(1/2*a)^5*tan(1/2*c)^5 - 10*tan(1/2*a)^5*tan(1/2*c)^3 - 25*tan(
1/2*a)^4*tan(1/2*c)^4 - 10*tan(1/2*a)^3*tan(1/2*c)^5 + tan(1/2*a)^5*tan(1/
2*c) + 34*tan(1/2*a)^4*tan(1/2*c)^2 + 76*tan(1/2*a)^3*tan(1/2*c)^3 + 34*ta
n(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)*tan(1/2*c)^5 - tan(1/2*a)^4 - 34*tan(
1/2*a)^3*tan(1/2*c) - 76*tan(1/2*a)^2*tan(1/2*c)^2 - 34*tan(1/2*a)*tan(1/2
*c)^3 - tan(1/2*c)^4 + 10*tan(1/2*a)^2 + 25*tan(1/2*a)*tan(1/2*c) + 10*tan
(1/2*c)^2 - 1)*log(abs(tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - tan(1/
2*b*x + 1/2*a) + tan(1/2*a) + tan(1/2*c)))/(tan(1/2*a)^5*tan(1/2*c)^5 + 2*
tan(1/2*a)^5*tan(1/2*c)^3 - tan(1/2*a)^4*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan
(1/2*c)^5 + tan(1/2*a)^5*tan(1/2*c) - 2*tan(1/2*a)^4*tan(1/2*c)^2 + 4*tan(
1/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*a)*tan(1/2*c
)^5 - tan(1/2*a)^4 + 2*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2*a)^2*tan(1/2*c)
^2 + 2*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 - 2*tan(1/2*a)^2 + tan(1/2*a
)*tan(1/2*c) - 2*tan(1/2*c)^2 - 1) - 4*(tan(1/2*a)^5*tan(1/2*c)^4 + tan(1/
2*a)^4*tan(1/2*c)^5 - 10*tan(1/2*a)^5*tan(1/2*c)^2 - 34*tan(1/2*a)^4*tan(1
/2*c)^3 - 34*tan(1/2*a)^3*tan(1/2*c)^4 - 10*tan(1/2*a)^2*tan(1/2*c)^5 + ta
n(1/2*a)^5 + 25*tan(1/2*a)^4*tan(1/2*c) + 76*tan(1/2*a)^3*tan(1/2*c)^2 + 7
6*tan(1/2*a)^2*tan(1/2*c)^3 + 25*tan(1/2*a)*tan(1/2*c)^4 + tan(1/2*c)^5 -
10*tan(1/2*a)^3 - 34*tan(1/2*a)^2*tan(1/2*c) - 34*tan(1/2*a)*tan(1/2*c)^2
- 10*tan(1/2*c)^3 + tan(1/2*a) + tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*...
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^3(c-bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)^2/sin(c - b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \cos^2(a+bx)\csc^3(c-bx)\,dx = -\left(\int \cos(bx+a)^2\csc(bx-c)^3\,dx\right)$$

input
```
int(-cos(b*x+a)^2*csc(b*x-c)^3,x)
```

output
```
- int(cos(a + b*x)**2*csc(b*x - c)**3,x)
```

3.402 $\int \cos^2(a+bx)\csc^4(c-bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \cos^2(a+bx)\csc^4(c-bx)\,dx = \frac{\cos^2(a+c)\cot(c-bx)}{b} - \frac{\cos(2(a+c))\cot(c-bx)}{b} + \frac{\cos^2(a+c)\cot^3(c-bx)}{3b} + \frac{\csc^2(c-bx)\sin(2(a+c))}{2b}$$

output
```
-cos(a+c)^2*cot(b*x-c)/b+cos(2*a+2*c)*cot(b*x-c)/b-1/3*cos(a+c)^2*cot(b*x-
c)^3/b+1/2*csc(b*x-c)^2*sin(2*a+2*c)/b
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \cos^2(a+bx)\csc^4(c-bx)\,dx = \frac{\csc(c)\csc^3(c-bx)(3\sin(bx)+\sin(2c-3bx)-\sin(2a+4c-3bx)+3\sin(2a+2c-bx)-3\sin(2a+bx}{12b}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[c - b*x]^4,x]
```

output
```
(Csc[c]*Csc[c - b*x]^3*(3*Sin[b*x] + Sin[2*c - 3*b*x] - Sin[2*a + 4*c - 3*
b*x] + 3*Sin[2*a + 2*c - b*x] - 3*Sin[2*a + b*x] + Sin[2*a + 3*b*x]))/(12*
b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^4(c-bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \cos^2(a+bx)\csc^4(c-bx)dx$$

input
```
Int[Cos[a + b*x]^2*Csc[c - b*x]^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{1}{3b(\cos(a)\cos(c)-\sin(a)\sin(c))(\tan(bx+a)\cos(a)\cos(c)-\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)-\cos(a)\sin(c))^3}$	58
parallelrisch	$\frac{\sec\left(\frac{bx}{2}-\frac{c}{2}\right)^3\csc\left(\frac{bx}{2}-\frac{c}{2}\right)^3(\cos(3bx-3c)-3\cos(bx-c)-2\cos(3bx+2a-c))}{96b}$	62
risch	$-\frac{2i\left(e^{8i(a+c)}-3\,e^{2i(bx+4a+3c)}-e^{6i(a+c)}+3\,e^{4i(bx+2a+c)}+3\,e^{2i(bx+3a+2c)}+e^{4i(a+c)}\right)}{3\left(e^{2i(a+c)}-e^{2i(bx+a)}\right)^3b}$	98

input
```
int(cos(b*x+a)^2*csc(b*x-c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(cos(a)*cos(c)-sin(a)*sin(c))/(tan(b*x+a)*cos(a)*cos(c)-tan(b*x+a)*
sin(a)*sin(c)-sin(a)*cos(c)-cos(a)*sin(c))^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(84) = 168$.

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.62

$$\int \cos^2(a+bx)\csc^4(c-bx)\,dx = -\frac{(16\cos(a+c)^5 - 24\cos(a+c)^3 + 9\cos(a+c))\cos(bx+a)^3 - (4\cos(a+c)^4 - (16\cos(a+c)^4 - }{3\left((b\cos(a+c)^3 - (4\,b\cos(a+c)^3 - 3\,b\cos(a+c))\cos(bx+\right.}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x-c)^4,x, algorithm="fricas")
```

output
```
-1/3*((16*cos(a + c)^5 - 24*cos(a + c)^3 + 9*cos(a + c))*cos(b*x + a)^3 -
(4*cos(a + c)^4 - (16*cos(a + c)^4 - 16*cos(a + c)^2 + 3)*cos(b*x + a)^2 -
 cos(a + c)^2)*sin(b*x + a)*sin(a + c) - 3*(4*cos(a + c)^5 - 5*cos(a + c)^
3 + cos(a + c))*cos(b*x + a))/((b*cos(a + c)^3 - (4*b*cos(a + c)^3 - 3*b*c
os(a + c))*cos(b*x + a)^2)*sin(b*x + a) + ((4*b*cos(a + c)^2 - b)*cos(b*x
+ a)^3 - 3*b*cos(b*x + a)*cos(a + c)^2)*sin(a + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^4(c-bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*csc(b*x-c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(84) = 168$.

Time = 0.05 (sec) , antiderivative size = 937, normalized size of antiderivative = 11.71

$$\int \cos^2(a+bx)\csc^4(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x-c)^4,x, algorithm="maxima")
```

output
```
-2/3*((3*sin(4*b*x + 4*a + 4*c) - 3*sin(2*b*x + 4*a + 6*c) + 3*sin(2*b*x +
 2*a + 4*c) + sin(4*a + 8*c) - sin(2*a + 6*c) + sin(4*c))*cos(6*b*x + 2*a)
 + 3*(3*sin(4*b*x + 2*a + 2*c) - 3*sin(2*b*x + 2*a + 4*c) + sin(2*a + 6*c)
)*cos(4*b*x + 4*a + 4*c) + 3*(3*sin(2*b*x + 4*a + 6*c) - 3*sin(2*b*x + 2*a
 + 4*c) - sin(4*a + 8*c) + sin(2*a + 6*c) - sin(4*c))*cos(4*b*x + 2*a + 2*
c) + 3*(3*sin(2*b*x + 2*a + 4*c) - sin(2*a + 6*c))*cos(2*b*x + 4*a + 6*c)
+ 3*(sin(4*a + 8*c) + sin(4*c))*cos(2*b*x + 2*a + 4*c) - (3*cos(4*b*x + 4*
a + 4*c) - 3*cos(2*b*x + 4*a + 6*c) + 3*cos(2*b*x + 2*a + 4*c) + cos(4*a +
 8*c) - cos(2*a + 6*c) + cos(4*c))*sin(6*b*x + 2*a) - 3*(3*cos(4*b*x + 2*a
 + 2*c) - 3*cos(2*b*x + 2*a + 4*c) + cos(2*a + 6*c))*sin(4*b*x + 4*a + 4*c
) - 3*(3*cos(2*b*x + 4*a + 6*c) - 3*cos(2*b*x + 2*a + 4*c) - cos(4*a + 8*c
) + cos(2*a + 6*c) - cos(4*c))*sin(4*b*x + 2*a + 2*c) - 3*(3*cos(2*b*x + 2
*a + 4*c) - cos(2*a + 6*c))*sin(2*b*x + 4*a + 6*c) - 3*(cos(4*a + 8*c) + c
os(4*c))*sin(2*b*x + 2*a + 4*c) - cos(2*a + 6*c)*sin(4*a + 8*c) + cos(4*a
+ 8*c)*sin(2*a + 6*c) + cos(4*c)*sin(2*a + 6*c) - cos(2*a + 6*c)*sin(4*c))
/(b*cos(6*b*x + 2*a)^2 + 9*b*cos(4*b*x + 2*a + 2*c)^2 + 9*b*cos(2*b*x + 2*
a + 4*c)^2 - 6*b*cos(2*b*x + 2*a + 4*c)*cos(2*a + 6*c) + b*cos(2*a + 6*c)^
2 + b*sin(6*b*x + 2*a)^2 + 9*b*sin(4*b*x + 2*a + 2*c)^2 + 9*b*sin(2*b*x +
2*a + 4*c)^2 - 6*b*sin(2*b*x + 2*a + 4*c)*sin(2*a + 6*c) + b*sin(2*a + 6*c
)^2 - 2*(3*b*cos(4*b*x + 2*a + 2*c) - 3*b*cos(2*b*x + 2*a + 4*c) + b*co...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(84) = 168$.

Time = 0.19 (sec) , antiderivative size = 441, normalized size of antiderivative = 5.51

$$\int \cos^2(a+bx)\csc^4(c-bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x-c)^4,x, algorithm="giac")
```

output
```
-1/3*(tan(1/2*a)^8*tan(1/2*c)^8 + 4*tan(1/2*a)^8*tan(1/2*c)^6 + 4*tan(1/2*
a)^6*tan(1/2*c)^8 + 6*tan(1/2*a)^8*tan(1/2*c)^4 + 16*tan(1/2*a)^6*tan(1/2*
c)^6 + 6*tan(1/2*a)^4*tan(1/2*c)^8 + 4*tan(1/2*a)^8*tan(1/2*c)^2 + 24*tan(
1/2*a)^6*tan(1/2*c)^4 + 24*tan(1/2*a)^4*tan(1/2*c)^6 + 4*tan(1/2*a)^2*tan(
1/2*c)^8 + tan(1/2*a)^8 + 16*tan(1/2*a)^6*tan(1/2*c)^2 + 36*tan(1/2*a)^4*t
an(1/2*c)^4 + 16*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*c)^8 + 4*tan(1/2*a)^6
 + 24*tan(1/2*a)^4*tan(1/2*c)^2 + 24*tan(1/2*a)^2*tan(1/2*c)^4 + 4*tan(1/2
*c)^6 + 6*tan(1/2*a)^4 + 16*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(1/2*c)^4 + 4
*tan(1/2*a)^2 + 4*tan(1/2*c)^2 + 1)/((tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)
^2 - tan(b*x + a)*tan(1/2*a)^2 - 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) + 2*
tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2
*c)^2 + tan(b*x + a) - 2*tan(1/2*a) - 2*tan(1/2*c))^3*(tan(1/2*a)^2*tan(1/
2*c)^2 - tan(1/2*a)^2 - 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^4(c-bx)\,dx = \text{Hanged}$$

input
```
int(cos(a + b*x)^2/sin(c - b*x)^4,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

$$\int \cos^2(a+bx)\csc^4(c-bx)\,dx$$
$$= \frac{-\cos(bx-c)\sin(bx-c)^2+\cos(bx-c)\sin(bx+a)^2-\cos(bx-c)+\cos(bx+a)\sin(bx-c)\sin(bx+}{3\sin(bx-c)^3 b}$$

input
```
int(cos(b*x+a)^2*csc(b*x-c)^4,x)
```

output
```
( - cos(b*x - c)*sin(b*x - c)**2 + cos(b*x - c)*sin(a + b*x)**2 - cos(b*x
- c) + cos(a + b*x)*sin(b*x - c)*sin(a + b*x))/(3*sin(b*x - c)**3*b)
```

3.403 $\int \tan(a+bx)\tan(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \tan(a+bx)\tan(c+bx)\,dx = -x-\frac{\cot(a-c)\log(\cos(a+bx))}{b}+\frac{\cot(a-c)\log(\cos(c+bx))}{b}$$

output
```
-x-cot(a-c)*ln(cos(b*x+a))/b+cot(a-c)*ln(cos(b*x+c))/b
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \tan(a+bx)\tan(c+bx)\,dx = -x+\frac{\cot(a-c)(-\log(\cos(a+bx))+\log(\cos(c+bx)))}{b}$$

input
```
Integrate[Tan[a + b*x]*Tan[c + b*x],x]
```

output
```
-x + (Cot[a - c]*(-Log[Cos[a + b*x]] + Log[Cos[c + b*x]]))/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5123, 5121, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)\tan(bx+c)\,dx$$

$$\Big\downarrow \text{5123}$$

$$\cos(a-c)\int \sec(a+bx)\sec(c+bx)dx-x$$

$$\Big\downarrow \text{5121}$$

$$\cos(a-c)(\csc(a-c)\int \tan(a+bx)dx-\csc(a-c)\int \tan(c+bx)dx)-x$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)(\csc(a-c)\int \tan(a+bx)dx-\csc(a-c)\int \tan(c+bx)dx)-x$$

$$\Big\downarrow \text{3956}$$

$$\cos(a-c)\left(\frac{\csc(a-c)\log(\cos(bx+c))}{b}-\frac{\csc(a-c)\log(\cos(a+bx))}{b}\right)-x$$

input
```
Int[Tan[a + b*x]*Tan[c + b*x],x]
```

output
```
-x + Cos[a - c]*(-((Csc[a - c]*Log[Cos[a + b*x]])/b) + (Csc[a - c]*Log[Cos
[c + b*x]])/b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5121
```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Csc[
(b*c - a*d)/d]   Int[Tan[a + b*x], x], x] + Simp[Csc[(b*c - a*d)/b]   Int[T
an[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b
*c - a*d, 0]
```

rule 5123
```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Simp[(b/d)*Cos[(b*c - a*d)/d]   Int[Sec[a + b*x]*Sec[c + d*x],
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 4.44

method	result	siz
risch	$-x-\frac{i \ln\left(\mathrm{e}^{2 i (b x+a)}+1\right) \mathrm{e}^{2 i a}}{b \left(\mathrm{e}^{2 i a}-\mathrm{e}^{2 i c}\right)}-\frac{i \ln\left(\mathrm{e}^{2 i (b x+a)}+1\right) \mathrm{e}^{2 i c}}{b \left(\mathrm{e}^{2 i a}-\mathrm{e}^{2 i c}\right)}+\frac{i \ln\left(\mathrm{e}^{2 i (b x+a)}+\mathrm{e}^{2 i (a-c)}\right) \mathrm{e}^{2 i a}}{b \left(\mathrm{e}^{2 i a}-\mathrm{e}^{2 i c}\right)}+\frac{i \ln\left(\mathrm{e}^{2 i (b x+a)}+\mathrm{e}^{2 i (a-c)}\right) \mathrm{e}^{2 i c}}{b \left(\mathrm{e}^{2 i a}-\mathrm{e}^{2 i c}\right)}$	17

input
```
int(tan(b*x+a)*tan(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-x-I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))+1)*exp(2*I*a)-I/b/(exp(
2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))+1)*exp(2*I*c)+I/b/(exp(2*I*a)-exp(2
*I*c))*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*exp(2*I*a)+I/b/(exp(2*I*a)-exp(
2*I*c))*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*exp(2*I*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(39) = 78$.

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.72

$$\int \tan(a+bx)\tan(c+bx)\,dx = -\frac{2\,bx\sin\left(-2\,a+2\,c\right)-\left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(-\frac{(\cos(-2\,a+2\,c)-1)\tan(bx+c)^2-2\,\sin(-2\,a+2\,c)\tan(bx+c)-\cos(-2}{(\cos(-2\,a+2\,c)+1)\tan(bx+c)^2+\cos(-2\,a+2\,c)+1}\right.}{2\,b\sin\left(-2\,a+2\,c\right)}$$

input
```
integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(2*b*x*sin(-2*a + 2*c) - (cos(-2*a + 2*c) + 1)*log(-((cos(-2*a + 2*c)
 - 1)*tan(b*x + c)^2 - 2*sin(-2*a + 2*c)*tan(b*x + c) - cos(-2*a + 2*c) -
1)/((cos(-2*a + 2*c) + 1)*tan(b*x + c)^2 + cos(-2*a + 2*c) + 1)) + (cos(-2
*a + 2*c) + 1)*log(1/(tan(b*x + c)^2 + 1)))/(b*sin(-2*a + 2*c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2020 vs. $2(31) = 62$.

Time = 3.37 (sec) , antiderivative size = 7672, normalized size of antiderivative = 196.72

$$\int \tan(a+bx)\tan(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(tan(b*x+a)*tan(b*x+c),x)
```

output
```
Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (-2*b*x*tan(c)/(2*b*tan(c)*
*2 + 2*b) - 2*log(tan(b*x) - 1/tan(c))/(2*b*tan(c)**2 + 2*b) + log(tan(b*x
)**2 + 1)/(2*b*tan(c)**2 + 2*b), Eq(a, 0)), (-2*b*x*tan(a)/(2*b*tan(a)**2
+ 2*b) - 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**2 + 2*b) + log(tan(b*x)**
2 + 1)/(2*b*tan(a)**2 + 2*b), Eq(c, 0)), (-4*b*x*tan(c)**2*tan(b*x)/(2*b*t
an(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2
 + 2*b*tan(c)*tan(b*x) - 2*b) + 4*b*x*tan(c)/(2*b*tan(c)**5*tan(b*x) - 2*b
*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x)
- 2*b) + 2*log(tan(b*x) - 1/tan(c))*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(
b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)
*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1/tan(c))*tan(c)**2/(2*b*tan(c)**5*tan
(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c
)*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1/tan(c))*tan(c)*tan(b*x)/(2*b*tan(c)
**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*
b*tan(c)*tan(b*x) - 2*b) + 2*log(tan(b*x) - 1/tan(c))/(2*b*tan(c)**5*tan(b
*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*
tan(b*x) - 2*b) - log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*t
an(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan
(c)*tan(b*x) - 2*b) + log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**5*tan(b*
x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(39) = 78$.

Time = 0.06 (sec) , antiderivative size = 371, normalized size of antiderivative = 9.51

$$\int \tan(a+bx)\tan(c+bx)\,dx = -\frac{\left(2\,b\cos\left(2\,a\right)\cos\left(2\,c\right)-b\cos\left(2\,c\right)^2+2\,b\sin\left(2\,a\right)\sin\left(2\,c\right)-b\sin\left(2\,c\right)^2-\left(\cos\left(2\,a\right)^2+\sin\left(2\,a\right)^2\right)b\right)x+ \cdots}{\cdots}$$

input
```
integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

output
```
-((2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*
c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)*x + (cos(2*a)^2 - cos(2*c)^2 + sin(2*a
)^2 - sin(2*c)^2)*arctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) -
(cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(2*b*x) - s
in(2*c), cos(2*b*x) + cos(2*c)) - (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*
log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*s
in(2*b*x)*sin(2*a) + sin(2*a)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))
*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*
sin(2*b*x)*sin(2*c) + sin(2*c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 +
 2*b*sin(2*a)*sin(2*c) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(39) = 78$.

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.08

$$\int \tan(a+bx)\tan(c+bx)\,dx = -x - \frac{(\tan(a)^2\tan(c)+\tan(a))\log(|\tan(bx)\tan(a)-1|)}{b\tan(a)^2-b\tan(a)\tan(c)} + \frac{(\tan(a)\tan(c)^2+\tan(c))\log(|\tan(bx)\tan(c)-1|)}{b\tan(a)\tan(c)-b\tan(c)^2}$$

input
```
integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

output
```
-x - (tan(a)^2*tan(c) + tan(a))*log(abs(tan(b*x)*tan(a) - 1))/(b*tan(a)^2
- b*tan(a)*tan(c)) + (tan(a)*tan(c)^2 + tan(c))*log(abs(tan(b*x)*tan(c) -
1))/(b*tan(a)*tan(c) - b*tan(c)^2)
```

Mupad [B] (verification not implemented)

Time = 21.78 (sec) , antiderivative size = 207, normalized size of antiderivative = 5.31

$$\int \tan(a+bx)\tan(c+bx)\,dx = -\frac{\frac{x}{2}+x\left(\sin\left(a-c\right)^2-\frac{1}{2}\right)}{\sin\left(a-c\right)^2} - \frac{\frac{\sin(2\,a-2\,c)\,\ln\left(\sin(2\,a-2\,c)^2\,2\mathrm{i}-\sin(a+b\,x)^2\,2\mathrm{i}+\sin(3\,a-2\,c+b\,x)^2\,2\mathrm{i}+\sin(4\,a-4\,c)+\sin(6\,a-4\,c+2\,b\,x)-\sin(2\,a+2\,b\,x)\right)}{2}-\frac{\sin(2\,a-2\,c)\,\ln}{}}{b\sin\left(a-c\right)^2}$$

input
```
int(tan(a + b*x)*tan(c + b*x),x)
```

output
```
- (x/2 + x*(sin(a - c)^2 - 1/2))/sin(a - c)^2 - ((sin(2*a - 2*c)*log(sin(4
*a - 4*c) + sin(6*a - 4*c + 2*b*x) - sin(2*a + 2*b*x) + sin(2*a - 2*c)^2*2
i - sin(a + b*x)^2*2i + sin(3*a - 2*c + b*x)^2*2i))/2 - (sin(2*a - 2*c)*lo
g(sin(4*a - 4*c) + sin(4*a - 2*c + 2*b*x) - sin(2*c + 2*b*x) + sin(2*a - 2
*c)^2*2i - sin(c + b*x)^2*2i + sin(2*a - c + b*x)^2*2i))/2)/(b*sin(a - c)^
2)
```

Reduce [F]

$$\int \tan(a+bx)\tan(c+bx)\,dx = \int \tan\left(bx+c\right)\tan\left(bx+a\right)dx$$

input
```
int(tan(b*x+a)*tan(b*x+c),x)
```

output
```
int(tan(b*x + c)*tan(a + b*x),x)
```

3.404 $\int \tan(c-bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \tan(c-bx)\tan(a+bx)\,dx = x - \frac{\cot(a+c)\log(\cos(c-bx))}{b} + \frac{\cot(a+c)\log(\cos(a+bx))}{b}$$

output
```
x-cot(a+c)*ln(cos(b*x-c))/b+cot(a+c)*ln(cos(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \tan(c-bx)\tan(a+bx)\,dx = x + \frac{\cot(a+c)(-\log(\cos(c-bx))+\log(\cos(a+bx)))}{b}$$

input
```
Integrate[Tan[c - b*x]*Tan[a + b*x],x]
```

output
```
x + (Cot[a + c]*(-Log[Cos[c - b*x]] + Log[Cos[a + b*x]]))/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5123, 5121, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)\tan(c-bx)\,dx$$

$$\Big\downarrow \text{5123}$$

$$x-\cos(a+c)\int \sec(c-bx)\sec(a+bx)dx$$

$$\Big\downarrow \text{5121}$$

$$x-\cos(a+c)\left(\csc(a+c)\int \tan(c-bx)dx+\csc(a+c)\int \tan(a+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$x-\cos(a+c)\left(\csc(a+c)\int \tan(c-bx)dx+\csc(a+c)\int \tan(a+bx)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$x-\cos(a+c)\left(\frac{\csc(a+c)\log(\cos(c-bx))}{b}-\frac{\csc(a+c)\log(\cos(a+bx))}{b}\right)$$

input
```
Int[Tan[c - b*x]*Tan[a + b*x],x]
```

output
```
x - Cos[a + c]*((Csc[a + c]*Log[Cos[c - b*x]])/b - (Csc[a + c]*Log[Cos[a +
 b*x]])/b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5121
```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Csc[
(b*c - a*d)/d]   Int[Tan[a + b*x], x], x] + Simp[Csc[(b*c - a*d)/b]   Int[T
an[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b
*c - a*d, 0]
```

rule 5123
```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Simp[(b/d)*Cos[(b*c - a*d)/d]   Int[Sec[a + b*x]*Sec[c + d*x],
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 4.26

method	result	size
risch	$x-\frac{i \ln\left(\mathrm{e}^{2 i (a+c)}+\mathrm{e}^{2 i (b x+a)}\right) \mathrm{e}^{2 i (a+c)}}{b \left(\mathrm{e}^{2 i (a+c)}-1\right)}-\frac{i \ln\left(\mathrm{e}^{2 i (a+c)}+\mathrm{e}^{2 i (b x+a)}\right)}{b \left(\mathrm{e}^{2 i (a+c)}-1\right)}+\frac{i \ln\left(\mathrm{e}^{2 i (b x+a)}+1\right) \mathrm{e}^{2 i (a+c)}}{b \left(\mathrm{e}^{2 i (a+c)}-1\right)}+\frac{i \ln\left(\mathrm{e}^{2 i (b x+a)}+1\right)}{b \left(\mathrm{e}^{2 i (a+c)}-1\right)}$	145

input
```
int(-tan(b*x-c)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x-I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(a+c))+exp(2*I*(b*x+a)))*exp(2*I*(a+c)
)-I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(a+c))+exp(2*I*(b*x+a)))+I/b/(exp(2*I*
(a+c))-1)*ln(exp(2*I*(b*x+a))+1)*exp(2*I*(a+c))+I/b/(exp(2*I*(a+c))-1)*ln(
exp(2*I*(b*x+a))+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(35) = 70.

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 4.26

$$\int \tan(c-bx)\tan(a+bx)\,dx$$
$$= \frac{2\,bx\sin\left(2\,a+2\,c\right)-\left(\cos\left(2\,a+2\,c\right)+1\right)\log\left(-\frac{(\cos(2\,a+2\,c)-1)\tan(bx+a)^2-2\,\sin(2\,a+2\,c)\tan(bx+a)-\cos(2\,a+2\,c)-1}{(\cos(2\,a+2\,c)+1)\tan(bx+a)^2+\cos(2\,a+2\,c)+1}\right)}{2\,b\sin\left(2\,a+2\,c\right)}$$

input
```
integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*sin(2*a + 2*c) - (cos(2*a + 2*c) + 1)*log(-((cos(2*a + 2*c) - 1
)*tan(b*x + a)^2 - 2*sin(2*a + 2*c)*tan(b*x + a) - cos(2*a + 2*c) - 1)/((c
os(2*a + 2*c) + 1)*tan(b*x + a)^2 + cos(2*a + 2*c) + 1)) + (cos(2*a + 2*c)
 + 1)*log(1/(tan(b*x + a)^2 + 1)))/(b*sin(2*a + 2*c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2021 vs. 2(31) = 62.

Time = 4.56 (sec) , antiderivative size = 7679, normalized size of antiderivative = 225.85

$$\int \tan(c-bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(-tan(b*x-c)*tan(b*x+a),x)
```

output
```
Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (-2*b*x*tan(c)/(2*b*tan(c)*
*2 + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**2 + 2*b) - log(tan(b*x
)**2 + 1)/(2*b*tan(c)**2 + 2*b), Eq(a, 0)), (2*b*x*tan(a)/(2*b*tan(a)**2 +
 2*b) + 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**2 + 2*b) - log(tan(b*x)**2
 + 1)/(2*b*tan(a)**2 + 2*b), Eq(c, 0)), (-4*b*x*tan(c)**2*tan(b*x)/(2*b*ta
n(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2
+ 2*b*tan(c)*tan(b*x) + 2*b) - 4*b*x*tan(c)/(2*b*tan(c)**5*tan(b*x) + 2*b*
tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) +
 2*b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b
*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*
tan(b*x) + 2*b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**2/(2*b*tan(c)**5*tan(
b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)
*tan(b*x) + 2*b) + 2*log(tan(b*x) + 1/tan(c))*tan(c)*tan(b*x)/(2*b*tan(c)*
*5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b
*tan(c)*tan(b*x) + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**5*tan(b*
x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*t
an(b*x) + 2*b) + log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*ta
n(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(
c)*tan(b*x) + 2*b) + log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**5*tan(b*x
) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(35) = 70$.

Time = 0.06 (sec) , antiderivative size = 290, normalized size of antiderivative = 8.53

$$\int \tan(c-bx)\tan(a+bx)\,dx$$
$$= \frac{\left(b\cos\left(2\,a+2\,c\right)^2+b\sin\left(2\,a+2\,c\right)^2-2\,b\cos\left(2\,a+2\,c\right)+b\right)x-\left(\cos\left(2\,a+2\,c\right)^2+\sin\left(2\,a+2\,c\right)^2-1\right)}{}$$

input
```
integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="maxima")
```

output
```
((b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)*x - (c
os(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(2*b*x) - sin(2*a), cos
(2*b*x) + cos(2*a)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(si
n(2*b*x) + sin(2*c), cos(2*b*x) + cos(2*c)) + log(cos(2*b*x)^2 + 2*cos(2*b
*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a
)^2)*sin(2*a + 2*c) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^
2 + sin(2*b*x)^2 + 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2)*sin(2*a + 2*c))/(b*
cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(35) = 70$.

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.38

$$\int \tan(c-bx)\tan(a+bx)\,dx = x - \frac{\left(\tan\left(a\right)^2\tan\left(c\right)-\tan\left(a\right)\right)\log\left(\left|\tan\left(bx\right)\tan\left(a\right)-1\right|\right)}{b\tan\left(a\right)^2+b\tan\left(a\right)\tan\left(c\right)} + \frac{\left(\tan\left(a\right)\tan\left(c\right)^2-\tan\left(c\right)\right)\log\left(\left|\tan\left(bx\right)\tan\left(c\right)+1\right|\right)}{b\tan\left(a\right)\tan\left(c\right)+b\tan\left(c\right)^2}$$

input
```
integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="giac")
```

output
```
x - (tan(a)^2*tan(c) - tan(a))*log(abs(tan(b*x)*tan(a) - 1))/(b*tan(a)^2 +
 b*tan(a)*tan(c)) + (tan(a)*tan(c)^2 - tan(c))*log(abs(tan(b*x)*tan(c) + 1
))/(b*tan(a)*tan(c) + b*tan(c)^2)
```

Mupad [B] (verification not implemented)

Time = 21.42 (sec) , antiderivative size = 196, normalized size of antiderivative = 5.76

$$\int \tan(c-bx)\tan(a+bx)\,dx = \frac{\frac{x}{2}+x\left(\sin\left(a+c\right)^2-\frac{1}{2}\right)}{\sin\left(a+c\right)^2} + \frac{\frac{\sin(2\,a+2\,c)\,\ln\left(\sin(2\,a+2\,c)^2\,2\mathrm{i}-\sin(a+b\,x)^2\,2\mathrm{i}+\sin(3\,a+2\,c+b\,x)^2\,2\mathrm{i}+\sin(4\,a+4\,c)+\sin(6\,a+4\,c+2\,b\,x)-\sin(2\,a+2\,b\,x)\right)}{2} - \frac{\sin(2\,a+2\,c)\,\ln}{}}{b\sin\left(a+c\right)^2}$$

input
```
int(tan(a + b*x)*tan(c - b*x),x)
```

output
```
(x/2 + x*(sin(a + c)^2 - 1/2))/sin(a + c)^2 + ((sin(2*a + 2*c)*log(sin(4*a
 + 4*c) + sin(6*a + 4*c + 2*b*x) - sin(2*a + 2*b*x) + sin(2*a + 2*c)^2*2i
- sin(a + b*x)^2*2i + sin(3*a + 2*c + b*x)^2*2i))/2 - (sin(2*a + 2*c)*log(
sin(4*a + 4*c) + sin(4*a + 2*c + 2*b*x) + sin(2*c - 2*b*x) + sin(2*a + c +
 b*x)^2*2i + sin(2*a + 2*c)^2*2i - sin(c - b*x)^2*2i))/2)/(b*sin(a + c)^2)
```

Reduce [F]

$$\int \tan(c-bx)\tan(a+bx)\,dx = -\left(\int \tan(bx-c)\tan(bx+a)\,dx\right)$$

input
```
int(-tan(b*x-c)*tan(b*x+a),x)
```

output
```
- int(tan(b*x - c)*tan(a + b*x),x)
```

3.405 $\int \cot(a+bx)\cot(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \cot(a+bx)\cot(c+bx)\,dx = -x-\frac{\cot(a-c)\log(\sin(a+bx))}{b}+\frac{\cot(a-c)\log(\sin(c+bx))}{b}$$

output
```
-x-cot(a-c)*ln(sin(b*x+a))/b+cot(a-c)*ln(sin(b*x+c))/b
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \cot(a+bx)\cot(c+bx)\,dx = -x+\frac{\cot(a-c)(-\log(\sin(a+bx))+\log(\sin(c+bx)))}{b}$$

input
```
Integrate[Cot[a + b*x]*Cot[c + b*x],x]
```

output
```
-x + (Cot[a - c]*(-Log[Sin[a + b*x]] + Log[Sin[c + b*x]]))/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5124, 5122, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\cot(bx+c)\,dx$$

$$\Big\downarrow \text{5124}$$

$$\cos(a-c)\int \csc(a+bx)\csc(c+bx)dx - x$$

$$\Big\downarrow \text{5122}$$

$$\cos(a-c)(\csc(a-c)\int \cot(c+bx)dx - \csc(a-c)\int \cot(a+bx)dx) - x$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\left(\csc(a-c)\int -\tan\left(c+bx+\frac{\pi}{2}\right)dx - \csc(a-c)\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx\right) - x$$

$$\Big\downarrow \text{25}$$

$$\cos(a-c)\left(\csc(a-c)\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx - \csc(a-c)\int \tan\left(\frac{1}{2}(2c+\pi)+bx\right)dx\right) - x$$

$$\Big\downarrow \text{3956}$$

$$\cos(a-c)\left(\frac{\csc(a-c)\log(-\sin(bx+c))}{b} - \frac{\csc(a-c)\log(-\sin(a+bx))}{b}\right) - x$$

input
```
Int[Cot[a + b*x]*Cot[c + b*x],x]
```

output
```
-x + Cos[a - c]*(-((Csc[a - c]*Log[-Sin[a + b*x]])/b) + (Csc[a - c]*Log[-S
in[c + b*x]])/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5122
```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[Csc[(
b*c - a*d)/b]   Int[Cot[a + b*x], x], x] - Simp[Csc[(b*c - a*d)/d]   Int[Co
t[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*
c - a*d, 0]
```

rule 5124
```
Int[Cot[(a_.) + (b_.)*(x_)]*Cot[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Simp[Cos[(b*c - a*d)/d]   Int[Csc[a + b*x]*Csc[c + d*x], x], x]
 /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 4.54

method	result	siz
risch	$-x-\frac{i\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)\mathrm{e}^{2ia}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}-\frac{i\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)\mathrm{e}^{2ic}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}+\frac{i\ln\left(\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{2ia}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}+\frac{i\ln\left(\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{2ic}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}$	17

input
```
int(cot(b*x+a)*cot(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-x-I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-1)*exp(2*I*a)-I/b/(exp(
2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-1)*exp(2*I*c)+I/b/(exp(2*I*a)-exp(2
*I*c))*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*exp(2*I*a)+I/b/(exp(2*I*a)-exp(
2*I*c))*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*exp(2*I*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(39) = 78$.

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.03

$$\int \cot(a+bx)\cot(c+bx)\,dx = -\frac{2\,bx\sin\left(-2\,a+2\,c\right) - \left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(-\frac{\cos(2\,bx+2\,c)\cos(-2\,a+2\,c)+\sin(2\,bx+2\,c)\sin(-2\,a+2\,c)-1}{\cos(-2\,a+2\,c)+1}\right) + (c}{2\,b\sin\left(-2\,a+2\,c\right)}$$

input
```
integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(2*b*x*sin(-2*a + 2*c) - (cos(-2*a + 2*c) + 1)*log(-(cos(2*b*x + 2*c)
*cos(-2*a + 2*c) + sin(2*b*x + 2*c)*sin(-2*a + 2*c) - 1)/(cos(-2*a + 2*c)
+ 1)) + (cos(-2*a + 2*c) + 1)*log(-1/2*cos(2*b*x + 2*c) + 1/2))/(b*sin(-2*
a + 2*c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1982 vs. $2(31) = 62$.

Time = 14.82 (sec) , antiderivative size = 7404, normalized size of antiderivative = 189.85

$$\int \cot(a+bx)\cot(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+a)*cot(b*x+c),x)
```

output
```
Piecewise((x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & E
q(a, atan(tan(c)) + pi*floor((c - pi/2)/pi))), (-b*x*tan(c)**5/(b*tan(c)**
5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(
c) + b*tan(b*x)) - b*x*tan(c)**4*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b
*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) + b*
x*tan(c)**3/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(
c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) + b*x*tan(c)**2*tan(b*x)/(b*tan(c)
**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*ta
n(c) + b*tan(b*x)) - 2*log(tan(c) + tan(b*x))*tan(c)**4/(b*tan(c)**5 + b*t
an(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*
tan(b*x)) - 2*log(tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*t
an(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*
tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**4/(b*tan(c)**5 + b*tan(c)**4*tan(
b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) + l
og(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x)
 + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - tan(c
)**6/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*t
an(b*x) + b*tan(c) + b*tan(b*x)) - tan(c)**4/(b*tan(c)**5 + b*tan(c)**4*ta
n(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)),
Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi))), (x/(cot(a)*cot(c) + zoo...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(39) = 78$.

Time = 0.08 (sec) , antiderivative size = 549, normalized size of antiderivative = 14.08

$$\int \cot(a+bx)\cot(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="maxima")
```

output
```
-((2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*
c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)*x + (cos(2*a)^2 - cos(2*c)^2 + sin(2*a
)^2 - sin(2*c)^2)*arctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + (cos(2*a
)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(b*x) - sin(a), cos
(b*x) + cos(a)) - (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arct
an2(sin(b*x) + sin(c), cos(b*x) - cos(c)) - (cos(2*a)^2 - cos(2*c)^2 + sin
(2*a)^2 - sin(2*c)^2)*arctan2(sin(b*x) - sin(c), cos(b*x) + cos(c)) - (cos
(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + c
os(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (cos(2*c)*sin(2*a)
- cos(2*a)*sin(2*c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b
*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(
2*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b
*x)*sin(c) + sin(c)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(cos(b
*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + si
n(c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b
*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(39) = 78.

Time = 0.15 (sec) , antiderivative size = 348, normalized size of antiderivative = 8.92

$$\int \cot(a+bx)\cot(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="giac")
```

output
```
-1/2*(2*b*x + (tan(1/2*a)^4*tan(1/2*c)^2 - tan(1/2*a)^4 + 4*tan(1/2*a)^3*t
an(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)^2 - 4*tan(1/2*a)*ta
n(1/2*c) + tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*a)^2 - tan(b*x) - 2*
tan(1/2*a)))/(tan(1/2*a)^4*tan(1/2*c) - tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/
2*a)^3 - 2*tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)
+ tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^2 +
 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + tan(1/2*a)^2 - 4*tan(1/2*a)*ta
n(1/2*c) + 2*tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*c)^2 - tan(b*x) -
2*tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 - tan(
1/2*a)^2*tan(1/2*c) + 2*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a
) + tan(1/2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 21.23 (sec) , antiderivative size = 207, normalized size of antiderivative = 5.31

$$\int \cot(a+bx)\cot(c+bx)\,dx = -\frac{\frac{x}{2}+x\left(\sin(a-c)^2-\frac{1}{2}\right)}{\sin(a-c)^2} - \frac{\frac{\sin(2a-2c)\ln\left(\sin(2a-2c)^2\,2\mathrm{i}+\sin(a+bx)^2\,2\mathrm{i}-\sin(3a-2c+bx)^2\,2\mathrm{i}+\sin(4a-4c)-\sin(6a-4c+2bx)+\sin(2a+2bx)\right)}{2} - \frac{\sin(2a-2c)\ln}{}}{b\sin(a-c)^2}$$

input
```
int(cot(a + b*x)*cot(c + b*x),x)
```

output
```
- (x/2 + x*(sin(a - c)^2 - 1/2))/sin(a - c)^2 - ((sin(2*a - 2*c)*log(sin(4
*a - 4*c) - sin(6*a - 4*c + 2*b*x) + sin(2*a + 2*b*x) + sin(2*a - 2*c)^2*2
i + sin(a + b*x)^2*2i - sin(3*a - 2*c + b*x)^2*2i))/2 - (sin(2*a - 2*c)*lo
g(sin(4*a - 4*c) - sin(4*a - 2*c + 2*b*x) + sin(2*c + 2*b*x) + sin(2*a - 2
*c)^2*2i + sin(c + b*x)^2*2i - sin(2*a - c + b*x)^2*2i))/2)/(b*sin(a - c)^
2)
```

Reduce [F]

$$\int \cot(a+bx)\cot(c+bx)\,dx = \int \cot(bx+c)\cot(bx+a)\,dx$$

input
```
int(cot(b*x+a)*cot(b*x+c),x)
```

output
```
int(cot(b*x + c)*cot(a + b*x),x)
```

3.406 $\int \cot(c-bx)\cot(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \cot(c-bx)\cot(a+bx)\,dx = x-\frac{\cot(a+c)\log(\sin(c-bx))}{b}+\frac{\cot(a+c)\log(\sin(a+bx))}{b}$$

output
```
x-cot(a+c)*ln(-sin(b*x-c))/b+cot(a+c)*ln(sin(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \cot(c-bx)\cot(a+bx)\,dx = x+\frac{\cot(a+c)(-\log(\sin(c-bx))+\log(-\sin(a+bx)))}{b}$$

input
```
Integrate[Cot[c - b*x]*Cot[a + b*x],x]
```

output
```
x + (Cot[a + c]*(-Log[Sin[c - b*x]] + Log[-Sin[a + b*x]]))/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5124, 5122, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\cot(c-bx)\,dx$$

$$\Big\downarrow \text{ 5124}$$

$$\cos(a+c)\int \csc(c-bx)\csc(a+bx)dx+x$$

$$\Big\downarrow \text{ 5122}$$

$$\cos(a+c)(\csc(a+c)\int \cot(c-bx)dx+\csc(a+c)\int \cot(a+bx)dx)+x$$

$$\Big\downarrow \text{ 3042}$$

$$\cos(a+c)\left(\csc(a+c)\int -\tan\left(c-bx+\frac{\pi}{2}\right)dx+\csc(a+c)\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx\right)+x$$

$$\Big\downarrow \text{ 25}$$

$$\cos(a+c)\left(-\csc(a+c)\int \tan\left(\frac{1}{2}(2c+\pi)-bx\right)dx-\csc(a+c)\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx\right)+x$$

$$\Big\downarrow \text{ 3956}$$

$$\cos(a+c)\left(\frac{\csc(a+c)\log(-\sin(a+bx))}{b}-\frac{\csc(a+c)\log(-\sin(c-bx))}{b}\right)+x$$

input
```
Int[Cot[c - b*x]*Cot[a + b*x],x]
```

output
```
x + Cos[a + c]*(-((Csc[a + c]*Log[-Sin[c - b*x]])/b) + (Csc[a + c]*Log[-Si
n[a + b*x]])/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5122
```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[Csc[(
b*c - a*d)/b]   Int[Cot[a + b*x], x], x] - Simp[Csc[(b*c - a*d)/d]   Int[Co
t[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*
c - a*d, 0]
```

rule 5124
```
Int[Cot[(a_.) + (b_.)*(x_)]*Cot[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Simp[Cos[(b*c - a*d)/d]   Int[Csc[a + b*x]*Csc[c + d*x], x], x]
 /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 4.38

method	result	si
risch	$x-\frac{i\ln\left(-\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)\mathrm{e}^{2i(a+c)}}{b\left(\mathrm{e}^{2i(a+c)}-1\right)}-\frac{i\ln\left(-\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)}{b\left(\mathrm{e}^{2i(a+c)}-1\right)}+\frac{i\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)\mathrm{e}^{2i(a+c)}}{b\left(\mathrm{e}^{2i(a+c)}-1\right)}+\frac{i\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)}{b\left(\mathrm{e}^{2i(a+c)}-1\right)}$	1

input
```
int(-cot(b*x-c)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x-I/b/(exp(2*I*(a+c))-1)*ln(-exp(2*I*(a+c))+exp(2*I*(b*x+a)))*exp(2*I*(a+c
))-I/b/(exp(2*I*(a+c))-1)*ln(-exp(2*I*(a+c))+exp(2*I*(b*x+a)))+I/b/(exp(2*
I*(a+c))-1)*ln(exp(2*I*(b*x+a))-1)*exp(2*I*(a+c))+I/b/(exp(2*I*(a+c))-1)*l
n(exp(2*I*(b*x+a))-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(37) = 74$.

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.47

$$\int \cot(c-bx)\cot(a+bx)\,dx = \frac{2\,bx\sin\left(2\,a+2\,c\right) - \left(\cos\left(2\,a+2\,c\right)+1\right)\log\left(-\frac{\cos(2\,bx+2\,a)\cos(2\,a+2\,c)+\sin(2\,bx+2\,a)\sin(2\,a+2\,c)-1}{\cos(2\,a+2\,c)+1}\right) + (\cos\left(2\,a+\right.}{2\,b\sin\left(2\,a+2\,c\right)}$$

input
```
integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*sin(2*a + 2*c) - (cos(2*a + 2*c) + 1)*log(-(cos(2*b*x + 2*a)*co
s(2*a + 2*c) + sin(2*b*x + 2*a)*sin(2*a + 2*c) - 1)/(cos(2*a + 2*c) + 1))
+ (cos(2*a + 2*c) + 1)*log(-1/2*cos(2*b*x + 2*a) + 1/2))/(b*sin(2*a + 2*c)
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1722 vs. $2(32) = 64$.

Time = 15.58 (sec) , antiderivative size = 7417, normalized size of antiderivative = 218.15

$$\int \cot(c-bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(-cot(b*x-c)*cot(b*x+a),x)
```

output
```
-Piecewise((x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) &
Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi))), (b*x*tan(c)**5/(-b*tan(c)
**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*ta
n(c) + b*tan(b*x)) - b*x*tan(c)**4*tan(b*x)/(-b*tan(c)**5 + b*tan(c)**4*ta
n(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) -
 b*x*tan(c)**3/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*
tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) + b*x*tan(c)**2*tan(b*x)/(-b*t
an(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) -
 b*tan(c) + b*tan(b*x)) - 2*log(-tan(c) + tan(b*x))*tan(c)**4/(-b*tan(c)**
5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(
c) + b*tan(b*x)) + 2*log(-tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(-b*tan(c)
**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*ta
n(c) + b*tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**4/(-b*tan(c)**5 + b*tan(
c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan
(b*x)) - log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(-b*tan(c)**5 + b*tan(c)*
*4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*
x)) - tan(c)**6/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b
*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - tan(c)**4/(-b*tan(c)**5 + b
*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) +
b*tan(b*x)), Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi))), (x/(-cot(...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(37) = 74$.

Time = 0.07 (sec) , antiderivative size = 432, normalized size of antiderivative = 12.71

$$\int \cot(c - bx) \cot(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="maxima")
```

output
```
((b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)*x - (c
os(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(b*x) + sin(a), cos(b*x
) - cos(a)) - (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(b*x) -
 sin(a), cos(b*x) + cos(a)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*ar
ctan2(sin(b*x) + sin(c), cos(b*x) + cos(c)) + (cos(2*a + 2*c)^2 + sin(2*a
+ 2*c)^2 - 1)*arctan2(sin(b*x) - sin(c), cos(b*x) - cos(c)) + log(cos(b*x)
^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a
)^2)*sin(2*a + 2*c) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(
b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2)*sin(2*a + 2*c) - log(cos(b*x)^2 + 2
*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*s
in(2*a + 2*c) - log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2
 - 2*sin(b*x)*sin(c) + sin(c)^2)*sin(2*a + 2*c))/(b*cos(2*a + 2*c)^2 + b*s
in(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(37) = 74$.

Time = 0.16 (sec) , antiderivative size = 345, normalized size of antiderivative = 10.15

$$\int \cot(c-bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="giac")
```

output
```
1/2*(2*b*x - (tan(1/2*a)^4*tan(1/2*c)^2 - tan(1/2*a)^4 - 4*tan(1/2*a)^3*ta
n(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)^2 + 4*tan(1/2*a)*tan
(1/2*c) + tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*a)^2 - tan(b*x) - 2*t
an(1/2*a)))/(tan(1/2*a)^4*tan(1/2*c) + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2
*a)^3 - 2*tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) +
 tan(1/2*c)) + (tan(1/2*a)^2*tan(1/2*c)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^2 -
4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + tan(1/2*a)^2 + 4*tan(1/2*a)*tan
(1/2*c) + 2*tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*c)^2 - tan(b*x) + 2
*tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 - tan(1
/2*a)^2*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*a)
 + tan(1/2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 21.85 (sec) , antiderivative size = 200, normalized size of antiderivative = 5.88

$$\int \cot(c-bx)\cot(a+bx)\,dx = \frac{\frac{x}{2}+x\left(\sin\left(a+c\right)^2-\frac{1}{2}\right)}{\sin\left(a+c\right)^2}$$

$$+\frac{\frac{\sin(2\,a+2\,c)\,\ln\left(\sin(2\,a+2\,c)^2\,2\mathrm{i}+\sin(a+b\,x)^2\,2\mathrm{i}-\sin(3\,a+2\,c+b\,x)^2\,2\mathrm{i}+\sin(4\,a+4\,c)-\sin(6\,a+4\,c+2\,b\,x)+\sin(2\,a+2\,b\,x)\right)}{2}-\frac{\sin(2\,a+2\,c)\,\ln}{}}{b\sin\left(a+c\right)^2}$$

input
```
int(cot(a + b*x)*cot(c - b*x),x)
```

output
```
(x/2 + x*(sin(a + c)^2 - 1/2))/sin(a + c)^2 + ((sin(2*a + 2*c)*log(sin(4*a
 + 4*c) - sin(6*a + 4*c + 2*b*x) + sin(2*a + 2*b*x) + sin(2*a + 2*c)^2*2i
+ sin(a + b*x)^2*2i - sin(3*a + 2*c + b*x)^2*2i))/2 - (sin(2*a + 2*c)*log(
sin(4*a + 4*c) - sin(4*a + 2*c + 2*b*x) - sin(2*c - 2*b*x) - sin(2*a + c +
 b*x)^2*2i + sin(2*a + 2*c)^2*2i + sin(c - b*x)^2*2i))/2)/(b*sin(a + c)^2)
```

Reduce [F]

$$\int \cot(c-bx)\cot(a+bx)\,dx = -\left(\int \cot\left(bx-c\right)\cot\left(bx+a\right)dx\right)$$

input
```
int(-cot(b*x-c)*cot(b*x+a),x)
```

output
```
- int(cot(b*x - c)*cot(a + b*x),x)
```

3.407 $\int \sec(a+bx)\sec(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \sec(a+bx)\sec(c+bx)\,dx=-\frac{\csc(a-c)\log(\cos(a+bx))}{b}+\frac{\csc(a-c)\log(\cos(c+bx))}{b}$$

output
```
-csc(a-c)*ln(cos(b*x+a))/b+csc(a-c)*ln(cos(b*x+c))/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \sec(a+bx)\sec(c+bx)\,dx=-\frac{\csc(a-c)(\log(\cos(a+bx))-\log(\cos(c+bx)))}{b}$$

input
```
Integrate[Sec[a + b*x]*Sec[c + b*x],x]
```

output
```
-((Csc[a - c]*(Log[Cos[a + b*x]] - Log[Cos[c + b*x]]))/b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5121, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \text{5121}$$

$$\csc(a-c)\int \tan(a+bx)dx-\csc(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a-c)\int \tan(a+bx)dx-\csc(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\csc(a-c)\log(\cos(bx+c))}{b}-\frac{\csc(a-c)\log(\cos(a+bx))}{b}$$

input
```
Int[Sec[a + b*x]*Sec[c + b*x],x]
```

output
```
-((Csc[a - c]*Log[Cos[a + b*x]])/b) + (Csc[a - c]*Log[Cos[c + b*x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5121
```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Csc[
(b*c - a*d)/d]   Int[Tan[a + b*x], x], x] + Simp[Csc[(b*c - a*d)/b]   Int[T
an[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b
*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\ln(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}{b(\sin(a)\cos(c)-\cos(a)\sin(c))}$	54
risch	$-\frac{2i\ln(e^{2i(bx+a)}+1)e^{i(a+c)}}{(e^{2ia}-e^{2ic})b}+\frac{2i\ln(e^{2i(bx+a)}+e^{2i(a-c)})e^{i(a+c)}}{(e^{2ia}-e^{2ic})b}$	90

input
```
int(sec(b*x+a)*sec(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b/(sin(a)*cos(c)-cos(a)*sin(c))*ln(tan(b*x+a)*sin(a)*cos(c)-tan(b*x+a)*c
os(a)*sin(c)+cos(a)*cos(c)+sin(a)*sin(c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.97

$$\int \sec(a+bx)\sec(c+bx)\,dx = -\frac{\log\left(\cos\left(bx+c\right)^2\right)-\log\left(\frac{4\left(2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(2\cos(-a+c)^2-1\right)\cos(bx+c)^2-\cos(-a+c)^2+1\right)}{\cos(-a+c)^2+2\cos(-a+c)+1}\right)}{2\,b\sin\left(-a+c\right)}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(log(cos(b*x + c)^2) - log(4*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)
*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)/(
cos(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*sin(-a + c))
```

Sympy [F]

$$\int \sec(a+bx)\sec(c+bx)\,dx = \int \sec(a+bx)\sec(bx+c)\,dx$$

input
```
integrate(sec(b*x+a)*sec(b*x+c),x)
```

output
```
Integral(sec(a + b*x)*sec(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(36) = 72.

Time = 0.14 (sec) , antiderivative size = 349, normalized size of antiderivative = 9.69

$$\int \sec(a+bx)\sec(c+bx)\,dx =$$

$$-\frac{2\left(\left(\cos\left(2a\right)-\cos\left(2c\right)\right)\cos\left(a+c\right)+\left(\sin\left(2a\right)-\sin\left(2c\right)\right)\sin\left(a+c\right)\right)\arctan\left(\sin\left(2bx\right)-\sin\left(2a\right),\text{co}\right.}{}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="maxima")
```

output
```
-(2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*
arctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - 2*((cos(2*a) - cos
(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(2*b*x) -
 sin(2*c), cos(2*b*x) + cos(2*c)) - ((sin(2*a) - sin(2*c))*cos(a + c) - (c
os(2*a) - cos(2*c))*sin(a + c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) +
 cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + ((sin(2
*a) - sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(2*b
*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*s
in(2*c) + sin(2*c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a
)*sin(2*c) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(36) = 72$.

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 4.75

$$\int \sec(a+bx)\sec(c+bx)\,dx = \frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1\right)\log\left(\left|2\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-2\tan\left(bx\right.\right.\right.}{2\left(\tan\left(\frac{1}{2}\right.\right.}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="giac")
```

output
```
1/2*(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(
2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2
*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)
^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 +
1))/((tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan
(1/2*c))*b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 249, normalized size of antiderivative = 6.92

$$\int \sec(a+bx)\sec(c+bx)\,dx = \frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}\left(\ln\left(-\frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}\,\left(4\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,4\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\right)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}-1\right)}+\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\,4\mathrm{i}\right)-\ln\left(-\cdots\right.}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-1\right)}$$

input
```
int(1/(cos(a + b*x)*cos(c + b*x)),x)
```

output
```
(2*(-exp(a*2i - c*2i))^(1/2)*(log(exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2
i)*4i - (2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(4*b*exp(a*2i)*exp(-c*2i) + 2*b*e
xp(a*2i)*exp(b*x*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))/(b*(exp(a*2i
)*exp(-c*2i) - 1))) - log(exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2i)*4i -
(2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*2i)
*exp(b*x*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))/(b - b*exp(a*2i)*exp
(-c*2i)))))/(b*(exp(a*2i - c*2i) - 1))
```

Reduce [F]

$$\int \sec(a+bx)\sec(c+bx)\,dx = \frac{4\left(\int \frac{1}{\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-\tan\left(\frac{bx}{2}+\frac{c}{2}\right)^2-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1}dx\right)b+\log\left(\tan\left(\frac{bx}{2}+\frac{c}{2}\right)-1\right)-\log\left(\tan\left(\frac{bx}{2}+\frac{c}{2}\right)+\cdots\right.}{b}$$

input
```
int(sec(b*x+a)*sec(b*x+c),x)
```

output
```
(4*int(1/(tan((b*x + c)/2)**2*tan((a + b*x)/2)**2 - tan((b*x + c)/2)**2 -
tan((a + b*x)/2)**2 + 1),x)*b + log(tan((b*x + c)/2) - 1) - log(tan((b*x +
 c)/2) + 1) + log(tan((a + b*x)/2) - 1) - log(tan((a + b*x)/2) + 1) - b*x)
/b
```

3.408 $\int \sec(c-bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \sec(c-bx)\sec(a+bx)\,dx=\frac{\csc(a+c)\log(\cos(c-bx))}{b}-\frac{\csc(a+c)\log(\cos(a+bx))}{b}$$

output
```
csc(a+c)*ln(cos(b*x-c))/b-csc(a+c)*ln(cos(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \sec(c-bx)\sec(a+bx)\,dx=\frac{\csc(a+c)(\log(\cos(c-bx))-\log(\cos(a+bx)))}{b}$$

input
```
Integrate[Sec[c - b*x]*Sec[a + b*x],x]
```

output
```
(Csc[a + c]*(Log[Cos[c - b*x]] - Log[Cos[a + b*x]]))/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5121, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\sec(c-bx)\,dx$$

$$\Big\downarrow \text{5121}$$

$$\csc(a+c)\int \tan(c-bx)dx+\csc(a+c)\int \tan(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+c)\int \tan(c-bx)dx+\csc(a+c)\int \tan(a+bx)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\csc(a+c)\log(\cos(c-bx))}{b}-\frac{\csc(a+c)\log(\cos(a+bx))}{b}$$

input
```
Int[Sec[c - b*x]*Sec[a + b*x],x]
```

output
```
(Csc[a + c]*Log[Cos[c - b*x]])/b - (Csc[a + c]*Log[Cos[a + b*x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5121
```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Csc[
(b*c - a*d)/d]   Int[Tan[a + b*x], x], x] + Simp[Csc[(b*c - a*d)/b]   Int[T
an[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b
*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

method	result	size
default	$\frac{\ln(\tan(bx+a)\sin(a)\cos(c)+\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)-\sin(a)\sin(c))}{b(\sin(a)\cos(c)+\cos(a)\sin(c))}$	53
risch	$-\frac{2i\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)\mathrm{e}^{i(a+c)}}{\left(\mathrm{e}^{2i(a+c)}-1\right)b}+\frac{2i\ln\left(\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)\mathrm{e}^{i(a+c)}}{\left(\mathrm{e}^{2i(a+c)}-1\right)b}$	80

input
```
int(sec(b*x-c)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b/(sin(a)*cos(c)+cos(a)*sin(c))*ln(tan(b*x+a)*sin(a)*cos(c)+tan(b*x+a)*c
os(a)*sin(c)+cos(a)*cos(c)-sin(a)*sin(c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.82

$$\int \sec(c-bx)\sec(a+bx)\,dx =$$
$$-\frac{\log\left(\cos\left(bx+a\right)^2\right)-\log\left(\frac{4\left(2\cos(bx+a)\cos(a+c)\sin(bx+a)\sin(a+c)+\left(2\cos(a+c)^2-1\right)\cos(bx+a)^2-\cos(a+c)^2+1\right)}{\cos(a+c)^2+2\cos(a+c)+1}\right)}{2\,b\sin\left(a+c\right)}$$

input
```
integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(log(cos(b*x + a)^2) - log(4*(2*cos(b*x + a)*cos(a + c)*sin(b*x + a)*
sin(a + c) + (2*cos(a + c)^2 - 1)*cos(b*x + a)^2 - cos(a + c)^2 + 1)/(cos(
a + c)^2 + 2*cos(a + c) + 1)))/(b*sin(a + c))
```

Sympy [F]

$$\int \sec(c-bx)\sec(a+bx)\,dx = \int \sec(a+bx)\sec(bx-c)\,dx$$

input
```
integrate(sec(b*x-c)*sec(b*x+a),x)
```

output
```
Integral(sec(a + b*x)*sec(b*x - c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(34) = 68$.

Time = 0.05 (sec) , antiderivative size = 322, normalized size of antiderivative = 9.76

$$\int \sec(c-bx)\sec(a+bx)\,dx = \frac{2\left(\cos\left(2a+2c\right)\cos\left(a+c\right)+\sin\left(2a+2c\right)\sin\left(a+c\right)-\cos\left(a+c\right)\right)\arctan\left(\sin\left(2bx\right)-\sin\left(2a\right),\cos\left(2\right.\right.}{}$$

input
```
integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="maxima")
```

output
```
(2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*ar
ctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - 2*(cos(2*a + 2*c)*co
s(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(2*b*x) + si
n(2*c), cos(2*b*x) + cos(2*c)) - (cos(a + c)*sin(2*a + 2*c) - cos(2*a + 2*
c)*sin(a + c) + sin(a + c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos
(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + (cos(a + c)
*sin(2*a + 2*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(2*b*x)^2
 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 + 2*sin(2*b*x)*sin(2*
c) + sin(2*c)^2))/(b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a +
 2*c) + b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(34) = 68$.

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 5.12

$$\int \sec(c-bx)\sec(a+bx)\,dx = -\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1\right)\log\left(\left|2\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+2\tan\right.\right.}{2\left(\tan\right.}$$

input
```
integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs
(2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) + 2*tan(b*x + a)*tan(1/2*a)*tan(1/
2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 - 2*tan(b*x + a)*tan(1/2*a) + tan(1/2*a
)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 -
 1))/((tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) - ta
n(1/2*c))*b)
```

Mupad [B] (verification not implemented)

Time = 25.22 (sec) , antiderivative size = 249, normalized size of antiderivative = 7.55

$$\int \sec(c-bx)\sec(a+bx)\,dx$$
$$=\frac{2\sqrt{-e^{a\,2i+c\,2i}}\left(\ln\left(-\frac{2\sqrt{-e^{a\,2i}\,e^{c\,2i}}\,\left(4\,b\,e^{a\,2i}\,e^{c\,2i}+2\,b\,e^{a\,2i}\,e^{b\,x\,2i}+2\,b\,e^{a\,4i}\,e^{c\,2i}\,e^{b\,x\,2i}\right)}{b\,\left(e^{a\,2i}\,e^{c\,2i}-1\right)}+e^{a\,1i}\,e^{a\,2i}\,e^{c\,1i}\,e^{b\,x\,2i}\,4i\right)-\ln\left(-\frac{2\sqrt{-e^{\cdots}}}{\cdots}\right.\right.}{b\,\left(e^{a\,2i+c\,2i}-1\right)}$$

input
```
int(1/(cos(a + b*x)*cos(c - b*x)),x)
```

output
```
(2*(-exp(a*2i + c*2i))^(1/2)*(log(exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(b*x*2i
)*4i - (2*(-exp(a*2i)*exp(c*2i))^(1/2)*(4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(
a*2i)*exp(b*x*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp(b*x*2i)))/(b*(exp(a*2i)*ex
p(c*2i) - 1))) - log(exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(b*x*2i)*4i - (2*(-e
xp(a*2i)*exp(c*2i))^(1/2)*(4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*2i)*exp(b*x
*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp(b*x*2i)))/(b - b*exp(a*2i)*exp(c*2i))))
)/(b*(exp(a*2i + c*2i) - 1))
```

Reduce [F]

$$\int \sec(c-bx)\sec(a+bx)\,dx$$
$$=\frac{4\left(\int \frac{1}{\tan\left(\frac{bx}{2}-\frac{c}{2}\right)^{2}\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-\tan\left(\frac{bx}{2}-\frac{c}{2}\right)^{2}-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}dx\right)b+\log\left(\tan\left(\frac{bx}{2}-\frac{c}{2}\right)-1\right)-\log\left(\tan\left(\frac{bx}{2}-\frac{c}{2}\right)+\cdots\right.}{b}$$

input
```
int(sec(b*x-c)*sec(b*x+a),x)
```

output
```
(4*int(1/(tan((b*x - c)/2)**2*tan((a + b*x)/2)**2 - tan((b*x - c)/2)**2 -
tan((a + b*x)/2)**2 + 1),x)*b + log(tan((b*x - c)/2) - 1) - log(tan((b*x -
 c)/2) + 1) + log(tan((a + b*x)/2) - 1) - log(tan((a + b*x)/2) + 1) - b*x)
/b
```

3.409 $\int \csc(a+bx)\csc(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \csc(a+bx)\csc(c+bx)\,dx = -\frac{\csc(a-c)\log(\sin(a+bx))}{b}+\frac{\csc(a-c)\log(\sin(c+bx))}{b}$$

output
```
-csc(a-c)*ln(sin(b*x+a))/b+csc(a-c)*ln(sin(b*x+c))/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \csc(a+bx)\csc(c+bx)\,dx = -\frac{\csc(a-c)(\log(\sin(a+bx))-\log(\sin(c+bx)))}{b}$$

input
```
Integrate[Csc[a + b*x]*Csc[c + b*x],x]
```

output
```
-((Csc[a - c]*(Log[Sin[a + b*x]] - Log[Sin[c + b*x]]))/b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5122, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \text{5122}$$

$$\csc(a-c)\int \cot(c+bx)dx-\csc(a-c)\int \cot(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a-c)\int -\tan\left(c+bx+\frac{\pi}{2}\right)dx-\csc(a-c)\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\csc(a-c)\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\csc(a-c)\int \tan\left(\frac{1}{2}(2c+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\csc(a-c)\log(-\sin(bx+c))}{b}-\frac{\csc(a-c)\log(-\sin(a+bx))}{b}$$

input
```
Int[Csc[a + b*x]*Csc[c + b*x],x]
```

output
```
-((Csc[a - c]*Log[-Sin[a + b*x]])/b) + (Csc[a - c]*Log[-Sin[c + b*x]])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5122
```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[Csc[(
b*c - a*d)/b]   Int[Cot[a + b*x], x], x] - Simp[Csc[(b*c - a*d)/d]   Int[Co
t[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*
c - a*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.56

method	result
risch	$-\frac{2i\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)\mathrm{e}^{i(a+c)}}{\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)b}+\frac{2i\ln\left(\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{i(a+c)}}{\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)b}$
default	$\frac{\frac{\ln(\tan(bx+a))}{-\sin(a)\cos(c)+\cos(a)\sin(c)}+\frac{(-\cos(a)\cos(c)-\sin(a)\sin(c))\ln(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))}{(-\sin(a)\cos(c)+\cos(a)\sin(c))(\cos(a)\cos(c)+\sin(a)\sin(c))}}{b}$

input
```
int(csc(b*x+a)*csc(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2*I*ln(exp(2*I*(b*x+a))-1)/(exp(2*I*a)-exp(2*I*c))/b*exp(I*(a+c))+2*I*ln(
exp(2*I*(b*x+a))-exp(2*I*(a-c)))/(exp(2*I*a)-exp(2*I*c))/b*exp(I*(a+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(36) = 72.

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.06

$$\int \csc(a+bx)\csc(c+bx)\,dx =$$

$$-\frac{\log\left(-\frac{1}{4}\cos\left(bx+c\right)^{2}+\frac{1}{4}\right)-\log\left(-\frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(2\cos(-a+c)^{2}-1\right)\cos(bx+c)^{2}-\cos(-a+c)^{2}}{\cos(-a+c)^{2}+2\cos(-a+c)+1}\right)}{2\,b\sin\left(-a+c\right)}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(log(-1/4*cos(b*x + c)^2 + 1/4) - log(-(2*cos(b*x + c)*cos(-a + c)*si
n(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c
)^2)/(cos(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*sin(-a + c))
```

Sympy [F]

$$\int \csc(a+bx)\csc(c+bx)\,dx = \int \csc\left(a+bx\right)\csc\left(bx+c\right)\,dx$$

input
```
integrate(csc(b*x+a)*csc(b*x+c),x)
```

output
```
Integral(csc(a + b*x)*csc(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 564, normalized size of antiderivative = 15.67

$$\int \csc(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="maxima")
```

output
```
-(2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*
arctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + 2*((cos(2*a) - cos(2*c))*c
os(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(b*x) - sin(a), c
os(b*x) + cos(a)) - 2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(
2*c))*sin(a + c))*arctan2(sin(b*x) + sin(c), cos(b*x) - cos(c)) - 2*((cos(
2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(si
n(b*x) - sin(c), cos(b*x) + cos(c)) - ((sin(2*a) - sin(2*c))*cos(a + c) -
(cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos
(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - ((sin(2*a) - sin(2*c)
)*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 - 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + ((sin(
2*a) - sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*
x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin
(c)^2) + ((sin(2*a) - sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a +
 c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*
x)*sin(c) + sin(c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a
)*sin(2*c) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(36) = 72$.

Time = 0.16 (sec) , antiderivative size = 396, normalized size of antiderivative = 11.00

$$\int \csc(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="giac")
```

output
```
1/2*((tan(1/2*a)^4*tan(1/2*c)^4 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)
^4 + 4*tan(1/2*a)^3*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4
+ 4*tan(1/2*a)*tan(1/2*c) + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c
)^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2
*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/
2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(1/
2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*tan(1/2*a
)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 -
 tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)^2 + ta
n(1/2*c)^3 + tan(1/2*a) - tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 + tan(1
/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)))/(tan(1/2*a)^2*tan(1/2*c
) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 25.21 (sec) , antiderivative size = 249, normalized size of antiderivative = 6.92

$$\int \csc(a+bx)\csc(c+bx)\,dx = \frac{2\sqrt{-e^{a\,2i-c\,2i}}\left(\ln\left(\frac{2\sqrt{-e^{a\,2i}\,e^{-c\,2i}}\left(-4\,b\,e^{a\,2i}\,e^{-c\,2i}+2\,b\,e^{a\,2i}\,e^{b\,x\,2i}+2\,b\,e^{a\,4i}\,e^{-c\,2i}\,e^{b\,x\,2i}\right)}{b\left(e^{a\,2i}\,e^{-c\,2i}-1\right)}-e^{a\,1i}\,e^{a\,2i}\,e^{-c\,1i}\,e^{b\,x\,2i}\,4i\right)-\ln\left(2\sqrt{\cdots}\right.\right.}{b\left(e^{a\,2i-c\,2i}-1\right)}$$

input
```
int(1/(sin(a + b*x)*sin(c + b*x)),x)
```

output
```
(2*(-exp(a*2i - c*2i))^(1/2)*(log((2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(2*b*ex
p(a*2i)*exp(b*x*2i) - 4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*4i)*exp(-c*2i)*
exp(b*x*2i)))/(b*(exp(a*2i)*exp(-c*2i) - 1)) - exp(a*1i)*exp(a*2i)*exp(-c*
1i)*exp(b*x*2i)*4i) - log((2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(2*b*exp(a*2i)*
exp(b*x*2i) - 4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*
2i)))/(b - b*exp(a*2i)*exp(-c*2i)) - exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*
x*2i)*4i)))/(b*(exp(a*2i - c*2i) - 1))
```

Reduce [F]

$$\int \csc(a+bx)\csc(c+bx)\,dx = \int \csc(bx+c)\csc(bx+a)\,dx$$

input
```
int(csc(b*x+a)*csc(b*x+c),x)
```

output
```
int(csc(b*x + c)*csc(a + b*x),x)
```

3.410 $\int \csc(c-bx)\csc(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \csc(c-bx)\csc(a+bx)\,dx = -\frac{\csc(a+c)\log(\sin(c-bx))}{b} + \frac{\csc(a+c)\log(\sin(a+bx))}{b}$$

output
```
-csc(a+c)*ln(-sin(b*x-c))/b+csc(a+c)*ln(sin(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \csc(c-bx)\csc(a+bx)\,dx = -\frac{\csc(a+c)(\log(\sin(c-bx)) - \log(-\sin(a+bx)))}{b}$$

input
```
Integrate[Csc[c - b*x]*Csc[a + b*x],x]
```

output
```
-((Csc[a + c]*(Log[Sin[c - b*x]] - Log[-Sin[a + b*x]]))/b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5122, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc(c-bx)\,dx$$

$$\Big\downarrow \text{5122}$$

$$\csc(a+c)\int \cot(c-bx)dx+\csc(a+c)\int \cot(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+c)\int -\tan\left(c-bx+\frac{\pi}{2}\right)dx+\csc(a+c)\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\csc(a+c)\int \tan\left(\frac{1}{2}(2c+\pi)-bx\right)dx-\csc(a+c)\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\csc(a+c)\log(-\sin(a+bx))}{b}-\frac{\csc(a+c)\log(-\sin(c-bx))}{b}$$

input
```
Int[Csc[c - b*x]*Csc[a + b*x],x]
```

output
```
-((Csc[a + c]*Log[-Sin[c - b*x]])/b) + (Csc[a + c]*Log[-Sin[a + b*x]])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5122
```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[Csc[(
b*c - a*d)/b]   Int[Cot[a + b*x], x], x] - Simp[Csc[(b*c - a*d)/d]   Int[Co
t[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*
c - a*d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(36) = 72$.

Time = 0.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.42

method	result	size
default	$-\frac{-\frac{\ln(\tan(bx+a))}{\sin(a)\cos(c)+\cos(a)\sin(c)}+\frac{\ln(\tan(bx+a)\cos(a)\cos(c)-\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)-\cos(a)\sin(c))}{\sin(a)\cos(c)+\cos(a)\sin(c)}}{b}$	80
risch	$-\frac{2i\ln\left(-e^{2i(a+c)}+e^{2i(bx+a)}\right)e^{i(a+c)}}{\left(e^{2i(a+c)}-1\right)b}+\frac{2i\ln\left(e^{2i(bx+a)}-1\right)e^{i(a+c)}}{\left(e^{2i(a+c)}-1\right)b}$	82

input
```
int(-csc(b*x-c)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(-1/(sin(a)*cos(c)+cos(a)*sin(c))*ln(tan(b*x+a))+1/(sin(a)*cos(c)+cos
(a)*sin(c))*ln(tan(b*x+a)*cos(a)*cos(c)-tan(b*x+a)*sin(a)*sin(c)-sin(a)*co
s(c)-cos(a)*sin(c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91

$$\int \csc(c-bx)\csc(a+bx)\,dx$$
$$= \frac{\log\left(-\frac{1}{4}\cos\left(bx+a\right)^2+\frac{1}{4}\right)-\log\left(-\frac{2\cos(bx+a)\cos(a+c)\sin(bx+a)\sin(a+c)+\left(2\cos(a+c)^2-1\right)\cos(bx+a)^2-\cos(a+c)^2}{\cos(a+c)^2+2\cos(a+c)+1}\right)}{2\,b\sin\left(a+c\right)}$$

input
```
integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(log(-1/4*cos(b*x + a)^2 + 1/4) - log(-(2*cos(b*x + a)*cos(a + c)*sin(
b*x + a)*sin(a + c) + (2*cos(a + c)^2 - 1)*cos(b*x + a)^2 - cos(a + c)^2)/
(cos(a + c)^2 + 2*cos(a + c) + 1)))/(b*sin(a + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1824 vs. $2(31) = 62$.

Time = 55.22 (sec) , antiderivative size = 1824, normalized size of antiderivative = 55.27

$$\int \csc(c-bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(-csc(b*x-c)*csc(b*x+a),x)
```

output
```
Piecewise((-log(-tan(c/2) + tan(b*x/2))*tan(c/2)/(2*b) - log(-tan(c/2) + t
an(b*x/2))/(2*b*tan(c/2)) - log(tan(b*x/2) + 1/tan(c/2))*tan(c/2)/(2*b) -
log(tan(b*x/2) + 1/tan(c/2))/(2*b*tan(c/2)) + log(tan(b*x/2))*tan(c/2)/(2*
b) + log(tan(b*x/2))/(2*b*tan(c/2)), Eq(a, 0)), (log(tan(a/2) + tan(b*x/2)
)*tan(a/2)/(2*b) + log(tan(a/2) + tan(b*x/2))/(2*b*tan(a/2)) + log(tan(b*x
/2) - 1/tan(a/2))*tan(a/2)/(2*b) + log(tan(b*x/2) - 1/tan(a/2))/(2*b*tan(a
/2)) - log(tan(b*x/2))*tan(a/2)/(2*b) - log(tan(b*x/2))/(2*b*tan(a/2)), Eq
(c, 0)), (-tan(c/2)**4*tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c
/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)) - 2*tan(
c/2)**2*tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/
2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)) - tan(b*x/2)/(-2*b*tan(
c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b
*tan(c/2)*tan(b*x/2)), Eq(a, 2*atan(1/tan(c/2)))), (tan(c/2)**4*tan(b*x/2)
/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/
2)**2 + 2*b*tan(c/2)*tan(b*x/2)) + 2*tan(c/2)**2*tan(b*x/2)/(-2*b*tan(c/2)
**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan
(c/2)*tan(b*x/2)) + tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)
**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)), Eq(a, -2*a
tan(tan(c/2)) - 2*pi*floor((c/2 - pi/2)/pi))), (x/(sin(a)*sin(c)), Eq(b, 0
)), (-log(tan(a/2) + tan(b*x/2))*tan(a/2)**2*tan(c/2)**2/(2*b*tan(a/2)*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 536, normalized size of antiderivative = 16.24

$$\int \csc(c - bx) \csc(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="maxima")
```

output
```
-(2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*a
rctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + 2*(cos(2*a + 2*c)*cos(a + c
) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(b*x) - sin(a), cos
(b*x) + cos(a)) - 2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c)
 - cos(a + c))*arctan2(sin(b*x) + sin(c), cos(b*x) + cos(c)) - 2*(cos(2*a
+ 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(b*
x) - sin(c), cos(b*x) - cos(c)) - (cos(a + c)*sin(2*a + 2*c) - cos(2*a + 2
*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2
 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (cos(a + c)*sin(2*a + 2*c)
 - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos
(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + (cos(a + c)*
sin(2*a + 2*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 +
2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)
+ (cos(a + c)*sin(2*a + 2*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log
(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c
) + sin(c)^2))/(b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*
c) + b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(36) = 72$.

Time = 0.16 (sec) , antiderivative size = 397, normalized size of antiderivative = 12.03

$$\int \csc(c - bx)\csc(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="giac")
```

output
```
1/2*((tan(1/2*a)^4*tan(1/2*c)^4 - 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)
^4 - 4*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4
- 4*tan(1/2*a)*tan(1/2*c) + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c
)^2 - tan(b*x + a)*tan(1/2*a)^2 - 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) + 2
*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/
2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) - 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(1/
2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) - 6*tan(1/2*a
)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 +
 tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*a)*tan(1/2*c)^2 + ta
n(1/2*c)^3 - tan(1/2*a) - tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 + tan(1
/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)))/(tan(1/2*a)^2*tan(1/2*c
) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) - tan(1/2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 25.41 (sec) , antiderivative size = 249, normalized size of antiderivative = 7.55

$$\int \csc(c-bx)\csc(a+bx)\,dx = \frac{2\sqrt{-e^{a\,2i+c\,2i}}\left(\ln\left(\frac{2\sqrt{-e^{a\,2i}\,e^{c\,2i}}\,\left(-4\,b\,e^{a\,2i}\,e^{c\,2i}+2\,b\,e^{a\,2i}\,e^{b\,x\,2i}+2\,b\,e^{a\,4i}\,e^{c\,2i}\,e^{b\,x\,2i}\right)}{b\,\left(e^{a\,2i}\,e^{c\,2i}-1\right)}+e^{a\,1i}\,e^{a\,2i}\,e^{c\,1i}\,e^{b\,x\,2i}\,4i\right)-\ln\left(\frac{2\sqrt{-e^{a\,2i}}\cdots}{\cdots}\right)\cdots\right)}{b\,\left(e^{a\,2i+c\,2i}-1\right)}$$

input
```
int(1/(sin(a + b*x)*sin(c - b*x)),x)
```

output
```
(2*(-exp(a*2i + c*2i))^(1/2)*(log((2*(-exp(a*2i)*exp(c*2i))^(1/2)*(2*b*exp
(a*2i)*exp(b*x*2i) - 4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp
(b*x*2i)))/(b*(exp(a*2i)*exp(c*2i) - 1)) + exp(a*1i)*exp(a*2i)*exp(c*1i)*e
xp(b*x*2i)*4i) - log((2*(-exp(a*2i)*exp(c*2i))^(1/2)*(2*b*exp(a*2i)*exp(b*
x*2i) - 4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp(b*x*2i)))/(b
 - b*exp(a*2i)*exp(c*2i)) + exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(b*x*2i)*4i))
)/(b*(exp(a*2i + c*2i) - 1))
```

Reduce [F]

$$\int \csc(c-bx)\csc(a+bx)\,dx = -\left(\int \csc\left(bx-c\right)\csc\left(bx+a\right)dx\right)$$

input
```
int(-csc(b*x-c)*csc(b*x+a),x)
```

output
```
- int(csc(b*x - c)*csc(a + b*x),x)
```

3.411 $\int \sin(a+bx)\sin^7(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx = \frac{128\sin^9(a+bx)}{9b}-\frac{384\sin^{11}(a+bx)}{11b}+\frac{384\sin^{13}(a+bx)}{13b}-\frac{128\sin^{15}(a+bx)}{15b}$$

output
```
128/9*sin(b*x+a)^9/b-384/11*sin(b*x+a)^11/b+384/13*sin(b*x+a)^13/b-128/15*
sin(b*x+a)^15/b
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx = \frac{4(8330+10755\cos(2(a+bx))+3366\cos(4(a+bx))+429\cos(6(a+bx)))\sin^9(a+bx)}{6435b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

output
```
(4*(8330 + 10755*Cos[2*(a + b*x)] + 3366*Cos[4*(a + b*x)] + 429*Cos[6*(a +
 b*x)])*Sin[a + b*x]^9)/(6435*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^7dx$$

$$\Big\downarrow \text{4776}$$

$$128\int \cos^7(a+bx)\sin^8(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$128\int \cos(a+bx)^7\sin(a+bx)^8dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{128\int \sin^8(a+bx)\left(1-\sin^2(a+bx)\right)^3d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{128\int \left(-\sin^{14}(a+bx)+3\sin^{12}(a+bx)-3\sin^{10}(a+bx)+\sin^8(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{128\left(-\frac{1}{15}\sin^{15}(a+bx)+\frac{3}{13}\sin^{13}(a+bx)-\frac{3}{11}\sin^{11}(a+bx)+\frac{1}{9}\sin^9(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

output
```
(128*(Sin[a + b*x]^9/9 - (3*Sin[a + b*x]^11)/11 + (3*Sin[a + b*x]^13)/13 -
 Sin[a + b*x]^15/15))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 25.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.51

method	result
parallelrisch	$\frac{5005\sin(9bx+9a)+19305\sin(7bx+7a)-4095\sin(11bx+11a)-75075\sin(3bx+3a)+429\sin(15bx+15a)-495\sin(13bx+13a)+}{823680b}$
default	$\frac{35\sin(bx+a)}{128b}-\frac{35\sin(3bx+3a)}{384b}-\frac{21\sin(5bx+5a)}{640b}+\frac{3\sin(7bx+7a)}{128b}+\frac{7\sin(9bx+9a)}{1152b}-\frac{7\sin(11bx+11a)}{1408b}-\frac{\sin(13b}{16}$
risch	$\frac{35\sin(bx+a)}{128b}-\frac{35\sin(3bx+3a)}{384b}-\frac{21\sin(5bx+5a)}{640b}+\frac{3\sin(7bx+7a)}{128b}+\frac{7\sin(9bx+9a)}{1152b}-\frac{7\sin(11bx+11a)}{1408b}-\frac{\sin(13b}{16}$
orering	Expression too large to display

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/823680*(5005*sin(9*b*x+9*a)+19305*sin(7*b*x+7*a)-4095*sin(11*b*x+11*a)-7
5075*sin(3*b*x+3*a)+429*sin(15*b*x+15*a)-495*sin(13*b*x+13*a)+225225*sin(b
*x+a)-27027*sin(5*b*x+5*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.36

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx$$
$$=\frac{128\left(429\cos(bx+a)^{14}-1518\cos(bx+a)^{12}+1854\cos(bx+a)^{10}-800\cos(bx+a)^{8}+5\cos(bx+a\right.}{6435\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

output
```
128/6435*(429*cos(b*x + a)^14 - 1518*cos(b*x + a)^12 + 1854*cos(b*x + a)^1
0 - 800*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*cos(b*x + a)^4 + 8*cos(b*x +
 a)^2 + 16)*sin(b*x + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(53) = 106$.

Time = 26.39 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.41

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{3838\sin(a+bx)\sin^6(2a+2bx)\cos(2a+2bx)}{6435b} - \frac{1648\sin(a+bx)\sin^4(2a+2bx)\cos^3(2a+2bx)}{1287b} - \frac{768\sin(a+bx)\sin^2(2a+2bx)\cos^5(2a+} \\ x\sin(a)\sin^7(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**7,x)
```

output
```
Piecewise((-3838*sin(a + b*x)*sin(2*a + 2*b*x)**6*cos(2*a + 2*b*x)/(6435*b
) - 1648*sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(2*a + 2*b*x)**3/(1287*b) - 7
68*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**5/(715*b) - 2048*sin
(a + b*x)*cos(2*a + 2*b*x)**7/(6435*b) + 1241*sin(2*a + 2*b*x)**7*cos(a +
b*x)/(6435*b) + 376*sin(2*a + 2*b*x)**5*cos(a + b*x)*cos(2*a + 2*b*x)**2/(
715*b) + 640*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**4/(1287*b)
 + 1024*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**6/(6435*b), Ne(b,
0)), (x*sin(a)*sin(2*a)**7, True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx$$
$$= \frac{429\sin(15bx+15a) - 495\sin(13bx+13a) - 4095\sin(11bx+11a) + 5005\sin(9bx+9a) + 19305}{823680b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

output
```
1/823680*(429*sin(15*b*x + 15*a) - 495*sin(13*b*x + 13*a) - 4095*sin(11*b*
x + 11*a) + 5005*sin(9*b*x + 9*a) + 19305*sin(7*b*x + 7*a) - 27027*sin(5*b
*x + 5*a) - 75075*sin(3*b*x + 3*a) + 225225*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx = -\frac{128\left(429\,\sin\left(bx+a\right)^{15}-1485\,\sin\left(bx+a\right)^{13}+1755\,\sin\left(bx+a\right)^{11}-715\,\sin\left(bx+a\right)^{9}\right)}{6435\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

output
```
-128/6435*(429*sin(b*x + a)^15 - 1485*sin(b*x + a)^13 + 1755*sin(b*x + a)^
11 - 715*sin(b*x + a)^9)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx = \frac{-\frac{128\sin(a+b\,x)^{15}}{15}+\frac{384\sin(a+b\,x)^{13}}{13}-\frac{384\sin(a+b\,x)^{11}}{11}+\frac{128\sin(a+b\,x)^{9}}{9}}{b}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^7,x)
```

output
```
((128*sin(a + b*x)^9)/9 - (384*sin(a + b*x)^11)/11 + (384*sin(a + b*x)^13)
/13 - (128*sin(a + b*x)^15)/15)/b
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.97

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx$$
$$= \frac{-462\cos(2bx+2a)\sin(2bx+2a)^6\sin(bx+a)-560\cos(2bx+2a)\sin(2bx+2a)^4\sin(bx+a)-768\,\mathrm{c}}{}$$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^7,x)
```

output
```
( - 462*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**6*sin(a + b*x) - 560*cos(2*a +
2*b*x)*sin(2*a + 2*b*x)**4*sin(a + b*x) - 768*cos(2*a + 2*b*x)*sin(2*a + 2
*b*x)**2*sin(a + b*x) - 2048*cos(2*a + 2*b*x)*sin(a + b*x) + 33*cos(a + b*
x)*sin(2*a + 2*b*x)**7 + 56*cos(a + b*x)*sin(2*a + 2*b*x)**5 + 128*cos(a +
 b*x)*sin(2*a + 2*b*x)**3 + 1024*cos(a + b*x)*sin(2*a + 2*b*x))/(6435*b)
```

3.412 $\int \sin(a+bx)\sin^6(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx=-\frac{64\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{3b}-\frac{192\cos^{11}(a+bx)}{11b}+\frac{64\cos^{13}(a+bx)}{13b}$$

output
```
-64/7*cos(b*x+a)^7/b+64/3*cos(b*x+a)^9/b-192/11*cos(b*x+a)^11/b+64/13*cos(
b*x+a)^13/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx=\frac{2\cos^7(a+bx)(-5230+6377\cos(2(a+bx))-1890\cos(4(a+bx))+231\cos(6(a+bx)))}{3003b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

output
```
(2*Cos[a + b*x]^7*(-5230 + 6377*Cos[2*(a + b*x)] - 1890*Cos[4*(a + b*x)] +
 231*Cos[6*(a + b*x)]))/(3003*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^6dx$$

$$\Big\downarrow \text{4776}$$

$$64\int \cos^6(a+bx)\sin^7(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$64\int \cos(a+bx)^6\sin(a+bx)^7dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{64\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)^3d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{64\int \left(-\cos^{12}(a+bx)+3\cos^{10}(a+bx)-3\cos^8(a+bx)+\cos^6(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{64\left(-\frac{1}{13}\cos^{13}(a+bx)+\frac{3}{11}\cos^{11}(a+bx)-\frac{1}{3}\cos^9(a+bx)+\frac{1}{7}\cos^7(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

output
```
(-64*(Cos[a + b*x]^7/7 - Cos[a + b*x]^9/3 + (3*Cos[a + b*x]^11)/11 - Cos[a
 + b*x]^13/13))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 14.59 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

method	result
parallelrisch	$\frac{-65536+2574\cos(7bx+7a)-2002\cos(9bx+9a)+231\cos(13bx+13a)-273\cos(11bx+11a)-15015\cos(3bx+3a)-60060\cos(bx}{192192b}$
default	$-\frac{5\cos(bx+a)}{16b}-\frac{5\cos(3bx+3a)}{64b}+\frac{3\cos(5bx+5a)}{64b}+\frac{3\cos(7bx+7a)}{224b}-\frac{\cos(9bx+9a)}{96b}-\frac{\cos(11bx+11a)}{704b}+\frac{\cos(13bx+}{832b}$
risch	$-\frac{5\cos(bx+a)}{16b}-\frac{5\cos(3bx+3a)}{64b}+\frac{3\cos(5bx+5a)}{64b}+\frac{3\cos(7bx+7a)}{224b}-\frac{\cos(9bx+9a)}{96b}-\frac{\cos(11bx+11a)}{704b}+\frac{\cos(13bx+}{832b}$
orering	Expression too large to display

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

output
```
1/192192*(-65536+2574*cos(7*b*x+7*a)-2002*cos(9*b*x+9*a)+231*cos(13*b*x+13
*a)-273*cos(11*b*x+11*a)-15015*cos(3*b*x+3*a)-60060*cos(b*x+a)+9009*cos(5*
b*x+5*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx$$
$$=\frac{64\left(231\cos\left(bx+a\right)^{13}-819\cos\left(bx+a\right)^{11}+1001\cos\left(bx+a\right)^{9}-429\cos\left(bx+a\right)^{7}\right)}{3003\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

output
```
64/3003*(231*cos(b*x + a)^13 - 819*cos(b*x + a)^11 + 1001*cos(b*x + a)^9 -
 429*cos(b*x + a)^7)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(53) = 106$.

Time = 11.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.85

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx = \begin{cases} -\frac{1084\sin(a+bx)\sin^5(2a+2bx)\cos(2a+2bx)}{3003b} - \frac{64\sin(a+bx)\sin^3(2a+2bx)\cos^3(2a+2bx)}{143b} - \frac{512\sin(a+bx)\sin(2a+2bx)\cos^5(2a+2bx)}{3003b} \\ x\sin(a)\sin^6(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**6,x)
```

output
```
Piecewise((-1084*sin(a + b*x)*sin(2*a + 2*b*x)**5*cos(2*a + 2*b*x)/(3003*b
) - 64*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**3/(143*b) - 512*
sin(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**5/(3003*b) - 835*sin(2*a +
 2*b*x)**6*cos(a + b*x)/(3003*b) - 2776*sin(2*a + 2*b*x)**4*cos(a + b*x)*c
os(2*a + 2*b*x)**2/(3003*b) - 2944*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*
a + 2*b*x)**4/(3003*b) - 1024*cos(a + b*x)*cos(2*a + 2*b*x)**6/(3003*b), N
e(b, 0)), (x*sin(a)*sin(2*a)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx = \frac{231\cos(13\,bx+13\,a) - 273\cos(11\,bx+11\,a) - 2002\cos(9\,bx+9\,a) + 2574\cos(7\,bx+7\,a) + 9009\cos}{192192\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

output
```
1/192192*(231*cos(13*b*x + 13*a) - 273*cos(11*b*x + 11*a) - 2002*cos(9*b*x
 + 9*a) + 2574*cos(7*b*x + 7*a) + 9009*cos(5*b*x + 5*a) - 15015*cos(3*b*x
+ 3*a) - 60060*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx$$
$$=\frac{231\,\cos\left(13\,bx+13\,a\right)-273\,\cos\left(11\,bx+11\,a\right)-2002\,\cos\left(9\,bx+9\,a\right)+2574\,\cos\left(7\,bx+7\,a\right)+9009\,\mathrm{co}}{192192\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

output
```
1/192192*(231*cos(13*b*x + 13*a) - 273*cos(11*b*x + 11*a) - 2002*cos(9*b*x
 + 9*a) + 2574*cos(7*b*x + 7*a) + 9009*cos(5*b*x + 5*a) - 15015*cos(3*b*x
+ 3*a) - 60060*cos(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx$$
$$=-\frac{-\frac{64\cos(a+b\,x)^{13}}{13}+\frac{192\cos(a+b\,x)^{11}}{11}-\frac{64\cos(a+b\,x)^{9}}{3}+\frac{64\cos(a+b\,x)^{7}}{7}}{b}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^6,x)
```

output
```
-((64*cos(a + b*x)^7)/7 - (64*cos(a + b*x)^9)/3 + (192*cos(a + b*x)^11)/11
 - (64*cos(a + b*x)^13)/13)/b
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.52

$$\int \sin(a+bx)\sin^6(2a+2bx)\,dx$$
$$= \frac{-1260\cos(2bx+2a)\sin(2bx+2a)^5\sin(bx+a) - 1600\cos(2bx+2a)\sin(2bx+2a)^3\sin(bx+a) - 25$$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^6,x)
```

output
```
( - 1260*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**5*sin(a + b*x) - 1600*cos(2*a
+ 2*b*x)*sin(2*a + 2*b*x)**3*sin(a + b*x) - 2560*cos(2*a + 2*b*x)*sin(2*a
+ 2*b*x)*sin(a + b*x) + 105*cos(a + b*x)*sin(2*a + 2*b*x)**6 + 200*cos(a +
 b*x)*sin(2*a + 2*b*x)**4 + 640*cos(a + b*x)*sin(2*a + 2*b*x)**2 - 5120*co
s(a + b*x) + 3696)/(15015*b)
```

3.413 $\int \sin(a+bx)\sin^5(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\sin^7(a+bx)}{7b} - \frac{64\sin^9(a+bx)}{9b} + \frac{32\sin^{11}(a+bx)}{11b}$$

output
```
32/7*sin(b*x+a)^7/b-64/9*sin(b*x+a)^9/b+32/11*sin(b*x+a)^11/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx = \frac{4(365+364\cos(2(a+bx))+63\cos(4(a+bx)))\sin^7(a+bx)}{693b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

output
```
(4*(365 + 364*Cos[2*(a + b*x)] + 63*Cos[4*(a + b*x)])*Sin[a + b*x]^7)/(693
*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^5dx$$

$$\Big\downarrow \text{ 4776}$$

$$32\int \cos^5(a+bx)\sin^6(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$32\int \cos(a+bx)^5\sin(a+bx)^6dx$$

$$\Big\downarrow \text{ 3044}$$

$$\frac{32\int \sin^6(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{32\int \left(\sin^{10}(a+bx)-2\sin^8(a+bx)+\sin^6(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{32\left(\frac{1}{11}\sin^{11}(a+bx)-\frac{2}{9}\sin^9(a+bx)+\frac{1}{7}\sin^7(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

output
```
(32*(Sin[a + b*x]^7/7 - (2*Sin[a + b*x]^9)/9 + Sin[a + b*x]^11/11))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 8.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.52

method	result	size
parallelrisch	$\frac{6930 \sin \left(b x+a\right)-2310 \sin \left(3 b x+3 a\right)-693 \sin \left(5 b x+5 a\right)+495 \sin \left(7 b x+7 a\right)+77 \sin \left(9 b x+9 a\right)-63 \sin \left(11 b x+11 a\right)}{22176 b}$	70
default	$\frac{5 \sin \left(b x+a\right)}{16 b}-\frac{5 \sin \left(3 b x+3 a\right)}{48 b}-\frac{\sin \left(5 b x+5 a\right)}{32 b}+\frac{5 \sin \left(7 b x+7 a\right)}{224 b}+\frac{\sin \left(9 b x+9 a\right)}{288 b}-\frac{\sin \left(11 b x+11 a\right)}{352 b}$	83
risch	$\frac{5 \sin \left(b x+a\right)}{16 b}-\frac{5 \sin \left(3 b x+3 a\right)}{48 b}-\frac{\sin \left(5 b x+5 a\right)}{32 b}+\frac{5 \sin \left(7 b x+7 a\right)}{224 b}+\frac{\sin \left(9 b x+9 a\right)}{288 b}-\frac{\sin \left(11 b x+11 a\right)}{352 b}$	83
orering	Expression too large to display	893

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/22176*(6930*sin(b*x+a)-2310*sin(3*b*x+3*a)-693*sin(5*b*x+5*a)+495*sin(7*
b*x+7*a)+77*sin(9*b*x+9*a)-63*sin(11*b*x+11*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\left(63\cos(bx+a)^{10}-161\cos(bx+a)^{8}+113\cos(bx+a)^{6}-3\cos(bx+a)^{4}-4\cos(bx+a)^{2}-8\right)}{693\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
-32/693*(63*cos(b*x + a)^10 - 161*cos(b*x + a)^8 + 113*cos(b*x + a)^6 - 3*
cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b*x + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(39) = 78.

Time = 4.99 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.28

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx = \begin{cases} -\frac{422\sin(a+bx)\sin^4(2a+2bx)\cos(2a+2bx)}{693b} - \frac{608\sin(a+bx)\sin^2(2a+2bx)\cos^3(2a+2bx)}{693b} - \frac{256\sin(a+bx)\cos^5(2a+2bx)}{693b} + \frac{151\sin^5 \\ x\sin(a)\sin^5(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**5,x)
```

output
```
Piecewise((-422*sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(2*a + 2*b*x)/(693*b)
- 608*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**3/(693*b) - 256*s
in(a + b*x)*cos(2*a + 2*b*x)**5/(693*b) + 151*sin(2*a + 2*b*x)**5*cos(a +
b*x)/(693*b) + 272*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**2/(6
93*b) + 128*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(693*b), Ne(
b, 0)), (x*sin(a)*sin(2*a)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx =$$
$$-\frac{63\sin(11\,bx+11\,a)-77\sin(9\,bx+9\,a)-495\sin(7\,bx+7\,a)+693\sin(5\,bx+5\,a)+2310\sin(3\,b\ldots}{22176\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
-1/22176*(63*sin(11*b*x + 11*a) - 77*sin(9*b*x + 9*a) - 495*sin(7*b*x + 7*
a) + 693*sin(5*b*x + 5*a) + 2310*sin(3*b*x + 3*a) - 6930*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(63\sin(bx+a)^{11}-154\sin(bx+a)^{9}+99\sin(bx+a)^{7}\right)}{693\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
32/693*(63*sin(b*x + a)^11 - 154*sin(b*x + a)^9 + 99*sin(b*x + a)^7)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\,\left(63\sin\left(a+b\,x\right)^{11}-154\sin\left(a+b\,x\right)^9+99\sin\left(a+b\,x\right)^7\right)}{693\,b}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^5,x)
```

output
```
(32*(99*sin(a + b*x)^7 - 154*sin(a + b*x)^9 + 63*sin(a + b*x)^11))/(693*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.91

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx = \frac{-70\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)^4\sin\left(bx+a\right)-96\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)^2\sin\left(bx+a\right)-256\cos}{}$$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^5,x)
```

output
```
( - 70*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**4*sin(a + b*x) - 96*cos(2*a + 2*
b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x) - 256*cos(2*a + 2*b*x)*sin(a + b*x)
+ 7*cos(a + b*x)*sin(2*a + 2*b*x)**5 + 16*cos(a + b*x)*sin(2*a + 2*b*x)**3
 + 128*cos(a + b*x)*sin(2*a + 2*b*x))/(693*b)
```

3.414 $\int \sin(a+bx)\sin^4(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx = -\frac{16\cos^5(a+bx)}{5b}+\frac{32\cos^7(a+bx)}{7b}-\frac{16\cos^9(a+bx)}{9b}$$

output
```
-16/5*cos(b*x+a)^5/b+32/7*cos(b*x+a)^7/b-16/9*cos(b*x+a)^9/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx = \frac{2\cos^5(a+bx)(-249+220\cos(2(a+bx))-35\cos(4(a+bx)))}{315b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

output
```
(2*Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(31
5*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^4dx$$

$$\Big\downarrow \text{4776}$$

$$16\int \cos^4(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$16\int \cos(a+bx)^4\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{16\int \cos^4(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{16\int \left(\cos^8(a+bx)-2\cos^6(a+bx)+\cos^4(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16\left(\frac{1}{9}\cos^9(a+bx)-\frac{2}{7}\cos^7(a+bx)+\frac{1}{5}\cos^5(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

output
```
(-16*(Cos[a + b*x]^5/5 - (2*Cos[a + b*x]^7)/7 + Cos[a + b*x]^9/9))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

method	result
parallelrisch	$\frac{-2048+45\cos(7bx+7a)-35\cos(9bx+9a)-420\cos(3bx+3a)-1890\cos(bx+a)+252\cos(5bx+5a)}{5040b}$
default	$-\frac{3\cos(bx+a)}{8b}-\frac{\cos(3bx+3a)}{12b}+\frac{\cos(5bx+5a)}{20b}+\frac{\cos(7bx+7a)}{112b}-\frac{\cos(9bx+9a)}{144b}$
risch	$-\frac{3\cos(bx+a)}{8b}-\frac{\cos(3bx+3a)}{12b}+\frac{\cos(5bx+5a)}{20b}+\frac{\cos(7bx+7a)}{112b}-\frac{\cos(9bx+9a)}{144b}$
orering	$-\frac{117469\left(b\cos(bx+a)\sin(2bx+2a)^4+8\sin(bx+a)\sin(2bx+2a)^3b\cos(2bx+2a)\right)}{99225b^2}-\frac{34562\left(-49b^3\cos(bx+a)\sin(2bx+2a)^4\right.}{}$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/5040*(-2048+45*cos(7*b*x+7*a)-35*cos(9*b*x+9*a)-420*cos(3*b*x+3*a)-1890*
cos(b*x+a)+252*cos(5*b*x+5*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx$$
$$= -\frac{16\left(35\,\cos\left(bx+a\right)^9-90\,\cos\left(bx+a\right)^7+63\,\cos\left(bx+a\right)^5\right)}{315\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
-16/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(39) = 78.

Time = 2.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.54

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{104\sin\left(a+bx\right)\sin^3\left(2a+2bx\right)\cos\left(2a+2bx\right)}{315b} - \frac{64\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos^3\left(2a+2bx\right)}{315b} - \frac{107\sin^4\left(2a+2bx\right)\cos\left(a+bx\right)}{315b} - 16\sin^2\left(2a\right. \\ x\sin\left(a\right)\sin^4\left(2a\right) \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**4,x)
```

output
```
Piecewise((-104*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)/(315*b)
- 64*sin(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(315*b) - 107*sin(2
*a + 2*b*x)**4*cos(a + b*x)/(315*b) - 16*sin(2*a + 2*b*x)**2*cos(a + b*x)*
cos(2*a + 2*b*x)**2/(21*b) - 128*cos(a + b*x)*cos(2*a + 2*b*x)**4/(315*b),
 Ne(b, 0)), (x*sin(a)*sin(2*a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx = -\frac{35\,\cos(9\,bx+9\,a)-45\,\cos(7\,bx+7\,a)-252\,\cos(5\,bx+5\,a)+420\,\cos(3\,bx+3\,a)+1890\,\cos(bx+}{5040\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
-1/5040*(35*cos(9*b*x + 9*a) - 45*cos(7*b*x + 7*a) - 252*cos(5*b*x + 5*a)
+ 420*cos(3*b*x + 3*a) + 1890*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx = -\frac{35\,\cos(9\,bx+9\,a)-45\,\cos(7\,bx+7\,a)-252\,\cos(5\,bx+5\,a)+420\,\cos(3\,bx+3\,a)+1890\,\cos(bx+}{5040\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
-1/5040*(35*cos(9*b*x + 9*a) - 45*cos(7*b*x + 7*a) - 252*cos(5*b*x + 5*a)
+ 420*cos(3*b*x + 3*a) + 1890*cos(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 18.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx = -\frac{16\left(35\cos(a+b\,x)^9-90\cos(a+b\,x)^7+63\cos(a+b\,x)^5\right)}{315\,b}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^4,x)
```

output
```
-(16*(63*cos(a + b*x)^5 - 90*cos(a + b*x)^7 + 35*cos(a + b*x)^9))/(315*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.33

$$\int \sin(a+bx)\sin^4(2a+2bx)\,dx = \frac{-120\cos(2bx+2a)\sin(2bx+2a)^3\sin(bx+a)-192\cos(2bx+2a)\sin(2bx+2a)\sin(bx+a)+15\cos}{945b}$$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^4,x)
```

output
```
( - 120*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**3*sin(a + b*x) - 192*cos(2*a +
2*b*x)*sin(2*a + 2*b*x)*sin(a + b*x) + 15*cos(a + b*x)*sin(2*a + 2*b*x)**4
 + 48*cos(a + b*x)*sin(2*a + 2*b*x)**2 - 384*cos(a + b*x) + 280)/(945*b)
```

3.415 $\int \sin(a+bx)\sin^3(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\sin^5(a+bx)}{5b} - \frac{8\sin^7(a+bx)}{7b}$$

output
```
8/5*sin(b*x+a)^5/b-8/7*sin(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx = \frac{4(9+5\cos(2(a+bx)))\sin^5(a+bx)}{35b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

output
```
(4*(9 + 5*Cos[2*(a + b*x)])*Sin[a + b*x]^5)/(35*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^3dx$$

$$\Big\downarrow \text{4776}$$

$$8\int \cos^3(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)^3\sin(a+bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{8\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{8\int \left(\sin^4(a+bx)-\sin^6(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8\left(\frac{1}{5}\sin^5(a+bx)-\frac{1}{7}\sin^7(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

output
```
(8*(Sin[a + b*x]^5/5 - Sin[a + b*x]^7/7))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77

method	result
default	$\frac{3\sin(bx+a)}{8b}-\frac{\sin(3bx+3a)}{8b}-\frac{\sin(5bx+5a)}{40b}+\frac{\sin(7bx+7a)}{56b}$
risch	$\frac{3\sin(bx+a)}{8b}-\frac{\sin(3bx+3a)}{8b}-\frac{\sin(5bx+5a)}{40b}+\frac{\sin(7bx+7a)}{56b}$
parallelrisch	$\frac{\left(-16\tan(bx+a)^5-40\tan(bx+a)^3-16\tan(bx+a)\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+\left(32\tan(bx+a)^6+80\tan(bx+a)^4-80\tan(bx+a)^2-32\right)\mathrm{ta}}{35b\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(\tan(bx+a)^2+1\right)^3}$
orering	$-\frac{12916\left(b\cos(bx+a)\sin(2bx+2a)^3+6\sin(bx+a)\sin(2bx+2a)^2b\cos(2bx+2a)\right)}{11025b^2}-\frac{94\left(-37\cos(bx+a)b^3\sin(2bx+2a)^3-186\right.}{}$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/8*sin(b*x+a)/b-1/8*sin(3*b*x+3*a)/b-1/40/b*sin(5*b*x+5*a)+1/56/b*sin(7*b
*x+7*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(5\cos(bx+a)^6-8\cos(bx+a)^4+\cos(bx+a)^2+2\right)\sin(bx+a)}{35\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
8/35*(5*cos(b*x + a)^6 - 8*cos(b*x + a)^4 + cos(b*x + a)^2 + 2)*sin(b*x +
a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(26) = 52.

Time = 0.83 (sec) , antiderivative size = 126, normalized size of antiderivative = 4.06

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx = \begin{cases} -\frac{22\sin(a+bx)\sin^2(2a+2bx)\cos(2a+2bx)}{35b} - \frac{16\sin(a+bx)\cos^3(2a+2bx)}{35b} + \frac{9\sin^3(2a+2bx)\cos(a+bx)}{35b} + \frac{8\sin(2a+2bx)\cos(a+bx)\,c}{35b} \\ x\sin(a)\sin^3(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**3,x)
```

output
```
Piecewise((-22*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)/(35*b) -
16*sin(a + b*x)*cos(2*a + 2*b*x)**3/(35*b) + 9*sin(2*a + 2*b*x)**3*cos(a +
 b*x)/(35*b) + 8*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(35*b),
 Ne(b, 0)), (x*sin(a)*sin(2*a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx = \frac{5\,\sin(7\,bx+7\,a)-7\,\sin(5\,bx+5\,a)-35\,\sin(3\,bx+3\,a)+105\,\sin(bx+a)}{280\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/280*(5*sin(7*b*x + 7*a) - 7*sin(5*b*x + 5*a) - 35*sin(3*b*x + 3*a) + 105
*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(5\,\sin(bx+a)^7-7\,\sin(bx+a)^5\right)}{35\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-8/35*(5*sin(b*x + a)^7 - 7*sin(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 18.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx=\frac{8\,\left(7\sin\left(a+b\,x\right)^5-5\sin\left(a+b\,x\right)^7\right)}{35\,b}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^3,x)
```

output
```
(8*(7*sin(a + b*x)^5 - 5*sin(a + b*x)^7))/(35*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.77

$$\int \sin(a+bx)\sin^3(2a+2bx)\,dx$$
$$=\frac{-6\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)^2\sin\left(bx+a\right)-16\cos\left(2bx+2a\right)\sin\left(bx+a\right)+\cos\left(bx+a\right)\sin\left(2bx+2\right.}{35b}$$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^3,x)
```

output
```
( - 6*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x) - 16*cos(2*a + 2*b
*x)*sin(a + b*x) + cos(a + b*x)*sin(2*a + 2*b*x)**3 + 8*cos(a + b*x)*sin(2
*a + 2*b*x))/(35*b)
```

3.416 $\int \sin(a+bx)\sin^2(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\cos^3(a+bx)}{3b}+\frac{4\cos^5(a+bx)}{5b}$$

output
```
-4/3*cos(b*x+a)^3/b+4/5*cos(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx = \frac{2\cos^3(a+bx)(-7+3\cos(2(a+bx)))}{15b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

output
```
(2*Cos[a + b*x]^3*(-7 + 3*Cos[2*(a + b*x)]))/(15*b)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(a+bx)\sin(2a+2bx)^2dx$$

$$\Big\downarrow \; 4776$$

$$4\int \cos^2(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$4\int \cos(a+bx)^2\sin(a+bx)^3dx$$

$$\Big\downarrow \; 3045$$

$$-\frac{4\int \cos^2(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 244$$

$$-\frac{4\int \left(\cos^2(a+bx)-\cos^4(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{4\left(\frac{1}{3}\cos^3(a+bx)-\frac{1}{5}\cos^5(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

output
```
(-4*(Cos[a + b*x]^3/3 - Cos[a + b*x]^5/5))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\cos(bx+a)}{2b}-\frac{\cos(3bx+3a)}{12b}+\frac{\cos(5bx+5a)}{20b}$
risch	$-\frac{\cos(bx+a)}{2b}-\frac{\cos(3bx+3a)}{12b}+\frac{\cos(5bx+5a)}{20b}$
parallelrisch	$\frac{\frac{4\left(4\tan(bx+a)^4+7\tan(bx+a)^2+4\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{15}+\frac{16\left(\tan(bx+a)^3-\tan(bx+a)\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{15}+\frac{4\tan(bx+a)^2}{15}}{b\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(\tan(bx+a)^2+1\right)^2}$
norman	$\frac{-\frac{16}{15b}-\frac{16\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan(bx+a)}{15b}+\frac{16\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan(bx+a)^3}{15b}-\frac{16\tan(bx+a)^4}{15b}-\frac{28\tan(bx+a)^2}{15b}-\frac{4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan(bx+a)^2}{15b}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(\tan(bx+a)^2+1\right)^2}$
orering	$-\frac{259\left(\cos(bx+a)b\sin(2bx+2a)^2+4b\sin(bx+a)\cos(2bx+2a)\sin(2bx+2a)\right)}{225b^2}-\frac{7\left(-25b^3\cos(bx+a)\sin(2bx+2a)^2-76b^3\sin(\right.}{}$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(b*x+a)/b-1/12*cos(3*b*x+3*a)/b+1/20*cos(5*b*x+5*a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx=\frac{4\left(3\cos\left(bx+a\right)^5-5\cos\left(bx+a\right)^3\right)}{15\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
4/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(26) = 52$.

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.97

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx = \begin{cases} -\frac{4\sin(a+bx)\sin(2a+2bx)\cos(2a+2bx)}{15b} - \frac{7\sin^2(2a+2bx)\cos(a+bx)}{15b} - \frac{8\cos(a+bx)\cos^2(2a+2bx)}{15b} & \text{for } b \neq 0 \\ x\sin(a)\sin^2(2a) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((-4*sin(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)/(15*b) - 7*si
n(2*a + 2*b*x)**2*cos(a + b*x)/(15*b) - 8*cos(a + b*x)*cos(2*a + 2*b*x)**2
/(15*b), Ne(b, 0)), (x*sin(a)*sin(2*a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx = \frac{3\cos(5bx+5a) - 5\cos(3bx+3a) - 30\cos(bx+a)}{60b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
1/60*(3*cos(5*b*x + 5*a) - 5*cos(3*b*x + 3*a) - 30*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx = \frac{3\cos(5bx+5a)-5\cos(3bx+3a)-30\cos(bx+a)}{60b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
1/60*(3*cos(5*b*x + 5*a) - 5*cos(3*b*x + 3*a) - 30*cos(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(5\cos(a+bx)^3-3\cos(a+bx)^5\right)}{15b}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^2,x)
```

output
```
-(4*(5*cos(a + b*x)^3 - 3*cos(a + b*x)^5))/(15*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \sin(a+bx)\sin^2(2a+2bx)\,dx$$
$$= \frac{-4\cos(2bx+2a)\sin(2bx+2a)\sin(bx+a)+\cos(bx+a)\sin(2bx+2a)^2-8\cos(bx+a)+6}{15b}$$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^2,x)
```

output
```
( - 4*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)*sin(a + b*x) + cos(a + b*x)*sin(2*
a + 2*b*x)**2 - 8*cos(a + b*x) + 6)/(15*b)
```

3.417 $\int \sin(a+bx)\sin(2a+2bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \sin(a+bx)\sin(2a+2bx)\,dx = \frac{\sin(a+bx)}{2b} - \frac{\sin(3a+3bx)}{6b}$$

output
```
1/2*sin(b*x+a)/b-1/6*sin(3*b*x+3*a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.50

$$\int \sin(a+bx)\sin(2a+2bx)\,dx = \frac{2\sin^3(a+bx)}{3b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x],x]
```

output
```
(2*Sin[a + b*x]^3)/(3*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)dx$$

$$\Big\downarrow \text{4770}$$

$$\frac{\sin(a+bx)}{2b}-\frac{\sin(3a+3bx)}{6b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x],x]
```

output
```
Sin[a + b*x]/(2*b) - Sin[3*a + 3*b*x]/(6*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result
parallelrisch	$\frac{-\sin(3bx+3a)+3\sin(bx+a)}{6b}$
default	$\frac{\sin(bx+a)}{2b}-\frac{\sin(3bx+3a)}{6b}$
risch	$\frac{\sin(bx+a)}{2b}-\frac{\sin(3bx+3a)}{6b}$
orering	$-\frac{10(b\cos(bx+a)\sin(2bx+2a)+2\sin(bx+a)b\cos(2bx+2a))}{9b^2}-\frac{-13b^3\cos(bx+a)\sin(2bx+2a)-14b^3\sin(bx+a)\cos(2bx+2a)}{9b^4}$
norman	$\frac{-\frac{4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{3b}+\frac{2\tan(bx+a)}{3b}+\frac{4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan(bx+a)^2}{3b}-\frac{2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan(bx+a)}{3b}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(\tan(bx+a)^2+1\right)}$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-sin(3*b*x+3*a)+3*sin(b*x+a))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \sin(a+bx)\sin(2a+2bx)\,dx=-\frac{2\left(\cos\left(bx+a\right)^2-1\right)\sin\left(bx+a\right)}{3\,b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-2/3*(cos(b*x + a)^2 - 1)*sin(b*x + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

$$\int \sin(a+bx)\sin(2a+2bx)\,dx = \begin{cases} -\frac{2\sin(a+bx)\cos(2a+2bx)}{3b} + \frac{\sin(2a+2bx)\cos(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin(a)\sin(2a) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a),x)
```

output
```
Piecewise((-2*sin(a + b*x)*cos(2*a + 2*b*x)/(3*b) + sin(2*a + 2*b*x)*cos(a
 + b*x)/(3*b), Ne(b, 0)), (x*sin(a)*sin(2*a), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \sin(a+bx)\sin(2a+2bx)\,dx = -\frac{\sin(3bx+3a)}{6b} + \frac{\sin(bx+a)}{2b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/6*sin(3*b*x + 3*a)/b + 1/2*sin(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.43

$$\int \sin(a+bx)\sin(2a+2bx)\,dx = \frac{2\sin(bx+a)^3}{3b}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
2/3*sin(b*x + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 18.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \sin(a+bx)\sin(2a+2bx)\,dx = \begin{cases} 2\,x\,\left(\cos\left(a\right)-{\cos\left(a\right)}^3\right) & \text{if}\ \ b=0\\ \frac{3\,\sin\left(a+b\,x\right)-\sin\left(3\,a+3\,b\,x\right)}{6\,b} & \text{if}\ \ b\neq 0 \end{cases}$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x),x)
```

output
```
piecewise(b == 0, 2*x*(cos(a) - cos(a)^3), b ~= 0, (3*sin(a + b*x) - sin(3
*a + 3*b*x))/(6*b))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \sin(a+bx)\sin(2a+2bx)\,dx = \frac{-2\cos\left(2bx+2a\right)\sin\left(bx+a\right)+\cos\left(bx+a\right)\sin\left(2bx+2a\right)}{3b}$$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a),x)
```

output
```
( - 2*cos(2*a + 2*b*x)*sin(a + b*x) + cos(a + b*x)*sin(2*a + 2*b*x))/(3*b)
```

3.418 $\int \csc(2a+2bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \csc(2a+2bx)\sin(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{2b}$$

output
```
1/2*arctanh(sin(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \csc(2a+2bx)\sin(a+bx)\,dx = \frac{\coth^{-1}(\sin(a+bx))}{2b}$$

input
```
Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x],x]
```

output
```
ArcCoth[Sin[a + b*x]]/(2*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4776, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{2}\int \sec(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{\text{arctanh}(\sin(a+bx))}{2b}$$

input
```
Int[Csc[2*a + 2*b*x]*Sin[a + b*x],x]
```

output
```
ArcTanh[Sin[a + b*x]]/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

method	result	size
default	$\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2b}$	20
risch	$\frac{\ln(e^{i(bx+a)}+i)}{2b}-\frac{\ln(e^{i(bx+a)}-i)}{2b}$	38

input
```
int(csc(2*b*x+2*a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \csc(2a+2bx)\sin(a+bx)\,dx = \frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)}{4\,b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \csc(2a + 2bx)\sin(a + bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(12) = 24$.

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 8.21

$$\int \csc(2a + 2bx)\sin(a + bx)\,dx = -\frac{\log\left(\frac{\cos(bx+2a)^2+\cos(a)^2-2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2+2\cos(bx+2a)\sin(a)+\sin(a)^2}{\cos(bx+2a)^2+\cos(a)^2+2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2-2\cos(bx+2a)\sin(a)+\sin(a)^2}\right)}{4b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x
+ 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^
2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) +
 sin(a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \csc(2a+2bx)\sin(a+bx)\,dx = \frac{\log(\sin(bx+a)+1) - \log(-\sin(bx+a)+1)}{4\,b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 18.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \csc(2a+2bx)\sin(a+bx)\,dx = \frac{\operatorname{atanh}(\sin(a+b\,x))}{2\,b}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x),x)
```

output
```
atanh(sin(a + b*x))/(2*b)
```

Reduce [F]

$$\int \csc(2a+2bx)\sin(a+bx)\,dx = \int \csc(2bx+2a)\sin(bx+a)\,dx$$

input
```
int(csc(2*b*x+2*a)*sin(b*x+a),x)
```

output
```
int(csc(2*a + 2*b*x)*sin(a + b*x),x)
```

3.419 $\int \csc^2(2a+2bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{4b}+\frac{\sec(a+bx)}{4b}$$

output
```
-1/4*arctanh(cos(b*x+a))/b+1/4*sec(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx=-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{4b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{4b}+\frac{\sec(a+bx)}{4b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x],x]
```

output
```
-1/4*Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/(4*b) + Sec[a + b*x]/
(4*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3042, 4776, 3042, 3102, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{4}\int \csc(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc(a+bx)\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\sec(a+bx)-\int \frac{1}{1-\sec^2(a+bx)}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(a+bx)-\text{arctanh}(\sec(a+bx))}{4b}$$

input
```
Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x],x]
```

output
```
(-ArcTanh[Sec[a + b*x]] + Sec[a + b*x])/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{4b}$	31
risch	$\frac{e^{i(bx+a)}}{2b(e^{2i(bx+a)}+1)}+\frac{\ln(e^{i(bx+a)}-1)}{4b}-\frac{\ln(e^{i(bx+a)}+1)}{4b}$	63

input
```
int(csc(2*b*x+2*a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*(1/cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(24) = 48.

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx$$
$$= -\frac{\cos(bx+a)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-\cos(bx+a)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-2}{8\,b\cos(bx+a)}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(cos(b*x + a)*log(1/2*cos(b*x + a) + 1/2) - cos(b*x + a)*log(-1/2*cos
(b*x + a) + 1/2) - 2)/(b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**2*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(24) = 48$.

Time = 0.05 (sec) , antiderivative size = 236, normalized size of antiderivative = 8.43

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx$$
$$= \frac{4\cos(2bx+2a)\cos(bx+a) - \left(\cos(2bx+2a)^2 + \sin(2bx+2a)^2 + 2\cos(2bx+2a) + 1\right)\log\left(\cos(b\right.}{}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/8*(4*cos(2*b*x + 2*a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a
)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + (cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(2*b*x
 + 2*a)*sin(b*x + a) + 4*cos(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x
 + 2*a)^2 + 2*b*cos(2*b*x + 2*a) + b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(24) = 48$.

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx = \frac{\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1} + \log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{8\,b}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
1/8*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + log(-(cos(b*x + a) -
1)/(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 18.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx = \frac{1}{4\,b\,\cos(a+b\,x)} - \frac{\mathrm{atanh}(\cos(a+b\,x))}{4\,b}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^2,x)
```

output
```
1/(4*b*cos(a + b*x)) - atanh(cos(a + b*x))/(4*b)
```

Reduce [F]

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx = \int \csc(2bx+2a)^2\sin(bx+a)\,dx$$

input
```
int(csc(2*b*x+2*a)^2*sin(b*x+a),x)
```

output
```
int(csc(2*a + 2*b*x)**2*sin(a + b*x),x)
```

3.420 $\int \csc^3(2a+2bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx = \frac{3\operatorname{arctanh}(\sin(a+bx))}{16b} - \frac{\csc(a+bx)}{8b} + \frac{\sec(a+bx)\tan(a+bx)}{16b}$$

output
```
3/16*arctanh(sin(b*x+a))/b-1/8*csc(b*x+a)/b+1/16*sec(b*x+a)*tan(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx = -\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\sin^2(a+bx)\right)}{8b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x],x]
```

output
```
-1/8*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4776, 3042, 3101, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{8}\int \csc^2(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)^2\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int \frac{\csc^4(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\int \frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\csc^2(a+bx)}d\csc(a+bx)-\csc(a+bx)\right)}{8b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}(\operatorname{arctanh}(\csc(a+bx))-\csc(a+bx))}{8b}$$

input
```
Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x],x]
```

output
```
-1/8*((-3*(ArcTanh[Csc[a + b*x]] - Csc[a + b*x]))/2 + Csc[a + b*x]^3/(2*(1
 - Csc[a + b*x]^2)))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\frac{1}{2\sin(bx+a)\cos(bx+a)^2}-\frac{3}{2\sin(bx+a)}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{2}}{8b}$	51
risch	$-\frac{i\left(3\,\mathrm{e}^{5i(bx+a)}+2\,\mathrm{e}^{3i(bx+a)}+3\,\mathrm{e}^{i(bx+a)}\right)}{8b\left(\mathrm{e}^{2i(bx+a)}+1\right)^2\left(\mathrm{e}^{2i(bx+a)}-1\right)}+\frac{3\ln\left(\mathrm{e}^{i(bx+a)}+i\right)}{16b}-\frac{3\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{16b}$	104

input
```
int(csc(2*b*x+2*a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(1/2/sin(b*x+a)/cos(b*x+a)^2-3/2/sin(b*x+a)+3/2*ln(sec(b*x+a)+tan(b*
x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(41) = 82$.

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.81

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx$$
$$= \frac{3\cos(bx+a)^2\log(\sin(bx+a)+1)\sin(bx+a)-3\cos(bx+a)^2\log(-\sin(bx+a)+1)\sin(bx+a)-}{32\,b\cos(bx+a)^2\sin(bx+a)}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/32*(3*cos(b*x + a)^2*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*cos(b*x + a)
^2*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 2)/(b*cos(b*x
+ a)^2*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**3*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(41) = 82$.

Time = 0.24 (sec) , antiderivative size = 808, normalized size of antiderivative = 17.19

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx=\text{Too large to display}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/32*(4*(3*sin(5*b*x + 5*a) + 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*b
*x + 6*a) - 12*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 4*
(2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(4*b*x + 4*a) - 3*(2*(cos(4*b*x +
 4*a) - cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + cos(6*b*x + 6*a)^2 - 2*(c
os(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + cos(2*b*x + 2
*a)^2 + 2*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + sin(6*b
*x + 6*a)^2 + sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + s
in(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log((cos(b*x + 2*a)^2 + cos(a)
^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a)
+ sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b
*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(3*cos(5*b*x + 5*a)
 + 2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a) + 12*(cos(4*b*x +
 4*a) - cos(2*b*x + 2*a) - 1)*sin(5*b*x + 5*a) - 4*(2*cos(3*b*x + 3*a) + 3
*cos(b*x + a))*sin(4*b*x + 4*a) - 8*(cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a
) + 8*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + 12*cos(b*x + a)*sin(2*b*x + 2*a)
 - 12*cos(2*b*x + 2*a)*sin(b*x + a) - 12*sin(b*x + a))/(b*cos(6*b*x + 6*a)
^2 + b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 +
b*sin(4*b*x + 4*a)^2 - 2*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x
 + 2*a)^2 + 2*(b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*
a) - 2*(b*cos(2*b*x + 2*a) + b)*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a)...
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx = -\frac{\frac{2\left(3\sin(bx+a)^2-2\right)}{\sin(bx+a)^3-\sin(bx+a)} - 3\log\left(\sin\left(bx+a\right)+1\right) + 3\log\left(-\sin\left(bx+a\right)+1\right)}{32\,b}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/32*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(si
n(b*x + a) + 1) + 3*log(-sin(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx = \frac{3\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{16\,b}+\frac{\frac{3\sin(a+b\,x)^2}{16}-\frac{1}{8}}{b\,\left(\sin\left(a+b\,x\right)-\sin\left(a+b\,x\right)^3\right)}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^3,x)
```

output
```
(3*atanh(sin(a + b*x)))/(16*b) + ((3*sin(a + b*x)^2)/16 - 1/8)/(b*(sin(a +
 b*x) - sin(a + b*x)^3))
```

Reduce [F]

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx = \int \csc\left(2bx+2a\right)^3\sin\left(bx+a\right)dx$$

input
```
int(csc(2*b*x+2*a)^3*sin(b*x+a),x)
```

output
```
int(csc(2*a + 2*b*x)**3*sin(a + b*x),x)
```

3.421 $\int \csc^4(2a+2bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx = -\frac{5\operatorname{arctanh}(\cos(a+bx))}{32b} - \frac{\cot(a+bx)\csc(a+bx)}{32b} + \frac{\sec(a+bx)}{8b} + \frac{\sec^3(a+bx)}{48b}$$

output
```
-5/32*arctanh(cos(b*x+a))/b-1/32*cot(b*x+a)*csc(b*x+a)/b+1/8*sec(b*x+a)/b+
1/48*sec(b*x+a)^3/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 205 vs. 2(62) = 124.

Time = 0.43 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.31

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx = \frac{\csc^8(a+bx)\left(22-40\cos(2(a+bx))+13\cos(3(a+bx))-30\cos(4(a+bx))+13\cos(5(a+bx))+15\,\mathrm{c}\right.}{}$$

input
```
Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x],x]
```

output
```
(Csc[a + b*x]^8*(22 - 40*Cos[2*(a + b*x)] + 13*Cos[3*(a + b*x)] - 30*Cos[4
*(a + b*x)] + 13*Cos[5*(a + b*x)] + 15*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/
2]] + 15*Cos[5*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 15*Cos[3*(a + b*x)]*Log[
Sin[(a + b*x)/2]] - 15*Cos[5*(a + b*x)]*Log[Sin[(a + b*x)/2]] + Cos[a + b*
x]*(-26 - 30*Log[Cos[(a + b*x)/2]] + 30*Log[Sin[(a + b*x)/2]])))/(24*b*(Cs
c[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4776, 3042, 3102, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{16}\int \csc^3(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^3\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int \frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int\left(-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)}{16b}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\left(\operatorname{arctanh}(\sec(a+bx))-\frac{1}{3}\sec^3(a+bx)-\sec(a+bx)\right)}{16b}$$

input
```
Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x],x]
```

output
```
(Sec[a + b*x]^5/(2*(1 - Sec[a + b*x]^2)) - (5*(ArcTanh[Sec[a + b*x]] - Sec
[a + b*x] - Sec[a + b*x]^3/3))/2)/(16*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)    Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\frac{1}{3 \sin(bx+a)^2 \cos(bx+a)^3}-\frac{5}{6 \sin(bx+a)^2 \cos(bx+a)}+\frac{5}{2 \cos(bx+a)}+\frac{5 \ln(\csc(bx+a)-\cot(bx+a))}{2}}{16b}$	71
risch	$\frac{15\,\mathrm{e}^{9i(bx+a)}+20\,\mathrm{e}^{7i(bx+a)}-22\,\mathrm{e}^{5i(bx+a)}+20\,\mathrm{e}^{3i(bx+a)}+15\,\mathrm{e}^{i(bx+a)}}{48b\left(\mathrm{e}^{2i(bx+a)}+1\right)^3\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}-\frac{5\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{32b}+\frac{5\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{32b}$	123

input
```
int(csc(2*b*x+2*a)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/16/b*(1/3/sin(b*x+a)^2/cos(b*x+a)^3-5/6/sin(b*x+a)^2/cos(b*x+a)+5/2/cos(
b*x+a)+5/2*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(54) = 108.

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx$$
$$= \frac{30\,\cos(bx+a)^4 - 20\,\cos(bx+a)^2 - 15\left(\cos(bx+a)^5 - \cos(bx+a)^3\right)\log\left(\frac{1}{2}\,\cos(bx+a)+\frac{1}{2}\right) + 15\,(\mathrm{c}}{192\left(b\cos(bx+a)^5 - b\cos(bx+a)^3\right)}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/192*(30*cos(b*x + a)^4 - 20*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - cos(b*
x + a)^3)*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 - cos(b*x + a)^
3)*log(-1/2*cos(b*x + a) + 1/2) - 4)/(b*cos(b*x + a)^5 - b*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**4*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2174 vs. $2(54) = 108$.

Time $= 0.15$ (sec) , antiderivative size $= 2174$, normalized size of antiderivative $= 35.06$

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/192*(4*(15*cos(9*b*x + 9*a) + 20*cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a)
+ 20*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) + 60*(cos(8*b*
x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)
*cos(9*b*x + 9*a) + 4*(20*cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) + 20*cos(
3*b*x + 3*a) + 15*cos(b*x + a))*cos(8*b*x + 8*a) - 80*(2*cos(6*b*x + 6*a)
+ 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(7*b*x + 7*a) + 8*(22*cos(
5*b*x + 5*a) - 20*cos(3*b*x + 3*a) - 15*cos(b*x + a))*cos(6*b*x + 6*a) + 8
8*(2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) - 40*(4*cos
(3*b*x + 3*a) + 3*cos(b*x + a))*cos(4*b*x + 4*a) + 80*(cos(2*b*x + 2*a) +
1)*cos(3*b*x + 3*a) + 60*cos(2*b*x + 2*a)*cos(b*x + a) - 15*(2*(cos(8*b*x
+ 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*c
os(10*b*x + 10*a) + cos(10*b*x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) + 2*cos(4
*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*x + 8*a)^2
+ 4*(2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 4*cos(6
*b*x + 6*a)^2 - 4*(cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*cos(4*b*x +
4*a)^2 + cos(2*b*x + 2*a)^2 + 2*(sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2
*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + sin(10*b*x + 10
*a)^2 - 2*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin
(8*b*x + 8*a) + sin(8*b*x + 8*a)^2 + 4*(2*sin(4*b*x + 4*a) - sin(2*b*x + 2
*a))*sin(6*b*x + 6*a) + 4*sin(6*b*x + 6*a)^2 + 4*sin(4*b*x + 4*a)^2 - 4...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(54) = 108$.

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.58

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx = -\frac{\frac{3\left(\frac{10(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}+\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{16\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{9(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+7\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}-30\log\left(-\frac{\cos(bx+a)-}{\cos(bx+a)+}\right.}{384\,b}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/384*(3*(10*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1
)/(cos(b*x + a) - 1) + 3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 16*(12*(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) + 9*(cos(b*x + a) - 1)^2/(cos(b*x + a)
 + 1)^2 + 7)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3 - 30*log(-(cos(
b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 18.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx = \frac{-\frac{5\cos(a+b\,x)^4}{32}+\frac{5\cos(a+b\,x)^2}{48}+\frac{1}{48}}{b\,\left(\cos\left(a+b\,x\right)^3-\cos\left(a+b\,x\right)^5\right)} - \frac{5\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{32\,b}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^4,x)
```

output
```
((5*cos(a + b*x)^2)/48 - (5*cos(a + b*x)^4)/32 + 1/48)/(b*(cos(a + b*x)^3
- cos(a + b*x)^5)) - (5*atanh(cos(a + b*x)))/(32*b)
```

Reduce [F]

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx$$

$$= \frac{-20\cos\left(2bx+2a\right)\cos\left(bx+a\right)\sin\left(2bx+2a\right)-50\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)^2\sin\left(bx+a\right)-20\cos(2}{}$$

input
```
int(csc(2*b*x+2*a)^4*sin(b*x+a),x)
```

output
```
( - 20*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x) - 50*cos(2*a + 2*b*x
)*sin(2*a + 2*b*x)**2*sin(a + b*x) - 20*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)
- 32*cos(2*a + 2*b*x)*sin(a + b*x) - 28*cos(a + b*x)*sin(2*a + 2*b*x) - 60
*int(tan(a + b*x)/(tan((a + b*x)/2)**2 + 1),x)*sin(2*a + 2*b*x)**3*b - 40*
int(1/(tan(a + b*x)**3*tan((a + b*x)/2)**2 + tan(a + b*x)**3),x)*sin(2*a +
 2*b*x)**3*b + 25*log(tan(a + b*x)**2 + 1)*sin(2*a + 2*b*x)**3 - 20*log(ta
n(a + b*x))*sin(2*a + 2*b*x)**3 + 24*sin(2*a + 2*b*x)**3 + 10*sin(2*a + 2*
b*x)**2*sin(a + b*x) - 20*sin(2*a + 2*b*x))/(192*sin(2*a + 2*b*x)**3*b)
```

3.422 $\int \csc^5(2a+2bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx = \frac{35\text{arctanh}(\sin(a+bx))}{256b} - \frac{3\csc(a+bx)}{32b} - \frac{\csc^3(a+bx)}{96b} + \frac{13\sec(a+bx)\tan(a+bx)}{256b} + \frac{\sec(a+bx)\tan^3(a+bx)}{128b}$$

output
```
35/256*arctanh(sin(b*x+a))/b-3/32*csc(b*x+a)/b-1/96*csc(b*x+a)^3/b+13/256*
sec(b*x+a)*tan(b*x+a)/b+1/128*sec(b*x+a)*tan(b*x+a)^3/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx = -\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},3,-\frac{1}{2},\sin^2(a+bx)\right)}{96b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x],x]
```

output
```
-1/96*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4776, 3042, 3101, 25, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{32}\int \csc^4(a+bx)\sec^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \csc(a+bx)^4\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\csc^8(a+bx)}{(1-\csc^2(a+bx))^3}d\csc(a+bx)}{32b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc^8(a+bx)}{(1-\csc^2(a+bx))^3}d\csc(a+bx)}{32b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{7}{4}\int \frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)-\frac{\csc^7(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{ 252}$$

$$-\frac{\frac{7}{4}\left(\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)\right)-\frac{\csc^7(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{ 254}$$

$$-\frac{\frac{7}{4}\left(\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \left(-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)\right)-\frac{\csc^7(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{7}{4}\left(\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\left(\text{arctanh}(\csc(a+bx))-\frac{1}{3}\csc^3(a+bx)-\csc(a+bx)\right)\right)-\frac{\csc^7(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

input
```
Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x],x]
```

output
```
-1/32*(-1/4*Csc[a + b*x]^7/(1 - Csc[a + b*x]^2)^2 + (7*(Csc[a + b*x]^5/(2*
(1 - Csc[a + b*x]^2)) - (5*(ArcTanh[Csc[a + b*x]] - Csc[a + b*x] - Csc[a +
 b*x]^3/3))/2))/4)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

method	result
default	$\frac{\frac{1}{4 \sin \left(b x +a \right)^{3} \cos \left(b x +a \right)^{4}}-\frac{7}{12 \sin \left(b x +a \right)^{3} \cos \left(b x +a \right)^{2}}+\frac{35}{24 \sin \left(b x +a \right) \cos \left(b x +a \right)^{2}}-\frac{35}{8 \sin \left(b x +a \right)}+\frac{35 \ln \left(\sec \left(b x +a \right)+\tan \left(b x +a \right)\right)}{8}}{32 b}$
risch	$-\frac{i \left(105 \,{\mathrm e}^{13 i \left(b x +a \right)}+70 \,{\mathrm e}^{11 i \left(b x +a \right)}-329 \,{\mathrm e}^{9 i \left(b x +a \right)}-204 \,{\mathrm e}^{7 i \left(b x +a \right)}-329 \,{\mathrm e}^{5 i \left(b x +a \right)}+70 \,{\mathrm e}^{3 i \left(b x +a \right)}+105 \,{\mathrm e}^{i \left(b x +a \right)}\right)}{384 b \left({\mathrm e}^{2 i \left(b x +a \right)}+1\right)^{4} \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)^{3}}+\frac{35 \ln \left({\mathrm e}^{i \left(b x +a \right)}+i\right)}{256 b}$

input
```
int(csc(2*b*x+2*a)^5*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(1/4/sin(b*x+a)^3/cos(b*x+a)^4-7/12/sin(b*x+a)^3/cos(b*x+a)^2+35/24
/sin(b*x+a)/cos(b*x+a)^2-35/8/sin(b*x+a)+35/8*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.69

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx =$$

$$-\frac{210\cos(bx+a)^6-280\cos(bx+a)^4-105\left(\cos(bx+a)^6-\cos(bx+a)^4\right)\log(\sin(bx+a)+1)\sin(}{1536\left(b\cos(bx+a)^6-\right.}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/1536*(210*cos(b*x + a)^6 - 280*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - c
os(b*x + a)^4)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*(cos(b*x + a)^6 -
cos(b*x + a)^4)*log(-sin(b*x + a) + 1)*sin(b*x + a) + 42*cos(b*x + a)^2 +
12)/((b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**5*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3088 vs. $2(73) = 146$.

Time = 0.41 (sec) , antiderivative size = 3088, normalized size of antiderivative = 37.20

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/1536*(4*(105*sin(13*b*x + 13*a) + 70*sin(11*b*x + 11*a) - 329*sin(9*b*x
+ 9*a) - 204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) + 70*sin(3*b*x + 3*a)
 + 105*sin(b*x + a))*cos(14*b*x + 14*a) - 420*(sin(12*b*x + 12*a) - 3*sin(
10*b*x + 10*a) - 3*sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4
*a) - sin(2*b*x + 2*a))*cos(13*b*x + 13*a) + 4*(70*sin(11*b*x + 11*a) - 32
9*sin(9*b*x + 9*a) - 204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) + 70*sin(
3*b*x + 3*a) + 105*sin(b*x + a))*cos(12*b*x + 12*a) + 280*(3*sin(10*b*x +
10*a) + 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + sin
(2*b*x + 2*a))*cos(11*b*x + 11*a) + 12*(329*sin(9*b*x + 9*a) + 204*sin(7*b
*x + 7*a) + 329*sin(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))
*cos(10*b*x + 10*a) - 1316*(3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) - 3*si
n(4*b*x + 4*a) + sin(2*b*x + 2*a))*cos(9*b*x + 9*a) + 12*(204*sin(7*b*x +
7*a) + 329*sin(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))*cos(
8*b*x + 8*a) + 816*(3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) - sin(2*b*x +
2*a))*cos(7*b*x + 7*a) - 84*(47*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) - 1
5*sin(b*x + a))*cos(6*b*x + 6*a) + 1316*(3*sin(4*b*x + 4*a) - sin(2*b*x +
2*a))*cos(5*b*x + 5*a) + 420*(2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(4*b
*x + 4*a) - 105*(2*(cos(12*b*x + 12*a) - 3*cos(10*b*x + 10*a) - 3*cos(8*b*
x + 8*a) + 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)
*cos(14*b*x + 14*a) + cos(14*b*x + 14*a)^2 - 2*(3*cos(10*b*x + 10*a) + ...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx = -\frac{\frac{6\left(11\sin(bx+a)^3-13\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2}+\frac{16\left(9\sin(bx+a)^2+1\right)}{\sin(bx+a)^3}-105\log\left(\sin\left(bx+a\right)+1\right)+105\log\left(-\sin\left(bx+a\right)+\right.}{1536\,b}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/1536*(6*(11*sin(b*x + a)^3 - 13*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 +
16*(9*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 105*log(sin(b*x + a) + 1) + 105
*log(-sin(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 18.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx = \frac{35\,\mathrm{atanh}(\sin(a+b\,x))}{256\,b} - \frac{\frac{35\sin(a+b\,x)^6}{256}-\frac{175\sin(a+b\,x)^4}{768}+\frac{7\sin(a+b\,x)^2}{96}+\frac{1}{96}}{b\,\left(\sin(a+b\,x)^7-2\sin(a+b\,x)^5+\sin(a+b\,x)^3\right)}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^5,x)
```

output
```
(35*atanh(sin(a + b*x)))/(256*b) - ((7*sin(a + b*x)^2)/96 - (175*sin(a + b
*x)^4)/768 + (35*sin(a + b*x)^6)/256 + 1/96)/(b*(sin(a + b*x)^3 - 2*sin(a
+ b*x)^5 + sin(a + b*x)^7))
```

Reduce [F]

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx$$

$$= \frac{280\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)+35\cos(2bx+2a)\sin(2bx+2a)^3-490\cos(2bx+2a)\,\mathrm{si}}{}$$

input
```
int(csc(2*b*x+2*a)^5*sin(b*x+a),x)
```

output
```
(280*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x) + 35*cos(2*a + 2*b*x)*
sin(2*a + 2*b*x)**3 - 490*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x
) + 280*cos(2*a + 2*b*x)*sin(2*a + 2*b*x) - 288*cos(2*a + 2*b*x)*sin(a + b
*x) - 980*cos(a + b*x)*sin(2*a + 2*b*x)**3 + 232*cos(a + b*x)*sin(2*a + 2*
b*x) - 630*int((tan(a + b*x)**2*tan((a + b*x)/2)**2)/(tan((a + b*x)/2)**2
+ 1),x)*sin(2*a + 2*b*x)**4*b + 420*int(1/(tan(a + b*x)**4*tan((a + b*x)/2
)**2 + tan(a + b*x)**4),x)*sin(2*a + 2*b*x)**4*b + 105*sin(2*a + 2*b*x)**4
*sin(a + b*x) + 630*sin(2*a + 2*b*x)**4*tan(a + b*x) + 105*sin(2*a + 2*b*x
)**4*a - 525*sin(2*a + 2*b*x)**4*b*x - 735*sin(2*a + 2*b*x)**3 - 70*sin(2*
a + 2*b*x)**2*sin(a + b*x) + 280*sin(2*a + 2*b*x))/(2304*sin(2*a + 2*b*x)*
*4*b)
```

3.423 $\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{4\sin^8(a+bx)}{b} - \frac{32\sin^{10}(a+bx)}{5b} + \frac{8\sin^{12}(a+bx)}{3b}$$

output
```
4*sin(b*x+a)^8/b-32/5*sin(b*x+a)^10/b+8/3*sin(b*x+a)^12/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx$$
$$= \frac{-600\cos(2(a+bx)) + 75\cos(4(a+bx)) + 100\cos(6(a+bx)) - 30\cos(8(a+bx)) - 12\cos(10(a+bx))}{3840b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

output
```
(-600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] + 100*Cos[6*(a + b*x)] - 30*C
os[8*(a + b*x)] - 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/(3840*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4776, 3042, 3044, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^5dx$$

$$\Big\downarrow \text{4776}$$

$$32\int \cos^5(a+bx)\sin^7(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$32\int \cos(a+bx)^5\sin(a+bx)^7dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{32\int \sin^7(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{16\int \sin^6(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin^2(a+bx)}{b}$$

$$\Big\downarrow \text{49}$$

$$\frac{16\int \left(\sin^{10}(a+bx)-2\sin^8(a+bx)+\sin^6(a+bx)\right)d\sin^2(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16\left(\frac{1}{6}\sin^{12}(a+bx)-\frac{2}{5}\sin^{10}(a+bx)+\frac{1}{4}\sin^8(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

output
```
(16*(Sin[a + b*x]^8/4 - (2*Sin[a + b*x]^10)/5 + Sin[a + b*x]^12/6))/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 13.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.68

method	result	size
parallelrisch	$\frac{75\cos(4bx+4a)+5\cos(12bx+12a)+462-30\cos(8bx+8a)-600\cos(2bx+2a)+100\cos(6bx+6a)-12\cos(10bx+10a)}{3840b}$	74
default	$-\frac{5\cos(2bx+2a)}{32b}+\frac{5\cos(4bx+4a)}{256b}+\frac{5\cos(6bx+6a)}{192b}-\frac{\cos(8bx+8a)}{128b}-\frac{\cos(10bx+10a)}{320b}+\frac{\cos(12bx+12a)}{768b}$	86
risch	$-\frac{5\cos(2bx+2a)}{32b}+\frac{5\cos(4bx+4a)}{256b}+\frac{5\cos(6bx+6a)}{192b}-\frac{\cos(8bx+8a)}{128b}-\frac{\cos(10bx+10a)}{320b}+\frac{\cos(12bx+12a)}{768b}$	86
orering	Expression too large to display	1391

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/3840*(75*cos(4*b*x+4*a)+5*cos(12*b*x+12*a)+462-30*cos(8*b*x+8*a)-600*cos
(2*b*x+2*a)+100*cos(6*b*x+6*a)-12*cos(10*b*x+10*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{4\left(10\cos(bx+a)^{12}-36\cos(bx+a)^{10}+45\cos(bx+a)^{8}-20\cos(bx+a)^{6}\right)}{15\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
4/15*(10*cos(b*x + a)^12 - 36*cos(b*x + a)^10 + 45*cos(b*x + a)^8 - 20*cos
(b*x + a)^6)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(37) = 74$.

Time = 11.58 (sec) , antiderivative size = 593, normalized size of antiderivative = 13.48

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

output
```
Piecewise((5*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**5/32 + 5*x*sin(a + b*x)**
2*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**2/16 + 5*x*sin(a + b*x)**2*sin(2*a
 + 2*b*x)*cos(2*a + 2*b*x)**4/32 + 5*x*sin(a + b*x)*sin(2*a + 2*b*x)**4*co
s(a + b*x)*cos(2*a + 2*b*x)/16 + 5*x*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(
a + b*x)*cos(2*a + 2*b*x)**3/8 + 5*x*sin(a + b*x)*cos(a + b*x)*cos(2*a + 2
*b*x)**5/16 - 5*x*sin(2*a + 2*b*x)**5*cos(a + b*x)**2/32 - 5*x*sin(2*a + 2
*b*x)**3*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 - 5*x*sin(2*a + 2*b*x)*cos
(a + b*x)**2*cos(2*a + 2*b*x)**4/32 - 65*sin(a + b*x)**2*sin(2*a + 2*b*x)*
*4*cos(2*a + 2*b*x)/(128*b) - 2*sin(a + b*x)**2*sin(2*a + 2*b*x)**2*cos(2*
a + 2*b*x)**3/(3*b) - 167*sin(a + b*x)**2*cos(2*a + 2*b*x)**5/(640*b) + 11
*sin(a + b*x)*sin(2*a + 2*b*x)**5*cos(a + b*x)/(64*b) + sin(a + b*x)*sin(2
*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**2/(4*b) + 19*sin(a + b*x)*si
n(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(192*b) + sin(2*a + 2*b*x)
**4*cos(a + b*x)**2*cos(2*a + 2*b*x)/(128*b) - 11*cos(a + b*x)**2*cos(2*a
+ 2*b*x)**5/(1920*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.64

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx$$
$$= \frac{5\cos(12\,bx+12\,a) - 12\cos(10\,bx+10\,a) - 30\cos(8\,bx+8\,a) + 100\cos(6\,bx+6\,a) + 75\cos(4\,bx+}{3840\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/3840*(5*cos(12*b*x + 12*a) - 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a)
 + 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) - 600*cos(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{4\left(10\sin\left(bx+a\right)^{12}-24\sin\left(bx+a\right)^{10}+15\sin\left(bx+a\right)^{8}\right)}{15\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
4/15*(10*sin(b*x + a)^12 - 24*sin(b*x + a)^10 + 15*sin(b*x + a)^8)/b
```

Mupad [B] (verification not implemented)

Time = 18.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx = -\frac{-\frac{8\cos(a+b\,x)^{12}}{3}+\frac{48\cos(a+b\,x)^{10}}{5}-12\cos\left(a+b\,x\right)^8+\frac{16\cos(a+b\,x)^6}{3}}{b}$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^5,x)
```

output
```
-((16*cos(a + b*x)^6)/3 - 12*cos(a + b*x)^8 + (48*cos(a + b*x)^10)/5 - (8*
cos(a + b*x)^12)/3)/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 250, normalized size of antiderivative = 5.68

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx$$

$$= \frac{300\cos(2bx+2a)\cos(bx+a)\sin(bx+a)\,bx - 100\cos(2bx+2a)\sin(2bx+2a)^4\sin(bx+a)^2 + 2\cos(\cdots}{\cdots}$$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x)
```

output
```
(300*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x)*b*x - 100*cos(2*a + 2*b*x)
*sin(2*a + 2*b*x)**4*sin(a + b*x)**2 + 2*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)
**4 - 150*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x)**2 + 11*cos(2*
a + 2*b*x)*sin(2*a + 2*b*x)**2 - 150*cos(2*a + 2*b*x)*sin(a + b*x)**2 - 53
*cos(2*a + 2*b*x) + 20*cos(a + b*x)*sin(2*a + 2*b*x)**5*sin(a + b*x) + 50*
cos(a + b*x)*sin(2*a + 2*b*x)**3*sin(a + b*x) + 300*sin(2*a + 2*b*x)*sin(a
 + b*x)**2*b*x - 150*sin(2*a + 2*b*x)*b*x + 3)/(960*b)
```

3.424 $\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{3x}{16} - \frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b} - \frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} - \frac{\sin^5(2a+2bx)}{20b}$$

output
```
3/16*x-3/32*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b-1/16*cos(2*b*x+2*a)*sin(2*b*x+
2*a)^3/b-1/20*sin(2*b*x+2*a)^5/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{120bx - 20\sin(2(a+bx)) - 40\sin(4(a+bx)) + 10\sin(6(a+bx)) + 5\sin(8(a+bx)) - 2\sin(10(a+bx)}{640b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

output
```
(120*b*x - 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] + 10*Sin[6*(a + b*x)]
 + 5*Sin[8*(a + b*x)] - 2*Sin[10*(a + b*x)])/(640*b)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 4774, 3042, 3044, 15, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^4dx$$

$$\Big\downarrow \text{4774}$$

$$\frac{1}{2}\int \sin^4(2a+2bx)dx-\frac{1}{2}\int \cos(2a+2bx)\sin^4(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^4dx-\frac{1}{2}\int \cos(2a+2bx)\sin(2a+2bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{1}{2}\int \sin(2a+2bx)^4dx-\frac{\int \sin^4(2a+2bx)d\sin(2a+2bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \sin(2a+2bx)^4dx-\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \sin^2(2a+2bx)dx-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)-\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \sin(2a+2bx)^2dx-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)-\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \textbf{3115}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)-\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \textbf{24}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)-\frac{\sin^5(2a+2bx)}{20b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

output
```
-1/20*Sin[2*a + 2*b*x]^5/b + (-1/8*(Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^3)/b
 + (3*(x/2 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b)))/4)/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4774
```
Int[sin[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Simp[1/2   Int[(g*Sin[c + d*x])^p, x], x] - Simp[1/2   Int[Cos[c + d*
x]*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d
, 0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Maple [A] (verified)

Time = 6.92 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{10\sin(6bx+6a)-40\sin(4bx+4a)+120bx-2\sin(10bx+10a)+5\sin(8bx+8a)-20\sin(2bx+2a)}{640b}$	66
default	$\frac{3x}{16}-\frac{\sin(2bx+2a)}{32b}-\frac{\sin(4bx+4a)}{16b}+\frac{\sin(6bx+6a)}{64b}+\frac{\sin(8bx+8a)}{128b}-\frac{\sin(10bx+10a)}{320b}$	75
risch	$\frac{3x}{16}-\frac{\sin(2bx+2a)}{32b}-\frac{\sin(4bx+4a)}{16b}+\frac{\sin(6bx+6a)}{64b}+\frac{\sin(8bx+8a)}{128b}-\frac{\sin(10bx+10a)}{320b}$	75
orering	Expression too large to display	2048

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/640*(10*sin(6*b*x+6*a)-40*sin(4*b*x+4*a)+120*b*x-2*sin(10*b*x+10*a)+5*si
n(8*b*x+8*a)-20*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{15\,bx - \left(128\,\cos\left(bx+a\right)^9 - 336\,\cos\left(bx+a\right)^7 + 248\,\cos\left(bx+a\right)^5 - 10\,\cos\left(bx+a\right)^3 - 15\,\cos\left(bx+a\right)}{80\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
1/80*(15*b*x - (128*cos(b*x + a)^9 - 336*cos(b*x + a)^7 + 248*cos(b*x + a)
^5 - 10*cos(b*x + a)^3 - 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(70) = 140$.

Time = 5.10 (sec) , antiderivative size = 434, normalized size of antiderivative = 5.71

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \begin{cases} \frac{3x\sin^2(a+bx)\sin^4(2a+2bx)}{16} + \frac{3x\sin^2(a+bx)\sin^2(2a+2bx)\cos^2(2a+2bx)}{8} + \frac{3x\sin^2(a+bx)\cos^4(2a+2bx)}{16} + \frac{3x\sin^4(2a+2bx)\cos^2(a}{16} \\ x\sin^2(a)\sin^4(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**4/16 + 3*x*sin(a + b*x)**
2*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*sin(a + b*x)**2*cos(2*a
+ 2*b*x)**4/16 + 3*x*sin(2*a + 2*b*x)**4*cos(a + b*x)**2/16 + 3*x*sin(2*a
+ 2*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*cos(a + b*x)**2*co
s(2*a + 2*b*x)**4/16 - 57*sin(a + b*x)**2*sin(2*a + 2*b*x)**3*cos(2*a + 2*
b*x)/(160*b) - 109*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(4
80*b) - sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(a + b*x)/(10*b) - 2*sin(a + b
*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(5*b) - 4*sin(a +
 b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(15*b) + 7*sin(2*a + 2*b*x)**3*cos(
a + b*x)**2*cos(2*a + 2*b*x)/(160*b) + 19*sin(2*a + 2*b*x)*cos(a + b*x)**2
*cos(2*a + 2*b*x)**3/(480*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{120\,bx - 2\,\sin\left(10\,bx + 10\,a\right) + 5\,\sin\left(8\,bx + 8\,a\right) + 10\,\sin\left(6\,bx + 6\,a\right) - 40\,\sin\left(4\,bx + 4\,a\right) - 20\,\sin\left(2\,bx\right.}{640\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/640*(120*b*x - 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) + 10*sin(6*b*x
+ 6*a) - 40*sin(4*b*x + 4*a) - 20*sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{120\,bx + 120\,a - 2\,\sin\left(10\,bx + 10\,a\right) + 5\,\sin\left(8\,bx + 8\,a\right) + 10\,\sin\left(6\,bx + 6\,a\right) - 40\,\sin\left(4\,bx + 4\,a\right) - 20}{640\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/640*(120*b*x + 120*a - 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) + 10*si
n(6*b*x + 6*a) - 40*sin(4*b*x + 4*a) - 20*sin(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 19.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.45

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{3\,x}{16} - \frac{-\frac{3\tan(a+b\,x)^9}{16}-\frac{7\tan(a+b\,x)^7}{8}+\frac{8\tan(a+b\,x)^5}{5}+\frac{7\tan(a+b\,x)^3}{8}+\frac{3\tan(a+b\,x)}{16}}{b\,\left(\tan\left(a+b\,x\right)^{10}+5\tan\left(a+b\,x\right)^8+10\tan\left(a+b\,x\right)^6+10\tan\left(a+b\,x\right)^4+5\tan\left(a+b\,x\right)^2+1\right)}$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^4,x)
```

output
```
(3*x)/16 - ((3*tan(a + b*x))/16 + (7*tan(a + b*x)^3)/8 + (8*tan(a + b*x)^5
)/5 - (7*tan(a + b*x)^7)/8 - (3*tan(a + b*x)^9)/16)/(b*(5*tan(a + b*x)^2 +
 10*tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^1
0 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.99

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{128\cos\left(2bx+2a\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-64\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)^3\sin\left(bx+a\right)^2+2\cos\left(2bx\right.}{}$$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x)
```

output
```
(128*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x) - 64*cos(2*a + 2*b*x)*sin(
2*a + 2*b*x)**3*sin(a + b*x)**2 + 2*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**3 -
 128*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)*sin(a + b*x)**2 + 19*cos(2*a + 2*b*
x)*sin(2*a + 2*b*x) + 16*cos(a + b*x)*sin(2*a + 2*b*x)**4*sin(a + b*x) + 6
4*cos(a + b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x) - 128*cos(a + b*x)*sin(a +
 b*x) + 128*sin(2*a + 2*b*x)*sin(a + b*x)**2 - 64*sin(2*a + 2*b*x) + 90*b*
x)/(480*b)
```

3.425 $\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx=\frac{4\sin^6(a+bx)}{3b}-\frac{\sin^8(a+bx)}{b}$$

output
```
4/3*sin(b*x+a)^6/b-sin(b*x+a)^8/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx =\frac{-72\cos(2(a+bx))+12\cos(4(a+bx))+8\cos(6(a+bx))-3\cos(8(a+bx))}{384b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-72*Cos[2*(a + b*x)] + 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] - 3*Cos[8
*(a + b*x)])/(384*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^3dx$$

$$\Big\downarrow \text{4776}$$

$$8\int \cos^3(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)^3\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{8\int \sin^5(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{8\int \left(\sin^5(a+bx)-\sin^7(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8\left(\frac{1}{6}\sin^6(a+bx)-\frac{1}{8}\sin^8(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

output
```
(8*(Sin[a + b*x]^6/6 - Sin[a + b*x]^8/8))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.79

method	result
parallelrisch	$\frac{12\cos(4bx+4a)+71-3\cos(8bx+8a)-72\cos(2bx+2a)+8\cos(6bx+6a)}{384b}$
default	$-\frac{3\cos(2bx+2a)}{16b}+\frac{\cos(4bx+4a)}{32b}+\frac{\cos(6bx+6a)}{48b}-\frac{\cos(8bx+8a)}{128b}$
risch	$-\frac{3\cos(2bx+2a)}{16b}+\frac{\cos(4bx+4a)}{32b}+\frac{\cos(6bx+6a)}{48b}-\frac{\cos(8bx+8a)}{128b}$
orering	$-\frac{205\left(2\sin(bx+a)\sin(2bx+2a)^3 b\cos(bx+a)+6\sin(bx+a)^2\sin(2bx+2a)^2 b\cos(2bx+2a)\right)}{576b^2}-$ 91(−80 sin(bx+a) sin(2bx+2

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/384*(12*cos(4*b*x+4*a)+71-3*cos(8*b*x+8*a)-72*cos(2*b*x+2*a)+8*cos(6*b*x
+6*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{3\,\cos(bx+a)^8 - 8\,\cos(bx+a)^6 + 6\,\cos(bx+a)^4}{3\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-1/3*(3*cos(b*x + a)^8 - 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(22) = 44$.

Time = 2.17 (sec) , antiderivative size = 359, normalized size of antiderivative = 12.38

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx$$
$$= \begin{cases} \frac{3x\sin^2(a+bx)\sin^3(2a+2bx)}{16} + \frac{3x\sin^2(a+bx)\sin(2a+2bx)\cos^2(2a+2bx)}{16} + \frac{3x\sin(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{8} + 3x \\ x\sin^2(a)\sin^3(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**3,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**3/16 + 3*x*sin(a + b*x)**
2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/16 + 3*x*sin(a + b*x)*sin(2*a + 2*b
*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)/8 + 3*x*sin(a + b*x)*cos(a + b*x)*cos
(2*a + 2*b*x)**3/8 - 3*x*sin(2*a + 2*b*x)**3*cos(a + b*x)**2/16 - 3*x*sin(
2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 - sin(a + b*x)**2*sin(
2*a + 2*b*x)**2*cos(2*a + 2*b*x)/(2*b) - 31*sin(a + b*x)**2*cos(2*a + 2*b*
x)**3/(96*b) + 3*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(a + b*x)/(16*b) + si
n(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(8*b) - cos(a
 + b*x)**2*cos(2*a + 2*b*x)**3/(96*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**3
, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx$$
$$= -\frac{3\cos(8\,bx+8\,a)-8\cos(6\,bx+6\,a)-12\cos(4\,bx+4\,a)+72\cos(2\,bx+2\,a)}{384\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
-1/384*(3*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) + 72
*cos(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{3\sin(bx+a)^8-4\sin(bx+a)^6}{3\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-1/3*(3*sin(b*x + a)^8 - 4*sin(b*x + a)^6)/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{\cos(a+b\,x)^4\left(\cos(a+b\,x)^4-\frac{8\cos(a+b\,x)^2}{3}+2\right)}{b}$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^3,x)
```

output
```
-(cos(a + b*x)^4*(cos(a + b*x)^4 - (8*cos(a + b*x)^2)/3 + 2))/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 5.93

$$\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx$$
$$= \frac{36\cos(2bx+2a)\cos(bx+a)\sin(bx+a)\,bx-18\cos(2bx+2a)\sin(2bx+2a)^2\sin(bx+a)^2+\cos(2bx}{}$$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x)
```

output
```
(36*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x)*b*x - 18*cos(2*a + 2*b*x)*s
in(2*a + 2*b*x)**2*sin(a + b*x)**2 + cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**2
 - 18*cos(2*a + 2*b*x)*sin(a + b*x)**2 - 7*cos(2*a + 2*b*x) + 6*cos(a + b*x
)*sin(2*a + 2*b*x)**3*sin(a + b*x) + 36*sin(2*a + 2*b*x)*sin(a + b*x)**2*b
*x - 18*sin(2*a + 2*b*x)*b*x + 1)/(96*b)
```

3.426 $\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx=\frac{x}{4}-\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}-\frac{\sin^3(2a+2bx)}{12b}$$

output
```
1/4*x-1/8*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b-1/12*sin(2*b*x+2*a)^3/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx =\frac{12bx-3\sin(2(a+bx))-3\sin(4(a+bx))+\sin(6(a+bx))}{48b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

output
```
(12*b*x - 3*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)])/(48*
b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4774, 3042, 3044, 15, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^2dx$$

$$\Big\downarrow \ 4774$$

$$\frac{1}{2}\int \sin^2(2a+2bx)dx-\frac{1}{2}\int \cos(2a+2bx)\sin^2(2a+2bx)dx$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\int \sin(2a+2bx)^2dx-\frac{1}{2}\int \cos(2a+2bx)\sin(2a+2bx)^2dx$$

$$\Big\downarrow \ 3044$$

$$\frac{1}{2}\int \sin(2a+2bx)^2dx-\frac{\int \sin^2(2a+2bx)d\sin(2a+2bx)}{4b}$$

$$\Big\downarrow \ 15$$

$$\frac{1}{2}\int \sin(2a+2bx)^2dx-\frac{\sin^3(2a+2bx)}{12b}$$

$$\Big\downarrow \ 3115$$

$$\frac{1}{2}\left(\frac{\int 1dx}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)-\frac{\sin^3(2a+2bx)}{12b}$$

$$\Big\downarrow \ 24$$

$$\frac{1}{2}\left(\frac{x}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)-\frac{\sin^3(2a+2bx)}{12b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

output
```
-1/12*Sin[2*a + 2*b*x]^3/b + (x/2 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4
*b))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4774
```
Int[sin[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Simp[1/2   Int[(g*Sin[c + d*x])^p, x], x] - Simp[1/2   Int[Cos[c + d*
x]*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d
, 0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{12bx-3\sin(2bx+2a)-3\sin(4bx+4a)+\sin(6bx+6a)}{48b}$
default	$\frac{x}{4}-\frac{\sin(2bx+2a)}{16b}-\frac{\sin(4bx+4a)}{16b}+\frac{\sin(6bx+6a)}{48b}$
risch	$\frac{x}{4}-\frac{\sin(2bx+2a)}{16b}-\frac{\sin(4bx+4a)}{16b}+\frac{\sin(6bx+6a)}{48b}$
orering	$x\sin(bx+a)^2\sin(2bx+2a)^2-\frac{49\left(2\cos(bx+a)\sin(2bx+2a)^2b\sin(bx+a)+4b\sin(bx+a)^2\cos(2bx+2a)\sin(2bx\right.}{144b^2}$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/48*(12*b*x-3*sin(2*b*x+2*a)-3*sin(4*b*x+4*a)+sin(6*b*x+6*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{3\,bx+\left(8\,\cos(bx+a)^5-14\,\cos(bx+a)^3+3\,\cos(bx+a)\right)\sin(bx+a)}{12\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
1/12*(3*b*x + (8*cos(b*x + a)^5 - 14*cos(b*x + a)^3 + 3*cos(b*x + a))*sin(
b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(41) = 82$.

Time = 0.85 (sec) , antiderivative size = 231, normalized size of antiderivative = 4.71

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx$$
$$= \begin{cases} \frac{x\sin^2(a+bx)\sin^2(2a+2bx)}{4} + \frac{x\sin^2(a+bx)\cos^2(2a+2bx)}{4} + \frac{x\sin^2(2a+2bx)\cos^2(a+bx)}{4} + \frac{x\cos^2(a+bx)\cos^2(2a+2bx)}{4} - \frac{7\sin^2(a\cdots \\ x\sin^2(a)\sin^2(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)**2/4 + x*sin(a + b*x)**2*cos
(2*a + 2*b*x)**2/4 + x*sin(2*a + 2*b*x)**2*cos(a + b*x)**2/4 + x*cos(a + b
*x)**2*cos(2*a + 2*b*x)**2/4 - 7*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a
+ 2*b*x)/(24*b) - sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)/(6*b) - si
n(a + b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(3*b) + sin(2*a + 2*b*x)*cos(a
 + b*x)**2*cos(2*a + 2*b*x)/(24*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**2, T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{12\,bx + \sin(6\,bx+6\,a) - 3\,\sin(4\,bx+4\,a) - 3\,\sin(2\,bx+2\,a)}{48\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
1/48*(12*b*x + sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))
/b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{12\,bx + 12\,a + \sin(6\,bx + 6\,a) - 3\,\sin(4\,bx + 4\,a) - 3\,\sin(2\,bx + 2\,a)}{48\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
1/48*(12*b*x + 12*a + sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 3*sin(2*b*x
+ 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 18.59 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{x}{4} - \frac{\frac{\sin(2\,a+2\,b\,x)}{16} + \frac{\sin(4\,a+4\,b\,x)}{16} - \frac{\sin(6\,a+6\,b\,x)}{48}}{b}$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^2,x)
```

output
```
x/4 - (sin(2*a + 2*b*x)/16 + sin(4*a + 4*b*x)/16 - sin(6*a + 6*b*x)/48)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.04

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{-8\cos(2bx+2a)\cos(bx+a)\sin(bx+a) - 8\cos(2bx+2a)\sin(2bx+2a)\sin(bx+a)^2 + \cos(2bx+2a)}{}$$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x)
```

output
```
( - 8*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x) - 8*cos(2*a + 2*b*x)*sin(
2*a + 2*b*x)*sin(a + b*x)**2 + cos(2*a + 2*b*x)*sin(2*a + 2*b*x) + 4*cos(a
 + b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x) - 8*cos(a + b*x)*sin(a + b*x) - 8
*sin(2*a + 2*b*x)*sin(a + b*x)**2 + 4*sin(2*a + 2*b*x) + 6*b*x)/(24*b)
```

3.427 $\int \sin^2(a+bx)\sin(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx=\frac{\sin^4(a+bx)}{2b}$$

output
```
1/2*sin(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx=\frac{\sin^4(a+bx)}{2b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

output
```
Sin[a + b*x]^4/(2*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4776, 3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)dx$$

$$\Big\downarrow \text{4776}$$

$$2\int \cos(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \cos(a+bx)\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{2\int \sin^3(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sin^4(a+bx)}{2b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

output
```
Sin[a + b*x]^4/(2*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

method	result
parallelrisch	$\frac{3+\cos(4bx+4a)-4\cos(2bx+2a)}{16b}$
default	$-\frac{\cos(2bx+2a)}{4b}+\frac{\cos(4bx+4a)}{16b}$
risch	$-\frac{\cos(2bx+2a)}{4b}+\frac{\cos(4bx+4a)}{16b}$
orering	$-\frac{5\left(2\sin(bx+a)\sin(2bx+2a)b\cos(bx+a)+2\sin(bx+a)^2 b\cos(2bx+2a)\right)}{16b^2}-\frac{12b^3\cos(2bx+2a)\cos(bx+a)^2-32\sin(2bx+2a)}{}$
norman	$\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3\tan(bx+a)^2+\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan(bx+a)}{b}-x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3-\frac{x\tan(bx+a)}{2}-x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan(bx+a}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/16*(3+cos(4*b*x+4*a)-4*cos(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx=\frac{\cos\left(bx+a\right)^4-2\,\cos\left(bx+a\right)^2}{2\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
1/2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(10) = 20.

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 8.73

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx$$
$$=\begin{cases}\frac{x\sin^2\left(a+bx\right)\sin\left(2a+2bx\right)}{4}+\frac{x\sin\left(a+bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{2}-\frac{x\sin\left(2a+2bx\right)\cos^2\left(a+bx\right)}{4}-\frac{\sin^2\left(a+bx\right)\cos\left(2a+2bx\right)}{2b}+\frac{\sin\left(a\right.}{} \\ x\sin^2\left(a\right)\sin\left(2a\right)\end{cases}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a),x)
```

output
```
Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)/4 + x*sin(a + b*x)*cos(a + b
*x)*cos(2*a + 2*b*x)/2 - x*sin(2*a + 2*b*x)*cos(a + b*x)**2/4 - sin(a + b*
x)**2*cos(2*a + 2*b*x)/(2*b) + sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)/
(4*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx = \frac{\cos(4\,bx+4\,a)-4\,\cos(2\,bx+2\,a)}{16\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
1/16*(cos(4*b*x + 4*a) - 4*cos(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx = \frac{\sin(bx+a)^4}{2\,b}$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
1/2*sin(b*x + a)^4/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx = \frac{\sin(a+b\,x)^4}{2\,b}$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x),x)
```

output
```
sin(a + b*x)^4/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 6.40

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx$$
$$= \frac{4\cos(2bx+2a)\cos(bx+a)\sin(bx+a)\,bx-2\cos(2bx+2a)\sin(bx+a)^2-\cos(2bx+2a)+4\sin(2bx$$
$$8b$$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a),x)
```

output
```
(4*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x)*b*x - 2*cos(2*a + 2*b*x)*sin
(a + b*x)**2 - cos(2*a + 2*b*x) + 4*sin(2*a + 2*b*x)*sin(a + b*x)**2*b*x -
 2*sin(2*a + 2*b*x)*b*x + 1)/(8*b)
```

3.428 $\int \csc(2a+2bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx=-\frac{\log(\cos(a+bx))}{2b}$$

output
```
-1/2*ln(cos(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx=-\frac{\log(\cos(a+bx))}{2b}$$

input
```
Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/2*Log[Cos[a + b*x]]/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4776, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{2}\int \tan(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \tan(a+bx)dx$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\log(\cos(a+bx))}{2b}$$

input
```
Int[Csc[2*a + 2*b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/2*Log[Cos[a + b*x]]/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{\ln(\cos(bx+a))}{2b}$	13
risch	$\frac{ix}{2}+\frac{ia}{b}-\frac{\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)}{2b}$	30

input
```
int(csc(2*b*x+2*a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(cos(b*x+a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\log\left(-\cos\left(bx+a\right)\right)}{2\,b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*log(-cos(b*x + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.93

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\log\left(\cos\left(2\,bx\right)^2+2\,\cos\left(2\,bx\right)\cos\left(2\,a\right)+\cos\left(2\,a\right)^2+\sin\left(2\,bx\right)^2-2\,\sin\left(2\,bx\right)\sin\left(2\,a\right)+\sin\left(2\,a\right)^2\right)}{4\,b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2
- 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\log\left(-\sin\left(bx+a\right)^2+1\right)}{4\,b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/4*log(-sin(b*x + a)^2 + 1)/b
```

Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\ln\left(\cos\left(a+b\,x\right)\right)}{2\,b}$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x),x)
```

output
```
-log(cos(a + b*x))/(2*b)
```

Reduce [F]

$$\int \csc(2a+2bx)\sin^2(a+bx)\,dx = \int \csc\left(2bx+2a\right)\sin\left(bx+a\right)^2dx$$

input
```
int(csc(2*b*x+2*a)*sin(b*x+a)^2,x)
```

output
```
int(csc(2*a + 2*b*x)*sin(a + b*x)**2,x)
```

3.429 $\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx = \frac{\tan(a+bx)}{4b}$$

output
```
1/4*tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx = \frac{\tan(a+bx)}{4b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^2,x]
```

output
```
Tan[a + b*x]/(4*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4776, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{4}\int \sec^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d(-\tan(a+bx))}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan(a+bx)}{4b}$$

input
```
Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^2,x]
```

output
```
Tan[a + b*x]/(4*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\tan(bx+a)}{4b}$	12
risch	$\frac{i}{2b\left(e^{2i(bx+a)}+1\right)}$	20

input
```
int(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*tan(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx = \frac{\sin(bx+a)}{4\,b\cos(bx+a)}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*sin(b*x + a)/(b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**2*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(11) = 22.

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 4.08

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx = \frac{\sin(2\,bx+2\,a)}{2\left(b\cos(2\,bx+2\,a)^2+b\sin(2\,bx+2\,a)^2+2\,b\cos(2\,bx+2\,a)+b\right)}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*sin(2*b*x + 2*a)/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 + 2*b*co
s(2*b*x + 2*a) + b)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx = \frac{\tan(bx+a)}{4\,b}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*tan(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 17.96 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx = \frac{\tan(a+b\,x)}{4\,b}$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^2,x)
```

output
```
tan(a + b*x)/(4*b)
```

Reduce [F]

$$\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx = \int \csc(2bx+2a)^2\sin(bx+a)^2\,dx$$

input
```
int(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x)
```

output
```
int(csc(2*a + 2*b*x)**2*sin(a + b*x)**2,x)
```

3.430 $\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx=\frac{\log(\tan(a+bx))}{8b}+\frac{\tan^2(a+bx)}{16b}$$

output
```
1/8*ln(tan(b*x+a))/b+1/16*tan(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx=\frac{1}{8}\left(-\frac{\log(\cos(a+bx))}{b}+\frac{\log(\sin(a+bx))}{b}+\frac{\sec^2(a+bx)}{2b}\right)$$

input
```
Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(-(Log[Cos[a + b*x]]/b) + Log[Sin[a + b*x]]/b + Sec[a + b*x]^2/(2*b))/8
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{8}\int \csc(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot(a+bx)\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{8b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int (\cot(a+bx)+\tan(a+bx))d\tan(a+bx)}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\tan^2(a+bx)+\log(\tan(a+bx))}{8b}$$

input
```
Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(Log[Tan[a + b*x]] + Tan[a + b*x]^2/2)/(8*b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result
default	$\frac{\frac{1}{2\cos\left(bx+a\right)^{2}}+\ln\left(\tan\left(bx+a\right)\right)}{8b}$
risch	$\frac{e^{2i(bx+a)}}{4b\left(e^{2i(bx+a)}+1\right)^{2}}-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{8b}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{8b}$
parallelrisch	$\frac{-\csc\left(bx+a\right)^{2}\left(\sec\left(bx+a\right)^{2}-2\right)\cos\left(2bx+2a\right)-2\sin\left(2bx+2a\right)\sec\left(bx+a\right)\csc\left(bx+a\right)+4\ln\left(\tan\left(bx+a\right)\right)+\csc\left(bx+a\right)^{2}\left(\sec\left(bx+a\right)\cdots\right.}{32b}$

input
```
int(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(1/2/cos(b*x+a)^2+ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\cos(bx+a)^2\log\left(\cos(bx+a)^2\right)-\cos(bx+a)^2\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-1}{16\,b\cos(bx+a)^2}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/16*(cos(b*x + a)^2*log(cos(b*x + a)^2) - cos(b*x + a)^2*log(-1/4*cos(b*
x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 641, normalized size of antiderivative = 21.37

$$\int \csc^3(2a + 2bx)\sin^2(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/16*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + 8*cos(2*b*x + 2*a)^2 - (2*(2*c
os(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x +
 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin
(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*
cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)
 + (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*c
os(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*
a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 + 2*cos
(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + (2*
(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b
*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4
*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*
cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b
*x + 4*a)*sin(2*b*x + 2*a) + 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b
*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*
sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(2*
b*x + 2*a) + b)*cos(4*b*x + 4*a) + 4*b*cos(2*b*x + 2*a) + b)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \csc^3(2a + 2bx)\sin^2(a + bx)\,dx = -\frac{\frac{1}{\sin(bx+a)^2-1} + \log\left(-\sin\left(bx + a\right)^2 + 1\right) - 2\,\log\left(\left|\sin\left(bx + a\right)\right|\right)}{16\,b}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/16*(1/(sin(b*x + a)^2 - 1) + log(-sin(b*x + a)^2 + 1) - 2*log(abs(sin(b
*x + a))))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx = \frac{\frac{\ln\left(\sin(a+b\,x)^2\right)}{16}-\frac{\ln(\cos(a+b\,x))}{8}+\frac{1}{16\cos(a+b\,x)^2}}{b}$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^3,x)
```

output
```
(log(sin(a + b*x)^2)/16 - log(cos(a + b*x))/8 + 1/(16*cos(a + b*x)^2))/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.60

$$\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx$$
$$= \frac{-2\cos(2bx+2a)\sin(bx+a)^2-2\cos(bx+a)\sin(2bx+2a)\sin(bx+a)+\log(\tan(bx+a))\sin(2bx+}{8\sin(2bx+2a)^2 b}$$

input
```
int(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x)
```

output
```
( - 2*cos(2*a + 2*b*x)*sin(a + b*x)**2 - 2*cos(a + b*x)*sin(2*a + 2*b*x)*s
in(a + b*x) + log(tan(a + b*x))*sin(2*a + 2*b*x)**2)/(8*sin(2*a + 2*b*x)**
2*b)
```

3.431 $\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx=-\frac{\cot(a+bx)}{16b}+\frac{\tan(a+bx)}{8b}+\frac{\tan^3(a+bx)}{48b}$$

output
```
-1/16*cot(b*x+a)/b+1/8*tan(b*x+a)/b+1/48*tan(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx=-\frac{\cot(a+bx)}{16b}+\frac{5\tan(a+bx)}{48b}+\frac{\sec^2(a+bx)\tan(a+bx)}{48b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^2,x]
```

output
```
-1/16*Cot[a + b*x]/b + (5*Tan[a + b*x])/(48*b) + (Sec[a + b*x]^2*Tan[a + b
*x])/(48*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{16}\int \csc^2(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^2\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^2(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{16b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^2(a+bx)+\tan^2(a+bx)+2\right)d\tan(a+bx)}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\tan^3(a+bx)+2\tan(a+bx)-\cot(a+bx)}{16b}$$

input
```
Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^2,x]
```

output
```
(-Cot[a + b*x] + 2*Tan[a + b*x] + Tan[a + b*x]^3/3)/(16*b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.52 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result	size
risch	$-\frac{i\left(2\,{\mathrm e}^{2 i \left(b x +a \right)}+1\right)}{3 b \left({\mathrm e}^{2 i \left(b x +a \right)}+1\right)^{3} \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)}$	46
default	$\frac{\frac{1}{3 \sin \left(b x +a \right) \cos \left(b x +a \right)^{3}}+\frac{4}{3 \sin \left(b x +a \right) \cos \left(b x +a \right)}-\frac{8 \cot \left(b x +a \right)}{3}}{16 b}$	51

input
```
int(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*(2*exp(2*I*(b*x+a))+1)/b/(exp(2*I*(b*x+a))+1)^3/(exp(2*I*(b*x+a))-1
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx = -\frac{8\cos(bx+a)^4 - 4\cos(bx+a)^2 - 1}{48\,b\cos(bx+a)^3\sin(bx+a)}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/48*(8*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3*sin(b*x
+ a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**4*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(36) = 72.

Time = 0.06 (sec) , antiderivative size = 308, normalized size of antiderivative = 7.33

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cos\left(8\,bx+8\,a\right)^2+4\,b\cos\left(6\,bx+6\,a\right)^2+4\,b\cos\left(2\,bx+2\,a\right)^2+b\sin\left(8\,bx+8\,a\right)^2+4\,b\sin\left(6\,bx+\right.\right.}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3*((2*cos(2*b*x + 2*a) + 1)*sin(8*b*x + 8*a) + 2*(2*cos(2*b*x + 2*a) +
1)*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) - 4*cos(6*b*x +
6*a)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*
b*cos(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b
*sin(6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(6
*b*x + 6*a) - 2*b*cos(2*b*x + 2*a) - b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*
x + 2*a) + b)*cos(6*b*x + 6*a) + 4*b*cos(2*b*x + 2*a) + 4*(b*sin(6*b*x + 6
*a) - b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx = \frac{\tan\left(bx+a\right)^3 - \frac{3}{\tan(bx+a)} + 6\,\tan\left(bx+a\right)}{48\,b}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/48*(tan(b*x + a)^3 - 3/tan(b*x + a) + 6*tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 18.88 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx = \frac{\tan(a+b\,x)^4 + 6\tan(a+b\,x)^2 - 3}{48\,b\tan\left(a+b\,x\right)}$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^4,x)
```

output
```
(6*tan(a + b*x)^2 + tan(a + b*x)^4 - 3)/(48*b*tan(a + b*x))
```

Reduce [F]

$$\int \csc^4(2a + 2bx)\sin^2(a + bx)\,dx = \int \csc(2bx + 2a)^4 \sin(bx + a)^2\,dx$$

input
```
int(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x)
```

output
```
int(csc(2*a + 2*b*x)**4*sin(a + b*x)**2,x)
```

3.432 $\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx = -\frac{\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}+\frac{3\tan^2(a+bx)}{64b}+\frac{\tan^4(a+bx)}{128b}$$

output
```
-1/64*cot(b*x+a)^2/b+3/32*ln(tan(b*x+a))/b+3/64*tan(b*x+a)^2/b+1/128*tan(b
*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx = -\frac{2\csc^2(a+bx)+12\log(\cos(a+bx))-12\log(\sin(a+bx))-4\sec^2(a+bx)-\sec^4(a+bx)}{128b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^2,x]
```

output
```
-1/128*(2*Csc[a + b*x]^2 + 12*Log[Cos[a + b*x]] - 12*Log[Sin[a + b*x]] - 4
*Sec[a + b*x]^2 - Sec[a + b*x]^4)/b
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4776, 3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{32}\int \csc^3(a+bx)\sec^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \csc(a+bx)^3\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan(a+bx)}{32b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \cot^2(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan^2(a+bx)}{64b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\cot^2(a+bx)+3\cot(a+bx)+\tan^2(a+bx)+3\right)d\tan^2(a+bx)}{64b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\tan^4(a+bx)+3\tan^2(a+bx)-\cot(a+bx)+3\log\left(\tan^2(a+bx)\right)}{64b}$$

input
```
Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^2,x]
```

output
```
(-Cot[a + b*x] + 3*Log[Tan[a + b*x]^2] + 3*Tan[a + b*x]^2 + Tan[a + b*x]^4
/2)/(64*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

method	result
default	$\frac{\frac{1}{4\sin(bx+a)^2\cos(bx+a)^4}+\frac{3}{4\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{2\sin(bx+a)^2}+3\ln(\tan(bx+a))}{32b}$
parallelrisch	$-\frac{\sec(bx+a)^4\csc(bx+a)^4(-64\cos(4bx+4a)+16\cos(8bx+8a)+66\cos(2bx+2a)-18\cos(6bx+6a)+36\ln(\tan(bx+a))\cos(4bx\cdots}{12288b}$
risch	$\frac{3\,e^{10i(bx+a)}+6\,e^{8i(bx+a)}-2\,e^{6i(bx+a)}+6\,e^{4i(bx+a)}+3\,e^{2i(bx+a)}}{16b(e^{2i(bx+a)}+1)^4(e^{2i(bx+a)}-1)^2}-\frac{3\ln(e^{2i(bx+a)}+1)}{32b}+\frac{3\ln(e^{2i(bx+a)}-1)}{32b}$

input
```
int(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(1/4/sin(b*x+a)^2/cos(b*x+a)^4+3/4/sin(b*x+a)^2/cos(b*x+a)^2-3/2/si
n(b*x+a)^2+3*ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(52) = 104.

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.87

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx = \frac{6\cos(bx+a)^4-3\cos(bx+a)^2-6\left(\cos(bx+a)^6-\cos(bx+a)^4\right)\log\left(\cos(bx+a)^2\right)+6\left(\cos(bx+a\cdots\right.}{128\left(b\cos(bx+a)^6-b\cos(bx+a)^4\right)}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/128*(6*cos(b*x + a)^4 - 3*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - cos(b*x +
 a)^4)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*
cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^6 - b*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(2a + 2bx) \sin^2(a + bx) \, dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**5*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3164 vs. $2(52) = 104$.

Time = 0.22 (sec) , antiderivative size = 3164, normalized size of antiderivative = 52.73

$$\int \csc^5(2a + 2bx) \sin^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/64*(4*(3*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) +
6*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b
*x + 8*a) - 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) + 12*cos(2*b*x + 2*a)
 + 3)*cos(10*b*x + 10*a) + 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a
) + 12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) - 6)*cos(8*b*x + 8*a) - 24*co
s(8*b*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) + 8*cos(2*b*x + 2*a) + 1)*cos(6*
b*x + 6*a) + 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) + 2)*cos(4*b*x
 + 4*a) - 24*cos(4*b*x + 4*a)^2 + 24*cos(2*b*x + 2*a)^2 - 3*(2*(2*cos(10*b
*x + 10*a) - cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) + 2*
cos(2*b*x + 2*a) + 1)*cos(12*b*x + 12*a) + cos(12*b*x + 12*a)^2 - 4*(cos(8
*b*x + 8*a) + 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) -
 1)*cos(10*b*x + 10*a) + 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) +
cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*x +
8*a)^2 + 8*(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) +
16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(
4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) - sin(8*b*
x + 8*a) - 4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) + sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) + 4*sin(6*b*x
 + 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 4*si
n(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 2*sin(2...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx$$
$$= -\frac{\frac{6\sin(bx+a)^4-9\sin(bx+a)^2+2}{\left(\sin(bx+a)^2-1\right)^2\sin(bx+a)^2} + 6\log\left(-\sin(bx+a)^2+1\right) - 12\log\left(|\sin(bx+a)|\right)}{128\,b}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/128*((6*sin(b*x + a)^4 - 9*sin(b*x + a)^2 + 2)/((sin(b*x + a)^2 - 1)^2*
sin(b*x + a)^2) + 6*log(-sin(b*x + a)^2 + 1) - 12*log(abs(sin(b*x + a))))/
b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx = \frac{3\,\ln\left(\sin\left(a+b\,x\right)^2\right)}{64\,b}-\frac{3\,\ln\left(\cos\left(a+b\,x\right)\right)}{32\,b}+\frac{-\frac{3\cos(a+b\,x)^4}{64}+\frac{3\cos(a+b\,x)^2}{128}+\frac{1}{128}}{b\,\left(\cos\left(a+b\,x\right)^4-\cos\left(a+b\,x\right)^6\right)}$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^5,x)
```

output
```
(3*log(sin(a + b*x)^2))/(64*b) - (3*log(cos(a + b*x)))/(32*b) + ((3*cos(a
+ b*x)^2)/128 - (3*cos(a + b*x)^4)/64 + 1/128)/(b*(cos(a + b*x)^4 - cos(a
+ b*x)^6))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.60

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx$$
$$= \frac{-16\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)^2\sin\left(bx+a\right)^2-\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)^2-12\cos\left(2bx+2a\right)\sin}{}$$

input
```
int(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x)
```

output
```
( - 16*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x)**2 - cos(2*a + 2*
b*x)*sin(2*a + 2*b*x)**2 - 12*cos(2*a + 2*b*x)*sin(a + b*x)**2 - 16*cos(a
+ b*x)*sin(2*a + 2*b*x)**3*sin(a + b*x) - 4*cos(a + b*x)*sin(2*a + 2*b*x)*
sin(a + b*x) + 9*log(tan(a + b*x))*sin(2*a + 2*b*x)**4)/(96*sin(2*a + 2*b*
x)**4*b)
```

3.433 $\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\sin^9(a+bx)}{9b} - \frac{64\sin^{11}(a+bx)}{11b} + \frac{32\sin^{13}(a+bx)}{13b}$$

output
```
32/9*sin(b*x+a)^9/b-64/11*sin(b*x+a)^11/b+32/13*sin(b*x+a)^13/b
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx = \frac{4(505+540\cos(2(a+bx))+99\cos(4(a+bx)))\sin^9(a+bx)}{1287b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

output
```
(4*(505 + 540*Cos[2*(a + b*x)] + 99*Cos[4*(a + b*x)])*Sin[a + b*x]^9)/(128
7*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\sin(2a+2bx)^5dx$$

$$\Big\downarrow \text{4776}$$

$$32\int \cos^5(a+bx)\sin^8(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$32\int \cos(a+bx)^5\sin(a+bx)^8dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{32\int \sin^8(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{32\int \left(\sin^{12}(a+bx)-2\sin^{10}(a+bx)+\sin^8(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{32\left(\frac{1}{13}\sin^{13}(a+bx)-\frac{2}{11}\sin^{11}(a+bx)+\frac{1}{9}\sin^9(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

output
```
(32*(Sin[a + b*x]^9/9 - (2*Sin[a + b*x]^11)/11 + Sin[a + b*x]^13/13))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(40) = 80$.

Time = 22.87 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.11

method	result
default	$\frac{5\sin(bx+a)}{32b} - \frac{25\sin(3bx+3a)}{384b} - \frac{\sin(5bx+5a)}{128b} + \frac{\sin(7bx+7a)}{64b} - \frac{\sin(9bx+9a)}{576b} - \frac{3\sin(11bx+11a)}{1408b} + \frac{\sin(13bx+13a)}{1664b}$
risch	$\frac{5\sin(bx+a)}{32b} - \frac{25\sin(3bx+3a)}{384b} - \frac{\sin(5bx+5a)}{128b} + \frac{\sin(7bx+7a)}{64b} - \frac{\sin(9bx+9a)}{576b} - \frac{3\sin(11bx+11a)}{1408b} + \frac{\sin(13bx+13a)}{1664b}$
parallelrisch	$\frac{\left(512\tan(bx+a)^7+1216\tan(bx+a)^5+512\tan(bx+a)^3\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6+\left(-3072\tan(bx+a)^8-4992\tan(bx+a)^6+4992\tan(bx+}{}$
orering	Expression too large to display

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
5/32*sin(b*x+a)/b-25/384*sin(3*b*x+3*a)/b-1/128/b*sin(5*b*x+5*a)+1/64/b*si
n(7*b*x+7*a)-1/576/b*sin(9*b*x+9*a)-3/1408/b*sin(11*b*x+11*a)+1/1664/b*sin
(13*b*x+13*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$$
$$= \frac{32\left(99\cos\left(bx+a\right)^{12}-360\cos\left(bx+a\right)^{10}+458\cos\left(bx+a\right)^{8}-212\cos\left(bx+a\right)^{6}+3\cos\left(bx+a\right)^{4}+4\right.}{1287\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
32/1287*(99*cos(b*x + a)^12 - 360*cos(b*x + a)^10 + 458*cos(b*x + a)^8 - 2
12*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*x + a)^2 + 8)*sin(b*x + a)/
b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(39) = 78.

Time = 26.38 (sec) , antiderivative size = 447, normalized size of antiderivative = 9.72

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{1366\sin^3(a+bx)\sin^4(2a+2bx)\cos(2a+2bx)}{3003b} - \frac{4960\sin^3(a+bx)\sin^2(2a+2bx)\cos^3(2a+2bx)}{9009b} - \frac{256\sin^3(a+bx)\cos^5(2a+2bx)}{1287b} - 27 \\ x\sin^3(a)\sin^5(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

output
```
Piecewise((-1366*sin(a + b*x)**3*sin(2*a + 2*b*x)**4*cos(2*a + 2*b*x)/(300
3*b) - 4960*sin(a + b*x)**3*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**3/(9009*
b) - 256*sin(a + b*x)**3*cos(2*a + 2*b*x)**5/(1287*b) - 271*sin(a + b*x)**
2*sin(2*a + 2*b*x)**5*cos(a + b*x)/(3003*b) - 48*sin(a + b*x)**2*sin(2*a +
 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**2/(143*b) - 640*sin(a + b*x)**2*
sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(3003*b) - 1388*sin(a +
b*x)*sin(2*a + 2*b*x)**4*cos(a + b*x)**2*cos(2*a + 2*b*x)/(3003*b) - 2944*
sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)**3/(3003
*b) - 512*sin(a + b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**5/(1001*b) + 2234
*sin(2*a + 2*b*x)**5*cos(a + b*x)**3/(9009*b) + 4544*sin(2*a + 2*b*x)**3*c
os(a + b*x)**3*cos(2*a + 2*b*x)**2/(9009*b) + 256*sin(2*a + 2*b*x)*cos(a +
 b*x)**3*cos(2*a + 2*b*x)**4/(1001*b), Ne(b, 0)), (x*sin(a)**3*sin(2*a)**5
, True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$$
$$= \frac{99\sin(13\,bx+13\,a)-351\sin(11\,bx+11\,a)-286\sin(9\,bx+9\,a)+2574\sin(7\,bx+7\,a)-1287\sin(5}{164736\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/164736*(99*sin(13*b*x + 13*a) - 351*sin(11*b*x + 11*a) - 286*sin(9*b*x +
 9*a) + 2574*sin(7*b*x + 7*a) - 1287*sin(5*b*x + 5*a) - 10725*sin(3*b*x +
3*a) + 25740*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$$
$$= \frac{32\left(99\sin(bx+a)^{13}-234\sin(bx+a)^{11}+143\sin(bx+a)^{9}\right)}{1287\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
32/1287*(99*sin(b*x + a)^13 - 234*sin(b*x + a)^11 + 143*sin(b*x + a)^9)/b
```

Mupad [B] (verification not implemented)

Time = 18.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\,\left(99\sin\left(a+b\,x\right)^{13}-234\sin\left(a+b\,x\right)^{11}+143\sin\left(a+b\,x\right)^{9}\right)}{1287\,b}$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^5,x)
```

output
```
(32*(143*sin(a + b*x)^9 - 234*sin(a + b*x)^11 + 99*sin(a + b*x)^13))/(1287
*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 345, normalized size of antiderivative = 7.50

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx = \frac{-4096\cos(2bx+2a)\cos(bx+a)\sin(bx+a)-990\cos(2bx+2a)\sin(2bx+2a)^4\sin(bx+a)^3+60\cos}{}$$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x)
```

output
```
( - 4096*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x) - 990*cos(2*a + 2*b*x)
*sin(2*a + 2*b*x)**4*sin(a + b*x)**3 + 60*cos(2*a + 2*b*x)*sin(2*a + 2*b*x
)**4*sin(a + b*x) - 1760*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x)
**3 + 384*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x) + 2816*cos(2*a
 + 2*b*x)*sin(a + b*x)**3 - 4608*cos(2*a + 2*b*x)*sin(a + b*x) + 297*cos(a
 + b*x)*sin(2*a + 2*b*x)**5*sin(a + b*x)**2 - 6*cos(a + b*x)*sin(2*a + 2*b
*x)**5 + 880*cos(a + b*x)*sin(2*a + 2*b*x)**3*sin(a + b*x)**2 - 64*cos(a +
 b*x)*sin(2*a + 2*b*x)**3 - 4224*cos(a + b*x)*sin(2*a + 2*b*x)*sin(a + b*x
)**2 + 2304*cos(a + b*x)*sin(2*a + 2*b*x) - 4096*sin(2*a + 2*b*x)*sin(a +
b*x)**2 + 2048*sin(2*a + 2*b*x))/(9009*b)
```

3.434 $\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx = -\frac{16\cos^5(a+bx)}{5b}+\frac{48\cos^7(a+bx)}{7b}-\frac{16\cos^9(a+bx)}{3b}+\frac{16\cos^{11}(a+bx)}{11b}$$

output
```
-16/5*cos(b*x+a)^5/b+48/7*cos(b*x+a)^7/b-16/3*cos(b*x+a)^9/b+16/11*cos(b*x
+a)^11/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx = \frac{\cos^5(a+bx)(-3042+3335\cos(2(a+bx))-910\cos(4(a+bx))+105\cos(6(a+bx)))}{2310b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

output
```
(Cos[a + b*x]^5*(-3042 + 3335*Cos[2*(a + b*x)] - 910*Cos[4*(a + b*x)] + 10
5*Cos[6*(a + b*x)]))/(2310*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(a+bx)^3\sin(2a+2bx)^4dx$$

$$\Big\downarrow \; 4776$$

$$16\int \cos^4(a+bx)\sin^7(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$16\int \cos(a+bx)^4\sin(a+bx)^7dx$$

$$\Big\downarrow \; 3045$$

$$-\frac{16\int \cos^4(a+bx)\left(1-\cos^2(a+bx)\right)^3d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 244$$

$$-\frac{16\int \left(-\cos^{10}(a+bx)+3\cos^8(a+bx)-3\cos^6(a+bx)+\cos^4(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{16\left(-\frac{1}{11}\cos^{11}(a+bx)+\frac{1}{3}\cos^9(a+bx)-\frac{3}{7}\cos^7(a+bx)+\frac{1}{5}\cos^5(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

output
```
(-16*(Cos[a + b*x]^5/5 - (3*Cos[a + b*x]^7)/7 + Cos[a + b*x]^9/3 - Cos[a +
 b*x]^11/11))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 12.53 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

method	result
parallelrisch	$\frac{-32768-165\cos(7bx+7a)-385\cos(9bx+9a)+105\cos(11bx+11a)-2310\cos(3bx+3a)-16170\cos(bx+a)+2541\cos(5bx+5a)}{73920b}$
default	$-\frac{7\cos(bx+a)}{32b}-\frac{\cos(3bx+3a)}{32b}+\frac{11\cos(5bx+5a)}{320b}-\frac{\cos(7bx+7a)}{448b}-\frac{\cos(9bx+9a)}{192b}+\frac{\cos(11bx+11a)}{704b}$
risch	$-\frac{7\cos(bx+a)}{32b}-\frac{\cos(3bx+3a)}{32b}+\frac{11\cos(5bx+5a)}{320b}-\frac{\cos(7bx+7a)}{448b}-\frac{\cos(9bx+9a)}{192b}+\frac{\cos(11bx+11a)}{704b}$
orering	Expression too large to display

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/73920*(-32768-165*cos(7*b*x+7*a)-385*cos(9*b*x+9*a)+105*cos(11*b*x+11*a)
-2310*cos(3*b*x+3*a)-16170*cos(b*x+a)+2541*cos(5*b*x+5*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx = \frac{16\left(105\cos(bx+a)^{11}-385\cos(bx+a)^{9}+495\cos(bx+a)^{7}-231\cos(bx+a)^{5}\right)}{1155\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
16/1155*(105*cos(b*x + a)^11 - 385*cos(b*x + a)^9 + 495*cos(b*x + a)^7 - 2
31*cos(b*x + a)^5)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(53) = 106$.

Time = 11.70 (sec) , antiderivative size = 366, normalized size of antiderivative = 6.00

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{472\sin^3(a+bx)\sin^3(2a+2bx)\cos(2a+2bx)}{1155b} - \frac{64\sin^3(a+bx)\sin(2a+2bx)\cos^3(2a+2bx)}{231b} - \frac{211\sin^2(a+bx)\sin^4(2a+2bx)\cos(a+bx)}{1155b} \\ x\sin^3(a)\sin^4(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

output
```
Piecewise((-472*sin(a + b*x)**3*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)/(1155
*b) - 64*sin(a + b*x)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(231*b) - 21
1*sin(a + b*x)**2*sin(2*a + 2*b*x)**4*cos(a + b*x)/(1155*b) - 304*sin(a +
b*x)**2*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(385*b) - 128
*sin(a + b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**4/(231*b) + 272*sin(a + b*
x)*sin(2*a + 2*b*x)**3*cos(a + b*x)**2*cos(2*a + 2*b*x)/(1155*b) + 256*sin
(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**3/(1155*b) -
46*sin(2*a + 2*b*x)**4*cos(a + b*x)**3/(165*b) - 192*sin(2*a + 2*b*x)**2*c
os(a + b*x)**3*cos(2*a + 2*b*x)**2/(385*b) - 256*cos(a + b*x)**3*cos(2*a +
 2*b*x)**4/(1155*b), Ne(b, 0)), (x*sin(a)**3*sin(2*a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{105\cos(11\,bx+11\,a) - 385\cos(9\,bx+9\,a) - 165\cos(7\,bx+7\,a) + 2541\cos(5\,bx+5\,a) - 2310\cos(3}{73920\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/73920*(105*cos(11*b*x + 11*a) - 385*cos(9*b*x + 9*a) - 165*cos(7*b*x + 7
*a) + 2541*cos(5*b*x + 5*a) - 2310*cos(3*b*x + 3*a) - 16170*cos(b*x + a))/
b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{105\cos(11bx+11a) - 385\cos(9bx+9a) - 165\cos(7bx+7a) + 2541\cos(5bx+5a) - 2310\cos(3}{73920b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/73920*(105*cos(11*b*x + 11*a) - 385*cos(9*b*x + 9*a) - 165*cos(7*b*x + 7
*a) + 2541*cos(5*b*x + 5*a) - 2310*cos(3*b*x + 3*a) - 16170*cos(b*x + a))/
b
```

Mupad [B] (verification not implemented)

Time = 16.74 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$= -\frac{-\frac{16\cos(a+bx)^{11}}{11} + \frac{16\cos(a+bx)^9}{3} - \frac{48\cos(a+bx)^7}{7} + \frac{16\cos(a+bx)^5}{5}}{b}$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^4,x)
```

output
```
-((16*cos(a + b*x)^5)/5 - (48*cos(a + b*x)^7)/7 + (16*cos(a + b*x)^9)/3 -
(16*cos(a + b*x)^11)/11)/b
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 288, normalized size of antiderivative = 4.72

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{-1512\cos(2bx+2a)\sin(2bx+2a)^3\sin(bx+a)^3+144\cos(2bx+2a)\sin(2bx+2a)^3\sin(bx+a)-51}{}$$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x)
```

output
```
( - 1512*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**3*sin(a + b*x)**3 + 144*cos(2*
a + 2*b*x)*sin(2*a + 2*b*x)**3*sin(a + b*x) - 5184*cos(2*a + 2*b*x)*sin(2*
a + 2*b*x)*sin(a + b*x)**3 + 2304*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)*sin(a
+ b*x) - 2304*cos(2*a + 2*b*x)*sin(a + b*x)**2 + 1152*cos(2*a + 2*b*x) + 5
67*cos(a + b*x)*sin(2*a + 2*b*x)**4*sin(a + b*x)**2 - 18*cos(a + b*x)*sin(
2*a + 2*b*x)**4 + 3888*cos(a + b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x)**2 -
576*cos(a + b*x)*sin(2*a + 2*b*x)**2 + 2304*cos(a + b*x)*sin(2*a + 2*b*x)*
sin(a + b*x) - 3456*cos(a + b*x)*sin(a + b*x)**2 - 2304*cos(a + b*x) - 328
0)/(10395*b)
```

3.435 $\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx=\frac{8\sin^7(a+bx)}{7b}-\frac{8\sin^9(a+bx)}{9b}$$

output
```
8/7*sin(b*x+a)^7/b-8/9*sin(b*x+a)^9/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx=\frac{4(11+7\cos(2(a+bx)))\sin^7(a+bx)}{63b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

output
```
(4*(11 + 7*Cos[2*(a + b*x)])*Sin[a + b*x]^7)/(63*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(a+bx)^3\sin(2a+2bx)^3dx$$

$$\Big\downarrow \ 4776$$

$$8\int \cos^3(a+bx)\sin^6(a+bx)dx$$

$$\Big\downarrow \ 3042$$

$$8\int \cos(a+bx)^3\sin(a+bx)^6dx$$

$$\Big\downarrow \ 3044$$

$$\frac{8\int \sin^6(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \ 244$$

$$\frac{8\int \left(\sin^6(a+bx)-\sin^8(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{8\left(\frac{1}{7}\sin^7(a+bx)-\frac{1}{9}\sin^9(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

output
```
(8*(Sin[a + b*x]^7/7 - Sin[a + b*x]^9/9))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 5.54 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77

method	result
default	$\frac{3\sin(bx+a)}{16b}-\frac{\sin(3bx+3a)}{12b}+\frac{3\sin(7bx+7a)}{224b}-\frac{\sin(9bx+9a)}{288b}$
risch	$\frac{3\sin(bx+a)}{16b}-\frac{\sin(3bx+3a)}{12b}+\frac{3\sin(7bx+7a)}{224b}-\frac{\sin(9bx+9a)}{288b}$
parallelrisch	$16\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{6}\tan(bx+a)^{3}+\left(-96\tan(bx+a)^{4}+96\tan(bx+a)^{2}\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{5}+\left(192\tan(bx+a)^{5}-720\tan(bx+a)^{3}+192\,\mathrm{t}\right.$
orering	$-\frac{4540\left(3\sin(bx+a)^{2}\sin(2bx+2a)^{3}b\cos(bx+a)+6\sin(bx+a)^{3}\sin(2bx+2a)^{2}b\cos(2bx+2a)\right)}{3969b^{2}}-\frac{754\left(-129\cos(bx+a)\sin(b\right.}{}$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/16*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b+3/224/b*sin(7*b*x+7*a)-1/288/b*sin
(9*b*x+9*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(7\cos\left(bx+a\right)^8-19\cos\left(bx+a\right)^6+15\cos\left(bx+a\right)^4-\cos\left(bx+a\right)^2-2\right)\sin\left(bx+a\right)}{63\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-8/63*(7*cos(b*x + a)^8 - 19*cos(b*x + a)^6 + 15*cos(b*x + a)^4 - cos(b*x
+ a)^2 - 2)*sin(b*x + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(26) = 52.

Time = 4.92 (sec) , antiderivative size = 284, normalized size of antiderivative = 9.16

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx = \begin{cases} -\frac{46\sin^3(a+bx)\sin^2(2a+2bx)\cos(2a+2bx)}{105b} - \frac{16\sin^3(a+bx)\cos^3(2a+2bx)}{63b} - \frac{13\sin^2(a+bx)\sin^3(2a+2bx)\cos(a+bx)}{105b} - \frac{8\sin^2(a+b \\ x\sin^3(a)\sin^3(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

output
```
Piecewise((-46*sin(a + b*x)**3*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)/(105*b
) - 16*sin(a + b*x)**3*cos(2*a + 2*b*x)**3/(63*b) - 13*sin(a + b*x)**2*sin
(2*a + 2*b*x)**3*cos(a + b*x)/(105*b) - 8*sin(a + b*x)**2*sin(2*a + 2*b*x)
*cos(a + b*x)*cos(2*a + 2*b*x)**2/(35*b) - 4*sin(a + b*x)*sin(2*a + 2*b*x)
**2*cos(a + b*x)**2*cos(2*a + 2*b*x)/(7*b) - 64*sin(a + b*x)*cos(a + b*x)*
*2*cos(2*a + 2*b*x)**3/(105*b) + 94*sin(2*a + 2*b*x)**3*cos(a + b*x)**3/(3
15*b) + 32*sin(2*a + 2*b*x)*cos(a + b*x)**3*cos(2*a + 2*b*x)**2/(105*b), N
e(b, 0)), (x*sin(a)**3*sin(2*a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx = -\frac{7\sin(9\,bx+9\,a)-27\sin(7\,bx+7\,a)+168\sin(3\,bx+3\,a)-378\sin(bx+a)}{2016\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
-1/2016*(7*sin(9*b*x + 9*a) - 27*sin(7*b*x + 7*a) + 168*sin(3*b*x + 3*a) -
 378*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(7\sin(bx+a)^9-9\sin(bx+a)^7\right)}{63\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-8/63*(7*sin(b*x + a)^9 - 9*sin(b*x + a)^7)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx=\frac{8\,\left(9\sin\left(a+b\,x\right)^7-7\sin\left(a+b\,x\right)^9\right)}{63\,b}$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^3,x)
```

output
```
(8*(9*sin(a + b*x)^7 - 7*sin(a + b*x)^9))/(63*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 241, normalized size of antiderivative = 7.77

$$\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx$$
$$=\frac{-144\cos\left(2bx+2a\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-70\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)^2\sin\left(bx+a\right)^3+12\cos\left(2\right.}{}$$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x)
```

output
```
( - 144*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x) - 70*cos(2*a + 2*b*x)*s
in(2*a + 2*b*x)**2*sin(a + b*x)**3 + 12*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)*
*2*sin(a + b*x) + 112*cos(2*a + 2*b*x)*sin(a + b*x)**3 - 192*cos(2*a + 2*b
*x)*sin(a + b*x) + 35*cos(a + b*x)*sin(2*a + 2*b*x)**3*sin(a + b*x)**2 - 2
*cos(a + b*x)*sin(2*a + 2*b*x)**3 - 168*cos(a + b*x)*sin(2*a + 2*b*x)*sin(
a + b*x)**2 + 96*cos(a + b*x)*sin(2*a + 2*b*x) - 144*sin(2*a + 2*b*x)*sin(
a + b*x)**2 + 72*sin(2*a + 2*b*x))/(315*b)
```

3.436 $\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx=-\frac{4\cos^3(a+bx)}{3b}+\frac{8\cos^5(a+bx)}{5b}-\frac{4\cos^7(a+bx)}{7b}$$

output
```
-4/3*cos(b*x+a)^3/b+8/5*cos(b*x+a)^5/b-4/7*cos(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx =\frac{\cos^3(a+bx)(-157+108\cos(2(a+bx))-15\cos(4(a+bx)))}{210b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
(Cos[a + b*x]^3*(-157 + 108*Cos[2*(a + b*x)] - 15*Cos[4*(a + b*x)]))/(210*
b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\sin(2a+2bx)^2dx$$

$$\Big\downarrow \text{4776}$$

$$4\int \cos^2(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \cos(a+bx)^2\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{4\int \cos^2(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{4\int \left(\cos^6(a+bx)-2\cos^4(a+bx)+\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4\left(\frac{1}{7}\cos^7(a+bx)-\frac{2}{5}\cos^5(a+bx)+\frac{1}{3}\cos^3(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
(-4*(Cos[a + b*x]^3/3 - (2*Cos[a + b*x]^5)/5 + Cos[a + b*x]^7/7))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

method	result
default	$-\frac{5\cos(bx+a)}{16b}-\frac{\cos(3bx+3a)}{48b}+\frac{3\cos(5bx+5a)}{80b}-\frac{\cos(7bx+7a)}{112b}$
risch	$-\frac{5\cos(bx+a)}{16b}-\frac{\cos(3bx+3a)}{48b}+\frac{3\cos(5bx+5a)}{80b}-\frac{\cos(7bx+7a)}{112b}$
parallelrisch	$\frac{24+\left(64\tan(bx+a)^4+176\tan(bx+a)^2+88\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6-192\tan(bx+a)^3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5+\left(384\tan(bx+a)^4-48\tan(bx+a)\right.}{105b\left(1+\tan\right.}$
orering	$-\frac{12916\left(3\sin(bx+a)^2\sin(2bx+2a)^2b\cos(bx+a)+4\sin(bx+a)^3\sin(2bx+2a)b\cos(2bx+2a)\right)}{11025b^2}-\frac{94\left(6b^3\cos(bx+a)^3\sin(2bx\right.}{}$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/16*cos(b*x+a)/b-1/48*cos(3*b*x+3*a)/b+3/80*cos(5*b*x+5*a)/b-1/112*cos(7
*b*x+7*a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(15\,\cos\left(bx+a\right)^7-42\,\cos\left(bx+a\right)^5+35\,\cos\left(bx+a\right)^3\right)}{105\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-4/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(39) = 78.

Time = 2.17 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.39

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx = \begin{cases} -\frac{12\sin^3(a+bx)\sin(2a+2bx)\cos(2a+2bx)}{35b} - \frac{11\sin^2(a+bx)\sin^2(2a+2bx)\cos(a+bx)}{35b} - \frac{24\sin^2(a+bx)\cos(a+bx)\cos^2(2a+2bx)}{35b} + 8 \\ x\sin^3(a)\sin^2(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((-12*sin(a + b*x)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)/(35*b) -
11*sin(a + b*x)**2*sin(2*a + 2*b*x)**2*cos(a + b*x)/(35*b) - 24*sin(a + b*
x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(35*b) + 8*sin(a + b*x)*sin(2*a + 2
*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)/(35*b) - 38*sin(2*a + 2*b*x)**2*cos
(a + b*x)**3/(105*b) - 32*cos(a + b*x)**3*cos(2*a + 2*b*x)**2/(105*b), Ne(
b, 0)), (x*sin(a)**3*sin(2*a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{15\cos(7bx+7a)-63\cos(5bx+5a)+35\cos(3bx+3a)+525\cos(bx+a)}{1680b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
-1/1680*(15*cos(7*b*x + 7*a) - 63*cos(5*b*x + 5*a) + 35*cos(3*b*x + 3*a) +
 525*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{15\cos(7bx+7a)-63\cos(5bx+5a)+35\cos(3bx+3a)+525\cos(bx+a)}{1680b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-1/1680*(15*cos(7*b*x + 7*a) - 63*cos(5*b*x + 5*a) + 35*cos(3*b*x + 3*a) +
 525*cos(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 17.71 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx$$
$$= -\frac{4\left(15\cos\left(a+b\,x\right)^7-42\cos\left(a+b\,x\right)^5+35\cos\left(a+b\,x\right)^3\right)}{105\,b}$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^2,x)
```

output
```
-(4*(35*cos(a + b*x)^3 - 42*cos(a + b*x)^5 + 15*cos(a + b*x)^7))/(105*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 184, normalized size of antiderivative = 4.00

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx$$
$$= \frac{-60\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)\sin\left(bx+a\right)^3+24\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)\sin\left(bx+a\right)-24\cos\left(2\right.}{}$$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x)
```

output
```
( - 60*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)*sin(a + b*x)**3 + 24*cos(2*a + 2*
b*x)*sin(2*a + 2*b*x)*sin(a + b*x) - 24*cos(2*a + 2*b*x)*sin(a + b*x)**2 +
 12*cos(2*a + 2*b*x) + 45*cos(a + b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x)**2
 - 6*cos(a + b*x)*sin(2*a + 2*b*x)**2 + 24*cos(a + b*x)*sin(2*a + 2*b*x)*s
in(a + b*x) - 40*cos(a + b*x)*sin(a + b*x)**2 - 32*cos(a + b*x) - 52)/(105
*b)
```

3.437 $\int \sin^3(a+bx)\sin(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx=\frac{2\sin^5(a+bx)}{5b}$$

output
```
2/5*sin(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx=\frac{2\sin^5(a+bx)}{5b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

output
```
(2*Sin[a + b*x]^5)/(5*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4776, 3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\sin(2a+2bx)dx$$

$$\Big\downarrow \text{4776}$$

$$2\int \cos(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \cos(a+bx)\sin(a+bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{2\int \sin^4(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2\sin^5(a+bx)}{5b}$$

input
```
Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

output
```
(2*Sin[a + b*x]^5)/(5*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(13) = 26$.

Time = 1.68 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.73

method	result
default	$\frac{\sin(bx+a)}{4b}-\frac{\sin(3bx+3a)}{8b}+\frac{\sin(5bx+5a)}{40b}$
risch	$\frac{\sin(bx+a)}{4b}-\frac{\sin(3bx+3a)}{8b}+\frac{\sin(5bx+5a)}{40b}$
parallelrisch	$\frac{\frac{8\tan(bx+a)}{5}-\frac{16\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{5}+\frac{16\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan(bx+a)^{2}}{5}-8\tan(bx+a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}}{b\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}\right)^{3}\left(\tan(bx+a)^{2}+1\right)}$
orering	$-\frac{259\left(3\sin(bx+a)^{2}\sin(2bx+2a)b\cos(bx+a)+2\sin(bx+a)^{3}b\cos(2bx+2a)\right)}{225b^{2}}-\frac{7\left(6b^{3}\cos(bx+a)^{3}\sin(2bx+2a)+36\sin(bx+\right.}{}$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(b*x+a)/b-1/8*sin(3*b*x+3*a)/b+1/40/b*sin(5*b*x+5*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(13) = 26$.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx = \frac{2\left(\cos\left(bx+a\right)^4 - 2\cos\left(bx+a\right)^2 + 1\right)\sin\left(bx+a\right)}{5\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
2/5*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sin(b*x + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(12) = 24$.

Time = 0.83 (sec) , antiderivative size = 117, normalized size of antiderivative = 7.80

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{2\sin^3(a+bx)\cos(2a+2bx)}{5b} - \frac{\sin^2(a+bx)\sin(2a+2bx)\cos(a+bx)}{5b} - \frac{4\sin(a+bx)\cos^2(a+bx)\cos(2a+2bx)}{5b} + \frac{2\sin(2a+2bx)\cos^3(a+}{5b} \\ x\sin^3(a)\sin(2a) \end{cases}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a),x)
```

output
```
Piecewise((-2*sin(a + b*x)**3*cos(2*a + 2*b*x)/(5*b) - sin(a + b*x)**2*sin
(2*a + 2*b*x)*cos(a + b*x)/(5*b) - 4*sin(a + b*x)*cos(a + b*x)**2*cos(2*a
+ 2*b*x)/(5*b) + 2*sin(2*a + 2*b*x)*cos(a + b*x)**3/(5*b), Ne(b, 0)), (x*s
in(a)**3*sin(2*a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx = \frac{\sin(5\,bx+5\,a) - 5\,\sin(3\,bx+3\,a) + 10\,\sin(bx+a)}{40\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
1/40*(sin(5*b*x + 5*a) - 5*sin(3*b*x + 3*a) + 10*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx = \frac{2\,\sin(bx+a)^5}{5\,b}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
2/5*sin(b*x + a)^5/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx = \frac{2\sin(a+b\,x)^5}{5\,b}$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x),x)
```

output
```
(2*sin(a + b*x)^5)/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 9.13

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx$$
$$= \frac{-4\cos(2bx+2a)\cos(bx+a)\sin(bx+a)+2\cos(2bx+2a)\sin(bx+a)^3-4\cos(2bx+2a)\sin(bx+a}{}$$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a),x)
```

output
```
( - 4*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x) + 2*cos(2*a + 2*b*x)*sin(
a + b*x)**3 - 4*cos(2*a + 2*b*x)*sin(a + b*x) - 3*cos(a + b*x)*sin(2*a + 2
*b*x)*sin(a + b*x)**2 + 2*cos(a + b*x)*sin(2*a + 2*b*x) - 4*sin(2*a + 2*b*
x)*sin(a + b*x)**2 + 2*sin(2*a + 2*b*x))/(5*b)
```

3.438 $\int \csc(2a+2bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{2b} - \frac{\sin(a+bx)}{2b}$$

output
```
1/2*arctanh(sin(b*x+a))/b-1/2*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = \frac{1}{2}\left(\frac{\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{\sin(a+bx)}{b}\right)$$

input
```
Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x]^3,x]
```

output
```
(ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b)/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{2}\int \sin(a+bx)\tan(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(a+bx)\tan(a+bx)dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{\sin^2(a+bx)}{1-\sin^2(a+bx)}d\sin(a+bx)}{2b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\int \frac{1}{1-\sin^2(a+bx)}d\sin(a+bx)-\sin(a+bx)}{2b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}(\sin(a+bx))-\sin(a+bx)}{2b}$$

input
```
Int[Csc[2*a + 2*b*x]*Sin[a + b*x]^3,x]
```

output
```
(ArcTanh[Sin[a + b*x]] - Sin[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{-\sin(bx+a)+\ln(\sec(bx+a)+\tan(bx+a))}{2b}$	29
risch	$\frac{i\,\mathrm{e}^{i(bx+a)}}{4b}-\frac{i\,\mathrm{e}^{-i(bx+a)}}{4b}-\frac{\ln(\mathrm{e}^{i(bx+a)}-i)}{2b}+\frac{\ln(\mathrm{e}^{i(bx+a)}+i)}{2b}$	68

input
```
int(csc(2*b*x+2*a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(-sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = \frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)-2\,\sin\left(bx+a\right)}{4\,b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1) - 2*sin(b*x + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(24) = 48$.

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 4.43

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = -\frac{\log\left(\frac{\cos(bx+2a)^2+\cos(a)^2-2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2+2\cos(bx+2a)\sin(a)+\sin(a)^2}{\cos(bx+2a)^2+\cos(a)^2+2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2-2\cos(bx+2a)\sin(a)+\sin(a)^2}\right)+2\sin(bx+a)}{4b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x
 + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)
^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a)
+ sin(a)^2)) + 2*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = \frac{\log(\sin(bx+a)+1)-\log(-\sin(bx+a)+1)-2\sin(bx+a)}{4b}$$

input
```
integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1) - 2*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 17.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = -\frac{\frac{\sin(a+b\,x)}{2}-\frac{\mathrm{atanh}(\sin(a+b\,x))}{2}}{b}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x),x)
```

output
```
-(sin(a + b*x)/2 - atanh(sin(a + b*x))/2)/b
```

Reduce [F]

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = \int \csc(2bx+2a)\sin(bx+a)^3\,dx$$

input
```
int(csc(2*b*x+2*a)*sin(b*x+a)^3,x)
```

output
```
int(csc(2*a + 2*b*x)*sin(a + b*x)**3,x)
```

3.439 $\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx=\frac{\sec(a+bx)}{4b}$$

output
```
1/4*sec(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx=\frac{\sec(a+bx)}{4b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^3,x]
```

output
```
Sec[a + b*x]/(4*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4776, 3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{ 4776}$$

$$\frac{1}{4}\int \sec(a+bx)\tan(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{4}\int \sec(a+bx)\tan(a+bx)dx$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{\int 1d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sec(a+bx)}{4b}$$

input
```
Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^3,x]
```

output
```
Sec[a + b*x]/(4*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{1}{4\cos(bx+a)b}$	14
risch	$\frac{e^{i(bx+a)}}{2b(e^{2i(bx+a)}+1)}$	28

input
```
int(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/cos(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx = \frac{1}{4\,b\cos\left(bx+a\right)}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4/(b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**2*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(11) = 22.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 6.38

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx = \frac{\cos\left(2\,bx+2\,a\right)\cos\left(bx+a\right)+\sin\left(2\,bx+2\,a\right)\sin\left(bx+a\right)+\cos\left(bx+a\right)}{2\left(b\cos\left(2\,bx+2\,a\right)^2+b\sin\left(2\,bx+2\,a\right)^2+2\,b\cos\left(2\,bx+2\,a\right)+b\right)}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b
*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 + 2*b*cos(2*b*x + 2*
a) + b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.15

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx = \frac{1}{2\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)}$$

input
```
integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx = \frac{1}{4\,b\,\cos\left(a+b\,x\right)}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^2,x)
```

output
```
1/(4*b*cos(a + b*x))
```

Reduce [F]

$$\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx = \int \csc(2bx+2a)^2\sin(bx+a)^3\,dx$$

input
```
int(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x)
```

output
```
int(csc(2*a + 2*b*x)**2*sin(a + b*x)**3,x)
```

3.440 $\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx=\frac{\text{arctanh}(\sin(a+bx))}{16b}+\frac{\sec(a+bx)\tan(a+bx)}{16b}$$

output
```
1/16*arctanh(sin(b*x+a))/b+1/16*sec(b*x+a)*tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx=\frac{1}{8}\left(\frac{\text{arctanh}(\sin(a+bx))}{2b}+\frac{\sec(a+bx)\tan(a+bx)}{2b}\right)$$

input
```
Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b))/8
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{8}\int \sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{8}\left(\frac{1}{2}\int \sec(a+bx)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{1}{2}\int \csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{8}\left(\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)$$

input
```
Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b))/8
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\frac{\sec(bx+a)\tan(bx+a)}{2}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2}}{8b}$	37
risch	$-\frac{i\left(\mathrm{e}^{3i(bx+a)}-\mathrm{e}^{i(bx+a)}\right)}{8b\left(\mathrm{e}^{2i(bx+a)}+1\right)^{2}}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{16b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\right)}{16b}$	78

input
```
int(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(1/2*sec(b*x+a)*tan(b*x+a)+1/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \csc^3(2a + 2bx)\sin^3(a + bx)\,dx = \frac{\cos(bx + a)^2\log(\sin(bx + a) + 1) - \cos(bx + a)^2\log(-\sin(bx + a) + 1) + 2\sin(bx + a)}{32\,b\cos(bx + a)^2}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/32*(cos(b*x + a)^2*log(sin(b*x + a) + 1) - cos(b*x + a)^2*log(-sin(b*x +
 a) + 1) + 2*sin(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(2a + 2bx)\sin^3(a + bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**3*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(30) = 60$.

Time = 0.26 (sec) , antiderivative size = 480, normalized size of antiderivative = 14.12

$$\int \csc^3(2a + 2bx)\sin^3(a + bx)\,dx = \frac{4(\sin(3bx + 3a) - \sin(bx + a))\cos(4bx + 4a) - (2(2\cos(2bx + 2a) + 1)\cos(4bx + 4a) + \cos(4bx}{}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/32*(4*(sin(3*b*x + 3*a) - sin(b*x + a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b
*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^
2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x
 + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*c
os(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)
^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a
)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(cos(3*b*x + 3*a) - cos(b*x
 + a))*sin(4*b*x + 4*a) + 4*(2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a) - 8*
cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + 8*cos(b*x + a)*sin(2*b*x + 2*a) - 8*co
s(2*b*x + 2*a)*sin(b*x + a) - 4*sin(b*x + a))/(b*cos(4*b*x + 4*a)^2 + 4*b*
cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(2*b*x + 2*a) + b)*cos(4*b*x
+ 4*a) + 4*b*cos(2*b*x + 2*a) + b)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx = -\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1} - \log(\sin(bx+a)+1) + \log(-\sin(bx+a)+1)}{32\,b}$$

input
```
integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/32*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(-
sin(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx=\frac{\operatorname{atanh}(\sin{(a+b\,x)})}{16\,b}-\frac{\sin{(a+b\,x)}}{16\,b\,\left({\sin{(a+b\,x)}}^2-1\right)}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^3,x)
```

output
```
atanh(sin(a + b*x))/(16*b) - sin(a + b*x)/(16*b*(sin(a + b*x)^2 - 1))
```

Reduce [F]

$$\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx=\int \csc{(2bx+2a)}^3\sin{(bx+a)}^3\,dx$$

input
```
int(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x)
```

output
```
int(csc(2*a + 2*b*x)**3*sin(a + b*x)**3,x)
```

3.441 $\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{16b}+\frac{\sec(a+bx)}{16b}+\frac{\sec^3(a+bx)}{48b}$$

output
```
-1/16*arctanh(cos(b*x+a))/b+1/16*sec(b*x+a)/b+1/48*sec(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx=\frac{1}{16}\left(-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}\right)$$

input
```
Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^3,x]
```

output
```
(-(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b + S
ec[a + b*x]^3/(3*b))/16
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4776, 3042, 3102, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{16}\int \csc(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\int \left(-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\text{arctanh}(\sec(a+bx))+\frac{1}{3}\sec^3(a+bx)+\sec(a+bx)}{16b}$$

input
```
Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^3,x]
```

output
```
(-ArcTanh[Sec[a + b*x]] + Sec[a + b*x] + Sec[a + b*x]^3/3)/(16*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 3.71 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\frac{1}{3\cos(bx+a)^3}+\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{16b}$	41
risch	$\frac{3\,e^{5i(bx+a)}+10\,e^{3i(bx+a)}+3\,e^{i(bx+a)}}{24b\left(e^{2i(bx+a)}+1\right)^3}-\frac{\ln\left(e^{i(bx+a)}+1\right)}{16b}+\frac{\ln\left(e^{i(bx+a)}-1\right)}{16b}$	88

input
```
int(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16/b*(1/3/cos(b*x+a)^3+1/cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx = -\frac{3\cos(bx+a)^3\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-3\cos(bx+a)^3\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-6\cos(bx+a)^2}{96\,b\cos(bx+a)^3}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/96*(3*cos(b*x + a)^3*log(1/2*cos(b*x + a) + 1/2) - 3*cos(b*x + a)^3*log
(-1/2*cos(b*x + a) + 1/2) - 6*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(2a + 2bx) \sin^3(a + bx) \, dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**4*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 987 vs. $2(37) = 74$.

Time $= 0.15$ (sec) , antiderivative size $= 987$, normalized size of antiderivative $= 22.95$

$$\int \csc^4(2a + 2bx) \sin^3(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/96*(4*(3*cos(5*b*x + 5*a) + 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*
b*x + 6*a) + 12*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x +
5*a) + 12*(10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(4*b*x + 4*a) + 40*(3*
cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) + 36*cos(2*b*x + 2*a)*cos(b*x + a)
- 3*(2*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) + co
s(6*b*x + 6*a)^2 + 6*(3*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 9*cos(4*b
*x + 4*a)^2 + 9*cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a
))*sin(6*b*x + 6*a) + sin(6*b*x + 6*a)^2 + 9*sin(4*b*x + 4*a)^2 + 18*sin(4
*b*x + 4*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) +
 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x
)*sin(a) + sin(a)^2) + 3*(2*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*
cos(6*b*x + 6*a) + cos(6*b*x + 6*a)^2 + 6*(3*cos(2*b*x + 2*a) + 1)*cos(4*b
*x + 4*a) + 9*cos(4*b*x + 4*a)^2 + 9*cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4
*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + sin(6*b*x + 6*a)^2 + 9*sin(4*b*
x + 4*a)^2 + 18*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2 +
 6*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + s
in(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(5*b*x + 5*a) + 10*sin
(3*b*x + 3*a) + 3*sin(b*x + a))*sin(6*b*x + 6*a) + 36*(sin(4*b*x + 4*a) +
sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 12*(10*sin(3*b*x + 3*a) + 3*sin(b*x +
 a))*sin(4*b*x + 4*a) + 120*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + 36*sin(...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(37) = 74$.

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.28

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx = \frac{\frac{8\left(\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+2\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}+3\log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{96\,b}$$

input
```
integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/96*(8*(3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 2)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3 +
3*log(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx = \frac{\frac{\cos(a+b\,x)^2}{16}+\frac{1}{48}}{b\cos\left(a+b\,x\right)^3}-\frac{\mathrm{atanh}(\cos\left(a+b\,x\right))}{16\,b}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^4,x)
```

output
```
(cos(a + b*x)^2/16 + 1/48)/(b*cos(a + b*x)^3) - atanh(cos(a + b*x))/(16*b)
```

Reduce [F]

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx = \int \csc\left(2bx+2a\right)^4\sin\left(bx+a\right)^3 dx$$

input
```
int(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x)
```

output
```
int(csc(2*a + 2*b*x)**4*sin(a + b*x)**3,x)
```

3.442 $\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = \frac{15\text{arctanh}(\sin(a+bx))}{256b} - \frac{\csc(a+bx)}{32b} + \frac{9\sec(a+bx)\tan(a+bx)}{256b} + \frac{\sec(a+bx)\tan^3(a+bx)}{128b}$$

output
```
15/256*arctanh(sin(b*x+a))/b-1/32*csc(b*x+a)/b+9/256*sec(b*x+a)*tan(b*x+a)
/b+1/128*sec(b*x+a)*tan(b*x+a)^3/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.43

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = -\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},\sin^2(a+bx)\right)}{32b}$$

input
```
Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^3,x]
```

output
```
-1/32*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 4776, 3042, 3101, 25, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{32}\int \csc^2(a+bx)\sec^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \csc(a+bx)^2\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^3}d\csc(a+bx)}{32b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^3}d\csc(a+bx)}{32b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{5}{4}\int \frac{\csc^4(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)-\frac{\csc^5(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \textbf{252}$$

$$-\frac{\frac{5}{4}\left(\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\int \frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)\right)-\frac{\csc^5(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \textbf{262}$$

$$-\frac{\frac{5}{4}\left(\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\csc^2(a+bx)}d\csc(a+bx)-\csc(a+bx)\right)\right)-\frac{\csc^5(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{\frac{5}{4}\left(\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\csc(a+bx))-\csc(a+bx))\right)-\frac{\csc^5(a+bx)}{4(1-\csc^2(a+bx))^2}}{32b}$$

input
```
Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^3,x]
```

output
```
-1/32*(-1/4*Csc[a + b*x]^5/(1 - Csc[a + b*x]^2)^2 + (5*((-3*(ArcTanh[Csc[a
 + b*x]] - Csc[a + b*x]))/2 + Csc[a + b*x]^3/(2*(1 - Csc[a + b*x]^2))))/4)
/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 8.44 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{\frac{1}{4\sin(bx+a)\cos(bx+a)^4}+\frac{5}{8\sin(bx+a)\cos(bx+a)^2}-\frac{15}{8\sin(bx+a)}+\frac{15\ln(\sec(bx+a)+\tan(bx+a))}{8}}{32b}$	69
risch	$-\frac{i\left(15\,\mathrm{e}^{9i(bx+a)}+40\,\mathrm{e}^{7i(bx+a)}+18\,\mathrm{e}^{5i(bx+a)}+40\,\mathrm{e}^{3i(bx+a)}+15\,\mathrm{e}^{i(bx+a)}\right)}{128b\left(\mathrm{e}^{2i(bx+a)}+1\right)^4\left(\mathrm{e}^{2i(bx+a)}-1\right)}+\frac{15\ln\left(\mathrm{e}^{i(bx+a)}+i\right)}{256b}-\frac{15\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{256b}$	126

input
```
int(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(1/4/sin(b*x+a)/cos(b*x+a)^4+5/8/sin(b*x+a)/cos(b*x+a)^2-15/8/sin(b
*x+a)+15/8*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.40

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx$$
$$= \frac{15\cos(bx+a)^4\log(\sin(bx+a)+1)\sin(bx+a) - 15\cos(bx+a)^4\log(-\sin(bx+a)+1)\sin(bx+a}{512\,b\cos(bx+a)^4\sin(bx+a)}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/512*(15*cos(b*x + a)^4*log(sin(b*x + a) + 1)*sin(b*x + a) - 15*cos(b*x +
 a)^4*log(-sin(b*x + a) + 1)*sin(b*x + a) - 30*cos(b*x + a)^4 + 10*cos(b*x
 + a)^2 + 4)/(b*cos(b*x + a)^4*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(2*b*x+2*a)**5*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1805 vs. 2(60) = 120.

Time = 0.44 (sec) , antiderivative size = 1805, normalized size of antiderivative = 26.54

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/512*(4*(15*sin(9*b*x + 9*a) + 40*sin(7*b*x + 7*a) + 18*sin(5*b*x + 5*a)
+ 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) - 60*(3*sin(8*
b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))
*cos(9*b*x + 9*a) + 12*(40*sin(7*b*x + 7*a) + 18*sin(5*b*x + 5*a) + 40*sin
(3*b*x + 3*a) + 15*sin(b*x + a))*cos(8*b*x + 8*a) - 160*(2*sin(6*b*x + 6*a
) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(18*sin(
5*b*x + 5*a) + 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(6*b*x + 6*a) + 7
2*(2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(8*sin(3
*b*x + 3*a) + 3*sin(b*x + a))*cos(4*b*x + 4*a) - 15*(2*(3*cos(8*b*x + 8*a)
 + 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) - 1)*cos(1
0*b*x + 10*a) + cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x
 + 4*a) - 3*cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + 9*cos(8*b*x + 8*a)^2
- 4*(2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) + 4*cos
(6*b*x + 6*a)^2 + 4*(3*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*cos(4*b*
x + 4*a)^2 + 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) + 2*sin(6*b*x +
6*a) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + sin(1
0*b*x + 10*a)^2 + 6*(2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x
 + 2*a))*sin(8*b*x + 8*a) + 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a) +
 3*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + 4*sin(6*b*x + 6*a)^2 + 4*sin(4*b*x
 + 4*a)^2 + 12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = -\frac{\frac{2\left(7\sin(bx+a)^3-9\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2}+\frac{16}{\sin(bx+a)}-15\log\left(\sin\left(bx+a\right)+1\right)+15\log\left(-\sin\left(bx+a\right)+1\right)}{512\,b}$$

input
```
integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/512*(2*(7*sin(b*x + a)^3 - 9*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 16/
sin(b*x + a) - 15*log(sin(b*x + a) + 1) + 15*log(-sin(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 18.45 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = \frac{15\,\mathrm{atanh}(\sin(a+b\,x))}{256\,b} - \frac{\frac{15\sin(a+b\,x)^4}{256}-\frac{25\sin(a+b\,x)^2}{256}+\frac{1}{32}}{b\,\left(\sin(a+b\,x)^5-2\sin(a+b\,x)^3+\sin(a+b\,x)\right)}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^5,x)
```

output
```
(15*atanh(sin(a + b*x)))/(256*b) - ((15*sin(a + b*x)^4)/256 - (25*sin(a +
b*x)^2)/256 + 1/32)/(b*(sin(a + b*x) - 2*sin(a + b*x)^3 + sin(a + b*x)^5))
```

Reduce [F]

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = \int \csc(2bx+2a)^5\sin(bx+a)^3\,dx$$

input
```
int(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x)
```

output
```
int(csc(2*a + 2*b*x)**5*sin(a + b*x)**3,x)
```

3.443 $\int \csc(a+bx)\sin^8(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = -\frac{256\cos^9(a+bx)}{9b}+\frac{768\cos^{11}(a+bx)}{11b}-\frac{768\cos^{13}(a+bx)}{13b}+\frac{256\cos^{15}(a+bx)}{15b}$$

output
```
-256/9*cos(b*x+a)^9/b+768/11*cos(b*x+a)^11/b-768/13*cos(b*x+a)^13/b+256/15
*cos(b*x+a)^15/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = -\frac{35\cos(a+bx)}{64b}-\frac{35\cos(3(a+bx))}{192b}+\frac{21\cos(5(a+bx))}{320b}+\frac{3\cos(7(a+bx))}{64b}-\frac{7\cos(9(a+bx))}{576b}-\frac{7\cos(11(a+bx))}{704b}+\frac{\cos(13(a+bx))}{832b}+\frac{\cos(15(a+bx))}{960b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^8,x]
```

output
```
(-35*Cos[a + b*x])/(64*b) - (35*Cos[3*(a + b*x)])/(192*b) + (21*Cos[5*(a +
 b*x)])/(320*b) + (3*Cos[7*(a + b*x)])/(64*b) - (7*Cos[9*(a + b*x)])/(576*
b) - (7*Cos[11*(a + b*x)])/(704*b) + Cos[13*(a + b*x)]/(832*b) + Cos[15*(a
 + b*x)]/(960*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^8}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$256\int \cos^8(a+bx)\sin^7(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$256\int \cos(a+bx)^8\sin(a+bx)^7dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{256\int \cos^8(a+bx)\left(1-\cos^2(a+bx)\right)^3d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{256\int \left(-\cos^{14}(a+bx)+3\cos^{12}(a+bx)-3\cos^{10}(a+bx)+\cos^8(a+bx)\right)d\cos(a+bx)}{b}$$

$$\downarrow \text{2009}$$

$$-\frac{256\left(-\frac{1}{15}\cos^{15}(a+bx)+\frac{3}{13}\cos^{13}(a+bx)-\frac{3}{11}\cos^{11}(a+bx)+\frac{1}{9}\cos^{9}(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^8,x]
```

output
```
(-256*(Cos[a + b*x]^9/9 - (3*Cos[a + b*x]^11)/11 + (3*Cos[a + b*x]^13)/13
- Cos[a + b*x]^15/15))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 13.58 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result
default	$\frac{\frac{256 \cos(bx+a)^{15}}{15}-\frac{768 \cos(bx+a)^{13}}{13}+\frac{768 \cos(bx+a)^{11}}{11}-\frac{256 \cos(bx+a)^{9}}{9}}{b}$
risch	$-\frac{35 \cos(bx+a)}{64b}+\frac{\cos(15bx+15a)}{960b}+\frac{\cos(13bx+13a)}{832b}-\frac{7 \cos(11bx+11a)}{704b}-\frac{7 \cos(9bx+9a)}{576b}+\frac{3 \cos(7bx+7a)}{64b}+\frac{21 \cos(5bx}{320b}$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^8,x,method=_RETURNVERBOSE)
```

output
```
256/b*(1/15*cos(b*x+a)^15-3/13*cos(b*x+a)^13+3/11*cos(b*x+a)^11-1/9*cos(b*
x+a)^9)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = \frac{256\left(429\cos\left(bx+a\right)^{15}-1485\cos\left(bx+a\right)^{13}+1755\cos\left(bx+a\right)^{11}-715\cos\left(bx+a\right)^{9}\right)}{6435\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

output
```
256/6435*(429*cos(b*x + a)^15 - 1485*cos(b*x + a)^13 + 1755*cos(b*x + a)^1
1 - 715*cos(b*x + a)^9)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = \frac{429 \cos(15\,bx+15\,a)+495 \cos(13\,bx+13\,a)-4095 \cos(11\,bx+11\,a)-5005 \cos(9\,bx+9\,a)+1930}{411840\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

output
```
1/411840*(429*cos(15*b*x + 15*a) + 495*cos(13*b*x + 13*a) - 4095*cos(11*b*
x + 11*a) - 5005*cos(9*b*x + 9*a) + 19305*cos(7*b*x + 7*a) + 27027*cos(5*b
*x + 5*a) - 75075*cos(3*b*x + 3*a) - 225225*cos(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(53) = 106.

Time = 0.17 (sec) , antiderivative size = 270, normalized size of antiderivative = 4.43

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = -\frac{8192\left(\frac{15(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{105(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{455(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{5070(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{30030(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{700}{(}\right.}{64}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

output
```
-8192/6435*(15*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 105*(cos(b*x + a) -
 1)^2/(cos(b*x + a) + 1)^2 + 455*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3
 + 5070*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 30030*(cos(b*x + a) -
1)^5/(cos(b*x + a) + 1)^5 + 70070*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^
6 + 115830*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 + 109395*(cos(b*x + a
) - 1)^8/(cos(b*x + a) + 1)^8 + 75075*(cos(b*x + a) - 1)^9/(cos(b*x + a) +
 1)^9 + 27027*(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 + 6435*(cos(b*x
+ a) - 1)^11/(cos(b*x + a) + 1)^11 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x +
a) + 1) - 1)^15)
```

Mupad [B] (verification not implemented)

Time = 18.72 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = -\frac{-\frac{256\cos(a+b\,x)^{15}}{15}+\frac{768\cos(a+b\,x)^{13}}{13}-\frac{768\cos(a+b\,x)^{11}}{11}+\frac{256\cos(a+b\,x)^{9}}{9}}{b}$$

input
```
int(sin(2*a + 2*b*x)^8/sin(a + b*x),x)
```

output
```
-((256*cos(a + b*x)^9)/9 - (768*cos(a + b*x)^11)/11 + (768*cos(a + b*x)^13
)/13 - (256*cos(a + b*x)^15)/15)/b
```

Reduce [F]

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = \int \csc(bx+a)\sin(2bx+2a)^8\,dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^8,x)
```

output
```
int(csc(a + b*x)*sin(2*a + 2*b*x)**8,x)
```

3.444 $\int \csc(a+bx)\sin^7(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx=\frac{128\sin^7(a+bx)}{7b}-\frac{128\sin^9(a+bx)}{3b}+\frac{384\sin^{11}(a+bx)}{11b}-\frac{128\sin^{13}(a+bx)}{13b}$$

output
```
128/7*sin(b*x+a)^7/b-128/3*sin(b*x+a)^9/b+384/11*sin(b*x+a)^11/b-128/13*si
n(b*x+a)^13/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx=\frac{128\sin^7(a+bx)}{7b}-\frac{128\sin^9(a+bx)}{3b}+\frac{384\sin^{11}(a+bx)}{11b}-\frac{128\sin^{13}(a+bx)}{13b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

output
```
(128*Sin[a + b*x]^7)/(7*b) - (128*Sin[a + b*x]^9)/(3*b) + (384*Sin[a + b*x
]^11)/(11*b) - (128*Sin[a + b*x]^13)/(13*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^7}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$128\int \cos^7(a+bx)\sin^6(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$128\int \cos(a+bx)^7\sin(a+bx)^6dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{128\int \sin^6(a+bx)\left(1-\sin^2(a+bx)\right)^3d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{128\int \left(-\sin^{12}(a+bx)+3\sin^{10}(a+bx)-3\sin^8(a+bx)+\sin^6(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{128\left(-\frac{1}{13}\sin^{13}(a+bx)+\frac{3}{11}\sin^{11}(a+bx)-\frac{1}{3}\sin^9(a+bx)+\frac{1}{7}\sin^7(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

output
```
(128*(Sin[a + b*x]^7/7 - Sin[a + b*x]^9/3 + (3*Sin[a + b*x]^11)/11 - Sin[a
 + b*x]^13/13))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 5.69 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result	si
default	$-\frac{128\left(\frac{\sin(bx+a)^{13}}{13}-\frac{3\sin(bx+a)^{11}}{11}+\frac{\sin(bx+a)^{9}}{3}-\frac{\sin(bx+a)^{7}}{7}\right)}{b}$	4
risch	$\frac{5\sin(bx+a)}{8b}-\frac{\sin(13bx+13a)}{416b}-\frac{\sin(11bx+11a)}{352b}+\frac{\sin(9bx+9a)}{48b}+\frac{3\sin(7bx+7a)}{112b}-\frac{3\sin(5bx+5a)}{32b}-\frac{5\sin(3bx+3a)}{32b}$	9

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

output
```
-128/b*(1/13*sin(b*x+a)^13-3/11*sin(b*x+a)^11+1/3*sin(b*x+a)^9-1/7*sin(b*x
+a)^7)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx =$$
$$-\frac{128\left(231\cos\left(bx+a\right)^{12}-567\cos\left(bx+a\right)^{10}+371\cos\left(bx+a\right)^{8}-5\cos\left(bx+a\right)^{6}-6\cos\left(bx+a\right)^{4}-\right.}{3003\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

output
```
-128/3003*(231*cos(b*x + a)^12 - 567*cos(b*x + a)^10 + 371*cos(b*x + a)^8
- 5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 16)*sin(b*x + a
)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx = -\frac{231\sin(13bx+13a)+273\sin(11bx+11a)-2002\sin(9bx+9a)-2574\sin(7bx+7a)+9009\sin}{96096\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

output
```
-1/96096*(231*sin(13*b*x + 13*a) + 273*sin(11*b*x + 11*a) - 2002*sin(9*b*x
 + 9*a) - 2574*sin(7*b*x + 7*a) + 9009*sin(5*b*x + 5*a) + 15015*sin(3*b*x
+ 3*a) - 60060*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx = -\frac{128\left(231\sin(bx+a)^{13}-819\sin(bx+a)^{11}+1001\sin(bx+a)^{9}-429\sin(bx+a)^{7}\right)}{3003\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

output
```
-128/3003*(231*sin(b*x + a)^13 - 819*sin(b*x + a)^11 + 1001*sin(b*x + a)^9
 - 429*sin(b*x + a)^7)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx = \frac{-\frac{128\sin(a+b\,x)^{13}}{13}+\frac{384\sin(a+b\,x)^{11}}{11}-\frac{128\sin(a+b\,x)^9}{3}+\frac{128\sin(a+b\,x)^7}{7}}{b}$$

input
```
int(sin(2*a + 2*b*x)^7/sin(a + b*x),x)
```

output
```
((128*sin(a + b*x)^7)/7 - (128*sin(a + b*x)^9)/3 + (384*sin(a + b*x)^11)/1
1 - (128*sin(a + b*x)^13)/13)/b
```

Reduce [F]

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx = \int \csc(bx+a)\sin(2bx+2a)^7\,dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^7,x)
```

output
```
int(csc(a + b*x)*sin(2*a + 2*b*x)**7,x)
```

3.445 $\int \csc(a+bx)\sin^6(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx = -\frac{64\cos^7(a+bx)}{7b}+\frac{128\cos^9(a+bx)}{9b}-\frac{64\cos^{11}(a+bx)}{11b}$$

output
```
-64/7*cos(b*x+a)^7/b+128/9*cos(b*x+a)^9/b-64/11*cos(b*x+a)^11/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.93

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx = -\frac{5\cos(a+bx)}{8b}-\frac{5\cos(3(a+bx))}{24b}+\frac{\cos(5(a+bx))}{16b}+\frac{5\cos(7(a+bx))}{112b}-\frac{\cos(9(a+bx))}{144b}-\frac{\cos(11(a+bx))}{176b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

output
```
(-5*Cos[a + b*x])/(8*b) - (5*Cos[3*(a + b*x)])/(24*b) + Cos[5*(a + b*x)]/(
16*b) + (5*Cos[7*(a + b*x)])/(112*b) - Cos[9*(a + b*x)]/(144*b) - Cos[11*(
a + b*x)]/(176*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^6}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$64\int \cos^6(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$64\int \cos(a+bx)^6\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{64\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{64\int \left(\cos^{10}(a+bx)-2\cos^8(a+bx)+\cos^6(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{64\left(\frac{1}{11}\cos^{11}(a+bx)-\frac{2}{9}\cos^9(a+bx)+\frac{1}{7}\cos^7(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

output
```
(-64*(Cos[a + b*x]^7/7 - (2*Cos[a + b*x]^9)/9 + Cos[a + b*x]^11/11))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{64\left(\frac{\cos(bx+a)^{11}}{11}-\frac{2\cos(bx+a)^{9}}{9}+\frac{\cos(bx+a)^{7}}{7}\right)}{b}$	37
risch	$-\frac{5\cos(bx+a)}{8b}-\frac{\cos(11bx+11a)}{176b}-\frac{\cos(9bx+9a)}{144b}+\frac{5\cos(7bx+7a)}{112b}+\frac{\cos(5bx+5a)}{16b}-\frac{5\cos(3bx+3a)}{24b}$	83

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

output
```
-64/b*(1/11*cos(b*x+a)^11-2/9*cos(b*x+a)^9+1/7*cos(b*x+a)^7)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx = -\frac{64\,\left(63\,\cos\left(bx+a\right)^{11}-154\,\cos\left(bx+a\right)^{9}+99\,\cos\left(bx+a\right)^{7}\right)}{693\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

output
```
-64/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx = -\frac{63\,\cos\left(11\,bx+11\,a\right)+77\,\cos\left(9\,bx+9\,a\right)-495\,\cos\left(7\,bx+7\,a\right)-693\,\cos\left(5\,bx+5\,a\right)+2310\,\cos\left(3\right.}{11088\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

output
```
-1/11088*(63*cos(11*b*x + 11*a) + 77*cos(9*b*x + 9*a) - 495*cos(7*b*x + 7*
a) - 693*cos(5*b*x + 5*a) + 2310*cos(3*b*x + 3*a) + 6930*cos(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(40) = 80$.

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.43

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx =$$

$$-\frac{1024\left(\frac{11\,(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{55\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - \frac{297\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} - \frac{1485\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} - \frac{2079\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5} - \frac{2541\,(}{(\cos}\right.}{693\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} - 1\right)^{11}}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

output
```
-1024/693*(11*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 55*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 - 297*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 -
 1485*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 2079*(cos(b*x + a) - 1)^
5/(cos(b*x + a) + 1)^5 - 2541*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 -
1155*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 462*(cos(b*x + a) - 1)^8/
(cos(b*x + a) + 1)^8 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^1
1)
```

Mupad [B] (verification not implemented)

Time = 17.71 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx = -\frac{64\left(63\cos\left(a+b\,x\right)^{11} - 154\cos\left(a+b\,x\right)^9 + 99\cos\left(a+b\,x\right)^7\right)}{693\,b}$$

input
```
int(sin(2*a + 2*b*x)^6/sin(a + b*x),x)
```

output
```
-(64*(99*cos(a + b*x)^7 - 154*cos(a + b*x)^9 + 63*cos(a + b*x)^11))/(693*b
)
```

Reduce [F]

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx = \int \csc(bx+a)\sin(2bx+2a)^6\,dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^6,x)
```

output
```
int(csc(a + b*x)*sin(2*a + 2*b*x)**6,x)
```

3.446 $\int \csc(a+bx)\sin^5(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx=\frac{32\sin^5(a+bx)}{5b}-\frac{64\sin^7(a+bx)}{7b}+\frac{32\sin^9(a+bx)}{9b}$$

output
```
32/5*sin(b*x+a)^5/b-64/7*sin(b*x+a)^7/b+32/9*sin(b*x+a)^9/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx=\frac{32\sin^5(a+bx)}{5b}-\frac{64\sin^7(a+bx)}{7b}+\frac{32\sin^9(a+bx)}{9b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

output
```
(32*Sin[a + b*x]^5)/(5*b) - (64*Sin[a + b*x]^7)/(7*b) + (32*Sin[a + b*x]^9
)/(9*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^5}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$32\int \cos^5(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$32\int \cos(a+bx)^5\sin(a+bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{32\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{32\int \left(\sin^8(a+bx)-2\sin^6(a+bx)+\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{32\left(\frac{1}{9}\sin^9(a+bx)-\frac{2}{7}\sin^7(a+bx)+\frac{1}{5}\sin^5(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

output
```
(32*(Sin[a + b*x]^5/5 - (2*Sin[a + b*x]^7)/7 + Sin[a + b*x]^9/9))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\frac{32 \sin(bx+a)^9}{9}-\frac{64 \sin(bx+a)^7}{7}+\frac{32 \sin(bx+a)^5}{5}}{b}$	37
risch	$\frac{3 \sin(bx+a)}{4b}+\frac{\sin(9bx+9a)}{72b}+\frac{\sin(7bx+7a)}{56b}-\frac{\sin(5bx+5a)}{10b}-\frac{\sin(3bx+3a)}{6b}$	69

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
32/b*(1/9*sin(b*x+a)^9-2/7*sin(b*x+a)^7+1/5*sin(b*x+a)^5)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(35\cos(bx+a)^8-50\cos(bx+a)^6+3\cos(bx+a)^4+4\cos(bx+a)^2+8\right)\sin(bx+a)}{315\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
32/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b
*x + a)^2 + 8)*sin(b*x + a)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = \frac{35\sin(9\,bx+9\,a)+45\sin(7\,bx+7\,a)-252\sin(5\,bx+5\,a)-420\sin(3\,bx+3\,a)+1890\sin(bx+a)}{2520\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/2520*(35*sin(9*b*x + 9*a) + 45*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) -
 420*sin(3*b*x + 3*a) + 1890*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(35\,\sin\left(bx+a\right)^9-90\,\sin\left(bx+a\right)^7+63\,\sin\left(bx+a\right)^5\right)}{315\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
32/315*(35*sin(b*x + a)^9 - 90*sin(b*x + a)^7 + 63*sin(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 17.62 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(35\sin\left(a+b\,x\right)^9-90\sin\left(a+b\,x\right)^7+63\sin\left(a+b\,x\right)^5\right)}{315\,b}$$

input
```
int(sin(2*a + 2*b*x)^5/sin(a + b*x),x)
```

output
```
(32*(63*sin(a + b*x)^5 - 90*sin(a + b*x)^7 + 35*sin(a + b*x)^9))/(315*b)
```

Reduce [F]

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = \int \csc(bx+a)\sin(2bx+2a)^5\,dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^5,x)
```

output
```
int(csc(a + b*x)*sin(2*a + 2*b*x)**5,x)
```

3.447 $\int \csc(a+bx)\sin^4(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx=-\frac{16\cos^5(a+bx)}{5b}+\frac{16\cos^7(a+bx)}{7b}$$

output
```
-16/5*cos(b*x+a)^5/b+16/7*cos(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx=-\frac{3\cos(a+bx)}{4b}-\frac{\cos(3(a+bx))}{4b}+\frac{\cos(5(a+bx))}{20b}+\frac{\cos(7(a+bx))}{28b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

output
```
(-3*Cos[a + b*x])/(4*b) - Cos[3*(a + b*x)]/(4*b) + Cos[5*(a + b*x)]/(20*b)
 + Cos[7*(a + b*x)]/(28*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^4}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$16\int \cos^4(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$16\int \cos(a+bx)^4\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{16\int \cos^4(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{16\int \left(\cos^4(a+bx)-\cos^6(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16\left(\frac{1}{5}\cos^5(a+bx)-\frac{1}{7}\cos^7(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

output
```
(-16*(Cos[a + b*x]^5/5 - Cos[a + b*x]^7/7))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\frac{16\cos(bx+a)^7}{7}-\frac{16\cos(bx+a)^5}{5}}{b}$	27
risch	$-\frac{3\cos(bx+a)}{4b}+\frac{\cos(7bx+7a)}{28b}+\frac{\cos(5bx+5a)}{20b}-\frac{\cos(3bx+3a)}{4b}$	55

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
16/b*(1/7*cos(b*x+a)^7-1/5*cos(b*x+a)^5)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = \frac{16\left(5\cos\left(bx+a\right)^7-7\cos\left(bx+a\right)^5\right)}{35\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
16/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = \frac{5\cos\left(7\,bx+7\,a\right)+7\cos\left(5\,bx+5\,a\right)-35\cos\left(3\,bx+3\,a\right)-105\cos\left(bx+a\right)}{140\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/140*(5*cos(7*b*x + 7*a) + 7*cos(5*b*x + 5*a) - 35*cos(3*b*x + 3*a) - 105
*cos(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(27) = 54$.

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.45

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = -\frac{64\left(\frac{7\,(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{14\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{70\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} + \frac{35\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} + \frac{35\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5} - 1\right)}{35\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} - 1\right)^7}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
-64/35*(7*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 14*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 70*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 35*(
cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 35*(cos(b*x + a) - 1)^5/(cos(b*
x + a) + 1)^5 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^7)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = -\frac{16\left(7\cos\left(a+b\,x\right)^5 - 5\cos\left(a+b\,x\right)^7\right)}{35\,b}$$

input
```
int(sin(2*a + 2*b*x)^4/sin(a + b*x),x)
```

output
```
-(16*(7*cos(a + b*x)^5 - 5*cos(a + b*x)^7))/(35*b)
```

Reduce [F]

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = \int \csc(bx+a)\sin(2bx+2a)^4\,dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^4,x)
```

output
```
int(csc(a + b*x)*sin(2*a + 2*b*x)**4,x)
```

3.448 $\int \csc(a+bx)\sin^3(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx=\frac{8\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}$$

output
```
8/3*sin(b*x+a)^3/b-8/5*sin(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx=\frac{8\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

output
```
(8*Sin[a + b*x]^3)/(3*b) - (8*Sin[a + b*x]^5)/(5*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^3}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$8\int \cos^3(a+bx)\sin^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)^3\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{8\int \sin^2(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{8\int \left(\sin^2(a+bx)-\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8\left(\frac{1}{3}\sin^3(a+bx)-\frac{1}{5}\sin^5(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

output
```
(8*(Sin[a + b*x]^3/3 - Sin[a + b*x]^5/5))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

method	result	size
risch	$\frac{\sin(bx+a)}{b}-\frac{\sin(5bx+5a)}{10b}-\frac{\sin(3bx+3a)}{6b}$	40
default	$\frac{-\frac{8\sin(bx+a)\cos(bx+a)^4}{5}+\frac{8\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{15}}{b}$	41

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
sin(b*x+a)/b-1/10/b*sin(5*b*x+5*a)-1/6*sin(3*b*x+3*a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(3\cos\left(bx+a\right)^4-\cos\left(bx+a\right)^2-2\right)\sin\left(bx+a\right)}{15\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-8/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = -\frac{3\sin\left(5\,bx+5\,a\right)+5\sin\left(3\,bx+3\,a\right)-30\sin\left(bx+a\right)}{30\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
-1/30*(3*sin(5*b*x + 5*a) + 5*sin(3*b*x + 3*a) - 30*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(3\sin\left(bx+a\right)^5-5\sin\left(bx+a\right)^3\right)}{15\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-8/15*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 18.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(5\sin\left(a+b\,x\right)^3-3\sin\left(a+b\,x\right)^5\right)}{15\,b}$$

input
```
int(sin(2*a + 2*b*x)^3/sin(a + b*x),x)
```

output
```
(8*(5*sin(a + b*x)^3 - 3*sin(a + b*x)^5))/(15*b)
```

Reduce [F]

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = \int \csc\left(bx+a\right)\sin\left(2bx+2a\right)^3dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^3,x)
```

output
```
int(csc(a + b*x)*sin(2*a + 2*b*x)**3,x)
```

3.449 $\int \csc(a+bx)\sin^2(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\cos^3(a+bx)}{3b}$$

output
```
-4/3*cos(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\cos^3(a+bx)}{3b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

output
```
(-4*Cos[a + b*x]^3)/(3*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4776, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^2}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$4\int \cos^2(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \cos(a+bx)^2\sin(a+bx)dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{4\int \cos^2(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{4\cos^3(a+bx)}{3b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

output
```
(-4*Cos[a + b*x]^3)/(3*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{4 \cos(bx+a)^{3}}{3b}$	14
risch	$-\frac{\cos(bx+a)}{b}-\frac{\cos(3bx+3a)}{3b}$	27

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/3*cos(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\cos(bx+a)^3}{3\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-4/3*cos(b*x + a)^3/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15030 vs. 2(14) = 28.

Time = 82.04 (sec) , antiderivative size = 104225, normalized size of antiderivative = 6948.33

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**2,x)
```

output
```
16*Piecewise((0, Eq(a, 0) & Eq(b, 0)), (-cos(b*x)**3/(3*b), Eq(a, 0)), (0,
 Eq(b, 0)), (12*log(tan(a/2) + tan(b*x/2))*tan(a/2)**6*tan(b*x/2)**6/(3*b*
tan(a/2)**8*tan(b*x/2)**6 + 9*b*tan(a/2)**8*tan(b*x/2)**4 + 9*b*tan(a/2)**
8*tan(b*x/2)**2 + 3*b*tan(a/2)**8 + 12*b*tan(a/2)**6*tan(b*x/2)**6 + 36*b*
tan(a/2)**6*tan(b*x/2)**4 + 36*b*tan(a/2)**6*tan(b*x/2)**2 + 12*b*tan(a/2)
**6 + 18*b*tan(a/2)**4*tan(b*x/2)**6 + 54*b*tan(a/2)**4*tan(b*x/2)**4 + 54
*b*tan(a/2)**4*tan(b*x/2)**2 + 18*b*tan(a/2)**4 + 12*b*tan(a/2)**2*tan(b*x
/2)**6 + 36*b*tan(a/2)**2*tan(b*x/2)**4 + 36*b*tan(a/2)**2*tan(b*x/2)**2 +
 12*b*tan(a/2)**2 + 3*b*tan(b*x/2)**6 + 9*b*tan(b*x/2)**4 + 9*b*tan(b*x/2)
**2 + 3*b) + 36*log(tan(a/2) + tan(b*x/2))*tan(a/2)**6*tan(b*x/2)**4/(3*b*
tan(a/2)**8*tan(b*x/2)**6 + 9*b*tan(a/2)**8*tan(b*x/2)**4 + 9*b*tan(a/2)**
8*tan(b*x/2)**2 + 3*b*tan(a/2)**8 + 12*b*tan(a/2)**6*tan(b*x/2)**6 + 36*b*
tan(a/2)**6*tan(b*x/2)**4 + 36*b*tan(a/2)**6*tan(b*x/2)**2 + 12*b*tan(a/2)
**6 + 18*b*tan(a/2)**4*tan(b*x/2)**6 + 54*b*tan(a/2)**4*tan(b*x/2)**4 + 54
*b*tan(a/2)**4*tan(b*x/2)**2 + 18*b*tan(a/2)**4 + 12*b*tan(a/2)**2*tan(b*x
/2)**6 + 36*b*tan(a/2)**2*tan(b*x/2)**4 + 36*b*tan(a/2)**2*tan(b*x/2)**2 +
 12*b*tan(a/2)**2 + 3*b*tan(b*x/2)**6 + 9*b*tan(b*x/2)**4 + 9*b*tan(b*x/2)
**2 + 3*b) + 36*log(tan(a/2) + tan(b*x/2))*tan(a/2)**6*tan(b*x/2)**2/(3*b*
tan(a/2)**8*tan(b*x/2)**6 + 9*b*tan(a/2)**8*tan(b*x/2)**4 + 9*b*tan(a/2)**
8*tan(b*x/2)**2 + 3*b*tan(a/2)**8 + 12*b*tan(a/2)**6*tan(b*x/2)**6 + 36...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = -\frac{\cos(3\,bx+3\,a)+3\,\cos(bx+a)}{3\,b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
-1/3*(cos(3*b*x + 3*a) + 3*cos(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(13) = 26$.

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 3.47

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = \frac{8\left(\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)}{3\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
8/3*(3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 1)/(b*((cos(b*x + a) -
1)/(cos(b*x + a) + 1) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\cos\left(a+b\,x\right)^3}{3\,b}$$

input
```
int(sin(2*a + 2*b*x)^2/sin(a + b*x),x)
```

output
```
-(4*cos(a + b*x)^3)/(3*b)
```

Reduce [F]

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = \int \csc\left(bx+a\right)\sin\left(2bx+2a\right)^2\,dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^2,x)
```

output
```
int(csc(a + b*x)*sin(2*a + 2*b*x)**2,x)
```

3.450 $\int \csc(a+bx)\sin(2a+2bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \csc(a+bx)\sin(2a+2bx)\,dx=\frac{2\sin(a+bx)}{b}$$

output
```
2*sin(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \csc(a+bx)\sin(2a+2bx)\,dx=2\left(\frac{\cos(bx)\sin(a)}{b}+\frac{\cos(a)\sin(bx)}{b}\right)$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x],x]
```

output
```
2*((Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4776, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$2\int \cos(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\frac{2\sin(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x],x]
```

output
```
(2*Sin[a + b*x])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{2\sin(bx+a)}{b}$	12
risch	$\frac{2\sin(bx+a)}{b}$	12

input
```
int(csc(b*x+a)*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
2*sin(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin(2a+2bx)\,dx = \frac{2\,\sin\left(bx+a\right)}{b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
2*sin(b*x + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. 2(8) = 16.

Time = 11.15 (sec) , antiderivative size = 3636, normalized size of antiderivative = 330.55

$$\int \csc(a+bx)\sin(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a),x)
```

output
```
4*Piecewise((x, Eq(a, 0) & Eq(b, 0)), (sin(b*x)/b, Eq(a, 0)), (0, Eq(b, 0)
), (2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**3*tan(b*x/2)**2/(b*tan(a/2)**4*
tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/
2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**3/(b
*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2
 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(a/2) + tan(b*x/2))*t
an(a/2)*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*t
an(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(
tan(a/2) + tan(b*x/2))*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)*
*4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b
) - 2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**3*tan(b*x/2)**2/(b*tan(a/2)**
4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(
a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**
3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2
)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - 1/tan(a
/2))*tan(a/2)*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 +
 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) +
2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*t
an(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2
)**2 + b) - 2*tan(a/2)**4*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*t...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin(2a+2bx)\,dx = \frac{2\,\sin{(bx+a)}}{b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
2*sin(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin(2a+2bx)\,dx = \frac{2\,\sin{(bx+a)}}{b}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
2*sin(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sin(2a+2bx)\,dx = \frac{2\,\sin{(a+b\,x)}}{b}$$

input
```
int(sin(2*a + 2*b*x)/sin(a + b*x),x)
```

output
```
(2*sin(a + b*x))/b
```

Reduce [F]

$$\int \csc(a+bx)\sin(2a+2bx)\,dx = \int \csc(bx+a)\sin(2bx+2a)\,dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a),x)
```

output
```
int(csc(a + b*x)*sin(2*a + 2*b*x),x)
```

3.451 $\int \csc(a+bx)\csc(2a+2bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 28

$$\int \csc(a+bx)\csc(2a+2bx)\,dx=\frac{\text{arctanh}(\sin(a+bx))}{2b}-\frac{\csc(a+bx)}{2b}$$

output
```
1/2*arctanh(sin(b*x+a))/b-1/2*csc(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \csc(a+bx)\csc(2a+2bx)\,dx = -\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(a+bx)\right)}{2b}$$

input
```
Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x],x]
```

output
```
-1/2*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 4776, 3042, 3101, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{ 4776}$$

$$\frac{1}{2}\int \csc^2(a+bx)\sec(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \csc(a+bx)^2\sec(a+bx)dx$$

$$\Big\downarrow \text{ 3101}$$

$$-\frac{\int -\frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{\csc(a+bx)-\int \frac{1}{1-\csc^2(a+bx)}d\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\csc(a+bx)-\operatorname{arctanh}(\csc(a+bx))}{2b}$$

input
```
Int[Csc[a + b*x]*Csc[2*a + 2*b*x],x]
```

output
```
-1/2*(-ArcTanh[Csc[a + b*x]] + Csc[a + b*x])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{-\frac{1}{\sin(bx+a)}+\ln(\sec(bx+a)+\tan(bx+a))}{2b}$	31
risch	$-\frac{i\,\mathrm{e}^{i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\right)}{2b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{2b}$	66

input
```
int(csc(b*x+a)*csc(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(24) = 48$.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \csc(a+bx)\csc(2a+2bx)\,dx = \frac{\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-2}{4\,b\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="fricas")
```

output
```
1/4*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
 a) - 2)/(b*sin(b*x + a))
```

Sympy [F]

$$\int \csc(a+bx)\csc(2a+2bx)\,dx=\int \csc\left(a+bx\right)\csc\left(2a+2bx\right)dx$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a),x)
```

output
```
Integral(csc(a + b*x)*csc(2*a + 2*b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(24) = 48$.

Time = 0.19 (sec) , antiderivative size = 233, normalized size of antiderivative = 8.32

$$\int \csc(a+bx)\csc(2a+2bx)\,dx =$$
$$-\frac{\left(\cos\left(2\,bx+2\,a\right)^2+\sin\left(2\,bx+2\,a\right)^2-2\,\cos\left(2\,bx+2\,a\right)+1\right)\log\left(\frac{\cos(bx+2\,a)^2+\cos(a)^2-2\,\cos(a)\sin(bx+2\,a)+\mathrm{si}}{\cos(bx+2\,a)^2+\cos(a)^2+2\,\cos(a)\sin(bx+2\,a)+\mathrm{si}}\right.}{4\left(b\cos\left(2\,bx+2\,a\right)^2+b\sin\right.}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/4*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*l
og((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)
^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*
cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a
)^2)) + 4*cos(b*x + a)*sin(2*b*x + 2*a) - 4*cos(2*b*x + 2*a)*sin(b*x + a)
+ 4*sin(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2
*b*x + 2*a) + b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \csc(a+bx)\csc(2a+2bx)\,dx$$
$$= -\frac{\frac{2}{\sin(bx+a)} - \log(\sin(bx+a)+1) + \log(-\sin(bx+a)+1)}{4\,b}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="giac")
```

output
```
-1/4*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 19.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \csc(a+bx)\csc(2a+2bx)\,dx = \frac{\operatorname{atanh}(\sin(a+b\,x))}{2\,b} - \frac{1}{2\,b\,\sin(a+b\,x)}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)),x)
```

output
```
atanh(sin(a + b*x))/(2*b) - 1/(2*b*sin(a + b*x))
```

Reduce [F]

$$\int \csc(a+bx)\csc(2a+2bx)\,dx = \int \csc(2bx+2a)\csc(bx+a)\,dx$$

input
```
int(csc(b*x+a)*csc(2*b*x+2*a),x)
```

output
```
int(csc(2*a + 2*b*x)*csc(a + b*x),x)
```

3.452 $\int \csc(a+bx)\csc^2(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = -\frac{3\text{arctanh}(\cos(a+bx))}{8b} - \frac{\cot(a+bx)\csc(a+bx)}{8b} + \frac{\sec(a+bx)}{4b}$$

output
```
-3/8*arctanh(cos(b*x+a))/b-1/8*cot(b*x+a)*csc(b*x+a)/b+1/4*sec(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(47) = 94.

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.04

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = \frac{\csc^4(a+bx)\left(2-6\cos(2(a+bx))+2\cos(3(a+bx))+3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-3\cos(3\cdots\right.}{8b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)\cdots\right.}$$

input
```
Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

output
```
(Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a
+ b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] +
 Cos[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(
8*b*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4776, 3042, 3102, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{4}\int \csc^3(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc(a+bx)^3\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int \frac{\sec^4(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\int \frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\sec^2(a+bx)}d\sec(a+bx)-\sec(a+bx)\right)}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}(\operatorname{arctanh}(\sec(a+bx))-\sec(a+bx))}{4b}$$

input
```
Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

output
```
((-3*(ArcTanh[Sec[a + b*x]] - Sec[a + b*x]))/2 + Sec[a + b*x]^3/(2*(1 - Se
c[a + b*x]^2)))/(4*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{-\frac{1}{2\sin(bx+a)^2\cos(bx+a)}+\frac{3}{2\cos(bx+a)}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{2}}{4b}$	53
risch	$\frac{3\,\mathrm{e}^{5i(bx+a)}-2\,\mathrm{e}^{3i(bx+a)}+3\,\mathrm{e}^{i(bx+a)}}{4b\left(\mathrm{e}^{2i(bx+a)}-1\right)^2\left(\mathrm{e}^{2i(bx+a)}+1\right)}+\frac{3\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{8b}-\frac{3\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{8b}$	101

input
```
int(csc(b*x+a)*csc(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/b*(-1/2/sin(b*x+a)^2/cos(b*x+a)+3/2/cos(b*x+a)+3/2*ln(csc(b*x+a)-cot(b
*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(41) = 82.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.04

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx$$
$$= \frac{6\cos(bx+a)^2 - 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) + 3\left(\cos(bx+a)^3 - \cos(bx+\right.}{16\left(b\cos(bx+a)^3 - b\cos(bx+a)\right)}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
1/16*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x
 + a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1
/2) - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [F]

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = \int \csc(a+bx)\csc^2(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)**2,x)
```

output
```
Integral(csc(a + b*x)*csc(2*a + 2*b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(41) = 82$.

Time = 0.07 (sec) , antiderivative size = 974, normalized size of antiderivative = 20.72

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
1/16*(4*(3*cos(5*b*x + 5*a) - 2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*b
*x + 6*a) - 12*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a)
+ 4*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 8*(cos(2*b*x
+ 2*a) - 1)*cos(3*b*x + 3*a) - 12*cos(2*b*x + 2*a)*cos(b*x + a) + 3*(2*(co
s(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)
^2 - 2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(
2*b*x + 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a)
- sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*co
s(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*
(2*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x
+ 6*a)^2 - 2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2
- cos(2*b*x + 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x +
 6*a) - sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2
- 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2
) + 4*(3*sin(5*b*x + 5*a) - 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(6*b*x
 + 6*a) - 12*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 4*(2
*sin(3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 8*sin(3*b*x + 3*a)*
sin(2*b*x + 2*a) - 12*sin(2*b*x + 2*a)*sin(b*x + a) + 12*cos(b*x + a))/...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(41) = 82$.

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.91

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = \frac{\frac{\frac{14(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+6\log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{32\,b}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
1/32*((14*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 + 1)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x
 + a) - 1)^2/(cos(b*x + a) + 1)^2) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1)
 + 6*log(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = -\frac{3\,\mathrm{atanh}(\cos(a+b\,x))}{8\,b} - \frac{\frac{3\cos(a+b\,x)^2}{8}-\frac{1}{4}}{b\,\left(\cos(a+b\,x)-\cos(a+b\,x)^3\right)}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^2),x)
```

output
```
- (3*atanh(cos(a + b*x)))/(8*b) - ((3*cos(a + b*x)^2)/8 - 1/4)/(b*(cos(a +
 b*x) - cos(a + b*x)^3))
```

Reduce [F]

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = \int \csc(2bx+2a)^2\csc(bx+a)\,dx$$

input
```
int(csc(b*x+a)*csc(2*b*x+2*a)^2,x)
```

output
```
int(csc(2*a + 2*b*x)**2*csc(a + b*x),x)
```

3.453 $\int \csc(a+bx)\csc^3(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx=\frac{5\text{arctanh}(\sin(a+bx))}{16b}-\frac{\csc(a+bx)}{4b}-\frac{\csc^3(a+bx)}{24b}+\frac{\sec(a+bx)\tan(a+bx)}{16b}$$

output
```
5/16*arctanh(sin(b*x+a))/b-1/4*csc(b*x+a)/b-1/24*csc(b*x+a)^3/b+1/16*sec(b
*x+a)*tan(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.50

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx =-\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},\sin^2(a+bx)\right)}{24b}$$

input
```
Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

output
```
-1/24*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3042, 4776, 3042, 3101, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{8}\int \csc^4(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)^4\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int \frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \left(-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))} - \frac{5}{2}\left(\operatorname{arctanh}(\csc(a+bx)) - \frac{1}{3}\csc^3(a+bx) - \csc(a+bx)\right)}{8b}$$

input
```
Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

output
```
-1/8*(Csc[a + b*x]^5/(2*(1 - Csc[a + b*x]^2)) - (5*(ArcTanh[Csc[a + b*x]]
- Csc[a + b*x] - Csc[a + b*x]^3/3))/2)/b
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{-\frac{1}{3\sin(bx+a)^3\cos(bx+a)^2}+\frac{5}{6\sin(bx+a)\cos(bx+a)^2}-\frac{5}{2\sin(bx+a)}+\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{2}}{8b}$	69
risch	$-\frac{i\left(15\,e^{9i(bx+a)}-20\,e^{7i(bx+a)}-22\,e^{5i(bx+a)}-20\,e^{3i(bx+a)}+15\,e^{i(bx+a)}\right)}{24b\left(e^{2i(bx+a)}-1\right)^3\left(e^{2i(bx+a)}+1\right)^2}-\frac{5\ln\left(e^{i(bx+a)}-i\right)}{16b}+\frac{5\ln\left(e^{i(bx+a)}+i\right)}{16b}$	126

input
```
int(csc(b*x+a)*csc(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)^2+5/6/sin(b*x+a)/cos(b*x+a)^2-5/2/sin(
b*x+a)+5/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(54) = 108.

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.10

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx =$$
$$-\frac{30\cos(bx+a)^4-15\left(\cos(bx+a)^4-\cos(bx+a)^2\right)\log(\sin(bx+a)+1)\sin(bx+a)+15\,(\cos(bx+}{96\left(b\cos(bx+a)^4-b\cos(bx+a)^2\right.}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-1/96*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b*
x + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(b
*x + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b*
cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F]

$$\int \csc(a + bx) \csc^3(2a + 2bx)\, dx = \int \csc(a + bx) \csc^3(2a + 2bx)\, dx$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)**3,x)
```

output
```
Integral(csc(a + b*x)*csc(2*a + 2*b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1780 vs. $2(54) = 108$.

Time = 0.27 (sec) , antiderivative size = 1780, normalized size of antiderivative = 28.71

$$\int \csc(a + bx) \csc^3(2a + 2bx)\, dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/96*(4*(15*sin(9*b*x + 9*a) - 20*sin(7*b*x + 7*a) - 22*sin(5*b*x + 5*a) -
 20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) + 60*(sin(8*b*x
 + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(
9*b*x + 9*a) + 4*(20*sin(7*b*x + 7*a) + 22*sin(5*b*x + 5*a) + 20*sin(3*b*x
 + 3*a) - 15*sin(b*x + a))*cos(8*b*x + 8*a) - 80*(2*sin(6*b*x + 6*a) - 2*s
in(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(22*sin(5*b*x + 5
*a) + 20*sin(3*b*x + 3*a) - 15*sin(b*x + a))*cos(6*b*x + 6*a) + 88*(2*sin(
4*b*x + 4*a) + sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(4*sin(3*b*x + 3*a)
 - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 15*(2*(cos(8*b*x + 8*a) + 2*cos(6*b*
x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) -
 cos(10*b*x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2
*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 4*(2*cos(4*b*x +
4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 - 4*(
cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(8*b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) -
 sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^2 - 2*(2*sin(6*
b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin
(8*b*x + 8*a)^2 + 4*(2*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*
a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log((cos(b*...
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx =$$

$$-\frac{\frac{6\sin(bx+a)}{\sin(bx+a)^2-1}+\frac{4\left(6\sin(bx+a)^2+1\right)}{\sin(bx+a)^3}-15\log\left(\sin\left(bx+a\right)+1\right)+15\log\left(-\sin\left(bx+a\right)+1\right)}{96\,b}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-1/96*(6*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(6*sin(b*x + a)^2 + 1)/sin(
b*x + a)^3 - 15*log(sin(b*x + a) + 1) + 15*log(-sin(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx = \frac{5\,\mathrm{atanh}(\sin(a+b\,x))}{16\,b} - \frac{-\frac{5\sin(a+b\,x)^4}{16}+\frac{5\sin(a+b\,x)^2}{24}+\frac{1}{24}}{b\,\left(\sin(a+b\,x)^3-\sin(a+b\,x)^5\right)}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^3),x)
```

output
```
(5*atanh(sin(a + b*x)))/(16*b) - ((5*sin(a + b*x)^2)/24 - (5*sin(a + b*x)^
4)/16 + 1/24)/(b*(sin(a + b*x)^3 - sin(a + b*x)^5))
```

Reduce [F]

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx = \int \csc(2bx+2a)^3\csc(bx+a)\,dx$$

input
```
int(csc(b*x+a)*csc(2*b*x+2*a)^3,x)
```

output
```
int(csc(2*a + 2*b*x)**3*csc(a + b*x),x)
```

3.454 $\int \csc(a+bx)\csc^4(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = -\frac{35\text{arctanh}(\cos(a+bx))}{128b} - \frac{13\cot(a+bx)\csc(a+bx)}{128b} - \frac{\cot^3(a+bx)\csc(a+bx)}{64b} + \frac{3\sec(a+bx)}{16b} + \frac{\sec^3(a+bx)}{48b}$$

output
```
-35/128*arctanh(cos(b*x+a))/b-13/128*cot(b*x+a)*csc(b*x+a)/b-1/64*cot(b*x+
a)^3*csc(b*x+a)/b+3/16*sec(b*x+a)/b+1/48*sec(b*x+a)^3/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 268 vs. 2(83) = 166.

Time = 0.45 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.23

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = -\frac{\csc^{10}(a+bx)\left(-204+658\cos(2(a+bx))-228\cos(3(a+bx))+140\cos(4(a+bx))-76\cos(5(a+b\right.}{}$$

input
```
Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

output
```
-1/384*(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)
] + 140*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76
*Cos[7*(a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5
*(a + b*x)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)
/2]] + 3*Cos[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b
*x)/2]]) + 315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x
)]*Log[Sin[(a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(b
*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4776, 3042, 3102, 25, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{16}\int \csc^5(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^5\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^8(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\int \frac{\sec^8(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \textbf{252}$$

$$\frac{\frac{7}{4}\int \frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{16b}$$

$$\Big\downarrow \textbf{252}$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{16b}$$

$$\Big\downarrow \textbf{254}$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \left(-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{16b}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\left(\text{arctanh}(\sec(a+bx))-\frac{1}{3}\sec^3(a+bx)-\sec(a+bx)\right)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{16b}$$

input
```
Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

output
```
(-1/4*Sec[a + b*x]^7/(1 - Sec[a + b*x]^2)^2 + (7*(Sec[a + b*x]^5/(2*(1 - S
ec[a + b*x]^2)) - (5*(ArcTanh[Sec[a + b*x]] - Sec[a + b*x] - Sec[a + b*x]^
3/3))/2))/4)/(16*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

method	result
default	$\frac{-\frac{1}{4\sin(bx+a)^4\cos(bx+a)^3}+\frac{7}{12\sin(bx+a)^2\cos(bx+a)^3}-\frac{35}{24\sin(bx+a)^2\cos(bx+a)}+\frac{35}{8\cos(bx+a)}+\frac{35\ln(\csc(bx+a)-\cot(bx+a))}{8}}{16b}$
risch	$\frac{105\,\mathrm{e}^{13i(bx+a)}-70\,\mathrm{e}^{11i(bx+a)}-329\,\mathrm{e}^{9i(bx+a)}+204\,\mathrm{e}^{7i(bx+a)}-329\,\mathrm{e}^{5i(bx+a)}-70\,\mathrm{e}^{3i(bx+a)}+105\,\mathrm{e}^{i(bx+a)}}{192b(\mathrm{e}^{2i(bx+a)}-1)^4(\mathrm{e}^{2i(bx+a)}+1)^3}-\frac{35\ln(\mathrm{e}^{i(bx+a)}+1)}{128b}+$

input
```
int(csc(b*x+a)*csc(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^3+7/12/sin(b*x+a)^2/cos(b*x+a)^3-35/2
4/sin(b*x+a)^2/cos(b*x+a)+35/8/cos(b*x+a)+35/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(73) = 146.

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.78

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx$$
$$=\frac{210\cos(bx+a)^6-350\cos(bx+a)^4+112\cos(bx+a)^2-105\left(\cos(bx+a)^7-2\cos(bx+a)^5+\cos(\right.}{768\left(b\cos(bx+a)^7\right.}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
1/768*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105*
(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a)
+ 1/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/2
*cos(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(
b*x + a)^3)
```

Sympy [F]

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = \int \csc(a+bx)\csc^4(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)**4,x)
```

output
```
Integral(csc(a + b*x)*csc(2*a + 2*b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3846 vs. $2(73) = 146$.

Time = 0.21 (sec) , antiderivative size = 3846, normalized size of antiderivative = 46.34

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/768*(4*(105*cos(13*b*x + 13*a) - 70*cos(11*b*x + 11*a) - 329*cos(9*b*x +
 9*a) + 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a)
+ 105*cos(b*x + a))*cos(14*b*x + 14*a) - 420*(cos(12*b*x + 12*a) + 3*cos(1
0*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*
a) + cos(2*b*x + 2*a) - 1)*cos(13*b*x + 13*a) + 4*(70*cos(11*b*x + 11*a) +
 329*cos(9*b*x + 9*a) - 204*cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*c
os(3*b*x + 3*a) - 105*cos(b*x + a))*cos(12*b*x + 12*a) + 280*(3*cos(10*b*x
 + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) +
cos(2*b*x + 2*a) - 1)*cos(11*b*x + 11*a) + 12*(329*cos(9*b*x + 9*a) - 204*
cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*b*x + 3*a) - 105*cos(b*
x + a))*cos(10*b*x + 10*a) - 1316*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a)
 - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(9*b*x + 9*a) + 12*(204*c
os(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) + 105*cos(b*x
 + a))*cos(8*b*x + 8*a) + 816*(3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - c
os(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) - 84*(47*cos(5*b*x + 5*a) + 10*cos(3
*b*x + 3*a) - 15*cos(b*x + a))*cos(6*b*x + 6*a) + 1316*(3*cos(4*b*x + 4*a)
 + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) + 420*(2*cos(3*b*x + 3*a) - 3*co
s(b*x + a))*cos(4*b*x + 4*a) + 280*(cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)
 - 420*cos(2*b*x + 2*a)*cos(b*x + a) + 105*(2*(cos(12*b*x + 12*a) + 3*cos(
10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(73) = 146$.

Time = 0.14 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.48

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx$$
$$= \frac{\frac{3\left(\frac{24(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{210(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}-\frac{72(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{256\left(\frac{9(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{6(\cos(bx\cdots}{(\cos(bx+\cdots}\right.}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}}{3072\,b}$$

input
```
integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/3072*(3*(24*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 210*(cos(b*x + a) -
1)^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 -
 72*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*
x + a) + 1)^2 + 256*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 6*(cos(b*x
+ a) - 1)^2/(cos(b*x + a) + 1)^2 + 5)/((cos(b*x + a) - 1)/(cos(b*x + a) +
1) + 1)^3 + 420*log(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 18.96 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = \frac{\frac{35\cos(a+b\,x)^6}{128}-\frac{175\cos(a+b\,x)^4}{384}+\frac{7\cos(a+b\,x)^2}{48}+\frac{1}{48}}{b\,\left(\cos\left(a+b\,x\right)^7-2\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)} - \frac{35\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{128\,b}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^4),x)
```

output
```
((7*cos(a + b*x)^2)/48 - (175*cos(a + b*x)^4)/384 + (35*cos(a + b*x)^6)/12
8 + 1/48)/(b*(cos(a + b*x)^3 - 2*cos(a + b*x)^5 + cos(a + b*x)^7)) - (35*a
tanh(cos(a + b*x)))/(128*b)
```

Reduce [F]

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = \int \csc\left(2bx+2a\right)^4\csc\left(bx+a\right)dx$$

input
```
int(csc(b*x+a)*csc(2*b*x+2*a)^4,x)
```

output
```
int(csc(2*a + 2*b*x)**4*csc(a + b*x),x)
```

3.455 $\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 155

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx = \frac{5x}{8}+\frac{5\cos(a+bx)\sin(a+bx)}{8b}+\frac{5\cos^3(a+bx)\sin(a+bx)}{12b}+\frac{\cos^5(a+bx)\sin(a+bx)}{3b}+\frac{2\cos^7(a+bx)\sin(a+bx)}{7b}-\frac{16\cos^9(a+bx)\sin(a+bx)}{7b}-\frac{160\cos^9(a+bx)\sin^3(a+bx)}{21b}-\frac{128\cos^9(a+bx)\sin^5(a+bx)}{7b}$$

output

```
5/8*x+5/8*cos(b*x+a)*sin(b*x+a)/b+5/12*cos(b*x+a)^3*sin(b*x+a)/b+1/3*cos(b
*x+a)^5*sin(b*x+a)/b+2/7*cos(b*x+a)^7*sin(b*x+a)/b-16/7*cos(b*x+a)^9*sin(b
*x+a)/b-160/21*cos(b*x+a)^9*sin(b*x+a)^3/b-128/7*cos(b*x+a)^9*sin(b*x+a)^5
/b
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.55

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx$$
$$= \frac{840a+840bx+105\sin(2(a+bx))-315\sin(4(a+bx))-63\sin(6(a+bx))+63\sin(8(a+bx))+21\sin}{1344b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^8,x]
```

output
```
(840*a + 840*b*x + 105*Sin[2*(a + b*x)] - 315*Sin[4*(a + b*x)] - 63*Sin[6*
(a + b*x)] + 63*Sin[8*(a + b*x)] + 21*Sin[10*(a + b*x)] - 7*Sin[12*(a + b*
x)] - 3*Sin[14*(a + b*x)])/(1344*b)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.850$, Rules used = {3042, 4776, 3042, 3048, 3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^8}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$256\int \cos^8(a+bx)\sin^6(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$256\int \cos(a+bx)^8\sin(a+bx)^6dx$$

$\downarrow$ 3048

$$256\left(\frac{5}{14}\int \cos^8(a+bx)\sin^4(a+bx)dx-\frac{\sin^5(a+bx)\cos^9(a+bx)}{14b}\right)$$

$\downarrow$ 3042

$$256\left(\frac{5}{14}\int \cos(a+bx)^8\sin(a+bx)^4dx-\frac{\sin^5(a+bx)\cos^9(a+bx)}{14b}\right)$$

$\downarrow$ 3048

$$256\left(\frac{5}{14}\left(\frac{1}{4}\int \cos^8(a+bx)\sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos^9(a+bx)}{12b}\right)-\frac{\sin^5(a+bx)\cos^9(a+bx)}{14b}\right)$$

$\downarrow$ 3042

$$256\left(\frac{5}{14}\left(\frac{1}{4}\int \cos(a+bx)^8\sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos^9(a+bx)}{12b}\right)-\frac{\sin^5(a+bx)\cos^9(a+bx)}{14b}\right)$$

$\downarrow$ 3048

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\int \cos^8(a+bx)dx-\frac{\sin(a+bx)\cos^9(a+bx)}{10b}\right)-\frac{\sin^3(a+bx)\cos^9(a+bx)}{12b}\right)-\frac{\sin^5(a+bx)\cos}{14b}\right.$$

$\downarrow$ 3042

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\int \sin\left(a+bx+\frac{\pi}{2}\right)^8dx-\frac{\sin(a+bx)\cos^9(a+bx)}{10b}\right)-\frac{\sin^3(a+bx)\cos^9(a+bx)}{12b}\right)-\frac{\sin^5(a+}{}\right.$$

$\downarrow$ 3115

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\int \cos^6(a+bx)dx+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin(a+bx)\cos^9(a+bx)}{10b}\right)-\frac{\sin^3(a+bx)}{12}\right.\right.$$

$\downarrow$ 3042

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\int \sin\left(a+bx+\frac{\pi}{2}\right)^6dx+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin(a+bx)\cos^9(a+bx)}{10b}\right)-\frac{\sin^3(a}{}\right.\right.$$

$\downarrow$ 3115

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \cos^4(a+bx)dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin(a+b}{}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin$$

$\Big\downarrow$ 3115

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)+\frac{\sin(a}{}$$

$\Big\downarrow$ 3042

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)+$$

$\Big\downarrow$ 3115

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a}{6b}$$

$\Big\downarrow$ 24

$$256\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{\sin(a+bx)\cos^7(a+bx)}{8b}+\frac{7}{8}\left(\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\text{si}}{}\right.\right.\right.\right.\right.\right.$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^8,x]
```

output
```
256*(-1/14*(Cos[a + b*x]^9*Sin[a + b*x]^5)/b + (5*(-1/12*(Cos[a + b*x]^9*S
in[a + b*x]^3)/b + (-1/10*(Cos[a + b*x]^9*Sin[a + b*x])/b + ((Cos[a + b*x]
^7*Sin[a + b*x])/(8*b) + (7*((Cos[a + b*x]^5*Sin[a + b*x])/(6*b) + (5*((Co
s[a + b*x]^3*Sin[a + b*x])/(4*b) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(
2*b)))/4))/6))/8)/10)/4))/14)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 29.48 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66

method	result
risch	$\frac{5x}{8}-\frac{\sin(14bx+14a)}{448b}-\frac{\sin(12bx+12a)}{192b}+\frac{\sin(10bx+10a)}{64b}+\frac{3\sin(8bx+8a)}{64b}-\frac{3\sin(6bx+6a)}{64b}-\frac{15\sin(4bx+4a)}{64b}+\frac{5\sin(2\ldots}{6\ldots}$
default	$\frac{-\frac{128\sin(bx+a)^5\cos(bx+a)^9}{7}-\frac{160\sin(bx+a)^3\cos(bx+a)^9}{21}-\frac{16\sin(bx+a)\cos(bx+a)^9}{7}+\frac{2\left(\cos(bx+a)^7+\frac{7\cos(bx+a)^5}{6}+\frac{35\cos(bx+a)^3}{24}+\frac{35\cos(b\ldots}{16}\ldots\right.}{7}\ldots}{b}$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x,method=_RETURNVERBOSE)
```

output
```
5/8*x-1/448/b*sin(14*b*x+14*a)-1/192/b*sin(12*b*x+12*a)+1/64/b*sin(10*b*x+
10*a)+3/64/b*sin(8*b*x+8*a)-3/64/b*sin(6*b*x+6*a)-15/64/b*sin(4*b*x+4*a)+5
/64*sin(2*b*x+2*a)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.56

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx = \frac{105\,bx-\left(3072\,\cos\left(bx+a\right)^{13}-7424\,\cos\left(bx+a\right)^{11}+4736\,\cos\left(bx+a\right)^{9}-48\,\cos\left(bx+a\right)^{7}-56\,\cos\left(b\right.\right.}{168\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

output
```
1/168*(105*b*x - (3072*cos(b*x + a)^13 - 7424*cos(b*x + a)^11 + 4736*cos(b
*x + a)^9 - 48*cos(b*x + a)^7 - 56*cos(b*x + a)^5 - 70*cos(b*x + a)^3 - 10
5*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.56

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx = \frac{840\,bx - 3\,\sin(14\,bx + 14\,a) - 7\,\sin(12\,bx + 12\,a) + 21\,\sin(10\,bx + 10\,a) + 63\,\sin(8\,bx + 8\,a) - 63\,\sin}{1344\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

output
```
1/1344*(840*b*x - 3*sin(14*b*x + 14*a) - 7*sin(12*b*x + 12*a) + 21*sin(10*
b*x + 10*a) + 63*sin(8*b*x + 8*a) - 63*sin(6*b*x + 6*a) - 315*sin(4*b*x +
4*a) + 105*sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.61

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx = \frac{105\,bx + 105\,a + \frac{105\,\tan(bx+a)^{13} + 700\,\tan(bx+a)^{11} + 1981\,\tan(bx+a)^9 + 3072\,\tan(bx+a)^7 - 1981\,\tan(bx+a)^5 - 700\,\tan(bx+a)^3 - 105\,\mathrm{t}}{\left(\tan(bx+a)^2+1\right)^7}}{168\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

output
```
1/168*(105*b*x + 105*a + (105*tan(b*x + a)^13 + 700*tan(b*x + a)^11 + 1981
*tan(b*x + a)^9 + 3072*tan(b*x + a)^7 - 1981*tan(b*x + a)^5 - 700*tan(b*x
+ a)^3 - 105*tan(b*x + a))/(tan(b*x + a)^2 + 1)^7)/b
```

Mupad [B] (verification not implemented)

Time = 20.58 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx = \frac{5\,x}{8} + \frac{\frac{5\tan(a+b\,x)^{13}}{8}+\frac{25\tan(a+b\,x)^{11}}{6}+\frac{283\tan(a+b\,x)^{9}}{24}+\frac{128\tan(a+b\,x)^{7}}{7}-\frac{283\tan(a+b\,x)^{5}}{24}-\frac{25\tan(a+}{6}}{b\,\left(\tan\left(a+b\,x\right)^{14}+7\tan\left(a+b\,x\right)^{12}+21\tan\left(a+b\,x\right)^{10}+35\tan\left(a+b\,x\right)^{8}+35\tan\left(a+b\,x\right)^{6}+21\,\mathrm{t}\right.}$$

input
```
int(sin(2*a + 2*b*x)^8/sin(a + b*x)^2,x)
```

output
```
(5*x)/8 + ((128*tan(a + b*x)^7)/7 - (25*tan(a + b*x)^3)/6 - (283*tan(a + b
*x)^5)/24 - (5*tan(a + b*x))/8 + (283*tan(a + b*x)^9)/24 + (25*tan(a + b*x
)^11)/6 + (5*tan(a + b*x)^13)/8)/(b*(7*tan(a + b*x)^2 + 21*tan(a + b*x)^4
+ 35*tan(a + b*x)^6 + 35*tan(a + b*x)^8 + 21*tan(a + b*x)^10 + 7*tan(a + b
*x)^12 + tan(a + b*x)^14 + 1))
```

Reduce [F]

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx = \int \csc\left(bx+a\right)^2\sin\left(2bx+2a\right)^8 dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x)
```

output
```
int(csc(a + b*x)**2*sin(2*a + 2*b*x)**8,x)
```

3.456 $\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx=-\frac{16\cos^8(a+bx)}{b}+\frac{128\cos^{10}(a+bx)}{5b}-\frac{32\cos^{12}(a+bx)}{3b}$$

output
```
-16*cos(b*x+a)^8/b+128/5*cos(b*x+a)^10/b-32/3*cos(b*x+a)^12/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx=\frac{64\sin^6(a+bx)}{3b}-\frac{48\sin^8(a+bx)}{b}+\frac{192\sin^{10}(a+bx)}{5b}-\frac{32\sin^{12}(a+bx)}{3b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^7,x]
```

output
```
(64*Sin[a + b*x]^6)/(3*b) - (48*Sin[a + b*x]^8)/b + (192*Sin[a + b*x]^10)/
(5*b) - (32*Sin[a + b*x]^12)/(3*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4776, 3042, 3045, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^7}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$128\int \cos^7(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$128\int \cos(a+bx)^7\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{128\int \cos^7(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{64\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos^2(a+bx)}{b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{64\int \left(\cos^{10}(a+bx)-2\cos^8(a+bx)+\cos^6(a+bx)\right)d\cos^2(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{64\left(\frac{1}{6}\cos^{12}(a+bx)-\frac{2}{5}\cos^{10}(a+bx)+\frac{1}{4}\cos^8(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^7,x]
```

output
```
(-64*(Cos[a + b*x]^8/4 - (2*Cos[a + b*x]^10)/5 + Cos[a + b*x]^12/6))/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 17.97 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{128\left(\frac{\sin(bx+a)^{12}}{12}-\frac{3\sin(bx+a)^{10}}{10}+\frac{3\sin(bx+a)^{8}}{8}-\frac{\sin(bx+a)^{6}}{6}\right)}{b}$	47
risch	$-\frac{\cos(12bx+12a)}{192b}-\frac{\cos(10bx+10a)}{80b}+\frac{\cos(8bx+8a)}{32b}+\frac{5\cos(6bx+6a)}{48b}-\frac{5\cos(4bx+4a)}{64b}-\frac{5\cos(2bx+2a)}{8b}$	86

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

output
```
-128/b*(1/12*sin(b*x+a)^12-3/10*sin(b*x+a)^10+3/8*sin(b*x+a)^8-1/6*sin(b*x
+a)^6)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx = -\frac{16\left(10\cos\left(bx+a\right)^{12}-24\cos\left(bx+a\right)^{10}+15\cos\left(bx+a\right)^{8}\right)}{15\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

output
```
-16/15*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.64

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx = -\frac{5\cos(12\,bx+12\,a)+12\cos(10\,bx+10\,a)-30\cos(8\,bx+8\,a)-100\cos(6\,bx+6\,a)+75\cos(4\,bx}{960\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

output
```
-1/960*(5*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a)
 - 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) + 600*cos(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx = -\frac{16\left(10\sin(bx+a)^{12}-36\sin(bx+a)^{10}+45\sin(bx+a)^{8}-20\sin(bx+a)^{6}\right)}{15\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

output
```
-16/15*(10*sin(b*x + a)^12 - 36*sin(b*x + a)^10 + 45*sin(b*x + a)^8 - 20*s
in(b*x + a)^6)/b
```

Mupad [B] (verification not implemented)

Time = 19.56 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx = -\frac{16\cos(a+b\,x)^8\,\left(10\cos(a+b\,x)^4-24\cos(a+b\,x)^2+15\right)}{15\,b}$$

input
```
int(sin(2*a + 2*b*x)^7/sin(a + b*x)^2,x)
```

output
```
-(16*cos(a + b*x)^8*(10*cos(a + b*x)^4 - 24*cos(a + b*x)^2 + 15))/(15*b)
```

Reduce [F]

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx = \int \csc(bx+a)^2\sin(2bx+2a)^7\,dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x)
```

output
```
int(csc(a + b*x)**2*sin(2*a + 2*b*x)**7,x)
```

3.457 $\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \frac{3x}{4}+\frac{3\cos(a+bx)\sin(a+bx)}{4b}+\frac{\cos^3(a+bx)\sin(a+bx)}{2b}+\frac{2\cos^5(a+bx)\sin(a+bx)}{5b}-\frac{12\cos^7(a+bx)\sin(a+bx)}{5b}-\frac{32\cos^7(a+bx)\sin^3(a+bx)}{5b}$$

output
```
3/4*x+3/4*cos(b*x+a)*sin(b*x+a)/b+1/2*cos(b*x+a)^3*sin(b*x+a)/b+2/5*cos(b*
x+a)^5*sin(b*x+a)/b-12/5*cos(b*x+a)^7*sin(b*x+a)/b-32/5*cos(b*x+a)^7*sin(b
*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \frac{120bx+20\sin(2(a+bx))-40\sin(4(a+bx))-10\sin(6(a+bx))+5\sin(8(a+bx))+2\sin(10(a+bx))}{160b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^6,x]
```

output
```
(120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
 + 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(160*b)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {3042, 4776, 3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^6}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$64\int \cos^6(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$64\int \cos(a+bx)^6\sin(a+bx)^4dx$$

$$\Big\downarrow \text{3048}$$

$$64\left(\frac{3}{10}\int \cos^6(a+bx)\sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}\right)$$

$$\Big\downarrow \text{3042}$$

$$64\left(\frac{3}{10}\int \cos(a+bx)^6\sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}\right)$$

$$\Big\downarrow \text{3048}$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\int \cos^6(a+bx)dx-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}\right)$$

$$\Big\downarrow \text{3042}$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\int \sin\left(a+bx+\frac{\pi}{2}\right)^6dx-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}\right)$$

$$\Big\downarrow \text{3115}$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \cos^4(a+bx)dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin^3(a+bx)\cos^7(a}{10b}$$

$$\Big\downarrow \text{3042}$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin^3(a+bx)}{1}$$

$$\Big\downarrow \text{3115}$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos}{8b}$$

$$\Big\downarrow \text{3042}$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\sin(a+b$$

$$\Big\downarrow \text{3115}$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{24}$$

$$64\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)\right)-\text{si}\right.\right.$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^6,x]
```

output
```
64*(-1/10*(Cos[a + b*x]^7*Sin[a + b*x]^3)/b + (3*(-1/8*(Cos[a + b*x]^7*Sin
[a + b*x])/b + ((Cos[a + b*x]^5*Sin[a + b*x])/(6*b) + (5*((Cos[a + b*x]^3*
Sin[a + b*x])/(4*b) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4))/6)
/8))/10)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 10.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{3x}{4}+\frac{\sin(10bx+10a)}{80b}+\frac{\sin(8bx+8a)}{32b}-\frac{\sin(6bx+6a)}{16b}-\frac{\sin(4bx+4a)}{4b}+\frac{\sin(2bx+2a)}{8b}$	75
default	$\frac{-\frac{32\sin(bx+a)^3\cos(bx+a)^7}{5}-\frac{12\sin(bx+a)\cos(bx+a)^7}{5}+\frac{2\left(\cos(bx+a)^5+\frac{5\cos(bx+a)^3}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{5}+\frac{3bx}{4}+\frac{3a}{4}}{b}$	83

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

output
```
3/4*x+1/80/b*sin(10*b*x+10*a)+1/32/b*sin(8*b*x+8*a)-1/16/b*sin(6*b*x+6*a)-
1/4/b*sin(4*b*x+4*a)+1/8*sin(2*b*x+2*a)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.59

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx$$
$$=\frac{15\,bx+\left(128\,\cos\left(bx+a\right)^9-176\,\cos\left(bx+a\right)^7+8\,\cos\left(bx+a\right)^5+10\,\cos\left(bx+a\right)^3+15\,\cos\left(bx+a\right)\right)\mathrm{s}}{20\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

output
```
1/20*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)^5
 + 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.59

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \frac{120\,bx + 2\,\sin\left(10\,bx + 10\,a\right) + 5\,\sin\left(8\,bx + 8\,a\right) - 10\,\sin\left(6\,bx + 6\,a\right) - 40\,\sin\left(4\,bx + 4\,a\right) + 20\,\sin\left(2\,bx}{160\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

output
```
1/160*(120*b*x + 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) - 10*sin(6*b*x
+ 6*a) - 40*sin(4*b*x + 4*a) + 20*sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \frac{15\,bx + 15\,a + \frac{15\,\tan(bx+a)^9 + 70\,\tan(bx+a)^7 + 128\,\tan(bx+a)^5 - 70\,\tan(bx+a)^3 - 15\,\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)^5}}{20\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

output
```
1/20*(15*b*x + 15*a + (15*tan(b*x + a)^9 + 70*tan(b*x + a)^7 + 128*tan(b*x
 + a)^5 - 70*tan(b*x + a)^3 - 15*tan(b*x + a))/(tan(b*x + a)^2 + 1)^5)/b
```

Mupad [B] (verification not implemented)

Time = 20.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \frac{3\,x}{4} + \frac{\frac{3\tan(a+b\,x)^9}{4}+\frac{7\tan(a+b\,x)^7}{2}+\frac{32\tan(a+b\,x)^5}{5}-\frac{7\tan(a+b\,x)^3}{2}-\frac{3\tan(a+b\,x)}{4}}{b\,\left(\tan\left(a+b\,x\right)^{10}+5\tan\left(a+b\,x\right)^8+10\tan\left(a+b\,x\right)^6+10\tan\left(a+b\,x\right)^4+5\tan\left(a+b\,x\right)^2+1\right)}$$

input
```
int(sin(2*a + 2*b*x)^6/sin(a + b*x)^2,x)
```

output
```
(3*x)/4 + ((32*tan(a + b*x)^5)/5 - (7*tan(a + b*x)^3)/2 - (3*tan(a + b*x))
/4 + (7*tan(a + b*x)^7)/2 + (3*tan(a + b*x)^9)/4)/(b*(5*tan(a + b*x)^2 + 1
0*tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^10
+ 1))
```

Reduce [F]

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = \int \csc\left(bx+a\right)^2\sin\left(2bx+2a\right)^6 dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x)
```

output
```
int(csc(a + b*x)**2*sin(2*a + 2*b*x)**6,x)
```

3.458 $\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx=-\frac{16\cos^6(a+bx)}{3b}+\frac{4\cos^8(a+bx)}{b}$$

output
```
-16/3*cos(b*x+a)^6/b+4*cos(b*x+a)^8/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx =\frac{-72\cos(2(a+bx))-12\cos(4(a+bx))+8\cos(6(a+bx))+3\cos(8(a+bx))}{96b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

output
```
(-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8
*(a + b*x)])/(96*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^5}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$32\int \cos^5(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$32\int \cos(a+bx)^5\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{32\int \cos^5(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{32\int \left(\cos^5(a+bx)-\cos^7(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{32\left(\frac{1}{6}\cos^6(a+bx)-\frac{1}{8}\cos^8(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

output
```
(-32*(Cos[a + b*x]^6/6 - Cos[a + b*x]^8/8))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 5.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{4\cos(bx+a)^8-\frac{16\cos(bx+a)^6}{3}}{b}$	27
risch	$\frac{\cos(8bx+8a)}{32b}+\frac{\cos(6bx+6a)}{12b}-\frac{\cos(4bx+4a)}{8b}-\frac{3\cos(2bx+2a)}{4b}$	58

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
32/b*(1/8*cos(b*x+a)^8-1/6*cos(b*x+a)^6)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{4\left(3\cos\left(bx+a\right)^8-4\cos\left(bx+a\right)^6\right)}{3b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
4/3*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{3\cos\left(8\,bx+8\,a\right)+8\cos\left(6\,bx+6\,a\right)-12\cos\left(4\,bx+4\,a\right)-72\cos\left(2\,bx+2\,a\right)}{96\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/96*(3*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) - 72*c
os(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{4\left(3\sin(bx+a)^8 - 8\sin(bx+a)^6 + 6\sin(bx+a)^4\right)}{3b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
4/3*(3*sin(b*x + a)^8 - 8*sin(b*x + a)^6 + 6*sin(b*x + a)^4)/b
```

Mupad [B] (verification not implemented)

Time = 18.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx = \frac{4\cos(a+b\,x)^6\left(3\cos(a+b\,x)^2-4\right)}{3\,b}$$

input
```
int(sin(2*a + 2*b*x)^5/sin(a + b*x)^2,x)
```

output
```
(4*cos(a + b*x)^6*(3*cos(a + b*x)^2 - 4))/(3*b)
```

Reduce [F]

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx = \int \csc(bx+a)^2\sin(2bx+2a)^5\,dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x)
```

output
```
int(csc(a + b*x)**2*sin(2*a + 2*b*x)**5,x)
```

3.459 $\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx = x+\frac{\cos(a+bx)\sin(a+bx)}{b}+\frac{2\cos^3(a+bx)\sin(a+bx)}{3b}-\frac{8\cos^5(a+bx)\sin(a+bx)}{3b}$$

output
```
x+cos(b*x+a)*sin(b*x+a)/b+2/3*cos(b*x+a)^3*sin(b*x+a)/b-8/3*cos(b*x+a)^5*s
in(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx = -\frac{-12bx-3\sin(2(a+bx))+3\sin(4(a+bx))+\sin(6(a+bx))}{12b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

output
```
-1/12*(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)
])/b
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.32, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3042, 4776, 3042, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^4}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$16\int \cos^4(a+bx)\sin^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$16\int \cos(a+bx)^4\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$16\left(\frac{1}{6}\int \cos^4(a+bx)dx-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{3042}$$

$$16\left(\frac{1}{6}\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{3115}$$

$$16\left(\frac{1}{6}\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{3042}$$

$$16\left(\frac{1}{6}\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{3115}$$

$$16\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{24}$$

$$16\left(\frac{1}{6}\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

output
```
16*(-1/6*(Cos[a + b*x]^5*Sin[a + b*x])/b + ((Cos[a + b*x]^3*Sin[a + b*x])/
(4*b) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4)/6)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

method	result	size
risch	$x-\frac{\sin(6bx+6a)}{12b}-\frac{\sin(4bx+4a)}{4b}+\frac{\sin(2bx+2a)}{4b}$	45
default	$\frac{-\frac{8\sin(bx+a)\cos(bx+a)^5}{3}+\frac{2\left(\cos(bx+a)^3+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{3}+bx+a}{b}$	55

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
x-1/12/b*sin(6*b*x+6*a)-1/4/b*sin(4*b*x+4*a)+1/4*sin(2*b*x+2*a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{3\,bx-\left(8\,\cos\left(bx+a\right)^5-2\,\cos\left(bx+a\right)^3-3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{3\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
1/3*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b*
x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.72

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{12\,bx - \sin\left(6\,bx+6\,a\right) - 3\,\sin\left(4\,bx+4\,a\right) + 3\,\sin\left(2\,bx+2\,a\right)}{12\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/12*(12*b*x - sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))
/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{3\,bx + 3\,a + \frac{3\,\tan(bx+a)^5 + 8\,\tan(bx+a)^3 - 3\,\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)^3}}{3\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/3*(3*b*x + 3*a + (3*tan(b*x + a)^5 + 8*tan(b*x + a)^3 - 3*tan(b*x + a))/
(tan(b*x + a)^2 + 1)^3)/b
```

Mupad [B] (verification not implemented)

Time = 19.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= x + \frac{\tan(a+b\,x)^5+\frac{8\tan(a+b\,x)^3}{3}-\tan(a+b\,x)}{b\,\left(\tan\left(a+b\,x\right)^6+3\tan\left(a+b\,x\right)^4+3\tan\left(a+b\,x\right)^2+1\right)}$$

input
```
int(sin(2*a + 2*b*x)^4/sin(a + b*x)^2,x)
```

output
```
x + ((8*tan(a + b*x)^3)/3 - tan(a + b*x) + tan(a + b*x)^5)/(b*(3*tan(a + b
*x)^2 + 3*tan(a + b*x)^4 + tan(a + b*x)^6 + 1))
```

Reduce [F]

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx = \int \csc\left(bx+a\right)^2\sin\left(2bx+2a\right)^4dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x)
```

output
```
int(csc(a + b*x)**2*sin(2*a + 2*b*x)**4,x)
```

3.460 $\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx=-\frac{2\cos^4(a+bx)}{b}$$

output
```
-2*cos(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx=-\frac{2\cos^4(a+bx)}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-2*Cos[a + b*x]^4)/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4776, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^3}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$8\int \cos^3(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)^3\sin(a+bx)dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{8\int \cos^3(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\cos^4(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-2*Cos[a + b*x]^4)/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{2\cos(bx+a)^4}{b}$	14
risch	$-\frac{\cos(4bx+4a)}{4b}-\frac{\cos(2bx+2a)}{b}$	30

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*cos(b*x+a)^4/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx=-\frac{2\cos(bx+a)^4}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-2*cos(b*x + a)^4/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx=\text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx=-\frac{\cos(4\,bx+4\,a)+4\,\cos(2\,bx+2\,a)}{4\,b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
-1/4*(cos(4*b*x + 4*a) + 4*cos(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{2\cos(bx+a)^4}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-2*cos(b*x + a)^4/b
```

Mupad [B] (verification not implemented)

Time = 18.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{2\cos(a+b\,x)^4}{b}$$

input
```
int(sin(2*a + 2*b*x)^3/sin(a + b*x)^2,x)
```

output
```
-(2*cos(a + b*x)^4)/b
```

Reduce [F]

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx = \int \csc(bx+a)^2\sin(2bx+2a)^3\,dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x)
```

output
```
int(csc(a + b*x)**2*sin(2*a + 2*b*x)**3,x)
```

3.461 $\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 21

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = 2x+\frac{2\cos(a+bx)\sin(a+bx)}{b}$$

output
```
2*x+2*cos(b*x+a)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{2(a+bx)+\sin(2(a+bx))}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

output
```
(2*(a + b*x) + Sin[2*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4776, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(2a+2bx)^2}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{ 4776}$$

$$4\int \cos^2(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$4\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 3115}$$

$$4\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 24}$$

$$4\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

output
```
4*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
risch	$2x+\frac{\sin \left(2 b x +2 a \right)}{b}$	18
default	$\frac{2 \sin \left(b x +a \right) \cos \left(b x +a \right)+2 b x +2 a}{b}$	28

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x+sin(2*b*x+2*a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{2\,(bx+\cos{(bx+a)}\sin{(bx+a)})}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
2*(b*x + cos(b*x + a)*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{2\,bx+\sin{(2\,bx+2\,a)}}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
(2*b*x + sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{2\left(bx+a+\frac{\tan(bx+a)}{\tan(bx+a)^2+1}\right)}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
2*(b*x + a + tan(b*x + a)/(tan(b*x + a)^2 + 1))/b
```

Mupad [B] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = 2\,x + \frac{\sin\left(2\,a+2\,b\,x\right)}{b}$$

input
```
int(sin(2*a + 2*b*x)^2/sin(a + b*x)^2,x)
```

output
```
2*x + sin(2*a + 2*b*x)/b
```

Reduce [F]

$$\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx = \int \csc\left(bx+a\right)^2\sin\left(2bx+2a\right)^2dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x)
```

output
```
int(csc(a + b*x)**2*sin(2*a + 2*b*x)**2,x)
```

3.462 $\int \csc^2(a+bx)\sin(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx=\frac{2\log(\sin(a+bx))}{b}$$

output
```
2*ln(sin(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx=\frac{2\log(\sin(a+bx))}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

output
```
(2*Log[Sin[a + b*x]])/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4776, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$2\int \cot(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-2\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{2\log(-\sin(a+bx))}{b}$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

output
```
(2*Log[-Sin[a + b*x]])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2\ln(\sin(bx+a))}{b}$	13
risch	$-2ix-\frac{4ia}{b}+\frac{2\ln\left(e^{2i(bx+a)}-1\right)}{b}$	30

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
2*ln(sin(b*x+a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx=\frac{2\log\left(\frac{1}{2}\sin\left(bx+a\right)\right)}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
2*log(1/2*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx=\text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(12) = 24.

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 6.75

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx$$
$$=\frac{\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\cos\left(a\right)+\cos\left(a\right)^2+\sin\left(bx\right)^2-2\,\sin\left(bx\right)\sin\left(a\right)+\sin\left(a\right)^2\right)+\log\left(\cos\left(bx\right)^2-\right.}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
(log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*s
in(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*
x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx = \frac{2\,\log\left(|\sin\left(bx+a\right)|\right)}{b}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
2*log(abs(sin(b*x + a)))/b
```

Mupad [B] (verification not implemented)

Time = 18.46 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx = \frac{\ln\left(\sin\left(a+b\,x\right)^2\right)}{b}$$

input
```
int(sin(2*a + 2*b*x)/sin(a + b*x)^2,x)
```

output
```
log(sin(a + b*x)^2)/b
```

Reduce [F]

$$\int \csc^2(a+bx)\sin(2a+2bx)\,dx = \int \csc(bx+a)^2 \sin(2bx+2a)\,dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a),x)
```

output
```
int(csc(a + b*x)**2*sin(2*a + 2*b*x),x)
```

3.463 $\int \csc^2(a+bx)\csc(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 30

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx=-\frac{\cot^2(a+bx)}{4b}+\frac{\log(\tan(a+bx))}{2b}$$

output
```
-1/4*cot(b*x+a)^2/b+1/2*ln(tan(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx=-\frac{\csc^2(a+bx)}{4b}-\frac{\log(\cos(a+bx))}{2b}+\frac{\log(\sin(a+bx))}{2b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

output
```
-1/4*Csc[a + b*x]^2/b - Log[Cos[a + b*x]]/(2*b) + Log[Sin[a + b*x]]/(2*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4776, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{ 4776}$$

$$\frac{1}{2}\int \csc^3(a+bx)\sec(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \csc(a+bx)^3\sec(a+bx)dx$$

$$\Big\downarrow \text{ 3100}$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{2b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left(\cot^3(a+bx)+\cot(a+bx)\right)d\tan(a+bx)}{2b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\log(\tan(a+bx))-\frac{1}{2}\cot^2(a+bx)}{2b}$$

input
```
Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

output
```
(-1/2*Cot[a + b*x]^2 + Log[Tan[a + b*x]])/(2*b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{-\frac{1}{2\sin(bx+a)^2}+\ln(\tan(bx+a))}{2b}$	24
risch	$\frac{e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^2}-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{2b}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{2b}$	62

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(-1/2/sin(b*x+a)^2+ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx = -\frac{\left(\cos(bx+a)^2-1\right)\log\left(\cos(bx+a)^2\right)-\left(\cos(bx+a)^2-1\right)\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-1}{4\left(b\cos(bx+a)^2-b\right)}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-1/4*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log(
-1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F]

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx = \int \csc^2(a+bx)\csc(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(2*b*x+2*a),x)
```

output
```
Integral(csc(a + b*x)**2*csc(2*a + 2*b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 656, normalized size of antiderivative = 21.87

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="maxima")
```

output
```
1/4*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) - 8*cos(2*b*x + 2*a)^2 + (2*(2*co
s(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x +
2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(
2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*c
os(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)
- (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*co
s(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(
b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*(
2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*
x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*
sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*c
os(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b*
x + 4*a)*sin(2*b*x + 2*a) - 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b*
cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*b
*x + 2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) + b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx$$
$$= -\frac{\frac{1}{\sin(bx+a)^2} + \log\left(-\sin(bx+a)^2+1\right) - 2\log\left(|\sin(bx+a)|\right)}{4b}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="giac")
```

output
```
-1/4*(1/sin(b*x + a)^2 + log(-sin(b*x + a)^2 + 1) - 2*log(abs(sin(b*x + a)
)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx=-\frac{\frac{\ln(\cos(a+b\,x))}{2}-\frac{\ln\left(\sin(a+b\,x)^2\right)}{4}+\frac{1}{4\sin(a+b\,x)^2}}{b}$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)),x)
```

output
```
-(log(cos(a + b*x))/2 - log(sin(a + b*x)^2)/4 + 1/(4*sin(a + b*x)^2))/b
```

Reduce [F]

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx=\int \csc\left(2bx+2a\right)\csc\left(bx+a\right)^2dx$$

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a),x)
```

output
```
int(csc(2*a + 2*b*x)*csc(a + b*x)**2,x)
```

3.464 $\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx=-\frac{\cot(a+bx)}{2b}-\frac{\cot^3(a+bx)}{12b}+\frac{\tan(a+bx)}{4b}$$

output
```
-1/2*cot(b*x+a)/b-1/12*cot(b*x+a)^3/b+1/4*tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx=-\frac{5\cot(a+bx)}{12b}-\frac{\cot(a+bx)\csc^2(a+bx)}{12b}+\frac{\tan(a+bx)}{4b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

output
```
(-5*Cot[a + b*x])/(12*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(12*b) + Tan[a +
b*x]/(4*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{4}\int \csc^4(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc(a+bx)^4\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^4(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{4b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^4(a+bx)+2\cot^2(a+bx)+1\right)d\tan(a+bx)}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan(a+bx)-\frac{1}{3}\cot^3(a+bx)-2\cot(a+bx)}{4b}$$

input
```
Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

output
```
(-2*Cot[a + b*x] - Cot[a + b*x]^3/3 + Tan[a + b*x])/(4*b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result	size
risch	$\frac{4i\left(2\,\mathrm{e}^{2i(bx+a)}-1\right)}{3b\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\left(\mathrm{e}^{2i(bx+a)}+1\right)}$	46
default	$\frac{-\frac{1}{3\sin(bx+a)^{3}\cos(bx+a)}+\frac{4}{3\sin(bx+a)\cos(bx+a)}-\frac{8\cot(bx+a)}{3}}{4b}$	51

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/3*I*(2*exp(2*I*(b*x+a))-1)/b/(exp(2*I*(b*x+a))-1)^3/(exp(2*I*(b*x+a))+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{8\,\cos(bx+a)^4-12\,\cos(bx+a)^2+3}{12\,(b\cos(bx+a)^3-b\cos(bx+a))\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-1/12*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*co
s(b*x + a))*sin(b*x + a))
```

Sympy [F]

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx = \int \csc^2(a+bx)\csc^2(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**2,x)
```

output
```
Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(36) = 72.

Time = 0.15 (sec) , antiderivative size = 308, normalized size of antiderivative = 7.33

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx$$

$$= \frac{4((}{3\,(b\cos(8\,bx+8\,a)^2+4\,b\cos(6\,bx+6\,a)^2+4\,b\cos(2\,bx+2\,a)^2+b\sin(8\,bx+8\,a)^2+4\,b\sin(6\,bx+6\,a}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
4/3*((2*cos(2*b*x + 2*a) - 1)*sin(8*b*x + 8*a) - 2*(2*cos(2*b*x + 2*a) - 1
)*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) + 4*cos(6*b*x + 6
*a)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b
*cos(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b*
sin(6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(6*
b*x + 6*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*x
 + 2*a) - b)*cos(6*b*x + 6*a) - 4*b*cos(2*b*x + 2*a) - 4*(b*sin(6*b*x + 6*
a) - b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\frac{6\,\tan(bx+a)^2+1}{\tan(bx+a)^3} - 3\,\tan(bx+a)}{12\,b}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-1/12*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 18.94 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx = \frac{\tan(a+b\,x)}{4\,b} - \frac{\frac{\tan(a+b\,x)^2}{2}+\frac{1}{12}}{b\tan\left(a+b\,x\right)^3}$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^2),x)
```

output
```
tan(a + b*x)/(4*b) - (tan(a + b*x)^2/2 + 1/12)/(b*tan(a + b*x)^3)
```

Reduce [F]

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx = \int \csc(2bx+2a)^2\csc(bx+a)^2\,dx$$

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x)
```

output
```
int(csc(2*a + 2*b*x)**2*csc(a + b*x)**2,x)
```

3.465 $\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx=-\frac{3\cot^2(a+bx)}{16b}-\frac{\cot^4(a+bx)}{32b}+\frac{3\log(\tan(a+bx))}{8b}+\frac{\tan^2(a+bx)}{16b}$$

output
```
-3/16*cot(b*x+a)^2/b-1/32*cot(b*x+a)^4/b+3/8*ln(tan(b*x+a))/b+1/16*tan(b*x
+a)^2/b
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx=-\frac{4\csc^2(a+bx)+\csc^4(a+bx)+12\log(\cos(a+bx))-12\log(\sin(a+bx))-2\sec^2(a+bx)}{32b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

output
```
-1/32*(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[S
in[a + b*x]] - 2*Sec[a + b*x]^2)/b
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4776, 3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{8}\int \csc^5(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)^5\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^5(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan(a+bx)}{8b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan^2(a+bx)}{16b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\cot^3(a+bx)+3\cot^2(a+bx)+3\cot(a+bx)+1\right)d\tan^2(a+bx)}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan^2(a+bx) - \frac{1}{2}\cot^2(a+bx) - 3\cot(a+bx) + 3\log\left(\tan^2(a+bx)\right)}{16b}$$

input
```
Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

output
```
(-3*Cot[a + b*x] - Cot[a + b*x]^2/2 + 3*Log[Tan[a + b*x]^2] + Tan[a + b*x]
^2)/(16*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{-\frac{1}{4\sin(bx+a)^4\cos(bx+a)^2}+\frac{3}{4\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{2\sin(bx+a)^2}+3\ln(\tan(bx+a))}{8b}$	62
risch	$\frac{3\,e^{10i(bx+a)}-6\,e^{8i(bx+a)}-2\,e^{6i(bx+a)}-6\,e^{4i(bx+a)}+3\,e^{2i(bx+a)}}{4b\left(e^{2i(bx+a)}-1\right)^4\left(e^{2i(bx+a)}+1\right)^2}-\frac{3\ln\left(e^{2i(bx+a)}+1\right)}{8b}+\frac{3\ln\left(e^{2i(bx+a)}-1\right)}{8b}$	123

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^2+3/4/sin(b*x+a)^2/cos(b*x+a)^2-3/2/si
n(b*x+a)^2+3*ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(52) = 104.

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.30

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx = \frac{6\cos(bx+a)^4-9\cos(bx+a)^2-6\left(\cos(bx+a)^6-2\cos(bx+a)^4+\cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right.}{32\left(b\cos(bx+a)^6-2\,b\cos(bx+a)^4\right.}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
1/32*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x
+ a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(b
*x + a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*x
 + a)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)
```

Sympy [F]

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx=\int \csc^2\left(a+bx\right)\csc^3\left(2a+2bx\right)dx$$

input
```
integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**3,x)
```

output
```
Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3188 vs. $2(52) = 104$.

Time = 0.24 (sec) , antiderivative size = 3188, normalized size of antiderivative = 53.13

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx=\text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/16*(4*(3*cos(10*b*x + 10*a) - 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) -
6*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b
*x + 8*a) + 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) - 12*cos(2*b*x + 2*a)
 + 3)*cos(10*b*x + 10*a) - 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a
) - 12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) + 6)*cos(8*b*x + 8*a) + 24*co
s(8*b*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) - 8*cos(2*b*x + 2*a) + 1)*cos(6*
b*x + 6*a) - 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) - 2)*cos(4*b*x
 + 4*a) + 24*cos(4*b*x + 4*a)^2 - 24*cos(2*b*x + 2*a)^2 + 3*(2*(2*cos(10*b
*x + 10*a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*
cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8
*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) -
 1)*cos(10*b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) -
cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x +
8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) -
16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(
4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*
x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x
 + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*si
n(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2...
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$$
$$= -\frac{\frac{6\sin(bx+a)^4-3\sin(bx+a)^2-1}{\left(\sin(bx+a)^2-1\right)\sin(bx+a)^4}+6\log\left(-\sin\left(bx+a\right)^2+1\right)-12\log\left(\left|\sin\left(bx+a\right)\right|\right)}{32\,b}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-1/32*((6*sin(b*x + a)^4 - 3*sin(b*x + a)^2 - 1)/((sin(b*x + a)^2 - 1)*sin
(b*x + a)^4) + 6*log(-sin(b*x + a)^2 + 1) - 12*log(abs(sin(b*x + a))))/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.37

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$$
$$=\frac{3\,\ln\left(\sin\left(a+b\,x\right)^2\right)}{16\,b}-\frac{3\,\ln\left(\cos\left(a+b\,x\right)\right)}{8\,b}$$
$$+\frac{\frac{3\cos(a+b\,x)^4}{16}-\frac{9\cos(a+b\,x)^2}{32}+\frac{1}{16}}{b\,\left(\cos\left(a+b\,x\right)^6-2\cos\left(a+b\,x\right)^4+\cos\left(a+b\,x\right)^2\right)}$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^3),x)
```

output
```
(3*log(sin(a + b*x)^2))/(16*b) - (3*log(cos(a + b*x)))/(8*b) + ((3*cos(a +
 b*x)^4)/16 - (9*cos(a + b*x)^2)/32 + 1/16)/(b*(cos(a + b*x)^2 - 2*cos(a +
 b*x)^4 + cos(a + b*x)^6))
```

Reduce [F]

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx=\int \csc\left(2bx+2a\right)^3\csc\left(bx+a\right)^2dx$$

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x)
```

output
```
int(csc(2*a + 2*b*x)**3*csc(a + b*x)**2,x)
```

3.466 $\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx=-\frac{3\cot(a+bx)}{8b}-\frac{\cot^3(a+bx)}{12b}-\frac{\cot^5(a+bx)}{80b}+\frac{\tan(a+bx)}{4b}+\frac{\tan^3(a+bx)}{48b}$$

output
```
-3/8*cot(b*x+a)/b-1/12*cot(b*x+a)^3/b-1/80*cot(b*x+a)^5/b+1/4*tan(b*x+a)/b
+1/48*tan(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx=-\frac{73\cot(a+bx)}{240b}-\frac{7\cot(a+bx)\csc^2(a+bx)}{120b}-\frac{\cot(a+bx)\csc^4(a+bx)}{80b}+\frac{11\tan(a+bx)}{48b}+\frac{\sec^2(a+bx)\tan(a+bx)}{48b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

output
```
(-73*Cot[a + b*x])/(240*b) - (7*Cot[a + b*x]*Csc[a + b*x]^2)/(120*b) - (Co
t[a + b*x]*Csc[a + b*x]^4)/(80*b) + (11*Tan[a + b*x])/(48*b) + (Sec[a + b*
x]^2*Tan[a + b*x])/(48*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{16}\int \csc^6(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^6\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^6(a+bx)\left(\tan^2(a+bx)+1\right)^4d\tan(a+bx)}{16b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^6(a+bx)+4\cot^4(a+bx)+6\cot^2(a+bx)+\tan^2(a+bx)+4\right)d\tan(a+bx)}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\tan^3(a+bx)+4\tan(a+bx)-\frac{1}{5}\cot^5(a+bx)-\frac{4}{3}\cot^3(a+bx)-6\cot(a+bx)}{16b}$$

input
```
Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

output
```
(-6*Cot[a + b*x] - (4*Cot[a + b*x]^3)/3 - Cot[a + b*x]^5/5 + 4*Tan[a + b*x
] + Tan[a + b*x]^3/3)/(16*b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

method	result	size
risch	$-\frac{16i\left(6\,{\mathrm e}^{6i(bx+a)}-2\,{\mathrm e}^{4i(bx+a)}-2\,{\mathrm e}^{2i(bx+a)}+1\right)}{15b\left({\mathrm e}^{2i(bx+a)}+1\right)^{3}\left({\mathrm e}^{2i(bx+a)}-1\right)^{5}}$	68
default	$\frac{-\frac{1}{5\sin\left(bx+a\right)^{5}\cos\left(bx+a\right)^{3}}+\frac{8}{15\sin\left(bx+a\right)^{3}\cos\left(bx+a\right)^{3}}-\frac{16}{15\sin\left(bx+a\right)^{3}\cos\left(bx+a\right)}+\frac{64}{15\sin\left(bx+a\right)\cos\left(bx+a\right)}-\frac{128\cot\left(bx+a\right)}{15}}{16b}$	87

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
-16/15*I*(6*exp(6*I*(b*x+a))-2*exp(4*I*(b*x+a))-2*exp(2*I*(b*x+a))+1)/b/(e
xp(2*I*(b*x+a))+1)^3/(exp(2*I*(b*x+a))-1)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx = -\frac{128\,\cos\left(bx+a\right)^{8}-320\,\cos\left(bx+a\right)^{6}+240\,\cos\left(bx+a\right)^{4}-40\,\cos\left(bx+a\right)^{2}-5}{240\,\left(b\cos\left(bx+a\right)^{7}-2\,b\cos\left(bx+a\right)^{5}+b\cos\left(bx+a\right)^{3}\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
-1/240*(128*cos(b*x + a)^8 - 320*cos(b*x + a)^6 + 240*cos(b*x + a)^4 - 40*
cos(b*x + a)^2 - 5)/((b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x +
a)^3)*sin(b*x + a))
```

Sympy [F]

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx = \int \csc^2(a+bx)\csc^4(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**4,x)
```

output
```
Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. $2(62) = 124$.

Time = 0.12 (sec) , antiderivative size = 1227, normalized size of antiderivative = 17.04

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
16/15*(2*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(16
*b*x + 16*a) - 4*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x + 2*a)
)*cos(14*b*x + 14*a) - 4*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*
x + 2*a))*cos(12*b*x + 12*a) + 12*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) -
 sin(2*b*x + 2*a))*cos(10*b*x + 10*a) - (6*cos(6*b*x + 6*a) - 2*cos(4*b*x
+ 4*a) - 2*cos(2*b*x + 2*a) + 1)*sin(16*b*x + 16*a) + 2*(6*cos(6*b*x + 6*a
) - 2*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*sin(14*b*x + 14*a) + 2*(6
*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*sin(12*b*
x + 12*a) - 6*(6*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a
) + 1)*sin(10*b*x + 10*a))/(b*cos(16*b*x + 16*a)^2 + 4*b*cos(14*b*x + 14*a
)^2 + 4*b*cos(12*b*x + 12*a)^2 + 36*b*cos(10*b*x + 10*a)^2 + 36*b*cos(6*b*
x + 6*a)^2 + 4*b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(16*b*
x + 16*a)^2 + 4*b*sin(14*b*x + 14*a)^2 + 4*b*sin(12*b*x + 12*a)^2 + 36*b*s
in(10*b*x + 10*a)^2 + 36*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2 + 8
*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos
(14*b*x + 14*a) + 2*b*cos(12*b*x + 12*a) - 6*b*cos(10*b*x + 10*a) + 6*b*co
s(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(16*b
*x + 16*a) + 4*(2*b*cos(12*b*x + 12*a) - 6*b*cos(10*b*x + 10*a) + 6*b*cos(
6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(14*b*x
 + 14*a) - 4*(6*b*cos(10*b*x + 10*a) - 6*b*cos(6*b*x + 6*a) + 2*b*cos(4...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx = \frac{5\,\tan(bx+a)^3 - \frac{90\,\tan(bx+a)^4+20\,\tan(bx+a)^2+3}{\tan(bx+a)^5} + 60\,\tan(bx+a)}{240\,b}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/240*(5*tan(b*x + a)^3 - (90*tan(b*x + a)^4 + 20*tan(b*x + a)^2 + 3)/tan(
b*x + a)^5 + 60*tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 18.80 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx = -\frac{-5\tan(a+b\,x)^8-60\tan(a+b\,x)^6+90\tan(a+b\,x)^4+20\tan(a+b\,x)^2+3}{240\,b\tan\left(a+b\,x\right)^5}$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^4),x)
```

output
```
-(20*tan(a + b*x)^2 + 90*tan(a + b*x)^4 - 60*tan(a + b*x)^6 - 5*tan(a + b*
x)^8 + 3)/(240*b*tan(a + b*x)^5)
```

Reduce [F]

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx = \int \csc\left(2bx+2a\right)^4\csc\left(bx+a\right)^2dx$$

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x)
```

output
```
int(csc(2*a + 2*b*x)**4*csc(a + b*x)**2,x)
```

3.467 $\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx = -\frac{5\cot^2(a+bx)}{32b}-\frac{5\cot^4(a+bx)}{128b}-\frac{\cot^6(a+bx)}{192b}+\frac{5\log(\tan(a+bx))}{16b}+\frac{5\tan^2(a+bx)}{64b}+\frac{\tan^4(a+bx)}{128b}$$

output
```
-5/32*cot(b*x+a)^2/b-5/128*cot(b*x+a)^4/b-1/192*cot(b*x+a)^6/b+5/16*ln(tan
(b*x+a))/b+5/64*tan(b*x+a)^2/b+1/128*tan(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx = -\frac{36\csc^2(a+bx)+9\csc^4(a+bx)+2\csc^6(a+bx)+120\log(\cos(a+bx))-120\log(\sin(a+bx))-24\,\mathrm{s}}{384b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

output
```
-1/384*(36*Csc[a + b*x]^2 + 9*Csc[a + b*x]^4 + 2*Csc[a + b*x]^6 + 120*Log[
Cos[a + b*x]] - 120*Log[Sin[a + b*x]] - 24*Sec[a + b*x]^2 - 3*Sec[a + b*x]
^4)/b
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4776, 3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{32}\int \csc^7(a+bx)\sec^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \csc(a+bx)^7\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^7(a+bx)\left(\tan^2(a+bx)+1\right)^5d\tan(a+bx)}{32b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \cot^4(a+bx)\left(\tan^2(a+bx)+1\right)^5d\tan^2(a+bx)}{64b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\cot^4(a+bx)+5\cot^3(a+bx)+10\cot^2(a+bx)+10\cot(a+bx)+\tan^2(a+bx)+5\right)d\tan^2(a+bx)}{64b}$$

$$\big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\tan^4(a+bx)+5\tan^2(a+bx)-\frac{1}{3}\cot^3(a+bx)-\frac{5}{2}\cot^2(a+bx)-10\cot(a+bx)+10\log\left(\tan^2(a+bx)\right)}{64b}$$

input
```
Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

output
```
(-10*Cot[a + b*x] - (5*Cot[a + b*x]^2)/2 - Cot[a + b*x]^3/3 + 10*Log[Tan[a
 + b*x]^2] + 5*Tan[a + b*x]^2 + Tan[a + b*x]^4/2)/(64*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

method	result
default	$\frac{-\frac{1}{6\sin(bx+a)^6\cos(bx+a)^4}+\frac{5}{12\sin(bx+a)^4\cos(bx+a)^4}-\frac{5}{6\sin(bx+a)^4\cos(bx+a)^2}+\frac{5}{2\sin(bx+a)^2\cos(bx+a)^2}-\frac{5}{\sin(bx+a)^2}+10\ln(\tan(bx+a)}{32b}$
risch	$\frac{15\,e^{18i(bx+a)}-30\,e^{16i(bx+a)}-40\,e^{14i(bx+a)}+110\,e^{12i(bx+a)}+18\,e^{10i(bx+a)}+110\,e^{8i(bx+a)}-40\,e^{6i(bx+a)}-30\,e^{4i(bx+a)}+15\,e^{2i(bx+a)}}{24b\left(e^{2i(bx+a)}-1\right)^6\left(e^{2i(bx+a)}+1\right)^4}$

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(-1/6/sin(b*x+a)^6/cos(b*x+a)^4+5/12/sin(b*x+a)^4/cos(b*x+a)^4-5/6/
sin(b*x+a)^4/cos(b*x+a)^2+5/2/sin(b*x+a)^2/cos(b*x+a)^2-5/sin(b*x+a)^2+10*
ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(78) = 156$.

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.16

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx$$
$$= \frac{60\cos(bx+a)^8-150\cos(bx+a)^6+110\cos(bx+a)^4-15\cos(bx+a)^2-60\left(\cos(bx+a)^{10}-3\cos\right.}{384\,(b}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
1/384*(60*cos(b*x + a)^8 - 150*cos(b*x + a)^6 + 110*cos(b*x + a)^4 - 15*co
s(b*x + a)^2 - 60*(cos(b*x + a)^10 - 3*cos(b*x + a)^8 + 3*cos(b*x + a)^6 -
 cos(b*x + a)^4)*log(cos(b*x + a)^2) + 60*(cos(b*x + a)^10 - 3*cos(b*x + a
)^8 + 3*cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*cos(b*x + a)^2 + 1/4) -
3)/(b*cos(b*x + a)^10 - 3*b*cos(b*x + a)^8 + 3*b*cos(b*x + a)^6 - b*cos(b*
x + a)^4)
```

Sympy [F]

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx = \int \csc^2{(a+bx)}\csc^5{(2a+2bx)}\,dx$$

input
```
integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**5,x)
```

output
```
Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7650 vs. $2(78) = 156$.

Time = 0.72 (sec) , antiderivative size = 7650, normalized size of antiderivative = 85.00

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/96*(4*(15*cos(18*b*x + 18*a) - 30*cos(16*b*x + 16*a) - 40*cos(14*b*x + 1
4*a) + 110*cos(12*b*x + 12*a) + 18*cos(10*b*x + 10*a) + 110*cos(8*b*x + 8*
a) - 40*cos(6*b*x + 6*a) - 30*cos(4*b*x + 4*a) + 15*cos(2*b*x + 2*a))*cos(
20*b*x + 20*a) + 4*(15*cos(16*b*x + 16*a) + 200*cos(14*b*x + 14*a) - 190*c
os(12*b*x + 12*a) - 216*cos(10*b*x + 10*a) - 190*cos(8*b*x + 8*a) + 200*co
s(6*b*x + 6*a) + 15*cos(4*b*x + 4*a) - 60*cos(2*b*x + 2*a) + 15)*cos(18*b*
x + 18*a) - 120*cos(18*b*x + 18*a)^2 - 12*(40*cos(14*b*x + 14*a) + 130*cos
(12*b*x + 12*a) - 102*cos(10*b*x + 10*a) + 130*cos(8*b*x + 8*a) + 40*cos(6
*b*x + 6*a) - 60*cos(4*b*x + 4*a) - 5*cos(2*b*x + 2*a) + 10)*cos(16*b*x +
16*a) + 360*cos(16*b*x + 16*a)^2 + 32*(100*cos(12*b*x + 12*a) + 78*cos(10*
b*x + 10*a) + 100*cos(8*b*x + 8*a) - 80*cos(6*b*x + 6*a) - 15*cos(4*b*x +
4*a) + 25*cos(2*b*x + 2*a) - 5)*cos(14*b*x + 14*a) - 1280*cos(14*b*x + 14*
a)^2 - 8*(642*cos(10*b*x + 10*a) - 220*cos(8*b*x + 8*a) - 400*cos(6*b*x +
6*a) + 195*cos(4*b*x + 4*a) + 95*cos(2*b*x + 2*a) - 55)*cos(12*b*x + 12*a)
 + 880*cos(12*b*x + 12*a)^2 - 24*(214*cos(8*b*x + 8*a) - 104*cos(6*b*x + 6
*a) - 51*cos(4*b*x + 4*a) + 36*cos(2*b*x + 2*a) - 3)*cos(10*b*x + 10*a) -
864*cos(10*b*x + 10*a)^2 + 40*(80*cos(6*b*x + 6*a) - 39*cos(4*b*x + 4*a) -
 19*cos(2*b*x + 2*a) + 11)*cos(8*b*x + 8*a) + 880*cos(8*b*x + 8*a)^2 - 160
*(3*cos(4*b*x + 4*a) - 5*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 1280*cos
(6*b*x + 6*a)^2 + 60*(cos(2*b*x + 2*a) - 2)*cos(4*b*x + 4*a) + 360*cos(...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.04

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx = -\frac{\frac{60\sin(bx+a)^8-90\sin(bx+a)^6+20\sin(bx+a)^4+5\sin(bx+a)^2+2}{\left(\sin(bx+a)^2-1\right)^2\sin(bx+a)^6}+60\log\left(-\sin\left(bx+a\right)^2+1\right)-120\log\left(\left|\sin\left(bx+a\right.\right.\right.}{384\,b}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
-1/384*((60*sin(b*x + a)^8 - 90*sin(b*x + a)^6 + 20*sin(b*x + a)^4 + 5*sin
(b*x + a)^2 + 2)/((sin(b*x + a)^2 - 1)^2*sin(b*x + a)^6) + 60*log(-sin(b*x
 + a)^2 + 1) - 120*log(abs(sin(b*x + a))))/b
```

Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.27

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx$$
$$= \frac{5\ln\left(\sin\left(a+b\,x\right)^2\right)}{32\,b}-\frac{5\ln\left(\cos\left(a+b\,x\right)\right)}{16\,b}+\frac{-\frac{5\cos(a+b\,x)^8}{32}+\frac{25\cos(a+b\,x)^6}{64}-\frac{55\cos(a+b\,x)^4}{192}+\frac{5\cos(a+b\,x)^2}{128}+\frac{1}{128}}{b\,\left(-\cos\left(a+b\,x\right)^{10}+3\cos\left(a+b\,x\right)^8-3\cos\left(a+b\,x\right)^6+\cos\left(a+b\,x\right)^4\right)}$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^5),x)
```

output
```
(5*log(sin(a + b*x)^2))/(32*b) - (5*log(cos(a + b*x)))/(16*b) + ((5*cos(a
+ b*x)^2)/128 - (55*cos(a + b*x)^4)/192 + (25*cos(a + b*x)^6)/64 - (5*cos(
a + b*x)^8)/32 + 1/128)/(b*(cos(a + b*x)^4 - 3*cos(a + b*x)^6 + 3*cos(a +
b*x)^8 - cos(a + b*x)^10))
```

Reduce [F]

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx = \int \csc\left(2bx+2a\right)^5\csc\left(bx+a\right)^2 dx$$

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x)
```

output
```
int(csc(2*a + 2*b*x)**5*csc(a + b*x)**2,x)
```

3.468 $\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 102

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx=-\frac{5\cot(a+bx)}{16b}-\frac{5\cot^3(a+bx)}{64b}-\frac{3\cot^5(a+bx)}{160b}-\frac{\cot^7(a+bx)}{448b}+\frac{15\tan(a+bx)}{64b}+\frac{\tan^3(a+bx)}{32b}+\frac{\tan^5(a+bx)}{320b}$$

output
```
-5/16*cot(b*x+a)/b-5/64*cot(b*x+a)^3/b-3/160*cot(b*x+a)^5/b-1/448*cot(b*x+
a)^7/b+15/64*tan(b*x+a)/b+1/32*tan(b*x+a)^3/b+1/320*tan(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.29

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx = -\frac{281\cot(a+bx)}{1120b} - \frac{53\cot(a+bx)\csc^2(a+bx)}{1120b} - \frac{27\cot(a+bx)\csc^4(a+bx)}{2240b} - \frac{\cot(a+bx)\csc^6(a+bx)}{448b} + \frac{33\tan(a+bx)}{160b} + \frac{\sec^2(a+bx)\tan(a+bx)}{40b} + \frac{\sec^4(a+bx)\tan(a+bx)}{320b}$$

input
```
Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^6,x]
```

output
```
(-281*Cot[a + b*x])/(1120*b) - (53*Cot[a + b*x]*Csc[a + b*x]^2)/(1120*b) -
 (27*Cot[a + b*x]*Csc[a + b*x]^4)/(2240*b) - (Cot[a + b*x]*Csc[a + b*x]^6)
/(448*b) + (33*Tan[a + b*x])/(160*b) + (Sec[a + b*x]^2*Tan[a + b*x])/(40*b
) + (Sec[a + b*x]^4*Tan[a + b*x])/(320*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^6}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{64}\int \csc^8(a+bx)\sec^6(a+bx)dx$$

$\Big\downarrow$ 3042

$$\frac{1}{64}\int \csc(a+bx)^8\sec(a+bx)^6dx$$

$\Big\downarrow$ 3100

$$\frac{\int \cot^8(a+bx)\left(\tan^2(a+bx)+1\right)^6d\tan(a+bx)}{64b}$$

$\Big\downarrow$ 244

$$\frac{\int \left(\cot^8(a+bx)+6\cot^6(a+bx)+15\cot^4(a+bx)+20\cot^2(a+bx)+\tan^4(a+bx)+6\tan^2(a+bx)+15\right)d\,\text{ta}}{64b}$$

$\Big\downarrow$ 2009

$$\frac{\frac{1}{5}\tan^5(a+bx)+2\tan^3(a+bx)+15\tan(a+bx)-\frac{1}{7}\cot^7(a+bx)-\frac{6}{5}\cot^5(a+bx)-5\cot^3(a+bx)-20\cot(a}{64b}$$

input
```
Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^6,x]
```

output
```
(-20*Cot[a + b*x] - 5*Cot[a + b*x]^3 - (6*Cot[a + b*x]^5)/5 - Cot[a + b*x]
^7/7 + 15*Tan[a + b*x] + 2*Tan[a + b*x]^3 + Tan[a + b*x]^5/5)/(64*b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

method	result
risch	$\frac{32 i \left(20 \,{\mathrm e}^{10 i \left(b x +a \right)}-5 \,{\mathrm e}^{8 i \left(b x +a \right)}-10 \,{\mathrm e}^{6 i \left(b x +a \right)}+4 \,{\mathrm e}^{4 i \left(b x +a \right)}+2 \,{\mathrm e}^{2 i \left(b x +a \right)}-1\right)}{35 b \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)^{7} \left({\mathrm e}^{2 i \left(b x +a \right)}+1\right)^{5}}$
default	$\frac{-\frac{1}{7 \sin \left(b x +a \right)^{7} \cos \left(b x +a \right)^{5}}+\frac{12}{35 \sin \left(b x +a \right)^{5} \cos \left(b x +a \right)^{5}}-\frac{24}{35 \sin \left(b x +a \right)^{5} \cos \left(b x +a \right)^{3}}+\frac{64}{35 \sin \left(b x +a \right)^{3} \cos \left(b x +a \right)^{3}}-\frac{128}{35 \sin \left(b x +a \right)^{3} \cos \left(b x +a \right)}+\frac{}{35}}{64 b}$

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

output
```
32/35*I*(20*exp(10*I*(b*x+a))-5*exp(8*I*(b*x+a))-10*exp(6*I*(b*x+a))+4*exp
(4*I*(b*x+a))+2*exp(2*I*(b*x+a))-1)/b/(exp(2*I*(b*x+a))-1)^7/(exp(2*I*(b*x
+a))+1)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.16

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx =$$
$$-\frac{1024 \cos\left(bx+a\right)^{12} - 3584 \cos\left(bx+a\right)^{10} + 4480 \cos\left(bx+a\right)^{8} - 2240 \cos\left(bx+a\right)^{6} + 280 \cos\left(bx+\right.}{2240 \left(b\cos\left(bx+a\right)^{11} - 3\,b\cos\left(bx+a\right)^{9} + 3\,b\cos\left(bx+a\right)^{7} - b\cos\left(bx+a\right)^{5}\right)\sin\left(\right.}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="fricas")
```

output
```
-1/2240*(1024*cos(b*x + a)^12 - 3584*cos(b*x + a)^10 + 4480*cos(b*x + a)^8
 - 2240*cos(b*x + a)^6 + 280*cos(b*x + a)^4 + 28*cos(b*x + a)^2 + 7)/((b*c
os(b*x + a)^11 - 3*b*cos(b*x + a)^9 + 3*b*cos(b*x + a)^7 - b*cos(b*x + a)^
5)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**6,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2710 vs. $2(88) = 176$.

Time = 0.25 (sec) , antiderivative size = 2710, normalized size of antiderivative = 26.57

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="maxima")
```

output
```
-32/35*((20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a)
+ 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(24*b*x + 24*a) - 2*(20*sin(
10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x +
4*a) + 2*sin(2*b*x + 2*a))*cos(22*b*x + 22*a) - 4*(20*sin(10*b*x + 10*a) -
 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b
*x + 2*a))*cos(20*b*x + 20*a) + 10*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x +
8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(
18*b*x + 18*a) + 5*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*
b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(16*b*x + 16*a) -
 20*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*
sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(14*b*x + 14*a) - (20*cos(10*b*x
 + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) +
 2*cos(2*b*x + 2*a) - 1)*sin(24*b*x + 24*a) + 2*(20*cos(10*b*x + 10*a) - 5
*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x
 + 2*a) - 1)*sin(22*b*x + 22*a) + 4*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x +
 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)
*sin(20*b*x + 20*a) - 10*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*
cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(18*b*x
 + 18*a) - 5*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x +
6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(16*b*x + 16*a) ...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx = \frac{7\tan(bx+a)^5+70\tan(bx+a)^3-\frac{700\tan(bx+a)^6+175\tan(bx+a)^4+42\tan(bx+a)^2+5}{\tan(bx+a)^7}+525\tan(bx+a)}{2240\,b}$$

input
```
integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="giac")
```

output
```
1/2240*(7*tan(b*x + a)^5 + 70*tan(b*x + a)^3 - (700*tan(b*x + a)^6 + 175*t
an(b*x + a)^4 + 42*tan(b*x + a)^2 + 5)/tan(b*x + a)^7 + 525*tan(b*x + a))/
b
```

Mupad [B] (verification not implemented)

Time = 18.49 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx$$
$$=\frac{15\tan(a+b\,x)}{64\,b}+\frac{\tan(a+b\,x)^3}{32\,b}+\frac{\tan(a+b\,x)^5}{320\,b}-\frac{\cot(a+b\,x)^7\left(\frac{5\tan(a+b\,x)^6}{16}+\frac{5\tan(a+b\,x)^4}{64}+\frac{3\tan(a+b\,x)^2}{160}+\frac{1}{448}\right)}{b}$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^6),x)
```

output
```
(15*tan(a + b*x))/(64*b) + tan(a + b*x)^3/(32*b) + tan(a + b*x)^5/(320*b)
- (cot(a + b*x)^7*((3*tan(a + b*x)^2)/160 + (5*tan(a + b*x)^4)/64 + (5*tan
(a + b*x)^6)/16 + 1/448))/b
```

Reduce [F]

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx=\int \csc(2bx+2a)^6\csc(bx+a)^2\,dx$$

input
```
int(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x)
```

output
```
int(csc(2*a + 2*b*x)**6*csc(a + b*x)**2,x)
```

3.469 $\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx = -\frac{1024\cos^{11}(a+bx)}{11b}+\frac{3072\cos^{13}(a+bx)}{13b}-\frac{1024\cos^{15}(a+bx)}{5b}+\frac{1024\cos^{17}(a+bx)}{17b}$$

output
```
-1024/11*cos(b*x+a)^11/b+3072/13*cos(b*x+a)^13/b-1024/5*cos(b*x+a)^15/b+10
24/17*cos(b*x+a)^17/b
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx = -\frac{35\cos(a+bx)}{32b}-\frac{7\cos(3(a+bx))}{16b}+\frac{7\cos(5(a+bx))}{80b}+\frac{\cos(7(a+bx))}{8b}-\frac{5\cos(11(a+bx))}{176b}-\frac{\cos(13(a+bx))}{208b}+\frac{\cos(15(a+bx))}{320b}+\frac{\cos(17(a+bx))}{1088b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^10,x]
```

output
```
(-35*Cos[a + b*x])/(32*b) - (7*Cos[3*(a + b*x)])/(16*b) + (7*Cos[5*(a + b*
x)])/(80*b) + Cos[7*(a + b*x)]/(8*b) - (5*Cos[11*(a + b*x)])/(176*b) - Cos
[13*(a + b*x)]/(208*b) + Cos[15*(a + b*x)]/(320*b) + Cos[17*(a + b*x)]/(10
88*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{10}(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{10}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$1024\int \cos^{10}(a+bx)\sin^7(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$1024\int \cos(a+bx)^{10}\sin(a+bx)^7dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{1024\int \cos^{10}(a+bx)\left(1-\cos^2(a+bx)\right)^3d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{1024\int \left(-\cos^{16}(a+bx)+3\cos^{14}(a+bx)-3\cos^{12}(a+bx)+\cos^{10}(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1024\left(-\frac{1}{17}\cos^{17}(a+bx)+\frac{1}{5}\cos^{15}(a+bx)-\frac{3}{13}\cos^{13}(a+bx)+\frac{1}{11}\cos^{11}(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^10,x]
```

output
```
(-1024*(Cos[a + b*x]^11/11 - (3*Cos[a + b*x]^13)/13 + Cos[a + b*x]^15/5 -
Cos[a + b*x]^17/17))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 106.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result
default	$\frac{\frac{1024\cos(bx+a)^{17}}{17}-\frac{1024\cos(bx+a)^{15}}{5}+\frac{3072\cos(bx+a)^{13}}{13}-\frac{1024\cos(bx+a)^{11}}{11}}{b}$
risch	$-\frac{35\cos(bx+a)}{32b}+\frac{\cos(17bx+17a)}{1088b}+\frac{\cos(15bx+15a)}{320b}-\frac{\cos(13bx+13a)}{208b}-\frac{5\cos(11bx+11a)}{176b}+\frac{\cos(7bx+7a)}{8b}+\frac{7\cos(5bx+}{80b}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x,method=_RETURNVERBOSE)
```

output
```
1024/b*(1/17*cos(b*x+a)^17-1/5*cos(b*x+a)^15+3/13*cos(b*x+a)^13-1/11*cos(b
*x+a)^11)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx = \frac{1024\left(715\cos(bx+a)^{17}-2431\cos(bx+a)^{15}+2805\cos(bx+a)^{13}-1105\cos(bx+a)^{11}\right)}{12155\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="fricas")
```

output
```
1024/12155*(715*cos(b*x + a)^17 - 2431*cos(b*x + a)^15 + 2805*cos(b*x + a)
^13 - 1105*cos(b*x + a)^11)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**10,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx$$
$$= \frac{715\cos(17\,bx+17\,a)+2431\cos(15\,bx+15\,a)-3740\cos(13\,bx+13\,a)-22100\cos(11\,bx+11\,a)+}{777920\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="maxima")
```

output
```
1/777920*(715*cos(17*b*x + 17*a) + 2431*cos(15*b*x + 15*a) - 3740*cos(13*b
*x + 13*a) - 22100*cos(11*b*x + 11*a) + 97240*cos(7*b*x + 7*a) + 68068*cos
(5*b*x + 5*a) - 340340*cos(3*b*x + 3*a) - 850850*cos(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(53) = 106.

Time = 0.23 (sec) , antiderivative size = 314, normalized size of antiderivative = 5.15

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx =$$
$$-\frac{32768\left(\frac{17(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{136(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{680(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{9775(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{71825(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+22\right.}{}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="giac")
```

output
```
-32768/12155*(17*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 136*(cos(b*x + a)
 - 1)^2/(cos(b*x + a) + 1)^2 + 680*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)
^3 + 9775*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 71825*(cos(b*x + a)
- 1)^5/(cos(b*x + a) + 1)^5 + 221000*(cos(b*x + a) - 1)^6/(cos(b*x + a) +
1)^6 + 486200*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 + 668525*(cos(b*x
+ a) - 1)^8/(cos(b*x + a) + 1)^8 + 692835*(cos(b*x + a) - 1)^9/(cos(b*x +
a) + 1)^9 + 466752*(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 + 233376*(c
os(b*x + a) - 1)^11/(cos(b*x + a) + 1)^11 + 65637*(cos(b*x + a) - 1)^12/(c
os(b*x + a) + 1)^12 + 12155*(cos(b*x + a) - 1)^13/(cos(b*x + a) + 1)^13 -
1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^17)
```

Mupad [B] (verification not implemented)

Time = 18.53 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx = -\frac{-\frac{1024\cos(a+b\,x)^{17}}{17}+\frac{1024\cos(a+b\,x)^{15}}{5}-\frac{3072\cos(a+b\,x)^{13}}{13}+\frac{1024\cos(a+b\,x)^{11}}{11}}{b}$$

input
```
int(sin(2*a + 2*b*x)^10/sin(a + b*x)^3,x)
```

output
```
-((1024*cos(a + b*x)^11)/11 - (3072*cos(a + b*x)^13)/13 + (1024*cos(a + b*
x)^15)/5 - (1024*cos(a + b*x)^17)/17)/b
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx = \int \csc(bx+a)^3\sin(2bx+2a)^{10}\,dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x)
```

output
```
int(csc(a + b*x)**3*sin(2*a + 2*b*x)**10,x)
```

3.470 $\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = \frac{512\sin^7(a+bx)}{7b} - \frac{2048\sin^9(a+bx)}{9b} + \frac{3072\sin^{11}(a+bx)}{11b} - \frac{2048\sin^{13}(a+bx)}{13b} + \frac{512\sin^{15}(a+bx)}{15b}$$

output
```
512/7*sin(b*x+a)^7/b-2048/9*sin(b*x+a)^9/b+3072/11*sin(b*x+a)^11/b-2048/13
*sin(b*x+a)^13/b+512/15*sin(b*x+a)^15/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = \frac{512\sin^7(a+bx)}{7b} - \frac{2048\sin^9(a+bx)}{9b} + \frac{3072\sin^{11}(a+bx)}{11b} - \frac{2048\sin^{13}(a+bx)}{13b} + \frac{512\sin^{15}(a+bx)}{15b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^9,x]
```

output
```
(512*Sin[a + b*x]^7)/(7*b) - (2048*Sin[a + b*x]^9)/(9*b) + (3072*Sin[a + b
*x]^11)/(11*b) - (2048*Sin[a + b*x]^13)/(13*b) + (512*Sin[a + b*x]^15)/(15
*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^9(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^9}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$512\int \cos^9(a+bx)\sin^6(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$512\int \cos(a+bx)^9\sin(a+bx)^6dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{512\int \sin^6(a+bx)\left(1-\sin^2(a+bx)\right)^4d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{512\int \left(\sin^{14}(a+bx)-4\sin^{12}(a+bx)+6\sin^{10}(a+bx)-4\sin^8(a+bx)+\sin^6(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{512\left(\frac{1}{15}\sin^{15}(a+bx)-\frac{4}{13}\sin^{13}(a+bx)+\frac{6}{11}\sin^{11}(a+bx)-\frac{4}{9}\sin^{9}(a+bx)+\frac{1}{7}\sin^{7}(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^9,x]
```

output
```
(512*(Sin[a + b*x]^7/7 - (4*Sin[a + b*x]^9)/9 + (6*Sin[a + b*x]^11)/11 - (
4*Sin[a + b*x]^13)/13 + Sin[a + b*x]^15/15))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 69.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result
default	$\frac{\frac{512 \sin(bx+a)^{15}}{15}-\frac{2048 \sin(bx+a)^{13}}{13}+\frac{3072 \sin(bx+a)^{11}}{11}-\frac{2048 \sin(bx+a)^{9}}{9}+\frac{512 \sin(bx+a)^{7}}{7}}{b}$
risch	$\frac{45 \sin(bx+a)}{32b}-\frac{\sin(15bx+15a)}{480b}-\frac{3 \sin(13bx+13a)}{416b}+\frac{3 \sin(11bx+11a)}{352b}+\frac{17 \sin(9bx+9a)}{288b}+\frac{3 \sin(7bx+7a)}{224b}-\frac{39 \sin(5bx+}{160b}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x,method=_RETURNVERBOSE)
```

output
```
512/b*(1/15*sin(b*x+a)^15-4/13*sin(b*x+a)^13+6/11*sin(b*x+a)^11-4/9*sin(b*
x+a)^9+1/7*sin(b*x+a)^7)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = -\frac{512\left(3003\cos(bx+a)^{14}-7161\cos(bx+a)^{12}+4473\cos(bx+a)^{10}-35\cos(bx+a)^{8}-40\cos(bx\right.}{45045\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="fricas")
```

output
```
-512/45045*(3003*cos(b*x + a)^14 - 7161*cos(b*x + a)^12 + 4473*cos(b*x + a
)^10 - 35*cos(b*x + a)^8 - 40*cos(b*x + a)^6 - 48*cos(b*x + a)^4 - 64*cos(
b*x + a)^2 - 128)*sin(b*x + a)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**9,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = -\frac{3003 \sin(15\,bx+15\,a)+10395 \sin(13\,bx+13\,a)-12285 \sin(11\,bx+11\,a)-85085 \sin(9\,bx+9\,a)}{14414}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="maxima")
```

output
```
-1/1441440*(3003*sin(15*b*x + 15*a) + 10395*sin(13*b*x + 13*a) - 12285*sin
(11*b*x + 11*a) - 85085*sin(9*b*x + 9*a) - 19305*sin(7*b*x + 7*a) + 351351
*sin(5*b*x + 5*a) + 375375*sin(3*b*x + 3*a) - 2027025*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = \frac{512\left(3003 \sin(bx+a)^{15}-13860 \sin(bx+a)^{13}+24570 \sin(bx+a)^{11}-20020 \sin(bx+a)^{9}+6435 \sin\right.}{45045\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="giac")
```

output
```
512/45045*(3003*sin(b*x + a)^15 - 13860*sin(b*x + a)^13 + 24570*sin(b*x +
a)^11 - 20020*sin(b*x + a)^9 + 6435*sin(b*x + a)^7)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = \frac{\frac{512\sin(a+b\,x)^{15}}{15}-\frac{2048\sin(a+b\,x)^{13}}{13}+\frac{3072\sin(a+b\,x)^{11}}{11}-\frac{2048\sin(a+b\,x)^9}{9}+\frac{512\sin(a+b\,x)^7}{7}}{b}$$

input
```
int(sin(2*a + 2*b*x)^9/sin(a + b*x)^3,x)
```

output
```
((512*sin(a + b*x)^7)/7 - (2048*sin(a + b*x)^9)/9 + (3072*sin(a + b*x)^11)
/11 - (2048*sin(a + b*x)^13)/13 + (512*sin(a + b*x)^15)/15)/b
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = \int \csc(bx+a)^3\sin(2bx+2a)^9\,dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x)
```

output
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x)
```

3.471 $\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx=-\frac{256\cos^9(a+bx)}{9b}+\frac{512\cos^{11}(a+bx)}{11b}-\frac{256\cos^{13}(a+bx)}{13b}$$

output
```
-256/9*cos(b*x+a)^9/b+512/11*cos(b*x+a)^11/b-256/13*cos(b*x+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 104 vs. 2(46) = 92.

Time = 0.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.26

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx=-\frac{5\cos(a+bx)}{4b}-\frac{25\cos(3(a+bx))}{48b}+\frac{\cos(5(a+bx))}{16b}+\frac{\cos(7(a+bx))}{8b}+\frac{\cos(9(a+bx))}{72b}-\frac{3\cos(11(a+bx))}{176b}-\frac{\cos(13(a+bx))}{208b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^8,x]
```

output
```
(-5*Cos[a + b*x])/(4*b) - (25*Cos[3*(a + b*x)])/(48*b) + Cos[5*(a + b*x)]/
(16*b) + Cos[7*(a + b*x)]/(8*b) + Cos[9*(a + b*x)]/(72*b) - (3*Cos[11*(a +
 b*x)])/(176*b) - Cos[13*(a + b*x)]/(208*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^8}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$256\int \cos^8(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$256\int \cos(a+bx)^8\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{256\int \cos^8(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{256\int \left(\cos^{12}(a+bx)-2\cos^{10}(a+bx)+\cos^8(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{256\left(\frac{1}{13}\cos^{13}(a+bx)-\frac{2}{11}\cos^{11}(a+bx)+\frac{1}{9}\cos^9(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^8,x]
```

output
```
(-256*(Cos[a + b*x]^9/9 - (2*Cos[a + b*x]^11)/11 + Cos[a + b*x]^13/13))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 42.89 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result
default	$-\frac{256\left(\frac{\cos(bx+a)^{13}}{13}-\frac{2\cos(bx+a)^{11}}{11}+\frac{\cos(bx+a)^{9}}{9}\right)}{b}$
risch	$-\frac{5\cos(bx+a)}{4b}-\frac{\cos(13bx+13a)}{208b}-\frac{3\cos(11bx+11a)}{176b}+\frac{\cos(9bx+9a)}{72b}+\frac{\cos(7bx+7a)}{8b}+\frac{\cos(5bx+5a)}{16b}-\frac{25\cos(3bx+3a)}{48b}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x,method=_RETURNVERBOSE)
```

output
```
-256/b*(1/13*cos(b*x+a)^13-2/11*cos(b*x+a)^11+1/9*cos(b*x+a)^9)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx = -\frac{256\left(99\cos(bx+a)^{13}-234\cos(bx+a)^{11}+143\cos(bx+a)^{9}\right)}{1287\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

output
```
-256/1287*(99*cos(b*x + a)^13 - 234*cos(b*x + a)^11 + 143*cos(b*x + a)^9)/
b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx = -\frac{99\,\cos(13\,bx+13\,a)+351\,\cos(11\,bx+11\,a)-286\,\cos(9\,bx+9\,a)-2574\,\cos(7\,bx+7\,a)-1287\,\mathrm{co}}{20592\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

output
```
-1/20592*(99*cos(13*b*x + 13*a) + 351*cos(11*b*x + 11*a) - 286*cos(9*b*x +
 9*a) - 2574*cos(7*b*x + 7*a) - 1287*cos(5*b*x + 5*a) + 10725*cos(3*b*x +
3*a) + 25740*cos(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(40) = 80$.

Time = 0.20 (sec) , antiderivative size = 248, normalized size of antiderivative = 5.39

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx = -\frac{4096\left(\frac{13(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{78(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-\frac{572(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{3718(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-\frac{7722(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}-\frac{13728}{(\mathrm{co}}\right.}{1287\,b\left(\frac{\cos(bx+a)}{\cos(bx+a)}\right.}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

output
```
-4096/1287*(13*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 78*(cos(b*x + a) -
1)^2/(cos(b*x + a) + 1)^2 - 572*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3
- 3718*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 7722*(cos(b*x + a) - 1)
^5/(cos(b*x + a) + 1)^5 - 13728*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6
- 12012*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 9009*(cos(b*x + a) - 1
)^8/(cos(b*x + a) + 1)^8 - 3003*(cos(b*x + a) - 1)^9/(cos(b*x + a) + 1)^9
- 858*(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 - 1)/(b*((cos(b*x + a) -
 1)/(cos(b*x + a) + 1) - 1)^13)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx = -\frac{256\,\left(99\cos\left(a+b\,x\right)^{13}-234\cos\left(a+b\,x\right)^{11}+143\cos\left(a+b\,x\right)^{9}\right)}{1287\,b}$$

input
```
int(sin(2*a + 2*b*x)^8/sin(a + b*x)^3,x)
```

output
```
-(256*(143*cos(a + b*x)^9 - 234*cos(a + b*x)^11 + 99*cos(a + b*x)^13))/(12
87*b)
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx = \int \csc\left(bx+a\right)^3\sin\left(2bx+2a\right)^8\,dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x)
```

output
```
int(csc(a + b*x)**3*sin(2*a + 2*b*x)**8,x)
```

3.472 $\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx=\frac{128\sin^5(a+bx)}{5b}-\frac{384\sin^7(a+bx)}{7b}+\frac{128\sin^9(a+bx)}{3b}-\frac{128\sin^{11}(a+bx)}{11b}$$

output
```
128/5*sin(b*x+a)^5/b-384/7*sin(b*x+a)^7/b+128/3*sin(b*x+a)^9/b-128/11*sin(
b*x+a)^11/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx=\frac{128\sin^5(a+bx)}{5b}-\frac{384\sin^7(a+bx)}{7b}+\frac{128\sin^9(a+bx)}{3b}-\frac{128\sin^{11}(a+bx)}{11b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^7,x]
```

output
```
(128*Sin[a + b*x]^5)/(5*b) - (384*Sin[a + b*x]^7)/(7*b) + (128*Sin[a + b*x
]^9)/(3*b) - (128*Sin[a + b*x]^11)/(11*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^7}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$128\int \cos^7(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$128\int \cos(a+bx)^7\sin(a+bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{128\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)^3d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{128\int \left(-\sin^{10}(a+bx)+3\sin^8(a+bx)-3\sin^6(a+bx)+\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{128\left(-\frac{1}{11}\sin^{11}(a+bx)+\frac{1}{3}\sin^9(a+bx)-\frac{3}{7}\sin^7(a+bx)+\frac{1}{5}\sin^5(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^7,x]
```

output
```
(128*(Sin[a + b*x]^5/5 - (3*Sin[a + b*x]^7)/7 + Sin[a + b*x]^9/3 - Sin[a +
 b*x]^11/11))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 25.44 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{128\left(\frac{\sin(bx+a)^{11}}{11}-\frac{\sin(bx+a)^{9}}{3}+\frac{3\sin(bx+a)^{7}}{7}-\frac{\sin(bx+a)^{5}}{5}\right)}{b}$	47
risch	$\frac{7\sin(bx+a)}{4b}+\frac{\sin(11bx+11a)}{88b}+\frac{\sin(9bx+9a)}{24b}-\frac{\sin(7bx+7a)}{56b}-\frac{11\sin(5bx+5a)}{40b}-\frac{\sin(3bx+3a)}{4b}$	83

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

output
```
-128/b*(1/11*sin(b*x+a)^11-1/3*sin(b*x+a)^9+3/7*sin(b*x+a)^7-1/5*sin(b*x+a
)^5)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx$$
$$= \frac{128\left(105\cos(bx+a)^{10}-140\cos(bx+a)^{8}+5\cos(bx+a)^{6}+6\cos(bx+a)^{4}+8\cos(bx+a)^{2}+16\right)\mathrm{s}}{1155\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

output
```
128/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*
cos(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx = \frac{105\sin(11\,bx+11\,a)+385\sin(9\,bx+9\,a)-165\sin(7\,bx+7\,a)-2541\sin(5\,bx+5\,a)-2310\sin(3\,b}{9240\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

output
```
1/9240*(105*sin(11*b*x + 11*a) + 385*sin(9*b*x + 9*a) - 165*sin(7*b*x + 7*
a) - 2541*sin(5*b*x + 5*a) - 2310*sin(3*b*x + 3*a) + 16170*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx = -\frac{128\left(105\sin(bx+a)^{11}-385\sin(bx+a)^{9}+495\sin(bx+a)^{7}-231\sin(bx+a)^{5}\right)}{1155\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

output
```
-128/1155*(105*sin(b*x + a)^11 - 385*sin(b*x + a)^9 + 495*sin(b*x + a)^7 -
 231*sin(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx = \frac{-\frac{128\sin(a+b\,x)^{11}}{11}+\frac{128\sin(a+b\,x)^9}{3}-\frac{384\sin(a+b\,x)^7}{7}+\frac{128\sin(a+b\,x)^5}{5}}{b}$$

input
```
int(sin(2*a + 2*b*x)^7/sin(a + b*x)^3,x)
```

output
```
((128*sin(a + b*x)^5)/5 - (384*sin(a + b*x)^7)/7 + (128*sin(a + b*x)^9)/3
- (128*sin(a + b*x)^11)/11)/b
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx = \int \csc(bx+a)^3\sin(2bx+2a)^7\,dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x)
```

output
```
int(csc(a + b*x)**3*sin(2*a + 2*b*x)**7,x)
```

3.473 $\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx = -\frac{64\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{9b}$$

output
```
-64/7*cos(b*x+a)^7/b+64/9*cos(b*x+a)^9/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx = \frac{32\cos^7(a+bx)(-11+7\cos(2(a+bx)))}{63b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^6,x]
```

output
```
(32*Cos[a + b*x]^7*(-11 + 7*Cos[2*(a + b*x)]))/(63*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^6}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$64\int \cos^6(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$64\int \cos(a+bx)^6\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{64\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{64\int \left(\cos^6(a+bx)-\cos^8(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{64\left(\frac{1}{7}\cos^7(a+bx)-\frac{1}{9}\cos^9(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^6,x]
```

output
```
(-64*(Cos[a + b*x]^7/7 - Cos[a + b*x]^9/9))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 14.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\frac{64\cos(bx+a)^9}{9}-\frac{64\cos(bx+a)^7}{7}}{b}$	27
risch	$-\frac{3\cos(bx+a)}{2b}+\frac{\cos(9bx+9a)}{36b}+\frac{3\cos(7bx+7a)}{28b}-\frac{2\cos(3bx+3a)}{3b}$	55

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

output
```
64/b*(1/9*cos(b*x+a)^9-1/7*cos(b*x+a)^7)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx = \frac{64\left(7\cos(bx+a)^9 - 9\cos(bx+a)^7\right)}{63\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

output
```
64/63*(7*cos(b*x + a)^9 - 9*cos(b*x + a)^7)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx = \frac{7\cos(9\,bx+9\,a)+27\cos(7\,bx+7\,a)-168\cos(3\,bx+3\,a)-378\cos(bx+a)}{252\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

output
```
1/252*(7*cos(9*b*x + 9*a) + 27*cos(7*b*x + 7*a) - 168*cos(3*b*x + 3*a) - 3
78*cos(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(27) = 54$.

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 5.87

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx =$$
$$-\frac{256\left(\frac{9\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{27\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{189\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{189\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{315\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{105\,(\cos(bx}{(\cos(bx+}\right.}{63\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^9}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

output
```
-256/63*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 27*(cos(b*x + a) - 1)^2
/(cos(b*x + a) + 1)^2 + 189*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 18
9*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 315*(cos(b*x + a) - 1)^5/(co
s(b*x + a) + 1)^5 + 105*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 63*(co
s(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*
x + a) + 1) - 1)^9)
```

Mupad [B] (verification not implemented)

Time = 19.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx = -\frac{64\left(9\cos\left(a+b\,x\right)^7-7\cos\left(a+b\,x\right)^9\right)}{63\,b}$$

input
```
int(sin(2*a + 2*b*x)^6/sin(a + b*x)^3,x)
```

output
```
-(64*(9*cos(a + b*x)^7 - 7*cos(a + b*x)^9))/(63*b)
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx = \int \csc(bx+a)^3 \sin(2bx+2a)^6\,dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x)
```

output
```
int(csc(a + b*x)**3*sin(2*a + 2*b*x)**6,x)
```

3.474 $\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx=\frac{32\sin^3(a+bx)}{3b}-\frac{64\sin^5(a+bx)}{5b}+\frac{32\sin^7(a+bx)}{7b}$$

output
```
32/3*sin(b*x+a)^3/b-64/5*sin(b*x+a)^5/b+32/7*sin(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx =\frac{4(157+108\cos(2(a+bx))+15\cos(4(a+bx)))\sin^3(a+bx)}{105b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

output
```
(4*(157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(105
*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4776, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^5}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$32\int \cos^5(a+bx)\sin^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$32\int \cos(a+bx)^5\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{32\int \sin^2(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{32\int \left(\sin^6(a+bx)-2\sin^4(a+bx)+\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{32\left(\frac{1}{7}\sin^7(a+bx)-\frac{2}{5}\sin^5(a+bx)+\frac{1}{3}\sin^3(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

output
```
(32*(Sin[a + b*x]^3/3 - (2*Sin[a + b*x]^5)/5 + Sin[a + b*x]^7/7))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 7.86 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\frac{32 \sin(bx+a)^7}{7}-\frac{64 \sin(bx+a)^5}{5}+\frac{32 \sin(bx+a)^3}{3}}{b}$	37
risch	$\frac{5 \sin(bx+a)}{2b}-\frac{\sin(7bx+7a)}{14b}-\frac{3 \sin(5bx+5a)}{10b}-\frac{\sin(3bx+3a)}{6b}$	55

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
32/b*(1/7*sin(b*x+a)^7-2/5*sin(b*x+a)^5+1/3*sin(b*x+a)^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\,\left(15\,\cos\left(bx+a\right)^6-3\,\cos\left(bx+a\right)^4-4\,\cos\left(bx+a\right)^2-8\right)\sin\left(bx+a\right)}{105\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
-32/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(
b*x + a)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx = -\frac{15\,\sin\left(7\,bx+7\,a\right)+63\,\sin\left(5\,bx+5\,a\right)+35\,\sin\left(3\,bx+3\,a\right)-525\,\sin\left(bx+a\right)}{210\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
-1/210*(15*sin(7*b*x + 7*a) + 63*sin(5*b*x + 5*a) + 35*sin(3*b*x + 3*a) -
525*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(15\sin\left(bx+a\right)^7-42\sin\left(bx+a\right)^5+35\sin\left(bx+a\right)^3\right)}{105\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
32/105*(15*sin(b*x + a)^7 - 42*sin(b*x + a)^5 + 35*sin(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 18.85 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx = \frac{32\left(15\sin\left(a+b\,x\right)^7-42\sin\left(a+b\,x\right)^5+35\sin\left(a+b\,x\right)^3\right)}{105\,b}$$

input
```
int(sin(2*a + 2*b*x)^5/sin(a + b*x)^3,x)
```

output
```
(32*(35*sin(a + b*x)^3 - 42*sin(a + b*x)^5 + 15*sin(a + b*x)^7))/(105*b)
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx = \int \csc(bx+a)^3 \sin(2bx+2a)^5\,dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x)
```

output
```
int(csc(a + b*x)**3*sin(2*a + 2*b*x)**5,x)
```

3.475 $\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 15

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx=-\frac{16\cos^5(a+bx)}{5b}$$

output
```
-16/5*cos(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx=-\frac{16\cos^5(a+bx)}{5b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

output
```
(-16*Cos[a + b*x]^5)/(5*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4776, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^4}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$16\int \cos^4(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$16\int \cos(a+bx)^4\sin(a+bx)dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{16\int \cos^4(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{16\cos^5(a+bx)}{5b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

output
```
(-16*Cos[a + b*x]^5)/(5*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{16 \cos (b x+a)^{5}}{5 b}$	14
risch	$-\frac{2 \cos (b x+a)}{b}-\frac{\cos (5 b x+5 a)}{5 b}-\frac{\cos (3 b x+3 a)}{b}$	41

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
-16/5*cos(b*x+a)^5/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx = -\frac{16\cos(bx+a)^5}{5b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
-16/5*cos(b*x + a)^5/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(13) = 26.

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx = -\frac{\cos(5bx+5a)+5\cos(3bx+3a)+10\cos(bx+a)}{5b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
-1/5*(cos(5*b*x + 5*a) + 5*cos(3*b*x + 3*a) + 10*cos(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(13) = 26$.

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 4.93

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx = \frac{32\left(\frac{10\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{5\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+1\right)}{5\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^5}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
32/5*(10*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 5*(cos(b*x + a) - 1)^
4/(cos(b*x + a) + 1)^4 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)
^5)
```

Mupad [B] (verification not implemented)

Time = 18.60 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx = -\frac{16\cos\left(a+b\,x\right)^5}{5\,b}$$

input
```
int(sin(2*a + 2*b*x)^4/sin(a + b*x)^3,x)
```

output
```
-(16*cos(a + b*x)^5)/(5*b)
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx = \int \csc\left(bx+a\right)^3\sin\left(2bx+2a\right)^4dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x)
```

output
```
int(csc(a + b*x)**3*sin(2*a + 2*b*x)**4,x)
```

3.476 $\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx=\frac{8\sin(a+bx)}{b}-\frac{8\sin^3(a+bx)}{3b}$$

output
```
8*sin(b*x+a)/b-8/3*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx=8\left(\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}\right)$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

output
```
8*(Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4776, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^3}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$8\int \cos^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \sin\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$-\frac{8\int \left(1-\sin^2(a+bx)\right)d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-8*(-Sin[a + b*x] + Sin[a + b*x]^3/3))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{8\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{3b}$	22
risch	$\frac{6\sin(bx+a)}{b}+\frac{2\sin(3bx+3a)}{3b}$	27

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
8/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(\cos(bx+a)^2+2\right)\sin(bx+a)}{3\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
8/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{2\left(\sin(3\,bx+3\,a)+9\,\sin(bx+a)\right)}{3\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
2/3*(sin(3*b*x + 3*a) + 9*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(\sin\left(bx+a\right)^3-3\,\sin\left(bx+a\right)\right)}{3\,b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-8/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(3\,\sin\left(a+b\,x\right)-\sin\left(a+b\,x\right)^3\right)}{3\,b}$$

input
```
int(sin(2*a + 2*b*x)^3/sin(a + b*x)^3,x)
```

output
```
(8*(3*sin(a + b*x) - sin(a + b*x)^3))/(3*b)
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx = \int \csc\left(bx+a\right)^3\sin\left(2bx+2a\right)^3\,dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x)
```

output
```
int(csc(a + b*x)**3*sin(2*a + 2*b*x)**3,x)
```

3.477 $\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\text{arctanh}(\cos(a+bx))}{b}+\frac{4\cos(a+bx)}{b}$$

output
```
-4*arctanh(cos(b*x+a))/b+4*cos(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx = 4\left(\frac{\cos(a+bx)}{b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}\right)$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
4*(Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4776, 3042, 25, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^2}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$4\int \cos(a+bx)\cot(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int -\sin\left(a+bx+\frac{\pi}{2}\right)\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-4\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3072}$$

$$-\frac{4\int \frac{\cos^2(a+bx)}{1-\cos^2(a+bx)}d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{4\left(\int \frac{1}{1-\cos^2(a+bx)}d\cos(a+bx)-\cos(a+bx)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{4(\operatorname{arctanh}(\cos(a+bx))-\cos(a+bx))}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
(-4*(ArcTanh[Cos[a + b*x]] - Cos[a + b*x]))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

method	result	size
default	$\frac{4\cos(bx+a)+4\ln(\csc(bx+a)-\cot(bx+a))}{b}$	29
risch	$\frac{2\,e^{i(bx+a)}}{b}+\frac{2\,e^{-i(bx+a)}}{b}-\frac{4\ln(e^{i(bx+a)}+1)}{b}+\frac{4\ln(e^{i(bx+a)}-1)}{b}$	64

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/b*(cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx = \frac{2\left(2\cos(bx+a)-\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)\right)}{b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
2*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a) +
1/2))/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(24) = 48$.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.83

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx$$
$$= \frac{2\left(2\cos(bx+a) - \log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right.\right.}{b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
2*(2*cos(b*x + a) - log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*
x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a)
+ cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{2\left(\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1} - \log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)\right)}{b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-2*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - log(-(cos(b*x + a) - 1
)/(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 18.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx=\frac{4\,\cos\left(a+b\,x\right)-4\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{b}$$

input
```
int(sin(2*a + 2*b*x)^2/sin(a + b*x)^3,x)
```

output
```
(4*cos(a + b*x) - 4*atanh(cos(a + b*x)))/b
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx=\int \csc\left(bx+a\right)^3\sin\left(2bx+2a\right)^2dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x)
```

output
```
int(csc(a + b*x)**3*sin(2*a + 2*b*x)**2,x)
```

3.478 $\int \csc^3(a+bx)\sin(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 11

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx = -\frac{2\csc(a+bx)}{b}$$

output
```
-2*csc(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx = -\frac{2\csc(a+bx)}{b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

output
```
(-2*Csc[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4776, 3042, 25, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$2\int \cot(a+bx)\csc(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int -\sec\left(a+bx-\frac{\pi}{2}\right)\tan\left(a+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-2\int \sec\left(\frac{1}{2}(2a-\pi)+bx\right)\tan\left(\frac{1}{2}(2a-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{2\int 1d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{2\csc(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

output
```
(-2*Csc[a + b*x])/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

method	result	size
default	$-\frac{2}{\sin(bx+a)b}$	14
risch	$-\frac{4ie^{i(bx+a)}}{b(e^{2i(bx+a)}-1)}$	29

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
-2/sin(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx = -\frac{2}{b\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-2/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(11) = 22.

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 7.64

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx = -\frac{4(\cos(bx+a)\sin(2bx+2a)-\cos(2bx+2a)\sin(bx+a)+\sin(bx+a))}{b\cos(2bx+2a)^2+b\sin(2bx+2a)^2-2b\cos(2bx+2a)+b}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-4*(cos(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*
x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x + 2*a
) + b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx=-\frac{2}{b\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
-2/(b*sin(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 18.99 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx=-\frac{2}{b\,\sin(a+b\,x)}$$

input
```
int(sin(2*a + 2*b*x)/sin(a + b*x)^3,x)
```

output
```
-2/(b*sin(a + b*x))
```

Reduce [F]

$$\int \csc^3(a+bx)\sin(2a+2bx)\,dx = \int \csc(bx+a)^3 \sin(2bx+2a)\,dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a),x)
```

output
```
int(csc(a + b*x)**3*sin(2*a + 2*b*x),x)
```

3.479 $\int \csc^3(a+bx)\csc(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}-\frac{\csc(a+bx)}{2b}-\frac{\csc^3(a+bx)}{6b}$$

output
```
1/2*arctanh(sin(b*x+a))/b-1/2*csc(b*x+a)/b-1/6*csc(b*x+a)^3/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx=-\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(a+bx)\right)}{6b}$$

input
```
Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

output
```
-1/6*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4776, 3042, 3101, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{2}\int \csc^4(a+bx)\sec(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc(a+bx)^4\sec(a+bx)dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\int \left(-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\text{arctanh}(\csc(a+bx))+\frac{1}{3}\csc^3(a+bx)+\csc(a+bx)}{2b}$$

input
```
Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

output
```
-1/2*(-ArcTanh[Csc[a + b*x]] + Csc[a + b*x] + Csc[a + b*x]^3/3)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{-\frac{1}{3\sin(bx+a)^3}-\frac{1}{\sin(bx+a)}+\ln(\sec(bx+a)+\tan(bx+a))}{2b}$	41
risch	$-\frac{i(3\,\mathrm{e}^{5i(bx+a)}-10\,\mathrm{e}^{3i(bx+a)}+3\,\mathrm{e}^{i(bx+a)})}{3b(\mathrm{e}^{2i(bx+a)}-1)^3}+\frac{\ln(\mathrm{e}^{i(bx+a)}+i)}{2b}-\frac{\ln(\mathrm{e}^{i(bx+a)}-i)}{2b}$	91

input
```
int(csc(b*x+a)^3*csc(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(-1/3/sin(b*x+a)^3-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(37) = 74.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.19

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx$$
$$=\frac{3\left(\cos\left(bx+a\right)^2-1\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-3\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\sin\left(bx+a\right)+1\right)\mathrm{si}}{12\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="fricas")
```

output
```
1/12*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b
*x + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)
/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F]

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx = \int \csc^3(a+bx)\csc(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**3*csc(2*b*x+2*a),x)
```

output
```
Integral(csc(a + b*x)**3*csc(2*a + 2*b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(37) = 74$.

Time = 0.38 (sec) , antiderivative size = 834, normalized size of antiderivative = 19.40

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="maxima")
```

output
```
1/12*(4*(3*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*
b*x + 6*a) + 36*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 1
2*(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 3*(2*(3*cos(4*
b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2
 + 6*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 9*
cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6
*a) - sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 + 18*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) - 1)*log((cos(b*x
 + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(
b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(
b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(
3*cos(5*b*x + 5*a) - 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a
) - 12*(3*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) - 12
*(10*cos(3*b*x + 3*a) - 3*cos(b*x + a))*sin(4*b*x + 4*a) - 40*(3*cos(2*b*x
 + 2*a) - 1)*sin(3*b*x + 3*a) + 120*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 36
*cos(b*x + a)*sin(2*b*x + 2*a) + 36*cos(2*b*x + 2*a)*sin(b*x + a) - 12*sin
(b*x + a))/(b*cos(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x
+ 2*a)^2 + b*sin(6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 - 18*b*sin(4*b*x
+ 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*cos(4*b*x + 4*a)
 - 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) - 6*(3*b*cos(2*b*x + 2*a)...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx = -\frac{\frac{2\left(3\sin(bx+a)^2+1\right)}{\sin(bx+a)^3} - 3\log\left(\sin\left(bx+a\right)+1\right) + 3\log\left(-\sin\left(bx+a\right)+1\right)}{12\,b}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="giac")
```

output
```
-1/12*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) +
 3*log(-sin(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx = \frac{\operatorname{atanh}(\sin(a+b\,x))}{2\,b} - \frac{\frac{\sin(a+b\,x)^2}{2}+\frac{1}{6}}{b\sin(a+b\,x)^3}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)),x)
```

output
```
atanh(sin(a + b*x))/(2*b) - (sin(a + b*x)^2/2 + 1/6)/(b*sin(a + b*x)^3)
```

Reduce [F]

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx = \int \csc(2bx+2a)\csc(bx+a)^3\,dx$$

input
```
int(csc(b*x+a)^3*csc(2*b*x+2*a),x)
```

output
```
int(csc(2*a + 2*b*x)*csc(a + b*x)**3,x)
```

3.480 $\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{15\text{arctanh}(\cos(a+bx))}{32b} - \frac{9\cot(a+bx)\csc(a+bx)}{32b} - \frac{\cot^3(a+bx)\csc(a+bx)}{16b} + \frac{\sec(a+bx)}{4b}$$

output
```
-15/32*arctanh(cos(b*x+a))/b-9/32*cot(b*x+a)*csc(b*x+a)/b-1/16*cot(b*x+a)^
3*csc(b*x+a)/b+1/4*sec(b*x+a)/b
```

Mathematica [A] (verified)

Time = 3.02 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.90

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{14\csc^2\left(\frac{1}{2}(a+bx)\right)+\csc^4\left(\frac{1}{2}(a+bx)\right)+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)\left(78+\cos(a+bx)\left(-8\left(8+15\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-15\log\left(\sin\left(\frac{1}{2}(a+bx\right.\right.\right.\right.\right.}{-1+\tan^2\left(\frac{1}{2}(a+bx)\right)}}{256b}$$

input
```
Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/256*(14*Csc[(a + b*x)/2]^2 + Csc[(a + b*x)/2]^4 + (Sec[(a + b*x)/2]^2*(
78 + Cos[a + b*x]*(-8*(8 + 15*Log[Cos[(a + b*x)/2]] - 15*Log[Sin[(a + b*x)
/2]]) + Sec[(a + b*x)/2]^4) - 14*Tan[(a + b*x)/2]^2))/(-1 + Tan[(a + b*x)/
2]^2))/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 4776, 3042, 3102, 25, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{4}\int \csc^5(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc(a+bx)^5\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{4b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{4}\int \frac{\sec^4(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{4b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\int \frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{4b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\sec^2(a+bx)}d\sec(a+bx)-\sec(a+bx)\right)\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\sec(a+bx))-\sec(a+bx))\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{4b}$$

input
```
Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

output
```
(-1/4*Sec[a + b*x]^5/(1 - Sec[a + b*x]^2)^2 + (5*((-3*(ArcTanh[Sec[a + b*x
]] - Sec[a + b*x]))/2 + Sec[a + b*x]^3/(2*(1 - Sec[a + b*x]^2))))/4)/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{-\frac{1}{4 \sin(bx+a)^{4} \cos(bx+a)}-\frac{5}{8 \sin(bx+a)^{2} \cos(bx+a)}+\frac{15}{8 \cos(bx+a)}+\frac{15 \ln(\csc(bx+a)-\cot(bx+a))}{8}}{4b}$	71
risch	$\frac{15 \,\mathrm{e}^{9 i (bx+a)}-40 \,\mathrm{e}^{7 i (bx+a)}+18 \,\mathrm{e}^{5 i (bx+a)}-40 \,\mathrm{e}^{3 i (bx+a)}+15 \,\mathrm{e}^{i (bx+a)}}{16 b \left(\mathrm{e}^{2 i (bx+a)}-1\right)^{4} \left(\mathrm{e}^{2 i (bx+a)}+1\right)}-\frac{15 \ln\left(\mathrm{e}^{i (bx+a)}+1\right)}{32 b}+\frac{15 \ln\left(\mathrm{e}^{i (bx+a)}-1\right)}{32 b}$	123

input
```
int(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)-5/8/sin(b*x+a)^2/cos(b*x+a)+15/8/cos(b
*x+a)+15/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(60) = 120$.

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.94

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$$
$$= \frac{30\cos(bx+a)^4 - 50\cos(bx+a)^2 - 15\left(\cos(bx+a)^5 - 2\cos(bx+a)^3 + \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx -\right.}{64\left(b\cos(bx+a)^5 - 2\,b\cos(bx -\right.}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
1/64*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(b
*x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5
 - 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b*
cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [F]

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = \int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**2,x)
```

output
```
Integral(csc(a + b*x)**3*csc(2*a + 2*b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2237 vs. $2(60) = 120$.

Time = 0.25 (sec) , antiderivative size = 2237, normalized size of antiderivative = 32.90

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
1/64*(4*(15*cos(9*b*x + 9*a) - 40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a) -
 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) - 60*(3*cos(8*b
*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) -
 1)*cos(9*b*x + 9*a) + 12*(40*cos(7*b*x + 7*a) - 18*cos(5*b*x + 5*a) + 40*
cos(3*b*x + 3*a) - 15*cos(b*x + a))*cos(8*b*x + 8*a) - 160*(2*cos(6*b*x +
6*a) + 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) + 8*(
18*cos(5*b*x + 5*a) - 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(6*b*x + 6
*a) + 72*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x + 5*a) -
40*(8*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 160*(3*cos(2*b
*x + 2*a) - 1)*cos(3*b*x + 3*a) - 180*cos(2*b*x + 2*a)*cos(b*x + a) + 15*(
2*(3*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*
b*x + 2*a) - 1)*cos(10*b*x + 10*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x
 + 6*a) + 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) -
9*cos(8*b*x + 8*a)^2 - 4*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos
(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*
x + 4*a) - 4*cos(4*b*x + 4*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x +
8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(1
0*b*x + 10*a) - sin(10*b*x + 10*a)^2 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x
 + 4*a) - 3*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*
(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(60) = 120$.

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.35

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = \frac{\frac{\left(\frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{90(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}-\frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{128}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1}+60\log\left(-\frac{\cos\ldots}{\cos\ldots}\right.}{256\,b}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
1/256*((16*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 90*(cos(b*x + a) - 1)^2
/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 16*
(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a)
 + 1)^2 + 128/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + 60*log(-(cos(b
*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = \frac{\frac{15\cos(a+b\,x)^4}{32}-\frac{25\cos(a+b\,x)^2}{32}+\frac{1}{4}}{b\,\left(\cos\left(a+b\,x\right)^5-2\cos\left(a+b\,x\right)^3+\cos\left(a+b\,x\right)\right)} - \frac{15\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{32\,b}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^2),x)
```

output
```
((15*cos(a + b*x)^4)/32 - (25*cos(a + b*x)^2)/32 + 1/4)/(b*(cos(a + b*x) -
 2*cos(a + b*x)^3 + cos(a + b*x)^5)) - (15*atanh(cos(a + b*x)))/(32*b)
```

Reduce [F]

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = \int \csc(2bx+2a)^2 \csc(bx+a)^3\,dx$$

input
```
int(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x)
```

output
```
int(csc(2*a + 2*b*x)**2*csc(a + b*x)**3,x)
```

3.481 $\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = \frac{7\text{arctanh}(\sin(a+bx))}{16b} - \frac{3\csc(a+bx)}{8b} - \frac{\csc^3(a+bx)}{12b} - \frac{\csc^5(a+bx)}{40b} + \frac{\sec(a+bx)\tan(a+bx)}{16b}$$

output
```
7/16*arctanh(sin(b*x+a))/b-3/8*csc(b*x+a)/b-1/12*csc(b*x+a)^3/b-1/40*csc(b
*x+a)^5/b+1/16*sec(b*x+a)*tan(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = -\frac{\csc^5(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},2,-\frac{3}{2},\sin^2(a+bx)\right)}{40b}$$

input
```
Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

output
```
-1/40*(Csc[a + b*x]^5*Hypergeometric2F1[-5/2, 2, -3/2, Sin[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4776, 3042, 3101, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4776}$$

$$\frac{1}{8}\int \csc^6(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)^6\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int \frac{\csc^8(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\csc^7(a+bx)}{2(1-\csc^2(a+bx))}-\frac{7}{2}\int \frac{\csc^6(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{8b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\frac{\csc^7(a+bx)}{2(1-\csc^2(a+bx))}-\frac{7}{2}\int\left(-\csc^4(a+bx)-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)}{8b}$$

$\downarrow$ 2009

$$-\frac{\frac{\csc^7(a+bx)}{2(1-\csc^2(a+bx))}-\frac{7}{2}\left(\text{arctanh}(\csc(a+bx))-\frac{1}{5}\csc^5(a+bx)-\frac{1}{3}\csc^3(a+bx)-\csc(a+bx)\right)}{8b}$$

input
```
Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

output
```
-1/8*(Csc[a + b*x]^7/(2*(1 - Csc[a + b*x]^2)) - (7*(ArcTanh[Csc[a + b*x]]
- Csc[a + b*x] - Csc[a + b*x]^3/3 - Csc[a + b*x]^5/5))/2)/b
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

method	result
default	$\frac{-\frac{1}{5\sin(bx+a)^{5}\cos(bx+a)^{2}}-\frac{7}{15\sin(bx+a)^{3}\cos(bx+a)^{2}}+\frac{7}{6\sin(bx+a)\cos(bx+a)^{2}}-\frac{7}{2\sin(bx+a)}+\frac{7\ln(\sec(bx+a)+\tan(bx+a))}{2}}{8b}$
risch	$-\frac{i\left(105\,\mathrm{e}^{13i(bx+a)}-350\,\mathrm{e}^{11i(bx+a)}+231\,\mathrm{e}^{9i(bx+a)}+412\,\mathrm{e}^{7i(bx+a)}+231\,\mathrm{e}^{5i(bx+a)}-350\,\mathrm{e}^{3i(bx+a)}+105\,\mathrm{e}^{i(bx+a)}\right)}{120b\left(\mathrm{e}^{2i(bx+a)}-1\right)^{5}\left(\mathrm{e}^{2i(bx+a)}+1\right)^{2}}-\frac{7\ln\left(\mathrm{e}^{i(bx+a)}-\right.}{16b}$

input
```
int(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-1/5/sin(b*x+a)^5/cos(b*x+a)^2-7/15/sin(b*x+a)^3/cos(b*x+a)^2+7/6/s
in(b*x+a)/cos(b*x+a)^2-7/2/sin(b*x+a)+7/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(67) = 134.

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.16

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx =$$
$$-\frac{210\cos(bx+a)^{6}-490\cos(bx+a)^{4}-105\left(\cos(bx+a)^{6}-2\cos(bx+a)^{4}+\cos(bx+a)^{2}\right)\log(\sin}{480\left(b\cos(bx\right.}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-1/480*(210*cos(b*x + a)^6 - 490*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - 2*
cos(b*x + a)^4 + cos(b*x + a)^2)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*
(cos(b*x + a)^6 - 2*cos(b*x + a)^4 + cos(b*x + a)^2)*log(-sin(b*x + a) + 1
)*sin(b*x + a) + 322*cos(b*x + a)^2 - 30)/((b*cos(b*x + a)^6 - 2*b*cos(b*x
 + a)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F]

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = \int \csc^3(a+bx)\csc^3(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**3,x)
```

output
```
Integral(csc(a + b*x)**3*csc(2*a + 2*b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3095 vs. $2(67) = 134$.

Time = 0.49 (sec) , antiderivative size = 3095, normalized size of antiderivative = 40.19

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/480*(4*(105*sin(13*b*x + 13*a) - 350*sin(11*b*x + 11*a) + 231*sin(9*b*x
+ 9*a) + 412*sin(7*b*x + 7*a) + 231*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a
) + 105*sin(b*x + a))*cos(14*b*x + 14*a) + 420*(3*sin(12*b*x + 12*a) - sin
(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) + 5*sin(6*b*x + 6*a) + sin(4*b*x + 4*
a) - 3*sin(2*b*x + 2*a))*cos(13*b*x + 13*a) + 12*(350*sin(11*b*x + 11*a) -
 231*sin(9*b*x + 9*a) - 412*sin(7*b*x + 7*a) - 231*sin(5*b*x + 5*a) + 350*
sin(3*b*x + 3*a) - 105*sin(b*x + a))*cos(12*b*x + 12*a) + 1400*(sin(10*b*x
 + 10*a) + 5*sin(8*b*x + 8*a) - 5*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) + 3*
sin(2*b*x + 2*a))*cos(11*b*x + 11*a) + 4*(231*sin(9*b*x + 9*a) + 412*sin(7
*b*x + 7*a) + 231*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a) + 105*sin(b*x +
a))*cos(10*b*x + 10*a) - 924*(5*sin(8*b*x + 8*a) - 5*sin(6*b*x + 6*a) - si
n(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*cos(9*b*x + 9*a) + 20*(412*sin(7*b*x
+ 7*a) + 231*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a) + 105*sin(b*x + a))*c
os(8*b*x + 8*a) + 1648*(5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 3*sin(2*b*
x + 2*a))*cos(7*b*x + 7*a) - 140*(33*sin(5*b*x + 5*a) - 50*sin(3*b*x + 3*a
) + 15*sin(b*x + a))*cos(6*b*x + 6*a) + 924*(sin(4*b*x + 4*a) - 3*sin(2*b*
x + 2*a))*cos(5*b*x + 5*a) + 140*(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*co
s(4*b*x + 4*a) + 105*(2*(3*cos(12*b*x + 12*a) - cos(10*b*x + 10*a) - 5*cos
(8*b*x + 8*a) + 5*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a)
 + 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 + 6*(cos(10*b*x + 10*a)...
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = -\frac{\frac{30\sin(bx+a)}{\sin(bx+a)^2-1}+\frac{4\left(45\sin(bx+a)^4+10\sin(bx+a)^2+3\right)}{\sin(bx+a)^5}-105\log\left(\sin\left(bx+a\right)+1\right)+105\log\left(-\sin\left(bx+a\right)+1\right)}{480\,b}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
-1/480*(30*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(45*sin(b*x + a)^4 + 10*s
in(b*x + a)^2 + 3)/sin(b*x + a)^5 - 105*log(sin(b*x + a) + 1) + 105*log(-s
in(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 19.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = \frac{7\,\mathrm{atanh}(\sin(a+b\,x))}{16\,b} - \frac{-\frac{7\sin(a+b\,x)^6}{16}+\frac{7\sin(a+b\,x)^4}{24}+\frac{7\sin(a+b\,x)^2}{120}+\frac{1}{40}}{b\,\left(\sin(a+b\,x)^5-\sin(a+b\,x)^7\right)}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^3),x)
```

output
```
(7*atanh(sin(a + b*x)))/(16*b) - ((7*sin(a + b*x)^2)/120 + (7*sin(a + b*x)
^4)/24 - (7*sin(a + b*x)^6)/16 + 1/40)/(b*(sin(a + b*x)^5 - sin(a + b*x)^7
))
```

Reduce [F]

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = \int \csc(2bx+2a)^3\csc(bx+a)^3\,dx$$

input
```
int(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x)
```

output
```
int(csc(2*a + 2*b*x)**3*csc(a + b*x)**3,x)
```

3.482 $\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx = -\frac{105\text{arctanh}(\cos(a+bx))}{256b} - \frac{55\cot(a+bx)\csc(a+bx)}{256b} - \frac{25\cot^3(a+bx)\csc(a+bx)}{384b} - \frac{\cot^5(a+bx)\csc(a+bx)}{96b} + \frac{\sec(a+bx)}{4b} + \frac{\sec^3(a+bx)}{48b}$$

output
```
-105/256*arctanh(cos(b*x+a))/b-55/256*cot(b*x+a)*csc(b*x+a)/b-25/384*cot(b
*x+a)^3*csc(b*x+a)/b-1/96*cot(b*x+a)^5*csc(b*x+a)/b+1/4*sec(b*x+a)/b+1/48*
sec(b*x+a)^3/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 278 vs. $2(104) = 208$.

Time = 0.74 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.67

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$$
$$= \frac{\csc^{12}(a+bx)\left(1150-4752\cos(2(a+bx))+1600\cos(3(a+bx))+504\cos(4(a+bx))+1680\cos(6(a+\right.}{}$$

input
```
Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

output
```
(Csc[a + b*x]^12*(1150 - 4752*Cos[2*(a + b*x)] + 1600*Cos[3*(a + b*x)] + 5
04*Cos[4*(a + b*x)] + 1680*Cos[6*(a + b*x)] - 600*Cos[7*(a + b*x)] - 630*C
os[8*(a + b*x)] + 200*Cos[9*(a + b*x)] + 2520*Cos[3*(a + b*x)]*Log[Cos[(a
+ b*x)/2]] - 945*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]] + 315*Cos[9*(a + b
*x)]*Log[Cos[(a + b*x)/2]] - 30*Cos[a + b*x]*(40 + 63*Log[Cos[(a + b*x)/2]
] - 63*Log[Sin[(a + b*x)/2]]) - 2520*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]
] + 945*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]] - 315*Cos[9*(a + b*x)]*Log[
Sin[(a + b*x)/2]]))/(3072*b*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 4776, 3042, 3102, 252, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(a+bx)^3\sin(2a+2bx)^4}dx$$
$$\Big\downarrow \text{4776}$$

$$\frac{1}{16}\int \csc^7(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^7\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int \frac{\sec^{10}(a+bx)}{(1-\sec^2(a+bx))^4}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\sec^9(a+bx)}{6(1-\sec^2(a+bx))^3}-\frac{3}{2}\int \frac{\sec^8(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\sec^9(a+bx)}{6(1-\sec^2(a+bx))^3}-\frac{3}{2}\left(\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}-\frac{7}{4}\int \frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)\right)}{16b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\sec^9(a+bx)}{6(1-\sec^2(a+bx))^3}-\frac{3}{2}\left(\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}-\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)\right)\right)}{16b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{\sec^9(a+bx)}{6(1-\sec^2(a+bx))^3}-\frac{3}{2}\Big(\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}-\frac{7}{4}\Big(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \left(-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sec^9(a+bx)}{6(1-\sec^2(a+bx))^3}-\frac{3}{2}\Big(\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}-\frac{7}{4}\Big(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\Big(\text{arctanh}(\sec(a+bx))-\frac{1}{3}\sec^3(a+bx)-\sec(a+bx}{16b}$$

input
```
Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

output
```
(Sec[a + b*x]^9/(6*(1 - Sec[a + b*x]^2)^3) - (3*(Sec[a + b*x]^7/(4*(1 - Se
c[a + b*x]^2)^2) - (7*(Sec[a + b*x]^5/(2*(1 - Sec[a + b*x]^2)) - (5*(ArcTa
nh[Sec[a + b*x]] - Sec[a + b*x] - Sec[a + b*x]^3/3))/2))/4))/2)/(16*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

method	result
default	$\frac{-\frac{1}{6 \sin(bx+a)^{6} \cos(bx+a)^{3}}-\frac{3}{8 \sin(bx+a)^{4} \cos(bx+a)^{3}}+\frac{7}{8 \sin(bx+a)^{2} \cos(bx+a)^{3}}-\frac{35}{16 \sin(bx+a)^{2} \cos(bx+a)}+\frac{105}{16 \cos(bx+a)}+\frac{105 \ln(\csc(bx+a)-}{16}}{16b}$
risch	$\frac{315\, \mathrm{e}^{17i(bx+a)}-840\, \mathrm{e}^{15i(bx+a)}-252\, \mathrm{e}^{13i(bx+a)}+2376\, \mathrm{e}^{11i(bx+a)}-1150\, \mathrm{e}^{9i(bx+a)}+2376\, \mathrm{e}^{7i(bx+a)}-252\, \mathrm{e}^{5i(bx+a)}-840\, \mathrm{e}^{3i(bx+a)}+31}{384b\left(\mathrm{e}^{2i(bx+a)}-1\right)^{6}\left(\mathrm{e}^{2i(bx+a)}+1\right)^{3}}$

input
```
int(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/b*(-1/6/sin(b*x+a)^6/cos(b*x+a)^3-3/8/sin(b*x+a)^4/cos(b*x+a)^3+7/8/s
in(b*x+a)^2/cos(b*x+a)^3-35/16/sin(b*x+a)^2/cos(b*x+a)+105/16/cos(b*x+a)+1
05/16*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(92) = 184$.

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.87

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$$
$$= \frac{630\cos(bx+a)^8 - 1680\cos(bx+a)^6 + 1386\cos(bx+a)^4 - 288\cos(bx+a)^2 - 315\left(\cos(bx+a)^9 - \right.}{1}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
1/1536*(630*cos(b*x + a)^8 - 1680*cos(b*x + a)^6 + 1386*cos(b*x + a)^4 - 2
88*cos(b*x + a)^2 - 315*(cos(b*x + a)^9 - 3*cos(b*x + a)^7 + 3*cos(b*x + a
)^5 - cos(b*x + a)^3)*log(1/2*cos(b*x + a) + 1/2) + 315*(cos(b*x + a)^9 -
3*cos(b*x + a)^7 + 3*cos(b*x + a)^5 - cos(b*x + a)^3)*log(-1/2*cos(b*x + a
) + 1/2) - 32)/(b*cos(b*x + a)^9 - 3*b*cos(b*x + a)^7 + 3*b*cos(b*x + a)^5
 - b*cos(b*x + a)^3)
```

Sympy [F]

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx = \int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$$

input
```
integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**4,x)
```

output
```
Integral(csc(a + b*x)**3*csc(2*a + 2*b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4268 vs. $2(92) = 184$.

Time = 0.42 (sec) , antiderivative size = 4268, normalized size of antiderivative = 41.04

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx = \text{Too large to display}$$

input

```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

output

```
1/1536*(4*(315*cos(17*b*x + 17*a) - 840*cos(15*b*x + 15*a) - 252*cos(13*b*
x + 13*a) + 2376*cos(11*b*x + 11*a) - 1150*cos(9*b*x + 9*a) + 2376*cos(7*b
*x + 7*a) - 252*cos(5*b*x + 5*a) - 840*cos(3*b*x + 3*a) + 315*cos(b*x + a)
)*cos(18*b*x + 18*a) - 1260*(3*cos(16*b*x + 16*a) - 8*cos(12*b*x + 12*a) +
 6*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*
b*x + 2*a) - 1)*cos(17*b*x + 17*a) + 12*(840*cos(15*b*x + 15*a) + 252*cos(
13*b*x + 13*a) - 2376*cos(11*b*x + 11*a) + 1150*cos(9*b*x + 9*a) - 2376*co
s(7*b*x + 7*a) + 252*cos(5*b*x + 5*a) + 840*cos(3*b*x + 3*a) - 315*cos(b*x
 + a))*cos(16*b*x + 16*a) - 3360*(8*cos(12*b*x + 12*a) - 6*cos(10*b*x + 10
*a) - 6*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*co
s(15*b*x + 15*a) - 1008*(8*cos(12*b*x + 12*a) - 6*cos(10*b*x + 10*a) - 6*c
os(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(13*b*x
+ 13*a) + 32*(2376*cos(11*b*x + 11*a) - 1150*cos(9*b*x + 9*a) + 2376*cos(7
*b*x + 7*a) - 252*cos(5*b*x + 5*a) - 840*cos(3*b*x + 3*a) + 315*cos(b*x +
a))*cos(12*b*x + 12*a) - 9504*(6*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) -
 8*cos(6*b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(11*b*x + 11*a) + 24*(115
0*cos(9*b*x + 9*a) - 2376*cos(7*b*x + 7*a) + 252*cos(5*b*x + 5*a) + 840*co
s(3*b*x + 3*a) - 315*cos(b*x + a))*cos(10*b*x + 10*a) + 4600*(6*cos(8*b*x
+ 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(9*b*x + 9*a) - 7
2*(792*cos(7*b*x + 7*a) - 84*cos(5*b*x + 5*a) - 280*cos(3*b*x + 3*a) + ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(92) = 184$.

Time = 0.16 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.58

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx =$$

$$-\frac{\frac{285\,(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{21\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} + \frac{\frac{18\,(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{225\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - \frac{2966\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} - \frac{3513\,(\cos(b}{(\cos(bx+}}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} + \frac{(\cos(bx+a}{(\cos(bx+a}\right.}}{6144\,b}$$

input
```
integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
-1/6144*(285*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 21*(cos(b*x + a) - 1)
^2/(cos(b*x + a) + 1)^2 + (cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + (18*
(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 225*(cos(b*x + a) - 1)^2/(cos(b*x
+ a) + 1)^2 - 2966*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 3513*(cos(b
*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 660*(cos(b*x + a) - 1)^5/(cos(b*x +
a) + 1)^5 + 1155*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 - 1)/((cos(b*x
+ a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2)^
3 - 1260*log(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 20.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx = \frac{-\frac{105\cos(a+b\,x)^8}{256} + \frac{35\cos(a+b\,x)^6}{32} - \frac{231\cos(a+b\,x)^4}{256} + \frac{3\cos(a+b\,x)^2}{16} + \frac{1}{48}}{b\,\left(-\cos\left(a+b\,x\right)^9 + 3\cos\left(a+b\,x\right)^7 - 3\cos\left(a+b\,x\right)^5 + \cos\left(a+b\,x\right)^3\right)} - \frac{105\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{256\,b}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^4),x)
```

output
```
((3*cos(a + b*x)^2)/16 - (231*cos(a + b*x)^4)/256 + (35*cos(a + b*x)^6)/32
 - (105*cos(a + b*x)^8)/256 + 1/48)/(b*(cos(a + b*x)^3 - 3*cos(a + b*x)^5
+ 3*cos(a + b*x)^7 - cos(a + b*x)^9)) - (105*atanh(cos(a + b*x)))/(256*b)
```

Reduce [F]

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx = \int \csc(2bx+2a)^4 \csc(bx+a)^3\,dx$$

input
```
int(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x)
```

output
```
int(csc(2*a + 2*b*x)**4*csc(a + b*x)**3,x)
```

3.483 $\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 136

$$\begin{aligned}\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx &= -\frac{5\arcsin(\cos(a+bx)-\sin(a+bx))}{32b} \\ &\quad+\frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{32b} \\ &\quad-\frac{5\cos(a+bx)\sqrt{\sin(2a+2bx)}}{16b} \\ &\quad+\frac{5\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b}-\frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\end{aligned}$$

output
```
-5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b+5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b-5/16*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b+5/24*sin(b*x+a)*s
in(2*b*x+2*a)^(3/2)/b-1/6*cos(b*x+a)*sin(2*b*x+2*a)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$=\frac{15\left(-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)-2(14\cdots}{96b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(15*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]]) - 2*(14*Cos[a + b*x] + 3*Cos[3*(a + b*x)] - 2
*Cos[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/(96*b)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4790, 3042, 4789, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin(a+bx)\sin(2a+2bx)^{5/2}dx$$
$$\Big\downarrow \text{ 4790}$$
$$\frac{5}{6}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{5}{6}\int \cos(a+bx)\sin(2a+2bx)^{3/2}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$

$\Big\downarrow$ 4789

$$\frac{5}{6}\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$

$\Big\downarrow$ 3042

$$\frac{5}{6}\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$

$\Big\downarrow$ 4790

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$

$\Big\downarrow$ 3042

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$

$\Big\downarrow$ 4793

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\frac{\sqrt{\sin(2a}}{}\right.\right.$$

$$\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}$$

input

```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/6*(Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2))/b + (5*((3*((-1/2*ArcSin[Cos[a
+ b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a
+ 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 + (S
in[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)))/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 126.18 (sec) , antiderivative size = 266160190, normalized size of antiderivative = 1957060.22

method	result	size
default	Expression too large to display	266160190

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(118) = 236.

Time = 0.09 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.14

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{8\sqrt{2}\left(32\cos(bx+a)^5-52\cos(bx+a)^3+5\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+30\arctan\left(-\frac{\sqrt{2}\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
1/384*(8*sqrt(2)*(32*cos(b*x + a)^5 - 52*cos(b*x + a)^3 + 5*cos(b*x + a))*
sqrt(cos(b*x + a)*sin(b*x + a)) + 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(co
s(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) - 30*arctan(-(2*sqrt(2)*s
qrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a
) - sin(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)
^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x +
a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/
b
```

Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a), x)
```

Giac [F]

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(a+b\,x\right)\,\sin\left(2\,a+2\,b\,x\right)^{5/2}\,dx$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^(5/2),x)
```

output
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^(5/2), x)
```

Reduce [F]

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sqrt{\sin\left(2bx+2a\right)}\,\sin\left(2bx+2a\right)^{2}\sin\left(bx+a\right)dx$$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*sin(2*a + 2*b*x)**2*sin(a + b*x),x)
```

3.484 $\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\begin{aligned}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))}{16b} \\ &\quad -\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{16b} \\ &\quad +\frac{3\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b}-\frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\end{aligned}$$

output
```
-3/16*arcsin(cos(b*x+a)-sin(b*x+a))/b-3/16*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b+3/8*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b-1/4*cos(b*x+a)*sin
(2*b*x+2*a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= \frac{-3\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+2\sqrt{\sin}}{16b}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(-3*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(2*Sin[a + b*x] - S
in[3*(a + b*x)]))/(16*b)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4790, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^{3/2}dx$$

$$\Big\downarrow \text{4790}$$

$$\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{4789}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{4794}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a+bx}{}\right.$$

$$\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(3*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a
 + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(2*b)))/4 - (Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 24.99 (sec) , antiderivative size = 94440518, normalized size of antiderivative = 858550.16

method	result	size
default	Expression too large to display	94440518

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(96) = 192.

Time = 0.09 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.55

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx =$$

$$-\frac{8\sqrt{2}\left(4\cos(bx+a)^2-3\right)\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\mathrm{co}}{\cos(bx+a)^2-}\right.}{}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/64*(8*sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a))*si
n(b*x + a) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x +
 a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b
*x + a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*
x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*
log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 +
 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*co
s(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a), x)
```

Giac [F]

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(a+b\,x\right)\,\sin\left(2\,a+2\,b\,x\right)^{3/2}\,dx$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^(3/2), x)
```

Reduce [F]

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sqrt{\sin\left(2bx+2a\right)}\,\sin\left(2bx+2a\right)\sin\left(bx+a\right)dx$$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*sin(2*a + 2*b*x)*sin(a + b*x),x)
```

3.485 $\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{4b} + \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b} - \frac{\cos(a+bx)\sqrt{\sin(2a+2bx)}}{2b}$$

output
```
-1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b+1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b-1/2*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\cos(a+b}{4b}$$

input
```
Integrate[Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] +
Sqrt[Sin[2*(a + b*x)]]] - 2*Cos[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{4790}$$

$$\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{4793}$$

$$\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}$$

input
```
Int[Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a + b
*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*
x]])/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 5.19 (sec) , antiderivative size = 20170856, normalized size of antiderivative = 240129.24

method	result	size
default	Expression too large to display	20170856

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(74) = 148$.

Time = 0.09 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.17

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx =$$

$$-\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a) - 2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
-1/16*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) - 2*arctan(-
(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + c
os(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) -
1)) + 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a)
- sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 +
4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(
b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x
 + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin\left(2\,bx+2\,a\right)}\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a), x)
```

Giac [F]

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin\left(2\,bx+2\,a\right)}\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sin\left(a+b\,x\right)\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^(1/2), x)
```

Reduce [F]

$$\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin(2bx+2a)}\,\sin(bx+a)\,dx$$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*sin(a + b*x),x)
```

3.486 $\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b} - \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{2b}$$

output
```
-1/2*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)}{2b}$$

input
```
Integrate[Sin[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/2*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]])/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$
$$\Big\downarrow \text{ 4794}$$
$$-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}$$

input
```
Int[Sin[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a + b*
x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.63 (sec) , antiderivative size = 980083, normalized size of antiderivative = 16897.98

method	result	size
default	Expression too large to display	980083

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(52) = 104.

Time = 0.08 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.14

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= \frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cdots}{\cos(bx+a)-\sin\cdots}\right.}{}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - si
n(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*
sin(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))
- cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*co
s(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*
x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)
^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin(bx+a)}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin(bx+a)}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin(a+b\,x)}{\sqrt{\sin(2\,a+2\,b\,x)}}\,dx$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\sin(bx+a)}{\sin(2bx+2a)}\,dx$$

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x))/sin(2*a + 2*b*x),x)
```

3.487 $\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

output
```
sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=\frac{\sin(a+bx)}{b\sqrt{\sin(2(a+bx))}}$$

input
```
Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
Sin[a + b*x]/(b*Sqrt[Sin[2*(a + b*x)]])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{ 4780}$$

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
Sin[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 13.72 (sec) , antiderivative size = 94317036, normalized size of antiderivative = 4100740.70

method	result	size
default	Expression too large to display	94317036

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}+\cos(bx+a)}{2\,b\cos(bx+a)}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + cos(b*x + a))/(b*cos(b*x +
a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2029 vs. $2(21) = 42$.

Time = 13.57 (sec) , antiderivative size = 2029, normalized size of antiderivative = 88.22

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)
^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2
*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(
1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1
/2*b*x) + tan(1/2*a))*((2*(sqrt(2)*tan(1/2*a)^25 + 10*sqrt(2)*tan(1/2*a)^2
3 + 44*sqrt(2)*tan(1/2*a)^21 + 110*sqrt(2)*tan(1/2*a)^19 + 165*sqrt(2)*tan
(1/2*a)^17 + 132*sqrt(2)*tan(1/2*a)^15 - 132*sqrt(2)*tan(1/2*a)^11 - 165*s
qrt(2)*tan(1/2*a)^9 - 110*sqrt(2)*tan(1/2*a)^7 - 44*sqrt(2)*tan(1/2*a)^5 -
 10*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))*tan(1/2*b*x)/(tan(1/2*a)^24
 + 12*tan(1/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan(1/2*a
)^16 + 792*tan(1/2*a)^14 + 924*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 495*tan
(1/2*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 + 1) + (s
qrt(2)*tan(1/2*a)^26 + 5*sqrt(2)*tan(1/2*a)^24 - 10*sqrt(2)*tan(1/2*a)^22
- 154*sqrt(2)*tan(1/2*a)^20 - 605*sqrt(2)*tan(1/2*a)^18 - 1353*sqrt(2)*tan
(1/2*a)^16 - 1980*sqrt(2)*tan(1/2*a)^14 - 1980*sqrt(2)*tan(1/2*a)^12 - 135
3*sqrt(2)*tan(1/2*a)^10 - 605*sqrt(2)*tan(1/2*a)^8 - 154*sqrt(2)*tan(1/2*a
)^6 - 10*sqrt(2)*tan(1/2*a)^4 + 5*sqrt(2)*tan(1/2*a)^2 + sqrt(2))/(tan(1/2
*a)^24 + 12*tan(1/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan
(1/2*a)^16 + 792*tan(1/2*a)^14 + 924*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 4
95*tan(1/2*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 ...
```

Mupad [B] (verification not implemented)

Time = 18.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\cos(a+b\,x)\,\sqrt{\sin(2\,a+2\,b\,x)}}{b\,(\cos(2\,a+2\,b\,x)+1)}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^(3/2),x)
```

output
```
(cos(a + b*x)*sin(2*a + 2*b*x)^(1/2))/(b*(cos(2*a + 2*b*x) + 1))
```

Reduce [F]

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\sin(bx+a)}{\sin(2bx+2a)^2}dx$$

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x))/sin(2*a + 2*b*x)**2,x)
```

3.488 $\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

output
```
1/3*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-2/3*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=\frac{\sqrt{\sin(2(a+bx))}\left(-\frac{1}{4}\csc(a+bx)+\frac{1}{12}\sec(a+bx)\tan(a+bx)\right)}{b}$$

input
```
Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(Sqrt[Sin[2*(a + b*x)]]*(-1/4*Csc[a + b*x] + (Sec[a + b*x]*Tan[a + b*x])/1
2))/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4792}$$

$$\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4779}$$

$$\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*Sqrt[Sin
[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 69.21 (sec) , antiderivative size = 597, normalized size of antiderivative = 11.26

method	result
default	$\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\left(6\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\,\mathrm{EllipticE}\left(\sqrt{\right.\right.$

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(6*(tan(1/2*a+1
/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*t
an(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticE((tan(1/2*
a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2-3*(tan(1/2*a+1/2*b*x
)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2
*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*
b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2+6*(tan(1/2*a+1/2*b*x)*(tan
(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2
*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticE((tan(1/2*a+1/2*b*x)+1
)^(1/2),1/2*2^(1/2))-3*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)
*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+
1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))+2*(tan
(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^4+2*(ta
n(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^4-2*(tan(1
/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^2-2*(tan(
1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2))/tan(1/2*a+1/2*b*x)/(tan(1/2*a+
1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)/(1+tan(1/2*a+1/2*b*x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{4\,\cos\left(bx+a\right)^{2}\sin\left(bx+a\right)+\sqrt{2}\left(4\,\cos\left(bx+a\right)^{2}-1\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}}{12\,b\cos\left(bx+a\right)^{2}\sin\left(bx+a\right)}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(4*cos(b*x + a)^2*sin(b*x + a) + sqrt(2)*(4*cos(b*x + a)^2 - 1)*sqrt
(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^2*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7875 vs. $2(45) = 90$.

Time = 47.10 (sec) , antiderivative size = 7875, normalized size of antiderivative = 148.58

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
-1/24*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a
)^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/
2*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan
(1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(
1/2*b*x) + tan(1/2*a))*((((((2*(sqrt(2)*tan(1/2*a)^56 + 23*sqrt(2)*tan(1/2
*a)^54 + 251*sqrt(2)*tan(1/2*a)^52 + 1725*sqrt(2)*tan(1/2*a)^50 + 8350*sqr
t(2)*tan(1/2*a)^48 + 30130*sqrt(2)*tan(1/2*a)^46 + 83490*sqrt(2)*tan(1/2*a
)^44 + 179630*sqrt(2)*tan(1/2*a)^42 + 297275*sqrt(2)*tan(1/2*a)^40 + 36052
5*sqrt(2)*tan(1/2*a)^38 + 264385*sqrt(2)*tan(1/2*a)^36 - 37145*sqrt(2)*tan
(1/2*a)^34 - 445740*sqrt(2)*tan(1/2*a)^32 - 742900*sqrt(2)*tan(1/2*a)^30 -
 742900*sqrt(2)*tan(1/2*a)^28 - 445740*sqrt(2)*tan(1/2*a)^26 - 37145*sqrt(
2)*tan(1/2*a)^24 + 264385*sqrt(2)*tan(1/2*a)^22 + 360525*sqrt(2)*tan(1/2*a
)^20 + 297275*sqrt(2)*tan(1/2*a)^18 + 179630*sqrt(2)*tan(1/2*a)^16 + 83490
*sqrt(2)*tan(1/2*a)^14 + 30130*sqrt(2)*tan(1/2*a)^12 + 8350*sqrt(2)*tan(1/
2*a)^10 + 1725*sqrt(2)*tan(1/2*a)^8 + 251*sqrt(2)*tan(1/2*a)^6 + 23*sqrt(2
)*tan(1/2*a)^4 + sqrt(2)*tan(1/2*a)^2)*tan(1/2*b*x)/(tan(1/2*a)^51 + 23*ta
n(1/2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*a)^43
+ 31878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 + 38936
7*tan(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 534888*tan
(1/2*a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888*tan(1...
```

Mupad [B] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.04

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{2\,e^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{e^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+e^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{3\,b\,\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^(5/2),x)
```

output
```
-(2*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*
1i)/2)^(1/2)*(exp(a*2i + b*x*2i)*1i + exp(a*4i + b*x*4i)*1i + 1i))/(3*b*(e
xp(a*2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i) + 1)^2)
```

Reduce [F]

$$\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\sin(bx+a)}{\sin(2bx+2a)^3}dx$$

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x))/sin(2*a + 2*b*x)**3,x)
```

3.489 $\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

output
```
1/5*sin(b*x+a)/b/sin(2*b*x+2*a)^(5/2)-4/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/
2)+8/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{(-5\cot(a+bx)\csc(a+bx)+3\sec(a+bx)\,(9+\sec^2(a+bx)))\,\sqrt{\sin(2(a+bx))}}{120b}$$

input
```
Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
((-5*Cot[a + b*x]*Csc[a + b*x] + 3*Sec[a + b*x]*(9 + Sec[a + b*x]^2))*Sqrt
[Sin[2*(a + b*x)]])/(120*b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4792, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4792}$$

$$\frac{4}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4791}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4780}$$

$$\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{5}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)$$

input
```
Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(4*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*S
qrt[Sin[2*a + 2*b*x]])))/5 + Sin[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 97.64 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.90

$$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)\left(5\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\ \mathrm{Ell}\right.}{}$$

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

output
```
-1/48/b*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/
2*b*x)^2-1)/tan(1/2*a+1/2*b*x)*(5*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2
*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*
b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x)^6+5*(ta
n(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*
b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+
1/2*b*x)-7*tan(1/2*a+1/2*b*x)^4+7*tan(1/2*a+1/2*b*x)^2+1)/(tan(1/2*a+1/2*b
*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)/(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*
x))^(1/2)/(1+tan(1/2*a+1/2*b*x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.11

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$
$$=\frac{32\cos(bx+a)^{5}-32\cos(bx+a)^{3}+\sqrt{2}\left(32\cos(bx+a)^{4}-24\cos(bx+a)^{2}-3\right)\sqrt{\cos(bx+a)\sin(bx}}{120\left(b\cos(bx+a)^{5}-b\cos(bx+a)^{3}\right)}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
1/120*(32*cos(b*x + a)^5 - 32*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4
- 24*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^
5 - b*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18022 vs. $2(67) = 134$.

Time = 157.24 (sec) , antiderivative size = 18022, normalized size of antiderivative = 228.13

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
1/480*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a
)^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/
2*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan
(1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(
1/2*b*x) + tan(1/2*a))*(((((((((2*(sqrt(2)*tan(1/2*a)^87 - 92*sqrt(2)*tan
(1/2*a)^85 - 3213*sqrt(2)*tan(1/2*a)^83 - 48008*sqrt(2)*tan(1/2*a)^81 - 44
3462*sqrt(2)*tan(1/2*a)^79 - 2875040*sqrt(2)*tan(1/2*a)^77 - 13907802*sqrt
(2)*tan(1/2*a)^75 - 51781432*sqrt(2)*tan(1/2*a)^73 - 149943911*sqrt(2)*tan
(1/2*a)^71 - 333456564*sqrt(2)*tan(1/2*a)^69 - 536442973*sqrt(2)*tan(1/2*a
)^67 - 482080288*sqrt(2)*tan(1/2*a)^65 + 316221080*sqrt(2)*tan(1/2*a)^63 +
 2190937152*sqrt(2)*tan(1/2*a)^61 + 4607763368*sqrt(2)*tan(1/2*a)^59 + 574
2984608*sqrt(2)*tan(1/2*a)^57 + 3316624962*sqrt(2)*tan(1/2*a)^55 - 3241815
576*sqrt(2)*tan(1/2*a)^53 - 11030972730*sqrt(2)*tan(1/2*a)^51 - 1471202712
0*sqrt(2)*tan(1/2*a)^49 - 10524179460*sqrt(2)*tan(1/2*a)^47 + 10524179460*
sqrt(2)*tan(1/2*a)^43 + 14712027120*sqrt(2)*tan(1/2*a)^41 + 11030972730*sq
rt(2)*tan(1/2*a)^39 + 3241815576*sqrt(2)*tan(1/2*a)^37 - 3316624962*sqrt(2
)*tan(1/2*a)^35 - 5742984608*sqrt(2)*tan(1/2*a)^33 - 4607763368*sqrt(2)*ta
n(1/2*a)^31 - 2190937152*sqrt(2)*tan(1/2*a)^29 - 316221080*sqrt(2)*tan(1/2
*a)^27 + 482080288*sqrt(2)*tan(1/2*a)^25 + 536442973*sqrt(2)*tan(1/2*a)^23
 + 333456564*sqrt(2)*tan(1/2*a)^21 + 149943911*sqrt(2)*tan(1/2*a)^19 + ...
```

Mupad [B] (verification not implemented)

Time = 22.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.66

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{4\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-3\,\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+2\,\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}+2\,\mathrm{e}^{a\,8\mathrm{i}+b\,x\,8\mathrm{i}}+2\right)}{15\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^(7/2),x)
```

output
```
(4*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2)*(2*exp(a*2i + b*x*2i) - 3*exp(a*4i + b*x*4i) + 2*exp(a*6i + b*
x*6i) + 2*exp(a*8i + b*x*8i) + 2))/(15*b*(exp(a*2i + b*x*2i) - 1)^2*(exp(a
*2i + b*x*2i) + 1)^3)
```

Reduce [F]

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\sin(bx+a)}{\sin(2bx+2a)^4}dx$$

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x))/sin(2*a + 2*b*x)**4,x)
```

3.490 $\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{16\cos(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

output
```
1/7*sin(b*x+a)/b/sin(2*b*x+2*a)^(7/2)-6/35*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/
2)+8/35*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-16/35*cos(b*x+a)/b/sin(2*b*x+2*a
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{(-5-10\cos(2(a+bx))+4\cos(4(a+bx))+4\cos(6(a+bx)))\csc^3(a+bx)\sec^4(a+bx)\sqrt{\sin(2(a+bx}{560b}$$

input
```
Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
((-5 - 10*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[
a + b*x]^3*Sec[a + b*x]^4*Sqrt[Sin[2*(a + b*x)]])/(560*b)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4792, 3042, 4791, 3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{9/2}}dx$$

$$\Big\downarrow \text{4792}$$

$$\frac{6}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{7/2}}dx+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4791}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4792}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 4779}$$

$$\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{6}{7}\left(\frac{4}{5}\left(\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)$$

input
```
Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(6*((4*(Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*
Sqrt[Sin[2*a + 2*b*x]])))/5 - Cos[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2))))/
7 + Sin[a + b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sin(bx+a)}{\sin(2bx+2a)^{\frac{9}{2}}}dx$$

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\left(128\cos(bx+a)^6-160\cos(bx+a)^4+20\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+128\,(c}{560\left(b\cos(bx+a)^6-b\cos(bx+a)^4\right)\sin(bx+a)}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
-1/560*(sqrt(2)*(128*cos(b*x + a)^6 - 160*cos(b*x + a)^4 + 20*cos(b*x + a)
^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - cos(b*x +
a)^4)*sin(b*x + a))/((b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(9/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 24.47 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.34

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,1i}{7\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^4} + \frac{16\,e^{a\,3i+b\,x\,3i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{35\,b\,\left(e^{a\,2i+b\,x\,2i}-1\right)\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)} - \frac{e^{a\,1i+b\,x\,1i}\left(\frac{1i}{7\,b}+\frac{e^{a\,2i+b\,x\,2i}\,8i}{35\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}-1\right)^2\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^2} - \frac{e^{a\,1i+b\,x\,1i}\left(\frac{16}{35\,b}-\frac{44\,e^{a\,2i+b\,x\,2i}}{35\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}-1\right)^3\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^3}$$

input
```
int(sin(a + b*x)/sin(2*a + 2*b*x)^(9/2),x)
```

output
```
(16*exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*
1i)/2)^(1/2))/(35*b*(exp(a*2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i)*1i + 1i))
 - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*
1i)/2)^(1/2)*1i)/(7*b*(exp(a*2i + b*x*2i)*1i + 1i)^4) - (exp(a*1i + b*x*1i
)*(1i/(7*b) + (exp(a*2i + b*x*2i)*8i)/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2
 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x*2i) - 1)^2*(exp(a*2i
 + b*x*2i)*1i + 1i)^2) - (exp(a*1i + b*x*1i)*(16/(35*b) - (44*exp(a*2i + b
*x*2i))/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^
(1/2))/((exp(a*2i + b*x*2i) - 1)^3*(exp(a*2i + b*x*2i)*1i + 1i)^3)
```

Reduce [F]

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\sin(bx+a)}{\sin(2bx+2a)^{5}}dx$$

input
```
int(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x))/sin(2*a + 2*b*x)**5,x)
```

3.491 $\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \frac{5\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{42b} - \frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b} - \frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b} - \frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

output
```
5/42*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))/b-5/42*cos(2*b*x+2*a)*sin(2*b*x
+2*a)^(1/2)/b-1/14*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(5/2)/b-1/18*sin(2*b*x+2*
a)^(9/2)/b
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= \frac{240\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2(a+bx))}-70\sin(2(a+bx))-156\sin(4(a+bx))+35\sin(6(a+b}{2016b\sqrt{\sin(2(a+bx))}}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(240*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] - 70*Sin[2*(a + b
*x)] - 156*Sin[4*(a + b*x)] + 35*Sin[6*(a + b*x)] + 18*Sin[8*(a + b*x)] -
7*Sin[10*(a + b*x)])/(2016*b*Sqrt[Sin[2*(a + b*x)]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4786, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^{7/2}dx$$

$$\Big\downarrow \text{4786}$$

$$\frac{1}{2}\int \sin^{\frac{7}{2}}(2a+2bx)dx-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^{7/2}dx-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{5}{7}\int \sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5}{7}\int \sin(2a+2bx)^{3/2}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{2}\left(\frac{5}{7}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
-1/18*Sin[2*a + 2*b*x]^(9/2)/b + ((5*(EllipticF[a - Pi/4 + b*x, 2]/(3*b) -
 (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b)))/7 - (Cos[2*a + 2*b*x]*S
in[2*a + 2*b*x]^(5/2))/(7*b))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

output
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

Fricas [F]

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
integral(((cos(b*x + a)^2 - 1)*cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + 1)*si
n(2*b*x + 2*a)^(3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^{\frac{7}{2}}\sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^(7/2)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^{\frac{7}{2}}\sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^(7/2)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx=\int \sin\left(a+b\,x\right)^2\sin\left(2\,a+2\,b\,x\right)^{7/2}\,dx$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2),x)
```

output
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2), x)
```

Reduce [F]

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx=\int \sqrt{\sin\left(2bx+2a\right)}\,\sin\left(2bx+2a\right)^3\sin\left(bx+a\right)^2dx$$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*sin(2*a + 2*b*x)**3*sin(a + b*x)**2,x)
```

3.492 $\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b} - \frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b} - \frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

output
```
-3/10*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b-1/10*cos(2*b*x+2*a)*sin(2*b*x
+2*a)^(3/2)/b-1/14*sin(2*b*x+2*a)^(7/2)/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{84E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sqrt{\sin(2(a+bx))}(-15\sin(2(a+bx))-14\sin(4(a+bx))+5\sin(6(a+bx)))}{280b}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(-15*Sin[2*(a +
b*x)] - 14*Sin[4*(a + b*x)] + 5*Sin[6*(a + b*x)]))/(280*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4786, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^{5/2}dx$$

$$\Big\downarrow \text{4786}$$

$$\frac{1}{2}\int \sin^{\frac{5}{2}}(2a+2bx)dx-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^{5/2}dx-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{2}\left(\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/14*Sin[2*a + 2*b*x]^(7/2)/b + ((3*EllipticE[a - Pi/4 + b*x, 2])/(5*b) -
 (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(3/2))/(5*b))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x)
```

output
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x)
```

Fricas [F]

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
integral(((cos(b*x + a)^2 - 1)*cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + 1)*sq
rt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sin\left(a+b\,x\right)^2\sin\left(2\,a+2\,b\,x\right)^{5/2}\,dx$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2),x)
```

output
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2), x)
```

Reduce [F]

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sqrt{\sin\left(2bx+2a\right)}\,\sin\left(2bx+2a\right)^2\sin\left(bx+a\right)^2dx$$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*sin(2*a + 2*b*x)**2*sin(a + b*x)**2,x)
```

3.493 $\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{6b}-\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

output
```
1/6*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))/b-1/6*cos(2*b*x+2*a)*sin(2*b*x+2
*a)^(1/2)/b-1/10*sin(2*b*x+2*a)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx =\frac{20\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2(a+bx))}-9\sin(2(a+bx))-10\sin(4(a+bx))+3\sin(6(a+bx))}{120b\sqrt{\sin(2(a+bx))}}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] - 9*Sin[2*(a + b*x
)] - 10*Sin[4*(a + b*x)] + 3*Sin[6*(a + b*x)])/(120*b*Sqrt[Sin[2*(a + b*x)
]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4786, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^{3/2}dx$$

$$\Big\downarrow \text{4786}$$

$$\frac{1}{2}\int \sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^{3/2}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{2}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(EllipticF[a - Pi/4 + b*x, 2]/(3*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b
*x]])/(3*b))/2 - Sin[2*a + 2*b*x]^(5/2)/(10*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 22.92 (sec) , antiderivative size = 84470649, normalized size of antiderivative = 1224212.30

method	result	size
default	Expression too large to display	84470649

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^(3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\int \sin\left(a+b\,x\right)^2\sin\left(2\,a+2\,b\,x\right)^{3/2}\,dx$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2), x)
```

Reduce [F]

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\int \sqrt{\sin\left(2bx+2a\right)}\,\sin\left(2bx+2a\right)\sin\left(bx+a\right)^2dx$$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*sin(2*a + 2*b*x)*sin(a + b*x)**2,x)
```

3.494 $\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx=\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

output
```
-1/2*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b-1/6*sin(2*b*x+2*a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx=-\frac{-3E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sin^{\frac{3}{2}}(2(a+bx))}{6b}$$

input
```
Integrate[Sin[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/6*(-3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2))/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4786, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(a+bx)^2\sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \ 4786$$

$$\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \ 3119$$

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

input
```
Int[Sin[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
EllipticE[a - Pi/4 + b*x, 2]/(2*b) - Sin[2*a + 2*b*x]^(3/2)/(6*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 4.66 (sec) , antiderivative size = 26159629, normalized size of antiderivative = 653990.72

method	result	size
default	Expression too large to display	26159629

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx=\int \sqrt{\sin(2\,bx+2\,a)}\sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx=\text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx=\int \sqrt{\sin(2\,bx+2\,a)}\sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^2, x)
```

Giac [F]

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin\left(2\,bx+2\,a\right)}\sin\left(bx+a\right)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sin\left(a+b\,x\right)^2\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2), x)
```

Reduce [F]

$$\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin\left(2bx+2a\right)}\,\sin\left(bx+a\right)^2dx$$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*sin(a + b*x)**2,x)
```

3.495 $\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \frac{\operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{2b} - \frac{\sqrt{\sin(2a+2bx)}}{2b}$$

output
```
1/2*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))/b-1/2*sin(2*b*x+2*a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.88

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{2\sqrt{\sin(2(a+bx))} + \frac{\sqrt{2}\operatorname{EllipticF}\left(\arcsin(\cos(a+bx)-\sin(a+bx)),\frac{1}{2}\right)(\cos(a+bx)+\sin(a+bx))}{\sqrt{1+\sin(2(a+bx))}}}{4b}$$

input
```
Integrate[Sin[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/4*(2*Sqrt[Sin[2*(a + b*x)]] + (Sqrt[2]*EllipticF[ArcSin[Cos[a + b*x] -
Sin[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a + b*x
)]])/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4786, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4786}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b}-\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

input
```
Int[Sin[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
EllipticF[a - Pi/4 + b*x, 2]/(2*b) - Sqrt[Sin[2*a + 2*b*x]]/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.41 (sec) , antiderivative size = 11960757, normalized size of antiderivative = 299018.92

method	result	size
default	Expression too large to display	11960757

input
```
int(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin(bx+a)^2}{\sqrt{\sin(2bx+2a)}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)/sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin(bx+a)^2}{\sqrt{\sin(2bx+2a)}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sqrt{\sin\left(2bx+2a\right)}\,\sin\left(bx+a\right)^2}{\sin\left(2bx+2a\right)}dx$$

input
```
int(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x)**2)/sin(2*a + 2*b*x),x)
```

3.496 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}+\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

output
```
1/2*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b+sin(b*x+a)^2/b/sin(2*b*x+2*a)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{-E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sqrt{\sin(2(a+bx))}\tan(a+bx)}{2b}$$

input
```
Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(-EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*Tan[a + b*x])/(2*b
)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4784, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{4784}$$

$$\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

input
```
Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-1/2*EllipticE[a - Pi/4 + b*x, 2]/b + Sin[a + b*x]^2/(b*Sqrt[Sin[2*a + 2*b
*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4784
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Sin[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && L
tQ[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[
2*m, 2*p]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.97 (sec) , antiderivative size = 22356405, normalized size of antiderivative = 496809.00

method	result	size
default	Expression too large to display	22356405

input
```
int(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.47

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{-i\sqrt{2i}\cos(bx+a)\,E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+i\sqrt{-2i}\cos(bx+a)\,E(\arcsin(\cos(bx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(-I*sqrt(2*I)*cos(b*x + a)*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x
+ a)), -1) + I*sqrt(-2*I)*cos(b*x + a)*elliptic_e(arcsin(cos(b*x + a) - I*
sin(b*x + a)), -1) + I*sqrt(2*I)*cos(b*x + a)*elliptic_f(arcsin(cos(b*x +
a) + I*sin(b*x + a)), -1) - I*sqrt(-2*I)*cos(b*x + a)*elliptic_f(arcsin(co
s(b*x + a) - I*sin(b*x + a)), -1) + 2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x +
a))*sin(b*x + a))/(b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\sin(bx+a)^2}{\sin(2bx+2a)^2}dx$$

input
```
int(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x)**2)/sin(2*a + 2*b*x)**2,x)
```

3.497 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=\frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{6b}+\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

output
```
1/6*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))/b+1/3*sin(b*x+a)^2/b/sin(2*b*x+2
*a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.73

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\sec^2(a+bx)\sqrt{\sin(2(a+bx))}-\frac{\sqrt{2}\,\text{EllipticF}\left(\arcsin(\cos(a+bx)-\sin(a+bx)),\frac{1}{2}\right)(\cos(a+bx)+\sin(a+bx))}{\sqrt{1+\sin(2(a+bx))}}}{12b}$$

input
```
Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(Sec[a + b*x]^2*Sqrt[Sin[2*(a + b*x)]] - (Sqrt[2]*EllipticF[ArcSin[Cos[a +
 b*x] - Sin[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*
(a + b*x)]])/(12*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4784, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4784}$$

$$\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{6b}$$

input
```
Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
EllipticF[a - Pi/4 + b*x, 2]/(6*b) + Sin[a + b*x]^2/(3*b*Sin[2*a + 2*b*x]^
(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4784
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Sin[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && L
tQ[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[
2*m, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(42) = 84$.

Time = 19.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.56

method	result
default	$\frac{\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\ \operatorname{EllipticF}\left(\sqrt{\sin(2bx+2a)+1}, \frac{\sqrt{2}}{2}\right)\sin(2bx+2a)-2\cos(2bx+2a)^{2}+2\cos(2}{12\sin(2bx+2a)^{\frac{3}{2}}\cos(2bx+2a)b}$

input
```
int(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/sin(2*b*x+2*a)^(3/2)/cos(2*b*x+2*a)*((sin(2*b*x+2*a)+1)^(1/2)*(-2*sin
(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^
(1/2),1/2*2^(1/2))*sin(2*b*x+2*a)-2*cos(2*b*x+2*a)^2+2*cos(2*b*x+2*a))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.96

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2i}\cos(bx+a)^2 F(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+\sqrt{-2i}\cos(bx+a)^2 F(\arcsin(\cos(b}{12\,b\cos(bx+a)^2}$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(2*I)*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x
 + a)), -1) + sqrt(-2*I)*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a) - I
*sin(b*x + a)), -1) - sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x
+ a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(5/2),x)
```

output
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\sin(bx+a)^2}{\sin(2bx+2a)^3}dx$$

input
```
int(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x)**2)/sin(2*a + 2*b*x)**3,x)
```

3.498 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)} \, dx = -\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b}+\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

output
```
3/10*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b+1/5*sin(b*x+a)^2/b/sin(2*b*x+2
*a)^(5/2)-3/10*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)} \, dx = -\frac{12E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\frac{4(1+6\cos(2(a+bx))+3\cos(4(a+bx)))\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2(a+bx))}}{40b}$$

input
```
Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
-1/40*(12*EllipticE[a - Pi/4 + b*x, 2] + (4*(1 + 6*Cos[2*(a + b*x)] + 3*Co
s[4*(a + b*x)])*Sin[a + b*x]^2)/Sin[2*(a + b*x)]^(5/2))/b
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4784, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)^2}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{ 4784}$$

$$\frac{3}{10}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{10}\int \frac{1}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{3}{10}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)+\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{10}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)+\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{3}{10}\left(-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)$$

input
```
Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(3*(-(EllipticE[a - Pi/4 + b*x, 2]/b) - Cos[2*a + 2*b*x]/(b*Sqrt[Sin[2*a +
 2*b*x]])))/10 + Sin[a + b*x]^2/(5*b*Sin[2*a + 2*b*x]^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4784
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Sin[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && L
tQ[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[
2*m, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(68) = 136.

Time = 147.19 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.95

method	result
default	$\frac{\sqrt{2}\left(\frac{8\sqrt{2}}{5\sin(2bx+2a)^{\frac{5}{2}}}+\frac{4\sqrt{2}\left(6\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\sin(2bx+2a)^{2}\,\operatorname{EllipticE}\left(\sqrt{\sin(2bx+2a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{\sin(2}}{5\sin(2b}}{32b}$

input
```
int(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*2^(1/2)*(8/5*2^(1/2)/sin(2*b*x+2*a)^(5/2)+4/5*2^(1/2)/sin(2*b*x+2*a)^
(5/2)*(6*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+
2*a))^(1/2)*sin(2*b*x+2*a)^2*EllipticE((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2
))-3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a)
)^(1/2)*sin(2*b*x+2*a)^2*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+6
*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-2)/cos(2*b*x+2*a))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.75

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{-6i\sqrt{2i}\cos{(bx+a)}^3 E(\arcsin{(\cos{(bx+a)}+i\sin{(bx+a)})}\,|-1)\sin{(bx+a)}+6i\sqrt{-2i}\cos{(bx+a)}^3}{}$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
1/40*(-6*I*sqrt(2*I)*cos(b*x + a)^3*elliptic_e(arcsin(cos(b*x + a) + I*sin
(b*x + a)), -1)*sin(b*x + a) + 6*I*sqrt(-2*I)*cos(b*x + a)^3*elliptic_e(ar
csin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + 6*I*sqrt(2*I)*cos(
b*x + a)^3*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x +
 a) - 6*I*sqrt(-2*I)*cos(b*x + a)^3*elliptic_f(arcsin(cos(b*x + a) - I*sin
(b*x + a)), -1)*sin(b*x + a) - sqrt(2)*(12*cos(b*x + a)^4 - 6*cos(b*x + a)
^2 - 1)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^3*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=\int \frac{{\sin \left (a+b\,x\right)}^2}{{\sin \left (2\,a+2\,b\,x\right)}^{7/2}} \,d x$$

input
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(7/2),x)
```

output
```
int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=\int \frac{\sqrt{\sin \left (2 b x +2 a \right)}\, \sin \left (b x +a \right)^{2}}{\sin \left (2 b x +2 a \right)^{4}}d x$$

input
```
int(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x)**2)/sin(2*a + 2*b*x)**4,x)
```

3.499 $\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{7\arcsin(\cos(a+bx)-\sin(a+bx))}{64b} - \frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{64b} + \frac{7\sin(a+bx)\sqrt{\sin(2a+2bx)}}{32b} - \frac{7\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b} - \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

output
```
-7/64*arcsin(cos(b*x+a)-sin(b*x+a))/b-7/64*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b+7/32*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b-7/48*cos(b*x+a)*s
in(2*b*x+2*a)^(3/2)/b-1/12*sin(b*x+a)*sin(2*b*x+2*a)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$=\frac{-7\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+\frac{2}{3}\sqrt{\sin}}{64b}$$

input
```
Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(-7*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(10*Sin[a + b*x] -
 9*Sin[3*(a + b*x)] + 2*Sin[5*(a + b*x)]))/3)/(64*b)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4786, 3042, 4790, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(a+bx)^3\sin(2a+2bx)^{3/2}dx$$
$$\Big\downarrow \text{4786}$$
$$\frac{7}{12}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{7}{12}\int \sin(a+bx)\sin(2a+2bx)^{3/2}dx-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

$$\Big\downarrow \text{4790}$$

$$\frac{7}{12}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{12}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

$$\Big\downarrow \text{4789}$$

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

$$\Big\downarrow \text{4794}$$

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a\right.\right.$$
$$\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}$$

input

```
Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-1/12*(Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2))/b + (7*((3*((-1/2*ArcSin[Cos[a
 + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a
 + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 - (
Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)))/12
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

output
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(118) = 236.

Time = 0.10 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.13

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{8\sqrt{2}\left(32\cos(bx+a)^4-60\cos(bx+a)^2+21\right)\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)+42\arctan\left(-\frac{\sqrt{}}{}\right.}{}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/768*(8*sqrt(2)*(32*cos(b*x + a)^4 - 60*cos(b*x + a)^2 + 21)*sqrt(cos(b*x
 + a)*sin(b*x + a))*sin(b*x + a) + 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*s
in(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(c
os(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) - 42*arctan(-(2*sqrt(2)*
sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x +
a) - sin(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a
)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x +
 a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))
/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^3, x)
```

Giac [F]

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sin\left(a+b\,x\right)^3\sin\left(2\,a+2\,b\,x\right)^{3/2}\,dx$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(3/2), x)
```

Reduce [F]

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sqrt{\sin\left(2bx+2a\right)}\,\sin\left(2bx+2a\right)\sin\left(bx+a\right)^3dx$$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*sin(2*a + 2*b*x)*sin(a + b*x)**3,x)
```

3.500 $\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{5\arcsin(\cos(a+bx)-\sin(a+bx))}{32b} + \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{32b} - \frac{5\cos(a+bx)\sqrt{\sin(2a+2bx)}}{16b} - \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$

output
```
-5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b+5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b-5/16*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b-1/8*sin(b*x+a)*si
n(2*b*x+2*a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{5\left(-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+2(-6\cos}{32b}$$

input
```
Integrate[Sin[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(5*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + 2*(-6*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[
Sin[2*(a + b*x)]])/(32*b)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4786, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(a+bx)^3\sqrt{\sin(2a+2bx)}dx$$
$$\Big\downarrow \text{4786}$$
$$\frac{5}{8}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{5}{8}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$

$$\downarrow \text{4790}$$

$$\frac{5}{8}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$

$$\downarrow \text{3042}$$

$$\frac{5}{8}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$

$$\downarrow \text{4793}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\frac{\sqrt{\sin(2a+}}{}\right.$$
$$\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$

input
```
Int[Sin[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(5*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a
 + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(2*b)))/8 - (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(8*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 29.68 (sec) , antiderivative size = 89676612, normalized size of antiderivative = 815241.93

method	result	size
default	Expression too large to display	89676612

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(96) = 192.

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.55

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{8\sqrt{2}\left(4\cos(bx+a)^3 - 9\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)} + 10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos}{\cos(bx+a)^2+}\right.}{}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/128*(8*sqrt(2)*(4*cos(b*x + a)^3 - 9*cos(b*x + a))*sqrt(cos(b*x + a)*sin
(b*x + a)) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x
+ a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(
b*x + a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(
b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) -
5*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2
 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*
cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin\left(2\,bx+2\,a\right)}\sin\left(bx+a\right)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sin\left(a+b\,x\right)^3\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

input
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(1/2), x)
```

Reduce [F]

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin\left(2bx+2a\right)}\,\sin\left(bx+a\right)^3 dx$$

input
```
int(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*sin(a + b*x)**3,x)
```

3.501 $\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))}{8b} - \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b} - \frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}$$

output
```
-3/8*arcsin(cos(b*x+a)-sin(b*x+a))/b-3/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b-1/4*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))+3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)+2\sin(a}{8b}$$

input
```
Integrate[Sin[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/8*(3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a +
 b*x] + Sqrt[Sin[2*(a + b*x)]]] + 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4786, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4786}$$

$$\frac{3}{4}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}$$

$$\Big\downarrow \text{4794}$$

$$\frac{3}{4}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)-\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}$$

input
```
Int[Sin[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(3*(-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a
+ b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)))/4 - (Sin[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(4*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4786
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Sin
[a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p
}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] &&
NeQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

output
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(74) = 148$.

Time = 0.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.19

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx =$$
$$-\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a) - 6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)}{}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
-1/32*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) - 6*arctan(-
(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + c
os(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) -
1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a)
- sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4
+ 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*co
s(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b
*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin(bx+a)^3}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sin(a+b\,x)^3}{\sqrt{\sin(2\,a+2\,b\,x)}}\,dx$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\sin(bx+a)^3}{\sin(2bx+2a)}dx$$

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x)**3)/sin(2*a + 2*b*x),x)
```

3.502 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{4b} - \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b} + \frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

output
```
1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b+sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\arcsin(\cos(a+bx)-\sin(a+bx)) - \log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right) + 2\sec(a+bx}{4b}$$

input
```
Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(ArcSin[Cos[a + b*x] - Sin[a + b*x]] - Log[Cos[a + b*x] + Sin[a + b*x] + S
qrt[Sin[2*(a + b*x)]]] + 2*Sec[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4782, 3042, 4796, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{4782}$$

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{4}\int \csc(a+bx)\sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{4}\int \frac{\sqrt{\sin(2a+2bx)}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4796}$$

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4793}$$

$$\frac{1}{2}\left(\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(2*b) - Log[Cos[a + b*x] + Sin[a + b*
x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + Sin[a + b*x]/(b*Sqrt[Sin[2*a + 2*b
*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4782
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
 + 1)/(2*b*g*(p + 1))), x] + Simp[e^4*((m + p - 1)/(4*g^2*(p + 1)))   Int[(e
*Sin[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c,
d, e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m,
 2] && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x)
```

output
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(73) = 146.

Time = 0.09 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.65

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)\cos(bx+a)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{c}}{}\right.}{}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/16*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) -
sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a
)*sin(b*x + a) - 1))*cos(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)
*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a)
))*cos(b*x + a) - cos(b*x + a)*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b
*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos
(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a)
 + 1) - 8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - 8*cos(b*x + a))/(b*cos
(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\sin\left(2\,a+2\,b\,x\right)^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin\left(2bx+2a\right)}\,\sin\left(bx+a\right)^3}{\sin\left(2bx+2a\right)^2}dx$$

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x)**3)/sin(2*a + 2*b*x)**2,x)
```

3.503 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=\frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

output
```
1/3*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=\frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2(a+bx))}$$

input
```
Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
Sin[a + b*x]^3/(3*b*Sin[2*(a + b*x)]^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4780}$$

$$\frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

input
```
Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
Sin[a + b*x]^3/(3*b*Sin[2*a + 2*b*x]^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 35.10 (sec) , antiderivative size = 727, normalized size of antiderivative = 25.96

method	result	size
default	Expression too large to display	727

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2*
b*x)^2-1)*(6*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*
(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticE((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(
1/2))*tan(1/2*a+1/2*b*x)^6-3*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/
2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+
1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^6+18*(tan(1/2*a+1/2*b*x)+1)^(1/2)
*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticE((ta
n(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^4-9*(tan(1/2*a+1
/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/
2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^
4+6*tan(1/2*a+1/2*b*x)^8+18*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2
*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticE((tan(1/2*a+1/2*b*x)+1
)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2-9*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(
-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(
1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2-2*tan(1/2*a+1/2*
b*x)^6+6*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-ta
n(1/2*a+1/2*b*x))^(1/2)*EllipticE((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2)
)-3*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2
*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))+10*
tan(1/2*a+1/2*b*x)^4-14*tan(1/2*a+1/2*b*x)^2)/(tan(1/2*a+1/2*b*x)*(tan(...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\cos(bx+a)^2-\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)}{12\,b\cos(bx+a)^2}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(cos(b*x + a)^2 - sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x +
a))/(b*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15648 vs. 2(24) = 48.

Time = 92.92 (sec) , antiderivative size = 15648, normalized size of antiderivative = 558.86

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)
^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2
*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(
1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1
/2*b*x) + tan(1/2*a))*((((2*(4*(2*(sqrt(2)*tan(1/2*a)^54 + 17*sqrt(2)*tan(
1/2*a)^52 + 132*sqrt(2)*tan(1/2*a)^50 + 612*sqrt(2)*tan(1/2*a)^48 + 1842*s
qrt(2)*tan(1/2*a)^46 + 3570*sqrt(2)*tan(1/2*a)^44 + 3668*sqrt(2)*tan(1/2*a
)^42 - 1292*sqrt(2)*tan(1/2*a)^40 - 11457*sqrt(2)*tan(1/2*a)^38 - 19057*sq
rt(2)*tan(1/2*a)^36 - 12920*sqrt(2)*tan(1/2*a)^34 + 7752*sqrt(2)*tan(1/2*a
)^32 + 27132*sqrt(2)*tan(1/2*a)^30 + 27132*sqrt(2)*tan(1/2*a)^28 + 7752*sq
rt(2)*tan(1/2*a)^26 - 12920*sqrt(2)*tan(1/2*a)^24 - 19057*sqrt(2)*tan(1/2*
a)^22 - 11457*sqrt(2)*tan(1/2*a)^20 - 1292*sqrt(2)*tan(1/2*a)^18 + 3668*sq
rt(2)*tan(1/2*a)^16 + 3570*sqrt(2)*tan(1/2*a)^14 + 1842*sqrt(2)*tan(1/2*a)
^12 + 612*sqrt(2)*tan(1/2*a)^10 + 132*sqrt(2)*tan(1/2*a)^8 + 17*sqrt(2)*ta
n(1/2*a)^6 + sqrt(2)*tan(1/2*a)^4)*tan(1/2*b*x)/(tan(1/2*a)^51 + 23*tan(1/
2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*a)^43 + 31
878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 + 389367*ta
n(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 534888*tan(1/2
*a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888*tan(1/2*a)^2
3 - 653752*tan(1/2*a)^21 - 572033*tan(1/2*a)^19 - 389367*tan(1/2*a)^17 ...
```

Mupad [B] (verification not implemented)

Time = 20.78 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.04

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,\left(2\,\sin\left(a+b\,x\right)+3\,\sin\left(3\,a+3\,b\,x\right)+\sin\left(5\,a+5\,b\,x\right)\right)}{6\,b\,\left(30\,\sin\left(a+b\,x\right)^2+12\,\sin\left(2\,a+2\,b\,x\right)^2+2\,\sin\left(3\,a+3\,b\,x\right)^2-32\right)}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(5/2),x)
```

output
```
-(sin(2*a + 2*b*x)^(1/2)*(2*sin(a + b*x) + 3*sin(3*a + 3*b*x) + sin(5*a +
5*b*x)))/(6*b*(12*sin(2*a + 2*b*x)^2 + 2*sin(3*a + 3*b*x)^2 + 30*sin(a + b
*x)^2 - 32))
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin\left(2bx+2a\right)}\,\sin\left(bx+a\right)^3}{\sin\left(2bx+2a\right)^3}dx$$

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x)**3)/sin(2*a + 2*b*x)**3,x)
```

3.504 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

output
```
1/5*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(5/2)+1/5*sin(b*x+a)/b/sin(2*b*x+2*a)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=\frac{\sec(a+bx)\left(4+\sec^2(a+bx)\right)\sqrt{\sin(2(a+bx))}}{40b}$$

input
```
Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(Sec[a + b*x]*(4 + Sec[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/(40*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4784, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4784}$$

$$\frac{1}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4780}$$

$$\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
Sin[a + b*x]^3/(5*b*Sin[2*a + 2*b*x]^(5/2)) + Sin[a + b*x]/(5*b*Sqrt[Sin[2
*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4784
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Sin[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && L
tQ[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[
2*m, 2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 161.41 (sec) , antiderivative size = 4684, normalized size of antiderivative = 85.16

method	result	size
default	Expression too large to display	4684

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*(8*tan(1/2*a+1/2*b*x)^3+56*tan(1/2*a+1/2*b*x)^7-8*tan(1/2*a+1/2*b*x)
^5-18*I*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan
(1/2*a+1/2*b*x))^(1/2)*EllipticPi((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2+1/2*I,1
/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2-32*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1
/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/
2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^6-96*(tan(1/2*a+1/2*b*x)+1
)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*Ellipt
icF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^4-96*(tan
(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b
*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1
/2*b*x)^2-56*tan(1/2*a+1/2*b*x)-18*I*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(
1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticPi((tan(1/2*a+
1/2*b*x)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^4+18*I*(tan(1/
2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x)
)^(1/2)*EllipticPi((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*tan
(1/2*a+1/2*b*x)^2+6*I*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+
2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticPi((tan(1/2*a+1/2*b*x)+1)^(1/
2),1/2-1/2*I,1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^6-6*I*(tan(1/2*a+1/2*b*x)+1)^
(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*Elliptic
Pi((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*tan(1/2*a+1/2*b*...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{4\,\cos{(bx+a)}^3+\sqrt{2}\big(4\,\cos{(bx+a)}^2+1\big)\sqrt{\cos{(bx+a)}\sin{(bx+a)}}}{40\,b\cos{(bx+a)}^3}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
1/40*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 + 1)*sqrt(cos(b*x + a)*
sin(b*x + a)))/(b*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(7/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 23.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.60

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,\left(3\,e^{a\,2i+b\,x\,2i}+e^{a\,4i+b\,x\,4i}+1\right)}{5\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)^3}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(7/2),x)
```

output
```
(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*(3*exp(a*2i + b*x*2i) + exp(a*4i + b*x*4i) + 1))/(5*b*(exp(a*2i
+ b*x*2i) + 1)^3)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\sin(bx+a)^3}{\sin(2bx+2a)^4}dx$$

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x)**3)/sin(2*a + 2*b*x)**4,x)
```

3.505 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx=\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

output
```
1/7*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(7/2)+2/21*sin(b*x+a)/b/sin(2*b*x+2*a)^(
3/2)-4/21*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{(5+12\cos(2(a+bx))+4\cos(4(a+bx)))\csc(a+bx)\sec^4(a+bx)\sqrt{\sin(2(a+bx))}}{336b}$$

input
```
Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
-1/336*((5 + 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]*Sec[a
+ b*x]^4*Sqrt[Sin[2*(a + b*x)]])/b
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4784, 3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^{9/2}}dx$$

$$\Big\downarrow \text{4784}$$

$$\frac{2}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx+\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx+\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4792}$$

$$\frac{2}{7}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4779}$$

$$\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{2}{7}\left(\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\right)$$

input
```
Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(2*(Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*Sqrt
[Sin[2*a + 2*b*x]])))/7 + Sin[a + b*x]^3/(7*b*Sin[2*a + 2*b*x]^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4784
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Sin[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && L
tQ[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[
2*m, 2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sin(bx+a)^3}{\sin(2bx+2a)^{\frac{9}{2}}}dx$$

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{32\,\cos(bx+a)^4\sin(bx+a)+\sqrt{2}\left(32\,\cos(bx+a)^4-8\,\cos(bx+a)^2-3\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{336\,b\cos(bx+a)^4\sin(bx+a)}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
-1/336*(32*cos(b*x + a)^4*sin(b*x + a) + sqrt(2)*(32*cos(b*x + a)^4 - 8*co
s(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^4*sin(b
*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(9/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 24.64 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.70

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = &-\frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,5i}{84\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^2}\\&+\frac{3\,e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{14\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^3}\\&-\frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,1i}{7\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^4}\\&+\frac{e^{a\,1i+b\,x\,1i}\left(\frac{5}{84\,b}+\frac{4\,e^{a\,2i+b\,x\,2i}}{21\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}-1\right)\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)}\end{aligned}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(9/2),x)
```

output
```
(3*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2))/(14*b*(exp(a*2i + b*x*2i)*1i + 1i)^3) - (exp(a*1i + b*x*1i)*(
(exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*5i)/(84*b*(
exp(a*2i + b*x*2i)*1i + 1i)^2) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i
)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*1i)/(7*b*(exp(a*2i + b*x*2i)*1i
 + 1i)^4) + (exp(a*1i + b*x*1i)*(5/(84*b) + (4*exp(a*2i + b*x*2i))/(21*b))
*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*
2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i)*1i + 1i))
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\sin(bx+a)^3}{\sin(2bx+2a)^5}dx$$

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x)**3)/sin(2*a + 2*b*x)**5,x)
```

3.506 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)} + \frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

output
```
1/9*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(9/2)+1/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(
5/2)-4/45*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+8/45*sin(b*x+a)/b/sin(2*b*x+2*
a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \frac{(-15\cot(a+bx)\csc(a+bx)+113\sec(a+bx)+17\sec^3(a+bx)+5\sec^5(a+bx))\sqrt{\sin(2(a+bx))}}{1440b}$$

input
```
Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

output
```
((-15*Cot[a + b*x]*Csc[a + b*x] + 113*Sec[a + b*x] + 17*Sec[a + b*x]^3 + 5
*Sec[a + b*x]^5)*Sqrt[Sin[2*(a + b*x)]])/(1440*b)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4784, 3042, 4792, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^{11/2}}dx$$

$$\Big\downarrow \text{4784}$$

$$\frac{1}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{7/2}}dx+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4792}$$

$$\frac{1}{3}\left(\frac{4}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{4}{5}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4791}$$

$$\frac{1}{3}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4780}$$

$$\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{1}{3}\left(\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{5}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)\right)$$

input
```
Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

output
```
((4*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*
Sqrt[Sin[2*a + 2*b*x]])))/5 + Sin[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2)))/3
 + Sin[a + b*x]^3/(9*b*Sin[2*a + 2*b*x]^(9/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4784
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Sin[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && L
tQ[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[
2*m, 2*p]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sin(bx+a)^3}{\sin(2bx+2a)^{\frac{11}{2}}}\,dx$$

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

output
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx$$
$$= \frac{128\cos(bx+a)^7 - 128\cos(bx+a)^5 + \sqrt{2}\left(128\cos(bx+a)^6 - 96\cos(bx+a)^4 - 12\cos(bx+a)^2 - 5}{1440\left(b\cos(bx+a)^7 - b\cos(bx+a)^5\right)}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="fricas")
```

output
```
1/1440*(128*cos(b*x + a)^7 - 128*cos(b*x + a)^5 + sqrt(2)*(128*cos(b*x + a
)^6 - 96*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x
 + a)))/(b*cos(b*x + a)^7 - b*cos(b*x + a)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \int \frac{\sin(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{11}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(11/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 26.88 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.58

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = &-\frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,1i}{60\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^3}\\&-\frac{2\,e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^4}\\&+\frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,1i}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^5}\\&+\frac{e^{a\,3i+b\,x\,3i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,8i}{45\,b\,\left(e^{a\,2i+b\,x\,2i}-1\right)\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)}\\&-\frac{e^{a\,1i+b\,x\,1i}\left(\frac{49}{180\,b}-\frac{19\,e^{a\,2i+b\,x\,2i}}{180\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}-1\right)^2\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^2}\end{aligned}$$

input
```
int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(11/2),x)
```

output
```
(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*1i)/(9*b*(exp(a*2i + b*x*2i)*1i + 1i)^5) - (2*exp(a*1i + b*x*1i)
*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(9*b*(ex
p(a*2i + b*x*2i)*1i + 1i)^4) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*
1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*1i)/(60*b*(exp(a*2i + b*x*2i)*1i
+ 1i)^3) + (exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i +
b*x*2i)*1i)/2)^(1/2)*8i)/(45*b*(exp(a*2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i
)*1i + 1i)) - (exp(a*1i + b*x*1i)*(49/(180*b) - (19*exp(a*2i + b*x*2i))/(1
80*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((
exp(a*2i + b*x*2i) - 1)^2*(exp(a*2i + b*x*2i)*1i + 1i)^2)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\sin(bx+a)^3}{\sin(2bx+2a)^6}\,dx$$

input
```
int(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*sin(a + b*x)**3)/sin(2*a + 2*b*x)**6,x)
```

3.507 $\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 136

$$\begin{aligned}\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx &= -\frac{5\arcsin(\cos(a+bx)-\sin(a+bx))}{16b} \\ &\quad -\frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{16b} \\ &\quad +\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b} \\ &\quad -\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{12b}+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}\end{aligned}$$

output
```
-5/16*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/16*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b+5/8*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b-5/12*cos(b*x+a)*si
n(2*b*x+2*a)^(3/2)/b+1/3*sin(b*x+a)*sin(2*b*x+2*a)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= \frac{-5\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+\frac{2}{3}\sqrt{\sin}}{16b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(14*Sin[a + b*x] -
 3*Sin[3*(a + b*x)] - 2*Sin[5*(a + b*x)]))/3)/(16*b)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4796, 3042, 4789, 3042, 4790, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{7}{2}}(2a+2bx)\csc(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(2a+2bx)^{7/2}}{\sin(a+bx)}dx$$
$$\Big\downarrow \text{4796}$$
$$2\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)dx$$
$$\Big\downarrow \text{3042}$$
$$2\int \cos(a+bx)\sin(2a+2bx)^{5/2}dx$$

$$\Big\downarrow \text{4789}$$

$$2\left(\frac{5}{6}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{5}{6}\int \sin(a+bx)\sin(2a+2bx)^{3/2}dx+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)$$

$$\Big\downarrow \text{4790}$$

$$2\left(\frac{5}{6}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{5}{6}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)$$

$$\Big\downarrow \text{4789}$$

$$2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx}{}\right.$$

$$\Big\downarrow \text{4794}$$

$$2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin}{}\right.\right.\right.$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
2*((Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2))/(6*b) + (5*((3*((-1/2*ArcSin[Cos[
a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*
a + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 -
(Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)))/6)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 10.90 (sec) , antiderivative size = 973, normalized size of antiderivative = 7.15

method	result	size
default	Expression too large to display	973

input

```
int(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
-16/5*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(6*(tan(1/2*a+1
/2*b*x)*(tan(1/2*a+1/2*b*x)-1)*(tan(1/2*a+1/2*b*x)+1))^(1/2)*(tan(1/2*a+1/
2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2
)*EllipticE((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^4
-3*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)-1)*(tan(1/2*a+1/2*b*x)+1))^(1/2
)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a
+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1
/2*a+1/2*b*x)^4+6*(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*
a+1/2*b*x)^6-12*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)-1)*(tan(1/2*a+1/2*
b*x)+1))^(1/2)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2
)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticE((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2
^(1/2))*tan(1/2*a+1/2*b*x)^2+6*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)-1)*
(tan(1/2*a+1/2*b*x)+1))^(1/2)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1
/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)
+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2-12*(tan(1/2*a+1/2*b*x)^3-tan(1
/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^4+6*(tan(1/2*a+1/2*b*x)*(tan(1/2*a
+1/2*b*x)-1)*(tan(1/2*a+1/2*b*x)+1))^(1/2)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-
2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticE((tan(1
/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))-3*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b
*x)-1)*(tan(1/2*a+1/2*b*x)+1))^(1/2)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*t...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(118) = 236.

Time = 0.10 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.13

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = -\frac{8\sqrt{2}\left(32\cos(bx+a)^4-12\cos(bx+a)^2-15\right)\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)-30\arctan\left(-\right.}{}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
-1/192*(8*sqrt(2)*(32*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 15)*sqrt(cos(b*
x + a)*sin(b*x + a))*sin(b*x + a) - 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*
sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(
cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 30*arctan(-(2*sqrt(2)
*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x +
 a) - sin(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x +
a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x
+ a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)
)/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx=\int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(7/2), x)
```

Giac [F]

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx=\int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx=\int \frac{\sin(2\,a+2\,b\,x)^{7/2}}{\sin(a+b\,x)}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x),x)
```

output
```
int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x), x)
```

Reduce [F]

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \sqrt{\sin(2bx+2a)}\,\csc(bx+a)\sin(2bx+2a)^3\,dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x)*sin(2*a + 2*b*x)**3,x)
```

3.508 $\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))}{8b} + \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b} - \frac{3\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b} + \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{2b}$$

output
```
-3/8*arcsin(cos(b*x+a)-sin(b*x+a))/b+3/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b-3/4*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b+1/2*sin(b*x+a)*sin(2
*b*x+2*a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{3\left(-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)-2(2\cos}{8b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(3*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) - 2*(2*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[S
in[2*(a + b*x)]])/(8*b)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4796, 3042, 4789, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{5}{2}}(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{5/2}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4796}$$

$$2\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \cos(a+bx)\sin(2a+2bx)^{3/2}dx$$

$$\Big\downarrow \text{4789}$$

$$2\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{4790}$$

$$2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{4793}$$

$$2\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\frac{\sqrt{\sin(2a}{}\right.\right.$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
2*((3*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Si
n[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a
 + 2*b*x]])/(2*b)))/4 + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 45.38 (sec) , antiderivative size = 99228795, normalized size of antiderivative = 902079.95

method	result	size
default	Expression too large to display	99228795

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(96) = 192$.

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.55

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx =$$
$$-\frac{8\sqrt{2}\left(4\cos\left(bx+a\right)^{3}-\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(}{\cos(bx+a)^{2}+2}\right.}{}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/32*(8*sqrt(2)*(4*cos(b*x + a)^3 - cos(b*x + a))*sqrt(cos(b*x + a)*sin(b
*x + a)) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a
) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
 + a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x
+ a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 3*lo
g(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1
)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(
b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

Giac [F]

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^{5/2}}{\sin(a+b\,x)}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x),x)
```

output
```
int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x), x)
```

Reduce [F]

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sqrt{\sin(2bx+2a)}\,\csc(bx+a)\sin(2bx+2a)^2\,dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x)*sin(2*a + 2*b*x)**2,x)
```

3.509 $\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b} - \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{2b} + \frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}$$

output
```
-1/2*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b+sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\sin(a+b}{2b}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-1/2*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]] - 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4796, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2a+2bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{3/2}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4796}$$

$$2\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{4789}$$

$$2\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{4794}$$

$$2\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a+bx$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
2*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a
+ b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a + 2
*b*x]])/(2*b))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 9.44 (sec) , antiderivative size = 24409652, normalized size of antiderivative = 301353.73

method	result	size
default	Expression too large to display	24409652

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(73) = 146.

Time = 0.09 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.28

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)+2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + 2*arctan(-(s
qrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos
(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)
) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) -
sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 + 4*
sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*
x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x +
 a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

Giac [F]

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^{3/2}}{\sin(a+b\,x)}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x),x)
```

output
```
int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x), x)
```

Reduce [F]

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sqrt{\sin(2bx+2a)}\,\csc(bx+a)\sin(2bx+2a)\,dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x)*sin(2*a + 2*b*x),x)
```

3.510 $\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{b} + \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{b}$$

output
```
-arcsin(cos(b*x+a)-sin(b*x+a))/b+ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*a)^(
1/2))/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{b} + \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)}{b}$$

input
```
Integrate[Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-(ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b) + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4796, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2a+2bx)}\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin(2a+2bx)}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4796}$$

$$2\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4793}$$

$$2\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)$$

input
```
Int[Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
2*(-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a +
 b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 4.74 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.96

method	result
default	$\frac{2\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\ \mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1},\right.}{b\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}}$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2*b*
x)^2-1)/(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2
*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)
/(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*
b*x)+1)^(1/2),1/2*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(51) = 102$.

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.57

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cdots}{\cos(bx+a)-\sin\cdots}\right.}{}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - si
n(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*
sin(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))
- cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - log(-32*co
s(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*
x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)
^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \csc(bx+a)\sqrt{\sin(2bx+2a)}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F]

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \csc(bx+a)\sqrt{\sin(2bx+2a)}\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2a+2bx)}}{\sin(a+bx)}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x),x)
```

output
```
int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x), x)
```

Reduce [F]

$$\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin(2bx+2a)}\,\csc(bx+a)\,dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x),x)
```

3.511 $\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\csc(a+bx)\sqrt{\sin(2a+2bx)}}{b}$$

output
```
-csc(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\csc(a+bx)\sqrt{\sin(2(a+bx))}}{b}$$

input
```
Integrate[Csc[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-((Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{ 4780}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-((Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.97 (sec) , antiderivative size = 308, normalized size of antiderivative = 12.83

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\left(2\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\,\mathrm{EllipticE}\left(\sqrt{\right.}\right.}{}$

input
```
int(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(2*(tan(1/2*a+1/2
*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan
(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticE((tan(1/2*a+
1/2*b*x)+1)^(1/2),1/2*2^(1/2))-(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1
))^(1/2)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-ta
n(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2)
)+(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^2-(ta
n(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2))/tan(1/2*a+1/2*b*x)/(tan(1/2*
a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+\sin\left(bx+a\right)}{b\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + sin(b*x + a))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\csc(bx+a)}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\csc(bx+a)}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [B] (verification not implemented)

Time = 19.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}{b\,\sin\left(a+b\,x\right)}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(1/2)),x)
```

output
```
-sin(2*a + 2*b*x)^(1/2)/(b*sin(a + b*x))
```

Reduce [F]

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sqrt{\sin\left(2bx+2a\right)}\,\csc\left(bx+a\right)}{\sin\left(2bx+2a\right)}dx$$

input
```
int(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*csc(a + b*x))/sin(2*a + 2*b*x),x)
```

3.512 $\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{4\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

output
```
-2/3*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+4/3*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\left(-\frac{1}{6}\cot(a+bx)\csc(a+bx)+\frac{1}{2}\sec(a+bx)\right)\sqrt{\sin(2(a+bx))}}{b}$$

input
```
Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
((-1/6*(Cot[a + b*x]*Csc[a + b*x]) + Sec[a + b*x]/2)*Sqrt[Sin[2*(a + b*x)]
])/b
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4796, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{4796}$$

$$2\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4791}$$

$$2\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{4780}$$

$$2\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)$$

input
```
Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
2*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*Sq
rt[Sin[2*a + 2*b*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 8.52 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.66

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)\left(2\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{b}{2}\right.}\right.\right.}{12b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}}$

input
```
int(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/b*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/
2*b*x)^2-1)/tan(1/2*a+1/2*b*x)*(2*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2
*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*
b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)-tan(1/2*a+1/2*b*x)^4+1)/(tan
(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)/(tan(1/2*a+1/2*b*x)^3-tan(
1/2*a+1/2*b*x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{4\,\cos\left(bx+a\right)^{3}+\sqrt{2}\left(4\,\cos\left(bx+a\right)^{2}-3\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-4\,\cos\left(bx+a\right)}{6\,\left(b\cos\left(bx+a\right)^{3}-b\cos\left(bx+a\right)\right)}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*s
in(b*x + a)) - 4*cos(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 22.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.94

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{4\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(1+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}{3\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(3/2)),x)
```

output
```
(4*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2)*(exp(a*4i + b*x*4i) - exp(a*2i + b*x*2i) + 1))/(3*b*(exp(a*2i
+ b*x*2i) - 1)^2*(exp(a*2i + b*x*2i) + 1))
```

Reduce [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=\int \frac{\sqrt{\sin\left(2bx+2a\right)}\,\csc\left(bx+a\right)}{\sin\left(2bx+2a\right)^2}dx$$

input
```
int(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*csc(a + b*x))/sin(2*a + 2*b*x)**2,x)
```

3.513 $\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

output
```
-2/5*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2)+8/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(3
/2)-16/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$=-\frac{\sqrt{\sin(2(a+bx))}(27\csc(a+bx)+3\csc^3(a+bx)-5\sec(a+bx)\tan(a+bx))}{60b}$$

input
```
Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/60*(Sqrt[Sin[2*(a + b*x)]]*(27*Csc[a + b*x] + 3*Csc[a + b*x]^3 - 5*Sec[
a + b*x]*Tan[a + b*x]))/b
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4796, 3042, 4791, 3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4796}$$

$$2\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4791}$$

$$2\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{4792}$$

$$2\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{4779}$$

$$2\left(\frac{4}{5}\left(\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)$$

input
```
Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
2*((4*(Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*S
qrt[Sin[2*a + 2*b*x]])))/5 - Cos[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 81.95 (sec) , antiderivative size = 481, normalized size of antiderivative = 6.09

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\left(24\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\ \mathrm{EllipticE}\right.}{}$

input
```
int(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/80/b*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)/tan(1/2*a+1/2
*b*x)^3*(24*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a
+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(
1/2)*EllipticE((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x
)^2-12*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2*
b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*
EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2+(
tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^6-(t
an(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^4+12*
(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^4-(tan(
1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^2-12*(ta
n(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^2+(tan(1/2
*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2))/(tan(1/2*a+1/2*b*x)^3-tan(1/2
*a+1/2*b*x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.30

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\left(32\,\cos\left(bx+a\right)^4-40\,\cos\left(bx+a\right)^2+5\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\,\left(\cos\left(bx+a\right)^4-\cos\left(bx+\right.\right.}{60\,\left(b\cos\left(bx+a\right)^4-b\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/60*(sqrt(2)*(32*cos(b*x + a)^4 - 40*cos(b*x + a)^2 + 5)*sqrt(cos(b*x +
a)*sin(b*x + a)) + 32*(cos(b*x + a)^4 - cos(b*x + a)^2)*sin(b*x + a))/((b*
cos(b*x + a)^4 - b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 22.91 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{8\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,2\mathrm{i}-\mathrm{e}^{a\,8\mathrm{i}+b\,x\,8\mathrm{i}}\,2\mathrm{i}-2\mathrm{i}\right)}{15\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^3\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^2}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(5/2)),x)
```

output
```
(8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2)*(exp(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*
x*6i)*2i - exp(a*8i + b*x*8i)*2i - 2i))/(15*b*(exp(a*2i + b*x*2i) - 1)^3*(
exp(a*2i + b*x*2i) + 1)^2)
```

Reduce [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\csc(bx+a)}{\sin(2bx+2a)^3}dx$$

input
```
int(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*csc(a + b*x))/sin(2*a + 2*b*x)**3,x)
```

3.514 $\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)} \, dx = -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{32\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

output
```
-2/7*cos(b*x+a)/b/sin(2*b*x+2*a)^(7/2)+12/35*sin(b*x+a)/b/sin(2*b*x+2*a)^(
5/2)-16/35*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+32/35*sin(b*x+a)/b/sin(2*b*x+
2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)} \, dx = \frac{(5-10\cos(2(a+bx))-4\cos(4(a+bx))+4\cos(6(a+bx)))\csc^4(a+bx)\sec^3(a+bx)\sqrt{\sin(2(a+bx))}}{280b}$$

input
```
Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
((5 - 10*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[a
 + b*x]^4*Sec[a + b*x]^3*Sqrt[Sin[2*(a + b*x)]])/(280*b)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4796, 3042, 4791, 3042, 4792, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4796}$$

$$2\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{9/2}}dx$$

$$\Big\downarrow \text{4791}$$

$$2\left(\frac{6}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{6}{7}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{7/2}}dx-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{4792}$$

$$2\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{6}{7}\left(\frac{4}{5}\int\frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{4791}$$

$$2\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

$$\Big\downarrow \text{4780}$$

$$2\left(\frac{6}{7}\left(\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{5}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)$$

input
```
Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
2*((6*((4*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])
/(3*b*Sqrt[Sin[2*a + 2*b*x]])))/5 + Sin[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/
2))))/7 - Cos[a + b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 111.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.11

$$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-1}}\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-1\right)\left(3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^8+40\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\sqrt{-\cdots}\right.}{1344b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}\cdots\right.\right.}}$$

input
```
int(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

output
```
1/1344/b*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1
/2*b*x)^2-1)/tan(1/2*a+1/2*b*x)^3*(3*tan(1/2*a+1/2*b*x)^8+40*(tan(1/2*a+1/
2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2
)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^3
-26*tan(1/2*a+1/2*b*x)^6+26*tan(1/2*a+1/2*b*x)^2-3)/(tan(1/2*a+1/2*b*x)*(t
an(1/2*a+1/2*b*x)^2-1))^(1/2)/(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{128\cos(bx+a)^7 - 256\cos(bx+a)^5 + 128\cos(bx+a)^3 + \sqrt{2}\big(128\cos(bx+a)^6 - 224\cos(bx+a)^4 + \cdots}{280\left(b\cos(bx+a)^7 - 2\,b\cos(bx+a)^5 + b\cos(bx+a)^3\cdots\right.}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
1/280*(128*cos(b*x + a)^7 - 256*cos(b*x + a)^5 + 128*cos(b*x + a)^3 + sqrt
(2)*(128*cos(b*x + a)^6 - 224*cos(b*x + a)^4 + 84*cos(b*x + a)^2 + 7)*sqrt
(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*co
s(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=\int \frac{\csc\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=\int \frac{\csc\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 23.51 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.33

$$\begin{aligned}\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=&-\frac{2\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{7\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^4}\\&+\frac{\mathrm{e}^{a\,3\mathrm{i}+b\,x\,3\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,32\mathrm{i}}{35\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}\\&-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{2}{7\,b}-\frac{16\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}{35\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}\\&+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{32\mathrm{i}}{35\,b}+\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,88\mathrm{i}}{35\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^3}\end{aligned}$$

input
```
int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(7/2)),x)
```

output
```
(exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*32i)/(35*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i)
) - (2*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2
i)*1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^4) - (exp(a*1i + b*x*1i
)*(2/(7*b) - (16*exp(a*2i + b*x*2i))/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2
- (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x*2i) + 1)^2*(exp(a*2i
+ b*x*2i)*1i - 1i)^2) + (exp(a*1i + b*x*1i)*(32i/(35*b) + (exp(a*2i + b*x*
2i)*88i)/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)
^(1/2))/((exp(a*2i + b*x*2i) + 1)^3*(exp(a*2i + b*x*2i)*1i - 1i)^3)
```

Reduce [F]

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\csc(bx+a)}{\sin(2bx+2a)^4}dx$$

input
```
int(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*csc(a + b*x))/sin(2*a + 2*b*x)**4,x)
```

3.515 $\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b} - \frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{5b} - \frac{2\cos(2a+2bx)\sin^{\frac{7}{2}}(2a+2bx)}{7b} + \frac{\csc^2(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{7b}$$

output
```
-6/5*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b-2/5*cos(2*b*x+2*a)*sin(2*b*x+2
*a)^(3/2)/b-2/7*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(7/2)/b+1/7*csc(b*x+a)^2*sin
(2*b*x+2*a)^(11/2)/b
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \frac{84E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)+\sqrt{\sin(2(a+bx))}(15\sin(2(a+bx))-14\sin(4(a+bx))-5\sin(6(a+bx)))}{70b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(15*Sin[2*(a + b
*x)] - 14*Sin[4*(a + b*x)] - 5*Sin[6*(a + b*x)]))/(70*b)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4788, 3042, 3115, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(2a+2bx)^{9/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \ 4788$$

$$\frac{18}{7}\int \sin^{\frac{9}{2}}(2a+2bx)dx+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

$$\Big\downarrow \ 3042$$

$$\frac{18}{7}\int \sin(2a+2bx)^{9/2}dx+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{18}{7}\left(\frac{7}{9}\int \sin^{\frac{5}{2}}(2a+2bx)dx-\frac{\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{9b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{18}{7}\left(\frac{7}{9}\int \sin(2a+2bx)^{5/2}dx-\frac{\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{9b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{18}{7}\left(\frac{7}{9}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)-\frac{\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{9b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{18}{7}\left(\frac{7}{9}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)-\frac{\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{9b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}+\frac{18}{7}\left(\frac{7}{9}\left(\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)-\frac{\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{9b}\right)$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(11/2))/(7*b) + (18*(-1/9*(Cos[2*a + 2*b*
x]*Sin[2*a + 2*b*x]^(7/2))/b + (7*((3*EllipticE[a - Pi/4 + b*x, 2])/(5*b)
- (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(3/2))/(5*b)))/9))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(93) = 186$.

Time = 49.23 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.92

method	result
default	$\frac{8\sqrt{2}\left(\frac{\sqrt{2}\,\sin(2bx+2a)^{\frac{7}{2}}}{56}-\frac{\sqrt{2}\left(6\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\operatorname{EllipticE}\left(\sqrt{\sin(2bx+2a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{\sin(2bx+2a)+1}\,\cdots}{80\cos(2bx+2a)\sqrt{\sin\cdots}}\right.}{b}$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
8*2^(1/2)*(1/56*2^(1/2)*sin(2*b*x+2*a)^(7/2)-1/80*2^(1/2)*(6*(sin(2*b*x+2*
a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticE(
(sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2
*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1
/2),1/2*2^(1/2))-2*sin(2*b*x+2*a)^4+2*sin(2*b*x+2*a)^2)/cos(2*b*x+2*a)/sin
(2*b*x+2*a)^(1/2))/b
```

Fricas [F]

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \int \csc\left(bx+a\right)^2 \sin\left(2\,bx+2\,a\right)^{\frac{9}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
integral((cos(2*b*x + 2*a)^4 - 2*cos(2*b*x + 2*a)^2 + 1)*csc(b*x + a)^2*sq
rt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \int \csc\left(bx+a\right)^2 \sin\left(2\,bx+2\,a\right)^{\frac{9}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(9/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \int \frac{\sin\left(2\,a+2\,b\,x\right)^{9/2}}{\sin\left(a+b\,x\right)^2}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^2,x)
```

output
```
int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \int \sqrt{\sin(2bx+2a)}\,\csc(bx+a)^2\sin(2bx+2a)^4\,dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**2*sin(2*a + 2*b*x)**4,x)
```

3.516 $\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \frac{2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{3b} - \frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{3b} - \frac{2\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{5b} + \frac{\csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{5b}$$

output
```
2/3*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))/b-2/3*cos(2*b*x+2*a)*sin(2*b*x+2
*a)^(1/2)/b-2/5*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(5/2)/b+1/5*csc(b*x+a)^2*sin
(2*b*x+2*a)^(9/2)/b
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$=\frac{20\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2(a+bx))}+9\sin(2(a+bx))-10\sin(4(a+bx))-3\sin(6(a+bx))}{30b\sqrt{\sin(2(a+bx))}}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 9*Sin[2*(a + b*x
)] - 10*Sin[4*(a + b*x)] - 3*Sin[6*(a + b*x)])/(30*b*Sqrt[Sin[2*(a + b*x)]
])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4788, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sin(2a+2bx)^{7/2}}{\sin(a+bx)^2}dx$$
$$\Big\downarrow \ 4788$$
$$\frac{14}{5}\int \sin^{\frac{7}{2}}(2a+2bx)dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}$$
$$\Big\downarrow \ 3042$$
$$\frac{14}{5}\int \sin(2a+2bx)^{7/2}dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{14}{5}\left(\frac{5}{7}\int \sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{14}{5}\left(\frac{5}{7}\int \sin(2a+2bx)^{3/2}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{14}{5}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{14}{5}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}+\frac{14}{5}\left(\frac{5}{7}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(9/2))/(5*b) + (14*((5*(EllipticF[a - Pi/
4 + b*x, 2]/(3*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b)))/7 -
(Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(7*b)))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [A] (verified)

Time = 32.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.31

method	result
default	$\frac{4\sqrt{2}\left(\frac{\sqrt{2}\,\sin(2bx+2a)^{\frac{5}{2}}}{20}+\frac{\sqrt{2}\left(\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\operatorname{EllipticF}\left(\sqrt{\sin(2bx+2a)+1},\frac{\sqrt{2}}{2}\right)+2\sin(2bx+2a)^{3}-2\sin\right.}{24\cos(2bx+2a)\sqrt{\sin(2bx+2a)}}\right.}{b}$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
4*2^(1/2)*(1/20*2^(1/2)*sin(2*b*x+2*a)^(5/2)+1/24*2^(1/2)*((sin(2*b*x+2*a)
+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((s
in(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+2*sin(2*b*x+2*a)^3-2*sin(2*b*x+2*a))/c
os(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

Fricas [F]

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \csc\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
integral(-(cos(2*b*x + 2*a)^2 - 1)*csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \csc\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

Giac [F]

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \csc\left(bx+a\right)^2 \sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \frac{\sin\left(2\,a+2\,b\,x\right)^{7/2}}{\sin\left(a+b\,x\right)^2}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^2,x)
```

output
```
int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \sqrt{\sin\left(2bx+2a\right)}\,\csc\left(bx+a\right)^2\sin\left(2bx+2a\right)^3 dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**2*sin(2*a + 2*b*x)**3,x)
```

3.517 $\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b} - \frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b} + \frac{\csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{3b}$$

output
```
-2*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b-2/3*cos(2*b*x+2*a)*sin(2*b*x+2*a
)^(3/2)/b+1/3*csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2)/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.45

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{2\left(3E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sin^{\frac{3}{2}}(2(a+bx))\right)}{3b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(2*(3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2)))/(3*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4788, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{5/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{10}{3}\int \sin^{\frac{5}{2}}(2a+2bx)dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{3}\int \sin(2a+2bx)^{5/2}dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{10}{3}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{3}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}+\frac{10}{3}\left(\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2))/(3*b) + (10*((3*EllipticE[a - Pi/4
 + b*x, 2])/(5*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(3/2))/(5*b)))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [A] (verified)

Time = 23.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.83

method	result
default	$\frac{2\sqrt{2}\left(\frac{\sqrt{2}\,\sin(2bx+2a)^{\frac{3}{2}}}{6}-\frac{\sqrt{2}\,\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{\sin(2bx+2a)+1},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{\sin(2bx\cdots}\right.\right.}{4\cos(2bx+2a)\sqrt{\sin(2bx+2a)}}\right.}{b}$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/2)*(1/6*2^(1/2)*sin(2*b*x+2*a)^(3/2)-1/4*2^(1/2)*(sin(2*b*x+2*a)+1)
^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*(2*EllipticE((s
in(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2
*2^(1/2)))/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

Fricas [F]

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^2\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
integral(-(cos(2*b*x + 2*a)^2 - 1)*csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx=\int \csc\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

Giac [F]

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx=\int \csc\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx=\int \frac{\sin\left(2\,a+2\,b\,x\right)^{5/2}}{\sin\left(a+b\,x\right)^2}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^2,x)
```

output
```
int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sqrt{\sin(2bx+2a)}\,\csc(bx+a)^2\sin(2bx+2a)^2\,dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**2*sin(2*a + 2*b*x)**2,x)
```

3.518 $\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{b} - \frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{b} + \frac{\csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b}$$

output
```
2*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))/b-2*cos(2*b*x+2*a)*sin(2*b*x+2*a)^
(1/2)/b+csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{2\left(1+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{\sin(2(a+bx))}}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(2*(1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x
]^2])*Sqrt[Sin[2*(a + b*x)]])/b
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4788, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2a+2bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^{3/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4788}$$

$$6\int \sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$6\int \sin(2a+2bx)^{3/2}dx+\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

$$\Big\downarrow \text{3115}$$

$$6\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$6\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}+6\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
6*(EllipticF[a - Pi/4 + b*x, 2]/(3*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2
*b*x]])/(3*b)) + (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2))/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [A] (verified)

Time = 7.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.59

method	result	size
default	$\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{\sin(2bx+2a)}+\frac{\sqrt{2}\,\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\operatorname{EllipticF}\left(\sqrt{\sin(2bx+2a)+1},\frac{\sqrt{2}}{2}\right)}{2\cos(2bx+2a)\sqrt{\sin(2bx+2a)}}\right)}{b}$	111

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*(2^(1/2)*sin(2*b*x+2*a)^(1/2)+1/2*2^(1/2)*(sin(2*b*x+2*a)+1)^(1/2)
*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+
2*a)+1)^(1/2),1/2*2^(1/2))/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

Fricas [F]

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \csc\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^2\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Giac [F]

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^2\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^{3/2}}{\sin(a+b\,x)^2}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^2,x)
```

output
```
int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\int \sqrt{\sin(2bx+2a)}\,\csc(bx+a)^2\sin(2bx+2a)\,dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**2*sin(2*a + 2*b*x),x)
```

3.519 $\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 44

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx=-\frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b}-\frac{\csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}$$

output
```
2*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b-csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2)
/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx=-\frac{2\left(E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\cot(a+bx)\sqrt{\sin(2(a+bx))}\right)}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-2*(EllipticE[a - Pi/4 + b*x, 2] + Cot[a + b*x]*Sqrt[Sin[2*(a + b*x)]]))/
b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4788, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2a+2bx)}\csc^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{\sin(2a+2bx)}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \ 4788$$

$$-2\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

$$\Big\downarrow \ 3042$$

$$-2\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

$$\Big\downarrow \ 3119$$

$$-\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-2*EllipticE[a - Pi/4 + b*x, 2])/b - (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/
2))/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(43) = 86$.

Time = 3.43 (sec) , antiderivative size = 176, normalized size of antiderivative = 4.00

method	result
default	$\frac{2\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\mathrm{EllipticE}\left(\sqrt{\sin(2bx+2a)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a}}{\cos(2bx+2a)\sqrt{\sin(2bx+2a)}\,b}$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2)*(2*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(
2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticE((sin(2*b*x+2*a)+1)^(
1/2),1/2*2^(1/2))-(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-s
in(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-2*cos
(2*b*x+2*a)^2-2*cos(2*b*x+2*a))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.52

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{-i\sqrt{2i}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a)+i\sqrt{-2i}E(\arcsin(\cos(bx+a)-i\sin}{}$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2*I)*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*
x + a) + I*sqrt(-2*I)*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1
)*sin(b*x + a) + I*sqrt(2*I)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x +
a)), -1)*sin(b*x + a) - I*sqrt(-2*I)*elliptic_f(arcsin(cos(b*x + a) - I*si
n(b*x + a)), -1)*sin(b*x + a) - 2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*
cos(b*x + a))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \csc(bx+a)^2\sqrt{\sin(2bx+2a)}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F]

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \csc(bx+a)^2\sqrt{\sin(2bx+2a)}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2a+2bx)}}{\sin(a+bx)^2}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x)^2,x)
```

output
```
int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin(2bx+2a)}\,\csc(bx+a)^2\,dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**2,x)
```

3.520 $\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \frac{2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{3b} - \frac{\csc^2(a+bx)\sqrt{\sin(2a+2bx)}}{3b}$$

output
```
2/3*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))/b-1/3*csc(b*x+a)^2*sin(2*b*x+2*a
)^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\left(\csc^2(a+bx) - 2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{\sin(2(a+bx))}}{3b}$$

input
```
Integrate[Csc[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/3*((Csc[a + b*x]^2 - 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2
]*Sqrt[Sec[a + b*x]^2])*Sqrt[Sin[2*(a + b*x)]])/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4788, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{2}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\csc^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\csc^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\csc^2(a+bx)}{3b}$$

input
```
Int[Csc[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(2*EllipticF[a - Pi/4 + b*x, 2])/(3*b) - (Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*
b*x]])/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(42) = 84$.

Time = 6.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.56

method	result
default	$\frac{\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\operatorname{EllipticF}\left(\sqrt{\sin(2bx+2a)+1},\frac{\sqrt{2}}{2}\right)\sin(2bx+2a)-2\cos(2bx+2a)^2-2\cos(2}{3\sin(2bx+2a)^{\frac{3}{2}}\cos(2bx+2a)b}$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/sin(2*b*x+2*a)^(3/2)/cos(2*b*x+2*a)*((sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(
2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(
1/2),1/2*2^(1/2))*sin(2*b*x+2*a)-2*cos(2*b*x+2*a)^2-2*cos(2*b*x+2*a))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.15

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\sqrt{2i}\left(\cos\left(bx+a\right)^2-1\right)F\left(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|\,-1\right)+\sqrt{-2i}\left(\cos\left(bx+a\right)^2-1\right)F(\text{ar}}{3\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(2*I)*(cos(b*x + a)^2 - 1)*elliptic_f(arcsin(cos(b*x + a) + I*si
n(b*x + a)), -1) + sqrt(-2*I)*(cos(b*x + a)^2 - 1)*elliptic_f(arcsin(cos(b
*x + a) - I*sin(b*x + a)), -1) - sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)))/
(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\csc\left(bx+a\right)^2}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\csc\left(bx+a\right)^2}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^2\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\csc(bx+a)^2}{\sin(2bx+2a)}dx$$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**2)/sin(2*a + 2*b*x),x)
```

3.521 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b} - \frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}} - \frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

output
```
6/5*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b-6/5*cos(2*b*x+2*a)/b/sin(2*b*x+
2*a)^(1/2)-1/5*csc(b*x+a)^2/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{-12E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\frac{2(1-6\cos(2(a+bx))+3\cos(4(a+bx)))\cot(a+bx)}{\sin^{\frac{3}{2}}(2(a+bx))}}{10b}$$

input
```
Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(-12*EllipticE[a - Pi/4 + b*x, 2] + (2*(1 - 6*Cos[2*(a + b*x)] + 3*Cos[4*(
a + b*x)])*Cot[a + b*x])/Sin[2*(a + b*x)]^(3/2))/(10*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4788, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{6}{5}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}\int \frac{1}{\sin(2a+2bx)^{3/2}}dx-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{6}{5}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6}{5}\left(-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(6*(-(EllipticE[a - Pi/4 + b*x, 2]/b) - Cos[2*a + 2*b*x]/(b*Sqrt[Sin[2*a +
 2*b*x]])))/5 - Csc[a + b*x]^2/(5*b*Sqrt[Sin[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(68) = 136$.

Time = 13.92 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.95

method	result
default	$\frac{\sqrt{2}\left(-\frac{8\sqrt{2}}{5\sin(2bx+2a)^{\frac{5}{2}}}+\frac{4\sqrt{2}\left(6\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\sin(2bx+2a)^{2}\,\mathrm{EllipticE}\left(\sqrt{\sin(2bx+2a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{\sin\cdots}\right.}{5\sin(\cdots}\right.}{8b}$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*2^(1/2)*(-8/5*2^(1/2)/sin(2*b*x+2*a)^(5/2)+4/5*2^(1/2)/sin(2*b*x+2*a)^
(5/2)*(6*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+
2*a))^(1/2)*sin(2*b*x+2*a)^2*EllipticE((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2
))-3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a)
)^(1/2)*sin(2*b*x+2*a)^2*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+6
*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-2)/cos(2*b*x+2*a))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.45

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{6\sqrt{2i}\left(i\cos(bx+a)^3 - i\cos(bx+a)\right)E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a)+6\,\cdots}{\cdots}$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(6*sqrt(2*I)*(I*cos(b*x + a)^3 - I*cos(b*x + a))*elliptic_e(arcsin(c
os(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 6*sqrt(-2*I)*(-I*cos(b*x
 + a)^3 + I*cos(b*x + a))*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a))
, -1)*sin(b*x + a) + 6*sqrt(2*I)*(-I*cos(b*x + a)^3 + I*cos(b*x + a))*elli
ptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 6*sqrt(-2
*I)*(I*cos(b*x + a)^3 - I*cos(b*x + a))*elliptic_f(arcsin(cos(b*x + a) - I
*sin(b*x + a)), -1)*sin(b*x + a) + sqrt(2)*(12*cos(b*x + a)^4 - 18*cos(b*x
 + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a)))/((b*cos(b*x + a)^3 - b*cos(b
*x + a))*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{3/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin\left(2bx+2a\right)}\,\csc\left(bx+a\right)^2}{\sin\left(2bx+2a\right)^2}dx$$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**2)/sin(2*a + 2*b*x)**2,x)
```

3.522 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=\frac{10\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{21b}-\frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

output
```
10/21*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))/b-10/21*cos(2*b*x+2*a)/b/sin(2
*b*x+2*a)^(3/2)-1/7*csc(b*x+a)^2/b/sin(2*b*x+2*a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=\frac{40\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)+\left(-13\csc^2(a+bx)-3\csc^4(a+bx)+7\sec^2(a+bx)\right)\sqrt{\sin(2(a+bx))}}{84b}$$

input
```
Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(40*EllipticF[a - Pi/4 + b*x, 2] + (-13*Csc[a + b*x]^2 - 3*Csc[a + b*x]^4
+ 7*Sec[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/(84*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4788, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{10}{7}\int \frac{1}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{7}\int \frac{1}{\sin(2a+2bx)^{5/2}}dx-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{10}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos(2a+2bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos(2a+2bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{10}{7}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\cos(2a+2bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

input
```
Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(10*(EllipticF[a - Pi/4 + b*x, 2]/(3*b) - Cos[2*a + 2*b*x]/(3*b*Sin[2*a +
2*b*x]^(3/2))))/7 - Csc[a + b*x]^2/(7*b*Sin[2*a + 2*b*x]^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(67) = 134$.

Time = 58.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.00

method	result
default	$\frac{\sqrt{2}\left(-\frac{16\sqrt{2}}{7\sin(2bx+2a)^{\frac{7}{2}}}+\frac{8\sqrt{2}\left(5\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\ \mathrm{EllipticF}\left(\sqrt{\sin(2bx+2a)+1},\frac{\sqrt{2}}{2}\right)\sin(2bx+2a)^{3}+10\sin(\right.}{21\sin(2bx+2a)^{\frac{7}{2}}\cos(2bx+2a)}}{16b}$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*2^(1/2)*(-16/7*2^(1/2)/sin(2*b*x+2*a)^(7/2)+8/21*2^(1/2)/sin(2*b*x+2*
a)^(7/2)*(5*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b
*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))*sin(2*b*x+2
*a)^3+10*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-6)/cos(2*b*x+2*a))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.32

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{20\sqrt{2i}\left(\cos(bx+a)^6-2\cos(bx+a)^4+\cos(bx+a)^2\right)F(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)}{}$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/84*(20*sqrt(2*I)*(cos(b*x + a)^6 - 2*cos(b*x + a)^4 + cos(b*x + a)^2)*e
lliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + 20*sqrt(-2*I)*(cos(
b*x + a)^6 - 2*cos(b*x + a)^4 + cos(b*x + a)^2)*elliptic_f(arcsin(cos(b*x
+ a) - I*sin(b*x + a)), -1) - sqrt(2)*(20*cos(b*x + a)^4 - 30*cos(b*x + a)
^2 + 7)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^6 - 2*b*cos(b*x +
 a)^4 + b*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\csc\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

Giac [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\csc\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=\int \frac{1}{\sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{5/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=\int \frac{\sqrt{\sin\left(2bx+2a\right)}\,\csc\left(bx+a\right)^2}{\sin\left(2bx+2a\right)^3}dx$$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**2)/sin(2*a + 2*b*x)**3,x)
```

3.523 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{14E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{15b}-\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{14\cos(2a+2bx)}{15b\sqrt{\sin(2a+2bx)}}$$

output
```
14/15*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b-14/45*cos(2*b*x+2*a)/b/sin(2*
b*x+2*a)^(5/2)-1/9*csc(b*x+a)^2/b/sin(2*b*x+2*a)^(5/2)-14/15*cos(2*b*x+2*a
)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.80

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{336E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\frac{(-9+98\cos(2(a+bx))-28\cos(4(a+bx))-42\cos(6(a+bx))+21\cos(8(a+bx)))\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2(a+bx))}}{360b}$$

input
```
Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
-1/360*(336*EllipticE[a - Pi/4 + b*x, 2] + ((-9 + 98*Cos[2*(a + b*x)] - 28
*Cos[4*(a + b*x)] - 42*Cos[6*(a + b*x)] + 21*Cos[8*(a + b*x)])*Csc[a + b*x
]^2)/Sin[2*(a + b*x)]^(5/2))/b
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4788, 3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{14}{9}\int \frac{1}{\sin^{\frac{7}{2}}(2a+2bx)}dx-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{14}{9}\int \frac{1}{\sin(2a+2bx)^{7/2}}dx-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{14}{9}\left(\frac{3}{5}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(2a+2bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{14}{9}\left(\frac{3}{5}\int \frac{1}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(2a+2bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{14}{9}\left(\frac{3}{5}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(2a+2bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{14}{9}\left(\frac{3}{5}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(2a+2bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{14}{9}\left(\frac{3}{5}\left(-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(2a+2bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

input
```
Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(14*((3*(-(EllipticE[a - Pi/4 + b*x, 2]/b) - Cos[2*a + 2*b*x]/(b*Sqrt[Sin[
2*a + 2*b*x]])))/5 - Cos[2*a + 2*b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2))))/9 - C
sc[a + b*x]^2/(9*b*Sin[2*a + 2*b*x]^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(93) = 186$.

Time = 245.86 (sec), antiderivative size = 240, normalized size of antiderivative = 2.26

method	result
default	$\sqrt{2}\left(-\frac{32\sqrt{2}}{9\sin(2bx+2a)^{\frac{9}{2}}}+\frac{16\sqrt{2}\left(42\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\sin(2bx+2a)^4\,\mathrm{EllipticE}\left(\sqrt{\sin(2bx+2a)+1},\frac{\sqrt{2}}{2}\right)-21\right.}{}\right.$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*2^(1/2)*(-32/9*2^(1/2)/sin(2*b*x+2*a)^(9/2)+16/45*2^(1/2)/sin(2*b*x+2
*a)^(9/2)*(42*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2
*b*x+2*a))^(1/2)*sin(2*b*x+2*a)^4*EllipticE((sin(2*b*x+2*a)+1)^(1/2),1/2*2
^(1/2))-21*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*
x+2*a))^(1/2)*sin(2*b*x+2*a)^4*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1
/2))+42*sin(2*b*x+2*a)^6-28*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-10)/cos(2*
b*x+2*a))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 346, normalized size of antiderivative = 3.26

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{168\sqrt{2i}\left(i\cos(bx+a)^7-2i\cos(bx+a)^5+i\cos(bx+a)^3\right)E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
-1/360*(168*sqrt(2*I)*(I*cos(b*x + a)^7 - 2*I*cos(b*x + a)^5 + I*cos(b*x +
 a)^3)*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a)
+ 168*sqrt(-2*I)*(-I*cos(b*x + a)^7 + 2*I*cos(b*x + a)^5 - I*cos(b*x + a)^
3)*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + 16
8*sqrt(2*I)*(-I*cos(b*x + a)^7 + 2*I*cos(b*x + a)^5 - I*cos(b*x + a)^3)*el
liptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 168*sqr
t(-2*I)*(I*cos(b*x + a)^7 - 2*I*cos(b*x + a)^5 + I*cos(b*x + a)^3)*ellipti
c_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + sqrt(2)*(336
*cos(b*x + a)^8 - 840*cos(b*x + a)^6 + 644*cos(b*x + a)^4 - 126*cos(b*x +
a)^2 - 9)*sqrt(cos(b*x + a)*sin(b*x + a)))/((b*cos(b*x + a)^7 - 2*b*cos(b*
x + a)^5 + b*cos(b*x + a)^3)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

Giac [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{1}{\sin(a+b\,x)^2\,\sin(2\,a+2\,b\,x)^{7/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\csc(bx+a)^2}{\sin(2bx+2a)^4}dx$$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**2)/sin(2*a + 2*b*x)**4,x)
```

3.524 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{30\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{77b} - \frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{30\cos(2a+2bx)}{77b\sin^{\frac{3}{2}}(2a+2bx)}$$

output
```
30/77*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))/b-18/77*cos(2*b*x+2*a)/b/sin(2
*b*x+2*a)^(7/2)-1/11*csc(b*x+a)^2/b/sin(2*b*x+2*a)^(7/2)-30/77*cos(2*b*x+2
*a)/b/sin(2*b*x+2*a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.81

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{480\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right) + \left(-141\csc^2(a+bx) - 32\csc^4(a+bx) - 7\csc^6(a+bx) + 11\sec^2(a+bx)\right.}{1232b}$$

input
```
Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(480*EllipticF[a - Pi/4 + b*x, 2] + (-141*Csc[a + b*x]^2 - 32*Csc[a + b*x]
^4 - 7*Csc[a + b*x]^6 + 11*Sec[a + b*x]^2*(9 + Sec[a + b*x]^2))*Sqrt[Sin[2
*(a + b*x)]])/(1232*b)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4788, 3042, 3116, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sin(2a+2bx)^{9/2}}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{18}{11}\int \frac{1}{\sin^{\frac{9}{2}}(2a+2bx)}dx-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{18}{11}\int \frac{1}{\sin(2a+2bx)^{9/2}}dx-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{18}{11}\left(\frac{5}{7}\int \frac{1}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos(2a+2bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{18}{11}\left(\frac{5}{7}\int \frac{1}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos(2a+2bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{18}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos(2a+2bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(2a+2bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{18}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos(2a+2bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(2a+2bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{18}{11}\left(\frac{5}{7}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\cos(2a+2bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(2a+2bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}\right)-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

input
```
Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(18*((5*(EllipticF[a - Pi/4 + b*x, 2]/(3*b) - Cos[2*a + 2*b*x]/(3*b*Sin[2*
a + 2*b*x]^(3/2))))/7 - Cos[2*a + 2*b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2))))/11
 - Csc[a + b*x]^2/(11*b*Sin[2*a + 2*b*x]^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [F(-1)]

Timed out.

$$\int \frac{\csc(bx+a)^2}{\sin(2bx+2a)^{\frac{9}{2}}}dx$$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.22

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{240\sqrt{2i}\left(\cos(bx+a)^{10} - 3\cos(bx+a)^8 + 3\cos(bx+a)^6 - \cos(bx+a)^4\right)F(\arcsin(\cos(bx+a)+i}{}$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
-1/1232*(240*sqrt(2*I)*(cos(b*x + a)^10 - 3*cos(b*x + a)^8 + 3*cos(b*x + a
)^6 - cos(b*x + a)^4)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1
) + 240*sqrt(-2*I)*(cos(b*x + a)^10 - 3*cos(b*x + a)^8 + 3*cos(b*x + a)^6
- cos(b*x + a)^4)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) -
sqrt(2)*(240*cos(b*x + a)^8 - 600*cos(b*x + a)^6 + 444*cos(b*x + a)^4 - 66
*cos(b*x + a)^2 - 11)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^10
- 3*b*cos(b*x + a)^8 + 3*b*cos(b*x + a)^6 - b*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\csc\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(9/2), x)
```

Giac [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx=\int \frac{\csc\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx=\int \frac{1}{\sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{9/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(9/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(9/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx=\int \frac{\sqrt{\sin\left(2bx+2a\right)}\,\csc\left(bx+a\right)^2}{\sin\left(2bx+2a\right)^5}dx$$

input
```
int(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**2)/sin(2*a + 2*b*x)**5,x)
```

3.525 $\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 190

$$\begin{aligned}\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx &= -\frac{7\arcsin(\cos(a+bx)-\sin(a+bx))}{8b} \\ &\quad+\frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b} \\ &\quad-\frac{7\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b}+\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{6b} \\ &\quad-\frac{14\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{15b} \\ &\quad+\frac{4\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{5b}+\frac{\csc^3(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{5b}\end{aligned}$$

output
```
-7/8*arcsin(cos(b*x+a)-sin(b*x+a))/b+7/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b-7/4*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b+7/6*sin(b*x+a)*sin(2
*b*x+2*a)^(3/2)/b-14/15*cos(b*x+a)*sin(2*b*x+2*a)^(5/2)/b+4/5*sin(b*x+a)*s
in(2*b*x+2*a)^(7/2)/b+1/5*csc(b*x+a)^3*sin(2*b*x+2*a)^(11/2)/b
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$$
$$=\frac{7\left(-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)-\frac{2}{3}(10\,\mathrm{c}}{8b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(7*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) - (2*(10*Cos[a + b*x] + 9*Cos[3*(a + b*x)] + 2
*Cos[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/3)/(8*b)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.636$, Rules used = {3042, 4788, 3042, 4796, 3042, 4789, 3042, 4790, 3042, 4789, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(2a+2bx)^{9/2}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{ 4788}$$

$$\frac{16}{5}\int \csc(a+bx)\sin^{\frac{9}{2}}(2a+2bx)dx+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{16}{5}\int \frac{\sin(2a+2bx)^{9/2}}{\sin(a+bx)}dx+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\Big\downarrow$ 4796

$$\frac{32}{5}\int \cos(a+bx)\sin^{\frac{7}{2}}(2a+2bx)dx+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\Big\downarrow$ 3042

$$\frac{32}{5}\int \cos(a+bx)\sin(2a+2bx)^{7/2}dx+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\Big\downarrow$ 4789

$$\frac{32}{5}\left(\frac{7}{8}\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)dx+\frac{\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{8b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\Big\downarrow$ 3042

$$\frac{32}{5}\left(\frac{7}{8}\int \sin(a+bx)\sin(2a+2bx)^{5/2}dx+\frac{\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{8b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\Big\downarrow$ 4790

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}\right)+\frac{\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{8b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\Big\downarrow$ 3042

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\int \cos(a+bx)\sin(2a+2bx)^{3/2}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}\right)+\frac{\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{8b}\right)+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\Big\downarrow$ 4789

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}\right)+\right.$$
$$\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\Big\downarrow$ 3042

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}\right)+\right.$$
$$\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\Big\downarrow$ 4790

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(\right.\right.\right.$$
$$\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\Big\downarrow$ 3042

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)-\frac{\sin^{\frac{5}{2}}(\right.\right.\right.$$
$$\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

$\Big\downarrow$ 4793

$$\frac{32}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\right.\right.\right.\right.$$
$$\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(11/2))/(5*b) + (32*((Sin[a + b*x]*Sin[2*
a + 2*b*x]^(7/2))/(8*b) + (7*(-1/6*(Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2))/b
 + (5*((3*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x]
+ Sin[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin
[2*a + 2*b*x]])/(2*b)))/4 + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)))/
6))/8))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 108.87 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.32

method	result
default	$-\frac{64\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\,\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\tan\left(\frac{a}{2}+\frac{b}{}\right.\right.}{}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-64/21*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*((tan(1/2*a+1/
2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2
)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^6
-3*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*
a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(
1/2*a+1/2*b*x)^4+2*tan(1/2*a+1/2*b*x)^7+3*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2
*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/
2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2+10*tan(1/2*a+1/2*b
*x)^5-(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1
/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))+1
0*tan(1/2*a+1/2*b*x)^3+2*tan(1/2*a+1/2*b*x))/(tan(1/2*a+1/2*b*x)*(tan(1/2*
a+1/2*b*x)^2-1))^(1/2)/(tan(1/2*a+1/2*b*x)-1)^2/(tan(1/2*a+1/2*b*x)^3-tan(
1/2*a+1/2*b*x))^(1/2)/(tan(1/2*a+1/2*b*x)+1)^2/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.53

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx =$$
$$-\frac{8\sqrt{2}\left(32\cos(bx+a)^5-4\cos(bx+a)^3-7\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}-42\arctan\left(-\dots\right.}{\dots}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
-1/96*(8*sqrt(2)*(32*cos(b*x + a)^5 - 4*cos(b*x + a)^3 - 7*cos(b*x + a))*s
qrt(cos(b*x + a)*sin(b*x + a)) - 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin
(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos
(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 42*arctan(-(2*sqrt(2)*sq
rt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a)
 - sin(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^
3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a
)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^3 \sin(2\,bx+2\,a)^{\frac{9}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(9/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^{9/2}}{\sin(a+b\,x)^3}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^3,x)
```

output
```
int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^3, x)
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \int \sqrt{\sin(2bx+2a)}\,\csc(bx+a)^3\sin(2bx+2a)^4\,dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**3*sin(2*a + 2*b*x)**4,x)
```

3.526 $\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 164

$$\begin{aligned}\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx &= -\frac{5\arcsin(\cos(a+bx)-\sin(a+bx))}{4b}\\ &\quad -\frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b}\\ &\quad +\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}-\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}\\ &\quad +\frac{4\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}+\frac{\csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{3b}\end{aligned}$$

output
```
-5/4*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b+5/2*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b-5/3*cos(b*x+a)*sin(2
*b*x+2*a)^(3/2)/b+4/3*sin(b*x+a)*sin(2*b*x+2*a)^(5/2)/b+1/3*csc(b*x+a)^3*s
in(2*b*x+2*a)^(9/2)/b
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.51

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= \frac{-5\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+2\sqrt{\sin}}{4b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(6*Sin[a + b*x] + S
in[3*(a + b*x)]))/(4*b)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4788, 3042, 4796, 3042, 4789, 3042, 4790, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(2a+2bx)^{7/2}}{\sin(a+bx)^3}dx$$
$$\Big\downarrow \text{4788}$$
$$4\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$
$$\Big\downarrow \text{3042}$$
$$4\int \frac{\sin(2a+2bx)^{7/2}}{\sin(a+bx)}dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{4796}$$

$$8\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)\sin(2a+2bx)^{5/2}dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{4789}$$

$$8\left(\frac{5}{6}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$8\left(\frac{5}{6}\int \sin(a+bx)\sin(2a+2bx)^{3/2}dx+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{4790}$$

$$8\left(\frac{5}{6}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$8\left(\frac{5}{6}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$$\Big\downarrow \text{4789}$$

$$8\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx}{}\right.$$
$$\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$\downarrow$ 3042

$$8\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)}{}\right.$$
$$\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

$\downarrow$ 4794

$$8\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin}{}\right.\right.\right.$$
$$\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2))/(3*b) + 8*((Sin[a + b*x]*Sin[2*a +
 2*b*x]^(5/2))/(6*b) + (5*((3*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b
 - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + (S
in[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 - (Cos[a + b*x]*Sin[2*a + 2*
b*x]^(3/2))/(4*b)))/6)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 100.34 (sec) , antiderivative size = 973, normalized size of antiderivative = 5.93

method	result	size
default	Expression too large to display	973

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
32/5*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(2*(tan(1/2*a+1/
2*b*x)*(tan(1/2*a+1/2*b*x)-1)*(tan(1/2*a+1/2*b*x)+1))^(1/2)*(tan(1/2*a+1/2
*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)
*EllipticE((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^4-
(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)-1)*(tan(1/2*a+1/2*b*x)+1))^(1/2)*(
tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/
2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*
a+1/2*b*x)^4+2*(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1
/2*b*x)^6-4*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)-1)*(tan(1/2*a+1/2*b*x)
+1))^(1/2)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-
tan(1/2*a+1/2*b*x))^(1/2)*EllipticE((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/
2))*tan(1/2*a+1/2*b*x)^2+2*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)-1)*(tan
(1/2*a+1/2*b*x)+1))^(1/2)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b
*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^
(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2-4*(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+
1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^4+2*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*
b*x)-1)*(tan(1/2*a+1/2*b*x)+1))^(1/2)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan
(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticE((tan(1/2*a+
1/2*b*x)+1)^(1/2),1/2*2^(1/2))-(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)-1)*
(tan(1/2*a+1/2*b*x)+1))^(1/2)*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.71

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \frac{8\sqrt{2}\left(4\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)+10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(}{\cos(bx+a)^2+2}\right.}{}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
1/16*(8*sqrt(2)*(4*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a))*sin
(b*x + a) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x +
 a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b
*x + a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b
*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 5
*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2
+ 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*c
os(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^3\sin(2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(7/2), x)
```

Giac [F]

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \csc\left(bx+a\right)^3 \sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \frac{\sin\left(2\,a+2\,b\,x\right)^{7/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^3,x)
```

output
```
int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^3, x)
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \sqrt{\sin\left(2bx+2a\right)}\,\csc\left(bx+a\right)^3 \sin\left(2bx+2a\right)^3 dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**3*sin(2*a + 2*b*x)**3,x)
```

3.527 $\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\begin{aligned}\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx &= -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))}{b} \\ &\quad+\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{b} \\ &\quad-\frac{6\cos(a+bx)\sqrt{\sin(2a+2bx)}}{b} \\ &\quad+\frac{4\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}+\frac{\csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{b}\end{aligned}$$

output
```
-3*arcsin(cos(b*x+a)-sin(b*x+a))/b+3*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*
a)^(1/2))/b-6*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b+4*sin(b*x+a)*sin(2*b*x+2*a
)^(3/2)/b+csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2)/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.55

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$= \frac{-3\arcsin(\cos(a+bx)-\sin(a+bx))+3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)+\csc(a+}{b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]] + Csc[a + b*x]*Sin[2*(a + b*x)]^(3/2))/b
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 4788, 3042, 4796, 3042, 4789, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(2a+2bx)^{5/2}}{\sin(a+bx)^3}dx$$
$$\Big\downarrow \text{4788}$$
$$8\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$
$$\Big\downarrow \text{3042}$$
$$8\int \frac{\sin(2a+2bx)^{5/2}}{\sin(a+bx)}dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{4796}$$

$$16\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$16\int \cos(a+bx)\sin(2a+2bx)^{3/2}dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{4789}$$

$$16\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$16\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{4790}$$

$$16\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$16\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{4793}$$

$$16\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\frac{\sqrt{\sin(2\cdots}}{}\right.\right.$$

$$\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(7/2))/b + 16*((3*((-1/2*ArcSin[Cos[a + b
*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a + 2
*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 + (Sin[
a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 103.65 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.91

method	result
default	$\frac{16\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right.}{3\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\,\sqrt{\mathrm{ta}}}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
16/3*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*((tan(1/2*a+1/2*
b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*
EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2-(
tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/
2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))-tan(1/2*
a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))/(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))
^(1/2)/(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(119) = 238$.

Time = 0.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.11

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$= \frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a) + 6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + 6*arctan(-(s
qrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos
(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)
) - 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) -
sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4 +
4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(
b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x
 + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \csc\left(bx+a\right)^3 \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(5/2), x)
```

Giac [F]

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \csc\left(bx+a\right)^3 \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \frac{\sin\left(2\,a+2\,b\,x\right)^{5/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^3,x)
```

output
```
int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^3, x)
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sqrt{\sin(2bx+2a)}\,\csc(bx+a)^3\sin(2bx+2a)^2\,dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**3*sin(2*a + 2*b*x)**2,x)
```

3.528 $\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\begin{aligned}\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= \frac{2\arcsin(\cos(a+bx)-\sin(a+bx))}{b} \\ &+ \frac{2\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{b} \\ &- \frac{4\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b} - \frac{\csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b}\end{aligned}$$

output
```
2*arcsin(cos(b*x+a)-sin(b*x+a))/b+2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*a
)^(1/2))/b-4*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b-csc(b*x+a)^3*sin(2*b*x+2*a)
^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$=\frac{2\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\csc(a+b\right.}{b}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(2*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]] - 2*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]]))/b
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4788, 3042, 4796, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2a+2bx)\csc^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(2a+2bx)^{3/2}}{\sin(a+bx)^3}dx$$
$$\Big\downarrow \text{4788}$$
$$-4\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$
$$\Big\downarrow \text{3042}$$
$$-4\int \frac{\sin(2a+2bx)^{3/2}}{\sin(a+bx)}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{4796}$$

$$-8\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-8\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{4789}$$

$$-8\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-8\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

$$\Big\downarrow \text{4794}$$

$$-8\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a+b}{}\right.$$
$$\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-8*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a
 + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(2*b)) - (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2))/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 9.77 (sec) , antiderivative size = 542, normalized size of antiderivative = 5.21

method	result
default	$\frac{4\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\left(4\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{b}{2}\right.}\right.}{}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(4*(tan(1/2*a+1/2*b
*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*(
tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2*b*x)*(ta
n(1/2*a+1/2*b*x)-1)*(tan(1/2*a+1/2*b*x)+1))^(1/2)*EllipticE((tan(1/2*a+1/2
*b*x)+1)^(1/2),1/2*2^(1/2))-2*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1
/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*(tan(1/2*a+1/2*b*x)*(tan(1/2*
a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)-1)*(tan(1/2
*a+1/2*b*x)+1))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))+
(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*(tan(1/2*a+1/2*b*x)*(tan(1
/2*a+1/2*b*x)-1)*(tan(1/2*a+1/2*b*x)+1))^(1/2)*tan(1/2*a+1/2*b*x)^2+2*tan(
1/2*a+1/2*b*x)^2*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(
1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)-(tan(1/2*a+1/2*b*x)^3-tan(1/2*a
+1/2*b*x))^(1/2)*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)-1)*(tan(1/2*a+1/2
*b*x)+1))^(1/2))/tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*
x))^(1/2)/(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)-1)*(tan(1/2*a+1/2*b*x)+1
))^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(98) = 196.

Time = 0.09 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.84

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx =$$

$$-\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)\sin(bx+a)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos}}{}\right.}{}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - s
in(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)
*sin(b*x + a) - 1))*sin(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*
sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))
)*sin(b*x + a) + log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4
*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(
b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)*sin(b*x
+ a) + 8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + 8*sin(b*x + a))/(b*sin(
b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^3\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

Giac [F]

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \csc(bx+a)^3\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^{3/2}}{\sin(a+b\,x)^3}\,dx$$

input
```
int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^3,x)
```

output
```
int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^3, x)
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sqrt{\sin(2bx+2a)}\,\csc(bx+a)^3\sin(2bx+2a)\,dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**3*sin(2*a + 2*b*x),x)
```

3.529 $\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}$$

output
```
-1/3*csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\csc^3(a+bx)\sin^{\frac{3}{2}}(2(a+bx))}{3b}$$

input
```
Integrate[Csc[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/3*(Csc[a + b*x]^3*Sin[2*(a + b*x)]^(3/2))/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2a+2bx)}\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin(2a+2bx)}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4780}$$

$$-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

input
```
Int[Csc[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/3*(Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2))/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 7.56 (sec) , antiderivative size = 192, normalized size of antiderivative = 6.86

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\,\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)\left(4\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\right.\right.}{3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\,b}$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2*b*
x)^2-1)/tan(1/2*a+1/2*b*x)*(4*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1
/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)
+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)+tan(1/2*a+1/2*b*x)^4-1)/(tan(1/2
*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)/(tan(1/2*a+1/2*b*x)^3-tan(1/2*
a+1/2*b*x))^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(24) = 48$.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{2\left(\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)+\cos(bx+a)^2-1\right)}{3\left(b\cos(bx+a)^2-b\right)}$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + cos(b*x + a)^2
 - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \csc(bx+a)^3\sqrt{\sin(2\,bx+2\,a)}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F]

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \csc(bx+a)^3\sqrt{\sin(2\,bx+2\,a)}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [B] (verification not implemented)

Time = 20.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.39

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{4\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,\left(4\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-6\sin\left(\frac{3\,a}{2}+\frac{3\,b\,x}{2}\right)^2+2\sin\left(\frac{5\,a}{2}+\frac{5\,b\,x}{2}\right)^2\right)}{3\,b\,\left(30\sin\left(a+b\,x\right)^2-12\sin\left(2\,a+2\,b\,x\right)^2+2\sin\left(3\,a+3\,b\,x\right)^2\right)}$$

input
```
int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x)^3,x)
```

output
```
(4*sin(2*a + 2*b*x)^(1/2)*(4*sin(a/2 + (b*x)/2)^2 - 6*sin((3*a)/2 + (3*b*x
)/2)^2 + 2*sin((5*a)/2 + (5*b*x)/2)^2))/(3*b*(2*sin(3*a + 3*b*x)^2 - 12*si
n(2*a + 2*b*x)^2 + 30*sin(a + b*x)^2))
```

Reduce [F]

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin\left(2bx+2a\right)}\,\csc\left(bx+a\right)^3 dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**3,x)
```

3.530 $\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx=-\frac{4\csc(a+bx)\sqrt{\sin(2a+2bx)}}{5b}-\frac{\csc^3(a+bx)\sqrt{\sin(2a+2bx)}}{5b}$$

output
```
-4/5*csc(b*x+a)*sin(2*b*x+2*a)^(1/2)/b-1/5*csc(b*x+a)^3*sin(2*b*x+2*a)^(1/
2)/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx=-\frac{\csc(a+bx)\left(4+\csc^2(a+bx)\right)\sqrt{\sin(2(a+bx))}}{5b}$$

input
```
Integrate[Csc[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/5*(Csc[a + b*x]*(4 + Csc[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4788, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{ 4788}$$

$$\frac{4}{5}\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\csc^3(a+bx)}{5b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{5}\int \frac{1}{\sin(a+bx)\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\csc^3(a+bx)}{5b}$$

$$\Big\downarrow \text{ 4780}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\csc^3(a+bx)}{5b}-\frac{4\sqrt{\sin(2a+2bx)}\csc(a+bx)}{5b}$$

input
```
Int[Csc[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-4*Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(5*b) - (Csc[a + b*x]^3*Sqrt[Sin[
2*a + 2*b*x]])/(5*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 10.22 (sec) , antiderivative size = 482, normalized size of antiderivative = 8.76

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\left(16\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\operatorname{EllipticE}\left(\right.}{}$

input
```
int(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)/tan(1/2*a+1/2*b*
x)^3*(16*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/
2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2
)*EllipticE((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2
-8*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2*b*x)
+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*Elli
pticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2-(tan(
1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^6+(tan(1
/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^4+8*(tan(
1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^4+(tan(1/2*a
+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^2-8*(tan(1/2*
a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^2-(tan(1/2*a+1/2
*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2))/(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2
*b*x))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\sqrt{2}\left(4\cos\left(bx+a\right)^2-5\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+4\left(\cos\left(bx+a\right)^2-1\right)\sin\left(bx+a\right)}{5\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(sqrt(2)*(4*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 4*(
cos(b*x + a)^2 - 1)*sin(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\csc(bx+a)^3}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\csc(bx+a)^3}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [B] (verification not implemented)

Time = 24.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.69

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= -\frac{8\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{5\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^(1/2)),x)
```

output
```
-(8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*
1i)/2)^(1/2)*(exp(a*4i + b*x*4i)*1i - exp(a*2i + b*x*2i)*3i + 1i))/(5*b*(e
xp(a*2i + b*x*2i) - 1)^3)
```

Reduce [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\csc(bx+a)^3}{\sin(2bx+2a)}dx$$

input
```
int(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**3)/sin(2*a + 2*b*x),x)
```

3.531 $\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}} + \frac{32\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

output
```
-16/21*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-1/7*csc(b*x+a)^3/b/sin(2*b*x+2*a)
^(1/2)+32/21*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{(5-12\cos(2(a+bx))+4\cos(4(a+bx)))\csc^4(a+bx)\sec(a+bx)\sqrt{\sin(2(a+bx))}}{42b}$$

input
```
Integrate[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
((5 - 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4*Sec[a + b*x
]*Sqrt[Sin[2*(a + b*x)]])/(42*b)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4788, 3042, 4796, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{8}{7}\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{7}\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^{3/2}}dx-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{4796}$$

$$\frac{16}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{16}{7}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{4791}$$

$$\frac{16}{7}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{16}{7}\left(\frac{2}{3}\int\frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{4780}$$

$$\frac{16}{7}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(16*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*
Sqrt[Sin[2*a + 2*b*x]])))/7 - Csc[a + b*x]^3/(7*b*Sqrt[Sin[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 72.24 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.74

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)\left(-3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{8}+16\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\,\mathrm{Ellipt}\right.}{336\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{b}{}\right.}}$

input
```
int(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/336*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2
*b*x)^2-1)/tan(1/2*a+1/2*b*x)^3*(-3*tan(1/2*a+1/2*b*x)^8+16*(tan(1/2*a+1/2
*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)
*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^3-
2*tan(1/2*a+1/2*b*x)^6+2*tan(1/2*a+1/2*b*x)^2+3)/(tan(1/2*a+1/2*b*x)*(tan(
1/2*a+1/2*b*x)^2-1))^(1/2)/(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)
/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.28

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{32\cos(bx+a)^5 - 64\cos(bx+a)^3 + \sqrt{2}\left(32\cos(bx+a)^4 - 56\cos(bx+a)^2 + 21\right)\sqrt{\cos(bx+a)\sin(b}}{42\left(b\cos(bx+a)^5 - 2b\cos(bx+a)^3 + b\cos(bx+a)\right)}$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/42*(32*cos(b*x + a)^5 - 64*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4 -
 56*cos(b*x + a)^2 + 21)*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)
)/(b*cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 24.24 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.73

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{10\,e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{21\,b\,(e^{a\,2i+b\,x\,2i}\,1i-i)^2} + \frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,12i}{7\,b\,(e^{a\,2i+b\,x\,2i}\,1i-i)^3} - \frac{8\,e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{7\,b\,(e^{a\,2i+b\,x\,2i}\,1i-i)^4} - \frac{e^{a\,1i+b\,x\,1i}\left(\frac{10i}{21\,b}-\frac{e^{a\,2i+b\,x\,2i}\,32i}{21\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{(e^{a\,2i+b\,x\,2i}+1)\,(e^{a\,2i+b\,x\,2i}\,1i-i)}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^(3/2)),x)
```

output
```
(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*12i)/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^3) - (10*exp(a*1i + b*x*1
i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(21*b*
(exp(a*2i + b*x*2i)*1i - 1i)^2) - (8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x
*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i
 - 1i)^4) - (exp(a*1i + b*x*1i)*(10i/(21*b) - (exp(a*2i + b*x*2i)*32i)/(21
*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((ex
p(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i))
```

Reduce [F]

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\csc(bx+a)^3}{\sin(2bx+2a)^2}dx$$

input
```
int(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**3)/sin(2*a + 2*b*x)**2,x)
```

3.532 $\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{64\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

output
```
-8/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2)-1/9*csc(b*x+a)^3/b/sin(2*b*x+2*a)^
(3/2)+32/45*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-64/45*cos(b*x+a)/b/sin(2*b*x
+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{\sin(2(a+bx))}(113\csc(a+bx)+17\csc^3(a+bx)+5\csc^5(a+bx)-15\sec(a+bx)\tan(a+bx))}{180b}$$

input
```
Integrate[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/180*(Sqrt[Sin[2*(a + b*x)]]*(113*Csc[a + b*x] + 17*Csc[a + b*x]^3 + 5*C
sc[a + b*x]^5 - 15*Sec[a + b*x]*Tan[a + b*x]))/b
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 4788, 3042, 4796, 3042, 4791, 3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4788}$$

$$\frac{4}{3}\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}\int \frac{1}{\sin(a+bx)\sin(2a+2bx)^{5/2}}dx-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4796}$$

$$\frac{8}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{7/2}}dx-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4791}$$

$$\frac{8}{3}\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{3}\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4792}$$

$$\frac{8}{3}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{3}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4779}$$

$$\frac{8}{3}\left(\frac{4}{5}\left(\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

input
```
Int[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(8*((4*(Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*
Sqrt[Sin[2*a + 2*b*x]])))/5 - Cos[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2))))/
3 - Csc[a + b*x]^3/(9*b*Sin[2*a + 2*b*x]^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4788
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Sin[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 197.37 (sec) , antiderivative size = 560, normalized size of antiderivative = 5.23

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\left(5\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{10}+192\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\sqrt{-\tan\right.}{}$

input
```
int(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2880*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)/tan(1/2*a+1/2
*b*x)^5*(5*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1
/2*b*x)^10+192*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2
)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticE((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2
^(1/2))*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*
b*x)^4-96*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-t
an(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2
))*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^
4-7*tan(1/2*a+1/2*b*x)^8*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/
2)+96*(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^6
+2*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^
6-96*(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^4+
2*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^4
-7*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^
2+5*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2))/(tan(1/2*a+1/2*b*
x)^3-tan(1/2*a+1/2*b*x))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.22

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\left(128\cos(bx+a)^6-288\cos(bx+a)^4+180\cos(bx+a)^2-15\right)\sqrt{\cos(bx+a)\sin(bx+a)}+128}{180\left(b\cos(bx+a)^6-2\,b\cos(bx+a)^4+b\cos(bx+a)\right.}$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/180*(sqrt(2)*(128*cos(b*x + a)^6 - 288*cos(b*x + a)^4 + 180*cos(b*x + a
)^2 - 15)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - 2*cos(b*
x + a)^4 + cos(b*x + a)^2)*sin(b*x + a))/((b*cos(b*x + a)^6 - 2*b*cos(b*x
+ a)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

Giac [F]

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\csc(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.70 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.58

$$\begin{aligned}\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = & -\frac{2\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{15\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)^3} \\ & -\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,16\mathrm{i}}{9\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)^4} \\ & +\frac{8\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{9\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)^5} \\ & +\frac{64\,\mathrm{e}^{a\,3\mathrm{i}+b\,x\,3\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{45\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)} \\ & -\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{98\mathrm{i}}{45\,b}+\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,38\mathrm{i}}{45\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}\end{aligned}$$

input
```
int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^(5/2)),x)
```

output
```
(8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2))/(9*b*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (exp(a*1i + b*x*1i)*((
exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*16i)/(9*b*(e
xp(a*2i + b*x*2i)*1i - 1i)^4) - (2*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2
i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(15*b*(exp(a*2i + b*x*2i)*1i
- 1i)^3) + (64*exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i
 + b*x*2i)*1i)/2)^(1/2))/(45*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i
)*1i - 1i)) - (exp(a*1i + b*x*1i)*(98i/(45*b) + (exp(a*2i + b*x*2i)*38i)/(
45*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((
exp(a*2i + b*x*2i) + 1)^2*(exp(a*2i + b*x*2i)*1i - 1i)^2)
```

Reduce [F]

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\csc(bx+a)^3}{\sin(2bx+2a)^3}dx$$

input
```
int(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*csc(a + b*x)**3)/sin(2*a + 2*b*x)**3,x)
```

3.533 $\int \sin^2(a+bx)\sin^q(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \sin^2(a+bx)\sin^q(2a+2bx)\,dx$$
$$= \frac{\cos^2(a+bx)^{\frac{1-q}{2}}\,\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{3+q}{2},\frac{5+q}{2},\sin^2(a+bx)\right)\sin^2(a+bx)\sin^q(2a+2bx)\tan(a+bx}{b(3+q)}$$

output
```
(cos(b*x+a)^2)^(1/2-1/2*q)*hypergeom([1/2-1/2*q, 3/2+1/2*q],[5/2+1/2*q],si
n(b*x+a)^2)*sin(b*x+a)^2*sin(2*b*x+2*a)^q*tan(b*x+a)/b/(3+q)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \sin^2(a+bx)\sin^q(2a+2bx)\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(2+q,\frac{3+q}{2},\frac{5+q}{2},-\tan^2(a+bx)\right)\sec^2(a+bx)^q\sin^q(2(a+bx))\tan^3(a+bx)}{b(3+q)}$$

input
```
Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^q,x]
```

output
```
(Hypergeometric2F1[2 + q, (3 + q)/2, (5 + q)/2, -Tan[a + b*x]^2]*(Sec[a +
b*x]^2)^q*Sin[2*(a + b*x)]^q*Tan[a + b*x]^3)/(b*(3 + q))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4798, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sin^q(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sin(2a+2bx)^q dx$$

$$\Big\downarrow \text{4798}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos^q(a+bx)\sin^{q+2}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos(a+bx)^q\sin(a+bx)^{q+2}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sin^2(a+bx)\tan(a+bx)\sin^q(2a+2bx)\cos^2(a+bx)^{\frac{1-q}{2}}\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{q+3}{2},\frac{q+5}{2},\sin^2(a+bx)\right)}{b(q+3)}$$

input
```
Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^q,x]
```

output
```
((Cos[a + b*x]^2)^((1 - q)/2)*Hypergeometric2F1[(1 - q)/2, (3 + q)/2, (5 +
 q)/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Sin[2*a + 2*b*x]^q*Tan[a + b*x])/(b*
(3 + q))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 4798
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p)   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
 d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sin(bx+a)^2 \sin(2bx+2a)^q \, dx$$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^q,x)
```

output
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^q,x)
```

Fricas [F]

$$\int \sin^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q \sin(bx+a)^2 \, dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^q,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^q, x)
```

Sympy [F]

$$\int \sin^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin^2(a+bx)\sin^q(2a+2bx)\,dx$$

input
```
integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**q,x)
```

output
```
Integral(sin(a + b*x)**2*sin(2*a + 2*b*x)**q, x)
```

Maxima [F]

$$\int \sin^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q \sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^q,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^q*sin(b*x + a)^2, x)
```

Giac [F]

$$\int \sin^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q \sin(bx+a)^2\,dx$$

input
```
integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^q,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^q*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin{(a+b\,x)}^2\,\sin{(2\,a+2\,b\,x)}^q\,dx$$

input
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^q,x)
```

output
```
int(sin(a + b*x)^2*sin(2*a + 2*b*x)^q, x)
```

Reduce [F]

$$\int \sin^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin{(2bx+2a)}^q \sin{(bx+a)}^2\,dx$$

input
```
int(sin(b*x+a)^2*sin(2*b*x+2*a)^q,x)
```

output
```
int(sin(2*a + 2*b*x)**q*sin(a + b*x)**2,x)
```

3.534 $\int \sin(a+bx)\sin^q(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 82

$$\int \sin(a+bx)\sin^q(2a+2bx)\,dx = \frac{\cos^2(a+bx)^{\frac{1-q}{2}}\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{2+q}{2},\frac{4+q}{2},\sin^2(a+bx)\right)\sin(a+bx)\sin^q(2a+2bx)\tan(a+bx)}{b(2+q)}$$

output
```
(cos(b*x+a)^2)^(1/2-1/2*q)*hypergeom([1+1/2*q, 1/2-1/2*q],[2+1/2*q],sin(b*
x+a)^2)*sin(b*x+a)*sin(2*b*x+2*a)^q*tan(b*x+a)/b/(2+q)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.91 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.39

$$\int \sin(a+bx)\sin^q(2a+2bx)\,dx = \frac{8(4+q)\,\text{AppellF1}\,(\ldots}{b(2+q)\left(2\left(q\,\text{AppellF1}\left(2+\frac{q}{2},1-q,2+2q,3+\frac{q}{2},\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2(1+q)\,\text{Ap}\ldots}$$

input
```
Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^q,x]
```

output
```
(8*(4 + q)*AppellF1[1 + q/2, -q, 2 + 2*q, 2 + q/2, Tan[(a + b*x)/2]^2, -Ta
n[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^4*Sin[(a + b*x)/2]^2*Sin[2*(a + b*x)]^q
)/(b*(2 + q)*(2*(q*AppellF1[2 + q/2, 1 - q, 2 + 2*q, 3 + q/2, Tan[(a + b*x
)/2]^2, -Tan[(a + b*x)/2]^2] + 2*(1 + q)*AppellF1[2 + q/2, -q, 3 + 2*q, 3
+ q/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a + b*x]) + (4
+ q)*AppellF1[1 + q/2, -q, 2 + 2*q, 2 + q/2, Tan[(a + b*x)/2]^2, -Tan[(a +
 b*x)/2]^2]*(1 + Cos[a + b*x])))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4798, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin^q(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sin(2a+2bx)^q dx$$

$$\Big\downarrow \text{4798}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos^q(a+bx)\sin^{q+1}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos(a+bx)^q\sin(a+bx)^{q+1}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sin(a+bx)\tan(a+bx)\sin^q(2a+2bx)\cos^2(a+bx)^{\frac{1-q}{2}}\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{q+2}{2},\frac{q+4}{2},\sin^2(a+bx)\right)}{b(q+2)}$$

input
```
Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^q,x]
```

output
```
((Cos[a + b*x]^2)^((1 - q)/2)*Hypergeometric2F1[(1 - q)/2, (2 + q)/2, (4 +
 q)/2, Sin[a + b*x]^2]*Sin[a + b*x]*Sin[2*a + 2*b*x]^q*Tan[a + b*x])/(b*(2
 + q))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 4798
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p)   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
 d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sin(bx+a)\sin(2bx+2a)^q\,dx$$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^q,x)
```

output
```
int(sin(b*x+a)*sin(2*b*x+2*a)^q,x)
```

Fricas [F]

$$\int \sin(a+bx)\sin^q(2a+2bx)\,dx=\int \sin\left(2\,bx+2\,a\right)^q\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^q,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^q*sin(b*x + a), x)
```

Sympy [F]

$$\int \sin(a+bx)\sin^q(2a+2bx)\,dx=\int \sin\left(a+bx\right)\sin^q\left(2a+2bx\right)dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)**q,x)
```

output
```
Integral(sin(a + b*x)*sin(2*a + 2*b*x)**q, x)
```

Maxima [F]

$$\int \sin(a+bx)\sin^q(2a+2bx)\,dx=\int \sin\left(2\,bx+2\,a\right)^q\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^q,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^q*sin(b*x + a), x)
```

Giac [F]

$$\int \sin(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q \sin(bx+a)\,\,dx$$

input
```
integrate(sin(b*x+a)*sin(2*b*x+2*a)^q,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^q*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(a+b\,x)\,\sin(2\,a+2\,b\,x)^q\,dx$$

input
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^q,x)
```

output
```
int(sin(a + b*x)*sin(2*a + 2*b*x)^q, x)
```

Reduce [F]

$$\int \sin(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2bx+2a)^q \sin(bx+a)\,dx$$

input
```
int(sin(b*x+a)*sin(2*b*x+2*a)^q,x)
```

output
```
int(sin(2*a + 2*b*x)**q*sin(a + b*x),x)
```

3.535 $\int \csc(a+bx)\sin^q(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 72

$$\int \csc(a+bx)\sin^q(2a+2bx)\,dx = \frac{\cos^2(a+bx)^{\frac{1-q}{2}}\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{q}{2},\frac{2+q}{2},\sin^2(a+bx)\right)\sec(a+bx)\sin^q(2a+2bx)}{bq}$$

output
```
(cos(b*x+a)^2)^(1/2-1/2*q)*hypergeom([1/2*q, 1/2-1/2*q],[1+1/2*q],sin(b*x+
a)^2)*sec(b*x+a)*sin(2*b*x+2*a)^q/b/q
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.84 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.53

$$\int \csc(a+bx)\sin^q(2a+2bx)\,dx = \frac{2(2+q)\,\text{AppellF1}\left(\frac{q}{2},-q,\right.}{bq\left((2+q)\,\text{AppellF1}\left(\frac{q}{2},-q,2q,\frac{2+q}{2},\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)(1+\cos(a+bx))-4q\left(\text{App}\right.\right.}$$

input
```
Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^q,x]
```

output
```
(2*(2 + q)*AppellF1[q/2, -q, 2*q, (2 + q)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
 b*x)/2]^2]*Cos[(a + b*x)/2]^2*Sin[2*(a + b*x)]^q)/(b*q*((2 + q)*AppellF1[
q/2, -q, 2*q, (2 + q)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos
[a + b*x]) - 4*q*(AppellF1[(2 + q)/2, 1 - q, 2*q, (4 + q)/2, Tan[(a + b*x)
/2]^2, -Tan[(a + b*x)/2]^2] + 2*AppellF1[(2 + q)/2, -q, 1 + 2*q, (4 + q)/2
, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sin[(a + b*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4798, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sin^q(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^q}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{4798}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos^q(a+bx)\sin^{q-1}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos(a+bx)^q\sin(a+bx)^{q-1}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sec(a+bx)\sin^q(2a+2bx)\cos^2(a+bx)^{\frac{1-q}{2}}\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{q}{2},\frac{q+2}{2},\sin^2(a+bx)\right)}{bq}$$

input
```
Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^q,x]
```

output
```
((Cos[a + b*x]^2)^((1 - q)/2)*Hypergeometric2F1[(1 - q)/2, q/2, (2 + q)/2,
 Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^q)/(b*q)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 4798
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p)   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
 d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \csc(bx+a)\sin(2bx+2a)^q\,dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^q,x)
```

output
```
int(csc(b*x+a)*sin(2*b*x+2*a)^q,x)
```

Fricas [F]

$$\int \csc(a+bx)\sin^q(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^q \csc\left(bx+a\right)\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^q,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^q*csc(b*x + a), x)
```

Sympy [F]

$$\int \csc(a+bx)\sin^q(2a+2bx)\,dx = \int \sin^q\left(2a+2bx\right)\csc\left(a+bx\right)dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)**q,x)
```

output
```
Integral(sin(2*a + 2*b*x)**q*csc(a + b*x), x)
```

Maxima [F]

$$\int \csc(a+bx)\sin^q(2a+2bx)\,dx = \int \sin\left(2\,bx+2\,a\right)^q \csc\left(bx+a\right)\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^q,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^q*csc(b*x + a), x)
```

Giac [F]

$$\int \csc(a+bx)\sin^q(2a+2bx)\,dx=\int \sin\left(2\,bx+2\,a\right)^q\csc\left(bx+a\right)\,dx$$

input
```
integrate(csc(b*x+a)*sin(2*b*x+2*a)^q,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^q*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\sin^q(2a+2bx)\,dx=\int \frac{\sin\left(2\,a+2\,b\,x\right)^q}{\sin\left(a+b\,x\right)}\,dx$$

input
```
int(sin(2*a + 2*b*x)^q/sin(a + b*x),x)
```

output
```
int(sin(2*a + 2*b*x)^q/sin(a + b*x), x)
```

Reduce [F]

$$\int \csc(a+bx)\sin^q(2a+2bx)\,dx=\int \sin\left(2bx+2a\right)^q\csc\left(bx+a\right)dx$$

input
```
int(csc(b*x+a)*sin(2*b*x+2*a)^q,x)
```

output
```
int(sin(2*a + 2*b*x)**q*csc(a + b*x),x)
```

3.536 $\int \csc^2(a+bx)\sin^q(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \csc^2(a+bx)\sin^q(2a+2bx)\,dx = -\frac{\cos^2(a+bx)^{\frac{1-q}{2}}\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{1}{2}(-1+q),\frac{1+q}{2},\sin^2(a+bx)\right)\sec(a+bx)\sin^q(2}{b(1-q)}$$

output
```
-(cos(b*x+a)^2)^(1/2-1/2*q)*csc(b*x+a)*hypergeom([1/2-1/2*q, -1/2+1/2*q],[
1/2+1/2*q],sin(b*x+a)^2)*sec(b*x+a)*sin(2*b*x+2*a)^q/b/(1-q)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\int \csc^2(a+bx)\sin^q(2a+2bx)\,dx = \frac{\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+q),q,\frac{1+q}{2},-\tan^2(a+bx)\right)\sec^2(a+bx)^q\sin^q(2(a+bx))}{b(-1+q)}$$

input
```
Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^q,x]
```

output
```
(Cot[a + b*x]*Hypergeometric2F1[(-1 + q)/2, q, (1 + q)/2, -Tan[a + b*x]^2]
*(Sec[a + b*x]^2)^q*Sin[2*(a + b*x)]^q)/(b*(-1 + q))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4798, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sin^q(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^q}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{4798}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos^q(a+bx)\sin^{q-2}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos(a+bx)^q\sin(a+bx)^{q-2}dx$$

$$\Big\downarrow \text{3057}$$

$$-\frac{\csc(a+bx)\sec(a+bx)\sin^q(2a+2bx)\cos^2(a+bx)^{\frac{1-q}{2}}\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{q-1}{2},\frac{q+1}{2},\sin^2(a+bx)\right)}{b(1-q)}$$

input
```
Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^q,x]
```

output
```
-(((Cos[a + b*x]^2)^((1 - q)/2)*Csc[a + b*x]*Hypergeometric2F1[(1 - q)/2,
(-1 + q)/2, (1 + q)/2, Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^q)/(b
*(1 - q)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 4798
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p)   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
 d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \csc(bx+a)^2 \sin(2bx+2a)^q \, dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^q,x)
```

output
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^q,x)
```

Fricas [F]

$$\int \csc^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q \csc(bx+a)^2 \, dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^q,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^q*csc(b*x + a)^2, x)
```

Sympy [F]

$$\int \csc^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin^q(2a+2bx)\csc^2(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**q,x)
```

output
```
Integral(sin(2*a + 2*b*x)**q*csc(a + b*x)**2, x)
```

Maxima [F]

$$\int \csc^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q \csc(bx+a)^2\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^q,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^q*csc(b*x + a)^2, x)
```

Giac [F]

$$\int \csc^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q \csc(bx+a)^2\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^q,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^q*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sin^q(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^q}{\sin(a+b\,x)^2}\,dx$$

input
```
int(sin(2*a + 2*b*x)^q/sin(a + b*x)^2,x)
```

output
```
int(sin(2*a + 2*b*x)^q/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int \csc^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2bx+2a)^q \csc(bx+a)^2\,dx$$

input
```
int(csc(b*x+a)^2*sin(2*b*x+2*a)^q,x)
```

output
```
int(sin(2*a + 2*b*x)**q*csc(a + b*x)**2,x)
```

3.537 $\int \csc^3(a+bx)\sin^q(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \csc^3(a+bx)\sin^q(2a+2bx)\,dx = -\frac{\cos^2(a+bx)^{\frac{1-q}{2}}\csc^2(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{1}{2}(-2+q),\frac{q}{2},\sin^2(a+bx)\right)\sec(a+bx)\sin^q(2a}{b(2-q)}$$

output
```
-(cos(b*x+a)^2)^(1/2-1/2*q)*csc(b*x+a)^2*hypergeom([-1+1/2*q, 1/2-1/2*q],[
1/2*q],sin(b*x+a)^2)*sec(b*x+a)*sin(2*b*x+2*a)^q/b/(2-q)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 12.58 (sec) , antiderivative size = 1257, normalized size of antiderivative = 14.79

$$\int \csc^3(a+bx)\sin^q(2a+2bx)\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^q,x]
```

output
```
(AppellF1[-1 + q/2, -q, 2*q, q/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
*Cos[(a + b*x)/2]^2*Cot[(a + b*x)/2]^2*Sin[2*(a + b*x)]^q)/(2*b*(-2 + q)*(
2*(AppellF1[q/2, 1 - q, 2*q, 1 + q/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2
]^2] + 2*AppellF1[q/2, -q, 1 + 2*q, 1 + q/2, Tan[(a + b*x)/2]^2, -Tan[(a +
 b*x)/2]^2])*(-1 + Cos[a + b*x]) + AppellF1[-1 + q/2, -q, 2*q, q/2, Tan[(a
 + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x]))) + ((4 + q)*AppellF
1[1 + q/2, -q, 2*q, 2 + q/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[
a + b*x]*Sin[(a + b*x)/2]^2*Sin[2*(a + b*x)]^q)/(2*b*(2 + q)*((4 + q)*Appe
llF1[1 + q/2, -q, 2*q, 2 + q/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(
1 + Sec[a + b*x]) - 4*q*(AppellF1[2 + q/2, 1 - q, 2*q, 3 + q/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*AppellF1[2 + q/2, -q, 1 + 2*q, 3 + q/2,
 Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sec[a + b*x]*Sin[(a + b*x)/2]^2
)) + ((4 + q)*AppellF1[1 + q/2, -q, 1 + 2*q, 2 + q/2, Tan[(a + b*x)/2]^2,
-Tan[(a + b*x)/2]^2]*Sin[a + b*x]^2*Sin[2*(a + b*x)]^q)/(4*b*(2 + q)*(2*(q
*AppellF1[2 + q/2, 1 - q, 1 + 2*q, 3 + q/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2] + (1 + 2*q)*AppellF1[2 + q/2, -q, 2 + 2*q, 3 + q/2, Tan[(a + b*
x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a + b*x]) + (4 + q)*AppellF1[1 +
q/2, -q, 1 + 2*q, 2 + q/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + C
os[a + b*x]))) + ((4 + q)*Cos[(a + b*x)/2]^2*Sin[2*(a + b*x)]^q*((2 + q)*A
ppellF1[q/2, -q, 2*q, 1 + q/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]...
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4798, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sin^q(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2a+2bx)^q}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{4798}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos^q(a+bx)\sin^{q-3}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int\cos(a+bx)^q\sin(a+bx)^{q-3}dx$$

$$\Big\downarrow \text{3057}$$

$$-\frac{\csc^2(a+bx)\sec(a+bx)\sin^q(2a+2bx)\cos^2(a+bx)^{\frac{1-q}{2}}\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{q-2}{2},\frac{q}{2},\sin^2(a+bx)\right)}{b(2-q)}$$

input
```
Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^q,x]
```

output
```
-(((Cos[a + b*x]^2)^((1 - q)/2)*Csc[a + b*x]^2*Hypergeometric2F1[(1 - q)/2
, (-2 + q)/2, q/2, Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^q)/(b*(2
- q)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 4798
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p)   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
 d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p]
```

Maple [F]

$$\int \csc (bx+a)^3 \sin (2bx+2a)^q \, dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^q,x)
```

output
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^q,x)
```

Fricas [F]

$$\int \csc^3(a+bx)\sin^q(2a+2bx)\,dx = \int \sin \left(2\,bx+2\,a\right)^{q} \csc \left(bx+a\right)^{3}\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^q,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^q*csc(b*x + a)^3, x)
```

Sympy [F]

$$\int \csc^3(a+bx)\sin^q(2a+2bx)\,dx = \int \sin^{q}\left(2a+2bx\right)\csc^{3}\left(a+bx\right)dx$$

input
```
integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**q,x)
```

output
```
Integral(sin(2*a + 2*b*x)**q*csc(a + b*x)**3, x)
```

Maxima [F]

$$\int \csc^3(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q \csc(bx+a)^3\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^q,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^q*csc(b*x + a)^3, x)
```

Giac [F]

$$\int \csc^3(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q \csc(bx+a)^3\,dx$$

input
```
integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^q,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^q*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sin^q(2a+2bx)\,dx = \int \frac{\sin(2\,a+2\,b\,x)^q}{\sin(a+b\,x)^3}\,dx$$

input
```
int(sin(2*a + 2*b*x)^q/sin(a + b*x)^3,x)
```

output
```
int(sin(2*a + 2*b*x)^q/sin(a + b*x)^3, x)
```

Reduce [F]

$$\int \csc^3(a+bx)\sin^q(2a+2bx)\,dx=\int \sin\left(2bx+2a\right)^q\csc\left(bx+a\right)^3\,dx$$

input
```
int(csc(b*x+a)^3*sin(2*b*x+2*a)^q,x)
```

output
```
int(sin(2*a + 2*b*x)**q*csc(a + b*x)**3,x)
```

3.538 $\int \cos(a+bx)\sin^7(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx = -\frac{128\cos^9(a+bx)}{9b}+\frac{384\cos^{11}(a+bx)}{11b} - \frac{384\cos^{13}(a+bx)}{13b}+\frac{128\cos^{15}(a+bx)}{15b}$$

output
```
-128/9*cos(b*x+a)^9/b+384/11*cos(b*x+a)^11/b-384/13*cos(b*x+a)^13/b+128/15
*cos(b*x+a)^15/b
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx = \frac{4\cos^9(a+bx)(-8330+10755\cos(2(a+bx))-3366\cos(4(a+bx))+429\cos(6(a+bx)))}{6435b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

output
```
(4*Cos[a + b*x]^9*(-8330 + 10755*Cos[2*(a + b*x)] - 3366*Cos[4*(a + b*x)]
+ 429*Cos[6*(a + b*x)]))/(6435*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4775, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(2a+2bx)^7\cos(a+bx)dx$$

$$\Big\downarrow \; 4775$$

$$128\int \cos^8(a+bx)\sin^7(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$128\int \cos(a+bx)^8\sin(a+bx)^7dx$$

$$\Big\downarrow \; 3045$$

$$-\frac{128\int \cos^8(a+bx)\left(1-\cos^2(a+bx)\right)^3d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 244$$

$$-\frac{128\int \left(-\cos^{14}(a+bx)+3\cos^{12}(a+bx)-3\cos^{10}(a+bx)+\cos^8(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{128\left(-\frac{1}{15}\cos^{15}(a+bx)+\frac{3}{13}\cos^{13}(a+bx)-\frac{3}{11}\cos^{11}(a+bx)+\frac{1}{9}\cos^9(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

output
```
(-128*(Cos[a + b*x]^9/9 - (3*Cos[a + b*x]^11)/11 + (3*Cos[a + b*x]^13)/13
- Cos[a + b*x]^15/15))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 25.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

method	result
parallelrisch	$\frac{262144+19305\cos(7bx+7a)-5005\cos(9bx+9a)+429\cos(15bx+15a)+495\cos(13bx+13a)-4095\cos(11bx+11a)-75075\cos(}{823680b}$
default	$-\frac{35\cos(bx+a)}{128b}-\frac{35\cos(3bx+3a)}{384b}+\frac{21\cos(5bx+5a)}{640b}+\frac{3\cos(7bx+7a)}{128b}-\frac{7\cos(9bx+9a)}{1152b}-\frac{7\cos(11bx+11a)}{1408b}+\cos$
risch	$-\frac{35\cos(bx+a)}{128b}-\frac{35\cos(3bx+3a)}{384b}+\frac{21\cos(5bx+5a)}{640b}+\frac{3\cos(7bx+7a)}{128b}-\frac{7\cos(9bx+9a)}{1152b}-\frac{7\cos(11bx+11a)}{1408b}+\cos$
orering	Expression too large to display

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/823680*(262144+19305*cos(7*b*x+7*a)-5005*cos(9*b*x+9*a)+429*cos(15*b*x+1
5*a)+495*cos(13*b*x+13*a)-4095*cos(11*b*x+11*a)-75075*cos(3*b*x+3*a)-22522
5*cos(b*x+a)+27027*cos(5*b*x+5*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int\cos(a+bx)\sin^7(2a+2bx)\,dx = \frac{128\left(429\cos\left(bx+a\right)^{15}-1485\cos\left(bx+a\right)^{13}+1755\cos\left(bx+a\right)^{11}-715\cos\left(bx+a\right)^{9}\right)}{6435\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

output
```
128/6435*(429*cos(b*x + a)^15 - 1485*cos(b*x + a)^13 + 1755*cos(b*x + a)^1
1 - 715*cos(b*x + a)^9)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(53) = 106$.

Time = 25.62 (sec) , antiderivative size = 270, normalized size of antiderivative = 4.43

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{1241\sin(a+bx)\sin^7(2a+2bx)}{6435b} - \frac{376\sin(a+bx)\sin^5(2a+2bx)\cos^2(2a+2bx)}{715b} - \frac{640\sin(a+bx)\sin^3(2a+2bx)\cos^4(2a+2bx)}{1287b} - \frac{1024\sin \\ x\sin^7(2a)\cos(a) \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**7,x)
```

output
```
Piecewise((-1241*sin(a + b*x)*sin(2*a + 2*b*x)**7/(6435*b) - 376*sin(a + b
*x)*sin(2*a + 2*b*x)**5*cos(2*a + 2*b*x)**2/(715*b) - 640*sin(a + b*x)*sin
(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**4/(1287*b) - 1024*sin(a + b*x)*sin(2*a
+ 2*b*x)*cos(2*a + 2*b*x)**6/(6435*b) - 3838*sin(2*a + 2*b*x)**6*cos(a + b
*x)*cos(2*a + 2*b*x)/(6435*b) - 1648*sin(2*a + 2*b*x)**4*cos(a + b*x)*cos(
2*a + 2*b*x)**3/(1287*b) - 768*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a +
2*b*x)**5/(715*b) - 2048*cos(a + b*x)*cos(2*a + 2*b*x)**7/(6435*b), Ne(b,
0)), (x*sin(2*a)**7*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx$$
$$= \frac{429\cos(15bx+15a) + 495\cos(13bx+13a) - 4095\cos(11bx+11a) - 5005\cos(9bx+9a) + 1930}{823680b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

output
```
1/823680*(429*cos(15*b*x + 15*a) + 495*cos(13*b*x + 13*a) - 4095*cos(11*b*
x + 11*a) - 5005*cos(9*b*x + 9*a) + 19305*cos(7*b*x + 7*a) + 27027*cos(5*b
*x + 5*a) - 75075*cos(3*b*x + 3*a) - 225225*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx$$
$$=\frac{128\left(429\cos\left(bx+a\right)^{15}-1485\cos\left(bx+a\right)^{13}+1755\cos\left(bx+a\right)^{11}-715\cos\left(bx+a\right)^{9}\right)}{6435\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

output
```
128/6435*(429*cos(b*x + a)^15 - 1485*cos(b*x + a)^13 + 1755*cos(b*x + a)^1
1 - 715*cos(b*x + a)^9)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx$$
$$=-\frac{-\frac{128\cos(a+b\,x)^{15}}{15}+\frac{384\cos(a+b\,x)^{13}}{13}-\frac{384\cos(a+b\,x)^{11}}{11}+\frac{128\cos(a+b\,x)^{9}}{9}}{b}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^7,x)
```

output
```
-((128*cos(a + b*x)^9)/9 - (384*cos(a + b*x)^11)/11 + (384*cos(a + b*x)^13
)/13 - (128*cos(a + b*x)^15)/15)/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.98

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx$$
$$= \frac{-16170\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)^6-19600\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)^4-}{}$$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^7,x)
```

output
```
( - 16170*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**6 - 19600*cos(2*
a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**4 - 26880*cos(2*a + 2*b*x)*cos(a
 + b*x)*sin(2*a + 2*b*x)**2 - 71680*cos(2*a + 2*b*x)*cos(a + b*x) - 1155*s
in(2*a + 2*b*x)**7*sin(a + b*x) - 1960*sin(2*a + 2*b*x)**5*sin(a + b*x) -
4480*sin(2*a + 2*b*x)**3*sin(a + b*x) - 35840*sin(2*a + 2*b*x)*sin(a + b*x
) - 54912)/(225225*b)
```

3.539 $\int \cos(a+bx)\sin^6(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx = \frac{64\sin^7(a+bx)}{7b} - \frac{64\sin^9(a+bx)}{3b} + \frac{192\sin^{11}(a+bx)}{11b} - \frac{64\sin^{13}(a+bx)}{13b}$$

output
```
64/7*sin(b*x+a)^7/b-64/3*sin(b*x+a)^9/b+192/11*sin(b*x+a)^11/b-64/13*sin(b
*x+a)^13/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx = \frac{2(5230+6377\cos(2(a+bx))+1890\cos(4(a+bx))+231\cos(6(a+bx)))\sin^7(a+bx)}{3003b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

output
```
(2*(5230 + 6377*Cos[2*(a + b*x)] + 1890*Cos[4*(a + b*x)] + 231*Cos[6*(a +
b*x)])*Sin[a + b*x]^7)/(3003*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4775, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^6\cos(a+bx)dx$$

$$\Big\downarrow \text{4775}$$

$$64\int \cos^7(a+bx)\sin^6(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$64\int \cos(a+bx)^7\sin(a+bx)^6dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{64\int \sin^6(a+bx)\left(1-\sin^2(a+bx)\right)^3d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{64\int \left(-\sin^{12}(a+bx)+3\sin^{10}(a+bx)-3\sin^8(a+bx)+\sin^6(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{64\left(-\frac{1}{13}\sin^{13}(a+bx)+\frac{3}{11}\sin^{11}(a+bx)-\frac{1}{3}\sin^9(a+bx)+\frac{1}{7}\sin^7(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

output
```
(64*(Sin[a + b*x]^7/7 - Sin[a + b*x]^9/3 + (3*Sin[a + b*x]^11)/11 - Sin[a
+ b*x]^13/13))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 14.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.33

method	result
parallelrisch	$\frac{60060\sin(bx+a)-15015\sin(3bx+3a)-9009\sin(5bx+5a)+2574\sin(7bx+7a)+2002\sin(9bx+9a)-273\sin(11bx+11a)-231\sin}{192192b}$
default	$\frac{5\sin(bx+a)}{16b}-\frac{5\sin(3bx+3a)}{64b}-\frac{3\sin(5bx+5a)}{64b}+\frac{3\sin(7bx+7a)}{224b}+\frac{\sin(9bx+9a)}{96b}-\frac{\sin(11bx+11a)}{704b}-\frac{\sin(13bx+13a}{832b}$
risch	$\frac{5\sin(bx+a)}{16b}-\frac{5\sin(3bx+3a)}{64b}-\frac{3\sin(5bx+5a)}{64b}+\frac{3\sin(7bx+7a)}{224b}+\frac{\sin(9bx+9a)}{96b}-\frac{\sin(11bx+11a)}{704b}-\frac{\sin(13bx+13a}{832b}$
orering	Expression too large to display

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

output
```
1/192192*(60060*sin(b*x+a)-15015*sin(3*b*x+3*a)-9009*sin(5*b*x+5*a)+2574*s
in(7*b*x+7*a)+2002*sin(9*b*x+9*a)-273*sin(11*b*x+11*a)-231*sin(13*b*x+13*a
))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx = -\frac{64\left(231\cos\left(bx+a\right)^{12}-567\cos\left(bx+a\right)^{10}+371\cos\left(bx+a\right)^{8}-5\cos\left(bx+a\right)^{6}-6\cos\left(bx+a\right)^{4}-}{3003\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

output
```
-64/3003*(231*cos(b*x + a)^12 - 567*cos(b*x + a)^10 + 371*cos(b*x + a)^8 -
 5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 16)*sin(b*x + a)
/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(53) = 106$.

Time = 11.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.82

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx = \begin{cases} \frac{835\sin(a+bx)\sin^6(2a+2bx)}{3003b} + \frac{2776\sin(a+bx)\sin^4(2a+2bx)\cos^2(2a+2bx)}{3003b} + \frac{2944\sin(a+bx)\sin^2(2a+2bx)\cos^4(2a+2bx)}{3003b} + \frac{1024\sin}{} \\ x\sin^6(2a)\cos(a) \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**6,x)
```

output
```
Piecewise((835*sin(a + b*x)*sin(2*a + 2*b*x)**6/(3003*b) + 2776*sin(a + b*
x)*sin(2*a + 2*b*x)**4*cos(2*a + 2*b*x)**2/(3003*b) + 2944*sin(a + b*x)*si
n(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**4/(3003*b) + 1024*sin(a + b*x)*cos(2*a
 + 2*b*x)**6/(3003*b) - 1084*sin(2*a + 2*b*x)**5*cos(a + b*x)*cos(2*a + 2*
b*x)/(3003*b) - 64*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**3/(1
43*b) - 512*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**5/(3003*b), Ne
(b, 0)), (x*sin(2*a)**6*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx = -\frac{231\sin(13bx+13a)+273\sin(11bx+11a)-2002\sin(9bx+9a)-2574\sin(7bx+7a)+9009\sin}{192192\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

output
```
-1/192192*(231*sin(13*b*x + 13*a) + 273*sin(11*b*x + 11*a) - 2002*sin(9*b*
x + 9*a) - 2574*sin(7*b*x + 7*a) + 9009*sin(5*b*x + 5*a) + 15015*sin(3*b*x
 + 3*a) - 60060*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx =$$
$$-\frac{231\,\sin\left(13\,bx+13\,a\right)+273\,\sin\left(11\,bx+11\,a\right)-2002\,\sin\left(9\,bx+9\,a\right)-2574\,\sin\left(7\,bx+7\,a\right)+9009\,\mathrm{s}}{192192\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

output
```
-1/192192*(231*sin(13*b*x + 13*a) + 273*sin(11*b*x + 11*a) - 2002*sin(9*b*
x + 9*a) - 2574*sin(7*b*x + 7*a) + 9009*sin(5*b*x + 5*a) + 15015*sin(3*b*x
 + 3*a) - 60060*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx = \frac{-\frac{64\sin(a+b\,x)^{13}}{13}+\frac{192\sin(a+b\,x)^{11}}{11}-\frac{64\sin(a+b\,x)^{9}}{3}+\frac{64\sin(a+b\,x)^{7}}{7}}{b}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^6,x)
```

output
```
((64*sin(a + b*x)^7)/7 - (64*sin(a + b*x)^9)/3 + (192*sin(a + b*x)^11)/11
- (64*sin(a + b*x)^13)/13)/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.51

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx$$

$$= \frac{-252\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)^5 - 320\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)^3 - 512\,\cdots}{}$$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^6,x)
```

output
```
( - 252*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**5 - 320*cos(2*a +
2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**3 - 512*cos(2*a + 2*b*x)*cos(a + b*x
)*sin(2*a + 2*b*x) - 21*sin(2*a + 2*b*x)**6*sin(a + b*x) - 40*sin(2*a + 2*
b*x)**4*sin(a + b*x) - 128*sin(2*a + 2*b*x)**2*sin(a + b*x) + 1024*sin(a +
 b*x))/(3003*b)
```

3.540 $\int \cos(a+bx)\sin^5(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{9b}-\frac{32\cos^{11}(a+bx)}{11b}$$

output
```
-32/7*cos(b*x+a)^7/b+64/9*cos(b*x+a)^9/b-32/11*cos(b*x+a)^11/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx = \frac{4\cos^7(a+bx)(-365+364\cos(2(a+bx))-63\cos(4(a+bx)))}{693b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

output
```
(4*Cos[a + b*x]^7*(-365 + 364*Cos[2*(a + b*x)] - 63*Cos[4*(a + b*x)]))/(69
3*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4775, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^5\cos(a+bx)dx$$

$$\Big\downarrow \text{4775}$$

$$32\int \cos^6(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$32\int \cos(a+bx)^6\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{32\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{32\int \left(\cos^{10}(a+bx)-2\cos^8(a+bx)+\cos^6(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{32\left(\frac{1}{11}\cos^{11}(a+bx)-\frac{2}{9}\cos^9(a+bx)+\frac{1}{7}\cos^7(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

output
```
(-32*(Cos[a + b*x]^7/7 - (2*Cos[a + b*x]^9)/9 + Cos[a + b*x]^11/11))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 7.85 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.54

method	result	size
parallelrisch	$\frac{8192+495\cos(7bx+7a)-77\cos(9bx+9a)-63\cos(11bx+11a)-2310\cos(3bx+3a)-6930\cos(bx+a)+693\cos(5bx+5a)}{22176b}$	71
default	$-\frac{5\cos(bx+a)}{16b}-\frac{5\cos(3bx+3a)}{48b}+\frac{\cos(5bx+5a)}{32b}+\frac{5\cos(7bx+7a)}{224b}-\frac{\cos(9bx+9a)}{288b}-\frac{\cos(11bx+11a)}{352b}$	83
risch	$-\frac{5\cos(bx+a)}{16b}-\frac{5\cos(3bx+3a)}{48b}+\frac{\cos(5bx+5a)}{32b}+\frac{5\cos(7bx+7a)}{224b}-\frac{\cos(9bx+9a)}{288b}-\frac{\cos(11bx+11a)}{352b}$	83
orering	Expression too large to display	894

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/22176*(8192+495*cos(7*b*x+7*a)-77*cos(9*b*x+9*a)-63*cos(11*b*x+11*a)-231
0*cos(3*b*x+3*a)-6930*cos(b*x+a)+693*cos(5*b*x+5*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\left(63\cos\left(bx+a\right)^{11}-154\cos\left(bx+a\right)^{9}+99\cos\left(bx+a\right)^{7}\right)}{693\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
-32/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(39) = 78.

Time = 4.79 (sec) , antiderivative size = 199, normalized size of antiderivative = 4.33

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx = \begin{cases} -\frac{151\sin(a+bx)\sin^5(2a+2bx)}{693b} - \frac{272\sin(a+bx)\sin^3(2a+2bx)\cos^2(2a+2bx)}{693b} - \frac{128\sin(a+bx)\sin(2a+2bx)\cos^4(2a+2bx)}{693b} - \frac{422\sin^4}{} \\ x\sin^5(2a)\cos(a) \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**5,x)
```

output
```
Piecewise((-151*sin(a + b*x)*sin(2*a + 2*b*x)**5/(693*b) - 272*sin(a + b*x
)*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**2/(693*b) - 128*sin(a + b*x)*sin(2
*a + 2*b*x)*cos(2*a + 2*b*x)**4/(693*b) - 422*sin(2*a + 2*b*x)**4*cos(a +
b*x)*cos(2*a + 2*b*x)/(693*b) - 608*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2
*a + 2*b*x)**3/(693*b) - 256*cos(a + b*x)*cos(2*a + 2*b*x)**5/(693*b), Ne(
b, 0)), (x*sin(2*a)**5*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx = -\frac{63\,\cos(11\,bx+11\,a)+77\,\cos(9\,bx+9\,a)-495\,\cos(7\,bx+7\,a)-693\,\cos(5\,bx+5\,a)+2310\,\cos(3}{22176\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
-1/22176*(63*cos(11*b*x + 11*a) + 77*cos(9*b*x + 9*a) - 495*cos(7*b*x + 7*
a) - 693*cos(5*b*x + 5*a) + 2310*cos(3*b*x + 3*a) + 6930*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\left(63\,\cos(bx+a)^{11}-154\,\cos(bx+a)^{9}+99\,\cos(bx+a)^{7}\right)}{693\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
-32/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\left(63\cos(a+b\,x)^{11}-154\cos(a+b\,x)^{9}+99\cos(a+b\,x)^{7}\right)}{693\,b}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^5,x)
```

output
```
-(32*(99*cos(a + b*x)^7 - 154*cos(a + b*x)^9 + 63*cos(a + b*x)^11))/(693*b
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.93

$$\int \cos(a+bx)\sin^5(2a+2bx)\,dx$$
$$= \frac{-350\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)^4 - 480\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)^2 - 128}{}$$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^5,x)
```

output
```
( - 350*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**4 - 480*cos(2*a +
2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**2 - 1280*cos(2*a + 2*b*x)*cos(a + b*
x) - 35*sin(2*a + 2*b*x)**5*sin(a + b*x) - 80*sin(2*a + 2*b*x)**3*sin(a +
b*x) - 640*sin(2*a + 2*b*x)*sin(a + b*x) - 1008)/(3465*b)
```

3.541 $\int \cos(a+bx)\sin^4(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx = \frac{16\sin^5(a+bx)}{5b} - \frac{32\sin^7(a+bx)}{7b} + \frac{16\sin^9(a+bx)}{9b}$$

output
```
16/5*sin(b*x+a)^5/b-32/7*sin(b*x+a)^7/b+16/9*sin(b*x+a)^9/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx = \frac{2(249+220\cos(2(a+bx))+35\cos(4(a+bx)))\sin^5(a+bx)}{315b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

output
```
(2*(249 + 220*Cos[2*(a + b*x)] + 35*Cos[4*(a + b*x)])*Sin[a + b*x]^5)/(315
*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4775, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^4\cos(a+bx)dx$$

$$\Big\downarrow \text{4775}$$

$$16\int \cos^5(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$16\int \cos(a+bx)^5\sin(a+bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{16\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{16\int \left(\sin^8(a+bx)-2\sin^6(a+bx)+\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16\left(\frac{1}{9}\sin^9(a+bx)-\frac{2}{7}\sin^7(a+bx)+\frac{1}{5}\sin^5(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

output
```
(16*(Sin[a + b*x]^5/5 - (2*Sin[a + b*x]^7)/7 + Sin[a + b*x]^9/9))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

method	result
default	$\frac{3\sin(bx+a)}{8b}-\frac{\sin(3bx+3a)}{12b}-\frac{\sin(5bx+5a)}{20b}+\frac{\sin(7bx+7a)}{112b}+\frac{\sin(9bx+9a)}{144b}$
risch	$\frac{3\sin(bx+a)}{8b}-\frac{\sin(3bx+3a)}{12b}-\frac{\sin(5bx+5a)}{20b}+\frac{\sin(7bx+7a)}{112b}+\frac{\sin(9bx+9a)}{144b}$
parallelrisch	$\frac{\left(-128\tan(bx+a)^{7}-448\tan(bx+a)^{5}+448\tan(bx+a)^{3}+128\tan(bx+a)\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+\left(256\tan(bx+a)^{8}+896\tan(bx+a)^{6}+\right.}{315b\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right.}$
orering	$-\frac{117469\left(-b\sin(bx+a)\sin(2bx+2a)^{4}+8\cos(bx+a)\sin(2bx+2a)^{3}b\cos(2bx+2a)\right)}{99225b^{2}}-\frac{34562\left(49b^{3}\sin(bx+a)\sin(2bx+2a)^{4}\right.}{}$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
3/8*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b-1/20/b*sin(5*b*x+5*a)+1/112/b*sin(7
*b*x+7*a)+1/144/b*sin(9*b*x+9*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx = \frac{16\left(35\cos\left(bx+a\right)^8-50\cos\left(bx+a\right)^6+3\cos\left(bx+a\right)^4+4\cos\left(bx+a\right)^2+8\right)\sin\left(bx+a\right)}{315\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
16/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b
*x + a)^2 + 8)*sin(b*x + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(39) = 78.

Time = 2.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.52

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx = \begin{cases} \frac{107\sin\left(a+bx\right)\sin^4\left(2a+2bx\right)}{315b}+\frac{16\sin\left(a+bx\right)\sin^2\left(2a+2bx\right)\cos^2\left(2a+2bx\right)}{21b}+\frac{128\sin\left(a+bx\right)\cos^4\left(2a+2bx\right)}{315b}-\frac{104\sin^3\left(2a+2bx\right)\cos\left(a+\right.}{315b} \\ x\sin^4\left(2a\right)\cos\left(a\right) \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**4,x)
```

output
```
Piecewise((107*sin(a + b*x)*sin(2*a + 2*b*x)**4/(315*b) + 16*sin(a + b*x)*
sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/(21*b) + 128*sin(a + b*x)*cos(2*a
+ 2*b*x)**4/(315*b) - 104*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x
)/(315*b) - 64*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**3/(315*b),
Ne(b, 0)), (x*sin(2*a)**4*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx = \frac{35\,\sin(9\,bx+9\,a)+45\,\sin(7\,bx+7\,a)-252\,\sin(5\,bx+5\,a)-420\,\sin(3\,bx+3\,a)+1890\,\sin(bx+a)}{5040\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/5040*(35*sin(9*b*x + 9*a) + 45*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) -
 420*sin(3*b*x + 3*a) + 1890*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx = \frac{35\,\sin(9\,bx+9\,a)+45\,\sin(7\,bx+7\,a)-252\,\sin(5\,bx+5\,a)-420\,\sin(3\,bx+3\,a)+1890\,\sin(bx+a)}{5040\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/5040*(35*sin(9*b*x + 9*a) + 45*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) -
 420*sin(3*b*x + 3*a) + 1890*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 20.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx$$
$$=\frac{16\,\left(35\sin\left(a+b\,x\right)^9-90\sin\left(a+b\,x\right)^7+63\sin\left(a+b\,x\right)^5\right)}{315\,b}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^4,x)
```

output
```
(16*(63*sin(a + b*x)^5 - 90*sin(a + b*x)^7 + 35*sin(a + b*x)^9))/(315*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.30

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx$$
$$=\frac{-40\cos\left(2bx+2a\right)\cos\left(bx+a\right)\sin\left(2bx+2a\right)^3-64\cos\left(2bx+2a\right)\cos\left(bx+a\right)\sin\left(2bx+2a\right)-5\sin\left(2b\right.}{315b}$$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^4,x)
```

output
```
( - 40*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**3 - 64*cos(2*a + 2*
b*x)*cos(a + b*x)*sin(2*a + 2*b*x) - 5*sin(2*a + 2*b*x)**4*sin(a + b*x) -
16*sin(2*a + 2*b*x)**2*sin(a + b*x) + 128*sin(a + b*x))/(315*b)
```

3.542 $\int \cos(a+bx)\sin^3(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\cos^5(a+bx)}{5b} + \frac{8\cos^7(a+bx)}{7b}$$

output
```
-8/5*cos(b*x+a)^5/b+8/7*cos(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx = \frac{4\cos^5(a+bx)(-9+5\cos(2(a+bx)))}{35b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

output
```
(4*Cos[a + b*x]^5*(-9 + 5*Cos[2*(a + b*x)]))/(35*b)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4775, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^3\cos(a+bx)dx$$

$$\Big\downarrow \text{4775}$$

$$8\int \cos^4(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(a+bx)^4\sin(a+bx)^3dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{8\int \cos^4(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{8\int \left(\cos^4(a+bx)-\cos^6(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8\left(\frac{1}{5}\cos^5(a+bx)-\frac{1}{7}\cos^7(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-8*(Cos[a + b*x]^5/5 - Cos[a + b*x]^7/7))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77

method	result
default	$-\frac{3\cos(bx+a)}{8b}-\frac{\cos(3bx+3a)}{8b}+\frac{\cos(5bx+5a)}{40b}+\frac{\cos(7bx+7a)}{56b}$
risch	$-\frac{3\cos(bx+a)}{8b}-\frac{\cos(3bx+3a)}{8b}+\frac{\cos(5bx+5a)}{40b}+\frac{\cos(7bx+7a)}{56b}$
parallelrisch	$\frac{\frac{8\left(\tan(bx+a)^4+11\tan(bx+a)^2+4\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{35}+\frac{16\left(-2\tan(bx+a)^5-5\tan(bx+a)^3-2\tan(bx+a)\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{35}+\frac{32\tan(bx+a)^6}{35}+\frac{88\,\mathrm{ta}}{}}{b\left(\tan(bx+a)^2+1\right)^3\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)}$
orering	$-\frac{12916\left(-b\sin(bx+a)\sin(2bx+2a)^3+6\cos(bx+a)\sin(2bx+2a)^2b\cos(2bx+2a)\right)}{11025b^2}-\frac{94\left(37b^3\sin(2bx+2a)^3\sin(bx+a)-186\right.}{}$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*cos(b*x+a)/b-1/8*cos(3*b*x+3*a)/b+1/40*cos(5*b*x+5*a)/b+1/56*cos(7*b*
x+7*a)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(5\,\cos\left(bx+a\right)^{7}-7\,\cos\left(bx+a\right)^{5}\right)}{35\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
8/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(26) = 52.

Time = 0.81 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.13

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{9\sin\left(a+bx\right)\sin^{3}\left(2a+2bx\right)}{35b}-\frac{8\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos^{2}\left(2a+2bx\right)}{35b}-\frac{22\sin^{2}\left(2a+2bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{35b}-\frac{16\cos\left(a+bx\right)c}{35} \\ x\sin^{3}\left(2a\right)\cos\left(a\right) \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**3,x)
```

output
```
Piecewise((-9*sin(a + b*x)*sin(2*a + 2*b*x)**3/(35*b) - 8*sin(a + b*x)*sin
(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/(35*b) - 22*sin(2*a + 2*b*x)**2*cos(a +
b*x)*cos(2*a + 2*b*x)/(35*b) - 16*cos(a + b*x)*cos(2*a + 2*b*x)**3/(35*b),
 Ne(b, 0)), (x*sin(2*a)**3*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx = \frac{5\cos(7\,bx+7\,a)+7\cos(5\,bx+5\,a)-35\cos(3\,bx+3\,a)-105\cos(bx+a)}{280\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/280*(5*cos(7*b*x + 7*a) + 7*cos(5*b*x + 5*a) - 35*cos(3*b*x + 3*a) - 105
*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(5\cos(bx+a)^7-7\cos(bx+a)^5\right)}{35\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
8/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(7\cos(a+b\,x)^5-5\cos(a+b\,x)^7\right)}{35\,b}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^3,x)
```

output
```
-(8*(7*cos(a + b*x)^5 - 5*cos(a + b*x)^7))/(35*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.84

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx$$
$$= \frac{-18\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)^2 - 48\cos(2bx+2a)\cos(bx+a) - 3\sin(2bx+2a)^3\sin(b}{105b}$$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^3,x)
```

output
```
( - 18*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**2 - 48*cos(2*a + 2*
b*x)*cos(a + b*x) - 3*sin(2*a + 2*b*x)**3*sin(a + b*x) - 24*sin(2*a + 2*b*
x)*sin(a + b*x) - 40)/(105*b)
```

3.543 $\int \cos(a+bx)\sin^2(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx=\frac{4\sin^3(a+bx)}{3b}-\frac{4\sin^5(a+bx)}{5b}$$

output
```
4/3*sin(b*x+a)^3/b-4/5*sin(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx=\frac{2(7+3\cos(2(a+bx)))\sin^3(a+bx)}{15b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

output
```
(2*(7 + 3*Cos[2*(a + b*x)])*Sin[a + b*x]^3)/(15*b)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4775, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^2\cos(a+bx)dx$$

$$\Big\downarrow \text{4775}$$

$$4\int \cos^3(a+bx)\sin^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \cos(a+bx)^3\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{4\int \sin^2(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{4\int \left(\sin^2(a+bx)-\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(\frac{1}{3}\sin^3(a+bx)-\frac{1}{5}\sin^5(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

output
```
(4*(Sin[a + b*x]^3/3 - Sin[a + b*x]^5/5))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

method	result
parallelrisch	$\frac{30\sin(bx+a)-5\sin(3bx+3a)-3\sin(5bx+5a)}{60b}$
default	$\frac{\sin(bx+a)}{2b}-\frac{\sin(3bx+3a)}{12b}-\frac{\sin(5bx+5a)}{20b}$
risch	$\frac{\sin(bx+a)}{2b}-\frac{\sin(3bx+3a)}{12b}-\frac{\sin(5bx+5a)}{20b}$
norman	$\frac{\frac{16\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{15b}-\frac{8\tan(bx+a)}{15b}+\frac{8\tan(bx+a)^3}{15b}+\frac{8\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan(bx+a)^2}{5b}+\frac{16\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan(bx+a)^4}{15b}+\frac{8\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan(bx+a)}{15b}-\cdots}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(\tan(bx+a)^2+1\right)^2}$
orering	$-\frac{259\left(-b\sin(2bx+2a)^2\sin(bx+a)+4\cos(2bx+2a)b\cos(bx+a)\sin(2bx+2a)\right)}{225b^2}-\frac{7\left(-24b^3\cos(2bx+2a)^2\sin(bx+a)-76b^3 \cdots\right.}{\cdots}$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*(30*sin(b*x+a)-5*sin(3*b*x+3*a)-3*sin(5*b*x+5*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(3\cos(bx+a)^4-\cos(bx+a)^2-2\right)\sin(bx+a)}{15\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-4/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(26) = 52.

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.90

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx$$
$$= \begin{cases} \frac{7\sin(a+bx)\sin^2(2a+2bx)}{15b} + \frac{8\sin(a+bx)\cos^2(2a+2bx)}{15b} - \frac{4\sin(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{15b} & \text{for } b \neq 0 \\ x\sin^2(2a)\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((7*sin(a + b*x)*sin(2*a + 2*b*x)**2/(15*b) + 8*sin(a + b*x)*cos(
2*a + 2*b*x)**2/(15*b) - 4*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)/
(15*b), Ne(b, 0)), (x*sin(2*a)**2*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx$$
$$= -\frac{3\,\sin\left(5\,bx+5\,a\right)+5\,\sin\left(3\,bx+3\,a\right)-30\,\sin\left(bx+a\right)}{60\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
-1/60*(3*sin(5*b*x + 5*a) + 5*sin(3*b*x + 3*a) - 30*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx$$
$$= -\frac{3\,\sin\left(5\,bx+5\,a\right)+5\,\sin\left(3\,bx+3\,a\right)-30\,\sin\left(bx+a\right)}{60\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-1/60*(3*sin(5*b*x + 5*a) + 5*sin(3*b*x + 3*a) - 30*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx = \frac{4\left(5\sin\left(a+b\,x\right)^3-3\sin\left(a+b\,x\right)^5\right)}{15\,b}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^2,x)
```

output
```
(4*(5*sin(a + b*x)^3 - 3*sin(a + b*x)^5))/(15*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \cos(a+bx)\sin^2(2a+2bx)\,dx$$
$$=\frac{-4\cos\left(2bx+2a\right)\cos\left(bx+a\right)\sin\left(2bx+2a\right)-\sin\left(2bx+2a\right)^2\sin\left(bx+a\right)+8\sin\left(bx+a\right)}{15b}$$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^2,x)
```

output
```
( - 4*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x) - sin(2*a + 2*b*x)**2
*sin(a + b*x) + 8*sin(a + b*x))/(15*b)
```

3.544 $\int \cos(a+bx)\sin(2a+2bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \cos(a+bx)\sin(2a+2bx)\,dx=-\frac{\cos(a+bx)}{2b}-\frac{\cos(3a+3bx)}{6b}$$

output
```
-1/2*cos(b*x+a)/b-1/6*cos(3*b*x+3*a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.50

$$\int \cos(a+bx)\sin(2a+2bx)\,dx=-\frac{2\cos^3(a+bx)}{3b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x],x]
```

output
```
(-2*Cos[a + b*x]^3)/(3*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)\cos(a+bx)dx$$

$$\Big\downarrow \text{4772}$$

$$-\frac{\cos(a+bx)}{2b}-\frac{\cos(3a+3bx)}{6b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x],x]
```

output
```
-1/2*Cos[a + b*x]/b - Cos[3*a + 3*b*x]/(6*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\cos(bx+a)}{2b}-\frac{\cos(3bx+3a)}{6b}$
risch	$-\frac{\cos(bx+a)}{2b}-\frac{\cos(3bx+3a)}{6b}$
parallelrisch	$\frac{4\tan(bx+a)^2-4\tan(bx+a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{3b\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(\tan(bx+a)^2+1\right)}$
norman	$\frac{-\frac{4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan(bx+a)}{3b}+\frac{4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{3b}+\frac{4\tan(bx+a)^2}{3b}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)\left(\tan(bx+a)^2+1\right)}$
orering	$-\frac{10(-b\sin(bx+a)\sin(2bx+2a)+2\cos(bx+a)b\cos(2bx+2a))}{9b^2}-\frac{13b^3\sin(bx+a)\sin(2bx+2a)-14b^3\cos(bx+a)\cos(2bx+2a)}{9b^4}$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(b*x+a)/b-1/6*cos(3*b*x+3*a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.43

$$\int \cos(a+bx)\sin(2a+2bx)\,dx=-\frac{2\,\cos\left(bx+a\right)^3}{3\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-2/3*cos(b*x + a)^3/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(24) = 48$.

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int \cos(a+bx)\sin(2a+2bx)\,dx = \begin{cases} -\frac{\sin(a+bx)\sin(2a+2bx)}{3b} - \frac{2\cos(a+bx)\cos(2a+2bx)}{3b} & \text{for } b \neq 0 \\ x\sin(2a)\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a),x)
```

output
```
Piecewise((-sin(a + b*x)*sin(2*a + 2*b*x)/(3*b) - 2*cos(a + b*x)*cos(2*a +
 2*b*x)/(3*b), Ne(b, 0)), (x*sin(2*a)*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin(2a+2bx)\,dx = -\frac{\cos(3\,bx+3\,a)}{6\,b} - \frac{\cos(bx+a)}{2\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/6*cos(3*b*x + 3*a)/b - 1/2*cos(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.43

$$\int \cos(a+bx)\sin(2a+2bx)\,dx = -\frac{2\,\cos(bx+a)^3}{3\,b}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
-2/3*cos(b*x + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 19.66 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \cos(a+bx)\sin(2a+2bx)\,dx = \begin{cases} x\,\left(2\,\sin\left(a\right)-2\,{\sin\left(a\right)}^3\right) & \text{if}\ \ b=0\\ -\frac{3\,\cos\left(a+b\,x\right)+\cos\left(3\,a+3\,b\,x\right)}{6\,b} & \text{if}\ \ b\neq 0 \end{cases}$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x),x)
```

output
```
piecewise(b == 0, x*(2*sin(a) - 2*sin(a)^3), b ~= 0, -(3*cos(a + b*x) + co
s(3*a + 3*b*x))/(6*b))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \cos(a+bx)\sin(2a+2bx)\,dx = \frac{-2\cos\left(2bx+2a\right)\cos\left(bx+a\right)-\sin\left(2bx+2a\right)\sin\left(bx+a\right)}{3b}$$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a),x)
```

output
```
( - 2*cos(2*a + 2*b*x)*cos(a + b*x) - sin(2*a + 2*b*x)*sin(a + b*x))/(3*b)
```

3.545 $\int \cos(a+bx)\csc(2a+2bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \cos(a+bx)\csc(2a+2bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{2b}$$

output
```
-1/2*arctanh(cos(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\csc(2a+2bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{2b}$$

input
```
Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x],x]
```

output
```
-1/2*ArcTanh[Cos[a + b*x]]/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4775, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{2}\int \csc(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc(a+bx)dx$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\text{arctanh}(\cos(a+bx))}{2b}$$

input
```
Int[Cos[a + b*x]*Csc[2*a + 2*b*x],x]
```

output
```
-1/2*ArcTanh[Cos[a + b*x]]/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

method	result	size
default	$\frac{\ln(\csc(bx+a)-\cot(bx+a))}{2b}$	22
risch	$\frac{\ln\left(e^{i(bx+a)}-1\right)}{2b}-\frac{\ln\left(e^{i(bx+a)}+1\right)}{2b}$	36

input
```
int(cos(b*x+a)*csc(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*ln(csc(b*x+a)-cot(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \cos(a+bx)\csc(2a+2bx)\,dx=-\frac{\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)}{4\,b}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-1/4*(log(1/2*cos(b*x + a) + 1/2) - log(-1/2*cos(b*x + a) + 1/2))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \cos(a+bx)\csc(2a+2bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 6.00

$$\int \cos(a+bx)\csc(2a+2bx)\,dx = -\frac{\log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right) - \log\left(\cos(bx)^2\right.}{4b}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/4*(log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b
*x)*sin(a) + sin(a)^2) - log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + s
in(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \cos(a+bx)\csc(2a+2bx)\,dx = -\frac{\log\left(\cos\left(bx+a\right)+1\right)-\log\left(-\cos\left(bx+a\right)+1\right)}{4\,b}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a),x, algorithm="giac")
```

output
```
-1/4*(log(cos(b*x + a) + 1) - log(-cos(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 19.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \cos(a+bx)\csc(2a+2bx)\,dx = -\frac{\operatorname{atanh}(\cos\left(a+b\,x\right))}{2\,b}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x),x)
```

output
```
-atanh(cos(a + b*x))/(2*b)
```

Reduce [F]

$$\int \cos(a+bx)\csc(2a+2bx)\,dx = \int \cos\left(bx+a\right)\csc\left(2bx+2a\right)dx$$

input
```
int(cos(b*x+a)*csc(2*b*x+2*a),x)
```

output
```
int(cos(a + b*x)*csc(2*a + 2*b*x),x)
```

3.546 $\int \cos(a+bx)\csc^2(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{4b} - \frac{\csc(a+bx)}{4b}$$

output
```
1/4*arctanh(sin(b*x+a))/b-1/4*csc(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(a+bx)\right)}{4b}$$

input
```
Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/4*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4775, 3042, 3101, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \; 4775$$

$$\frac{1}{4}\int \csc^2(a+bx)\sec(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int \csc(a+bx)^2\sec(a+bx)dx$$

$$\Big\downarrow \; 3101$$

$$-\frac{\int -\frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{4b}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{4b}$$

$$\Big\downarrow \; 262$$

$$-\frac{\csc(a+bx)-\int \frac{1}{1-\csc^2(a+bx)}d\csc(a+bx)}{4b}$$

$$\Big\downarrow \; 219$$

$$-\frac{\csc(a+bx)-\text{arctanh}(\csc(a+bx))}{4b}$$

input
```
Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/4*(-ArcTanh[Csc[a + b*x]] + Csc[a + b*x])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{-\frac{1}{\sin(bx+a)}+\ln(\sec(bx+a)+\tan(bx+a))}{4b}$	31
risch	$-\frac{i e^{i(bx+a)}}{2b\left(e^{2i(bx+a)}-1\right)}+\frac{\ln\left(e^{i(bx+a)}+i\right)}{4b}-\frac{\ln\left(e^{i(bx+a)}-i\right)}{4b}$	66

input
```
int(cos(b*x+a)*csc(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/b*(-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(24) = 48.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = \frac{\log(\sin(bx+a)+1)\sin(bx+a)-\log(-\sin(bx+a)+1)\sin(bx+a)-2}{8\,b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
1/8*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
 a) - 2)/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(24) = 48.

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 8.32

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\left(\cos\left(2\,bx+2\,a\right)^2+\sin\left(2\,bx+2\,a\right)^2-2\,\cos\left(2\,bx+2\,a\right)+1\right)\log\left(\frac{\cos(bx+2\,a)^2+\cos(a)^2-2\,\cos(a)\sin(bx+2\,a)+\mathrm{si}}{\cos(bx+2\,a)^2+\cos(a)^2+2\,\cos(a)\sin(bx+2\,a)+\mathrm{si}}\right.}{8\left(b\cos\left(2\,bx+2\,a\right)^2+b\,\mathrm{sin}\right.}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
-1/8*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*l
og((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)
^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*
cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a
)^2)) + 4*cos(b*x + a)*sin(2*b*x + 2*a) - 4*cos(2*b*x + 2*a)*sin(b*x + a)
+ 4*sin(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2
*b*x + 2*a) + b)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx$$
$$= -\frac{\frac{2}{\sin(bx+a)} - \log(\sin(bx+a)+1) + \log(-\sin(bx+a)+1)}{8\,b}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-1/8*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 19.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = \frac{\operatorname{atanh}(\sin(a+b\,x))}{4\,b} - \frac{1}{4\,b\,\sin(a+b\,x)}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^2,x)
```

output
```
atanh(sin(a + b*x))/(4*b) - 1/(4*b*sin(a + b*x))
```

Reduce [F]

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = \int \cos(bx+a)\csc(2bx+2a)^2\,dx$$

input
```
int(cos(b*x+a)*csc(2*b*x+2*a)^2,x)
```

output
```
int(cos(a + b*x)*csc(2*a + 2*b*x)**2,x)
```

3.547 $\int \cos(a+bx)\csc^3(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx = -\frac{3\text{arctanh}(\cos(a+bx))}{16b} - \frac{\cot(a+bx)\csc(a+bx)}{16b} + \frac{\sec(a+bx)}{8b}$$

output
```
-3/16*arctanh(cos(b*x+a))/b-1/16*cot(b*x+a)*csc(b*x+a)/b+1/8*sec(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. $2(47) = 94$.

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.04

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx = \frac{\csc^4(a+bx)\left(2-6\cos(2(a+bx))+2\cos(3(a+bx))+3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-3\cos(3\cdots\right.}{16b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)\cdots\right.}$$

input
```
Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

output
```
(Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a
+ b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] +
 Cos[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(
16*b*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4775, 3042, 3102, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{8}\int \csc^3(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)^3\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int \frac{\sec^4(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)}{8b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\int \frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{8b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\sec^2(a+bx)}d\sec(a+bx)-\sec(a+bx)\right)}{8b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}(\operatorname{arctanh}(\sec(a+bx))-\sec(a+bx))}{8b}$$

input
```
Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

output
```
((-3*(ArcTanh[Sec[a + b*x]] - Sec[a + b*x]))/2 + Sec[a + b*x]^3/(2*(1 - Se
c[a + b*x]^2)))/(8*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{-\frac{1}{2 \sin(bx+a)^2 \cos(bx+a)}+\frac{3}{2 \cos(bx+a)}+\frac{3 \ln(\csc(bx+a)-\cot(bx+a))}{2}}{8b}$	53
risch	$\frac{3 \,\mathrm{e}^{5i(bx+a)}-2 \,\mathrm{e}^{3i(bx+a)}+3 \,\mathrm{e}^{i(bx+a)}}{8b\left(\mathrm{e}^{2i(bx+a)}-1\right)^2 \left(\mathrm{e}^{2i(bx+a)}+1\right)}+\frac{3 \ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{16b}-\frac{3 \ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{16b}$	101

input
```
int(cos(b*x+a)*csc(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-1/2/sin(b*x+a)^2/cos(b*x+a)+3/2/cos(b*x+a)+3/2*ln(csc(b*x+a)-cot(b
*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(41) = 82$.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.04

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx$$
$$= \frac{6\cos(bx+a)^2 - 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) + 3\left(\cos(bx+a)^3 - \cos(bx+\right.}{32\left(b\cos(bx+a)^3 - b\cos(bx+a)\right)}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
1/32*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x
 + a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1
/2) - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(41) = 82$.

Time = 0.05 (sec) , antiderivative size = 974, normalized size of antiderivative = 20.72

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/32*(4*(3*cos(5*b*x + 5*a) - 2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*b
*x + 6*a) - 12*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a)
+ 4*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 8*(cos(2*b*x
+ 2*a) - 1)*cos(3*b*x + 3*a) - 12*cos(2*b*x + 2*a)*cos(b*x + a) + 3*(2*(co
s(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)
^2 - 2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(
2*b*x + 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a)
- sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*co
s(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*
(2*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x
+ 6*a)^2 - 2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2
- cos(2*b*x + 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x +
 6*a) - sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2
- 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2
) + 4*(3*sin(5*b*x + 5*a) - 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(6*b*x
 + 6*a) - 12*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 4*(2
*sin(3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 8*sin(3*b*x + 3*a)*
sin(2*b*x + 2*a) - 12*sin(2*b*x + 2*a)*sin(b*x + a) + 12*cos(b*x + a))/...
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx = \frac{\frac{2\left(3\cos(bx+a)^2-2\right)}{\cos(bx+a)^3-\cos(bx+a)} - 3\log\left(\cos\left(bx+a\right)+1\right) + 3\log\left(-\cos\left(bx+a\right)+1\right)}{32\,b}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
1/32*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos
(b*x + a) + 1) + 3*log(-cos(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx = -\frac{3\,\mathrm{atanh}(\cos(a+b\,x))}{16\,b} - \frac{\frac{3\cos(a+b\,x)^2}{16}-\frac{1}{8}}{b\,\left(\cos(a+b\,x)-\cos(a+b\,x)^3\right)}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^3,x)
```

output
```
- (3*atanh(cos(a + b*x)))/(16*b) - ((3*cos(a + b*x)^2)/16 - 1/8)/(b*(cos(a
 + b*x) - cos(a + b*x)^3))
```

Reduce [F]

$$\int \cos(a+bx)\csc^3(2a+2bx)\,dx$$

$$= \frac{-12\cos(2bx+2a)\cos(bx+a)+2\cos(2bx+2a)\sin(2bx+2a)\sin(bx+a)-4\cos(2bx+2a)-4\cos(}{}$$

input
```
int(cos(b*x+a)*csc(2*b*x+2*a)^3,x)
```

output
```
( - 12*cos(2*a + 2*b*x)*cos(a + b*x) + 2*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)
*sin(a + b*x) - 4*cos(2*a + 2*b*x) - 4*cos(a + b*x) + 12*int(tan(a + b*x)/
(tan((a + b*x)/2)**2 + 1),x)*sin(2*a + 2*b*x)**2*b - 8*int(1/(tan(a + b*x)
**3*tan((a + b*x)/2)**2 + tan(a + b*x)**3),x)*sin(2*a + 2*b*x)**2*b - log(
tan(a + b*x)**2 + 1)*sin(2*a + 2*b*x)**2 - 4*log(tan(a + b*x))*sin(2*a + 2
*b*x)**2 + 4*sin(2*a + 2*b*x)**2 + 6*sin(2*a + 2*b*x)*sin(a + b*x) - 4)/(3
2*sin(2*a + 2*b*x)**2*b)
```

3.548 $\int \cos(a+bx)\csc^4(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx=\frac{5\text{arctanh}(\sin(a+bx))}{32b}-\frac{\csc(a+bx)}{8b}-\frac{\csc^3(a+bx)}{48b}+\frac{\sec(a+bx)\tan(a+bx)}{32b}$$

output
```
5/32*arctanh(sin(b*x+a))/b-1/8*csc(b*x+a)/b-1/48*csc(b*x+a)^3/b+1/32*sec(b
*x+a)*tan(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.50

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx=-\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},\sin^2(a+bx)\right)}{48b}$$

input
```
Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

output
```
-1/48*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4775, 3042, 3101, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{16}\int \csc^4(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^4\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int \frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)}{16b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{16b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \left(-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)}{16b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\left(\operatorname{arctanh}(\csc(a+bx))-\frac{1}{3}\csc^3(a+bx)-\csc(a+bx)\right)}{16b}$$

input
```
Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

output
```
-1/16*(Csc[a + b*x]^5/(2*(1 - Csc[a + b*x]^2)) - (5*(ArcTanh[Csc[a + b*x]]
 - Csc[a + b*x] - Csc[a + b*x]^3/3))/2)/b
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{-\frac{1}{3\sin(bx+a)^3\cos(bx+a)^2}+\frac{5}{6\sin(bx+a)\cos(bx+a)^2}-\frac{5}{2\sin(bx+a)}+\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{2}}{16b}$	69
risch	$-\frac{i\left(15\,\mathrm{e}^{9i(bx+a)}-20\,\mathrm{e}^{7i(bx+a)}-22\,\mathrm{e}^{5i(bx+a)}-20\,\mathrm{e}^{3i(bx+a)}+15\,\mathrm{e}^{i(bx+a)}\right)}{48b\left(\mathrm{e}^{2i(bx+a)}-1\right)^3\left(\mathrm{e}^{2i(bx+a)}+1\right)^2}+\frac{5\ln\left(\mathrm{e}^{i(bx+a)}+i\right)}{32b}-\frac{5\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{32b}$	126

input
```
int(cos(b*x+a)*csc(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)^2+5/6/sin(b*x+a)/cos(b*x+a)^2-5/2/sin
(b*x+a)+5/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(54) = 108.

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.10

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx = -\frac{30\cos(bx+a)^4-15\left(\cos(bx+a)^4-\cos(bx+a)^2\right)\log(\sin(bx+a)+1)\sin(bx+a)+15\left(\cos(bx+\right.}{192\left(b\cos(bx+a)^4-b\cos(bx+a)\right.}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
-1/192*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b
*x + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(
b*x + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b
*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1780 vs. $2(54) = 108$.

Time = 0.20 (sec) , antiderivative size = 1780, normalized size of antiderivative = 28.71

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/192*(4*(15*sin(9*b*x + 9*a) - 20*sin(7*b*x + 7*a) - 22*sin(5*b*x + 5*a)
- 20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) + 60*(sin(8*b*
x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos
(9*b*x + 9*a) + 4*(20*sin(7*b*x + 7*a) + 22*sin(5*b*x + 5*a) + 20*sin(3*b*
x + 3*a) - 15*sin(b*x + a))*cos(8*b*x + 8*a) - 80*(2*sin(6*b*x + 6*a) - 2*
sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(22*sin(5*b*x +
5*a) + 20*sin(3*b*x + 3*a) - 15*sin(b*x + a))*cos(6*b*x + 6*a) + 88*(2*sin
(4*b*x + 4*a) + sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(4*sin(3*b*x + 3*a
) - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 15*(2*(cos(8*b*x + 8*a) + 2*cos(6*b
*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a)
- cos(10*b*x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(
2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 4*(2*cos(4*b*x +
 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 - 4*
(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 - cos(2*b*x
 + 2*a)^2 + 2*(sin(8*b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a)
- sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^2 - 2*(2*sin(6
*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - si
n(8*b*x + 8*a)^2 + 4*(2*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6
*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log((cos(b...
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx = -\frac{\frac{6\sin(bx+a)}{\sin(bx+a)^2-1}+\frac{4\left(6\sin(bx+a)^2+1\right)}{\sin(bx+a)^3}-15\log\left(\sin\left(bx+a\right)+1\right)+15\log\left(-\sin\left(bx+a\right)+1\right)}{192\,b}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
-1/192*(6*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(6*sin(b*x + a)^2 + 1)/sin
(b*x + a)^3 - 15*log(sin(b*x + a) + 1) + 15*log(-sin(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx = \frac{5\,\mathrm{atanh}(\sin(a+b\,x))}{32\,b} - \frac{-\frac{5\sin(a+b\,x)^4}{32}+\frac{5\sin(a+b\,x)^2}{48}+\frac{1}{48}}{b\,\left(\sin(a+b\,x)^3-\sin(a+b\,x)^5\right)}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^4,x)
```

output
```
(5*atanh(sin(a + b*x)))/(32*b) - ((5*sin(a + b*x)^2)/48 - (5*sin(a + b*x)^
4)/32 + 1/48)/(b*(sin(a + b*x)^3 - sin(a + b*x)^5))
```

Reduce [F]

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx$$

$$= \frac{-88\cos(2bx+2a)\cos(bx+a)+85\cos(2bx+2a)\sin(2bx+2a)^2+10\cos(2bx+2a)\sin(2bx+2a)\sin}{}$$

input
```
int(cos(b*x+a)*csc(2*b*x+2*a)^4,x)
```

output
```
( - 88*cos(2*a + 2*b*x)*cos(a + b*x) + 85*cos(2*a + 2*b*x)*sin(2*a + 2*b*x
)**2 + 10*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)*sin(a + b*x) - 40*cos(2*a + 2*
b*x) + 20*cos(a + b*x)*sin(2*a + 2*b*x)**2 - 40*cos(a + b*x) - 90*int((tan
(a + b*x)**2*tan((a + b*x)/2)**2)/(tan((a + b*x)/2)**2 + 1),x)*sin(2*a + 2
*b*x)**3*b - 60*int(1/(tan(a + b*x)**4*tan((a + b*x)/2)**2 + tan(a + b*x)*
*4),x)*sin(2*a + 2*b*x)**3*b + 75*sin(2*a + 2*b*x)**3*sin(a + b*x) + 90*si
n(2*a + 2*b*x)**3*tan(a + b*x) - 15*sin(2*a + 2*b*x)**3*b*x + 15*sin(2*a +
 2*b*x)**2 + 22*sin(2*a + 2*b*x)*sin(a + b*x) - 40)/(288*sin(2*a + 2*b*x)*
*3*b)
```

3.549 $\int \cos(a+bx)\csc^5(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = -\frac{35\text{arctanh}(\cos(a+bx))}{256b} - \frac{13\cot(a+bx)\csc(a+bx)}{256b} - \frac{\cot^3(a+bx)\csc(a+bx)}{128b} + \frac{3\sec(a+bx)}{32b} + \frac{\sec^3(a+bx)}{96b}$$

output
```
-35/256*arctanh(cos(b*x+a))/b-13/256*cot(b*x+a)*csc(b*x+a)/b-1/128*cot(b*x
+a)^3*csc(b*x+a)/b+3/32*sec(b*x+a)/b+1/96*sec(b*x+a)^3/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 268 vs. 2(83) = 166.

Time = 0.46 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.23

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = -\frac{\csc^{10}(a+bx)\left(-204+658\cos(2(a+bx))-228\cos(3(a+bx))+140\cos(4(a+bx))-76\cos(5(a+b\right.}{}$$

input
```
Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^5,x]
```

output
```
-1/768*(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)
] + 140*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76
*Cos[7*(a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5
*(a + b*x)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)
/2]] + 3*Cos[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b
*x)/2]]) + 315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x
)]*Log[Sin[(a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(b
*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4775, 3042, 3102, 25, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{32}\int \csc^5(a+bx)\sec^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \csc(a+bx)^5\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^8(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{32b}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$-\frac{\int \frac{\sec^8(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{32b}$$

$$\Big\downarrow \textcolor{blue}{252}$$

$$\frac{\frac{7}{4}\int \frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \textcolor{blue}{252}$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \textcolor{blue}{254}$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \left(-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\left(\text{arctanh}(\sec(a+bx))-\frac{1}{3}\sec^3(a+bx)-\sec(a+bx)\right)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

input
```
Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^5,x]
```

output
```
(-1/4*Sec[a + b*x]^7/(1 - Sec[a + b*x]^2)^2 + (7*(Sec[a + b*x]^5/(2*(1 - S
ec[a + b*x]^2)) - (5*(ArcTanh[Sec[a + b*x]] - Sec[a + b*x] - Sec[a + b*x]^
3/3))/2))/4)/(32*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 5.78 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

method	result
default	$\frac{-\frac{1}{4\sin(bx+a)^4\cos(bx+a)^3}+\frac{7}{12\sin(bx+a)^2\cos(bx+a)^3}-\frac{35}{24\sin(bx+a)^2\cos(bx+a)}+\frac{35}{8\cos(bx+a)}+\frac{35\ln(\csc(bx+a)-\cot(bx+a))}{8}}{32b}$
risch	$\frac{105\,e^{13i(bx+a)}-70\,e^{11i(bx+a)}-329\,e^{9i(bx+a)}+204\,e^{7i(bx+a)}-329\,e^{5i(bx+a)}-70\,e^{3i(bx+a)}+105\,e^{i(bx+a)}}{384b\left(e^{2i(bx+a)}-1\right)^4\left(e^{2i(bx+a)}+1\right)^3}-\frac{35\ln\left(e^{i(bx+a)}+1\right)}{256b}+$

input
```
int(cos(b*x+a)*csc(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^3+7/12/sin(b*x+a)^2/cos(b*x+a)^3-35/2
4/sin(b*x+a)^2/cos(b*x+a)+35/8/cos(b*x+a)+35/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(73) = 146.

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.78

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx$$
$$=\frac{210\cos(bx+a)^6-350\cos(bx+a)^4+112\cos(bx+a)^2-105\left(\cos(bx+a)^7-2\cos(bx+a)^5+\cos(\right.}{1536\left(b\cos(bx+a)^7\right.}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
1/1536*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105
*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a)
 + 1/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/
2*cos(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos
(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3846 vs. $2(73) = 146$.

Time = 0.15 (sec) , antiderivative size = 3846, normalized size of antiderivative = 46.34

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/1536*(4*(105*cos(13*b*x + 13*a) - 70*cos(11*b*x + 11*a) - 329*cos(9*b*x
+ 9*a) + 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a)
 + 105*cos(b*x + a))*cos(14*b*x + 14*a) - 420*(cos(12*b*x + 12*a) + 3*cos(
10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4
*a) + cos(2*b*x + 2*a) - 1)*cos(13*b*x + 13*a) + 4*(70*cos(11*b*x + 11*a)
+ 329*cos(9*b*x + 9*a) - 204*cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*
cos(3*b*x + 3*a) - 105*cos(b*x + a))*cos(12*b*x + 12*a) + 280*(3*cos(10*b*
x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) +
 cos(2*b*x + 2*a) - 1)*cos(11*b*x + 11*a) + 12*(329*cos(9*b*x + 9*a) - 204
*cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*b*x + 3*a) - 105*cos(b
*x + a))*cos(10*b*x + 10*a) - 1316*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a
) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(9*b*x + 9*a) + 12*(204*
cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) + 105*cos(b*
x + a))*cos(8*b*x + 8*a) + 816*(3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) -
cos(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) - 84*(47*cos(5*b*x + 5*a) + 10*cos(
3*b*x + 3*a) - 15*cos(b*x + a))*cos(6*b*x + 6*a) + 1316*(3*cos(4*b*x + 4*a
) + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) + 420*(2*cos(3*b*x + 3*a) - 3*c
os(b*x + a))*cos(4*b*x + 4*a) + 280*(cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a
) - 420*cos(2*b*x + 2*a)*cos(b*x + a) + 105*(2*(cos(12*b*x + 12*a) + 3*cos
(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = \frac{\frac{6\left(11\cos(bx+a)^3-13\cos(bx+a)\right)}{\left(\cos(bx+a)^2-1\right)^2}+\frac{16\left(9\cos(bx+a)^2+1\right)}{\cos(bx+a)^3}-105\log\left(\cos\left(bx+a\right)+1\right)+105\log\left(-\cos\left(bx+a\right)+1\right)}{1536\,b}$$

input
```
integrate(cos(b*x+a)*csc(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
1/1536*(6*(11*cos(b*x + a)^3 - 13*cos(b*x + a))/(cos(b*x + a)^2 - 1)^2 + 1
6*(9*cos(b*x + a)^2 + 1)/cos(b*x + a)^3 - 105*log(cos(b*x + a) + 1) + 105*
log(-cos(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 19.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = \frac{\frac{35\cos(a+b\,x)^6}{256}-\frac{175\cos(a+b\,x)^4}{768}+\frac{7\cos(a+b\,x)^2}{96}+\frac{1}{96}}{b\,\left(\cos\left(a+b\,x\right)^7-2\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)} - \frac{35\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{256\,b}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^5,x)
```

output
```
((7*cos(a + b*x)^2)/96 - (175*cos(a + b*x)^4)/768 + (35*cos(a + b*x)^6)/25
6 + 1/96)/(b*(cos(a + b*x)^3 - 2*cos(a + b*x)^5 + cos(a + b*x)^7)) - (35*a
tanh(cos(a + b*x)))/(256*b)
```

Reduce [F]

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx$$

$$= \frac{-1540\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)^2+96\cos(2bx+2a)\cos(bx+a)+294\cos(2bx+2a)\mathrm{s}}{}$$

input
```
int(cos(b*x+a)*csc(2*b*x+2*a)^5,x)
```

output
```
( - 1540*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**2 + 96*cos(2*a +
2*b*x)*cos(a + b*x) + 294*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**3*sin(a + b*x
) - 672*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**2 - 112*cos(2*a + 2*b*x)*sin(2*
a + 2*b*x)*sin(a + b*x) + 672*cos(2*a + 2*b*x) - 1036*cos(a + b*x)*sin(2*a
 + 2*b*x)**2 + 672*cos(a + b*x) + 1260*int(tan(a + b*x)/(tan((a + b*x)/2)*
*2 + 1),x)*sin(2*a + 2*b*x)**4*b + 672*int(1/(tan(a + b*x)**5*tan((a + b*x
)/2)**2 + tan(a + b*x)**5),x)*sin(2*a + 2*b*x)**4*b - 147*log(tan(a + b*x)
**2 + 1)*sin(2*a + 2*b*x)**4 - 336*log(tan(a + b*x))*sin(2*a + 2*b*x)**4 +
 518*sin(2*a + 2*b*x)**4 + 770*sin(2*a + 2*b*x)**3*sin(a + b*x) - 1008*sin
(2*a + 2*b*x)**2 - 16*sin(2*a + 2*b*x)*sin(a + b*x) + 672)/(4608*sin(2*a +
 2*b*x)**4*b)
```

3.550 $\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx=-\frac{4\cos^8(a+bx)}{b}+\frac{32\cos^{10}(a+bx)}{5b}-\frac{8\cos^{12}(a+bx)}{3b}$$

output
```
-4*cos(b*x+a)^8/b+32/5*cos(b*x+a)^10/b-8/3*cos(b*x+a)^12/b
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx = -\frac{600\cos(2(a+bx))+75\cos(4(a+bx))-100\cos(6(a+bx))-30\cos(8(a+bx))+12\cos(10(a+bx)}{3840b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

output
```
-1/3840*(600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] - 100*Cos[6*(a + b*x)]
 - 30*Cos[8*(a + b*x)] + 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4775, 3042, 3045, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^5\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4775}$$

$$32\int \cos^7(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$32\int \cos(a+bx)^7\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{32\int \cos^7(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{16\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos^2(a+bx)}{b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{16\int \left(\cos^{10}(a+bx)-2\cos^8(a+bx)+\cos^6(a+bx)\right)d\cos^2(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16\left(\frac{1}{6}\cos^{12}(a+bx)-\frac{2}{5}\cos^{10}(a+bx)+\frac{1}{4}\cos^8(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

output
```
(-16*(Cos[a + b*x]^8/4 - (2*Cos[a + b*x]^10)/5 + Cos[a + b*x]^12/6))/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 13.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.68

method	result	siz
parallelrisch	$\frac{-75\cos(4bx+4a)-5\cos(12bx+12a)+30\cos(8bx+8a)-600\cos(2bx+2a)+100\cos(6bx+6a)-1662-12\cos(10bx+10a)}{3840b}$	74
default	$-\frac{5\cos(2bx+2a)}{32b}-\frac{5\cos(4bx+4a)}{256b}+\frac{5\cos(6bx+6a)}{192b}+\frac{\cos(8bx+8a)}{128b}-\frac{\cos(10bx+10a)}{320b}-\frac{\cos(12bx+12a)}{768b}$	86
risch	$-\frac{5\cos(2bx+2a)}{32b}-\frac{5\cos(4bx+4a)}{256b}+\frac{5\cos(6bx+6a)}{192b}+\frac{\cos(8bx+8a)}{128b}-\frac{\cos(10bx+10a)}{320b}-\frac{\cos(12bx+12a)}{768b}$	86
orering	Expression too large to display	13

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/3840*(-75*cos(4*b*x+4*a)-5*cos(12*b*x+12*a)+30*cos(8*b*x+8*a)-600*cos(2*
b*x+2*a)+100*cos(6*b*x+6*a)-1662-12*cos(10*b*x+10*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx = -\frac{4\left(10\cos(bx+a)^{12}-24\cos(bx+a)^{10}+15\cos(bx+a)^{8}\right)}{15\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
-4/15*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(37) = 74.

Time = 11.77 (sec) , antiderivative size = 597, normalized size of antiderivative = 13.57

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

output
```
Piecewise((-5*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**5/32 - 5*x*sin(a + b*x)*
*2*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**2/16 - 5*x*sin(a + b*x)**2*sin(2*
a + 2*b*x)*cos(2*a + 2*b*x)**4/32 - 5*x*sin(a + b*x)*sin(2*a + 2*b*x)**4*c
os(a + b*x)*cos(2*a + 2*b*x)/16 - 5*x*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos
(a + b*x)*cos(2*a + 2*b*x)**3/8 - 5*x*sin(a + b*x)*cos(a + b*x)*cos(2*a +
2*b*x)**5/16 + 5*x*sin(2*a + 2*b*x)**5*cos(a + b*x)**2/32 + 5*x*sin(2*a +
2*b*x)**3*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 + 5*x*sin(2*a + 2*b*x)*co
s(a + b*x)**2*cos(2*a + 2*b*x)**4/32 - 125*sin(a + b*x)**2*sin(2*a + 2*b*x
)**4*cos(2*a + 2*b*x)/(384*b) - 2*sin(a + b*x)**2*sin(2*a + 2*b*x)**2*cos(
2*a + 2*b*x)**3/(3*b) - 217*sin(a + b*x)**2*cos(2*a + 2*b*x)**5/(640*b) +
95*sin(a + b*x)*sin(2*a + 2*b*x)**5*cos(a + b*x)/(192*b) + 13*sin(a + b*x)
*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**2/(12*b) + 109*sin(a +
 b*x)*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(192*b) - 67*sin(2
*a + 2*b*x)**4*cos(a + b*x)**2*cos(2*a + 2*b*x)/(384*b) + 139*cos(a + b*x)
**2*cos(2*a + 2*b*x)**5/(1920*b), Ne(b, 0)), (x*sin(2*a)**5*cos(a)**2, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.64

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx = -\frac{5\cos(12bx+12a)+12\cos(10bx+10a)-30\cos(8bx+8a)-100\cos(6bx+6a)+75\cos(4bx}{3840b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
-1/3840*(5*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a
) - 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) + 600*cos(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx = -\frac{4\left(10\cos\left(bx+a\right)^{12}-24\cos\left(bx+a\right)^{10}+15\cos\left(bx+a\right)^{8}\right)}{15\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
-4/15*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

Mupad [B] (verification not implemented)

Time = 19.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx = -\frac{4\cos\left(a+b\,x\right)^8\,\left(10\cos\left(a+b\,x\right)^4-24\cos\left(a+b\,x\right)^2+15\right)}{15\,b}$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^5,x)
```

output
```
-(4*cos(a + b*x)^8*(10*cos(a + b*x)^4 - 24*cos(a + b*x)^2 + 15))/(15*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 250, normalized size of antiderivative = 5.68

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx$$
$$= \frac{-300\cos(2bx+2a)\cos(bx+a)\sin(bx+a)\,bx+100\cos(2bx+2a)\sin(2bx+2a)^4\sin(bx+a)^2-98\,\mathrm{c}}{}$$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x)
```

output
```
( - 300*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x)*b*x + 100*cos(2*a + 2*b
*x)*sin(2*a + 2*b*x)**4*sin(a + b*x)**2 - 98*cos(2*a + 2*b*x)*sin(2*a + 2*
b*x)**4 + 150*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x)**2 - 139*c
os(2*a + 2*b*x)*sin(2*a + 2*b*x)**2 + 150*cos(2*a + 2*b*x)*sin(a + b*x)**2
 - 203*cos(2*a + 2*b*x) - 20*cos(a + b*x)*sin(2*a + 2*b*x)**5*sin(a + b*x)
 - 50*cos(a + b*x)*sin(2*a + 2*b*x)**3*sin(a + b*x) - 300*sin(2*a + 2*b*x)
*sin(a + b*x)**2*b*x + 150*sin(2*a + 2*b*x)*b*x + 253)/(960*b)
```

3.551 $\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{3x}{16}-\frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b}-\frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b}+\frac{\sin^5(2a+2bx)}{20b}$$

output
```
3/16*x-3/32*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b-1/16*cos(2*b*x+2*a)*sin(2*b*x+
2*a)^3/b+1/20*sin(2*b*x+2*a)^5/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{120bx+20\sin(2(a+bx))-40\sin(4(a+bx))-10\sin(6(a+bx))+5\sin(8(a+bx))+2\sin(10(a+bx)}{640b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

output
```
(120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
 + 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(640*b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3042, 4773, 3042, 3044, 15, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^4\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4773}$$

$$\frac{1}{2}\int \sin^4(2a+2bx)dx+\frac{1}{2}\int \cos(2a+2bx)\sin^4(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^4dx+\frac{1}{2}\int \cos(2a+2bx)\sin(2a+2bx)^4dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^4(2a+2bx)d\sin(2a+2bx)}{4b}+\frac{1}{2}\int \sin(2a+2bx)^4dx$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \sin(2a+2bx)^4dx+\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \sin^2(2a+2bx)dx-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)+\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \sin(2a+2bx)^2dx-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)+\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \textbf{3115}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)+\frac{\sin^5(2a+2bx)}{20b}$$

$$\Big\downarrow \textbf{24}$$

$$\frac{\sin^5(2a+2bx)}{20b}+\frac{1}{2}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{8b}\right)$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

output
```
Sin[2*a + 2*b*x]^5/(20*b) + (-1/8*(Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^3)/b
+ (3*(x/2 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b)))/4)/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4773
```
Int[cos[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Simp[1/2   Int[(g*Sin[c + d*x])^p, x], x] + Simp[1/2   Int[Cos[c + d*
x]*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d
, 0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Maple [A] (verified)

Time = 7.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{120bx+20\sin(2bx+2a)-40\sin(4bx+4a)-10\sin(6bx+6a)+5\sin(8bx+8a)+2\sin(10bx+10a)}{640b}$	66
default	$\frac{3x}{16}+\frac{\sin(2bx+2a)}{32b}-\frac{\sin(4bx+4a)}{16b}-\frac{\sin(6bx+6a)}{64b}+\frac{\sin(8bx+8a)}{128b}+\frac{\sin(10bx+10a)}{320b}$	75
risch	$\frac{3x}{16}+\frac{\sin(2bx+2a)}{32b}-\frac{\sin(4bx+4a)}{16b}-\frac{\sin(6bx+6a)}{64b}+\frac{\sin(8bx+8a)}{128b}+\frac{\sin(10bx+10a)}{320b}$	75
orering	Expression too large to display	2048

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/640*(120*b*x+20*sin(2*b*x+2*a)-40*sin(4*b*x+4*a)-10*sin(6*b*x+6*a)+5*sin
(8*b*x+8*a)+2*sin(10*b*x+10*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$=\frac{15\,bx+\left(128\,\cos\left(bx+a\right)^9-176\,\cos\left(bx+a\right)^7+8\,\cos\left(bx+a\right)^5+10\,\cos\left(bx+a\right)^3+15\,\cos\left(bx+a\right)\right)\mathrm{s}}{80\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
1/80*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)^5
 + 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(70) = 140$.

Time = 5.16 (sec) , antiderivative size = 434, normalized size of antiderivative = 5.71

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \begin{cases} \frac{3x\sin^2(a+bx)\sin^4(2a+2bx)}{16} + \frac{3x\sin^2(a+bx)\sin^2(2a+2bx)\cos^2(2a+2bx)}{8} + \frac{3x\sin^2(a+bx)\cos^4(2a+2bx)}{16} + \frac{3x\sin^4(2a+2bx)\cos^2(a}{16} \\ x\sin^4(2a)\cos^2(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**4/16 + 3*x*sin(a + b*x)**
2*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*sin(a + b*x)**2*cos(2*a
+ 2*b*x)**4/16 + 3*x*sin(2*a + 2*b*x)**4*cos(a + b*x)**2/16 + 3*x*sin(2*a
+ 2*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*cos(a + b*x)**2*co
s(2*a + 2*b*x)**4/16 + 7*sin(a + b*x)**2*sin(2*a + 2*b*x)**3*cos(2*a + 2*b
*x)/(160*b) + 19*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(480
*b) + sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(a + b*x)/(10*b) + 2*sin(a + b*x
)*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(5*b) + 4*sin(a + b
*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(15*b) - 57*sin(2*a + 2*b*x)**3*cos(a
 + b*x)**2*cos(2*a + 2*b*x)/(160*b) - 109*sin(2*a + 2*b*x)*cos(a + b*x)**2
*cos(2*a + 2*b*x)**3/(480*b), Ne(b, 0)), (x*sin(2*a)**4*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{120\,bx + 2\,\sin\left(10\,bx + 10\,a\right) + 5\,\sin\left(8\,bx + 8\,a\right) - 10\,\sin\left(6\,bx + 6\,a\right) - 40\,\sin\left(4\,bx + 4\,a\right) + 20\,\sin\left(2\,bx\right.}{640\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/640*(120*b*x + 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) - 10*sin(6*b*x
+ 6*a) - 40*sin(4*b*x + 4*a) + 20*sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{120\,bx + 120\,a + 2\,\sin\left(10\,bx + 10\,a\right) + 5\,\sin\left(8\,bx + 8\,a\right) - 10\,\sin\left(6\,bx + 6\,a\right) - 40\,\sin\left(4\,bx + 4\,a\right) + 20}{640\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/640*(120*b*x + 120*a + 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) - 10*si
n(6*b*x + 6*a) - 40*sin(4*b*x + 4*a) + 20*sin(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 20.84 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.43

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{3\,x}{16} + \frac{\frac{3\tan(a+b\,x)^9}{16}+\frac{7\tan(a+b\,x)^7}{8}+\frac{8\tan(a+b\,x)^5}{5}-\frac{7\tan(a+b\,x)^3}{8}-\frac{3\tan(a+b\,x)}{16}}{b\,\left(\tan\left(a+b\,x\right)^{10}+5\tan\left(a+b\,x\right)^8+10\tan\left(a+b\,x\right)^6+10\tan\left(a+b\,x\right)^4+5\tan\left(a+b\,x\right)^2+1\right)}$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^4,x)
```

output
```
(3*x)/16 + ((8*tan(a + b*x)^5)/5 - (7*tan(a + b*x)^3)/8 - (3*tan(a + b*x))
/16 + (7*tan(a + b*x)^7)/8 + (3*tan(a + b*x)^9)/16)/(b*(5*tan(a + b*x)^2 +
 10*tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^1
0 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.99

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{-128\cos\left(2bx+2a\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+64\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)^3\sin\left(bx+a\right)^2-62\cos\left(2\right.}{}$$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x)
```

output
```
( - 128*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x) + 64*cos(2*a + 2*b*x)*s
in(2*a + 2*b*x)**3*sin(a + b*x)**2 - 62*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)*
*3 + 128*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)*sin(a + b*x)**2 - 109*cos(2*a +
 2*b*x)*sin(2*a + 2*b*x) - 16*cos(a + b*x)*sin(2*a + 2*b*x)**4*sin(a + b*x
) - 64*cos(a + b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x) + 128*cos(a + b*x)*si
n(a + b*x) - 128*sin(2*a + 2*b*x)*sin(a + b*x)**2 + 64*sin(2*a + 2*b*x) +
90*b*x)/(480*b)
```

3.552 $\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 28

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx=-\frac{4\cos^6(a+bx)}{3b}+\frac{\cos^8(a+bx)}{b}$$

output
```
-4/3*cos(b*x+a)^6/b+cos(b*x+a)^8/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx =\frac{-72\cos(2(a+bx))-12\cos(4(a+bx))+8\cos(6(a+bx))+3\cos(8(a+bx))}{384b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8
*(a + b*x)])/(384*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4775, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(2a+2bx)^3\cos(a+bx)^2dx$$

$$\Big\downarrow \ 4775$$

$$8\int \cos^5(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \ 3042$$

$$8\int \cos(a+bx)^5\sin(a+bx)^3dx$$

$$\Big\downarrow \ 3045$$

$$-\frac{8\int \cos^5(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \ 244$$

$$-\frac{8\int \left(\cos^5(a+bx)-\cos^7(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{8\left(\frac{1}{6}\cos^6(a+bx)-\frac{1}{8}\cos^8(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-8*(Cos[a + b*x]^6/6 - Cos[a + b*x]^8/8))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

method	result
parallelrisch	$\frac{-12\cos(4bx+4a)-199+3\cos(8bx+8a)-72\cos(2bx+2a)+8\cos(6bx+6a)}{384b}$
default	$-\frac{3\cos(2bx+2a)}{16b}-\frac{\cos(4bx+4a)}{32b}+\frac{\cos(6bx+6a)}{48b}+\frac{\cos(8bx+8a)}{128b}$
risch	$-\frac{3\cos(2bx+2a)}{16b}-\frac{\cos(4bx+4a)}{32b}+\frac{\cos(6bx+6a)}{48b}+\frac{\cos(8bx+8a)}{128b}$
orering	$-\frac{205\left(-2\cos(bx+a)\sin(2bx+2a)^3b\sin(bx+a)+6\cos(bx+a)^2\sin(2bx+2a)^2b\cos(2bx+2a)\right)}{576b^2}-\frac{91\left(80\cos(bx+a)\sin(2bx+2\right.}{}$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/384*(-12*cos(4*b*x+4*a)-199+3*cos(8*b*x+8*a)-72*cos(2*b*x+2*a)+8*cos(6*b
*x+6*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx = \frac{3\cos(bx+a)^8 - 4\cos(bx+a)^6}{3b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
1/3*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(22) = 44.

Time = 2.20 (sec) , antiderivative size = 362, normalized size of antiderivative = 12.93

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{3x\sin^2(a+bx)\sin^3(2a+2bx)}{16} - \frac{3x\sin^2(a+bx)\sin(2a+2bx)\cos^2(2a+2bx)}{16} - \frac{3x\sin(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{8} - \\ x\sin^3(2a)\cos^2(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**3,x)
```

output
```
Piecewise((-3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**3/16 - 3*x*sin(a + b*x)*
*2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/16 - 3*x*sin(a + b*x)*sin(2*a + 2*
b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)/8 - 3*x*sin(a + b*x)*cos(a + b*x)*co
s(2*a + 2*b*x)**3/8 + 3*x*sin(2*a + 2*b*x)**3*cos(a + b*x)**2/16 + 3*x*sin
(2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 - sin(a + b*x)**2*sin
(2*a + 2*b*x)**2*cos(2*a + 2*b*x)/(2*b) - 49*sin(a + b*x)**2*cos(2*a + 2*b
*x)**3/(96*b) + 13*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(a + b*x)/(16*b) +
7*sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(8*b) + 1
7*cos(a + b*x)**2*cos(2*a + 2*b*x)**3/(96*b), Ne(b, 0)), (x*sin(2*a)**3*co
s(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx = \frac{3\cos(8bx+8a)+8\cos(6bx+6a)-12\cos(4bx+4a)-72\cos(2bx+2a)}{384b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/384*(3*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) - 72*
cos(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx = \frac{3\cos(bx+a)^8-4\cos(bx+a)^6}{3b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
1/3*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

Mupad [B] (verification not implemented)

Time = 19.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx = -\frac{\frac{4\cos(a+b\,x)^6}{3}-\cos\left(a+b\,x\right)^8}{b}$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^3,x)
```

output
```
-((4*cos(a + b*x)^6)/3 - cos(a + b*x)^8)/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 6.18

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx$$
$$= \frac{-36\cos\left(2bx+2a\right)\cos\left(bx+a\right)\sin\left(bx+a\right)bx+18\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)^2\sin\left(bx+a\right)^2-17\cos}{}$$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x)
```

output
```
( - 36*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x)*b*x + 18*cos(2*a + 2*b*x
)*sin(2*a + 2*b*x)**2*sin(a + b*x)**2 - 17*cos(2*a + 2*b*x)*sin(2*a + 2*b*
x)**2 + 18*cos(2*a + 2*b*x)*sin(a + b*x)**2 - 25*cos(2*a + 2*b*x) - 6*cos(
a + b*x)*sin(2*a + 2*b*x)**3*sin(a + b*x) - 36*sin(2*a + 2*b*x)*sin(a + b*
x)**2*b*x + 18*sin(2*a + 2*b*x)*b*x + 31)/(96*b)
```

3.553 $\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx=\frac{x}{4}-\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}+\frac{\sin^3(2a+2bx)}{12b}$$

output
```
1/4*x-1/8*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b+1/12*sin(2*b*x+2*a)^3/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx = -\frac{-12bx-3\sin(2(a+bx))+3\sin(4(a+bx))+\sin(6(a+bx))}{48b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

output
```
-1/48*(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)
])/b
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4773, 3042, 3044, 15, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^2\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4773}$$

$$\frac{1}{2}\int \sin^2(2a+2bx)dx+\frac{1}{2}\int \cos(2a+2bx)\sin^2(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^2dx+\frac{1}{2}\int \cos(2a+2bx)\sin(2a+2bx)^2dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^2(2a+2bx)d\sin(2a+2bx)}{4b}+\frac{1}{2}\int \sin(2a+2bx)^2dx$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \sin(2a+2bx)^2dx+\frac{\sin^3(2a+2bx)}{12b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{\int 1dx}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)+\frac{\sin^3(2a+2bx)}{12b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin^3(2a+2bx)}{12b}+\frac{1}{2}\left(\frac{x}{2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b}\right)$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

output
```
Sin[2*a + 2*b*x]^3/(12*b) + (x/2 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*
b))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4773
```
Int[cos[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Simp[1/2   Int[(g*Sin[c + d*x])^p, x], x] + Simp[1/2   Int[Cos[c + d*
x]*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d
, 0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{12bx+3\sin(2bx+2a)-3\sin(4bx+4a)-\sin(6bx+6a)}{48b}$
default	$\frac{x}{4}+\frac{\sin(2bx+2a)}{16b}-\frac{\sin(4bx+4a)}{16b}-\frac{\sin(6bx+6a)}{48b}$
risch	$\frac{x}{4}+\frac{\sin(2bx+2a)}{16b}-\frac{\sin(4bx+4a)}{16b}-\frac{\sin(6bx+6a)}{48b}$
orering	$x\cos(bx+a)^2\sin(2bx+2a)^2-\frac{49\left(-2\cos(bx+a)\sin(2bx+2a)^2 b\sin(bx+a)+4\cos(bx+a)^2\cos(2bx+2a)b\sin(2b\right.}{144b^2}$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/48*(12*b*x+3*sin(2*b*x+2*a)-3*sin(4*b*x+4*a)-sin(6*b*x+6*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{3\,bx-\left(8\,\cos(bx+a)^5-2\,\cos(bx+a)^3-3\,\cos(bx+a)\right)\sin(bx+a)}{12\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
1/12*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b
*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(41) = 82$.

Time = 0.81 (sec) , antiderivative size = 231, normalized size of antiderivative = 4.71

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx$$
$$= \begin{cases} \frac{x\sin^2(a+bx)\sin^2(2a+2bx)}{4} + \frac{x\sin^2(a+bx)\cos^2(2a+2bx)}{4} + \frac{x\sin^2(2a+2bx)\cos^2(a+bx)}{4} + \frac{x\cos^2(a+bx)\cos^2(2a+2bx)}{4} + \frac{\sin^2(a+ \\ x\sin^2(2a)\cos^2(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)**2/4 + x*sin(a + b*x)**2*cos
(2*a + 2*b*x)**2/4 + x*sin(2*a + 2*b*x)**2*cos(a + b*x)**2/4 + x*cos(a + b
*x)**2*cos(2*a + 2*b*x)**2/4 + sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a +
2*b*x)/(24*b) + sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)/(6*b) + sin(
a + b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(3*b) - 7*sin(2*a + 2*b*x)*cos(a
 + b*x)**2*cos(2*a + 2*b*x)/(24*b), Ne(b, 0)), (x*sin(2*a)**2*cos(a)**2, T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{12\,bx - \sin(6\,bx + 6\,a) - 3\,\sin(4\,bx + 4\,a) + 3\,\sin(2\,bx + 2\,a)}{48\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
1/48*(12*b*x - sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))
/b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx$$
$$=\frac{12\,bx+12\,a-\sin\left(6\,bx+6\,a\right)-3\,\sin\left(4\,bx+4\,a\right)+3\,\sin\left(2\,bx+2\,a\right)}{48\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
1/48*(12*b*x + 12*a - sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x
+ 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 19.70 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx=\frac{x}{4}-\frac{\frac{\sin(4\,a+4\,b\,x)}{16}-\frac{\sin(2\,a+2\,b\,x)}{16}+\frac{\sin(6\,a+6\,b\,x)}{48}}{b}$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^2,x)
```

output
```
x/4 - (sin(4*a + 4*b*x)/16 - sin(2*a + 2*b*x)/16 + sin(6*a + 6*b*x)/48)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.06

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx$$
$$=\frac{8\cos\left(2bx+2a\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+8\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)\sin\left(bx+a\right)^2-7\cos\left(2bx+2a\right)}{}$$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x)
```

output

```
(8*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x) + 8*cos(2*a + 2*b*x)*sin(2*a
 + 2*b*x)*sin(a + b*x)**2 - 7*cos(2*a + 2*b*x)*sin(2*a + 2*b*x) - 4*cos(a
+ b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x) + 8*cos(a + b*x)*sin(a + b*x) + 8*
sin(2*a + 2*b*x)*sin(a + b*x)**2 - 4*sin(2*a + 2*b*x) + 6*b*x)/(24*b)
```

3.554 $\int \cos^2(a+bx)\sin(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx = -\frac{\cos^4(a+bx)}{2b}$$

output
```
-1/2*cos(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx = -\frac{\cos^4(a+bx)}{2b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

output
```
-1/2*Cos[a + b*x]^4/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4775, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4775}$$

$$2\int \cos^3(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \cos(a+bx)^3\sin(a+bx)dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{2\int \cos^3(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\cos^4(a+bx)}{2b}$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

output
```
-1/2*Cos[a + b*x]^4/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.

Time = 0.96 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

method	result
default	$-\frac{\cos(2bx+2a)}{4b}-\frac{\cos(4bx+4a)}{16b}$
risch	$-\frac{\cos(2bx+2a)}{4b}-\frac{\cos(4bx+4a)}{16b}$
parallelrisch	$\frac{-4\cos(2bx+2a)+5-\cos(4bx+4a)}{16b}$
orering	$-\frac{5\left(-2\cos(bx+a)\sin(2bx+2a)b\sin(bx+a)+2\cos(bx+a)^2b\cos(2bx+2a)\right)}{16b^2}-\frac{12b^3\cos(2bx+2a)\sin(bx+a)^2+32\sin(2bx+2a}{}$
norman	$\frac{x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3+x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\tan(bx+a)^2+\frac{3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan(bx+a)}{b}-x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+\frac{x\tan(bx+a)}{2}-3x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\tan(bx}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right.}$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(2*b*x+2*a)/b-1/16*cos(4*b*x+4*a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx=-\frac{\cos(bx+a)^4}{2\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-1/2*cos(b*x + a)^4/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(12) = 24.

Time = 0.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 8.87

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx$$
$$=\begin{cases}-\frac{x\sin^2(a+bx)\sin(2a+2bx)}{4}-\frac{x\sin(a+bx)\cos(a+bx)\cos(2a+2bx)}{2}+\frac{x\sin(2a+2bx)\cos^2(a+bx)}{4}-\frac{\sin^2(a+bx)\cos(2a+2bx)}{2b}+\frac{3\,\mathrm{s}}{} \\ x\sin(2a)\cos^2(a)\end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a),x)
```

output
```
Piecewise((-x*sin(a + b*x)**2*sin(2*a + 2*b*x)/4 - x*sin(a + b*x)*cos(a +
b*x)*cos(2*a + 2*b*x)/2 + x*sin(2*a + 2*b*x)*cos(a + b*x)**2/4 - sin(a + b
*x)**2*cos(2*a + 2*b*x)/(2*b) + 3*sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*
x)/(4*b), Ne(b, 0)), (x*sin(2*a)*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx = -\frac{\cos(4\,bx+4\,a)+4\,\cos(2\,bx+2\,a)}{16\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/16*(cos(4*b*x + 4*a) + 4*cos(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx = -\frac{\cos(bx+a)^4}{2\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
-1/2*cos(b*x + a)^4/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx = -\frac{\cos(a+b\,x)^4}{2\,b}$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x),x)
```

output
```
-cos(a + b*x)^4/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 6.40

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx$$
$$= \frac{-4\cos(2bx+2a)\cos(bx+a)\sin(bx+a)\,bx + 2\cos(2bx+2a)\sin(bx+a)^2 - 3\cos(2bx+2a) - 4\sin(}{8b}$$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a),x)
```

output
```
( - 4*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x)*b*x + 2*cos(2*a + 2*b*x)*
sin(a + b*x)**2 - 3*cos(2*a + 2*b*x) - 4*sin(2*a + 2*b*x)*sin(a + b*x)**2*
b*x + 2*sin(2*a + 2*b*x)*b*x + 3)/(8*b)
```

3.555 $\int \cos^2(a+bx)\csc(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx=\frac{\log(\sin(a+bx))}{2b}$$

output
```
1/2*ln(sin(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx=\frac{\log(\sin(a+bx))}{2b}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

output
```
Log[Sin[a + b*x]]/(2*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4775, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)}dx$$

$$\Big\downarrow \; 4775$$

$$\frac{1}{2}\int \cot(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{2}\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \; 3956$$

$$\frac{\log(-\sin(a+bx))}{2b}$$

input
```
Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

output
```
Log[-Sin[a + b*x]]/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(\sin(bx+a))}{2b}$	13
risch	$-\frac{ix}{2}-\frac{ia}{b}+\frac{\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)}{2b}$	30

input
```
int(cos(b*x+a)^2*csc(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(sin(b*x+a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx = \frac{\log\left(\frac{1}{2}\sin\left(bx+a\right)\right)}{2\,b}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="fricas")
```

output
```
1/2*log(1/2*sin(b*x + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(cos(b*x+a)**2*csc(2*b*x+2*a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(12) = 24.

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 5.86

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx = \frac{\log\left(\cos\left(bx\right)^2 + 2\,\cos\left(bx\right)\cos\left(a\right) + \cos\left(a\right)^2 + \sin\left(bx\right)^2 - 2\,\sin\left(bx\right)\sin\left(a\right) + \sin\left(a\right)^2\right) + \log\left(\cos\left(bx\right)^2 -\right.}{4\,b}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="maxima")
```

output
```
1/4*(log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*
x)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + si
n(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx = \frac{\log(|\sin(bx+a)|)}{2b}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="giac")
```

output
```
1/2*log(abs(sin(b*x + a)))/b
```

Mupad [B] (verification not implemented)

Time = 19.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx = \frac{\ln\left(\sin(a+b\,x)^2\right)}{4\,b}$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x),x)
```

output
```
log(sin(a + b*x)^2)/(4*b)
```

Reduce [F]

$$\int \cos^2(a+bx)\csc(2a+2bx)\,dx = \int \cos{(bx+a)}^2 \csc{(2bx+2a)}\,dx$$

input
```
int(cos(b*x+a)^2*csc(2*b*x+2*a),x)
```

output
```
int(cos(a + b*x)**2*csc(2*a + 2*b*x),x)
```

3.556 $\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx=-\frac{\cot(a+bx)}{4b}$$

output
```
-1/4*cot(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx=-\frac{\cot(a+bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/4*Cot[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4775, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{4}\int \csc^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc(a+bx)^2dx$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d\cot(a+bx)}{4b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\cot(a+bx)}{4b}$$

input
```
Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/4*Cot[a + b*x]/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{\cot(bx+a)}{4b}$	12
risch	$-\frac{i}{2b\left(e^{2i(bx+a)}-1\right)}$	20

input
```
int(cos(b*x+a)^2*csc(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*cot(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\cos(bx+a)}{4\,b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-1/4*cos(b*x + a)/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*csc(2*b*x+2*a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(11) = 22.

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 4.08

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\sin(2\,bx+2\,a)}{2\,\left(b\cos(2\,bx+2\,a)^2+b\sin(2\,bx+2\,a)^2-2\,b\cos(2\,bx+2\,a)+b\right)}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
-1/2*sin(2*b*x + 2*a)/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*c
os(2*b*x + 2*a) + b)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{1}{4\,b\tan\left(bx+a\right)}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-1/4/(b*tan(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 19.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\cot(a+b\,x)}{4\,b}$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^2,x)
```

output
```
-cot(a + b*x)/(4*b)
```

Reduce [F]

$$\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx = \int \cos\left(bx+a\right)^2\csc\left(2bx+2a\right)^2 dx$$

input
```
int(cos(b*x+a)^2*csc(2*b*x+2*a)^2,x)
```

output
```
int(cos(a + b*x)**2*csc(2*a + 2*b*x)**2,x)
```

3.557 $\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx=-\frac{\cot^2(a+bx)}{16b}+\frac{\log(\tan(a+bx))}{8b}$$

output
```
-1/16*cot(b*x+a)^2/b+1/8*ln(tan(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx=\frac{1}{8}\left(-\frac{\csc^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}+\frac{\log(\sin(a+bx))}{b}\right)$$

input
```
Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

output
```
(-1/2*Csc[a + b*x]^2/b - Log[Cos[a + b*x]]/b + Log[Sin[a + b*x]]/b)/8
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4775, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{8}\int \csc^3(a+bx)\sec(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)^3\sec(a+bx)dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{8b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^3(a+bx)+\cot(a+bx)\right)d\tan(a+bx)}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(\tan(a+bx))-\frac{1}{2}\cot^2(a+bx)}{8b}$$

input
```
Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

output
```
(-1/2*Cot[a + b*x]^2 + Log[Tan[a + b*x]])/(8*b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{-\frac{1}{2\sin(bx+a)^{2}}+\ln(\tan(bx+a))}{8b}$	24
risch	$\frac{e^{2i(bx+a)}}{4b\left(e^{2i(bx+a)}-1\right)^{2}}-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{8b}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{8b}$	63
parallelrisch	$-\frac{\sec(bx+a)^{2}\csc(bx+a)^{2}\left(\ln\left(\sqrt{\tan(bx+a)}\right)\cos(4bx+4a)+\ln\left(\frac{1}{\sqrt{\tan(bx+a)}}\right)+\cos(2bx+2a)+\cos(4bx+4a)\right)}{32b}$	69

input
```
int(cos(b*x+a)^2*csc(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-1/2/sin(b*x+a)^2+ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx = -\frac{(\cos(bx+a)^2-1)\log(\cos(bx+a)^2)-(\cos(bx+a)^2-1)\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-1}{16\left(b\cos(bx+a)^2-b\right)}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-1/16*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log
(-1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*csc(2*b*x+2*a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(26) = 52$.

Time = 0.06 (sec) , antiderivative size = 656, normalized size of antiderivative = 21.87

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/16*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) - 8*cos(2*b*x + 2*a)^2 + (2*(2*c
os(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x +
 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin
(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*
cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)
 - (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*c
os(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*
a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos
(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*
(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b
*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4
*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*
cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b
*x + 4*a)*sin(2*b*x + 2*a) - 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b
*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*
sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*
b*x + 2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) + b)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx = \frac{\frac{1}{\cos(bx+a)^2-1} + \log\left(-\cos\left(bx+a\right)^2+1\right) - 2\log\left(\left|\cos\left(bx+a\right)\right|\right)}{16\,b}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
1/16*(1/(cos(b*x + a)^2 - 1) + log(-cos(b*x + a)^2 + 1) - 2*log(abs(cos(b*
x + a))))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx = -\frac{\frac{\ln(\cos(a+b\,x))}{8}-\frac{\ln\left(\sin(a+b\,x)^2\right)}{16}+\frac{1}{16\sin(a+b\,x)^2}}{b}$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^3,x)
```

output
```
-(log(cos(a + b*x))/8 - log(sin(a + b*x)^2)/16 + 1/(16*sin(a + b*x)^2))/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.97

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx$$
$$= \frac{2\cos(2bx+2a)\sin(bx+a)^2-2\cos(2bx+2a)+2\cos(bx+a)\sin(2bx+2a)\sin(bx+a)+\log(\tan(bx}{8\sin(2bx+2a)^2 b}$$

input
```
int(cos(b*x+a)^2*csc(2*b*x+2*a)^3,x)
```

output
```
(2*cos(2*a + 2*b*x)*sin(a + b*x)**2 - 2*cos(2*a + 2*b*x) + 2*cos(a + b*x)*
sin(2*a + 2*b*x)*sin(a + b*x) + log(tan(a + b*x))*sin(2*a + 2*b*x)**2)/(8*
sin(2*a + 2*b*x)**2*b)
```

3.558 $\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx=-\frac{\cot(a+bx)}{8b}-\frac{\cot^3(a+bx)}{48b}+\frac{\tan(a+bx)}{16b}$$

output
```
-1/8*cot(b*x+a)/b-1/48*cot(b*x+a)^3/b+1/16*tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx=-\frac{5\cot(a+bx)}{48b}-\frac{\cot(a+bx)\csc^2(a+bx)}{48b}+\frac{\tan(a+bx)}{16b}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

output
```
(-5*Cot[a + b*x])/(48*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(48*b) + Tan[a + 
b*x]/(16*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4775, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{16}\int \csc^4(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^4\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^4(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{16b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^4(a+bx)+2\cot^2(a+bx)+1\right)d\tan(a+bx)}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan(a+bx)-\frac{1}{3}\cot^3(a+bx)-2\cot(a+bx)}{16b}$$

input
```
Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

output
```
(-2*Cot[a + b*x] - Cot[a + b*x]^3/3 + Tan[a + b*x])/(16*b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.73 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result	size
risch	$\frac{i\left(2\,{\mathrm e}^{2 i (b x +a)}-1\right)}{3 b \left({\mathrm e}^{2 i (b x +a)}-1\right)^{3} \left({\mathrm e}^{2 i (b x +a)}+1\right)}$	46
default	$\frac{-\frac{1}{3 \sin \left(b x +a \right)^{3} \cos \left(b x +a \right)}+\frac{4}{3 \sin \left(b x +a \right) \cos \left(b x +a \right)}-\frac{8 \cot \left(b x +a \right)}{3}}{16 b}$	51

input
```
int(cos(b*x+a)^2*csc(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*I*(2*exp(2*I*(b*x+a))-1)/b/(exp(2*I*(b*x+a))-1)^3/(exp(2*I*(b*x+a))+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx = -\frac{8\,\cos(bx+a)^4-12\,\cos(bx+a)^2+3}{48\,\left(b\cos(bx+a)^3-b\cos(bx+a)\right)\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
-1/48*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*co
s(b*x + a))*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*csc(2*b*x+2*a)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(36) = 72.

Time = 0.05 (sec) , antiderivative size = 308, normalized size of antiderivative = 7.33

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx$$

$$= \frac{(2}{3\,\left(b\cos(8\,bx+8\,a)^2+4\,b\cos(6\,bx+6\,a)^2+4\,b\cos(2\,bx+2\,a)^2+b\sin(8\,bx+8\,a)^2+4\,b\sin(6\,bx+6\,a\right.}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/3*((2*cos(2*b*x + 2*a) - 1)*sin(8*b*x + 8*a) - 2*(2*cos(2*b*x + 2*a) - 1
)*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) + 4*cos(6*b*x + 6
*a)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b
*cos(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b*
sin(6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(6*
b*x + 6*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*x
 + 2*a) - b)*cos(6*b*x + 6*a) - 4*b*cos(2*b*x + 2*a) - 4*(b*sin(6*b*x + 6*
a) - b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx = -\frac{\frac{6\,\tan(bx+a)^2+1}{\tan(bx+a)^3} - 3\,\tan(bx+a)}{48\,b}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
-1/48*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 19.99 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx = \frac{\tan(a+b\,x)}{16\,b} - \frac{\frac{\tan(a+b\,x)^2}{8}+\frac{1}{48}}{b\tan(a+b\,x)^3}$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^4,x)
```

output
```
tan(a + b*x)/(16*b) - (tan(a + b*x)^2/8 + 1/48)/(b*tan(a + b*x)^3)
```

Reduce [F]

$$\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx = \int \cos{(bx+a)}^2\csc{(2bx+2a)}^4\,dx$$

input
```
int(cos(b*x+a)^2*csc(2*b*x+2*a)^4,x)
```

output
```
int(cos(a + b*x)**2*csc(2*a + 2*b*x)**4,x)
```

3.559 $\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx=-\frac{3\cot^2(a+bx)}{64b}-\frac{\cot^4(a+bx)}{128b}+\frac{3\log(\tan(a+bx))}{32b}+\frac{\tan^2(a+bx)}{64b}$$

output
```
-3/64*cot(b*x+a)^2/b-1/128*cot(b*x+a)^4/b+3/32*ln(tan(b*x+a))/b+1/64*tan(b
*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx=-\frac{4\csc^2(a+bx)+\csc^4(a+bx)+12\log(\cos(a+bx))-12\log(\sin(a+bx))-2\sec^2(a+bx)}{128b}$$

input
```
Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

output
```
-1/128*(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[
Sin[a + b*x]] - 2*Sec[a + b*x]^2)/b
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4775, 3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{32}\int \csc^5(a+bx)\sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \csc(a+bx)^5\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^5(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan(a+bx)}{32b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan^2(a+bx)}{64b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\cot^3(a+bx)+3\cot^2(a+bx)+3\cot(a+bx)+1\right)d\tan^2(a+bx)}{64b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan^2(a+bx) - \frac{1}{2}\cot^2(a+bx) - 3\cot(a+bx) + 3\log\left(\tan^2(a+bx)\right)}{64b}$$

input
```
Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

output
```
(-3*Cot[a + b*x] - Cot[a + b*x]^2/2 + 3*Log[Tan[a + b*x]^2] + Tan[a + b*x]
^2)/(64*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

method	result
default	$\frac{-\frac{1}{4\sin(bx+a)^4\cos(bx+a)^2}+\frac{3}{4\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{2\sin(bx+a)^2}+3\ln(\tan(bx+a))}{32b}$
parallelrisch	$-\frac{\sec(bx+a)^4\csc(bx+a)^4(36\ln(\tan(bx+a))\cos(4bx+4a)-9\ln(\tan(bx+a))\cos(8bx+8a)+66\cos(2bx+2a)-18\cos(6bx+6a)-}{12288b}$
risch	$\frac{3\,e^{10i(bx+a)}-6\,e^{8i(bx+a)}-2\,e^{6i(bx+a)}-6\,e^{4i(bx+a)}+3\,e^{2i(bx+a)}}{16b(e^{2i(bx+a)}-1)^4(e^{2i(bx+a)}+1)^2}-\frac{3\ln(e^{2i(bx+a)}+1)}{32b}+\frac{3\ln(e^{2i(bx+a)}-1)}{32b}$

input
```
int(cos(b*x+a)^2*csc(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^2+3/4/sin(b*x+a)^2/cos(b*x+a)^2-3/2/s
in(b*x+a)^2+3*ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(52) = 104.

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.30

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx$$
$$=\frac{6\cos(bx+a)^4-9\cos(bx+a)^2-6\left(\cos(bx+a)^6-2\cos(bx+a)^4+\cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right.}{128\left(b\cos(bx+a)^6-2\,b\cos(bx+a)\right.}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
1/128*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x
 + a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(
b*x + a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*
x + a)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*csc(2*b*x+2*a)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3188 vs. $2(52) = 104$.

Time = 0.11 (sec) , antiderivative size = 3188, normalized size of antiderivative = 53.13

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/64*(4*(3*cos(10*b*x + 10*a) - 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) -
6*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b
*x + 8*a) + 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) - 12*cos(2*b*x + 2*a)
 + 3)*cos(10*b*x + 10*a) - 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a
) - 12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) + 6)*cos(8*b*x + 8*a) + 24*co
s(8*b*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) - 8*cos(2*b*x + 2*a) + 1)*cos(6*
b*x + 6*a) - 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) - 2)*cos(4*b*x
 + 4*a) + 24*cos(4*b*x + 4*a)^2 - 24*cos(2*b*x + 2*a)^2 + 3*(2*(2*cos(10*b
*x + 10*a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*
cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8
*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) -
 1)*cos(10*b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) -
cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x +
8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) -
16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(
4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*
x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x
 + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*si
n(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx = \frac{\frac{6\cos(bx+a)^4-9\cos(bx+a)^2+2}{\left(\cos(bx+a)^2-1\right)^2\cos(bx+a)^2}+6\log\left(-\cos(bx+a)^2+1\right)-12\log\left(|\cos(bx+a)|\right)}{128\,b}$$

input
```
integrate(cos(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
1/128*((6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 + 2)/((cos(b*x + a)^2 - 1)^2*c
os(b*x + a)^2) + 6*log(-cos(b*x + a)^2 + 1) - 12*log(abs(cos(b*x + a))))/b
```

Mupad [B] (verification not implemented)

Time = 20.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.37

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx$$
$$= \frac{3\ln\left(\sin\left(a+b\,x\right)^2\right)}{64\,b} - \frac{3\ln\left(\cos\left(a+b\,x\right)\right)}{32\,b} + \frac{\frac{3\cos(a+b\,x)^4}{64} - \frac{9\cos(a+b\,x)^2}{128} + \frac{1}{64}}{b\,\left(\cos\left(a+b\,x\right)^6 - 2\cos\left(a+b\,x\right)^4 + \cos\left(a+b\,x\right)^2\right)}$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^5,x)
```

output
```
(3*log(sin(a + b*x)^2))/(64*b) - (3*log(cos(a + b*x)))/(32*b) + ((3*cos(a
+ b*x)^4)/64 - (9*cos(a + b*x)^2)/128 + 1/64)/(b*(cos(a + b*x)^2 - 2*cos(a
 + b*x)^4 + cos(a + b*x)^6))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.78

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx$$
$$= \frac{16\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)^2\sin\left(bx+a\right)^2 - 17\cos\left(2bx+2a\right)\sin\left(2bx+2a\right)^2 + 12\cos\left(2bx+2a\right)\sin}{}$$

input
```
int(cos(b*x+a)^2*csc(2*b*x+2*a)^5,x)
```

output
```
(16*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x)**2 - 17*cos(2*a + 2*
b*x)*sin(2*a + 2*b*x)**2 + 12*cos(2*a + 2*b*x)*sin(a + b*x)**2 - 12*cos(2*
a + 2*b*x) + 16*cos(a + b*x)*sin(2*a + 2*b*x)**3*sin(a + b*x) + 4*cos(a +
b*x)*sin(2*a + 2*b*x)*sin(a + b*x) + 9*log(tan(a + b*x))*sin(2*a + 2*b*x)*
*4)/(96*sin(2*a + 2*b*x)**4*b)
```

3.560 $\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx=-\frac{32\cos^9(a+bx)}{9b}+\frac{64\cos^{11}(a+bx)}{11b}-\frac{32\cos^{13}(a+bx)}{13b}$$

output
```
-32/9*cos(b*x+a)^9/b+64/11*cos(b*x+a)^11/b-32/13*cos(b*x+a)^13/b
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx =\frac{4\cos^9(a+bx)(-505+540\cos(2(a+bx))-99\cos(4(a+bx)))}{1287b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

output
```
(4*Cos[a + b*x]^9*(-505 + 540*Cos[2*(a + b*x)] - 99*Cos[4*(a + b*x)]))/(12
87*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4775, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(2a+2bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^5\cos(a+bx)^3dx$$

$$\Big\downarrow \text{4775}$$

$$32\int \cos^8(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$32\int \cos(a+bx)^8\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{32\int \cos^8(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{32\int \left(\cos^{12}(a+bx)-2\cos^{10}(a+bx)+\cos^8(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{32\left(\frac{1}{13}\cos^{13}(a+bx)-\frac{2}{11}\cos^{11}(a+bx)+\frac{1}{9}\cos^9(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

output
```
(-32*(Cos[a + b*x]^9/9 - (2*Cos[a + b*x]^11)/11 + Cos[a + b*x]^13/13))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(40) = 80$.

Time = 23.77 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.11

method	result
default	$-\frac{5\cos(bx+a)}{32b}-\frac{25\cos(3bx+3a)}{384b}+\frac{\cos(5bx+5a)}{128b}+\frac{\cos(7bx+7a)}{64b}+\frac{\cos(9bx+9a)}{576b}-\frac{3\cos(11bx+11a)}{1408b}-\frac{\cos(13bx+}{1664b}$
risch	$-\frac{5\cos(bx+a)}{32b}-\frac{25\cos(3bx+3a)}{384b}+\frac{\cos(5bx+5a)}{128b}+\frac{\cos(7bx+7a)}{64b}+\frac{\cos(9bx+9a)}{576b}-\frac{3\cos(11bx+11a)}{1408b}-\frac{\cos(13bx+}{1664b}$
parallelrisch	$\frac{\left(1632\tan(bx+a)^6+53408\tan(bx+a)^4+27520\tan(bx+a)^2+5504\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6+\left(-13056\tan(bx+a)^7-44736\tan(bx+a)^5\right.}{}$
orering	Expression too large to display

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
-5/32*cos(b*x+a)/b-25/384*cos(3*b*x+3*a)/b+1/128*cos(5*b*x+5*a)/b+1/64*cos
(7*b*x+7*a)/b+1/576*cos(9*b*x+9*a)/b-3/1408*cos(11*b*x+11*a)/b-1/1664*cos(
13*b*x+13*a)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\left(99\cos\left(bx+a\right)^{13}-234\cos\left(bx+a\right)^{11}+143\cos\left(bx+a\right)^{9}\right)}{1287\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
-32/1287*(99*cos(b*x + a)^13 - 234*cos(b*x + a)^11 + 143*cos(b*x + a)^9)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(39) = 78.

Time = 25.34 (sec) , antiderivative size = 447, normalized size of antiderivative = 9.72

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx = \begin{cases} -\frac{2234\sin^3(a+bx)\sin^5(2a+2bx)}{9009b} - \frac{4544\sin^3(a+bx)\sin^3(2a+2bx)\cos^2(2a+2bx)}{9009b} - \frac{256\sin^3(a+bx)\sin(2a+2bx)\cos^4(2a+2bx)}{1001b} - 13 \\ x\sin^5(2a)\cos^3(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

output
```
Piecewise((-2234*sin(a + b*x)**3*sin(2*a + 2*b*x)**5/(9009*b) - 4544*sin(a
 + b*x)**3*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**2/(9009*b) - 256*sin(a +
b*x)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**4/(1001*b) - 1388*sin(a + b*x)*
*2*sin(2*a + 2*b*x)**4*cos(a + b*x)*cos(2*a + 2*b*x)/(3003*b) - 2944*sin(a
 + b*x)**2*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**3/(3003*b) -
 512*sin(a + b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**5/(1001*b) + 271*sin(a
 + b*x)*sin(2*a + 2*b*x)**5*cos(a + b*x)**2/(3003*b) + 48*sin(a + b*x)*sin
(2*a + 2*b*x)**3*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/(143*b) + 640*sin(a +
 b*x)*sin(2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**4/(3003*b) - 1366
*sin(2*a + 2*b*x)**4*cos(a + b*x)**3*cos(2*a + 2*b*x)/(3003*b) - 4960*sin(
2*a + 2*b*x)**2*cos(a + b*x)**3*cos(2*a + 2*b*x)**3/(9009*b) - 256*cos(a +
 b*x)**3*cos(2*a + 2*b*x)**5/(1287*b), Ne(b, 0)), (x*sin(2*a)**5*cos(a)**3
, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx = -\frac{99\cos(13\,bx+13\,a)+351\cos(11\,bx+11\,a)-286\cos(9\,bx+9\,a)-2574\cos(7\,bx+7\,a)-1287\,\mathrm{co}}{164736\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
-1/164736*(99*cos(13*b*x + 13*a) + 351*cos(11*b*x + 11*a) - 286*cos(9*b*x
+ 9*a) - 2574*cos(7*b*x + 7*a) - 1287*cos(5*b*x + 5*a) + 10725*cos(3*b*x +
 3*a) + 25740*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\left(99\cos(bx+a)^{13}-234\cos(bx+a)^{11}+143\cos(bx+a)^{9}\right)}{1287\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
-32/1287*(99*cos(b*x + a)^13 - 234*cos(b*x + a)^11 + 143*cos(b*x + a)^9)/b
```

Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx = -\frac{32\left(99\cos\left(a+b\,x\right)^{13}-234\cos\left(a+b\,x\right)^{11}+143\cos\left(a+b\,x\right)^9\right)}{1287\,b}$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^5,x)
```

output
```
-(32*(143*cos(a + b*x)^9 - 234*cos(a + b*x)^11 + 99*cos(a + b*x)^13))/(128
7*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 346, normalized size of antiderivative = 7.52

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx = \frac{4950\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)^4\sin(bx+a)^2-4650\cos(2bx+2a)\cos(bx+a)\sin(2bx+$$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x)
```

output
```
(4950*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**4*sin(a + b*x)**2 -
4650*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**4 + 8800*cos(2*a + 2*
b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x)**2 - 6880*cos(2*a + 2*b
*x)*cos(a + b*x)*sin(2*a + 2*b*x)**2 - 14080*cos(2*a + 2*b*x)*cos(a + b*x)
*sin(a + b*x)**2 - 8960*cos(2*a + 2*b*x)*cos(a + b*x) - 9600*cos(2*a + 2*b
*x)*sin(a + b*x)**2 + 4800*cos(2*a + 2*b*x) + 9600*cos(a + b*x)*sin(2*a +
2*b*x)*sin(a + b*x) + 1485*sin(2*a + 2*b*x)**5*sin(a + b*x)**3 - 1455*sin(
2*a + 2*b*x)**5*sin(a + b*x) + 4400*sin(2*a + 2*b*x)**3*sin(a + b*x)**3 -
4080*sin(2*a + 2*b*x)**3*sin(a + b*x) - 21120*sin(2*a + 2*b*x)*sin(a + b*x
)**3 + 9600*sin(2*a + 2*b*x)*sin(a + b*x) - 12944)/(45045*b)
```

3.561 $\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx=\frac{16\sin^5(a+bx)}{5b}-\frac{48\sin^7(a+bx)}{7b}+\frac{16\sin^9(a+bx)}{3b}-\frac{16\sin^{11}(a+bx)}{11b}$$

output
```
16/5*sin(b*x+a)^5/b-48/7*sin(b*x+a)^7/b+16/3*sin(b*x+a)^9/b-16/11*sin(b*x+
a)^11/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx=\frac{(3042+3335\cos(2(a+bx))+910\cos(4(a+bx))+105\cos(6(a+bx)))\sin^5(a+bx)}{2310b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

output
```
((3042 + 3335*Cos[2*(a + b*x)] + 910*Cos[4*(a + b*x)] + 105*Cos[6*(a + b*x
)])*Sin[a + b*x]^5)/(2310*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4775, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(2a+2bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(2a+2bx)^4\cos(a+bx)^3dx$$

$$\Big\downarrow \; 4775$$

$$16\int \cos^7(a+bx)\sin^4(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$16\int \cos(a+bx)^7\sin(a+bx)^4dx$$

$$\Big\downarrow \; 3044$$

$$\frac{16\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)^3d\sin(a+bx)}{b}$$

$$\Big\downarrow \; 244$$

$$\frac{16\int \left(-\sin^{10}(a+bx)+3\sin^8(a+bx)-3\sin^6(a+bx)+\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{16\left(-\frac{1}{11}\sin^{11}(a+bx)+\frac{1}{3}\sin^9(a+bx)-\frac{3}{7}\sin^7(a+bx)+\frac{1}{5}\sin^5(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

output
```
(16*(Sin[a + b*x]^5/5 - (3*Sin[a + b*x]^7)/7 + Sin[a + b*x]^9/3 - Sin[a +
b*x]^11/11))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 12.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

method	result	size
parallelrisch	$\frac{16170\sin(bx+a)-2310\sin(3bx+3a)-2541\sin(5bx+5a)-165\sin(7bx+7a)+385\sin(9bx+9a)+105\sin(11bx+11a)}{73920b}$	70
default	$\frac{7\sin(bx+a)}{32b}-\frac{\sin(3bx+3a)}{32b}-\frac{11\sin(5bx+5a)}{320b}-\frac{\sin(7bx+7a)}{448b}+\frac{\sin(9bx+9a)}{192b}+\frac{\sin(11bx+11a)}{704b}$	83
risch	$\frac{7\sin(bx+a)}{32b}-\frac{\sin(3bx+3a)}{32b}-\frac{11\sin(5bx+5a)}{320b}-\frac{\sin(7bx+7a)}{448b}+\frac{\sin(9bx+9a)}{192b}+\frac{\sin(11bx+11a)}{704b}$	83
orering	Expression too large to display	1585

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/73920*(16170*sin(b*x+a)-2310*sin(3*b*x+3*a)-2541*sin(5*b*x+5*a)-165*sin(
7*b*x+7*a)+385*sin(9*b*x+9*a)+105*sin(11*b*x+11*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx = \frac{16\left(105\cos(bx+a)^{10}-140\cos(bx+a)^8+5\cos(bx+a)^6+6\cos(bx+a)^4+8\cos(bx+a)^2+16\right)\mathrm{si}}{1155\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
16/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*c
os(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(53) = 106$.

Time = 11.16 (sec) , antiderivative size = 366, normalized size of antiderivative = 6.00

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \begin{cases} \frac{46\sin^3(a+bx)\sin^4(2a+2bx)}{165b} + \frac{192\sin^3(a+bx)\sin^2(2a+2bx)\cos^2(2a+2bx)}{385b} + \frac{256\sin^3(a+bx)\cos^4(2a+2bx)}{1155b} + \frac{272\sin^2(a+bx)\sin^3(2 \\ x\sin^4(2a)\cos^3(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

output
```
Piecewise((46*sin(a + b*x)**3*sin(2*a + 2*b*x)**4/(165*b) + 192*sin(a + b*
x)**3*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/(385*b) + 256*sin(a + b*x)**
3*cos(2*a + 2*b*x)**4/(1155*b) + 272*sin(a + b*x)**2*sin(2*a + 2*b*x)**3*c
os(a + b*x)*cos(2*a + 2*b*x)/(1155*b) + 256*sin(a + b*x)**2*sin(2*a + 2*b*
x)*cos(a + b*x)*cos(2*a + 2*b*x)**3/(1155*b) + 211*sin(a + b*x)*sin(2*a +
2*b*x)**4*cos(a + b*x)**2/(1155*b) + 304*sin(a + b*x)*sin(2*a + 2*b*x)**2*
cos(a + b*x)**2*cos(2*a + 2*b*x)**2/(385*b) + 128*sin(a + b*x)*cos(a + b*x
)**2*cos(2*a + 2*b*x)**4/(231*b) - 472*sin(2*a + 2*b*x)**3*cos(a + b*x)**3
*cos(2*a + 2*b*x)/(1155*b) - 64*sin(2*a + 2*b*x)*cos(a + b*x)**3*cos(2*a +
 2*b*x)**3/(231*b), Ne(b, 0)), (x*sin(2*a)**4*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{105\sin(11bx+11a) + 385\sin(9bx+9a) - 165\sin(7bx+7a) - 2541\sin(5bx+5a) - 2310\sin(3}{73920b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/73920*(105*sin(11*b*x + 11*a) + 385*sin(9*b*x + 9*a) - 165*sin(7*b*x + 7
*a) - 2541*sin(5*b*x + 5*a) - 2310*sin(3*b*x + 3*a) + 16170*sin(b*x + a))/
b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{105\sin(11bx+11a)+385\sin(9bx+9a)-165\sin(7bx+7a)-2541\sin(5bx+5a)-2310\sin(3b}{73920b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
1/73920*(105*sin(11*b*x + 11*a) + 385*sin(9*b*x + 9*a) - 165*sin(7*b*x + 7
*a) - 2541*sin(5*b*x + 5*a) - 2310*sin(3*b*x + 3*a) + 16170*sin(b*x + a))/
b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{-\frac{16\sin(a+bx)^{11}}{11}+\frac{16\sin(a+bx)^9}{3}-\frac{48\sin(a+bx)^7}{7}+\frac{16\sin(a+bx)^5}{5}}{b}$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^4,x)
```

output
```
((16*sin(a + b*x)^5)/5 - (48*sin(a + b*x)^7)/7 + (16*sin(a + b*x)^9)/3 - (
16*sin(a + b*x)^11)/11)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.61

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$$
$$= \frac{168\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)^3\sin(bx+a)^2 - 152\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)^3 \cdots}{}$$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x)
```

output
```
(168*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**3*sin(a + b*x)**2 - 1
52*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**3 + 576*cos(2*a + 2*b*x
)*cos(a + b*x)*sin(2*a + 2*b*x)*sin(a + b*x)**2 - 320*cos(2*a + 2*b*x)*cos
(a + b*x)*sin(2*a + 2*b*x) + 544*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x
) + 63*sin(2*a + 2*b*x)**4*sin(a + b*x)**3 - 61*sin(2*a + 2*b*x)**4*sin(a
+ b*x) + 432*sin(2*a + 2*b*x)**2*sin(a + b*x)**3 - 368*sin(2*a + 2*b*x)**2
*sin(a + b*x) + 544*sin(2*a + 2*b*x)*sin(a + b*x)**2 - 272*sin(2*a + 2*b*x
) - 384*sin(a + b*x)**3 + 640*sin(a + b*x))/(1155*b)
```

3.562 $\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx=-\frac{8\cos^7(a+bx)}{7b}+\frac{8\cos^9(a+bx)}{9b}$$

output
```
-8/7*cos(b*x+a)^7/b+8/9*cos(b*x+a)^9/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx=\frac{4\cos^7(a+bx)(-11+7\cos(2(a+bx)))}{63b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

output
```
(4*Cos[a + b*x]^7*(-11 + 7*Cos[2*(a + b*x)]))/(63*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4775, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2a+2bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(2a+2bx)^3\cos(a+bx)^3dx$$

$$\Big\downarrow \text{ 4775}$$

$$8\int \cos^6(a+bx)\sin^3(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$8\int \cos(a+bx)^6\sin(a+bx)^3dx$$

$$\Big\downarrow \text{ 3045}$$

$$-\frac{8\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{8\int \left(\cos^6(a+bx)-\cos^8(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{8\left(\frac{1}{7}\cos^7(a+bx)-\frac{1}{9}\cos^9(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

output
```
(-8*(Cos[a + b*x]^7/7 - Cos[a + b*x]^9/9))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 5.96 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

method	result
parallelrisch	$\frac{4864+135\cos(7bx+7a)+35\cos(9bx+9a)-840\cos(3bx+3a)-1890\cos(bx+a)}{10080b}$
default	$-\frac{3\cos(bx+a)}{16b}-\frac{\cos(3bx+3a)}{12b}+\frac{3\cos(7bx+7a)}{224b}+\frac{\cos(9bx+9a)}{288b}$
risch	$-\frac{3\cos(bx+a)}{16b}-\frac{\cos(3bx+3a)}{12b}+\frac{3\cos(7bx+7a)}{224b}+\frac{\cos(9bx+9a)}{288b}$
orering	$-\frac{4540\left(-3\cos(bx+a)^2\sin(2bx+2a)^3b\sin(bx+a)+6\cos(bx+a)^3\sin(2bx+2a)^2b\cos(2bx+2a)\right)}{3969b^2}-\frac{754\left(-6b^3\sin(bx+a)^3\sin\right.}{}$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10080*(4864+135*cos(7*b*x+7*a)+35*cos(9*b*x+9*a)-840*cos(3*b*x+3*a)-1890
*cos(b*x+a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(7\cos\left(bx+a\right)^9 - 9\cos\left(bx+a\right)^7\right)}{63\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
8/63*(7*cos(b*x + a)^9 - 9*cos(b*x + a)^7)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(26) = 52$.

Time = 4.85 (sec) , antiderivative size = 284, normalized size of antiderivative = 9.16

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{94\sin^3(a+bx)\sin^3(2a+2bx)}{315b} - \frac{32\sin^3(a+bx)\sin(2a+2bx)\cos^2(2a+2bx)}{105b} - \frac{4\sin^2(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{7b} - \\ x\sin^3(2a)\cos^3(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

output
```
Piecewise((-94*sin(a + b*x)**3*sin(2*a + 2*b*x)**3/(315*b) - 32*sin(a + b*
x)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/(105*b) - 4*sin(a + b*x)**2*sin
(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)/(7*b) - 64*sin(a + b*x)**2*
cos(a + b*x)*cos(2*a + 2*b*x)**3/(105*b) + 13*sin(a + b*x)*sin(2*a + 2*b*x
)**3*cos(a + b*x)**2/(105*b) + 8*sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x
)**2*cos(2*a + 2*b*x)**2/(35*b) - 46*sin(2*a + 2*b*x)**2*cos(a + b*x)**3*c
os(2*a + 2*b*x)/(105*b) - 16*cos(a + b*x)**3*cos(2*a + 2*b*x)**3/(63*b), N
e(b, 0)), (x*sin(2*a)**3*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{7\cos(9\,bx+9\,a)+27\cos(7\,bx+7\,a)-168\cos(3\,bx+3\,a)-378\cos(bx+a)}{2016\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/2016*(7*cos(9*b*x + 9*a) + 27*cos(7*b*x + 7*a) - 168*cos(3*b*x + 3*a) -
378*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx = \frac{8\left(7\cos(bx+a)^9-9\cos(bx+a)^7\right)}{63\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
8/63*(7*cos(b*x + a)^9 - 9*cos(b*x + a)^7)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx = -\frac{8\left(9\cos(a+b\,x)^7-7\cos(a+b\,x)^9\right)}{63\,b}$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^3,x)
```

output
```
-(8*(9*cos(a + b*x)^7 - 7*cos(a + b*x)^9))/(63*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 242, normalized size of antiderivative = 7.81

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx$$
$$= \frac{70\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)^2\sin(bx+a)^2-58\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a}{}$$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x)
```

output
```
(70*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x)**2 - 58
*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)**2 - 112*cos(2*a + 2*b*x)*
cos(a + b*x)*sin(a + b*x)**2 - 80*cos(2*a + 2*b*x)*cos(a + b*x) - 72*cos(2
*a + 2*b*x)*sin(a + b*x)**2 + 36*cos(2*a + 2*b*x) + 72*cos(a + b*x)*sin(2*
a + 2*b*x)*sin(a + b*x) + 35*sin(2*a + 2*b*x)**3*sin(a + b*x)**3 - 33*sin(
2*a + 2*b*x)**3*sin(a + b*x) - 168*sin(2*a + 2*b*x)*sin(a + b*x)**3 + 72*s
in(2*a + 2*b*x)*sin(a + b*x) - 116)/(315*b)
```

3.563 $\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx=\frac{4\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}+\frac{4\sin^7(a+bx)}{7b}$$

output
```
4/3*sin(b*x+a)^3/b-8/5*sin(b*x+a)^5/b+4/7*sin(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx=\frac{(157+108\cos(2(a+bx))+15\cos(4(a+bx)))\sin^3(a+bx)}{210b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
((157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(210*b
)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4775, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(2a+2bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^2\cos(a+bx)^3dx$$

$$\Big\downarrow \text{4775}$$

$$4\int \cos^5(a+bx)\sin^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \cos(a+bx)^5\sin(a+bx)^2dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{4\int \sin^2(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{4\int \left(\sin^6(a+bx)-2\sin^4(a+bx)+\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(\frac{1}{7}\sin^7(a+bx)-\frac{2}{5}\sin^5(a+bx)+\frac{1}{3}\sin^3(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
(4*(Sin[a + b*x]^3/3 - (2*Sin[a + b*x]^5)/5 + Sin[a + b*x]^7/7))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

method	result
default	$\frac{5\sin(bx+a)}{16b}-\frac{\sin(3bx+3a)}{48b}-\frac{3\sin(5bx+5a)}{80b}-\frac{\sin(7bx+7a)}{112b}$
risch	$\frac{5\sin(bx+a)}{16b}-\frac{\sin(3bx+3a)}{48b}-\frac{3\sin(5bx+5a)}{80b}-\frac{\sin(7bx+7a)}{112b}$
parallelrisch	$\frac{144\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{6}\tan(bx+a)+\left(-24\tan(bx+a)^{2}+288\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{5}+\left(48\tan(bx+a)^{3}-768\tan(bx+a)\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}+\left(-\right.}{}$
orering	$-\frac{12916\left(-3\cos(bx+a)^{2}\sin(2bx+2a)^{2}b\sin(bx+a)+4\cos(bx+a)^{3}\sin(2bx+2a)b\cos(2bx+2a)\right)}{11025b^{2}}-\frac{94\left(-6b^{3}\sin(bx+a)^{3}\sin(\right.}{}$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
5/16*sin(b*x+a)/b-1/48*sin(3*b*x+3*a)/b-3/80/b*sin(5*b*x+5*a)-1/112/b*sin(
7*b*x+7*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(15\cos(bx+a)^6-3\cos(bx+a)^4-4\cos(bx+a)^2-8\right)\sin(bx+a)}{105\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-4/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b
*x + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(39) = 78.

Time = 2.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.39

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx = \begin{cases} \frac{38\sin^3(a+bx)\sin^2(2a+2bx)}{105b} + \frac{32\sin^3(a+bx)\cos^2(2a+2bx)}{105b} + \frac{8\sin^2(a+bx)\sin(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{35b} + \frac{11\sin(a+bx)\sin \\ x\sin^2(2a)\cos^3(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((38*sin(a + b*x)**3*sin(2*a + 2*b*x)**2/(105*b) + 32*sin(a + b*x
)**3*cos(2*a + 2*b*x)**2/(105*b) + 8*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(
a + b*x)*cos(2*a + 2*b*x)/(35*b) + 11*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos
(a + b*x)**2/(35*b) + 24*sin(a + b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/
(35*b) - 12*sin(2*a + 2*b*x)*cos(a + b*x)**3*cos(2*a + 2*b*x)/(35*b), Ne(b
, 0)), (x*sin(2*a)**2*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx$$
$$= -\frac{15\sin(7\,bx+7\,a)+63\sin(5\,bx+5\,a)+35\sin(3\,bx+3\,a)-525\sin(bx+a)}{1680\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
-1/1680*(15*sin(7*b*x + 7*a) + 63*sin(5*b*x + 5*a) + 35*sin(3*b*x + 3*a) -
 525*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx$$
$$= -\frac{15\sin(7\,bx+7\,a)+63\sin(5\,bx+5\,a)+35\sin(3\,bx+3\,a)-525\sin(bx+a)}{1680\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-1/1680*(15*sin(7*b*x + 7*a) + 63*sin(5*b*x + 5*a) + 35*sin(3*b*x + 3*a) -
 525*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx = \frac{4\left(15\sin\left(a+b\,x\right)^7-42\sin\left(a+b\,x\right)^5+35\sin\left(a+b\,x\right)^3\right)}{105\,b}$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^2,x)
```

output
```
(4*(35*sin(a + b*x)^3 - 42*sin(a + b*x)^5 + 15*sin(a + b*x)^7))/(105*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.85

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx = \frac{60\cos\left(2bx+2a\right)\cos\left(bx+a\right)\sin\left(2bx+2a\right)\sin\left(bx+a\right)^2-36\cos\left(2bx+2a\right)\cos\left(bx+a\right)\sin\left(2bx+2a\right)}{}$$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x)
```

output
```
(60*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)*sin(a + b*x)**2 - 36*co
s(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x) - 72*cos(2*a + 2*b*x)*cos(a +
 b*x)*sin(a + b*x) + 45*sin(2*a + 2*b*x)**2*sin(a + b*x)**3 - 39*sin(2*a +
 2*b*x)**2*sin(a + b*x) - 72*sin(2*a + 2*b*x)*sin(a + b*x)**2 + 36*sin(2*a
 + 2*b*x) - 40*sin(a + b*x)**3 + 72*sin(a + b*x))/(105*b)
```

3.564 $\int \cos^3(a+bx)\sin(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx=-\frac{2\cos^5(a+bx)}{5b}$$

output
```
-2/5*cos(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx=-\frac{2\cos^5(a+bx)}{5b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

output
```
(-2*Cos[a + b*x]^5)/(5*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4775, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)\cos(a+bx)^3dx$$

$$\Big\downarrow \text{4775}$$

$$2\int \cos^4(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \cos(a+bx)^4\sin(a+bx)dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{2\int \cos^4(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\cos^5(a+bx)}{5b}$$

input
```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

output
```
(-2*Cos[a + b*x]^5)/(5*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(13) = 26.

Time = 1.81 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.73

method	result
default	$-\frac{\cos(bx+a)}{4b}-\frac{\cos(3bx+3a)}{8b}-\frac{\cos(5bx+5a)}{40b}$
risch	$-\frac{\cos(bx+a)}{4b}-\frac{\cos(3bx+3a)}{8b}-\frac{\cos(5bx+5a)}{40b}$
parallelrisch	$\frac{-\frac{4}{5}+4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{5}\tan(bx+a)+4\left(-\tan(bx+a)^{2}+1\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}-8\tan(bx+a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}-\frac{12\tan(bx+a)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{5}}{b\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}\right)^{3}\left(\tan(bx+a)^{2}+1\right)}$
orering	$-\frac{259\left(-3\cos(bx+a)^{2}\sin(2bx+2a)b\sin(bx+a)+2\cos(bx+a)^{3}b\cos(2bx+2a)\right)}{225b^{2}}-\frac{7\left(-6b^{3}\sin(bx+a)^{3}\sin(2bx+2a)+36\cos(b\right.}{}$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(b*x+a)/b-1/8*cos(3*b*x+3*a)/b-1/40*cos(5*b*x+5*a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx = -\frac{2\cos\left(bx+a\right)^5}{5\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-2/5*cos(b*x + a)^5/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(14) = 28.

Time = 0.80 (sec) , antiderivative size = 117, normalized size of antiderivative = 7.80

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{2\sin^3(a+bx)\sin(2a+2bx)}{5b} - \frac{4\sin^2(a+bx)\cos(a+bx)\cos(2a+2bx)}{5b} + \frac{\sin(a+bx)\sin(2a+2bx)\cos^2(a+bx)}{5b} - \frac{2\cos^3(a+bx)\cos(2a+}{5b} \\ x\sin(2a)\cos^3(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a),x)
```

output
```
Piecewise((-2*sin(a + b*x)**3*sin(2*a + 2*b*x)/(5*b) - 4*sin(a + b*x)**2*c
os(a + b*x)*cos(2*a + 2*b*x)/(5*b) + sin(a + b*x)*sin(2*a + 2*b*x)*cos(a +
 b*x)**2/(5*b) - 2*cos(a + b*x)**3*cos(2*a + 2*b*x)/(5*b), Ne(b, 0)), (x*s
in(2*a)*cos(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx = -\frac{\cos(5\,bx+5\,a)+5\,\cos(3\,bx+3\,a)+10\,\cos(bx+a)}{40\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/40*(cos(5*b*x + 5*a) + 5*cos(3*b*x + 3*a) + 10*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx = -\frac{2\,\cos(bx+a)^5}{5\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
-2/5*cos(b*x + a)^5/b
```

Mupad [B] (verification not implemented)

Time = 20.55 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx = -\frac{2\cos(a+b\,x)^5}{5\,b}$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x),x)
```

output
```
-(2*cos(a + b*x)^5)/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 9.20

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx$$
$$= \frac{-12\cos(2bx+2a)\cos(bx+a)\sin(bx+a)^2 - 12\cos(2bx+2a)\cos(bx+a) - 6\cos(2bx+2a)\sin(bx-}{}$$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a),x)
```

output
```
( - 12*cos(2*a + 2*b*x)*cos(a + b*x)*sin(a + b*x)**2 - 12*cos(2*a + 2*b*x)
*cos(a + b*x) - 6*cos(2*a + 2*b*x)*sin(a + b*x)**2 + 3*cos(2*a + 2*b*x) +
6*cos(a + b*x)*sin(2*a + 2*b*x)*sin(a + b*x) - 18*sin(2*a + 2*b*x)*sin(a +
 b*x)**3 + 6*sin(2*a + 2*b*x)*sin(a + b*x) - 25)/(30*b)
```

3.565 $\int \cos^3(a+bx)\csc(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{\cos(a+bx)}{2b}$$

output
```
-1/2*arctanh(cos(b*x+a))/b+1/2*cos(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx=\frac{1}{2}\left(\frac{\cos(a+bx)}{b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}\right)$$

input
```
Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

output
```
(Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b)/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3042, 4775, 3042, 25, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)}dx$$

$$\Big\downarrow \ 4775$$

$$\frac{1}{2}\int \cos(a+bx)\cot(a+bx)dx$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\int -\sin\left(a+bx+\frac{\pi}{2}\right)\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{2}\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \ 3072$$

$$-\frac{\int \frac{\cos^2(a+bx)}{1-\cos^2(a+bx)}d\cos(a+bx)}{2b}$$

$$\Big\downarrow \ 262$$

$$-\frac{\int \frac{1}{1-\cos^2(a+bx)}d\cos(a+bx)-\cos(a+bx)}{2b}$$

$$\Big\downarrow \ 219$$

$$-\frac{\text{arctanh}(\cos(a+bx))-\cos(a+bx)}{2b}$$

input
```
Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

output
```
-1/2*(ArcTanh[Cos[a + b*x]] - Cos[a + b*x])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\cos(bx+a)+\ln(\csc(bx+a)-\cot(bx+a))}{2b}$	29
risch	$\frac{e^{i(bx+a)}}{4b}+\frac{e^{-i(bx+a)}}{4b}+\frac{\ln(e^{i(bx+a)}-1)}{2b}-\frac{\ln(e^{i(bx+a)}+1)}{2b}$	64

input
```
int(cos(b*x+a)^3*csc(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx = \frac{2\cos(bx+a)-\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)}{4b}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="fricas")
```

output
```
1/4*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a)
+ 1/2))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(cos(b*x+a)**3*csc(2*b*x+2*a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(24) = 48$.

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.29

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx$$
$$= \frac{2\cos(bx+a) - \log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right) -}{4b}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="maxima")
```

output
```
1/4*(2*cos(b*x + a) - log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(
b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a
) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx$$
$$= \frac{2\cos(bx+a) - \log(\cos(bx+a)+1) + \log(-\cos(bx+a)+1)}{4b}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="giac")
```

output
```
1/4*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(-cos(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx = \frac{\frac{\cos(a+b\,x)}{2}-\frac{\operatorname{atanh}(\cos(a+b\,x))}{2}}{b}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x),x)
```

output
```
(cos(a + b*x)/2 - atanh(cos(a + b*x))/2)/b
```

Reduce [F]

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx = \int \cos(bx+a)^3\csc(2bx+2a)\,dx$$

input
```
int(cos(b*x+a)^3*csc(2*b*x+2*a),x)
```

output
```
int(cos(a + b*x)**3*csc(2*a + 2*b*x),x)
```

3.566 $\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\csc(a+bx)}{4b}$$

output
```
-1/4*csc(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\csc(a+bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/4*Csc[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4775, 3042, 25, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^2}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{4}\int \cot(a+bx)\csc(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int -\sec\left(a+bx-\frac{\pi}{2}\right)\tan\left(a+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{4}\int \sec\left(\frac{1}{2}(2a-\pi)+bx\right)\tan\left(\frac{1}{2}(2a-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\int 1d\csc(a+bx)}{4b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\csc(a+bx)}{4b}$$

input
```
Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

output
```
-1/4*Csc[a + b*x]/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{1}{4\sin(bx+a)b}$	14
risch	$-\frac{i e^{i(bx+a)}}{2b\left(e^{2i(bx+a)}-1\right)}$	29

input
```
int(cos(b*x+a)^3*csc(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/sin(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{1}{4\,b\sin\left(bx+a\right)}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

output
```
-1/4/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*csc(2*b*x+2*a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(11) = 22.

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 6.46

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{\cos\left(bx+a\right)\sin\left(2\,bx+2\,a\right)-\cos\left(2\,bx+2\,a\right)\sin\left(bx+a\right)+\sin\left(bx+a\right)}{2\,\left(b\cos\left(2\,bx+2\,a\right)^2+b\sin\left(2\,bx+2\,a\right)^2-2\,b\cos\left(2\,bx+2\,a\right)+b\right)}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

output
```
-1/2*(cos(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)*sin(b*x + a) + sin(
b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x + 2
*a) + b)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{1}{4\,b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-1/4/(b*sin(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx = -\frac{1}{4\,b\,\sin(a+b\,x)}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^2,x)
```

output
```
-1/(4*b*sin(a + b*x))
```

Reduce [F]

$$\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx=\int \cos(bx+a)^3\csc(2bx+2a)^2\,dx$$

input
```
int(cos(b*x+a)^3*csc(2*b*x+2*a)^2,x)
```

output
```
int(cos(a + b*x)**3*csc(2*a + 2*b*x)**2,x)
```

3.567 $\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{16b}-\frac{\cot(a+bx)\csc(a+bx)}{16b}$$

output
```
-1/16*arctanh(cos(b*x+a))/b-1/16*cot(b*x+a)*csc(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(34) = 68.

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx=\frac{1}{8}\left(-\frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}\right)$$

input
```
Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

output
```
(-1/8*Csc[(a + b*x)/2]^2/b - Log[Cos[(a + b*x)/2]]/(2*b) + Log[Sin[(a + b*
x)/2]]/(2*b) + Sec[(a + b*x)/2]^2/(8*b))/8
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4775, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^3}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{8}\int \csc^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \csc(a+bx)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{8}\left(\frac{1}{2}\int \csc(a+bx)dx-\frac{\cot(a+bx)\csc(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{1}{2}\int \csc(a+bx)dx-\frac{\cot(a+bx)\csc(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{8}\left(-\frac{\text{arctanh}(\cos(a+bx))}{2b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}\right)$$

input
```
Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

output
```
(-1/2*ArcTanh[Cos[a + b*x]]/b - (Cot[a + b*x]*Csc[a + b*x])/(2*b))/8
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 5.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{-\frac{\csc(bx+a)\cot(bx+a)}{2}+\frac{\ln(\csc(bx+a)-\cot(bx+a))}{2}}{8b}$	39
risch	$\frac{e^{3i(bx+a)}+e^{i(bx+a)}}{8b\left(e^{2i(bx+a)}-1\right)^{2}}-\frac{\ln\left(e^{i(bx+a)}+1\right)}{16b}+\frac{\ln\left(e^{i(bx+a)}-1\right)}{16b}$	73

input
```
int(cos(b*x+a)^3*csc(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/b*(-1/2*csc(b*x+a)*cot(b*x+a)+1/2*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.12

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx = -\frac{\left(\cos\left(bx+a\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-2\cos\left(\right.}{32\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

output
```
-1/32*((cos(b*x + a)^2 - 1)*log(1/2*cos(b*x + a) + 1/2) - (cos(b*x + a)^2
- 1)*log(-1/2*cos(b*x + a) + 1/2) - 2*cos(b*x + a))/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*csc(2*b*x+2*a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(30) = 60$.

Time = 0.05 (sec) , antiderivative size = 558, normalized size of antiderivative = 16.41

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

output
```
1/32*(4*(cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) - 4*(2*cos(2*b*
x + 2*a) - 1)*cos(3*b*x + 3*a) - 8*cos(2*b*x + 2*a)*cos(b*x + a) + (2*(2*c
os(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x +
 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin
(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(
a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*(2*cos(2*b
*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^
2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x
 + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + c
os(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(sin(3*b*x + 3*a)
 + sin(b*x + a))*sin(4*b*x + 4*a) - 8*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) -
8*sin(2*b*x + 2*a)*sin(b*x + a) + 4*cos(b*x + a))/(b*cos(4*b*x + 4*a)^2 +
4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*b*x + 2*a) - b)*cos(4*
b*x + 4*a) - 4*b*cos(2*b*x + 2*a) + b)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx = \frac{\frac{2\cos(bx+a)}{\cos(bx+a)^2-1} - \log(\cos(bx+a)+1) + \log(-\cos(bx+a)+1)}{32\,b}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

output
```
1/32*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log(-c
os(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx = \frac{\cos(a+b\,x)}{16\,b\,\left(\cos(a+b\,x)^2-1\right)} - \frac{\mathrm{atanh}(\cos(a+b\,x))}{16\,b}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^3,x)
```

output
```
cos(a + b*x)/(16*b*(cos(a + b*x)^2 - 1)) - atanh(cos(a + b*x))/(16*b)
```

Reduce [F]

$$\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx = \int \cos(bx+a)^3\csc(2bx+2a)^3\,dx$$

input
```
int(cos(b*x+a)^3*csc(2*b*x+2*a)^3,x)
```

output
```
int(cos(a + b*x)**3*csc(2*a + 2*b*x)**3,x)
```

3.568 $\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{16b} - \frac{\csc(a+bx)}{16b} - \frac{\csc^3(a+bx)}{48b}$$

output
```
1/16*arctanh(sin(b*x+a))/b-1/16*csc(b*x+a)/b-1/48*csc(b*x+a)^3/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = -\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(a+bx)\right)}{48b}$$

input
```
Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

output
```
-1/48*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4775, 3042, 3101, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^4}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{16}\int \csc^4(a+bx)\sec(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{16}\int \csc(a+bx)^4\sec(a+bx)dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{16b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{16b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\int \left(-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)}{16b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\text{arctanh}(\csc(a+bx))+\frac{1}{3}\csc^3(a+bx)+\csc(a+bx)}{16b}$$

input
```
Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

output
```
-1/16*(-ArcTanh[Csc[a + b*x]] + Csc[a + b*x] + Csc[a + b*x]^3/3)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 10.95 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{-\frac{1}{3\sin(bx+a)^3}-\frac{1}{\sin(bx+a)}+\ln(\sec(bx+a)+\tan(bx+a))}{16b}$	41
risch	$-\frac{i(3\,\mathrm{e}^{5i(bx+a)}-10\,\mathrm{e}^{3i(bx+a)}+3\,\mathrm{e}^{i(bx+a)})}{24b(\mathrm{e}^{2i(bx+a)}-1)^3}+\frac{\ln(\mathrm{e}^{i(bx+a)}+i)}{16b}-\frac{\ln(\mathrm{e}^{i(bx+a)}-i)}{16b}$	91

input
```
int(cos(b*x+a)^3*csc(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/b*(-1/3/sin(b*x+a)^3-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(37) = 74.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.19

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx$$
$$=\frac{3\left(\cos\left(bx+a\right)^2-1\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-3\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\sin\left(bx+a\right)+1\right)\mathrm{si}}{96\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

output
```
1/96*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b
*x + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)
/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*csc(2*b*x+2*a)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(37) = 74$.

Time = 0.17 (sec) , antiderivative size = 834, normalized size of antiderivative = 19.40

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

output
```
1/96*(4*(3*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*
b*x + 6*a) + 36*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 1
2*(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 3*(2*(3*cos(4*
b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2
 + 6*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 9*
cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6
*a) - sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 + 18*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) - 1)*log((cos(b*x
 + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(
b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(
b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(
3*cos(5*b*x + 5*a) - 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a
) - 12*(3*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) - 12
*(10*cos(3*b*x + 3*a) - 3*cos(b*x + a))*sin(4*b*x + 4*a) - 40*(3*cos(2*b*x
 + 2*a) - 1)*sin(3*b*x + 3*a) + 120*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 36
*cos(b*x + a)*sin(2*b*x + 2*a) + 36*cos(2*b*x + 2*a)*sin(b*x + a) - 12*sin
(b*x + a))/(b*cos(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x
+ 2*a)^2 + b*sin(6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 - 18*b*sin(4*b*x
+ 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*cos(4*b*x + 4*a)
 - 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) - 6*(3*b*cos(2*b*x + 2*a)...
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = -\frac{\frac{2\left(3\sin(bx+a)^2+1\right)}{\sin(bx+a)^3} - 3\log\left(\sin\left(bx+a\right)+1\right) + 3\log\left(-\sin\left(bx+a\right)+1\right)}{96\,b}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

output
```
-1/96*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) +
 3*log(-sin(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = \frac{\operatorname{atanh}(\sin(a+b\,x))}{16\,b} - \frac{\frac{\sin(a+b\,x)^2}{16}+\frac{1}{48}}{b\sin(a+b\,x)^3}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^4,x)
```

output
```
atanh(sin(a + b*x))/(16*b) - (sin(a + b*x)^2/16 + 1/48)/(b*sin(a + b*x)^3)
```

Reduce [F]

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = \int \cos(bx+a)^3\csc(2bx+2a)^4\,dx$$

input
```
int(cos(b*x+a)^3*csc(2*b*x+2*a)^4,x)
```

output
```
int(cos(a + b*x)**3*csc(2*a + 2*b*x)**4,x)
```

3.569 $\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = -\frac{15\text{arctanh}(\cos(a+bx))}{256b} - \frac{9\cot(a+bx)\csc(a+bx)}{256b} - \frac{\cot^3(a+bx)\csc(a+bx)}{128b} + \frac{\sec(a+bx)}{32b}$$

output
```
-15/256*arctanh(cos(b*x+a))/b-9/256*cot(b*x+a)*csc(b*x+a)/b-1/128*cot(b*x+
a)^3*csc(b*x+a)/b+1/32*sec(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 195 vs. $2(68) = 136$.

Time = 0.53 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.87

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = -\frac{7\csc^2\left(\frac{1}{2}(a+bx)\right)}{1024b} - \frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{2048b} - \frac{15\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{256b} + \frac{15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{256b} + \frac{7\sec^2\left(\frac{1}{2}(a+bx)\right)}{1024b} + \frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{2048b} + \frac{\sin\left(\frac{1}{2}(a+bx)\right)}{32b\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)} - \frac{\sin\left(\frac{1}{2}(a+bx)\right)}{32b\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}$$

input
```
Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^5,x]
```

output
```
(-7*Csc[(a + b*x)/2]^2)/(1024*b) - Csc[(a + b*x)/2]^4/(2048*b) - (15*Log[C
os[(a + b*x)/2]])/(256*b) + (15*Log[Sin[(a + b*x)/2]])/(256*b) + (7*Sec[(a
 + b*x)/2]^2)/(1024*b) + Sec[(a + b*x)/2]^4/(2048*b) + Sin[(a + b*x)/2]/(3
2*b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - Sin[(a + b*x)/2]/(32*b*(Cos[(
a + b*x)/2] + Sin[(a + b*x)/2]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 4775, 3042, 3102, 25, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^5}dx$$

$$\Big\downarrow \; 4775$$

$$\frac{1}{32}\int \csc^5(a+bx)\sec^2(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{32}\int \csc(a+bx)^5\sec(a+bx)^2dx$$

$$\Big\downarrow \; 3102$$

$$\frac{\int -\frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{32b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{32b}$$

$$\Big\downarrow \; 252$$

$$\frac{\frac{5}{4}\int \frac{\sec^4(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \; 252$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\int \frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \; 262$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\sec^2(a+bx)}d\sec(a+bx)-\sec(a+bx)\right)\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\sec(a+bx))-\sec(a+bx))\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{32b}$$

input
```
Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^5,x]
```

output
```
(-1/4*Sec[a + b*x]^5/(1 - Sec[a + b*x]^2)^2 + (5*((-3*(ArcTanh[Sec[a + b*x
]] - Sec[a + b*x]))/2 + Sec[a + b*x]^3/(2*(1 - Sec[a + b*x]^2))))/4)/(32*b
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 20.96 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{-\frac{1}{4 \sin(bx+a)^{4} \cos(bx+a)}-\frac{5}{8 \sin(bx+a)^{2} \cos(bx+a)}+\frac{15}{8 \cos(bx+a)}+\frac{15 \ln(\csc(bx+a)-\cot(bx+a))}{8}}{32b}$	71
risch	$\frac{15\, e^{9i(bx+a)}-40\, e^{7i(bx+a)}+18\, e^{5i(bx+a)}-40\, e^{3i(bx+a)}+15\, e^{i(bx+a)}}{128b\left(e^{2i(bx+a)}-1\right)^{4}\left(e^{2i(bx+a)}+1\right)}-\frac{15\ln\left(e^{i(bx+a)}+1\right)}{256b}+\frac{15\ln\left(e^{i(bx+a)}-1\right)}{256b}$	123

input
```
int(cos(b*x+a)^3*csc(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)-5/8/sin(b*x+a)^2/cos(b*x+a)+15/8/cos(
b*x+a)+15/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(60) = 120.

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.94

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx$$
$$= \frac{30\cos(bx+a)^4-50\cos(bx+a)^2-15\left(\cos(bx+a)^5-2\cos(bx+a)^3+\cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx-\right.}{512\left(b\cos(bx+a)^5-2b\cos(bx\right.}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a)^5,x, algorithm="fricas")
```

output
```
1/512*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(
b*x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^
5 - 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b
*cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*csc(2*b*x+2*a)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2237 vs. $2(60) = 120$.

Time = 0.10 (sec) , antiderivative size = 2237, normalized size of antiderivative = 32.90

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a)^5,x, algorithm="maxima")
```

output
```
1/512*(4*(15*cos(9*b*x + 9*a) - 40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a)
- 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) - 60*(3*cos(8*
b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a)
- 1)*cos(9*b*x + 9*a) + 12*(40*cos(7*b*x + 7*a) - 18*cos(5*b*x + 5*a) + 40
*cos(3*b*x + 3*a) - 15*cos(b*x + a))*cos(8*b*x + 8*a) - 160*(2*cos(6*b*x +
 6*a) + 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) + 8*
(18*cos(5*b*x + 5*a) - 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(6*b*x +
6*a) + 72*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x + 5*a) -
 40*(8*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 160*(3*cos(2*
b*x + 2*a) - 1)*cos(3*b*x + 3*a) - 180*cos(2*b*x + 2*a)*cos(b*x + a) + 15*
(2*(3*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2
*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*
x + 6*a) + 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) -
 9*cos(8*b*x + 8*a)^2 - 4*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*co
s(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b
*x + 4*a) - 4*cos(4*b*x + 4*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x +
 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(
10*b*x + 10*a) - sin(10*b*x + 10*a)^2 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*
x + 4*a) - 3*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4
*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = \frac{\frac{2\left(7\cos(bx+a)^3-9\cos(bx+a)\right)}{\left(\cos(bx+a)^2-1\right)^2}+\frac{16}{\cos(bx+a)}-15\log\left(\cos\left(bx+a\right)+1\right)+15\log\left(-\cos\left(bx+a\right)+1\right)}{512\,b}$$

input
```
integrate(cos(b*x+a)^3*csc(2*b*x+2*a)^5,x, algorithm="giac")
```

output
```
1/512*(2*(7*cos(b*x + a)^3 - 9*cos(b*x + a))/(cos(b*x + a)^2 - 1)^2 + 16/c
os(b*x + a) - 15*log(cos(b*x + a) + 1) + 15*log(-cos(b*x + a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 19.97 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = \frac{\frac{15\cos(a+b\,x)^4}{256}-\frac{25\cos(a+b\,x)^2}{256}+\frac{1}{32}}{b\,\left(\cos\left(a+b\,x\right)^5-2\cos\left(a+b\,x\right)^3+\cos\left(a+b\,x\right)\right)} - \frac{15\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{256\,b}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^5,x)
```

output
```
((15*cos(a + b*x)^4)/256 - (25*cos(a + b*x)^2)/256 + 1/32)/(b*(cos(a + b*x
) - 2*cos(a + b*x)^3 + cos(a + b*x)^5)) - (15*atanh(cos(a + b*x)))/(256*b)
```

Reduce [F]

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = \int \cos\left(bx+a\right)^3\csc\left(2bx+2a\right)^5dx$$

input
```
int(cos(b*x+a)^3*csc(2*b*x+2*a)^5,x)
```

output
```
int(cos(a + b*x)**3*csc(2*a + 2*b*x)**5,x)
```

3.570 $\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 136

$$\begin{aligned}\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx &= -\frac{5\arcsin(\cos(a+bx)-\sin(a+bx))}{32b} \\ &\quad -\frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{32b} \\ &\quad +\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b} \\ &\quad -\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b}+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}\end{aligned}$$

output
```
-5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b+5/16*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b-5/24*cos(b*x+a)*s
in(2*b*x+2*a)^(3/2)/b+1/6*sin(b*x+a)*sin(2*b*x+2*a)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$= \frac{-5\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+\frac{2}{3}\sqrt{\sin}}{32b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(14*Sin[a + b*x] -
 3*Sin[3*(a + b*x)] - 2*Sin[5*(a + b*x)]))/3)/(32*b)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4789, 3042, 4790, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin(2a+2bx)^{5/2}\cos(a+bx)dx$$
$$\Big\downarrow \text{ 4789}$$
$$\frac{5}{6}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{5}{6}\int \sin(a+bx)\sin(2a+2bx)^{3/2}dx+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{4790}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{4789}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}\right)+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{4794}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a\cdots}{}\right.\right.$$

$$\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2))/(6*b) + (5*((3*((-1/2*ArcSin[Cos[a +
 b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a +
 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 - (Co
s[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)))/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x)
```

output
```
int(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(118) = 236$.

Time = 0.10 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.13

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = -\frac{8\sqrt{2}\left(32\cos(bx+a)^4-12\cos(bx+a)^2-15\right)\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)-30\arctan\left(-\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/384*(8*sqrt(2)*(32*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 15)*sqrt(cos(b*
x + a)*sin(b*x + a))*sin(b*x + a) - 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*
sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(
cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 30*arctan(-(2*sqrt(2)
*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x +
 a) - sin(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x +
a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x
+ a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)
)/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \cos(bx+a)\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

Giac [F]

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \cos(bx+a)\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \cos(a+b\,x)\,\sin(2\,a+2\,b\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^(5/2),x)
```

output
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^(5/2), x)
```

Reduce [F]

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sqrt{\sin(2bx+2a)}\,\cos(bx+a)\sin(2bx+2a)^2\,dx$$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*cos(a + b*x)*sin(2*a + 2*b*x)**2,x)
```

3.571 $\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))}{16b} + \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{16b} - \frac{3\cos(a+bx)\sqrt{\sin(2a+2bx)}}{8b} + \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}$$

output
```
-3/16*arcsin(cos(b*x+a)-sin(b*x+a))/b+3/16*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b-3/8*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b+1/4*sin(b*x+a)*sin
(2*b*x+2*a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$=\frac{3\left(-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)-2(2\cos}{16b}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(3*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) - 2*(2*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[S
in[2*(a + b*x)]])/(16*b)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4789, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(2a+2bx)^{3/2}\cos(a+bx)dx$$
$$\Big\downarrow \text{4789}$$
$$\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}$$

$$\Big\downarrow \text{4790}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}$$

$$\Big\downarrow \text{4793}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\frac{\sqrt{\sin(2a+}}{}\right.$$

$$\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(3*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a
 + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(2*b)))/4 + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 44.79 (sec) , antiderivative size = 138159568, normalized size of antiderivative = 1255996.07

method	result	size
default	Expression too large to display	138159568

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(96) = 192.

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.55

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx =$$

$$-\frac{8\sqrt{2}\left(4\cos\left(bx+a\right)^{3}-\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(}{\cos(bx+a)^{2}+2}\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/64*(8*sqrt(2)*(4*cos(b*x + a)^3 - cos(b*x + a))*sqrt(cos(b*x + a)*sin(b
*x + a)) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a
) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
 + a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x
+ a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 3*lo
g(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1
)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(
b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos\left(bx+a\right)\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

Giac [F]

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(bx+a)\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(a+b\,x)\,\sin(2\,a+2\,b\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^(3/2), x)
```

Reduce [F]

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sqrt{\sin(2bx+2a)}\,\cos(bx+a)\sin(2bx+2a)\,dx$$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*cos(a + b*x)*sin(2*a + 2*b*x),x)
```

3.572 $\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{4b} - \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b} + \frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}$$

output
```
-1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b+1/2*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx)) + \log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right) - 2\sin(a+b}{4b}$$

input
```
Integrate[Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/4*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]] - 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2a+2bx)}\cos(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{\sin(2a+2bx)}\cos(a+bx)dx$$

$$\Big\downarrow \ 4789$$

$$\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}$$

$$\Big\downarrow \ 4794$$

$$\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}$$

input
```
Int[Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a + b
*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*
x]])/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 6.14 (sec) , antiderivative size = 24398096, normalized size of antiderivative = 290453.52

method	result	size
default	Expression too large to display	24398096

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(74) = 148$.

Time = 0.09 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.17

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a) + 2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/16*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + 2*arctan(-(
sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + co
s(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1
)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) -
 sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 + 4
*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b
*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x
+ a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos(bx+a)\,\sqrt{\sin(2\,bx+2\,a)}\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F]

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos(bx+a)\,\sqrt{\sin(2\,bx+2\,a)}\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos(a+b\,x)\,\sqrt{\sin(2\,a+2\,b\,x)}\,dx$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^(1/2), x)
```

Reduce [F]

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin(2bx+2a)}\,\cos(bx+a)\,dx$$

input

```
int(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x)
```

output

```
int(sqrt(sin(2*a + 2*b*x))*cos(a + b*x),x)
```

3.573 $\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b} + \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{2b}$$

output
```
-1/2*arcsin(cos(b*x+a)-sin(b*x+a))/b+1/2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \frac{-\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)}{2b}$$

input
```
Integrate[Cos[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] +
Sqrt[Sin[2*(a + b*x)]]])/(2*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{ 4793}$$

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}$$

input
```
Int[Cos[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a + b*
x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.07 (sec) , antiderivative size = 18464991, normalized size of antiderivative = 318361.91

method	result	size
default	Expression too large to display	18464991

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(52) = 104.

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.17

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= \frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cdots}{\cos(bx+a)-\mathrm{si}\cdots}\right.\cdots}{\cdots}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - si
n(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*
sin(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))
- cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - log(-32*co
s(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*
x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)
^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos(bx+a)}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F]

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos\left(bx+a\right)}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sqrt{\sin\left(2bx+2a\right)}\,\cos\left(bx+a\right)}{\sin\left(2bx+2a\right)}dx$$

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x))/sin(2*a + 2*b*x),x)
```

3.574 $\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

output
```
-cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{b\sqrt{\sin(2(a+bx))}}$$

input
```
Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-(Cos[a + b*x]/(b*Sqrt[Sin[2*(a + b*x)]]))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx$$
$$\Big\downarrow \text{ 4779}$$
$$-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-(Cos[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 7.04 (sec) , antiderivative size = 57930080, normalized size of antiderivative = 2413753.33

method	result	size
default	Expression too large to display	57930080

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}+\sin(bx+a)}{2\,b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + sin(b*x + a))/(b*sin(b*x +
 a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2029 vs. $2(22) = 44$.

Time = 13.31 (sec) , antiderivative size = 2029, normalized size of antiderivative = 84.54

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)
^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2
*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(
1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1
/2*b*x) + tan(1/2*a))*(((sqrt(2)*tan(1/2*a)^26 + 5*sqrt(2)*tan(1/2*a)^24 -
 10*sqrt(2)*tan(1/2*a)^22 - 154*sqrt(2)*tan(1/2*a)^20 - 605*sqrt(2)*tan(1/
2*a)^18 - 1353*sqrt(2)*tan(1/2*a)^16 - 1980*sqrt(2)*tan(1/2*a)^14 - 1980*s
qrt(2)*tan(1/2*a)^12 - 1353*sqrt(2)*tan(1/2*a)^10 - 605*sqrt(2)*tan(1/2*a)
^8 - 154*sqrt(2)*tan(1/2*a)^6 - 10*sqrt(2)*tan(1/2*a)^4 + 5*sqrt(2)*tan(1/
2*a)^2 + sqrt(2))*tan(1/2*b*x)/(tan(1/2*a)^24 + 12*tan(1/2*a)^22 + 66*tan(
1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan(1/2*a)^16 + 792*tan(1/2*a)^14 + 92
4*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 495*tan(1/2*a)^8 + 220*tan(1/2*a)^6
+ 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 + 1) - 8*(sqrt(2)*tan(1/2*a)^25 + 10*s
qrt(2)*tan(1/2*a)^23 + 44*sqrt(2)*tan(1/2*a)^21 + 110*sqrt(2)*tan(1/2*a)^1
9 + 165*sqrt(2)*tan(1/2*a)^17 + 132*sqrt(2)*tan(1/2*a)^15 - 132*sqrt(2)*ta
n(1/2*a)^11 - 165*sqrt(2)*tan(1/2*a)^9 - 110*sqrt(2)*tan(1/2*a)^7 - 44*sqr
t(2)*tan(1/2*a)^5 - 10*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))/(tan(1/2
*a)^24 + 12*tan(1/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan
(1/2*a)^16 + 792*tan(1/2*a)^14 + 924*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 4
95*tan(1/2*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 ...
```

Mupad [B] (verification not implemented)

Time = 19.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}{2\,b\,\sin\left(a+b\,x\right)}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^(3/2),x)
```

output
```
-sin(2*a + 2*b*x)^(1/2)/(2*b*sin(a + b*x))
```

Reduce [F]

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\cos(bx+a)}{\sin(2bx+2a)^2}dx$$

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x))/sin(2*a + 2*b*x)**2,x)
```

3.575 $\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

output
```
-1/3*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+2/3*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\left(-\frac{1}{12}\cot(a+bx)\csc(a+bx)+\frac{1}{4}\sec(a+bx)\right)\sqrt{\sin(2(a+bx))}}{b}$$

input
```
Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
((-1/12*(Cot[a + b*x]*Csc[a + b*x]) + Sec[a + b*x]/4)*Sqrt[Sin[2*(a + b*x)
]])/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx$$

$$\Big\downarrow \text{4791}$$

$$\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx - \frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx - \frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4780}$$

$$\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}} - \frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*Sqrt[
Sin[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 71.53 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.66

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\,\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)\left(2\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\right.\right.}{24b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}}$

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/b*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/
2*b*x)^2-1)/tan(1/2*a+1/2*b*x)*(2*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2
*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*
b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)-tan(1/2*a+1/2*b*x)^4+1)/(tan
(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)/(tan(1/2*a+1/2*b*x)^3-tan(
1/2*a+1/2*b*x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{4\cos(bx+a)^3+\sqrt{2}\left(4\cos(bx+a)^2-3\right)\sqrt{\cos(bx+a)\sin(bx+a)}-4\cos(bx+a)}{12\left(b\cos(bx+a)^3-b\cos(bx+a)\right)}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*
sin(b*x + a)) - 4*cos(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)}{\sin(2bx+2a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7875 vs. $2(45) = 90$.

Time = 46.02 (sec) , antiderivative size = 7875, normalized size of antiderivative = 148.58

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output

```
1/48*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)
^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2
*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(
1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1
/2*b*x) + tan(1/2*a))*(((((((sqrt(2)*tan(1/2*a)^57 + 18*sqrt(2)*tan(1/2*a)
^55 + 132*sqrt(2)*tan(1/2*a)^53 + 374*sqrt(2)*tan(1/2*a)^51 - 1375*sqrt(2)
*tan(1/2*a)^49 - 19620*sqrt(2)*tan(1/2*a)^47 - 108560*sqrt(2)*tan(1/2*a)^4
5 - 399740*sqrt(2)*tan(1/2*a)^43 - 1096755*sqrt(2)*tan(1/2*a)^41 - 2340250
*sqrt(2)*tan(1/2*a)^39 - 3941740*sqrt(2)*tan(1/2*a)^37 - 5204670*sqrt(2)*t
an(1/2*a)^35 - 5163155*sqrt(2)*tan(1/2*a)^33 - 3268760*sqrt(2)*tan(1/2*a)^
31 + 3268760*sqrt(2)*tan(1/2*a)^27 + 5163155*sqrt(2)*tan(1/2*a)^25 + 52046
70*sqrt(2)*tan(1/2*a)^23 + 3941740*sqrt(2)*tan(1/2*a)^21 + 2340250*sqrt(2)
*tan(1/2*a)^19 + 1096755*sqrt(2)*tan(1/2*a)^17 + 399740*sqrt(2)*tan(1/2*a)
^15 + 108560*sqrt(2)*tan(1/2*a)^13 + 19620*sqrt(2)*tan(1/2*a)^11 + 1375*sq
rt(2)*tan(1/2*a)^9 - 374*sqrt(2)*tan(1/2*a)^7 - 132*sqrt(2)*tan(1/2*a)^5 -
 18*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))*tan(1/2*b*x)/(tan(1/2*a)^51
 + 23*tan(1/2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/
2*a)^43 + 31878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37
 + 389367*tan(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 53
4888*tan(1/2*a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 5348...
```

Mupad [B] (verification not implemented)

Time = 22.77 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.96

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{2\sqrt{\sin(2a+2bx)}\,(3\cos(a+bx)-6\cos(3a+3bx)+4\cos(5a+5bx)-\cos(7a+7bx))}{3b\,(4\cos(2a+2bx)+4\cos(4a+4bx)-4\cos(6a+6bx)+\cos(8a+8bx)-5)}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^(5/2),x)
```

output
```
-(2*sin(2*a + 2*b*x)^(1/2)*(3*cos(a + b*x) - 6*cos(3*a + 3*b*x) + 4*cos(5*
a + 5*b*x) - cos(7*a + 7*b*x)))/(3*b*(4*cos(2*a + 2*b*x) + 4*cos(4*a + 4*b
*x) - 4*cos(6*a + 6*b*x) + cos(8*a + 8*b*x) - 5))
```

Reduce [F]

$$\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\cos(bx+a)}{\sin(2bx+2a)^3}dx$$

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x))/sin(2*a + 2*b*x)**3,x)
```

3.576 $\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

output
```
-1/5*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2)+4/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(3
/2)-8/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{\sin(2(a+bx))}\left(27\csc(a+bx)+3\csc^3(a+bx)-5\sec(a+bx)\tan(a+bx)\right)}{120b}$$

input
```
Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
-1/120*(Sqrt[Sin[2*(a + b*x)]]*(27*Csc[a + b*x] + 3*Csc[a + b*x]^3 - 5*Sec
[a + b*x]*Tan[a + b*x]))/b
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4791, 3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4791}$$

$$\frac{4}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4792}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4779}$$

$$\frac{4}{5}\left(\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(4*(Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*Sqrt
[Sin[2*a + 2*b*x]])))/5 - Cos[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 82.13 (sec) , antiderivative size = 481, normalized size of antiderivative = 6.09

$$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\,\left(24\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\,\sqrt{-}\right.}{}$$

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

output
```
-1/160/b*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)/tan(1/2*a+1/
2*b*x)^3*(24*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*
a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^
(1/2)*EllipticE((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*
x)^2-12*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2
*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)
*EllipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2+
(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^6-(
tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^4+12
*(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^4-(tan
(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^2-12*(t
an(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^2+(tan(1/
2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2))/(tan(1/2*a+1/2*b*x)^3-tan(1/
2*a+1/2*b*x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.30

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\left(32\cos(bx+a)^{4}-40\cos(bx+a)^{2}+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\left(\cos(bx+a)^{4}-\cos(bx\right.}{120\left(b\cos(bx+a)^{4}-b\cos(bx+a)^{2}\right)\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
-1/120*(sqrt(2)*(32*cos(b*x + a)^4 - 40*cos(b*x + a)^2 + 5)*sqrt(cos(b*x +
 a)*sin(b*x + a)) + 32*(cos(b*x + a)^4 - cos(b*x + a)^2)*sin(b*x + a))/((b
*cos(b*x + a)^4 - b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18022 vs. 2(67) = 134.

Time = 161.21 (sec) , antiderivative size = 18022, normalized size of antiderivative = 228.13

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
-1/960*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*
a)^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1
/2*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*ta
n(1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan
(1/2*b*x) + tan(1/2*a))*((((((((((3*sqrt(2)*tan(1/2*a)^88 + 221*sqrt(2)*t
an(1/2*a)^86 + 4529*sqrt(2)*tan(1/2*a)^84 + 46575*sqrt(2)*tan(1/2*a)^82 +
267014*sqrt(2)*tan(1/2*a)^80 + 611706*sqrt(2)*tan(1/2*a)^78 - 3503654*sqrt
(2)*tan(1/2*a)^76 - 44470106*sqrt(2)*tan(1/2*a)^74 - 259557037*sqrt(2)*tan
(1/2*a)^72 - 1054367027*sqrt(2)*tan(1/2*a)^70 - 3278963927*sqrt(2)*tan(1/2
*a)^68 - 8089589961*sqrt(2)*tan(1/2*a)^66 - 16006283224*sqrt(2)*tan(1/2*a)
^64 - 25186632744*sqrt(2)*tan(1/2*a)^62 - 30337876456*sqrt(2)*tan(1/2*a)^6
0 - 24685712920*sqrt(2)*tan(1/2*a)^58 - 5629982106*sqrt(2)*tan(1/2*a)^56 +
 19969391706*sqrt(2)*tan(1/2*a)^54 + 37658626338*sqrt(2)*tan(1/2*a)^52 + 3
6190152990*sqrt(2)*tan(1/2*a)^50 + 18717018180*sqrt(2)*tan(1/2*a)^48 + 204
0819900*sqrt(2)*tan(1/2*a)^46 + 2040819900*sqrt(2)*tan(1/2*a)^44 + 1871701
8180*sqrt(2)*tan(1/2*a)^42 + 36190152990*sqrt(2)*tan(1/2*a)^40 + 376586263
38*sqrt(2)*tan(1/2*a)^38 + 19969391706*sqrt(2)*tan(1/2*a)^36 - 5629982106*
sqrt(2)*tan(1/2*a)^34 - 24685712920*sqrt(2)*tan(1/2*a)^32 - 30337876456*sq
rt(2)*tan(1/2*a)^30 - 25186632744*sqrt(2)*tan(1/2*a)^28 - 16006283224*sqrt
(2)*tan(1/2*a)^26 - 8089589961*sqrt(2)*tan(1/2*a)^24 - 3278963927*sqrt(...
```

Mupad [B] (verification not implemented)

Time = 23.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{4\,{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,2{}\mathrm{i}+{\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,3{}\mathrm{i}+{\mathrm{e}}^{a\,6{}\mathrm{i}+b\,x\,6{}\mathrm{i}}\,2{}\mathrm{i}-{\mathrm{e}}^{a\,8{}\mathrm{i}+b\,x\,8{}\mathrm{i}}\,2{}\mathrm{i}-2{}\mathrm{i}\right)}{15\,b\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-1\right)}^3\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)}^2}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^(7/2),x)
```

output
```
(4*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2)*(exp(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*
x*6i)*2i - exp(a*8i + b*x*8i)*2i - 2i))/(15*b*(exp(a*2i + b*x*2i) - 1)^3*(
exp(a*2i + b*x*2i) + 1)^2)
```

Reduce [F]

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\cos(bx+a)}{\sin(2bx+2a)^4}dx$$

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x))/sin(2*a + 2*b*x)**4,x)
```

3.577 $\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{16\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

output
```
-1/7*cos(b*x+a)/b/sin(2*b*x+2*a)^(7/2)+6/35*sin(b*x+a)/b/sin(2*b*x+2*a)^(5
/2)-8/35*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+16/35*sin(b*x+a)/b/sin(2*b*x+2*
a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{(5-10\cos(2(a+bx))-4\cos(4(a+bx))+4\cos(6(a+bx)))\csc^4(a+bx)\sec^3(a+bx)\sqrt{\sin(2(a+bx))}}{560b}$$

input
```
Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
((5 - 10*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[a
 + b*x]^4*Sec[a + b*x]^3*Sqrt[Sin[2*(a + b*x)]])/(560*b)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4791, 3042, 4792, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{9/2}}dx$$

$$\Big\downarrow \text{ 4791}$$

$$\frac{6}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{6}{7}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{7/2}}dx-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 4792}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{ 4791}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$\Big\downarrow$ 3042

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)+\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$\Big\downarrow$ 4780

$$\frac{6}{7}\left(\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{5}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)\right)-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(6*((4*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3
*b*Sqrt[Sin[2*a + 2*b*x]])))/5 + Sin[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2))
))/7 - Cos[a + b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Maple [F(-1)]

Timed out.

$$\int \frac{\cos (bx+a)}{\sin (2bx+2a)^{\frac{9}{2}}}dx$$

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{128\cos(bx+a)^7-256\cos(bx+a)^5+128\cos(bx+a)^3+\sqrt{2}\big(128\cos(bx+a)^6-224\cos(bx+a)^4+}{560\left(b\cos(bx+a)^7-2\,b\cos(bx+a)^5+b\cos(bx+a)^3\right.}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
1/560*(128*cos(b*x + a)^7 - 256*cos(b*x + a)^5 + 128*cos(b*x + a)^3 + sqrt
(2)*(128*cos(b*x + a)^6 - 224*cos(b*x + a)^4 + 84*cos(b*x + a)^2 + 7)*sqrt
(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*co
s(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(9/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 23.22 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.33

$$\begin{aligned}\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = &-\frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{7\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4}\\&+\frac{e^{a\,3i+b\,x\,3i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,16i}{35\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}\\&-\frac{e^{a\,1i+b\,x\,1i}\left(\frac{1}{7\,b}-\frac{8\,e^{a\,2i+b\,x\,2i}}{35\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}+1\right)^2\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2}\\&+\frac{e^{a\,1i+b\,x\,1i}\left(\frac{16i}{35\,b}+\frac{e^{a\,2i+b\,x\,2i}\,44i}{35\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}+1\right)^3\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3}\end{aligned}$$

input
```
int(cos(a + b*x)/sin(2*a + 2*b*x)^(9/2),x)
```

output
```
(exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*16i)/(35*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i)
) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)
*1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^4) - (exp(a*1i + b*x*1i)*
(1/(7*b) - (8*exp(a*2i + b*x*2i))/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (
exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x*2i) + 1)^2*(exp(a*2i + b
*x*2i)*1i - 1i)^2) + (exp(a*1i + b*x*1i)*(16i/(35*b) + (exp(a*2i + b*x*2i)
*44i)/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1
/2))/((exp(a*2i + b*x*2i) + 1)^3*(exp(a*2i + b*x*2i)*1i - 1i)^3)
```

Reduce [F]

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\cos(bx+a)}{\sin(2bx+2a)^{5}}dx$$

input
```
int(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x))/sin(2*a + 2*b*x)**5,x)
```

3.578 $\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \frac{5\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{42b} - \frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b} - \frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b} + \frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

output
```
5/42*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))/b-5/42*cos(2*b*x+2*a)*sin(2*b*x
+2*a)^(1/2)/b-1/14*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(5/2)/b+1/18*sin(2*b*x+2*
a)^(9/2)/b
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$

$$= \frac{240\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2(a+bx))}+70\sin(2(a+bx))-156\sin(4(a+bx))-35\sin(6(a+b}{2016b\sqrt{\sin(2(a+bx))}}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(240*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 70*Sin[2*(a + b
*x)] - 156*Sin[4*(a + b*x)] - 35*Sin[6*(a + b*x)] + 18*Sin[8*(a + b*x)] +
7*Sin[10*(a + b*x)])/(2016*b*Sqrt[Sin[2*(a + b*x)]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4785, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{7}{2}}(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^{7/2}\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{1}{2}\int \sin^{\frac{7}{2}}(2a+2bx)dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^{7/2}dx+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{5}{7}\int \sin^{\frac{3}{2}}(2a+2bx)dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5}{7}\int \sin(2a+2bx)^{3/2}dx-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{2}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

$$\Big\downarrow \; 3120$$

$$\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{1}{2}\left(\frac{5}{7}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{7b}\right)$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
Sin[2*a + 2*b*x]^(9/2)/(18*b) + ((5*(EllipticF[a - Pi/4 + b*x, 2]/(3*b) -
(Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b)))/7 - (Cos[2*a + 2*b*x]*Si
n[2*a + 2*b*x]^(5/2))/(7*b))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

output
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

Fricas [F]

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx=\int \cos\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
integral(-(cos(2*b*x + 2*a)^2*cos(b*x + a)^2 - cos(b*x + a)^2)*sin(2*b*x +
 2*a)^(3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx=\text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx=\int \cos{(bx+a)}^2\sin{(2\,bx+2\,a)}^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx=\text{Timed out}$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \cos(a+b\,x)^2\sin(2\,a+2\,b\,x)^{7/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(7/2),x)
```

output
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(7/2), x)
```

Reduce [F]

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \int \sqrt{\sin(2bx+2a)}\,\cos(bx+a)^2\sin(2bx+2a)^3\,dx$$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**2*sin(2*a + 2*b*x)**3,x)
```

3.579 $\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b} - \frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b} + \frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

output
```
-3/10*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b-1/10*cos(2*b*x+2*a)*sin(2*b*x
+2*a)^(3/2)/b+1/14*sin(2*b*x+2*a)^(7/2)/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{84E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sqrt{\sin(2(a+bx))}(15\sin(2(a+bx))-14\sin(4(a+bx))-5\sin(6(a+bx)))}{280b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
(84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(15*Sin[2*(a + b
*x)] - 14*Sin[4*(a + b*x)] - 5*Sin[6*(a + b*x)]))/(280*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4785, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{5}{2}}(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^{5/2}\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{1}{2}\int \sin^{\frac{5}{2}}(2a+2bx)dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^{5/2}dx+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{3}{5}\int \sqrt{\sin(2a+2bx)}dx-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{1}{2}\left(\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}\right)$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
Sin[2*a + 2*b*x]^(7/2)/(14*b) + ((3*EllipticE[a - Pi/4 + b*x, 2])/(5*b) -
(Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(3/2))/(5*b))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 153.26 (sec) , antiderivative size = 185010647, normalized size of antiderivative = 2681313.72

method	result	size
default	Expression too large to display	185010647

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \cos(bx+a)^2 \sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
integral(-(cos(2*b*x + 2*a)^2*cos(b*x + a)^2 - cos(b*x + a)^2)*sqrt(sin(2*
b*x + 2*a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \cos\left(bx+a\right)^2 \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

Giac [F]

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \cos\left(bx+a\right)^2 \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \cos\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(5/2), x)
```

Reduce [F]

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \int \sqrt{\sin(2bx+2a)}\,\cos(bx+a)^2\sin(2bx+2a)^2\,dx$$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**2*sin(2*a + 2*b*x)**2,x)
```

3.580 $\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{6b}-\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

output
```
1/6*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))/b-1/6*cos(2*b*x+2*a)*sin(2*b*x+2
*a)^(1/2)/b+1/10*sin(2*b*x+2*a)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\frac{20\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2(a+bx))}+9\sin(2(a+bx))-10\sin(4(a+bx))-3\sin(6(a+bx))}{120b\sqrt{\sin(2(a+bx))}}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 9*Sin[2*(a + b*x
)] - 10*Sin[4*(a + b*x)] - 3*Sin[6*(a + b*x)])/(120*b*Sqrt[Sin[2*(a + b*x)
]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4785, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^{3/2}\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{1}{2}\int \sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(2a+2bx)^{3/2}dx+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{1}{2}\left(\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}\right)$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(EllipticF[a - Pi/4 + b*x, 2]/(3*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b
*x]])/(3*b))/2 + Sin[2*a + 2*b*x]^(5/2)/(10*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 22.22 (sec) , antiderivative size = 91497799, normalized size of antiderivative = 1326055.06

method	result	size
default	Expression too large to display	91497799

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Giac [F]

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\int \cos\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\int \cos\left(a+b\,x\right)^2\sin\left(2\,a+2\,b\,x\right)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(3/2), x)
```

Reduce [F]

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\int \sqrt{\sin\left(2bx+2a\right)}\,\cos\left(bx+a\right)^2\sin\left(2bx+2a\right)dx$$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**2*sin(2*a + 2*b*x),x)
```

3.581 $\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx=\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}+\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

output
```
-1/2*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b+1/6*sin(2*b*x+2*a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx=\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sin^{\frac{3}{2}}(2(a+bx))}{6b}$$

input
```
Integrate[Cos[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2))/(6*b)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4785, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2a+2bx)}\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(2a+2bx)}\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx + \frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx + \frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b} + \frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

input
```
Int[Cos[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
EllipticE[a - Pi/4 + b*x, 2]/(2*b) + Sin[2*a + 2*b*x]^(3/2)/(6*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 4.77 (sec) , antiderivative size = 24543726, normalized size of antiderivative = 613593.15

method	result	size
default	Expression too large to display	24543726

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos\left(bx+a\right)^2\sqrt{\sin\left(2\,bx+2\,a\right)}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos\left(bx+a\right)^2\sqrt{\sin\left(2\,bx+2\,a\right)}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F]

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos\left(bx+a\right)^2\sqrt{\sin\left(2\,bx+2\,a\right)}\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos\left(a+b\,x\right)^2\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(1/2), x)
```

Reduce [F]

$$\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin\left(2bx+2a\right)}\,\cos\left(bx+a\right)^2\,dx$$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**2,x)
```

3.582 $\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{2b}+\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

output
```
1/2*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))/b+1/2*sin(2*b*x+2*a)^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \frac{\left(1+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{\sin(2(a+bx))}}{2b}$$

input
```
Integrate[Cos[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
((1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^
2])*Sqrt[Sin[2*(a + b*x)]])/(2*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4785, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^2}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\sin(2a+2bx)}}{2b}+\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b}$$

input
```
Int[Cos[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
EllipticF[a - Pi/4 + b*x, 2]/(2*b) + Sqrt[Sin[2*a + 2*b*x]]/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.52 (sec) , antiderivative size = 12988217, normalized size of antiderivative = 324705.42

method	result	size
default	Expression too large to display	12988217

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos\left(bx+a\right)^2}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos\left(bx+a\right)^2}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F]

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos\left(bx+a\right)^2}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sqrt{\sin\left(2bx+2a\right)}\,\cos\left(bx+a\right)^2}{\sin\left(2bx+2a\right)}dx$$

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**2)/sin(2*a + 2*b*x),x)
```

3.583 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=-\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

output
```
1/2*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b-cos(b*x+a)^2/b/sin(2*b*x+2*a)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=-\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\cot(a+bx)\sqrt{\sin(2(a+bx))}}{2b}$$

input
```
Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-1/2*(EllipticE[a - Pi/4 + b*x, 2] + Cot[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/
b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4783, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{4783}$$

$$-\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
-1/2*EllipticE[a - Pi/4 + b*x, 2]/b - Cos[a + b*x]^2/(b*Sqrt[Sin[2*a + 2*b
*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4783
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Cos[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.45 (sec) , antiderivative size = 20591474, normalized size of antiderivative = 447640.74

method	result	size
default	Expression too large to display	20591474

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.39

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= \frac{-i\sqrt{2i}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a)+i\sqrt{-2i}E(\arcsin(\cos(bx+a)-i\sin}{}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(-I*sqrt(2*I)*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*si
n(b*x + a) + I*sqrt(-2*I)*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a))
, -1)*sin(b*x + a) + I*sqrt(2*I)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*
x + a)), -1)*sin(b*x + a) - I*sqrt(-2*I)*elliptic_f(arcsin(cos(b*x + a) -
I*sin(b*x + a)), -1)*sin(b*x + a) - 2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x +
a))*cos(b*x + a))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos(a+b\,x)^2}{\sin(2\,a+2\,b\,x)^{3/2}}\,dx$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\cos(bx+a)^2}{\sin(2bx+2a)^2}dx$$

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**2)/sin(2*a + 2*b*x)**2,x)
```

3.584 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx=\frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)}{6b}-\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

output
```
1/6*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))/b-1/3*cos(b*x+a)^2/b/sin(2*b*x+2
*a)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.53 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\left(\csc^2(a+bx)-2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{\sin(2(a+bx))}}{12b}$$

input
```
Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/12*((Csc[a + b*x]^2 - 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^
2]*Sqrt[Sec[a + b*x]^2])*Sqrt[Sin[2*(a + b*x)]])/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4783, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^{5/2}}dx$$
$$\Big\downarrow \text{ 4783}$$
$$\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$
$$\Big\downarrow \text{ 3120}$$
$$\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{6b}-\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
EllipticF[a - Pi/4 + b*x, 2]/(6*b) - Cos[a + b*x]^2/(3*b*Sin[2*a + 2*b*x]^
(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4783
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Cos[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(42) = 84$.

Time = 40.87 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.56

method	result
default	$\frac{\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\text{EllipticF}\left(\sqrt{\sin(2bx+2a)+1},\frac{\sqrt{2}}{2}\right)\sin(2bx+2a)-2\cos(2bx+2a)^{2}-2\cos(2}{12\sin(2bx+2a)^{\frac{3}{2}}\cos(2bx+2a)b}$

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/sin(2*b*x+2*a)^(3/2)/cos(2*b*x+2*a)*((sin(2*b*x+2*a)+1)^(1/2)*(-2*sin
(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^
(1/2),1/2*2^(1/2))*sin(2*b*x+2*a)-2*cos(2*b*x+2*a)^2-2*cos(2*b*x+2*a))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.15

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2i}\left(\cos\left(bx+a\right)^2-1\right)F(\arcsin\left(\cos\left(bx+a\right)+i\sin\left(bx+a\right)\right)|-1)+\sqrt{-2i}\left(\cos\left(bx+a\right)^2-1\right)F(\mathrm{ar}}{12\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(2*I)*(cos(b*x + a)^2 - 1)*elliptic_f(arcsin(cos(b*x + a) + I*s
in(b*x + a)), -1) + sqrt(-2*I)*(cos(b*x + a)^2 - 1)*elliptic_f(arcsin(cos(
b*x + a) - I*sin(b*x + a)), -1) - sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)))
/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\cos(a+b\,x)^2}{\sin(2\,a+2\,b\,x)^{5/2}}\,dx$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\cos(bx+a)^2}{\sin(2bx+2a)^3}dx$$

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**2)/sin(2*a + 2*b*x)**3,x)
```

3.585 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=-\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b}-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

output
```
3/10*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b-1/5*cos(b*x+a)^2/b/sin(2*b*x+2
*a)^(5/2)-3/10*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx=\frac{-12E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\frac{2(1-6\cos(2(a+bx))+3\cos(4(a+bx)))\cot(a+bx)}{\sin^{\frac{3}{2}}(2(a+bx))}}{40b}$$

input
```
Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(-12*EllipticE[a - Pi/4 + b*x, 2] + (2*(1 - 6*Cos[2*(a + b*x)] + 3*Cos[4*(
a + b*x)])*Cot[a + b*x])/Sin[2*(a + b*x)]^(3/2))/(40*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4783, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^2}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4783}$$

$$\frac{3}{10}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\int \frac{1}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{3}{10}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\left(-\int \sqrt{\sin(2a+2bx)}dx-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{3}{10}\left(-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\cos(2a+2bx)}{b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
(3*(-(EllipticE[a - Pi/4 + b*x, 2]/b) - Cos[2*a + 2*b*x]/(b*Sqrt[Sin[2*a +
 2*b*x]])))/10 - Cos[a + b*x]^2/(5*b*Sin[2*a + 2*b*x]^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4783
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Cos[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(68) = 136$.

Time = 134.03 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.95

method	result
default	$\sqrt{2}\left(-\frac{8\sqrt{2}}{5\sin(2bx+2a)^{\frac{5}{2}}}+\frac{4\sqrt{2}\left(6\sqrt{\sin(2bx+2a)+1}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\sin(2bx+2a)^{2}\,\text{EllipticE}\left(\sqrt{\sin(2bx+2a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{\sin\cdots}\right.}{5\sin(\cdots}\right. \quad / \quad 32b$

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*2^(1/2)*(-8/5*2^(1/2)/sin(2*b*x+2*a)^(5/2)+4/5*2^(1/2)/sin(2*b*x+2*a)
^(5/2)*(6*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x
+2*a))^(1/2)*sin(2*b*x+2*a)^2*EllipticE((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/
2))-3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a
))^(1/2)*sin(2*b*x+2*a)^2*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+
6*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-2)/cos(2*b*x+2*a))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.45

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{6\sqrt{2i}\left(i\cos{(bx+a)}^3 - i\cos{(bx+a)}\right)E(\arcsin{(\cos{(bx+a)}+i\sin{(bx+a)})}\,|-1)\sin{(bx+a)} + 6\,\sqrt{}}{}$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
-1/40*(6*sqrt(2*I)*(I*cos(b*x + a)^3 - I*cos(b*x + a))*elliptic_e(arcsin(c
os(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 6*sqrt(-2*I)*(-I*cos(b*x
 + a)^3 + I*cos(b*x + a))*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a))
, -1)*sin(b*x + a) + 6*sqrt(2*I)*(-I*cos(b*x + a)^3 + I*cos(b*x + a))*elli
ptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 6*sqrt(-2
*I)*(I*cos(b*x + a)^3 - I*cos(b*x + a))*elliptic_f(arcsin(cos(b*x + a) - I
*sin(b*x + a)), -1)*sin(b*x + a) + sqrt(2)*(12*cos(b*x + a)^4 - 18*cos(b*x
 + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a)))/((b*cos(b*x + a)^3 - b*cos(b
*x + a))*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

Giac [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{7/2}}\,dx$$

input
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(7/2),x)
```

output
```
int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin\left(2bx+2a\right)}\,\cos\left(bx+a\right)^2}{\sin\left(2bx+2a\right)^4}dx$$

input
```
int(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**2)/sin(2*a + 2*b*x)**4,x)
```

3.586 $\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\begin{aligned}\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= -\frac{7\arcsin(\cos(a+bx)-\sin(a+bx))}{64b} \\ &\quad+\frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{64b} \\ &\quad-\frac{7\cos(a+bx)\sqrt{\sin(2a+2bx)}}{32b} \\ &\quad+\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b}+\frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}\end{aligned}$$

output
```
-7/64*arcsin(cos(b*x+a)-sin(b*x+a))/b+7/64*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b-7/32*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b+7/48*sin(b*x+a)*s
in(2*b*x+2*a)^(3/2)/b+1/12*cos(b*x+a)*sin(2*b*x+2*a)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.73

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= \frac{-7\arcsin(\cos(a+bx)-\sin(a+bx))+7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-\frac{2}{3}(10\cos}{64b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 7*Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]] - (2*(10*Cos[a + b*x] + 9*Cos[3*(a + b*x)] + 2
*Cos[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/3)/(64*b)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4785, 3042, 4789, 3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2a+2bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2a+2bx)^{3/2}\cos(a+bx)^3dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{7}{12}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)dx+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{12}\int \cos(a+bx)\sin(2a+2bx)^{3/2}dx+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$

$$\Big\downarrow \text{4789}$$

$$\frac{7}{12}\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{12}\left(\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$

$$\Big\downarrow \text{4790}$$

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}\right)+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$

$$\Big\downarrow \text{4793}$$

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)-\frac{\sqrt{\sin(2\ldots}}{\ldots}\right.\right.$$
$$\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}$$

input

```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2))/(12*b) + (7*((3*((-1/2*ArcSin[Cos[a
+ b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a
+ 2*b*x]]]/(2*b))/2 - (Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b)))/4 + (S
in[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)))/12
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 202.21 (sec) , antiderivative size = 375719658, normalized size of antiderivative = 2762644.54

method	result	size
default	Expression too large to display	375719658

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(118) = 236.

Time = 0.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.14

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx =$$
$$-\frac{8\sqrt{2}\left(32\cos(bx+a)^5-4\cos(bx+a)^3-7\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}-42\arctan\left(-\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/768*(8*sqrt(2)*(32*cos(b*x + a)^5 - 4*cos(b*x + a)^3 - 7*cos(b*x + a))*
sqrt(cos(b*x + a)*sin(b*x + a)) - 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(co
s(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 42*arctan(-(2*sqrt(2)*s
qrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a
) - sin(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)
^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x +
a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/
b
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(bx+a)^3 \sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

Giac [F]

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos(bx+a)^3 \sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \cos\left(a+b\,x\right)^3\,\sin\left(2\,a+2\,b\,x\right)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(3/2), x)
```

Reduce [F]

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \int \sqrt{\sin\left(2bx+2a\right)}\,\cos\left(bx+a\right)^3\sin\left(2bx+2a\right)dx$$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**3*sin(2*a + 2*b*x),x)
```

3.587 $\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{5\arcsin(\cos(a+bx)-\sin(a+bx))}{32b} - \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{32b} + \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b} + \frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}$$

output
```
-5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*
b*x+2*a)^(1/2))/b+5/16*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b+1/8*cos(b*x+a)*si
n(2*b*x+2*a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{-5\left(\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+2\sqrt{\sin}}{32b}$$

input
```
Integrate[Cos[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(6*Sin[a + b*x] + S
in[3*(a + b*x)]))/(32*b)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4785, 3042, 4789, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2a+2bx)}\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(2a+2bx)}\cos(a+bx)^3dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{5}{8}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{8}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}dx+\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{ 4789}$$

$$\frac{5}{8}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)+\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{8}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\sin(a+bx)}{2b}\right)+\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{ 4794}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)+\frac{\sin(a+bx}{}\right.$$

$$\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}$$

input
```
Int[Cos[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(5*((-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a
 + b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 + (Sin[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(2*b)))/8 + (Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(8*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

rule 4789
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*(
g/(2*p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 39.62 (sec) , antiderivative size = 114426295, normalized size of antiderivative = 1040239.04

method	result	size
default	Expression too large to display	114426295

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(96) = 192$.

Time = 0.09 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.55

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{8\sqrt{2}\left(4\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)+10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(}{\cos(bx+a)^2+2}\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/128*(8*sqrt(2)*(4*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a))*si
n(b*x + a) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x
+ a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(
b*x + a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(
b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) +
5*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2
 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*
cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos(bx+a)^3\sqrt{\sin(2\,bx+2\,a)}\,dx$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \cos\left(a+b\,x\right)^3\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(1/2), x)
```

Reduce [F]

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \int \sqrt{\sin\left(2bx+2a\right)}\,\cos\left(bx+a\right)^3dx$$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x)
```

output
```
int(sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**3,x)
```

3.588 $\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{3\arcsin(\cos(a+bx)-\sin(a+bx))}{8b} + \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b} + \frac{\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b}$$

output
```
-3/8*arcsin(cos(b*x+a)-sin(b*x+a))/b+3/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*
x+2*a)^(1/2))/b+1/4*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \frac{-3\arcsin(\cos(a+bx)-\sin(a+bx))+3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)+\csc(a+}{8b}$$

input
```
Integrate[Cos[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]] + Csc[a + b*x]*Sin[2*(a + b*x)]^(3/2))/(8*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4785, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{4785}$$

$$\frac{3}{4}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}dx+\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{4793}$$

$$\frac{3}{4}\left(\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}\right)+\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}$$

input
```
Int[Cos[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

output
```
(3*(-1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a
+ b*x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)))/4 + (Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(4*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4785
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Simp[e^2*((m + p - 1)/(m + 2*p))   Int[(e*Cos[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 19.34 (sec) , antiderivative size = 74647250, normalized size of antiderivative = 888657.74

method	result	size
default	Expression too large to display	74647250

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(74) = 148.

Time = 0.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.19

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)+6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

output
```
1/32*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + 6*arctan(-(
sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + co
s(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1
)) - 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) -
 sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4 +
 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos
(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*
x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos\left(bx+a\right)^3}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

Giac [F]

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\cos\left(bx+a\right)^3}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{{\cos\left(a+b\,x\right)}^3}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\cos(bx+a)^3}{\sin(2bx+2a)}dx$$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**3)/sin(2*a + 2*b*x),x)
```

3.589 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{4b} + \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b} - \frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

output
```
1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b+1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b-cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\arcsin(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\csc(a+bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + S
qrt[Sin[2*(a + b*x)]]] - 2*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4781, 3042, 4795, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^{3/2}}dx$$

$$\Big\downarrow \text{4781}$$

$$-\frac{1}{4}\int \sec(a+bx)\sqrt{\sin(2a+2bx)}dx-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4}\int \frac{\sqrt{\sin(2a+2bx)}}{\cos(a+bx)}dx-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{4795}$$

$$-\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}dx-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{4794}$$

$$\frac{1}{2}\left(\frac{\arcsin(\cos(a+bx)-\sin(a+bx))}{2b}+\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}\right)-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

output
```
(ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(2*b) + Log[Cos[a + b*x] + Sin[a + b*
x] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b))/2 - Cos[a + b*x]/(b*Sqrt[Sin[2*a + 2*b
*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4781
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(p + 1))), x] + Simp[e^4*((m + p - 1)/(4*g^2*(p + 1)))   Int[(e*Co
s[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d,
e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
 && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4795
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/cos[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 8.30 (sec) , antiderivative size = 69948460, normalized size of antiderivative = 853030.00

method	result	size
default	Expression too large to display	69948460

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(74) = 148.

Time = 0.09 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.60

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)\sin(bx+a)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos}}{}\right.}{}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/16*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) -
sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a
)*sin(b*x + a) - 1))*sin(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)
*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a)
))*sin(b*x + a) + log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (
4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin
(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)*sin(b*x
 + a) + 8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + 8*sin(b*x + a))/(b*sin
(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\sin\left(2\,a+2\,b\,x\right)^{3/2}}\,dx$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin\left(2bx+2a\right)}\,\cos\left(bx+a\right)^3}{\sin\left(2bx+2a\right)^2}dx$$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**3)/sin(2*a + 2*b*x)**2,x)
```

3.590 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\cos^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

output
```
-1/3*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\csc^3(a+bx)\sin^{\frac{3}{2}}(2(a+bx))}{24b}$$

input
```
Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/24*(Csc[a + b*x]^3*Sin[2*(a + b*x)]^(3/2))/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^{5/2}}dx$$
$$\Big\downarrow \text{ 4779}$$
$$-\frac{\cos^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

output
```
-1/3*Cos[a + b*x]^3/(b*Sin[2*a + 2*b*x]^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 19.98 (sec) , antiderivative size = 192, normalized size of antiderivative = 6.86

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\,\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)\left(4\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\,\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\right.\right.}{24\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\,b}$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2*b
*x)^2-1)/tan(1/2*a+1/2*b*x)*(4*(tan(1/2*a+1/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+
1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*EllipticF((tan(1/2*a+1/2*b*x
)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)+tan(1/2*a+1/2*b*x)^4-1)/(tan(1/
2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)/(tan(1/2*a+1/2*b*x)^3-tan(1/2
*a+1/2*b*x))^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)+\cos(bx+a)^2-1}{12\left(b\cos(bx+a)^2-b\right)}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + cos(b*x + a)^
2 - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15292 vs. $2(24) = 48$.

Time = 91.67 (sec) , antiderivative size = 15292, normalized size of antiderivative = 546.14

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

output
```
1/48*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)
^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2
*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(
1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1
/2*b*x) + tan(1/2*a))*(((((((sqrt(2)*tan(1/2*a)^57 + 50*sqrt(2)*tan(1/2*a)
^55 + 516*sqrt(2)*tan(1/2*a)^53 + 1750*sqrt(2)*tan(1/2*a)^51 - 5215*sqrt(2
)*tan(1/2*a)^49 - 77796*sqrt(2)*tan(1/2*a)^47 - 386576*sqrt(2)*tan(1/2*a)^
45 - 1186876*sqrt(2)*tan(1/2*a)^43 - 2515251*sqrt(2)*tan(1/2*a)^41 - 37599
30*sqrt(2)*tan(1/2*a)^39 - 3812844*sqrt(2)*tan(1/2*a)^37 - 2196894*sqrt(2)
*tan(1/2*a)^35 - 36499*sqrt(2)*tan(1/2*a)^33 + 824296*sqrt(2)*tan(1/2*a)^3
1 - 824296*sqrt(2)*tan(1/2*a)^27 + 36499*sqrt(2)*tan(1/2*a)^25 + 2196894*s
qrt(2)*tan(1/2*a)^23 + 3812844*sqrt(2)*tan(1/2*a)^21 + 3759930*sqrt(2)*tan
(1/2*a)^19 + 2515251*sqrt(2)*tan(1/2*a)^17 + 1186876*sqrt(2)*tan(1/2*a)^15
 + 386576*sqrt(2)*tan(1/2*a)^13 + 77796*sqrt(2)*tan(1/2*a)^11 + 5215*sqrt(
2)*tan(1/2*a)^9 - 1750*sqrt(2)*tan(1/2*a)^7 - 516*sqrt(2)*tan(1/2*a)^5 - 5
0*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))*tan(1/2*b*x)/(tan(1/2*a)^51 +
 23*tan(1/2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*
a)^43 + 31878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 +
 389367*tan(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 5348
88*tan(1/2*a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888...
```

Mupad [B] (verification not implemented)

Time = 21.92 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.36

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\sqrt{\sin(2\,a+2\,b\,x)}\,\left(\frac{2\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{3}-\sin\left(\frac{3\,a}{2}+\frac{3\,b\,x}{2}\right)^2+\frac{\sin\left(\frac{5\,a}{2}+\frac{5\,b\,x}{2}\right)^2}{3}\right)}{b\,\left(30\sin\left(a+b\,x\right)^2-12\sin\left(2\,a+2\,b\,x\right)^2+2\sin\left(3\,a+3\,b\,x\right)^2\right)}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(5/2),x)
```

output
```
(sin(2*a + 2*b*x)^(1/2)*((2*sin(a/2 + (b*x)/2)^2)/3 - sin((3*a)/2 + (3*b*x
)/2)^2 + sin((5*a)/2 + (5*b*x)/2)^2/3))/(b*(2*sin(3*a + 3*b*x)^2 - 12*sin(
2*a + 2*b*x)^2 + 30*sin(a + b*x)^2))
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\cos(bx+a)^3}{\sin(2bx+2a)^3}dx$$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**3)/sin(2*a + 2*b*x)**3,x)
```

3.591 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{\cos(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

output
```
-1/5*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(5/2)-1/5*cos(b*x+a)/b/sin(2*b*x+2*a)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\csc(a+bx)\left(4+\csc^2(a+bx)\right)\sqrt{\sin(2(a+bx))}}{40b}$$

input
```
Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
-1/40*(Csc[a + b*x]*(4 + Csc[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4783, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^{7/2}}dx$$

$$\Big\downarrow \text{4783}$$

$$\frac{1}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4779}$$

$$-\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

output
```
-1/5*Cos[a + b*x]^3/(b*Sin[2*a + 2*b*x]^(5/2)) - Cos[a + b*x]/(5*b*Sqrt[Si
n[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4783
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Cos[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 169.19 (sec) , antiderivative size = 482, normalized size of antiderivative = 8.76

method	result
default	$\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1}}\left(16\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}-1\right)}\sqrt{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}\,\text{EllipticE}\Big(\sqrt{\right.$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/160*(-tan(1/2*a+1/2*b*x)/(tan(1/2*a+1/2*b*x)^2-1))^(1/2)/tan(1/2*a+1/2*b
*x)^3*(16*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1
/2*b*x)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/
2)*EllipticE((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^
2-8*(tan(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*(tan(1/2*a+1/2*b*x
)+1)^(1/2)*(-2*tan(1/2*a+1/2*b*x)+2)^(1/2)*(-tan(1/2*a+1/2*b*x))^(1/2)*Ell
ipticF((tan(1/2*a+1/2*b*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*b*x)^2-(tan
(1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^6+(tan(
1/2*a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^4+8*(tan
(1/2*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^4+(tan(1/2*
a+1/2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2)*tan(1/2*a+1/2*b*x)^2-8*(tan(1/2
*a+1/2*b*x)^3-tan(1/2*a+1/2*b*x))^(1/2)*tan(1/2*a+1/2*b*x)^2-(tan(1/2*a+1/
2*b*x)*(tan(1/2*a+1/2*b*x)^2-1))^(1/2))/(tan(1/2*a+1/2*b*x)^3-tan(1/2*a+1/
2*b*x))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\left(4\cos(bx+a)^2-5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+4\left(\cos(bx+a)^2-1\right)\sin(bx+a)}{40\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

output
```
-1/40*(sqrt(2)*(4*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 4*
(cos(b*x + a)^2 - 1)*sin(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(7/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 23.98 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.69

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{5\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(7/2),x)
```

output
```
-(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2)*(exp(a*4i + b*x*4i)*1i - exp(a*2i + b*x*2i)*3i + 1i))/(5*b*(exp
(a*2i + b*x*2i) - 1)^3)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\cos(bx+a)^3}{\sin(2bx+2a)^4}dx$$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**3)/sin(2*a + 2*b*x)**4,x)
```

3.592 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{4\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

output
```
-1/7*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(7/2)-2/21*cos(b*x+a)/b/sin(2*b*x+2*a)^
(3/2)+4/21*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{(5-12\cos(2(a+bx))+4\cos(4(a+bx)))\csc^4(a+bx)\sec(a+bx)\sqrt{\sin(2(a+bx))}}{336b}$$

input
```
Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
((5 - 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4*Sec[a + b*x
]*Sqrt[Sin[2*(a + b*x)]])/(336*b)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4783, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^{9/2}}dx$$

$$\Big\downarrow \text{4783}$$

$$\frac{2}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4791}$$

$$\frac{2}{7}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\left(\frac{2}{3}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{3/2}}dx-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4780}$$

$$\frac{2}{7}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

output
```
(2*(-1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*S
qrt[Sin[2*a + 2*b*x]])))/7 - Cos[a + b*x]^3/(7*b*Sin[2*a + 2*b*x]^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4783
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Cos[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Maple [F(-1)]

Timed out.

$$\int \frac{\cos (bx+a)^3}{\sin (2bx+2a)^{\frac{9}{2}}}dx$$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.28

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\, dx = \frac{32 \cos (bx+a)^5 - 64 \cos (bx+a)^3 + \sqrt{2}(32 \cos (bx+a)^4 - 56 \cos (bx+a)^2 + 21)\sqrt{\cos (bx+a) \sin (b}}{336 \left(b\cos (bx+a)^5 - 2\,b\cos (bx+a)^3 + b\cos (bx+a)\right)}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

output
```
1/336*(32*cos(b*x + a)^5 - 64*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4
- 56*cos(b*x + a)^2 + 21)*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a
))/(b*cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\, dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(9/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 23.69 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.73

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{5\,e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{84\,b\,(e^{a\,2i+b\,x\,2i}\,1i-i)^2} + \frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,3i}{14\,b\,(e^{a\,2i+b\,x\,2i}\,1i-i)^3} - \frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{7\,b\,(e^{a\,2i+b\,x\,2i}\,1i-i)^4} - \frac{e^{a\,1i+b\,x\,1i}\left(\frac{5i}{84\,b}-\frac{e^{a\,2i+b\,x\,2i}\,4i}{21\,b}\right)\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{(e^{a\,2i+b\,x\,2i}+1)\,(e^{a\,2i+b\,x\,2i}\,1i-i)}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(9/2),x)
```

output
```
(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*3i)/(14*b*(exp(a*2i + b*x*2i)*1i - 1i)^3) - (5*exp(a*1i + b*x*1i
)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(84*b*(
exp(a*2i + b*x*2i)*1i - 1i)^2) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i
)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i -
1i)^4) - (exp(a*1i + b*x*1i)*(5i/(84*b) - (exp(a*2i + b*x*2i)*4i)/(21*b))*
((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2
i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i))
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx=\int \frac{\sqrt{\sin(2bx+2a)}\,\cos(bx+a)^3}{\sin(2bx+2a)^5}dx$$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**3)/sin(2*a + 2*b*x)**5,x)
```

3.593 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)} - \frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{8\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

output
```
-1/9*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(9/2)-1/15*cos(b*x+a)/b/sin(2*b*x+2*a)^
(5/2)+4/45*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-8/45*cos(b*x+a)/b/sin(2*b*x+2
*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{\sin(2(a+bx))}\left(113\csc(a+bx)+17\csc^3(a+bx)+5\csc^5(a+bx)-15\sec(a+bx)\tan(a+bx)\right)}{1440b}$$

input
```
Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

output
```
-1/1440*(Sqrt[Sin[2*(a + b*x)]]*(113*Csc[a + b*x] + 17*Csc[a + b*x]^3 + 5*
Csc[a + b*x]^5 - 15*Sec[a + b*x]*Tan[a + b*x]))/b
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4783, 3042, 4791, 3042, 4792, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^3}{\sin(2a+2bx)^{11/2}}dx$$

$$\Big\downarrow \text{4783}$$

$$\frac{1}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{7/2}}dx-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4791}$$

$$\frac{1}{3}\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{4}{5}\int \frac{\sin(a+bx)}{\sin(2a+2bx)^{5/2}}dx-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4792}$$

$$\frac{1}{3}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\cos(a+bx)}{\sin(2a+2bx)^{3/2}}dx+\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

$$\Big\downarrow \text{4779}$$

$$\frac{1}{3}\left(\frac{4}{5}\left(\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\right)-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}\right)-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}$$

input
```
Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

output
```
((4*(Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*Sqr
t[Sin[2*a + 2*b*x]])))/5 - Cos[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2)))/3 -
Cos[a + b*x]^3/(9*b*Sin[2*a + 2*b*x]^(9/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4783
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Simp[e^2*((m + 2*p + 2)/(4*g^2*(p + 1)))   Int[(e*Cos[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4792
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + S
imp[(2*p + 3)/(2*g*(p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !
IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Maple [F(-1)]

Timed out.

$$\int \frac{\cos(bx+a)^3}{\sin(2bx+2a)^{\frac{11}{2}}}dx$$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

output
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.22

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = -\frac{\sqrt{2}\left(128\,\cos\left(bx+a\right)^6-288\,\cos\left(bx+a\right)^4+180\,\cos\left(bx+a\right)^2-15\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+128}{1440\,\left(b\cos\left(bx+a\right)^6-2\,b\cos\left(bx+a\right)^4+b\cos\left(bx+a\right)}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="fricas")
```

output
```
-1/1440*(sqrt(2)*(128*cos(b*x + a)^6 - 288*cos(b*x + a)^4 + 180*cos(b*x +
a)^2 - 15)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - 2*cos(b
*x + a)^4 + cos(b*x + a)^2)*sin(b*x + a))/((b*cos(b*x + a)^6 - 2*b*cos(b*x
 + a)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \int \frac{\cos\left(bx+a\right)^3}{\sin\left(2\,bx+2\,a\right)^{\frac{11}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(11/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 24.68 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.58

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = -\frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{60\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3} - \frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,2i}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4} + \frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^5} + \frac{8\,e^{a\,3i+b\,x\,3i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{45\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)} - \frac{e^{a\,1i+b\,x\,1i}\,\left(\frac{49i}{180\,b}+\frac{e^{a\,2i+b\,x\,2i}\,19i}{180\,b}\right)\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{\left(e^{a\,2i+b\,x\,2i}+1\right)^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2}$$

input
```
int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(11/2),x)
```

output
```
(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2))/(9*b*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (exp(a*1i + b*x*1i)*((ex
p(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*2i)/(9*b*(exp(
a*2i + b*x*2i)*1i - 1i)^4) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i
)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(60*b*(exp(a*2i + b*x*2i)*1i - 1i)
^3) + (8*exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x
*2i)*1i)/2)^(1/2))/(45*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i -
 1i)) - (exp(a*1i + b*x*1i)*(49i/(180*b) + (exp(a*2i + b*x*2i)*19i)/(180*b
))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(
a*2i + b*x*2i) + 1)^2*(exp(a*2i + b*x*2i)*1i - 1i)^2)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \int \frac{\sqrt{\sin(2bx+2a)}\,\cos(bx+a)^3}{\sin(2bx+2a)^6}dx$$

input
```
int(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

output
```
int((sqrt(sin(2*a + 2*b*x))*cos(a + b*x)**3)/sin(2*a + 2*b*x)**6,x)
```

3.594 $\int \frac{\cos(x)}{\sqrt{\sin(2x)}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}} \, dx = -\frac{1}{2}\arcsin(\cos(x)-\sin(x))+\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

output
```
-1/2*arcsin(cos(x)-sin(x))+1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}} \, dx = \frac{1}{2}\left(-\arcsin(\cos(x)-\sin(x))+\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\right)$$

input
```
Integrate[Cos[x]/Sqrt[Sin[2*x]],x]
```

output
```
(-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx$$

$$\Big\downarrow \text{4793}$$

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\arcsin(\cos(x)-\sin(x))$$

input
```
Int[Cos[x]/Sqrt[Sin[2*x]],x]
```

output
```
-1/2*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

method	result	size
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}}\,\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)^{3}-\tan\left(\frac{x}{2}\right)}}$	98

input
```
int(cos(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1/2
*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*
2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(25) = 50.

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.42

$$
\begin{aligned}
\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx
&= \frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^{2}+2\cos(x)\sin(x)-1}\right) \\
&\quad -\frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right)-\frac{1}{8}\log\Big(-32\cos(x)^{4} \\
&\quad +4\sqrt{2}\left(4\cos(x)^{3}-\left(4\cos(x)^{2}+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)} \\
&\quad +32\cos(x)^{2}+16\cos(x)\sin(x)+1\Big)
\end{aligned}
$$

input
```
integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x)
)/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*s
in(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/8*log(-32*cos(x)^4 + 4*sq
rt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)
) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)/sin(2*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos\left(x\right)}{\sqrt{\sin\left(2\,x\right)}}\,dx$$

input
```
integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(x)/sqrt(sin(2*x)), x)
```

Giac [F]

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

input
```
integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(x)/sqrt(sin(2*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

input
```
int(cos(x)/sin(2*x)^(1/2),x)
```

output
```
int(cos(x)/sin(2*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\sqrt{\sin(2x)}\cos(x)}{\sin(2x)}dx$$

input
```
int(cos(x)/sin(2*x)^(1/2),x)
```

output
```
int((sqrt(sin(2*x))*cos(x))/sin(2*x),x)
```

3.595 $\int \csc(x)\sqrt{\sin(2x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = -\arcsin(\cos(x)-\sin(x)) + \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

output
```
-arcsin(cos(x)-sin(x))+ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = -\arcsin(\cos(x)-\sin(x)) + \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

input
```
Integrate[Csc[x]*Sqrt[Sin[2*x]],x]
```

output
```
-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4796, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2x)}\csc(x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{\sin(2x)}}{\sin(x)}dx$$

$$\Big\downarrow \ 4796$$

$$2\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx$$

$$\Big\downarrow \ 3042$$

$$2\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx$$

$$\Big\downarrow \ 4793$$

$$2\left(\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\arcsin(\cos(x)-\sin(x))\right)$$

input
```
Int[Csc[x]*Sqrt[Sin[2*x]],x]
```

output
```
2*(-1/2*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.96

method	result	size
default	$\frac{2\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}}\,\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)^{3}-\tan\left(\frac{x}{2}\right)}}$	99

input
```
int(csc(x)*sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1
/2*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x
))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/
2*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(23) = 46$.

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.48

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = \frac{1}{2}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right) - \frac{1}{2}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right) - \frac{1}{4}\log\left(-32\cos(x)^4 + 4\sqrt{2}\left(4\cos(x)^3-\left(4\cos(x)^2+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2+16\cos(x)\sin(x)+1\right)$$

input
```
integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x)
)/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/2*arctan(-(2*sqrt(2)*sqrt(cos(x)*s
in(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/4*log(-32*cos(x)^4 + 4*sq
rt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)
) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = \text{Timed out}$$

input
```
integrate(csc(x)*sin(2*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = \int \csc(x)\,\sqrt{\sin(2\,x)}\,dx$$

input
```
integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(x)*sqrt(sin(2*x)), x)
```

Giac [F]

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = \int \csc(x)\,\sqrt{\sin(2\,x)}\,dx$$

input
```
integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(x)*sqrt(sin(2*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = \int \frac{\sqrt{\sin(2\,x)}}{\sin(x)}\,dx$$

input
```
int(sin(2*x)^(1/2)/sin(x),x)
```

output
```
int(sin(2*x)^(1/2)/sin(x), x)
```

Reduce [F]

$$\int \csc(x)\sqrt{\sin(2x)}\,dx = \int \sqrt{\sin(2x)}\,\csc(x)\,dx$$

input
```
int(csc(x)*sin(2*x)^(1/2),x)
```

output
```
int(sqrt(sin(2*x))*csc(x),x)
```

3.596 $\int \cos^3(a+bx)\sin^q(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \cos^3(a+bx)\sin^q(2a+2bx)\,dx =$$
$$-\frac{\cos^3(a+bx)\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{4+q}{2},\frac{6+q}{2},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-q}{2}}\sin^q(2a+2}{b(4+q)}$$

output
```
-cos(b*x+a)^3*cot(b*x+a)*hypergeom([2+1/2*q, 1/2-1/2*q],[3+1/2*q],cos(b*x+
a)^2)*(sin(b*x+a)^2)^(1/2-1/2*q)*sin(2*b*x+2*a)^q/b/(4+q)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.83 (sec) , antiderivative size = 1801, normalized size of antiderivative = 21.19

$$\int \cos^3(a+bx)\sin^q(2a+2bx)\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^q,x]
```

output
```
(2*(6*AppellF1[(1 + q)/2, -q, 2*(1 + q), (3 + q)/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] + 8*AppellF1[(1 + q)/2, -q, 2*(2 + q), (3 + q)/2, Tan[(
a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 + q)/2, -q, 1 + 2*q, (3
+ q)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*AppellF1[(1 + q)/2,
-q, 3 + 2*q, (3 + q)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[a +
b*x]^3*Sin[2*(a + b*x)]^q*Tan[(a + b*x)/2])/(b*((6*AppellF1[(1 + q)/2, -q,
 2*(1 + q), (3 + q)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*Appell
F1[(1 + q)/2, -q, 2*(2 + q), (3 + q)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)
/2]^2] - AppellF1[(1 + q)/2, -q, 1 + 2*q, (3 + q)/2, Tan[(a + b*x)/2]^2, -
Tan[(a + b*x)/2]^2] - 12*AppellF1[(1 + q)/2, -q, 3 + 2*q, (3 + q)/2, Tan[(
a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sec[(a + b*x)/2]^2 + q*(6*AppellF1[(1
 + q)/2, -q, 2*(1 + q), (3 + q)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2
] + 8*AppellF1[(1 + q)/2, -q, 2*(2 + q), (3 + q)/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] - AppellF1[(1 + q)/2, -q, 1 + 2*q, (3 + q)/2, Tan[(a +
b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*AppellF1[(1 + q)/2, -q, 3 + 2*q, (3 +
 q)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[2*(a + b*x)]*Sec[(a +
 b*x)/2]^2*Sec[a + b*x] + 4*q*(6*AppellF1[(1 + q)/2, -q, 2*(1 + q), (3 + q
)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + q)/2, -q,
2*(2 + q), (3 + q)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - AppellF1[
(1 + q)/2, -q, 1 + 2*q, (3 + q)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2...
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4797, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\sin^q(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)^3\sin(2a+2bx)^q dx$$

$$\Big\downarrow \text{4797}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos^{q+3}(a+bx)\sin^q(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int\cos(a+bx)^{q+3}\sin(a+bx)^q dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\cos^3(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-q}{2}}\sin^q(2a+2bx)\,\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{q+4}{2},\frac{q+6}{2},\cos^2(a+bx)\right)}{b(q+4)}$$

input
```
Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^q,x]
```

output
```
-((Cos[a + b*x]^3*Cot[a + b*x]*Hypergeometric2F1[(1 - q)/2, (4 + q)/2, (6
+ q)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - q)/2)*Sin[2*a + 2*b*x]^q)/(
b*(4 + q)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 4797
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p)   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \cos (bx+a)^3 \sin (2bx+2a)^q \, dx$$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^q,x)
```

output
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^q,x)
```

Fricas [F]

$$\int \cos^3(a+bx)\sin^q(2a+2bx) \, dx = \int \sin(2\,bx+2\,a)^q\cos(bx+a)^3 \, dx$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^q,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^q*cos(b*x + a)^3, x)
```

Sympy [F]

$$\int \cos^3(a+bx)\sin^q(2a+2bx) \, dx = \int \sin^q(2a+2bx)\cos^3(a+bx) \, dx$$

input
```
integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**q,x)
```

output
```
Integral(sin(2*a + 2*b*x)**q*cos(a + b*x)**3, x)
```

Maxima [F]

$$\int \cos^3(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q\cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^q,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^q*cos(b*x + a)^3, x)
```

Giac [F]

$$\int \cos^3(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q\cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^q,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^q*cos(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sin^q(2a+2bx)\,dx = \int \cos(a+b\,x)^3\sin(2\,a+2\,b\,x)^q\,dx$$

input
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^q,x)
```

output
```
int(cos(a + b*x)^3*sin(2*a + 2*b*x)^q, x)
```

Reduce [F]

$$\int \cos^3(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2bx+2a)^q\cos(bx+a)^3\,dx$$

input
```
int(cos(b*x+a)^3*sin(2*b*x+2*a)^q,x)
```

output
```
int(sin(2*a + 2*b*x)**q*cos(a + b*x)**3,x)
```

3.597 $\int \cos^2(a+bx)\sin^q(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \cos^2(a+bx)\sin^q(2a+2bx)\,dx = -\frac{\cos^2(a+bx)\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{3+q}{2},\frac{5+q}{2},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-q}{2}}\sin^q(2a+2}{b(3+q)}$$

output
```
-cos(b*x+a)^2*cot(b*x+a)*hypergeom([1/2-1/2*q, 3/2+1/2*q],[5/2+1/2*q],cos(
b*x+a)^2)*(sin(b*x+a)^2)^(1/2-1/2*q)*sin(2*b*x+2*a)^q/b/(3+q)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\int \cos^2(a+bx)\sin^q(2a+2bx)\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1+q}{2},2+q,\frac{3+q}{2},-\tan^2(a+bx)\right)\sec^2(a+bx)^q\sin^q(2(a+bx))\tan(a+bx)}{b(1+q)}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^q,x]
```

output
```
(Hypergeometric2F1[(1 + q)/2, 2 + q, (3 + q)/2, -Tan[a + b*x]^2]*(Sec[a +
b*x]^2)^q*Sin[2*(a + b*x)]^q*Tan[a + b*x])/(b*(1 + q))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4797, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sin^q(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)^2\sin(2a+2bx)^q dx$$

$$\Big\downarrow \text{4797}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos^{q+2}(a+bx)\sin^q(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos(a+bx)^{q+2}\sin(a+bx)^q dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\cos^2(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-q}{2}}\sin^q(2a+2bx)\,\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{q+3}{2},\frac{q+5}{2},\cos^2(a+bx)\right)}{b(q+3)}$$

input
```
Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^q,x]
```

output
```
-((Cos[a + b*x]^2*Cot[a + b*x]*Hypergeometric2F1[(1 - q)/2, (3 + q)/2, (5
+ q)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - q)/2)*Sin[2*a + 2*b*x]^q)/(
b*(3 + q)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 4797
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p)   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \cos(bx+a)^2 \sin(2bx+2a)^q \, dx$$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^q,x)
```

output
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^q,x)
```

Fricas [F]

$$\int \cos^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q \cos(bx+a)^2 \,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^q,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^q*cos(b*x + a)^2, x)
```

Sympy [F]

$$\int \cos^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin^q(2a+2bx)\cos^2(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**q,x)
```

output
```
Integral(sin(2*a + 2*b*x)**q*cos(a + b*x)**2, x)
```

Maxima [F]

$$\int \cos^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q\cos(bx+a)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^q,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^q*cos(b*x + a)^2, x)
```

Giac [F]

$$\int \cos^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q\cos(bx+a)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^q,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^q*cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sin^q(2a+2bx)\,dx = \int \cos\left(a+b\,x\right)^2\sin\left(2\,a+2\,b\,x\right)^q\,dx$$

input
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^q,x)
```

output
```
int(cos(a + b*x)^2*sin(2*a + 2*b*x)^q, x)
```

Reduce [F]

$$\int \cos^2(a+bx)\sin^q(2a+2bx)\,dx = \int \sin\left(2bx+2a\right)^q\cos\left(bx+a\right)^2dx$$

input
```
int(cos(b*x+a)^2*sin(2*b*x+2*a)^q,x)
```

output
```
int(sin(2*a + 2*b*x)**q*cos(a + b*x)**2,x)
```

3.598 $\int \cos(a+bx)\sin^q(2a+2bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \cos(a+bx)\sin^q(2a+2bx)\,dx = -\frac{\cos(a+bx)\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{2+q}{2},\frac{4+q}{2},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-q}{2}}\sin^q(2a+2b}{b(2+q)}$$

output
```
-cos(b*x+a)*cot(b*x+a)*hypergeom([1+1/2*q, 1/2-1/2*q],[2+1/2*q],cos(b*x+a)
^2)*(sin(b*x+a)^2)^(1/2-1/2*q)*sin(2*b*x+2*a)^q/b/(2+q)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 3.32 (sec) , antiderivative size = 567, normalized size of antiderivative = 6.83

$$\int \cos(a+bx)\sin^q(2a+2bx)\,dx$$

$$= \frac{}{2b(1+q)\left(2(3+q)\,\text{AppellF1}\left(\frac{1+q}{2},-q,2(1+q),\frac{3+q}{2},\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-(3+q)\,\text{Ap}}$$

input
```
Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^q,x]
```

output
```
((3 + q)*(2*AppellF1[(1 + q)/2, -q, 2*(1 + q), (3 + q)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 + q)/2, -q, 1 + 2*q, (3 + q)/2, Tan
[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sin[2*(a + b*x)]^(1 + q))/(2*b*(1 +
 q)*(2*(3 + q)*AppellF1[(1 + q)/2, -q, 2*(1 + q), (3 + q)/2, Tan[(a + b*x)
/2]^2, -Tan[(a + b*x)/2]^2] - (3 + q)*AppellF1[(1 + q)/2, -q, 1 + 2*q, (3
+ q)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*(-2*q*AppellF1[(3 + q
)/2, 1 - q, 2*(1 + q), (5 + q)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
 + q*AppellF1[(3 + q)/2, 1 - q, 1 + 2*q, (5 + q)/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] + AppellF1[(3 + q)/2, -q, 2*(1 + q), (5 + q)/2, Tan[(a
+ b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*q*AppellF1[(3 + q)/2, -q, 2*(1 + q),
 (5 + q)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[(3 + q)/
2, -q, 3 + 2*q, (5 + q)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*q*
AppellF1[(3 + q)/2, -q, 3 + 2*q, (5 + q)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2])*Tan[(a + b*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4797, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sin^q(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)\sin(2a+2bx)^q dx$$

$$\Big\downarrow \text{4797}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos^{q+1}(a+bx)\sin^q(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-q}(a+bx)\sin^q(2a+2bx)\cos^{-q}(a+bx)\int \cos(a+bx)^{q+1}\sin(a+bx)^q dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\cos(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-q}{2}}\sin^q(2a+2bx)\,\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{q+2}{2},\frac{q+4}{2},\cos^2(a+bx)\right)}{b(q+2)}$$

input
```
Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^q,x]
```

output
```
-((Cos[a + b*x]*Cot[a + b*x]*Hypergeometric2F1[(1 - q)/2, (2 + q)/2, (4 +
q)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - q)/2)*Sin[2*a + 2*b*x]^q)/(b*
(2 + q)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 4797
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p)   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \cos(bx+a)\sin(2bx+2a)^q\,dx$$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^q,x)
```

output
```
int(cos(b*x+a)*sin(2*b*x+2*a)^q,x)
```

Fricas [F]

$$\int \cos(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q\cos(bx+a)\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^q,x, algorithm="fricas")
```

output
```
integral(sin(2*b*x + 2*a)^q*cos(b*x + a), x)
```

Sympy [F]

$$\int \cos(a+bx)\sin^q(2a+2bx)\,dx = \int \sin^q(2a+2bx)\cos(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)**q,x)
```

output
```
Integral(sin(2*a + 2*b*x)**q*cos(a + b*x), x)
```

Maxima [F]

$$\int \cos(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q\cos(bx+a)\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^q,x, algorithm="maxima")
```

output
```
integrate(sin(2*b*x + 2*a)^q*cos(b*x + a), x)
```

Giac [F]

$$\int \cos(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2\,bx+2\,a)^q\cos(bx+a)\,dx$$

input
```
integrate(cos(b*x+a)*sin(2*b*x+2*a)^q,x, algorithm="giac")
```

output
```
integrate(sin(2*b*x + 2*a)^q*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\sin^q(2a+2bx)\,dx = \int \cos(a+b\,x)\,\sin(2\,a+2\,b\,x)^q\,dx$$

input
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^q,x)
```

output
```
int(cos(a + b*x)*sin(2*a + 2*b*x)^q, x)
```

Reduce [F]

$$\int \cos(a+bx)\sin^q(2a+2bx)\,dx = \int \sin(2bx+2a)^q \cos(bx+a)\,dx$$

input
```
int(cos(b*x+a)*sin(2*b*x+2*a)^q,x)
```

output
```
int(sin(2*a + 2*b*x)**q*cos(a + b*x),x)
```

3.599 $\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 46

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\cos^5(a+bx)}{5b}+\frac{8\cos^7(a+bx)}{7b}-\frac{4\cos^9(a+bx)}{9b}$$

output
```
-4/5*cos(b*x+a)^5/b+8/7*cos(b*x+a)^7/b-4/9*cos(b*x+a)^9/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = \frac{\cos^5(a+bx)(-249+220\cos(2(a+bx))-35\cos(4(a+bx)))}{630b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
(Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(630*
b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4800, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\sin(2a+2bx)^2\cos(a+bx)^2dx$$

$$\Big\downarrow \text{4800}$$

$$4\int \cos^4(a+bx)\sin^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \cos(a+bx)^4\sin(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{4\int \cos^4(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{4\int \left(\cos^8(a+bx)-2\cos^6(a+bx)+\cos^4(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4\left(\frac{1}{9}\cos^9(a+bx)-\frac{2}{7}\cos^7(a+bx)+\frac{1}{5}\cos^5(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^2*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

output
```
(-4*(Cos[a + b*x]^5/5 - (2*Cos[a + b*x]^7)/7 + Cos[a + b*x]^9/9))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4800
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((f_.)*sin[(a_.) + (b_.)*(x_)])^(
n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_Symbol] :> Simp[2^p/(e^p*f^p)   Int[(
e*Cos[a + b*x])^(m + p)*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 10.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

method	result	size
parallelrisch	$\frac{-2048+45\cos(7bx+7a)-35\cos(9bx+9a)-420\cos(3bx+3a)-1890\cos(bx+a)+252\cos(5bx+5a)}{20160b}$	60
default	$-\frac{3\cos(bx+a)}{32b}-\frac{\cos(3bx+3a)}{48b}+\frac{\cos(5bx+5a)}{80b}+\frac{\cos(7bx+7a)}{448b}-\frac{\cos(9bx+9a)}{576b}$	69
risch	$-\frac{3\cos(bx+a)}{32b}-\frac{\cos(3bx+3a)}{48b}+\frac{\cos(5bx+5a)}{80b}+\frac{\cos(7bx+7a)}{448b}-\frac{\cos(9bx+9a)}{576b}$	69
orering	Expression too large to display	1219

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/20160*(-2048+45*cos(7*b*x+7*a)-35*cos(9*b*x+9*a)-420*cos(3*b*x+3*a)-1890
*cos(b*x+a)+252*cos(5*b*x+5*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx$$
$$= -\frac{4\left(35\cos\left(bx+a\right)^9 - 90\cos\left(bx+a\right)^7 + 63\cos\left(bx+a\right)^5\right)}{315\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas"
)
```

output
```
-4/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(39) = 78$.

Time = 11.17 (sec) , antiderivative size = 318, normalized size of antiderivative = 6.91

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx$$
$$= \begin{cases} -\frac{8\sin^5(a+bx)\sin(2a+2bx)\cos(2a+2bx)}{315b} + \frac{16\sin^4(a+bx)\sin^2(2a+2bx)\cos(a+bx)}{315b} - \frac{16\sin^4(a+bx)\cos(a+bx)\cos^2(2a+2bx)}{315b} + \frac{44 \\ x\sin^3(a)\sin^2(2a)\cos^2(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

output
```
Piecewise((-8*sin(a + b*x)**5*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)/(315*b) +
16*sin(a + b*x)**4*sin(2*a + 2*b*x)**2*cos(a + b*x)/(315*b) - 16*sin(a + b
*x)**4*cos(a + b*x)*cos(2*a + 2*b*x)**2/(315*b) + 44*sin(a + b*x)**3*sin(2
*a + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)/(315*b) - 113*sin(a + b*x)**2
*sin(2*a + 2*b*x)**2*cos(a + b*x)**3/(315*b) + 8*sin(a + b*x)**2*cos(a + b
*x)**3*cos(2*a + 2*b*x)**2/(315*b) - 88*sin(a + b*x)*sin(2*a + 2*b*x)*cos(
a + b*x)**4*cos(2*a + 2*b*x)/(315*b) - 2*sin(2*a + 2*b*x)**2*cos(a + b*x)*
*5/(63*b) - 32*cos(a + b*x)**5*cos(2*a + 2*b*x)**2/(315*b), Ne(b, 0)), (x*
sin(a)**3*sin(2*a)**2*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{35\cos(9\,bx+9\,a)-45\cos(7\,bx+7\,a)-252\cos(5\,bx+5\,a)+420\cos(3\,bx+3\,a)+1890\cos(bx+a)}{20160\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima"
)
```

output
```
-1/20160*(35*cos(9*b*x + 9*a) - 45*cos(7*b*x + 7*a) - 252*cos(5*b*x + 5*a)
 + 420*cos(3*b*x + 3*a) + 1890*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(35\cos(bx+a)^9-90\cos(bx+a)^7+63\cos(bx+a)^5\right)}{315\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

output
```
-4/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = -\frac{4\left(35\cos\left(a+b\,x\right)^9-90\cos\left(a+b\,x\right)^7+63\cos\left(a+b\,x\right)^5\right)}{315\,b}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*sin(2*a + 2*b*x)^2,x)
```

output
```
-(4*(63*cos(a + b*x)^5 - 90*cos(a + b*x)^7 + 35*cos(a + b*x)^9))/(315*b)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 343, normalized size of antiderivative = 7.46

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = \frac{880\cos(2bx+2a)\cos(bx+a)\sin(2bx+2a)\sin(bx+a)^3-440\cos(2bx+2a)\cos(bx+a)\sin(2bx+2$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x)
```

output
```
(880*cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)*sin(a + b*x)**3 - 440*
cos(2*a + 2*b*x)*cos(a + b*x)*sin(2*a + 2*b*x)*sin(a + b*x) - 700*cos(2*a
+ 2*b*x)*sin(2*a + 2*b*x)*sin(a + b*x)**5 + 1100*cos(2*a + 2*b*x)*sin(2*a
+ 2*b*x)*sin(a + b*x)**3 - 440*cos(2*a + 2*b*x)*sin(2*a + 2*b*x)*sin(a + b
*x) + 875*cos(a + b*x)*sin(2*a + 2*b*x)**2*sin(a + b*x)**4 - 825*cos(a + b
*x)*sin(2*a + 2*b*x)**2*sin(a + b*x)**2 + 110*cos(a + b*x)*sin(2*a + 2*b*x
)**2 - 280*cos(a + b*x)*sin(a + b*x)**4 + 360*cos(a + b*x)*sin(a + b*x)**2
 - 160*cos(a + b*x) + 880*sin(2*a + 2*b*x)**2*sin(a + b*x)**4 - 880*sin(2*
a + 2*b*x)**2*sin(a + b*x)**2 + 110*sin(2*a + 2*b*x)**2 - 440*sin(a + b*x)
**4 + 440*sin(a + b*x)**2 + 512)/(1575*b)
```

3.600 $\int \sin^{10}(a+bx)\sin(12(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \sin^{10}(a+bx)\sin(12(a+bx))\,dx=\frac{\sin^{11}(a+bx)\sin(11(a+bx))}{11b}$$

output
```
1/11*sin(b*x+a)^11*sin(11*b*x+11*a)/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sin^{10}(a+bx)\sin(12(a+bx))\,dx=\frac{\sin^{11}(a+bx)\sin(11(a+bx))}{11b}$$

input
```
Integrate[Sin[a + b*x]^10*Sin[12*(a + b*x)],x]
```

output
```
(Sin[a + b*x]^11*Sin[11*(a + b*x)])/(11*b)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(23) = 46.

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 7.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{10}(a+bx)\sin(12(a+bx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^{10}\sin(12a+12bx)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(-\frac{\sin(2a+2bx)}{1024}+\frac{5}{512}\sin(4a+4bx)-\frac{45\sin(6a+6bx)}{1024}+\frac{15}{128}\sin(8a+8bx)-\frac{105}{512}\sin(10a+10bx)+\frac{63}{256}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos(2a+2bx)}{2048b}-\frac{5\cos(4a+4bx)}{2048b}+\frac{15\cos(6a+6bx)}{2048b}-\frac{15\cos(8a+8bx)}{1024b}+\frac{21\cos(10a+10bx)}{1024b}-\frac{21\cos(12a+12bx)}{1024b}+\frac{15\cos(14a+14bx)}{1024b}-\frac{15\cos(16a+16bx)}{2048b}+\frac{5\cos(18a+18bx)}{2048b}-\frac{\cos(20a+20bx)}{2048b}+\frac{\cos(22a+22bx)}{22528b}$$

input
```
Int[Sin[a + b*x]^10*Sin[12*(a + b*x)],x]
```

output
```
Cos[2*a + 2*b*x]/(2048*b) - (5*Cos[4*a + 4*b*x])/(2048*b) + (15*Cos[6*a +
6*b*x])/(2048*b) - (15*Cos[8*a + 8*b*x])/(1024*b) + (21*Cos[10*a + 10*b*x]
)/(1024*b) - (21*Cos[12*a + 12*b*x])/(1024*b) + (15*Cos[14*a + 14*b*x])/(1
024*b) - (15*Cos[16*a + 16*b*x])/(2048*b) + (5*Cos[18*a + 18*b*x])/(2048*b
) - Cos[20*a + 20*b*x]/(2048*b) + Cos[22*a + 22*b*x]/(22528*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(22) = 44$.

Time = 106.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 5.52

method	result
parallelrisch	$\frac{1-55\cos(4bx+4a)-462\cos(12bx+12a)-165\cos(16bx+16a)-330\cos(8bx+8a)+11\cos(2bx+2a)+\cos(22bx+22a)-11\cos(20}{22528b}$
default	$\frac{\cos(2bx+2a)}{2048b}-\frac{5\cos(4bx+4a)}{2048b}+\frac{15\cos(6bx+6a)}{2048b}-\frac{15\cos(8bx+8a)}{1024b}+\frac{21\cos(10bx+10a)}{1024b}-\frac{21\cos(12bx+12a)}{1024b}+15$
risch	$\frac{\cos(2bx+2a)}{2048b}-\frac{5\cos(4bx+4a)}{2048b}+\frac{15\cos(6bx+6a)}{2048b}-\frac{15\cos(8bx+8a)}{1024b}+\frac{21\cos(10bx+10a)}{1024b}-\frac{21\cos(12bx+12a)}{1024b}+15$
orering	Expression too large to display

input
```
int(sin(b*x+a)^10*sin(12*b*x+12*a),x,method=_RETURNVERBOSE)
```

output
```
1/22528*(1-55*cos(4*b*x+4*a)-462*cos(12*b*x+12*a)-165*cos(16*b*x+16*a)-330
*cos(8*b*x+8*a)+11*cos(2*b*x+2*a)+cos(22*b*x+22*a)-11*cos(20*b*x+20*a)+55*
cos(18*b*x+18*a)+165*cos(6*b*x+6*a)+330*cos(14*b*x+14*a)+462*cos(10*b*x+10
*a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(22) = 44$.

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 5.04

$$\int \sin^{10}(a+bx)\sin(12(a+bx))\,dx$$
$$= \frac{1024\,\cos(bx+a)^{22} - 8448\,\cos(bx+a)^{20} + 30976\,\cos(bx+a)^{18} - 66352\,\cos(bx+a)^{16} + 91740\,\cos(b$$

input
```
integrate(sin(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="fricas")
```

output
```
1/11*(1024*cos(b*x + a)^22 - 8448*cos(b*x + a)^20 + 30976*cos(b*x + a)^18
- 66352*cos(b*x + a)^16 + 91740*cos(b*x + a)^14 - 85305*cos(b*x + a)^12 +
53834*cos(b*x + a)^10 - 22671*cos(b*x + a)^8 + 6072*cos(b*x + a)^6 - 935*c
os(b*x + a)^4 + 66*cos(b*x + a)^2)/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin^{10}(a+bx)\sin(12(a+bx))\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**10*sin(12*b*x+12*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \sin^{10}(a+bx)\sin(12(a+bx))\,dx = \frac{\cos(22\,bx+22\,a) - 11\,\cos(20\,bx+20\,a) + 55\,\cos(18\,bx+18\,a) - 165\,\cos(16\,bx+16\,a) + 330\,\cos(14$$

input
```
integrate(sin(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="maxima")
```

output
```
1/22528*(cos(22*b*x + 22*a) - 11*cos(20*b*x + 20*a) + 55*cos(18*b*x + 18*a
) - 165*cos(16*b*x + 16*a) + 330*cos(14*b*x + 14*a) - 462*cos(12*b*x + 12*
a) + 462*cos(10*b*x + 10*a) - 330*cos(8*b*x + 8*a) + 165*cos(6*b*x + 6*a)
- 55*cos(4*b*x + 4*a) + 11*cos(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$.

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.87

$$\int \sin^{10}(a+bx)\sin(12(a+bx))\,dx = -\frac{1024\,\sin(bx+a)^{22} - 2816\,\sin(bx+a)^{20} + 2816\,\sin(bx+a)^{18} - 1232\,\sin(bx+a)^{16} + 220\,\sin(bx+}{11\,b}$$

input
```
integrate(sin(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="giac")
```

output
```
-1/11*(1024*sin(b*x + a)^22 - 2816*sin(b*x + a)^20 + 2816*sin(b*x + a)^18
- 1232*sin(b*x + a)^16 + 220*sin(b*x + a)^14 - 11*sin(b*x + a)^12)/b
```

Mupad [B] (verification not implemented)

Time = 20.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 6.26

$$\int \sin^{10}(a+bx)\sin(12(a+bx))\,dx = \frac{6\cos{(a+b\,x)}^2}{b} - \frac{85\cos{(a+b\,x)}^4}{b} + \frac{552\cos{(a+b\,x)}^6}{b} - \frac{2061\cos{(a+b\,x)}^8}{b} + \frac{4894\cos{(a+b\,x)}^{10}}{b} - \frac{7755\cos{(a+b\,x)}^{12}}{b} + \frac{8340\cos{(a+b\,x)}^{14}}{b} - \frac{6032\cos{(a+b\,x)}^{16}}{b} + \frac{2816\cos{(a+b\,x)}^{18}}{b} - \frac{768\cos{(a+b\,x)}^{20}}{b} + \frac{1024\cos{(a+b\,x)}^{22}}{11\,b}$$

input
```
int(sin(a + b*x)^10*sin(12*a + 12*b*x),x)
```

output
```
(6*cos(a + b*x)^2)/b - (85*cos(a + b*x)^4)/b + (552*cos(a + b*x)^6)/b - (2
061*cos(a + b*x)^8)/b + (4894*cos(a + b*x)^10)/b - (7755*cos(a + b*x)^12)/
b + (8340*cos(a + b*x)^14)/b - (6032*cos(a + b*x)^16)/b + (2816*cos(a + b*
x)^18)/b - (768*cos(a + b*x)^20)/b + (1024*cos(a + b*x)^22)/(11*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 10.17

$$\int \sin^{10}(a+bx)\sin(12(a+bx))\,dx = \frac{-6144\cos{(12bx+12a)}\sin{(bx+a)}^{10} + 6912\cos{(12bx+12a)}\sin{(bx+a)}^{8} - 3584\cos{(12bx+12a)}\sin}{}$$

input
```
int(sin(b*x+a)^10*sin(12*b*x+12*a),x)
```

output
```
( - 6144*cos(12*a + 12*b*x)*sin(a + b*x)**10 + 6912*cos(12*a + 12*b*x)*sin
(a + b*x)**8 - 3584*cos(12*a + 12*b*x)*sin(a + b*x)**6 + 840*cos(12*a + 12
*b*x)*sin(a + b*x)**4 - 72*cos(12*a + 12*b*x)*sin(a + b*x)**2 + cos(12*a +
 12*b*x) + 5120*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x)**9 - 4608*cos
(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x)**7 + 1792*cos(a + b*x)*sin(12*a
+ 12*b*x)*sin(a + b*x)**5 - 280*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*
x)**3 + 12*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x) - 1)/(22528*b)
```

3.601 $\int \sin^{10}(a+bx)\sin(6(2a+2bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \sin^{10}(a+bx)\sin(6(2a+2bx))\,dx = \frac{\sin^{11}(a+bx)\sin(11(a+bx))}{11b}$$

output
```
1/11*sin(b*x+a)^11*sin(11*b*x+11*a)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sin^{10}(a+bx)\sin(6(2a+2bx))\,dx = \frac{\sin^{11}(a+bx)\sin(11(a+bx))}{11b}$$

input
```
Integrate[Sin[a + b*x]^10*Sin[6*(2*a + 2*b*x)],x]
```

output
```
(Sin[a + b*x]^11*Sin[11*(a + b*x)])/(11*b)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(23) = 46$.

Time = 0.36 (sec) , antiderivative size = 177, normalized size of antiderivative = 7.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{10}(a+bx)\sin(6(2a+2bx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^{10}\sin(12a+12bx)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(-\frac{\sin(2a+2bx)}{1024}+\frac{5}{512}\sin(4a+4bx)-\frac{45\sin(6a+6bx)}{1024}+\frac{15}{128}\sin(8a+8bx)-\frac{105}{512}\sin(10a+10bx)+\frac{63}{256}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos(2a+2bx)}{2048b}-\frac{5\cos(4a+4bx)}{2048b}+\frac{15\cos(6a+6bx)}{2048b}-\frac{15\cos(8a+8bx)}{1024b}+\frac{21\cos(10a+10bx)}{1024b}-\frac{21\cos(12a+12bx)}{1024b}+\frac{15\cos(14a+14bx)}{1024b}-\frac{15\cos(16a+16bx)}{2048b}+\frac{5\cos(18a+18bx)}{2048b}-\frac{\cos(20a+20bx)}{2048b}+\frac{\cos(22a+22bx)}{22528b}$$

input
```
Int[Sin[a + b*x]^10*Sin[6*(2*a + 2*b*x)],x]
```

output
```
Cos[2*a + 2*b*x]/(2048*b) - (5*Cos[4*a + 4*b*x])/(2048*b) + (15*Cos[6*a +
6*b*x])/(2048*b) - (15*Cos[8*a + 8*b*x])/(1024*b) + (21*Cos[10*a + 10*b*x]
)/(1024*b) - (21*Cos[12*a + 12*b*x])/(1024*b) + (15*Cos[14*a + 14*b*x])/(1
024*b) - (15*Cos[16*a + 16*b*x])/(2048*b) + (5*Cos[18*a + 18*b*x])/(2048*b
) - Cos[20*a + 20*b*x]/(2048*b) + Cos[22*a + 22*b*x]/(22528*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(22) = 44$.

Time = 88.50 (sec) , antiderivative size = 127, normalized size of antiderivative = 5.52

method	result
parallelrisch	$\frac{1-55\cos(4bx+4a)-462\cos(12bx+12a)-165\cos(16bx+16a)-330\cos(8bx+8a)+11\cos(2bx+2a)+\cos(22bx+22a)-11\cos(20}{22528b}$
default	$\frac{\cos(2bx+2a)}{2048b}-\frac{5\cos(4bx+4a)}{2048b}+\frac{15\cos(6bx+6a)}{2048b}-\frac{15\cos(8bx+8a)}{1024b}+\frac{21\cos(10bx+10a)}{1024b}-\frac{21\cos(12bx+12a)}{1024b}+15$
risch	$\frac{\cos(2bx+2a)}{2048b}-\frac{5\cos(4bx+4a)}{2048b}+\frac{15\cos(6bx+6a)}{2048b}-\frac{15\cos(8bx+8a)}{1024b}+\frac{21\cos(10bx+10a)}{1024b}-\frac{21\cos(12bx+12a)}{1024b}+15$
orering	Expression too large to display

input
```
int(sin(b*x+a)^10*sin(12*b*x+12*a),x,method=_RETURNVERBOSE)
```

output
```
1/22528*(1-55*cos(4*b*x+4*a)-462*cos(12*b*x+12*a)-165*cos(16*b*x+16*a)-330
*cos(8*b*x+8*a)+11*cos(2*b*x+2*a)+cos(22*b*x+22*a)-11*cos(20*b*x+20*a)+55*
cos(18*b*x+18*a)+165*cos(6*b*x+6*a)+330*cos(14*b*x+14*a)+462*cos(10*b*x+10
*a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(22) = 44.

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 5.04

$$\int \sin^{10}(a+bx)\sin(6(2a+2bx))\,dx$$
$$= \frac{1024 \cos{(bx+a)}^{22} - 8448 \cos{(bx+a)}^{20} + 30976 \cos{(bx+a)}^{18} - 66352 \cos{(bx+a)}^{16} + 91740 \cos{(b}$$

input
```
integrate(sin(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="fricas")
```

output
```
1/11*(1024*cos(b*x + a)^22 - 8448*cos(b*x + a)^20 + 30976*cos(b*x + a)^18
- 66352*cos(b*x + a)^16 + 91740*cos(b*x + a)^14 - 85305*cos(b*x + a)^12 +
53834*cos(b*x + a)^10 - 22671*cos(b*x + a)^8 + 6072*cos(b*x + a)^6 - 935*c
os(b*x + a)^4 + 66*cos(b*x + a)^2)/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin^{10}(a+bx)\sin(6(2a+2bx))\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**10*sin(12*b*x+12*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \sin^{10}(a+bx)\sin(6(2a+2bx))\,dx = \frac{\cos(22\,bx+22\,a) - 11\,\cos(20\,bx+20\,a) + 55\,\cos(18\,bx+18\,a) - 165\,\cos(16\,bx+16\,a) + 330\,\cos(14$$

input
```
integrate(sin(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="maxima")
```

output
```
1/22528*(cos(22*b*x + 22*a) - 11*cos(20*b*x + 20*a) + 55*cos(18*b*x + 18*a
) - 165*cos(16*b*x + 16*a) + 330*cos(14*b*x + 14*a) - 462*cos(12*b*x + 12*
a) + 462*cos(10*b*x + 10*a) - 330*cos(8*b*x + 8*a) + 165*cos(6*b*x + 6*a)
- 55*cos(4*b*x + 4*a) + 11*cos(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$.

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.87

$$\int \sin^{10}(a+bx)\sin(6(2a+2bx))\,dx = -\frac{1024\,\sin(bx+a)^{22} - 2816\,\sin(bx+a)^{20} + 2816\,\sin(bx+a)^{18} - 1232\,\sin(bx+a)^{16} + 220\,\sin(bx+}{11\,b}$$

input
```
integrate(sin(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="giac")
```

output
```
-1/11*(1024*sin(b*x + a)^22 - 2816*sin(b*x + a)^20 + 2816*sin(b*x + a)^18
- 1232*sin(b*x + a)^16 + 220*sin(b*x + a)^14 - 11*sin(b*x + a)^12)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 6.26

$$\int \sin^{10}(a+bx)\sin(6(2a+2bx))\,dx = \frac{6\cos(a+bx)^2}{b} - \frac{85\cos(a+bx)^4}{b} + \frac{552\cos(a+bx)^6}{b} - \frac{2061\cos(a+bx)^8}{b} + \frac{4894\cos(a+bx)^{10}}{b} - \frac{7755\cos(a+bx)^{12}}{b} + \frac{8340\cos(a+bx)^{14}}{b} - \frac{6032\cos(a+bx)^{16}}{b} + \frac{2816\cos(a+bx)^{18}}{b} - \frac{768\cos(a+bx)^{20}}{b} + \frac{1024\cos(a+bx)^{22}}{11\,b}$$

input
```
int(sin(a + b*x)^10*sin(12*a + 12*b*x),x)
```

output
```
(6*cos(a + b*x)^2)/b - (85*cos(a + b*x)^4)/b + (552*cos(a + b*x)^6)/b - (2
061*cos(a + b*x)^8)/b + (4894*cos(a + b*x)^10)/b - (7755*cos(a + b*x)^12)/
b + (8340*cos(a + b*x)^14)/b - (6032*cos(a + b*x)^16)/b + (2816*cos(a + b*
x)^18)/b - (768*cos(a + b*x)^20)/b + (1024*cos(a + b*x)^22)/(11*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 10.17

$$\int \sin^{10}(a+bx)\sin(6(2a+2bx))\,dx$$
$$= \frac{-6144\cos(12bx+12a)\sin(bx+a)^{10} + 6912\cos(12bx+12a)\sin(bx+a)^8 - 3584\cos(12bx+12a)\sin}{}$$

input
```
int(sin(b*x+a)^10*sin(12*b*x+12*a),x)
```

output
```
( - 6144*cos(12*a + 12*b*x)*sin(a + b*x)**10 + 6912*cos(12*a + 12*b*x)*sin
(a + b*x)**8 - 3584*cos(12*a + 12*b*x)*sin(a + b*x)**6 + 840*cos(12*a + 12
*b*x)*sin(a + b*x)**4 - 72*cos(12*a + 12*b*x)*sin(a + b*x)**2 + cos(12*a +
 12*b*x) + 5120*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x)**9 - 4608*cos
(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x)**7 + 1792*cos(a + b*x)*sin(12*a
+ 12*b*x)*sin(a + b*x)**5 - 280*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*
x)**3 + 12*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x) - 1)/(22528*b)
```

3.602 $\int \sin^{10}(a+bx)\sin(3(4a+4bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \sin^{10}(a+bx)\sin(3(4a+4bx))\,dx=\frac{\sin^{11}(a+bx)\sin(11(a+bx))}{11b}$$

output
```
1/11*sin(b*x+a)^11*sin(11*b*x+11*a)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sin^{10}(a+bx)\sin(3(4a+4bx))\,dx=\frac{\sin^{11}(a+bx)\sin(11(a+bx))}{11b}$$

input
```
Integrate[Sin[a + b*x]^10*Sin[3*(4*a + 4*b*x)],x]
```

output
```
(Sin[a + b*x]^11*Sin[11*(a + b*x)])/(11*b)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(23) = 46$.

Time = 0.35 (sec) , antiderivative size = 177, normalized size of antiderivative = 7.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{10}(a+bx)\sin(3(4a+4bx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^{10}\sin(12a+12bx)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(-\frac{\sin(2a+2bx)}{1024}+\frac{5}{512}\sin(4a+4bx)-\frac{45\sin(6a+6bx)}{1024}+\frac{15}{128}\sin(8a+8bx)-\frac{105}{512}\sin(10a+10bx)+\frac{63}{256}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos(2a+2bx)}{2048b}-\frac{5\cos(4a+4bx)}{2048b}+\frac{15\cos(6a+6bx)}{2048b}-\frac{15\cos(8a+8bx)}{1024b}+\frac{21\cos(10a+10bx)}{1024b}-\frac{21\cos(12a+12bx)}{1024b}+\frac{15\cos(14a+14bx)}{1024b}-\frac{15\cos(16a+16bx)}{2048b}+\frac{5\cos(18a+18bx)}{2048b}-\frac{\cos(20a+20bx)}{2048b}+\frac{\cos(22a+22bx)}{22528b}$$

input
```
Int[Sin[a + b*x]^10*Sin[3*(4*a + 4*b*x)],x]
```

output
```
Cos[2*a + 2*b*x]/(2048*b) - (5*Cos[4*a + 4*b*x])/(2048*b) + (15*Cos[6*a +
6*b*x])/(2048*b) - (15*Cos[8*a + 8*b*x])/(1024*b) + (21*Cos[10*a + 10*b*x]
)/(1024*b) - (21*Cos[12*a + 12*b*x])/(1024*b) + (15*Cos[14*a + 14*b*x])/(1
024*b) - (15*Cos[16*a + 16*b*x])/(2048*b) + (5*Cos[18*a + 18*b*x])/(2048*b
) - Cos[20*a + 20*b*x]/(2048*b) + Cos[22*a + 22*b*x]/(22528*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(22) = 44$.

Time = 92.56 (sec) , antiderivative size = 127, normalized size of antiderivative = 5.52

method	result
parallelrisch	$\frac{1-55\cos(4bx+4a)-462\cos(12bx+12a)-165\cos(16bx+16a)-330\cos(8bx+8a)+11\cos(2bx+2a)+\cos(22bx+22a)-11\cos(20}{22528b}$
default	$\frac{\cos(2bx+2a)}{2048b}-\frac{5\cos(4bx+4a)}{2048b}+\frac{15\cos(6bx+6a)}{2048b}-\frac{15\cos(8bx+8a)}{1024b}+\frac{21\cos(10bx+10a)}{1024b}-\frac{21\cos(12bx+12a)}{1024b}+$ 15
risch	$\frac{\cos(2bx+2a)}{2048b}-\frac{5\cos(4bx+4a)}{2048b}+\frac{15\cos(6bx+6a)}{2048b}-\frac{15\cos(8bx+8a)}{1024b}+\frac{21\cos(10bx+10a)}{1024b}-\frac{21\cos(12bx+12a)}{1024b}+$ 15
orering	Expression too large to display

input
```
int(sin(b*x+a)^10*sin(12*b*x+12*a),x,method=_RETURNVERBOSE)
```

output
```
1/22528*(1-55*cos(4*b*x+4*a)-462*cos(12*b*x+12*a)-165*cos(16*b*x+16*a)-330
*cos(8*b*x+8*a)+11*cos(2*b*x+2*a)+cos(22*b*x+22*a)-11*cos(20*b*x+20*a)+55*
cos(18*b*x+18*a)+165*cos(6*b*x+6*a)+330*cos(14*b*x+14*a)+462*cos(10*b*x+10
*a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(22) = 44.

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 5.04

$$\int \sin^{10}(a+bx)\sin(3(4a+4bx))\,dx$$
$$= \frac{1024\cos(bx+a)^{22} - 8448\cos(bx+a)^{20} + 30976\cos(bx+a)^{18} - 66352\cos(bx+a)^{16} + 91740\cos(b$$

input
```
integrate(sin(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="fricas")
```

output
```
1/11*(1024*cos(b*x + a)^22 - 8448*cos(b*x + a)^20 + 30976*cos(b*x + a)^18
- 66352*cos(b*x + a)^16 + 91740*cos(b*x + a)^14 - 85305*cos(b*x + a)^12 +
53834*cos(b*x + a)^10 - 22671*cos(b*x + a)^8 + 6072*cos(b*x + a)^6 - 935*c
os(b*x + a)^4 + 66*cos(b*x + a)^2)/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin^{10}(a+bx)\sin(3(4a+4bx))\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**10*sin(12*b*x+12*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \sin^{10}(a+bx)\sin(3(4a+4bx))\,dx = \frac{\cos(22\,bx+22\,a) - 11\,\cos(20\,bx+20\,a) + 55\,\cos(18\,bx+18\,a) - 165\,\cos(16\,bx+16\,a) + 330\,\cos(14}{}$$

input
```
integrate(sin(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="maxima")
```

output
```
1/22528*(cos(22*b*x + 22*a) - 11*cos(20*b*x + 20*a) + 55*cos(18*b*x + 18*a
) - 165*cos(16*b*x + 16*a) + 330*cos(14*b*x + 14*a) - 462*cos(12*b*x + 12*
a) + 462*cos(10*b*x + 10*a) - 330*cos(8*b*x + 8*a) + 165*cos(6*b*x + 6*a)
- 55*cos(4*b*x + 4*a) + 11*cos(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$.

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.87

$$\int \sin^{10}(a+bx)\sin(3(4a+4bx))\,dx = -\frac{1024\,\sin(bx+a)^{22} - 2816\,\sin(bx+a)^{20} + 2816\,\sin(bx+a)^{18} - 1232\,\sin(bx+a)^{16} + 220\,\sin(bx+}{11\,b}$$

input
```
integrate(sin(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="giac")
```

output
```
-1/11*(1024*sin(b*x + a)^22 - 2816*sin(b*x + a)^20 + 2816*sin(b*x + a)^18
- 1232*sin(b*x + a)^16 + 220*sin(b*x + a)^14 - 11*sin(b*x + a)^12)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 6.26

$$\int \sin^{10}(a+bx)\sin(3(4a+4bx))\,dx = \frac{6\cos(a+b\,x)^2}{b} - \frac{85\cos(a+b\,x)^4}{b} + \frac{552\cos(a+b\,x)^6}{b} - \frac{2061\cos(a+b\,x)^8}{b} + \frac{4894\cos(a+b\,x)^{10}}{b} - \frac{7755\cos(a+b\,x)^{12}}{b} + \frac{8340\cos(a+b\,x)^{14}}{b} - \frac{6032\cos(a+b\,x)^{16}}{b} + \frac{2816\cos(a+b\,x)^{18}}{b} - \frac{768\cos(a+b\,x)^{20}}{b} + \frac{1024\cos(a+b\,x)^{22}}{11\,b}$$

input
```
int(sin(a + b*x)^10*sin(12*a + 12*b*x),x)
```

output
```
(6*cos(a + b*x)^2)/b - (85*cos(a + b*x)^4)/b + (552*cos(a + b*x)^6)/b - (2
061*cos(a + b*x)^8)/b + (4894*cos(a + b*x)^10)/b - (7755*cos(a + b*x)^12)/
b + (8340*cos(a + b*x)^14)/b - (6032*cos(a + b*x)^16)/b + (2816*cos(a + b*
x)^18)/b - (768*cos(a + b*x)^20)/b + (1024*cos(a + b*x)^22)/(11*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 234, normalized size of antiderivative = 10.17

$$\int \sin^{10}(a+bx)\sin(3(4a+4bx))\,dx$$

$$= \frac{-6144\cos(12bx+12a)\sin(bx+a)^{10} + 6912\cos(12bx+12a)\sin(bx+a)^8 - 3584\cos(12bx+12a)\sin}{}$$

input
```
int(sin(b*x+a)^10*sin(12*b*x+12*a),x)
```

output
```
( - 6144*cos(12*a + 12*b*x)*sin(a + b*x)**10 + 6912*cos(12*a + 12*b*x)*sin
(a + b*x)**8 - 3584*cos(12*a + 12*b*x)*sin(a + b*x)**6 + 840*cos(12*a + 12
*b*x)*sin(a + b*x)**4 - 72*cos(12*a + 12*b*x)*sin(a + b*x)**2 + cos(12*a +
 12*b*x) + 5120*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x)**9 - 4608*cos
(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x)**7 + 1792*cos(a + b*x)*sin(12*a
+ 12*b*x)*sin(a + b*x)**5 - 280*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*
x)**3 + 12*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x) - 1)/(22528*b)
```

3.603 $\int (e\sin(a+bx))^m \sin((2+m)(a+bx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int (e\sin(a+bx))^m \sin((2+m)(a+bx))\,dx = \frac{(e\sin(a+bx))^{1+m}\sin((1+m)(a+bx))}{be(1+m)}$$

output
```
(e*sin(b*x+a))^(1+m)*sin((1+m)*(b*x+a))/b/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int (e\sin(a+bx))^m \sin((2+m)(a+bx))\,dx = \frac{\sin(a+bx)(e\sin(a+bx))^m\sin((1+m)(a+bx))}{b(1+m)}$$

input
```
Integrate[(e*Sin[a + b*x])^m*Sin[(2 + m)*(a + b*x)],x]
```

output
```
(Sin[a + b*x]*(e*Sin[a + b*x])^m*Sin[(1 + m)*(a + b*x)])/(b*(1 + m))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 172, normalized size of antiderivative = 5.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {7271, 7281, 5064, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin((m+2)(a+bx))(e\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{7271}$$

$$\sin^{-m}(a+bx)(e\sin(a+bx))^m\int \sin^m(a+bx)\sin((m+2)(a+bx))dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\sin^{-m}(a+bx)(e\sin(a+bx))^m\int \sin^m(a+bx)\sin((m+2)(a+bx))d(a+bx)}{b}$$

$$\Big\downarrow \text{5064}$$

$$\frac{2^{-m-1}\sin^{-m}(a+bx)(e\sin(a+bx))^m\int \left(ie^{-i(m+2)(a+bx)}\left(ie^{-i(a+bx)}-ie^{i(a+bx)}\right)^m-ie^{i(m+2)(a+bx)}\left(ie^{-i(a+bx)}-i\right.\right.}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2^{-m-1}\left(\frac{\left(1-e^{2i(a+bx)}\right)e^{im(a+bx)}\left(ie^{-i(a+bx)}-ie^{i(a+bx)}\right)^m}{2(m+1)}-\frac{\left(1-e^{2i(a+bx)}\right)e^{-i(m+2)(a+bx)}\left(ie^{-i(a+bx)}-ie^{i(a+bx)}\right)^m}{2(m+1)}\right)\sin^{-m}(a+bx)(}{b}$$

input
```
Int[(e*Sin[a + b*x])^m*Sin[(2 + m)*(a + b*x)],x]
```

output
```
(2^(-1 - m)*((E^(I*m*(a + b*x))*(I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^m*
(1 - E^((2*I)*(a + b*x))))/(2*(1 + m)) - ((I/E^(I*(a + b*x)) - I*E^(I*(a +
 b*x)))^m*(1 - E^((2*I)*(a + b*x))))/(2*E^(I*(2 + m)*(a + b*x))*(1 + m)))*
(e*Sin[a + b*x])^m)/(b*Sin[a + b*x]^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5064
```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c +
d*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a
, b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.11 (sec) , antiderivative size = 964, normalized size of antiderivative = 28.35

method	result	size
risch	Expression too large to display	964

input
```
int((e*sin(b*x+a))^m*sin((2+m)*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(1+m)*(exp(2*I*(b*x+a))-1)^(1+m)/b*exp(I*(b*x+a))^(-m)*e^m*(1/2)^m*ex
p(1/2*I*m*(Pi*csgn(I*(exp(2*I*(b*x+a))-1))*csgn(I*exp(-I*(b*x+a)))*csgn(si
n(b*x+a))+Pi*csgn(I*(exp(2*I*(b*x+a))-1))*csgn(sin(b*x+a))^2+Pi*csgn(I*exp
(-I*(b*x+a)))*csgn(sin(b*x+a))^2-Pi*csgn(I*e)*csgn(sin(b*x+a))*csgn(e*sin(
b*x+a))+Pi*csgn(I*e)*csgn(e*sin(b*x+a))^2+Pi*csgn(sin(b*x+a))^3-Pi*csgn(si
n(b*x+a))*csgn(e*sin(b*x+a))^2+Pi*csgn(e*sin(b*x+a))^3-Pi*csgn(e*sin(b*x+a
))*csgn(I*e*sin(b*x+a))^2-Pi*csgn(I*e*sin(b*x+a))^3+Pi*csgn(e*sin(b*x+a))*
csgn(I*e*sin(b*x+a))+Pi*csgn(I*e*sin(b*x+a))^2+2*b*x-Pi+2*a))+1/4*(exp(2*I
*(b*x+a))-1)^m/(1+m)/b/(exp(I*(b*x+a))^m)/(2^m)*e^m*exp(-1/2*I*m*(-Pi*csgn
(I*(exp(2*I*(b*x+a))-1))*csgn(I*exp(-I*(b*x+a)))*csgn(sin(b*x+a))-Pi*csgn(
I*(exp(2*I*(b*x+a))-1))*csgn(sin(b*x+a))^2-Pi*csgn(I*exp(-I*(b*x+a)))*csgn
(sin(b*x+a))^2-Pi*csgn(sin(b*x+a))^3+Pi*csgn(I*e)*csgn(sin(b*x+a))*csgn(e*
sin(b*x+a))+Pi*csgn(sin(b*x+a))*csgn(e*sin(b*x+a))^2-Pi*csgn(I*e)*csgn(e*s
in(b*x+a))^2-Pi*csgn(e*sin(b*x+a))^3+Pi*csgn(e*sin(b*x+a))*csgn(I*e*sin(b*
x+a))^2+Pi*csgn(I*e*sin(b*x+a))^3-Pi*csgn(e*sin(b*x+a))*csgn(I*e*sin(b*x+a
))-Pi*csgn(I*e*sin(b*x+a))^2+2*b*x+Pi+2*a))-1/4*(exp(2*I*(b*x+a))-1)^m/(1+
m)/b/(exp(I*(b*x+a))^m)/(2^m)*e^m*exp(-1/2*I*(-Pi*m*csgn(I*(exp(2*I*(b*x+a
))-1))*csgn(I*exp(-I*(b*x+a)))*csgn(sin(b*x+a))-Pi*m*csgn(I*(exp(2*I*(b*x+
a))-1))*csgn(sin(b*x+a))^2-Pi*m*csgn(I*exp(-I*(b*x+a)))*csgn(sin(b*x+a))^2
-Pi*m*csgn(sin(b*x+a))^3+Pi*m*csgn(I*e)*csgn(sin(b*x+a))*csgn(e*sin(b*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.26

$$\int (e\sin(a+bx))^m \sin((2+m)(a+bx))\,dx$$
$$= \frac{\left(\cos\left(bx+a\right)\sin\left(am+(bm+2\,b)x+2\,a\right)\sin\left(bx+a\right)+\left(\cos\left(bx+a\right)^2-1\right)\cos\left(am+(bm+2\,b)x+2\right.}{bm+b}$$

input
```
integrate((e*sin(b*x+a))^m*sin((2+m)*(b*x+a)),x, algorithm="fricas")
```

output
```
(cos(b*x + a)*sin(a*m + (b*m + 2*b)*x + 2*a)*sin(b*x + a) + (cos(b*x + a)^
2 - 1)*cos(a*m + (b*m + 2*b)*x + 2*a))*(e*sin(b*x + a))^m/(b*m + b)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sin(a+bx))^m \sin((2+m)(a+bx))\,dx = \text{Timed out}$$

input
```
integrate((e*sin(b*x+a))**m*sin((2+m)*(b*x+a)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\sin(a+bx))^m \sin((2+m)(a+bx))\,dx = \int (e\sin(bx+a))^m \sin((bx+a)(m+2))\,dx$$

input
```
integrate((e*sin(b*x+a))^m*sin((2+m)*(b*x+a)),x, algorithm="maxima")
```

output
```
integrate((e*sin(b*x + a))^m*sin((b*x + a)*(m + 2)), x)
```

Giac [F]

$$\int (e\sin(a+bx))^m \sin((2+m)(a+bx))\,dx = \int (e\sin(bx+a))^m \sin((bx+a)(m+2))\,dx$$

input
```
integrate((e*sin(b*x+a))^m*sin((2+m)*(b*x+a)),x, algorithm="giac")
```

output
```
integrate((e*sin(b*x + a))^m*sin((b*x + a)*(m + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 19.94 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int (e\sin(a+bx))^m \sin((2+m)(a+bx))\,dx = \frac{(e\,\sin\left(a+b\,x\right))^m\,\left(\cos\left(m\,\left(a+b\,x\right)\right)-\cos\left(\left(m+2\right)\,\left(a+b\,x\right)\right)\right)}{2\,b\,\left(m+1\right)}$$

input
```
int(sin((m + 2)*(a + b*x))*(e*sin(a + b*x))^m,x)
```

output
```
((e*sin(a + b*x))^m*(cos(m*(a + b*x)) - cos((m + 2)*(a + b*x))))/(2*b*(m +
 1))
```

Reduce [F]

$$\int (e\sin(a+bx))^m \sin((2+m)(a+bx))\,dx = e^m\left(\int \sin\left(bx+a\right)^m \sin\left(bmx+am+2bx+2a\right)dx\right)$$

input
```
int((e*sin(b*x+a))^m*sin((2+m)*(b*x+a)),x)
```

output
```
e**m*int(sin(a + b*x)**m*sin(a*m + 2*a + b*m*x + 2*b*x),x)
```

3.604 $\int (e\sin(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int (e\sin(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = \frac{(e\sin(a+bx))^{1+m}\sin((1+m)(a+bx))}{be(1+m)}$$

output
```
(e*sin(b*x+a))^(1+m)*sin((1+m)*(b*x+a))/b/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int (e\sin(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = \frac{\sin(a+bx)(e\sin(a+bx))^m\sin((1+m)(a+bx))}{b(1+m)}$$

input
```
Integrate[(e*Sin[a + b*x])^m*Sin[a*(2 + m) + b*(2 + m)*x],x]
```

output
```
(Sin[a + b*x]*(e*Sin[a + b*x])^m*Sin[(1 + m)*(a + b*x)])/(b*(1 + m))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 220, normalized size of antiderivative = 6.47, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7271, 5064, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a(m+2)+b(m+2)x)(e\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{7271}$$

$$\sin^{-m}(a+bx)(e\sin(a+bx))^m\int \sin^m(a+bx)\sin(a(m+2)+bx(m+2))dx$$

$$\Big\downarrow \text{5064}$$

$$2^{-m-1}\sin^{-m}(a+bx)(e\sin(a+bx))^m\int \left(ie^{-ia(m+2)-ibx(m+2)}\left(ie^{-i(a+bx)}-ie^{i(a+bx)}\right)^m-ie^{ia(m+2)+ibx(m+2)}\left(ie^{-i(a+bx)}-ie^{i(a+bx)}\right)^m\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2^{-m-1}\left(\frac{\left(1-e^{2ia+2ibx}\right)e^{im(a+bx)}\left(ie^{-i(a+bx)}-ie^{i(a+bx)}\right)^m}{2b(m+1)}-\frac{\left(1-e^{2ia+2ibx}\right)^{-m}\left(ie^{-i(a+bx)}-ie^{i(a+bx)}\right)^m\left(1-e^{2i\cdots}\right.}{2b\cdots}\right.$$
$$bx)(e\sin(a+bx))^m$$

input
```
Int[(e*Sin[a + b*x])^m*Sin[a*(2 + m) + b*(2 + m)*x],x]
```

output
```
(2^(-1 - m)*((E^(I*m*(a + b*x))*(1 - E^((2*I)*a + (2*I)*b*x))*(I/E^(I*(a +
 b*x)) - I*E^(I*(a + b*x)))^m)/(2*b*(1 + m)) - (E^((-2*I)*a*(1 + m) - (2*I
)*b*(1 + m)*x + I*m*(a + b*x))*(I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^m*(
1 - E^((2*I)*(a + b*x)))^(1 + m))/(2*b*(1 - E^((2*I)*a + (2*I)*b*x))^m*(1
+ m)))*(e*Sin[a + b*x])^m)/Sin[a + b*x]^m
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5064
```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c +
d*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a
, b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.71 (sec) , antiderivative size = 964, normalized size of antiderivative = 28.35

method	result	size
risch	Expression too large to display	964

input
```
int((e*sin(b*x+a))^m*sin(a*(2+m)+b*(2+m)*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(1+m)*(exp(2*I*(b*x+a))-1)^(1+m)/b*exp(I*(b*x+a))^(-m)*e^m*(1/2)^m*ex
p(1/2*I*m*(Pi*csgn(I*(exp(2*I*(b*x+a))-1))*csgn(I*exp(-I*(b*x+a)))*csgn(si
n(b*x+a))+Pi*csgn(I*(exp(2*I*(b*x+a))-1))*csgn(sin(b*x+a))^2+Pi*csgn(I*exp
(-I*(b*x+a)))*csgn(sin(b*x+a))^2-Pi*csgn(I*e)*csgn(sin(b*x+a))*csgn(e*sin(
b*x+a))+Pi*csgn(I*e)*csgn(e*sin(b*x+a))^2+Pi*csgn(sin(b*x+a))^3-Pi*csgn(si
n(b*x+a))*csgn(e*sin(b*x+a))^2+Pi*csgn(e*sin(b*x+a))^3-Pi*csgn(e*sin(b*x+a
))*csgn(I*e*sin(b*x+a))^2-Pi*csgn(I*e*sin(b*x+a))^3+Pi*csgn(e*sin(b*x+a))*
csgn(I*e*sin(b*x+a))+Pi*csgn(I*e*sin(b*x+a))^2+2*b*x-Pi+2*a))+1/4*(exp(2*I
*(b*x+a))-1)^m/(1+m)/b/(exp(I*(b*x+a))^m)/(2^m)*e^m*exp(-1/2*I*m*(-Pi*csgn
(I*(exp(2*I*(b*x+a))-1))*csgn(I*exp(-I*(b*x+a)))*csgn(sin(b*x+a))-Pi*csgn(
I*(exp(2*I*(b*x+a))-1))*csgn(sin(b*x+a))^2-Pi*csgn(I*exp(-I*(b*x+a)))*csgn
(sin(b*x+a))^2-Pi*csgn(sin(b*x+a))^3+Pi*csgn(I*e)*csgn(sin(b*x+a))*csgn(e*
sin(b*x+a))+Pi*csgn(sin(b*x+a))*csgn(e*sin(b*x+a))^2-Pi*csgn(I*e)*csgn(e*s
in(b*x+a))^2-Pi*csgn(e*sin(b*x+a))^3+Pi*csgn(e*sin(b*x+a))*csgn(I*e*sin(b*
x+a))^2+Pi*csgn(I*e*sin(b*x+a))^3-Pi*csgn(e*sin(b*x+a))*csgn(I*e*sin(b*x+a
))-Pi*csgn(I*e*sin(b*x+a))^2+2*b*x+Pi+2*a))-1/4*(exp(2*I*(b*x+a))-1)^m/(1+
m)/b/(exp(I*(b*x+a))^m)/(2^m)*e^m*exp(-1/2*I*(-Pi*m*csgn(I*(exp(2*I*(b*x+a
))-1))*csgn(I*exp(-I*(b*x+a)))*csgn(sin(b*x+a))-Pi*m*csgn(I*(exp(2*I*(b*x+
a))-1))*csgn(sin(b*x+a))^2-Pi*m*csgn(I*exp(-I*(b*x+a)))*csgn(sin(b*x+a))^2
-Pi*m*csgn(sin(b*x+a))^3+Pi*m*csgn(I*e)*csgn(sin(b*x+a))*csgn(e*sin(b*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.26

$$\int (e\sin(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx$$
$$= \frac{\left(\cos\left(bx+a\right)\sin\left(am+\left(bm+2\,b\right)x+2\,a\right)\sin\left(bx+a\right)+\left(\cos\left(bx+a\right)^{2}-1\right)\cos\left(am+\left(bm+2\,b\right)x+2\right.}{bm+b}$$

input
```
integrate((e*sin(b*x+a))^m*sin(a*(2+m)+b*(2+m)*x),x, algorithm="fricas")
```

output
```
(cos(b*x + a)*sin(a*m + (b*m + 2*b)*x + 2*a)*sin(b*x + a) + (cos(b*x + a)^
2 - 1)*cos(a*m + (b*m + 2*b)*x + 2*a))*(e*sin(b*x + a))^m/(b*m + b)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sin(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = \text{Timed out}$$

input
```
integrate((e*sin(b*x+a))**m*sin(a*(2+m)+b*(2+m)*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\sin(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = \int (e\sin(bx+a))^m \sin(b(m+2)x+a(m+2))\,dx$$

input
```
integrate((e*sin(b*x+a))^m*sin(a*(2+m)+b*(2+m)*x),x, algorithm="maxima")
```

output
```
integrate((e*sin(b*x + a))^m*sin(b*(m + 2)*x + a*(m + 2)), x)
```

Giac [F]

$$\int (e\sin(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = \int (e\sin(bx+a))^m \sin(b(m+2)x+a(m+2))\,dx$$

input
```
integrate((e*sin(b*x+a))^m*sin(a*(2+m)+b*(2+m)*x),x, algorithm="giac")
```

output
```
integrate((e*sin(b*x + a))^m*sin(b*(m + 2)*x + a*(m + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 19.63 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int (e\sin(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = \frac{(e\,\sin(a+b\,x))^m\,(\cos(m\,(a+b\,x))-\cos((m+2)\,(a+b\,x)))}{2\,b\,(m+1)}$$

input
```
int(sin(a*(m + 2) + b*x*(m + 2))*(e*sin(a + b*x))^m,x)
```

output
```
((e*sin(a + b*x))^m*(cos(m*(a + b*x)) - cos((m + 2)*(a + b*x))))/(2*b*(m +
 1))
```

Reduce [F]

$$\int (e\sin(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = e^m\left(\int \sin(bx+a)^m \sin(bmx+am+2bx+2a)\,dx\right)$$

input
```
int((e*sin(b*x+a))^m*sin(a*(2+m)+b*(2+m)*x),x)
```

output
```
e**m*int(sin(a + b*x)**m*sin(a*m + 2*a + b*m*x + 2*b*x),x)
```

3.605 $\int \cos(12(a+bx))\sin^{10}(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \cos(12(a+bx))\sin^{10}(a+bx)\,dx = \frac{\cos(11(a+bx))\sin^{11}(a+bx)}{11b}$$

output
```
1/11*cos(11*b*x+11*a)*sin(b*x+a)^11/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos(12(a+bx))\sin^{10}(a+bx)\,dx = \frac{\cos(11(a+bx))\sin^{11}(a+bx)}{11b}$$

input
```
Integrate[Cos[12*(a + b*x)]*Sin[a + b*x]^10,x]
```

output
```
(Cos[11*(a + b*x)]*Sin[a + b*x]^11)/(11*b)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(23) = 46$.

Time = 0.36 (sec) , antiderivative size = 177, normalized size of antiderivative = 7.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{10}(a+bx)\cos(12(a+bx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^{10}\cos(12a+12bx)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(-\frac{\cos(2a+2bx)}{1024}+\frac{5}{512}\cos(4a+4bx)-\frac{45\cos(6a+6bx)}{1024}+\frac{15}{128}\cos(8a+8bx)-\frac{105}{512}\cos(10a+10bx)+\frac{63}{25\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(2a+2bx)}{2048b}+\frac{5\sin(4a+4bx)}{2048b}-\frac{15\sin(6a+6bx)}{2048b}+\frac{15\sin(8a+8bx)}{1024b}-\frac{21\sin(10a+10bx)}{1024b}+\frac{21\sin(12a+12bx)}{1024b}-\frac{15\sin(14a+14bx)}{1024b}+\frac{15\sin(16a+16bx)}{2048b}-\frac{5\sin(18a+18bx)}{2048b}+\frac{\sin(20a+20bx)}{2048b}-\frac{\sin(22a+22bx)}{22528b}$$

input
```
Int[Cos[12*(a + b*x)]*Sin[a + b*x]^10,x]
```

output
```
-1/2048*Sin[2*a + 2*b*x]/b + (5*Sin[4*a + 4*b*x])/(2048*b) - (15*Sin[6*a +
 6*b*x])/(2048*b) + (15*Sin[8*a + 8*b*x])/(1024*b) - (21*Sin[10*a + 10*b*x
])/(1024*b) + (21*Sin[12*a + 12*b*x])/(1024*b) - (15*Sin[14*a + 14*b*x])/(
1024*b) + (15*Sin[16*a + 16*b*x])/(2048*b) - (5*Sin[18*a + 18*b*x])/(2048*
b) + Sin[20*a + 20*b*x]/(2048*b) - Sin[22*a + 22*b*x]/(22528*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(22) = 44$.

Time = 103.74 (sec) , antiderivative size = 128, normalized size of antiderivative = 5.57

method	result
parallelrisch	$\frac{-11 \sin(2bx+2a)+55 \sin(4bx+4a)-165 \sin(6bx+6a)+330 \sin(8bx+8a)-462 \sin(10bx+10a)+462 \sin(12bx+12a)-330 \sin(14}{22528b}$
default	$-\frac{\sin(2bx+2a)}{2048b}+\frac{5\sin(4bx+4a)}{2048b}-\frac{15\sin(6bx+6a)}{2048b}+\frac{15\sin(8bx+8a)}{1024b}-\frac{21\sin(10bx+10a)}{1024b}+\frac{21\sin(12bx+12a)}{1024b}-$
risch	$-\frac{\sin(2bx+2a)}{2048b}+\frac{5\sin(4bx+4a)}{2048b}-\frac{15\sin(6bx+6a)}{2048b}+\frac{15\sin(8bx+8a)}{1024b}-\frac{21\sin(10bx+10a)}{1024b}+\frac{21\sin(12bx+12a)}{1024b}-$
orering	Expression too large to display

input
```
int(cos(12*b*x+12*a)*sin(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/22528*(-11*sin(2*b*x+2*a)+55*sin(4*b*x+4*a)-165*sin(6*b*x+6*a)+330*sin(8
*b*x+8*a)-462*sin(10*b*x+10*a)+462*sin(12*b*x+12*a)-330*sin(14*b*x+14*a)+1
65*sin(16*b*x+16*a)-55*sin(18*b*x+18*a)+11*sin(20*b*x+20*a)-sin(22*b*x+22*
a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(22) = 44$.

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.22

$$\int \cos(12(a+bx))\sin^{10}(a+bx)\,dx = -\frac{\left(1024\cos(bx+a)^{21} - 7936\cos(bx+a)^{19} + 27136\cos(bx+a)^{17} - 53712\cos(bx+a)^{15} + 67820\cos\right.}{}$$

input
```
integrate(cos(12*b*x+12*a)*sin(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/11*(1024*cos(b*x + a)^21 - 7936*cos(b*x + a)^19 + 27136*cos(b*x + a)^17
 - 53712*cos(b*x + a)^15 + 67820*cos(b*x + a)^13 - 56695*cos(b*x + a)^11 +
 31471*cos(b*x + a)^9 - 11286*cos(b*x + a)^7 + 2442*cos(b*x + a)^5 - 275*c
os(b*x + a)^3 + 11*cos(b*x + a))*sin(b*x + a)/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos(12(a+bx))\sin^{10}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(12*b*x+12*a)*sin(b*x+a)**10,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos(12(a+bx))\sin^{10}(a+bx)\,dx = -\frac{\sin(22\,bx+22\,a)-11\,\sin(20\,bx+20\,a)+55\,\sin(18\,bx+18\,a)-165\,\sin(16\,bx+16\,a)+330\,\sin(14$$

input
```
integrate(cos(12*b*x+12*a)*sin(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/22528*(sin(22*b*x + 22*a) - 11*sin(20*b*x + 20*a) + 55*sin(18*b*x + 18*
a) - 165*sin(16*b*x + 16*a) + 330*sin(14*b*x + 14*a) - 462*sin(12*b*x + 12
*a) + 462*sin(10*b*x + 10*a) - 330*sin(8*b*x + 8*a) + 165*sin(6*b*x + 6*a)
 - 55*sin(4*b*x + 4*a) + 11*sin(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 0.63 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos(12(a+bx))\sin^{10}(a+bx)\,dx = -\frac{\sin(22\,bx+22\,a)-11\,\sin(20\,bx+20\,a)+55\,\sin(18\,bx+18\,a)-165\,\sin(16\,bx+16\,a)+330\,\sin(14$$

input
```
integrate(cos(12*b*x+12*a)*sin(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/22528*(sin(22*b*x + 22*a) - 11*sin(20*b*x + 20*a) + 55*sin(18*b*x + 18*
a) - 165*sin(16*b*x + 16*a) + 330*sin(14*b*x + 14*a) - 462*sin(12*b*x + 12
*a) + 462*sin(10*b*x + 10*a) - 330*sin(8*b*x + 8*a) + 165*sin(6*b*x + 6*a)
 - 55*sin(4*b*x + 4*a) + 11*sin(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 20.58 (sec) , antiderivative size = 127, normalized size of antiderivative = 5.52

$$\int \cos(12(a+bx))\sin^{10}(a+bx)\,dx = -\frac{\frac{\sin(2a+2bx)}{2048}-\frac{5\sin(4a+4bx)}{2048}+\frac{15\sin(6a+6bx)}{2048}-\frac{15\sin(8a+8bx)}{1024}+\frac{21\sin(10a+10bx)}{1024}-\frac{21\sin(12a+12bx)}{1024}+\frac{15\sin(14}{10}}{b}$$

input
```
int(sin(a + b*x)^10*cos(12*a + 12*b*x),x)
```

output
```
-(sin(2*a + 2*b*x)/2048 - (5*sin(4*a + 4*b*x))/2048 + (15*sin(6*a + 6*b*x)
)/2048 - (15*sin(8*a + 8*b*x))/1024 + (21*sin(10*a + 10*b*x))/1024 - (21*s
in(12*a + 12*b*x))/1024 + (15*sin(14*a + 14*b*x))/1024 - (15*sin(16*a + 16
*b*x))/2048 + (5*sin(18*a + 18*b*x))/2048 - sin(20*a + 20*b*x)/2048 + sin(
22*a + 22*b*x)/22528)/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 10.22

$$\int \cos(12(a+bx))\sin^{10}(a+bx)\,dx = \frac{5120\cos(12bx+12a)\cos(bx+a)\sin(bx+a)^9-4608\cos(12bx+12a)\cos(bx+a)\sin(bx+a)^7+179}{}$$

input
```
int(cos(12*b*x+12*a)*sin(b*x+a)^10,x)
```

output
```
(5120*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x)**9 - 4608*cos(12*a + 12
*b*x)*cos(a + b*x)*sin(a + b*x)**7 + 1792*cos(12*a + 12*b*x)*cos(a + b*x)*
sin(a + b*x)**5 - 280*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x)**3 + 12
*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x) + 6144*sin(12*a + 12*b*x)*si
n(a + b*x)**10 - 6912*sin(12*a + 12*b*x)*sin(a + b*x)**8 + 3584*sin(12*a +
 12*b*x)*sin(a + b*x)**6 - 840*sin(12*a + 12*b*x)*sin(a + b*x)**4 + 72*sin
(12*a + 12*b*x)*sin(a + b*x)**2 - sin(12*a + 12*b*x))/(22528*b)
```

3.606 $\int \cos(6(2a+2bx))\sin^{10}(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \cos(6(2a+2bx))\sin^{10}(a+bx)\,dx=\frac{\cos(11(a+bx))\sin^{11}(a+bx)}{11b}$$

output
```
1/11*cos(11*b*x+11*a)*sin(b*x+a)^11/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos(6(2a+2bx))\sin^{10}(a+bx)\,dx=\frac{\cos(11(a+bx))\sin^{11}(a+bx)}{11b}$$

input
```
Integrate[Cos[6*(2*a + 2*b*x)]*Sin[a + b*x]^10,x]
```

output
```
(Cos[11*(a + b*x)]*Sin[a + b*x]^11)/(11*b)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(23) = 46$.

Time = 0.35 (sec) , antiderivative size = 177, normalized size of antiderivative = 7.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{10}(a+bx)\cos(6(2a+2bx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^{10}\cos(12a+12bx)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(-\frac{\cos(2a+2bx)}{1024}+\frac{5}{512}\cos(4a+4bx)-\frac{45\cos(6a+6bx)}{1024}+\frac{15}{128}\cos(8a+8bx)-\frac{105}{512}\cos(10a+10bx)+\frac{63}{25\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(2a+2bx)}{2048b}+\frac{5\sin(4a+4bx)}{2048b}-\frac{15\sin(6a+6bx)}{2048b}+\frac{15\sin(8a+8bx)}{1024b}-\frac{21\sin(10a+10bx)}{1024b}+\frac{21\sin(12a+12bx)}{1024b}-\frac{15\sin(14a+14bx)}{1024b}+\frac{15\sin(16a+16bx)}{2048b}-\frac{5\sin(18a+18bx)}{2048b}+\frac{\sin(20a+20bx)}{2048b}-\frac{\sin(22a+22bx)}{22528b}$$

input
```
Int[Cos[6*(2*a + 2*b*x)]*Sin[a + b*x]^10,x]
```

output
```
-1/2048*Sin[2*a + 2*b*x]/b + (5*Sin[4*a + 4*b*x])/(2048*b) - (15*Sin[6*a +
 6*b*x])/(2048*b) + (15*Sin[8*a + 8*b*x])/(1024*b) - (21*Sin[10*a + 10*b*x
])/(1024*b) + (21*Sin[12*a + 12*b*x])/(1024*b) - (15*Sin[14*a + 14*b*x])/(
1024*b) + (15*Sin[16*a + 16*b*x])/(2048*b) - (5*Sin[18*a + 18*b*x])/(2048*
b) + Sin[20*a + 20*b*x]/(2048*b) - Sin[22*a + 22*b*x]/(22528*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(22) = 44$.

Time = 86.64 (sec) , antiderivative size = 128, normalized size of antiderivative = 5.57

method	result
parallelrisch	$\frac{-11\sin(2bx+2a)+55\sin(4bx+4a)-165\sin(6bx+6a)+330\sin(8bx+8a)-462\sin(10bx+10a)+462\sin(12bx+12a)-330\sin(14}{22528b}$
default	$-\frac{\sin(2bx+2a)}{2048b}+\frac{5\sin(4bx+4a)}{2048b}-\frac{15\sin(6bx+6a)}{2048b}+\frac{15\sin(8bx+8a)}{1024b}-\frac{21\sin(10bx+10a)}{1024b}+\frac{21\sin(12bx+12a)}{1024b}-$
risch	$-\frac{\sin(2bx+2a)}{2048b}+\frac{5\sin(4bx+4a)}{2048b}-\frac{15\sin(6bx+6a)}{2048b}+\frac{15\sin(8bx+8a)}{1024b}-\frac{21\sin(10bx+10a)}{1024b}+\frac{21\sin(12bx+12a)}{1024b}-$
orering	Expression too large to display

input
```
int(cos(12*b*x+12*a)*sin(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/22528*(-11*sin(2*b*x+2*a)+55*sin(4*b*x+4*a)-165*sin(6*b*x+6*a)+330*sin(8
*b*x+8*a)-462*sin(10*b*x+10*a)+462*sin(12*b*x+12*a)-330*sin(14*b*x+14*a)+1
65*sin(16*b*x+16*a)-55*sin(18*b*x+18*a)+11*sin(20*b*x+20*a)-sin(22*b*x+22*
a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(22) = 44.

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.22

$$\int \cos(6(2a+2bx))\sin^{10}(a+bx)\,dx = -\frac{\left(1024\cos\left(bx+a\right)^{21}-7936\cos\left(bx+a\right)^{19}+27136\cos\left(bx+a\right)^{17}-53712\cos\left(bx+a\right)^{15}+67820\cos\right.}{}$$

input
```
integrate(cos(12*b*x+12*a)*sin(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/11*(1024*cos(b*x + a)^21 - 7936*cos(b*x + a)^19 + 27136*cos(b*x + a)^17
 - 53712*cos(b*x + a)^15 + 67820*cos(b*x + a)^13 - 56695*cos(b*x + a)^11 +
 31471*cos(b*x + a)^9 - 11286*cos(b*x + a)^7 + 2442*cos(b*x + a)^5 - 275*c
os(b*x + a)^3 + 11*cos(b*x + a))*sin(b*x + a)/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos(6(2a+2bx))\sin^{10}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(12*b*x+12*a)*sin(b*x+a)**10,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(22) = 44.

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos(6(2a+2bx))\sin^{10}(a+bx)\,dx = -\frac{\sin(22\,bx+22\,a)-11\,\sin(20\,bx+20\,a)+55\,\sin(18\,bx+18\,a)-165\,\sin(16\,bx+16\,a)+330\,\sin(14$$

input
```
integrate(cos(12*b*x+12*a)*sin(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/22528*(sin(22*b*x + 22*a) - 11*sin(20*b*x + 20*a) + 55*sin(18*b*x + 18*
a) - 165*sin(16*b*x + 16*a) + 330*sin(14*b*x + 14*a) - 462*sin(12*b*x + 12
*a) + 462*sin(10*b*x + 10*a) - 330*sin(8*b*x + 8*a) + 165*sin(6*b*x + 6*a)
 - 55*sin(4*b*x + 4*a) + 11*sin(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(22) = 44.

Time = 0.60 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos(6(2a+2bx))\sin^{10}(a+bx)\,dx = -\frac{\sin(22\,bx+22\,a)-11\,\sin(20\,bx+20\,a)+55\,\sin(18\,bx+18\,a)-165\,\sin(16\,bx+16\,a)+330\,\sin(14$$

input
```
integrate(cos(12*b*x+12*a)*sin(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/22528*(sin(22*b*x + 22*a) - 11*sin(20*b*x + 20*a) + 55*sin(18*b*x + 18*
a) - 165*sin(16*b*x + 16*a) + 330*sin(14*b*x + 14*a) - 462*sin(12*b*x + 12
*a) + 462*sin(10*b*x + 10*a) - 330*sin(8*b*x + 8*a) + 165*sin(6*b*x + 6*a)
 - 55*sin(4*b*x + 4*a) + 11*sin(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 127, normalized size of antiderivative = 5.52

$$\int \cos(6(2a+2bx))\sin^{10}(a+bx)\,dx = -\frac{\frac{\sin(2\,a+2\,b\,x)}{2048} - \frac{5\,\sin(4\,a+4\,b\,x)}{2048} + \frac{15\,\sin(6\,a+6\,b\,x)}{2048} - \frac{15\,\sin(8\,a+8\,b\,x)}{1024} + \frac{21\,\sin(10\,a+10\,b\,x)}{1024} - \frac{21\,\sin(12\,a+12\,b\,x)}{1024} + \frac{15\,\sin(14}{10}}{b}$$

input
```
int(sin(a + b*x)^10*cos(12*a + 12*b*x),x)
```

output
```
-(sin(2*a + 2*b*x)/2048 - (5*sin(4*a + 4*b*x))/2048 + (15*sin(6*a + 6*b*x)
)/2048 - (15*sin(8*a + 8*b*x))/1024 + (21*sin(10*a + 10*b*x))/1024 - (21*s
in(12*a + 12*b*x))/1024 + (15*sin(14*a + 14*b*x))/1024 - (15*sin(16*a + 16
*b*x))/2048 + (5*sin(18*a + 18*b*x))/2048 - sin(20*a + 20*b*x)/2048 + sin(
22*a + 22*b*x)/22528)/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 10.22

$$\int \cos(6(2a+2bx))\sin^{10}(a+bx)\,dx = \frac{5120\cos(12bx+12a)\cos(bx+a)\sin(bx+a)^9 - 4608\cos(12bx+12a)\cos(bx+a)\sin(bx+a)^7 + 179}{}$$

input
```
int(cos(12*b*x+12*a)*sin(b*x+a)^10,x)
```

output
```
(5120*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x)**9 - 4608*cos(12*a + 12
*b*x)*cos(a + b*x)*sin(a + b*x)**7 + 1792*cos(12*a + 12*b*x)*cos(a + b*x)*
sin(a + b*x)**5 - 280*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x)**3 + 12
*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x) + 6144*sin(12*a + 12*b*x)*si
n(a + b*x)**10 - 6912*sin(12*a + 12*b*x)*sin(a + b*x)**8 + 3584*sin(12*a +
 12*b*x)*sin(a + b*x)**6 - 840*sin(12*a + 12*b*x)*sin(a + b*x)**4 + 72*sin
(12*a + 12*b*x)*sin(a + b*x)**2 - sin(12*a + 12*b*x))/(22528*b)
```

3.607 $\int \cos(3(4a+4bx))\sin^{10}(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \cos(3(4a+4bx))\sin^{10}(a+bx)\,dx = \frac{\cos(11(a+bx))\sin^{11}(a+bx)}{11b}$$

output
```
1/11*cos(11*b*x+11*a)*sin(b*x+a)^11/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos(3(4a+4bx))\sin^{10}(a+bx)\,dx = \frac{\cos(11(a+bx))\sin^{11}(a+bx)}{11b}$$

input
```
Integrate[Cos[3*(4*a + 4*b*x)]*Sin[a + b*x]^10,x]
```

output
```
(Cos[11*(a + b*x)]*Sin[a + b*x]^11)/(11*b)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(23) = 46$.

Time = 0.35 (sec) , antiderivative size = 177, normalized size of antiderivative = 7.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{10}(a+bx)\cos(3(4a+4bx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^{10}\cos(12a+12bx)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(-\frac{\cos(2a+2bx)}{1024}+\frac{5}{512}\cos(4a+4bx)-\frac{45\cos(6a+6bx)}{1024}+\frac{15}{128}\cos(8a+8bx)-\frac{105}{512}\cos(10a+10bx)+\frac{63}{25\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(2a+2bx)}{2048b}+\frac{5\sin(4a+4bx)}{2048b}-\frac{15\sin(6a+6bx)}{2048b}+\frac{15\sin(8a+8bx)}{1024b}-\frac{21\sin(10a+10bx)}{1024b}+\frac{21\sin(12a+12bx)}{1024b}-\frac{15\sin(14a+14bx)}{1024b}+\frac{15\sin(16a+16bx)}{2048b}-\frac{5\sin(18a+18bx)}{2048b}+\frac{\sin(20a+20bx)}{2048b}-\frac{\sin(22a+22bx)}{22528b}$$

input
```
Int[Cos[3*(4*a + 4*b*x)]*Sin[a + b*x]^10,x]
```

output
```
-1/2048*Sin[2*a + 2*b*x]/b + (5*Sin[4*a + 4*b*x])/(2048*b) - (15*Sin[6*a +
 6*b*x])/(2048*b) + (15*Sin[8*a + 8*b*x])/(1024*b) - (21*Sin[10*a + 10*b*x
])/(1024*b) + (21*Sin[12*a + 12*b*x])/(1024*b) - (15*Sin[14*a + 14*b*x])/(
1024*b) + (15*Sin[16*a + 16*b*x])/(2048*b) - (5*Sin[18*a + 18*b*x])/(2048*
b) + Sin[20*a + 20*b*x]/(2048*b) - Sin[22*a + 22*b*x]/(22528*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(22) = 44.

Time = 78.02 (sec) , antiderivative size = 128, normalized size of antiderivative = 5.57

method	result
parallelrisch	$\frac{-11\sin(2bx+2a)+55\sin(4bx+4a)-165\sin(6bx+6a)+330\sin(8bx+8a)-462\sin(10bx+10a)+462\sin(12bx+12a)-330\sin(14}{22528b}$
default	$-\frac{\sin(2bx+2a)}{2048b}+\frac{5\sin(4bx+4a)}{2048b}-\frac{15\sin(6bx+6a)}{2048b}+\frac{15\sin(8bx+8a)}{1024b}-\frac{21\sin(10bx+10a)}{1024b}+\frac{21\sin(12bx+12a)}{1024b}-$
risch	$-\frac{\sin(2bx+2a)}{2048b}+\frac{5\sin(4bx+4a)}{2048b}-\frac{15\sin(6bx+6a)}{2048b}+\frac{15\sin(8bx+8a)}{1024b}-\frac{21\sin(10bx+10a)}{1024b}+\frac{21\sin(12bx+12a)}{1024b}-$
orering	Expression too large to display

input
```
int(cos(12*b*x+12*a)*sin(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/22528*(-11*sin(2*b*x+2*a)+55*sin(4*b*x+4*a)-165*sin(6*b*x+6*a)+330*sin(8
*b*x+8*a)-462*sin(10*b*x+10*a)+462*sin(12*b*x+12*a)-330*sin(14*b*x+14*a)+1
65*sin(16*b*x+16*a)-55*sin(18*b*x+18*a)+11*sin(20*b*x+20*a)-sin(22*b*x+22*
a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(22) = 44$.

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.22

$$\int \cos(3(4a+4bx))\sin^{10}(a+bx)\,dx = -\frac{\left(1024\cos\left(bx+a\right)^{21} - 7936\cos\left(bx+a\right)^{19} + 27136\cos\left(bx+a\right)^{17} - 53712\cos\left(bx+a\right)^{15} + 67820\cos\right.}{}$$

input
```
integrate(cos(12*b*x+12*a)*sin(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/11*(1024*cos(b*x + a)^21 - 7936*cos(b*x + a)^19 + 27136*cos(b*x + a)^17
 - 53712*cos(b*x + a)^15 + 67820*cos(b*x + a)^13 - 56695*cos(b*x + a)^11 +
 31471*cos(b*x + a)^9 - 11286*cos(b*x + a)^7 + 2442*cos(b*x + a)^5 - 275*c
os(b*x + a)^3 + 11*cos(b*x + a))*sin(b*x + a)/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos(3(4a+4bx))\sin^{10}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(12*b*x+12*a)*sin(b*x+a)**10,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos(3(4a+4bx))\sin^{10}(a+bx)\,dx =$$
$$-\frac{\sin(22\,bx+22\,a) - 11\,\sin(20\,bx+20\,a) + 55\,\sin(18\,bx+18\,a) - 165\,\sin(16\,bx+16\,a) + 330\,\sin(1}{}$$

input
```
integrate(cos(12*b*x+12*a)*sin(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/22528*(sin(22*b*x + 22*a) - 11*sin(20*b*x + 20*a) + 55*sin(18*b*x + 18*
a) - 165*sin(16*b*x + 16*a) + 330*sin(14*b*x + 14*a) - 462*sin(12*b*x + 12
*a) + 462*sin(10*b*x + 10*a) - 330*sin(8*b*x + 8*a) + 165*sin(6*b*x + 6*a)
 - 55*sin(4*b*x + 4*a) + 11*sin(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 0.60 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos(3(4a+4bx))\sin^{10}(a+bx)\,dx =$$
$$-\frac{\sin(22\,bx+22\,a) - 11\,\sin(20\,bx+20\,a) + 55\,\sin(18\,bx+18\,a) - 165\,\sin(16\,bx+16\,a) + 330\,\sin(1}{}$$

input
```
integrate(cos(12*b*x+12*a)*sin(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/22528*(sin(22*b*x + 22*a) - 11*sin(20*b*x + 20*a) + 55*sin(18*b*x + 18*
a) - 165*sin(16*b*x + 16*a) + 330*sin(14*b*x + 14*a) - 462*sin(12*b*x + 12
*a) + 462*sin(10*b*x + 10*a) - 330*sin(8*b*x + 8*a) + 165*sin(6*b*x + 6*a)
 - 55*sin(4*b*x + 4*a) + 11*sin(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 127, normalized size of antiderivative = 5.52

$$\int \cos(3(4a+4bx))\sin^{10}(a+bx)\,dx = -\frac{\frac{\sin(2a+2bx)}{2048}-\frac{5\sin(4a+4bx)}{2048}+\frac{15\sin(6a+6bx)}{2048}-\frac{15\sin(8a+8bx)}{1024}+\frac{21\sin(10a+10bx)}{1024}-\frac{21\sin(12a+12bx)}{1024}+\frac{15\sin(14}{10}}{b}$$

input
```
int(sin(a + b*x)^10*cos(12*a + 12*b*x),x)
```

output
```
-(sin(2*a + 2*b*x)/2048 - (5*sin(4*a + 4*b*x))/2048 + (15*sin(6*a + 6*b*x)
)/2048 - (15*sin(8*a + 8*b*x))/1024 + (21*sin(10*a + 10*b*x))/1024 - (21*s
in(12*a + 12*b*x))/1024 + (15*sin(14*a + 14*b*x))/1024 - (15*sin(16*a + 16
*b*x))/2048 + (5*sin(18*a + 18*b*x))/2048 - sin(20*a + 20*b*x)/2048 + sin(
22*a + 22*b*x)/22528)/b
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 235, normalized size of antiderivative = 10.22

$$\int \cos(3(4a+4bx))\sin^{10}(a+bx)\,dx = \frac{5120\cos(12bx+12a)\cos(bx+a)\sin(bx+a)^9-4608\cos(12bx+12a)\cos(bx+a)\sin(bx+a)^7+179}{}$$

input
```
int(cos(12*b*x+12*a)*sin(b*x+a)^10,x)
```

output
```
(5120*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x)**9 - 4608*cos(12*a + 12
*b*x)*cos(a + b*x)*sin(a + b*x)**7 + 1792*cos(12*a + 12*b*x)*cos(a + b*x)*
sin(a + b*x)**5 - 280*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x)**3 + 12
*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x) + 6144*sin(12*a + 12*b*x)*si
n(a + b*x)**10 - 6912*sin(12*a + 12*b*x)*sin(a + b*x)**8 + 3584*sin(12*a +
 12*b*x)*sin(a + b*x)**6 - 840*sin(12*a + 12*b*x)*sin(a + b*x)**4 + 72*sin
(12*a + 12*b*x)*sin(a + b*x)**2 - sin(12*a + 12*b*x))/(22528*b)
```

3.608 $\int \cos((2+m)(a+bx))(e\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \cos((2+m)(a+bx))(e\sin(a+bx))^m\,dx=\frac{\cos((1+m)(a+bx))(e\sin(a+bx))^{1+m}}{be(1+m)}$$

output
```
cos((1+m)*(b*x+a))*(e*sin(b*x+a))^(1+m)/b/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \cos((2+m)(a+bx))(e\sin(a+bx))^m\,dx=\frac{\cos((1+m)(a+bx))\sin(a+bx)(e\sin(a+bx))^m}{b(1+m)}$$

input
```
Integrate[Cos[(2 + m)*(a + b*x)]*(e*Sin[a + b*x])^m,x]
```

output
```
(Cos[(1 + m)*(a + b*x)]*Sin[a + b*x]*(e*Sin[a + b*x])^m)/(b*(1 + m))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 176, normalized size of antiderivative = 5.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {7271, 7281, 5067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos((m+2)(a+bx))(e\sin(a+bx))^m\,dx$$

$$\Big\downarrow \; 7271$$

$$\sin^{-m}(a+bx)(e\sin(a+bx))^m\int \cos((m+2)(a+bx))\sin^m(a+bx)dx$$

$$\Big\downarrow \; 7281$$

$$\frac{\sin^{-m}(a+bx)(e\sin(a+bx))^m\int \cos((m+2)(a+bx))\sin^m(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \; 5067$$

$$\frac{2^{-m-1}\sin^{-m}(a+bx)(e\sin(a+bx))^m\int \left(e^{-i(m+2)(a+bx)}\left(ie^{-i(a+bx)}-ie^{i(a+bx)}\right)^m+e^{i(m+2)(a+bx)}\left(ie^{-i(a+bx)}-ie\right.\right.}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{2^{-m-1}\left(\frac{i\left(1-e^{2i(a+bx)}\right)e^{im(a+bx)}\left(ie^{-i(a+bx)}-ie^{i(a+bx)}\right)^m}{2(m+1)}+\frac{i\left(1-e^{2i(a+bx)}\right)e^{-i(m+2)(a+bx)}\left(ie^{-i(a+bx)}-ie^{i(a+bx)}\right)^m}{2(m+1)}\right)\sin^{-m}(a+bx)}{b}$$

input
```
Int[Cos[(2 + m)*(a + b*x)]*(e*Sin[a + b*x])^m,x]
```

output
```
(2^(-1 - m)*(((I/2)*E^(I*m*(a + b*x))*(I/E^(I*(a + b*x)) - I*E^(I*(a + b*x
)))^m*(1 - E^((2*I)*(a + b*x))))/(1 + m) + ((I/2)*(I/E^(I*(a + b*x)) - I*E
^(I*(a + b*x)))^m*(1 - E^((2*I)*(a + b*x))))/(E^(I*(2 + m)*(a + b*x))*(1 +
 m))))*(e*Sin[a + b*x])^m)/(b*Sin[a + b*x]^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5067
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c +
d*x)))^q, (E^((-I)*(a + b*x)) + E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.07 (sec) , antiderivative size = 967, normalized size of antiderivative = 28.44

method	result	size
risch	Expression too large to display	967

input
```
int(cos((2+m)*(b*x+a))*(e*sin(b*x+a))^m,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/(1+m)*(exp(2*I*(b*x+a))-1)^(1+m)/b*exp(I*(b*x+a))^(-m)*e^m*(1/2)^m*
exp(1/2*I*m*(Pi*csgn(I*(exp(2*I*(b*x+a))-1))*csgn(I*exp(-I*(b*x+a)))*csgn(
sin(b*x+a))+Pi*csgn(I*(exp(2*I*(b*x+a))-1))*csgn(sin(b*x+a))^2+Pi*csgn(I*e
xp(-I*(b*x+a)))*csgn(sin(b*x+a))^2-Pi*csgn(I*e)*csgn(sin(b*x+a))*csgn(e*si
n(b*x+a))+Pi*csgn(I*e)*csgn(e*sin(b*x+a))^2+Pi*csgn(sin(b*x+a))^3-Pi*csgn(
sin(b*x+a))*csgn(e*sin(b*x+a))^2+Pi*csgn(e*sin(b*x+a))^3-Pi*csgn(e*sin(b*x
+a))*csgn(I*e*sin(b*x+a))^2-Pi*csgn(I*e*sin(b*x+a))^3+Pi*csgn(e*sin(b*x+a)
)*csgn(I*e*sin(b*x+a))+Pi*csgn(I*e*sin(b*x+a))^2+2*b*x-Pi+2*a))-1/4*I*(exp
(2*I*(b*x+a))-1)^m/(1+m)/b/(exp(I*(b*x+a))^m)/(2^m)*e^m*exp(-1/2*I*m*(-Pi*
csgn(I*(exp(2*I*(b*x+a))-1))*csgn(I*exp(-I*(b*x+a)))*csgn(sin(b*x+a))-Pi*c
sgn(I*(exp(2*I*(b*x+a))-1))*csgn(sin(b*x+a))^2-Pi*csgn(I*exp(-I*(b*x+a)))*
csgn(sin(b*x+a))^2-Pi*csgn(sin(b*x+a))^3+Pi*csgn(I*e)*csgn(sin(b*x+a))*csg
n(e*sin(b*x+a))+Pi*csgn(sin(b*x+a))*csgn(e*sin(b*x+a))^2-Pi*csgn(I*e)*csgn
(e*sin(b*x+a))^2-Pi*csgn(e*sin(b*x+a))^3+Pi*csgn(e*sin(b*x+a))*csgn(I*e*si
n(b*x+a))^2+Pi*csgn(I*e*sin(b*x+a))^3-Pi*csgn(e*sin(b*x+a))*csgn(I*e*sin(b
*x+a))-Pi*csgn(I*e*sin(b*x+a))^2+2*b*x+Pi+2*a))+1/4*I*(exp(2*I*(b*x+a))-1)
^m*e^m/(1+m)/b/(exp(I*(b*x+a))^m)/(2^m)*exp(-1/2*I*(-Pi*m*csgn(I*(exp(2*I*
(b*x+a))-1))*csgn(I*exp(-I*(b*x+a)))*csgn(sin(b*x+a))-Pi*m*csgn(I*(exp(2*I
*(b*x+a))-1))*csgn(sin(b*x+a))^2-Pi*m*csgn(I*exp(-I*(b*x+a)))*csgn(sin(b*x
+a))^2-Pi*m*csgn(sin(b*x+a))^3+Pi*m*csgn(I*e)*csgn(sin(b*x+a))*csgn(e*s...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.29

$$\int \cos((2+m)(a+bx))(e\sin(a+bx))^m\,dx$$
$$= \frac{\left(\cos\left(am+(bm+2\,b)x+2\,a\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-\left(\cos\left(bx+a\right)^2-1\right)\sin\left(am+(bm+2\,b)x+2\right.}{bm+b}$$

input
```
integrate(cos((2+m)*(b*x+a))*(e*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
(cos(a*m + (b*m + 2*b)*x + 2*a)*cos(b*x + a)*sin(b*x + a) - (cos(b*x + a)^
2 - 1)*sin(a*m + (b*m + 2*b)*x + 2*a))*(e*sin(b*x + a))^m/(b*m + b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos((2+m)(a+bx))(e\sin(a+bx))^m\,dx = \text{Timed out}$$

input
```
integrate(cos((2+m)*(b*x+a))*(e*sin(b*x+a))**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos((2+m)(a+bx))(e\sin(a+bx))^m\,dx = \int (e\sin(bx+a))^m\cos((bx+a)(m+2))\,dx$$

input
```
integrate(cos((2+m)*(b*x+a))*(e*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate((e*sin(b*x + a))^m*cos((b*x + a)*(m + 2)), x)
```

Giac [F]

$$\int \cos((2+m)(a+bx))(e\sin(a+bx))^m\,dx = \int (e\sin(bx+a))^m\cos((bx+a)(m+2))\,dx$$

input
```
integrate(cos((2+m)*(b*x+a))*(e*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate((e*sin(b*x + a))^m*cos((b*x + a)*(m + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 18.92 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \cos((2+m)(a+bx))(e\sin(a+bx))^m\,dx = -\frac{(e\,\sin\left(a+b\,x\right))^m\,\left(\sin\left(m\,\left(a+b\,x\right)\right)-\sin\left(\left(m+2\right)\,\left(a+b\,x\right)\right)\right)}{2\,b\,\left(m+1\right)}$$

input
```
int(cos((m + 2)*(a + b*x))*(e*sin(a + b*x))^m,x)
```

output
```
-((e*sin(a + b*x))^m*(sin(m*(a + b*x)) - sin((m + 2)*(a + b*x))))/(2*b*(m
+ 1))
```

Reduce [F]

$$\int \cos((2+m)(a+bx))(e\sin(a+bx))^m\,dx = e^m\left(\int \sin\left(bx+a\right)^m\cos\left(bmx+am+2bx+2a\right)dx\right)$$

input
```
int(cos((2+m)*(b*x+a))*(e*sin(b*x+a))^m,x)
```

output
```
e**m*int(sin(a + b*x)**m*cos(a*m + 2*a + b*m*x + 2*b*x),x)
```

3.609 $\int \cos(a(2+m)+b(2+m)x)(e\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int \cos(a(2+m)+b(2+m)x)(e\sin(a+bx))^m\,dx = \frac{\cos((1+m)(a+bx))(e\sin(a+bx))^{1+m}}{be(1+m)}$$

output
```
cos((1+m)*(b*x+a))*(e*sin(b*x+a))^(1+m)/b/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \cos(a(2+m)+b(2+m)x)(e\sin(a+bx))^m\,dx = \frac{\cos((1+m)(a+bx))\sin(a+bx)(e\sin(a+bx))^m}{b(1+m)}$$

input
```
Integrate[Cos[a*(2 + m) + b*(2 + m)*x]*(e*Sin[a + b*x])^m,x]
```

output
```
(Cos[(1 + m)*(a + b*x)]*Sin[a + b*x]*(e*Sin[a + b*x])^m)/(b*(1 + m))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 224, normalized size of antiderivative = 6.59, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7271, 5067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a(m+2)+b(m+2)x)(e\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{7271}$$

$$\sin^{-m}(a+bx)(e\sin(a+bx))^m\int \cos(a(m+2)+bx(m+2))\sin^m(a+bx)dx$$

$$\Big\downarrow \text{5067}$$

$$2^{-m-1}\sin^{-m}(a+bx)(e\sin(a+bx))^m\int \left(e^{-ia(m+2)-ibx(m+2)}\left(ie^{-i(a+bx)}-ie^{i(a+bx)}\right)^m+e^{ia(m+2)+ibx(m+2)}\left(ie^{-i(a+bx)}-ie^{i(a+bx)}\right)^m\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2^{-m-1}\left(\frac{i\left(ie^{-i(a+bx)}-ie^{i(a+bx)}\right)^m\left(1-e^{2i(a+bx)}\right)^{m+1}\left(1-e^{2ia+2ibx}\right)^{-m}\exp(im(a+bx)-2ia(m+1)-2ib(m+\ldots}{2b(m+1)}\right.$$
$$bx)(e\sin(a+bx))^m$$

input
```
Int[Cos[a*(2 + m) + b*(2 + m)*x]*(e*Sin[a + b*x])^m,x]
```

output
```
(2^(-1 - m)*(((I/2)*E^(I*m*(a + b*x))*(1 - E^((2*I)*a + (2*I)*b*x))*(I/E^(
I*(a + b*x)) - I*E^(I*(a + b*x)))^m)/(b*(1 + m)) + ((I/2)*E^((-2*I)*a*(1 +
 m) - (2*I)*b*(1 + m)*x + I*m*(a + b*x))*(I/E^(I*(a + b*x)) - I*E^(I*(a +
b*x)))^m*(1 - E^((2*I)*(a + b*x)))^(1 + m))/(b*(1 - E^((2*I)*a + (2*I)*b*x
))^m*(1 + m)))*(e*Sin[a + b*x])^m)/Sin[a + b*x]^m
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5067
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c +
d*x)))^q, (E^((-I)*(a + b*x)) + E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.99 (sec) , antiderivative size = 967, normalized size of antiderivative = 28.44

method	result	size
risch	Expression too large to display	967

input
```
int(cos(a*(2+m)+b*(2+m)*x)*(e*sin(b*x+a))^m,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/(1+m)*(exp(2*I*(b*x+a))-1)^(1+m)/b*exp(I*(b*x+a))^(-m)*e^m*(1/2)^m*
exp(1/2*I*m*(Pi*csgn(I*(exp(2*I*(b*x+a))-1))*csgn(I*exp(-I*(b*x+a)))*csgn(
sin(b*x+a))+Pi*csgn(I*(exp(2*I*(b*x+a))-1))*csgn(sin(b*x+a))^2+Pi*csgn(I*e
xp(-I*(b*x+a)))*csgn(sin(b*x+a))^2-Pi*csgn(I*e)*csgn(sin(b*x+a))*csgn(e*si
n(b*x+a))+Pi*csgn(I*e)*csgn(e*sin(b*x+a))^2+Pi*csgn(sin(b*x+a))^3-Pi*csgn(
sin(b*x+a))*csgn(e*sin(b*x+a))^2+Pi*csgn(e*sin(b*x+a))^3-Pi*csgn(e*sin(b*x
+a))*csgn(I*e*sin(b*x+a))^2-Pi*csgn(I*e*sin(b*x+a))^3+Pi*csgn(e*sin(b*x+a)
)*csgn(I*e*sin(b*x+a))+Pi*csgn(I*e*sin(b*x+a))^2+2*b*x-Pi+2*a))-1/4*I*(exp
(2*I*(b*x+a))-1)^m/(1+m)/b/(exp(I*(b*x+a))^m)/(2^m)*e^m*exp(-1/2*I*m*(-Pi*
csgn(I*(exp(2*I*(b*x+a))-1))*csgn(I*exp(-I*(b*x+a)))*csgn(sin(b*x+a))-Pi*c
sgn(I*(exp(2*I*(b*x+a))-1))*csgn(sin(b*x+a))^2-Pi*csgn(I*exp(-I*(b*x+a)))*
csgn(sin(b*x+a))^2-Pi*csgn(sin(b*x+a))^3+Pi*csgn(I*e)*csgn(sin(b*x+a))*csg
n(e*sin(b*x+a))+Pi*csgn(sin(b*x+a))*csgn(e*sin(b*x+a))^2-Pi*csgn(I*e)*csgn
(e*sin(b*x+a))^2-Pi*csgn(e*sin(b*x+a))^3+Pi*csgn(e*sin(b*x+a))*csgn(I*e*si
n(b*x+a))^2+Pi*csgn(I*e*sin(b*x+a))^3-Pi*csgn(e*sin(b*x+a))*csgn(I*e*sin(b
*x+a))-Pi*csgn(I*e*sin(b*x+a))^2+2*b*x+Pi+2*a))+1/4*I*(exp(2*I*(b*x+a))-1)
^m*e^m/(1+m)/b/(exp(I*(b*x+a))^m)/(2^m)*exp(-1/2*I*(-Pi*m*csgn(I*(exp(2*I*
(b*x+a))-1))*csgn(I*exp(-I*(b*x+a)))*csgn(sin(b*x+a))-Pi*m*csgn(I*(exp(2*I
*(b*x+a))-1))*csgn(sin(b*x+a))^2-Pi*m*csgn(I*exp(-I*(b*x+a)))*csgn(sin(b*x
+a))^2-Pi*m*csgn(sin(b*x+a))^3+Pi*m*csgn(I*e)*csgn(sin(b*x+a))*csgn(e*s...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.29

$$\int \cos(a(2+m)+b(2+m)x)(e\sin(a+bx))^m\,dx$$
$$= \frac{\left(\cos\left(am+(bm+2\,b)x+2\,a\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-\left(\cos\left(bx+a\right)^2-1\right)\sin\left(am+(bm+2\,b)x+2\right.}{bm+b}$$

input
```
integrate(cos(a*(2+m)+b*(2+m)*x)*(e*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
(cos(a*m + (b*m + 2*b)*x + 2*a)*cos(b*x + a)*sin(b*x + a) - (cos(b*x + a)^
2 - 1)*sin(a*m + (b*m + 2*b)*x + 2*a))*(e*sin(b*x + a))^m/(b*m + b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(a(2+m)+b(2+m)x)(e\sin(a+bx))^m\,dx = \text{Timed out}$$

input
```
integrate(cos(a*(2+m)+b*(2+m)*x)*(e*sin(b*x+a))**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(a(2+m)+b(2+m)x)(e\sin(a+bx))^m\,dx = \int (e\sin(bx+a))^m\cos(b(m+2)x+a(m+2))\,dx$$

input
```
integrate(cos(a*(2+m)+b*(2+m)*x)*(e*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate((e*sin(b*x + a))^m*cos(b*(m + 2)*x + a*(m + 2)), x)
```

Giac [F]

$$\int \cos(a(2+m)+b(2+m)x)(e\sin(a+bx))^m\,dx = \int (e\sin(bx+a))^m\cos(b(m+2)x+a(m+2))\,dx$$

input
```
integrate(cos(a*(2+m)+b*(2+m)*x)*(e*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate((e*sin(b*x + a))^m*cos(b*(m + 2)*x + a*(m + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 18.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \cos(a(2+m)+b(2+m)x)(e\sin(a+bx))^m\,dx$$
$$=-\frac{(e\,\sin\left(a+b\,x\right))^m\,\left(\sin\left(m\,\left(a+b\,x\right)\right)-\sin\left(\left(m+2\right)\,\left(a+b\,x\right)\right)\right)}{2\,b\,\left(m+1\right)}$$

input
```
int(cos(a*(m + 2) + b*x*(m + 2))*(e*sin(a + b*x))^m,x)
```

output
```
-((e*sin(a + b*x))^m*(sin(m*(a + b*x)) - sin((m + 2)*(a + b*x))))/(2*b*(m
+ 1))
```

Reduce [F]

$$\int \cos(a(2+m)+b(2+m)x)(e\sin(a+bx))^m\,dx$$
$$=e^m\left(\int \sin\left(bx+a\right)^m\cos\left(bmx+am+2bx+2a\right)dx\right)$$

input
```
int(cos(a*(2+m)+b*(2+m)*x)*(e*sin(b*x+a))^m,x)
```

output
```
e**m*int(sin(a + b*x)**m*cos(a*m + 2*a + b*m*x + 2*b*x),x)
```

3.610 $\int \cos^{10}(a+bx)\sin(12(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \cos^{10}(a+bx)\sin(12(a+bx))\,dx = -\frac{\cos^{11}(a+bx)\cos(11(a+bx))}{11b}$$

output
```
-1/11*cos(b*x+a)^11*cos(11*b*x+11*a)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos^{10}(a+bx)\sin(12(a+bx))\,dx = -\frac{\cos^{11}(a+bx)\cos(11(a+bx))}{11b}$$

input
```
Integrate[Cos[a + b*x]^10*Sin[12*(a + b*x)],x]
```

output
```
-1/11*(Cos[a + b*x]^11*Cos[11*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3042, 4819}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(12(a+bx))\cos^{10}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(12a+12bx)\cos(a+bx)^{10}dx$$

$$\Big\downarrow \text{4819}$$

$$-\frac{\cos^{11}(a+bx)\cos(11(a+bx))}{11b}$$

input
```
Int[Cos[a + b*x]^10*Sin[12*(a + b*x)],x]
```

output
```
-1/11*(Cos[a + b*x]^11*Cos[11*(a + b*x)])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4819
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol
] :> Simp[(-(m + 2))*(e*Cos[a + b*x])^(m + 1)*(Cos[(m + 1)*(a + b*x)]/(d*e*
(m + 1))), x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d
/b, Abs[m + 2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(22) = 44$.

Time = 105.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 5.61

method	result
parallelrisch	$\frac{-2047-55\cos(4bx+4a)-462\cos(12bx+12a)-165\cos(16bx+16a)-330\cos(8bx+8a)-11\cos(2bx+2a)-\cos(22bx+22a)-11\cos}{22528b}$
default	$-\frac{\cos(2bx+2a)}{2048b}-\frac{5\cos(4bx+4a)}{2048b}-\frac{15\cos(6bx+6a)}{2048b}-\frac{15\cos(8bx+8a)}{1024b}-\frac{21\cos(10bx+10a)}{1024b}-\frac{21\cos(12bx+12a)}{1024b}-$
risch	$-\frac{\cos(2bx+2a)}{2048b}-\frac{5\cos(4bx+4a)}{2048b}-\frac{15\cos(6bx+6a)}{2048b}-\frac{15\cos(8bx+8a)}{1024b}-\frac{21\cos(10bx+10a)}{1024b}-\frac{21\cos(12bx+12a)}{1024b}-$
orering	Expression too large to display

input
```
int(cos(b*x+a)^10*sin(12*b*x+12*a),x,method=_RETURNVERBOSE)
```

output
```
1/22528*(-2047-55*cos(4*b*x+4*a)-462*cos(12*b*x+12*a)-165*cos(16*b*x+16*a)
-330*cos(8*b*x+8*a)-11*cos(2*b*x+2*a)-cos(22*b*x+22*a)-11*cos(20*b*x+20*a)
-55*cos(18*b*x+18*a)-165*cos(6*b*x+6*a)-330*cos(14*b*x+14*a)-462*cos(10*b*
x+10*a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$.

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.87

$$\int \cos^{10}(a+bx)\sin(12(a+bx))\,dx = -\frac{1024\cos(bx+a)^{22}-2816\cos(bx+a)^{20}+2816\cos(bx+a)^{18}-1232\cos(bx+a)^{16}+220\cos(bx-}{11\,b}$$

input
```
integrate(cos(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="fricas")
```

output
```
-1/11*(1024*cos(b*x + a)^22 - 2816*cos(b*x + a)^20 + 2816*cos(b*x + a)^18
- 1232*cos(b*x + a)^16 + 220*cos(b*x + a)^14 - 11*cos(b*x + a)^12)/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{10}(a+bx)\sin(12(a+bx))\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**10*sin(12*b*x+12*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos^{10}(a+bx)\sin(12(a+bx))\,dx = -\frac{\cos(22\,bx+22\,a)+11\,\cos(20\,bx+20\,a)+55\,\cos(18\,bx+18\,a)+165\,\cos(16\,bx+16\,a)+330\,\cos(}{}$$

input
```
integrate(cos(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="maxima")
```

output
```
-1/22528*(cos(22*b*x + 22*a) + 11*cos(20*b*x + 20*a) + 55*cos(18*b*x + 18*
a) + 165*cos(16*b*x + 16*a) + 330*cos(14*b*x + 14*a) + 462*cos(12*b*x + 12
*a) + 462*cos(10*b*x + 10*a) + 330*cos(8*b*x + 8*a) + 165*cos(6*b*x + 6*a)
 + 55*cos(4*b*x + 4*a) + 11*cos(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$.

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.87

$$\int \cos^{10}(a+bx)\sin(12(a+bx))\,dx = -\frac{1024\cos(bx+a)^{22} - 2816\cos(bx+a)^{20} + 2816\cos(bx+a)^{18} - 1232\cos(bx+a)^{16} + 220\cos(bx+}{11\,b}$$

input
```
integrate(cos(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="giac")
```

output
```
-1/11*(1024*cos(b*x + a)^22 - 2816*cos(b*x + a)^20 + 2816*cos(b*x + a)^18
- 1232*cos(b*x + a)^16 + 220*cos(b*x + a)^14 - 11*cos(b*x + a)^12)/b
```

Mupad [B] (verification not implemented)

Time = 18.67 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.39

$$\int \cos^{10}(a+bx)\sin(12(a+bx))\,dx = \frac{\cos(a+b\,x)^{12}}{b} - \frac{20\cos(a+b\,x)^{14}}{b} + \frac{112\cos(a+b\,x)^{16}}{b} - \frac{256\cos(a+b\,x)^{18}}{b} + \frac{256\cos(a+b\,x)^{20}}{b} - \frac{1024\cos(a+b\,x)^{22}}{11\,b}$$

input
```
int(cos(a + b*x)^10*sin(12*a + 12*b*x),x)
```

output
```
cos(a + b*x)^12/b - (20*cos(a + b*x)^14)/b + (112*cos(a + b*x)^16)/b - (25
6*cos(a + b*x)^18)/b + (256*cos(a + b*x)^20)/b - (1024*cos(a + b*x)^22)/(1
1*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 236, normalized size of antiderivative = 10.26

$$\int \cos^{10}(a+bx)\sin(12(a+bx))\,dx$$
$$= \frac{6144\cos(12bx+12a)\sin(bx+a)^{10}-23808\cos(12bx+12a)\sin(bx+a)^{8}+37376\cos(12bx+12a)\sin}{}$$

input
```
int(cos(b*x+a)^10*sin(12*b*x+12*a),x)
```

output
```
(6144*cos(12*a + 12*b*x)*sin(a + b*x)**10 - 23808*cos(12*a + 12*b*x)*sin(a
 + b*x)**8 + 37376*cos(12*a + 12*b*x)*sin(a + b*x)**6 - 29880*cos(12*a + 1
2*b*x)*sin(a + b*x)**4 + 12216*cos(12*a + 12*b*x)*sin(a + b*x)**2 - 2047*c
os(12*a + 12*b*x) - 5120*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x)**9 +
 15872*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x)**7 - 18688*cos(a + b*x
)*sin(12*a + 12*b*x)*sin(a + b*x)**5 + 9960*cos(a + b*x)*sin(12*a + 12*b*x
)*sin(a + b*x)**3 - 2036*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x) + 20
47)/(22528*b)
```

3.611 $\int \cos^{10}(a+bx)\sin(6(2a+2bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \cos^{10}(a+bx)\sin(6(2a+2bx))\,dx=-\frac{\cos^{11}(a+bx)\cos(11(a+bx))}{11b}$$

output
```
-1/11*cos(b*x+a)^11*cos(11*b*x+11*a)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos^{10}(a+bx)\sin(6(2a+2bx))\,dx=-\frac{\cos^{11}(a+bx)\cos(11(a+bx))}{11b}$$

input
```
Integrate[Cos[a + b*x]^10*Sin[6*(2*a + 2*b*x)],x]
```

output
```
-1/11*(Cos[a + b*x]^11*Cos[11*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 4819}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(6(2a+2bx))\cos^{10}(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(12a+12bx)\cos(a+bx)^{10}dx$$

$$\Big\downarrow \; 4819$$

$$-\frac{\cos^{11}(a+bx)\cos(11(a+bx))}{11b}$$

input
```
Int[Cos[a + b*x]^10*Sin[6*(2*a + 2*b*x)],x]
```

output
```
-1/11*(Cos[a + b*x]^11*Cos[11*(a + b*x)])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4819
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol
] :> Simp[(-(m + 2))*(e*Cos[a + b*x])^(m + 1)*(Cos[(m + 1)*(a + b*x)]/(d*e*
(m + 1))), x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d
/b, Abs[m + 2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(22) = 44.

Time = 81.97 (sec) , antiderivative size = 129, normalized size of antiderivative = 5.61

method	result
parallelrisch	$\frac{-2047-55\cos(4bx+4a)-462\cos(12bx+12a)-165\cos(16bx+16a)-330\cos(8bx+8a)-11\cos(2bx+2a)-\cos(22bx+22a)-11\cos}{22528b}$
default	$-\frac{\cos(2bx+2a)}{2048b}-\frac{5\cos(4bx+4a)}{2048b}-\frac{15\cos(6bx+6a)}{2048b}-\frac{15\cos(8bx+8a)}{1024b}-\frac{21\cos(10bx+10a)}{1024b}-\frac{21\cos(12bx+12a)}{1024b}-$
risch	$-\frac{\cos(2bx+2a)}{2048b}-\frac{5\cos(4bx+4a)}{2048b}-\frac{15\cos(6bx+6a)}{2048b}-\frac{15\cos(8bx+8a)}{1024b}-\frac{21\cos(10bx+10a)}{1024b}-\frac{21\cos(12bx+12a)}{1024b}-$
orering	Expression too large to display

input
```
int(cos(b*x+a)^10*sin(12*b*x+12*a),x,method=_RETURNVERBOSE)
```

output
```
1/22528*(-2047-55*cos(4*b*x+4*a)-462*cos(12*b*x+12*a)-165*cos(16*b*x+16*a)
-330*cos(8*b*x+8*a)-11*cos(2*b*x+2*a)-cos(22*b*x+22*a)-11*cos(20*b*x+20*a)
-55*cos(18*b*x+18*a)-165*cos(6*b*x+6*a)-330*cos(14*b*x+14*a)-462*cos(10*b*
x+10*a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(22) = 44.

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.87

$$\int \cos^{10}(a+bx)\sin(6(2a+2bx))\,dx = -\frac{1024\cos(bx+a)^{22}-2816\cos(bx+a)^{20}+2816\cos(bx+a)^{18}-1232\cos(bx+a)^{16}+220\cos(bx-}{11\,b}$$

input
```
integrate(cos(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="fricas")
```

output
```
-1/11*(1024*cos(b*x + a)^22 - 2816*cos(b*x + a)^20 + 2816*cos(b*x + a)^18
- 1232*cos(b*x + a)^16 + 220*cos(b*x + a)^14 - 11*cos(b*x + a)^12)/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{10}(a+bx)\sin(6(2a+2bx))\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**10*sin(12*b*x+12*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos^{10}(a+bx)\sin(6(2a+2bx))\,dx =$$
$$-\frac{\cos(22\,bx+22\,a)+11\,\cos(20\,bx+20\,a)+55\,\cos(18\,bx+18\,a)+165\,\cos(16\,bx+16\,a)+330\,\cos(}{}$$

input
```
integrate(cos(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="maxima")
```

output
```
-1/22528*(cos(22*b*x + 22*a) + 11*cos(20*b*x + 20*a) + 55*cos(18*b*x + 18*
a) + 165*cos(16*b*x + 16*a) + 330*cos(14*b*x + 14*a) + 462*cos(12*b*x + 12
*a) + 462*cos(10*b*x + 10*a) + 330*cos(8*b*x + 8*a) + 165*cos(6*b*x + 6*a)
 + 55*cos(4*b*x + 4*a) + 11*cos(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$.

Time = 0.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.87

$$\int \cos^{10}(a+bx)\sin(6(2a+2bx))\,dx =$$
$$-\frac{1024\cos(bx+a)^{22} - 2816\cos(bx+a)^{20} + 2816\cos(bx+a)^{18} - 1232\cos(bx+a)^{16} + 220\cos(bx -}{11\,b}$$

input
```
integrate(cos(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="giac")
```

output
```
-1/11*(1024*cos(b*x + a)^22 - 2816*cos(b*x + a)^20 + 2816*cos(b*x + a)^18
- 1232*cos(b*x + a)^16 + 220*cos(b*x + a)^14 - 11*cos(b*x + a)^12)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.39

$$\int \cos^{10}(a+bx)\sin(6(2a+2bx))\,dx = \frac{\cos(a+b\,x)^{12}}{b} - \frac{20\cos(a+b\,x)^{14}}{b} + \frac{112\cos(a+b\,x)^{16}}{b} - \frac{256\cos(a+b\,x)^{18}}{b} + \frac{256\cos(a+b\,x)^{20}}{b} - \frac{1024\cos(a+b\,x)^{22}}{11\,b}$$

input
```
int(cos(a + b*x)^10*sin(12*a + 12*b*x),x)
```

output
```
cos(a + b*x)^12/b - (20*cos(a + b*x)^14)/b + (112*cos(a + b*x)^16)/b - (25
6*cos(a + b*x)^18)/b + (256*cos(a + b*x)^20)/b - (1024*cos(a + b*x)^22)/(1
1*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 10.26

$$\int \cos^{10}(a+bx)\sin(6(2a+2bx))\,dx$$
$$= \frac{6144\cos(12bx+12a)\sin(bx+a)^{10} - 23808\cos(12bx+12a)\sin(bx+a)^{8} + 37376\cos(12bx+12a)\sin}{}$$

input
```
int(cos(b*x+a)^10*sin(12*b*x+12*a),x)
```

output
```
(6144*cos(12*a + 12*b*x)*sin(a + b*x)**10 - 23808*cos(12*a + 12*b*x)*sin(a
 + b*x)**8 + 37376*cos(12*a + 12*b*x)*sin(a + b*x)**6 - 29880*cos(12*a + 1
2*b*x)*sin(a + b*x)**4 + 12216*cos(12*a + 12*b*x)*sin(a + b*x)**2 - 2047*c
os(12*a + 12*b*x) - 5120*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x)**9 +
 15872*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x)**7 - 18688*cos(a + b*x
)*sin(12*a + 12*b*x)*sin(a + b*x)**5 + 9960*cos(a + b*x)*sin(12*a + 12*b*x
)*sin(a + b*x)**3 - 2036*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x) + 20
47)/(22528*b)
```

3.612 $\int \cos^{10}(a+bx)\sin(3(4a+4bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \cos^{10}(a+bx)\sin(3(4a+4bx))\,dx = -\frac{\cos^{11}(a+bx)\cos(11(a+bx))}{11b}$$

output
```
-1/11*cos(b*x+a)^11*cos(11*b*x+11*a)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos^{10}(a+bx)\sin(3(4a+4bx))\,dx = -\frac{\cos^{11}(a+bx)\cos(11(a+bx))}{11b}$$

input
```
Integrate[Cos[a + b*x]^10*Sin[3*(4*a + 4*b*x)],x]
```

output
```
-1/11*(Cos[a + b*x]^11*Cos[11*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 4819}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3(4a+4bx))\cos^{10}(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(12a+12bx)\cos(a+bx)^{10}dx$$

$$\Big\downarrow \text{ 4819}$$

$$-\frac{\cos^{11}(a+bx)\cos(11(a+bx))}{11b}$$

input
```
Int[Cos[a + b*x]^10*Sin[3*(4*a + 4*b*x)],x]
```

output
```
-1/11*(Cos[a + b*x]^11*Cos[11*(a + b*x)])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4819
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol
] :> Simp[(-(m + 2))*(e*Cos[a + b*x])^(m + 1)*(Cos[(m + 1)*(a + b*x)]/(d*e*
(m + 1))), x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d
/b, Abs[m + 2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(22) = 44$.

Time = 73.38 (sec), antiderivative size = 129, normalized size of antiderivative = 5.61

method	result
parallelrisch	$\frac{-2047-55\cos(4bx+4a)-462\cos(12bx+12a)-165\cos(16bx+16a)-330\cos(8bx+8a)-11\cos(2bx+2a)-\cos(22bx+22a)-11\cos}{22528b}$
default	$-\frac{\cos(2bx+2a)}{2048b}-\frac{5\cos(4bx+4a)}{2048b}-\frac{15\cos(6bx+6a)}{2048b}-\frac{15\cos(8bx+8a)}{1024b}-\frac{21\cos(10bx+10a)}{1024b}-\frac{21\cos(12bx+12a)}{1024b}-$
risch	$-\frac{\cos(2bx+2a)}{2048b}-\frac{5\cos(4bx+4a)}{2048b}-\frac{15\cos(6bx+6a)}{2048b}-\frac{15\cos(8bx+8a)}{1024b}-\frac{21\cos(10bx+10a)}{1024b}-\frac{21\cos(12bx+12a)}{1024b}-$
orering	Expression too large to display

input
```
int(cos(b*x+a)^10*sin(12*b*x+12*a),x,method=_RETURNVERBOSE)
```

output
```
1/22528*(-2047-55*cos(4*b*x+4*a)-462*cos(12*b*x+12*a)-165*cos(16*b*x+16*a)
-330*cos(8*b*x+8*a)-11*cos(2*b*x+2*a)-cos(22*b*x+22*a)-11*cos(20*b*x+20*a)
-55*cos(18*b*x+18*a)-165*cos(6*b*x+6*a)-330*cos(14*b*x+14*a)-462*cos(10*b*
x+10*a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$.

Time = 0.13 (sec), antiderivative size = 66, normalized size of antiderivative = 2.87

$$\int \cos^{10}(a+bx)\sin(3(4a+4bx))\,dx = -\frac{1024\cos(bx+a)^{22}-2816\cos(bx+a)^{20}+2816\cos(bx+a)^{18}-1232\cos(bx+a)^{16}+220\cos(bx-}{11\,b}$$

input
```
integrate(cos(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="fricas")
```

output
```
-1/11*(1024*cos(b*x + a)^22 - 2816*cos(b*x + a)^20 + 2816*cos(b*x + a)^18
- 1232*cos(b*x + a)^16 + 220*cos(b*x + a)^14 - 11*cos(b*x + a)^12)/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{10}(a+bx)\sin(3(4a+4bx))\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**10*sin(12*b*x+12*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos^{10}(a+bx)\sin(3(4a+4bx))\,dx = -\frac{\cos(22\,bx+22\,a)+11\,\cos(20\,bx+20\,a)+55\,\cos(18\,bx+18\,a)+165\,\cos(16\,bx+16\,a)+330\,\cos(}{}$$

input
```
integrate(cos(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="maxima")
```

output
```
-1/22528*(cos(22*b*x + 22*a) + 11*cos(20*b*x + 20*a) + 55*cos(18*b*x + 18*
a) + 165*cos(16*b*x + 16*a) + 330*cos(14*b*x + 14*a) + 462*cos(12*b*x + 12
*a) + 462*cos(10*b*x + 10*a) + 330*cos(8*b*x + 8*a) + 165*cos(6*b*x + 6*a)
 + 55*cos(4*b*x + 4*a) + 11*cos(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$.

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.87

$$\int \cos^{10}(a+bx)\sin(3(4a+4bx))\,dx = -\frac{1024\cos(bx+a)^{22} - 2816\cos(bx+a)^{20} + 2816\cos(bx+a)^{18} - 1232\cos(bx+a)^{16} + 220\cos(bx-}{11\,b}$$

input
```
integrate(cos(b*x+a)^10*sin(12*b*x+12*a),x, algorithm="giac")
```

output
```
-1/11*(1024*cos(b*x + a)^22 - 2816*cos(b*x + a)^20 + 2816*cos(b*x + a)^18
- 1232*cos(b*x + a)^16 + 220*cos(b*x + a)^14 - 11*cos(b*x + a)^12)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.39

$$\int \cos^{10}(a+bx)\sin(3(4a+4bx))\,dx = \frac{\cos(a+b\,x)^{12}}{b} - \frac{20\cos(a+b\,x)^{14}}{b} + \frac{112\cos(a+b\,x)^{16}}{b} - \frac{256\cos(a+b\,x)^{18}}{b} + \frac{256\cos(a+b\,x)^{20}}{b} - \frac{1024\cos(a+b\,x)^{22}}{11\,b}$$

input
```
int(cos(a + b*x)^10*sin(12*a + 12*b*x),x)
```

output
```
cos(a + b*x)^12/b - (20*cos(a + b*x)^14)/b + (112*cos(a + b*x)^16)/b - (25
6*cos(a + b*x)^18)/b + (256*cos(a + b*x)^20)/b - (1024*cos(a + b*x)^22)/(1
1*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 236, normalized size of antiderivative = 10.26

$$\int \cos^{10}(a+bx)\sin(3(4a+4bx))\,dx$$
$$= \frac{6144\cos(12bx+12a)\sin(bx+a)^{10} - 23808\cos(12bx+12a)\sin(bx+a)^{8} + 37376\cos(12bx+12a)\sin}{}$$

input
```
int(cos(b*x+a)^10*sin(12*b*x+12*a),x)
```

output
```
(6144*cos(12*a + 12*b*x)*sin(a + b*x)**10 - 23808*cos(12*a + 12*b*x)*sin(a
 + b*x)**8 + 37376*cos(12*a + 12*b*x)*sin(a + b*x)**6 - 29880*cos(12*a + 1
2*b*x)*sin(a + b*x)**4 + 12216*cos(12*a + 12*b*x)*sin(a + b*x)**2 - 2047*c
os(12*a + 12*b*x) - 5120*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x)**9 +
 15872*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x)**7 - 18688*cos(a + b*x
)*sin(12*a + 12*b*x)*sin(a + b*x)**5 + 9960*cos(a + b*x)*sin(12*a + 12*b*x
)*sin(a + b*x)**3 - 2036*cos(a + b*x)*sin(12*a + 12*b*x)*sin(a + b*x) + 20
47)/(22528*b)
```

3.613 $\int (e\cos(a+bx))^m \sin((2+m)(a+bx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int (e\cos(a+bx))^m \sin((2+m)(a+bx))\,dx = -\frac{(e\cos(a+bx))^{1+m}\cos((1+m)(a+bx))}{be(1+m)}$$

output
```
-(e*cos(b*x+a))^(1+m)*cos((1+m)*(b*x+a))/b/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int (e\cos(a+bx))^m \sin((2+m)(a+bx))\,dx = -\frac{\cos(a+bx)(e\cos(a+bx))^m\cos((1+m)(a+bx))}{b(1+m)}$$

input
```
Integrate[(e*Cos[a + b*x])^m*Sin[(2 + m)*(a + b*x)],x]
```

output
```
-((Cos[a + b*x]*(e*Cos[a + b*x])^m*Cos[(1 + m)*(a + b*x)])/(b*(1 + m)))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.34, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {7271, 7281, 5066, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin((m+2)(a+bx))(e\cos(a+bx))^m\,dx$$

$$\Big\downarrow \; 7271$$

$$\cos^{-m}(a+bx)(e\cos(a+bx))^m\int \cos^m(a+bx)\sin((m+2)(a+bx))dx$$

$$\Big\downarrow \; 7281$$

$$\frac{\cos^{-m}(a+bx)(e\cos(a+bx))^m\int \cos^m(a+bx)\sin((m+2)(a+bx))d(a+bx)}{b}$$

$$\Big\downarrow \; 5066$$

$$\frac{2^{-m-1}\cos^{-m}(a+bx)(e\cos(a+bx))^m\int \Big(ie^{-i(m+2)(a+bx)}\left(e^{-i(a+bx)}+e^{i(a+bx)}\right)^m-ie^{i(m+2)(a+bx)}\left(e^{-i(a+bx)}+e^{i(\cdots}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{2^{-m-1}\left(-\frac{\left(1+e^{2i(a+bx)}\right)e^{im(a+bx)}\left(e^{-i(a+bx)}+e^{i(a+bx)}\right)^m}{2(m+1)}-\frac{\left(1+e^{2i(a+bx)}\right)e^{-i(m+2)(a+bx)}\left(e^{-i(a+bx)}+e^{i(a+bx)}\right)^m}{2(m+1)}\right)\cos^{-m}(a+bx)(e\cdots}{b}$$

input
```
Int[(e*Cos[a + b*x])^m*Sin[(2 + m)*(a + b*x)],x]
```

output
```
(2^(-1 - m)*(-1/2*(E^(I*m*(a + b*x))*(E^((-I)*(a + b*x)) + E^(I*(a + b*x))
)^m*(1 + E^((2*I)*(a + b*x))))/(1 + m) - ((E^((-I)*(a + b*x)) + E^(I*(a +
b*x)))^m*(1 + E^((2*I)*(a + b*x))))/(2*E^(I*(2 + m)*(a + b*x))*(1 + m)))*(
e*Cos[a + b*x])^m)/(b*Cos[a + b*x]^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5066
```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.90 (sec) , antiderivative size = 792, normalized size of antiderivative = 22.63

method	result	size
risch	Expression too large to display	792

input
```
int((e*cos(b*x+a))^m*sin((2+m)*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(1+m)*(exp(2*I*(b*x+a))+1)^(1+m)/b/(exp(I*(b*x+a))^m)*e^m/(2^m)*exp(1
/2*I*m*(-Pi*csgn(I*exp(-I*(b*x+a)))*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*co
s(b*x+a))+Pi*csgn(I*exp(-I*(b*x+a)))*csgn(I*cos(b*x+a))^2+Pi*csgn(I*(exp(2
*I*(b*x+a))+1))*csgn(I*cos(b*x+a))^2-Pi*csgn(I*e)*csgn(I*cos(b*x+a))*csgn(
I*e*cos(b*x+a))+Pi*csgn(I*e)*csgn(I*e*cos(b*x+a))^2-Pi*csgn(I*cos(b*x+a))^
3+Pi*csgn(I*cos(b*x+a))*csgn(I*e*cos(b*x+a))^2-Pi*csgn(I*e*cos(b*x+a))^3+2
*b*x+2*a))-1/4/(b*m+b)*(1/2)^m*e^m*exp(I*(b*x+a))^(-m)*(exp(2*I*(b*x+a))+1
)^m*exp(-1/2*I*(Pi*m*csgn(I*exp(-I*(b*x+a)))*csgn(I*(exp(2*I*(b*x+a))+1))*
csgn(I*cos(b*x+a))-Pi*m*csgn(I*exp(-I*(b*x+a)))*csgn(I*cos(b*x+a))^2-Pi*m*
csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))^2+Pi*m*csgn(I*cos(b*x+a))^
3+Pi*m*csgn(I*e)*csgn(I*cos(b*x+a))*csgn(I*e*cos(b*x+a))-Pi*m*csgn(I*cos(b
*x+a))*csgn(I*e*cos(b*x+a))^2-Pi*m*csgn(I*e)*csgn(I*e*cos(b*x+a))^2+Pi*m*c
sgn(I*e*cos(b*x+a))^3+2*b*m*x+2*a*m+4*b*x+4*a))-1/4*(exp(2*I*(b*x+a))+1)^m
*exp(I*(b*x+a))^(-m)*e^m*(1/2)^m/(1+m)/b*exp(-1/2*I*m*(Pi*csgn(I*exp(-I*(b
*x+a)))*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))-Pi*csgn(I*exp(-I*(
b*x+a)))*csgn(I*cos(b*x+a))^2-Pi*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b
*x+a))^2+Pi*csgn(I*cos(b*x+a))^3+Pi*csgn(I*e)*csgn(I*cos(b*x+a))*csgn(I*e*
cos(b*x+a))-Pi*csgn(I*cos(b*x+a))*csgn(I*e*cos(b*x+a))^2-Pi*csgn(I*e)*csgn
(I*e*cos(b*x+a))^2+Pi*csgn(I*e*cos(b*x+a))^3+2*b*x+2*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.17

$$\int (e\cos(a+bx))^m \sin((2+m)(a+bx))\,dx = -\frac{\left(\cos\left(am+(bm+2\,b)x+2\,a\right)\cos\left(bx+a\right)^2+\cos\left(bx+a\right)\sin\left(am+(bm+2\,b)x+2\,a\right)\sin\left(bx+a\right)\right)(}{bm+b}$$

input
```
integrate((e*cos(b*x+a))^m*sin((2+m)*(b*x+a)),x, algorithm="fricas")
```

output
```
-(cos(a*m + (b*m + 2*b)*x + 2*a)*cos(b*x + a)^2 + cos(b*x + a)*sin(a*m + (
b*m + 2*b)*x + 2*a)*sin(b*x + a))*(e*cos(b*x + a))^m/(b*m + b)
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(a+bx))^m \sin((2+m)(a+bx))\,dx = \text{Timed out}$$

input
```
integrate((e*cos(b*x+a))**m*sin((2+m)*(b*x+a)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\cos(a+bx))^m \sin((2+m)(a+bx))\,dx = \int (e\cos(bx+a))^m \sin((bx+a)(m+2))\,dx$$

input
```
integrate((e*cos(b*x+a))^m*sin((2+m)*(b*x+a)),x, algorithm="maxima")
```

output
```
integrate((e*cos(b*x + a))^m*sin((b*x + a)*(m + 2)), x)
```

Giac [F]

$$\int (e\cos(a+bx))^m \sin((2+m)(a+bx))\,dx = \int (e\cos(bx+a))^m \sin((bx+a)(m+2))\,dx$$

input
```
integrate((e*cos(b*x+a))^m*sin((2+m)*(b*x+a)),x, algorithm="giac")
```

output
```
integrate((e*cos(b*x + a))^m*sin((b*x + a)*(m + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 18.87 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int (e\cos(a+bx))^m \sin((2+m)(a+bx))\,dx = -\frac{(\cos(m(a+bx))+\cos((m+2)(a+bx)))\,(e\cos(a+bx))^m}{2b(m+1)}$$

input
```
int(sin((m + 2)*(a + b*x))*(e*cos(a + b*x))^m,x)
```

output
```
-((cos(m*(a + b*x)) + cos((m + 2)*(a + b*x)))*(e*cos(a + b*x))^m)/(2*b*(m
+ 1))
```

Reduce [F]

$$\int (e\cos(a+bx))^m \sin((2+m)(a+bx))\,dx = e^m\left(\int \cos(bx+a)^m \sin(bmx+am+2bx+2a)\,dx\right)$$

input
```
int((e*cos(b*x+a))^m*sin((2+m)*(b*x+a)),x)
```

output
```
e**m*int(cos(a + b*x)**m*sin(a*m + 2*a + b*m*x + 2*b*x),x)
```

3.614 $\int (e\cos(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int (e\cos(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = -\frac{(e\cos(a+bx))^{1+m}\cos((1+m)(a+bx))}{be(1+m)}$$

output
```
-(e*cos(b*x+a))^(1+m)*cos((1+m)*(b*x+a))/b/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int (e\cos(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = -\frac{\cos(a+bx)(e\cos(a+bx))^m\cos((1+m)(a+bx))}{b(1+m)}$$

input
```
Integrate[(e*Cos[a + b*x])^m*Sin[a*(2 + m) + b*(2 + m)*x],x]
```

output
```
-((Cos[a + b*x]*(e*Cos[a + b*x])^m*Cos[(1 + m)*(a + b*x)])/(b*(1 + m)))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 198, normalized size of antiderivative = 5.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7271, 5066, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a(m+2)+b(m+2)x)(e\cos(a+bx))^m\,dx$$

$$\Big\downarrow \; 7271$$

$$\cos^{-m}(a+bx)(e\cos(a+bx))^m\int \cos^m(a+bx)\sin(a(m+2)+bx(m+2))dx$$

$$\Big\downarrow \; 5066$$

$$2^{-m-1}\cos^{-m}(a+bx)(e\cos(a+bx))^m\int \left(ie^{-ia(m+2)-ibx(m+2)}\left(e^{-i(a+bx)}+e^{i(a+bx)}\right)^m-ie^{ia(m+2)+ibx(m+2)}\left(e^{-i(a+bx)}+e^{i(a+bx)}\right)^m\right)dx$$

$$\Big\downarrow \; 2009$$

$$2^{-m-1}\left(-\frac{\left(e^{-i(a+bx)}+e^{i(a+bx)}\right)^m\left(1+e^{2i(a+bx)}\right)^{m+1}\left(1+e^{2ia+2ibx}\right)^{-m}\exp(im(a+bx)-2ia(m+1)-2ib(m+ \right.$$

$$bx)(e\cos(a+bx))^m$$

input
```
Int[(e*Cos[a + b*x])^m*Sin[a*(2 + m) + b*(2 + m)*x],x]
```

output
```
(2^(-1 - m)*(-1/2*(E^(I*m*(a + b*x))*(1 + E^((2*I)*a + (2*I)*b*x))*(E^((-I
)*(a + b*x)) + E^(I*(a + b*x)))^m)/(b*(1 + m)) - (E^((-2*I)*a*(1 + m) - (2
*I)*b*(1 + m)*x + I*m*(a + b*x))*(E^((-I)*(a + b*x)) + E^(I*(a + b*x)))^m*
(1 + E^((2*I)*(a + b*x)))^(1 + m))/(2*b*(1 + E^((2*I)*a + (2*I)*b*x))^m*(1
 + m)))*(e*Cos[a + b*x])^m)/Cos[a + b*x]^m
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5066
```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.42 (sec) , antiderivative size = 792, normalized size of antiderivative = 22.63

method	result	size
risch	Expression too large to display	792

input
```
int((e*cos(b*x+a))^m*sin(a*(2+m)+b*(2+m)*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(1+m)*(exp(2*I*(b*x+a))+1)^(1+m)/b/(exp(I*(b*x+a))^m)*e^m/(2^m)*exp(1
/2*I*m*(-Pi*csgn(I*exp(-I*(b*x+a)))*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*co
s(b*x+a))+Pi*csgn(I*exp(-I*(b*x+a)))*csgn(I*cos(b*x+a))^2+Pi*csgn(I*(exp(2
*I*(b*x+a))+1))*csgn(I*cos(b*x+a))^2-Pi*csgn(I*e)*csgn(I*cos(b*x+a))*csgn(
I*e*cos(b*x+a))+Pi*csgn(I*e)*csgn(I*e*cos(b*x+a))^2-Pi*csgn(I*cos(b*x+a))^
3+Pi*csgn(I*cos(b*x+a))*csgn(I*e*cos(b*x+a))^2-Pi*csgn(I*e*cos(b*x+a))^3+2
*b*x+2*a))-1/4/(b*m+b)*(1/2)^m*e^m*exp(I*(b*x+a))^(-m)*(exp(2*I*(b*x+a))+1
)^m*exp(-1/2*I*(Pi*m*csgn(I*exp(-I*(b*x+a)))*csgn(I*(exp(2*I*(b*x+a))+1))*
csgn(I*cos(b*x+a))-Pi*m*csgn(I*exp(-I*(b*x+a)))*csgn(I*cos(b*x+a))^2-Pi*m*
csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))^2+Pi*m*csgn(I*cos(b*x+a))^
3+Pi*m*csgn(I*e)*csgn(I*cos(b*x+a))*csgn(I*e*cos(b*x+a))-Pi*m*csgn(I*cos(b
*x+a))*csgn(I*e*cos(b*x+a))^2-Pi*m*csgn(I*e)*csgn(I*e*cos(b*x+a))^2+Pi*m*c
sgn(I*e*cos(b*x+a))^3+2*b*m*x+2*a*m+4*b*x+4*a))-1/4*(exp(2*I*(b*x+a))+1)^m
*exp(I*(b*x+a))^(-m)*e^m*(1/2)^m/(1+m)/b*exp(-1/2*I*m*(Pi*csgn(I*exp(-I*(b
*x+a)))*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))-Pi*csgn(I*exp(-I*(
b*x+a)))*csgn(I*cos(b*x+a))^2-Pi*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b
*x+a))^2+Pi*csgn(I*cos(b*x+a))^3+Pi*csgn(I*e)*csgn(I*cos(b*x+a))*csgn(I*e*
cos(b*x+a))-Pi*csgn(I*cos(b*x+a))*csgn(I*e*cos(b*x+a))^2-Pi*csgn(I*e)*csgn
(I*e*cos(b*x+a))^2+Pi*csgn(I*e*cos(b*x+a))^3+2*b*x+2*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.17

$$\int (e\cos(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = -\frac{\left(\cos\left(am+(bm+2\,b)x+2\,a\right)\cos\left(bx+a\right)^2+\cos\left(bx+a\right)\sin\left(am+(bm+2\,b)x+2\,a\right)\sin\left(bx+a\right)\right)(}{bm+b}$$

input
```
integrate((e*cos(b*x+a))^m*sin(a*(2+m)+b*(2+m)*x),x, algorithm="fricas")
```

output
```
-(cos(a*m + (b*m + 2*b)*x + 2*a)*cos(b*x + a)^2 + cos(b*x + a)*sin(a*m + (
b*m + 2*b)*x + 2*a)*sin(b*x + a))*(e*cos(b*x + a))^m/(b*m + b)
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = \text{Timed out}$$

input
```
integrate((e*cos(b*x+a))**m*sin(a*(2+m)+b*(2+m)*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\cos(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = \int (e\cos(bx+a))^m \sin(b(m+2)x+a(m+2))\,dx$$

input
```
integrate((e*cos(b*x+a))^m*sin(a*(2+m)+b*(2+m)*x),x, algorithm="maxima")
```

output
```
integrate((e*cos(b*x + a))^m*sin(b*(m + 2)*x + a*(m + 2)), x)
```

Giac [F]

$$\int (e\cos(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = \int (e\cos(bx+a))^m \sin(b(m+2)x+a(m+2))\,dx$$

input
```
integrate((e*cos(b*x+a))^m*sin(a*(2+m)+b*(2+m)*x),x, algorithm="giac")
```

output
```
integrate((e*cos(b*x + a))^m*sin(b*(m + 2)*x + a*(m + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 20.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int (e\cos(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = -\frac{(\cos(m(a+bx))+\cos((m+2)(a+bx)))(e\cos(a+bx))^m}{2b(m+1)}$$

input
```
int(sin(a*(m + 2) + b*x*(m + 2))*(e*cos(a + b*x))^m,x)
```

output
```
-((cos(m*(a + b*x)) + cos((m + 2)*(a + b*x)))*(e*cos(a + b*x))^m)/(2*b*(m
+ 1))
```

Reduce [F]

$$\int (e\cos(a+bx))^m \sin(a(2+m)+b(2+m)x)\,dx = e^m\left(\int \cos(bx+a)^m \sin(bmx+am+2bx+2a)\,dx\right)$$

input
```
int((e*cos(b*x+a))^m*sin(a*(2+m)+b*(2+m)*x),x)
```

output
```
e**m*int(cos(a + b*x)**m*sin(a*m + 2*a + b*m*x + 2*b*x),x)
```

3.615 $\int \cos^{10}(a+bx)\cos(12(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \cos^{10}(a+bx)\cos(12(a+bx))\,dx=\frac{\cos^{11}(a+bx)\sin(11(a+bx))}{11b}$$

output
```
1/11*cos(b*x+a)^11*sin(11*b*x+11*a)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos^{10}(a+bx)\cos(12(a+bx))\,dx=\frac{\cos^{11}(a+bx)\sin(11(a+bx))}{11b}$$

input
```
Integrate[Cos[a + b*x]^10*Cos[12*(a + b*x)],x]
```

output
```
(Cos[a + b*x]^11*Sin[11*(a + b*x)])/(11*b)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(23) = 46$.

Time = 0.35 (sec) , antiderivative size = 177, normalized size of antiderivative = 7.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{10}(a+bx)\cos(12(a+bx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)^{10}\cos(12a+12bx)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(\frac{\cos(2a+2bx)}{1024}+\frac{5}{512}\cos(4a+4bx)+\frac{45\cos(6a+6bx)}{1024}+\frac{15}{128}\cos(8a+8bx)+\frac{105}{512}\cos(10a+10bx)+\frac{63}{256}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(2a+2bx)}{2048b}+\frac{5\sin(4a+4bx)}{2048b}+\frac{15\sin(6a+6bx)}{2048b}+\frac{15\sin(8a+8bx)}{1024b}+\frac{21\sin(10a+10bx)}{1024b}+\frac{21\sin(12a+12bx)}{1024b}+\frac{15\sin(14a+14bx)}{1024b}+\frac{15\sin(16a+16bx)}{2048b}+\frac{5\sin(18a+18bx)}{2048b}+\frac{\sin(20a+20bx)}{2048b}+\frac{\sin(22a+22bx)}{22528b}$$

input
```
Int[Cos[a + b*x]^10*Cos[12*(a + b*x)],x]
```

output
```
Sin[2*a + 2*b*x]/(2048*b) + (5*Sin[4*a + 4*b*x])/(2048*b) + (15*Sin[6*a +
6*b*x])/(2048*b) + (15*Sin[8*a + 8*b*x])/(1024*b) + (21*Sin[10*a + 10*b*x]
)/(1024*b) + (21*Sin[12*a + 12*b*x])/(1024*b) + (15*Sin[14*a + 14*b*x])/(1
024*b) + (15*Sin[16*a + 16*b*x])/(2048*b) + (5*Sin[18*a + 18*b*x])/(2048*b
) + Sin[20*a + 20*b*x]/(2048*b) + Sin[22*a + 22*b*x]/(22528*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 105.72 (sec) , antiderivative size = 126, normalized size of antiderivative = 5.48

method	result
parallelrisch	$\frac{11\sin(2bx+2a)+55\sin(4bx+4a)+165\sin(6bx+6a)+330\sin(8bx+8a)+462\sin(10bx+10a)+462\sin(12bx+12a)+330\sin(14bx}{225286}$
default	$\frac{\sin(2bx+2a)}{2048b}+\frac{5\sin(4bx+4a)}{2048b}+\frac{15\sin(6bx+6a)}{2048b}+\frac{15\sin(8bx+8a)}{1024b}+\frac{21\sin(10bx+10a)}{1024b}+\frac{21\sin(12bx+12a)}{1024b}+15\,s$
risch	$\frac{\sin(2bx+2a)}{2048b}+\frac{5\sin(4bx+4a)}{2048b}+\frac{15\sin(6bx+6a)}{2048b}+\frac{15\sin(8bx+8a)}{1024b}+\frac{21\sin(10bx+10a)}{1024b}+\frac{21\sin(12bx+12a)}{1024b}+15\,s$
orering	Expression too large to display

input
```
int(cos(b*x+a)^10*cos(12*b*x+12*a),x,method=_RETURNVERBOSE)
```

output
```
1/22528*(11*sin(2*b*x+2*a)+55*sin(4*b*x+4*a)+165*sin(6*b*x+6*a)+330*sin(8*
b*x+8*a)+462*sin(10*b*x+10*a)+462*sin(12*b*x+12*a)+330*sin(14*b*x+14*a)+16
5*sin(16*b*x+16*a)+55*sin(18*b*x+18*a)+11*sin(20*b*x+20*a)+sin(22*b*x+22*a
))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(22) = 44$.

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.13

$$\int \cos^{10}(a+bx)\cos(12(a+bx))\,dx = \frac{(1024\cos(bx+a)^{21} - 2304\cos(bx+a)^{19} + 1792\cos(bx+a)^{17} - 560\cos(bx+a)^{15} + 60\cos(bx+a)}{11\,b}$$

input
```
integrate(cos(b*x+a)^10*cos(12*b*x+12*a),x, algorithm="fricas")
```

output
```
1/11*(1024*cos(b*x + a)^21 - 2304*cos(b*x + a)^19 + 1792*cos(b*x + a)^17 -
 560*cos(b*x + a)^15 + 60*cos(b*x + a)^13 - cos(b*x + a)^11)*sin(b*x + a)/
b
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{10}(a+bx)\cos(12(a+bx))\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**10*cos(12*b*x+12*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos^{10}(a+bx)\cos(12(a+bx))\,dx = \frac{\sin(22\,bx+22\,a) + 11\sin(20\,bx+20\,a) + 55\sin(18\,bx+18\,a) + 165\sin(16\,bx+16\,a) + 330\sin(14\,b}{}$$

input
```
integrate(cos(b*x+a)^10*cos(12*b*x+12*a),x, algorithm="maxima")
```

output
```
1/22528*(sin(22*b*x + 22*a) + 11*sin(20*b*x + 20*a) + 55*sin(18*b*x + 18*a
) + 165*sin(16*b*x + 16*a) + 330*sin(14*b*x + 14*a) + 462*sin(12*b*x + 12*
a) + 462*sin(10*b*x + 10*a) + 330*sin(8*b*x + 8*a) + 165*sin(6*b*x + 6*a)
+ 55*sin(4*b*x + 4*a) + 11*sin(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 149.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos^{10}(a+bx)\cos(12(a+bx))\,dx = \frac{\sin(22\,bx+22\,a)+11\,\sin(20\,bx+20\,a)+55\,\sin(18\,bx+18\,a)+165\,\sin(16\,bx+16\,a)+330\,\sin(14\,b\ldots}{\ldots}$$

input
```
integrate(cos(b*x+a)^10*cos(12*b*x+12*a),x, algorithm="giac")
```

output
```
1/22528*(sin(22*b*x + 22*a) + 11*sin(20*b*x + 20*a) + 55*sin(18*b*x + 18*a
) + 165*sin(16*b*x + 16*a) + 330*sin(14*b*x + 14*a) + 462*sin(12*b*x + 12*
a) + 462*sin(10*b*x + 10*a) + 330*sin(8*b*x + 8*a) + 165*sin(6*b*x + 6*a)
+ 55*sin(4*b*x + 4*a) + 11*sin(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 19.80 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.43

$$\int \cos^{10}(a+bx)\cos(12(a+bx))\,dx = -\frac{-\frac{1024\,\sin(a+b\,x)\,\cos(a+b\,x)^{21}}{11}+\frac{2304\,\sin(a+b\,x)\,\cos(a+b\,x)^{19}}{11}-\frac{1792\,\sin(a+b\,x)\,\cos(a+b\,x)^{17}}{11}+\frac{560\,\sin(a+b\,x)\,\cos(a+b\,x)^{15}}{11}-\ldots}{b}$$

input
```
int(cos(a + b*x)^10*cos(12*a + 12*b*x),x)
```

output
```
-((cos(a + b*x)^11*sin(a + b*x))/11 - (60*cos(a + b*x)^13*sin(a + b*x))/11
 + (560*cos(a + b*x)^15*sin(a + b*x))/11 - (1792*cos(a + b*x)^17*sin(a + b
*x))/11 + (2304*cos(a + b*x)^19*sin(a + b*x))/11 - (1024*cos(a + b*x)^21*s
in(a + b*x))/11)/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 235, normalized size of antiderivative = 10.22

$$\int \cos^{10}(a+bx)\cos(12(a+bx))\,dx$$
$$= \frac{-5120\cos(12bx+12a)\cos(bx+a)\sin(bx+a)^9 + 15872\cos(12bx+12a)\cos(bx+a)\sin(bx+a)^7 - 1}{}$$

input
```
int(cos(b*x+a)^10*cos(12*b*x+12*a),x)
```

output
```
( - 5120*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x)**9 + 15872*cos(12*a
+ 12*b*x)*cos(a + b*x)*sin(a + b*x)**7 - 18688*cos(12*a + 12*b*x)*cos(a +
b*x)*sin(a + b*x)**5 + 9960*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x)**
3 - 2036*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x) - 6144*sin(12*a + 12
*b*x)*sin(a + b*x)**10 + 23808*sin(12*a + 12*b*x)*sin(a + b*x)**8 - 37376*
sin(12*a + 12*b*x)*sin(a + b*x)**6 + 29880*sin(12*a + 12*b*x)*sin(a + b*x)
**4 - 12216*sin(12*a + 12*b*x)*sin(a + b*x)**2 + 2047*sin(12*a + 12*b*x))/
(22528*b)
```

3.616 $\int \cos^{10}(a+bx)\cos(6(2a+2bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \cos^{10}(a+bx)\cos(6(2a+2bx))\,dx=\frac{\cos^{11}(a+bx)\sin(11(a+bx))}{11b}$$

output
```
1/11*cos(b*x+a)^11*sin(11*b*x+11*a)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos^{10}(a+bx)\cos(6(2a+2bx))\,dx=\frac{\cos^{11}(a+bx)\sin(11(a+bx))}{11b}$$

input
```
Integrate[Cos[a + b*x]^10*Cos[6*(2*a + 2*b*x)],x]
```

output
```
(Cos[a + b*x]^11*Sin[11*(a + b*x)])/(11*b)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(23) = 46$.

Time = 0.35 (sec) , antiderivative size = 177, normalized size of antiderivative = 7.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{10}(a+bx)\cos(6(2a+2bx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)^{10}\cos(12a+12bx)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(\frac{\cos(2a+2bx)}{1024}+\frac{5}{512}\cos(4a+4bx)+\frac{45\cos(6a+6bx)}{1024}+\frac{15}{128}\cos(8a+8bx)+\frac{105}{512}\cos(10a+10bx)+\frac{63}{256}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(2a+2bx)}{2048b}+\frac{5\sin(4a+4bx)}{2048b}+\frac{15\sin(6a+6bx)}{2048b}+\frac{15\sin(8a+8bx)}{1024b}+\frac{21\sin(10a+10bx)}{1024b}+\frac{21\sin(12a+12bx)}{1024b}+\frac{15\sin(14a+14bx)}{1024b}+\frac{15\sin(16a+16bx)}{2048b}+\frac{5\sin(18a+18bx)}{2048b}+\frac{\sin(20a+20bx)}{2048b}+\frac{\sin(22a+22bx)}{22528b}$$

input
```
Int[Cos[a + b*x]^10*Cos[6*(2*a + 2*b*x)],x]
```

output
```
Sin[2*a + 2*b*x]/(2048*b) + (5*Sin[4*a + 4*b*x])/(2048*b) + (15*Sin[6*a +
6*b*x])/(2048*b) + (15*Sin[8*a + 8*b*x])/(1024*b) + (21*Sin[10*a + 10*b*x]
)/(1024*b) + (21*Sin[12*a + 12*b*x])/(1024*b) + (15*Sin[14*a + 14*b*x])/(1
024*b) + (15*Sin[16*a + 16*b*x])/(2048*b) + (5*Sin[18*a + 18*b*x])/(2048*b
) + Sin[20*a + 20*b*x]/(2048*b) + Sin[22*a + 22*b*x]/(22528*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 89.67 (sec) , antiderivative size = 126, normalized size of antiderivative = 5.48

method	result
parallelrisch	$\frac{11\sin(2bx+2a)+55\sin(4bx+4a)+165\sin(6bx+6a)+330\sin(8bx+8a)+462\sin(10bx+10a)+462\sin(12bx+12a)+330\sin(14bx}{22528b}$
default	$\frac{\sin(2bx+2a)}{2048b}+\frac{5\sin(4bx+4a)}{2048b}+\frac{15\sin(6bx+6a)}{2048b}+\frac{15\sin(8bx+8a)}{1024b}+\frac{21\sin(10bx+10a)}{1024b}+\frac{21\sin(12bx+12a)}{1024b}+\frac{15\,\mathrm{s}}{}$
risch	$\frac{\sin(2bx+2a)}{2048b}+\frac{5\sin(4bx+4a)}{2048b}+\frac{15\sin(6bx+6a)}{2048b}+\frac{15\sin(8bx+8a)}{1024b}+\frac{21\sin(10bx+10a)}{1024b}+\frac{21\sin(12bx+12a)}{1024b}+\frac{15\,\mathrm{s}}{}$
orering	Expression too large to display

input
```
int(cos(b*x+a)^10*cos(12*b*x+12*a),x,method=_RETURNVERBOSE)
```

output
```
1/22528*(11*sin(2*b*x+2*a)+55*sin(4*b*x+4*a)+165*sin(6*b*x+6*a)+330*sin(8*
b*x+8*a)+462*sin(10*b*x+10*a)+462*sin(12*b*x+12*a)+330*sin(14*b*x+14*a)+16
5*sin(16*b*x+16*a)+55*sin(18*b*x+18*a)+11*sin(20*b*x+20*a)+sin(22*b*x+22*a
))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(22) = 44$.

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.13

$$\int \cos^{10}(a+bx)\cos(6(2a+2bx))\,dx$$
$$= \frac{\left(1024\,\cos\left(bx+a\right)^{21} - 2304\,\cos\left(bx+a\right)^{19} + 1792\,\cos\left(bx+a\right)^{17} - 560\,\cos\left(bx+a\right)^{15} + 60\,\cos\left(bx+a\right)}{11\,b}$$

input
```
integrate(cos(b*x+a)^10*cos(12*b*x+12*a),x, algorithm="fricas")
```

output
```
1/11*(1024*cos(b*x + a)^21 - 2304*cos(b*x + a)^19 + 1792*cos(b*x + a)^17 -
 560*cos(b*x + a)^15 + 60*cos(b*x + a)^13 - cos(b*x + a)^11)*sin(b*x + a)/
b
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{10}(a+bx)\cos(6(2a+2bx))\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**10*cos(12*b*x+12*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos^{10}(a+bx)\cos(6(2a+2bx))\,dx$$
$$= \frac{\sin\left(22\,bx+22\,a\right) + 11\,\sin\left(20\,bx+20\,a\right) + 55\,\sin\left(18\,bx+18\,a\right) + 165\,\sin\left(16\,bx+16\,a\right) + 330\,\sin\left(14\,b\right.}{}$$

input
```
integrate(cos(b*x+a)^10*cos(12*b*x+12*a),x, algorithm="maxima")
```

output
```
1/22528*(sin(22*b*x + 22*a) + 11*sin(20*b*x + 20*a) + 55*sin(18*b*x + 18*a
) + 165*sin(16*b*x + 16*a) + 330*sin(14*b*x + 14*a) + 462*sin(12*b*x + 12*
a) + 462*sin(10*b*x + 10*a) + 330*sin(8*b*x + 8*a) + 165*sin(6*b*x + 6*a)
+ 55*sin(4*b*x + 4*a) + 11*sin(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 149.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos^{10}(a+bx)\cos(6(2a+2bx))\,dx = \frac{\sin(22\,bx+22\,a)+11\,\sin(20\,bx+20\,a)+55\,\sin(18\,bx+18\,a)+165\,\sin(16\,bx+16\,a)+330\,\sin(14\,b}{}$$

input
```
integrate(cos(b*x+a)^10*cos(12*b*x+12*a),x, algorithm="giac")
```

output
```
1/22528*(sin(22*b*x + 22*a) + 11*sin(20*b*x + 20*a) + 55*sin(18*b*x + 18*a
) + 165*sin(16*b*x + 16*a) + 330*sin(14*b*x + 14*a) + 462*sin(12*b*x + 12*
a) + 462*sin(10*b*x + 10*a) + 330*sin(8*b*x + 8*a) + 165*sin(6*b*x + 6*a)
+ 55*sin(4*b*x + 4*a) + 11*sin(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.43

$$\int \cos^{10}(a+bx)\cos(6(2a+2bx))\,dx =$$

$$-\frac{-\frac{1024\sin(a+b\,x)\cos(a+b\,x)^{21}}{11}+\frac{2304\sin(a+b\,x)\cos(a+b\,x)^{19}}{11}-\frac{1792\sin(a+b\,x)\cos(a+b\,x)^{17}}{11}+\frac{560\sin(a+b\,x)\cos(a+b\,x)^{15}}{11}-6}{b}$$

input
```
int(cos(a + b*x)^10*cos(12*a + 12*b*x),x)
```

output
```
-((cos(a + b*x)^11*sin(a + b*x))/11 - (60*cos(a + b*x)^13*sin(a + b*x))/11
 + (560*cos(a + b*x)^15*sin(a + b*x))/11 - (1792*cos(a + b*x)^17*sin(a + b
*x))/11 + (2304*cos(a + b*x)^19*sin(a + b*x))/11 - (1024*cos(a + b*x)^21*s
in(a + b*x))/11)/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 235, normalized size of antiderivative = 10.22

$$\int \cos^{10}(a+bx)\cos(6(2a+2bx))\,dx = \frac{-5120\cos(12bx+12a)\cos(bx+a)\sin(bx+a)^9+15872\cos(12bx+12a)\cos(bx+a)\sin(bx+a)^7-1}{}$$

input
```
int(cos(b*x+a)^10*cos(12*b*x+12*a),x)
```

output
```
( - 5120*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x)**9 + 15872*cos(12*a
+ 12*b*x)*cos(a + b*x)*sin(a + b*x)**7 - 18688*cos(12*a + 12*b*x)*cos(a +
b*x)*sin(a + b*x)**5 + 9960*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x)**
3 - 2036*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x) - 6144*sin(12*a + 12
*b*x)*sin(a + b*x)**10 + 23808*sin(12*a + 12*b*x)*sin(a + b*x)**8 - 37376*
sin(12*a + 12*b*x)*sin(a + b*x)**6 + 29880*sin(12*a + 12*b*x)*sin(a + b*x)
**4 - 12216*sin(12*a + 12*b*x)*sin(a + b*x)**2 + 2047*sin(12*a + 12*b*x))/
(22528*b)
```

3.617 $\int \cos^{10}(a+bx)\cos(3(4a+4bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \cos^{10}(a+bx)\cos(3(4a+4bx))\,dx = \frac{\cos^{11}(a+bx)\sin(11(a+bx))}{11b}$$

output
```
1/11*cos(b*x+a)^11*sin(11*b*x+11*a)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos^{10}(a+bx)\cos(3(4a+4bx))\,dx = \frac{\cos^{11}(a+bx)\sin(11(a+bx))}{11b}$$

input
```
Integrate[Cos[a + b*x]^10*Cos[3*(4*a + 4*b*x)],x]
```

output
```
(Cos[a + b*x]^11*Sin[11*(a + b*x)])/(11*b)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(23) = 46$.

Time = 0.35 (sec) , antiderivative size = 177, normalized size of antiderivative = 7.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{10}(a+bx)\cos(3(4a+4bx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)^{10}\cos(12a+12bx)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(\frac{\cos(2a+2bx)}{1024}+\frac{5}{512}\cos(4a+4bx)+\frac{45\cos(6a+6bx)}{1024}+\frac{15}{128}\cos(8a+8bx)+\frac{105}{512}\cos(10a+10bx)+\frac{63}{256}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(2a+2bx)}{2048b}+\frac{5\sin(4a+4bx)}{2048b}+\frac{15\sin(6a+6bx)}{2048b}+\frac{15\sin(8a+8bx)}{1024b}+\frac{21\sin(10a+10bx)}{1024b}+\frac{21\sin(12a+12bx)}{1024b}+\frac{15\sin(14a+14bx)}{1024b}+\frac{15\sin(16a+16bx)}{2048b}+\frac{5\sin(18a+18bx)}{2048b}+\frac{\sin(20a+20bx)}{2048b}+\frac{\sin(22a+22bx)}{22528b}$$

input
```
Int[Cos[a + b*x]^10*Cos[3*(4*a + 4*b*x)],x]
```

output
```
Sin[2*a + 2*b*x]/(2048*b) + (5*Sin[4*a + 4*b*x])/(2048*b) + (15*Sin[6*a +
6*b*x])/(2048*b) + (15*Sin[8*a + 8*b*x])/(1024*b) + (21*Sin[10*a + 10*b*x]
)/(1024*b) + (21*Sin[12*a + 12*b*x])/(1024*b) + (15*Sin[14*a + 14*b*x])/(1
024*b) + (15*Sin[16*a + 16*b*x])/(2048*b) + (5*Sin[18*a + 18*b*x])/(2048*b
) + Sin[20*a + 20*b*x]/(2048*b) + Sin[22*a + 22*b*x]/(22528*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 73.95 (sec) , antiderivative size = 126, normalized size of antiderivative = 5.48

method	result
parallelrisch	$\frac{11\sin(2bx+2a)+55\sin(4bx+4a)+165\sin(6bx+6a)+330\sin(8bx+8a)+462\sin(10bx+10a)+462\sin(12bx+12a)+330\sin(14bx}{225286}$
default	$\frac{\sin(2bx+2a)}{2048b}+\frac{5\sin(4bx+4a)}{2048b}+\frac{15\sin(6bx+6a)}{2048b}+\frac{15\sin(8bx+8a)}{1024b}+\frac{21\sin(10bx+10a)}{1024b}+\frac{21\sin(12bx+12a)}{1024b}+\frac{15\,\mathrm{s}}{}$
risch	$\frac{\sin(2bx+2a)}{2048b}+\frac{5\sin(4bx+4a)}{2048b}+\frac{15\sin(6bx+6a)}{2048b}+\frac{15\sin(8bx+8a)}{1024b}+\frac{21\sin(10bx+10a)}{1024b}+\frac{21\sin(12bx+12a)}{1024b}+\frac{15\,\mathrm{s}}{}$
orering	Expression too large to display

input
```
int(cos(b*x+a)^10*cos(12*b*x+12*a),x,method=_RETURNVERBOSE)
```

output
```
1/22528*(11*sin(2*b*x+2*a)+55*sin(4*b*x+4*a)+165*sin(6*b*x+6*a)+330*sin(8*
b*x+8*a)+462*sin(10*b*x+10*a)+462*sin(12*b*x+12*a)+330*sin(14*b*x+14*a)+16
5*sin(16*b*x+16*a)+55*sin(18*b*x+18*a)+11*sin(20*b*x+20*a)+sin(22*b*x+22*a
))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(22) = 44$.

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.13

$$\int \cos^{10}(a+bx)\cos(3(4a+4bx))\,dx$$
$$= \frac{\left(1024\,\cos\left(bx+a\right)^{21} - 2304\,\cos\left(bx+a\right)^{19} + 1792\,\cos\left(bx+a\right)^{17} - 560\,\cos\left(bx+a\right)^{15} + 60\,\cos\left(bx+a\right)\right.}{11\,b}$$

input
```
integrate(cos(b*x+a)^10*cos(12*b*x+12*a),x, algorithm="fricas")
```

output
```
1/11*(1024*cos(b*x + a)^21 - 2304*cos(b*x + a)^19 + 1792*cos(b*x + a)^17 -
 560*cos(b*x + a)^15 + 60*cos(b*x + a)^13 - cos(b*x + a)^11)*sin(b*x + a)/
b
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{10}(a+bx)\cos(3(4a+4bx))\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**10*cos(12*b*x+12*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(22) = 44$.

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos^{10}(a+bx)\cos(3(4a+4bx))\,dx$$
$$= \frac{\sin\left(22\,bx+22\,a\right) + 11\,\sin\left(20\,bx+20\,a\right) + 55\,\sin\left(18\,bx+18\,a\right) + 165\,\sin\left(16\,bx+16\,a\right) + 330\,\sin\left(14\,b\right.}{}$$

input
```
integrate(cos(b*x+a)^10*cos(12*b*x+12*a),x, algorithm="maxima")
```

output
```
1/22528*(sin(22*b*x + 22*a) + 11*sin(20*b*x + 20*a) + 55*sin(18*b*x + 18*a
) + 165*sin(16*b*x + 16*a) + 330*sin(14*b*x + 14*a) + 462*sin(12*b*x + 12*
a) + 462*sin(10*b*x + 10*a) + 330*sin(8*b*x + 8*a) + 165*sin(6*b*x + 6*a)
+ 55*sin(4*b*x + 4*a) + 11*sin(2*b*x + 2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(22) = 44.

Time = 148.37 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.43

$$\int \cos^{10}(a+bx)\cos(3(4a+4bx))\,dx = \frac{\sin(22\,bx+22\,a)+11\,\sin(20\,bx+20\,a)+55\,\sin(18\,bx+18\,a)+165\,\sin(16\,bx+16\,a)+330\,\sin(14\,b \cdots}{\cdots}$$

input
```
integrate(cos(b*x+a)^10*cos(12*b*x+12*a),x, algorithm="giac")
```

output
```
1/22528*(sin(22*b*x + 22*a) + 11*sin(20*b*x + 20*a) + 55*sin(18*b*x + 18*a
) + 165*sin(16*b*x + 16*a) + 330*sin(14*b*x + 14*a) + 462*sin(12*b*x + 12*
a) + 462*sin(10*b*x + 10*a) + 330*sin(8*b*x + 8*a) + 165*sin(6*b*x + 6*a)
+ 55*sin(4*b*x + 4*a) + 11*sin(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.43

$$\int \cos^{10}(a+bx)\cos(3(4a+4bx))\,dx = -\frac{-\frac{1024\,\sin(a+b\,x)\,\cos(a+b\,x)^{21}}{11}+\frac{2304\,\sin(a+b\,x)\,\cos(a+b\,x)^{19}}{11}-\frac{1792\,\sin(a+b\,x)\,\cos(a+b\,x)^{17}}{11}+\frac{560\,\sin(a+b\,x)\,\cos(a+b\,x)^{15}}{11}-6\cdots}{b}$$

input
```
int(cos(a + b*x)^10*cos(12*a + 12*b*x),x)
```

output
```
-((cos(a + b*x)^11*sin(a + b*x))/11 - (60*cos(a + b*x)^13*sin(a + b*x))/11
 + (560*cos(a + b*x)^15*sin(a + b*x))/11 - (1792*cos(a + b*x)^17*sin(a + b
*x))/11 + (2304*cos(a + b*x)^19*sin(a + b*x))/11 - (1024*cos(a + b*x)^21*s
in(a + b*x))/11)/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 235, normalized size of antiderivative = 10.22

$$\int \cos^{10}(a+bx)\cos(3(4a+4bx))\,dx$$
$$= \frac{-5120\cos(12bx+12a)\cos(bx+a)\sin(bx+a)^9+15872\cos(12bx+12a)\cos(bx+a)\sin(bx+a)^7-1}{}$$

input
```
int(cos(b*x+a)^10*cos(12*b*x+12*a),x)
```

output
```
( - 5120*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x)**9 + 15872*cos(12*a
+ 12*b*x)*cos(a + b*x)*sin(a + b*x)**7 - 18688*cos(12*a + 12*b*x)*cos(a +
b*x)*sin(a + b*x)**5 + 9960*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x)**
3 - 2036*cos(12*a + 12*b*x)*cos(a + b*x)*sin(a + b*x) - 6144*sin(12*a + 12
*b*x)*sin(a + b*x)**10 + 23808*sin(12*a + 12*b*x)*sin(a + b*x)**8 - 37376*
sin(12*a + 12*b*x)*sin(a + b*x)**6 + 29880*sin(12*a + 12*b*x)*sin(a + b*x)
**4 - 12216*sin(12*a + 12*b*x)*sin(a + b*x)**2 + 2047*sin(12*a + 12*b*x))/
(22528*b)
```

3.618 $\int (e\cos(a+bx))^m \cos((2+m)(a+bx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int (e\cos(a+bx))^m \cos((2+m)(a+bx))\,dx = \frac{(e\cos(a+bx))^{1+m}\sin((1+m)(a+bx))}{be(1+m)}$$

output
```
(e*cos(b*x+a))^(1+m)*sin((1+m)*(b*x+a))/b/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int (e\cos(a+bx))^m \cos((2+m)(a+bx))\,dx = \frac{\cos(a+bx)(e\cos(a+bx))^m\sin((1+m)(a+bx))}{b(1+m)}$$

input
```
Integrate[(e*Cos[a + b*x])^m*Cos[(2 + m)*(a + b*x)],x]
```

output
```
(Cos[a + b*x]*(e*Cos[a + b*x])^m*Sin[(1 + m)*(a + b*x)])/(b*(1 + m))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.59, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {7271, 7281, 5065, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos((m+2)(a+bx))(e\cos(a+bx))^m\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cos^{-m}(a+bx)(e\cos(a+bx))^m\int \cos^m(a+bx)\cos((m+2)(a+bx))dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\cos^{-m}(a+bx)(e\cos(a+bx))^m\int \cos^m(a+bx)\cos((m+2)(a+bx))d(a+bx)}{b}$$

$$\Big\downarrow \text{5065}$$

$$\frac{2^{-m-1}\cos^{-m}(a+bx)(e\cos(a+bx))^m\int \left(e^{-i(m+2)(a+bx)}\left(e^{-i(a+bx)}+e^{i(a+bx)}\right)^m+e^{i(m+2)(a+bx)}\left(e^{-i(a+bx)}+e^{i(a+\cdots}\right.\right.}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2^{-m-1}\left(\frac{i\left(1+e^{2i(a+bx)}\right)e^{-i(m+2)(a+bx)}\left(e^{-i(a+bx)}+e^{i(a+bx)}\right)^m}{2(m+1)}-\frac{i\left(1+e^{2i(a+bx)}\right)e^{im(a+bx)}\left(e^{-i(a+bx)}+e^{i(a+bx)}\right)^m}{2(m+1)}\right)\cos^{-m}(a+bx)(e\cdots}{b}$$

input
```
Int[(e*Cos[a + b*x])^m*Cos[(2 + m)*(a + b*x)],x]
```

output
```
(2^(-1 - m)*(((-1/2*I)*E^(I*m*(a + b*x))*(E^((-I)*(a + b*x)) + E^(I*(a + b
*x)))^m*(1 + E^((2*I)*(a + b*x))))/(1 + m) + ((I/2)*(E^((-I)*(a + b*x)) +
E^(I*(a + b*x)))^m*(1 + E^((2*I)*(a + b*x))))/(E^(I*(2 + m)*(a + b*x))*(1
+ m)))*(e*Cos[a + b*x])^m)/(b*Cos[a + b*x]^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5065
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Cos[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (E^((-I)*(a + b*x)) + E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.15 (sec) , antiderivative size = 795, normalized size of antiderivative = 23.38

method	result	size
risch	Expression too large to display	795

input
```
int((e*cos(b*x+a))^m*cos((2+m)*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/(1+m)*(exp(2*I*(b*x+a))+1)^(1+m)/b/(exp(I*(b*x+a))^m)*e^m/(2^m)*exp
(1/2*I*m*(-Pi*csgn(I*exp(-I*(b*x+a)))*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*
cos(b*x+a))+Pi*csgn(I*exp(-I*(b*x+a)))*csgn(I*cos(b*x+a))^2+Pi*csgn(I*(exp
(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))^2-Pi*csgn(I*e)*csgn(I*cos(b*x+a))*csg
n(I*e*cos(b*x+a))+Pi*csgn(I*e)*csgn(I*e*cos(b*x+a))^2-Pi*csgn(I*cos(b*x+a)
)^3+Pi*csgn(I*cos(b*x+a))*csgn(I*e*cos(b*x+a))^2-Pi*csgn(I*e*cos(b*x+a))^3
+2*b*x+2*a))+1/4*I/(b*m+b)*(1/2)^m*e^m*exp(I*(b*x+a))^(-m)*(exp(2*I*(b*x+a
))+1)^m*exp(-1/2*I*(Pi*m*csgn(I*exp(-I*(b*x+a)))*csgn(I*(exp(2*I*(b*x+a))+
1))*csgn(I*cos(b*x+a))-Pi*m*csgn(I*exp(-I*(b*x+a)))*csgn(I*cos(b*x+a))^2-P
i*m*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))^2+Pi*m*csgn(I*cos(b*x+
a))^3+Pi*m*csgn(I*e)*csgn(I*cos(b*x+a))*csgn(I*e*cos(b*x+a))-Pi*m*csgn(I*c
os(b*x+a))*csgn(I*e*cos(b*x+a))^2-Pi*m*csgn(I*e)*csgn(I*e*cos(b*x+a))^2+Pi
*m*csgn(I*e*cos(b*x+a))^3+2*b*m*x+2*a*m+4*b*x+4*a))+1/4*I*(exp(2*I*(b*x+a)
)+1)^m*exp(I*(b*x+a))^(-m)*e^m*(1/2)^m/(1+m)/b*exp(-1/2*I*m*(Pi*csgn(I*exp
(-I*(b*x+a)))*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))-Pi*csgn(I*ex
p(-I*(b*x+a)))*csgn(I*cos(b*x+a))^2-Pi*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I
*cos(b*x+a))^2+Pi*csgn(I*cos(b*x+a))^3+Pi*csgn(I*e)*csgn(I*cos(b*x+a))*csg
n(I*e*cos(b*x+a))-Pi*csgn(I*cos(b*x+a))*csgn(I*e*cos(b*x+a))^2-Pi*csgn(I*e
)*csgn(I*e*cos(b*x+a))^2+Pi*csgn(I*e*cos(b*x+a))^3+2*b*x+2*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int (e\cos(a+bx))^m \cos((2+m)(a+bx))\,dx$$
$$= \frac{\left(\cos\left(bx+a\right)^2 \sin\left(am+(bm+2\,b)x+2\,a\right) - \cos\left(am+(bm+2\,b)x+2\,a\right)\cos\left(bx+a\right)\sin\left(bx+a\right)\right)(e\,c}{bm+b}$$

input
```
integrate((e*cos(b*x+a))^m*cos((2+m)*(b*x+a)),x, algorithm="fricas")
```

output
```
(cos(b*x + a)^2*sin(a*m + (b*m + 2*b)*x + 2*a) - cos(a*m + (b*m + 2*b)*x +
 2*a)*cos(b*x + a)*sin(b*x + a))*(e*cos(b*x + a))^m/(b*m + b)
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(a+bx))^m \cos((2+m)(a+bx))\,dx = \text{Timed out}$$

input
```
integrate((e*cos(b*x+a))**m*cos((2+m)*(b*x+a)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\cos(a+bx))^m \cos((2+m)(a+bx))\,dx = \int (e\cos(bx+a))^m \cos((bx+a)(m+2))\,dx$$

input
```
integrate((e*cos(b*x+a))^m*cos((2+m)*(b*x+a)),x, algorithm="maxima")
```

output
```
integrate((e*cos(b*x + a))^m*cos((b*x + a)*(m + 2)), x)
```

Giac [F]

$$\int (e\cos(a+bx))^m \cos((2+m)(a+bx))\,dx = \int (e\cos(bx+a))^m \cos((bx+a)(m+2))\,dx$$

input
```
integrate((e*cos(b*x+a))^m*cos((2+m)*(b*x+a)),x, algorithm="giac")
```

output
```
integrate((e*cos(b*x + a))^m*cos((b*x + a)*(m + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 19.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int (e\cos(a+bx))^m\cos((2+m)(a+bx))\,dx = \frac{(e\,\cos(a+b\,x))^m\,(\sin(m\,(a+b\,x))+\sin((m+2)\,(a+b\,x)))}{2\,b\,(m+1)}$$

input
```
int(cos((m + 2)*(a + b*x))*(e*cos(a + b*x))^m,x)
```

output
```
((e*cos(a + b*x))^m*(sin(m*(a + b*x)) + sin((m + 2)*(a + b*x))))/(2*b*(m +
 1))
```

Reduce [F]

$$\int (e\cos(a+bx))^m\cos((2+m)(a+bx))\,dx = e^m\left(\int \cos(bx+a)^m\cos(bmx+am+2bx+2a)\,dx\right)$$

input
```
int((e*cos(b*x+a))^m*cos((2+m)*(b*x+a)),x)
```

output
```
e**m*int(cos(a + b*x)**m*cos(a*m + 2*a + b*m*x + 2*b*x),x)
```

3.619 $\int (e\cos(a+bx))^m \cos(a(2+m)+b(2+m)x)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int (e\cos(a+bx))^m \cos(a(2+m)+b(2+m)x)\,dx = \frac{(e\cos(a+bx))^{1+m}\sin((1+m)(a+bx))}{be(1+m)}$$

output
```
(e*cos(b*x+a))^(1+m)*sin((1+m)*(b*x+a))/b/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int (e\cos(a+bx))^m \cos(a(2+m)+b(2+m)x)\,dx = \frac{\cos(a+bx)(e\cos(a+bx))^m\sin((1+m)(a+bx))}{b(1+m)}$$

input
```
Integrate[(e*Cos[a + b*x])^m*Cos[a*(2 + m) + b*(2 + m)*x],x]
```

output
```
(Cos[a + b*x]*(e*Cos[a + b*x])^m*Sin[(1 + m)*(a + b*x)])/(b*(1 + m))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 202, normalized size of antiderivative = 5.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7271, 5065, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a(m+2)+b(m+2)x)(e\cos(a+bx))^m\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cos^{-m}(a+bx)(e\cos(a+bx))^m\int \cos^m(a+bx)\cos(a(m+2)+bx(m+2))dx$$

$$\Big\downarrow \text{5065}$$

$$2^{-m-1}\cos^{-m}(a+bx)(e\cos(a+bx))^m\int \left(e^{-ia(m+2)-ibx(m+2)}\left(e^{-i(a+bx)}+e^{i(a+bx)}\right)^m+e^{ia(m+2)+ibx(m+2)}\left(e^{-i(a+bx)}+e^{i(a+bx)}\right)^m\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2^{-m-1}\left(\frac{i\left(1+e^{2ia+2ibx}\right)^{-m}\left(e^{-i(a+bx)}+e^{i(a+bx)}\right)^m\left(1+e^{2i(a+bx)}\right)^{m+1}\exp(im(a+bx)-2ia(m+1)-2ib(m+}{2b(m+1)}\right.$$
$$bx)(e\cos(a+bx))^m$$

input
```
Int[(e*Cos[a + b*x])^m*Cos[a*(2 + m) + b*(2 + m)*x],x]
```

output
```
(2^(-1 - m)*(((-1/2*I)*E^(I*m*(a + b*x))*(1 + E^((2*I)*a + (2*I)*b*x))*(E^
((-I)*(a + b*x)) + E^(I*(a + b*x)))^m)/(b*(1 + m)) + ((I/2)*E^((-2*I)*a*(1
 + m) - (2*I)*b*(1 + m)*x + I*m*(a + b*x))*(E^((-I)*(a + b*x)) + E^(I*(a +
 b*x)))^m*(1 + E^((2*I)*(a + b*x)))^(1 + m))/(b*(1 + E^((2*I)*a + (2*I)*b*
x))^m*(1 + m))))*(e*Cos[a + b*x])^m)/Cos[a + b*x]^m
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5065
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Cos[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Simp[1/2^(p + q)   Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d
*x)))^q, (E^((-I)*(a + b*x)) + E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.29 (sec) , antiderivative size = 795, normalized size of antiderivative = 23.38

method	result	size
risch	Expression too large to display	795

input
```
int((e*cos(b*x+a))^m*cos(a*(2+m)+b*(2+m)*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/(1+m)*(exp(2*I*(b*x+a))+1)^(1+m)/b/(exp(I*(b*x+a))^m)*e^m/(2^m)*exp
(1/2*I*m*(-Pi*csgn(I*exp(-I*(b*x+a)))*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*
cos(b*x+a))+Pi*csgn(I*exp(-I*(b*x+a)))*csgn(I*cos(b*x+a))^2+Pi*csgn(I*(exp
(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))^2-Pi*csgn(I*e)*csgn(I*cos(b*x+a))*csg
n(I*e*cos(b*x+a))+Pi*csgn(I*e)*csgn(I*e*cos(b*x+a))^2-Pi*csgn(I*cos(b*x+a)
)^3+Pi*csgn(I*cos(b*x+a))*csgn(I*e*cos(b*x+a))^2-Pi*csgn(I*e*cos(b*x+a))^3
+2*b*x+2*a))+1/4*I/(b*m+b)*(1/2)^m*e^m*exp(I*(b*x+a))^(-m)*(exp(2*I*(b*x+a
))+1)^m*exp(-1/2*I*(Pi*m*csgn(I*exp(-I*(b*x+a)))*csgn(I*(exp(2*I*(b*x+a))+
1))*csgn(I*cos(b*x+a))-Pi*m*csgn(I*exp(-I*(b*x+a)))*csgn(I*cos(b*x+a))^2-P
i*m*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))^2+Pi*m*csgn(I*cos(b*x+
a))^3+Pi*m*csgn(I*e)*csgn(I*cos(b*x+a))*csgn(I*e*cos(b*x+a))-Pi*m*csgn(I*c
os(b*x+a))*csgn(I*e*cos(b*x+a))^2-Pi*m*csgn(I*e)*csgn(I*e*cos(b*x+a))^2+Pi
*m*csgn(I*e*cos(b*x+a))^3+2*b*m*x+2*a*m+4*b*x+4*a))+1/4*I*(exp(2*I*(b*x+a)
)+1)^m*exp(I*(b*x+a))^(-m)*e^m*(1/2)^m/(1+m)/b*exp(-1/2*I*m*(Pi*csgn(I*exp
(-I*(b*x+a)))*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))-Pi*csgn(I*ex
p(-I*(b*x+a)))*csgn(I*cos(b*x+a))^2-Pi*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I
*cos(b*x+a))^2+Pi*csgn(I*cos(b*x+a))^3+Pi*csgn(I*e)*csgn(I*cos(b*x+a))*csg
n(I*e*cos(b*x+a))-Pi*csgn(I*cos(b*x+a))*csgn(I*e*cos(b*x+a))^2-Pi*csgn(I*e
)*csgn(I*e*cos(b*x+a))^2+Pi*csgn(I*e*cos(b*x+a))^3+2*b*x+2*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int (e\cos(a+bx))^m \cos(a(2+m)+b(2+m)x)\,dx = \frac{\left(\cos(bx+a)^2 \sin(am+(bm+2b)x+2a) - \cos(am+(bm+2b)x+2a)\cos(bx+a)\sin(bx+a)\right)(e}{bm+b}$$

input
```
integrate((e*cos(b*x+a))^m*cos(a*(2+m)+b*(2+m)*x),x, algorithm="fricas")
```

output
```
(cos(b*x + a)^2*sin(a*m + (b*m + 2*b)*x + 2*a) - cos(a*m + (b*m + 2*b)*x +
 2*a)*cos(b*x + a)*sin(b*x + a))*(e*cos(b*x + a))^m/(b*m + b)
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(a+bx))^m \cos(a(2+m)+b(2+m)x)\,dx = \text{Timed out}$$

input
```
integrate((e*cos(b*x+a))**m*cos(a*(2+m)+b*(2+m)*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\cos(a+bx))^m \cos(a(2+m)+b(2+m)x)\,dx = \int (e\cos(bx+a))^m \cos(b(m+2)x+a(m+2))\,dx$$

input
```
integrate((e*cos(b*x+a))^m*cos(a*(2+m)+b*(2+m)*x),x, algorithm="maxima")
```

output
```
integrate((e*cos(b*x + a))^m*cos(b*(m + 2)*x + a*(m + 2)), x)
```

Giac [F]

$$\int (e\cos(a+bx))^m \cos(a(2+m)+b(2+m)x)\,dx = \int (e\cos(bx+a))^m \cos(b(m+2)x+a(m+2))\,dx$$

input
```
integrate((e*cos(b*x+a))^m*cos(a*(2+m)+b*(2+m)*x),x, algorithm="giac")
```

output
```
integrate((e*cos(b*x + a))^m*cos(b*(m + 2)*x + a*(m + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 19.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int (e\cos(a+bx))^m\cos(a(2+m)+b(2+m)x)\,dx = \frac{(e\,\cos(a+b\,x))^m\,(\sin(m\,(a+b\,x))+\sin((m+2)\,(a+b\,x)))}{2\,b\,(m+1)}$$

input
```
int(cos(a*(m + 2) + b*x*(m + 2))*(e*cos(a + b*x))^m,x)
```

output
```
((e*cos(a + b*x))^m*(sin(m*(a + b*x)) + sin((m + 2)*(a + b*x))))/(2*b*(m +
 1))
```

Reduce [F]

$$\int (e\cos(a+bx))^m\cos(a(2+m)+b(2+m)x)\,dx = e^m\left(\int \cos(bx+a)^m\cos(bmx+am+2bx+2a)\,dx\right)$$

input
```
int((e*cos(b*x+a))^m*cos(a*(2+m)+b*(2+m)*x),x)
```

output
```
e**m*int(cos(a + b*x)**m*cos(a*m + 2*a + b*m*x + 2*b*x),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                   ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file